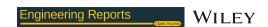
### RESEARCH ARTICLE



# Study of mechanical properties and enhancing auxetic mechanism of composite auxetic structures

 $Zengqin~Shi^1 \perp Qing~Wang^{1,2} \\ @ \perp Yunfeng~Li^1 \perp Ning~Wang^1 \perp Lulu~Lei^1 \perp Xiaodong~Li^1$ 

<sup>1</sup>Institute of Nano Engineering, College of Civil Engineering and Architecture, Shandong University of Science and Technology, Qingdao, China

<sup>2</sup>College of Mechanical and Architectural Engineering, Taishan University, Tai'an, China

#### Correspondence

Qing Wang, Institute of Nano Engineering, College of Civil Engineering and Architecture, Shandong University of Science and Technology, Qingdao, 266590 Shandong, China.

Email: qwang@sdust.edu.cn

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#### **Abstract**

The auxetic structures attract much attention because of their good properties, such as enhanced energy absorption capacity and buckling resistance. It is an important topic to combine the auxetic structures with other materials or structures to improve their mechanical properties. In this paper, a composite auxetic structure was proposed, in which the embedded structure and re-entrant structure can deform independently to adjust the mechanical properties. Under uniaxial compression, the effects of changes in the embedded structure and re-entrant structure on Poisson's ratio, relative Young's modulus, and energy absorption of the composite auxetic structure were studied. By introducing the rate of change of mechanical properties, the effects of the embedded structure and re-entrant structure on the composite auxetic structure were studied. The results show that changing the embedded structure for Poisson's ratio and relative Young's modulus and changing the re-entrant structure for energy absorption can maximize the material utilization. And changing the re-entrant structure can make the mechanical properties of the composite auxetic structure reach a larger adjustable range. This study can provide a new way for the combination of future auxetic structures to meet the needs of different applications in civil engineering.

#### **KEYWORDS**

 $auxetic\ mechanism,\ embedded\ structure,\ flexible\ deformation,\ mechanical\ properties,\ re-entrant\ structure$ 

## 1 | INTRODUCTION

Different from conventional materials, the auxetic structures (also termed metamaterials) have the unique property of negative Poisson's ratio. The auxetic structures contract (expand) laterally when they are vertically compressed (stretched). In 1987, Lakes<sup>1</sup> reported the first artificial auxetic structure in his experiment. Then researchers began to pay more and more attention to the auxetic structures. Many good properties were studied, such as enhanced buckling resistance,<sup>2</sup> shear modulus,<sup>3</sup> indentation resistance,<sup>4</sup> fracture toughness,<sup>5,6</sup> and energy absorption capacity.<sup>7</sup> These good properties are reported to be beneficial for many practical applications.<sup>8,9</sup>

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Up to now, many kinds of the auxetic structures have been developed, mainly including the re-entrant structures, 10 rotating polygonal structures, 11 and chiral structures. 12 It is an important topic to combine the auxetic structures with other materials or structures to improve mechanical properties of the auxetic structures. 13-15 In order to improve the stiffness of the auxetic structures, Fu et al. 16,17 proposed two kinds of the composite auxetic structures that embed the x-structure and rhombic structure into the re-entrant structures, in which the effects of adjusting the angle of the proposed composite auxetic structures on Poisson's ratio and Young's modulus were studied. Chen et al. 18 designed a combined enhanced honeycomb structure based on two existing embedded enhanced honeycombs. Li et al. 19 investigated the yield strengths of the re-entrant structure and hexagonal structure with the embedded horizontal structure under the coupling effect of the shear and axial stress. They also found that the Young's modulus and strength of the auxetic structures can be designed independently with its Poisson's ratio by studying the re-entrant structure and double arrowhead structure with the embedded vertical structure.<sup>20</sup> When the auxetic structures are combined with other structures, their auxetic mechanism is often reduced. In order to reduce the loss of auxetic mechanism and increase stiffness, Baran et al.<sup>21</sup> obliquely embedded the rod into the traditional re-entrant structure. The mechanical properties of the re-entrant structure were studied by changing the geometric and material parameters. Chen et al.<sup>22</sup> proposed a lightweight auxetic structure by embedding the support rod into the rotating square structure. The combination of the auxetic structures with other structures affects the energy absorption capability.<sup>23,24</sup> It was found that the combination of the re-entrant structure and hexagonal structure has better energy absorption capability than that of the hexagonal structure.<sup>25</sup> To improve the energy absorption capability, three kinds of the auxetic structures were compared by embedding different ribs into the classic re-entrant structures.<sup>26</sup>

To achieve better integrity, sufficient constraints are often imposed between the embedded structure and auxetic structure for the composite auxetic structure. Adjusting mechanical properties often requires changing the shape of the structure. However, due to the existence of too many constraints, the embedded structure or auxetic structure cannot be deformed independently. When one structure is changed to adjust mechanical properties, other structures are often changed as well. In this paper, the independent deformation of the embedded structure and re-entrant structure was achieved in the proposed composite auxetic structure with flexible deformation (CASwFD). The effects of the embedded structure and re-entrant structure on the mechanical properties of the proposed composite auxetic structure were studied by the using rate of change of mechanical properties. This study found that the auxetic mechanism of the proposed auxetic structure was improved due to the reduction of the constraints between the embedded structure and re-entrant structure.

