**Question 1**

The variables $y$ and $x$ are related as follows:

$$y = x^2 \cos(2x)$$

a) Find $\frac{dy}{dx}$.

**(A)**

Let $u = x^2$ and $v = \cos(2x)$

such that $\frac{du}{dx} = 2x$.

Also, let $w = 2x$ such that $v = \cos(w)$

and $\frac{dw}{dx} = 2$ while $\frac{dv}{dw} = -\sin(w)$

We therefore have that

$$\frac{dv}{dx} = \frac{dv}{dw}\frac{dw}{dx}$$

$$= -\sin(w) \times 2$$

$$= -2\sin(2x)$$

furthermore, the product rules states that

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$= \cos(2x) \times 2x + x^2(-2\sin(2x))$$

$$= 2x\cos(2x) - 2x^2\sin(2x)$$

| $\frac{dy}{dx} =$ | $2x\cos(2x) - 2x^2\sin(2x)$ |
|---|---|

Evidence of checking

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Comment on evidence

*Replace this text and use this space to explain why you believe your check confirms your result. Make this row as deep as is necessary to contain your work.*

b) Evaluate $\int_1^2 (3x^2 - 6x + 7)dx$.

**(A)**

$$\int_1^2 3x^2 - 6x + 7 \, dx = \left[x^3 - 3x^2 + 7x\right]_1^2$$

$$= (2^3 - 3\times 2^2 + 7\times 2) - (1^3 - 3\times 1^2 + 7\times 1)$$

$$= 10 - 5$$

$$= 5$$

| | |
|---|---|
| $\int_1^2 (3x^2 - 6x + 7)dx =$ | **5** |

Evidence of checking

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Comment on evidence

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c) Evaluate $\int_1^2 x^2 \ln(x)\, dx$.

**(B)**

Let $u = \ln(x)$ and $\frac{dv}{dx} = x^2$

such that $\frac{du}{dx} = \frac{1}{x}$ and $v = \frac{x^3}{3}$

We therefore have

$$\int x^2 \ln(x)\, dx = uv - \int v \frac{du}{dx}\, dx$$

$$= \frac{x^3 \ln(x)}{3} - \int \frac{x^3}{3x}\, dx$$

$$= \frac{x^3 \ln(x)}{3} - \frac{1}{3}\int x^2\,dx$$

$$= \frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C$$

hence

$$\int_1^2 x^2 \ln(x)\,dx = \left[\frac{x^3 \ln(x)}{3} - \frac{x^3}{9}\right]_1^2$$

$$= \left(\frac{2^3 \ln(2)}{3} - \frac{2^3}{9}\right) - \left(\frac{1^3 \ln(1)}{3} - \frac{1^3}{9}\right)$$

$$= \frac{8\ln(2)}{3} - \frac{8}{9} + \frac{1}{9}$$

$$= \frac{8\ln(2)}{3} - \frac{7}{9}$$

$$\simeq 1.07$$

| $\int_1^2 x^2 \ln(x)\,dx =$ | **1.07** |
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Evidence of checking

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Comment on evidence

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