PennState

The Pennsylvania State University
Department of Engineering Science and Mechanics

ESC 407

Computational Methods in Engineering Science

Exam 1

Start: 8:00 a.m. on Saturday, October 24
End: 4:00 p.m. on Monday, October 26, 2020

Problem 1	*Problem 2*	*Problem 3*	**Total**

By signing below, I pledge that:

✓ I will not consult any external resources for any reason. I will not use the internet for *anything*, materials from other classes or books or any notes you have taken in other classes, etc.

✓ I will not consult with or talk to any other person about the exam. I will not check my exam answers with any person.

✓ If I have a question or need a clarification about the exam, the only person I will ask is Prof. Gray, either on Zoom or by email.

Signature: ____

If you do not sign this pledge and return it with your exam, you will receive zero credit on the exam.

General Instructions

- This exam is open book and open notes, which means you are permitted to use any materials handed out in class, your own notes from the course, and anything on our Canvas website.
- You may use MATLAB (or other Python or whatever) as a calculator or to perform repetitive calculations, but you must show all detailed work written by hand, either with pencil/pen and paper or on a tablet.
- Submit the exam in Canvas, with the problems in the correct order, before 4:00 p.m. on Monday, October 26.
- *Failure to show **all** of your work on any give problem will result in a zero on that problem!*

Problem 1

(30 pts)

Find the fourth Taylor polynomial $P_4(x)$ for the function $f(x) = xe^{x^2}$ about $x_0 = 0$.

(a) Find an upper bound for $|f(x) - P_4(x)|$ on $x \in [0, 0.4]$.

(b) Approximate $\int_0^{0.4} f(x)\,dx$ using $\int_0^{0.4} P_4(x)\,dx$.

(c) Find an upper bound on the error of the integral in part (b) when using $P_4(x)$ to approximate $f(x)$.

(d) Approximate $f'(0.2)$ using $P_4'(0.2)$ and the find true error.

Problem 2

(30 pts)

Answer the following questions using the bisection method and its error term.

(a) Find an approximation to $\sqrt[3]{25}$ correct to within an approximate relative error of 10^{-4}. Use the two integers nearest to the actual root as your initial bracket.

(b) Find the number of bisection iterations needed for accuracy 0.000001 using the initial bracket you used in part (a). *Do not do the iterations.*

Problem 3

(40 pts)

An object of mass m is placed a distance h above a nonlinear spring whose force-displacement relationship is given by

$$F_s = k_1\delta + k_2\delta^{7/4},$$

where δ is the deflection of the spring. Newton's second law or the work-energy principle can be used to show that the maximum deflection of the spring δ_m is given by the solution to

$$\frac{4}{11}k_2\delta_m^{11/4} + \frac{1}{2}k_1\delta_m^2 - mg(\delta_m + h) = 0.$$

Using $k_1 = 40\,\mathrm{N/n}$, $k_2 = 10\,\mathrm{N/m^{7/4}}$, $m = 3\,\mathrm{kg}$, $h = 0.5\,\mathrm{m}$, and $g = 9.81\,\mathrm{m/s^2}$, do the following.

(a) Use Newton's method starting with $\delta_{m0} = 1.4$ to determine δ_m to within an approximate relative error of 10^{-4}. Report your approximation of the root, the approximate relative error, and the number of iterations required.

(b) Now use the secant method with the same number of iterations used in part (a) to estimate the root, and with starting guesses $x_0 = 0.1$ and $x_1 = 2$. Report the estimate of the root and the approximate relative error.

Show all your work!