Railway Engineering Mathematics
Tutorial Sheet 2
Solutions

1. Evaluate the following by hand where possible, and using technology otherwise:

(a) 7^3

(b) 3^7

(c) 5^{-2}

(d) $2^{5/2}$

(e) $(-2)^6$

(f) -2^6

(g) $(1.234)^8$

(h) 2.345^{-5}

Solution:

(a) $7^3 = 343$

(b) $3^7 = 2187$

(c) $5^{-2} = \dfrac{1}{25} = 0.04$

(d) $2^{5/2} = \sqrt{2^5} = 4\sqrt{2} = 5.657$

(e) $(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$

(f) $-2^6 = -2 \times 2 \times 2 \times 2 \times 2 \times 2 = -64$

(g) $(1.234)^8 = 5.377$

(h) $2.345^{-5} = 0.014$

2. Express each of the following as a single power:

(a) $2^3 \times 2^5 \times 2^7$

(b) $3^3 \times 3^{-4}$

(c) $4^6 \times \dfrac{4^{-7}}{4^{-5}}$

(d) $\dfrac{5^5 \times 5^{-3}}{5^4 \times 5^{-2} \times 5^{-7}}$

(e) $(2^{-3})^4$

(f) $3^{2/3} \times 3^{-2}$

(g) $\dfrac{6^7 \times 6^{-7} \times \sqrt{6}}{6 \times \sqrt[3]{6^4}}$

(h) $\dfrac{5^{-8} \times 5^2 \times \dfrac{1}{\sqrt[3]{5^2}}}{5^{-2} \times (5^3)^4}$

Solution:

(a) $2^3 \times 2^5 \times 2^7 = 2^{3+5+7} = 2^{15}$

(b) $3^3 \times 3^{-4} = 3^{3+-4}$

(c) $4^6 \times \dfrac{4^{-7}}{4^{-5}} = 4^6 \times 4^{-7-(-5)} = 4^6 \times 4^{-2} = 4^{6-2} = 4^4$

(d) $\dfrac{5^5 \times 5^{-3}}{5^4 \times 5^{-2} \times 5^{-7}} = \dfrac{5^{5+(-3)}}{5^{4+(-2)+(-7)}} = \dfrac{5^2}{5^{-5}} = 5^{2-(-5)} = 5^7$

(e) $(2^{-3})^4 = 2^{-3\times 4} = 2^{-12}$

(f) $3^{2/3} \times 3^{-2} = 3^{\frac{2}{3}+(-2)} = 3^{-4/3}$

(g)
$$\begin{aligned}\frac{6^7 \times 6^{-7} \times \sqrt{6}}{6 \times \sqrt[3]{6^4}} &= \frac{6^7 \times 6^{-7} \times 6^{1/2}}{6^1 \times 6^{4/3}} \\ &= \frac{6^{7+(-7)+1/2}}{6^{1+4/3}} \\ &= \frac{6^{1/2}}{6^{7/3}} \\ &= 6^{-11/6}\end{aligned}$$

(h)
$$\begin{aligned}\frac{5^{-8} \times 5^2 \times \dfrac{1}{\sqrt[3]{5^2}}}{5^{-2} \times (5^3)^4} &= \frac{5^{-8} \times 5^2 \times 5^{-2/3}}{5^{-2} \times 5^{12}} \\ &= \frac{5^{-20/3}}{5^{10}} \\ &= 5^{-50/3}\end{aligned}$$

3. Simplify the following:

(a) $y^5 \times y^8$

(b) $\dfrac{x^{10}}{x^7}$

(c) $x^3 \div x$

(d) $(a^2)^3$

(e) $3x^2 \times 4x^7 \times 2x^{-3}$

(f) $6x \times 7y^2 \times x^5$

(g) $\dfrac{3y^4 \times 4y^2}{6x^2 \times y^8}$

(h) $\dfrac{(3x^4)^2 \times 5\sqrt[3]{8x^2}}{15x^2 \times y^7}$

Solution:

(a) $y^5 \times y^8 = y^{13}$

(b) $\dfrac{x^{10}}{x^7} = x^3$

(c) $x^3 \div x = x^{3-1} = x^2$

(d) $(a^2)^3 = a^6$

(e) $3x^2 \times 4x^7 \times 2x^{-3} = 3 \times 4 \times 2 \times x^{2+7-3} = 24x^6$

(f) $6x \times 7y^2 \times x^5 = 42x^6y^2$

(g) $$\frac{3y^4 \times 4y^2}{6x^2 \times y^8} = \frac{3 \times 4 \times y^{4+2}}{6x^2y^8}$$
$$= \frac{12y^6}{6x^2y^8}$$
$$= \frac{2}{x^2y^2}$$

Or:

$$2x^{-2}y^{-2}$$

(h) $$\frac{(3x^4)^2 \times 5\sqrt[3]{8x^2}}{15x^2 \times y^7} = \frac{9x^8 \times 5 \times (8x^2)^{1/3}}{15x^2 \times y^7}$$
$$= \frac{9x^8 \times 5 \times 2x^{2/3}}{15x^2y^7}$$
$$= 6x^{20/3}y^{-7}$$

4. Change the following to the specified base:

(a) 25^3 to the base 5

(b) 8^6 to the base 2

(c) 9^4 to the base 3

(d) 81^5 to the base 3

Solution:

(a) $25^3 = (5^2)^3 = 5^6$

(b) $8^6 = (2^3)^6 = 2^{18}$

(c) $9^4 = (3^2)^4 = 3^8$

(d) $81^5 = (3^4)^5 = 3^{20}$

5. Determine the value of y:

$$16^{\frac{1}{4}} \times 2^y = 8^{\frac{3}{4}}$$

Solution:

$$\begin{aligned} (2^4)^{\frac{1}{4}} \times 2^y &= (2^3)^{\frac{3}{4}} \\ 2^1 \times 2^y &= 2^{\frac{9}{4}} \\ 2^{1+y} &= 2^{\frac{9}{4}} \end{aligned}$$

Therefore,

$$\begin{aligned} 1 + y &= \frac{9}{4} \\ y &= \frac{9}{4} - 1 \\ y &= \frac{5}{4} \end{aligned}$$

6. A patient has a disease. They have 4^3 body cells affected on day 1. The number of affected cells doubles every day. The disease becomes serious when 2^{10} body cells are affected. On which day does the disease become serious?

Solution:

$$\begin{aligned} 4^3 \times 2^0 &\quad \text{on day 1} \\ 4^3 \times 2^1 &\quad \text{on day 2} \\ \text{or } (2^2)^3 \times 2^{n-1} &\quad \text{on day n} \\ \text{or, more simply } 2^6 \times 2^{n-1} &\quad \text{on day n} \end{aligned}$$

Therefore, to determine the value of n for which the disease becomes serious:

$$\begin{aligned} 2^6 \times 2^{n-1} &\geq 2^{10} \\ 2^{6+n-1} &\geq 2^{10} \\ 2^{n+5} &\geq 2^{10} \end{aligned}$$

So, the value of n is

$$\begin{aligned} n + 5 &\geq 10 \\ n &\geq 5 \end{aligned}$$

i.e. day 5.

7. The area of a rectangle is $125^{\frac{1}{4}}$ cm^2. The lengths of the rectangle are 5^{x+1} cm and $25^{\frac{1}{2}}$ cm. Work out the value of x.

Solution:

$$
\begin{aligned}
5^{x+1} \times 25^{\frac{1}{2}} &= 125^{\frac{1}{4}} \\
5^{x+1} \times (5^2)^{\frac{1}{2}} &= (5^3)^{\frac{1}{4}} \\
5^{x+1} \times 5^1 &= 5^{\frac{3}{4}} \\
5^{x+1+1} &= 5^{\frac{3}{4}}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
x + 2 &= \frac{3}{4} \\
x &= \frac{3}{4} - 2 \\
x &= -\frac{5}{4}
\end{aligned}
$$