CS917 Seminar Sheet 2 - Probability and Statistics Answers

1.a . There are 4 throws, each of the throws can be either H or T. Therefore:

HHHH
HHHT
HHTH
HHTT
HTHH
HTHT
HTTH
HTTT
THHH
THHT
THTH
THTT
TTHH
TTHT
TTTH
TTTT

1.b . There are 10 outcomes of at least 3 same-value tosses: {HHHH, HHHT, HHTH, HTHH, HTTT, THHH, THTT, TTHT, TTTH, TTTT}.

1.c . There are 2 outcomes with all 4 equal values : {HHHH, TTTT}.

2 . The outcomes are $\{(1,5),(5,1),(2,4),(4,2),(3,3)\}$.

3 . The number of subsets of k of the set {1, 2, . . . , n} is $\binom{n}{k}$.

4 . One course

$$= \binom{5}{1} + \binom{10}{1} + \binom{4}{1} = 19$$

Two courses

$$= \left(\binom{5}{1} \times \binom{10}{1}\right) + \left(\binom{10}{1} \times \binom{4}{1}\right) + \left(\binom{5}{1} \times \binom{4}{1}\right) = 110$$

Three courses

$$= \binom{5}{1} \times \binom{10}{1} \times \binom{4}{1} = 220$$

Therefore, the total numbers of possibilities = 19 + 110 + 200 = 329

5 . A fair die has a set of events {1, 2, 3, 4, 5, 6} with equal possibility, and a fair coin's events {H, T} both with equal probability of occurring. Therefore:

(a) Rolling a six = $\frac{1}{6}$.

(b) Rolling an odd number = {1,3,5} out of 6 possibilities = $\frac{3}{6} = \frac{1}{2}$.

(c) Rolling a six or an odd number = {1, 3,5,6} out of six possibilities = $\frac{4}{6} = \frac{2}{3}$.

(d) Rolling a number less than five on the die and getting a head when tossing the coin =

$$P(\{1,2,3,4\}) = \frac{4}{6} = \frac{2}{3}$$

on the die and

$$P(H) = \frac{1}{2}$$

on the coin. Multiply both together for $\frac{1}{3}$.

6 . The coin is the same as before, but the die is now biased with events {2, 2, 3, 4, 6, 6} where

$$P(2) = P(6) = \frac{1}{3}, P(3) = P(4) = \frac{1}{6}$$

Therefore:

(a) Rolling a six = $\frac{2}{6} = \frac{1}{3}$.

(b) Rolling an odd number = {3} out of 6 possibilities = $\frac{1}{6}$.

(c) Rolling a six or an odd number = {3, 6,6} out of six possibilities = $\frac{3}{6} = \frac{1}{2}$.

(d) Rolling a number less than five on the die and getting a head when tossing the coin =

$$P(\{2,2,3,4\}) = \frac{4}{6} = \frac{2}{3}$$

on the die and

$$P(H) = \frac{1}{2}$$

on the coin. Multiply both together for $\frac{1}{3}$.

7 . (a) Using the values quoted and a bit of logic:

$$\begin{aligned} P(Fall) =& P(Fall \cap Rain) + P(Fall \cap \overline{Rain}) \\ =& P(Rain)P(Fall|Rain) + P(\overline{Rain})P(Fall|\overline{Rain}) \\ =& \frac{1}{5} \times \frac{1}{10} + \frac{4}{5} \times \frac{1}{20} \\ =& \frac{6}{100} = 0.06 \end{aligned}$$

(b) Using Bayes' Theorem and your answer from part (a):

$$\begin{aligned}
P(Rain|Fall) =& \frac{P(Rain \cap Fall)}{P(Fall)} \\
=& \frac{P(Rain)P(Fall|Rain)}{P(Fall)} \\
=& \frac{\frac{1}{5} \times \frac{1}{10}}{\frac{6}{100}} \\
=& \frac{1}{3} = 0.3333
\end{aligned}$$

8 . Suppose
$Pos =$ the event that a patient is tested positive for Meningitis,

$Neg =$ the event that a patient is tested negative for Meningitis

$M =$ the event that a patient has Meningitis,

$\overline{M} =$ the event that a patient does not have Meningitis.

