Building an Efficient Portfolio

For our portfolio, we settled on investing in US stocks; thus we use the NASDAQ Composite as our market index. The stocks we've decided to include in our portfolio are Keurig Dr. Pepper Inc, Starbucks Corporation, Kellogg Company, Equinix, Inc., Pfizer Inc., The Kroger Co., Paramount Global, Pioneer Natural Resources Company, and Harley-Davidson, Inc. Further, we focused on data for these stocks for the most recent five years. For the US, the 5-year bond yield is currently 3.52%, with a frequency of 252 (there are 252 trading days in a year). We use this 5-year bond yield to decide whether we should invest in our portfolio for five years or buy a US bond for five years.

1. **For each stock, we calculate their Betas and use CAPM to estimate their cost of capital.**
   a.) Using Yahoo Finance, we download historical daily prices (closing prices) for each of our stocks and our market index for the past five years.

   b.) To calculate the daily returns for our stocks and the market index, we use the formula of subtracting today's closing price from the last day's closing price and dividing that as a whole by the last day's closing price. We do this for all of our historical data for each of our stocks and the market index. We get incredibly small values because these are daily returns, not annual.

   c.) Next, we calculate the covariance of each of our stocks with the market, as well as the covariance of the market with itself, which is essentially the variance of the market. To calculate these, we find the covariance of the daily returns of each of the stocks against the daily returns of the market. The betas for each stock can be calculated by dividing the covariance of each stock with the market divided by the variance of the market.

   d.) To plot a volatility-returns graph, we must find the annual volatility and expected annual returns for each of our stocks. To find the expected annual returns of each of our stocks, we calculate the average of the daily returns of each stock and multiply it by our frequency, 252 (which expresses our expected annual returns in terms of APR). To find the annual volatility of each of our stocks, we calculate the variance of the daily returns of each of our stocks times the frequency and take its square root(which expresses our annual volatility in terms of APR). Using a scatterplot, we can then plot each of our stocks according to their expected annual returns and annual volatility in a volatility-returns graph.

   e.) To find the cost of capital of each of our stocks, we use the Capital Asset Pricing Model (CAPM) formula (cost of capital_i = risk-free interest rate + beta_i * { expected annual returns of the market - risk-free interest rate}). Our risk-free interest rate is the US 5-year bond yield, 3.52%, beta_i refers to the beta of stock i (the covariance of each of our stocks against the variance of the market), and we simply take our previously

calculated value of the expected annual returns of our market (using NASDAQ Composite).

f.) The CAPM pricing formula helps investors calculate the expected return on an investment based on its level of risk. The intuition behind the CAPM pricing formula is that investors require some sort of compensation for the time value of money, as well as the risk they are taking by investing in a particular security. In contrast, other valuation models, such as the Dividend Discount Model (DDM), focus on the present value of a stock's future dividends, while the Discounted Cash Flow Model (DFCFM) considers the future cash flows that a company is expected to generate. While these other valuation models are useful for valuing stocks that pay dividends or have predictable cash flows, they do not take into account the systematic risks that the CAPM pricing formula does. The CAPM pricing formula tells us that an investor should expect to earn a higher return on a riskier investment, but only if that risk is systematic (market-wide). Thus the CAPM provides valuable insights into the expected returns and risks of individual stocks, allowing investors to make informed decisions about their portfolio composition and performance.

The CAPM pricing formula allows us to calculate a stock's beta coefficient, which is a measure of how much a stock's expected return is affected by market-wide risk compared to the overall market. Thus, for an investor, this information can be used to select stocks for a portfolio based on the investors' risk tolerance and investment goals. A stock with a beta of 1.0 has the same risk as the overall market, while a stock with a beta greater than 1 is considered riskier than the overall market, and a stock with a beta less than 1 is considered less risky. Hence, investors with a high-risk tolerance may use this CAPM pricing formula to choose which high-beta stocks to invest in. From our selected stocks, only the Harley-Davidson Inc stock would interest risk-loving investors, with a beta of 1.01 as they have the potential for high returns, but also higher volatility. Investors with a low-risk tolerance, on the other hand, would likely invest in Keurig Dr Pepper, Kelloggs, Pfizer, and Kroger stocks, which are less volatile but also have lower potential returns. Further, the CAPM can be used to evaluate the performance of individual stocks in a portfolio by comparing the expected return of stocks (as calculated by CAPM) with their actual returns; this determines if a stock is overvalued or undervalued. A higher actual return of a stock relative to its expected return would be considered overvalued and should be sold. Conversely, a lower actual return of a stock relative to its expected return would be considered undervalued and would prompt be bought up at the lower price.