## 2 | THE PROPOSED COMPOSITE AUXETIC STRUCTURE

As shown in Figure 1(A), there are four fixed constraints ( $u_x = u_y = u_z = 0$  and  $rot_x = rot_y = rot_z = 0$ ) between the embedded structure and re-entrant structure of the rhombic grid embedded enhanced honeycomb (RGEEH).<sup>17</sup> When the shape of the re-entrant structure changes, it drives the embedded structure to deform (comparing the light and dark structure). Similarly, when the shape of the embedded structure changes, the re-entrant structure also changes. This makes the adjustment of mechanical properties complicated. To solve the above question, the top and bottom fixed constraints of the RGEEH were removed. And a CASwFD was proposed in Figure 1(B). It can be seen from Figure 1(C)-(E) that the embedded structure and re-entrant structure can deform independently. In this way, the effects of the embedded structure and re-entrant structure on the mechanical properties of the overall structure can be separated.

# 3 | THEORETICAL ANALYSIS

## 3.1 | Simplification

The CASwFD under uniaxial compression loading was displayed on Figure 2(A). The lengths of the horizontal and slant rods of the re-entrant structure are  $h_0$  and  $l_0$ , respectively. And their thicknesses are  $t_0$ . The length and thickness of the slant rod of the embedded structure are  $l_1$  and  $t_1$ , respectively. The angles  $\theta_0$  and  $\theta_1$  are formed by slant rods of the re-entrant structure and embedded structure with X direction, respectively.

Based on the geometrical and loading symmetry of the CASwFD, the rods AB, BC, and CD were adopted as the representative structure to analyze the mechanical properties of the CASwFD. As shown in Figure 2(B), because the points

FIGURE 1 (A) The rhombic grid embedded enhanced honeycomb18; (B) The composite auxetic structure with flexible deformation (CASwFD); (C) CASwFD-1: the embedded structure (the red structure) is changed and the re-entrant structure remains unchanged (the blue structure); (D) CASwFD-2: The re-entrant structure is changed (the length of the horizontal rod is changed, and the length of the slant rod remains the same) and the embedded structure remains unchanged; (E) CASwFD-3: The re-entrant structure is changed (the length of the horizontal rod remains unchanged, the length of the slant rod is changed) and the embedded structure remains unchanged

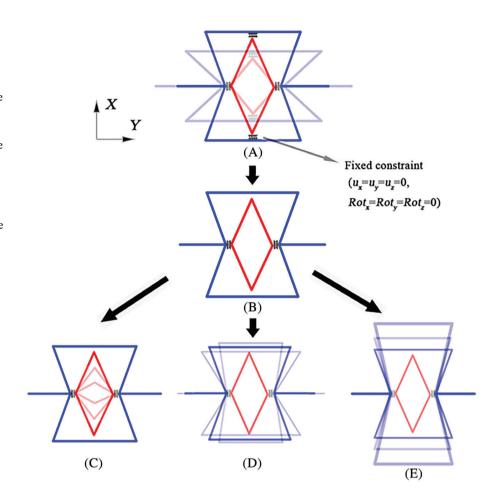
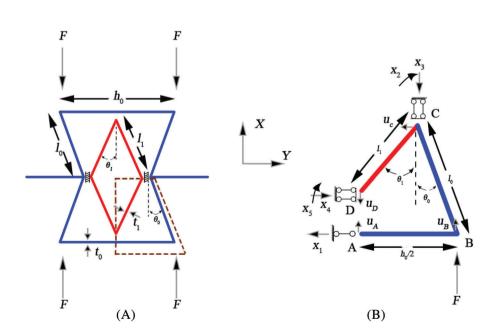


FIGURE 2 (A) Geometric parameters of the CASwFD; (B) the representative structure



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A and D are on the left–right symmetry plane of the CASwFD, under the compression of the vertical force F, they only cause vertical displacement  $u_A$  and  $u_D$ . And the point C is on the upper and lower symmetry plane of the CASwFD, which only causes horizontal displacement  $u_C$ . In the deformation process of the rod AB, the displacement of point A is equal to that of the point B ( $u_A = u_B$ ). Thus, the rod AB is only subjected to axial force  $x_1$ . Due to the deformations of the rods BC and CD dominated by bending, the point C is subjected to bending moment  $x_2$  and vertical reaction force  $x_3$ . And the point D is subjected to bending moment  $x_5$  and horizontal reaction force  $x_4$ .

