CSCE 221 Cover Page

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Please list all sources in the table below including web pages which you used to solve or implement the current homework. If you fail to cite sources you can get a lower number of points or even zero, read more: Aggie Honor System Office

Type of sources		
People		
Web pages (provide URL)		
Printed material	Fourth Edition Data Structures and Algorithm Analysis in C++	
Other Sources		

I certify that I have listed all the sources that I used to develop the solutions/codes to the submitted work.

"*On my honor as an Aggie, I have neither given nor received any unauthorized help on this academic work.*"

Tianlan Li 9/16/2023

Homework 1

Check the Canvas calendar for the deadliness.
The homework submission to Gradescope only.

Typeset your solutions to the homework problems listed below using LaTeX (or LyX). See the class webpage for information about their installation and tutorials.

Homework 1 Objectives:

1. Developing the C++ programming skills by using
   (a) templated dynamic arrays and STL vectors
   (b) tests for checking correctness of a program.
2. Comparing theory with a computation experiment in order to classify algorithm.
3. Preparing reports/documents using the professional software LaTeX or LyX.
4. Understanding the definition of the big-O asymptotic notation.
5. Classifying algorithms based on pseudocode.

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1. (25 points) Include your C++ code in the problem solution—**do not use attachments or screenshots.**
   (a) (10 points) Use the STL class `vector<int>` to write two C++ functions that return true if there exist **two** elements of the vector such that their **product** is divisible by 12, and return false otherwise. The efficiency of the first function should be $O(n)$ and the efficiency of second one should be $O(n^2)$ where n is the size of the vector.

Solution 1:
```
bool productDivisibleBy12_O_N(const std::vector<int>& nums) {
    int countDivBy2 = 0;
    int countDivBy3 = 0;
    int countDivBy4 = 0;
    int countDivBy6 = 0;

    for (int num : nums) {
        if (num % 12 == 0 && nums.size() >= 2)
            return true;
        else if (num % 6 == 0)
            countDivBy6;
        else if (num % 4 == 0)
            countDivBy4++;
        else if (num % 3 == 0)
            countDivBy3++;
        else if (num % 2 == 0)
            countDivBy2++;
    }
    // 3*4 2*6
    if (countDivBy4 >= 1 && countDivBy3 >= 1)
        return true;
    if (countDivBy2 >= 1 && countDivBy6 >= 1)
        return true;

    return false;
}
\end{lslisting}
\item[\textbf{Solution 2:}]
```

```
\begin{lstlisting}[language={C++},basicstyle={\small\ttfamily},showstringspaces=false]
bool productDivisibleBy12_O_N2(const std::vector<int>& nums) {
    int n = nums.size();

    for (int i = 0; i < n; i++) {
        for (int j = i + 1; j < n; j++) {
            if (nums[i] * nums[j] % 12 == 0)
                return true;
        }
    }
    return false;
}
```

(b) (6 points) Justify your answer by writing the running time functions in terms of n for both the algorithms and their classification in terms of big-O asymptotic notation.

Solution: The $O(n)$ approach is classified as $O(n)$, indicating a linear time complexity. The $O(n^2)$ approach is classified as $O(n^2)$, indicating a quadratic time complexity.

(c) (2 points) What do you consider as an operation for each algorithm?

Solution: In the $O(n)$ approach, operation includes modulus operation, arithmetic operations, and comparison operations. Meanwhile in the $O(n^2)$ approach, only the comparison operation combined with modulus was used.

(d) (2 points) Are the best and worst cases for both the algorithms the same in terms of big-O notation? Justify your answer.

Solution: For the $O(n)$ algorithm, the best-case scenario for this algorithm occurs when the first elements of the vector is a multiple of 12 and the size of the array is larger than 2, which the function returns true after examining just one elements, so it performs in constant time, O(1). In the worst-case scenario is when none of the pairs of elements in the vector have a product divisible by 12, which the function iterates through all 'n' elements of the vector, performing the constant-time operations during each iteration. Therefore, the worst-case time complexity is $O(n)$.
For the $O(n^2)$ algorithm, best-case scenario for this algorithm is also when the first pair of elements in the vector has a product divisible by 12, which the function returns true after examining just one pair of elements, so it performs in constant time, O(1). In the worst-case scenario occurs when none of the pairs of elements in the vector have a product divisible by 12, which the function needs to compare all possible pairs of 'n' elements in the vector using two nested loops. Therefore, the worst-case time complexity is $O(n^2)$.

(e) (5 points) Describe the situations of getting the best and worst cases, give the samples of the input for each case, and check if your running time functions match the number of operations.

Solution: For the $O(n)$ algorithm, the best-case scenario for this algorithm can be archieve using vec = {12, 1}, which is has $O(1)$ time complexity. Worst case can be archieved by vec = {1, 2, 3, 5, 7, 9, 11} which has a size of 7, and the runtime would be O(n) with n = 7.
For the $O(n^2)$ algorithm, best-case scenario can be also archieve by vec = {12, 1} with checking 1 pair of combination results in $O(1)$ time complexity. For worst-case, using the same example vec = {1, 2, 3, 5, 7, 9, 11}, the function evaluates all possible combinations which leads to a run-time of $O(n * (n - 1))/2$, or $O(21)$ operation in this case.

