

# Stewart platform with fixed rotary actuators: a low cost design study

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## Abstract

This work presents a design example of a generic six-degree-of-freedom parallel manipulator commonly known as the Stewart platform. It is meant as a practical guideline covering the basic theory of Stewart platforms and the actual low cost realization suitable for rapid prototyping. The inverse kinematics solution and a coarse-grained evaluation are provided for the actually constructed prototype. Additionally, the application of generic Stewart platforms as tool holders in the context of minimally invasive robotic surgery is discussed and a proposal for a surgical robot given.

## Index Terms

Stewart platform, low cost design, rotary actuator, minimally invasive surgery, surgical robot

## I. INTRODUCTION

The Stewart platform (SP) has been a popular research topic in robotics since its first appearance on the scientific agenda in 1965 in the renown work by Stewart [1]. Many publications concerning its kinematics [2, 3, 4], dynamics [5, 6], work space estimation [7, 8], path planning [9] and force sensing applications [10, 11] have been published since the time of Stewart's original publication. For an extensive review of the literature the reader is referred to [12]. Much fewer works covering the practical design issues have followed the theoretical debate with some prominent exceptions including [2, 13, 14]. Despite its many potential advantages over serial manipulators like higher end effector accuracy, rigidity, load-to-weight ratio [15] and force sensing capacity as well as Stewart's original design aims to achieve the most simple and cohesive design for a wide range of applications, the SP has found relatively little resonance outside the scientific community. Most practical designs are constrained to the so called 6-UPS form with the natural application in flight simulators, CNC machining centers or SMT placement machines. Ji [16] attributes this to the lack of rational synthesis tools for a practical design. However, given the rapid development of computational capabilities and efficient CAD design tools over the last 10 years the situation is on the best way to a change. Many applications in the field of medicine [17] including eye [7] and skull surgery [18] are conceivable. This development paralleled by a rapid development of minimally invasive surgical (MIS) robots and is of special interest for this work. This article is further structured as follows. The next section gives a short overview of the state-of-art MIS robots and is succeeded by a discussion of a theoretical MIS system employing the SP as a laparoscopic tool holder. Sec. II covers the fundamentals of SP architectures and introduces the relevant mathematical notation. Sec. III presents a complete design example of a low cost SP with a crude evaluation of its work area. The last two sections discuss the exemplary design and indicate the necessary adjustments for a possible application of a SP in the context of MIS robots.

### A. MIS robots

The history of MIS robotic systems probably dates back to the research done by NASA in the 1980s in which the possibility of remote treatment of injured soldiers (teleoperation) was considered. The first robotic manipulator for surgery known to the author was developed at the Stanford Research Institute

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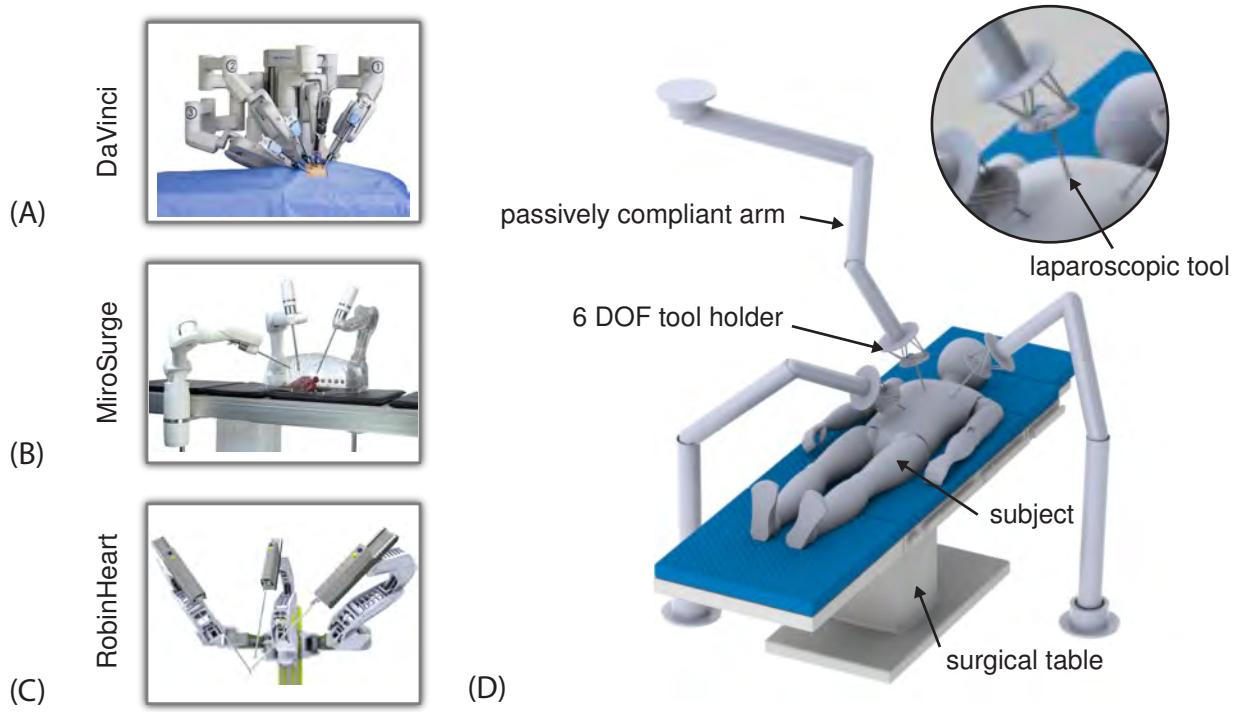