# 3.2 | Model analysis

The force method and Mohr's theorem were applied to analyze this problem. The unknown reaction force  $x_1$  and bending moment  $x_2$  were employed to set up canonical equation of force method:

$$\delta_{11}x_1 + \delta_{12}x_2 + V_{1p} = 0, (1)$$

$$\delta_{21}x_1 + \delta_{22}x_2 + V_{2p} = 0. (2)$$

The  $\delta_{ij}$  is the displacement along the  $x_i$  direction caused by the unit force  $x_j = 1$ .  $V_{ip}$  is the displacement along  $x_i$  direction generated by the force F. Because the deformations of the robs BC and CD are mainly dominated by bending, their shear and axial deformations can be ignored. Therefore,  $\delta_{11}$ ,  $\delta_{22}$ ,  $\delta_{12}$ ,  $\delta_{21}$ ,  $V_{1p}$  and  $V_{1p}$  in Equations (1) and (2) can be expressed as

$$\delta_{11} = \sum \int \frac{\overline{M}_{1}^{2}}{EI} ds + \frac{\overline{F}_{N1}^{2} l}{EA} = \frac{l_{0}^{3} \cos^{2} \theta_{0}}{3EI_{0}} + \frac{l_{1} l_{0}^{2} \cos^{2} \theta_{0} - l_{1}^{2} l_{0} \cos^{2} \theta_{1} \cos^{2} \theta_{0} + (1/3) l_{1}^{3} \cos^{2} \theta_{1}}{EI_{1}} + \frac{h_{0}}{2EA_{0}}$$
(3)

$$\delta_{22} = \sum \int \frac{\overline{M}_2^2}{EI} ds = \frac{l_1}{EI_1},\tag{4}$$

$$\delta_{12} = \delta_{21} = \sum_{l} \int \frac{\overline{M}_{1} \overline{M}_{2}}{EI} ds = \frac{l_{1} l_{0} \cos \theta_{0} - (1/2) l_{1}^{2} \cos \theta_{1}}{EI_{1}}, \tag{5}$$

$$V_{1p} = \sum \int \frac{\overline{M}_1 M_p}{EI} ds = \frac{-(1/3)F l_0^3 \cos \theta_0 \sin \theta_0}{EI_0} - \frac{F l_1 l_0^2 \cos \theta_0 \sin \theta_0 - (1/2)F l_1^2 l_0 \sin \theta_0 \cos \theta_1}{EI_1}$$
 (6)

and

$$V_{2p} = \sum \int \frac{\overline{M}_2 M_p}{EI} ds = \frac{-Fl_1 l_0 \sin \theta_0}{EI_1},\tag{7}$$

respectively.  $\overline{M}_i$  is the bending moment generated by unit force  $x_i = 1$  at any section of the basic structure.  $M_p$  is the bending moment generated by the force F in the basic structure. E is the Young's modulus of the material.  $I_i = (bt_i^3)/12$  is second moment of area where b is the thickness of structure in out-plane direction. And  $A_0 = bt_0$  is the cross-section area of the rods AB and BC. By substituting Equations (3)–(7) into Equations (1) and (2),  $x_1$  and  $x_2$  are obtained as

$$x_1 = \frac{(Fl_0^3 \cos \theta_0 \sin \theta_0)/(3EI_0)}{(l_0^3 \cos^2 \theta_0)/(3EI_0) + (l_1^3 \cos^2 \theta_1)/(12EI_1) + h_0/(2EA_0)},$$
(8)

and

$$x_2 = Fl_0 \sin \theta_0 - \frac{(1/EI_0)(1/3)Fl_0^3 \cos \theta_0 \sin \theta_0 [l_0 l_1 \cos \theta_0 - (1/2)l_1^2 \cos \theta_1]}{(l_1 l_0^3 \cos^2 \theta_0)/(3EI_0) + (l_1^4 \cos^2 \theta_1)/(12EI_1) + (l_1 h_0)/(2EA_0)},$$
(9)

respectively. According to the superposition principle, the distribution of moment M of the representative structure in Figure 2(B) can be obtained as

$$M = \overline{M}_1 x_1 + \overline{M}_2 x_2 + M_p. \tag{10}$$

$$u_c = \sum \int \frac{\overline{M}_6 M}{EI} ds = \frac{3F l_1^2 l_0 \sin \theta_0 - 3x_1 l_1^2 l_0 \cos \theta_0 + 2x_1 l_1^3 \cos \theta_1 - 3x_2 l_1^2}{6EI_1}$$
(11)

