3.4. Scrambling Time

Scrambling time → is the time it takes for information dropped into a black hole to reappear in the Hawking Radiation.

The island rule effectively asserts that the information in the island at time x_Q^- is available at I^+ at the canonically associated retarded time x_0^- in the eqn:

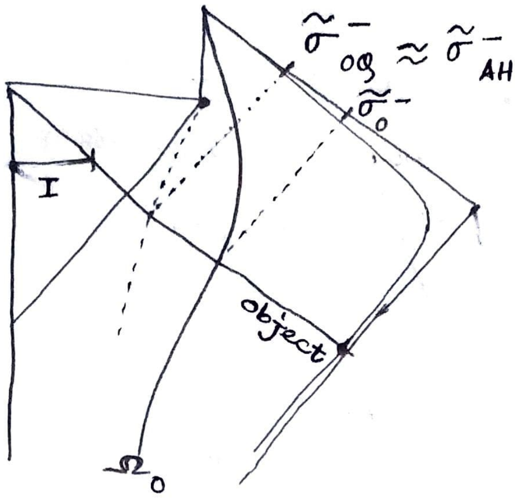
$$x_0^- = x_Q^- \, e^{\frac{4(x_Q^- + M)}{M}} \quad \cdots\cdots (3.9)$$

The dropped information hence becomes available when it hits the QES ~~surface~~ curve.

The position of the observer: The observer should be far from the black hole, ~~so that~~ but close enough so that the blackhole is effectively stationary for the whole experiment. The observer is at rest in the inertial coordinates near infinity,

$$\tilde{\sigma}^+ = t + y \quad ; \quad \tilde{\sigma}^- = t - y \quad \cdots\cdots (3.18)$$

After ~~T~~ the Page time, an object is dropped into the blackhole by the observer.



The observer collects the Hawking radiation along I^+.
Let $\tilde{\sigma}^-_0$ be the retarded time when the object reaches a few Schwarzschild radii from the black hole and ~~And~~ $\tilde{\sigma}^-_{oq}$ be the retarded time associated via

$$x_0^- = x_q^- \, e^{\frac{4(x_q^- + M)}{M}} \quad \cdots\cdots (3.9)$$

to the object crossing the QES curve and entering the island. Then the scrambling time is defined by

$$t_{scr} = \tilde{\sigma}^-_{oq} - \tilde{\sigma}^-_0 \quad \cdots\cdots (3.19)$$

First the object falls to near the blackhole in time t_{fall}, then it is scrambled, then the Hawking radiation carrying information about object takes the sametime t_{fall} to return from the near horizon to the observer.

$$\Delta t = t_{scr} + 2t_{fall} \quad \cdots\cdots (3.20)$$

$\because$ Observer is stationary.

$$\therefore \tilde{\sigma}_0^- = t_{obj} + 2t_{fall} - y$$

Now we define 'near' the blackhole.

$$\Omega \lesssim \Omega_0 \equiv (1+A)\Omega_H \quad \cdots\cdots (3.21)$$

$\Omega_H \Rightarrow \Omega$ at horizon

$A(>0)$ is an $O(1)$ constant. A is larger so that blackhole related redshift can be neglected. But A is parametrically small compared to N or M. Thus we define $\tilde{\sigma}_0^-$ to be retarded time when the object reaches the curve $\Omega = \Omega_0$. This is when the observer along I^+ starts the timer to define the scrambling time.

We take the object to fall on a null trajectory with $x^+ = x^+_{obj}$.

$$\Omega \approx -x^+(x^- + M) + \frac{6S_{BH}}{N} \quad \cdots\cdots (3.22)$$

where $\dfrac{6S_{BH}}{N} = M$.

The horizon is at $x^- = -M$ as x^+ cannot be zero.

$$\therefore \Omega = \frac{6S_{BH}}{N}.$$

$$\therefore \Omega_0 = (1+A)\frac{6S_{BH}}{N}$$

$$\Rightarrow -x_0^+(x_0^- + M) + \frac{6S_{BH}}{N} = (1+A)\frac{6S_{BH}}{N}$$

$$\Rightarrow -x^+_{obj}\left(-e^{-\tilde{\sigma}_0^-}\right) = \frac{6AS_{BH}}{N}$$

$$\Rightarrow \tilde{\sigma}_0^- = \ln\left(\frac{N x^+_{obj}}{6 A S_{BH}}\right) \quad \ldots\ldots\ldots (3.23)$$

According to (3.16),—

$$x^+_{oq} \approx \frac{3}{4} e^{\tilde{\sigma}^-_{oq}}$$

$$\Rightarrow \tilde{\sigma}^-_{oq} \approx \ln\left(\frac{4x^+_{obj}}{3}\right)$$

$\therefore$ The object is seen at $\mathcal{I}^+$ to enter the island at $\tilde{\sigma}^-_{oq}$.

$$\therefore \tilde{\sigma}^-_{oq} - \tilde{\sigma}^-_0 = \ln\left(\frac{4x^+_{obj}}{3} \cdot \frac{6 A S_{BH}}{N x^+_{obj}}\right)$$

$$= \ln\left(\frac{8 A S_{BH}}{N}\right)$$

$$\Rightarrow \tilde{\sigma}^-_{oq} = \tilde{\sigma}^-_0 + \ln\left(\frac{8AS_{BH}}{N}\right) \quad \ldots\ldots\ldots (3.26)$$

$$\Rightarrow \tilde{\sigma}^-_{oq} = \tilde{\sigma}_0 + \ln\left(\frac{S_{BH}}{N}\right) + \ln(8A)$$

The $\ln(8A)$ term is subleading, so for the scrambling time we find leading order

$$t_{scr} = \frac{\beta}{2\pi} \ln\left(\frac{S_{BH}}{N}\right)$$

where the inverse temperature of the black hole is $\beta = 2\pi$.