Railway Engineering Mathematics
Tutorial Sheet 3
Solutions

General practice of solving equations

1. Solve the following equations:

   (a) $6x - 4 = 8$

   (b) $6x - 4 = 9x + 8$

   (c) $8q + 6 = 4q - 14$

   (d) $14 + 13y = 20y - 21$

   (e) $-15b + 21 + 5b = -19$

   (f) $-10x + 90 = -21x + 2$

   Solutions:

   (a) Move all constants to the right-hand side (RHS):

$$\begin{aligned} 6x - 4 &= 8 \qquad &\text{[add 4 to both sides]} \\ 6x &= 12 \qquad &\text{[divide both sides by 2]} \\ \therefore \quad x &= \frac{12}{6} = 2 \end{aligned}$$

(b) Gather the 'x' terms and the constant terms together on different sides of the equation:

$$
\begin{aligned}
6x - 4 &= 9x + 8 && \text{[subtract } 9x \text{ from both sides]} \\
-3x - 4 &= 8 && \text{[add 4 to both sides]} \\
-3x &= 12 && \text{[divide both sides by -3]} \\
\therefore \quad x &= -4
\end{aligned}
$$

(c) Same process as above, but this demonstrates that it doesn't matter what side we take the x to (try it the other way to confirm that you get the same answer:

$$
\begin{aligned}
8q + 6 &= 4q - 14 && \text{[subtract } 8q \text{ from both sides]} \\
6 &= -4q - 14 && \text{[add 14 to both sides]} \\
20 &= -4q && \text{[divide both sides by -4]} \\
\therefore \quad q &= -5 && \text{[note that this is the same as } -5 = q\text{]}
\end{aligned}
$$

(d) Gather 'like' terms:

$$
\begin{aligned}
14 + 13y &= 20y - 21 && \text{[subtract } 20y \text{ from both sides]} \\
14 - 7y &= -21 && \text{[subtract 14 from both sides]} \\
-7y &= -35 && \text{[divide both sides by -4]} \\
\therefore \quad y &= 5
\end{aligned}
$$

(e) Gather 'like' terms:

$$\begin{aligned} -15b + 21 + 5b &= -19 \quad \text{[subtract 21 from both sides]} \\ -10b &= -40 \quad \text{[divide both sides by -10]} \\ \therefore \qquad b &= 4 \end{aligned}$$

(f) Gather 'like' terms:

$$\begin{aligned} -10x + 90 &= -21x + 2 \quad \text{[add } 21x \text{ to both sides]} \\ 11x + 90 &= 2 \quad \text{[subtract 90 from both sides]} \\ 11x &= -88 \quad \text{[divide both sides by 11]} \\ \therefore \qquad x &= -8 \end{aligned}$$

Solving equations with brackets

2. Solve the following equations:

   (a) $3(x + 10) = 63$

   (b) $2(x + 4) = x + 10$

   (c) $-3(7p + 5) = 27$

   (d) $7 - (5t - 13) = -25$

   (e) $3(x - 4) = 2(-2x + 1)$

   (f) $2(h + 4) = 3(h + 10) - 2$

(g) $3\big(x+2(x-2)\big)-2\big(x-3(x-1)\big)=0$

(h) $4(x^2+2)=44$

Solutions:

(a) Multiply out the brackets first, as the variable we wish to solve for is inside the bracket:

$$\begin{aligned} 3(x+10) &= 63 \\ 3x+30 &= 63 \quad \text{[subtract 30 from both sides]} \\ 3x &= 33 \quad \text{[divide both sides by 3]} \\ \therefore \qquad x &= 11 \end{aligned}$$

(b)

$$\begin{aligned} 2(x+4) &= x+10 \quad \text{[multiply out the brackets]} \\ 2x+8 &= x+10 \quad \text{[subtract } x \text{ from both sides]} \\ x+8 &= 10 \quad \text{[subtract 8 from both sides]} \\ \therefore \qquad x &= 2 \end{aligned}$$

(c)

$$\begin{aligned} -3(7p+5) &= 27 \quad \text{[multiply out the brackets]} \\ -21p-15 &= 27 \quad \text{[add 15 to both sides]} \\ -21p &= 42 \quad \text{[divide both sides by -21]} \\ \therefore \qquad p &= -2 \end{aligned}$$

(d)

$$
\begin{aligned}
7-(5t-13) &= -25 && \text{[multiply out the brackets]}\\
7-5t+13 &= -25 && \text{[simplify]}\\
-5t+20 &= -25 && \text{[subtract 20 from both sides]}\\
-5t &= -45 && \text{[divide both sides by -5]}\\
\therefore \quad t &= 9
\end{aligned}
$$

