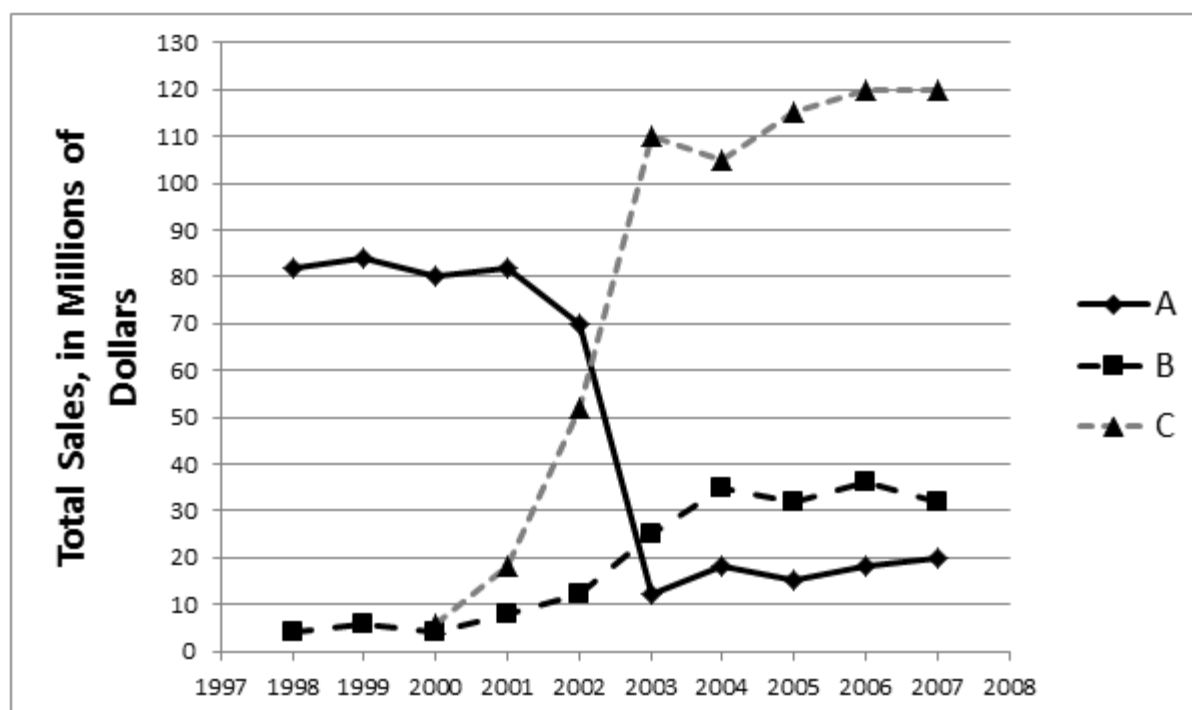


Data Interpretation-附解析



The graph above shows the total sales, in millions of dollars, for three companies, A & B & C, in a particular sector for the years 1998 through 2007 inclusive. Assume these companies are the only three companies active in this particular sector. Company A is represented by a solid black line; Company B is represented by the black line with large dashes; Company C is represented by the gray line with small dashes. Companies A & B existed since the 1980s, although only data from 1998 is shown. Company C's first year in existence was 2000.

1 Total sales, by all companies in the sector, increased by approximately what percent from 1998 to 2007?

- ☐ 46%
- ☐ 72%
- ☐ 100%
- ☐ 153%
- ☐ 300%

We know these three companies are the only companies in the sector, so the numbers shown on the chart account for all sales in the sector. In 1998, company B was at about 5M, and company A was about 81M, for a total of 86M. In 2007, A was at 20M, B was at 32M, and C was at 120M, for a total of 172M. Approximate $172/86$ as $170/85 = 2$ --- sales in the sector more or less doubled, which is about 100% increase. Answer = (C).

A more precise calculation:

$$\text{percentage change} = \frac{\text{end value} - \text{start value}}{\text{start value}} \times 100\%$$

$$\text{percentage change} = \frac{172 - 86}{86} \times 100\% = 100\%$$

Answer = **(C)**.

2 company C was responsible for approximately what percent of total sales in the sector in 2007?

- ☐ 24%
- ☐ 40%
- ☐ 55%
- ☐ 70%
- ☐ 93%

In 2007, A was at 20M, B was at 32M, and C was at 120M, for a total of 172M. What percent of 172 is 120? Let's approximate this: 10% of 172 is approximately 17, so 20% is 34 and 30% is 51. Well, 51 is approximately equal to the sum of A & B's sales --- if they account for 30%, that must mean C accounts for 70%. Answer = **(D)**.

A more precise calculation:

$$\text{percent} = \frac{120}{172} \times 100\% = 69.767$$

which we round to 70%. Answer = **(D)**.

3 From 2002 to 2003, Company C had what percentage increase in its total sales?

- ☐ 26%
- ☐ 53%
- ☐ 78%
- ☐ 90%
- ☐ 112%

Company C, the gray-dashed line with triangles, had an approximate value a little \$50M in sales in 2002, and about \$110M in 2003. That's more than double, which is more than 100% increase. Only **(E)** is in that range, so that's the answer.

