$$u = -\frac{r r_{,u}}{4m^2} e^{\frac{r}{2m}} \qquad \text{Similarly,} \quad v = -\frac{r r_{,u}}{4m^2} e^{\frac{r}{2m}}$$

$$g = r^2 \Rightarrow g_{,u} = 2 r r_{,u} = 2 \cdot \left(-4m^2 v e^{-\frac{r}{2m}}\right)$$

$$\Rightarrow g_{,uv} = -8m^2 e^{-\frac{r}{2m}} \left(-u \frac{r_{,u}}{2m} + 1\right)$$

$$= -8m^2 e^{-\frac{r}{2m}} \left(-u \cdot \left(-\frac{2mv}{r}\right) e^{-\frac{r}{2m}} + 1\right)$$

$$= -8m^2 e^{-\frac{r}{2m}} \left(uv \cdot \left(\frac{2m}{r} e^{-\frac{r}{2m}}\right) + 1\right)$$

$$= -8m^2 e^{-\frac{r}{2m}} \left(-\left(\frac{r}{2m} - 1\right) \cancel{e^{\frac{r}{2m}}} \cdot \frac{2m}{r} \cancel{e^{-\frac{r}{2m}}} + 1\right)$$

$$= -8m^2 e^{-\frac{r}{2m}} \cdot \frac{2m}{r}$$

$$= -\frac{16m^3}{r} e^{-\frac{r}{2m}} = A \Rightarrow \boxed{g_{,uv} = A}$$

$$\frac{A}{g} \Delta f - \frac{g_{,uv}}{g} f = 32\pi p A^2 \delta(p)$$

$$\Rightarrow \frac{A}{g} (\Delta f - f) = 32\pi p A^2 \delta(p)$$

$$\Rightarrow (\Delta f - f) = 32\pi p A^2 g \, \delta(p)$$

Calculating energy momentum tensor of a massless particle:

At first lets assume, there is a particle of mass m moving with velocity u^{μ}_{1} and situated at x'. Then the energy momentum tensor of the particle is

$$T^{\mu\nu} = m u^{\mu} u^{\nu} \delta^{(4)}(x - x')$$

In our case, —

$$T^{\mu\nu} = \lim_{m\to 0} m u^{\mu} u^{\nu} \delta^{(4)}(x - x')$$

$$= \lim_{\beta\to\infty} m u^{\mu} u^{\nu} \delta^{(4)}(x - x')$$

$$\boxed{\begin{array}{l} \lim_{\beta\to\infty} p^{\mu} = 2p\delta^{\mu}_{v} \\ \lim_{\beta\to\infty} u^{\mu} = 2\delta^{\mu}_{v} \end{array}}$$

$$= 2p\,\delta^{\mu}_{v} \cdot 2\delta^{\nu}_{v}\, \delta^{(4)}(x - x')$$

$$= 4p\,\delta^{(4)}(x - x')\,\delta^{\mu}_{v}\,\delta^{\nu}_{v}$$

The stress energy tensor for a massless particle located at origin $\rho = 0$ of the (x^i) 2-surface and $u = 0$

$$T^{ab} = 4p\,\delta(\rho)\,\delta(u)\,\delta^{a}_{v}\,\delta^{b}_{v}$$

$$\therefore \quad T_{\alpha\sigma} = g_{\alpha a} g_{\sigma b} T^{ab}$$

$$= 4p\,\delta(\rho)\,\delta(u)\,(\delta^{a}_{v} g_{\alpha a})(\delta^{b}_{v} g_{\sigma b})$$

$$= 4p\, g_{\alpha v} g_{\sigma v}\,\delta(\rho)\,\delta(u)$$

Only nonzero component is $T_{uu} = 4pA^2\,\delta(\rho)\,\delta(u)$

Minkowski metric in spherical polar coordinates is

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2 \quad ; \quad d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$$

Now lets introduce coordinates

$$u = t - r \qquad \therefore -dt^2 + dr^2 = -du\,dv$$

$$v = t + r$$

$\therefore \quad r = \dfrac{v-u}{2}$ and the metric transforms into

$$ds^2 = -du\,dv + \frac{1}{4}(v-u)^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

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$$ds^2 = -\left(1-\frac{r_g}{r}\right)dt^2 - \frac{dr^2}{1-\frac{r_g}{r}} - r^2 d\Omega^2$$

Lets assume,—

$$\boxed{r = \left(1+\frac{r_g}{4\rho}\right)^2 \rho}$$

$$\Rightarrow ds^2 = -\left(\frac{1-\frac{r_g}{4\rho}}{1+\frac{r_g}{4\rho}}\right)^2 dt^2 + \left(1+\frac{r_g}{4\rho}\right)^4 \left(d\rho^2+\rho^2 d\Omega^2\right)$$

$$\boxed{\rho^2 = x^2+y^2+z^2}$$

$$\Rightarrow ds^2 = -\left(\frac{1-A}{1+A}\right)^2 dt^2 + (1+A)^4\left(dx^2+dy^2+dz^2\right)$$

$$\boxed{\text{where } A = \frac{r_g}{4\rho}}$$

Now, ~~boost~~ lets boost the rest frame with rapidity β.

