Probability & Counting – 附解析

Section1 Probability

1 A box contains 6 black balls and 4 white balls. If two balls are selected at random without replacement, what is the probability that both balls are white?

- ○ 7/90
- ○ 3/25
- ○ 2/15
- ○ 4/25
- ○ 4/9

A box contains 6 black balls and 4 white balls. If two balls are selected at random without replacement, what is the probability that both balls are white?

(A) $\frac{7}{90}$

(B) $\frac{3}{25}$

(C) $\frac{2}{15}$

(D) $\frac{4}{25}$

(E) $\frac{4}{9}$

$$P(\text{both white}) = P(W_1 \text{ AND } W_2) = P(W_1) \times P(W_2) = \frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$$

given W_1

2 The probability that event A occurs is 0.4, and the probability that events A and B both occur is 0.25. If the probability that either event A or event B occurs is 0.6, what is the probability that event B will occur?

- ○ 0.05
- ○ 0.15
- ○ 0.45
- ○ 0.50
- ○ 0.55

The probability that event A occurs is 0.4, and the probability that events A and B both occur is 0.25. If the probability that either event A or event B occurs is 0.6, what is the probability that event B will occur?

(A) 0.05

(B) 0.15

(C) 0.45

(D) 0.50

(E) 0.55

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$$

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ 0.6 &= 0.4 + P(B) - 0.25 \\ 0.6 &= 0.15 + P(B) \\ 0.45 &= P(B) \end{aligned}$$

3 If four numbers are randomly selected without replacement from set {1, 2, 3, 4}, what is the probability that the four numbers are selected in ascending order?

- 1/256
- 1/64
- 1/48
- 1/24
- 1/12

If four numbers are randomly selected without replacement from set {1, 2, 3, 4}, what is the probability that the four numbers are selected in ascending order?

(A) $\frac{1}{256}$

(B) $\frac{1}{64}$

(C) $\frac{1}{48}$

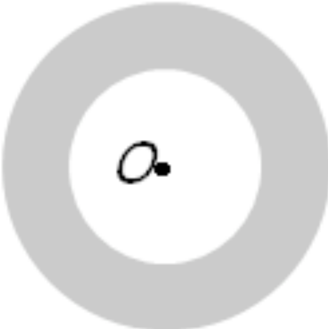
(D) $\frac{1}{24}$

(E) $\frac{1}{12}$

$$\begin{aligned} & P(1 \to 2 \to 3 \to 4) \\ &= P(1_{\text{first}} \text{ AND } 2_{\text{second}} \text{ AND } 3_{\text{third}} \text{ AND } 4_{\text{fourth}}) \\ &= P(1_{\text{first}}) \times P(2_{\text{second}}) \times P(3_{\text{third}}) \times P(4_{\text{fourth}}) \\ &= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times 1 \\ &= \frac{1}{24} \end{aligned}$$

4 Each circle has center O. The radius of the smaller circle is 2 and the radius of the larger circle is 6. If a point is selected at random from the larger circular region, what is the probability that the point will lie in the shaded region?

- 1/9
- 1/6
- 2/3
- 5/6
- 8/9



Each circle has center *O*. The radius of the smaller circle is 2 and the radius of the larger circle is 6. If a point is selected at random from the larger circular region, what is the probability that the point will lie in the shaded region?

(A) $\frac{1}{9}$

(B) $\frac{1}{6}$

(C) $\frac{2}{3}$

(D) $\frac{5}{6}$

(E) $\frac{8}{9}$

$$\boxed{Area = \pi r^2}$$

Small circle: $Area = \pi(2)^2 = 4\pi$

Large circle: $Area = \pi(6)^2 = 36\pi$

$$\text{Unshaded portion} = \frac{4\pi}{36\pi} = \frac{1}{9}$$

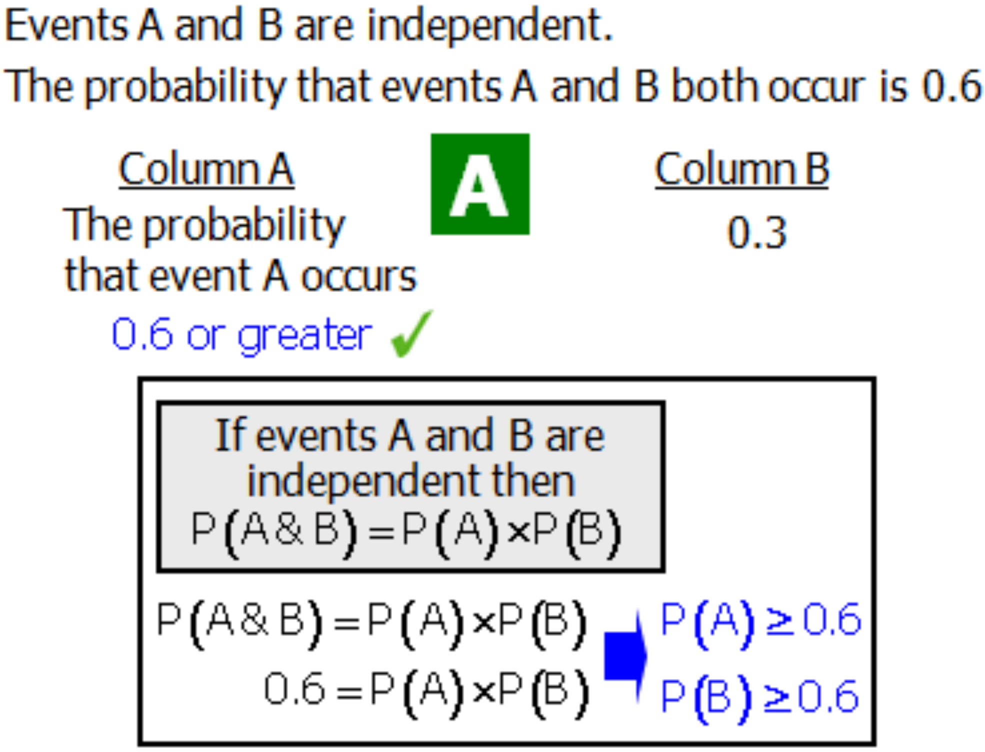
$$\text{Shaded portion} = \frac{8}{9}$$

5

Events A and B are independent.

The probability that events A and B both occur is 0.6

Column A	Column B
The probability that event A occurs	0.3



6 Set A: {1, 3, 4, 6, 9, 12, 15}

If three numbers are randomly selected from set A without replacement, what is the probability that the sum of the three numbers is divisible by 3?

