Problem 1

1. $\exists x(P(x) \wedge Q(x))$
2. $\forall x\neg(P(x) \vee Q(x))$
3. $\forall x(P(x) \rightarrow Q(x))$
4. $\forall x(P(x) \vee Q(x))$

Problem 2

a) $(\forall xP(x)) \vee A \equiv \forall xP(x) \vee \forall xA \equiv \forall x(P(x) \vee A)$

b) $(\exists xP(x)) \vee A \equiv \exists xP(x) \vee \exists xA \equiv \exists x(P(x) \vee A)$

Problem 3

a) $\exists !xP(x) \rightarrow \exists xP(x) \equiv 1$

b) 当论域只含一个元素时真值为 1，否则真值为 0.

c) $\exists !x\neg P(x) \rightarrow \exists x\neg P(x) \equiv 1$, $\exists x\neg P(x) \equiv \neg\forall xP(x)$, $\exists !x\neg P(x) \rightarrow \neg\forall xP(x) \equiv 1$.

Problem 4

$\exists x\forall a\forall b\forall c((x>0) \wedge (x \neq a^2+b^2+c^2))$ // x, a, b, c 的论域为整数

Problem 5

a) $P(1, 1) \wedge P(1, 2) \wedge P(1, 3) \wedge P(2, 1) \wedge P(2, 2) \wedge P(2, 3) \wedge P(3, 1) \wedge P(3, 2) \wedge P(3, 3)$

b) $(P(1, 1) \vee P(2, 1) \vee P(3, 1)) \wedge (P(1, 2) \vee P(2, 2) \vee P(3, 2)) \wedge (P(1, 3) \vee P(2, 3) \vee P(3, 3))$

Problem 6

{1, 2}使逻辑公式为真, {1, 2, 3}使逻辑公式为假.

Problem 7

$\neg\exists x\forall yP(x, y) \equiv \forall x\neg\forall yP(x, y) \equiv \forall x\exists y\neg P(x, y)$

Problem 8

因为$\exists x\neg P(x)$ //前提

根据存在例示，有某个 a, $\neg P(a)$成立.

$\forall x(P(x) \vee Q(x))$ //前提

根据存在例示，有某个 a, $P(a) \vee Q(a)$成立.

根据析取三段论，得到 Q(a).

$\forall x(\neg Q(x) \vee S(x))$ //前提

根据存在例示，有某个 a, $\neg Q(a) \vee S(a)$成立.

根据析取三段论，得到 S(a).

$\forall x(R(x) \rightarrow \neg S(x)) \equiv \forall x(\neg R(x) \vee \neg S(x))$ //前提

根据存在例示，有某个 a, $\neg R(a) \vee \neg S(a)$成立.

根据析取三段论，得到$\neg R(a)$.

根据存在生成，得到$\exists x\neg R(x)$.