FLA (Fall 2021) – Assignment 1

Name: ________________ Dept: ________________

Grade: ________________ ID: ________________

Due: Oct. 26, 2021

Problem 1

Provide DFAs and REs of the following languages. In all parts, the alphabet $\Sigma = \{0, 1\}$ and $|v|_\omega$ means the number of substring v occurrences in string ω.

a. $\{\omega \mid |101|_\omega = 0\ \}$

b. $\{\omega \mid |0|_\omega \bmod 3 \equiv 0 \wedge |1|_\omega \bmod 2 \equiv 0\ \}$ (DFA only)

c. $\{\omega \mid |01|_\omega = |10|_\omega\ \}$

d. $\{\omega \mid$ every four consecutive symbols in ω contains at least two 0 $\}$ (DFA only)

e. $\{\omega \mid |01|_\omega \bmod 2\ \equiv\ 0\ \}$

Solution.

Problem 2

Let $R = (\mathbf{a} + \mathbf{b})^*(\mathbf{b} + \mathbf{c})^*\mathbf{ab}(\mathbf{a} + \mathbf{c})^*$.

a. Convert R to an ϵ-NFA

b. Convert the ϵ-NFA to a DFA by subset construction

Solution.

Problem 3

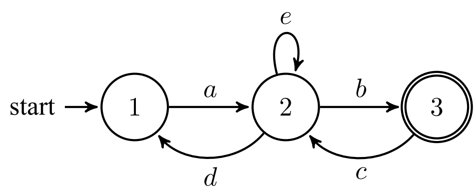
Give a DFA as figure below, please give the regular expression for the following R_{ij}^{k}, and try to simplify the expressions as much as possible.

a. All the REs R_{ij}^{0}

b. All the REs R_{ij}^{1}

c. All the REs R_{ij}^{2}

d. The RE for this DFA



Solution.

Problem 4

Prove that the following languages are not regular. You may use the pumping lemma and the closure properties of the class of regular languages.

a. $\{\omega 2\omega \mid \omega \in \{0,\ 1\}^*\ \}$

b. $\{0^a 1^b 2^c \mid a,\ b,\ c \geq 0 \wedge$ if a = 1, then b = c $\}$

c. $\{0^a 1^b \mid \gcd(a,\ b) = 2 \wedge a,\ b \geq 0\ \}$

d. $\{0^a 1 0^b 1 0^{max(a,\ b)} \mid a,\ b \in \mathbb{Z}\ \}$

Proof.

Problem 5

We define an operation ***three*** on strings as $three(c1c2c3c4c5c6...) = c3c6...$ then the above-described definition is extended to languages. Prove that the class of regular languages is closed under this operation.

Proof.

Problem 6

We define an operation **min** for language L to be

$$min(L) = \{w | w \; is \; in \; L, but \; no \; proper \; prefix \; of \; w \; is \; in \; L\}$$

In other words, $min(L)$ is the set of strings in L, and for each string $w \in L$, there is no $u \in L$, $v \in \Sigma^+$, such that $w = uv$. For example, if $L = ab*$, then $min(L) = a$. If $L = a * b$, then $min(L) = a * b$. If $L = a * b*, min(L) = \lambda$. Prove that the class of regular languages is closed under min operation.

Proof.

Problem 7

Prove or disprove the following statement:

a. If A is a language over alphabet Σ, h is a homomorphism on Σ and A is not regular, then $h(A)$ is not regular.

b. If A and B are not regular languages and C is a language such that $A \subseteq C \subseteq B$, then C is not regular.

Solution.

Problem 8

Let A and B be languages over $\Sigma = \{0, 1\}$. Define $N_0(w)$ is the number of 0s that string w contains and $N_1(w)$ is the number of 1s that string w contains. Define

$$A \sim_0 B = \{a \in A \mid \text{for some } b \in B,\, N_0(a) = N_0(b)\}$$
$$A \sim_{01} B = \{a \in A \mid \text{for some } b \in B,\, N_0(a) = N_0(b) \text{ and } N_1(a) = N_1(b)\}$$

a. Show that the class of regular languages is closed under $\sim_0$ operation.

b. Show that the class of regular languages is not closed under $\sim_{01}$ operation.

Proof.