Linear Functions

Railway Engineering Mathematics

Sheffield Hallam University

Lecture 5

Learning Outcomes

- Recognise polynomials
- Interpret linear equations
- Recognise typical shapes of polynomial graphs (constant and linear)
- Plot linear graphs using Excel

Function notation

A **function** tells us how one variable depends on (possibly multiple) others. For example, $y = 3x^2 + 8x - 7$. Here the **independent variable** is x and the **dependent variable** is y; we say that y is dependent upon x. That is, the value of y depends on the value of x that we put in.

We could also express the function as:

$y(x) = 3x^2 + 8x - 7,$

$f(x) = 3x^2 + 8x - 7,$

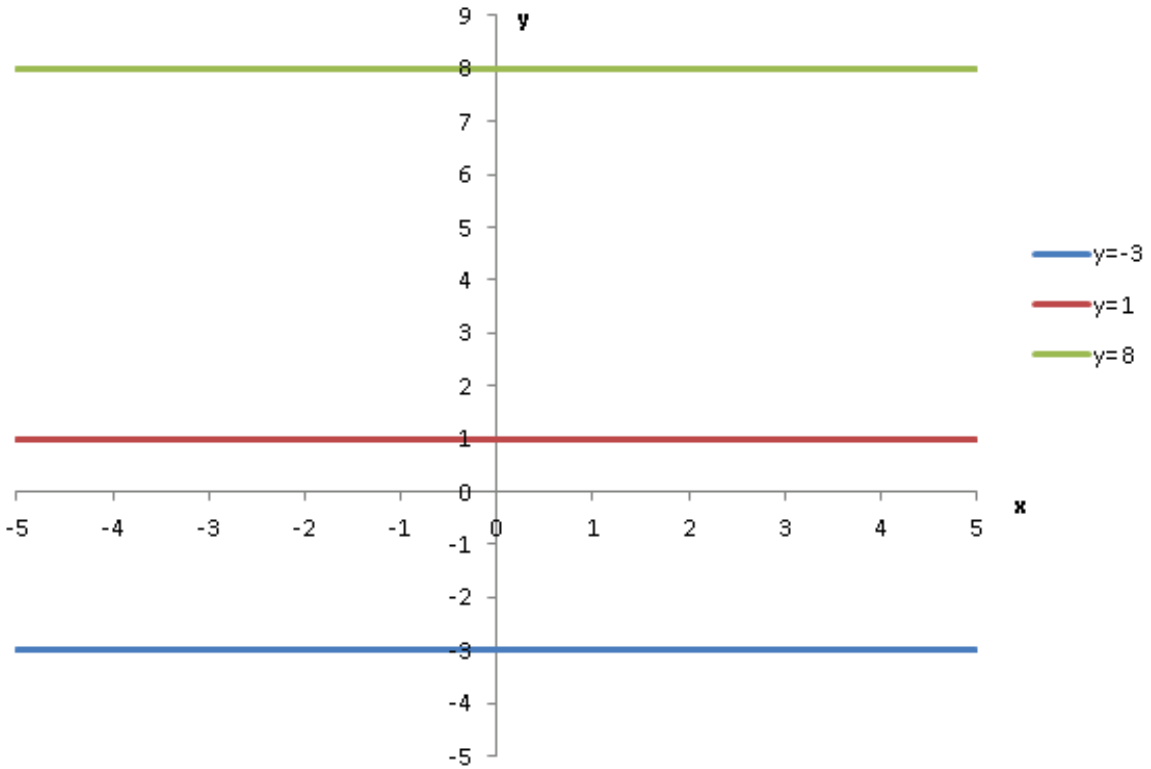
$g(x) = 3x^2 + 8x - 7,$ etc

Here the independent variable is explicitly x, and the function is named y, f, or g

