Railway Engineering Mathematics

Tutorial Sheet 6

Analysing and sketching quadratics

1. Consider the quadratic function $y = x^2 + 3x - 4$

   (a) Where does $y = x^2 + 3x - 4$ cross the y-axis?

   (b) Where does $y = x^2 + 3x - 4$ cross the x-axis?

   (c) Is it $\cup$-shaped, or $\cap$-shaped?

   (d) Sketch the graph of this function.

2. Repeat the process of Question 1 for the functions:

   (i) $y = 16 - x^2$

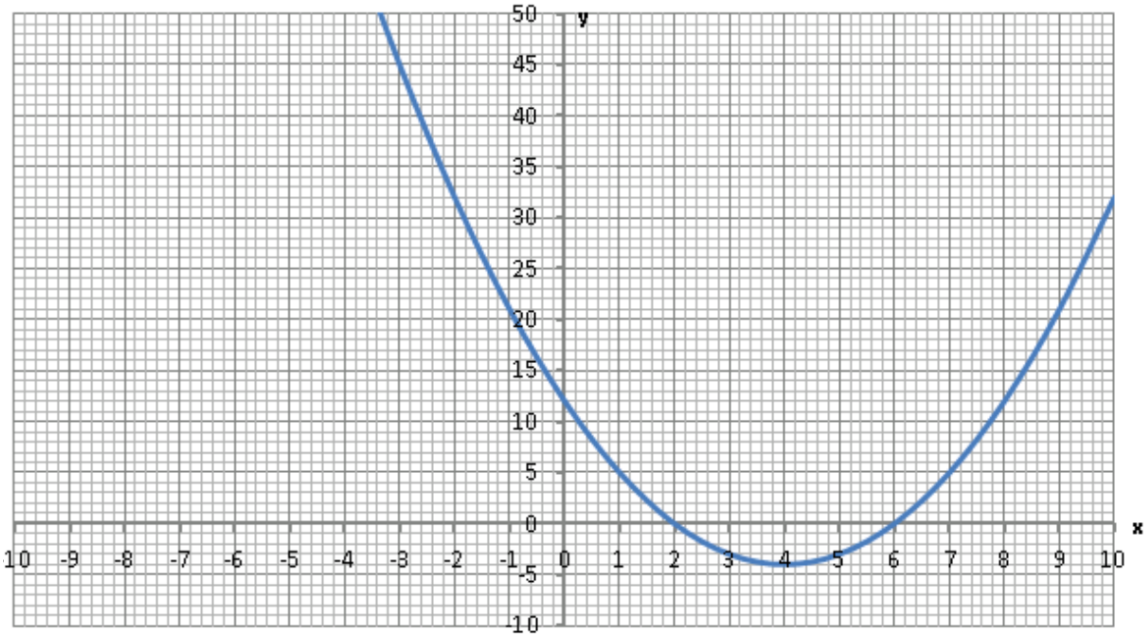
   (ii) $y = 2x^2 + 4x - 20$

   Check your results by plotting the curves in EXCEL or other software (such as Desmos or GeoGebra).

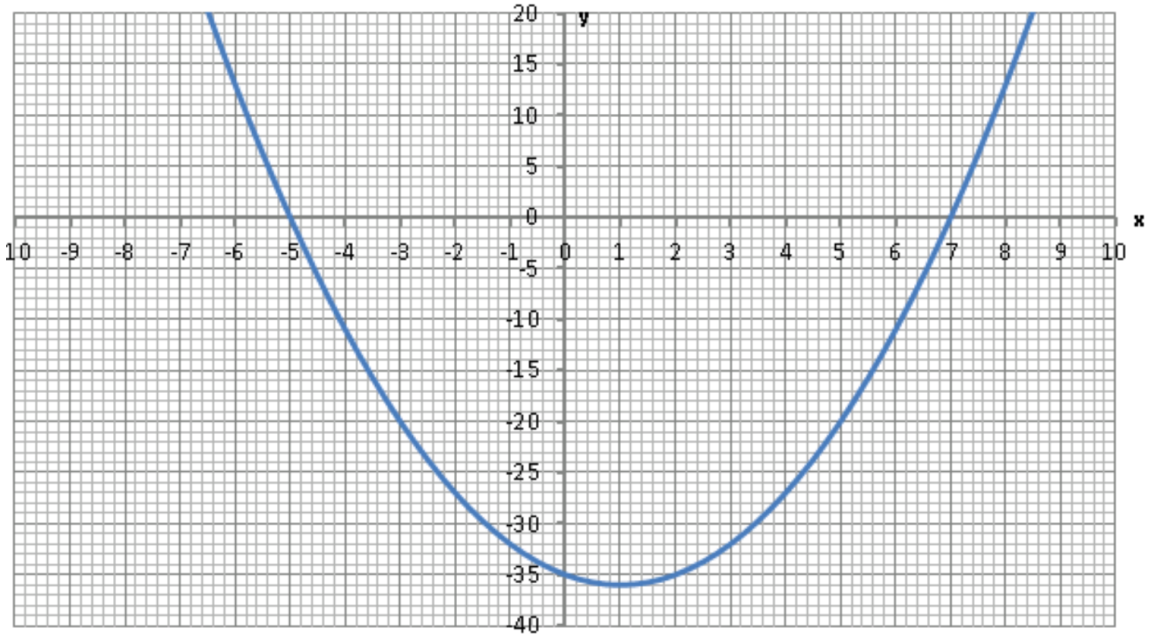
Determining the equation of a parabola

3. Determine the quadratic equation of the following graphs, given that they are in the form of either $y = x^2 + bx + c$ or $y = -x^2 + bx + c$ (i.e. note that the x^2 term is already known to have a coefficient equal to either one or minus one).

