Question 2:

Given:

- Hopfield w/ 5 neurons
- 3 input patterns

  X_1, X_2, X_3

Find:

- 5x5 weight matrix
- randomly update, and show all inputs satisfy stopping condition
- Investigate performance where

  $X_2 = [1, 1, -1, 1, -1]^T$

Inputs/outputs:

- $X_1 = [1,1,1,1,1]^T$
- $X_2 = [1,-1,-1,1,-1]^T$
- $X_3 = [-1,1,-1,1,1]^T$

Training algorithm:

First, we will initialize the 5x5 weight matrix with

$$W_{ij} = \sum_p S_i(p) S_j(p) \text{ when } i \neq j$$

Then, since hopfield is an iterative associative net we will iterate to determine activation values. We will first set initial activations y_i = to the input, X_i. Then, we will update y_i's elements in a random order with 2 steps. First, we will compare the net input with $y_{in} = x_i \sum_j y_j w_{ji}$ then apply a bipolar step activation function.

$$f(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

Finally, we will broadcast y_i to the activation vector and go to the next element in the random order. The net will continue to iterate until the activation vector stops changing and matches the the inputs. In other words, when $\Delta y = 0$ and $y_i = x_i$

Calculating weights!

* will be zeroing diagonal at the end

Pattern 1:

$$W\Delta_1 = S(1)\,S(1)^T$$

$$W\Delta = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Pattern 2:

$$W\Delta 2 = S(2)\,S(2)^T$$

$$W\Delta 2 = \begin{bmatrix} 1 & -1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 \end{bmatrix}$$

Pattern 3:

$$W\Delta 3 = S(3)\,S(3)^T$$

$$W\Delta 3 = \begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ -1 & 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 & 1 \end{bmatrix}$$

$W_{ij} = \sum_p s_i(p)\,s_j(p)$ should result in the same thing as $W = \sum_p S(p)\,S(p)^T$

$$W = \Delta w1 + \Delta w2 + \Delta w3$$

$$W = \begin{bmatrix} 0 & -1 & 1 & 1 & -1 \\ -1 & 0 & 1 & 1 & 3 \\ 1 & 1 & 0 & -1 & 1 \\ 1 & 1 & -1 & 0 & 1 \\ -1 & 3 & 1 & 1 & 0 \end{bmatrix}$$

Set the diagonal to zero to improve accuracy

Finding activation values:

* order determined by matlab random permutation command

Pattern 1:

$y_{1i} = x_{1i} = [1,1,1,1,1]^T$

order = 3, 1, 4, 5, 2

$i = 3$

$y_{in_3} = (1) + \sum_j y_j w_{j3}$

$y_{in_3} = 1 + 1 + 0 + (-1) + 1 + 1$

$y_{in_3} = 3$

$3 > 0$, so

$y_{13} = 1$

$i = 1$

$y_{in_1} = (1) + \sum_j y_j w_{j1}$

$y_{in_1} = 1 + 0 + (-1) + 1 + 1 + (-1)$

$y_{in_1} = 1$

$1 > 0$, so

$y_{11} = 1$

$i = 4$

$y_{in_4} = (1) + \sum_j y_j w_{j1}$

$y_{in_4} = 1 + 1 + 1 + (-1) + 0 + 1$

$y_{in_4} = 3$

$3 > 0$, so

$y_{14} = 1$

$i = 5$

$y_{in_5} = (1) + \sum_j y_j w_{j1}$

$y_{in_5} = 1 + (-1) + (3) + (1) + 1 + 0$

$y_{in_5} = 5$

$5 > 0$, so

$y_{i5} = 1$

$i = 2$

$y_{in_2} = (1) + \sum_j y_j w_{j1}$

$y_{in_2} = 1 + (-1) + 0 + 1 + 1 + 3$

$y_{in_2} = 5$

$5 > 0$, so

$y_{i2} = 1$

$$\Delta y_1 = \underbrace{[1\ 1\ 1\ 1\ 1]}_{\text{init}} - \underbrace{[1\ 1\ 1\ 1\ 1]}_{\text{after 1 round}}$$

$$\Delta y_1 = 0$$

$$y_1 = x_1$$

Therefore, y_1 has converged

Pattern 2:

$y_{2i} = x_{2i}$: $[1, -1, -1, 1, -1]^T$

order $= 2, 4, 3, 5, 1$

$i = 2$

$y_{in_2} = (-1) + \sum_j y_j w_{j2}$

$y_{in_2} = (-1) + (1)(-1) + (-1)(0) + (-1)(1) + (1)(1) + (-1)(3)$

$y_{in_2} = (-1) + (-1) + (-1) + 1 + (-3)$

$y_{in_2} = -5$

$-5 < 0$, so

$y_{22} = -1$

$i = 4$

$y_{in_4} = (1) + \sum_j y_j w_{j4}$

$y_{in_4} = (1) + (1)(1) + (-1)(1) + (-1)(-1) + (1)(0) + (-1)(1)$

$y_{in_4} =$

$1 > 0, \leq 0$

$y_{24} = 1$

$i = 3$

$$y_{in} = (-1) + \sum_j y_j w_{j3}$$

$$y_{in_3} = (-1) + (1)(1) + (-1)(1) + (-1)(0) + (1)(-1) + (-1)(1)$$

$$y_{in_3} = -3$$

$-3 < 0$, so

$$y_{2_3} = -1$$

$i = 5$

$$y_{in_5} = (-1) + \sum_j y_j w_{j5}$$

$$y_{in_5} = (-1) + (1)(-1) + (-1)(3) + (-1)(1) + (1)(1) + (-1)(0)$$

$$y_{in_5} = -5$$

$-5 < 0$, so

$$y_{25} = -1$$

$i = 1$

$$y_{in_1} = (1) + \sum_j y_j w_{j1}$$

$$y_{in_1} = (1) + (1)(0) + (-1)(-1) + (-1)(1) + (1)(1) + (-1)(-1)$$

$$y_{in_1} = 3$$

$3 > 0$, so

$$y_{21} = 1$$

$$\Delta y_2 = \underbrace{[1,-1,-1,1,-1]}_{\text{init}} - \underbrace{[1,-1,-1,1,-1]}_{\text{after 1 round}}$$