A more precise calculation:

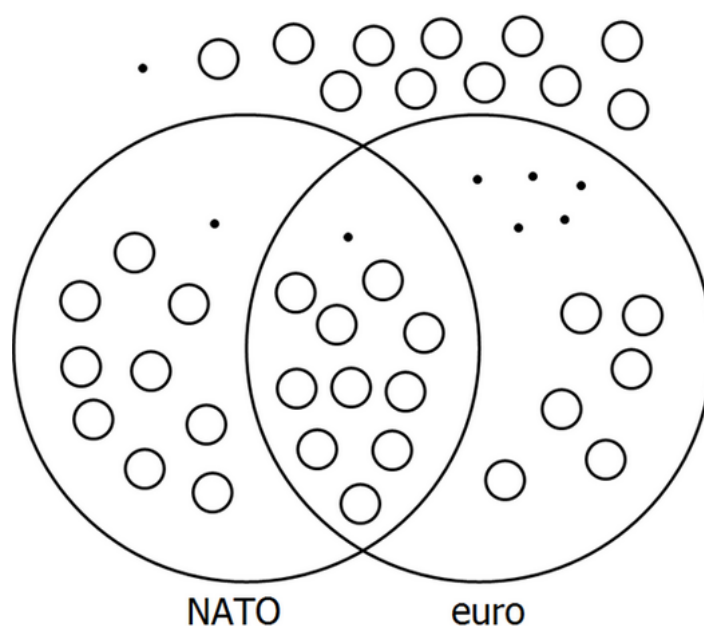
$$\text{percentage change} = \frac{\text{end value} - \text{start value}}{\text{start value}} \times 100\%$$

$$\text{percentage change} = \frac{110 - 52}{52} \times 100\% = 112\%$$

Answer = (E).



The diagram shows the 44 nations that occupy the continent of Europe. (The diagram excludes Russia, which occupies both Europe & Asia.) Every dot is a smaller nation, with a national population less than 500,000; the circles are nations each with more than half a million people. Those nations in the "NATO" circle, as of 2013, are members of the NATO military alliance. Those nations in the "euro" circle, as of 2013, use the euro as their primary currency.



1 Approximately what percent of nations in Europe have a national population less than 500,000 people?

- ☐ 18%
- ☐ 22%
- ☐ 27%
- ☐ 36%
- ☐ 50%

We know there are 44 countries --- that's the "whole." We easily count eight dots on the

diagram --- those are the countries with populations less than 500,000, so 8 is the "part". We'll estimate --- $8/44$ is smaller than $8/40$ (when the denominator gets smaller, the fraction gets larger), and $8/40 = 1/5 = 20\%$, so we need a percent slightly less than 20%. The only possibility is **(A)**. In a more precise calculation:

$$\text{percent} = \frac{8}{44} \times 100\% = 18.181818 \rightarrow 18\%$$

Answer = **(A)**.

2 Of the nations with national populations more than half a million people, approximately what percent of European nations are neither members of NATO nor primary users of the euro?

- ☐ 25%
- ☐ 27%
- ☐ 31%
- ☐ 47%
- ☐ 75%

The circles are the countries with more than half a million people. We know there are 44 countries altogether, and we easily count 8 dots, so there must be $44 - 8 = 36$ circles. That's the "whole". The "part" consists of the 11 circles outside both the NATO & euro Venn circles.

We'll estimate ---- $11/36$ is less than $11/33$ (when the denominator gets smaller, the fraction gets larger), and $11/33 = 1/3 = 33\%$, so $11/36$ must be slightly less than 33%. Answer **(C)** is perfect.

In a more precise calculation:

$$\text{percent} = \frac{11}{36} \times 100\% = 30.5555 \rightarrow 31\%$$

Answer = **(C)**.

3 Of all the nations with populations greater than 500,000 who are not members of NATO, approximately what percent of them use the euro as their primary currency?

- ☐ 26%
- ☐ 35%
- ☐ 41%
- ☐ 48%
- ☐ 63%

The circles are the countries with more than half a million people. If we count all the circles, not dots, that are outside the NATO circle, we find 6 are in the euro circle, and 11 are above the two Venn circles. That's a total of 17, the "whole", and the "part" consists of those six inside the euro circle.

We'll estimate ---- $6/17$ is slightly more than $6/18$ (when the denominator gets bigger, the fraction gets smaller), and $6/18 = 1/3 = 33\%$, so we expect a percent slightly larger than 33% . Answer **(B)** is perfect.

In a more precise calculation:

$$\text{percent} = \frac{6}{17} \times 100\% = 35.2941 \rightarrow 35\%$$

Answer = **(B)**.

4 How many nations in Europe have a population more than 500,000, are members of NATO, and do not use the euro as their primary currency?

Here, we have to count not dots, only circles (i.e. countries with populations greater than 500,000) that are inside the NATO circle but outside the euro circle. These are the circles in the crescent on the far left in the diagram --- there are 9 circles there, so there are 9 countries with a population more than 500,000 that are members of NATO and do not use the euro as their primary currency.

5 What is the least number of nations currently not members of NATO that would have to join NATO so that more than 50% of the nations in Europe would be members of NATO?

- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4
- ☐ 5

Inside the NATO circle, counting both circles & dots, there are 21 countries. There are 44 altogether, so outside the NATO circle there must be $44 - 21 = 23$ countries. Right now, NATO = 21 and non-NATO = 23. If one country not in NATO joined NATO, the count would be NATO = 22 and non-NATO = 22 --- the two segments would be equal, but we still would not meet the "more than 50%" condition. If from the starting configuration, two countries join NATO from outside, the count would be NATO = 23 and non-NATO = 21 --- under those conditions, more than 50% of the countries would be NATO members. We need at least two to switch, so that's the answer. Answer = **(B)**.

6 Consider the nation represented by the single dot inside the left circle but outside the right circle. This dot has to represent which of the following nations?

- Iceland (population = 103,000); NATO member; primary currency = krona
- Latvia (population = 2,067,900); NATO member; primary currency = lats
- Luxembourg (population = 448,569); NATO member; primary currency = euro
- Montenegro (population = 616,258); not a NATO member; primary currency = euro
- Vatican City (population = 900); not a NATO member; primary currency = euro

That country is inside the NATO circle, so the countries with "not a NATO member" (choices (D) & (E)) must be wrong. That country is outside the euro circle, so any country with "primary currency = euro" is wrong --- we also can eliminate (C). Finally, that country is a dot, not a circle, so it has a population less than 500,000, so any country with a population greater than 500,000 is wrong --- we also can eliminate (B). This leaves (A) as the only possibility, the beautiful country of Iceland.



