Railway Engineering Mathematics
Tutorial Sheet 22

Given the following matrices:

$$\underline{A} = \begin{pmatrix} 7 & 2 \\ -3 & 8 \end{pmatrix} \qquad \underline{B} = \begin{pmatrix} 4 & -1 & 7 \\ 5 & -3 & -2 \end{pmatrix}$$

$$\underline{C} = \begin{pmatrix} 6 & 1 & 9 \\ -2 & 0 & -8 \end{pmatrix} \qquad \underline{D} = \begin{pmatrix} -5 & -4 \\ 1 & 6 \end{pmatrix} \qquad \underline{F} = \begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix}$$

1. Determine if the following operations are possible, and if so then evaluate them:

   (a) $|\underline{A}|$

   (b) $|\underline{C}|$

   (c) $\underline{D}^{-1}$

   (d) $\underline{B}^{-1}$

   (e) $\underline{F}^{-1}$

   (f) $\underline{A}^{T}$

   (g) $\underline{B}^{T}$

2. Solve the following systems of linear simultaneous equations for x and y using a matrix method:

   (a)
   $$5x + 3y = -23$$
   $$-4x - 9y = 58$$

   (b)
   $$x_2 = 7.1 + 7x_1$$
   $$2x_1 + 3x_2 = 28.2$$

3. The *UltraEmissions* car manufacturer has two models currently in production: the 1-litre *Despondent* front-engine front-wheel drive 6-seater people carrier[1], and the ever-popular *Merciless* rear-wheel drive V8 roadster. Two shipping containers containing cars from this brand are delivered to the port, where you have been employed by HM Revenue and Customs to assist the port authorities in issuing import duty.According to the manifest, one of the containers holds five of the *Despondent* model and eight of the *Merciless*, while the second container is storing ten *Despondent* and six of the *Merciless* model. The first container weighs 35 tonnes, and the second weighs 52 tonnes. The shipping company informs you that an empty shipping container weighs exactly three tonnes.

   The table below lists the import duty per vehicle, dependent on the weight class of that vehicle. How much is the total duty to be paid by the *UltraEmissions* company for this particular delivery?

Weight (tonnes)	Duty (£/tonne)
0-1	200
1.01-2	220
2.01-3	250
3.01-4	300
4.01-5	340
5.01-6	400

Table 1: Import duty on vehicle weight classes

[1]Only available in beige.