Homework 2 Raiid Ahmed

3(a)(i): $f(x) = x^2 \sin(x) \quad x_0 = 3, 30$

$0^2\sin(0) + 0(2\sin(0) + 0\cos(0))(x)$

$+ (4(0)\cos(0) - (0^2-2)\sin(0))(x^2)(1/2)$

$-((0^2-6)\cos(0) + 6(0)\sin(0))(x^3)(1/6) = 27, 27000$

$+((0^2-12)\sin(0) - 8(0)\sin(0))(x^4)(1/24)$

$+((0^2-20)\cos(0) + 10(0)\sin(0))(x^5)(1/120) = -40.5, -4050000$

$+(12(0)\cos(0) - (0^2-30)\sin(0)(x^6))(1/720)$

$-((0^2-42)\cos(0) + 14(0)\sin(0))(x^7)(1/5040) = 18.225, 18225000$

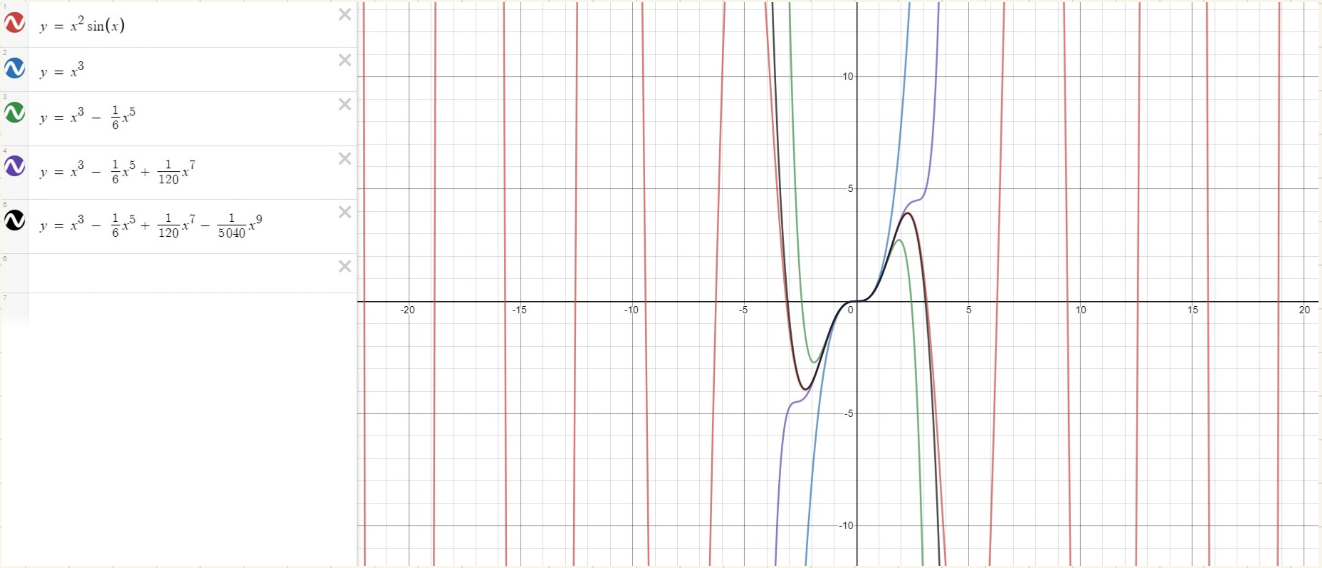
$+((x^2-56)\sin(0) - 16(0)\cos(0))(x^8)(1/40320)$ -3.905357

$+((x^2-72)\cos(0) - 18(0)\sin(0))(x^9)(1/362880) = -3905357$

3(a)(ii): Simplifying: $x^3 - \frac{1}{6}x^5 + \frac{1}{120}x^7 - \frac{1}{5040}x^9 \ldots$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2+(2n+1)}}{(2n+1)}$$

3(a)(iii):



3(b)(i): $f(x) = \cos(3x^2)$ $\quad x_0 = 2$ $\quad$ * only showing nonzero expressions to save space

$$\cos(3(0)^2) = 1$$

$$+ \left(108\left(12(0)^2 \sin(3(0)^2) + (12(0)-1)\cos(3(0)^2)\right)\right)(2^4)\left(\frac{1}{24}\right) = -72$$

$$+ \left(3888\left(168(12(0)^4 - 5)(0)^2 \sin(3(0)^2) + (432(0)^8 - 2520(0)^4 + 35)\cos(3x^2)\right)\right) \rightarrow$$

$$\rightarrow (2^8)(1/40320) = 864$$

$$+ \left(1259712\left(396(48(0)^8 - 280(0)^4 + 35)(0)^2 \sin(3(0)^2) +\right.\right.$$