Number of televisions	Percent of households
0	16%
1	8%
2	24%
3	20%
4	20%
5 or more	12%

The table identifies the percentage of households in Townville that have a certain number of televisions.

1 If k is the number of households with exactly 2 televisions, then the number of households with exactly 4 televisions is

- $k - 4$
- $2k$
- $k/100 - 4$
- $5k/6$
- $500k/6$

If k is the number of households with exactly 2 televisions, then the number of households with exactly 4 televisions is

(A) $k - 4$

(B) $2k$

(C) $\frac{k}{100} - 4$

(D) $\frac{5k}{6}$

(E) $\frac{500k}{6}$

$$\begin{aligned} \frac{\text{\# of households}}{\text{\% of households}} &: \frac{k}{24} = \frac{?}{20} \\ 24? &= 20k \\ ? &= \frac{20k}{24} \\ ? &= \frac{5k}{6} \end{aligned}$$

Number of televisions	Percent of households
0	16%
1	8%
2	24%
3	20%
4	20%
5 or more	12%

The table identifies the percentage of households in Townville that have a certain number of televisions.

2 What is the median number of televisions per household?

- ☐ Cannot be determined
- ☐ 1
- ☐ 2
- ☐ 2.5
- ☐ 3

What is the median number of televisions per household?

(A) Cannot be determined

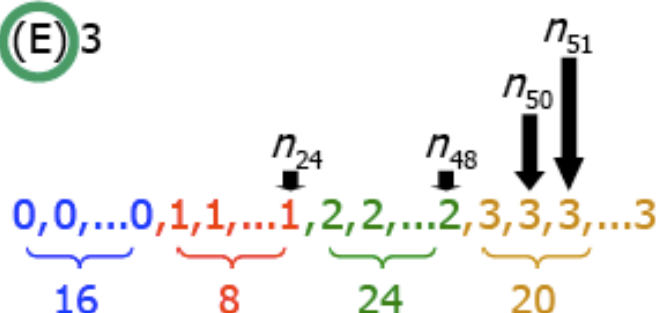
(B) 1

(C) 2

(D) 2.5

(E) 3

$$\frac{n_{50} + n_{51}}{2} = \frac{3 + 3}{2} = 3$$



Number of televisions	Percent of households
0	16%
1	8%
2	24%
3	20%
4	20%
5 or more	12%

The table identifies the percentage of households in Townville that have a certain number of televisions.

3 What is the average (arithmetic mean) number of televisions per household?

- Cannot be determined
 ○ 2.1
 ○ 2.3
 ○ 2.5
 ○ 2.7

What is the average (arithmetic mean) number of televisions per household?

- (A) Cannot be determined
 (B) 2.1
 (C) 2.3
 (D) 2.5
 (E) 2.7

Total # of televisions

100

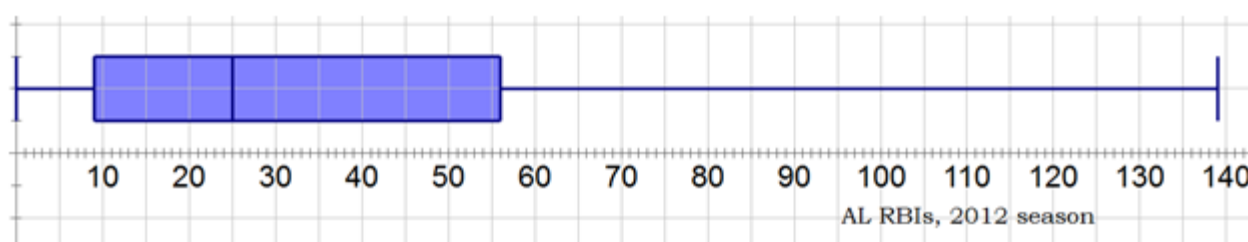
$$= \frac{???}{100}$$

Number of televisions	Percent of households
0	16%
1	8%
2	24%
3	20%
4	20%
5 or more	12%

The table identifies the percentage of households in Townville that have a certain number of televisions.

四

The following boxplot shows the 2012 season runs batted in (RBIs) of 280 American League (AL) batters (the top 280 batters in terms of number of plate appearances).



Five-Number Summary for AL RBIs in 2012:

Minimum = 0

First Quartile = 10

Median = 25

Third Quartile = 56

Maximum = 139

1 What is the size of the Interquartile Range (IQR) of this distribution?

- ☐ 25
- ☐ 47
- ☐ 56
- ☐ 83
- ☐ 139

IQR stands for Interquartile Range. The IQR is the range between the two quartiles, (third quartile value) – (first quartile value). The "box" in the middle of the boxplot represents the IQR. From the five-number summary, we see First Quartile = 9 and Third Quartile = 56, so the difference between these is $56 - 9 = 47$. Answer = **(B)**.

2 If no batter hit exactly 25 RBIs, then how many AL hitters hit more than 25 RBIs in 2012?

- ☐ 9
- ☐ 56
- ☐ 83
- ☐ 114
- ☐ 140

Notice that 25 is the median value. Half the population lies above the median and half, below. The size of this "population" is 280 batters, so half of that is 140. Answer = **(E)**.

3 B. J. Upton, who played on the Tampa Bay Rays that season, hit 78 RBIs in 2012; this is the 90th percentile value on this chart. How many players hit between 56 and 78 RBIs?

- ☐ 14
- ☐ 22
- ☐ 28
- ☐ 34
- ☐ 42

90th percentile value = 78. Notice that 56 is the Third Quartile value, i.e. the 75th percentile value. Well, between the 75th percentile and the 90th percentile is $90 - 75 = 15\%$ of the population. What is 15% of 280? We don't need a calculator for this. Clearly,

10% of 280 = 28

Take half of that.