- 3/14
- 2/7
- 9/14
- 5/7
- 11/14

Set A: {1, 3, 4, 6, 9, 12, 15}

If three numbers are randomly selected from set A without replacement, what is the probability that the sum of the three numbers is divisible by 3?

(A) $\frac{3}{14}$

(B) $\frac{2}{7}$

(C) $\frac{9}{14}$

(D) $\frac{5}{7}$

(E) $\frac{11}{14}$

$$\begin{aligned} & P(\text{divisible by } 3) \\ &= P(1 \text{ or } 4 \text{ are not selected}) \\ &= \frac{\#\text{ outcomes where 1 or 4 are not selected}}{\text{total }\#\text{ of outcomes}} \\ &= \frac{{}_5C_3}{{}_7C_3} \\ &= \frac{10}{35} \\ &= \frac{2}{7} \end{aligned}$$

$$\boxed{{}_nC_r = \frac{n!}{r!(n-r)!}}$$

7 A box contains 4 red chips and 2 blue chips. If two chips are selected at random without replacement, what is the probability that the chips are different colors?

- 1/2
- 8/15
- 7/12
- 2/3
- 7/10

A box contains 4 red chips and 2 blue chips. If two chips are selected at random without replacement, what is the probability that the chips are different colors?

(A) $\frac{1}{2}$

(B) $\frac{8}{15}$

(C) $\frac{7}{12}$

(D) $\frac{2}{3}$

(E) $\frac{7}{10}$

$$\begin{aligned} P(\text{different colors}) &= P(1_R \ \& \ 2_B \text{ OR } 1_B \ \& \ 2_R) \\ &= P(1_R \ \& \ 2_B) + P(1_B \ \& \ 2_R) \\ &= \left(\frac{4}{6}\right)\left(\frac{2}{5}\right) + \left(\frac{2}{6}\right)\left(\frac{4}{5}\right) \\ &= \frac{8}{15} \end{aligned}$$

8 A bag contains x blue chips and y red chips. If the probability of selecting a red chip at random is 3/7, then x/y =

- 7/11
- 3/4
- 7/4
- 4/3
- 11/7

A bag contains x blue chips and y red chips. If the probability of selecting a red chip at random is $\frac{3}{7}$, then $\frac{x}{y} =$

(A) $\frac{7}{11}$

(B) $\frac{3}{4}$

(C) $\frac{7}{4}$

(D) $\frac{4}{3}$

(E) $\frac{11}{7}$

$$P(\text{select red chip}) = \frac{y}{x+y} = \frac{3}{7}$$

$$7y = 3(x+y)$$

$$7y = 3x + 3y$$

$$4y = 3x$$

$$4 = \frac{3x}{y}$$

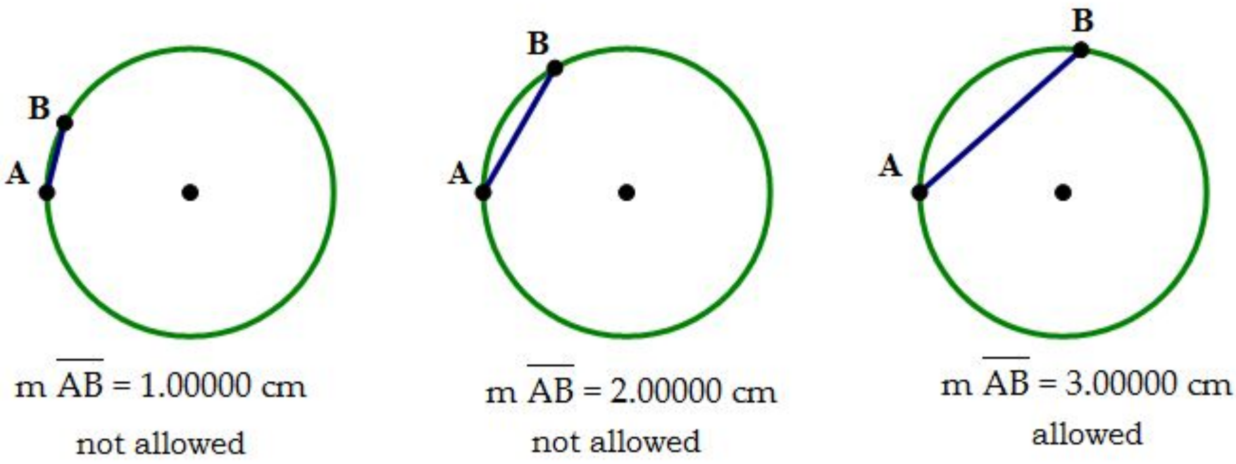
$$\frac{4}{3} = \frac{x}{y}$$

9

If points A and B are randomly placed on the circumference of a circle with radius 2, what is the probability that the length of chord AB is greater than 2?

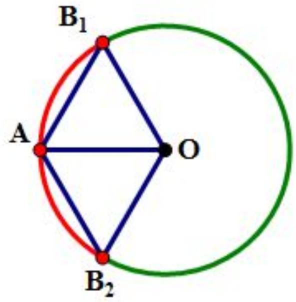
- 1/4
- 1/3
- 1/2
- 2/3
- 3/4

First of all, the circle has what is called rotational symmetry, and we will take advantage of this. Because of rotational symmetry, we can pick, at random, any location we want for point A, and just consider how far away randomly chosen point B locations would be. If the locations of points A & B that are, say, 3 cm apart, it will not make any difference to the problem where A and B are individually on the circle --- the only thing that matters to the problem is the distance between them. That's why we can simply select an arbitration location for point A and consider only the random possibilities for the location of point B.



Because we are interested in an inequality (chord > 2 cm), we will employ a standard mathematical strategy of *solving the equation first* (chord = 2 cm). Even though this particular chord would not satisfy the condition, this chord or location on the circle will form, as it were, a "boundary" between the "allowed" region and the "not allowed" region. Again, this is a standard mathematic strategy, often used in algebraic inequalities for example: turn it into an equation and solve the equation first --- the solutions to the equation form what mathematicians call the "boundary conditions" for the solution to the inequality.

Also, keep in mind ---- because point B could be randomly located anywhere on the circle, it could be on either side of point A --- clockwise from point A, or counterclockwise from point A. Therefore, we have to consider the (chord = 2) case on both sides of the circle.

