FLA (Fall 2021) – Assignment 3

Name: ______________ Dept: ______________

Grade: ______________ ID: ______________

Due: 30 Dec 2021

Problem 1

Consider the (deterministic) Turing machine M given by

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_2\})$$

which has exactly four transitions defined in it, as described below.

1. $\delta(q_0, a) = (q_0, B, R)$
2. $\delta(q_0, b) = (q_1, B, R)$
3. $\delta(q_1, b) = (q_1, B, R)$
4. $\delta(q_1, B) = (q_2, B, R)$

Please answer the following questions:

a. Specify the execution trace of M on the input string aab.

b. Provide a regular expression for the language of the Turing machine.

c. Suppose we added the transition $\delta(q_1, a) = (q_0, B, R)$ to the above machine, provide a regular expression for the language of the resulting Turing machine.

Solution.

Problem 2

a. For a string $s \in \{0,1,2\}^*$ and a symbol $d \in \{1,2,3\}$. Let #(s,d) denote the number of times d appears in s. For example, #(0120012, 0) = 3. Consider the language:

$$L = \{u\#w | u, w \in \{0,1,2\}^*, \#(u,0) \leq \#(w,0), \#(u,1) \leq \#(w,1), \#(u,2) \leq \#(w,2)\}.$$

For example, 202102#0011222 $\in L$. Construct a TM M_1 that decides this language.

b. Design a TM M_2 to decides the languages $\{uu^R | u \in a, b^*\}$

Solution.

Problem 3

Prove or disprove the following languages are decidable(not use Rice's Theorem):

a. The set L of codes for TM's that never make a move left on any input.

b. The set L of codes for TM's that, when started with the blank tape will eventually write some nonblank symbol on its tape.

Proof.

Problem 4

We label the regular languages as A, the context-free languages as B, the recursive languages as C, the recursively enumerable languages as D, and the all possible languages as E. Please answer the most appropriate label of following languages. For example, the language of balanced parentheses can be labeled as E, but the correct answer is B.

a. The difference between a context-free language and a regular language can be labeled as:

b. The difference between a language in $\mathcal{P}$ and a recursive language can be labeled as:

c. If L_1 is recursive and L_2 is recursively enumerable, then $L_1 \cup$ $the\ complement\ of\ L_2$ can be labeled as:

d. If L_1 is recursively enumerable and L_2 is recursive then $L_1 \cup$ $the\ complement\ of\ L_2$ can be labeled as:

e. $\{ a^i b^j c^k d^l \mid i = j$ and $j = k \}$ can be labeled as:

f. $\{ a^i b^j c^k d^l \mid i = l$ and $j = k \}$ can be labeled as.

Solution.

Problem 5

As classes of languages, $\mathcal{P}$ and $\mathcal{NP}$ each have certain closure properties. Prove or disprove that $\mathcal{P}$ and $\mathcal{NP}$ are closed under each of the following operations:

a. Concatenation.

b. Complementation. ($\mathcal{P}$ only)

Solution.

Problem 6

Two space aliens walk into your home, both claiming to be oracles for the Boolean Satisfiability (SAT) decision problem. They both always give a yes/no answer in constant time for any SAT instance, and are each self-consistent (i.e. each always gives the same answer for the same instance). However, one is a true oracle and the other is a shameless impostor, and you have a large SAT instance F upon which they disagree (i.e. one claims that this SAT instance F satisfiable and the other one claims that F is not satisfiable). Show that it is possible to expose the impostor within time polynomial in the size of that SAT instance F.

Problem 7

Let A and B be two disjoint languages (i.e. their intersection is empty). A language C is said to separate languages A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that if $\overline{A}$ and $\overline{B}$ are recursively enumerable, then A and B are separable by some decidable language.

Proof.