θ, ϕ are mutually disconnected.

De Sitter and Anti-de Sitter:

Homogeneous spacetime with $\Lambda \neq 0$

$\Lambda \neq 0$, $T_{\mu\nu} = 0$

We will discuss only ground states solution of Einstein's equation or vacuum ~~state~~ solutions of Einstein's solution.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} \Lambda g_{\mu\nu}$$

$$\boxed{T_{\mu\nu} = \rho u_\mu u_\nu + p(u_\mu u_\nu - g_{\mu\nu}), \quad p = -\rho, \quad w = -1 \\ \text{then } T_{\mu\nu} = \rho g_{\mu\nu} \quad \text{where } \rho = \text{const.}}$$

$$G_{\mu\nu} = \pm \frac{(D-1)(D-2)}{2} H^2 g_{\mu\nu}$$

$\mu, \nu = 0, \ldots, D-1$

D-dim spacetime of const. curvature

$H \Rightarrow$ Hubble constant

$$\Lambda = \frac{(D-1)(D-2)}{2} H^2$$

"+" $\Rightarrow$ +ve curvature $\longrightarrow$ De Sitter space

"−" $\Rightarrow$ −ve " $\longrightarrow$ Anti de-Sitter space

De Sitter space can be expressed as following hypersurface

$$-\eta_{AB} X^A X^B \equiv -(X^0)^2 + (X^1)^2 + \cdots + (X^{D})^2 = H^{-2}$$

$$ds^2 = \eta_{AB}\, dX^A dX^B$$

$SO(D,1)$

$(x^0, x^1, x^2, \ldots x^D) = (0, \frac{1}{H}, 0, 0, \ldots 0)$

$\longrightarrow$ Minkowskian in D+1 dim spacetime

(1) $X^0 \longrightarrow i X^{D+1}$

~~$SO(D+1)$~~

$$-\eta_{AB} X^A X^B = \sum_{A=1}^{D+1} (X^A)^2 = H^{-2}$$

Radius of sphere $= \frac{1}{H}$

$\eta_{AB} \rightarrow -\delta_{AB}$

And this does solve $G_{\mu\nu} = \pm \frac{(D-1)(D-2)}{2} H^2 g_{\mu\nu}$

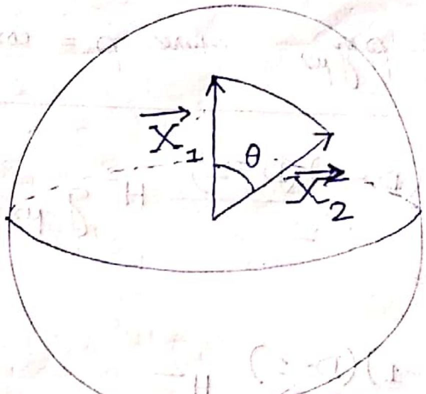
in Euclidean Signature. The sphere will ~~sh~~ solve this with "+" sign.

(2)

$\underline{SO(D,1)}$

$$\frac{SO(D,1)}{SO(D-1,1)} \qquad \frac{SO(D+1)}{SO(D)}$$

$$\vec{X}_1 \cdot \vec{X}_2 = R^2 \cos\left(\frac{e_{12}}{R}\right)$$



$$-\eta_{AB} X^A_{1,2} X^B_{1,2} = H^{-2}$$

$$\underbrace{-\eta_{AB} X^A_1 X^B_2}_{Z_{12}} = \frac{\cos(HL_{12})}{H^2}$$

$R\theta = e_{12}$

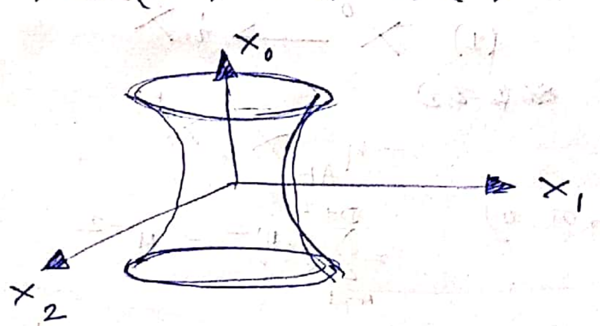
$R = \frac{1}{H}$

$\vec{X}_1^2 = \vec{X}_2^2 = R^2$

when $D = 2$

$$-(X^0)^2 + (X^1)^2 + (X^2)^2 = H^{-2}$$

$$(X^1)^2 + (X^2)^2 = H^{-2} + (X^0)^2$$



$$-(X^0)^2 + (X^1)^2 + \cdots + (X^D)^2 = H^{-2}$$

$$ds^2 = (dX^0)^2 - (dX^1)^2 - \cdots - (dX^D)^2$$

$$X^0 = \frac{\sinh(Ht)}{H} \; ; \; X^i = \frac{w_i \cosh(Ht)}{H} \qquad i = 1, \ldots D$$