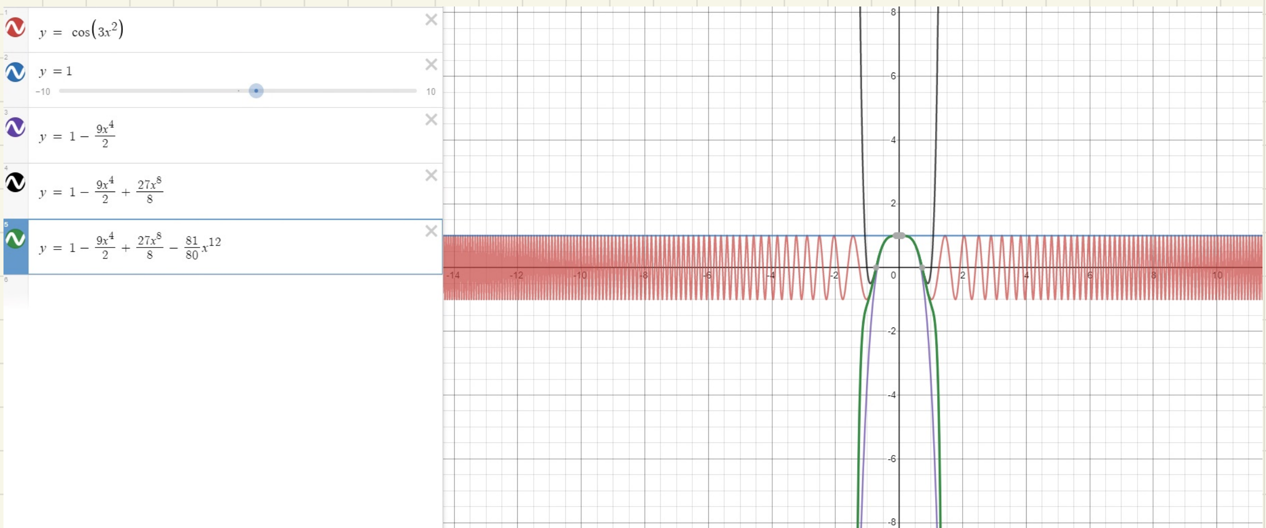
$$\left.\left.(1728(0)^{12} - 71280(0)^8 + 69300(0)^4 - 385)\cos(3(0)^2)\right)\right)(2^{12})\left(\frac{1}{12!}\right) =$$

$$-4147.2$$

3(b)(ii): Simplifying: $1 - \frac{9x^4}{2} + \frac{27x^8}{8} - \frac{81x^{12}}{80} \dots$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-9)^n (x^2)^{2n}}{(2n)!}$$

3(b)(iii):



3(c)(i): $f(x) = \int_0^x \cos \xi^2 \, d\xi \quad x_0 = 1$ * only showing nozero expressions to save space

Convert to Fresnel C integral

* wikipedia.com/wiki/Fresnel_integral
quora.com/How-do-you-take-the-integral-of-cos-x-2

$$\cos(\xi^2) = \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{4n}}{(2n)!}$$ Maclaurin expansion from $\cos(x)$

$$\int_0^x \cos(\xi^2)\, d\xi = \int_0^x \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{4n}}{(2n)!} d\xi = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \int_0^x \xi^{4n} \, d\xi$$

$$\int_0^x \xi^{4n} d\xi = \left[\frac{\xi^{4n+1}}{4n+1} \right]_0^x = \frac{x^{4n+1}}{4n+1} - \frac{0^{4n+1}}{1} = \frac{x^{4n+1}}{4n+1}$$

3(c)(ii)! $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \, x^{4n+1}}{(2n)!(4n+1)}$

$f(0) = 1$

3(c)(iii):

$f(1) = -\frac{1}{10}$

$f(2) = \frac{1}{216}$

$f(3) = -\frac{1}{9360}$

3(d)(i): $f(x) = \frac{1}{(1+x^2)}$ $x_0 = .5,\ x_0 = 2$

$$\frac{1}{1+(0)^2} = 1, 1$$

$$\left(\frac{8(0)^2}{((0)^2+1)^3} - \frac{2}{((0)^2+1)^2}\right)(x^2)\left(\frac{1}{2}\right) = -x^2 = -.25, -4$$

$$\left(-\frac{288(0)^2}{((0^2)+1)^4} + \frac{24}{((0)^2+1)^3} + \frac{384(0)^4}{((0)^2+1)^5}\right)(x^4)\left(\frac{1}{24}\right) = x^4 = .0625, 16$$

$$\left(\frac{17280(0)^2}{((0)^2+1)^5} - \frac{720}{((0)^2+1)^4} + \frac{46080(0)^6}{((0)^2+1)^7} - \frac{57600(0)^4}{((0)^2+1)^6}\right)(x^6)\left(\frac{1}{720}\right) = -x^6 = -.015625,$$

-64

3(d)(ii): simplifying: $1 - x^2 + x^4 - x^6 \ldots$

$$f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

3(d)(iii):