

Department of Computer Science CS917
Foundations of Computing - Maths and Stats
Assignment

This assignment is due **at noon on Monday 11th December, 2023**. The submission is on Tabula, and should comprise scanned copies of written work.

The work that you submit should be **your own work** and please show **full working** where appropriate, as this is necessary to gain full marks.

Marks for each question are indicated. The total marks you can get is 100.

If you have any questions then do please email me at long.tran-thanh@warwick.ac.uk.

1 Discrete Mathematics

1. For each of the following formulae, find a logically equivalent formula in which \wedge , \implies and \iff do not occur: (i) $\neg(p \implies q)$; (ii) $((p \wedge q) \vee r)$; (iii) $\neg((p \wedge q) \iff r)$. Use the truth table to show that the proposed solution is indeed equivalent with the original one. [12 marks]
2. Write out paraphrases of the following, using \forall , \exists and $=$ (i) Frodo has a ring; (ii) Sauron does not have any rings; (iii) The One Ring rules all the other rings; (iv) The ring that Frodo has is the One Ring; (v) whoever wears the ring, becomes invisible (i.e., no other ordinary human can see that person); (vi) Bombadil Tom can see the ring-wearer, hence he is no ordinary human (use the previous statement from (v) to formulate this one). Auxiliary clauses: $\text{has}(x, y)$: x has/is in possession of y ; $\text{rules}(x, y)$: x rules y ; $\text{wears}(x, y)$: x wears y ; $\text{sees}(x, y)$: x can see y ; and $\text{ordinary}(x)$: x is an ordinary human. [12 marks]
3. Write predicate logic formulae which state that the relation expressed by $R_{x,y}$ has the following properties: (i) $R_{x,y}$ is irreflexive; (ii) $R_{x,y}$ is intransitive; (iii) $R_{x,y}$ is not a partial order. Note that this formulation is a bit different from the one in the slides. To make this consistent, think about $R_{x,y}$ as R_p with relation p between x and y (you can assume that both x and y are from the same set A). So you can use $R_{x,y}$ as p in your

formulations. Therefore, in your answer, you can use both notations, just be consistent (i.e., if you choose 1 notation, then use the same for all your answers). [9 marks]

4. Determine which of the following functions are injective and which are surjective (please provide explanations as well):
 - (i) $f: \mathbb{Z} \rightarrow \mathbb{N}$, where $\forall n \in \mathbb{Z}: f(n) = n^{2022} + 1$;
 - (ii) $g: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$, where $\forall (n, k) \in \mathbb{N} \times \mathbb{N}: g(n, k) = 2^n 3^k 5^{n+k}$;
 - (iii) $h: \mathbf{P}(\mathbb{N}) \rightarrow \mathbf{P}(\mathbb{N})$, where $\forall A \in \mathbf{P}(\mathbb{N}): h(A) = \mathbb{N} \setminus A$ (recall what $\mathbf{P}(\mathbb{N})$ means);
 - (iv) $k: \mathbb{N} \rightarrow \mathbb{Z}$, where $\forall n \in \mathbb{N}: k(n) = (-1)^n$.
 [12 marks].

2 Statistical Analysis

Question 1 has two parts, each is marked out of 5. Both Question 2 and 3 are marked out of 10. Question 4 is marked out of 20.

1. A fair coin is tossed twice and in each toss H and T appears with equal probability.
 - (i) Determine the probability that one toss yields H and the other toss yields T given that the first toss yields H.
 - (ii) Determine the probability that both tosses yield H given that at least one the tosses yield H
2. Among all the computers supplied, company A, B and C supply 40%, 30% and 30% respectively. Of their supplies 5%, 3% and 2.5% are defective computers respectively. A computer is chosen at random and it is found to be defective. What is the probability that it is supplied by Company A?
3. There are 3 white and 5 black balls, from them 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and it is found to be white. What is the probability that out of 4 balls transferred 3 are white and 1 is black?
4. It is shown that conventional fertilizers can help one particular variety of mangoes grow by an average of 20 gm. A new company claims that their fertilizer can cause the growth more than 20 gm on the average. Check this claim using the p-value test with the significance level 0.05. You are given a sample of 12 mangoes which received the new fertilizer and their sample mean weight is 20.175 gm and sample standard deviation 3.0211 gm. Consult the t-distribution table as given below for estimating the p-value.

	90%	95%	97.5%	99%	99.5%	99.95%	1-Tail Confidence Level
	80%	90%	95%	98%	99%	99.9%	2-Tail Confidence Level
	0.100	0.050	0.025	0.010	0.005	0.0005	1-Tail Alpha
df	0.20	0.10	0.05	0.02	0.01	0.001	2-Tail Alpha
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9200	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3649	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.4370	
12	1.3562	1.7823	2.1788	2.6810	3.0545	4.3178	
13	1.3502	1.7709	2.1604	2.6503	3.0123	4.2208	
14	1.3450	1.7613	2.1448	2.6245	2.9768	4.1405	
15	1.3406	1.7531	2.1314	2.6025	2.9467	4.0728	
16	1.3368	1.7459	2.1199	2.5835	2.9208	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.9651	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.9216	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.8834	
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.8495	
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.8193	
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.7921	
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.7676	
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.7454	
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.7251	
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.7066	
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.6896	
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.6739	
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.6594	
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.6460	