$$R_{\mu\nu} - \cancel{\tfrac{1}{2} g R}\ \frac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu}$$

$$\Rightarrow \quad g^{\mu\nu} R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} g_{\mu\nu} R = -8\pi g^{\mu\nu} T_{\mu\nu}$$

$$\Rightarrow \quad R - \frac{1}{2} d R = -8\pi T^{\lambda}{}_{\lambda}$$

$$\Rightarrow \quad R\left(\frac{d-2}{2}\right) = 8\pi T^{\lambda}{}_{\lambda}$$

$$\Rightarrow \quad \frac{R}{2} = \frac{1}{d-2}\, 8\pi T^{\lambda}{}_{\lambda}$$

$$\therefore \quad R_{\mu\nu} = -8\pi T_{\mu\nu} + g_{\mu\nu} \frac{R}{2}$$

$$= -8\pi T_{\mu\nu} + 8\pi \frac{1}{d-2} T^{\lambda}{}_{\lambda}$$

$$= -8\pi \underbrace{\left(T_{\mu\nu} - \frac{1}{d-2} T^{\lambda}{}_{\lambda}\right)}_{= \tilde{T}_{\mu\nu}} \qquad (2.8)$$

From (A.2)

$$R_{uv} = \left(\frac{A_{,u} A_{,v}}{A^2} - \frac{A_{,uv}}{A} + \frac{d-2}{4} \frac{g_{,u} g_{,v}}{g^2} - \frac{d-2}{2} \frac{g_{,uv}}{g}\right)$$

$$+ \left(\frac{A_{,v}^2}{A^2} - \frac{\cancel{A_{,v} A}\ A_{,vv}}{A} - \frac{d-2}{2} \frac{g_{,v} A_{,v}}{g A}\right) \delta f$$

$$\therefore \quad R_{uv}\big|_{u=0} = -\frac{A_{,uv}}{A} - \frac{d-2}{2} \frac{g_{,uv}}{g}$$

At $u=0$

$$R_{uv} = -8\pi \tilde{T}_{uv}$$

$$\Rightarrow -\frac{A_{,uv}}{A} - \frac{d-2}{2}\frac{g_{,uv}}{g} = -8\pi \tilde{T}_{uv}$$

$$\Rightarrow \frac{A_{,uv}}{A} = -\frac{d-2}{2}\frac{g_{,uv}}{g} + 8\pi \tilde{T}_{p\,uv} \qquad (2.11)$$

$$\cancel{R_{uu}\big|_{u=0} = \frac{d-2}{2}\left(\frac{g_{,u}A_{,u}}{gA} - \frac{g_{,uu}}{g} + \frac{g_{,u}^2}{2g^2}\right)}$$

$$\cancel{= \frac{d-2}{2}\left(-\frac{g_{,uu}}{g}\right) + \frac{A}{g}\,\delta\,\Delta_{h_{ij}} f + 2\frac{A_{,uv}}{A}\delta f = -8\pi \tilde{T}_{uu}}$$

$$ds^2 = -\lambda(r)\,dt^2 + \lambda^{-1}(r)\,dr^2 + r^2 d\Omega^2_{(d-2)}$$

$$\lambda(r) = 1 - \frac{2C}{r^{d-3}} + \frac{E^2}{r^{2(d-3)}}$$

$$u = e^{t/\alpha} F(r) \Rightarrow du = \frac{1}{\alpha} e^{t/\alpha} F(r)\,dt + e^{t/\alpha} F'(r)\,dr$$

$$v = e^{-t/\alpha} F(r) \Rightarrow dv = -\frac{1}{\alpha} e^{-t/\alpha} F(r)\,dt + e^{-t/\alpha} F'(r)\,dr$$

$$\therefore\ du\,dv = \left(\frac{1}{\alpha}F(r)\,dt + F'(r)\,dr\right)\left(-\frac{1}{\alpha}F(r)\,dt + F'(r)\,dr\right)$$

$$= -\frac{1}{\alpha^2}F^2(r)\,dt^2 + (F'(r))^2\,dr^2$$

$$F(r) = e^{\frac{1}{\alpha}\int dr\,\lambda^{-1}(r)}$$

$$\Rightarrow \ln F(r) = \frac{1}{\alpha}\int dr\,\lambda^{-1}(r)$$

$$\Rightarrow \frac{F'(r)}{F(r)} = \frac{1}{\alpha}\lambda^{-1}(r)$$

$$\Rightarrow \boxed{\lambda(r) = \frac{1}{\alpha}\frac{F(r)}{F'(r)}}$$

Hence, the metric takes the form,—

$$\cancel{ds^2 = \frac{1}{\alpha}\frac{F(r)}{F'(r)}\,dt^2 + \alpha\frac{F'(r)}{F(r)}\,dr^2 + r^2 d\Omega^2_{(d-2)}}$$

$$\cancel{= \frac{1}{2}\alpha\Big(}$$

$$ds^2 = 2 \cdot \frac{1}{2}\alpha^2 \lambda(r) \frac{1}{F^2(r)} \left(- \frac{1}{\alpha^2} F^2(r)\, dt^2 + (F'(r))^2 dr^2 \right)$$

$$+ \; r^2 d\Omega^2_{(d-2)}$$

$$= 2 \cdot \frac{1}{2}\alpha^2 \lambda(r) e^{-\frac{2}{\alpha}\int dr\, \lambda^{-1}(r)} du\, dv + r^2 d\Omega^2_{(d-2)}$$

$$\therefore \; A(u,v) = \frac{1}{2}\alpha^2 \lambda(r) e^{-\frac{2}{\alpha}\int dr\, \lambda^{-1}(r)} \;;$$

$$g(u,v) = r^2$$

at $u=0$; $g_{,v} = A_{,v} = 0$

$$\kappa = (r_+ - r_-)^{-(r_-^2/r_+^2)/2} \, e^{(r_+ - r_-)/2r_+}$$

$$A = \frac{2r_+}{d-3}\kappa^{-2}\left(1 - \left(\frac{r_-}{r_+}\right)^{d-3}\right)^{-1}$$

$$\cancel{uv = F^2(r) \Rightarrow uv|_{r=0} = \kappa^2 r_+^2}$$

$$uv \approx \kappa^2 (r - r_+)^2 \Rightarrow r = r_+ + \kappa^{-1}\sqrt{uv}$$

$$\therefore \; g = (r_+ + \kappa^{-1}\sqrt{uv})^2 \Rightarrow g|_{u=0} = r_+^2$$

$$\therefore \; g_{,uv} = \frac{\kappa^{-1} r_+}{2\sqrt{uv}} + \kappa^{-2}$$

$$\Delta_{h_{ij}} f - \frac{d-2}{2}\frac{g_{,uv}}{A} f = 32\pi p g A\, \delta^{(d-2)}(x)$$

$$\Rightarrow \Delta_{(d-2)} f - a(d, r_+, r_-) f = 2\pi b(d, r_+, r_-)\, \delta^{(d-2)}(x)$$

Where, $a(d, r_+, r_-) = \frac{d-2}{2}\,\frac{2r_+ x^{-2}}{2r_+ x^{-2}}\,(d-3)\left(1-\left(\frac{r_-}{r_+}\right)^{(d-3)}\right)$

$$= \frac{1}{2}(d-2)(d-3)\left(1-\left(\frac{r_-}{r_+}\right)^{(d-3)}\right)$$

$$b(d, r_+, r_-) = 32\, p r_+^3 (d-3)^{-1} x^{-2}\left(1-\left(\frac{r_-}{r_+}\right)^{(d-3)}\right)^{-1}$$