2. **Create a portfolio of the stocks in Q1 and estimate its expected return and its volatility.**

   a.) For a covariance matrix of our stocks, the covariance of each stock with itself, that is, the variance of each stock, lies on the diagonal, while the rest of the cells are filled with the covariance of each stock with all the other stocks in the portfolio. For a correlation

matrix on the other hand, the correlation of each stock with itself lies on the diagonal, which will always be 1, and the rest of the cells are filled with the values of the covariances of each stock with all the other stocks in our portfolio. We can easily use the data analysis toolpack to calculate these matrices, using the data for the daily expected returns of each stock.

b.) We now create a portfolio and name it Portfolio_1, and estimate some portfolio weights, x_i, for our stocks. We find this portfolio's daily expected returns by multiplying the expected daily returns of each individual stock against the weights of the respective stocks in that portfolio. To find the annual portfolio expected return, we simply multiply this value by our frequency, 252. Further, to calculate this portfolio's daily volatility, we require the covariance matrix we previously created and the stock weights in our portfolio. To find the annual portfolio volatility, we multiply this value by the square root of the frequency.

c.) To find this portfolio's beta, we use the weighted average of its component's betas. That is, we multiply our portfolio weights with our respective stock betas. We also calculate the Sharpe Ratio, which is our portfolio's expected annual returns minus the market premium (3.52%), divided by our portfolio's annual volatility.

d.) Once we calculate the annual expected return and volatility of our portfolio, we can include this Portfolio_1 plot point in our previous volatility-returns graph. Our portfolio appears to have both a lower volatility, but also a lower expected annual return compared to our market.

e.) Diversification is a strategy used by investors to reduce the risk of their portfolio by spreading their investments across different classes and industries of assets. The idea behind diversification is that by holding a variety of investments, any losses in one asset class or sector can be offset by gains in another, thus leading to a more stable and consistent return on investment. Diversification helps investors reduce the overall risk of their portfolio, which leads to more consistent returns in the long run and it also allows investors to take advantage of opportunities in different markets or sectors, which may perform better or worse than others depending on the economic environment. By diversifying the stocks in our portfolio, we see how the volatility of our portfolio is much lower than the stocks' individual volatilities. With an annual volatility of 20.54%, our portfolio allows us to face less risk compared to if we had invested in individual stocks. In fact, the next lowest annual volatility among our stocks is approximately 3 percentage points higher than our annual portfolio volatility (this is the annual volatility of Kelloggs stocks, a 23.53% of risk). However, diversification in our portfolio 1 doesn't give us the highest expected return in comparison to some of the stocks in our portfolio; it still allows us to earn a higher expected return relative to its level of risk, given investing in individual stock would have meant a higher risk to earnings ratio.

Further, drawing conclusions from historical data entails inaccurate reflections of future market conditions, as economic and political factors are bound to change rapidly. Additionally, historical data can potentially be biased toward certain asset classes or simply due to unusual events during certain periods, leading to investors making decisions based on incomplete or inaccurate information. According to historical data, we would expect Pfizer to do well, but according to our data, we observe Pfizer to have a lower expected return relative to its volatility. This expectation of a good performance was induced as a result of Pfizer stocks being classified under the healthcare industry, especially in light of the recent pandemic and its effects on market conditions. However, even though historically, Pfizer stocks have been performing well in the market, the pandemic has been slowing down and this change in economic conditions has reduced the performance of Pfizer stocks; its unforeseen outcomes like these make historical data an unreliable predictor of future stock performance.

f.) We plot the Efficient Market Frontier with different combinations of expected returns and their respective volatilities.

3. **Find the tangent portfolio. (using the 3-5 stocks you picked in Q1)**

a.) Using the Solver extension in Excel, we aim to maximize the Sharpe Ratio by only changing the weights of the portfolio. We must ensure that none of the weights are negative, i.e. they are equal to zero or more, and that the sum of the weights are equal to 1. This essentially gives us the optimal weights in our portfolio to maximize our Sharpe Ratio.

b.) Thus, working backward, we can find the beta of this new portfolio with adjusted weights. When our Sharpe Ratio is maximized, we essentially have a tangent portfolio. Thus, the beta for our tangent portfolio is calculated using the weighted average of the betas of our stocks using the new weights we just calculated, which allows us to maximize our Sharpe Ratio.

