2. Gravitational Shockwaves:

$$ds^2 = -du\,dv + f(\vec{x})\,\delta(u-u_0)\,du^2 + \delta_{ij}\,dx^i dx^j \qquad (2.1)$$

$$X \equiv (t, z, \vec{x})$$

$$u = t - z$$

$$v = t + z$$

$\vec{x}$ or x^i, $i \in \{2,3, \ldots D-2\}$ denote the transverse directions (coords on wavefront). The wavefront is localized at $u = u_0$.

$$T_{uu} = \delta(u-u_0)\,\rho(\vec{x}) \qquad (2.2)$$

$$\Delta f(\vec{x}) = -16\pi G \rho(\vec{x}) \qquad (2.3)$$

where $\Delta = \delta^{ij}\partial_i \partial_j$

$$f(\vec{x}) = -4GP\log\left(\frac{|\vec{x}|}{x_0}\right) \qquad (2.4)$$

$$f(\vec{x}) = -\vec{x}\cdot A\cdot\vec{x} = -\sum_i a_i (x_i)^2 \qquad (2.5)$$

where A is an antisymmetric matrix.

$$\rho = \frac{\mathrm{Tr}(A)}{8\pi G} \qquad (2.6)$$

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Introduction of new coord.:

$$\hat{v} = v - \Theta(u - u_0) f(\vec{x}) \qquad (2.7)$$

$$ds^2 = -du\, d\hat{v} - \Theta(u - u_0)\, \partial_i f(\vec{x})\, du\, dx^i + d\vec{x}^2 \qquad (2.8)$$

3. Quantum field theory in shockwave geometry:

3.1. Klein Gordon eqn and its soln.

$$\Box \phi = 0 \qquad (3.1)$$

∂_v is a K.V. of (2.1). So, we take ansatz for plane wave modes in ∂_v dirn.

$$\phi_{k_-} = e^{-ik_- v}\, \psi(u, \vec{x})$$

where $k_\mu = (k_t, k_z, \vec{k})$ and $k_\pm = \frac{1}{2}(k_t \pm k_z)$

$$i\partial_u \psi = -\left(\frac{\Delta}{4k_-} + f(\vec{x})\, k_-\, \delta(u - u_0)\right)\psi \qquad (3.2)$$

in region: $u < u_0$

out region: $u > u_0$

The soln-s are simple plane waves that provide a complete basis for quantization. Thus, we have two mode expansions at our disposal $\phi^{in}_{k_-,\vec{k}}$ and $\phi^{out}_{k_+,\vec{k}}$ for which we require that the mode fn-s reduce ~~completely~~ to plane waves in the in and out regions, respectively.

$$\phi^{in}_{k_-,\vec{k}}\Big|_{u<u_0} = e^{-ik_- v}\,\psi_<(u,\vec{x})$$

$$\psi_<(u,\vec{x}) = N_{k_-}\, e^{-i\left(k_+(u-u_0) - \vec{k}\cdot\vec{x}\right)} \qquad (3.3)$$

where $N_{k_-} = \left([2\pi]^{D-1}\, 2k_-\right)^{-1/2}$ and $k_+ = \dfrac{\vec{k}^2}{4k_-}$

Right after shockwave,—

$$\phi^{in}_{k_-,\vec{k}}\Big|_{u=u_0^+} = N_{k_-}\, e^{-ik_- v}\, e^{i(\vec{k}\cdot\vec{x} + k_- f(x))} \qquad (3.4)$$

$$\cancel{\psi_>(u,\vec{x}) = N_{k_-}\, e^{-ik_- v}\, e^{i\vec{k}\cdot\vec{x}} \int \frac{d\vec{k}'\,d\vec{x}'}{(2\pi)^{D-2}}}$$

$$\cancel{e^{i(\vec{k}-\vec{k}')(\vec{x}'-\vec{x})}}$$

$$\psi_{>}(u,\vec{x}) = N_{k_-} e^{-ik_- v} e^{i\vec{k}\cdot\vec{x}} \int \frac{dk'\, dx'}{(2\pi)^{D-2}} \; e^{i(\vec{k}-\vec{k}')(\vec{x}'-\vec{x})} \; e^{-\frac{ik'^2}{4k_-}(u-u_0) + ik_- f(\vec{x}')}$$

(3.5)

Putting this together, we can write the full 'in' mode as

$$\phi^{in}_{k_-,\vec{k}} = N_{k_-} e^{-ik_- v} e^{i\vec{k}\cdot\vec{x}} \int \frac{dx'\, dk'}{(2\pi)^{D-2}} \, e^{\left[i(\vec{k}-\vec{k}')(\vec{x}'-\vec{x}) - \frac{i\vec{k}'^2}{4k_-}(u-u_0) + ik_- \theta(u-u_0) f(\vec{x}')\right]}$$

(3.6)

In order to obtain out modes we use the fact ~~that~~ the eqn (3.2) is symmetric under

$$u \to -u, \quad v \to -v, \quad k_- \to -k_-, \quad k_+ \to -k_+,$$

$$u_0 \to -u_0$$

(3.7)

Applying this we get,

$$\phi^{out}_{k_-,\vec{k}} = N_{k_-} e^{-ik_- v} e^{i\vec{k}\cdot\vec{x}} \int \frac{d\vec{x}'\, d\vec{k}'}{(2\pi)^{D-2}} \, e^{\left[i(\vec{k}-\vec{k}')(\vec{x}'-\vec{x}) - \frac{i\vec{k}'}{4k_-}(u-u_0) - ik_- \theta(u_0-u) f(\vec{x}')\right]}$$

(3.8)