# Sheffield Hallam University

Mathematics

Formula Sheet

# Differentiation

y	$\frac{\mathrm{d}y}{\mathrm{d}x}$	
a (any constant)	0	
ax	a	
$ax^n$	$n \times ax^{n-1}$	
$ae^{nx}$	$n \times ae^{nx}$	
$a \ln(nx)$	$\frac{a}{x}$	
$a\sin(nx)$	$n \times a\cos(nx)$	
$a\cos(nx)$	$-n \times a\sin(nx)$	
$a \sinh(nx)$	$n \times a \cosh(nx)$	
$a \cosh(nx)$	$n \times a \sinh(nx)$	

# **Product Rule**

If 
$$y = uv$$
 then  $\frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$ 

## Quotient Rule

If 
$$y = \frac{u}{v}$$
 then  $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ 

## Chain Rule

If 
$$y = f(u)$$
 and  $u = g(x)$  then  $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$ 

# Integration by Parts

$$\int u \frac{\mathrm{d}v}{\mathrm{d}x} \, \mathrm{d}x = uv - \int v \frac{\mathrm{d}u}{\mathrm{d}x} \, \mathrm{d}x$$

# Integration

y	$\int y  dx$	
a (any constant)	ax + C	
$ax^n$	$\frac{ax^{n+1}}{n+1} + C  (n \neq -1)$	
$ae^{nx}$	$\frac{ae^{nx}}{n} + C$	
$\frac{a}{x}$	$a \ln x + C$	
$\frac{a}{nx+b}$	$\frac{a\ln(nx+b)}{n} + C$	
$a\sin(nx)$	$\frac{-a\cos(nx)}{n} + C$	
$a\cos(nx)$	$\frac{a\sin(nx)}{n} + C$	
$a \sinh(nx)$	$\frac{a\cosh(nx)}{n} + C$	
$a \cosh(nx)$	$\frac{a\sinh(nx)}{n} + C$	

#### The Quadratic Equation

For the quadratic equation  $ax^2 + bx + c = 0$ :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Completing the Square

For the quadratic equation  $ax^2 + bx + c = 0$ , where a = 1:

$$\left(x+\frac{b}{2}\right)^2+c-\left(\frac{b}{2}\right)^2$$

#### Modulus and Argument

Given a complex number in the form z = x + jb, the modulus, r, and argument,  $\theta$  can be determined by:

$$r = \sqrt{x^2 + y^2}$$
  $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ 

#### Volume of Revolution

$$V = \int_{a}^{b} \pi y^2 \, \mathrm{d}x$$

# Vector Analysis

Given the matrix:

$$\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

then the determinant:

$$\det \underline{A} = |\underline{A}| = ad - bc$$

and the inverse:

$$\operatorname{inv}\underline{A} = \underline{A}^{-1} = \frac{1}{|\underline{A}|} \times \operatorname{adj}\underline{A} = \frac{1}{|\underline{A}|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

## **Rules of Indices**

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$$

$$\frac{A}{a^n} = Aa^{-n}$$

$$a^0 = 1$$

$$a^1 = a$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$(a^m)^n = a^{m \times n}$$

## **Rules for Fractions**

# **Multiplying Fractions**

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

# **Dividing Fractions**

$$\frac{\frac{a}{\overline{b}}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

## **Triange Geometry**

#### Right angled triangles

Pythagoras' theorem:

$$a^2 = b^2 + c^2$$

where a is the hypotenuse and b, c are the other two sides.

#### General rules for triangles

Sine rule:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Cosine rule:

$$a^{2} = b^{2} + c^{2} - 2bc\cos(A)$$
 or  $\cos(A) = \frac{b^{2} + c^{2} - a^{2}}{2bc}$ 

# Numerical Prefixes

Prefix	Symbol	Meaning
Giga	G	$\times 10^9$
Mega	M	$\times 10^6$
Kilo	k	$\times 10^3$
Milli	m	$\times 10^{-3}$
Micro	$\mu$	$\times 10^{-6}$
Nano	n	$\times 10^{-9}$