Question 6

let CLAIM(n) be $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for all n ∈ N.

Step 1: CLAIM(1) is $1^3 = \frac{1^2(1+1)^2}{4}$

LHS = 1; RHS = 1;

Therefore, LHS = RHS and so CLAIM(1) is true.

Step 2: Assume CLAIM(k) is true for some k ∈ N

Assume CLAIM(k) is true for some, prove that CLAIM(k+1) is also true which means to prove that $1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{(k+1)^2((k+1)+1)^2}{4}$

that is, to prove that $1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$

LHS of CLAIM(k+1) = $1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3$

= $\frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4}$

= $\frac{(k+1)^2(k^2+4k+4))}{4}$

= $\frac{(k+1)^2(k+2)^2}{4}$

= RHS of CLAIM(k+1)

So, CLAIM(k) implies CLAIM(k+1), completing Step 2. Therefore, by mathematical induction, CLAIM(n) is true for all n ∈ N.