Control With a Known Model

Raihan Seraj

***Abstract*— In this report control on a known base model have been applied. The known model is a standard cartpole model, where two controllers, PID and LQR are implemented and the performance of these models are compared.**

I. INTRODUCTION

The task of balancing a cartpole which is an inverted pendulum robot that is balanced by a moving cart along a horizontal track is a classical problem in the field of Control. Two models are implemented in order to stabilize the pole at the top. Given the dynamics of the CartPole system which is non linear, we present a PID controller and a Linear Quadratic Regular (LQR) in order to stabilize the CartPole. We give detailed analysis and the tunable parameters and suitable gain for both of the controllers. In the following section we discuss the dynamics of the CartPole problem followed by the justification of the use of PID and LQR controller. We further provide the results for our experiments and show the stability properties for both the controllers.

II. MODEL DYNAMICS

The system consists of an inverted pendulum mounted on top of a motorized cart. The system is unstable if a controller is not applied and hence the pendulum fall off if the cart is not moved in order to balance it.Balancing of the pendulum is obtained by applying force to the cart which brings the pendulum at equilibrium.The system has two equilibrium- stable equilibrium where the pole is balanced in an upright position and the unstable equilibrium- where the pole position is vertically inverted. This therefore makes the system inherently non linear due to existence of multiple equilibrium points.

The control in to the given system, comprises of the applied force which moves the cart in a particular direction. For the system provided to us the output of the system for the given control input consists of the position of the cart, the sin and cosine of the angle of the pendulum, the velocity of the cart and the angular velocity of the pendulum. The following table gives the parameters for the CartPole system that we are provided with.

TABLE I
MY CAPTION

Parameter	Symbol	Value	Unit
Mass of Pendulum	M	0.5	Kg
Mass of Cart	m	0.5	Kg
Length of Pendulum	l	0.5	m
Cart Friction Coefficient	b	0.5	Ns/m
Acceleration due to gravity	g	9.82	m/s^2

A. Control Schemes

In order to stabilize the CartPole, with in the region of the simulation window ranging from -1.5 to 1.5, two independent controllers have been implemented.

B. PID Controller

A PID (Proportional Integral and Differential) controller is a closed loop control mechanism. A PID controller continuously calculate the error between a desired set point and the output of the system process.The PID controller has three different modes.

- Proportional mode: Where the control of the output is proportionally changed with the error governed by the following equation
$$K_p \times error \tag{1}$$
where Kp is the proportional gain
- Integral mode: This term is included at the point where the system process has some errors other than the set point error. The integral gain continuously change the output of the system as long as there are some errors present in the system. Given enough time, integral action will drive the controller output far enough to reduce the error to zero.
- Derivative mode: The derivative control mode produces an output based on the rate of change of the error. Derivative mode is sometimes called Rate. The derivative mode produces more control action if the error changes at a faster rate.

For the cartpole model provided,we use a single PID controller for tuning the parameter. Only the angle of the output for the CartPole was considered and was fed into the PID controller to balance the pole. We set a negative value for both all the proportional and integral and the differential gains. The following table shows the values used for proportional, integral and differential gains. The tuning of the PID was done manually.

TABLE II
GAIN VALUES OF PID CONTROLLER

Gain	Value
Kp	-11
Kd	-0.0017
Ki	-0.0076

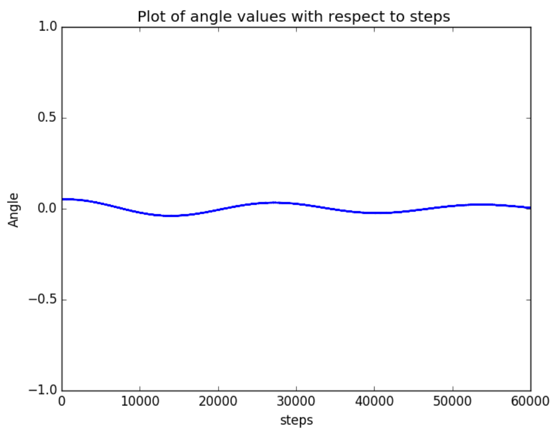

Fig. 1. angle values with number of steps

Figure 1 shows the variation of the angle values with the number of steps. The figure explains that the system is stabilized with the controller when the system starts with the pendulum in upright position. The cart moves in order to minimize the error associated with the angle of the pole, here the reference angle is considered as 0 (i,e the pole in upright vertical position).

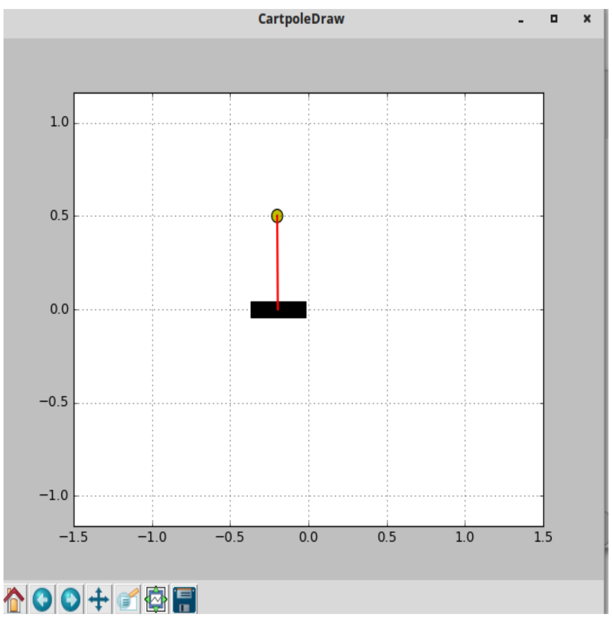

Fig. 2. CartPole stabilized with PID controller

C. LQR (Linear Quadratic Regulator)

The given system is non-linear, hence in order to use LQR for the control of the system, we linearize the dynamics of the cartPole system and apply LQR control. LQR is an optimal control regulator that better tracks a reference trajectory compared against traditional controllers such as PID.