$$\Delta y_2 = 0$$

$$y_2 = x_2$$

Therefore, y_2 has converged

Pattern 3:

$y_{3i} = x_{3i} = [-1, 1, -1, 1, 1]^T$

order = 3, 5, 2, 1, 4

$i = 3$

$y_{in_3} = (-1) + \sum_j y_j w_{j3}$

$y_{in_3} = (-1) + (-1)(1) + (1)(1) + (-1)(0) + (1)(-1) + (1)(1)$

$y_{in_3} = -1$

$-1 < 0$, so

$y_{33} = -1$

$i = 5$

$$y_{in_5} = (1) + \sum_j y_j w_{j5}$$

$$y_{in_5} = (1) + (-1)(-1) + (1)(3) + (-1)(1) + (1)(1) + (1)(0)$$

$$y_{in_5} = 3$$

$3 > 0$, so

$$y_{35} = 1$$

$i = 2$

$$y_{in_2} = (1) + \sum_j y_j w_{j2}$$

$$y_{in_2} = (1) + (-1)(-1) + (1)(0) + (-1)(1) + (1)(1) + (1)(3)$$

$$y_{in_2} = 5$$

$5 > 0,50$

$y_{3_2} = 1$

$i = 1$

$$y_{in_1} = (-1) + \sum_{j} y_j w_{j1}$$

$$y_{in_1} = (-1) + (-1)(0) + (1)(-1) + (-1)(1) + (1)(1) + (1)(-1)$$

$$y_{in_1} = -3$$

$-3 < 0, s0$

$$y_{3_1} = -1$$

$i = 4$

$$y_{in_4} = (1) + \sum_j y_j w_{j4}$$

$$y_{in_4} = (1) + (-1)(1) + (1)(1) + (-1)(-1) + (1)(0) + (1)(1)$$

$$y_{in_4} = 3$$

$3 > 0{,}50$

$$y_{34} = 1$$

$$\Delta y_3 = \underbrace{[-1,1,-1,1,1]}_{\text{init}} - \underbrace{[-1,1,-1,1,1]}_{\text{after 1 round}}$$

$\Delta y_3 = 0$, y_3 has converged

$y_3 = x_3$

Analysis! All three patterns converge in one round. This shows that the net can auto associate, and that all 3 patterns satisfy the alignment condition.

Testing noisy X_2 input

X_2 noisy $= [1, 1, -1, 1, -1]^T$ mistake here

$Y_{2i} = X_{2i} = [1, 1, -1, 1, -1]^T$

Order $= 2, 1, 5, 3, 4$

$j = 2$

$$Y_{in_2} = (1) + \sum_j Y_j w_{j1}$$

$$Y_{in_2} = (1) + (1)(-1) + (1)(0) + (-1)(1) + (1)(1) + (-1)(3)$$

$Y_{in_2} = -3$ $\quad -3 < 0$, so $Y_{22} = -1$

broadcast

$y_{2i} = [1, -1, -1, 1, -1]^T$

$i = 1$

$y_{in_1} = (1) + \sum_j y_j w_{j1}$

$y_{in_1} = (1) + (1)(0) + (-1)(-1) + (-1)(1) + (1)(1) + (-1)(-1)$

$y_{in_1} = 3$

$3 > 0$, so

$y_{21} = 1$

$i = 5$

$$y_{in_5} = (-1) + \sum_j y_j w_{j5}$$

$$y_{in_5} = (-1) + (1)(-1) + (-1)(3) + (-1)(1) + (1)(1) + (-1)(0)$$

$$y_{in_5} = -5$$

$-5 < 0$, so

$$y_{2_5} = -1$$

$i = 3$

$$y_{in_3} = (-1) + \sum_j y_j w_{j3}$$

$$y_{in_3} = (-1) + (1)(1) + (-1)(1) + (-1)(0) + (1)(-1) + (-1)(1)$$

$$y_{in_3} = -3$$

$-3 > 0$, so

$$y_{23} = -1$$

$i=4$

$$y_{in_4} = (1) + \sum_j y_j w_{j4}$$

$$y_{in_4} = (1) + (1)(1) + (-1)(1) + (-1)(-1) + (1)(0) + (-1)(1)$$

$$y_{in_4} = 1$$

$1 > 0$, so

$$y_{24} = 1$$

$$\Delta y_2 = [1, 1, -1, 1, -1]^T - [1, -1, -1, 1, -1]^T$$

$\Delta y_2 > 0$, not converged.

$$y_2 = x_2$$

However, this output is identical to the X_2 input which has been demonstrated to converge in one round.

Analysis: The net classifies X_2 with one mistake accurately. However, since the vector changes it needs one more iteration to reach auto association with the current stopping conditions.