Polynomials

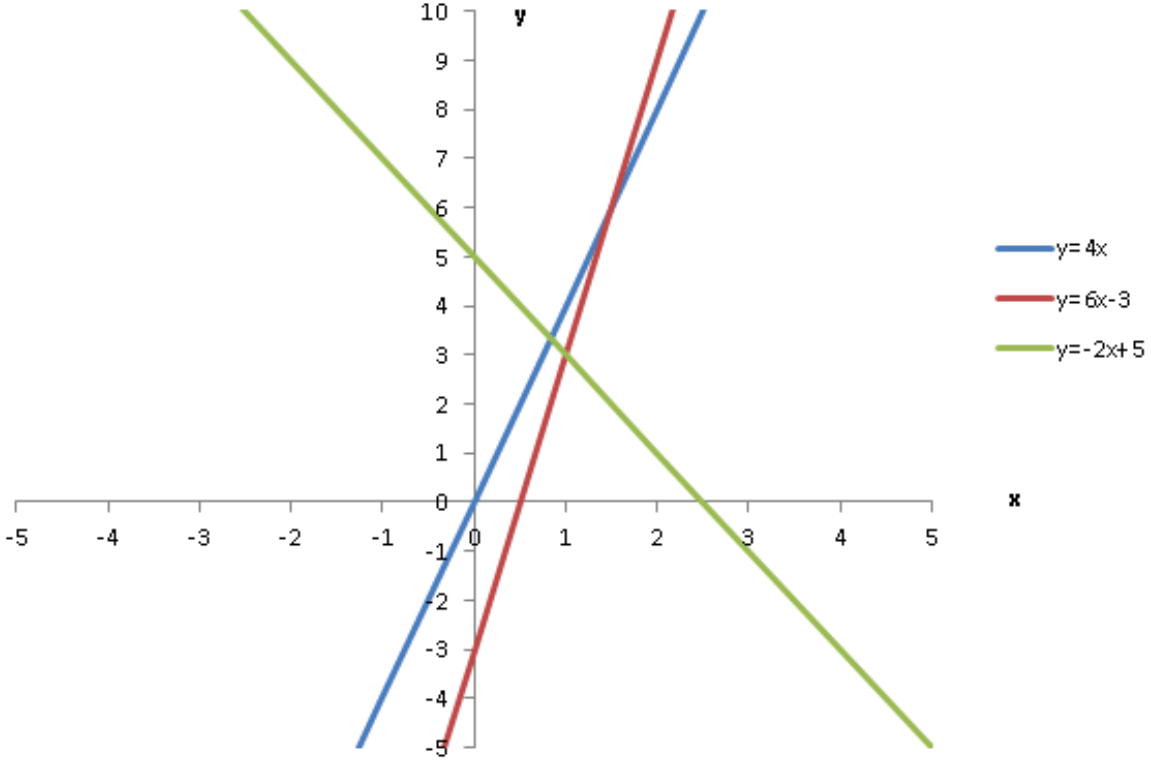
- A **polynomial function** is one that *only* involves non-negative integer powers of x, for example:
  - $y = 7x + 4$ (polynomial of order/degree 1, linear)
  - $y = 3x^2 - 5x - 1$ (polynomial of order/degree 2, quadratic)
  - $y = -x^3 + 5x^2 - 7x + 12.01$ (poly. of order/degree 3, cubic)
- Functions containing negative or non-integer powers, or other functions, (such as trigonometric) are *not* polynomials, e.g.
  - $y = x^2 + 4\sqrt{x} - 5$
  - $y = \dfrac{5}{x^2} - 7x^3 + 6x - 4$
  - $y = x^2 + \sin(x)$

Graphs of Polynomials: Constant functions



Graphs of **constant functions** (no dependency on x) are always straight, horizontal lines.

Graphs of Polynomials: Linear functions



Graphs of **linear functions** are always straight lines.
(We will look at quadratics later.)

Graphs of Polynomials - Linear

To plot graphs manually we first have to define the x range, if not already specified. Then we need to calculate the value of the function, y, for the specific values of x.

Plot the function $y = 5x + 3$ in the range $-1 \leq x \leq 5$.

x	y
-1	
0	
1	
2	
3	
4	
5	

Graphs of Polynomials - Linear

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Plot the function $y = 5x + 3$ in the range $-1 \leq x \leq 5$.

x	y
-1	5(-1)+3 = -2
0	
1	
2	
3	
4	
5	

Graphs of Polynomials - Linear

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Plot the function $y = 5x + 3$ in the range $-1 \leq x \leq 5$.