(e)

$$
\begin{aligned}
3(x-4) &= 2(-2x+1) && \text{[multiply out the brackets]}\\
3x-12 &= -4x+2 && \text{[add } 4x \text{ to both sides]}\\
7x-12 &= 2 && \text{[add 12 to both sides]}\\
7x &= 14 && \text{[divide both sides by 7]}\\
\therefore \quad x &= 2
\end{aligned}
$$

(f)

$$
\begin{aligned}
2(h+4) &= 3(h+10)-2 && \text{[multiply out the brackets]}\\
2h+8 &= 3h+30-2 && \text{[simplify]}\\
2h+8 &= 3h+28 && \text{[subtract } 3h \text{ from both sides]}\\
-h+8 &= 28 && \text{[subtract 8 from both sides]}\\
-h &= 20 && \text{[divide both sides by -1]}\\
\therefore \quad h &= -20
\end{aligned}
$$

(g)

$$\begin{aligned}
3\big(x+2(x-2)\big)-2\big(x-3(x-1)\big) &= 0 && \text{[multiply out the inner brackets first]}\\
3\big(x+2x-4\big)-2\big(x-3x+3\big) &= 0 && \text{[multiply out the outer brackets next]}\\
3x+6x-12-2x+6x-6 &= 0 && \text{[simplify]}\\
13x-18 &= 0 && \text{[add 18 to both sides]}\\
13x &= 18 && \text{[divide both sides by 13]}\\
\therefore \qquad x &= \frac{18}{13}
\end{aligned}$$

(h)

$$\begin{aligned}
4(x^2+2) &= 44 && \text{[divide both sides by 4]}\\
x^2+2 &= 11 && \text{[subtract 2 from both sides]}\\
x^2 &= 9 && \text{[square root both sides]}\\
\therefore \qquad x &= \pm 3
\end{aligned}$$

Solving equations with fractions

3. Solve the following equations:

(a) $\dfrac{-8-3k}{2} = 11$

(b) $9 = \dfrac{p+4}{p+12}$

(c) $\dfrac{5b+10}{5} = -b+10$

(d) $\dfrac{3y+2}{2} = 6y + 4$

(e) $\dfrac{3\delta+9}{6} = \dfrac{2\delta+10}{3}$

(f) $\dfrac{7x}{4} - 3 = 2 + \dfrac{9x}{2}$

(g) $\dfrac{3c+8}{3} = \dfrac{1}{2} + \dfrac{c}{4}$

(h) $3\left(a - \dfrac{2}{3}\right) = \dfrac{3a}{4} + \dfrac{9}{4}$

Solutions:

(a)

$$\begin{aligned} \frac{-8-3k}{2} &= 11 \quad \text{[muliply both sides by 2]} \\ -8-3k &= 22 \quad \text{[add 8 to both sides]} \\ -3k &= 30 \quad \text{[divide both sides by -3]} \\ \therefore \qquad k &= -10 \end{aligned}$$

(b)

$$\begin{aligned}
9 &= \frac{p+4}{p+12} && \text{[multiply both sides by } p+12\text{]} \\
9(p+12) &= p+4 && \text{[multiply out the brackets]} \\
9p+108 &= p+4 && \text{[subtract } p \text{ from both sides]} \\
8p+108 &= 4 && \text{[subtract 108 from both sides]} \\
8p &= -104 && \text{[divide both sides by 8]} \\
\therefore \quad p &= -13
\end{aligned}$$

(c)

$$\begin{aligned}
\frac{5b+10}{5} &= -b+10 && \text{[multiply both sides by 5]} \\
5b+10 &= 5(-b+10) && \text{[multiply out the brackets]} \\
5b+10 &= -5b+50 && \text{[add } 5b \text{ to both sides]} \\
10b+10 &= 50 && \text{[subtract 10 from both sides]} \\
10b &= 40 && \text{[divide both sides by 10]} \\
\therefore \quad b &= 4
\end{aligned}$$

(d)

$$\begin{aligned}
\frac{3y+2}{2} &= 6y+4 && \text{[multiply both sides by 2]} \\
3y+2 &= 2(6y+4) && \text{[multiply out the brackets]} \\
3y+2 &= 12y+8 && \text{[subtract } 12y \text{ from both sides]} \\
-9y+2 &= 8 && \text{[subtract 12 from both sides]} \\
-9y &= 6 && \text{[divide both sides by -9]} \\
\therefore \qquad y &= \frac{6}{-9} = -\frac{2}{3}
\end{aligned}$$

