# **MATH 1170 - Operational Research: Linear Programming**

### **Coursework 1**

### **Group U**

First name	Surname	Student ID	
Barney	Clarke	001301673	
Ruben	Coello Solorzano	001283292	
Nate	Jermy	001302646	
Arian	Kabir	001312324	
Ahnaf Farhan	Shameem	001235779	
Abdul Raihan	Tanzim	001341954	

### **Question 1**

The first thing we need to do to solve this problem is to find every possible set of boards that we can make from a single 2400mm board, or "pattern". Each pattern will be formatted in letter-number pairs, representing how many of each board length is made in that pattern (e.g. A1C1 would be making 1 board A and 1 board C).

Pattern	Pattern	Number of each board made			Total	Waste	
number	name	1200mm	900mm	500mm	350mm	used	(mm)
						(mm)	
1	A1	1				1200	1200
2	A1B1	1	1			2100	300
3	A1C1	1		1		1700	700
4	A1C2	1		2		2200	200
5	A1C1D1	1		1	1	2050	350
6	A1D1	1			1	1550	850
7	A1D2	1			2	1900	500
8	A1D3	1			3	2250	150
9	B1		1			900	1500
10	B1D1		1		1	1250	1150
11	B1D2		1		2	1600	800
12	B1D3		1		3	1950	450

14       B1C1       1       1       1 400       10         15       B1C1D1       1       1       1       1750       6         16       B1C1D2       1       1       2       2100       3         17       B1C2       1       2       1900       5         18       B1C2D1       1       2       1       250       1         19       B2       2       1800       6         20       B2C1       2       1       2300       1         21       B2D1       2       1       2150       2         22       C1       1       500       19         23       C1D1       1       1       850       15         24       C1D2       1       2       1200       12         25       C1D3       1       3       1550       8	00 000 50 00 00 50 00 00 50
15       B1C1D1       1       1       1       1750       6         16       B1C1D2       1       1       2       2100       3         17       B1C2       1       2       1900       5         18       B1C2D1       1       2       1       2250       1         19       B2       2       1800       6         20       B2C1       2       1       2300       1         21       B2D1       2       1       2150       2         22       C1       1       500       19         23       C1D1       1       1       850       15         24       C1D2       1       2       1200       12         25       C1D3       1       3       1550       8	50 00 00 50 00 00 50
16       B1C1D2       1       1       2       2100       3         17       B1C2       1       2       1900       5         18       B1C2D1       1       2       1       2250       1         19       B2       2       1800       6         20       B2C1       2       1       2300       1         21       B2D1       2       1       2150       2         22       C1       1       500       19         23       C1D1       1       1       850       19         24       C1D2       1       2       1200       12         25       C1D3       1       3       1550       8	00 00 50 00 00 50
17     B1C2     1     2     1900     5       18     B1C2D1     1     2     1     2250     1       19     B2     2     1800     6       20     B2C1     2     1     2300     1       21     B2D1     2     1     2150     2       22     C1     1     500     19       23     C1D1     1     1     850     15       24     C1D2     1     2     1200     12       25     C1D3     1     3     1550     8	00 50 00 00 50
18     B1C2D1     1     2     1     2250     1       19     B2     2     1800     6       20     B2C1     2     1     2300     1       21     B2D1     2     1     2150     2       22     C1     1     500     19       23     C1D1     1     1     850     15       24     C1D2     1     2     1200     12       25     C1D3     1     3     1550     8	50 00 00 50
19     B2     2     1800     6       20     B2C1     2     1     2300     1       21     B2D1     2     1     2150     2       22     C1     1     500     19       23     C1D1     1     1     850     15       24     C1D2     1     2     1200     12       25     C1D3     1     3     1550     8	00 00 50 000
20     B2C1     2     1     2300     1       21     B2D1     2     1     2150     2       22     C1     1     500     19       23     C1D1     1     1     850     19       24     C1D2     1     2     1200     12       25     C1D3     1     3     1550     8	00 50 000
21     B2D1     2     1     2150     2       22     C1     1     500     19       23     C1D1     1     1     850     15       24     C1D2     1     2     1200     12       25     C1D3     1     3     1550     8	50 000
22     C1     1     500     19       23     C1D1     1     1     850     15       24     C1D2     1     2     1200     12       25     C1D3     1     3     1550     8	000
23     C1D1     1     1     850     15       24     C1D2     1     2     1200     12       25     C1D3     1     3     1550     8	
24     C1D2     1     2     1200     12       25     C1D3     1     3     1550     8	ГΩ
25 C1D3 1 3 1550 8	550
	200
	50
26 C1D4 1 4 1900 5	00
27 C1D5 1 5 2250 1	50
28 C2 2 1000 14	100
29 C2D1 2 1350 10	50
30 C2D2 2 1700 7	00
31 C2D3 2 3 2050 3	50
32 C3 3 1500 9	00
33 C3D1 3 1 1850 5	50
34 C3D2 3 2 2200 2	00
35 C4 4 2000 4	00
36 C4D1 4 1 2350 5	0
37 D1 1 350 20	50
38 D2 2 700 17	'00
39 D3 3 1050 13	50
40 D4 4 1400 10	000
41 D5 5 1750 6	
42 D6 6 2100 3	50