x	y
-1	-2
0	3
1	8
2	
3	
4	
5	

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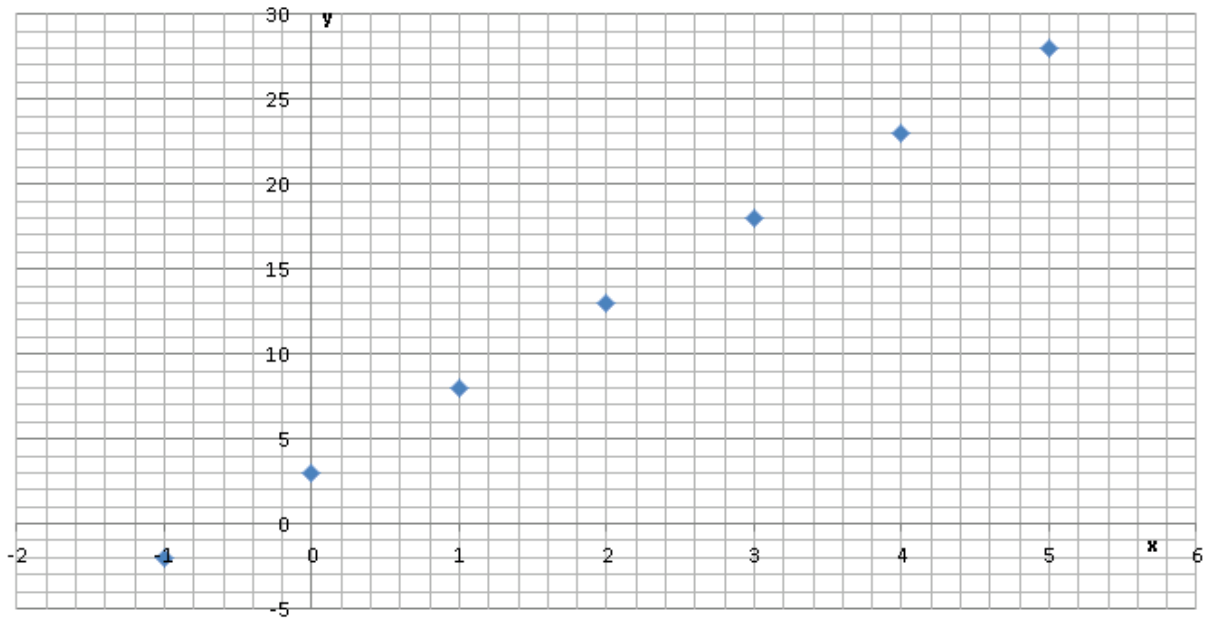
x	y
-1	-2
0	3
1	8
2	13
3	18
4	23
5	28

Graphs of Polynomials - Linear

We can then plot the (x, y) coordinates on a graph:

x	y
-1	-2
0	3
1	8
2	13
3	18
4	23
5	28



Graphs of Polynomials - Linear

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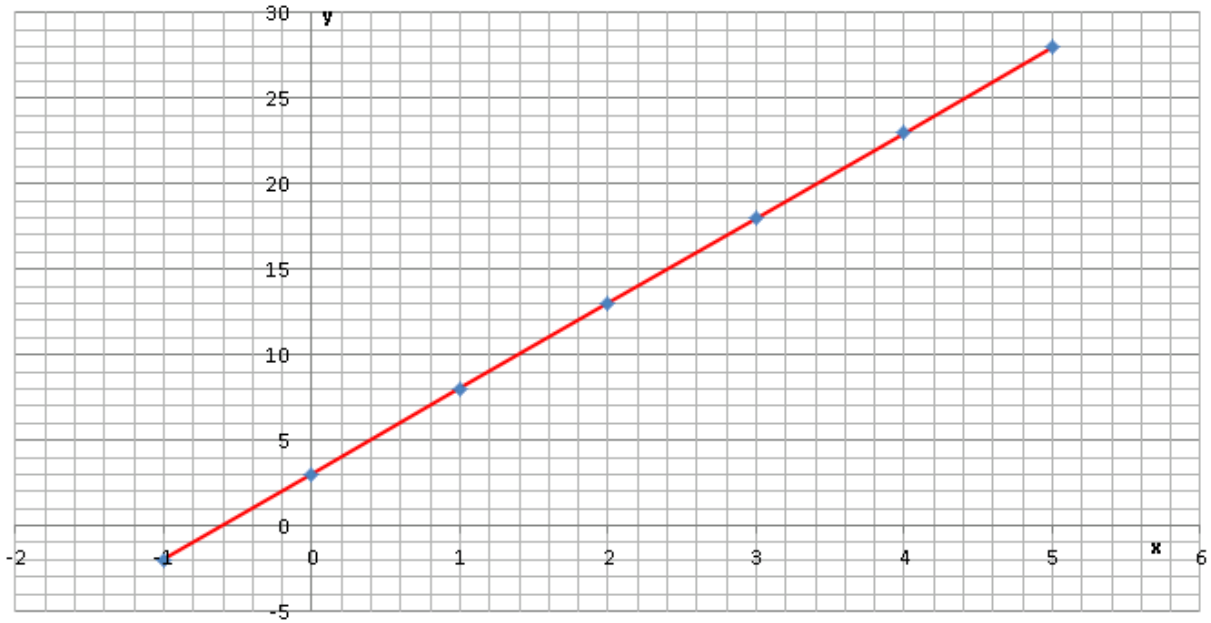