and

$$u_{B} = \sum \int \frac{\overline{M}_{7}M}{EI} ds = \frac{[l_{0}^{2} \sin \theta_{0}(Fl_{0} \sin \theta_{0} - x_{1}l_{0} \cos \theta_{0})]}{3EI_{0}} + \frac{[l_{1}l_{0} \sin \theta_{0}(Fl_{0} \sin \theta_{0} - x_{1}l_{0} \cos \theta_{0} + (x_{1}l_{1} \cos \theta_{1})/2 - x_{2})]}{EI_{1}}, (12)$$

respectively. The strains of the representative structure in Y and X directions are

$$\varepsilon_y = \frac{u_c}{(h_0/2) + l_1 \sin \theta_1} \tag{13}$$

and

$$\varepsilon_{x} = \frac{u_{B}}{l_{0}\cos\theta_{0}}.\tag{14}$$

respectively. The stress caused by the force F on the representative structure is

$$\sigma = \frac{2F}{bh_0}. (15)$$

The Poisson's ratio and Young's modulus of the CASwFD can be obtained as

$$\mu = -\frac{\varepsilon_y}{\varepsilon_x} \tag{16}$$

and

$$E_0 = \sigma \varepsilon_x, \tag{17}$$

respectively.

## 4 | FINITE ELEMENT MODEL

Figure 3 shows the finite element model and boundary conditions of the CASwFD for elastic analysis. The CASwFD was modeled using SOLID186 which is a higher order 3D 20-node solid element that exhibits quadratic displacement behavior. The calculation of the Poisson's ratio and Young's modulus of the CASwFD did not consider large deformation. The parameters of matrix material were used as: the Young's modulus E = 69 GPa, Poisson's ratio  $\mu_0 = 0.33$ , yield strength  $\sigma_{ys} = 76$  Mpa and density  $\rho_s = 2700 \, \text{kg/m}^3$ . The mesh size (0.08 mm) made the average mesh quality reach 0.99, which is enough to prove the accuracy of the results. The compressive force  $F = 0.1 \, N$  was applied to the two end points of the top surface of the CASwFD. The application position of the force F was the same as that of the theoretical analysis. Fixed constraint was imposed on the bottom of the CASwFD. The degree of freedom in the E direction of the CASwFD was restricted. Under the compressive force E, distance E0 between points E1 and E2 in the E1 direction were used to calculate Poisson's ratio and relative Young's modulus.

Dynamic simulation was used to study energy absorption capability. Figure 4 shows the finite element model and boundary conditions of the CASwFD for dynamic analysis. As shown in the Figure 4, 8\*8 cell model was established. The impact velocity v(5 m/s) along the X direction was applied to the upper rigid plate, and the bottom of the model was fixed. In order to prevent out-of-plane expansion, the displacement of the model in the Z direction was constrained. In the compression process, single-surface contact was used between the model walls, and surface-to-surface contact between the model and the rigid plate was used. The effect of friction between the model walls was not considered. The Shell 163

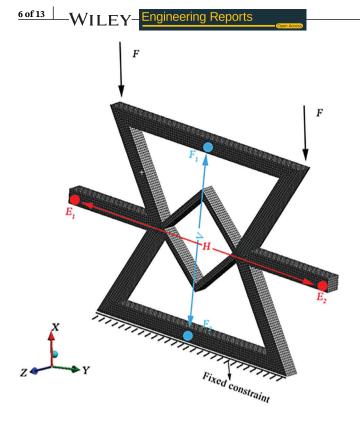


FIGURE 3 Finite element model of the CASwFD for elastic analysis

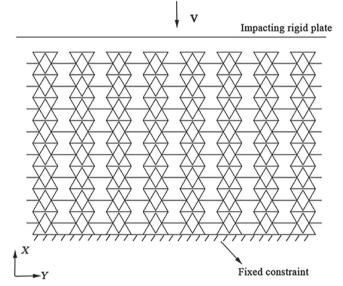


FIGURE 4 Finite element model of the CASwFD for dynamic analysis

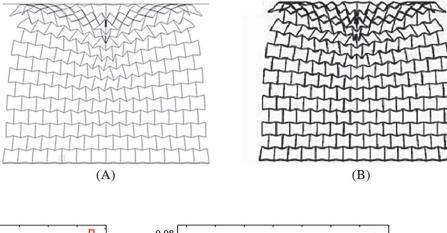
unit for dynamic analysis was applied to the model.<sup>27</sup> The parameters of material were the same as those of the elastic analysis, and the model was assumed to be ideal elastoplastic.