$$\boxed{w_i^2 = 1}$$

$$ds^2 = dt^2 - \frac{\cosh^2(Ht)}{H^2} d\Omega^2_{D-1}$$

$$d\Omega^2_{D-1} = \sum_{j=1}^{D-1} \prod_{i=1}^{D-1} \sin^2\theta_i \, d\theta_j^2$$

$w_1 = \cos\theta_1 \qquad -\frac{\pi}{2} \le \theta_1 \le \frac{\pi}{2}$

$w_2 = \sin\theta_1 \cos\theta_2 \qquad -\frac{\pi}{2} \le \theta_2 \le \frac{\pi}{2}$

$\vdots$

$w_{D-1} = \sin\theta_1 \sin\theta_2 \cdots \sin\theta_{D-2} \cos\theta_{D-1}$

$-\pi \le \theta_{D-1} \le \pi$

$w_D = \sin\theta_1 \sin\theta_2 \cdots \sin\theta_{D-1}$

$$Ht \longrightarrow i\left(\theta_D - \frac{\pi}{2}\right) \longrightarrow \frac{1}{H^2} d\Omega^2_D$$

$$Z_{12} = -\sinh(Ht_1)\sinh(Ht_2) + \cosh(Ht_1)\cosh(Ht_2)\cos\omega$$

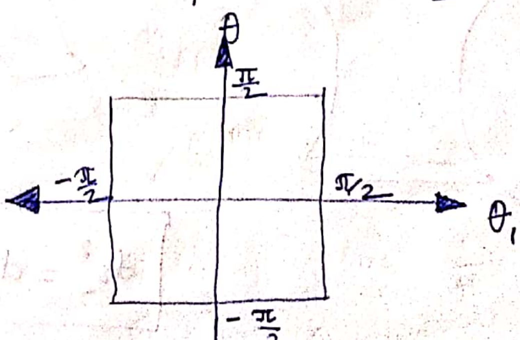
where $\cos\omega = (\vec{w}_1, \vec{w}_2)$

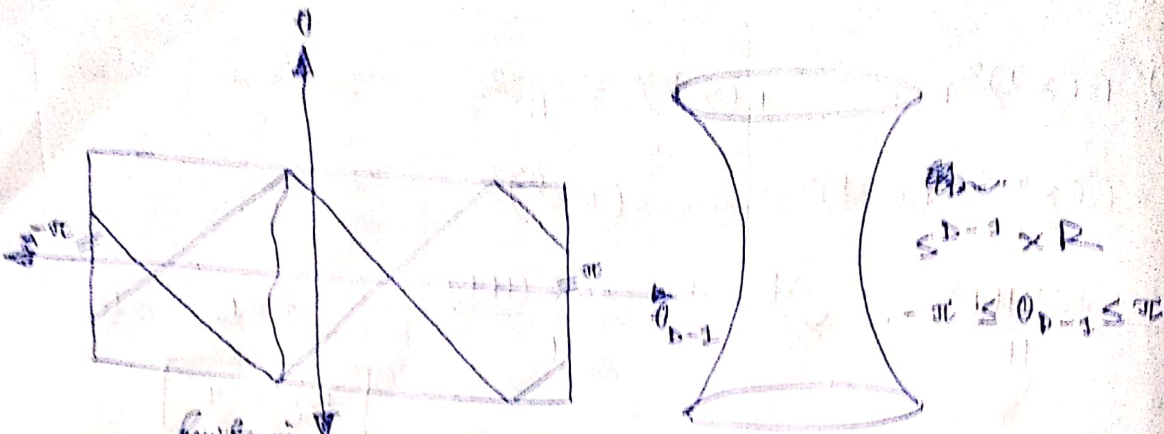
$$ds^2 = dt^2 - \frac{\cosh^2(Ht)}{H^2} d\Omega^2_{D-1}$$