Determining the Linear Equation

Equation of a straight line

$$y = mx + c$$

where m and c are constants, represents a straight line.

m is the **gradient** (slope) of the line and can be calculated as

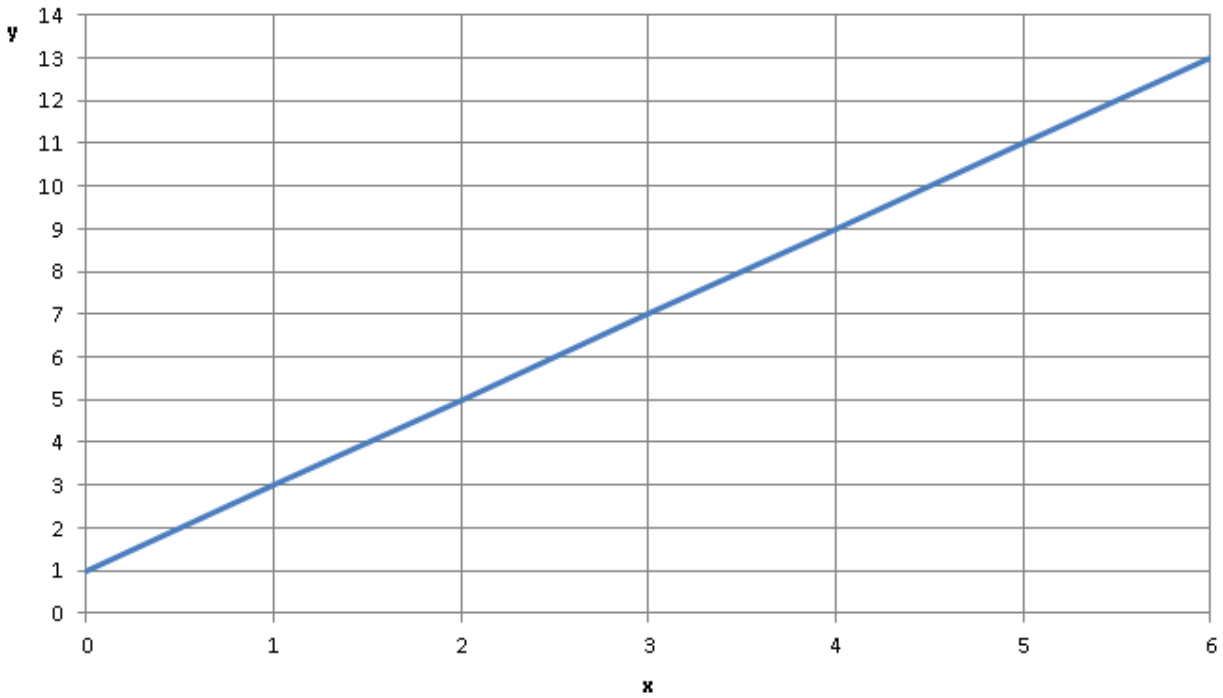
$$m = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}} = \frac{\Delta y}{\Delta x}$$

c is the value of y when the line crosses the y-axis (at $x = 0$), known as the y-**intercept**.

Note: to find where the line crosses the x-axis, simply let $y = 0$.