This problem is a relatively large one, using 42 different decision variables, and is therefore difficult to represent concisely and readably using a normal linear equation, so we will need to define a set of variables to use in the first version of our objective function.

For all variables listed, *i* is an integer between 1 and 42 (the total number of possible patterns, excluding a null pattern where no boards are made), inclusive. Also, each board length is referred to with a letter, with A, B, C and D corresponding to boards of length 1200mm, 900mm, 500mm and 350mm respectively.

 $x_i$  is used to define each pattern, and  $x_i^A$  through  $x_i^D$  are defined as the number of a specific board length made in pattern  $x_i$ .

 $w_i$  is defined as the waste of a given pattern  $x_i$ .

 $n_i$  is defined as the number of times a given pattern  $x_i$  is used. These will be the decision variables.  $n_i$  must be an integer for all values of i, as having fractional patterns would either result in a different pattern (e.g. half of D2 is just D1) or having boards of the wrong length in the case where there isn't an even number of boards being made.

w is defined as the total waste generated from a given solution, this is the value we will be minimising.

### **Objective function**

There are two different kinds/categories of sources of waste. The first is the inherent waste that each pattern leaves after being cut, which can be represented by  $\sum n_i w_i$  to remove the need to write out each of the 42 variables with their corresponding waste. The second is from cut boards that exceed the amount that was ordered. This can be incorporated into the function by taking the sum of the number of times each pattern is used  $(n_i)$ , multiplied by the number of a specific board made in each pattern (e.g.  $x_i^A$ ), minus the number of that board that was ordered and finally all multiplied by that board's length. For board A, that would be  $1200(\sum n_i x_i^A - 40)$ , which we can repeat for B, C and D. The complete objective function is shown below.

$$\min w = \sum n_i w_i + 1200 \left( \sum n_i x_i^A - 40 \right) + 900 \left( \sum n_i x_i^B - 50 \right) + 500 \left( \sum n_i x_i^C - 100 \right) + 350 \left( \sum n_i x_i^D - 75 \right)$$

However, we can't use this equation in its current form to solve the problem, as there is more than one term for each decision variable. So, we need to expand the sums and then collect like terms until we have one term of each decision variable. For the sake of brevity, the fully expanded sums will not be included here, only the final simplified version. The new objective function is shown below.

$$\begin{aligned} \min w &= 2400n_1 + 2400n_2 + 2400n_3 + 2400n_4 + 2400n_5 + 2400n_6 + 2400n_7 + 2400n_8 \\ &\quad + 2400n_9 + 2400n_{10} + 2400n_{11} + 2400n_{12} + 2400n_{13} + 2400n_{14} \\ &\quad + 2400n_{15} + 2400n_{16} + 2400n_{17} + 2400n_{18} + 2400n_{19} + 2400n_{20} \\ &\quad + 2400n_{21} + 2400n_{22} + 2400n_{23} + 2400n_{24} + 2400n_{25} + 2400n_{26} \\ &\quad + 2400n_{27} + 2400n_{28} + 2400n_{29} + 2400n_{30} + 2400n_{31} + 2400n_{32} \\ &\quad + 2400n_{33} + 2400n_{34} + 2400n_{35} + 2400n_{36} + 2400n_{37} + 2400n_{38} \\ &\quad + 2400n_{39} + 2400n_{40} + 2400n_{41} + 2400n_{42} - 169250 \end{aligned}$$

