Railway Engineering Mathematics
Tutorial Sheet 4

Solving equations with fractions

1. Solve the following for x:

   (a) $\dfrac{x+11}{2x-5} = 2$

   (b) $\dfrac{5x-24}{x-4} = 3$

   (c) $4y - 9 = \dfrac{2y+3}{3}$

   (d) $\dfrac{x+4}{3} + \dfrac{x+1}{2} = 1$

   (e) $\dfrac{2x-5}{7} - \dfrac{2x-1}{2} = 3$

   (f) $\dfrac{x+1}{2} + \dfrac{2x-1}{4} + \dfrac{x+2}{3} = 1$

   (g) $\dfrac{z-7}{3} + \dfrac{2z-1}{4} = \dfrac{z+3}{6}$

   (h) $\dfrac{x}{2} - \dfrac{2+3x}{5} = 1 + \dfrac{1}{x}$

General practice of transposition

2. Transpose the following formulae for the variable stated in the brackets:

   (a) $P = \dfrac{mRT}{V} + \dfrac{mRT_0}{V}$ $\qquad (m)$ and (V)

   (b) $z = 13x - 6 + \alpha x$ $\qquad (x)$

   (c) $\dfrac{2h}{3h - p} = 5p$ $\qquad (h)$

   (d) $5(3m - 2) = 8mk - 9$ $\qquad (m)$

   (e) $x = \dfrac{12}{y}$ $\qquad (y)$

   (f) $a = \dfrac{4}{b} + 2c$ $\qquad (b)$

   (g) $y = \dfrac{7}{2x + 3}$ $\qquad (x)$

Powers and roots

3. Transpose the following formulae for the variable stated in the brackets:

   (a) $V = \dfrac{4}{3}\pi r^3$ $\qquad (r)$

   (b) $y = 5 + \sqrt{x - 2}$ $\qquad (x)$

   (c) $9x + \dfrac{3}{P} = \dfrac{1}{2r^2}$ $\qquad (r)$

   (d) $M = 7t - \sqrt{\dfrac{2}{1 - r}}$ $\qquad (r)$

4. Make e the subject of:

$$T = \frac{2v}{g}\left(\frac{1}{1-e}\right)$$

5. Given that:

$$U = V\left(1 - \frac{C}{D\sqrt{N}}\right),$$

find N in terms of U, V, D and C.

6. Make b the subject of:

$$a(3b-1) = 2b+2$$

7. Make r the subject of:

$$P = \frac{P_0}{1-r^2}$$

8. Make x the subject of:

$$y = a + \frac{1}{1-x}$$

9. Make y the subject of:

$$\frac{y}{y+x} + 5 = x$$

10. The equations for a battery with e.m.f. E and an internal resistance r, connected across a resistor R can be expressed as:

$$E = \frac{V(R+r)}{R}$$

where V is the voltage. Find an expression for R.

11. The impedance, Z, of a circuit containing a resistor of resistance R, a capacitor of capacitance C and an inductor of inductance L is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2},$$

where $X_L = 2\pi f L$ and $X_C = \dfrac{1}{2\pi f C}$.

Determine and expression for C in terms of f, L, R and Z.

12. A system with feedback β and gain A has an output voltage v_{in} given by

$$v_{in} = \left(\frac{1}{A} - \beta\right) v_{out}$$

where v_{out} is the output voltage. Determine the ratio of the output voltage to the input voltage.

13. As shown in Figure 1, one end of a light inextensible string of length l is attached to a fixed point A and a particle P is attached to the other end. The ends of a second string of the same length are attached to P and to a fixed point B at a distance h $(< 2l)$ vertically below A. The particle moves in a horizontal circle with uniform angular speed ω.

The tension in the second string can be expressed as:

$$T_2 = \frac{ml}{h}\left(h\omega^2 - 2g\right).$$

Given that $T_1 > T_2$, both strings will be taut if $T_2 \geq 0$. Determine the least value of

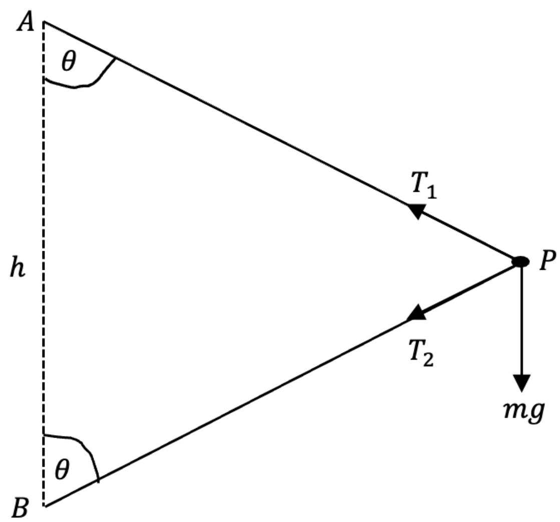

Figure 1

ω for which both strings are taut.

14. In aerodynamics, minimum drag on an aircraft occurs when the lift coefficient L satisfies:

$$kL^2 = Z$$

where Z is the zero lift coefficient and k is a constant. The velocity v of an aircraft satisfies:

$$w = \frac{1}{2}\rho v^2 LA$$

where w is weight, ρ is the density, and A is area. Show that:

$$v^4 = \left(\frac{2w}{\rho A}\right)^2 \cdot \frac{k}{Z}$$

15. In thermodynamics, the exit velocity u of a fluid from a nozzle is given by:

$$u = \left\{\frac{2\gamma P_1 V_1}{\gamma - 1}\left[1 - \frac{P_2 V_2}{P_1 V_1}\right]\right\}^{\frac{1}{2}}$$

where P_1, V_1 represent the entrance pressure and the specific volume respectively, and P_2, V_2 represent the exit pressure and specific volume respectively. γ is the ratio of specific heat capacities. Given that:

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

show that:

$$u^2 = \frac{2\gamma P_1 V_1}{\gamma - 1}\left[1 - \left(\frac{P_2}{P_1}\right)^{1-1/\gamma}\right]$$

then calculate u (correct to 1 d.p.), given the following:

- $\gamma = 1.39$
- $P_1 = 5.2 \times 10^6 \text{ N/m}^2$
- $V_1 = 3.1 \times 10^{-3} \text{ m}^3/\text{kg}$
- $V_2 = 5 \times 10^{-3} \text{ m}^3/\text{kg}$