第7章 上下文无关语言的性质

上下文无关语言的泵引理

For every context-free language L
There is an integer n, such that
For every string z in L of length ≥ n
There exists z = uvwxy such that:

1. $|vwx| \leq n$.
2. $|vx| > 0$.
3. For all $i \geq 0$, uv^iwx^iy is in L.

注意：v和x至少一个不为空

一个常见的非上下文无关语言：$L_1 = \{0^n1^n2^n \mid n \geq 1\}$

上下文无关语言的判定性质

非空性

给出其上下文无关文法，检验S是否可产生终结符字符串

成员性

动态规划的CYK算法

- Let $w = a_1 \ldots a_n$.
- We construct an n-by-n triangular array of sets of variables.
- X_{ij} = {variables A | A =>* $a_i \ldots a_j$}.
- Induction on j–i+1.
  - The length of the derived string.
- Finally, ask if S is in X_{1n}.

- Basis: X_{ii} = {A | A -> a_i is a production}.
- Induction: X_{ij} = {A | there is a production A -> BC and an integer k, with $i \leq k < j$, such that B is in X_{ik} and C is in $X_{k+1,j}$.

有穷性

- The idea is essentially the same as for regular languages.
- Use the pumping lemma constant n.
- If there is a string in the language of length between n and 2n-1, then the language is infinite; otherwise not.

为什么n到2n-1？泵引理

将这些字符串一个个用CYK算法试

上下文无关语言的封闭性

- CFL’s are closed under union, concatenation, and Kleene closure.
- Also, under reversal, homomorphisms and inverse homomorphisms.
- But not under intersection or difference.

前5个用构造文法的方法，∩举反例，差可以转化为∩的形式，所以也不封闭

逆同态是最难的：构造PDA，核心在构造一个缓冲区

1. δ’([q, ϵ], a, X) = {([q, h(a)], X)} for any input symbol *a* of P’ and any stack symbol X.
   - When the buffer is empty, P’ can reload it.
2. δ’([q, bw], ϵ, X) contains ([p, w], α) if δ(q, b, X) contains (p, α), where b is either an input symbol of P or ϵ.
   - Simulate P from the buffer.