Railway Engineering Mathematics
Tutorial Sheet 14

1. An alternating current, i amperes, is given by:

   $$i = 10\sin(2\pi ft)$$

   where f is the frequency in hertz and t the time in seconds.

   Determine the rate of change of the current when $t = 20$ ms, given that $f = 150$ Hz.

2. The distance x metres moved by a car in a time t seconds is given by:

   $$x = 3t^3 - 2t^2 + 4t - 1$$

   Determine the velocity and acceleration when:

   (a) $t = 0$
   (b) $t = 1.5$ s

3. Find the number of revolutions per minute n which gives the minimum frictional couple F on a bearing where:

   $$F = \frac{180000 - 1200n - 45n^2 + n^3}{80000}$$

   Check your answer by plotting an appropriate graph in EXCEL.

4. A wooden packing case is made in the form of a rectangular block without a lid and standing on a square base. Its volume is 0.2 metres cubed. If x (in metres) is the length of a side of the base, find the value of x for which the case's surface area is a minimum.

5. The formula:

$$x = \frac{20t^3}{3} - \frac{23t^2}{2} + 6t + 5$$

represents the distance x, in metres, moved by a body in t seconds.

Determine:

(a) The velocity and acceleration at the initial time.

(b) The velocity and acceleration when $t = 3$ s

(c) The values of t when the body is at rest.

6. A manufacturer develops a formula to determine the demand for its product depending on the price in dollars. The formula is:

$$D = 2000 + 120P - 6P^2$$

where P is the price per unit, and D is the number of units in demand.

What is the maximum demand possible?

7. The downward deflection y of a cantilever of length L with a load W is given by the equation:

$$y = \frac{W}{6EI}(3L^2x - x^3)$$

where the constants E and I are related to the physical properties of the cantilever.

Find the maximum deflection as x varies.

8. While on annual leave, Gavin holds up an armoured car, and attempts to flee the scene using a jetpack of his own creation. In the initial movement, the vertical height s (metres) of the jetpack at time t (seconds) is given by the function:

$$s(t) = 2t^3 + t^2$$

It is estimated that Gavin needs to achieve an acceleration of at least 40m/s^2 after 3 seconds, in order to escape the large nets employed by the security guards.

Does he succeed?

9. A mechanical component vibrates such that its vertical displacement y (m) at time t (s) obeys:

$$y(t) = t^3 - 9t^2 + 24t + 5$$

valid for the range $0 < t < 10$.

(i) Determine the value(s) of displacement that occur at stationary points(s) within this range.

(ii) Are the values identified in part (i) local maxima or minima?

(iii) Are the values identified in part (i) the most extreme that occur in the range? Plot the function $y(t)$ in EXCEL to find out.

10. The displacement s cm of the end of a stiff spring at time t seconds is given by:

$$s = \mu\, \mathrm{e}^{-kt} \sin(2\pi f t)$$

Determine the velocity of the end of the spring after one second if $\mu = 2$, $k = 0.9$ and $f = 5$.

11. A missile fired from ground level rises x metres vertically upwards in t seconds and

$$x = 100t - \frac{25}{2}t^2$$

Find:

(a) The initial velocity of the missile;

(b) The time when the height of the missile is a maximum;

(c) The maximum height reached;

(d) The velocity with which the missile strikes the ground.

12. At any time t seconds, the displacement x metres of a particle moving in a straight line from a fixed point is given by:

$$x = 4t + \ln(1 - t)$$

Determine the acceleration of the particle after 1.5 seconds.