(a)



(b)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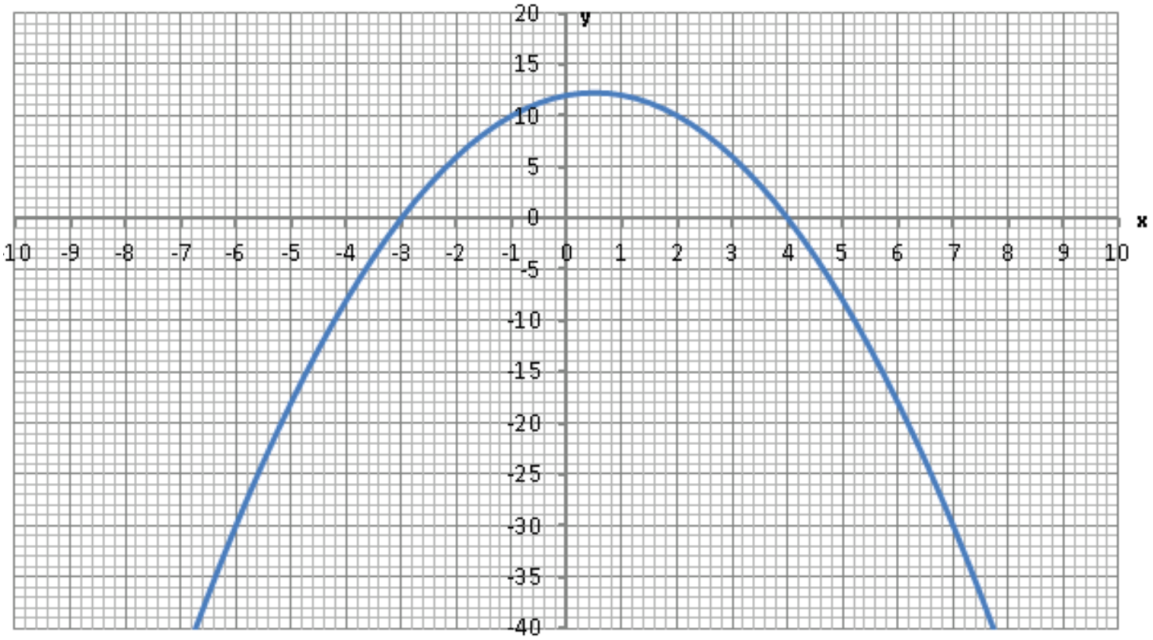
(c)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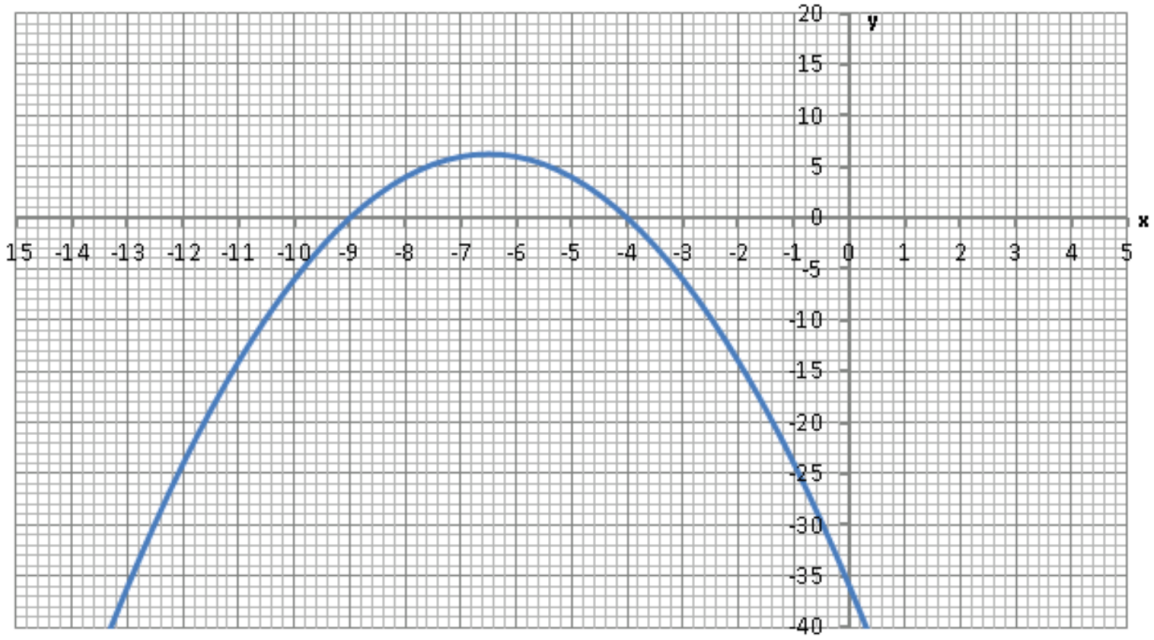
(d)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Using the quadratic formula

