Railway Engineering Mathematics
Tutorial Sheet 18

1. Find the area between the curve

   $$y = 4x - x^2$$

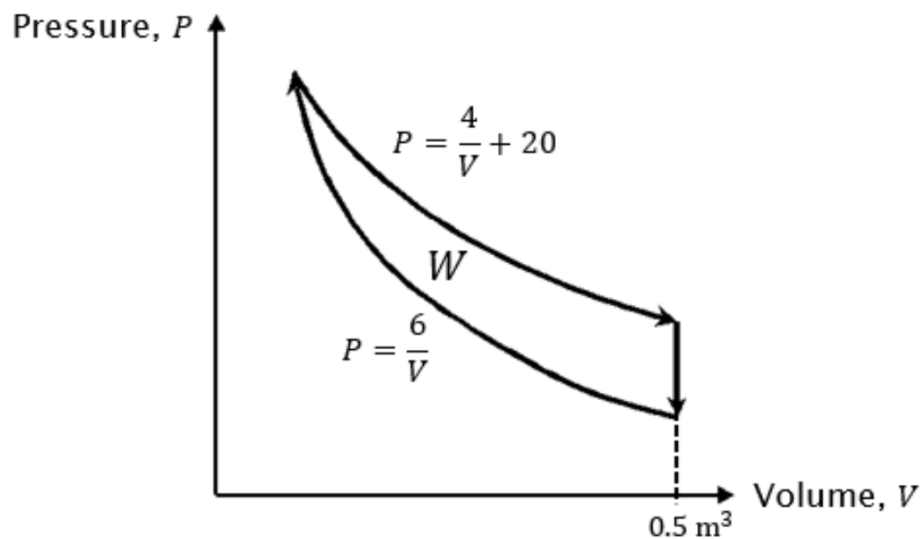   and the x-axis.

2. The velocity v of a vehicle (in m/s) t seconds after a certain instant is given by:

   $$v = 3t^2 + 4$$

   Determine how far it moves in the interval from $t = 1$ s to $t = 5$ s.

3. The work done W by a thermodynamic process in one cycle is equal to the area enclosed by the curves shown below.

   Calculate the value of W.

4. Find the area enclosed by the x-axis, the curve

$$y^2 = 4x - 2$$

and the line $x = 5$.

5. The acceleration of a particle is given by $a = 4t^3$ and has an initial velocity of 3 m/s. Find the velocity after 1.5 s.

6. Find the area enclosed between the curve $y = \sin(x)$ and the x-axis between $x = 0$ and $x = 2\pi$.

7. Consider a thin rod orientated on the x-axis over the interval $\left[\frac{\pi}{2}, \pi\right]$. The density of the rod is given by $\rho(x) = \sin(x)$.

   Determine the mass of the rod given that:

   $$m = \int_a^b \rho(x)\, dx$$

   where m is the mass, a the lower bound and b the upper bound of the interval.

8. Find the area enclosed between the curve

$$y = x^2 + 8x + 15$$

and the x-axis.

9. Evaluate the following expression for x:

$$x = \int 3t^2 - 7t + 6 \, dt$$

given that when $t = 0$, $x = 2$.

10. Evaluate this expression for T:

$$T = \int 5\,e^{-2x} - 3x \, dx$$

given that $T(x = 0) = \dfrac{5}{3}$.

11. Alex attempts to steal a diamond from a bank vault.

   (a) She needs to get out the door of the bank vault, located 4m from the jewel's pedestal, within 3 seconds of snatching the diamond or she will be trapped in the vault by the automatically-closing doors. Following simulations in a replica vault, it is determined that whilst dodging the vault's laser security system, her velocity (in the direction of the vault door) is given by:

$$v(t) = (6t^2 - 8t)\sin(t^3 - 2t^2)$$

   Will she make it out? (Remember to use radians, not degrees.)

   (b) At the sentencing, the judge decides that the sentences (in years) should be given as the area between the curve $y = -x^2 + x + 12$ and the x-axis between $x = -5$ and $x = 6$.

   How long is she going to prison for?

   Your solution should include a sketch of the curve.

12. Starting from a stationary position, a car undergoes a period of constant acceleration at 2 ms^{-2} for 5 seconds. It then maintains speed for a further 12 seconds. What is the final displacement of the car from its initial location?

13. Evaluate the following integral:

$$y = \int_0^{10} x(t)\, dt$$

where $x(t) = 6t^2$ if $0 < t < 3$, and $x(t) = \dfrac{108}{t}$ if $3 < t < 10$.