ASTR 507 Winter 2022
Homework 2, Due Feb. 8, 2022

1. [2 pt] Show that $C_P - C_V = Nk_B$ and that $C_P/C_V = \gamma$.

2. [3 pt] The hot interstellar phase in the disk of our galaxy contains ionized hydrogen at $\sim 10^6$ K with a density of $n \sim 0.01$ cm^{-3}. If the volume is ~ 40 kpc^3, assuming this gas obeys the Maxwell-Boltzmann distribution, what speed would you expect the fastest electron to be moving? (Hint: $\int_x^\infty y^2 e^{-y^2} dy \sim xe^{-x^2}/2$ for $x >> 1$). How does this compare to the velocity of cosmic rays, $v \sim 0.99c$? Can cosmic rays be of thermal origin?

3. Using the Maxwell-Boltzmann distribution, derive an estimate for the rate at which neutral molecules or atoms of mass m escape from a planet atmosphere, as follows:

   (a) [2 pt] Assuming a particle cross section of σ, compute the number density, n_{esc}, at which the mean free path of particles equals the local scale height, $H = kT/(mg)$, where g is the gravitational acceleration (above this layer is referred to as the "exosphere").

   (b) [1 pt] For particles with speeds between v and $v + dv$ moving in direction (θ, ϕ) relative to the outward radial direction, $f(v)dv/(4\pi)$, compute the flux of particles in the upwards direction, $\phi(v)dv$, by integrating

$$\phi(v)dv = \frac{f(v)}{4\pi} dv \int_0^{2\pi} d\phi \int_0^{\pi/2} v \cos\theta \sin\theta d\theta. \tag{1}$$

   (c) [4 pt] Assume that at the height in the atmosphere with n_{esc} all particles moving upwards with $v > v_{esc}$ will escape. Compute the total flux of escaping particles, $\phi = \int_{v_{esc}}^\infty \phi(v)dv$ (ϕ has units of particles cm^{-2} s^{-1}). Express your answer in terms of n_{esc}, $v_s = \sqrt{2kT/m}$, and $\lambda_{esc} = \left(\frac{v_{esc}}{v_s}\right)^2$.

   (d) [3 pt] Apply your formula to the early Earth for molecular hydrogen, H_2. Assume that the temperature at the exosphere is 1000 K at a height above the Earth's surface that is small compared to the Earth's radius, and assume the molecular cross section is π Å^2. Compute the rate of loss of hydrogen over the entire area of the exosphere. How much hydrogen would be lost thermally over 1 Gyr? How does this compare to the current hydrogen content of the Earth's atmosphere (locked up in water) of $\sim 10^{43}$ atoms?

(e) [1 pt] Repeat this calculation for molecular oxygen. How does the rate of loss of oxygen compare to hydrogen (assume oxygen molecules are the same size as hydrogen)? What do you conclude about the possible change in abundance of atmospheric oxygen relative to hydrogen with time (assuming no other sources or sinks)?