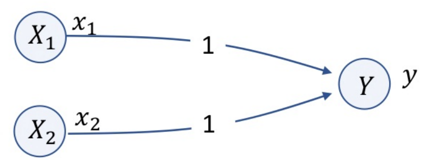
Rasid Ahmed Homework 1 9/3/20

1.5

Problem 1:

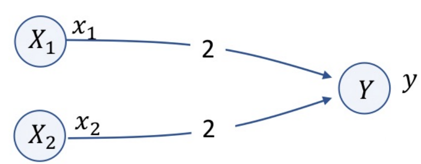
Source material:

Basic M-P neuron:
Logical AND



x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

Basic M-P neuron:
Logical OR



x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

Given:

M and P Neurons

Find:

a) 3 input OR

b) 3 input And

Analysis:

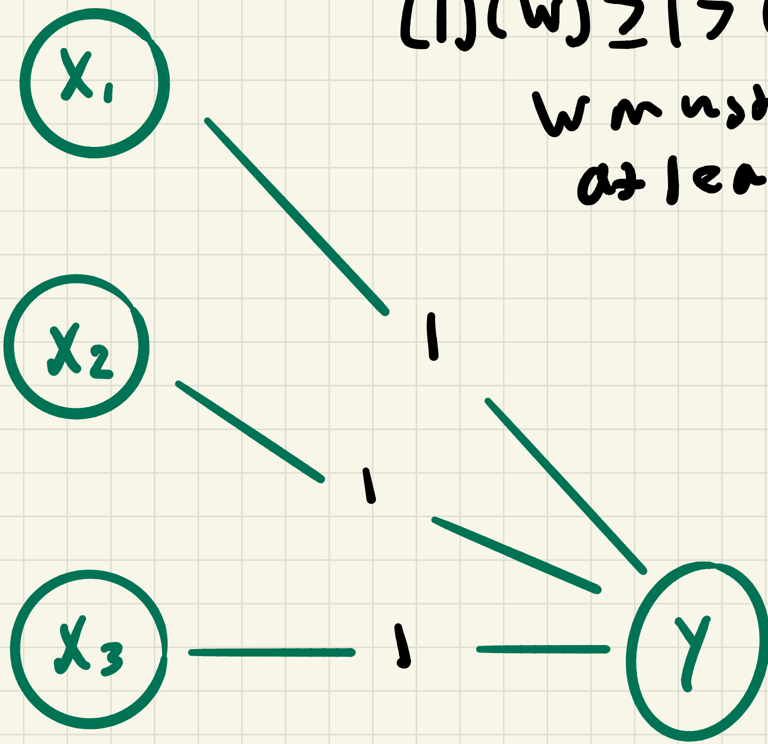
$Y: \theta = 1$

a)

$kw \geq \theta > (k-1)w$

$(1)(w) \geq 1 > 0$

w must be at least 1



x_1	x_2	x_3	y
0	0	1	1
0	1	0	1
1	0	0	1
1	1	1	1
0	0	0	0
1	1	0	1
0	1	1	1

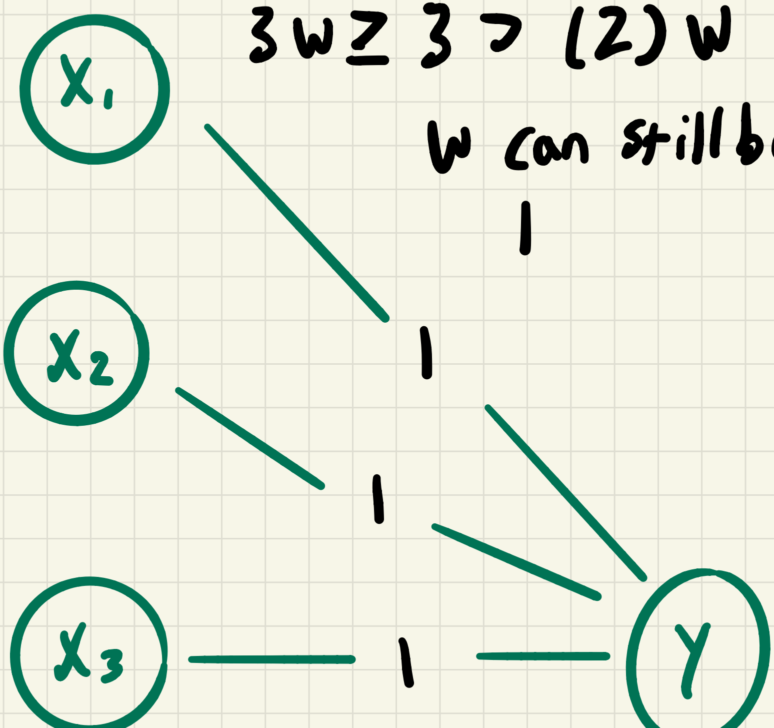
$Y: x_1(t-1)$ or $x_2(t-1)$ or $x_3(t-1)$