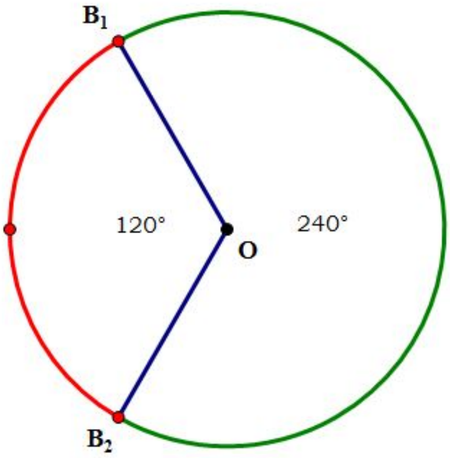


In that diagram, chords AB1 and AB2 both have a length of exactly 2 cm, so those points, and anything closer to point A than those points would be a "not allowed" location, a place with a chord less than or equal to 2 cm. This is the red region on the circle. The places where the chord would be greater than 2 cm is the green region of the circle. The probability question really reduces to a geometric question: what percent of the whole circle is the green arc?

Notice there's a most fortuitous set of connections in that diagram. There are five chord ---- AO, AB1, AB2, OB1, & OB2 ---- that all have the length of 2 cm. The segments AB1 & AB2, were chosen to have this length, and the other three are all radii of the circle, and we are told that the length of a radius is 2 cm.

This means the two triangles --- triangle OAB1 & triangle OAB2 ---- are both equilateral triangles. (In fact, any time you have chord = radius, and you connect radii to the endpoints of the chord, you will get an equilateral triangle). This means that each of angles is 60°. Since angle AOB1 = 60° and angle AOB2 = 60°, we know that angle B1OB2 = 120°. This is the central angle associated with the red arc (see the lesson on "Pieces of Pi" below).

There are 360° in a circle altogether. Since there are 120° in the angle associated with the red arc, the green arc must have an angle of 360° – 120° = 240°:



If point B lands anywhere on the green arc, it meets the condition (chord > 2 cm), and if point B lands anywhere on the red arc, it fails to meet the condition (chord ≤ 2 cm). The green arc takes up 240° of the full 360° of the circle.

$$\text{Probability} = \frac{240}{360} = \frac{24}{36} = \frac{2}{3}$$

Answer = D

10 The probability is 0.6 that an "unfair" coin will turn up tails on any given toss. If the coin is tossed 3 times, what is the probability that at least one of the tosses will turn up tails?

- ○ 0.064
- ○ 0.36
- ○ 0.64
- ○ 0.784
- ○ 0.936

The probability is 0.6 that an "unfair" coin will turn up tails on any given toss. If the coin is tossed 3 times, what is the probability that <u>at least</u> one of the tosses will turn up tails?

(A) 0.064
(B) 0.36
(C) 0.64
(D) 0.784
(E) 0.936

$$P(\text{tails}) = 0.6 \Rightarrow P(\text{heads}) = 1 - 0.6 = 0.4$$

$$\begin{aligned} P(\text{at least 1 tails}) &= 1 - P(\text{no tails}) \\ &= 1 - P(\text{all heads}) \\ &= 1 - P(H_1 \text{ AND } H_2 \text{ AND } H_3) \\ &= 1 - [P(H_1) \times P(H_2) \times P(H_3)] \\ &= 1 - [0.4 \times 0.4 \times 0.4] \\ &= 1 - [.064] \\ &= 0.936 \end{aligned}$$

11 An integer is randomly selected from the integers from 200 to 900 inclusive.

Column A	Column B
Probability that the number is either even or prime.	14/13

An integer is randomly selected from the integers from 200 to 900 inclusive.

<u>Column A</u>	B	<u>Column B</u>
Probability that the number is either even or prime.		$\frac{14}{13}$

$\boxed{0 \le probability \le 1}$

The probability that event A will occur is 0.5

The probability that event B will occur is 0.4

The probability that event A or B will occur is 0.8

Column A	Column B
Probability that A and B both occur	0.1

The probability that event A will occur is 0.5
The probability that event B will occur is 0.4
The probability that event A or B will occur is 0.8

Column A		Column B
Probability that A and B both occur	**C**	0.1

0.1

$$\boxed{P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})}$$

$$\begin{aligned} P(\text{A and B}) &= P(\text{A}) + P(\text{B}) - P(\text{A or B}) \\ &= 0.5 + 0.4 - 0.8 \\ &= 0.1 \end{aligned}$$

13 A box contains 10 balls numbered from 1 to 10 inclusive. If Ann removes a ball at random and replaces it, and then Jane removes a ball at random, what is the probability that both women removed the same ball?

- 1/100
- 1/90
- 1/45
- 1/10
- 41/45

A box contains 10 balls numbered from 1 to 10 inclusive. If Ann removes a ball at random and replaces it, and then Jane removes a ball at random, what is the probability that both women removed the same ball?

