UTM
UNIVERSITI TEKNOLOGI MALAYSIA

Faculty of Computing
SECI1013 - Discrete Structure

Prepared for: Dr Shukor Talib

ASSIGNMENT 2

Prepared by:

No.	Name	Matric Number
1.	Siti Sarah binti Muhammad Hafizam	A24CS0193
2.	Nurain Nabilah binti Azman	A25CS0324
3.	Nor Ain Fahira bintivMuhammad Fariq	A25CS0295

Question 1

$D = \{1, 3, 5\}$, $x, y \in D$, xRy , $3x + y$ is multiple of 6

i) $x = 1, y = 1 : 3(1) + 1 = 4$
$x = 1, y = 3 : 3(1) + 3 = 6$
$x = 1, y = 5 : 3(1) + 5 = 8$

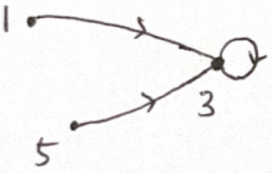
$x = 3, y = 1 : 3(3) + 1 = 10$
$x = 3, y = 3 : 3(3) + 3 = 12$
$x = 3, y = 5 : 3(3) + 5 = 14$

$x = 5, y = 1 : 3(5) + 1 = 16$
$x = 5, y = 3 : 3(5) + 3 = 18$
$x = 5, y = 5 : 3(5) + 5 = 20$

$R = \{(1, 3), (3, 3), (5, 3)\}$

ii) Domain of $R = \{1, 3, 5\}$
Range of $R = \{3\}$

iii)



iv) R is not asymmetric because it contains ordered pair which is $(3, 3)$.

Question 2

$A = \{x, y, z\}$ $(x, y) \in R$ $(y, z) \in R$

equivalence relation must reflexive, symmetric and transitive

$R = \{(x,x), (y,y), (z,z), (x,y), (y,x), (x,z), (z,x), (y,z), (z,y)\}$

Reflexive requires $(a,a) \in R$. In this case, (x,x), (y,y) and (z,z) are element of R.

Symmetry requires $(a,b) \in R$ and $(b,a) \in R$. In this case, we have (x,y), (y,x), (x,z), (z,x), (y,z), (z,y).

Transitive requires $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$. In R, we have (x,y), (y,z) and (x,z).

Question 3

$B = \{u, v, w, y\}$ $R = \{(u,u), (u,w), (v,v), (v,w), (w,w), (w,y), (y,u), (y,v), (y,y)\}$

i) $M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

ii)

	in-degrees	out-degrees
u	2	2
v	2	2
w	3	2
y	2	3

iii) partial order relation must reflexive, antisymmetric and transitive.

For R it is reflexive because it have all loops.
R not antisymmetric because it doesn't fulfill $(a,b) \in R$ and $a \neq b$ then $(b,a) \notin R$.
R not transitive because $(y,u) \in R$, $(u,w) \in R$ but $(y,w) \notin R$.

Thus, R is not a partial order relation.

Question 4

Let

$$f: [1, \infty) \to [0, \infty), \; f(x) = (x-1)^2$$

Determined whether the function f is one-to-one, onto or bijective.
Show full working and justify your answer.

one-to-one

let $f(x_1) = f(x_2)$

$$\sqrt{(x_1-1)^2} = \sqrt{(x_2-1)^2}$$

$$x_1 - 1 = x_2 - 1$$

$$x_1 = x_2$$

$\therefore x_1 = x_2$, therefore f is one-to-one

Domain $A = [1, \infty)$, Codomain $B = [0, \infty)$

$$(x-1)^2 = y$$

$$x - 1 = \sqrt{y}$$

$$x = 1 + \sqrt{y}$$

check $x = 1$ and $x = 2$

$$1 = 1 + \sqrt{y} \qquad 2 = 1 + \sqrt{y}$$

$$\sqrt{y} = 0 \qquad \sqrt{y} = 1$$

$$y = 0 \qquad y = 1$$

Since for each $y \in [0, \infty)$, therefore $x = 1 + \sqrt{y}$ in the domain. Thus f is onto.

$\therefore$ Since the function is both one-to-one and onto, therefore it is bijective.