$\cosh^2(Ht) = \frac{1}{\cos^2\theta}$ (assume) where $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$ds^2 = \frac{1}{H^2\cos^2\theta}\left[d\theta^2 - d\Omega^2_{D-1}\right] \; ; \; d\Omega^2_{D-1} = d\theta_1^2 - \sin^2\theta \, d\Omega^2_{D-2}$$

$d\theta - d\theta_1^2 = ds^2 \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$





Every diamond is causally connected

$$-\eta_{AB}X^A X^B = H^{-2} \Rightarrow \boxed{ds_+^2 = d\tau_+^2 - e^{2H\tau_+}\, d\vec{x}_+^{\,2}}$$

$$-(HX^0)^2 + (HX^D)^2 = 1 - (Hx_+^i)^2 e^{2H\tau_+}$$

$$(HX^1)^2 + \cdots + (HX^{D-1})^2 = (Hx_+^i)^2 e^{2H\tau_+}$$

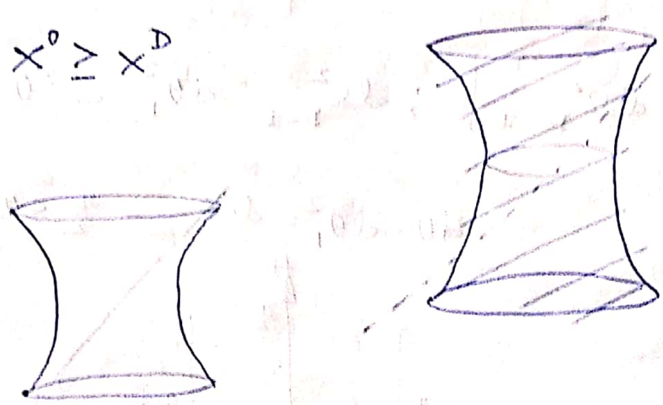
$$HX^0 = \sinh(H\tau_+) + \frac{(Hx_+^i)^2}{2} e^{H\tau_+}$$

$$HX^i = Hx_+^i e^{H\tau_+} \qquad i = 1, \ldots, D-1$$

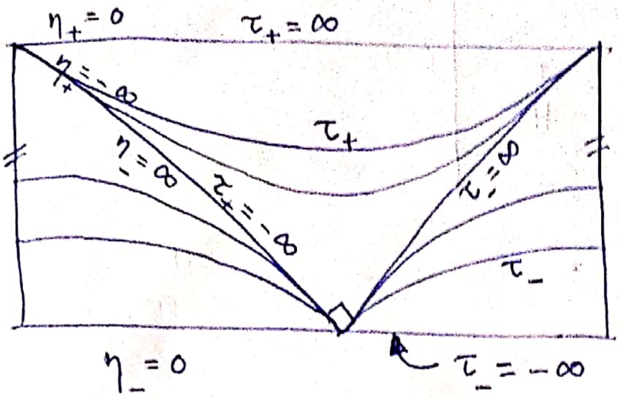
$$HX^D = -\cosh(H\tau_+) + \frac{(Hx_+^i)}{2} e^{H\tau_+}$$

$$-X^0 + X^D = -\frac{1}{H} e^{H\tau_+} \le 0$$

$$X^0 \ge X^D$$



$$\boxed{ds_-^2 = d\tau_-^2 - e^{-2H\tau_-}\, dx_-^2}$$



__Conformal time:__

$$H\eta_\pm = e^{\mp H\tau_\pm}$$

$$ds^2_\pm = \frac{1}{(H\eta_\pm)^2}\left[d\eta^2_\pm - d\vec{x}^{\,2}_\pm\right]$$ Poincare coordinates

$$z_{12} = 1 + \frac{(\eta_1-\eta_2)^2 - |(\vec{x}_1-\vec{x}_2)|^2}{2\eta_1\eta_2}$$

