Raiid Ahmed Homework 5

Project 4.1:

Given:

Kohonen self organizing
2 input units
50 cluster units
linear topology for cluster
winners and neighbors learn
start with $\alpha = .5$, reduce gradually to $\alpha = .01$ over 100 epochs
initial weights are random between -1 and 1
for each component of the weight vector for each unit

Training set

- Start with 2 random numbers X_1 and X_2 in range of -.5 to .5, (X_1, X_2) denote a point
- if $X_1^2 + X_2^2 < .25$, put X_1 and X_2 in training set
- repeat until we have 100 training points

- graph cluster units every 10 epochs
  - use weight vector as a position in a 2D Euclidean plane
  - graph line from point to point

Net construction:

Initialization

Training set:

- pick 2 random #s using rand and bound range to (-.5, .5)
- check if X_1, X_2 satisfies $X_1^2 + X_2^2 < .25$
- if so, add X_1, X_2 to 100x2 vector, repeat
- if not, repeat
- repeat until training vector is full

Weights!

- Generate a 2 x 50 matrix using rand command, then bound range to (-1, 1)

learning rates!

- use matlab matrix sequence to generate vector for α
- iterate from .5 to .01 in 100 equal steps

Radius:

- R stays at 1, direct neighbors change but only the neighbors

Algorithm!

- Randomly select input X from training set
  - use bounded rand command to select a random index
- Calculate $O(j) = \sum_i (w_{ij} - x_i)^2$
  - winning output J is index of min(O)
- Update weights of winning nodes and neighbors

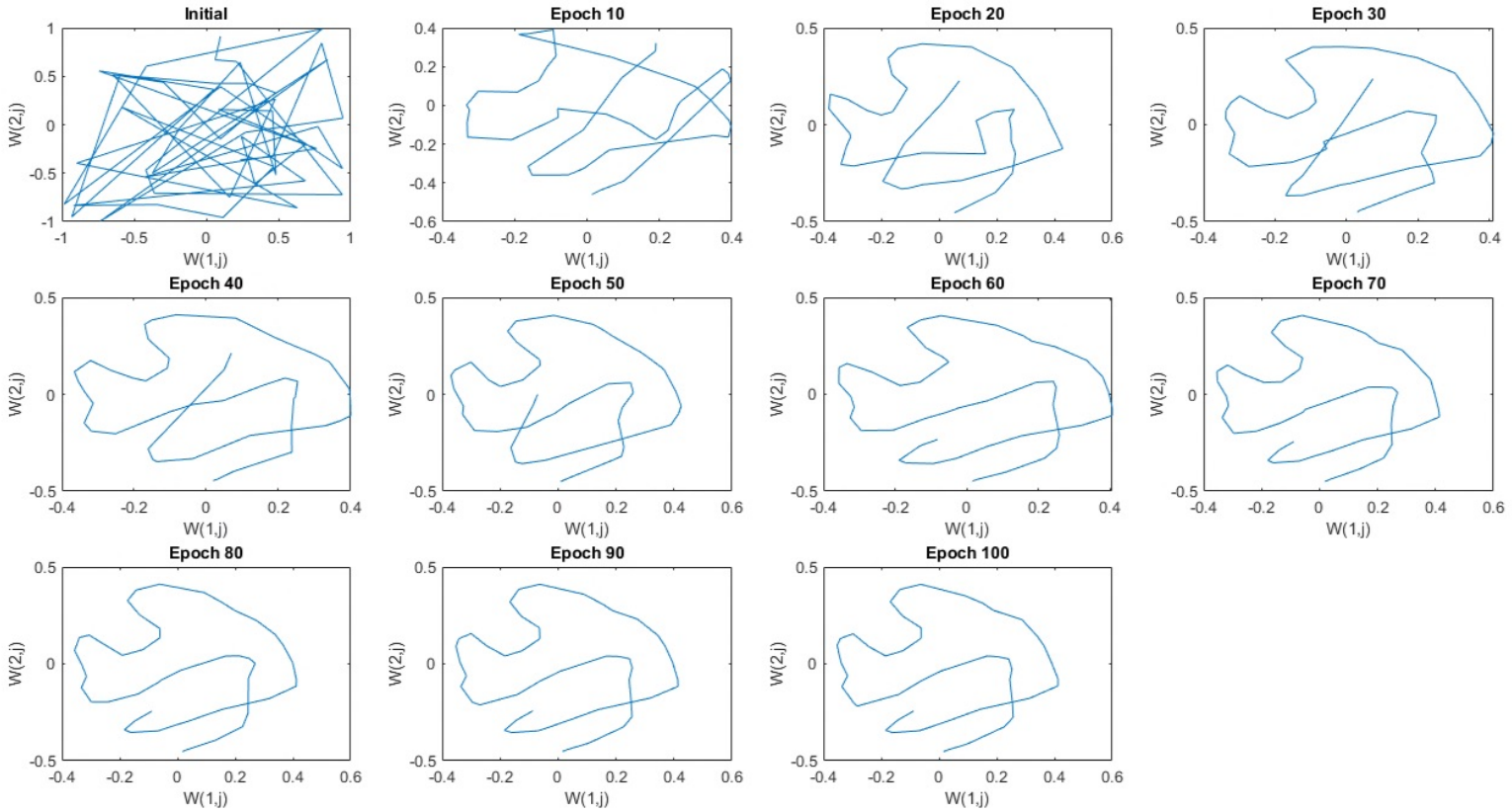
$$w_{ij}(new) = w_{ij}(old) + \alpha(x - w_{ij})$$