Fig. 1. Robotic minimally invasive surgical systems including (A) the commercially available DaVinci [19] telerobotic system and a selection of research projects: (B) the MiroSurge [20] of DLR (Germany) and (C) the RobinHeart mc<sup>2</sup> [21] of the Foundation for Cardiac Surgery Development (Poland). (D) is the conception of a surgical robot consisting of passively positionable arms and 6 DOF platforms for holding and adjusting the positions of laparoscopic tools.

and licensed to the company Intuitive Surgical Inc. (USA) in 1994 [22, 23]. Since that time a variety of research projects have been started all around the globe. These include the only, to date, commercially available Da Vinci surgical system [24, 19], the Raven [25] (University of Washington, USA) the MIRO of DLR [20] (Germany) and the RobinHeart of the Foundation for Cardiac Surgery Development [21] (Poland) to name a few. Fig. 1(A)-(C) shows a selection of these systems. One of the most important technical challenges that each of these systems has to deal with is how to keep the entry point (incision point) to subject's body constant. The solutions range from the employment of a passive joint at the end-effector through a remote center of motion mechanism to a virtually programmable center of motion.

### B. A surgical scenario

Each of these robotic MIS systems consists of several robotic arms each of which directly holds a laparoscopic tool or an endoscopic camera. Any change of orientation or penetration depth of the tool, except of the passive joint variant, affects to some degree the configuration of the whole arm. If multiple arms are employed and/or medical personnel need access to the patient this can possibly lead to collisions and thus health-threatening hazards. This problem is mediated e.g. by pre-operational planning and/or use of redundant arms. Both solutions depend on an increased complexity either on the hardware or the software side and do not support a more intuitive approach to the surgery. Fig. 1(D) shows the proposal of a theoretical MIS robotic system which could possibly alleviate the above mentioned problem. The system consists as before of a few robotic arms each of which now holds a Stewart platform to which an actual tool is attached. In this setup the arms function mainly as passive holders for the SPs and only the latter are responsible for orientation or penetration depth change of the tools. The main advantage of this setup lies in the absence of any large or unintuitive movements of the arms. In fact, a completely passive system with only a few degrees of freedom (DOFs) whose position could be fixed at a suitable location

