CS917 Foundations of Computing - Maths and Stats Problem sheet 2 - Probability and statistics

1. Consider the random experiment of throwing a coin 4 times.
    a. Enumerate all the outcomes.
    b. In how many of these outcomes are there at least 3 tosses with the same value?
    c. In how many of these outcomes are there 4 tosses having the same value?
2. Enumerate all the outcomes of the random experiment of rolling two dice where the sum of the values is 6.
3. How many subsets of size k are there of the set $\{1, 2, \ldots, n\}$?
4. A restaurant offers 5 choices of starter, 10 choices of main meal and 4 choices of dessert. A customer can choose to eat just one course, or two different courses, or all three courses. Assuming all choices are available, how many different possible meals does the restaurant offer?
5. Calculate the probabilities of the events below, assuming that the die and coin are fair/unbiased:
    a Rolling a six on the die
    b Rolling an odd number on the die
    c Rolling a six or an odd number on the die
    d Rolling a number less than five on the die and getting a head when tossing the coin.
6. Calculate the probabilities of the events in the previous question, assuming the coin are fair/unbiased, and the die is balanced but has sides [2, 2, 3, 4, 6, 6]:
7. A frog climbing out of a well is affected by the weather. When it rains, he falls back down the well with a probability of 10 1 . In dry weather, he only falls back down with probability of 20 1 . The probability of rain is 15.
    a Find the probability he falls on any given day.

b Find the probability that it was raining, after finding out the frog fell.

8. A screening test for meningitis is known to provide a positive result 95% of the time when a patient with meningitis is tested, which it gives a negative result for 70% of patients tested who are not suffering from the disease. National statistics suggest that 5% of the population have the disease.

   a Given that a patient has tested positive, does the patient has meningitis or not?

   b If the doctor orders a second test that returns a negative result, how would this affect the probabilities associated with having the disease or not?

9. Suppose that I have two coins in my pocket. One ordinary, fair coin and one strange coin with two heads. I pick a random coin from my pocket, toss it, and a heads comes up.

   a What is the probability I have thrown the fair coin?

   b If I throw the same coin again and heads comes up again, what is the probability that I have thrown the fair coin?

10. Devices produced in a factory are believed to have average weight 20 gm. A new technique is proposed that claims to lower the average weight below 20 gm. Can you check the claim using the p-value test with significance level 0.05? You are given a sample of 20 devices with sample mean 19.8 gm and sample standard deviation 3.1 gm. Consult the t-distribution table as given below for estimating the p-value.

	90%	95%	97.5%	99%	99.5%	99.95%	1-Tail Confidence Level
	80%	90%	95%	98%	99%	99.9%	2-Tail Confidence Level
	0.100	0.050	0.025	0.010	0.005	0.0005	1-Tail Alpha
df	0.20	0.10	0.05	0.02	0.01	0.001	2-Tail Alpha
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9200	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3649	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.4370	
12	1.3562	1.7823	2.1788	2.6810	3.0545	4.3178	
13	1.3502	1.7709	2.1604	2.6503	3.0123	4.2208	
14	1.3450	1.7613	2.1448	2.6245	2.9768	4.1405	
15	1.3406	1.7531	2.1314	2.6025	2.9467	4.0728	
16	1.3368	1.7459	2.1199	2.5835	2.9208	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.9651	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.9216	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.8834	
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.8495	
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.8193	
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.7921	
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.7676	
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.7454	
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.7251	
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.7066	
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.6896	
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.6739	
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.6594	
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.6460	