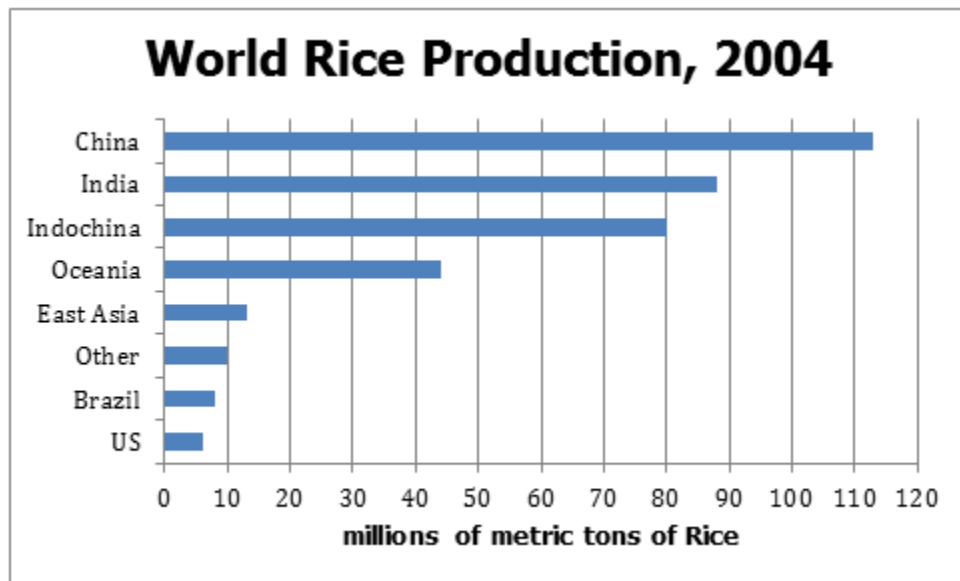
5% of 280 = 14

Now, add those two.

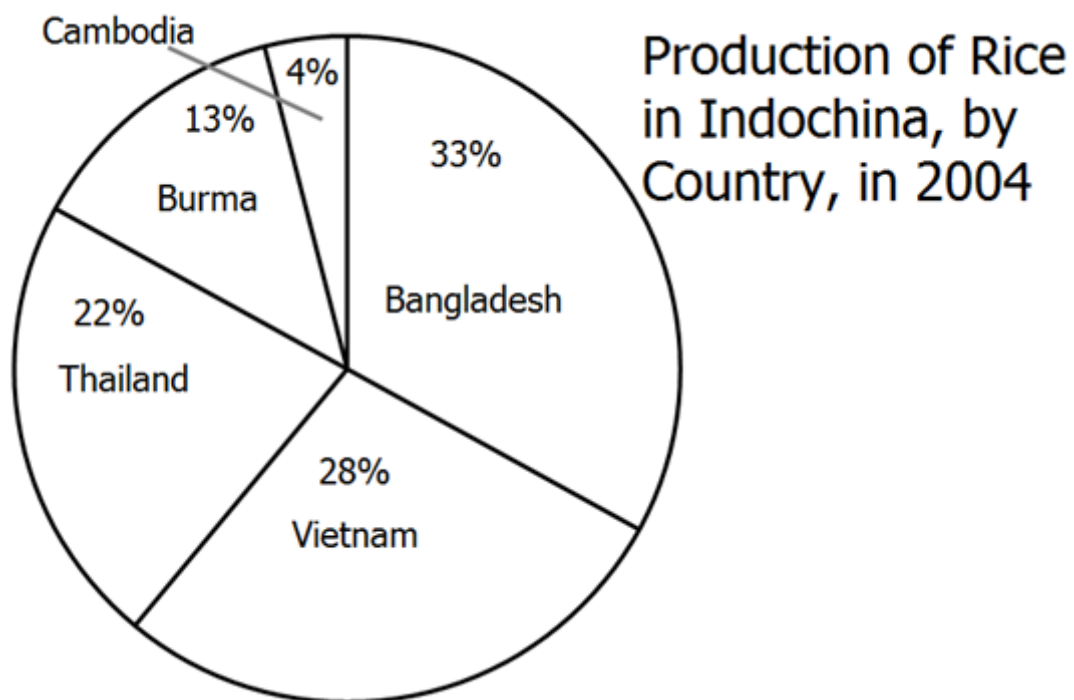
15% of 280 = 42

Answer = **(E)**.

五.



World's Total Rice Production in 2004 = 368,080,000 metric tons



1 China accounted for approximately what percent of world rice production in 2004?

- ☐ 9.6%
- ☐ 24.0%
- ☐ 30.7%
- ☐ 40.1%

○ 65.3%

China's production is approximately 112M tons, and world production is approximate 368M tons. These are the "part" and "whole" respectively. Approximate with the first two digits – 11/36. Well, $12/36 = 1/3 = 33\%$, so this percent should be slightly less than 33%. Answer **C** is perfect.

In a more precise calculation, China's production is approximately 112,000,000 metric tons; that's the "part". The world production is 368,080,000. That's the "whole." Then

$$\text{percent} = \frac{110,000,000}{360,000,000} \times 100\% = 30.6\%$$

Answer = **C**.

2 Which bars on the graph, if they increased by 50% from 2004 to 2005, would be equal to or greater than the current 2004 value of the bar immediately above it on the graph?

- ☐ India
- ☐ Indochina
- ☐ Oceania
- ☐ East Asia
- ☐ Other
- ☐ Brazil
- ☐ US

This is a real visual question. What you have to do, for each bar: mentally divide it in half, add a half-length to the right side, and compare that to the length of the bar above it. The visual approach is definitely the fastest way to answer this question. You are making a serious strategic error if you try to do this whole question by means of numerical calculations.

