Railway Engineering Mathematics

Tutorial Sheet 8

1. A rectangular football pitch has its length equal to twice its width and a perimeter of 360 m. Find its length and width.

2. (a) Given that $y = \sqrt{x}$, show that the equation

$$\sqrt{x} + \frac{10}{\sqrt{x}} = 7 \tag{1}$$

maybe written as

$$y^2 - 7y + 10 = 0$$

(b) Hence solve equation (1).

3. A person drove from the Dead Sea up to Amman, and their altitude increased at a constant rate. When they began driving, their altitude was 400 metres below sea level. When they arrived in Amman 2 hours later, their altitude was 1000 metres above sea level.

After how many hours does the person take to reach an altitude of 1750 metres?

4. The manager of a factory finds that it costs £2200 to produce 100 parts in one day and £4800 to produce 300 parts in one day.

(a) Express the cost as a function of the number of parts produced, assuming that it is linear.

(b) Plot a graph of the function from part (a), using any software.

(c) What does the gradient of the graph represent?

(d) What does the y-intercept represent?

5. A bacterial culture starts with 500 bacteria and doubles in size every half hour.

   (a) How many bacteria are there after 3 hours?

   (b) How many bacteria are there after t hours?

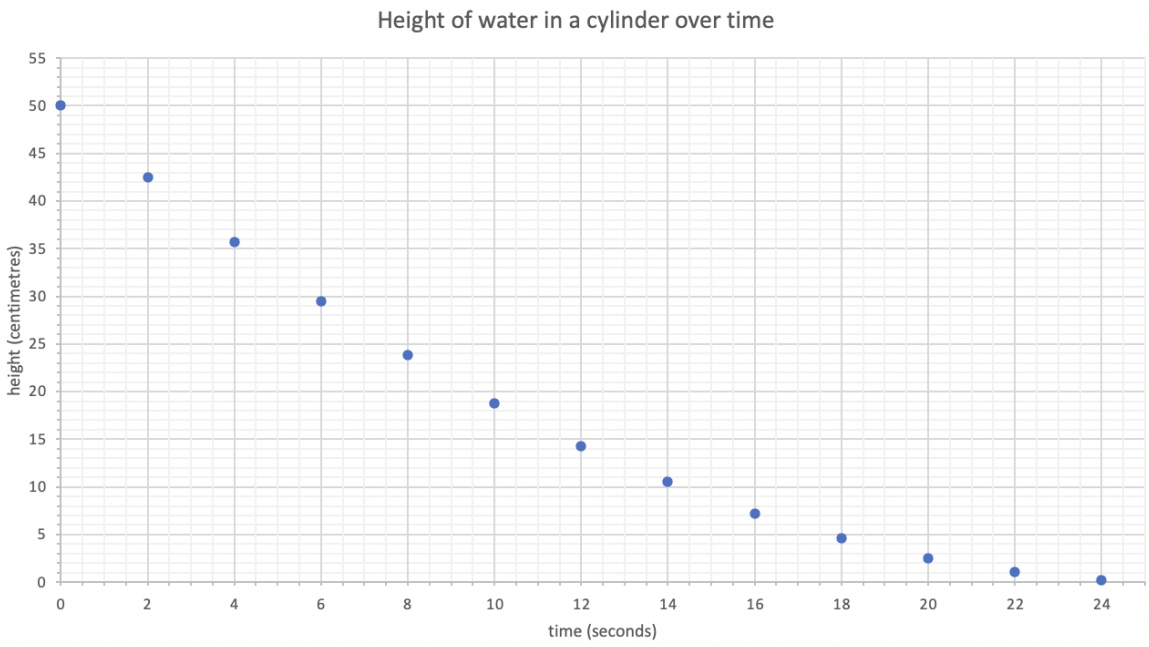
   (c) How many bacteria are there after 40 minutes?

   (d) Graph the population function and estimate the time for the population to reach 100,000.

6. A cylinder is filled with water to a height of 50 cm. The cylinder has a hole at the bottom which is covered with a stopper. The stopper is released at time $t = 0$ seconds and is allowed to empty. The following data shows the height of the water in the cylinder at different times:

Time (seconds)	Height (cm)
0	50
2	42.5
4	35.7
6	29.5
8	23.8
10	18.8
12	14.3
14	10.5
16	7.2
18	4.6
20	2.5
22	1.1
24	0.2

Depicted in the graph below is the same data:



Assume that the relationship between the height of the water and the time is linear.

(a) With a ruler, draw a linear line of best fit on the graph and determine the equation of the line.

(b) Using the equation obtained in part (a) calculate the height of the water at $t = 1\,\text{s}$, $t = 12\,\text{s}$ and $t = 24\,\text{s}$.

(c) What can you conclude about the linear fit in comparison to the original data?

(d) Suggest a better model for the data, determine the equation and test its viability plot plotting the equation and original data on the same axes.