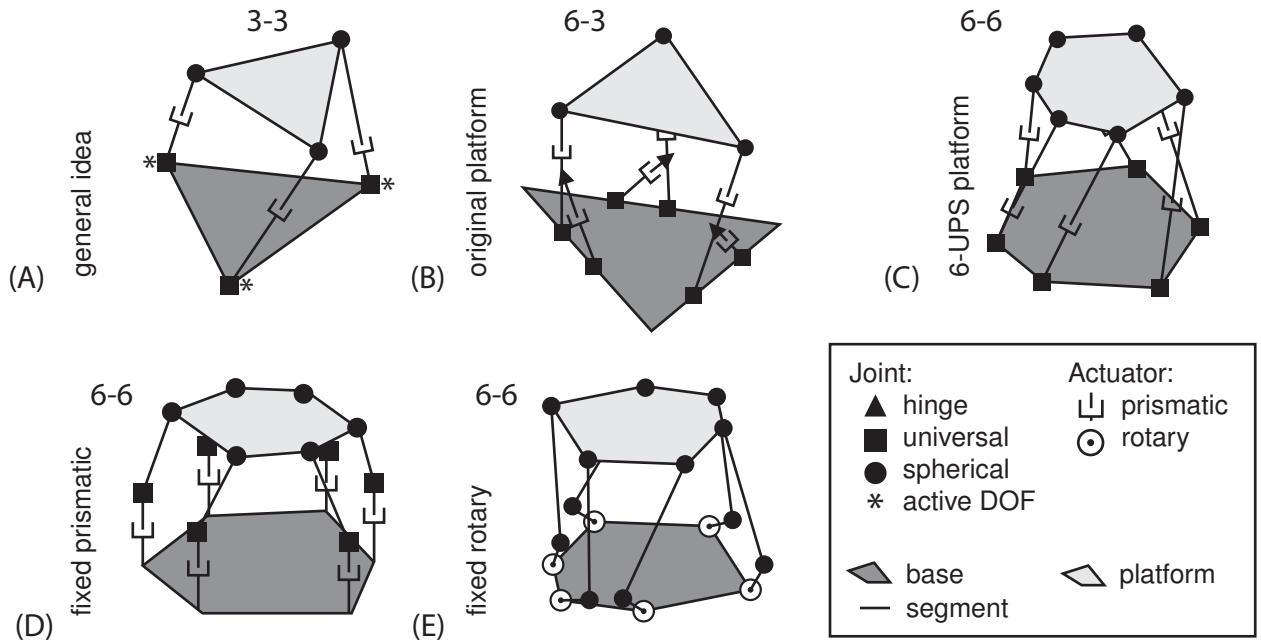


Fig. 2. Simplified depictions of several GSP architectures. (A) the original idea by Stewart and (B) its actual realization with 6 prismatic (hydraulic) actuators [1]. (C) the most typical realization of a 6 DOF platform commonly known as 'the Stewart platform' or a hexapod robot. (D) and (E) show examples of Stewart platforms with, respectively, prismatic and rotary actuators fixed at their bases. The latter platform is further elaborated on in this work.

close to the incision point would be sufficient. Any DOFs required for the tool holder are covered by the SP directly at the point of interest. The surgeon can shape the passive or actively compliant [26] arm into a suitable ergonomic configuration without the need of any special configuration procedure. From this point on any additional movement of a relatively small magnitude is performed by the SP directly at the patient's body. Other advantages follow from the properties of the SP. A light-weight and strong design capable of carrying much heavier tools is possible, the end point precision is improved and a 6-dimensional force sensing capability at the trocar can be gained easily. The main disadvantage lies in the increased size of the tool holders. However, considering the variety of possible SP designs (see sec. II-A) and the flexibility of fixing the tool either to the upper or lower part [17] of the platform an appropriate design can be achieved. Moreover, as only 4 DOFs are actively used by the SP at the trocar a reduced design with smaller size and lower weight is conceivable.

## II. STEWART PLATFORMS

The literature on SP is abundant in its definitions. The only agreement seems to concern the fact that it is a parallel manipulator. In his original article [1], Stewart defined the SP as a mechanism which has 6 DOFs controlled in any combination by 6 motors each having a ground abutment. Xiao defines in [4] the generalized SP (GSP) as a 6 DOFs parallel manipulator consisting of two rigid bodies connected with 6 distance or/and angular constrains between 6 pairs of points, lines, and/or planes in the base and platform, respectively. With this definition there are 3850 possible forms or architectures of GSP. Without a further reference to Xiao's article or definition of a GSP the following section presents several GSP architecture examples. Sec. II-B introduces the mathematical notation used throughout this work.

### A. Generalized architectures

Parallel manipulators are often classified according to the number of connections between the lower (base) and the upper platform (in following simply platform). Stewart's original construction was a 6-3

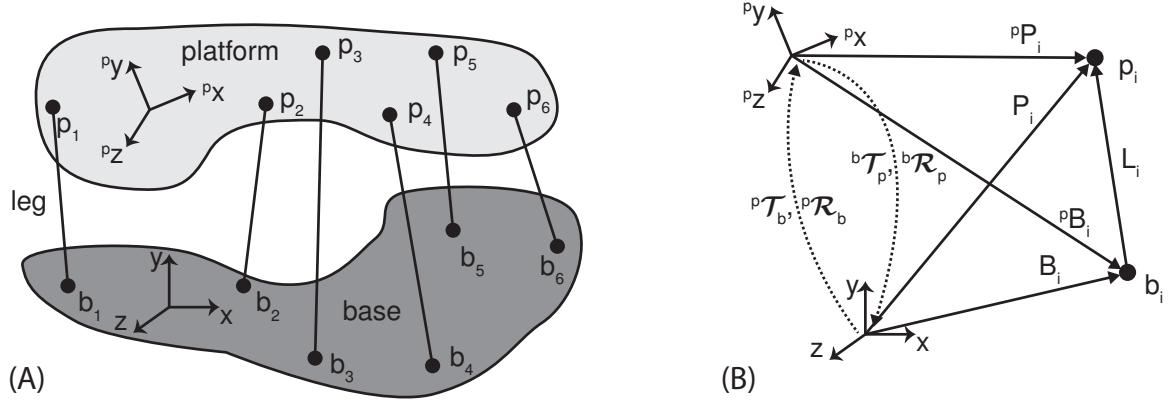


Fig. 3. Schematic illustration of a SP indicating the mathematical notation used throughout this work. (A) shows the upper platform and the lower base with their corresponding coordinate systems and the attachment points of the legs (after [2]). (B) shows the transformations between the two coordinate systems.