Determining the Linear Equation

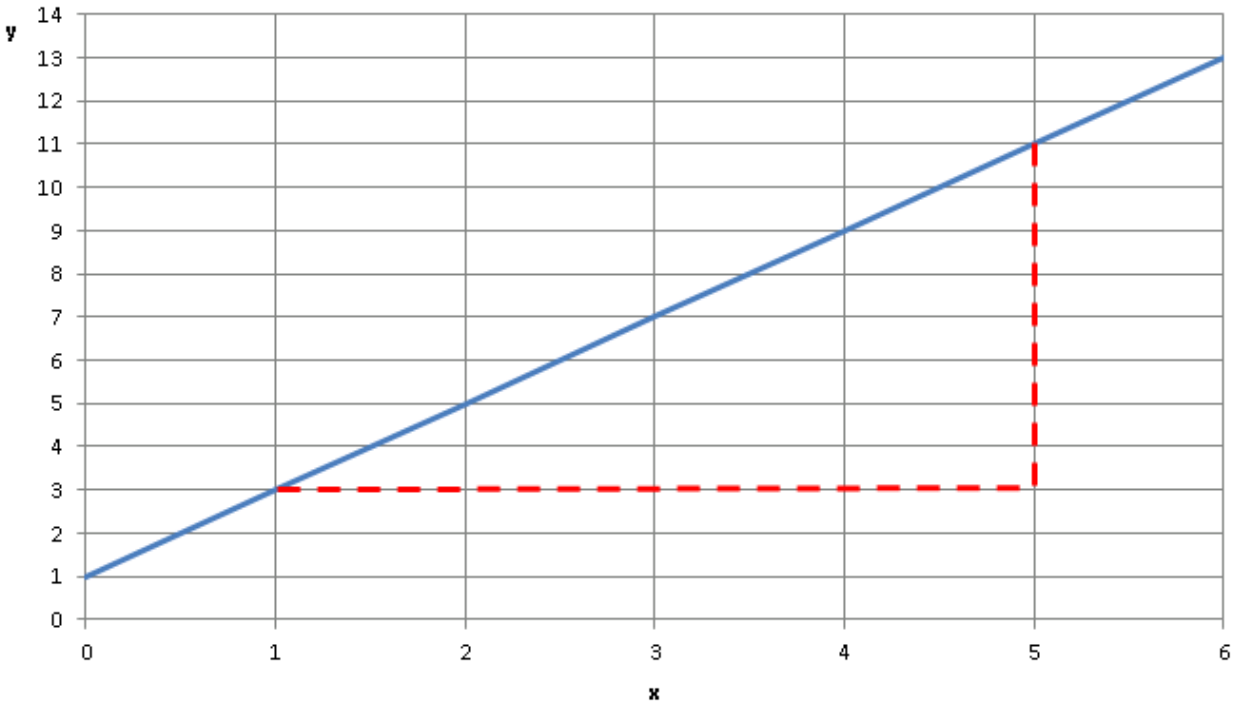
Example: Find the equation of this line:



First, we can see that $c = 1$ as this is the height where the y-axis is crossed.

Determining the Linear Equation

Example: Find the equation of this line:



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Determining the Linear Equation

