

CS412 Exercise sheet 3

Functions

1. If $f_1 \in X \rightarrow Y, f_2 \in X \rightarrow Y, f_3 \in Y \rightarrow Z$ which of the following are necessarily functions? Give a brief justification or a counterexample as appropriate.
 - (a) $f_1 \cup f_2$
 - (b) $f_1 \cap f_2$
 - (c) f_1^{-1}
 - (d) $f_1; f_3$
 - (e) $f_1; (f_2^{-1})$
2. As with relations, functions are simply sets of pairs. Set notation can therefore be used to give the value of a function, either by enumeration (for a small sized function) or by an intensional definition.
 - (a) List the elements of $\{x, y \mid x \in \mathbb{N} \wedge y \in \mathbb{N} \wedge x \leq 4 \wedge y = x * x\}$
 - (b) Give a set definition for a function which maps each element of \mathbb{N}_1 to its predecessor.
 - (c) Give a set definition for a function which maps each even element of \mathbb{N} to the set of numbers less than or equal to it.
3. A university keeps a database of all registered students. A specification of the system uses the deferred sets *NAME* for student names (not necessarily unique), *IDENTIFIER* for the set of identifiers used to uniquely reference students, *DEGREE* for the set of possible degree courses.
 - (a) Suppose a function *sname* is used to map the identifier of each registered student to the name of that student. The function *sdeg* is used to map the identifier of each registered student to the degree that student is taking. Each registered student must have a degree subject recorded in *sdeg*. Write an appropriate invariant for the system.
 - (b) Write a suitable initialisation.
 - (c) Write an operation to output the name of student with identifier *ss*.
 - (d) Write an operation to output all degrees for which noone is currently registered.
 - (e) Write an operation to add a new student name and the degree they are taking. The system should select and output a suitable identifier.
 - (f) Write an operation to output the names of all students taking degree *dd*.
4. A delivery company has a number of lorries which are loaded with goods to be delivered. Delivery of each order is made to the appropriate address. Once a lorry is empty, it may return to be loaded for another round of deliveries. Use an Abstract Machine to specify the state of the delivery system and to include operations:

- to input to the system the next collection of deliveries a particular lorry should make (this might in practice be provided by interfacing to another part of the system dealing with orders etc);
- to output the total goods a lorry should be loaded with at the start of a round;
- to represent the delivery of an order being made by a lorry.

5. Check out your answers to the previous questions in AtelierB.