

# Learning Uncertainty models for Reliable Operation of Autonomous Underwater Vehicles

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# Introduction

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# Introduction

The paper discusses problems of learning uncertainty models of ocean processes to assist the operation of Autonomous underwater vehicles.

Existing methods although provide predictions of ocean currents incorporating large scale measurements from satellite images and HF radar without providing any accuracy estimates of these predictions.

The paper proposes the use of interpolation variance as an alternative to GP variance and integrate the confidence measure into the action model of the probabilistic planner.

The key contributions of this paper are

- The application of interpolation variance to the prediction of ocean currents
- The integration of interpolation variance into probabilistic planning
- The field implementation of these techniques on an AUV.

## Related Work

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# Error Subspace Statistical Estimation:

## Error Subspace Statistical Estimation

This is a rigorous computational method for the quantification, prediction and estimation of uncertainties. This method requires the knowledge of the approximating governing equations for the time-evolving error covariance bases.

## Gaussian Process

GPs have been successfully used in large scale terrain modeling, underwater habitat mapping, navigation under uncertainty. However GP variance predictions do not essentially capture the uncertainty inherent in prediction of ocean processes

# Methods and Algorithms

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# Gaussian Process Regression

The ocean currents at a given latitude and longitude and time is represented as a vector

$c(lat, long, t) = u, v$  where  $u$  and  $v$  are the components of the current along the latitude and longitude.

The historical data for times  $t = \{T - 1, T - 2, \dots\}$  and the prediction from ROMs at times  $t = \{T + 1, T + 2, T + 3 \dots\}$  are taken to provide better prediction and the confidence bounds for these predictions.

# Modeling Of Ocean Currents

The ocean currents are modeled using non-parametric Bayesian regression in the form of Gaussian Process. The Gp models a noisy process  $z_i = f(x_i) + \epsilon$  where  $z_i \in R$  and  $x_i \in R^d$  and  $\epsilon$  is Gaussian noise

The dataset  $D$  consists of  $[(x_1, z_1), (x_2, z_2), \dots, (x_n, z_n)]$  This results in a matrix  $X$  of size  $d \times n$  where  $d$  is the number of instances.

# Kernel Functions

In order to define the Gaussian Process fully, the covariance function is chosen that relates points to each other within the data. Data points close to each other have high correlations among themselves and points separated by 12 hours have also correlations due to periodic effect created by the tidal process.

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## Kernel

The following kernel has been used based on squared exponential

$$\begin{aligned} k(x_i, x_j) = \sigma^2 \exp[ & -w_{lat}(x_i^{lat} - x_j^{lat})^2 \\ & -w_{lon}(x_i^{lon} - x_j^{lon})^2 \\ & -w_t(x_i^t - x_j^t)^2 \\ & -w_p \sin^2(\pi(x_i^t - x_j^t)/12)] \end{aligned}$$

The the kernel matrix  $K$  is computed combining the covariance values and upon adding the observation noise  $\sigma^2$  which is a hyperparameter, The following model is obtained.  $K_z = K + \sigma^2 I$  Thus for a test point  $x_*$  the mean and variance is calculated as follows

$$\mu(X_*) = k_*^T (K + \sigma^2 I)^{-1} z \quad (1)$$

$$V_{gp} = k(x_*, x_*) - k_*^T (K + \sigma_n^2 I)^{-1} k_* \quad (2)$$

where  $k_*$  is the covariance vector between the selected point  $x_*$  and the training input  $X$

# Proposed Variance Measures

The Gaussian Process variance described so far gives an uncertainty estimate of the predictions so far based on the data sparsity around the point and the estimated hyperparameters.

This however does not take into account the variations of ocean currents around that point which is an important factor for measurement of confidence bounds. Therefore the paper proposes the variance measure of the process as follows

$$V_{gp}(x_*) = k_*^T (K + \sigma_n^2 I)^{-1} (z - \mu(x_*))^T (z - \mu(x_*)) \quad (3)$$

This variance measure takes into account the the correlations between the data learned using the GP framework as well as the variability of the correlated data. To determine the predicted variance.

# Local Approximation and Probabilistic Planning

The problem of using GP is that the computation cost of scaling this measure to higher dimensions is quite high ( $O(n^3)$ ). To alleviate this problem the authors use a subset of data that corresponding to the points that are expected to be mostly correlated. The data is then stored in a **KD-tree**

The hyperparameters of the kernel (i.e  $\sigma^2, w_{lat}, w_{lon}, w_p, w_t$ ) are estimated using the maximum likelihood of the measurements given the data and the hyperparameters as follows.

$$\log(pz|X, \theta) = \frac{1}{2}z^TK_z^{-1}z - \frac{1}{2}\log|K_z| - \frac{n}{2}\log 2\pi \quad (4)$$

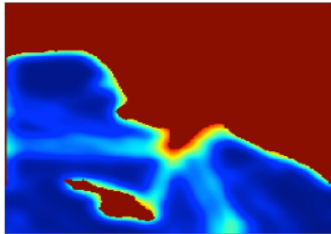
where  $K_z = K + \sigma^2I$  and  $\theta$  is the hyperparameters. Separate hyperparameters are being learned for each day for ROMS dataset.

