Solving and Transposing Equations

Railway Engineering Mathematics

Sheffield Hallam University

Lecture 3

Learning Outcomes

- Solve equations.
- Transpose equations.

Solving and Transposing Equations

Transposition is the process of rearranging an equation into a different form using logically valid steps.

All steps in transposition originate from a single logical principle:

If two initially equal things are changed in an identical manner, then they must still be equal *after* the change.

This means we can make changes to, say, the left-hand side (LHS) of an equation, provided we make precisely the **same change** to the right-hand side (RHS) also.

Transposing Equations

General principles are to:

- Get rid of fractions by multiplying.
- Get rid of brackets by expanding.
- Gather all terms with the unknown to one side by addition/subtraction.
- Remove everything else to the other side by addition/subtraction.
- Use division to leave the unknown by itself.

But this is a skill mainly acquired by practising many examples.

Example 1

Solve $2x - 4 = 10$ for x.

First, we add $+4$ to both sides to remove the -4 term on the LHS:

$$2x - 4 + 4 = 10 + 4$$

$$2x = 14$$ Now $2x$ is alone.

$$\frac{2x}{2} = \frac{14}{2}$$ Remove the factor of 2 by division.

$$x = 7$$

Example 2

Solve $3x + 4 = 31$.

$3x + 4 - 4 = 31 - 4$ Subtract away the $+4$ on the LHS.

$3x = 27$ Now the only x-term is alone.

$$\frac{3x}{3} = \frac{27}{3}$$

$$x = 9$$

Example 3

Solve $5x - 6 = 3x - 8$.

$$5x - 6 - 3x = 3x - 8 - 3x \quad \text{Gather the } x\text{-terms on LHS.}$$

$$2x - 6 = -8$$

$$2x - 6 + 6 = -8 + 6 \quad \text{Remove the other term.}$$

$$2x = -2$$

$$\frac{2x}{2} = \frac{-2}{2} \quad \text{Divide away the factor of 2.}$$

$$x = -1$$

Example 4 - Part I

Solve $2(3x-7)+4(3x+2)=6(5x+9)+3$.

First, always expand all the brackets. Then we will gather the x-terms and the constants together:

$$\begin{aligned}
6x-14+12x+8 &= 30x+54+3 \\
18x-6 &= 30x+57 \\
18x-6{\color{red}-30x} &= 30x+57{\color{red}-30x} \\
-12x-6 &= 57
\end{aligned}$$

Example 4 - Part II

Now proceeding as in previous examples:

$$-12x - 6 = 57$$

$$-12x - 6 + 6 = 57 + 6$$

$$-12x = 63$$

$$\frac{-12x}{-12} = \frac{63}{-12}$$

$$\therefore x = -\frac{63}{12} \quad \text{or} \quad -5.25$$

Example 5

Solve $\mu = u + at$ for a.

$$\mu - u = u + at - u$$

$$\mu - u = at$$

$$\frac{\mu - u}{t} = \frac{at}{t}$$

$$\frac{\mu - u}{t} = a$$

Or rather:

$$a = \frac{\mu - u}{t}$$

Example 6 - Part I/III

Solve $\dfrac{2+t}{3} = 2(t-k)$ for t.

$$3\times\frac{(2+t)}{3} = 3\times 2(t-k)$$

$$2+t = 6(t-k)$$

$$2+t = 6t-6k$$

$$2+t-6t = 6t-6k-6t$$

$$2-5t = -6k$$

Example 6 - Part II/III

Solve $\dfrac{2+t}{3} = 2(t-k)$ for t.

$$2 - 5t = -6k$$

$$2 - 5t - 2 = -6k - 2$$

$$-5t = -6k - 2$$

$$\frac{-5t}{-5} = \frac{-6k-2}{-5}$$

$$t = \frac{-6k-2}{-5}$$

Example 6 - Part III/III

The solution could also be written as:

$$t = \frac{6k + 2}{5}$$

Or:

$$t = \frac{2(3k + 1)}{5}$$

Or:

$$t = \frac{2}{5}(3k + 1)$$

There are often multiple ways to present the solution.

Example 7 - Part I/IV

Solve $\frac{1}{f} = \frac{1}{u} + \frac{1}{\mu}$ for μ.

$$\frac{1}{f} - \frac{1}{u} = \frac{1}{u} + \frac{1}{\mu} - \frac{1}{u}$$

$$\frac{1}{f} - \frac{1}{u} = \frac{1}{\mu}$$

$$\mu\left(\frac{1}{f} - \frac{1}{u}\right) = \mu\frac{1}{\mu}$$

$$\mu\left(\frac{1}{f} - \frac{1}{u}\right) = 1$$

Example 7 - Part II/IV

Continuing...

$$\mu\left(\frac{1}{f}-\frac{1}{u}\right)=1$$

$$\frac{\mu\left(\frac{1}{f}-\frac{1}{u}\right)}{\frac{1}{f}-\frac{1}{u}}=\frac{1}{\frac{1}{f}-\frac{1}{u}}$$

$$\mu=\frac{1}{\frac{1}{f}-\frac{1}{u}}$$

Example 7 - Part III/IV

Let's see an alternative approach to solving $\frac{1}{f} = \frac{1}{u} + \frac{1}{\mu}$ for μ.

$$\mu \times \frac{1}{f} = \mu \times \frac{1}{u} + \mu \times \frac{1}{\mu}$$

$$\frac{\mu}{f} = \frac{\mu}{u} + 1$$

$$\frac{\mu}{f} - \frac{\mu}{u} = \frac{\mu}{u} + 1 - \frac{\mu}{u}$$

$$\frac{\mu}{f} - \frac{\mu}{u} = 1$$

Example 7 - Part IV/IV

Continuing...

$$\frac{\mu}{f} - \frac{\mu}{u} = 1$$

$$\mu\left(\frac{1}{f} - \frac{1}{u}\right) = 1$$

$$\frac{\mu\left(\frac{1}{f} - \frac{1}{u}\right)}{\frac{1}{f} - \frac{1}{u}} = \frac{1}{\frac{1}{f} - \frac{1}{u}}$$

$$\mu = \frac{1}{\frac{1}{f} - \frac{1}{u}} = \frac{1}{\frac{u-f}{fu}} = \frac{fu}{u-f}$$

Example 8

Solve $A = 2\pi r^2 + 2\pi r h$ for h.

$$A - 2\pi r^2 = 2\pi r^2 + 2\pi r h - 2\pi r^2$$

$$A - 2\pi r^2 = 2\pi r h$$

$$\frac{A - 2\pi r^2}{2\pi r} = \frac{2\pi r h}{2\pi r}$$

$$\frac{A - 2\pi r^2}{2\pi r} = h, \quad \text{so} \quad h = \frac{A - 2\pi r^2}{2\pi r}$$

Example 9 - Part I

Solve $T = 2\pi\sqrt{\frac{L}{g}}$ for L.

$$\frac{T}{\color{red}{2\pi}} = \frac{2\pi}{\color{red}{2\pi}}\sqrt{\frac{L}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{L}{g}}$$

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{L}{g}}\right)^2$$

$$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g}$$

Example 9 - Part II

$$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g}$$

$$\left(\frac{T}{2\pi}\right)^2 \times g = \frac{L}{g} \times g$$

$$g\left(\frac{T}{2\pi}\right)^2 = L$$

Hence,

$$L = g\left(\frac{T}{2\pi}\right)^2$$