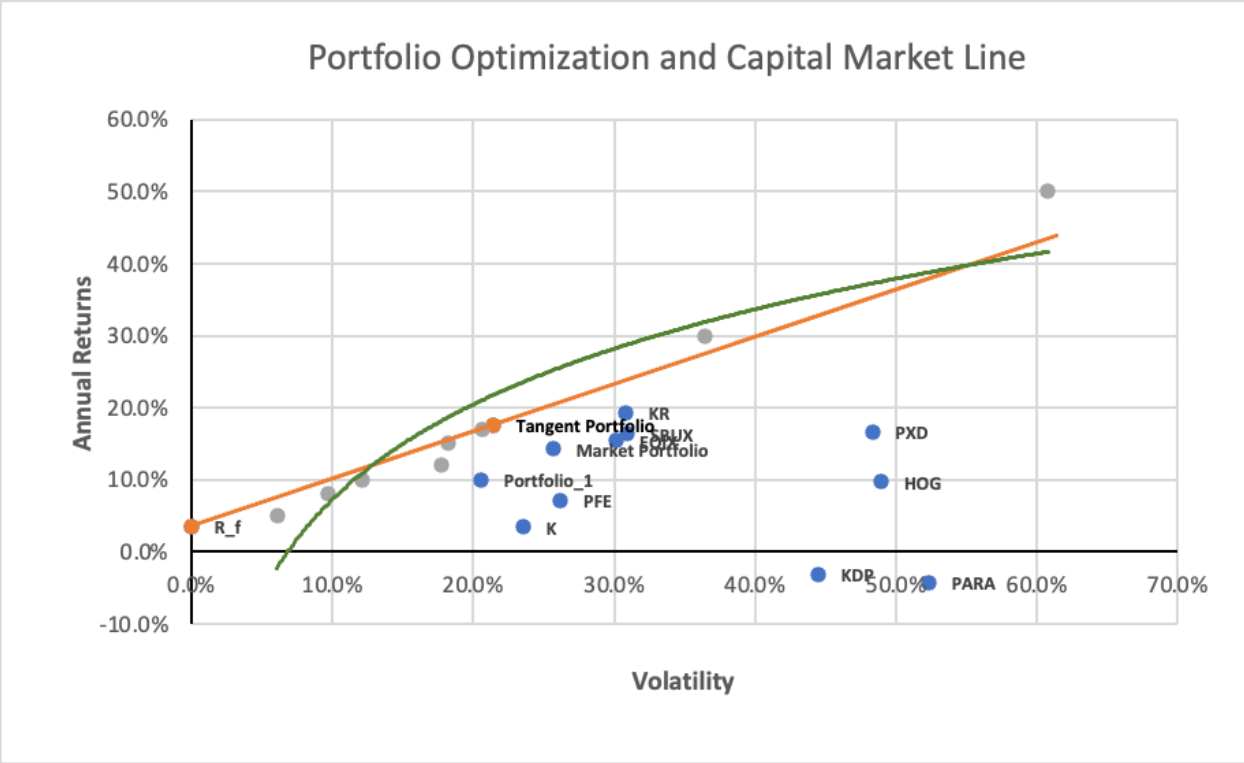
c.) Once we calculate the annual expected return and volatility of our tangent portfolio, we can include this in our previous volatility-returns graph. Our tangent portfolio has low volatility (the second lowest in comparison to the individual volatilities of the stocks in our portfolio) and a relatively high expected return (also the second highest expected annual return in comparison to the individual expected annual returns of our portfolio stocks). While minimizing the volatility, it maximizes the expected returns. The tangent portfolio is placed above the market index; the volatility of our tangent portfolio is approximately 4 percentage points lower than our market index and the expected annual returns of our tangent portfolio is approximately 3 percentage points higher than our market index.

d.) Our tangent portfolio appears to be performing the best. The portfolio we began with (portfolio 1) has the lowest volatility (20.54%) between the tangent and market portfolios, but it also incurs the lowest expected annual returns (10.04%) in comparison to the other two. This makes sense as low volatility means a low risk is taken to earn potentially high

returns. Our market portfolio faces the highest volatility (25.66%) among the three, yet does not incur the highest expected annual returns (only 14.30%); the expected annual returns of the market portfolio just lag behind the tangent portfolio's annual expected returns by 3.3 percentage points. The tangent portfolio not only sustains volatility (21.40%) lower than the market portfolio (25.66%) but also an annual expected return (17.60%) higher than the market portfolio's annual expected return. Thus, we can conclude that maximizing the Sharpe Ratio earns us a portfolio that incurs the highest expected annual returns for its given volatility, as it appears to be performing better than even the market portfolio.

