

1.

p	q	$P \rightarrow q$	$P \rightarrow \neg q$	Final (OR)
0	0	T	T	T
0	1	T	T	T
1	0	F	T	T
1	1	T	F	T

Answer: It's valid

2.

p	q	r	$(\neg p) \vee q$	$q \rightarrow (\neg r) \wedge (\neg p)$	$p \vee r$	Final (AND)
0	0	0	T	T	F	F
0	0	1	T	T	T	T
0	1	0	T	T	F	F
0	1	1	T	F	T	F
1	0	0	F	T	T	F
1	0	1	F	T	T	F
1	1	0	T	F	T	F
1	1	1	T	F	T	F

Answer: satisfiable:  $p = 0, q = 0, r = 1$

3.

Red: Freedom is behind this door

Blue: Freedom is not behind this door

Green: Freedom is not behind the blue door

Constraints:

- At least 1 statement is True and at least 1 is False.
- Only 1 is the door to freedom

Solution:

Step 1: check if Red is true (incorrect, why?)

Step 2: Red is false. Check if Blue is true (correct).

Step 3: Red is false. Check if Blue is false (incorrect, why?).

No more cases.

Answer: red is false, blue is also false, green is true  $\rightarrow$  freedom is behind the green door.

4.

a) Forall x:  $K(x, \text{Son Goku})$

b) Forall x: Exists y:  $K(x, y)$

c) Exists y: Forall x:  $K(x, y)$

d) Exists y:  $\neg K(\text{Son Goku}, y)$

e) Exists y: Forall x: \Not K(x,y)

5.

$$5) a) \quad \forall x, y \in \mathbb{N}; \exists z \in \mathbb{N} : x > z > y$$

$$b) \quad \text{Try } x=1, y=2, z=?$$

No results,  $\therefore$  False.

6.

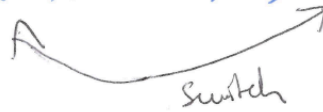
$$6) a) i) \quad \neg \forall x \in \mathbb{Z} : p(x) \vee r(x)$$

$$\text{or, } \exists x \in \mathbb{Z} : \neg p(x) \wedge \neg r(x) \quad (\text{via De Morgan's})$$

$$ii) \quad \forall x \in \mathbb{Z} : q(x) \wedge s(x) \Rightarrow r(x)$$

$$b) i) \quad \text{True, e.g. } \underline{1}$$

$$ii) \quad \text{False, e.g. } \underline{6}$$

 Switch

7.

$$7) a) \quad \{ x \in \mathbb{Z} : x^2 \leq 200 \}$$

$$b) \quad \{ n \in \mathbb{N} : n \bmod 3 = 0 \} \wedge n \in \mathbb{N}$$

Many solutions to these!