CS412 Exercise sheet 1

Using basic logic and set notation

1. (a) Show that $P \Rightarrow Q$ is equivalent to $\neg P \vee Q$

   (b) Show that $(P \wedge Q) \Rightarrow R$ is equivalent to $P \Rightarrow (Q \Rightarrow R)$

   (c) Let $X = P \vee (Q \wedge R)$ and $Y = \neg Q \Rightarrow P$. Show that Y is a logical consequence of X

   (d) Show that Q is a logical consequence of $P \wedge \neg P$. That is, anything follows from a contradiction.

   (e) Suppose Q is a *contradiction* (that is, it's a proposition that's always false such as $R \wedge \neg R$). Show that $P \wedge Q$ is itself a contradiction for any P. Show also that $P \vee Q$ is equivalent to P.

   If Q is a *tautology* (that is, it's a proposition that's always true such as $R \vee \neg R$) what can you say about $P \wedge Q$ and $P \vee Q$ for any P?

   (f) Simplify: $(Q \vee P) \wedge (P \Rightarrow (Q \wedge R)) \wedge \neg P$

   (g) Give an example to show that $\forall x \bullet (\exists y \bullet P(x, y))$ is not equivalent to $\exists y \bullet (\forall x \bullet P(x, y))$

2. If $X = \{1, 2, 3\}$ what are:

   (a) $X \times X$

   (b) $\mathbb{P} X$

   (c) $\mathbb{P}(X \times \{a\})$

3. With $\mathbb{N}$ as the set of natural numbers, use logic and set notation to describe the following.

   (a) The set of all numbers between 100 and 200 inclusive.

   (b) The set of all prime numbers.

(c) The set of all finite sets of numbers which contain their own cardinality (size) as a member.

4. Which of the following are true for all sets S, T and U? Justify your answer.

   (a) $(S \cap T) \cup U = (S \cup U) \cap (T \cup U)$

   (b) $S \cup T \neq S$

   (c) $S - (T \cap U) = (S - T) \cap (S - U)$

5. Suppose a specification uses the set, PID of person identifiers and declares the variables:
   $club1, club2 \subseteq PID$
   $committee1, committee2 \subseteq PID$

   (a) Write logical statements to express the following.
      i. For both clubs, the committee must be comprised of members of that club.
      ii. Members of club2 are not allowed to serve on club1's committee.
      iii. Some people are not members of either club1 or club2.

   (b) Write expressions to denote:
      i. The number of ordinary (ie: non committee) members of club1.
      ii. The set of people who belong to one club but not both.
      iii. The set of pairs in which the first of each pair is a person and the second is the set of clubs they belong to.

   (c) A sign in the clubroom for club1 says "If you are a member of club1 then you are entitled to free coffee".
      i. How might a non-member of club1 be expected to interpret this natural language statement?
      ii. If it were to be expressed formally as

$$\forall\, xx \bullet (xx \in club1 \Rightarrow \mathit{freecoffee}(xx))$$

      what could be deduced about teh entitlement of non-members?

6. The typed versions of the quantifiers are:

$$\forall\, xx \bullet (xx \in T \Rightarrow P)$$
$$\exists\, xx \bullet (xx \in T \wedge P)$$

   Check that you understand the need to use implication in one case and conjunction in the other. Give examples to show that conjunction would not be appropriate to use with $\forall$ and similarly that implication would not be appropriate with $\exists$.