India's bar, plus an extra half, is greater than China's bar. Yes to **[A]**.

Indochina's bar, plus an extra half, is greater than India's bar. Yes to **[B]**.

Oceania's bar, plus an extra half, is *not* greater than Indochina's bar. No to **[C]**.

East Asia's bar, plus an extra half, is *not* greater than Oceania's bar. No to **[D]**.

Other's bar, plus an extra half, is greater than East Asia's bar. Yes to **[E]**.

Brazil's bar, plus an extra half, is greater than Other's bar. Yes to **[F]**.

The US's bar, plus an extra half, is greater than Brazil's bar. Yes to **[G]**.

3 Bangladesh accounted for what percent of world rice production in 2004?

- ☐ 1.4%
- ☐ 7.2%
- ☐ 11.6%
- ☐ 33%
- ☐ 47.6%

Bangladesh is 33% of the Indochina circle, about 1/3 of 80M, or around 26M. Well, world production is approximately 368M --- this means, 1% would be 3.68M, and 10% would be 36.8M. Approximate this as 36M --- half of that, 5%, would be around 18M. We see that 26M is clearly less than 10%, but greater than 5%. The answer must be **B**.

In a more precise calculation,

$$33\% \text{ of } 80,000,000 = (0.33) * 80,000,000 = 26,400,000$$

That's the "part", and the "whole" is world rice production:

$$\text{percent} = \frac{26,400,000}{368,080,000} \times 100\% = 7.2\%$$

Answer = **B**.

4 If from 2004 to 2005, Vietnam's rice production increases by 25%, and all the other countries in the "Indochina" group maintain the same levels of production, then the rice production of the Indochina group would increase by what percent?

- ☐ 2.8%
- ☐ 4%
- ☐ 5.6%
- ☐ 7%
- ☐ 12.5%

Since everything here is a ratio, we can just treat the percent values directly. Vietnam is 28% of Indochina's production. If this increases by 25%, by 1/4, that's an additional 7%. Everything else stays the same, so Indochina's production overall would increase by 7%. Answer = **D**.

5 What was the approximate rice production of Vietnam in 2004?

- ☐ 8,400,000
- ☐ 22,400,000
- ☐ 43,700,000
- ☐ 80,000,000

☐ 103,060,000

The answers for this are widely spread, which gives us permission to do some very rough approximations. Vietnam is 28% of the Indochina circle --- approximate that as 30%. All of Indochina had a production of 80M. Well, 10% of 80 is 8, and if we multiply that by 3, 30% of 80 is 24. Our approximate answer is 24M, and **B** is by far the closest.

In a more precise calculation,

$$28\% \text{ of } 80,000,000 = (0.28) * 80,000,000 = 22,400,000.$$

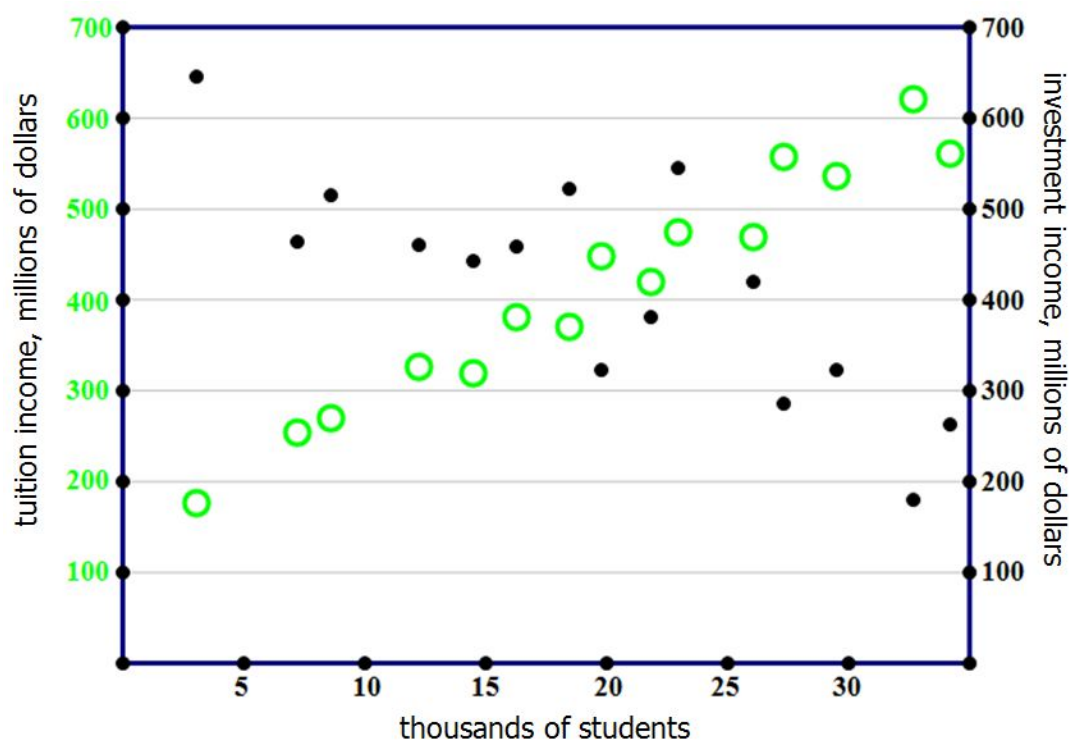
Answer = **B**

6 Suppose China's production remains more or less constant from 2004 to 2006. Suppose India is able to sustain the same percent increase in both of those years. By what percent would India's rice production have to increase from 2004 to 2005 and again from 2005 to 2006 so that it equaled China in rice production in 2006?