b) $Y: \theta = 3$

$kw \geq \theta > (k-1)w$

$3w \geq 3 > (2)w$

w can still be 1



x_1	x_2	x_3	y
0	0	1	0
0	1	0	0
1	0	0	0
1	1	1	1
0	0	0	0
1	1	0	0
0	1	1	0

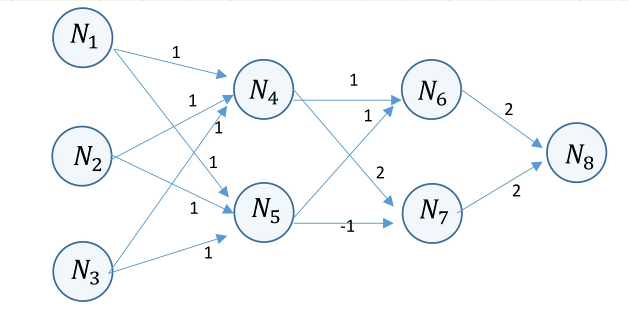
$Y: X_1(t-1)$ and $X_2(t-1)$ and $X_3(t-1)$

Problem 2:

Given:

N_n θ is 2

N_5 θ is 3



Find:

response of output at t based on input neuron activation

Analysis:

a)

Needs to capture $N_1(t-3)$ & $N_3(t-3)$

$$N_4(t-2) = N_1(t-3) \text{ or } N_2(t-3) \text{ and } N_3(t-3)$$

$$\text{or}$$

$$N_1(t-3) \text{ and } N_2(t-3) \text{ or } N_3(t-3)$$

$$N_5(t-2) = N_1(t-3) \text{ and } N_2(t-3) \text{ and } N_3(t-3)$$

$$N_6(t-1) = N_4(t-2) \text{ and } N_5(t-2)$$

$$N_7(t-1) = N_4(t-2) \text{ and not } N_5(t-2)$$

$$N_8(t) = N_6(t-1) \text{ or } N_7(t-1)$$

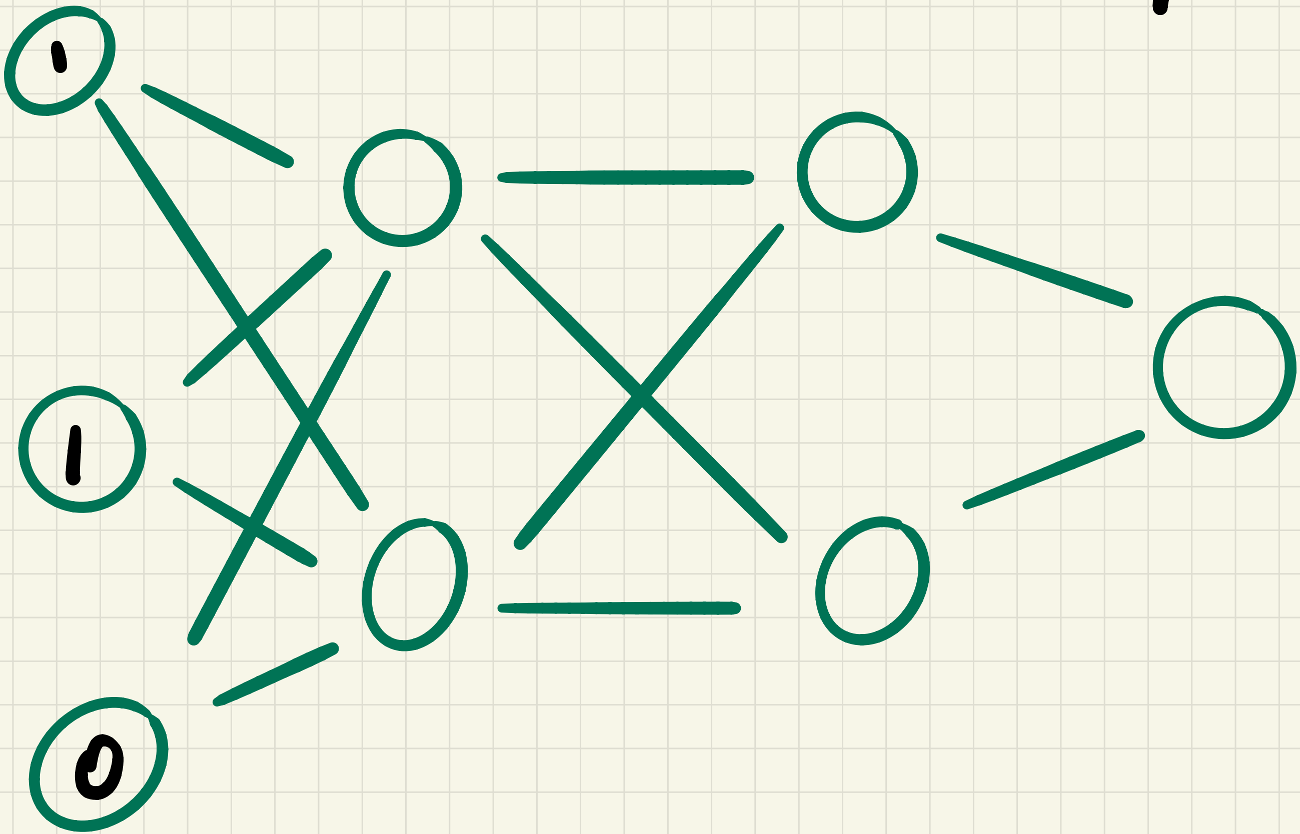
X Tracing back the expression for N_8 reveals that N_4 determines the output