(A) $\frac{1}{100}$

(B) $\frac{1}{90}$

(C) $\frac{1}{45}$

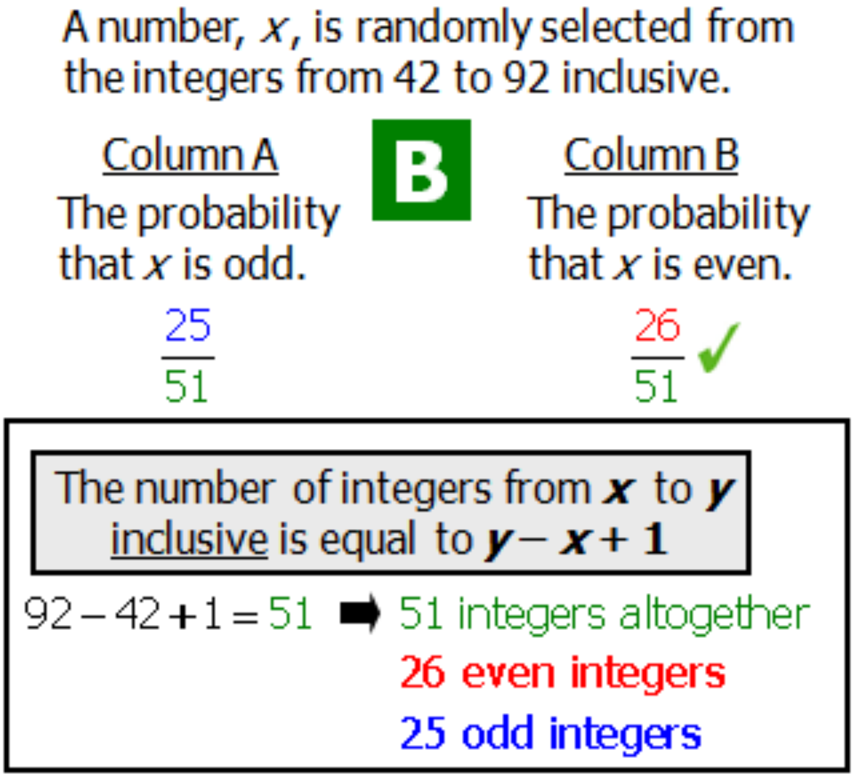
(D) $\frac{1}{10}$

(E) $\frac{41}{45}$

$$P(\text{Ann selects \# AND Jane selects same \#})$$
$$= P(\text{Ann selects \#}) \times P(\text{Jane selects same \#})$$
$$= 1 \times \frac{1}{10}$$
$$= \frac{1}{10}$$

14 A number, x, is randomly selected from the integers from 42 to 92 inclusive.

Column A	Column B
The probability that x is odd.	The probability that x is even.



15 When a coin is flipped, the probability of getting heads is 0.5, and the probability of getting tails is 0.5

A coin is flipped 5 times

Column A	Column B
Probability of getting exactly 2 heads	Probability of getting exactly 3 heads

This is a deeply conceptual question and, like many on the GRE, it really doesn't involve a single calculation. It requires imagination and the ability to shift perspectives.

Scenario A is a result of 2 H's & 3 T's.
Scenario B is a result of 3 H's & 2 T's.
If we were to do a calculation, we would use combinations, which are discussed the lesson video below.

nCr = the number of ways to choose r items from a pool of n items or another way to think about it
nCr = the number of ways r identical items can be placed in n possible positions

Let's arbitrarily say that getting a single H when we flip the coin is a "success." Then the probability of one success is 1/2. To figure out the number of ways exactly 2 successes could be distributed among the five trials, we would use combinations.

5C2 = the number of ways two successes could be distributed among 5 trials

For the purposes of this question, though, we don't even need that numerical value. We just need to know (a) there are five trials; (b) the probability of success in each trial is exactly 1/2; (c) we want exactly two successes in five trials, so there are 5C2 ways these successes could be distributed among the five trials. That's enough to allow us to calculate P, the probability of getting exactly two H's in five tosses. P is the value in Column A.

But wait! Notice, at the very beginning, I said: pretend H is a "success." That was completely arbitrary. I could have equally well said that T was a "success." If we treat T as a success, then conditions (a) & (b) & (c) in the previous paragraph would remain true, and the probability that results from those conditions, P, now would be the probability of getting exactly two T's in five tosses. Well, notice, if we toss the coin five times, and get exactly two T's, then we would have to get exactly 3 H's. Thus, P is also the value in Column B, and the columns are equal.

Answer = C

16 A box contains 10 pairs of shoes (20 shoes in total). If two shoes are selected at random, what is the probability that they are matching shoes?

- 1/190
- 1/20
- 1/19
- 1/10
- 1/9

A box contains 10 pairs of shoes (20 shoes in total). If two shoes are selected at random, what it is the probability that they are matching shoes?

(A) $\frac{1}{190}$

(B) $\frac{1}{20}$

(C) $\frac{1}{19}$

(D) $\frac{1}{10}$

(E) $\frac{1}{9}$

$$
\begin{aligned}
&P(\text{matching shoes}) \\
&= P(\text{selecting 1st shoe AND 2nd shoe matches 1st}) \\
&= P(\text{selecting 1st shoe}) \times P(\text{2nd shoe matches 1st}) \\
&= 1 \times \frac{1}{19} \\
&= \frac{1}{19}
\end{aligned}
$$

19 A: {71,73,79,83,87} B:{57,59,61,67}

If one number is selected at random from set A, and one number is selected at random from set B, what is the probability that both numbers are prime?

- ○ 9/20
- ○ 3/5
- ○ 3/4
- ○ 4/5
- ○ 1

$A:\{71,73,79,83,87\}$ $B:\{57,59,61,67\}$

If one number is selected at random from set A, and one number is selected at random from set B, what is the probability that both numbers are prime?

(A) $\frac{9}{20}$

(B) $\frac{3}{5}$

(C) $\frac{3}{4}$

(D) $\frac{4}{5}$

(E) 1

$A:\{71,73,79,83,87\}$ $B:\{57,59,61,67\}$

$$\begin{aligned} P(\text{both prime}) &= P(\text{A is prime AND B is prime}) \\ &= P(\text{A is prime}) \times P(\text{B is prime}) \\ &= \frac{4}{5} \times \frac{3}{4} \\ &= \frac{3}{5} \end{aligned}$$

18 When a certain coin is flipped, the probability of heads is 0.5. If the coin is flipped 6 times, what is the probability that there are exactly 3 heads?

- 1/4
- 1/3
- 5/16
- 31/64
- 1/2

When a certain coin is flipped, the probability of heads is 0.5
If the coin is flipped 6 times, what is the probability that there are exactly 3 heads?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{5}{16}$

(D) $\frac{31}{64}$

(E) $\frac{1}{2}$

$$\text{Probability} = \frac{\text{\# outcomes with 3 heads}}{\text{total \# of outcomes}} = \frac{20}{64} = \frac{5}{16}$$

$$\underset{\#1}{\underline{2}} \times \underset{\#2}{\underline{2}} \times \underset{\#3}{\underline{2}} \times \underset{\#4}{\underline{2}} \times \underset{\#5}{\underline{2}} \times \underset{\#6}{\underline{2}} = 64$$

$$\underset{\#1}{\underline{\quad}} \quad \underset{\#2}{\underline{\quad}} \quad \underset{\#3}{\underline{\quad}} \quad \underset{\#4}{\underline{\quad}} \quad \underset{\#5}{\underline{\quad}} \quad \underset{\#6}{\underline{\quad}}$$

Ways to select 3 out of 6 tosses $= {}_6C_3 = 20$

$$\boxed{{}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

Counting

1 From a group of 8 people, it is possible to create 56 different k-person committees. Which of the following could be the value of k ?

Indicate all such values.

☐ 1
☐ 2
☐ 3
☐ 4
☐ 5
☐ 6
☐ 7

From a group of 8 people, it is possible to create 56 different k-person committees. Which of the following could be the value of k?

$${}_8C_k = 56$$

Indicate <u>all</u> such values.