- ☐ 10.0%
- ☐ 12.8%
- ☐ 16.3%
- ☐ 21.5%
- ☐ 27.4%

India's production in 2004 is approximately 88,000,000 tons. That's our starting value, our "whole." China's production is approximately 112,000,000. If r is the annual percentage increase, written as a decimal, then the multiplier for an r percent increase is $(1 + r)$. India's value of 88M must get multiplied by this multiplier twice in order to equal China's output. Therefore:

So $r = 0.128 = 12.8\%$. Answer = **B**



On the above diagram, each of fifteen private colleges is represented by a circle and a dot. The light green circle, read against the green scale on the left, gives the college's annual gross tuition income in 2008; the data point is the very center of the circle. The black dot, directly above or below the center of the green circle and read against the right scale, gives the college's annual income in 2008 from investments such as endowments.

1 For the 15 colleges shown, the graph supports which of the following statements

- I. tuition income is positively correlated with student enrollment
- II. investment income is negatively correlated with student enrollment
- III. all colleges with over 20,000 student have less than \$500 million in investment income

- ☐ I only
- ☐ I and II only
- ☐ I and III only
- ☐ II and III only
- ☐ I, II, and III

Statement I: As we move across from left to right, student enrollment increases, and the green dots, tuition income, clearly go up. As one goes up, the other goes up. We can find individual dot exceptions, but this is the overall pattern of all the green dots together. That's a positive correlation. Therefore, tuition income is positively correlated with student enrollment. The graph supports I.

Statement II: As we move across from right to left, student enrollment increases, and the red dots go down--as one increases, the other decreases. We can find individual dot exceptions, but this is the

overall pattern of all the red dots together. That's a negative correlation. Therefore, investment income is negatively correlated with student enrollment. The graph supports II.

Statement III: At the dot 20 on the horizontal scale, which represents 20,000 students, imagine a vertical line--we are considering all colleges to the right of that line. For investment income, we are looking at red dots. Six of the seven red dots are below the \$500 million line, but one dot, the college with approximately 22,000 students, has an investment income of \$570 million. Therefore, we can't say all these schools have investment income of less than \$500 million. The graph does not support III.

The graph supports I & II only. Answer = **(B)**

2 The college that is drawing the most investment income in 2008 takes in approximately how much in mean total income per student in 2008? (Total income = tuition + investments)

- ☐ \$5,600
- ☐ \$28,000
- ☐ \$36,000
- ☐ \$56,000
- ☐ \$237,000

The "college that draws the most investment income" --- we are looking at the dots on the extreme left side of the graph. This college's investment income is about \$650M, and its tuition income is about \$190M; the sum is \$840M. That's the gross income. That college has about 3000 or 4000 students. With 3000 students, we get:

$$\text{mean income} = \frac{\$840,000,000}{3000} = \frac{\$840,000}{3} = \$280,000.$$

With 4000 students, we get:

$$\text{mean income} = \frac{\$840,000,000}{4000} = \frac{\$840,000}{4} = \$210,000.$$

Since the student body of this college is somewhere in the 3000-4000 range, the mean income would have to be between those two numbers. The only one that qualifies is \$237,000, answer = **(E)**.

3 If the tuition income at a college exceeds its investment income, then that college is said to be "tuition driven." How many colleges shown here were tuition driven in 2008?

- ☐ four
- ☐ five
- ☐ six
- ☐ seven
- ☐ eight

If the tuition income is greater than the investment income, this means the green dot is higher than the red dot. For how many colleges is the green dot higher than the red dot? I count seven, answer = **(D)**. Sometimes, when you interpret what a graph question is asking, it's something so easy that a kindergartener could do it. Don't be put off by that--don't automatically think you are doing something wrong because it's easy! The whole point of graphs is to make information easier to see!

4 The college shown with the highest tuition income in 2008 has how much investment income?

- ☐ \$190 million
- ☐ \$340 million
- ☐ \$590 million
- ☐ \$610 million
- ☐ \$640 million

The college with the highest tuition income is the highest green dot, the second from the far right. The red dot directly below is at a height of about \$190M--that's its investment income. Answer = **(A)**.

5 For how many colleges shown is the investment income in 2008 more than double the same college's tuition income in 2008?

- ☐ none
- ☐ one
- ☐ two
- ☐ three
- ☐ four

Colleges' investment incomes are the red dots, and the tuition incomes are the green dots. For how many colleges is the red dot twice as high as the green dot? Well, for the far left college, that red dot is very high, and the green dot is relatively low, so that red dot is definitely more than twice as high as that green. Take a look at the next--red dot at around \$460M and green at around \$250M, so the red dot falls short of twice as high as the green, so this one doesn't count. Take a look at the next--red dot at around \$520M and green at around \$260M, so arguably, the red dot is almost exactly twice as high as the green, but not more than twice as high, so this one doesn't count either. Then for the next

schools, the green dots are considerably higher, more than half the height of the red dot, and after, the remaining schools have green dots higher than the red dot--none of these can possibly count. Therefore, the only college to satisfy this condition is the one on the far left. Answer = (B).

6 The college with the highest 2008 mean investment income per student enrolled generates how much annual tuition income in 2008.

- ☐ \$190 million
- ☐ \$360 million
- ☐ \$450 million
- ☐ \$560 million
- ☐ \$620 million

Mean investment income per student = (total investment income)/(# of students). The way to make a fraction as big as possible is to make the numerator as large as possible and the denominator as small as possible. Well, the college on the far left has the largest total investment income (the highest red dot) and the smallest student enrollment, so it must have the highest mean investment income per student. This college has a tuition income (green dot) of about \$190M. Answer = (A).

七

The following tables show the revenues & costs, in thousands of dollars, for a small company in the year 2007.

