4. Constant Curvature:

$$ds^2_{DS} = -\left(1 - \frac{r^2}{a^2}\right) dt^2 + \left(1 - \frac{r^2}{a^2}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

$$\cancel{u = e^{t} F(r) \Rightarrow du = e^{t} F(r)\, dt + e^{t} F'(r)\, dr}$$

$$\cancel{v = e^{-t} F(r) \Rightarrow dv = -e^{-t} F(r)\, dt + e^{-t} F'(r)\, dr}$$

$$\cancel{du\,dv = -F^2(r)\, dt^2 + (F'(r))^2 dr^2}$$

$$F(r) = \left(\pm \frac{a-r}{r+a}\right)^{1/2}$$

$$A(u,v) = \frac{1}{2}(r+a)^2 \quad ; \quad g(u,v) = r^2$$

$$u = e^{t/a} F(r) \Rightarrow du = \frac{1}{a} e^{t/a} F(r)\, dt + e^{t/a} F'(r)\, dr$$

$$v = e^{-t/a} F(r) \Rightarrow dv = -\frac{1}{a} e^{-t/a} F(r)\, dt + e^{-t/a} F'(r)\, dr$$

$$uv = F^2(r) = \pm \frac{a-r}{r+a}$$

When we take the case for + sign, —

$$uv = + \frac{a-r}{r+a}$$

$$\Rightarrow \frac{uv+1}{uv-1} = \frac{a}{-r}$$

$$\Rightarrow r = \frac{1-uv}{1+uv} a \qquad \therefore g_{,uv} = -4a^2$$

$$\therefore A|_{u=0} = 2a^2 \qquad \therefore \frac{g_{,uv}}{A} = -2$$

when we take the case for $-$ sign, —

$$uv = \frac{r-a}{r+a}$$

$$\Rightarrow \frac{uv+1}{1-uv} = \frac{r}{a}$$

$$\Rightarrow r = \frac{1}{a} \cdot \frac{1-uv}{1+uv} \qquad \therefore g_{uv} = -\frac{4}{a^2}$$

$$\therefore A|_{u=0} = \frac{1}{a} \qquad \therefore \frac{g_{uv}}{A} = +2$$

$$\partial_u \partial_v \left(\frac{1-uv}{1+uv}\right)^2 = 2\,\partial_u \left[\left(\frac{1-uv}{1+uv}\right) \partial_v \left(\frac{1-uv}{1+uv}\right)\right]$$

$$= 2\,\partial_u \left[\frac{1-uv}{1+uv} \cdot \frac{(1+uv)(-u) - (1-uv)u}{(1+uv)^2}\right]$$

$$= 4\,\partial_u \left(\frac{(uv-1)u}{(1+uv)^3}\right)$$

$$= 4\,\frac{(1+uv)^3(2uv-1) - (uv-1)u \cdot 3(1+uv)^2 v}{(1+uv)^6}$$

when $u = 0$, this reduces to

$$= -4$$

Substituting the value of A_1 and η in the expression of k

we get

$$k = 32pa^4$$