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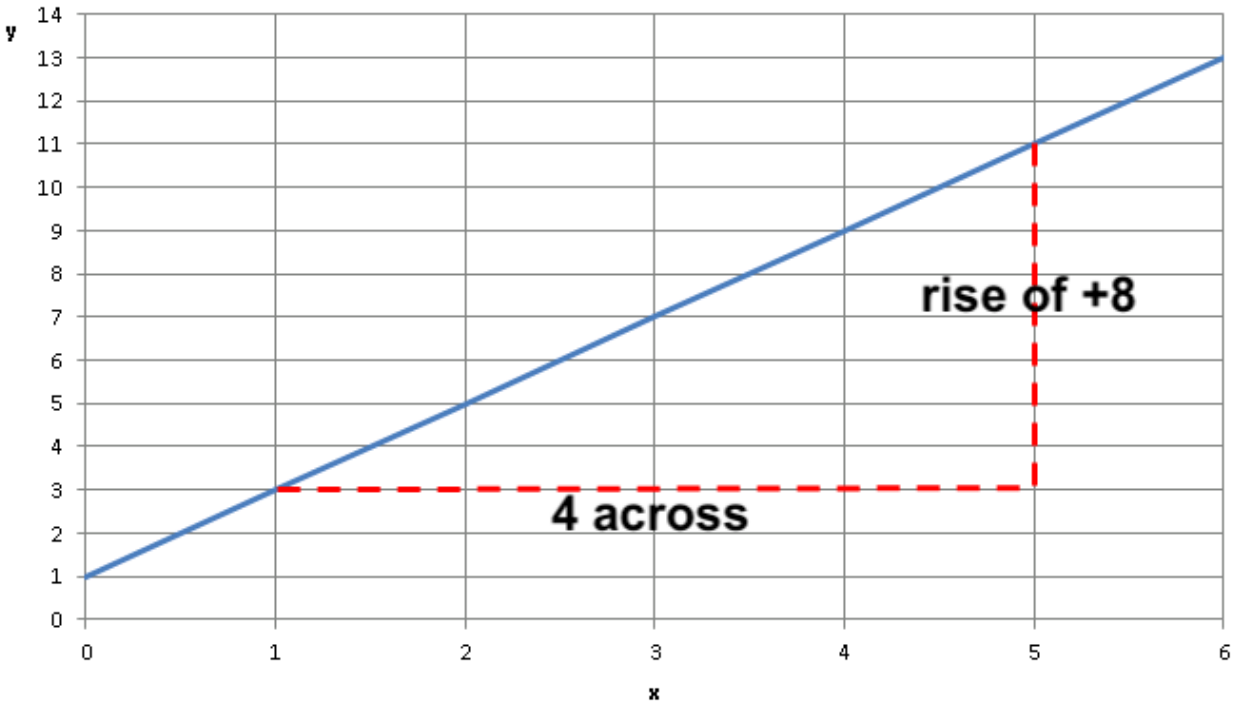


First, we can see that $c = 1$ as this is the height where the y-axis is crossed.

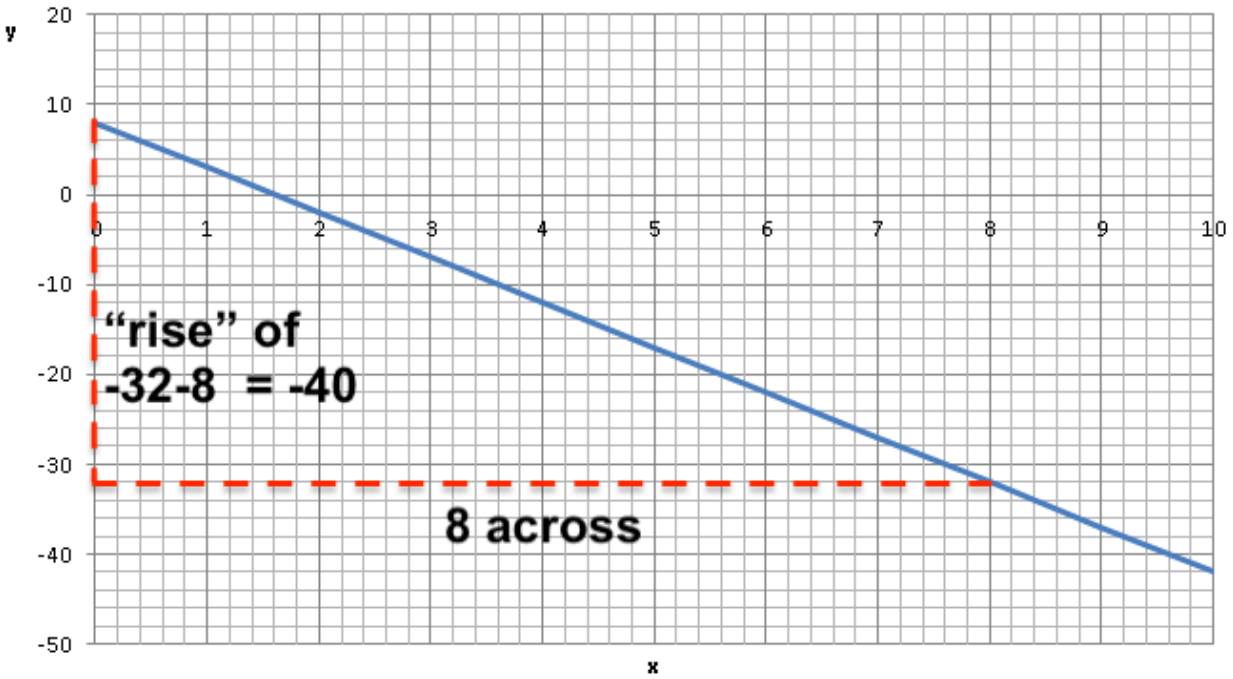
and $m = \dfrac{\Delta y}{\Delta x} = \dfrac{8}{4} = 2$

$\therefore \quad y = 2x + 1$

Determining the Linear Equation

Note that if the straight line graph is *decreasing* then we expect a **negative gradient**.