The constant -169250 at the end of the equation is a result of the need to take the ordered amount of each board from the total of each made, which introduces

constants. This number won't be included when we solve for the optimal solution, as it won't change the found solution, but will be added back in afterwards to find the actual minimum waste.

### Simplifying the objective function

Since this is a relatively large problem to solve, it would be useful to look for ways to simplify it. The best way of doing this would be to remove decision variables that generate a lot of waste and don't make a lot of boards, that are therefore very unlikely to be part of an optimal solution.

We can do this by removing all patterns where the waste generated is above a certain threshold. Since the lower the waste, the more boards are made per pattern, we can place the threshold quite low, as the larger a pattern we use, the less we have to use it and the less waste we make per use. If the threshold is 1000m, then we can remove a total of 13 patterns, cutting the total number of variables by just under a third. If the threshold is instead 750mm, we can cut 17 patterns, which reduces the total variables by 40%. If the threshold is instead 500mm, the amount of removed patterns jumps to 26, cutting our total variables by almost two-thirds down to just 16. The new set of patterns and new objective function after removing those 26 patterns are shown below.

Pattern	Pattern	Number of each board made			Total	Waste	
number	name	1200mm	900mm	500mm	350mm	used	(mm)
						(mm)	
1	A1B1	1	1			2100	300
2	A1C2	1		2		2200	200
3	A1C1D1	1		1	1	2050	350
4	A1D3	1			3	2250	150
5	B1D3		1		3	1950	450
6	B1D4		1		4	2300	100
7	B1C1D2		1	1	2	2100	300
8	B1C2D1		1	2	1	2250	150
9	B2C1		2	1		2300	100
10	B2D1		2		1	2150	250

11	C1D5	1	5	2250	150
12	C2D3	2	3	2050	350
13	C3D2	3	2	2200	200
14	C4	4		2000	400
15	C4D1	4	1	2350	50
16	D6		6	2100	300

$$\begin{aligned} \min w &= 2400n_1 + 2400n_2 + 2400n_3 + 2400n_4 + 2400n_5 + 2400n_6 + 2400n_7 + 2400n_8 \\ &\quad + 2400n_9 + 2400n_{10} + 2400n_{11} + 2400n_{12} + 2400n_{13} + 2400n_{14} \\ &\quad + 2400n_{15} + 2400n_{16} - 169250 \end{aligned}$$

#### **Problem constraints**

The constraints for our objective function require that we make at least the required amount of each board type, and that the waste of any given pattern must be greater than 0 (due to there being no way to perfectly cut a 2400mm board into two 1200mm boards, for example), and that the number of times a given pattern is used must not be negative.

$$\sum n_i x_i^A \ge 40$$

$$\sum n_i x_i^B \ge 50$$

$$\sum n_i x_i^C \ge 100$$

$$\sum n_i x_i^D \ge 75$$

$$w_i > 0$$

$$n_i \geq 0$$

### **Optimal Solution**

In this solution, each pattern's number will be taken from the simplified objective function with 16 variables instead of 42.

```
> library(lpsolve)
> objectiveCoefficients <- c(2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2400, 2
```

The above image is the R console output showing the optimal solution and the decision variables used in said solution. As stated previously, the equation used to calculate this solution doesn't include the constant at the end, so the real minimum waste is 180000 - 169250 = 10750.

Pattern	Times used	Waste created	Total waste
		(mm)	created (mm)
A1 C2 (n <sub>2</sub> )	19	200	3800
A1 D3 (n <sub>4</sub> )	21	150	3150
B1 D4 (n <sub>6</sub> )	1	100	100
B1 C2 D1 (n <sub>8</sub> )	1	150	150
B2 C1 (n <sub>9</sub> )	24	100	2400
C4 D1 (n <sub>15</sub> )	9	50	450

This solution makes 40 boards of length 1200mm, 50 boards of length 900mm, 100 boards of length 500mm, and 77 boards of length 350mm, creating an excess of 2 350mm boards. This could be fixed by reducing the number of pattern C4D1 to 7 and making 2 of pattern C4, but the resulting waste would be the exact same due to the excess boards being counted as waste.