__Anti-de Sitter Space:__

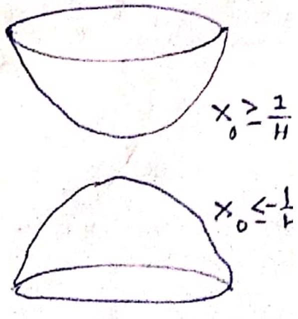
$$G_{\mu\nu} = -\frac{(D-1)(D-2)}{2} H^2 g_{\mu\nu} \quad ; \quad ds^2 = dx_0^2 - \sum_{j=1}^{D-1} dx_j^2 + dx_D^2$$

$$(+, -, \ldots, -, +)$$

$$-x_0^2 + \sum_{j=1}^{D-1} dx_j^2 - x_D^2 = -\frac{1}{H^2}$$

$$\boxed{x_D \longrightarrow i x_D} \Rightarrow -x_0^2 + \sum_{j=1}^{D} x_j^2 = -\frac{1}{H^2}$$

$$\Rightarrow \boxed{\sum_{j=1}^{D} x_j^2 = x_0^2 - \frac{1}{H^2}}$$



$$\boxed{H \longrightarrow iH}$$

$$S^D \longleftrightarrow L^D \longleftrightarrow dS^D \longleftrightarrow AdS^D$$

$$\boxed{-X_0^2 + \sum_{j=1}^{D-1} X_j^2 - X_D^2 = -\frac{1}{H^2}}$$

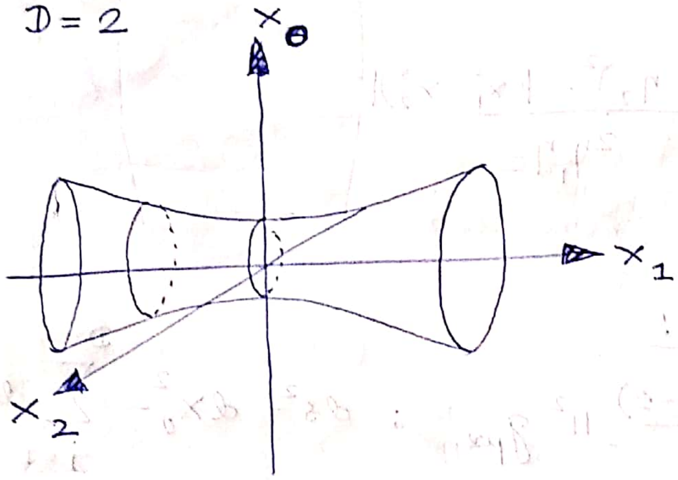
$$X_0 = \frac{1}{H} \cosh \rho \cos \tau$$

$$X_D = \frac{1}{H} \cosh \rho \sin \tau$$

$$X_j = \frac{1}{H} \sinh \rho \; w_j \qquad \boxed{w_j^2 = 1} \qquad j = 1, \dots, D-1$$

$$ds^2 = \frac{1}{H^2} \left[\cosh^2 \rho \, d\tau^2 - d\rho^2 - \sinh^2 \rho \, d\Omega^2_{D-2} \right]$$

For $D = 2$



$$\tau \in [0, 2\pi)$$

$$\boxed{\tau \in (-\infty, \infty)}$$

$$D = 2 \Leftrightarrow \rho \in (-\infty, \infty)$$

$D > 2 \quad ; \quad \rho \in [0, +\infty)$

$$dl^2 = \frac{1}{H^2}\left[d\rho^2 + \sinh^2\rho \, d\Omega^2_{D-2}\right]$$

$(D-1)$ dim Lobachowski Space

$H = 1, \quad \rho \longleftrightarrow x$

$R \times S^{D-2}$

$$ds^2 = \frac{1}{H^2}\left[\cosh^2\rho \, d\tau^2 - d\rho^2 - \sinh^2\rho \, d\Omega^2_{D-2}\right]$$

