PTC (Fall 2021) – Assignment 2

Name: ______________ Dept: ______________

Grade: ______________ ID: ______________

Due: Nov. 30, 2021

Problem 1

Give context free grammars that generate the following languages, and give a brief description of the functionality of each variable in your grammars (in natural language).

a. $\{w \in \{0,1\}^* \mid w = w^R \ and \ |w| \ is \ even\}$

b. $\{w \in \{a,b\}^* | in \ w, \ the \ number \ of \ b \ = \ 1 + \ the \ number \ of \ a\}.$

c. $\{a^i b^j c^k \mid i, j, k \geq 0, \ and \ i = j \ or \ i = k\}$

Problem 2

Let the language L consist of all the regular expressions over the alphabet $\{0,\ 1\}$. Thus, L is a language over the alphabet $\{0,\ 1,\ \epsilon,\ \emptyset,\ +,\ \circ,\ *,\ (,\ )\}$ and includes, for example, the string $(1 \circ (0 + 0 \circ 0))^*$. (For simplicity let us assume that concatenation is always written out explicitly using the symbol $\circ$)

Please construct two context-free grammars for L, one ambiguous and one unambiguous. For the ambiguous one, please give an example of a string with two different parse trees, and the corresponding left derivations.

Problem 3

Consider the following context free grammar: G = ({ S, B }, { 0, 1 }, P, S), where P consists of:

$$S \to 0BB$$

$$B \to 0S|1S|0$$

a. For the string 010000, give its parse tree and rightmost derivation according to G.

b. Provide a nondeterministic PDA P that accepts the language $L(G)$ by empty stack.

Problem 4

Begin with the grammer:

$$S \to aAa \mid bBb \mid \epsilon$$

$$A \to C \mid a$$

$$B \to C \mid b$$

$$C \to CD \mid \epsilon$$

$$D \to A \mid B \mid ab$$

1. Eliminate ϵ-productions.
2. Eliminate any unit productions in the resulting grammar of (1.).
3. Eliminate any useless symbols in the resulting grammar of (2.).
4. Put the resulting grammar of (3.) into Chomsky normal form.

Problem 5

Given grammer G:

$$S \to AB \mid BC$$

$$A \to BA \mid a$$

$$B \to CC \mid b$$

$$C \to AB \mid a$$

Please use CYK algorithm to decide whether string $aaaaba$ belongs to $L(G)$.

Problem 6

Use the CFL pumping lemma to show each of these languages are not context free.

a. L = { 0^p | p is a prime }

b. L = { $0^p 1^q 2^p 3^{(p+q)}$ | $p,\ q \geq 0$ }

c. L = { $\omega \in \{0,\ 1,\ 2\}^*$ | ω has equal number of 0’s, 1’s and 2’s. }

Problem 7

For a language L, let **prefix**(L) be the language $\{w \mid wv \in L, \text{ for some } v \in \Sigma^*\}$

a. Show that if L is context-free, so is **prefix**(L).

b. Show that if **prefix**$(L) \subseteq L$, L is infinite and L is context-free, then there is an infinite regular subset of L.

Proof.

Problem 8

We define an operation $\bowtie$ for language A and B to be

$$A \bowtie B = \{w \mid w = a_1 b_1 a_2 b_2 \cdots a_n b_n,\ \text{where } a_1 a_2 \cdots a_n \in A \text{ and } b_1 b_2 \cdots b_n \in B,\ \text{each } a_i, b_i \in \Sigma\}$$

a. Show that if A is context-free and B is regular, then $A \bowtie B$ is context-free.

b. Show that the class of CFL is not closed under $\bowtie$ operation.

Proof.