[A] 1
[B] 2
[C] 3
[D] 4
[E] 5
[F] 6
[G] 7

$$\boxed{{}_nC_r = \frac{\text{first } r \text{ values of } n!}{r!}}$$

$$\boxed{{}_nC_r = {}_nC_{n-r}}$$

$${}_{10}C_3 = {}_{10}C_7$$

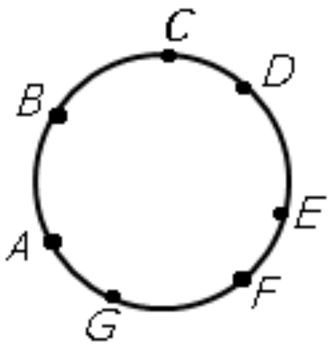
$${}_8C_1 = \frac{8}{1} = 8 \Rightarrow {}_8C_7 = 8$$

$${}_8C_2 = \frac{8\times 7}{2\times 1} = 28 \Rightarrow {}_8C_6 = 28$$

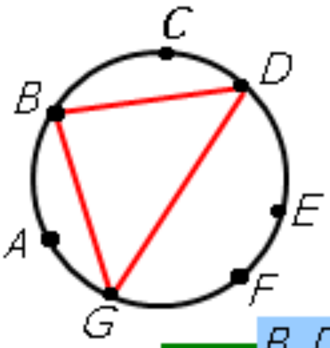
$${}_8C_3 = \frac{8\times 7\times 6}{3\times 2\times 1} = 56 \Rightarrow {}_8C_5 = 56$$

$${}_8C_4 = \frac{8\times 7\times 6\times 5}{4\times 3\times 2\times 1} = 70$$

2



Column A	Column B
Number of different triangles possible using the given points as vertices.	42



$B,D,G = D,B,G$

B

Column A	Column B
Number of different triangles possible using the given points as vertices.	42 ✓

$${}_7C_3 = \frac{7!}{3!\,(7-3)!}$$
$$= 35$$

$$\boxed{{}_nC_r = \frac{n!}{r!\,(n-r)!}}$$

3 An office has 6 employees. The manager must create a committee consisting of 3 employees.

Column A	Column B
Number of different committees possible.	40

An office has 6 employees. The manager must create a committee consisting of 3 employees.

B

Column A	Column B
Number of different committees possible.	40 ✓

$$\boxed{{}_nC_r = \frac{n!}{r!(n-r)!}}$$

$${}_6C_3 = \frac{6!}{3!(6-3)!}$$
$$= 20$$

4 $\dfrac{10! - 8!}{7!} =$

- 232
- 352
- 472
- 552
- 712

$\dfrac{10! - 8!}{7!} =$

(A) 232

(B) 352

(C) 472

(D) 552

(E) 712

$$\boxed{\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}}$$

$$\frac{10! - 8!}{7!} = \frac{10!}{7!} - \frac{8!}{7!} = 720 - 8 = 712$$

$$\frac{10!}{7!} = \frac{10\times9\times8\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}} = 720$$

$$\frac{8!}{7!} = \frac{8\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}} = 8$$

5 Joan is allowed to invite 3 of her friends to join her on a family camping trip. If Joan has 10 friends, in how many ways can she invite 3 of them?

- 27
- 120
- 240
- 360
- 720

Joan is allowed to invite 3 of her friends to join her on a family camping trip. If Joan has 10 friends, in how many ways can she invite 3 of them?

(A) 27

(B) 120

(C) 240

(D) 360

(E) 720

Question: Does order matter? No → combination

A, B, C, D, E, F, G, H, I, J

A, D, H $\overset{?}{=}$ D, A, H

$$\boxed{{}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

$$\begin{aligned}
{}_{10}C_3 &= \frac{10!}{3!(10-3)!} \\
&= \frac{10!}{3!(7)!} \\
&= \frac{10\times 9\times 8\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times \cancel{1}}{(3\times 2\times 1)(\cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times \cancel{1})} \\
&= \frac{10\times 9\times 8}{3\times 2\times 1} \\
&= 120
\end{aligned}$$

6 Car X can come with any of these 5 additional features: sunroof, stereo, tinted windows, leather seats and cruise control.

Column A	Column B
Number of different combinations possible	25

Car X can come with any of these 5 additional features: sunroof, stereo, tinted windows, leather seats and cruise control.

Column A		Column B
Number of different combinations possible	**A**	25
32 ✓		

$$\underbrace{2}_{\text{Sunroof}} \times \underbrace{2}_{\text{Stereo}} \times \underbrace{2}_{\text{Tinted windows}} \times \underbrace{2}_{\text{Leather seats}} \times \underbrace{2}_{\text{Cruise control}} = 32$$

7 From a total of 5 boys and 4 girls, how many 4-person committees can be selected if the committee must have exactly 2 boys and 2 girls?

- ○ 16
- ○ 24
- ○ 60
- ○ 120
- ○ 240

From a total of 5 boys and 4 girls, how many 4-person committees can be selected if the committee must have exactly 2 boys and 2 girls?

(A) 16

(B) 24

(C) 60

(D) 120

(E) 240

Boys	Girls
A, B, C, D, E	**F, G, H, I**

Question: Does order matter? No → combination

$$\mathbf{B, E} \overset{?}{=} \mathbf{E, B}$$

$$\underset{\text{select 2 boys}}{\underline{{}_5C_2}} \times \underset{\text{select 2 girls}}{\underline{{}_4C_2}}$$

$$\boxed{{}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

$$ {}_5C_2 \times {}_4C_2 = \frac{5!}{2!3!} \times \frac{4!}{2!2!}$$

$$= 10 \times 6$$

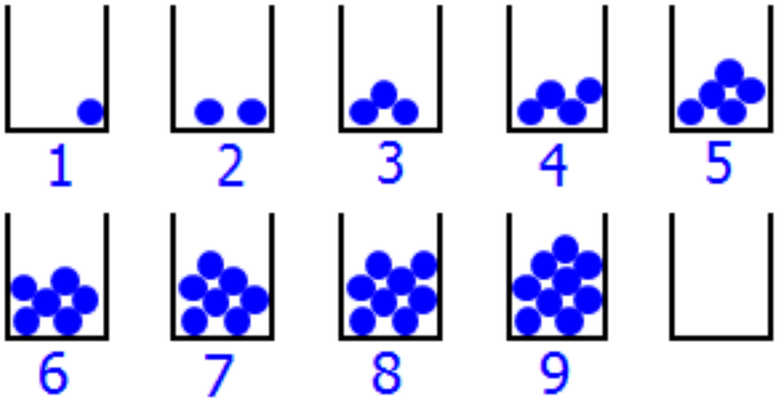
$$= 60$$

8 Joan has 100 candies to distribute among 10 children. If each child receives at least 1 candy and no two children receive the same number of candies, what is the maximum number of candies that a child can receive?

