CS412 Exercise sheet 4

A bit of wp and some sequences

1. What are:

   (a) $(x > y)[3, 2/x, y]$

   (b) $(x > y)[x + y, 2x/x, y]$

   (c) $(\forall\, x, y \bullet (x \in \mathbb{N} \wedge y \in \mathbb{N} \Rightarrow P(x, y)))[3/x]$

   (d) $(\forall\, x \bullet (x \in \mathbb{N} \Rightarrow (P(x, y) \wedge \exists\, y \bullet (y \in \mathbb{N} \wedge Q(x, y)))))[3/y]$

2. Calculate and simplify:

   (a) $[x, y := x + y, 2x]x > y$

   (b) For $n.m : \mathbb{N}$:
   $[x := 1 \,..\, n \;\;||\;\; y := 1 \,..\, m]\,\exists\, x \bullet (x \subseteq \mathbb{N} \wedge x \subset y)$

   (c) $\left[\begin{array}{l} \textbf{IF} \quad x < y \\ \textbf{THEN} \quad x := y \\ \textbf{END} \end{array}\right] x = y$

   (d) $\left[\begin{array}{l} \textbf{PRE} \quad x < y \\ \textbf{THEN} \quad x := y \\ \textbf{END} \end{array}\right] x = y$

   (e) $\left[\begin{array}{l} \textbf{CASE} \quad \mathit{clearance} \quad \textbf{OF} \\ \textbf{EITHER} \quad \mathit{unk} \quad \textbf{THEN} \quad \mathit{action} := \mathit{alert} \\ \textbf{OR} \quad \mathit{conf} \quad \textbf{THEN} \quad \mathit{permission} := \mathit{denied} \\ \textbf{OR} \quad \mathit{sec} \quad \textbf{THEN} \quad \mathit{permission} := \mathit{denied} \\ \textbf{ELSE} \quad \mathit{permission} := \mathit{granted} \\ \textbf{END} \end{array}\right] \mathit{permission} = \mathit{granted}$

3. Let $s_1 = [1, 1, 0]$ and $s_2 = [1, 0, 1]$.

   (a) Write s_1 and s_2 using set notation.

   (b) What are:

      i. $s_1 \mathbin{^\frown} s_2$

      ii. $s_1 \cup s_2$

      iii. $s_1 \cap s_2$

      iv. $s_1(2)$

      v. $s_1 \lhd\!\!\!\!+ s_2$

      vi. $\mathit{rev}\ (s_1 \mathbin{^\frown} s_2)$

      vii. $s_1 \rhd \{1\}$

viii. $s_1 \rhd \{1\}$

ix. $head\ (tail\ (s_2))$

x. $\lambda\, i \bullet (i \in \mathbb{N}_1 \mid i + 1)$

4. Show how each of the following can be defined using the old set, relation, function notation.

   (a) $last(ss)$

   (b) $tail(ss)$

   (c) $perm(X)$ which includes all items from set X exactly once

5. Suppose we wish to specify the action of a “lossy” communication medium. Users attempt to communicate with one another, but messages may be lost in transport. Messages that do arrive are stored sequentially in the user’s mailbox. Messages may also ”leap-frog” one another so there is no guarantee that messages will arrive in the order they are sent. How could this be represented in a single abstract machine?