2. (50 points + bonus) The binary search algorithm problem.

   (a) (5 points) Implement a templated C++ function for the binary search algorithm based on the set of the lecture slides *"Analysis of Algorithms"*.

```
int Binary_Search(vector<int> &v, int x) {
   int mid, low = 0;
   int high = (int) v.size()-1;
   while (low < high) {
      mid = (low+high)/2;
      if (num_comp++, v[mid] < x) low = mid+1;
      else high = mid;
   }
   if (num_comp++, x == v[low]) return low; //OK: found
   return -1; //not found
}
```

   Be sure that before calling `Binary_Search`, elements of the vector `v` are arranged in **ascending** order. The function should also keep track of the number of comparisons used to find the target `x`. The (global) variable `num_comp` keeps the number of comparisons and initially should be set to zero.

Solution:

```
int Binary_Search(std::vector<int> &v, int x, int &num_comp) {
     int mid, low = 0;
     int high = (int) v.size() - 1;
     while (low < high) {
          mid = (low + high) / 2;

          if (num_comp++, v[mid] < x)
               low = mid+1;
          else
               high = mid;
          }
     if (num_comp++, x == v[low]) return low; //OK: found
     return -1; //not found
}
```

   (b) (10 points) Test your algorithm for correctness using a vector of data with 16 elements sorted in ascending order. An error message should be printed or exception should be thrown when the input vector is unsorted.
   What is the value of `num_comp` in the cases when

   i. the target `x` is the first element of the vector `v`

```
v = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16};
Element 1 found at index 0
Number of comparisons: 5
```

   ii. the target `x` is the last element of the vector `v`

```
v = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16};
Element 1 found at index 15
Number of comparisons: 5
```

   iii. the target `x` is in the middle of the vector `v`

```
v = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16};
Element 8 found at index 7
Number of comparisons: 5
```

   What is your conclusion from the testing for $n = 16$?

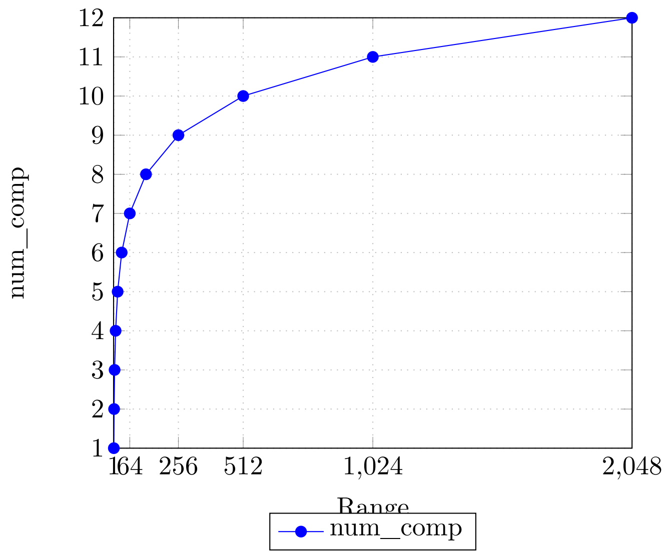
Solution: In the best-case, run time for n = 16 is $O(1)$. Worst case is $O(logn)$, or $O(log16)$

   (c) (10 points) Test your program using vectors of size $n = 2^k$ where $k = 0, 1, 2, \ldots, 11$ populated with consecutive increasing integers in these ranges: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048.

Select the target as the last element in the vector. Record the value of `num_comp` for each vector size in the table below.

Range [1,n]	Target	num_comp
[1, 1]	1	1
[1, 2]	2	2
[1, 4]	4	3
[1, 8]	8	4
[1, 16]	16	5
[1, 32]	32	6
[1, 64]	64	7
[1, 128]	128	8
[1, 256]	256	9
[1, 512]	512	10
[1, 1024]	1024	11
[1, 2048]	2048	12

(d) (5 points) Plot the number of comparisons for the vector size $n = 2^k$, $k = 0, 1, 2, \ldots, 11$. You can use a spreadsheet or any graphical package.



(e) (5 points) Provide a mathematical formula/function which takes n as an argument, where n is the vector size, and returns as its value the number of comparisons. Does your formula match the computed output for any input? Justify your answer.

Solution: $num_comp = \log_2(n)+1$ Yes, the formula matches the computed output for any input since running the code with a sorted_vector of size 1048576 only uses 21 comparisons, which matches the result using the calculated formula.

(f) (5 points) How can you modify your formula/function if the largest number in a vector is not the exact power of two? Test your program using input in ranges from 1 to $n = 2^k - 1$, $k = 0, 1, 2, \ldots, 11$ and plot the number of comparisons vs. the size of the vector.

