191220154 AsLambda

1. $(\lambda f.\, \lambda x.\, f(f\, x))(\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)(\lambda z.\, \lambda w.\, z)$

(a) the normal order reduction sequence

$(\lambda f.\, \lambda x.\, f(f\, x))(\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)(\lambda z.\, \lambda w.\, z)$

$\to\ (\lambda x.\, (\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)((\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)\, x)))(\lambda z.\, \lambda w.\, z)$

$\to\ (\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)((\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)(\lambda z.\, \lambda w.\, z))$

$\to\ \lambda x.\, \lambda y.\, ((\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)(\lambda z.\, \lambda w.\, z))\, y\, x$ *

$\to\ \lambda x.\, \lambda y.\, (\lambda x.\, \lambda y.\, (\lambda z.\, \lambda w.\, z)\, y\, x)\, y\, x$

$\to\ \lambda x.\, \lambda y.\, (\lambda u.\, (\lambda z.\, \lambda w.\, z)\, u\, y)\, x$

$\to\ \lambda x.\, \lambda y.\, ((\lambda z.\, \lambda w.\, z)\, x\, y)$

$\to\ \lambda x.\, \lambda y.\, ((\lambda w.\, x)\, y)$

$\to\ \lambda x.\, \lambda y.\, x$

(b) the first canonical form is reached at mark *

$\to\ \lambda x.\, \lambda y.\, ((\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)(\lambda z.\, \lambda w.\, z))\, y\, x$

(c) the eager evaluation sequence

$(\lambda f.\, \lambda x.\, f(f\, x))(\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)(\lambda z.\, \lambda w.\, z)$

$(\lambda f.\, \lambda x.\, f(f\, x))(\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)$

$\lambda f.\, \lambda x.\, f(f\, x)\ \Rightarrow_E\ \lambda f.\, \lambda x.\, f(f\, x)$

$\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x\ \Rightarrow_E\ \lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x$

$\lambda x.\, (\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)((\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)\, x))$

$\Rightarrow_E\ \lambda x.\, (\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)((\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)\, x))$

$\Rightarrow_E\ \lambda x.\, (\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)((\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)\, x))$

$\lambda z.\, \lambda w.\, z\ \Rightarrow_E\ \lambda z.\, \lambda w.\, z$

$(\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)((\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)\, (\lambda z.\, \lambda w.\, z))$

$\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x\ \Rightarrow_E\ \lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x$

$(\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x)\, (\lambda z.\, \lambda w.\, z)$

$\lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x\ \Rightarrow_E\ \lambda b.\, \lambda x.\, \lambda y.\, b\, y\, x$

$\lambda z.\, \lambda w.\, z\ \Rightarrow_E\ \lambda z.\, \lambda w.\, z$

$\lambda x.\, \lambda y.\, (\lambda z.\, \lambda w.\, z)\, y\, x\ \Rightarrow_E\ \lambda x.\, \lambda y.\, (\lambda z.\, \lambda w.\, z)\, y\, x$

$\Rightarrow_E\ \lambda x.\, \lambda y.\, (\lambda z.\, \lambda w.\, z)\, y\, x$

$\lambda x.\, \lambda y.\, (\lambda x.\, \lambda y.\, (\lambda z.\, \lambda w.\, z)\, y\, x)\, y\, x$

$\Rightarrow_E\ \lambda x.\, \lambda y.\, (\lambda x.\, \lambda y.\, (\lambda z.\, \lambda w.\, z)\, y\, x)\, y\, x$

$\Rightarrow_E\ \lambda x.\, \lambda y.\, (\lambda x.\, \lambda y.\, (\lambda z.\, \lambda w.\, z)\, y\, x)\, y\, x$

$\Rightarrow_E\ \lambda x.\, \lambda y.\, (\lambda x.\, \lambda y.\, (\lambda z.\, \lambda w.\, z)\, y\, x)\, y\, x$

$(\lambda f.\, f\, f)(\lambda f.\, \lambda x.\, f\, (f\, x))$

(a) the normal order reduction sequence

$(\lambda f.\, f\, f)(\lambda f.\, \lambda x.\, f\, (f\, x))$

$\rightarrow\ (\lambda f.\, \lambda x.\, f\, (f\, x))(\lambda f.\, \lambda x.\, f\, (f\, x))$

$\rightarrow\ \lambda y.\, (\lambda f.\, \lambda x.\, f\, (f\, x))((\lambda f.\, \lambda x.\, f\, (f\, x))\, y)$ *

