Department of Computer Science

CS917 Foundations of Computing - Discrete Maths and Statistical Analysis

Problem sheet 1 - Discrete Maths

We have covered Propositional and Predicate Logic in the lectures and this problem sheet is designed to ensure that you are comfortable with these topics before we move on to sets, relations and functions.

Do feel free to work together on these problems: This is a good opportunity to do this as you will need to complete the assignment independently. The TAs will be available during the seminar to help you if you get stuck, but you should be able to get through these exercises in the hour provided.

Propositional Logic

1. Using truth tables, determine whether $(p \implies q) \vee (p \implies \neg q)$ is *valid*, that is all possible assignments to p and q compute to *true*.

2. Using the same technique, determine whether $(\neg p \vee q) \wedge (q \implies \neg r \wedge \neg p) \wedge (p \vee r)$ is *satisfiable*, that is there exists at least one assignment to p, q and r that computes to *true*.

3. Feng, Indranil and Long find themselves trapped in the Computing Lab one New Year's evening (they were staying late to finish a research paper) and security appear to have locked up and gone to the pub. After a while the three Professors find three doors, one red, one blue and one green; however, they remember from their Health and Safety induction that one of the doors leads to the exit (and freedom), whilst the other two lead to mathematical purgatory (the dreaded 'infinite loop').

   On each door there is an inscription: The red door contains the words "Freedom is behind this door"; on the blue door is written "Freedom is not behind this door", and on the green door is written "Freedom is not behind the blue door".

   Given that at least one of the three statements on the doors is true, and at least one is false, which door would lead the three Professors to the exit?

Predicate Logic

4. Let x and y denote people, and $K(x, y)$ denotes x *knows* y. Translate the following into prepositional logic: (i) Everyone knows Son Goku; (ii) Everyone knows someone; (iii) There is someone who everyone knows; (iv) There is someone who Son Goku does not know; (v) There is someone whom no one knows.

Sets

5. For every pair of natural numbers x and y such that $x > y$ there is a natural number z such that $x > z > y$.

(a) Write the statement above in mathematical notation, using symbols for quantifiers.

(b) Show that the statement is false by giving a counter example.

6. For the universe of all integers ($\mathbb{Z}$), let p(x), q(x) r(x) and s(x) denote the following statements:
$p(x)$: x is even
$q(x)$: x is divisible by 5
$r(x)$: x is divisible by 3
Write the following statements in symbolic form:

   (a) Not all integers are divisible by 3 or are even.

   (b) if x is even and divisible by 3 then x is divisible by 5.

7. Define the following sets formally using the notation with quantifiers and arithmetical operators.

   (a) A = The complement of the set of all integers whose square is greater than 200.

   (b) B = The set of all natural numbers which are perfect cubes. A perfect cube, x, is a number that can be expressed as x^n, where n is divisible by 3.