Railway Engineering Mathematics
Tutorial Sheet 15
Solutions

1. Evaluate the following indefinite integrals.

   Note: As these are *indefinite*, we always require one constant of integration "$+c$"

   (a) $$\int 4x - 3 \, \mathrm{d}x$$

   Solution:

   To make clear how the rules of integration apply, let's write the first term explicitly as a power of x:

   $$\begin{aligned} \int 4x - 3 \, \mathrm{d}x &= \int 4x^1 - 3 \, \mathrm{d}x \\ &= \frac{4}{1+1} x^{1+1} - 3x + c \\ &= 2x^2 - 3x + c \end{aligned}$$

   In this example, we used the following standard integrals, for a polynomial and a constant term respectively:

   $$\int ax^n \, \mathrm{d}x = \frac{a}{n+1} x^{n+1} + c$$

   and

   $$\int a \, \mathrm{d}x = ax + c$$

   where a and n are any constant values in each case.

(b) $$\int 5x^2 + 2x + 1 \, \mathrm{d}x$$

Solution:

$$\begin{aligned}
\int 5x^2 + 2x + 1 \, \mathrm{d}x &= \int 5x^2 + 2x^1 + 1 \, \mathrm{d}x \\
&= \frac{5}{2+1}x^{2+1} + \frac{2}{1+1}x^{1+1} + 1x + c \\
&= \frac{5}{3}x^3 + x^2 + x + c
\end{aligned}$$

(c) $$\int 6x^{-2} + 4\cos(9x) - 6\,\mathrm{e}^{5x} \; \mathrm{d}x$$

Solution:

$$\begin{aligned}
\int 6x^{-2} + 4\cos(9x) - 6\,\mathrm{e}^{5x} \; \mathrm{d}x &= \frac{6x^{-2+1}}{-2+1} + \frac{4\sin(9x)}{9} - \frac{6}{5}\,\mathrm{e}^{5x} + c \\
&= -6x^{-1} + \frac{4\sin(9x)}{9} - \frac{6}{5}\,\mathrm{e}^{5x} + c
\end{aligned}$$

Or:

$$\int 6x^{-2} + 4\cos(9x) - 6\,\mathrm{e}^{5x} \; \mathrm{d}x = \frac{-6}{x} + \frac{4}{9}\sin(9x) - \frac{6}{5}\,\mathrm{e}^{5x} + c$$

(d) $$\int \frac{5}{x^3} - 7\,\mathrm{e}^{-2x} + 2x^{-1}\,\mathrm{d}x$$

Solution:

Before integrating, we will need to re-write the first term in index form:

$$\begin{aligned}
\int \frac{5}{x^3} - 7\,\mathrm{e}^{-2x} + 2x^{-1}\,\mathrm{d}x &= \int 5x^{-3} - 7\,\mathrm{e}^{-2x} + 2\frac{1}{x}\,\mathrm{d}x \\
&= \frac{5}{-2}x^{-2} - \frac{7}{-2}\,\mathrm{e}^{-2x} + 2\ln(x) + c \\
&= -\frac{5}{2}x^{-2} + \frac{7}{2}\,\mathrm{e}^{-2x} + 2\ln(x) + c
\end{aligned}$$

(e) $$\int \frac{11x^3}{2} + \frac{3\sin(2x)}{5} - 6\cos\left(\frac{7x}{4}\right)\mathrm{d}x$$

Solution:

To make working with this large function more manageable, let's assign it a variable name:

$$\text{Let}\quad I = \int \frac{11x^3}{2} + \frac{3\sin(2x)}{5} - 6\cos\left(\frac{7x}{4}\right)\mathrm{d}x$$

Then:

$$\begin{aligned}
I &= \int \frac{11}{2}x^3 + \frac{3}{5}\sin(2x) - 6\cos\left(\frac{7}{4}x\right)\mathrm{d}x \\
&= \frac{11}{2}\cdot\frac{x^4}{4} + \frac{3}{5}\left(-\frac{1}{2}\cos(2x)\right) - 6\frac{1}{7/4}\sin\left(\frac{7}{4}x\right) + c \\
&= \frac{11}{8}x^4 - \frac{3}{10}\cos(2x) - \frac{24}{7}\sin\left(\frac{7x}{4}\right) + c
\end{aligned}$$

2. Evaluate the following definite integrals.

Note: As these are *definite*, we do not need to include the constant of integration "$+c$", and instead evaluate the resulting integral at the upper and lower limits, and take the difference. We should always obtain a *value* rather than a function.