4. Calculate the roots of the following polynomials:

   (a) $y = x^2 + 6x - 27$

   (b) $y = x^2 + 2x - 48$

   (c) $y = 3x^2 + 11x + 8$

   (d) $y = 6x^2 - 23x + 20$

5. If the roots of the quadratic $y = ax^2 + bx + c$ are x_1 and x_2 then it is possible to rewrite the quadratic in its factorised form as follows:

   $$y = a(x - x_1)(x - x_2)$$

   (a) Use this information to rewrite the four quadratics that appear in Question 4.

   (b) Expand the brackets of your new form in each case to demonstrate that it is equivalent to the original form.

Trickier questions

6. Solve $5x^2 + 2x = 4$, giving the solutions correct to 3 significant figures.

7. Solve $x^2 = 4(x - 3)^2$

8. Solve $\dfrac{5}{x+2} + \dfrac{9}{x-2} = 2$

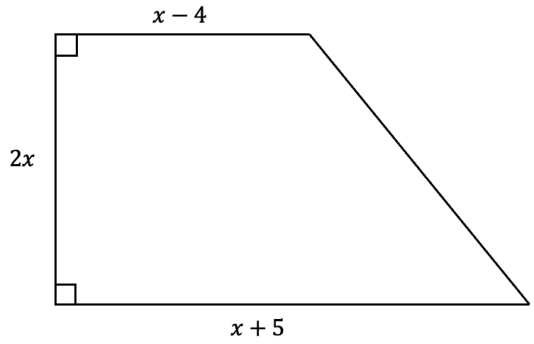
9. The quadratic formula is being used to solve a quadratic equation. After substitution of the coefficients into the formula, we have

$$x = \frac{-7 \pm \sqrt{17}}{4}$$

Work out the original quadratic equation, giving your answer in the form $ax^2 + bx + c = 0$.

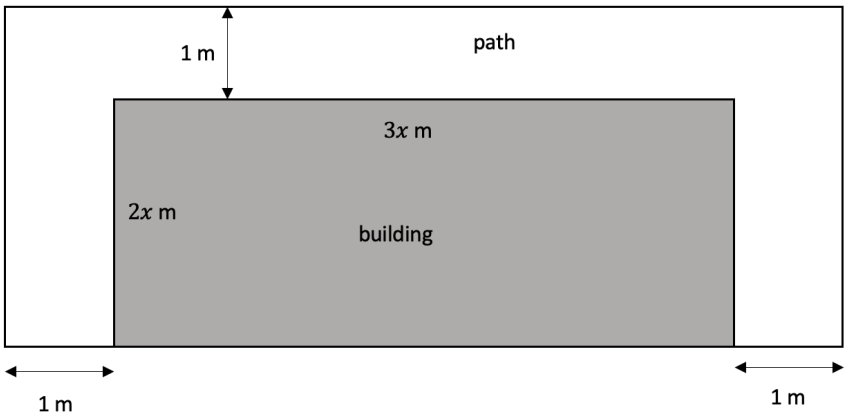
10. Solve the equation $\dfrac{2^{(n^2)}}{2^n \times 2^6} = 1$

11. Shown in the diagram below is a trapezium (not drawn to scale) where all measurements are in centimetres.



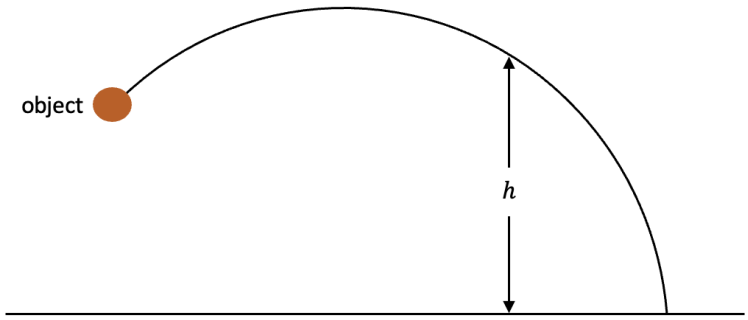
The area of the trapezium is 351 m^2. Determine the value of x.

12. A rectangular building has a length of $3x$ metres and a width of $2x$ metres. The building has a path of width 1 metre on three of its sides, as depicted in the diagram below. Note that the diagram is not drawn to scale.



Given that the total area of the building and path is 100 m^2 calculate the area of the building.

13. An object is being launched from an initial height of 2 m above the ground. The object follows a parabola in the form $h = 2 + 6t - 5t^2$, where h is the height in metres above the ground t seconds after it has been launched.



The object hits the ground after T seconds. Calculate the value of T, giving the answer to 2 decimal places.