

Question 4

Let $m^2 - 2m - 3$ be even.

Suppose m is even.

Then $m = 2k$ for some $k \in \mathbb{Z}$.

$$\begin{aligned}\text{Now, } m^2 - 2m - 3 &= (2k)^2 - 2(2k) - 3 \\ &= 4k^2 - 4k - 3 \\ &= 2x - 3 \text{ where } x = (2k^2 - 2k)\end{aligned}$$

This implies that $m^2 - 2m - 3$ is odd because any even number subtracted by an odd number, will always result in an odd number.

Therefore, we have a contradiction, and so, m is odd.