Problem 1

a) $P(A|B) = P(A\cap B) / P(B) = 1/3$

b) $P(B|A) = P(A\cap B) / P(A) = 1/5$

c) $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.7$

$P(A|A\cup B) = P(A\cap (A\cup B)) / P(A\cup B) = P(A) / P(A\cup B) = 5/7$

d) $P(A|A\cap B) = P(A\cap (A\cap B)) / P(A\cap B) = P(A\cap B) / P(A\cap B) = 1$

e) $P(A\cap B|A\cup B) = P(A\cap B) / P(A\cup B) = 1/8$

Problem 2

a) $p(E1) = 1/2, p(E2) = 1/2, p(E1\cap E2) = 1/4 = p(E1)\cdot p(E2)$, 是

b) $p(E1) = 1/2, p(E2) = 3/8, p(E1\cap E2) = 1/8 \neq p(E1)\cdot p(E2)$, 不是

c) $p(E1) = 1/2, p(E2) = 3/8, p(E1\cap E2) = 1/8 \neq p(E1)\cdot p(E2)$, 不是

Problem 3

a) E1: 5 名志愿者血液中均含恒河因子, $P(-E1) = 1 - P(E1) = 1 - 80\%^5 = 0.67232$

b) E2: 5 名志愿者血液中均含恒河因子, $P(-E2) = 1 - P(E2) = 1 - 80\%^5 = 0.67232$

c) E3: n 名志愿者中至多 4 名血液含 Rh 因子, $P(-E3) = 1 - P(E3)$

$P(E3) = \sum_{k=0}^{4} C(n, k)\times 0.8^k\times 0.2^{(n-k)} < 1 - 90\% = 0.1$, 又 $n\in N$

代入数据得 n=7 时 $P(E3) > 0.1$, n=8 时 $P(E3) < 0.1$ 最少志愿者人数是 8.

Problem 4

a) $C(n, 2)\ (n-2)! / n! = 1/2$

b) $C(n, 4)\ C(4, 2)\ (n-4)! / n! = 1/4$

c) $C(n, 2)\ (n-2)! / n! = 1/2$

d) $2\ C(n, 3)\ (n-3)! / n! = 1/3$

e) $(n-1)! / n! = 1/n$

Problem 5

a) $0.97\times 0.04 / (0.97\times 0.04 + 0.02\times 0.96) \approx 0.669$

b) $1 - 0.669 = 0.331$

c) $0.03\times 0.04 / (0.03\times 0.04 + 0.98\times 0.96) \approx 0.001$

d) $1 - 0.001 = 0.999$

Problem 6

E: Remesh 迟到, A: Remesh 骑自行车, B: Remesh 开车, C: Remsesh 坐公共汽车.

则有 $P(E|A) = 5\%, P(E|B) = 50\%, P(E|C) = 20\%$.

a) $P(A) = P(B) = P(C) = 1/3$

$P(B|E) = P(E|B)\cdot P(B) / P(E|A)\cdot P(A) + P(E|B)\cdot P(B) + P(E|C)\cdot P(C) = 2/3$

b) $P(A) = 60\%, P(B) = 30\%, P(C) = 10\%$

$P(B|E) = P(E|B)\cdot P(B) / P(E|A)\cdot P(A) + P(E|B)\cdot P(B) + P(E|C)\cdot P(C) = 3/4$

Problem 7

$\sum_{i=0}^{4} C(9, 2i) \cdot (1/2)^9 = (1+36+126+84+9)\cdot (1/2)^9 = 256 / 2^9 = 1/2$

Problem 8

假设 100 个座位随机选择 n 个座位，所选的连续座位对的期望是 P(n)

$P(1) = 0, P(2) = 1\times C(99, 1)/C(100, 2) = 1/50$

$P(3) = 2\times C(98, 1)/C(100, 3) + 1\times C(98, 2)/C(100, 3) = 101/3300$

$P(n) = \sum_{i=1}^{n-1} i\cdot C(100-n+1, n-i)/C(100, n)$

$P(25) = \sum_{i=1}^{24} i\cdot C(76, 25-i) / C(100, 25)$

Problem 9

$npq = 10\times(1/6)\times(1-1/6) = 25/18$

Problem 10

E: 它不会被拒绝(没有观察到次品), $P(E) = C(16, 5) / C(20, 5) = 91/323$

$P(-E) = 1 - P(E) = 1 - 91/323 = 232/323 \approx 0.72$

样本大小为 5 的采样中次品的预期数量:

$\sum_{i=0}^{4} i\cdot C(4, i)C(16, 5-i) / C(20, 5) = 15504/15504 = 1$

样本大小为 5 的采样中次品数量的方差是多少

$\sum_{i=0}^{4} (i-1)^2\cdot C(4, i)C(16, 5-i) / C(20, 5) = 13152/15504 \approx 0.85$