

## 2. Shock Wave: an example



Gravitational field of a massless particle in Minkowski space is described by the metric

$$ds^2 = -d\hat{u} (d\hat{v} + 4p \ln(p^2) \delta(\hat{u}) d\hat{u}) + dx^2 + dy^2$$

where  $p^2 = x^2 + y^2$

The particle moves in the  $\hat{u}$  direction with momentum  $p$ . By calculating geodesics which ~~crossing~~ cross the shock wave which is located at  $u=0$ , we obtain the following two physical effects of such a shock wave (see Appendix A): geodesics have a discontinuity  $\Delta\hat{v}$  at  $u=0$  and are refracted in the transverse direction. The shift  $\Delta\hat{v}$  is given by

$$\Delta\hat{v} = -\frac{4Gp}{c^3} \ln \frac{p_0^2}{l_{Pl}^2} \quad (2a)$$

which, for a photon, is

$$\Delta\hat{v} = -\frac{4l_{Pl}^2}{c} \ln \frac{p_0^2}{l_{Pl}^2} \quad (2b)$$

Where  $E = pc = h\nu$  and  $p = p(u)$  and  $p(0) = p_0$ .

There is also a refraction effect described by

$$\cot \alpha + \cot \beta = \frac{4Gp}{c^3 p_0} \quad (3a)$$

For a proton, is

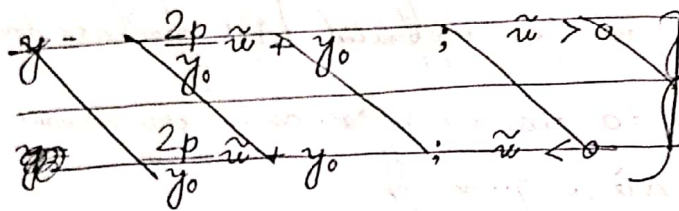
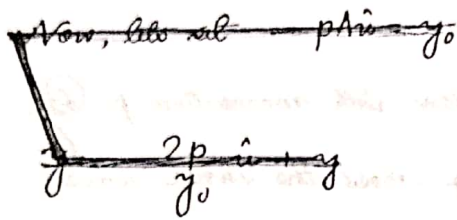
$$\cot \alpha + \cot \beta = \frac{4l_{Pl}^2}{c p_0} \quad (3b)$$

For  $m \rightarrow 0$ , —

$$\frac{\partial y}{\partial \hat{u}} = -\frac{2p}{y_0} \operatorname{sgn} \hat{u} - pA \Rightarrow \left. \frac{\partial y}{\partial \hat{u}} \right|_{\epsilon} - \left. \frac{\partial y}{\partial \hat{u}} \right|_{-\epsilon} = -\frac{4p}{y_0}$$

$$\Rightarrow y = -\frac{2p}{y_0} \hat{u} - pA \hat{u}^{+\chi}; \quad \hat{u} > 0$$

$$\frac{2p}{y_0} \hat{u} - pA \hat{u}^{+\chi}; \quad \hat{u} < 0$$



$\therefore$  Difference b/w the inclinations is  $-\frac{4p}{y_0}$

### General Result

Consider a solution of the vacuum Einstein field eqns of the form

$$ds^2 = 2A(u, v) du dv + g(u, v) h_{ij}(x^i) dx^i dx^j \quad (4)$$

Under what conditions

$$A_{,v} = 0 = g_{,v}$$

$$\frac{A}{g} \Delta f - \frac{g_{,uv}}{g} f = ~~32\pi p~~ 32\pi p A^2 \delta(p) \quad (5)$$

where  $f = f(x^i)$  represents the shift in  $v$ ,  $\Delta f$  is the Laplacian of  $f$  wrt. the 2 metric  $h_{ij}$  and the resulting metric described by (B.2) or (B.4). Eqn(5) represents our main result. We now turn to specific examples.

For a plane wave due to a photon in Minkowski Space we have

$$ds^2 = - du dv + dx^2 + dy^2 \quad (6a)$$

and thus  $A = -\frac{1}{2}$  (6b)

$$g = 1$$

$$\therefore A_{,v} = g_{,v} = 0 \quad ; \quad \Delta f = -16\pi p \delta(p) \quad (7)$$

Where  $\rho^2 = x^2 + y^2$ . The solution of this equation, unique up to sol<sup>n</sup> of homogeneous eqn, is

$$f = -4p \ln \rho^2 \quad (8)$$

which agrees precisely with (2) and (A.26).



For a spherical wave in Minkowski space we write the metric in the form

$$ds^2 = - du dv + \frac{1}{4} (v-u)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (10 a)$$

so that

$$A = -\frac{1}{2}$$

$$g = r^2 = \frac{1}{4} (v-u)^2$$

(10 b)

But  $g, v|_{u=0} \neq 0$ . Thus  $\nexists$  spherical waves of this form in Minkowski Space.

Kruskal - Seckers coord.:

$$ds^2 = - \frac{32m^3}{r} e^{-\frac{r}{2m}} du dv + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (11 a)$$

$$\therefore A = - \frac{16m^3}{r} e^{-\frac{r}{2m}} ; g = r^2 \quad (11 b)$$

$$uv = - \left( \frac{r}{2m} - 1 \right) e^{\frac{r}{2m}} \quad (11 c)$$

$$\Rightarrow u = - \left( \frac{r, v}{2m} \right) e^{\frac{r}{2m}} - \left( \frac{r}{2m} - 1 \right) \frac{r, v}{2m} e^{\frac{r}{2m}} = \frac{r, v}{4m^2} e^{\frac{r}{2m}}$$

$$\Rightarrow r, v \propto u$$

So that all  $v$ -derivatives of  $r$  are proportional to  $u$ . Thus, the conditions on the metric coeff.  $A$  and  $g$  are satisfied at  $u=0$ .

Furthermore, since  $g_{rr} \equiv A$  the condition  $\frac{\delta}{\delta r}$  on  $f$  becomes

$$\begin{aligned}\Delta f - f &= 32\pi p g A|_{u=0} \delta(\theta) \\ &= -2\pi\kappa\delta(\theta)\end{aligned}\quad (12)$$

where  $\kappa = 2^9 m^4 p e^{-\frac{2m}{r}}$  and where we have arranged the coordinates so that the photon is at  $\theta=0=u$ .