Problem 1

	自反	对称	反对称	传递
a	×	×	√	√
b	√	√	×	√
c	√	√	×	√
d	√	√	×	×

Problem 2

“取元素 b∈A 使得{a, b}∈R”，可能 A 中不存在这样的 b.

Problem 3

a) $M_{R1\cup R2}$ =
$M_{R1} \vee M_{R2}$ =
[0 1 0]
[1 1 1]
[1 1 1]

b) $M_{R1\cap R2}$ =
$M_{R1} \wedge M_{R2}$ =
[0 1 0]
[0 1 1]
[1 0 0]

c) $M_{R2\circ R1}$ =
$M_{R1} \odot M_{R2}$ =
[0 1 1]
[1 1 1]
[0 1 0]

d) $M_{R1\circ R2}$ =
$M_{R1} \odot M_{R1}$ =
[1 1 1]
[1 1 1]
[0 1 0]

e) $M_{R1\oplus R2}$ =
$M_{R1} \oplus M_{R2}$ =
[0 0 0]
[1 0 0]
[0 1 1]

Problem 4

a) W0 =	W1 =	W2 =	W3 =	W4 =	W5 =
[0 0 1 0 0]	[0 0 1 0 0]	[0 0 1 0 0]	[1 0 1 0 0]	[1 0 1 0 0]	[1 0 1 0 0]
[0 0 0 1 0]	[0 0 0 1 0]	[0 0 0 1 0]	[0 0 0 1 0]	[0 1 0 1 0]	[0 1 0 1 0]
[1 0 0 0 0]	[1 0 1 0 0]	[1 0 1 0 0]	[1 0 1 0 0]	[1 0 1 0 0]	[1 0 1 0 0]
[0 1 0 0 0]	[0 1 0 0 0]	[0 1 0 1 0]	[0 1 0 1 0]	[0 1 0 1 0]	[0 1 0 1 0]
[0 0 0 1 0]	[0 0 0 1 0]	[0 0 0 1 0]	[0 0 0 1 0]	[0 1 0 1 0]	[0 1 0 1 0]

{(a, a), (a, c), (b, b), (b, d), (c, a), (c, c), (d, b), (d, d), (e, b), (e, d)}

b) W0 =	W1 =	W2 =	W3 =	W4 =	W5 =
[0 0 0 0 0]	[0 0 0 0 0]	[0 0 0 0 0]	[0 0 0 0 0]	[0 0 0 0 0]	[0 0 0 0 0]
[0 0 1 0 1]	[0 0 1 0 1]	[0 0 1 0 1]	[0 0 1 0 1]	[0 0 1 0 1]	[0 1 1 0 1]
[0 0 0 0 1]	[0 0 0 0 1]	[0 0 0 0 1]	[0 0 1 0 1]	[0 0 1 0 1]	[0 1 1 0 1]
[1 0 0 0 0]	[1 0 0 0 0]	[1 0 0 0 0]	[1 0 0 0 0]	[1 0 0 0 0]	[1 0 0 0 0]
[0 1 1 0 0]	[0 1 1 0 0]	[0 1 1 0 1]	[0 1 1 0 1]	[0 1 1 0 1]	[0 1 1 0 1]

{(b, b), (b, c), (b, e), (c, b), (c, c), (c, e), (d, a), (e, b), (e, c), (e, e)}

c) W0 =	W1 =	W2 =	W3 =	W4 =	W5 =
[0 1 1 0 1]	[0 1 1 0 1]	[1 1 1 0 1]	[1 1 1 0 1]	[1 1 1 0 1]	[1 1 1 1 1]

[1 0 1 0 0]   [1 1 1 0 1]   [1 1 1 0 1]   [1 1 1 0 1]   [1 1 1 0 1]   [1 1 1 1 1]
[1 1 0 0 0]   [1 1 1 0 1]   [1 1 1 0 1]   [1 1 1 0 1]   [1 1 1 0 1]   [1 1 1 1 1]
[1 0 0 0 0]   [1 1 1 0 1]   [1 1 1 0 1]   [1 1 1 0 1]   [1 1 1 1 1]   [1 1 1 1 1]
[0 0 0 1 0]   [0 0 0 1 0]   [0 0 0 1 0]   [0 0 0 1 0]   [1 1 1 1 1]   [1 1 1 1 1]
{(a, a), (a, b), (a, c), (a, d), (a, e), (b, a), (b, b), (b, c), (b, d), (b, e), (c, a), (c, b), (c, c), (c, d), (c, e), (d, a), (d, b), (d, c) (d, d), (d, e), (e, a), (e, b), (e, c) (e, d), (e, e)}