(a) $$\int_1^4 7x + 2 \, \mathrm{d}x$$

Solution:

$$\begin{aligned}
\int_1^4 7x + 2 \, \mathrm{d}x &= \left[\frac{7}{2}x^2 + 2x\right]_1^4 \\
&= \left(\frac{7}{2}(4)^2 + 2(4)\right) - \left(\frac{7}{2}(1)^2 + 2(1)\right) \\
&= 64 - \frac{11}{2} \\
&= 58.5
\end{aligned}$$

(b) $$\int_{-2}^3 2x^2 - x + 5 \, \mathrm{d}x$$

Solution:

$$\begin{aligned}
\int_{-2}^3 2x^2 - x + 5 \, \mathrm{d}x &= \left[\frac{2}{3}x^3 - \frac{1}{2}x^2 + 5x\right]_{-2}^3 \\
&= \left(\frac{2}{3}(3)^3 - \frac{1}{2}(3)^2 + 5(3)\right) - \left(\frac{2}{3}(-2)^3 - \frac{1}{2}(-2)^2 + 5(-2)\right) \\
&= \frac{57}{2} - \left(-\frac{52}{3}\right) \\
&= \frac{275}{6} \quad \text{or} \quad 45.83 \quad (2\text{ d.p.})
\end{aligned}$$

(c) $$\int_{-\pi}^{2\pi} 3\cos(1.5t)\ \mathrm{d}t$$

Solution:

Ensure that your calculator is set to radian mode when evaluating the trigonometric term.

$$\begin{aligned}
\int_{-\pi}^{2\pi} 3\cos(1.5t)\ \mathrm{d}t &= \left[\frac{3}{1.5}\sin(1.5t)\right]_{-\pi}^{2\pi} \\
&= \big[2\sin(1.5t)\big]_{-\pi}^{2\pi} \\
&= \big(2\sin(1.5\times 2\pi)\big) - \big(2\sin(1.5\times(-\pi))\big) \\
&= \big(2\sin(3\pi)\big) - \left(2\sin\left(-\frac{3}{2}\pi\right)\right) \\
&= (0) - (2) \\
&= -2
\end{aligned}$$

(d) $$\int_{2}^{7} \frac{5}{2x}\ \mathrm{d}x$$

Solution:

$$\begin{aligned}
\int_{2}^{7} \frac{5}{2x}\ \mathrm{d}x &= \int_{2}^{7} \frac{5}{2}\cdot\frac{1}{x}\ \mathrm{d}x \\
&= \left[\frac{5}{2}\ln(x)\right]_{2}^{7} \\
&= \frac{5}{2}\ln(7) - \frac{5}{2}\ln(2) \\
&= 3.13 \quad \text{(2 d.p.)}
\end{aligned}$$

(e) $$\int_{-0.5}^{2.3} -2\,\mathrm{e}^{3\theta}\ \mathrm{d}\theta$$

Solution:

$$\begin{aligned}
\int_{-0.5}^{2.3} -2\,\mathrm{e}^{3\theta}\ \mathrm{d}\theta &= \left[-2\frac{1}{3}\,\mathrm{e}^{3\theta}\right]_{-0.5}^{2.3} \\
&= \left[-\frac{2}{3}\,\mathrm{e}^{3\theta}\right]_{-0.5}^{2.3} \\
&= \left(-\frac{2}{3}\,\mathrm{e}^{3\times 2.3}\right) - \left(-\frac{2}{3}\,\mathrm{e}^{3\times(-0.5)}\right) \\
&= -661.37 \quad (2\text{ d.p.})
\end{aligned}$$