ESC 407

Computational Methods in Engineering Science

Homework 6 Supplement

Pseudocode for Runge-Kutta Fehlberg

Maybe some text here?

function RKF45($\vec{f}$, tspan: a and b, y_0, tol, $h_{\max}$, $h_{\min}$)
  $t_1 \leftarrow a$
  $y_1 \leftarrow y_0$ ▷ first row of the solution array is y_0
  $h_1 \leftarrow h_{\max}$
  flag $\leftarrow 1$ ▷ stop flag
  $i = 1$ ▷ initialize the array index
  while flag $= 1$ **do**
    $k_1 \leftarrow h_i f\left(t_i, y_i\right)$ ▷ calculate all the k's
    $k_2 \leftarrow h_i f\left(t_i + \frac{1}{4}h_i, y_i + \frac{1}{4}k_1\right)$
    $k_3 \leftarrow h_i f\left(t_i + \frac{3}{8}h_i, y_i + \frac{3}{32}k_1 + \frac{9}{32}k_2\right)$
    $k_4 \leftarrow h_i f\left(t_i + \frac{12}{13}h_i, y_i + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3\right)$
    $k_5 \leftarrow h_i f\left(t_i + h_i, y_i + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right)$
    $k_6 \leftarrow h_i f\left(t_i + \frac{1}{2}h_i, y_i - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5\right)$
    $e \leftarrow \min\left(\frac{1}{h}\left|\frac{1}{360}k_1 - \frac{128}{4275}k_3 - \frac{2197}{75240}k_4 + \frac{1}{50}k_5 + \frac{2}{55}k_6\right|\right)$ ▷ error between RK4 and RK5
    if $e \leq$ tol **then** ▷ the error is acceptable
      $i \leftarrow i + 1$ ▷ increment the array index
      $h_i \leftarrow h_{i-1}$ ▷ the step-size is unchanged
      $t_i \leftarrow t_{i-1} + h_i$ ▷ the next time step
      $y_i \leftarrow y_{i-1} + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4104}k_4 - \frac{1}{5}k_5$ ▷ the RK4 step
    end if
    $d \leftarrow 0.84\left(\text{tol}/e\right)^{1/4}$ ▷ compute the factor for scaling the step-size
    if $d \leq 0.1$ **then** ▷ step-size change is too small
      $h_i \leftarrow 0.1h_i$
    else if $d \geq 4$ **then** ▷ step-size change is too large
      $h_i \leftarrow 4h_i$
    else ▷ step-size change is just right!
      $h_i = dh_i$
    end if
    if $h_i > h_{\max}$ **then** ▷ if the new h is too large, set it to $h_{\max}$
      $h_i \leftarrow h_{\max}$
    end if
    if $t_i \geq b$ **then**
      flag $= 0$ ▷ we've reached b, so we are done
    else if $t_i + h_i > b$ **then**

35: $h_i = b - t_i$ ▷ if the next step goes past b, take next step exactly to b
36: **else if** $h < h_{\min}$ **then**
37: `flag` $= 0$ ▷ required step-size is too small so quit
38: display minimum h exceeded
39: **end if**
40: **end while**
41: **end function**

Output: solution array $[y]$, vector of solution times $\vec{t}$, vector of time steps $\vec{h}$