Problem 1

$(A\cap B)\times(A\cup B)=\varnothing$, 若$(A\cap B)\neq\varnothing$ 且$(A\cup B)\neq\varnothing$, $\exists x\in A$, $\exists y\in B$,

则$\exists(x, y)\in(A\cap B)\times(A\cup B)$, 矛盾，可知$((A\cap B)=\varnothing)\vee((A\cup B)=\varnothing)$.

假设$(A\cap B)\neq\varnothing$, 根据析取三段论, $(A\cup B)=\varnothing$.

$A=\varnothing$, $B=\varnothing$, $(A\cap B)=\varnothing$, 矛盾，故$(A\cap B)=\varnothing$.

Problem 2

a) $\{x\in Z \mid x\geqslant 1\}$ b) $\varnothing$ c) $\{x\in Z \mid x<0 \wedge x>1\}=\{x\in Z \mid x\neq 0 \wedge x\neq 1\}$

Problem 3

a) 不能，取 $A=\{0\}$, $B=\{1\}$, $C=U$, $A\cup C=U$, $B\cup C=U$, $A\cup C=B\cup C$, $A\neq B$.

b) 不能，取 $A=\{0\}$, $B=\{1\}$, $C=\varnothing$, $A\cap C=\varnothing$, $B\cap C=\varnothing$, $A\cap C=B\cap C$, $A\neq B$.

c) 能. 取 $x\in A$, $x\in A\cup C$, 又 $A\cup C=B\cup C$, 则 $x\in B\cup C$.

1° 若 $x\in C$, 得 $x\in A\cap C$, 又 $A\cap C=B\cap C$, 则 $x\in B\cap C$, $x\in B$.

2° 若 $x\notin C$, 由 $x\in B\cup C$ 得 $x\in B$.

综上所述可知 $A\subseteq B$, 同理取 $y\in B$ 可证 $B\subseteq A$, $A=B$.

Problem 4

$A\subseteq B$ 可表示为$\forall x(x\in A\rightarrow x\in B)$.

$\forall x(x\in A\rightarrow x\in B)\equiv\forall x(x\notin A\vee x\in B)\equiv\forall x(x\notin\sim B\vee x\in\sim A)\equiv\forall x(x\in\sim B\rightarrow x\in\sim A)$

由$\forall x(x\in\sim B\rightarrow x\in\sim A)$可得$\sim B\subseteq\sim A$.

Problem 5

a) $A\oplus A=(A\cup A)-(A\cap A)=A-A=A\cap\sim A=\varnothing$

b) $A\oplus U=(A\cup U)-(A\cap U)=U-A=U\cap\sim A=\sim A$

Problem 6

对 $Ai=\{x \mid x\in Z \wedge x\leqslant i\}$, $Ai\subseteq Ai+1$, 则:

a) $\cup n$ $i=1$ $Ai=A1\cup A2\cup\cdots\cup Ai=Ai=\{x \mid x\in Z\wedge x\leqslant i\}$

b) $\cap n$ $i=1$ $Ai=A1\cap A2\cap\cdots\cap Ai=A1=\{x \mid x\in Z\wedge x\leqslant 1\}$

Problem 7

$\rho(A)=\{\varnothing, \{\varnothing\}, \{\{1\}\}, \{\{1\}, \varnothing\}, \{\{1, 2\}\}, \{\{1, 2\}, \varnothing\}, \{\{1, 2\}, \{1\}\}, \{\{1, 2\}, \{1\}, \varnothing\}\}$

a) $\cup\rho(A)=\varnothing\cup\{\varnothing\}\cup\{\{1\}\}\cup\{\{1\}, \varnothing\}\cup\{\{1, 2\}\}\cup\{\{1, 2\}, \varnothing\}\cup\{\{1, 2\}, \{1\}\}\cup\{\{1, 2\}, \{1\}, \varnothing\}$

$\cup\rho(A)=\{\{1, 2\}, \{1\}, \varnothing\}=A$

b) $\cap\cup\rho(A)=\cap A=\{1, 2\}\cap\{1\}\cap\varnothing=\varnothing$