This is because the "rise" will actually be a fall - a decrease in y.



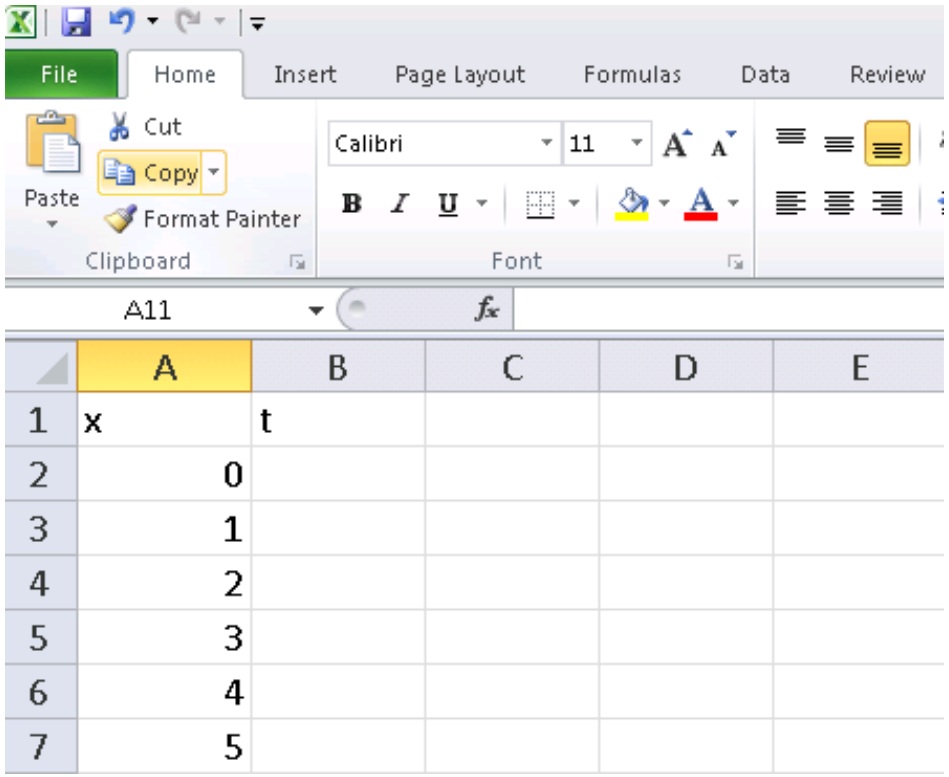
This time the gradient is negative as there is a decrease from left to right:

$$m = \frac{\Delta y}{\Delta x} = \frac{-40}{8} = -5$$

Using Excel to Plot Polynomials

Using Excel to plot a function allows us to automate the process.

To plot the linear function $y = 7x - 4$ in the range $0 \leq x \leq 5$:



	A	B	C	D	E
1	x	t			
2	0				
3	1				
4	2				
5	3				
6	4				
7	5				

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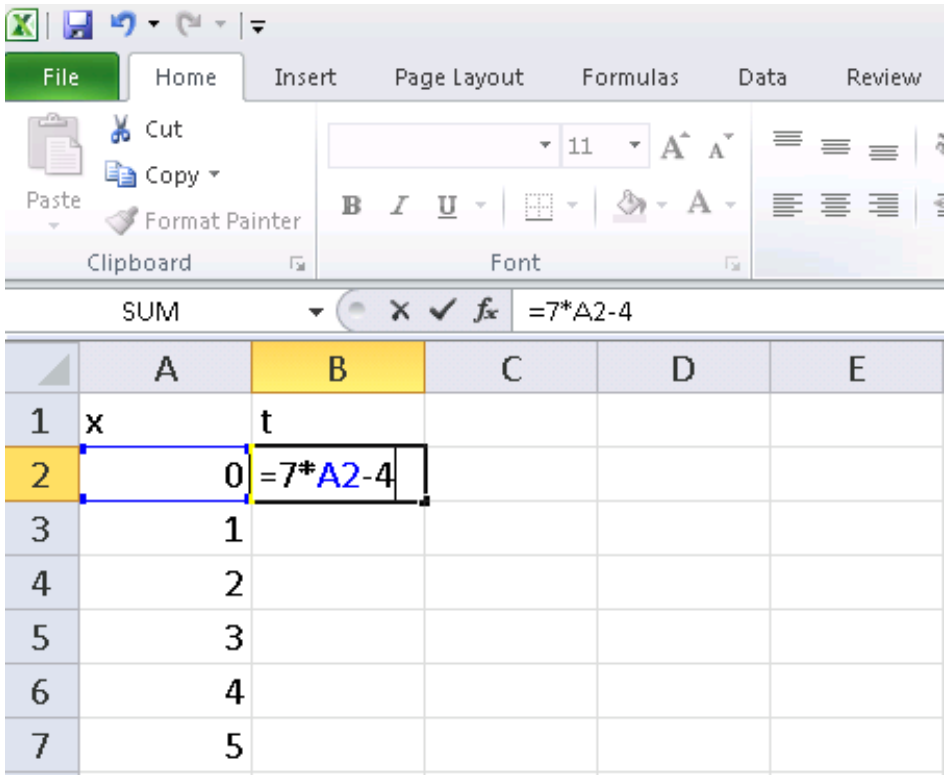

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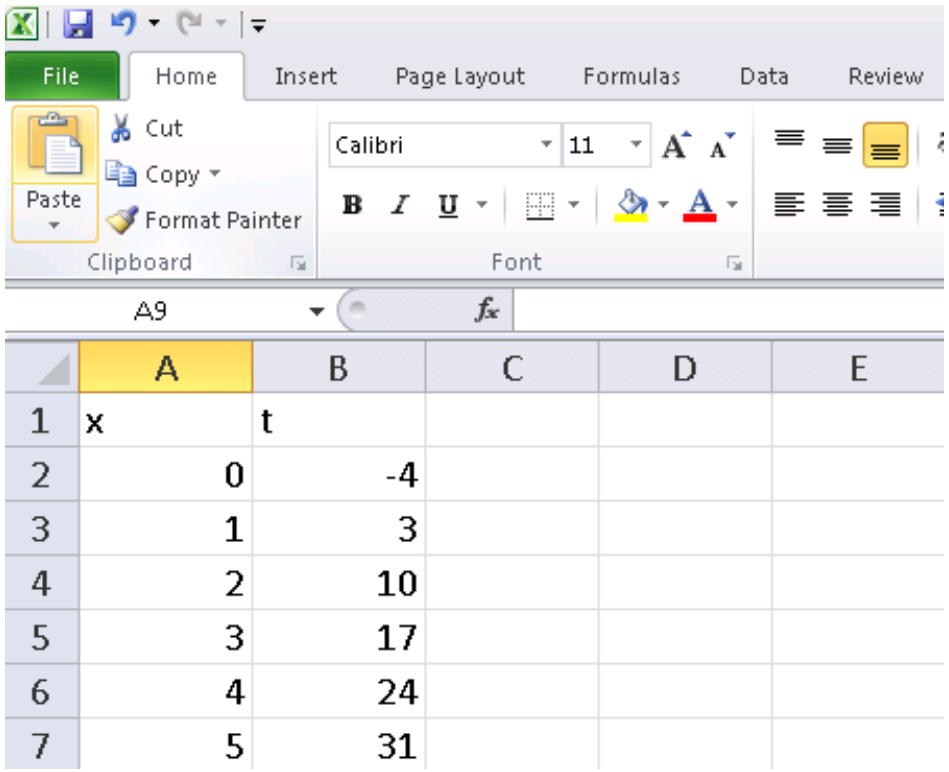

	A	B	C	D	E
1	x	t			
2	0	-4			
3	1	3			
4	2	10			
5	3	17			
6	4	24			
7	5	31			

Using Excel to Plot Polynomials

Using Excel to plot a function allows us to automate the process.

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