Railway Engineering Mathematics
Tutorial Sheet 7

1. Download the `Picturebook.xlsx` from the module Blackboard site. Navigate to the "logarithmic" tab, featuring the general logarithmic equation:

   $$y = A\ln(x) + B$$

   Determine how the two parameters affect the shape of the graph:

   (a) A

   (b) B

   What happens when you try to calculate $\ln(0)$ or log of a negative number?

2. Use the laws of logarithms to match each expression on the left with an equivalent expression on the right:

$\ln(4) + \ln(7)$	$-\ln(3)$
$\ln(e^5)$	$\ln(49)$
$\ln(8) - \ln(2)$	$\ln(28)$
$\ln\left(\frac{9}{27}\right)$	0
$2\ln(7)$	5
$\ln(1)$	$\ln(4)$

3. Download the `Picturebook.xlsx` from the module Blackboard site. Navigate to the "exponential" tab, featuring the general exponential function:

$$y = A\,\mathrm{e}^{Bx} + C$$

Determine how the two parameters affect the shape of the graph:

(a) A

(b) B

(c) C

What conditions must be satisfied by B in order to obtain exponential growth, or to obtain exponential decay?

4. Solve the following exponential equations for x:

(a) $\mathrm{e}^x = 9.5$

(b) $5\,\mathrm{e}^x = 50.6$

(c) $27.3\,\mathrm{e}^x - 12 = 112.7$

(d) $-14.6\,\mathrm{e}^x + 7.5 = 4.2$

(e) $29 - 6.3\,\mathrm{e}^x = 15.4$

(f) $34.7\,\mathrm{e}^{4.2x} + 4.3 = 25.1$

(g) $117.1\,\mathrm{e}^{-0.4x} - 15.7 = -3.8$

5. Solve the following logarithmic equations for x:

(a) $\ln(x) = 0.6$

(b) $15 - \ln(2x) = 10.3$

(c) $2.8 = \log_{10}(0.3x + 6)$

6. The pressure p pascals at height h metres above ground level is given by:

$$p = p_0 \, \mathrm{e}^{-\frac{h}{C}}$$

where p_0 is the pressure at ground level and C is a constant.

Find the pressure p when $p_0 = 1.012 \times 10^5$ Pa, height is $h = 1420$m and $C = 71500$.

7. The current i flowing in a capacitor at time t is given by:

$$i = 12.5\left(1 - \mathrm{e}^{-\frac{t}{CR}}\right)$$

where $R = 30$ kilohms, and the capacitance C is 20 micro-farads.

Determine:

(a) the current flowing after 0.5 seconds.

(b) the time for the current to reach 10 amperes.

8. The resistance R of an electrical conductor at temperature $\theta^\circ C$ is given by $R = R_0 \, \mathrm{e}^{\alpha\theta}$, where α is a constant and $R_0 = 5 \times 10^3$ ohms.

Determine the value of α, correct to 4 significant figures, when $R = 6 \times 10^3$ ohms and $\theta = 1500^\circ C$.

Also, find the temperature, correct to the nearest degree, when the resistance R is 5.4×10^3 ohms.

9. A research group models the population P (in millions) of a particular country by following formula:

$$P = A\,\mathrm{e}^{kt},$$

where t is the time (in years) since 1980, the value of k is 0.0241, and A is a constant that is yet to be determined.

In the year 2000, the population of the country was recorded as 11 million.

(a) What is the population projected to be in 2020?

(b) When is the population forecast to exceed 25 million?

(c) Plot the projected population in EXCEL between the years 1980 and 2050. From the graph, describe the behaviour of this exponential function.

(d) From the graph, determine when the population is equal to 27 million. Indicate clearly on the graph how you obtain your solution.

(e) Your colleague uses this formula to predict the population of the country in the year 2500. Discuss the reliability of this prediction.

10. The temperatures θ_1 and θ_2 of a pipe with inner radius r_1 and outer radius r_2 are given by:

$$\theta_1 = -\frac{Q}{2\pi kL}\ln(r_1) \quad \text{and} \quad \theta_2 = -\frac{Q}{2\pi kL}\ln(r_2)$$

where Q is the heat transfer rate, L is the length of the pipe and k is the thermal conductivity. Show that:

$$Q = \frac{2\pi kL(\theta_1 - \theta_2)}{\ln\left(\dfrac{r_2}{r_1}\right)}$$

11. The voltage v and current i of an inductor is given by:

$$i = 5\big(\,\mathrm{e}^{-200t} - \mathrm{e}^{-800t}\,\big) \quad \text{and} \quad v = \mathrm{e}^{-200t} + 400\,\mathrm{e}^{-800t}$$

Find an expression for the power $p = vi$ of the inductor.