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1. False. Let $f = \lambda x \in \boldsymbol{N}.\, x+1$, $\emptyset \in \mathcal{P}(f)$ and $\{(0,1)\} \in \mathcal{P}(f)$. If there exists $(A \subseteq \boldsymbol{N})$ and $(B \subseteq \boldsymbol{N})$ that satisfy $\mathcal{P}(f) \subseteq A \to B$, there must be $\emptyset \in A \to B$ and $\{(0,1)\} \in A \to B$. Consider $A \to B$ represent the set of all *total* functions from A to B, $dom(A \to B) = A$, $A = \emptyset$ and $A = \{0\}$ cannot both be true, thus such A, B do not exist. In fact, for any $f \neq \emptyset$, such A and B cannot be found.

2. False. Suppose for f there is $f(0) = 42$, for h there is $h(1) = 42$, and for g, there exists no such y so that $g(y) = 42$, so for f and g, $\forall x, y.\, (f(x) = 42) \wedge (g(y) = 42) \implies (x = y)$, thus $f \bowtie g$, for g and h, $\forall x, y.\, (g(x) = 42) \wedge (h(y) = 42) \implies (x = y)$, thus $g \bowtie h$, but for f and h, there exists $f(0) = 42$ and $h(1) = 42$, let $x = 0$ and $y = 1$, $x \neq y$, thus $f \bowtie h$ is not true.

3. False. Consider H being the set of all functions from S to $\boldsymbol{N}$ where S is a finite subset of $\boldsymbol{N}$, let $h_1 = \{(0,0)\}$ and $h_2 = \{(0,1)\}$, so that $(h_1 \in H)$ and $(h_2 \in H)$, however $(h_1 \cup h_2) = \{(0,0),(0,1)\}$ is not even a function, thus $(h_1 \cup h_2) \notin H$.

4. True. Consider any $h \in H$, for any x, $hclosed(h(x \rightsquigarrow 42))$ implies $closed(dom(h) \cup \{x\}, h(x \rightsquigarrow 42)$, that is to say, $\forall l, l'.\, (l \in dom(h) \cup \{x\}) \wedge (l' = h\{x \rightsquigarrow 42\}(l)) \implies l' \in dom(h) \cup \{x\}$. If $hclosed(h)$ is not true, there exist h_0, l_0, l_0' so that $l_0 \in dom(h_0)$, $l_0' = h_0(l_0)$ and $l_0' \notin dom(h_0)$. Suppose this $l_0' = 42$, that is to say, $42 \notin dom(h_0)$, let $l = x$ and $l' = h_0\{x \rightsquigarrow 42\}(x) = 42$, $l' \in dom(h_0) \cup \{x\}$ is not true for any x but only when $x = 42$. Suppose this $l_0' \neq 42$, let $l = l_0$ and $l' = h\{x \rightsquigarrow 42\}(x) = l_0'$, $l' \in dom(h) \cup \{x\}$ is only true when $x = l_0'$. Either ways such h_0, l_0, l_0' do not exist, so $hclosed(h)$.