We have
$P(Pos|M) = 0.95$

This implies $P(Neg|M) = 1 - 0.95 = 0.05$

$P(Neg|\overline{M}) = 0.7$

This implies $P(Pos|\overline{M}) = 1 - 0.7 = 0.3$

$P(M) = 0.05$

This implies $P(\overline{M}) = 1 - 0.05 = 0.95$

(a) We want $P(M|Pos)$

$$\begin{aligned}
P(Pos) &= P(Pos \cap M) + P(Pos \cap \overline{M}) \\
&= P(Pos|M)P(M) + P(Pos|\overline{M})P(\overline{M}) \\
&= 0.95 \times 0.05 + 0.3 \times 0.95 \\
&= 0.3325
\end{aligned}$$

Using Bayes' Theorem:

$$
\begin{aligned}
P(M|Pos) &= \frac{P(Pos \cap M)}{P(Pos)} \\
&= \frac{P(Pos|M)P(M)}{P(Pos)} \\
&= \frac{0.95 \times 0.05}{0.3325} = 0.14285
\end{aligned}
$$

(b)

$$
\begin{aligned}
P(Neg) &= P(Neg \cap M) + P(Neg \cap \overline{M}) \\
&= P(Neg|M)P(M) + P(Neg|\overline{M})P(\overline{M}) \\
&= 0.05 \times 0.05 + 0.7 \times 0.95 \\
&= 0.6675
\end{aligned}
$$

Using Bayes' Theorem:

$$
\begin{aligned}
P(M|Neg) &= \frac{P(M \cap Neg)}{P(Neg)} \\
&= \frac{P(Neg|M)P(M)}{P(Neg)} \\
&= \frac{0.05 \times 0.05}{0.6675} \\
&= 0.0037
\end{aligned}
$$

Let (Pos, Neg) be the event that first test is positive and the second one is negative.
Then $(Pos, Neg) = P(Pos)P(Neg)$

$$
\begin{aligned}
P(M|(Pos, Neg)) &= \frac{P((Pos, Neg)|M)P(M)}{P((Pos, Neg))} \\
&= \frac{P(Pos|M)P(Neg|M)P(M)}{P((Pos, Neg))} \\
&\quad \text{Using Conditional Independence} \\
&= \frac{P(Pos|M)P(Neg|M)P(M)}{P(Pos)P(Neg)} \\
&= \frac{0.95 \times 0.05 \times 0.05}{0.335 \times 0.6675} \\
&= 0.0107
\end{aligned}
$$

Hence, after the second test coming negative, the probability of getting Meningitis decreases a bit.

9 .

(a) It's the sum of the products of the chance of pulling the coin out and the result of the flip being heads:

$$\begin{aligned} P(H) &= P(H \cap Coin_{unbiased}) + P(H \cap Coin_{baised}) \\ &= P(Coin_{unbiased})P(H|Coin_{unbiased}) + P(Coin_{biased})P(H|Coin_{biased}) \\ &= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times 1\right) = \frac{3}{4} \end{aligned}$$

$$\begin{aligned} P(Coin_{unbiased}|H) &= \frac{P(H \cap Coin_{unbiased})}{P(H)} \\ &= \frac{P(Coin_{unbiased})P(H|Coin_{unbiased})}{P(H)} \\ &= \frac{\left(\frac{1}{2} \times \frac{1}{2}\right)}{\frac{3}{4}} = \frac{1}{3} \end{aligned}$$

(b) . We want to find $P(Coin_{unbiased}|HH)$, but we need to find $P(HH)$ first:

$$\begin{aligned} P(HH) &= P(HH \cap Coin_{unbiased}) + P(HH \cap Coin_{baised}) \\ &= P(Coin_{unbiased})P(HH|Coin_{unbiased}) + P(Coin_{biased})P(HH|Coin_{biased}) \\ &= \left(\frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times 1\right) = \frac{5}{8} \end{aligned}$$

So:

$$\begin{aligned} P(Coin_{unbiased}|HH) &= \frac{P(Coin_{unbiased} \cap HH)}{P(HH)} \\ &= \frac{\frac{1}{2} \times \frac{1}{4}}{\frac{5}{8}} = \frac{1}{5} \end{aligned}$$

10 STEP 1: Given
Sample size $n = 20$
Sample mean $\overline{X} = 19.8$ gm
Sample standard deviation $S = 3.1$ gm

STEP 2: Then we form the null hypothesis H_0 and the alternative hypothesis H_a

H_0 : population mean $\mu \geq 20$
H_a : population mean $\mu < 20$

STEP 3: Find the test statistics $t = \frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{19.8 - 20}{\frac{3.1}{\sqrt{20}}} = -0.2885$

STEP 4: Find the p-value for the test statistics
Refer to the t-Distribution table and go to $(n-1)$-th row which is for the degrees of freedom. In this case 20 - 1 = 19. In that row find where the absolute value of $t = 0.2885$ resides.

We see that $0.2885 < 1.3277$

Since this is one-sided test, so p-value > 0.1

STEP 5: Conclusion: Since, p-value $\not< 0.05$, we cannot reject H_0. So we do not have data to accept this claim.