An LQR is based on the receding horizon concept such that future outputs are predicted at every time step in order to minimize a global criterion/cost function. By estimating future outputs based on past outputs, we are able to better regulate offset in tracking. For applying LQR, the we provide the following matrix that incorporates the parameters for the linearized models. We define the A and B matrix for LQR

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g*m/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \text{(M+m)*g/(l*M)} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ 0/(l*M) \end{bmatrix}$$

The Q and R matrix are empirically tuned and we obtained the following parameters which stabilizes the CartPole model.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 15 & 0 \\ 0 & 0 & 0 & 11 \end{bmatrix}$$

R=5 which is the penalty that we would incur. We used a package lqr available in python control library. Since negative values with in the Q matrix somehow incorporates error in terms of proper eigen values not being present. We feed in the negative of the postition and the angle to the controller so that intrinsically the gain applied to these two terms are negative. We finally obtain an appropriate value of gain which produces the desired control for stabilizing the pole at the top.

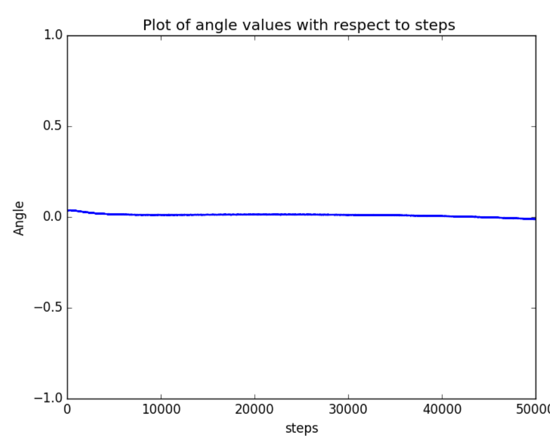

Fig. 3. angle values with number of steps for LQR

We further show the change is position of the cartpole with respect to the number of steps.

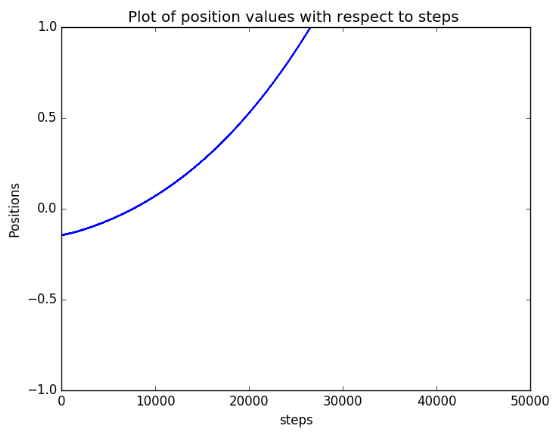

Fig. 4. Position values with number of steps for LQR

The figure above shows that although with the use of LQR the pole can be balanced, however the cart itself goes out of the window while moving and trying to balance the pendulum.

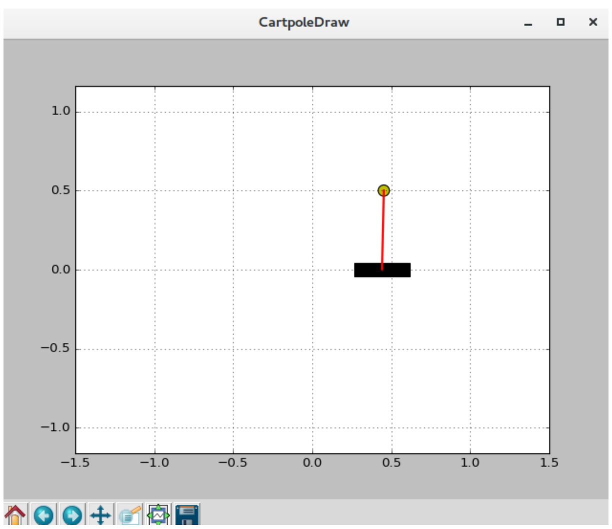

Fig. 5. CartPole stabilized with LQR

III. Conclusion

We study the use of two control algorithms for stabilizing the control dynamics of a CartPole system which is non-linear. We mention the way in which the dynamics of the CartPole system is linearized for using LQR. For PID we saw that a very small integral gain stabilizes the model. Overall although the implementation of these algorithms are relatively simple, a lot of time is invested in tuning the gains for achieving the desired output. In both the cases it is assumed that the cartpole starts from upright and the controller is applied in order to prevent the pole from falling. It would be an interesting approach to swing up the pendulum if it is vertically downward and then try and achieve is to control with PID or LQR. Also use of learning algorithms such as TRPO will ease us with the laborious task of tuning the parameters by hand. This can be achieved by feeding the errors at each time step with as a reward signal, and use Reinforcement learning algorithms that optimizes the policy such as Trust Region Policy Optimization (TRPO).

Appendix

The codes for the report can be found at `https://github.com/Raihan-Seraj/Control-of-Cartpole`