Problem 1

设存在元素 y 使得 $x \vee y=1$ 且 $x \wedge y=0$，则

$y = y \wedge 1 = y \wedge (x \vee \bar{x}) = (y \wedge x) \vee (y \wedge \bar{x}) = (x \wedge y) \vee (y \wedge \bar{x}) = 0 \vee (y \wedge \bar{x}) =$

$y \wedge \bar{x} = (\bar{x} \wedge y) \vee 0 = (\bar{x} \wedge y) \vee (x \wedge \bar{x}) = \bar{x} \wedge (y \vee x) = \bar{x} \wedge (x \vee y) = \bar{x} \wedge 1 = \bar{x}$

存在唯一的元素 $y= \bar{x}$ 使得 $x \vee y=x \vee \bar{x}=1$, $x \wedge y=x \wedge \bar{x}=0$

Problem 2

a) 成立，列出真值表可证

x y z	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
$x \oplus (y \oplus z)$	0	1	1	0	1	0	0	1
$(x \oplus y) \oplus z$	0	1	1	0	1	1	0	1

b) 不成立，取 x=1, y=1, z=1, $1+(1 \oplus 1)=1+0=1$, $(1+1) \oplus (1+1)=1 \oplus 1=0$

c) 不成立，取 x=1, y=1, z=0, $1 \oplus (1+0)=1 \oplus 1=0$, $(1 \oplus 1)+(1 \oplus 0)=0+1=1$

Problem 3

$a \leqslant b \Leftrightarrow a \wedge b=a$，则 $a \wedge b' = (a \wedge b) \wedge b' = a \wedge (b \wedge b') = a \wedge 0 = 0$

$a \wedge b'=0 \Leftrightarrow a' \vee b = (a \wedge b')' = 1$

$a' \vee b=1 \Leftrightarrow a = a \wedge 1 = a \wedge (a' \vee b) = (a \wedge a') \vee (a \wedge b) = 0 \vee (a \wedge b) = a \wedge b) \Leftrightarrow a \leqslant b$

Problem 4

易见$\oplus$运算在 B 上封闭，又对任意 x, y, z∈B，有

$(x \oplus y) \oplus z = (((x \wedge y') \vee (x' \wedge y)) \wedge z') \vee (((x \wedge y') \vee (x' \wedge y))' \wedge z)$

$x \oplus (y \oplus z) = (x \wedge ((y \wedge z') \vee (y' \wedge z))') \vee (x' \wedge ((y \wedge z') \vee (y' \wedge z)))$

则$(x \oplus y) \oplus z = x \oplus (y \oplus z) = (x \wedge y \wedge z) \vee (x \wedge y' \wedge z')$，$\oplus$满足结合性

又对任意 x∈B 有 $x \oplus 0 = 0 \oplus x = 0$，则 0 是单位元

对任意 x∈B 有 $x \oplus x = x \oplus x = 0$，任意 x 是自身的逆元, $\langle B, \oplus \rangle$构成群

Problem 5

对任意 a, b, c∈B，若 $a \leqslant c$ 则有 $a \vee c=c$, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) = (a \vee b) \wedge c$

Problem 6

运用数学归纳法，当 n=2 时$(a1 \vee a2)'=a1' \wedge a2'$, $(a1 \wedge a2)'=a1' \vee a2'$，即德摩根律

假设对于 n=k 命题成立，则对 n=k+1 有

1) $(a1 \vee a2 \vee \cdots \vee ak+1)' = ((a1 \vee a2 \vee \cdots \vee ak) \vee ak+1)' =$
   $(a1' \wedge a2' \wedge \cdots \wedge ak') \wedge ak+1' = a1' \wedge a2' \wedge \cdots \wedge ak' \wedge ak+1'$

2) $(a1 \wedge a2 \wedge \cdots \wedge ak+1)' = ((a1 \wedge a2 \wedge \cdots \wedge ak) \wedge ak+1)' =$
   $(a1' \vee a2' \vee \cdots \vee ak') \vee ak+1' = a1' \vee a2' \vee \cdots \vee ak' \vee ak+1'$

命题对 n=k+1 也成立，由数学归纳法 n 对全部 n∈N 且 $n \geqslant 2$ 恒成立，证毕

Problem 7

对任意 a, b∈B，若 $a \leqslant b$，有 $a \wedge b=a$, $a' \vee b' = (a \wedge b)' = a'$，则 $b' \leqslant a'$

反之若 $b' \leqslant a'$，有 $a' \wedge b' = b'$, $a \vee b = (a' \wedge b')' = (b')' = b$，即 $a \leqslant b$

Problem 8

任一有限布尔代数 B 同构于 B 中所有的原子构成集合 A 的幂集代数系统 P(A)

则两个有限布尔代数同构的充分必要条件是元素个数相同

已知 $B1 \cong B2, B2 \cong B3$，则 $|B1| = |B2|, |B1| = |B3|$，则 $B2 \cong B3$

Problem 9

1) 0-7 之间的斐波那契数有 1, 2, 3, 5，则 F 的真值表为

x y z	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
F	0	1	1	1	0	1	0	0

2) 由真值表可得 $F = \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z$

3) 根据真值表作卡诺图如下，则有 $F = \bar{x}y + \bar{y}z$

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x				1
$\bar{x}$	1	1		1