- 10
- 34
- 39
- 45
- 55

Joan has 100 candies to distribute among 10 children. If each child receives at least 1 candy and no two children receive the same number of candies, what is the maximum number of candies that a child can receive?

(A) 10
(B) 34
(C) 39
(D) 45
(E) 55



$$1+2+3+\ldots+n=\frac{n\times(n+1)}{2}$$

$$1+2+3+4+5+6+7+8+9=\frac{9\times 10}{2}$$

$$=45$$

$$\text{Candies in last bag}=100-45=55$$

9 In how many ways can Ann, Bob, Chuck, Don and Ed be seated in a row such that Ann and Bob are not seated next to each other?

- 24
- 48
- 56
- 72
- 96

In how many ways can Ann, Bob, Chuck, Don and Ed be seated in a row such that Ann and Bob are not seated next to each other?

(A) 24
(B) 48
(C) 56
(D) 72
(E) 96

\# without restrictions − # with A&B together

A, B, C, D, E

$$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$$

$\boxed{AB}, C, D, E$

$$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$$

$\boxed{BA}, C, D, E$

$$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$$

$$120 - 24 - 24 = 72$$

10 How many integers between 1 and 1021 are such that the sum of their digits is 2?

- 190
- 210
- 211
- 230
- 231

How many integers between 1 and 10^{21} are such that the sum of their digits is 2?

(A) 190

(B) 210

(C) 211

(D) 230

(E) 231

$$10^{21} = 1,000,000,000,000,000,000,000$$

$$000,000,000,000,010,100,000 = 10,100,000$$

$$000,000,000,000,000,020,000 = 20,000$$

Case I : two 1's ➡ ${}_{21}C_2$ ways

➡ 210 ways

$$\boxed{{}_nC_r = \frac{n!}{r!(n-r)!}}$$

_ _ _ , _ _ _ , _ _ _ , _ _ _ , _ _ _ , _ _ _ , _ _ _

Case II : one 2 ➡ 21 ways

_ _ _ , _ _ _ , _ _ _ , _ _ _ , _ _ _ , _ _ _ , _ _ _

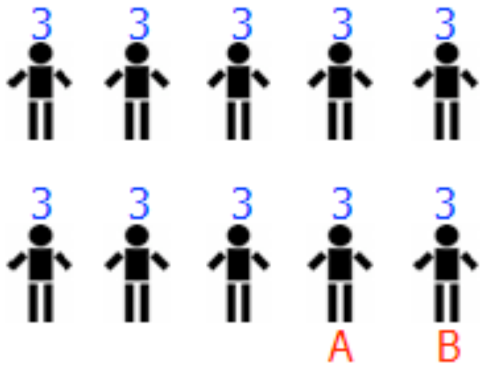
$$\text{Total} = 210 + 21 = 231$$

11 There are 10 people in a room. If each person shakes hands with exactly 3 other people, what is the total number of handshakes?

- 15
- 30
- 45
- 60
- 120

There are 10 people in a room. If each person shakes hands with exactly 3 other people, what is the total number of handshakes?

(A) 15
(B) 30
(C) 45
(D) 60
(E) 120



$$\frac{3+3+3+3+3+3+3+3+3+3}{2}=\frac{30}{2}=15$$

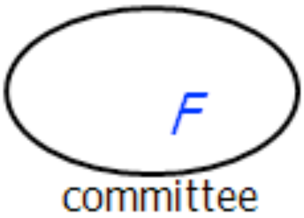
12 An office has 6 employees; there are 5 female employees and 1 male employee. In how many ways can a 3-person committee be created if the committee must include the male employee?

- 10
- 12
- 15
- 24
- 30

An office has 6 employees; there are 5 female employees and 1 male employee. In how many ways can a 3-person committee be created if the committee must include the male employee?

(A) 10
(B) 12
(C) 15
(D) 24
(E) 30

A, B, C, D, E



$${}_{5}C_{2} = 10$$

$$\boxed{{}_{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

13 How many multiples of 5 are there between 81 and 358?

- 54
- 55
- 56
- 57
- 58

How many multiples of 5 are there between 81 and 358?

(A) 54
(B) 55
(C) 56
(D) 57
(E) 58

$$\left.\begin{aligned} 85 &= 17 \times 5 \\ 90 &= 18 \times 5 \\ 95 &= 19 \times 5 \\ &\ \vdots \\ 350 &= 70 \times 5 \\ 355 &= 71 \times 5 \end{aligned}\right\} \text{how many?}$$

$$\boxed{\begin{array}{c}\text{The number of integers} \\ \text{from } \boldsymbol{x} \text{ to } \boldsymbol{y} \text{ \underline{inclusive}} \\ \text{is equal to } \boldsymbol{y} - \boldsymbol{x} + 1\end{array}}$$

$$\begin{aligned} \# &= 71 - 17 + 1 \\ &= 55 \end{aligned}$$

14 N equals the number of positive 3-digit numbers that contain odd digits only.

Column A	Column B
N	125

N equals the number of 3-digit numbers that contain odd digits only.

Column A		Column B
N	C	125
125		

1,3,5,7,9

$$\underset{1^{st}\text{digit}}{\underline{\;5\;}} \times \underset{2^{nd}\text{digit}}{\underline{\;5\;}} \times \underset{3^{rd}\text{digit}}{\underline{\;5\;}} = 125$$

15 A certain restaurant offers 8 different salads, 5 different main courses, 6 different desserts. If customers choose one salad, one main course and two desserts for their meal, how many different meals are possible?

- 120
- 240
- 480
- 600
- 1200

A certain restaurant offers 8 different salads, 5 different main courses, 6 different desserts. If customers choose one salad, one main course and two desserts for their meal, how many different meals are possible?