$$\bar{t} = \gamma t + \gamma V z \qquad \gamma = \frac{1}{\sqrt{1-V^2}}$$

$$\bar{z} = \gamma V t + \gamma z$$

If $\gamma \to \infty$, $r_g \to 0$, $\frac{r_g \gamma}{2} = $ constant, $\frac{r_g \gamma}{2} = \kappa p$

$$\cancel{ds^2 = -2du\,dv - 2H(u, x_\perp)\,du^2 + dx_\perp^2} \qquad x_\perp = (x, y)$$

$$\cancel{u = \frac{t-z}{\sqrt{2}}\ ;\ v = \frac{t+z}{\sqrt{2}}}$$

$$ds^2 = -du\,dv - H(u, x_\perp)\,du^2 + dx_\perp^2 \qquad x_\perp = (x, y)$$

$$u = t - z \quad ; \quad v = t + z$$

$$H(u, x_\perp) = \kappa p\,\delta(u)\,\log|x_\perp|$$

$$R_{uu} = \Delta_{x_\perp} H(u, x_\perp) \qquad \Delta_{x_\perp} = \partial_x^2 + \partial_y^2$$

$$\Delta_{x_\perp} \log|x_\perp| = 2\pi\,\delta^{(2)}(x_\perp) \Rightarrow T_{uu} = p\,\delta(u)\,\delta^2(x_\perp)$$

$$\boxed{\text{pp-wave} : \; u = 0 \; ; \; x_\perp = 0}$$

As we are dealing with massless the geodesic will be of null type.

$$0 = ds^2 = -du\,dv - H(u, x_\perp)\,du^2 + dx_\perp^2$$

Lets set, $x_\perp = \text{constant} = \vec{a}$ $\therefore$ $dx_\perp = 0$

$$\Rightarrow du\,(dv + H(u, \vec{a})\,du) = 0$$

$\Rightarrow du = 0 \Rightarrow u = \text{constant}$ | $\Rightarrow dv = -\kappa p\,\delta(u)\,\cancel{\delta^2(\vec{a})}\,\log|\vec{a}|\,du$

When $u \neq 0$ then $dv = 0 \Rightarrow v = \text{constant}$.

$$\Delta v = \int dv = \int_{-\varepsilon}^{\varepsilon} du\;\kappa p\,\delta(u)\,\log|\vec{a}| = \kappa p \log|\vec{a}|$$

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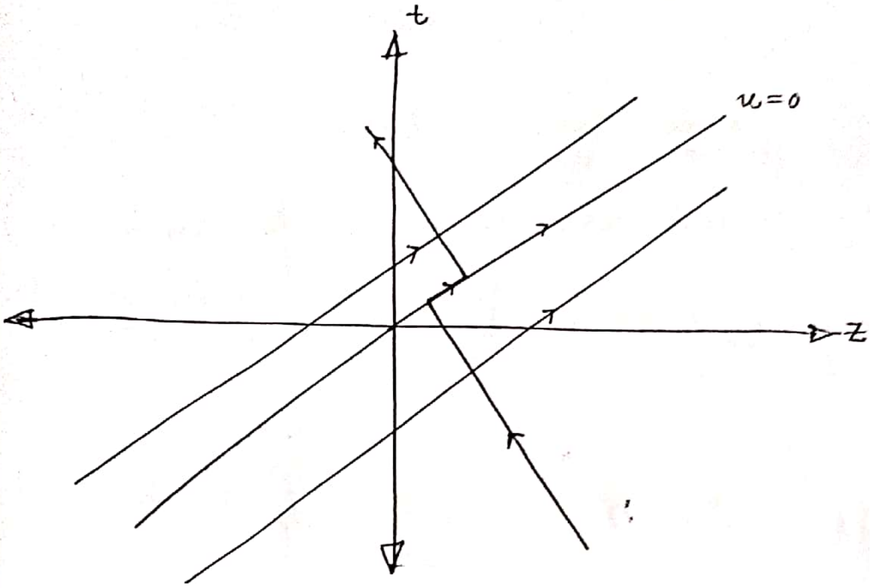

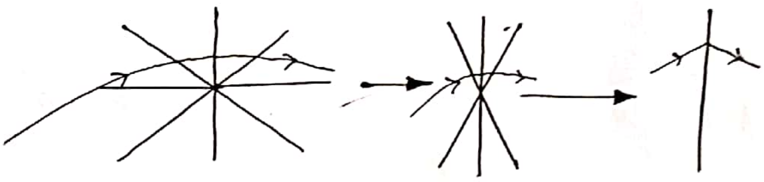

Birrel and Davis.