d) W0 =   W1 =   W2 =   W3 =   W4 =   W5 =
[0 0 0 0 1]   [0 0 0 0 1]   [0 0 0 0 1]   [0 0 0 0 1]   [0 0 0 0 1]   [1 1 1 1 1]
[1 0 0 1 0]   [1 0 0 1 1]   [1 1 0 1 1]   [1 0 0 1 1]   [1 0 1 1 1]   [1 1 1 1 1]
[0 0 0 1 0]   [0 0 0 1 0]   [0 0 0 1 0]   [0 0 0 1 0]   [1 0 1 1 1]   [1 1 1 1 1]
[1 0 1 0 0]   [1 0 1 0 1]   [1 0 1 0 1]   [1 0 1 1 1]   [1 0 1 1 1]   [1 1 1 1 1]
[1 1 1 0 1]   [1 1 1 0 1]   [1 1 1 1 1]   [1 1 1 1 1]   [1 1 1 1 1]   [0 1 1 1 1]
{(a, a), (a, b), (a, c), (a, d), (a, e), (b, a), (b, b), (b, c), (b, d), (b, e), (c, a), (c, b), (c, c), (c, d), (c, e), (d, a), (d, b), (d, c) (d, d), (d, e), (e, a), (e, b), (e, c) (e, d), (e, e)}

Problem 5
a+b = b+a，则((a, b), (a, b))∈R, R 是自反的.
((a, b), (c, d))∈R, a+d = b+c，即 c+b = d+a, ((c, d), (a, b))∈R, R 是对称的.
((a, b), (c, d)), ((c, d), (e, f))∈R, a+d = b+c 且 c+f = d+e,
则 a+d+c+f = b+c+d+e, a+f = b+e, ((a, b), (e, f))∈R, R 是传递的，则 R 是等价关系.

Problem 6
x 与 x 的前三位相同, (x, x)∈R, R 自反.
(x, y)∈R, x 与 y 的前三位相同, y 与 x 的前三位相同, (y, x)∈R, R 对称.
(x, y), (y, z)∈R, x 与 y 的前三位相同, y 与 z 的前三位相同,
则 x 与 z 的前三位相同, (x, z)∈R, R 传递，则 R 是等价关系.

Problem 7
n 元素集合上有 2^(n^2)个关系，其中自反的有 2^(n^2-n)
a) 对称的有 2^n×2^C(n, 2) = 2^(n + n(n-1)/2) = 2^(n(n+1)/2)
b) 反对称的有 2^n×3^C(n, 2) = 2^n×3^(n(n-1)/2)
c) 非对称的有 2^(n^2) − 2^(n(n+1)/2)
d) 反自反的有 2^(n^2-n)
e) 自反的和对称的有 2^C(n, 2) = 2^(n(n-1)/2)
f) 既不自反也不对称的有 2^(n^2) − 2^(n^2-n) − 2^(n(n+1)/2) + 2^(n(n-1)/2)
= 2^n(2^n–2^(n-1)–2^(n+1 / 2)+2^(n-1 / 2) = 2^n×(2^(n-1)–2^(n-1 / 2))

Problem 8
a) 关系 R 的对称闭包为(R∪R^-1)，它的自反闭包为(R∪R^-1)∪I_A
关系 R 的自反闭包为(R∪I_A)，它的为对称闭包为(R∪I_A)∪(R∪I_A)^-1
= (R∪I_A)∪(R^-1∪I_A^-1) =(R∪I_A)∪(R^-1∪I_A) = (R∪R^-1)∪I_A
b) 假设(a, b)∈s(t(R))，则(a, b)∈t(R)或(b, a)∈t(R).
则 R 中存在一条从 a 到 b 或从 b 到 a 的路径(步数不限),

则 $s(R)$中存在从 a 到 b 和从 b 到 a 的两条路径, $(a, b)\in t(s(R))$.

Problem 9

a) 否，可能不传递，如{(1, 1), (1, 2), (2, 1), (2, 2)}与{(1, 1), (1, 3), (3, 1), (3, 3)}

b) 是, $x\in S_{(R1\cap R2)}$, $x\in S1$ 且 $x\in S2$, $(x, x)\in R1$ 且$(x, x)\in R2$, $(x, x)\in R1\cap R2$

$(x, y)\in R1\cap R2$, $(x, y)\in S1$ 且$(x, y)\in S2$, $(y, x)\in R1$ 且$(y, x)\in R2$, $(y, x)\in R1\cap R2$

$(x, y), (y, z)\in R1\cap R2$, $(x, y), (y, z)\in S1$ 且$(x, y), (y, z)\in S2$,

$(x, z)\in R1$ 且$(x, z)\in R2$, $(x, z)\in R1\cap R2$, $R1\cap R2$ 自反，对称且传递.

c) 否，可能不自反，如{(1, 1)}与{(1, 1), (1, 2), (2, 1), (2, 2)}

Problem 10

否，如{1, 2, 3}上的关系{(1, 2), (3, 2)}，传递闭包为{(1, 2), (3, 2)},

传递闭包的自反闭包为{(1, 1), (1, 2), (2, 2), (3, 2), (3, 3)},

传递闭包的自反闭包的对称闭包为{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)},

(1, 2), (2, 3)都属于这个闭包，但(1, 3)不属于，闭包不传递，不是等价关系.