architecture and rather a special design according to the generalizing modifications it underwent in the course of time. Fig. 2 shows a schematic depiction of several GSP architectures. Besides the spatial configuration (locations of the connections), the type of these connections (joints) and of the employed actuators are the most important design aspects. Although a variety of different architectural designs is clearly possible, only one of them has gained widespread popularity - the so called 6-UPS (universal-prismatic-spherical) SP which is often referred to as 'the Stewart platform'. Interestingly, Stewart came up with the idea of this platform in his original work when he discussed the possibility of linear coordinate control as opposed to the polar coordinates he employed in his actual design. The reasons for the popularity of the 6-UPS platform are certainly manifold ranging from the similarity of the first designs following and even preceding [27] Stewart's original work to the ease of construction and employment of standard components. SPs are usually realized with help of prismatic actuators which constitute the length-varying elements (legs) between the base and the platform but a GSP can be realized with any type of prismatic or rotary actuators. Together with the design and quality of the joints this gives the engineer a large playground for finding a compromise between the technical requirements (size, weight, work area, speed etc.) and the available budget. Fig. 2(E) shows a GSP which can be realized with simple servo motors and which is further described in sec. III.

### B. Basic notation

This section introduces the mathematical notation used in this work in order to describe the kinematics of SPs. The notation is based mostly on [2]. Although the SP lends itself to the description in the framework of screw theory, the mathematical treatment in this work only assumes the basic knowledge of linear transformations. Fig. 3(A) shows a schematic depiction of a SP consisting of a base and a platform with their corresponding right-handed coordinate systems (CSs). The base and the platform are connected by means of 6 (length-varying) legs which are attached to them at some arbitrary locations \$b\_i\$ on the base and \$p\_i\$ on the platform surface (\$i \in \{1, \dots, 6\}\$). For the sake of a clear mathematical treatment, the attachment points are assumed to be 3 DOFs spherical joints with no constraints on their rotations. The transformations between the platform, the base CS and the inverse transformations are realized by means of three successive Euler rotations in the \$x - y - z\$ convention and a subsequent translation with the rotation matrix defined as

$$\mathcal{R} = \mathcal{R}_z(\gamma)\mathcal{R}_y(\beta)\mathcal{R}_x(\alpha) \quad (1)$$

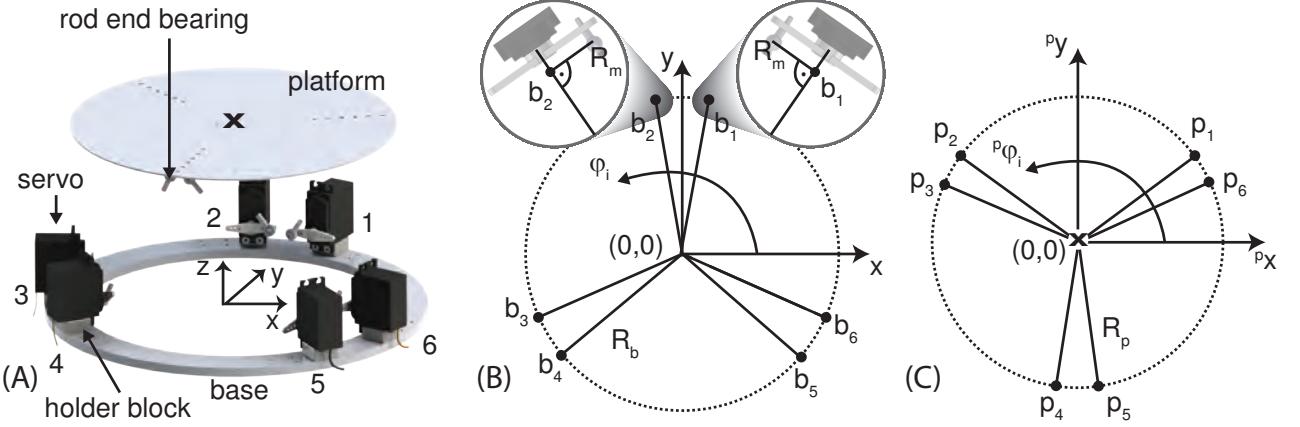


Fig. 4. Exemplary SP design with fixed rotary actuators. (A) is the isometric view of the arrangement of servo motors; connecting rods are not shown for the sake of clarity. (B) and (C) show the geometrical arrangement of leg attachment points in agreement with the notation from Fig. 3. Platform's origin is marked with an x.