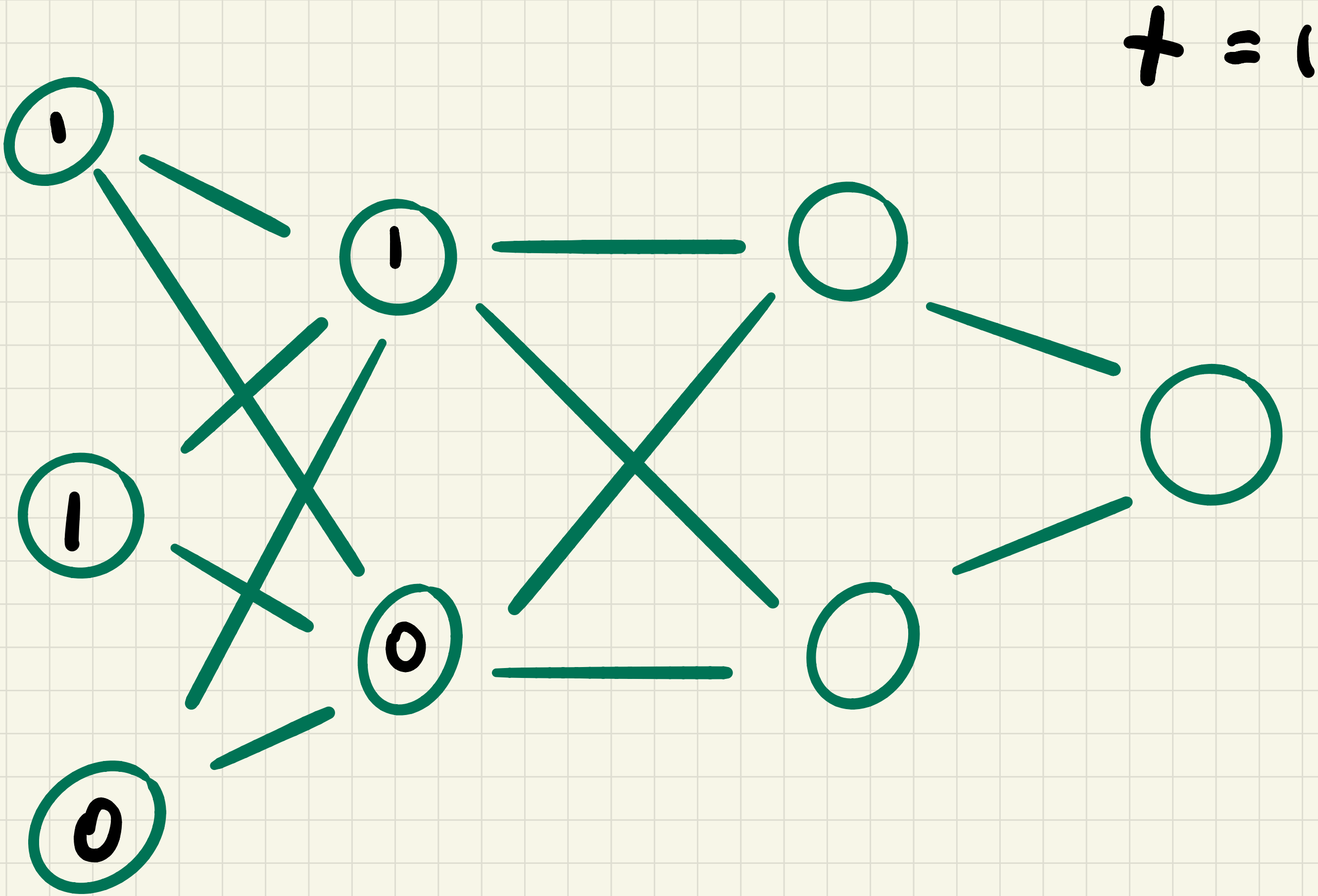
$$y(t) = N_8(t) = N_4(t-2) = \{X_1(t-3) \,\&\, X_2(t-3)\} \,||\, \{X_1(t-3) \,\&\, X_3(t-3)\} \,||\, \{X_2(t-3) \,\&\, X_3(t-3)\}$$