The overall waste created by this solution is 10750mm off a total of 75 2400mm boards used, or 180,000mm, for a total waste percentage of 5.97%.

#### **Question 2**

### **Linear Programming in Telecommunications – Network Optimization**

Over The past decades, Telecommunication companies have encountered significant growth in their services: a rise in the demand of their internet services, network connectivity in addition to a digital transformation. However, this growth brought several challenges that needed to be addressed to continue running effectively. Some of the challenges were network connection issues, bandwidth management, infrastructure expansion, cybersecurity threats, network expansion and environmental and energy challenges, which had a lot of costs. Companies such AT&T and Verizon in the USA decided to use Linear programming to find the best and most optimal solution to minimise the costs and ensure smooth operations. By using an LP mathematical model, the company could determine the best way to route the data, assign bandwidth and make decisions about network expansions. The objective function is the cost minimisation. This represents the main goal of the companies is to address the challenges that have been raised. Efficient data flow between nodes, bandwidth allocation, better infrastructure investment choices are the focus for the decision variables to ensure the most ultimate answer to mitigate the issues. The companies aim to keep the models as realistic as possible, to do that, they have included network capacity limits preventing overload, flow conservation so there is no data loss, users' needs met, and user restriction limiting investment cost and that was the constraints the companies used to work out the linear programming model. By solving this optimisation problem, these communication companies can improve service reliability, reduce operational costs and plan for future network expansions. This problem is interesting because it directly impacts millions of users, ensuring better internet speeds and connectivity while allowing network companies to operate efficiently. Our group collaborated by researching real-world applications, formulating the mathematical model, and refining explanations to create a clear and practical summary of how LP optimises telecommunications networks.

- 1. **Machado, M.A.S. and Gassenferth, W.** (2015) 'An application for efficient telecommunication networks provisioning using linear programming', *Independent Journal of Management & Production*, **6**(1), pp. 30–43. doi: 10.14807/ijmp.v6i1.247.
- Rardin, R.L. (2014) Linear Programming and Algorithms for Communication Networks: A Practical Guide to Network Design, Control, and Management. Springer. Available at: <a href="https://www.researchgate.net/publication/267077770">https://www.researchgate.net/publication/267077770</a> Linear Programming and Algorithms for Communication Networks A Practical Guide to Network Design Control and Management (Accessed: 3 March 2025).

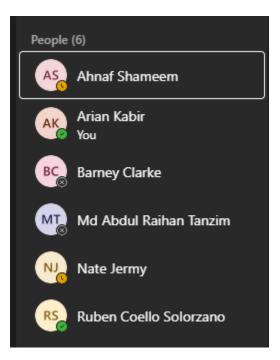
### The Role of Linear Programming in Flight Scheduling

