BIDMAS and Fractions

Railway Engineering Mathematics

Sheffield Hallam University

Lecture 1

Learning Outcomes

- Communicate calculations to technology.
- Understand the rules of BIDMAS.
- Perform calculations (addition, subtraction, multiplication, division) with fractions.

Communicating with Technology

In order to effectively communicate with technology (calculators, Excel, computer algebra systems (CAS), etc.) you need to know how it interprets the information you provide.

For example, how would you tell a calculator or Excel to compute the following?

$$\frac{9-3}{5} \times 2^4$$

BIDMAS

BIDMAS is an acronym to help you remember the correct **order of operations**. This is the unambiguous way in which software evaluates a complicated calculation. It stands for:

- Brackets
- Index (powers and roots)
- Division and Multiplication,
- Addition and Subtraction.

Highest priority is given to brackets while lowest priority is given to addition and subtraction. Division and multiplication have the same priority level, as do addition and subtraction.

Example of BIDMAS

If we wish to evaluate:

$$10 + 5 \times 4$$

We must first perform the multiplication, then the addition:

$$\begin{aligned} & 10 + 5 \times 4 \\ = \;& 10 + 20 \\ = \;& 30 \end{aligned}$$

A common error is to perform the sum from left to right, i.e. $15 \times 4 = 60$. But say four people visited a theme park. It cost £5 each to enter and there's a fixed car park fee of £10. What would you expect to pay in total for entry? Not £60!

Communicating with Technology

BIDMAS means that we can use brackets to tell our technology to do a particular calculation first, e.g.

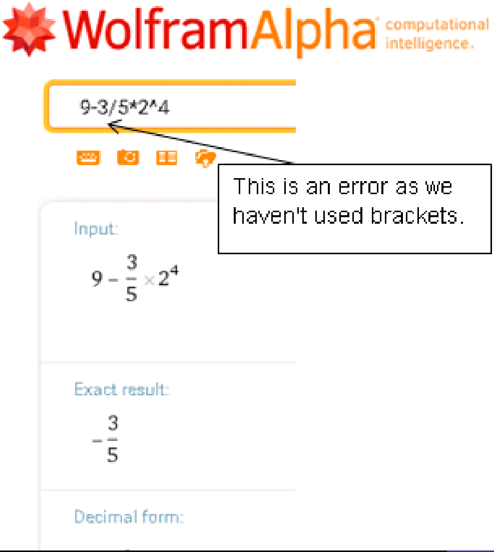
$$\frac{9-3}{5} \times 2^4 \tag{1}$$

should be written in a calculator or Excel, etc. as:

$$(9-3)/5 \times 2^4 \;=\; \frac{6}{5} \times 2^4 \;=\; \frac{6}{5} \times 16 \;=\; 19.2$$

Note that modern calculators have a fraction button which means that the sum can be input as in (1).

Communicating with Technology



The web-based CAS Wolfram Alpha gives an interpretation of your entry (under “input”) in addition to the answer to your calculation.

Exercise

How should this calculation be input in a calculator?

$$\frac{1}{3+7} \times 4 - \frac{3 \times 2}{6}$$

1. $1 \div 3 + 7 \times 4 - 3 \times 2 \div 6$
2. $1 \div (3 + 7) \times 4 - 3 \times 2 \div 6$
3. $1 \div 3 + 7 \times (4 - 3) \times 2 \div 6$
4. $1 \div 3 + 7 \times 4 - (3 \times 2) \div 6$

What are fractions?

A fraction (or “quotient”) can be thought of as the ratio of two numbers, it is equivalent to **division** of the top number by the bottom number.

So the following are *completely identical* objects:

$$5 \div 9 \qquad \frac{5}{9} \qquad \frac{5.0}{9.0}$$

The top number (in this 5) is the **numerator** and the bottom number is the **denominator**.

Any number is equivalent to itself divided by 1, so for example:

$$5 = \frac{5}{1}$$

Working with Fractions: Addition and Subtraction

Addition and Subtraction

Fractions may be added/subtracted if every denominator is the same, e.g.

$$\frac{4}{7} + \frac{2}{7} = \frac{6}{7}$$

and

$$\frac{1}{3} - \frac{2}{3} + \frac{5}{3} = \frac{4}{3}$$

Working with Fractions: Addition and Subtraction

If the denominators are **not** the same, then we must first make them so, by scaling both numerator and denominator of one or both of the fractions by an appropriate amount.

Example 1:

$$\begin{aligned}\frac{3}{5}-\frac{1}{2} &= \frac{3\times 2}{5\times 2}-\frac{1\times 5}{2\times 5}\\ &= \frac{6}{10}-\frac{5}{10}\\ &= \frac{1}{10}\end{aligned}$$

Working with Fractions: Addition and Subtraction

Example 2:

$$\begin{aligned}
\frac{2}{5} + \frac{7}{15} &= \frac{2 \times 3}{5 \times 3} + \frac{7}{15} \\
&= \frac{6}{15} + \frac{7}{15} \\
&= \frac{13}{15}
\end{aligned}$$

Working with Fractions: Mixed Fractions

We may also have to deal with **mixed fractions** (a combination of a whole number and a fraction less than 1). In this case, we can convert it to an "improper" or "top-heavy" fraction:

$$\begin{aligned}
4\frac{5}{6} &= 4 + \frac{5}{6} \\
&= \frac{4}{1} + \frac{5}{6} \\
&= \frac{4 \times 6}{1 \times 6} + \frac{5}{6} \quad = \quad \frac{24}{6} + \frac{5}{6} \\
&= \frac{29}{6}
\end{aligned}$$

Working with Fractions: Multiplication

Multiplication

The following rule can be used to deal with all multiplications:

Multiplying fractions

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

For example:

$$\frac{4}{9} \times \frac{3}{5} = \frac{4 \times 3}{9 \times 5} = \frac{12}{45} = \frac{4}{15}$$

Working with Fractions: Division

Division

The following rule can be used to deal with all divisions:

Fraction Division

$$\frac{a}{b} \div \frac{c}{d} \quad \text{or} \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

For example:

$$\frac{\frac{3}{7}}{\frac{2}{5}} = \frac{3}{7} \times \frac{5}{2} = \frac{3 \times 5}{7 \times 2} = \frac{15}{14}$$