(e)

$$\begin{aligned}
\frac{3\delta+9}{6} &= \frac{2\delta+10}{3} && \text{[multiply both sides by 6]} \\
3\delta+9 &= \frac{6(2\delta+10)}{3} && \text{[there's still a fraction on the RHS, multiply both sides by 3]} \\
3(3\delta+9) &= 6(2\delta+10) && \text{[multiply out the brackets]} \\
9\delta+27 &= 12\delta+60 && \text{[subtract } 12\delta \text{ from both sides]} \\
-3\delta+27 &= 60 && \text{[subtract 27 from both sides]} \\
-3\delta &= 33 && \text{[divide both sides by -3]} \\
\therefore \qquad \delta &= -11
\end{aligned}$$

(f) Here the fraction doesn't take up the whole of either the LHS or RHS, so when we multiply both sides by a number, it means multiplying every single term on each side:

$$
\begin{aligned}
\frac{7x}{4} - 3 &= 2 + \frac{9x}{2} && \text{[multiply both sides by 4]} \\
4\left(\frac{7x}{4} - 3\right) &= 4\left(2 + \frac{9x}{2}\right) && \text{[multiply out the brackets]} \\
7x - 12 &= 8 + 18x && \text{[Here } 4 \times 9x \text{ is } 36x\text{, but then we divide by 2]}
\end{aligned}
$$

If the denominator still remained, we would still have a fraction on the RHS. If that is the case, then multiplying by the denominator would be necessary, as in part (e). Although in that case, the 6 was divisible by 3, so we could have simplified the fraction.

$$
\begin{aligned}
7x - 12 &= 8 + 18x && \text{[subtract } 18x \text{ from both sides]} \\
-11x - 12 &= 8 && \text{[add 12 to both sides]} \\
-11x &= 20 && \text{[divide both sides by -11]} \\
\therefore \qquad x &= -\frac{20}{11}
\end{aligned}
$$

(g)

$$\begin{aligned}
\frac{3c+8}{3} &= \frac{1}{2}+\frac{c}{4} && \text{[multiply both sides by 3]}\\
3\left(\frac{3c+8}{3}\right) &= 3\left(\frac{1}{2}+\frac{c}{4}\right) && \text{[multiply out the brackets]}\\
3c+8 &= \frac{3}{2}+\frac{3c}{4} && \text{[multiply both sides by 4]}\\
4\,(3c+8) &= 4\left(\frac{3}{2}+\frac{3c}{4}\right) && \text{[multiply out the brackets]}\\
12c+32 &= 6+3c && \text{[subtract } 3c \text{ from both sides]}\\
9c+32 &= 6 && \text{[subtract 32 from both sides]}\\
9c &= -26 && \text{[divide both sides by 9]}\\
\therefore \qquad c &= -\frac{26}{9}
\end{aligned}$$

(h) There are a number of ways to solve this equation and you may have used different steps.

$$\begin{aligned}
3\left(a-\frac{2}{3}\right) &= \frac{3a}{4}+\frac{9}{4} && \text{[write the RHS over the same denominator]}\\
3\left(a-\frac{2}{3}\right) &= \frac{3a+9}{4} && \text{[multiply both sides by 4]}\\
12\left(a-\frac{2}{3}\right) &= 3a+9 && \text{[multiply out the brackets]}\\
12a-8 &= 3a+9 && \text{[subtract } 3a \text{ from both sides]}\\
9a-8 &= 9 && \text{[add 8 to both sides]}\\
9a &= 17 && \text{[divide both sides by 9]}\\
\therefore \qquad a &= \frac{17}{9}
\end{aligned}$$

General practice of transposition

4. Manipulate the equation $PV = RT$ to obtain a formula for:

(a) V

(b) R

(c) T

(d) P

Solutions:

(a)

$$\begin{aligned} PV &= RT \quad \text{[divide both sides by } P\text{]} \\ V &= \frac{RT}{P} \end{aligned}$$

(b)

$$\begin{aligned} PV &= RT \quad \text{[divide both sides by } T\text{]} \\ R &= \frac{PV}{T} \end{aligned}$$

(c)

$$\begin{aligned} PV &= RT \quad \text{[divide both sides by } R\text{]} \\ T &= \frac{PV}{R} \end{aligned}$$

(d)

$$PV = RT \quad \text{[divide both sides by } V\text{]}$$

$$P = \frac{RT}{V}$$

5. Transpose the following formulae for the variable stated in the brackets:

   (a) $v^2 = u^2 + 2as$ (u)

   (b) $s = ut + \frac{1}{2}at^2$ (u)

   (c) $m = k\sqrt{a(1-x)}$ (x)

   (d) $V = \pi r^2 l + \frac{1}{3}\pi r^2 h$ (l)

   (e) $P = \mu_1 c_1 + \mu_2 c_2$ (c_1)

   (f) $\rho = \frac{M}{V}$ (M)

   (g) $T = \frac{V}{A} + d$ (V)

   (h) $F = \frac{x}{k} + E$ (x)

   (i) $V = \frac{jI}{\omega C} + V_1$ (I)

Solutions:

(a)

$$v^2 = u^2 + 2as \quad \text{[subtract } 2as \text{ from both sides]}$$

$$u^2 = v^2 - 2as \quad \text{[square root both sides]}$$

$$\therefore\ u = \sqrt{v^2 - 2as}$$

(b)

$$\begin{aligned}
s &= ut + \frac{1}{2}at^2 && \text{[subtract } \tfrac{1}{2}at^2 \text{ from both sides]} \\
ut &= s - \frac{1}{2}at^2 && \text{[divide both sides by } t\text{]} \\
u &= \frac{s - \frac{1}{2}at^2}{t} && \text{[split the numerator]} \\
u &= \frac{s}{t} - \frac{\frac{1}{2}at^2}{t} && \text{[simplify]} \\
\therefore u &= \frac{s}{t} - \frac{1}{2}at && \text{[this could also be written as]} \\
u &= \frac{s}{t} - \frac{at}{2}
\end{aligned}$$

(c) As with the majority of equations, there is more than one way to transpose this equation.

$$
\begin{aligned}
m &= k\sqrt{a(1-x)} && \text{[divide both sides by } k\text{]}\\
\frac{m}{k} &= \sqrt{a(1-x)} && \text{[square both sides]}\\
\left(\frac{m}{k}\right)^2 &= a(1-x) && \text{[we can square each element]}\\
\frac{m^2}{k^2} &= a(1-x) && \text{[divide both sides by } a\text{]}\\
\frac{\frac{m^2}{k^2}}{a} &= 1-x && \text{[which can be written as]}\\
\frac{m^2}{ak^2} &= 1-x && \text{[add } x \text{ to both sides]}\\
\frac{m^2}{ak^2}+x &= 1 && \text{[subtract } \frac{m^2}{ak^2} \text{ from both sides]}\\
\therefore \qquad x &= 1-\frac{m^2}{ak^2}
\end{aligned}
$$

(d)

$$
\begin{aligned}
V &= \pi r^2 l + \frac{1}{3}\pi r^2 h && \text{[subtract } \frac{1}{3}\pi r^2 h \text{ from both sides]}\\
\pi r^2 l &= V - \frac{1}{3}\pi r^2 h && \text{[divide both sides by } \pi r^2\text{]}\\
l &= \frac{V-\frac{1}{3}\pi r^2 h}{\pi r^2} && \text{[split the numerator]}\\
l &= \frac{V}{\pi r^2} - \frac{\frac{1}{3}\pi r^2 h}{\pi r^2} && \text{[simplify]}\\
\therefore l &= \frac{V}{\pi r^2} - \frac{h}{3}
\end{aligned}
$$

(e)

$$P = \mu_1 c_1 + \mu_2 c_2 \quad \text{[subtract } \mu_2 c_2 \text{ from both sides]}$$

$$P - \mu_2 c_2 = \mu_1 c_1 \quad \text{[divide both sides by } \mu_1\text{]}$$

$$\therefore \quad c_1 = \frac{P - \mu_2 c_2}{\mu_1}$$

(f)

$$\rho = \frac{M}{V} \quad \text{[multiply both sides by } V\text{]}$$

$$\therefore M = \rho V$$

(g)

$$T = \frac{V}{A} + d \quad \text{[subtract } d \text{ from both sides]}$$

$$\frac{V}{A} = T - d \quad \text{[multiply both sides by } A\text{]}$$

$$\therefore V = A(T - d)$$

Note that this could have been transposed differently. Some people wish to get rid of the fraction first: This is fine to do, but remember that *both sides* should be multiplied by A. For example:

$$T = \frac{V}{A} + d \quad \text{[multiply both sides by } A\text{]}$$

$$AT = A\left(\frac{V}{A} + d\right) \quad \text{[multiply out the brackets]}$$

$$AT = V + Ad \quad \text{[subtract } Ad \text{ from both sides]}$$

$$V = AT - Ad \quad \text{[factorise the right-hand side]}$$

$$\therefore \quad V = A(T - d)$$

(h)

$$\begin{aligned} F &= \frac{x}{k} + E && \text{[subtract } E \text{ from both sides]} \\ F - E &= \frac{x}{k} && \text{[multiply both sides by } k\text{]} \\ \therefore \quad x &= k(F - E) \end{aligned}$$

(i)

$$\begin{aligned} V &= \frac{jI}{\omega C} + V_1 && \text{[subtract } V_1 \text{ from both sides]} \\ \frac{jI}{\omega C} &= V - V_1 && \text{[multiply both sides by } \omega C\text{]} \\ jI &= \omega C\,(V - V_1) && \text{[divide both sides by } j\text{]} \\ \therefore \quad I &= \frac{\omega C\,(V - V_1)}{j} \end{aligned}$$