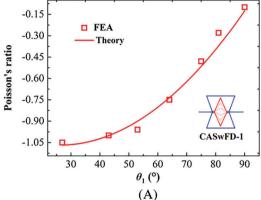
In order to verify the accuracy of the dynamic analysis, the same numerical model as Reference 27 was built for comparison. Figure 5 shows deformation of the re-entrant structure at impact velocity v = 7 m/s. It can be seen from Figure 5 that the deformation of the model established in this paper was basically the same as that of the model in Ref.<sup>27</sup>

## 5 | RESULTS AND DISCUSSION

# 5.1 | Changing the embedded structure

In this section, the mechanical properties of the CASwFD-1 were studied. The change of the embedded structure was defined by the change of angle  $\theta_1$ . The geometric parameters of the unchanging re-entrant structure with  $h_0 = 9.5$  mm,





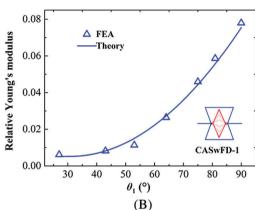


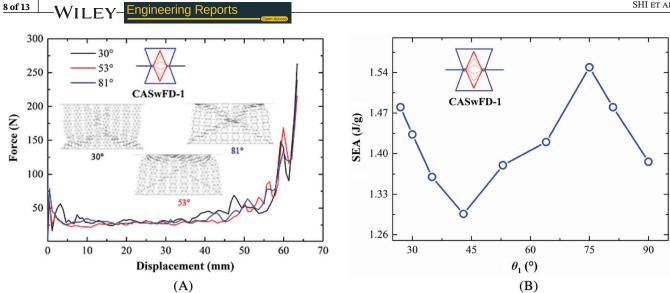
FIGURE 6 Effects of the angle  $\theta_1$  on the mechanical properties of the CASwFD-1: (A) Poisson's ratio and (B) relative Young's modulus

 $l_0 = 5$  mm,  $t_0 = 0.5$  mm, and  $\theta_0 = 30^\circ$  were determined. And the thickness  $t_1$  of the embedded structure is 0.2 mm. Figure 6 shows the effects of the angle  $\theta_1$  on the Poisson's ratio and relative Young's modulus  $(E_0/E)$  of the CASwFD-1. From Figure 6, the finite element results are in good agreement with the theoretical analysis. As shown in Figure 6(A), as the angle  $\theta_1$  increased, the Poisson's ratio of the CASwFD-1 increased monotonously. It was obtained that the increase of angle  $\theta_1$  leads to the decrease of auxetic mechanism. From Figure 6(B), as the angle  $\theta_1$  increased, the relative Young's modulus of the CASwFD-1 increased monotonously. This is because as the angle  $\theta_1$  increases, the Young's modulus of the embedded structure in the *Y* direction can increase, resulting in the higher stiffness and worse auxetic mechanism of the CASwFD-1.

Specific energy absorption (SEA) as the standard to measure the energy absorption capability was employed.<sup>25</sup> It is expressed as.<sup>28</sup>

$$SEA = \frac{EA}{m} = \frac{\int_0^L F(x)dx}{m} \tag{18}$$

where L is the length from the initial point to the beginning of the densification. F is the force corresponding to the length L. m is the mass of the structure. The force–displacement and SEA curves of the CASwFD-1 with various the angle  $\theta_1$  were shown in Figure 7. The deformation figures inserted in Figure 7(A) show the deformation of the CASwFD-1 with angle  $\theta_1 = 30^\circ$ , 53°, and 81° in the platform stage. It can be seen from the deformation figures that the three structures all presented a V-shaped deformation band at the bottom end,  $^{29}$  but the deformations of their upper parts were different. The force–displacement curves of the three structures were not much different, because the change of the embedded structure has little effect on the CASwFD-1. As shown in Figure 7(B), when the angle  $\theta_1$  was between about 27° to 43° and 75° to 90°, the SEA of the CASwFD-1 showed a decreasing trend. When the angle  $\theta_1$  was between 43° and 75°, the SEA of the CASwFD-1 showed an increasing trend.



(A) The force–displacement and (B) SEA curves of the CASwFD-1 with various the angle  $\theta_1$ 

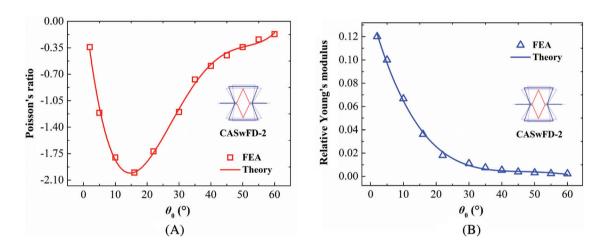


FIGURE 8 Effects of the angle  $\theta_0$  on the mechanical properties of the CASwFD-2: (A) Poisson's ratio and (B) relative Young's modulus

#### 5.2 Changing re-entrant structure

The mechanical properties of the CASwFD-2 and CASwFD-3 were studied. The change of the re-entrant structure was defined by the change of angle  $\theta_0$ . The geometric parameters of the unchanging embedded structure with  $l_1 = 3.18$  mm,  $\theta_1 = 45^{\circ}$ ,  $t_1 = 0.2$  mm were determined.