- Run until all input vectors are tested
  - iterate for 100 epochs, switching α to a lower value each time
- Every 10 iterations, generate a map of Cluster units with w_{ij} as position for each unit

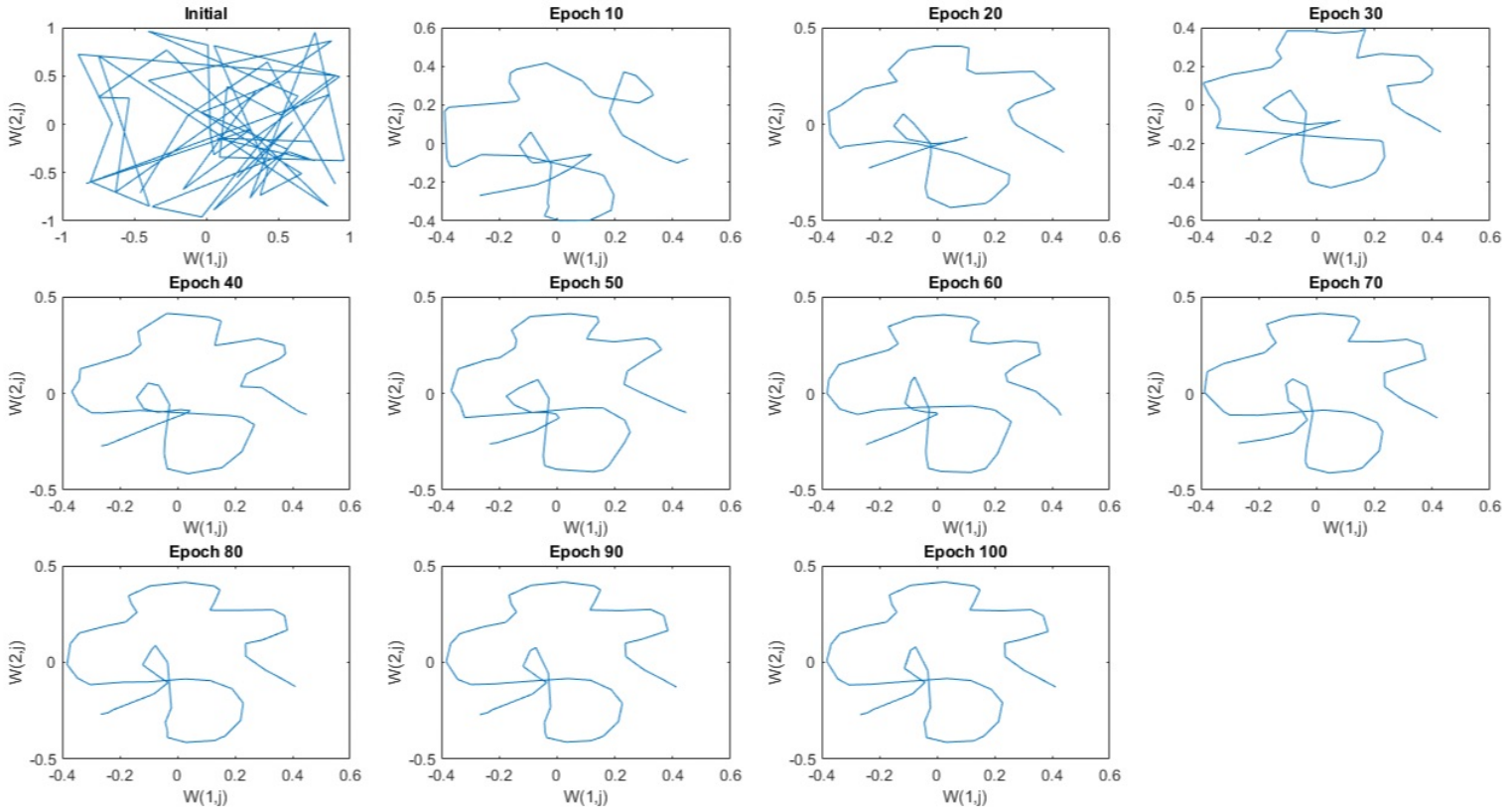
Test!