-.2

b)

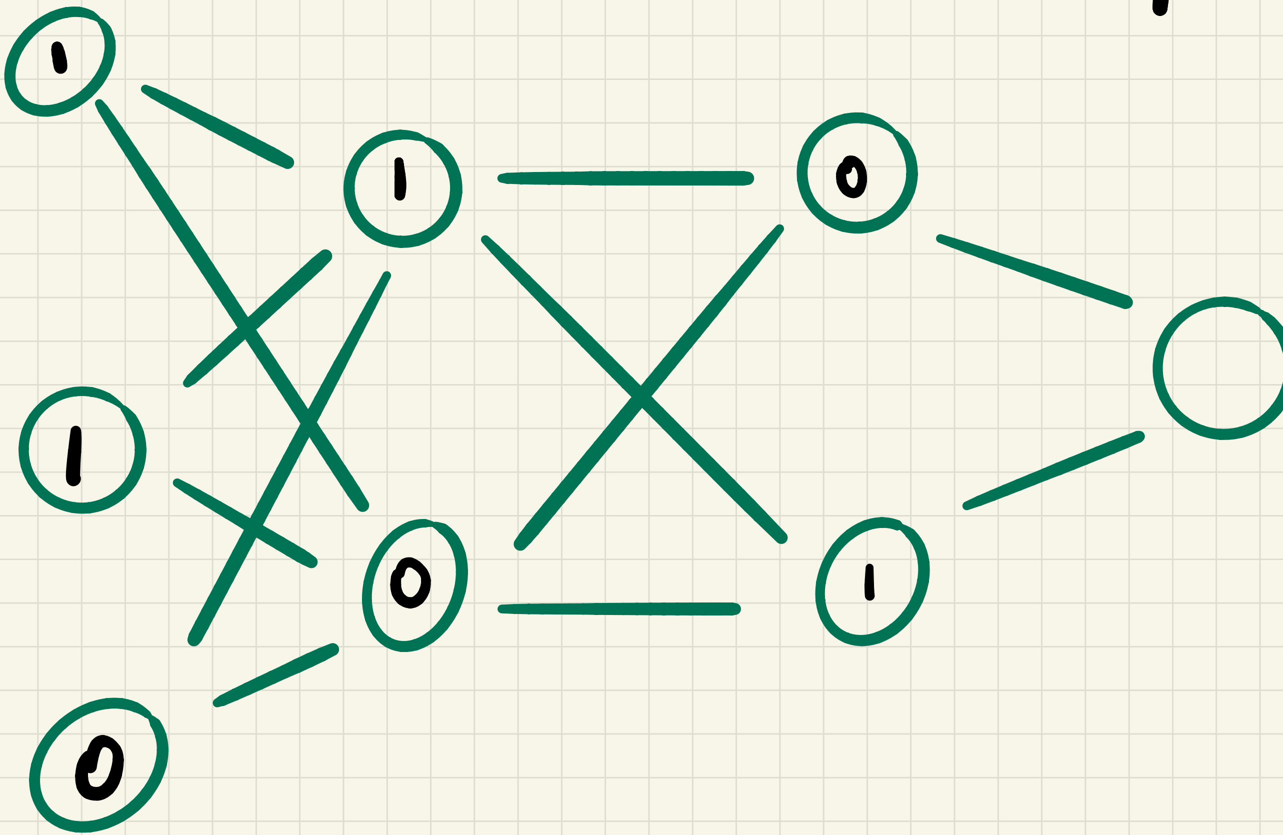
$t = 0$



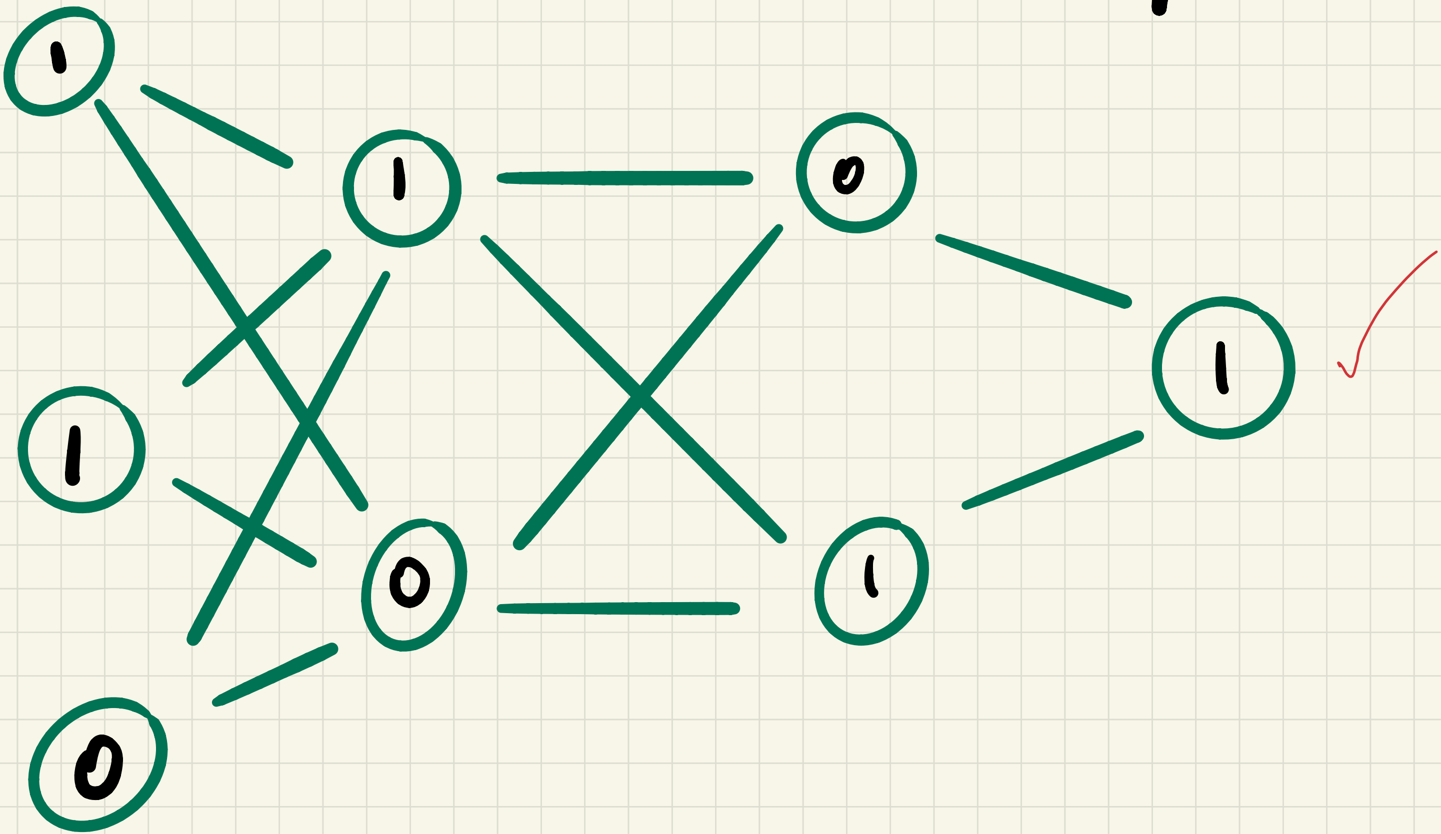


t = 1
1
1
0
1
0

$t = 2$

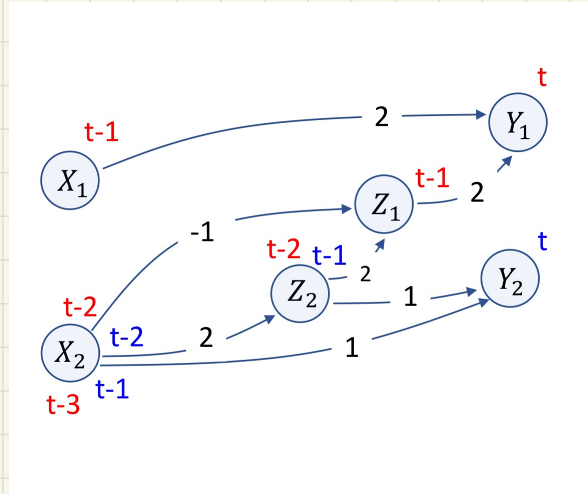


$t = 3$



Problem 3:

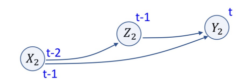
Source material:



✓

Cold Y_2: $x_2(t-1)\ AND\ x_2(t-1) = x_1(t-1)\ AND\ Z_2(t-1)$

$x_2(t-2)$	$x_2(t-1)$	Y_2
1	1	1
1	0	0
0	1	0
0	0	0



Given:

Hot and cold model

Cold stimulus must be there at t-4

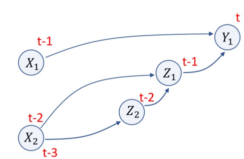
Find:

Model w/ these conditions

Hot physiology

Heat Y_1: $x_1(t-1)OR\left\{\overline{x_2(t-2)}\ AND\ x_2(t-3)\right\} = x_1(t-1)OR\ Z_1(t-1)$

$x_2(t-3)$	$x_2(t-2)$	$x_1(t-1)$	Y_1
1	1	1	1
1	0	1	1
0	1	1	1
0	0	1	1
1	1	0	0
1	0	0	1
0	1	0	0
0	0	0	0



Analysis:

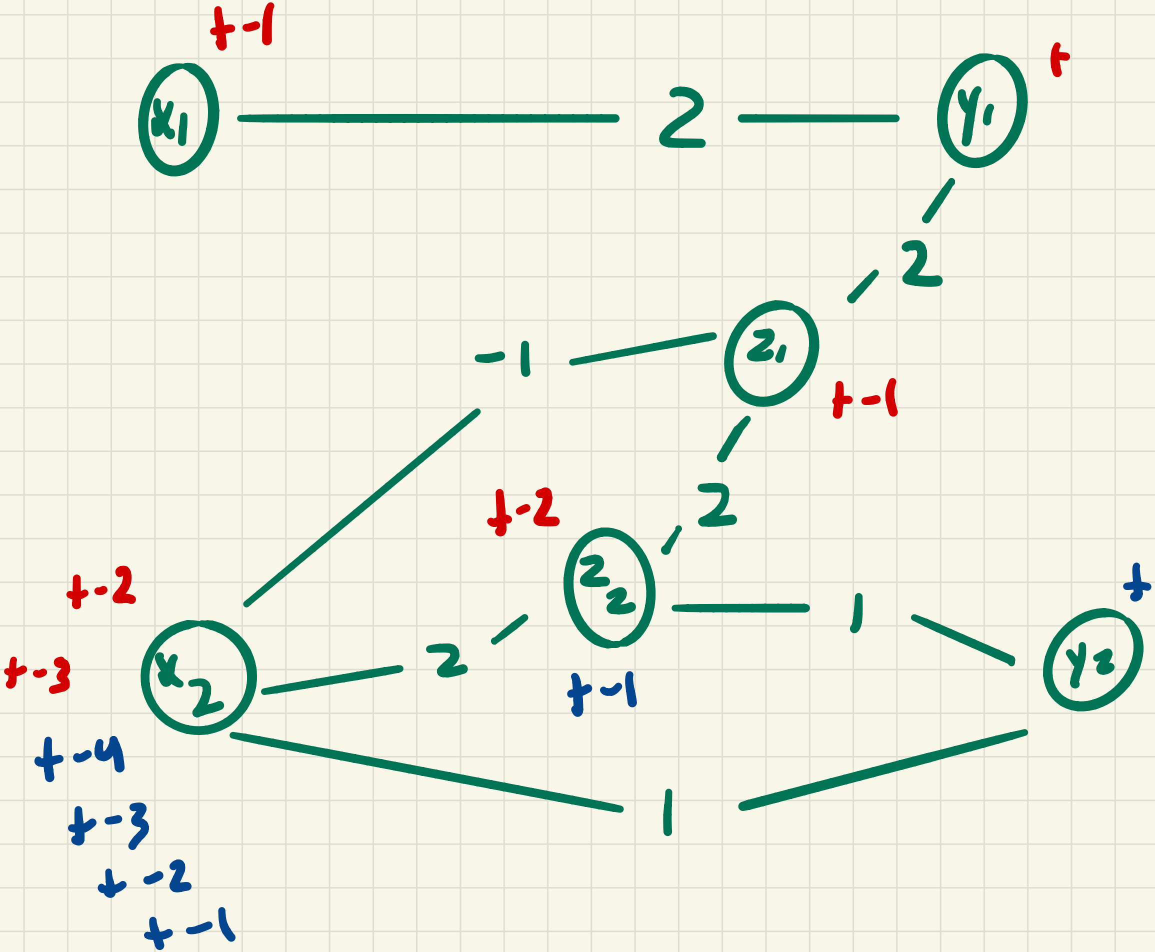
with current conditions cold is percieved if stimulated for ≥ 2 time steps

Changing it to ≥ 4 time steps

Necessary Changes:

Z_2 activation must be dependent on

$X_2(t-4, 3, 2, 1)$



✗ This model is correct for the 'hot' perceptron, but the model gives:

$y_2(t) = x_2(t-1) \& x_2(t-2)$

i.e. There is no architecture shown to execute the

$x_2(t-1) \& x_2(t-2) \&$
$x_2(t-3) \& x_4(t-4)$

requirement.