(A) 120

(B) 240

(C) 480

(D) 600

(E) 1200

$$\underset{\text{select salad}}{\underline{\quad 8 \quad}} \times \underset{\text{select main}}{\underline{\quad 5 \quad}} \times \underset{\text{select desserts}}{\underline{\quad {}_6C_2 \quad}}$$

$$\boxed{{}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

$$\text{Total meals} = 8 \times 5 \times {}_6C_2 = 8 \times 5 \times 15 = 600$$

16 Hal has 4 girl friends and 5 boy friends. In how many different ways can Hal invite 2 girls and 2 boys to his birthday party?

- 54
- 60
- 72
- 120
- 240

Hal has 4 girl friends and 5 boy friends. In how many different ways can Hal invite 2 girls and 2 boys to his birthday party?

(A) 54
(B) 60
(C) 72
(D) 120
(E) 240

$$\underbrace{{}_4C_2}_{\text{invite 2 girls}} \times \underbrace{{}_5C_2}_{\text{invite 2 boys}}$$

$$6 \times 10 = 60$$

$$\boxed{{}_nC_r = \frac{n!}{r!(n-r)!}}$$

17 A knockoff website requires users to create a password using letters from the word MAGOSH. If each password must have at least 4 letters and no repeated letters are allowed, how many different passwords are possible?

A knockoff website requires users to create a password using letters from the word MAGOSH. If each password must have at least 4 letters and no repeated letters are allowed, how many different passwords are possible? $\boxed{1800}$

MAGOSH

4 letters: $\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} = 360$

5 letters: $\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} = 720$

6 letters: $\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 720$

Total $= 360 + 720 + 720 = 1800$

18 If k is the greatest positive integer such that 3k is a divisor of 15! then k =

If k is the greatest positive integer such that 3^k is a divisor of 15! then $k =$

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

$$\begin{aligned}15! &= 1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11\times12\times13\times14\times15\\ &= 1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11\times12\times13\times14\times15\\ &= 1\times2\times3\times4\times5\times(3\times2)\times7\times8\times(3\times3)\times10\times11\times(3\times4)\times13\times14\times(3\times5)\\ &= 3^6(1\times2\times4\times5\times2\times7\times8\times10\times11\times4\times13\times14\times5)\end{aligned}$$

19 Main course: Chicken, Beef, Tofu

Side dish: Rice, Salad, Soup, Pasta

Dessert: Pie, Cake

A meal at a certain restaurant consists of 1 main course, 2 different side dishes and 1 dessert.

Column A	Column B
Number of different meals possible	36

Main course	Side dish	Dessert
Chicken	Rice	Pie
Beef	Salad	Cake
Tofu	Soup	
	Pasta	

A meal at a certain restaurant consists of 1 main course, 2 different side dishes and 1 dessert.

Column A		Column B
Number of different meals possible	C	36

36

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

$$ \underset{\text{Main}}{\underline{3}} \times \underset{\text{Side}}{\underline{6}} \times \underset{\text{Dessert}}{\underline{2}} = 36 $$

${}_4C_2$

20 How many three-digit numbers are there such that all three digits are different and the first digit is not zero?

- 504
- 648
- 720
- 729
- 810

How many three-digit numbers are there such that all three digits are different and the first digit is not zero?

(A) 504
(B) 648
(C) 720
(D) 729
(E) 810

$$ \underset{\text{select 1st digit}}{\underline{9}} \times \underset{\text{select 2nd digit}}{\underline{9}} \times \underset{\text{select 3rd digit}}{\underline{8}} = 648 $$

21

In a certain sock drawer, there are 4 pairs of black socks, 3 pairs of gray socks and 2 pairs of orange socks. If socks are removed at random without replacement, what is the minimum number of socks that must be removed in order to ensure that two socks of the same color have been removed?

- ○ 4
- ○ 7
- ○ 9
- ○ 10
- ○ 11

In a certain sock drawer, there are 4 pairs of black socks, 3 pairs of gray socks and 2 pairs of orange socks. If socks are removed at random without replacement, what is the minimum number of socks that must be removed in order to ensure that two socks of the same color have been removed?

(A) 4
(B) 7
(C) 9
(D) 10
(E) 11

22 In how many different ways can 3 boys and 3 girls be seated in a row of 6 chairs such that the girls are not separated, and the boys are not separated?

- ○ 24
- ○ 36
- ○ 72
- ○ 144
- ○ 288

In how many different ways can 3 boys and 3 girls be seated in a row of 6 chairs such that the girls are not separated, and the boys are not separated?

(A) 24

(B) 36

(C) 72

(D) 144

(E) 288

ABC DEF

Boys: ABC Girls: DEF

$$\underline{3} \times \underline{2} \times \underline{1} \times \underline{3} \times \underline{2} \times \underline{1} = 36$$

Girls: DEF Boys: ABC

$$\underline{3} \times \underline{2} \times \underline{1} \times \underline{3} \times \underline{2} \times \underline{1} = 36$$

$$\text{Total} = 36 + 36 = 72$$

23 In how many ways can 16 different gifts be divided among four children such that each child receives exactly four gifts?

- ○ 164
- ○ (4!)4
- ○ 16!/(4!)4
- ○ 16!/4!
- ○ 416

In how many ways can 16 different gifts be divided among four children such that each child receives exactly four gifts?

$$\text{}_nC_r = \frac{n!}{r!(n-r)!}$$

A) 16^4

B) $(4!)^4$

C) $\frac{16!}{(4!)^4}$

D) $\frac{16!}{4!}$

E) 4^{16}

$$\underbrace{{}_{16}C_4}_{\substack{1\text{st}\\ \text{child}}} \times \underbrace{{}_{12}C_4}_{\substack{2\text{nd}\\ \text{child}}} \times \underbrace{{}_{8}C_4}_{\substack{3\text{rd}\\ \text{child}}} \times \underbrace{{}_{4}C_4}_{\substack{4\text{th}\\ \text{child}}}$$

$$= \frac{16!}{4!12!} \times \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times 1$$

$$= \frac{16!}{4!\cancel{12!}} \times \frac{\cancel{12!}}{4!\cancel{8!}} \times \frac{\cancel{8!}}{4!4!} \times 1$$

$$= \frac{16!}{4!4!4!4!}$$

$$= \frac{16!}{(4!)^4}$$

24 Kim is taking a math class, and the teacher gives a multiple choice test consisting of 8 questions. If each question has 5 answer choices, and Kim answers every question, in how many different ways can she complete the test?

- 40
- 400
- 58
- 85
- 4040

Kim is writing a multiple choice test consisting of 8 questions. If each question has 5 answer choices, and Kim answers every question, in how many different ways can she complete the test?