When selecting 100 testing points randomly and iterating for 100 epochs we get sets of 11 graphs showing how the weights update in the Kohonen map. First graph shows the initial weights, second graph shows the weights after 10 epochs, third for 20 epochs, and so on so forth. 3 individual runs with unique input sets are shown here.

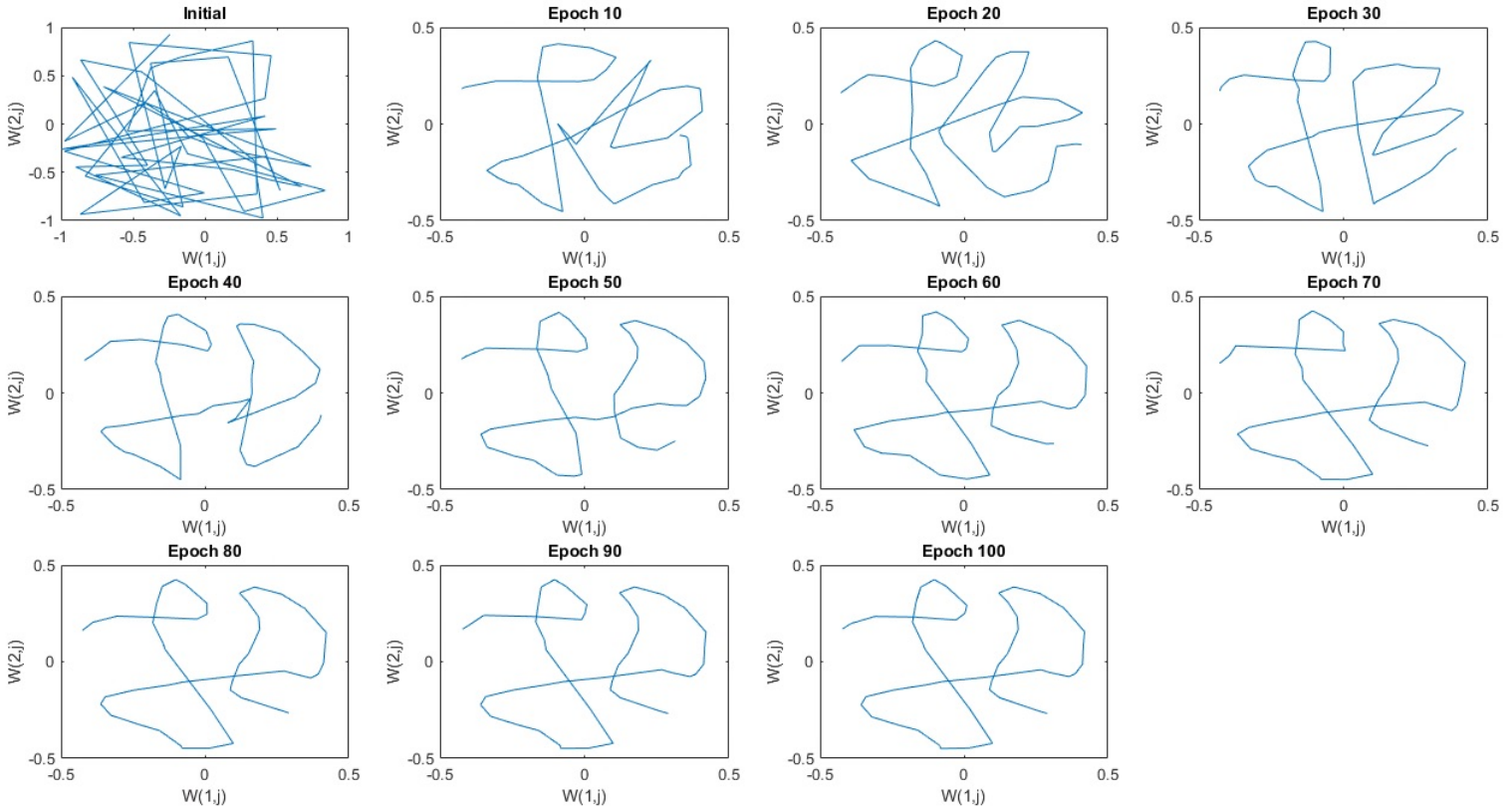
Run 1:


Initial
Epoch 10
Epoch 20
Epoch 30
Epoch 40
Epoch 50
Epoch 60
Epoch 70
Epoch 80
Epoch 90
Epoch 100
W(1,j)
W(2,j)

Run 2:


Initial
Epoch 10
Epoch 20
Epoch 30
Epoch 40
Epoch 50
Epoch 60
Epoch 70
Epoch 80
Epoch 90
Epoch 100
W(1,j)
W(2,j)

Run 3:


Initial
Epoch 10
Epoch 20
Epoch 30
Epoch 40
Epoch 50
Epoch 60
Epoch 70
Epoch 80
Epoch 90
Epoch 100
W(1,j)
W(2,j)

Analysis:

The initial graphs are a complete mess as expected. However, the first 10 epochs bring a massive improvement to the smoothness of the map.

Further training seems to provide incrementally decreasing gains, as all 3 runs were well-fitted after 50 epochs.

I would like to see if it possible to train further and analyze the general rand function.

Experiment:

I want to see if we can create a map to approximate matlabs rand function.

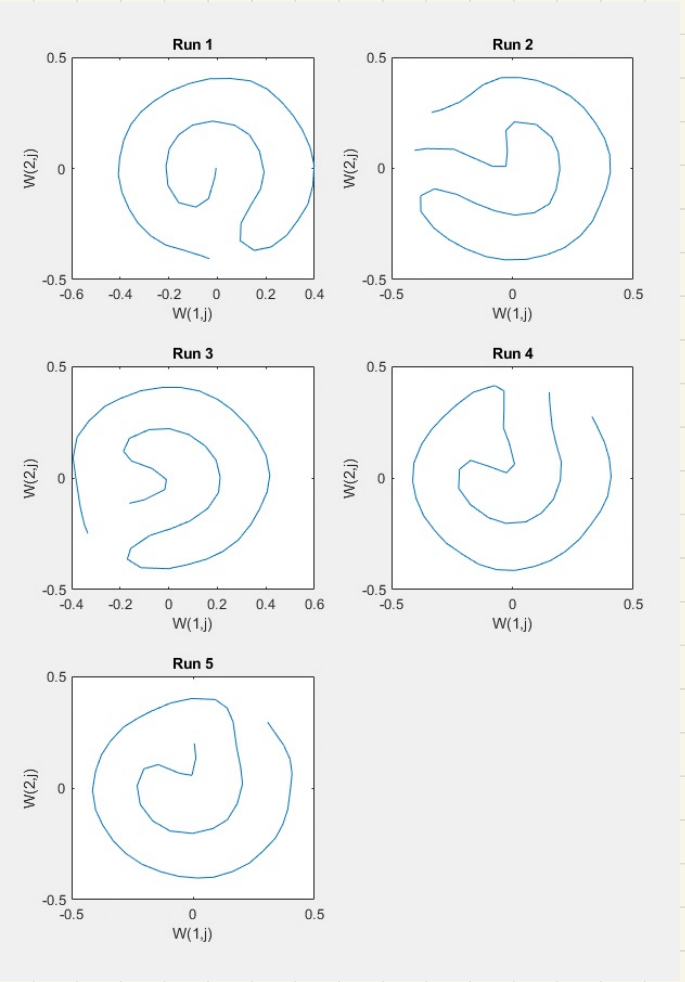
Modifications:

- Train for 1000000 epochs
- Reduce input to a single 2 element vector that is changed every epoch with matlab's rand function
- graph weights after each run of 1000000 epochs

Test!

Ran algorithm for 5 runs of 1000000 epochs each, all with random and "unique" inputs.

Graphs show W after each run.



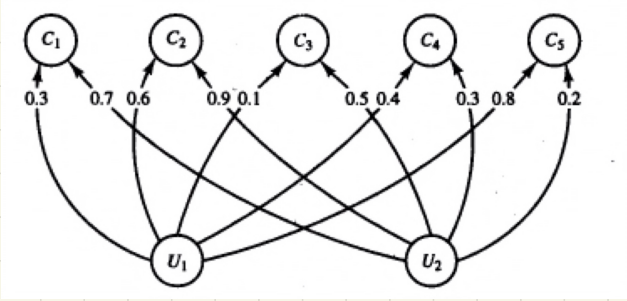
Analysis:

After training for 1000000 epochs, the resulting final cluster maps look very similar. They seem to all be permutations of the same set of values. We can say that the patterns are very similar Overtime for the rand function when generating only 2 inputs.