Figure 8 shows the effects of angle  $\theta_0$  on the Poisson's ratio and relative Young's modulus  $(E_0/E)$  of the CASwFD-2  $(l_0 = 5 \text{ mm})$ . From Figure 8, the finite element results are in good agreement with the theoretical analysis. As shown in Figure 8(A), as the angle  $\theta_0$  increased, the Poisson's ratio of the CASwFD-2 decreased first and then increased. When the angle  $\theta_0$  was about 15°, the Poisson's ratio of the CASwFD-2 reached the minimum value -2, in which the auxetic mechanism is the best. In Figure 8(B), as the angle  $\theta_0$  increased, the relative Young's modulus of the CASwFD-2 decreased monotonously. For energy absorption capability of the CASwFD-2, the force-displacement and SEA curves of the structure with various the angle  $\theta_0$  were shown in Figure 9. Similar to the CASwFD-1, the deformation of the CASwFD-2 with angle  $\theta_0 = 30^\circ$ ,  $40^\circ$ , and  $55^\circ$  in the platform stage was shown in Figure 9(A). It can be seen from Figure 9(A) that during the deformation of the platform, the bottoms of the CASwFD-2 with angle  $\theta_0 = 30^{\circ}$  and  $40^{\circ}$  also showed V-shaped deformation band, while its upper part showed dense collapse. But dense collapse occurred in the lower part of the CASwFD-2 with  $\theta_0 = 55^{\circ}$ . In Figure 9(A), as the angle  $\theta_0$  of the CASwFD-2 increased, its platform stage became shorter, but the

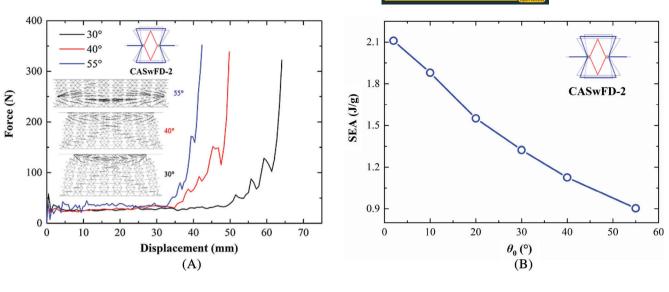


FIGURE 9 (A) The force-displacement and (B) SEA curves of the CASwFD-2 with various the angle  $\theta_0$ 

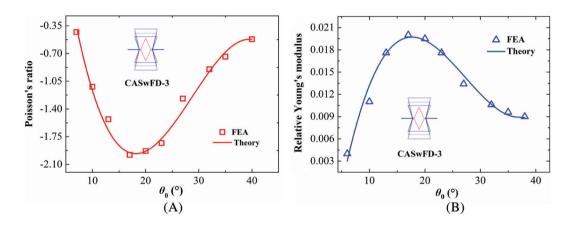


FIGURE 10 Effects of the angle  $\theta_0$  on the mechanical properties of the CASwFD-3: (A) Poisson's ratio and (B) relative Young's modulus

value of the platform force was not much different. The platform stage has an important effect on the energy absorption of the auxetic structures.<sup>27</sup> As shown in Figure 9(B), as the angle  $\theta_0$  increased, the SEA of the CASwFD-2 decreased monotonically.

Figure 10 shows the effects of the angle  $\theta_0$  on the Poisson's ratio and relative Young's modulus  $(E_0/E)$  of the CASwFD-3  $(h_0 = 9.5 \,\mathrm{mm})$ . From Figure 10, the finite element results are in good agreement with the theoretical analysis. As shown in Figure 10(A), as the angle  $\theta_0$  increased, the Poisson's ratio of the CASwFD-3 decreased first and then increased. When the angle  $\theta_0$  was about 18°, the Poisson's ratio of the CASwFD-3 reached the minimum value -1.98. In contrast, from Figure 10(B), with increase of the angle  $\theta_0$ , the relative Young's modulus of the CASwFD-3 increased first and then decreased, which reached a maximum value 0.02 at angle  $\theta_0 = 18^\circ$ . For energy absorption capability of the CASwFD-3, the force-displacement and SEA curves of the structure with various the angle  $\theta_0$  were shown in Figure 11. As shown in Figure 11(A), the deformation of the CASwFD-3 with angle  $\theta_0 = 23^\circ$ , 30°, and 40° at the platform stage was similar to that of the CASwFD-2 with angle  $\theta_0 = 30^\circ$ , 40°, and 55°, respectively. And as the angle  $\theta_0$  increased, the platform stage of the CASwFD-3 also became shorter. In Figure 11(B), similar to Figure 10(B), as the angle  $\theta_0$  increased, the SEA of the CASwFD-3 monotonically decreased.