# Variance Measure in Probabilistic Planning

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# Probabilistic Planning

The variance measure is incorporated into the probabilistic path planning of AUV to improve the safety and reliability of the operation. The operation of AUVs are improved by avoiding areas with high probability of encountering ships.



**Figure 1:** Risk Map built for Southern California coastal ocean



# Probabilistic Planning

The path planners devise action models taking into account the ocean current predictions. Two probabilistic path planners have been devised.

## MDP planner

This assume that the stochasticity in the ROMs prediction is uniform throughout

## IV-MDP planner

This incorporates the spatio-temporal interpolation variance estimates from Gaussian Process

The transition model involves a discrete grid of states for the AUV in the ocean. The actions involve moving from one state to any of the 8-connected neighbors.

# Simulation and Transition Model

- The grid is built by performing performing number of simulations traversing each pair of grid location under the influence of ocean current with different starting times for each simulation
- An additive gaussian noise proportional to the predicted noise is put
- For naive MDP planner the noise distribution is constant through the entire map and for IV-MDP the noise distribution is based upon the interpolation variance measures
- To simulate the ocean current model, the simulated noise is drawn from Normal distribution.

# Simulation and Transition Model

The ocean currents are simulated as follows.

$$\begin{aligned} & N_u(0, \sigma_u^2) \text{ and } N_v(0, \sigma_v^2) \\ u_{sim}(x, y, t) &= u_{pred}(x, y, t) + N_u \\ v_{sim}(x, y, t) &= v_{pred}(x, y, t) + N_v \end{aligned}$$

The IV-MDP planner takes the maximum interpolation variance from state  $s$  and  $s'$  at each transition edge  $e(s, s')$  over every time interval  $t \in [t_1, t_2]$  to generate the transition model  $T(s''|s, a(s, s'))$ . The value iteration algorithm yields the following recursive Bellman update

$$U_{i+1}(s) \leftarrow -R(s) + \operatorname{argmax}_a \sum_{s''} T(s''|s, a(s, s')) U_i(s') \quad (5)$$

The transition models of IV-MDP is more representative of the true errors than the models using a constant prediction noise value over the entire planning path.

## Results

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# Results

The correlation coefficients (R values) between the interpolation variance and the standard variance of the Gaussian process with the true prediction error is compared.

The following figure shows that interpolation variance has a positive correlation coefficient with true prediction error where as standard GP variance essentially shows no correlation.

CORRELATION COEFFICIENTS (R-VALUES) FOR GAUSSIAN PROCESS  
VARIANCE AND INTERPOLATION VARIANCE RELATIVE TO TRUE  
PREDICTION ERROR (SHOWN SEPARATELY FOR N/S COMPONENT AND  
E/W COMPONENTS OF THE OCEAN CURRENT VECTORS)

Month (2012)	June	July	August
GP variance R-value (E/W)	-0.0519	-0.0178	-0.0277
Interp. variance R-value (E/W)	0.1383	0.1260	0.1655
GP variance R-value (N/S)	-0.0652	-0.0245	-0.0271
Interp. variance R-value (N/S)	0.1745	0.1400	0.1323

Figure 2: Comparison of R values for three months

# Risk Aware Planning

A more complicated simulation environment has been used where by adding additional islands and shipping lanes in a virtual archipelago. The following results were obtained.

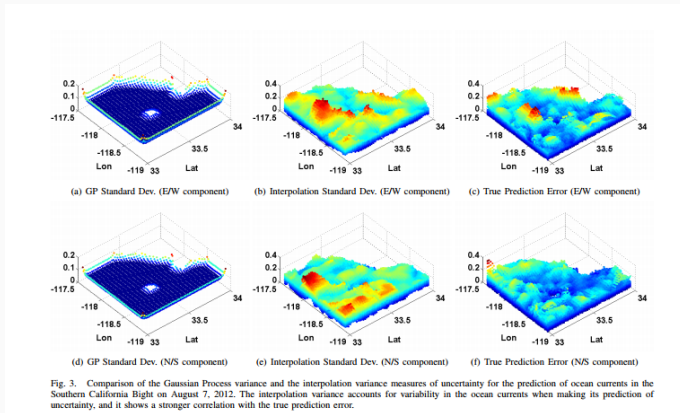


Figure 3: Comparison of Different variance models with ground truth

# Risk Aware Planning

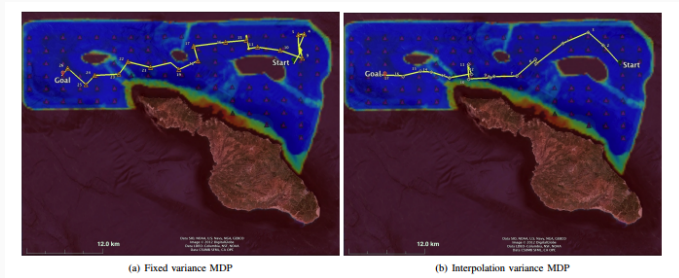


Figure 4: Path Planning for MDP and IV-MDP

Thank You  
Questions?