Revenues	
Sales	753
Investments	53
Subsidiaries	246
TOTAL	1052

Costs	
Materials & Resource	83
Production	16
Payroll & Benefits	452
Insurance & Plant	123
Research & Development (R & D)	75
TOTAL	749

1 The costs associated with insurance and the physical plant are what percentage of total costs?

- ☐ 6.8%
- ☐ 16.4%
- ☐ 27.2%
- ☐ 83.6%
- ☐ 90.0%

The "part" is the cost of Insurance & Plant, \$123,000, and the "whole" is sum for the total costs \$749,000. We can approximate this as $123/749$, close to $12/72 = 1/6$, which is about 16%. In a more precise calculation:

Answer = B.

2 Profit = Revenue – Costs. If costs remain constant from 2007 to 2008, and if revenues increase by 10% in that same period, by what percent will profits increase from 2007 to 2008?

- ☐ 11.6%
- ☐ 25.8%
- ☐ 34.7%
- ☐ 71.2%
- ☐ 116.3%

Current profit is about \$300K --- that's the "whole" of the percent change. If revenues increase by 10%, approximately \$100K, all of which is added to the profits because costs remain fixed. That's the "part". Percent = $100/300 = 1/3 = 33.3\%$, very close to C.

In a more precise calculation, current profit is $\$1,052,000 - \$749,000 = \$303,000$. If revenues increase by 10%, that's an additional \$105,200.

$$\text{Percent increase} = \frac{\$105,200}{\$303,000} \times 100\% = 34.7\%$$

Answer = C.

3 Investments and Subsidiary revenues combined constitute what percent of total revenue?

- ☐ 5.0%
- ☐ 14.2%
- ☐ 22.6%
- ☐ 28.4%
- ☐ 39.9%

The sum of Subsidiaries + Investment is approximately \$300K: that's the "part". The total revenues are approximately \$1,000K: that's the "whole". Percent = $300/1000 = .3 = 30\%$. But revenues are a little more than \$1,000K, which makes the denominator bigger, which makes the whole fraction smaller, so we expect answer just under 30% --- D is perfect.

In a more precise calculation, the sum of Subsidiaries + Investment = $\$246,000 + \$53,000 = \$299,000$ is the "part", and the total revenues, \$1,052,000, is the "whole".

Answer = D.

4 Suppose in the following year, 2008, the sales are the same value, and half of those sales are directly due to the 2007 investment in R & D. The revenue received from these sales would be what percent greater than the money invested in R & D?

- ☐ 85%
- ☐ 110%
- ☐ 200%
- ☐ 402%
- ☐ 503%

Notice that the value of the sale is approximately 10x bigger than the value of the R & D budget. If R & D is responsible for half the sales, that's a value 5x bigger. An increase from 100% to five times bigger, 500%, is an increase of 400%. Answer D is the closest. A very quick approximation leads to the answer!

In a more precise calculation, the 2008 sales are the same, so half of those sales, the part due to R & D, would be

The "whole" is the 2007 R&D budget, \$75,000, and the "part" would be the difference between

\$75,000 and \$376,500 --- that dollar amount is the "return on the investment."

Answer = D.

5 The CEO has promised that any increase in revenues from investments in 2008 will go toward increasing the R&D budget. Assume that revenues from investments increase by 40% from 2007 to 2008, and that these additional funds are the only change to the R & D budget. By what percent does the R & D budget increase?

- ☐ 2.8%
- ☐ 4.0%
- ☐ 11.5%
- ☐ 28.3%
- ☐ 56.6%

Investments start at \$53,000 --- approximate that at \$50K. We know that 40% is $\frac{2}{5}$, and $(\frac{2}{5})$ of 50K is 20K --- that's what's added to R & D.

The R & D budget starts at \$75,000, so we can estimate

10% of \$75K = 7.5K

20% of \$75K = 15K

30% of \$75K = 22.5K

Well, \$20K is between 20% and 30%, much closer to 30%. There's only one answer, **D**, that matches this.

In a more precise calculation:

$0.4 * \$53,000 = \$21,200$

$$\text{Percent increase} = \frac{\$21,200}{\$75,000} \times 100\% = 28.3\%$$

Answer = **D**.

6 Profit is the difference between revenues and costs. If profits increase 20% from 2006 to 2007, difference between this year's profit and last year's profit is how much?

- ☐ \$50,500
- ☐ \$60,600
- ☐ \$149,800
- ☐ \$175,330

○ \$210,400

The profit in 2007 = \$1,052,000 - \$749,000 = \$303,000. The profit in 2006 increased 20% to this amount. Let the profit in 2006 be P. We can approximate profit as \$300K. We know that 20% = 1/5, so an increase of 20% means the original was "five parts" and the new value, \$300K, is "six parts." Therefore, the original is 5/6 of the new value --- $5/6 * 300 = 250K$. The increase from one year's \$250K to the next year's \$300K was an increase of \$50K.

In a more precise calculation, let P be the profits in 2006, which increased 20% to \$303,000:

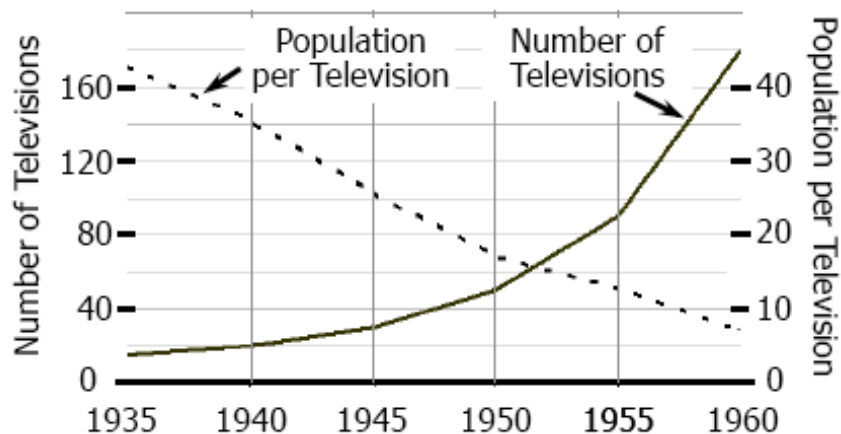
$$P * 1.2 = \$303,000$$

$$P = \frac{\$303,000}{1.2} = \$252,500$$

The difference between profits in 2006 & 2007 is \$303,000 - \$252,500 = \$50,500. Answer = A.

