Kerr's rotating black hole :

$$ds^2 = \frac{\Delta - a^2\sin^2\theta}{\rho^2}\,dt^2 + \frac{2\kappa Ma}{\rho^2}\,r\sin^2\theta\,d\phi\,dt - \frac{\rho^2}{\Delta}dr^2$$
$$- \rho^2 d\theta^2 - \frac{A\sin^2\theta}{\rho^2}\,d\phi^2$$

$(t, r, \theta, \phi) \longrightarrow$ coordinate choice

$\Delta = r^2 - 2\kappa M r + a^2$

$\rho^2 = r^2 + a^2\cos^2\theta$

$A = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta$

M and a are the parameters on which the black hole depends.

When $a = 0$, this metric reduces to Schwarzschild metric.

As $r \to \infty$, $ds^2 \approx \left(1 - \frac{2\kappa M}{r}\right)dt^2 + \underbrace{\frac{2\kappa Ma\sin^2\theta}{r}\,d\phi\,dt}_{\text{rotational term}}$
$$- \left(1 + \frac{2\kappa M}{r}\right)dr^2 - r^2 d\Omega^2$$

this metric $t \rightarrow t + a$ ~~does not change~~ invariant

but not $t \rightarrow -t$ invariant due to the presence of $d\phi\,dt$ term.

So, this metric is not static but stationary.

$J = Ma \longrightarrow$ total angular momentum

$a \longrightarrow$ angular momentum per unit mass

$\Delta = 0 \Rightarrow r_{\pm} = \kappa M \pm \sqrt{(\kappa M)^2 - a^2}$

$r_+ \longrightarrow$ radius of horizon

when $a \longrightarrow 0$, $r_+ \longrightarrow r_g$

when $r = r_+$, $(r_\mu)^2 = 0$. So at $r = r_+$ the escape velocity is the speed of the light

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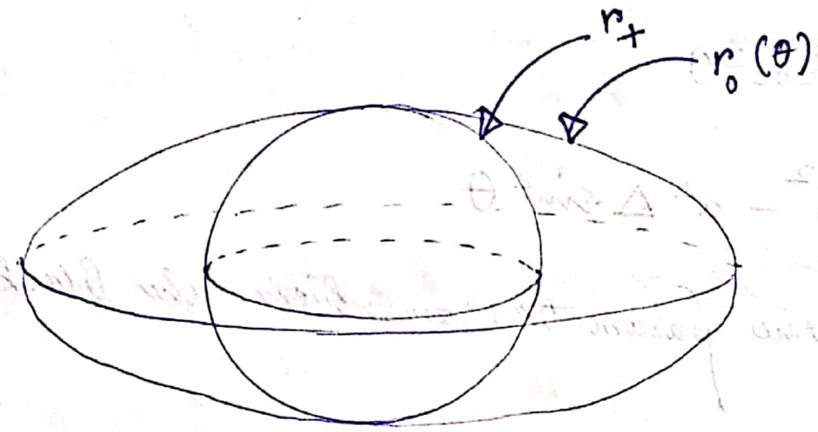
$$ds^2 = \underbrace{\frac{\Delta - a^2 \sin^2\theta}{\rho^2}}_{=0} dt^2 + \cdots\cdots \Rightarrow r_0(\theta) = \varkappa M \pm \sqrt{(\varkappa M)^2 - a^2\cos^2\theta}$$

→ Ergo region.

$$r_0(\theta) \geq r_+$$



When $r = r_0(\theta)$ then $g_{00} = 0$

$$\begin{cases} r_\pm = \varkappa M \pm \sqrt{(\varkappa M)^2 - a^2} \\ r_0(\theta) = \varkappa M \pm \sqrt{(\varkappa M)^2 - a^2\cos^2\theta} \end{cases}$$

→ both $r_\pm$ and $r_0(\theta)$ is real when $\varkappa M > a$