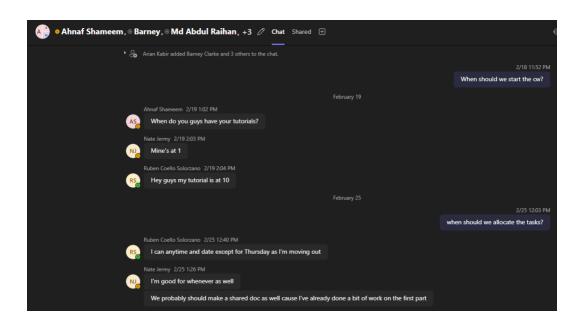
Airlines like American Airlines, Delta, and Lufthansa use Linear Programming to optimise their flight schedules, crew assignments, and fuel management while minimising costs and maximising efficiency. Managing airline operations is highly complex due to factors such as aircraft availability, crew working hours, airport capacity, fuel consumption, and passenger demand. Without optimisation, airlines face increased delays, high operational costs, and inefficient use of their resources. The objective function of LP in airline scheduling aims to minimise total operational costs, including fuel expenses, employee wages, airport fees, and aircraft maintenance costs. To achieve this, airlines must determine efficient departure times, ensuring that flights are scheduled in a way that reduces congestion and maximises aircraft usage. They also need to assign aircraft to specific flights while considering factors such as seating capacity, refuelling requirements and any other maintenance needed. Another important decision is crew assignment, where pilots and cabin crew must be scheduled in compliance with labour laws and their contracts. Additionally, maintenance scheduling plays a crucial role, as aircraft must undergo necessary servicing, including cleaning and technical inspections, before being put back into operation. The Linear Programme is bound by multiple constraints that airlines must consider, including aircraft availability (each aircraft can only operate a limited number of flights per day), crew work-hour limits (ensuring compliance with labour laws and preventing fatigue), airport capacity (limited landing and take-off slots), budget restrictions (ensuring operational costs remain within financial plans), and passenger demand (adjusting flight frequency and seat allocation to meet customer needs). Delta Air Lines uses LP for its Dynamic Scheduling Model to adjust flight schedules based on demand, crew availability, and weather conditions. Additionally, LP is applied in dynamic ticket pricing to reduce wasted products while maintaining revenue. By balancing these constraints, airlines will optimise their schedules. One of the most significant benefits of using LP in airline scheduling is its ability to improve fuel efficiency, which is crucial in reducing both costs and environmental impact. As fuel prices fluctuate and environmental regulations tighten, airlines must amend operations to remain competitive. The use of a Linear Programming model helps airlines manage these shifts by generating cost-effective schedules while maintaining reliability and regulatory compliance. Air travel is a highly interconnected and fast-paced industry where even small inefficiencies can cause major disruptions. Delays on one flight can create a domino effect, impacting multiple routes and increasing costs. Linear Programming helps airlines manage these complexities by generating the most cost-effective schedules while maintaining reliability and regulatory compliance. Additionally, with the rise of low-cost carriers and increased competition, airlines must continuously seek ways to improve efficiency and maximise revenue without compromising service quality.

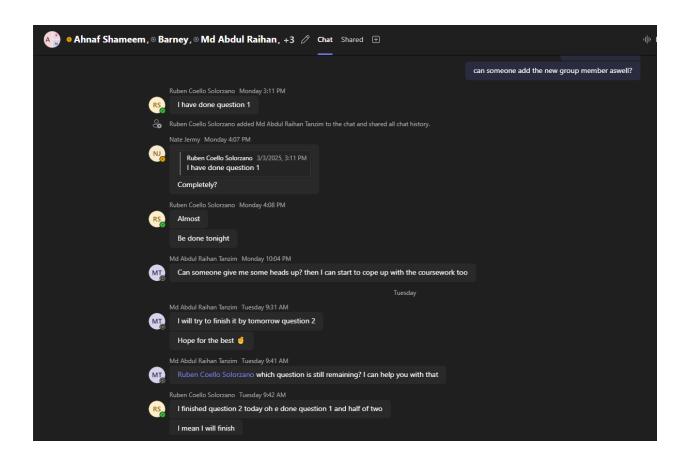
Our group found Airlines' optimisation of flight schedules using Linear Programmes to be interesting, as in a fast-paced environment such as an airport, there are an incredible number of factors that could be considered when scheduling flights to such a degree where the problem seems intractable when approached by a human. A computational solution that is coherent with such large vessels as aircraft on the tight schedule that airports uphold seems revolutionary and an excellent example of how Linear Programmes can be used to solve real problems.