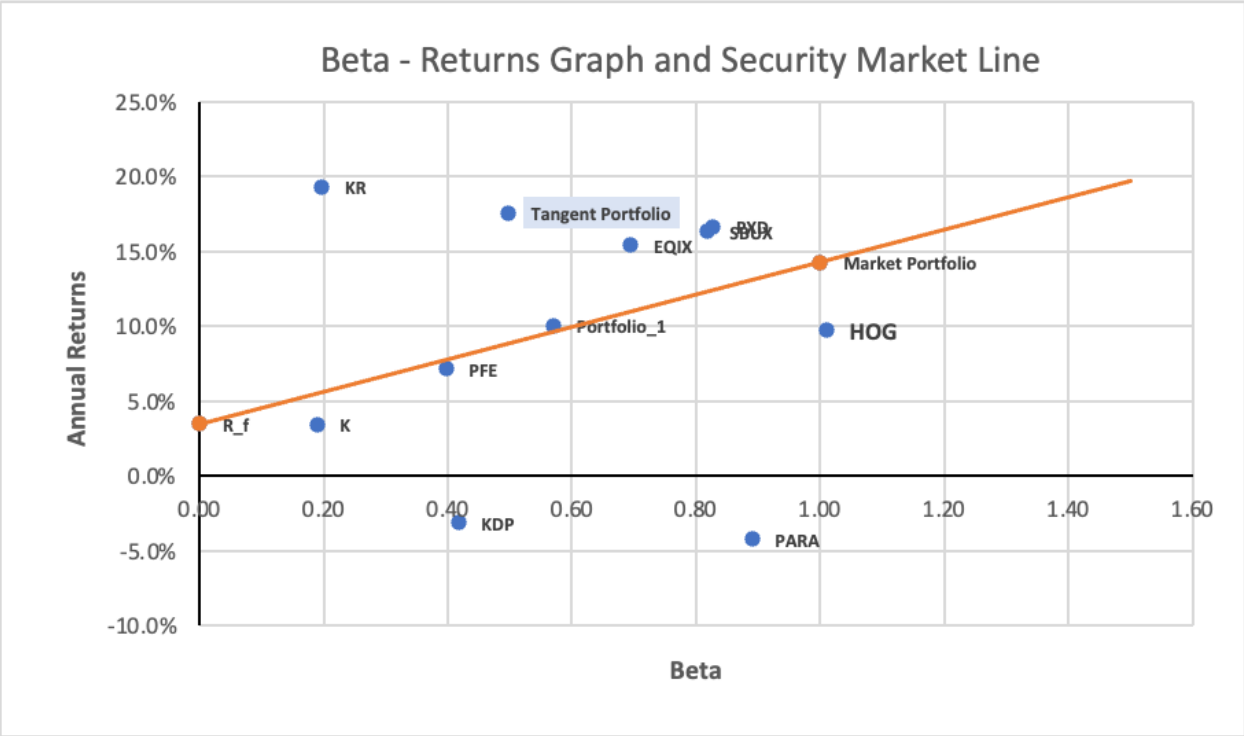
4. **Plot the Capital Market Line and Securities Market Line**

a.) By drawing a line between our risk-free rate (3.52%) and our tangent portfolio, we are essentially plotting the Capital Market Line (CML). The CML is tangent to the efficient frontier and meets it at the Market Portfolio. The CML is a graphical representation of the relationship between risk and returns for a portfolio that includes both risky assets and risk-free assets. Further, it's a useful tool for investors to determine the optimal portfolio of risky and risk-free assets that will provide the highest expected return for a given level of risk.



b.) We plot a graph of expected returns against betas with our risk-free rate, individual stocks, portfolio 1, our tangent portfolio, and our market portfolio. To add to this, we add a Security Market Line (SML). The SML is a graphical representation of the Capital Asset Pricing Model (CAPM) that shows the relationship between the expected return

and the risk of an individual security or portfolio of securities. The SML is a straight line that runs through the market portfolio the risk-free investment. It plots the expected return of an asset or portfolio against its systematic risk, which is measured by its beta.



c.) NASDAQ, our market index, is below our efficient frontier because we don't have a portfolio that can reach that expected return. Our tangent portfolio appears to be at a higher Sharpe Ratio than our market portfolio. Given that only our tangent portfolio falls in line with the CML, it is the only portfolio that is as efficient as possible for the given amount of risk we are taking. Each of the stocks falls below this line, along with our portfolio 1 and the market portfolio.

d.) The SML is plotted on our Betas vs. Returns graph. With the Beta, we're saying that the correlation that the stocks have with the market should match the annual return they're being compensated for. The SML is based on the CAPM. The SML is the line for which if the expected return of the CAPM were true, the expected return should align as a linear form of the betas. That is, as the betas increase, the annual returns of portfolios should fall on the SML. However, we find only Pfizer stocks being close to that line and surprisingly, our portfolio 1 falls just above the SML. The SML does not accurately predict the returns of individual stocks, though it can still be used as a helpful framework for evaluating the risk-return tradeoff of individual stocks or portfolios.