3.3. A taming of the particle concept: particle detectors

Part of the reason for the nebulousness of the particle concept is its global nature. The modes are defined on the whole of spacetime (or at least a large patch), so the ~~particle~~ particular observer's specification of the field mode decomposition, and hence the number ~~of the field mode decompositions,~~ operator describing the response of a particle detector carried by him, will depend, for example, on the observer's entire past history. To obtain a more objective probe of the state of a field ~~must be constructed~~ one must construct locally defined quantities, such as $\langle \psi | T_{\mu\nu} | \psi \rangle$, which assumes a particle value at a point x of spacetime.

If $\langle \psi | T_{\mu\nu} | \psi \rangle = 0$ for one observer then it will vanish for all observers.

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_p \rightsquigarrow c\, m(\tau)\, \phi(x(\tau))$$

~~Whenever the~~ When c is sufficiently small,

~~By~~ First order perturbation theory amplitude of transition can be given by

$$ic \langle E, \psi | \int_{-\infty}^{\infty} m(\tau)\, \phi(x(\tau))\, d\tau\, | 0_M, E_0 \rangle$$

Using time evolution of $m(\tau)$,

$$m(\tau) = e^{iH_0\tau}\, m(0)\, e^{-iH_0\tau}$$

where $H_0 |E\rangle = E|E\rangle$

transition amplitude factorizes to give

$$ic \langle E | m(0) | E_0 \rangle \int_{-\infty}^{\infty} e^{i(E-E_0)\tau} \langle \psi | \phi(x) | 0_M \rangle\, d\tau$$

(3.50)

$|\psi\rangle = |1_k\rangle$ where $\omega = (|\vec{k}|^2 + m^2)^{1/2}$ for some $\vec{k}$.

$$\langle 1_k | \phi(x) | 0_M \rangle = \int d^3k' \,(16\pi^3\omega')^{-1/2} \langle 1_{\vec{k}} | a^{+}_{\vec{k}'} | 0_M \rangle\, e^{-i\vec{k}'\cdot\vec{x} + i\omega' t}$$

$$= (16\pi^3\omega)^{-1/2}\, e^{-i\vec{k}\cdot\vec{x} + i\omega t}$$

(3.51)

x as in (3.51) is not an independent variable but is determined by the detector's trajectory.

If $\vec{x} = \vec{x}_0 + \vec{v}t = \vec{x}_0 + \vec{v}\tau(1-v^2)^{-1/2}$ (3.52)

then (3.50) becomes

$$(16\pi^3\omega)^{-1/2}\, e^{-i\vec{k}\cdot\vec{x}_0}\int_{-\infty}^{\infty} e^{i(E-E_0)\tau}\, e^{i\tau(\omega-\vec{k}\cdot\vec{v})(1-v^2)^{-1/2}}\, d\tau$$

$$= (4\pi\omega)^{-1/2}\, e^{-i\vec{k}\cdot\vec{x}_0}\,\delta\left(E-E_0+(\omega-\vec{k}\cdot\vec{v})(1-v^2)^{-1/2}\right) \qquad (3.53)$$

where $\vec{x}_0$ = constant, $\vec{v}$ = constant, $|\vec{v}|<1$ with $|\psi\rangle = |1_{\vec{k}}\rangle$

As $\vec{k}\cdot\vec{v} \le |\vec{k}|\,|\vec{v}| < \omega$ and $E > E_0$. Argument of δ-fn is +ve and the transition amplitude vanishes.

Squaring the modulus of (3.50) and summing over E and the complete set ψ, we get

$$c^2\sum_{E} |\langle E|m(0)|E_0\rangle|^2\, \mathcal{F}(E-E_0) \qquad (3.54)$$

where

$$\mathcal{F}(E) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau'\, e^{-iE(\tau-\tau')}\, G^+(x(\tau), x(\tau')) \qquad (3.55)$$

detector response fn

In case of detector trajectories in Minkowski space for which

$$G^{(1)}\left(x(\tau), x'(\tau)\right) = g(\Delta\tau) \tag{3.56}$$

$$\Delta\tau \equiv \tau - \tau' \tag{3.57}$$

Considering instead the transition probability for unit proper time:

$$c^2 \sum_E \left| \langle E | m(0) | E_0 \rangle \right|^2 \int_{-\infty}^{\infty} d(\Delta\tau)\, e^{-i(E-E_0)\Delta\tau} G^{+}(\Delta\tau) \tag{3.58}$$