Sheffield Hallam University

Mathematics

Formula Sheet

Differentiation

y	$\frac{\mathrm{d}y}{\mathrm{d}x}$
a (any constant)	0
ax	a
ax^n	$n \times ax^{n-1}$
ae^{nx}	$n \times ae^{nx}$
$a\ln(nx)$	$\frac{a}{x}$
$a\sin(nx)$	$n \times a\cos(nx)$
$a\cos(nx)$	$-n \times a\sin(nx)$
$a\sinh(nx)$	$n \times a\cosh(nx)$
$a\cosh(nx)$	$n \times a\sinh(nx)$

Product Rule

If $y = uv$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = u\dfrac{\mathrm{d}v}{\mathrm{d}x} + v\dfrac{\mathrm{d}u}{\mathrm{d}x}$

Quotient Rule

If $y = \dfrac{u}{v}$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Chain Rule

If $y = f(u)$ and $u = g(x)$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u} \times \dfrac{\mathrm{d}u}{\mathrm{d}x}$

Integration by Parts

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Integration

y	$\int y \, \mathrm{d}x$
a (any constant)	$ax + C$
ax^n	$\dfrac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$
ae^{nx}	$\dfrac{ae^{nx}}{n} + C$
$\dfrac{a}{x}$	$a \ln\, x + C$
$\dfrac{a}{nx+b}$	$\dfrac{a \ln(nx+b)}{n} + C$
$a \sin(nx)$	$\dfrac{-a \cos(nx)}{n} + C$
$a \cos(nx)$	$\dfrac{a \sin(nx)}{n} + C$
$a \sinh(nx)$	$\dfrac{a \cosh(nx)}{n} + C$
$a \cosh(nx)$	$\dfrac{a \sinh(nx)}{n} + C$

The Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the Square

For the quadratic equation $ax^2 + bx + c = 0$, where $a = 1$:

$$\left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$$

Modulus and Argument

Given a complex number in the form $z = x + jb$, the modulus, r, and argument, θ can be determined by:

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Volume of Revolution

$$\mathrm{V} = \int_a^b \pi y^2 \, \mathrm{d}x$$

Vector Analysis

Given the matrix:

$$\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

then the determinant:

$$\mathrm{det}\underline{A} = |\underline{A}| = ad - bc$$

and the inverse:

$$\mathrm{inv}\underline{A} = \underline{A}^{-1} = \frac{1}{|\underline{A}|} \times \mathrm{adj}\,\underline{A} = \frac{1}{|\underline{A}|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Rules of Indices

$a^m \times a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$

$\frac{A}{a^n} = Aa^{-n}$

$a^0 = 1$

$a^1 = a$

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$(a^m)^n = a^{m \times n}$

Rules for Fractions

Multiplying Fractions

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Dividing Fractions

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Triange Geometry

Right angled triangles

Pythagoras' theorem:

$$a^2 = b^2 + c^2$$

where a is the hypotenuse and b, c are the other two sides.

General rules for triangles

Sine rule:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos(A) \quad \text{or} \quad \cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Numerical Prefixes

Prefix	Symbol	Meaning
Giga	G	$\times\ 10^{9}$
Mega	M	$\times\ 10^{6}$
Kilo	k	$\times\ 10^{3}$
Milli	m	$\times\ 10^{-3}$
Micro	μ	$\times\ 10^{-6}$
Nano	n	$\times\ 10^{-9}$