4.5. Quantum Field Theory in Rindler Space

Lets, consider 2-dim Minkowski Space with metric (3.8) & (3.10) i.e.,

$$ds^2 = d\bar{u}\, d\bar{v} = dt^2 - dx^2 \qquad (4.66)$$

Coordinate transform^n

$$\begin{cases} t = a^{-1} e^{a\xi} \sinh a\eta & (4.67) \\ x = a^{-1} e^{a\xi} \cosh a\eta & (4.68) \end{cases}$$

a = constant > 0 and $-\infty < \eta, \xi < \infty$ or equivalently

$$\bar{u} = -a^{-1} e^{-au} \qquad (4.69)$$

$$\bar{v} = a^{-1} e^{av} \qquad (4.70)$$

where $u = \eta - \xi$ and $v = \eta + \xi$, then (4.66) becomes

$$ds^2 = e^{2a\xi} du\, dv = e^{2a\xi}(d\eta^2 - d\xi^2)$$

The coordinates (η, ξ) cover only a qudrant of

Minkowski space, namely the wedge $x > |t|$ shown in fig. 13. Lines of constant η are straight ($x \propto t$) while lines of constant ξ are hyperbolae:

$$x^2 - t^2 = a^{-2} e^{2a\xi} = \text{constant} \qquad (4.72)$$

They therefore represent the world lines of uniformly accelerated observers treated in § 3.3. Comparison of (3.62) with (4.72) shows that

$$a e^{-a\xi} = \alpha^{-1} = \text{proper acceleration} \qquad (4.73)$$

Thus, lines of large positive ξ (far from $x = t = 0$) represent weakly accelerated observers, while the hyperbolae that closely approach $x = t = 0$ have large $-ve$ ξ and hence a higher proper acceleration. All the hyperbolae are asymptotically to the null rays $\bar{u} = 0$, $\bar{v} = 0$ (or $\bar{u} = \infty$; $\bar{v} = -\infty$), which means that the accelerated observers approach the speed of light as $\eta \to \pm\infty$. These observer's proper time τ is related to ξ and η by

$$\tau = e^{a\xi}\eta \qquad (4.74)$$

Lets consider the quantization of massless scalar field ϕ in 2-d Minkowski spacetime. The wave eqn.

$$\Box\phi = \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\phi \equiv \frac{\partial^2\phi}{\partial u\,\partial v} = 0 \qquad (4.75)$$

posses standard orthonormal modes soln.

$$\bar{u}_k = (4\pi\omega)^{-1/2}\, e^{i(\vec{k}\cdot\vec{x} - \omega t)} \qquad (4.76)$$

where $\omega = |k| > 0$ and $k \in (-\infty, \infty)$.

$$\mathcal{L}_{\partial_t}\bar{u}_k = -i\omega\,\bar{u}_k \qquad (4.77)$$

The modes with $k > 0$ consist of right moving wave

$$(4\pi\omega)^{-1/2}\, e^{-i\omega \bar{u}} \qquad (4.78)$$

left moving

$$\bar{u}_k = (4\pi\omega)^{-1/2} e^{-i\omega v} \qquad (4.79)$$

The metric (4.71) is conformal to the whole of Minkowski space under C.T. $g_{\mu\nu} \longrightarrow e^{-2a\xi} g_{\mu\nu}$, reduces to $d\eta^2 - d\xi^2$ with $-\infty < \eta, \xi < \infty$.

$$e^{2a\xi} \Box\phi = \left(\frac{\partial^2}{\partial\eta^2} - \frac{\partial^2}{\partial\xi^2}\right)\phi = \frac{\partial^2\phi}{\partial u\, \partial v} = 0 \qquad (4.80)$$

the mode solns, ——

$$u_k = (4\pi\omega)^{-1/2} e^{i(k\xi \pm \omega\eta)} \qquad (4.81)$$

$$\omega = |k| > 0 \quad , -\infty < k < \infty$$

$$\mathcal{L}_{\partial_\eta} u_k = -i\omega u_k \qquad (4.82)$$

$$\phi = \sum_{k=-\infty}^{\infty} \left(a_k \bar{u}_k + a_k^{+} \bar{u}_k^{*} \right) \qquad (4.85)$$

$$\phi = \sum_{k=-\infty}^{\infty} \left(b_k^{(1)}\, {}^{L}u_k + b_k^{(1)+}\, {}^{L}u_k^{*} + b_k^{(2)}\, {}^{R}u_k + b_k^{(2)+}\, {}^{R}u_k^{*} \right)$$

(4.86)

$$a_k |0_M\rangle = 0 \qquad (4.87)$$

$$b_k^{(1)} |0_R\rangle = b^{(2)}{}_k |0_R\rangle = 0 \qquad (4.88)$$