and the translation vector being

$$\mathcal{T} = (t_x \quad t_y \quad t_z)^T. \quad (2)$$

The kind of transformation is indicated by amending two designators (*b* and *p*) to the corresponding transformation, thus  ${}^p\mathcal{R}_b = \mathcal{R}$  and  ${}^p\mathcal{T}_b = \mathcal{T}$  mean the rotation and position of the platform relative to the base and

$${}^b\mathcal{R}_p = ({}^p\mathcal{R}_b)^{-1} = ({}^p\mathcal{R}_b)^T \quad (3a)$$

$${}^b\mathcal{T}_p = -({}^p\mathcal{T}_b) \quad (3b)$$

are the inverse relations. Vectors are written in uppercase and become the prefix *p* only if they are expressed in the platform CS. In any other case the base CS is assumed (see Fig. 3(B)). According to the above definitions the leg vector  $L_i$  of leg *i* in base CS becomes

$$L_i = {}^p\mathcal{T}_b + {}^p\mathcal{R}_b^p P_i - B_i = P_i - B_i. \quad (4)$$

The length of the leg is defined as the Euclidean norm of this vector

$$|L_i| = \|L_i\|_2. \quad (5)$$

The above equations are used to compute the lengths of virtual legs in the design example of the following section.

### III. DESIGN EXAMPLE

The purpose of this section is to demonstrate a low cost design example of a SP. The design is not meant for any particular application but is rather supposed to serve as a reference and guideline for rapid prototyping of GSPs. The low cost example is a 6-6 SP actuated by 6 standard analog servo motors fixed at the base. Fixed-length rods are used as connections between the servo horns and the platform. The attachment is realized by means of rod end bearings which play the role of low cost spherical joints. Fig. 4(A) shows the CAD rendering of this design. The servo motors are mounted on cuboid-shaped holder blocks and fixed to a circular base. Rod end bearings are attached to the servo horns as well as to holder blocks (not visible) fixed to the platform. The connecting rods were hidden for the sake of clarity. The numbering of the motors and the base CS are indicated. Fig. 4(B) and (C) show this arrangement schematically. Note that the base attachment points  $b_i$  are invariant under servo rotation and defined as

the projections of joint centers on the corresponding axes of rotation. The platform attachment points  $p_i$  are coincident with the centers of the corresponding joints attached to the platform. Both sets of points are easily found to be

$$\mathbf{b}_i = (x_i \quad y_i \quad z_i)^T = (R_b \cos(\gamma_i) \quad R_b \sin(\gamma_i) \quad 0)^T \quad (6a)$$

$$\mathbf{p}_i = ({}^p x_i \quad {}^p y_i \quad {}^p z_i)^T = (R_p \cos({}^p \gamma_i) \quad R_p \sin({}^p \gamma_i) \quad 0)^T \quad (6b)$$

with  $R_b$  and  $R_p$  being the radii of the circles on which, respectively,  $\mathbf{b}_i$  and  $\mathbf{p}_i$  lay. The corresponding angles are shown in Table I. Note that although the angles are provided in *degrees* all actual computations need to be performed in *rad*.

TABLE I  
ANGULAR COORDINATES OF BASE AND PLATFORM ATTACHMENT POINTS.

$i$	1	2	3	4	5	6
$\gamma_i$	77°	103°	197°	223°	317°	343°
${}^p \gamma_i$	37.5°	142.5°	157.5°	262.5°	277.5°	22.5°

### A. Inverse kinematics

The basic objective in the control of SPs is to solve the inverse kinematics (IK) problem – to find the lengths of all legs for a given desired position and orientation of the platform. The problem has a unique analytic solution in contrast to the forward kinematics problem which is highly nonlinear and usually requires either iterative approaches or additional sensory information. The general solution to the IK problem is already contained in Eq.(4). However, as rotary actuators and fixed-length rods are employed the GSP of this section does not have any real variable-length legs. Sticking to the definition of a leg from sec. II-B which is just a connecting element between  $\mathbf{b}_i$  and  $\mathbf{p}_i$ , the variability in length is achieved virtually by changing the locations of  $\mathbf{p}_i$  in the base CS through the rotation of servo  $i$ . This is shown schematically in Fig. 5. Each servo motor has a local CS with the origin at  $\mathbf{b}_i$  and the axis of rotation  ${}^m z$  pointing toward the origin of the base CS. The center of the joint attached to the servo horn  $m_i$  changes in dependency of the rotation angle  $\Delta_i$ . With  $R_m$  being the radius at which the joint is attached to the servo horn,  $D$  the fixed rod length and  $M_i$  the vector from the origin to  $m_i$  in base CS it holds