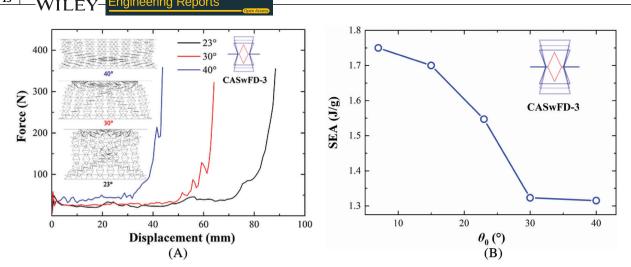


FIGURE 11 (A) The force-displacement and (B) SEA curves of the CASwFD-3 with various the angle  $\theta_0$ 

Structure	$\Delta \mu$	$\Delta m(g)$	$\Delta(E_0/E)$	$\Delta m(g)$	$\Delta EA(J)$	$\Delta m(g)$
CASwFD-1	0.95	0.00124	0.072	0.00124	0.184	0.448
					0.253	0.363
					0.163	0.284
CASwFD-2	1.66	0.0056	0.118	0.0164	0.957	0.855
	1.83	0.0108				
CASwFD-3	1.55	0.0437	0.016	0.0437	6.926	3.522
	1.46	0.0125	0.011	0.0125		

**TABLE 1**  $\Delta\mu$ ,  $\Delta(E_0/E)$ ,  $\Delta EA$ , and corresponding  $\Delta m$  of CASwFD-1, 2, and 3

# 5.3 | Comparison of the embedded and re-entrant structures

In order to compare the effects of embedded and re-entrant structures on the CASwFD, the change rate of mechanical properties was calculated using following Equations (19)–(21):

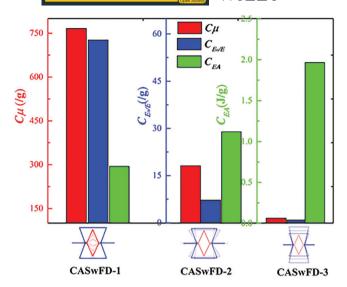
$$C_{\mu} = \frac{\Delta \mu}{\Delta m},\tag{19}$$

$$C_{E_0/E} = \frac{\Delta\left(\frac{E_0}{E}\right)}{\Delta m},\tag{20}$$

and

$$C_{EA} = \frac{\Delta EA}{\Delta m}. (21)$$

 $C_{\mu}$ ,  $C_{E_0/E}$ , and  $C_{EA}$ , respectively, represent the change value in Poisson's ratio, the relative Young's modulus, and energy absorption caused by changing unit mass, respectively.  $\Delta \mu$ ,  $\Delta \left(\frac{E_0}{E}\right)$ , and  $\Delta EA$  are the variation values of Poisson's ratio, relative Young's modulus and energy absorption of the embedded or re-entrant structures in the variable range, respectively.  $\Delta m$  is the corresponding mass change. Table 1 shows the data calculated from Figures 6–11. In Table 1, the energy absorption of the CASwFD-1 was divided into three parts. The Poisson's ratio of the CASwFD-2, Poisson's ratio and relative Young's modulus of the CASwFD-3 were divided into two parts in Table 1. Only the maximum values of them were obtained.



According to the data in Table 1, Figure 12 shows the values of  $C_{\mu}$ ,  $C_{E_0/E}$  and  $C_{EA}$  of the CASwFD-1, 2, 3. As shown in Figure 12, for the  $C_{\mu}$  and  $C_{E_0/E}$ , the values of the CASwFD-1 were the largest, which means that changing every unit mass of the CASwFD-1 can cause the largest variation range of Poisson's ratio and relative Young's modulus. For the  $C_{EA}$ , the value of the CASwFD-3 was the largest, which means that changing every unit mass of the CASwFD-3 can cause the largest variation range of energy absorption. Therefore, this also means that the material utilization rates of the CASwFD-1 for Poisson's ratio and relative Young's modulus and the CASwFD-3 for energy absorption reach maximum. The  $C_{\mu}$  of the CASwFD-1, the  $C_{E_0/E}$ , and  $C_{E_0}$  of the CASwFD-3 were the smallest. However, in Table 1,  $\Delta \mu$  and  $\Delta (E_0/E)$  of the CASwFD-2 were the largest, indicating that the CASwFD-2 has the largest adjustable range for Poisson's ratio and relative Young's modulus. The CASwFD-1 had the smallest adjustable range for relative Young's modulus and energy absorption. The  $\Delta EA$  of the CASwFD-3 was the largest, indicating that the CASwFD-3 has the largest adjustable range for energy absorption. The CASwFD-1 had the smallest adjustable range for Poisson's ratio and energy absorption. And the CASwFD-2 had the smallest adjustable range for Poisson's ratio and energy absorption. And the CASwFD-2 had the smallest adjustable range for Poisson's ratio and energy absorption. And the CASwFD-2 had the smallest adjustable range for Poisson's ratio and energy absorption. And the CASwFD-2 had the smallest adjustable range for Poisson's ratio and energy absorption. And the CASwFD-2 had the smallest adjustable range for Poisson's ratio and energy absorption. And the CASwFD-2 had the smallest adjustable range for Poisson's ratio and energy absorption.