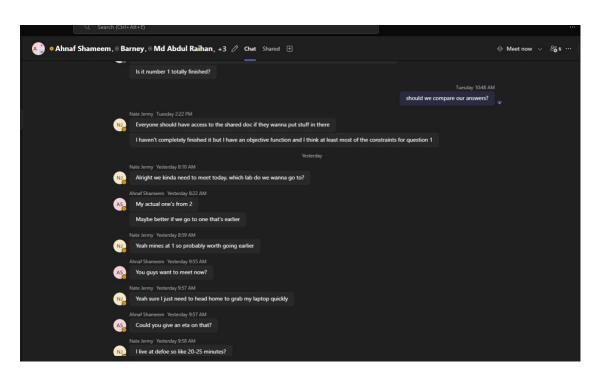
- 1. A Linear Programming Model for Airline Schedule Recovery after Unanticipated System Disruption
  - Jakob Kotas, (2022) 'A Linear Programming Model for Airline Schedule Recovery after Unanticipated System Disruption', *International Journal of Operational Research*, 45(3), pp. 378-396. Available at: https://ideas.repec.org/a/ids/ijores/v45y2022i3p378-396.html
- 2. Aircraft Routing and Scheduling: Airline Α Case Study in an Company Karaoglan, A.D., Gonen, D., and Ucmus, E. (2011) 'Aircraft Routing and Scheduling: a Case Study in an Airline Company', International Journal of Contemporary Mathematical Sciences, 6(27), pp. 1345-1358. Available at: https://www.researchgate.net/publication/314571820 Aircraft Routing and Scheduling a Ca se\_Study\_in\_an\_Airline\_Company
- 3. An Airline Crew Scheduling for Optimality Rauf, K., Nyor, N., Kanu, R.U., and Omolehin, J.O. (2016) 'An Airline Crew Scheduling for Optimality', *International Journal of Mathematical and Computational Sciences*, 10(2), pp. 188-193. Available at: https://future-in-tech.net/11.2/R-Rauf.pdf

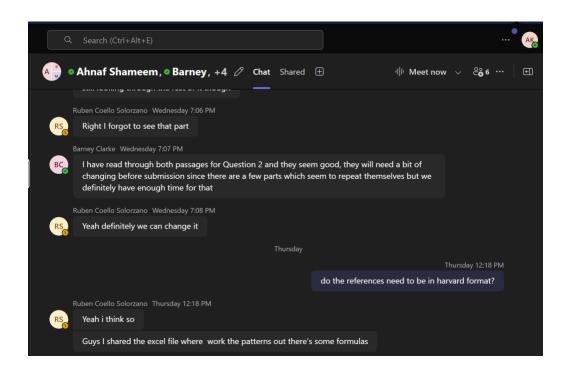
## **Communication Log**

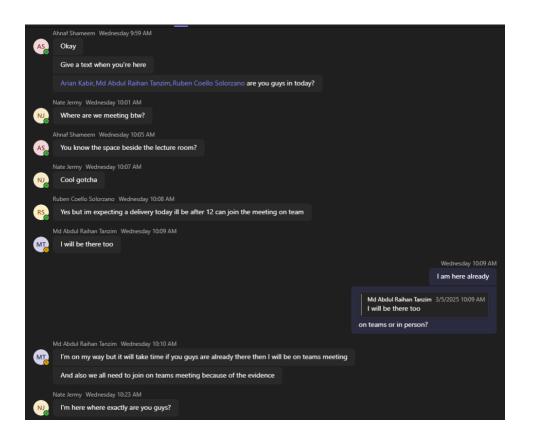


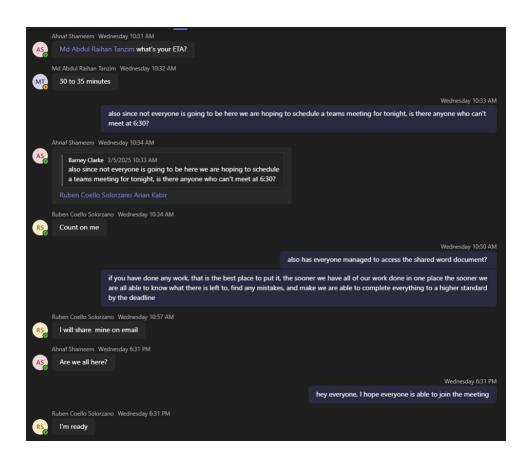














- Ruben Coello Solorzano,
- Md Abdul Raihan Tanzim,
- Ahnaf Shameem,
- Nate Jermy







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## 5 March 2025 ♥ Outgoing 18:33 Made by Barney Clarke Call ended 18:44 10m 33s Total call time