Exercise 4.2

Given!

Kohonen Map



Find:

a) Cluster unit C_J closest to vector (.5, .2)

b) new weights for C_J

$\alpha = .2$

c) new weights for C_{J-1}, C_{J+1}

Calculations!

a)

$$D(j) = \sum_i (w_{ij} - x_i)^2$$

$x_1 = .5$

$x_2 = .2$

$$D(1) = (.3 - .5)^2 + (.7 - .2)^2$$

$$D(1) = .29$$

$$D(2) = (.6 - .5)^2 + (.9 - .2)^2$$

$$D(2) = .5$$

$$D(3) = (.1 - .5)^2 + (.5 - .2)^2$$

$$D(3) = .25$$

$$D(4) = (.4 - .5)^2 + (.3 - .2)^2$$

$$D(4) = .02$$

$$D(5) = (.8 - .5)^2 + (.2 - .2)^2$$

$$D(5) = .09$$

C_4 is closest to input vector

b) $w_{14} = .4 + (.2)(.5 - .4)$

$w_{14} = .42$

$w_{24} = .3 + (.2)(.2 - .3)$

$w_{24} = .28$

C)

$$w_{13} = .1 + (.2)(.5 - .1)$$

$$w_{13} = .18$$

$$w_{23} = .5 + (.2)(.2 - .5)$$

$$w_{23} = .44$$

$$w_{15} = .8 + (.2)(.5 - .8)$$

$$w_{15} = .74$$

$$w_{25} = .2 + (.2)(.2 - .2)$$

$$w_{25} = .2$$

Exercise 4.3

Given!

Exercise 4.2

$a = .1$

$(.5, .5)$

Find:

4.2 a, b, c

Calculations:

a) $D(j) = \sum_i (w_{ij} - x_i)^2$

$x_1 = .5$

$x_2 = .5$

$D(1) = (.3 - .5)^2 + (.7 - .5)^2$

$D(1) = .08$

$D(2) = (.6 - .5)^2 + (.9 - .5)^2$

$D(2) = .17$

$D(3) = (.1 - .5)^2 + (.5 - .5)^2$

$D(3) = .16$

$D(4) = (.4 - .5)^2 + (.3 - .5)^2$

$D(4) = .05$

$D(5) = (.8 - .5)^2 + (.2 - .5)$

$D(5) = .18$

b)

$$w_{14} = .4 + (.1)(.5 - .4)$$

$$w_{14} = .41$$

$$w_{24} = .3 + (.1)(.5 - .3)$$

$$w_{24} = .32$$

c)

$$w_{13} = .1 + (.1)(.5 - .1)$$

$$w_{13} = .14$$

$$w_{23} = .5 + (.1)(.5 - .5)$$

$$w_{23} = .5$$

$$w_{15} = .8 + (.1)(.5 - .8)$$

$$w_{15} = .77$$

$$w_{25} = .2 + (.1)(.5 - .2)$$

$$w_{25} = .23$$

Exercise 4.4

Given:

Kohonen

2 clusters

5 inputs, $\alpha = .2$

$W_1 = (1.0, .8, .6, .4, .2)$

$W_2 = (.2, .4, .6, .8, 1.0)$

$x = (.5, 1.0, .5, 0, 0)$

Find:

- winning cluster
- update weights

Calculations!

$$D(j) = \sum_i (w_{ij} - x_i)^2$$

$$X = (.5, 1.0, .5, 0, 0)$$

$$D(1) = (1-.5)^2 + (.8-1.0)^2 + (.6-.5)^2 + (.4-0)^2 + (.2-0)^2$$

$$D(1) = .5$$

$$D(2) = (.2 - .5)^2 + (.4 - 1.0)^2 + (.6 - .5)^2 +$$
$$(.8 - 0)^2 + (1 - 0)^2$$

$$D(2) = 2.1$$

C_1 wins

$$W_1 = \begin{bmatrix} 1 \\ .8 \\ .6 \\ .4 \\ .2 \end{bmatrix} + (.2) \begin{bmatrix} .5 - 1 \\ 1 - .8 \\ .5 - .6 \\ 0 - .4 \\ 0 - .2 \end{bmatrix} = \begin{bmatrix} .9 \\ .84 \\ .58 \\ .32 \\ .16 \end{bmatrix}$$