## 5.4 | Comparison of auxetic mechanism

The auxetic mechanisms of the CASwFD were compared with the reported RGEEH.<sup>17</sup> Because of the unchanged re-entrant structure in the CASwFD-1, there is only one the RGEEH corresponding to the CASwFD-1. By calculation, the Poisson's ratio  $\mu$  of the RGEEH is 0.13. Compared to Figure 6(A), Poisson's ratio  $\mu$  of the RGEEH is larger. The comparisons of the Poisson's ratio of the CASwFD-2 and CASwFD-3 with the RGEEH at different angles  $\theta_0$  were shown in Figure 13. As shown in Figure 13(A), with increase of the angle  $\theta_0$ , the Poisson's ratio of the RGEEH was positive, while that of the CASwFD-2 was negative. From Figure 13(B), the Poisson's ratio of the CASwFD-3 was larger than that of the RGEEH when the angle  $\theta_0$  was less than 12°. These results show that the CASwFD has better auxetic mechanism than RGEEH. Based on Maxwell's stability criterion,<sup>30,31</sup> as shown in the following Equation (22)

$$M = n - 2j + 3, (22)$$

the Maxwell's stability factor M of the RGEEH is 0. Thus, the RGEEH is the deformation dominated by stretching. When the top and bottom constraints between the embedded and the re-entrant structures of the RGEEH are removed to obtain the CASwFD, the Maxwell's stability factor M of the CASwFD is less than 0. The deformation of the structure is transformed into bending-dominant deformation, which enhances the shrinkage of the structure in the Y direction. This means that the auxetic mechanism of the structure is improved.

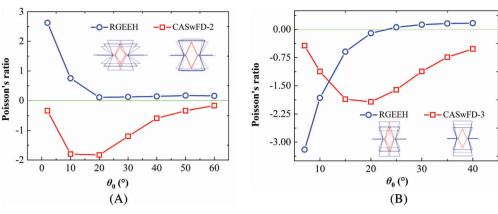


FIGURE 13
Comparison of the Poisson's ratio: (A) RGEEH versus
CASwFD-2; (B) RGEEH versus
CASwFD-3

# 6 | CONCLUSION

A CASwFD was proposed, in which the embedded and the re-entrant structures could deform independently to adjust the mechanical properties. By using theoretical and numerical analysis, the effects of changes in embedded and re-entrant structures on the overall structure's Poisson's ratio, relative Young's modulus, and energy absorption were studied. The results showed that as the angle  $\theta_1$  (CASwFD-1) of the embedded structure increased, the Poisson's ratio and relative Young's modulus of the structure monotonically increased. The energy absorption first decreased, then increased and finally decreased. When the angle  $\theta_0$  (CASwFD-2,3) of the re-entrant structure increased, the Poisson's ratio of the structure first increased and then decreased. The energy absorption decreased monotonically. The relative Young's modulus of the CASwFD-2 decreased monotonically, while the relative Young's modulus of the CASwFD-3 increased first and then decreased. In addition, the rate of change of mechanical properties was studied to compare the effects of embedded and re-entrant structures on the overall structure. It was found that the utilization rate of the material can reach an optimal value by changing the embedded structure for Poisson's ratio and relative Young's modulus and changing the re-entrant structure for energy absorption. By changing the re-entrant structure, the mechanical properties of the overall structure can be adjusted in a wider range. Finally, based on the Maxwell's stability criterion, the auxetic mechanisms of the CASwFD were compared with reported auxetic structure (RGEEH). It was found that the CASwFD dominated by bending has better auxetic mechanism than the RGEEH. It is expected that the proposed structure may provide a new way for the design of the composite auxetic structure.

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## CONFLICT OF INTEREST

The authors declare no competing financial interest.

## **AUTHOR CONTRIBUTIONS**

Zengqin Shi: Data curation; formal analysis; writing-original draft. Qing Wang: Supervision; writing-review & editing. Yunfeng Li: Data curation; formal analysis. Ning Wang: Data curation; formal analysis. Lulu Lei: Data curation; formal analysis. Xiaodong Li: Data curation; formal analysis.

### DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

#### ORCID

Qing Wang https://orcid.org/0000-0001-7228-6472

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