$$R_m = R_{mi} = |M_i - B_i| \quad (7a)$$

$$D = D_i = |P_i - M_i|. \quad (7b)$$

The end point of the vector  $M_i$  is found through the transformation

$$M_i = (x_{mi} \quad y_{mi} \quad z_{mi})^T = {}^{mi} \mathcal{T}_b + {}^{mi} \mathcal{R}_b (R_m \quad 0 \quad 0)^T \quad (8)$$

where

$${}^{mi} \mathcal{T}_b = (x_i \quad y_i \quad z_i)^T, \quad (9a)$$

$${}^{mi} \mathcal{R}_b = \mathcal{R}_z(\gamma_i - \frac{\pi}{2}) \mathcal{R}_y(-\Delta_i). \quad (9b)$$

The above rotation matrix is valid for the even-numbered servos as the one depicted in Fig. 5. Odd-numbered motors have their joints attached to the opposite sides of the servo horns. This is due to the fact that with such an arrangement a smaller distance between the neighboring joints is possible. This distance can be used as a design parameter [28, 16] for a more rigid or singularity-free design. The resulting coordinates for odd  $m_i$  require a sign change in the rotation-dependent terms of  $x_{mi}$  and  $y_{mi}$ . In summary

$$\begin{pmatrix} x_{mi} \\ y_{mi} \\ z_{mi} \end{pmatrix} = R_m \begin{pmatrix} \pm \cos(\Delta_i) \sin(\gamma_i) + x_i \\ \mp \cos(\Delta_i) \cos(\gamma_i) + y_i \\ \sin(\Delta_i) + z_i \end{pmatrix} \quad (10)$$

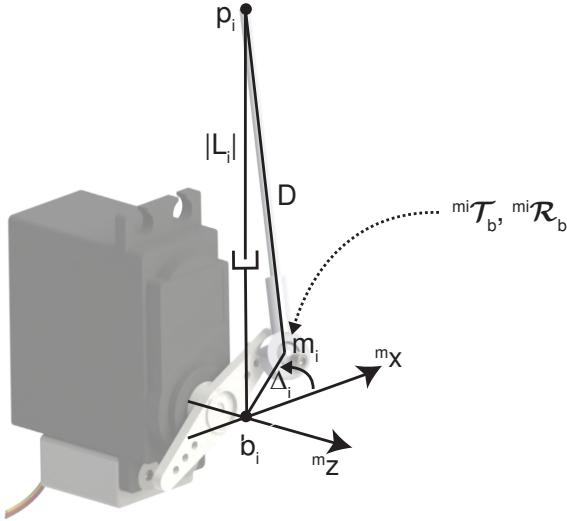


Fig. 5. Local coordinate system of a particular servo motor ( $i$ ) ( $i$  even) with indication of variables and constants involved in the computation of the length of a virtual leg  $L_i$ .

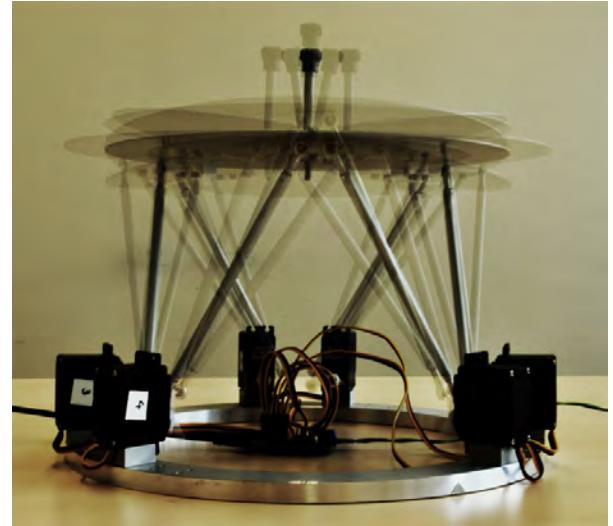


Fig. 6. Photograph of the low-cost Stewart platform overlaid with a series of images indicating the range of platform's motion.

