**CS9320_A**

UNIVERSITY OF WARWICK

## Paper Details

**Paper code:** CS9320_A

**Paper title:** Formal Systems Development

**Exam period:** Summer 2022

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## Exam Rubric

**Time Allowed:** 2 hours

**Exam Type:** Standard Examination

**Approved calculators:** may be used in this paper.

**Additional Stationary:**

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## Instructions

Answer Question 1 and any **TWO** from questions 2 to 5.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer booklet.

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# B maths notation: syntax summary

## 1 Predicates

1. Conjunction: $P \wedge Q$
2. Disjunction: $P \vee Q$
3. Implication: $P \Rightarrow Q$
4. Equivalence: $P \Leftrightarrow Q$
5. Negation: $\neg\, P$
6. Universal quant: $\forall\, z \bullet (P \Rightarrow Q)$
7. Existential quant: $\exists\, z \bullet (P \wedge Q)$
8. Substitution: $[G]P$
9. Equality: $E = F$
10. Inequality: $E \neq F$

## 2 Sets

1. Singleton set: $\{E\}$
2. Set enumeration: $\{E, F\}$
3. Empty set: $\{\}$
4. Set comprehension: $\{z \mid P\}$
5. Union: $S \cup T$
6. Intersection: $S \cap T$
7. Difference: $S - T$
8. Ordered pair: $E \mapsto F$
9. Cartesian product: $S \times T$
10. Powerset: $\mathbb{P}(S)$
11. Non-empty subsets: $\mathbb{P}_1(S)$
12. Finite subsets: $\mathbb{F}(S)$
13. Non-empty finite subsets: $\mathbb{F}_1(S)$
14. Cardinality: $card(S)$
15. Generalised union: $UNION(S)$
16. Generalised intersection: $inter(S)$
17. Generalised union: $\bigcup z \bullet (P \mid E)$
18. Generlsd. intersn.: $\bigcap z \bullet (P \mid E)$

### 2.1 Set predicates

1. Set membership: $E \in S$
2. Non-membership: $E \notin S$
3. Subset: $S \subseteq T$
4. Not a subset: $S \not\subseteq T$
5. Proper subset: $S \subset T$
6. Not proper subset: $S \not\subset T$

## 3 Numbers

1. Natural numbers: $\mathbb{N}$
2. Positive natural numbers: $\mathbb{N}1$
3. Minimum: $\min(S)$
4. Maximum: $\max(S)$
5. Sum: $m + n$
6. Difference: $m - n$
7. Product: $m \times n$
8. Quotient: $m/n$
9. Remainder: $m \bmod n$
10. Interval: $m \,..\, n$
11. Set summation: $\Sigma z \bullet (P \mid E)$
12. Set product: $\Pi z \bullet (P \mid E)$

### 3.1 Number predicates

1. Greater: $m > n$
2. Less: $m < n$
3. Greater or equal: $m \geq n$
4. Less or equal: $m \leq n$

# 4 Relations

1. Relations: $S \leftrightarrow T$
2. Domain: $\mathrm{dom}(R)$
3. Range: $\mathrm{ran}(R)$
4. Forward composition: $p \fatsemi q$
5. Backward composition: $p \circ q$
6. Identity: $\mathrm{id}(S)$
7. Domain restriction: $S \lhd r$
8. Domain subtraction: $S \lhd\!\!\!- r$
9. Range restriction: $r \rhd T$
10. Range subtraction: $r \rhd\!\!\!- T$
11. Inverse : $r^{\sim}$ or $r^{-1}$
12. Relational image: $r[S]$
13. Right overriding: $r_1 \leftplus r_2$
14. Left overriding: $r_1 \plusright r_2$
15. Direct product: $p \otimes q$
16. Parallel product: $p \parallel q$
17. Iteration: $r^n$
18. Closure: $r^*$
19. Projection1: $prj1(S, T)$
20. Projection2: $prj2(S, T)$

# 5 Functions

1. Partial functions: $S \pfun T$
2. Total functions: $S \fun T$
3. Partial injections: $S \pinj T$
4. Total injections: $S \inj T$
5. Partial surjections: $S \psur T$
6. Total surjections: $S \sur T$
7. Bijections: $S \bij T$
8. Lambda abstractn.: $\lambda\, z \bullet (P \mid E)$
9. Function application: $f(E)$

# 6 Sequences

1. Empty sequence: $[]$
2. Finite sequences: $\mathrm{seq}(S)$
3. Non-empty finite sequences: $\mathrm{seq}_1(S)$
4. Set of injective sequences: $\mathrm{iseq}(S)$
5. Permutations: $perm(S)$
6. Concatenation: $s \frown t$
7. Prepend element: $E \rightarrow s$
8. Append element: $s \leftarrow E$
9. Singleton sequence: $[E]$
10. Sequence construction: $[E, F]$
11. Size: $size(s)$
12. Reverse: $rev(s)$
13. Take: $s \uparrow n$
14. Drop: $s \downarrow n$
15. First element: $first(s)$
16. Last element: $last(s)$
17. Tail: $tail(s)$
18. Front: $front(s)$
19. Generalised concatenation: $conc(ss)$