Appendix B. Backgrounds ~~for~~ with covariantly constant null killing vector

Consider the metric with a covariantly ~~not~~ null killing vector.

$$ds^2 = 2\,du\,dv + g_{ij}(u,x)\,dx^i dx^j \qquad (B.1)$$

where $(i, j = 1, 2, \ldots, d-2)$. The most general matter field energy-tensor consistent with the non-vanishing ~~constant~~ components of the Ricci tensor corresponding to (B.1) has the form

$$T = T_{uu}(u,x)\,du^2 + 2\,T_{ui}(u,x)\,du\,dx^i + T_{ij}(u,x)\,dx^i dx^j \qquad (B.2)$$

The analog of (2.6) is

$$ds^2 = 2\,d\hat{u}\,d\hat{v} + \hat{F}\,du^2 + \hat{g}_{ij}\,d\hat{x}^i d\hat{x}^j \qquad (B.3)$$

$$\hat{F} = F(\hat{u}, \hat{x}) = -2\hat{f}\,\hat{\delta}$$

Appendix C. :

$$2A\dot{u}\dot{v} + g h_{ij}\dot{x}_i\dot{x}_j - 2f\delta A\dot{u}^2 = 0 \qquad (C.3)$$

$$\Rightarrow \quad 2A\dot{u}\dot{v} - 2f\delta A\dot{u}^2 = 0$$

$$\Rightarrow \quad \dot{v} = f\delta\dot{u}$$

$$\Rightarrow \quad \Delta v = f(0)$$

$$\ddot{x}^i + \Gamma^i_{jk}\dot{x}^j\dot{x}^k + \frac{g_{,u}}{g}u\dot{x}^i + \frac{g_{,v}}{g}v\dot{x}^i + \frac{A}{g}\delta f_{,j}h^{ij}\dot{u}^2 = 0$$

$$\ddot{x}^i + \frac{A}{g}\delta f_{,j}h^{ij}\dot{u}^2 = 0$$

$$\Rightarrow \quad \dot{x}^i\big|_{u=0^-} - \dot{x}^i\big|_{u=0^+} = \frac{A}{g}\Big|_{u=0} f_{,j}h^{ij}\dot{u}$$

$$\ddot{u} + \frac{A_{,u}}{A}\dot{u}^2 - \frac{g_{,v}}{2A}h_{ij}\dot{x}^i\dot{x}^j + f\frac{A_{,v}}{A}\delta u^2 = 0$$

At $u=0$; $g_{,v} = A_{,v} = 0$

$$\Rightarrow \left(\ddot{u} + \frac{A_{,u}}{A}\dot{u}^2\right)\Big|_{u=0} = 0$$

$$\Rightarrow \frac{\ddot{u}}{\dot{u}} = -\frac{A_{,u}}{A}\dot{u} \Rightarrow \frac{\ddot{u}}{\dot{u}} = -\frac{\dot{A}}{A}$$

$$\Rightarrow \ln\dot{u} = -\ln A \Rightarrow \dot{u} = \frac{1}{A}$$

$$\dot{x}\big|_{u=0^-} - \dot{x}\big|_{u=0^+} = \frac{A}{g}\Big|_{u=0} f_{,j}\, h^{ij}\, \dot{u}$$

$$\Rightarrow \frac{dx}{dw}\Big|_{u=0^-} - \frac{dx}{dw}\Big|_{u=0^+} = \frac{A}{g}\Big|_{u=0} f_{,j}\, h^{ij}$$