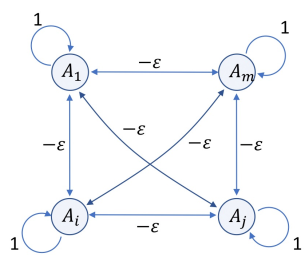
$-\frac{1}{2}$

Z_2: $X_2(t-4)$ and $X_2(t-3)$ and $X_2(t-2)$

Y_2: $X_2(t-4)$ and $X_2(t-3)$ and $X_2(t-2)$ and $X_2(t-1) = X_1(t-1)$ and $Z_2(t-1)$

Problem 3!

Given!



Competitive Layer
(Winner-take-all)

Find!

a) update activation for $j = 1:4$ w/

$$a_j(new) = f\left[a_j(old) - \epsilon \sum_{j \neq k} a_k(old)\right]$$

$$a_j(old) = a_j(new), j = 1:4$$

b) stop if only one node has positive activation value, otherwise re update

$a_j(new)$ = Activation of A_j

$a_j(0)$ = init value of node, j 1:4

$W_{i,j} = \begin{cases} 1 & \text{if } i = j \\ -\varepsilon & \text{if } i \neq j \end{cases}$

Nodes activate if $X > 0$, $\varepsilon = .2$ ✓

$a_1(0) = .3$, $a_2(0) = 1.2$

$a_3(0) = .6$, $a_4(0) = .9$

Analysis:

$$a_1(new) = f\left(a_1(old) - \varepsilon \sum_{1 \neq k} a_k(old)\right)$$

$$a_1(new) = f(.3 - .2(1.2 + .6 + .9))$$

$$a_1(new) = f(-.24)$$

$$a_1(new) = 0$$

$a_2(\text{new}) = f(1.2 - .2(.3 + .6 + .9))$

$a_2(\text{new}) = f(.84) = .84$ ✓

$a_3(\text{new}) = f(.6 - .2(.3 + 1.2 + .9))$

$a_3(\text{new}) = f(.12) = .12$ ✓

$a_4(\text{new}) = f(.9 - .2(.3 + 1.2 + .6))$

$a_4(\text{new}) = f(.48) = .48$ ✓

a_1 nodeis dropped due to the negative value b)

$a_2 (new) = f(.84 - .2(.12 + .48))$

$a_2 (new) = f(.72) = .72$ ✓

$a_3 (new) = f(.12 - .2(.84 + .48))$

$a_3 (new) = f(-.144) = 0$ ✓

$a_4 (new) = f(.48 - .2(.84 + .12))$

$a_4 (new) = f(.288) = .288$

a_3 node dropped

$a_2(\text{new}) = f(.72 - .2(.288))$

$a_2(\text{new}) = f(.6624) = .6624$

$a_4(\text{new}) = f(.288 - .2(.72))$

$a_4(\text{new} = f(.144) = .144$

No nodes dropped

$$a_2(\text{new}) = f(.6624 - .2(.144))$$

$$a_2(\text{new}) = f(.6336) = .6336$$

$$a_4(\text{new}) = f(.144 - .2(.6624))$$

$$a_4(\text{new}) = f(.01152) = .01152$$

No nodes dropped

$a_2(\text{new}) = f(.6336 - .2(.01152))$

$a_2(\text{new}) = f(.631296) = .631296$

$a_4(\text{new}) = f(.01152 - .2(.631296))$

$a_4(\text{new}) = f(-.1147392) = -.1147392$

a_4 dropped

a_2 is the winner at $t=5$

t	0	1	2	3	4	5
a_1	.3	0	0	0		
a_2	1.2	.84	.72	.6624	.6336	.631296
a_3	.6	.12	0	0		
a_4	.9	.48	.288	.144	.01152	0

Problem 5:

Source material:

Given:

XOR architecture

Find:

an alternate representation

MuCulloch & Pitts (M-P)

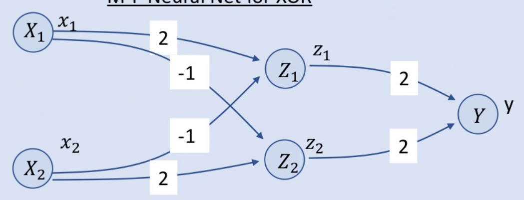
Applications (XOR)

$$y(t) = x_1(t-2)\ XOR\ x_2(t-2)$$

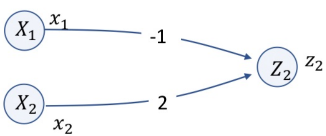
$$= z_1(t-1)\ OR\ z_2(t-1)$$

$$= \{x_1(t-2)\ And\ NOT\ x_2(t-2)\}$$
$$OR$$
$$\{x_2(t-2)\ And\ NOT\ x_1(t-2)\}$$