$\rightarrow\ \lambda y.\, \lambda z.\, ((\lambda f.\, \lambda x.\, f\, (f\, x))\, y)(((\lambda f.\, \lambda x.\, f\, (f\, x))\, y)\, z)$

$\rightarrow\ \lambda y.\, \lambda z.\, (\lambda x.\, y\, (y\, x))(((\lambda f.\, \lambda x.\, f\, (f\, x))\, y)\, z)$

$\rightarrow\ \lambda y.\, \lambda z.\, y\, (y\, (((\lambda f.\, \lambda x.\, f\, (f\, x))\, y)\, z))$

$\rightarrow\ \lambda y.\, \lambda z.\, y\, (y\, ((\lambda x.\, y\, (y\, x))\, z))$

$\rightarrow\ \lambda y.\, \lambda z.\, y\, (y\, (y\, (y\, z)))$

(b) the first canonical form is reached at mark *

$\rightarrow\ \lambda y.\, (\lambda f.\, \lambda x.\, f\, (f\, x))((\lambda f.\, \lambda x.\, f\, (f\, x))\, y)$

(c) the eager evaluation sequence

$(\lambda f.\, f\, f)(\lambda f.\, \lambda x.\, f\, (f\, x))$

$\quad \lambda f.\, f\, f\ \Rightarrow_E\ \lambda f.\, f\, f$

$\quad \lambda f.\, \lambda x.\, f\, (f\, x)\ \Rightarrow_E\ \lambda f.\, \lambda x.\, f\, (f\, x)$

$\quad (\lambda f.\, \lambda x.\, f\, (f\, x))(\lambda f.\, \lambda x.\, f\, (f\, x))$

$\quad\quad \lambda f.\, \lambda x.\, f\, (f\, x)\ \Rightarrow_E\ \lambda f.\, \lambda x.\, f\, (f\, x)$

$\quad\quad \lambda f.\, \lambda x.\, f\, (f\, x)\ \Rightarrow_E\ \lambda f.\, \lambda x.\, f\, (f\, x)$

$\quad \lambda y.\, (\lambda f.\, \lambda x.\, f\, (f\, x))((\lambda f.\, \lambda x.\, f\, (f\, x))\, y)$

$\quad \Rightarrow\ \lambda y.\, (\lambda f.\, \lambda x.\, f\, (f\, x))((\lambda f.\, \lambda x.\, f\, (f\, x))\, y)$

$\Rightarrow\ \lambda y.\, (\lambda f.\, \lambda x.\, f\, (f\, x))((\lambda f.\, \lambda x.\, f\, (f\, x))\, y)$

2. (a) 3 new typing rules, one for each new form of term

$$\frac{}{\mathit{\Gamma} \vdash \mathit{None} : \mathit{option}\ \tau} \qquad \frac{\mathit{\Gamma} \vdash M : \tau}{\mathit{\Gamma} \vdash \mathit{Some}\ M : \mathit{option}\ \tau} \qquad \frac{\mathit{\Gamma} \vdash M : \mathit{option}\ \tau}{\mathit{\Gamma} \vdash \mathit{get}\ M : \tau}$$

(b) consider preservation and progress theorem for each case

i. remove the ($GET - M$) rule

Preservation yes, progress no.

For example, for $\mathit{\Gamma} \vdash y : \tau\,,\ \mathit{\Gamma} \vdash \mathit{get}\ ((\lambda x : \tau.\, \mathit{Some}\ x) y) : \tau$

however, without $GET - M$, lambda reduction inside $get()$ is not allowed.

ii. remove both the ($SOME$) rule and the ($GET - M$) rule

Preservation yes, progress no.

Counterexample same as (i).

iii. add the rule ($GET - V$)

Preservation yes, progress yes.

iv. change the ($GET - SOME$) rule to ($GET - SOME'$)

Preservation yes, progress yes.

v. change the ($GET - SOME$) rule to ($GET - SOME''$)

Preservation no, progress yes.

For example, for $\Gamma \vdash x : \tau$, $\Gamma \vdash get\,(Some\, x) : \tau$

however, $Some\,(get\, x)$ will not have the same type τ.

vi. change the above ($GET - NONE$) rule to ($GET - NONE'$)

Preservation no, progress yes.

For example, suppose $None$ has a type σ (whatever that is)

cosider $get\,(None) \;\rightarrow\; None$, $get\,(None)$ should have exactly the same type

however, for $get\,(None) : \sigma$, $None$ should have $option\,\sigma$

there is no way σ and $option\,\sigma$ could be the same type.