ASTR 507 Homework #3
Due Feb. 22, 2022

1. Download the COBE spectrum of the CMB from

`Dropbox/ASTR507W22/firas_monopole_spec_v1.txt`

From these data, estimate the maximum allowed value of the chemical potential of photons (in units of kT_{CMB}) by allowing both the temperature and chemical potential to vary, finding where the χ^2 of your fit increases to the point that the fit is poor (see Numerical Recipes, Chapter 15, esp. 15.6, for advice on computing confidence intervals; available in course Dropbox). Please state the confidence of your upper limit. Note: the variable definitions and units can be tricky for this problem - please let me know if you need help interpreting.

2. (a) Derive the entropy for an ideal, non-relativistic Fermi gas in terms of V, z and T. You may leave your answer in terms of Fermi-Dirac integrals. (b) Expanding $P/(nkT)$ as a series in z keeping first order terms in z, show that electron degeneracy *increases* the pressure of the gas.

3. The center of a brown dwarf has density $\rho_c = 325$ g cm^{-3} and temperature 6×10^6 K. Assuming that the helium atom to hydrogen atom number density ratio is 0.1 throughout the star (and ignoring heavy elements), estimate the (a) electron fugacity (suggestion: plot Fermi-Dirac function versus z and match to central conditions of star, or solve numerically) and (b) electron gas pressure in the center of this star using the partially degenerate, non-relativistic Fermi equation of state for electrons. (c) Are the electrons in fact non-relativistic? (d) Demonstrate that degeneracy contributes significantly the pressure by computing the pressure were the gas a classical ideal gas. (e) Estimate the mass and radius of the star with the polytrope relations $\rho_c = 8.44 M/R^3$ g cm^{-3} and $RM^{1/3} = 0.13/(\mu_e F_{1/2}(z))^{2/3}$, where M & R are the mass & radius of the star in solar units, and μ_e is the mass per electron in units of the proton mass.

Approximate analytic formulas for the Fermi-Dirac functions are given in a paper in the course Dropbox (FermiDiracPressureIntegralApproximation.pdf).