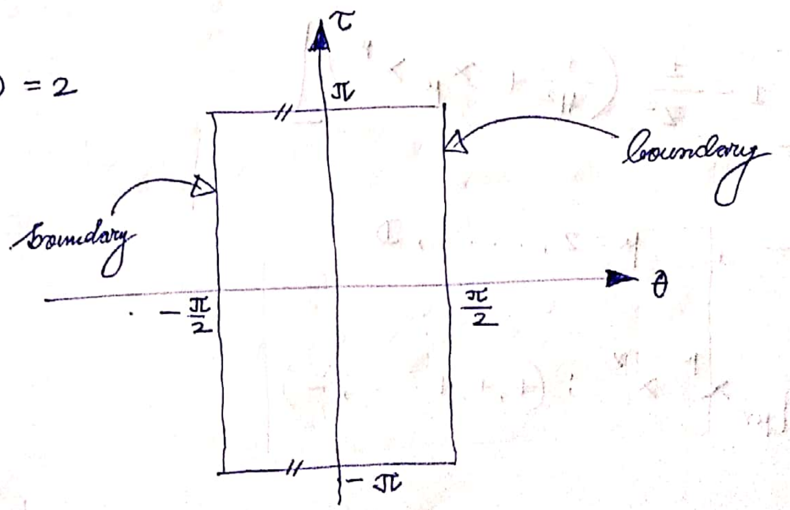
$$\tan\theta = \sinh\rho \quad \begin{cases} D = 2, & \rho \in (-\infty, \infty) \\ D > 2, & \rho \in [0, \infty) \end{cases}$$

$$ds^2 = \frac{1}{H^2\cos^2\theta}\left[d\tau^2 - d\theta^2 - \sin^2\theta \, d\Omega^2_{D-2}\right]$$

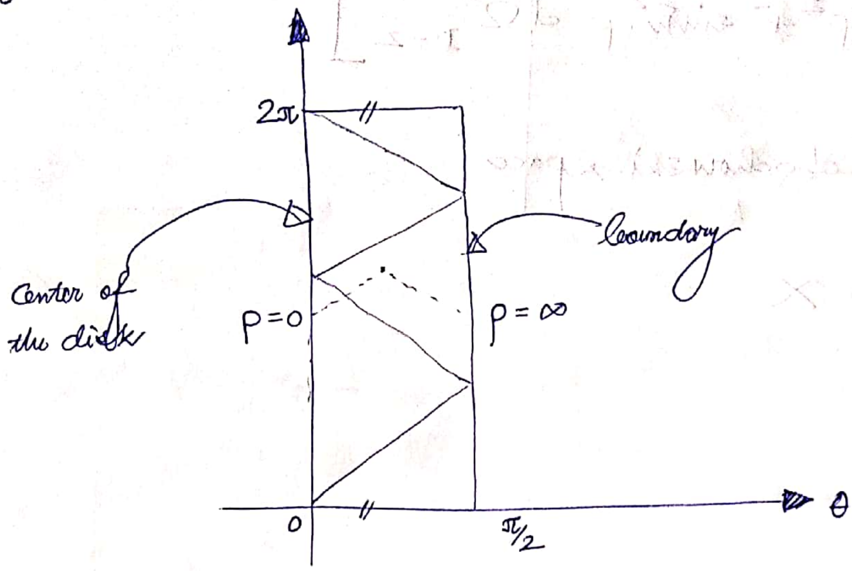
for $D = 2, \quad \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

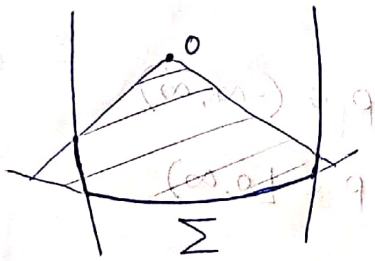
$D > 2, \quad \theta \in [0, \frac{\pi}{2}]$

for $D = 2$



for $D > 2$





absence of global hyperbolicity

$$-(X^0)^2 + \sum_{j=1}^{D-1} X_j^2 - X_D^2 = -\frac{1}{H^2}$$

$$X^0 = \frac{z}{2}\left[1 + \frac{1}{z^2}\left(\frac{1}{H^2} - x_\mu x^\mu\right)\right], \quad z \geq 0$$

$$X^1 = \frac{z}{2}\left[1 - \frac{1}{z^2}\left(\frac{1}{H^2} + x_\mu x^\mu\right)\right]$$

$$X^\mu = \frac{x^\mu}{H}, \quad \mu = 2, \dots, D$$

$$x^\mu x_\mu = \eta_{\mu\nu} x^\mu x^\nu \ ; \ (+,+,+,\cdots,-)$$

$$\boxed{x_0 - x_1 = \frac{1}{(Hz)^2} \geq 0}$$

$$ds^2 = \frac{1}{(Hz)^2}\left[dz^2 + dx_\mu dx^\mu\right]$$

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