(A) 40

(B) 400

(C) 5^8

(D) 8^5

(E) 40^{40}

$$\underset{\#1}{\underline{5}} \times \underset{\#2}{\underline{5}} \times \underset{\#3}{\underline{5}} \times \underset{\#4}{\underline{5}} \times \underset{\#5}{\underline{5}} \times \underset{\#6}{\underline{5}} \times \underset{\#7}{\underline{5}} \times \underset{\#8}{\underline{5}} = 5^8$$

25 Sid intended to type a seven-digit number, but the two 3's he meant to type did not appear. What appeared instead was the five-digit number 52115. How many different seven-digit numbers could Sid have meant to type?

- 10
- 16
- 21
- 24
- 27

First of all, notice that the numerals 5-2-1-1-5 must be in that order --- the order may be broken at any point but one or two 3's, but ignoring the 3's, those five must have that order.

As is often the case, in this counting problem we have a variety of ways to frame the question. I believe the most straightforward is as follows. Consider the seven "blank spaces" into which we will write the seven digits of the original number.

_ _ _ _ _ _ _

Now, select any two of those spaces, and put the two 3's there. The two 3's could be next to each other or apart. Suppose, for example, we put them here:

_ _ 3 _ _3 _

At this point, notice that everything else about the number is determined: we simply will put the digits 5-2-1-1-2 in order in the remaining spaces. Just picking two of the seven spaces for the location of the two 3's is enough to determine the entire original number. Well, there are 7C2 ways of selecting two slots from seven, so 7C2 must be the number of possible original numbers that Sid intended to write.

See this post for various techniques on calculating combinations.

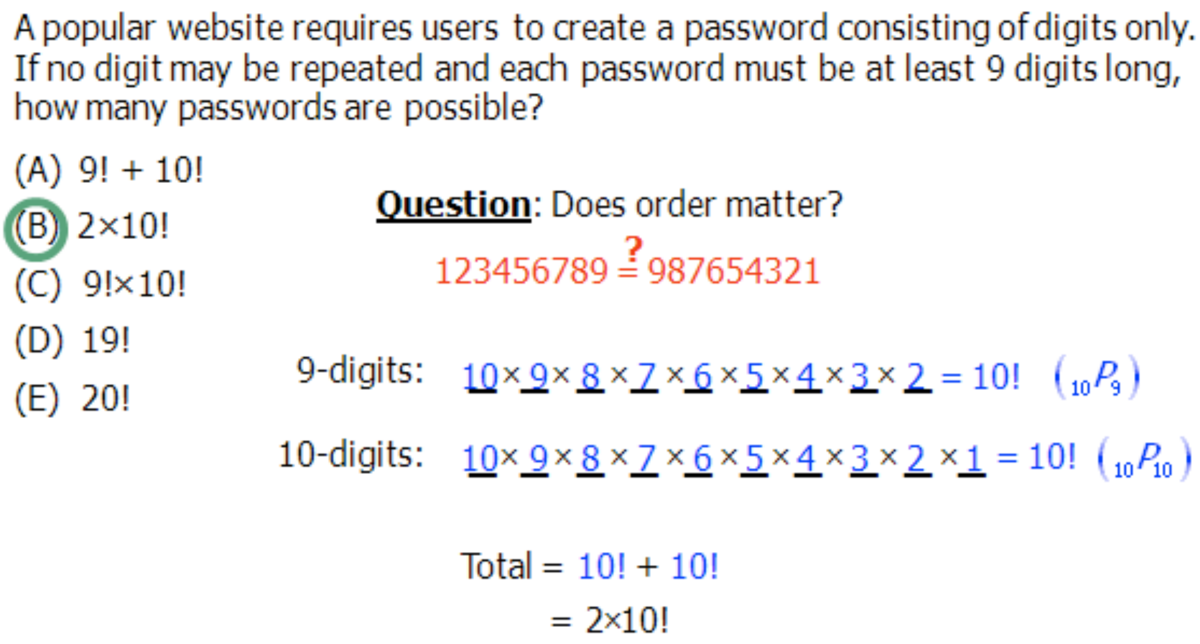
7C2 = 21

So there are 21 possible original numbers that Sid could have intended.

Answer = C

26 A popular website requires users to create a password consisting of digits only. If no digit may be repeated and each password must be at least 9 digits long, how many passwords are possible?

- 9! + 10!
- 2 x 10!
- 9! x 10!
- 19!
- 20!

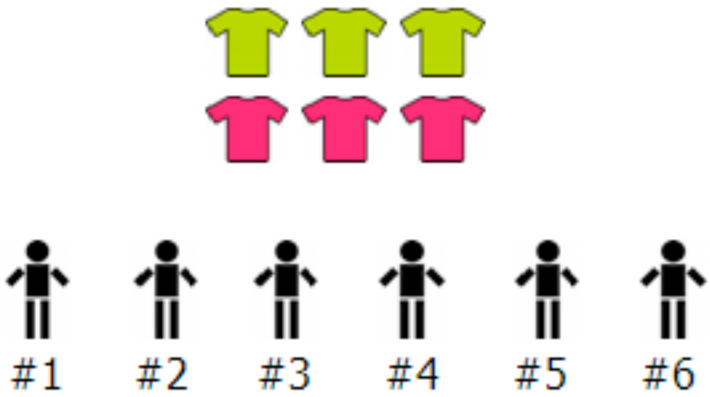


27 In how many different ways can 3 identical green shirts and 3 identical red shirts be distributed among 6 children such that each child receives a shirt?

- 20
- 40
- 216
- 720
- 729

In how many different ways can 3 identical green shirts and 3 identical red shirts be distributed among 6 children such that each child receives a shirt?

(A) 20
(B) 40
(C) 216
(D) 720
(E) 729



$$\boxed{{}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}} \quad {}_6C_3 = 20$$

28 How many positive integers less than 10,000 are such that the product of their digits is 210?

How many positive integers less than 10,000 are such that the product of their digits is 210?

(A) 24
(B) 30
(C) 48
(D) 54
(E) 72

$210 = 2 \times 3 \times 5 \times 7$

Case i: 4-digit number with 2, 3, 5, and 7
→ 4! (24) possibilities

Case ii: 3-digit number with 5, 6, and 7
→ 3! (6) possibilities

Case iii: 4-digit number with 1, 5, 6, and 7
→ 4! (24) possibilities

Total = 24 + 6 + 24 = **<u>54</u>**