Question 5

Let f and g be functions from the positive integers to the positive integers defined by

$$f(x) = 9x + 4 \quad , \quad g(x) = \frac{3}{2}x - 1$$

a) Find the inverse of $g(x)$.

let $g(x) = y$

$$y = \frac{3}{2}x - 1$$

$$y + 1 = \frac{3}{2}x$$

$$\frac{2}{3}(y+1) = x$$

$$g^{-1}(x) = \frac{2}{3}(x+1)$$

b) Find the composition $(g \circ f)(x)$.

$$g[f(x)] = \frac{3}{2}(9x + 4) - 1$$

$$= \frac{27}{2}x + 6 - 1$$

$$= \frac{27}{2}x + 5$$

c) Find the composition $(f \circ g)(x)$.

$$f[g(x)] = 9\left(\frac{3}{2}x - 1\right) + 4$$

$$= \frac{27}{2}x - 9 + 4$$

$$= \frac{27}{2}x - 5$$

d) Find the composition $(f \circ g \circ g)(x)$.

$$fg[g(x)] = \frac{27}{2}\left(\frac{3}{2}x - 1\right) - 5$$

$$= \frac{81}{4}x - \frac{27}{2} - 5$$

$$= \frac{81}{4}x - \frac{37}{2}$$

Question 6

(a) $P_n = P_{n-1} + \frac{1}{4}P_{n-2}$, $n \geq 2$

$P_0 = 4.0$, $P_1 = 5.0$

(b) $P_2 = P_1 + \frac{1}{4}P_0 = 5 + \frac{1}{4}(4) = 5 + 1 = 6$

$P_3 = P_2 + \frac{1}{4}P_1 = 6 + \frac{1}{4}(5) = 6 + 1.25 = 7.25$

$P_4 = P_3 = \frac{1}{4}P_2 = 7.25 + \frac{1}{4}(6) = 7.25 + \frac{6}{4} = 8.75$

$P_5 = P_4 + \frac{1}{4}P_3 = \frac{35}{4} + \frac{1}{4}(7.25) = \frac{39}{4} + \frac{29}{16} = \frac{140}{16} + \frac{29}{16} = \frac{169}{16} = 10.5625$

$P_6 = P_5 + \frac{1}{4}P_4 = \frac{169}{16} + \frac{1}{4}\left(\frac{35}{4}\right) = \frac{169}{16} + \frac{35}{16} = \frac{204}{16} = \frac{51}{4} = 12.75$

$\therefore$ $P_0 = 4.0$ $P_1 = 5.0$ $P_2 = 6$ $P_3 = 7.25$ $P_4 = 8.75$ $P_5 = 10.5625$ $P_6 = 12.75$

Question 7

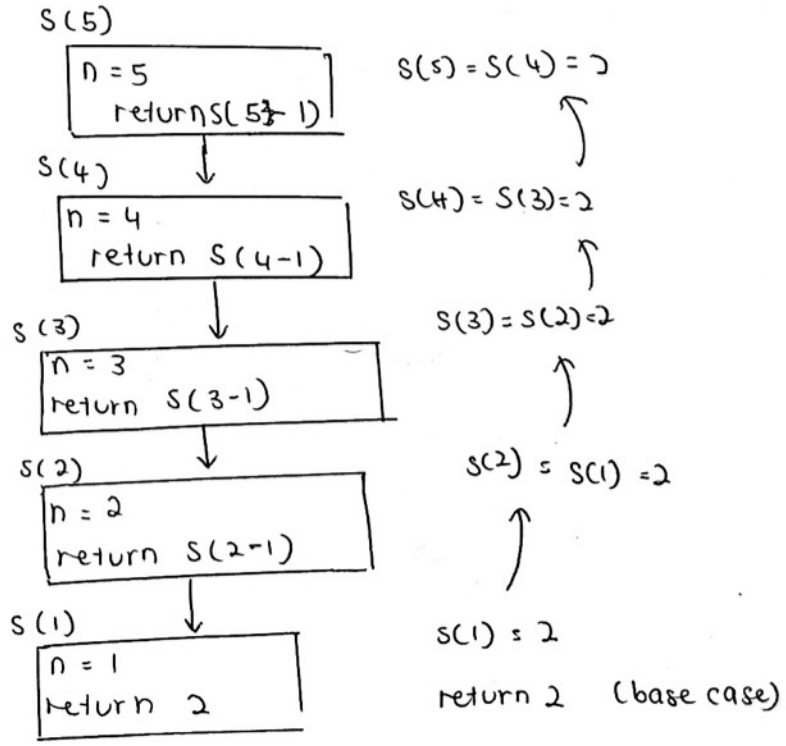
$S_1 = 2$, $S_n = S_{n-1}$ for $n \geq 2$

```
Funtion S(n)
    if n=1 then
        return 2
    else
        return S(n-1)
    end if
End function
```

~~S4~~ ~~S(5)~~

~~n=5~~

~~n-1 = 5-1 =~~



answer : S4 = 2