with the upper sign corresponding to the solution for even and the lower sign for odd servos. For any desired change in position/orientation of the platform a new set of vectors  $P_i$  and subsequently virtual leg lengths  $L_i$  is obtained. The solution to the IK problem consists now in finding the set  $\Delta_i$  which satisfies

$$R_m^2 = (M_i(\Delta_i) - B_i)^T(M_i(\Delta_i) - B_i) \quad (11a)$$

$$D^2 = (P_i - M_i(\Delta_i))^T(P_i - M_i(\Delta_i)) \quad (11b)$$

$$|L_i|^2 = (P_i - B_i)^T(P_i - B_i) \quad (11c)$$

for all  $i \in \{1, \dots, 6\}$ . Combining the above equations leads to

$$|L_i|^2 - D^2 + R_m^2 = 2(B_i - M_i(\Delta_i))^T(B_i - P_i) \quad (12)$$

which after substituting from (8) resolves into

$$\pm(|L_i|^2 - D^2 + R_m^2) = 2R_m(z_{pi} - z_i)\sin(\Delta_i) + 2R_m[\sin(\gamma_i)(x_{pi} - x_i) - \cos(\gamma_i)(y_{pi} - y_i)]\cos(\Delta_i) \quad (13)$$

with the upper sign corresponding to even and the lower sign to odd servos as before. This equality is a linear combination of sine functions. Using the trigonometric identity

$$a\sin(\phi) + b\cos(\phi) = \sqrt{a^2 + b^2}\sin(\phi + \varphi) \quad \text{with } \varphi = \arctan\left(\frac{b}{a}\right) + \begin{cases} 0 & , a \geq 0 \\ \pi & , a < 0 \end{cases}$$

having

$$a_i = 2R_m(z_{pi} - z_i) \quad (14a)$$

$$b_i = 2R_m[\sin(\gamma_i)(x_{pi} - x_i) - \cos(\gamma_i)(y_{pi} - y_i)] \quad (14b)$$

$$c_i = |L_i|^2 - D^2 + R_m^2 \quad (14c)$$

and assuming  $a_i$  positive the servo angles are found to be

$$\Delta_i = \arcsin\left(\frac{\pm c_i}{\sqrt{(a_i^2 + b_i^2)}}\right) - \arctan\left(\frac{b_i}{a_i}\right). \quad (15)$$

Assuming joints with a sufficiently large angular range of motion, the platform can reach the desired position and orientation if a real solution to (15) exists for all  $i$ .

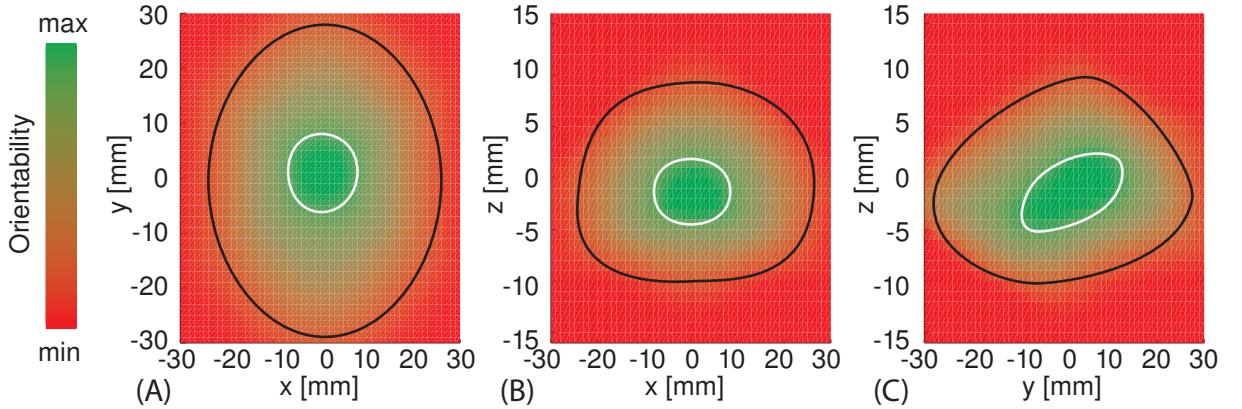


Fig. 7. Diagrams illustrating the work area of the low cost platform. Any position within the regions encircled by a black curve can be attained. Color-coded is the ability of the platform to change its orientation at a given location. Regions encircled by white curves allow the platform to change its orientation most freely.

### B. Design evaluation

The capabilities of the presented GSP with fixed rotary actuators strongly depend on the quality of the components used. Only low cost components were chosen for the prototype and these are summarized in Table II. Components for which no price is given were own manufactured. It is possible to arrive at

TABLE II  
PRICE OF THE COMPONENTS IN THE LOW COST SP DESIGN.