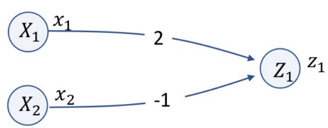
M-P Neural Net for XOR



$z_2(t) = x_2(t-1)\ AND\ \overline{x_1(t-1)}$



$z_1(t) = x_1(t-1)\ AND\ \overline{x_2(t-1)}$



XOR

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

Analysis:

Need to construct architecture where either X_1 or X_2 can activate Y, but not both

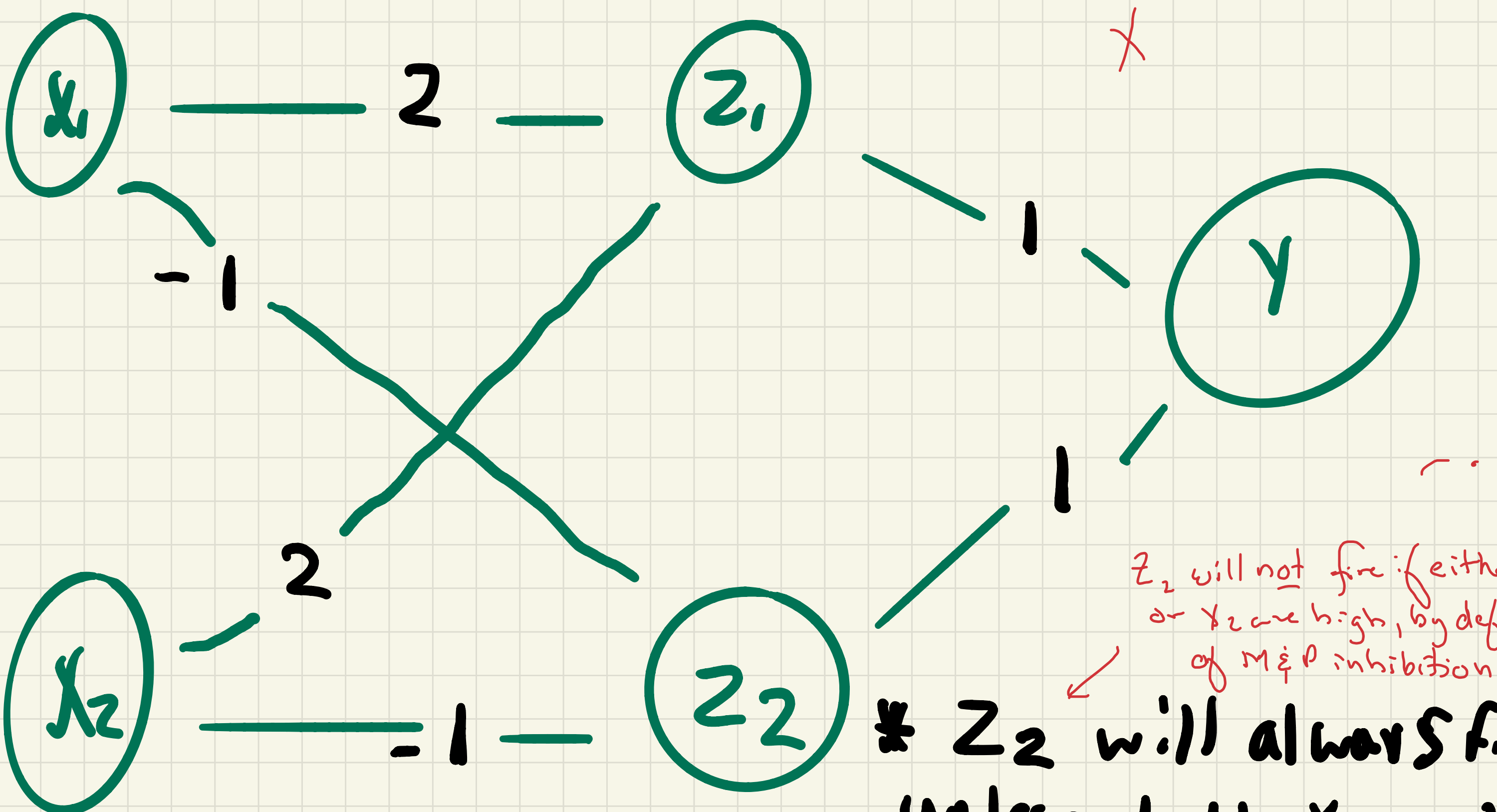
have $Z_1 = X_1$ or X_2

$Z_2 = X_1$ not and X_2

$Y = Z_1$ and Z_2

Θ for Z_1, Y is 2

Θ for Z_2 is -2



z_2 will not fire if either x_1 or x_2 are high, by definition of M&P inhibition reg.

* Z_2 will always fire unless both X_2 and X_1 are triggered