Solution:

Range [1,n]	Target	num_comp
[1, 1]	1	1
[1, 3]	3	2
[1, 7]	7	3
[1, 15]	15	4
[1, 31]	31	5
[1, 63]	63	6
[1, 127]	127	7
[1, 255]	255	8
[1, 511]	511	9
[1, 1023]	1023	10
[1, 2047]	2047	11

(g) (5 points) Do you think the number of comparisons in the experiment above are the same for a vector of strings or a vector of doubles? Justify your answer.

Solution: No, it will not be the same for a vector of strings or doubles since those 2 data structures are not in constant-time operation. Strings sorting will entirely depends on the length of strings and size of the vector, doubles are floating point decimals which are entirely based on the order of sorting. Therefore the number of operations would not be the same.

(h) (5 points) Use the big-O asymptotic notation to classify binary search algorithm and justify your answer.

Solution: The binary search algorithm is classified as having a time complexity of O(log n), where "n" is the number of elements in the sorted array. The basis is to halve the Search Space, thefore causing a logarithmic growth in the number of comparisons as the n increases, and it efficiently divides the search space in half with each iteration. This classification is justified by the algorithm's behavior and its worst-case performance.

(i) (Bonus 10 points) Read the sections 1.6.3 and 1.6.4 from the textbook and modify the algorithm using a functional object to compare vector elements. How can you modify the binary search algorithm to handle the vector of descending elements? What will be the value of `num_comp`? Repeat the search experiment for the smallest number in the integer arrays. Tabulate the results and write a conclusion of the experiment with your justification.

Range [1,n]	Target	num_comp
[1, 1]	1	1
[2, 1]	1	2
[4, 1]	1	3
[8, 1]	1	4
[16, 1]	1	5
[32, 1]	1	6
[64, 1]	1	7
[128, 1]	1	8
[256, 1]	1	9
[512, 1]	1	10
[1024, 1]	1	11
[2048, 1]	1	12

Solution: Despite the descending order, the number of comparisons remains consistent with the worst-case time complexity of O(log n). This is because the binary search algorithm effectively divides the search space by half in each step, regardless of the order of elements.

3. (25 points) Find running time functions for the algorithms below and write their classification using big-O asymptotic notation in terms of n. A running time function should provide a formula on the number of arithmetic operations and assignments performed on the variables s, t, or c. Note that array indices start from 0.

   (a) **Algorithm Ex1(A):**

```
    Input: An array A storing $n \geq 1$ integers.
    Output: The sum of the elements in A.
$s \leftarrow A[0]$
for $i \leftarrow 1$ to $n-1$ do
    $s \leftarrow s + A[i]$
end for
return $s$
```

Solution: The function has a running time of $O(n)$. The function is excuted $O(1+(n-1))$ iterations on s.

   (b) **Algorithm Ex2(A):**

```
    Input: An array A storing $n \geq 1$ integers.
    Output: The sum of the elements at even positions in A.
$s \leftarrow A[0]$
for $i \leftarrow 2$ to $n-1$ by increments of 2 do
    $s \leftarrow s + A[i]$
end for
return $s$
```

Solution: The function has a running time of $O(n/2)$. The function is excuted $O(1+((n-2)/2)$ iterations on s.

   (c) **Algorithm Ex3(A):**

```
    Input: An array A storing $n \geq 1$ integers.
    Output: The sum of the partial sums in A.
$s \leftarrow 0$
for $i \leftarrow 0$  to $n-1$ do
    $s \leftarrow s + A[0]$
    for $j \leftarrow 1$ to $i$ do
        $s \leftarrow s + A[j]$
    end for
end for
    return $s$
```

Solution: The function has a running time of $O(n^2)$. The function is excuted $O((n-1)*(n-1))$ iterations on s.

(d) **Algorithm Ex4(A):**

```
        **Input:** An array A storing $n \geq 1$ integers.
        **Output:** The sum of the partial sums in A.
    $t \leftarrow A[0]$
    $s \leftarrow A[0]$
    for $i \leftarrow 1$ to $n-1$ do
        $s \leftarrow s + A[i]$
        $t \leftarrow t + s$
    end for
    return $t$
```

Solution: The function has a running time of $O(n)$. The function is excuted $O(1+(2n-1))$ iterations on s and t.

(e) **Algorithm Ex5(A, B):**

```
        **Input:** Arrays A and B storing $n \geq 1$ integers.
        **Output:** The number of elements in B equal to the partial sums in A.
    $c \leftarrow 0$ //counter
    for $i \leftarrow 0$ to $n-1$ do
        $s \leftarrow 0$ //partial sum
        for $j \leftarrow 0$ to $n-1$ do
            $s \leftarrow s + A[0]$
            for $k \leftarrow 1$ to $j$ do
                $s \leftarrow s + A[k]$
            end for
        end for
        if $B[i] = s$ then
            $c \leftarrow c + 1$
        end if
    end for
    return $c$
```

Solution: The function has a running time of $O(n^3)$. The function is excuted $O(1+(n-1))$ iterations on c and $O(n*(n*(n-1)))$ iterations on s.