Component	Quantity	Unit price [USD]
Analog servo motor	6	10-20
Servo controller	1	20-40
Rod end bearing, size M2	12	4-20
Connecting rod	6	–
Holder block	9	–
Platform	1	–
Base	1	–

a final design not exceeding the budget of 200 USD. The photograph of the actually built SP is shown in Fig. 6. The photograph also indicates the range of motion of this platform. As there is no sensory feedback from the servos and no external measurements were carried out only a coarse-grained evaluation can be given here. In this context, the platform was commanded to move in  $x$ ,  $y$  and  $z$  direction as well as to perform positive and negative rotations around these axes as far as possible. The platform did not carry any additional load and the current consumption was monitored to see if the commanded position/orientation was reached - an increased current flow would indicate that one or more servos could not reach the commanded position. Since the employed servos are able to perform continuous rotation and the angular limits in the joints are large enough the platform was able to reach all the commanded positions/orientations according to the real solutions of (15). However, this pleasing result is to be ascribed to the limited variability in the length of the virtual legs and thus a relatively small work area. Fig. 7 shows the real-valued range of motion with help of 3 diagrams. The range of motion is approximately  $\pm 25$ ,  $\pm 28$  and  $\pm 15$  mm for the motion along the  $x$ ,  $y$  and  $z$  axes, respectively. Color-coded is the ability to change the orientation of the platform at a given position. This orientability was computed by sampling the intervals between the extreme rotations around all the axes at linearly equally spaced points and checking how many of these rotations can be attained at a given reachable location. A green color was assigned to the maximum number of attainable rotations and a red color was assigned to 0 – if no orientation change was possible. Any intermediate number of orientations was mapped onto a linear color ramp between

green and red. The highest orientability of the platform is at the origin where rotations of  $\pm 9^\circ$ ,  $\pm 9^\circ$  and  $\pm 15^\circ$  are possible around the  $x$ ,  $y$  and  $z$  axes, respectively.

The precision of the platform is limited mainly by the resolution of the employed servos whose angular position can be changed in the smallest increment of  $0.1^\circ$  only. The accuracy is mostly affected by manufacturing tolerances [29] and is not further discussed here. The operating speed of the servos is  $0.15 \text{ sec}/60^\circ$  which corresponds to approximately  $40 \text{ mm/s}$  for the platform. The overall weight of the low cost SP together with the servo controller and cabling is less than  $0.5 \text{ kg}$ . The platform can carry payloads of approximately a few kilograms which are much heavier than its own weight. However, this is not further elaborated on here as the load-carrying capacity depends on the current limit and would need to be evaluated with respect to a particular movement quality criterion.

#### IV. DISCUSSION

The purpose of this work was to provide a practical guideline for the construction of GSPs with further references for an interested reader. The basic theory together with the IK solution for the particular case of a GSP with fixed rotary actuators was presented. The constructed prototype was intentionally built out of commercially available low cost building blocks in order to arrive at the final design quickly. No further effort was put into a precise evaluation of the prototype. A coarse-grained evaluation of the workspace, orientability, speed and load-carrying capacity is given in the previous section. The achieved design is not meant for any particular application but is supposed to serve as a rapid prototyping example of a successful low cost and fully operational GSP construction. At the current stage the prototype is certainly not precise enough for MIS robotics in whose context the application of the SPs was discussed (see sec. I-B). Speed, precision and load-carrying capacity can all be improved easily by replacing the analog servos with digital ones. However, for an application in medical robotics a different architectural design with prismatic piezoelectric actuators is considered (see next section).

15 years ago, Ji argued in [16] that the variety of possible applications of the SPs is hampered by the lack of rational synthesis tools for a practical design. Today the situation has changed dramatically and with the availability of powerful CAD design tools incorporating simulation environments and rapid prototyping techniques like 3D ink-jet printing, laser sintering, fused deposition modeling or carbon fiber composite stitching his argument does not hold anymore. The gap between the concept and the final product fulfilling the specifications has never been so small. This should lead to a boom not only in various customized applications of GSPs but also to the appearance of competitive and commercially available surgical robots which are an interesting target group for Stewart's parallel manipulator.

#### V. FUTURE WORK

A new design of a SP is considered as a tool holder for the MIS robot RobinHeart [21]. The design objectives are small overall size and weight, minimal payload of  $5 \text{ kg}$ , high precision in the lower  $\mu\text{m}$  range and an easy integration with a variety of surgical tools. Piezoelectric prismatic actuators are considered in the new design. These actuators are highly precise with positioning capabilities in the lower nm range, can generate large displacements, do not require a gear and develop forces up to  $20 \text{ N}$  at velocities in the cm/s range [30]. The new design will be equipped with multiple sensors including force and position sensors in the legs and angular position sensors in the joints. The sensory information will be used to obtain a direct solution to the forward kinematics problem and thus provide the SP with self-calibration capabilities. Moreover, the newly designed SP will be used as a force sensor. The interaction forces with patient's body at the trocar will be reconstructed from individual force measurements in the legs in order to avoid any excessive stress on the tissue.

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