八

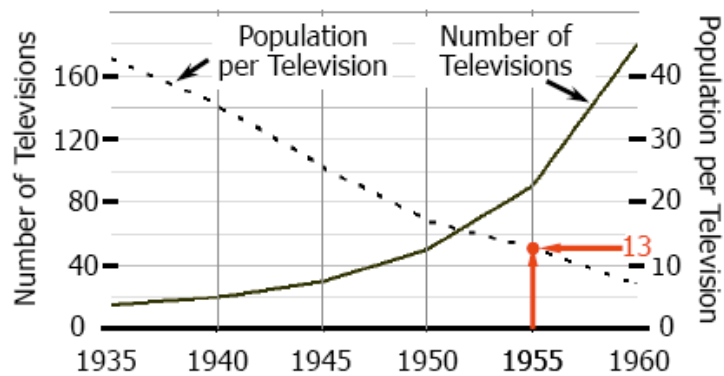
TELEVISIONS IN TOWN X, AND POPULATION PER TELEVISION



1 In 1955, the ratio of the number of televisions to the number of people was approximately

- 1 to 13
- 1 to 23
- 1 to 26
- 1 to 50
- 1 to 90

TELEVISIONS IN TOWN X, AND POPULATION PER TELEVISION



In 1955, the ratio of the number of televisions to the number of people was approximately

- (A) 1 to 13
- (B) 1 to 23
- (C) 1 to 26
- (D) 1 to 50
- (E) 1 to 90

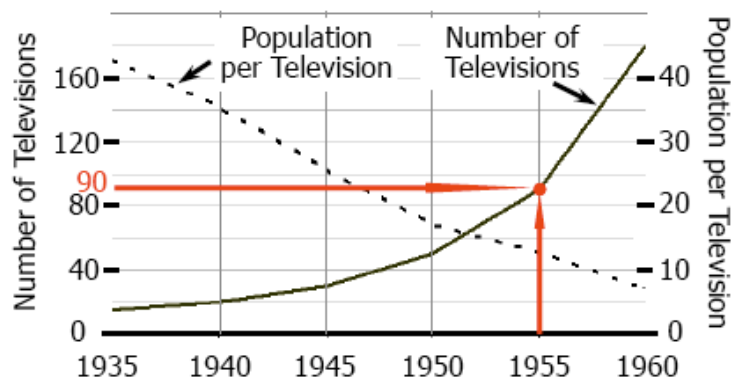
people to televisions \approx 13 to 1

→ televisions to people \approx 1 to 13

2 From 1940 to 1955, the percent increase in the number of televisions was closest to

- ☐ 30
- ☐ 130
- ☐ 350
- ☐ 450
- ☐ 650

TELEVISIONS IN TOWN X, AND POPULATION PER TELEVISION



From 1940 to 1955, the percent increase in the number of televisions was closest to

- (A) 30
- (B) 130
- (C) 350
- (D) 450
- (E) 650

televisions in 1940 \approx 20

televisions in 1955 \approx 90

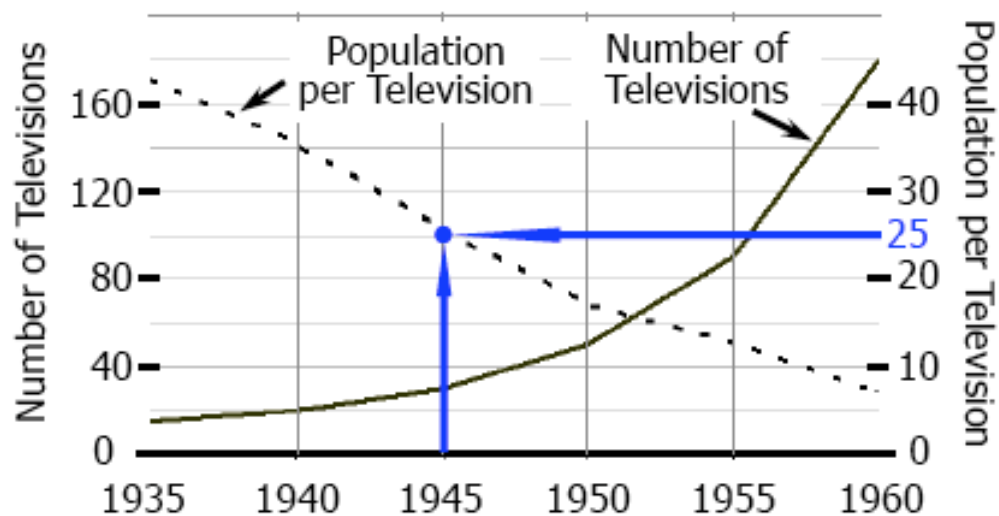
→ percent increase $\approx \frac{90 - 20}{20}$

$$= \frac{70}{20} = \frac{350}{100} = 350\%$$

3 What was the approximate population of Town X in 1945?

- ☐ 150
- ☐ 750
- ☐ 1500
- ☐ 3000
- ☐ 6000

TELEVISIONS IN TOWN X AND POPULATION PER TELEVISION



What was the approximate population of Town X in 1945?

- (A) 150
- ☒ (B) 750
- (C) 1500
- (D) 3000
- (E) 6000

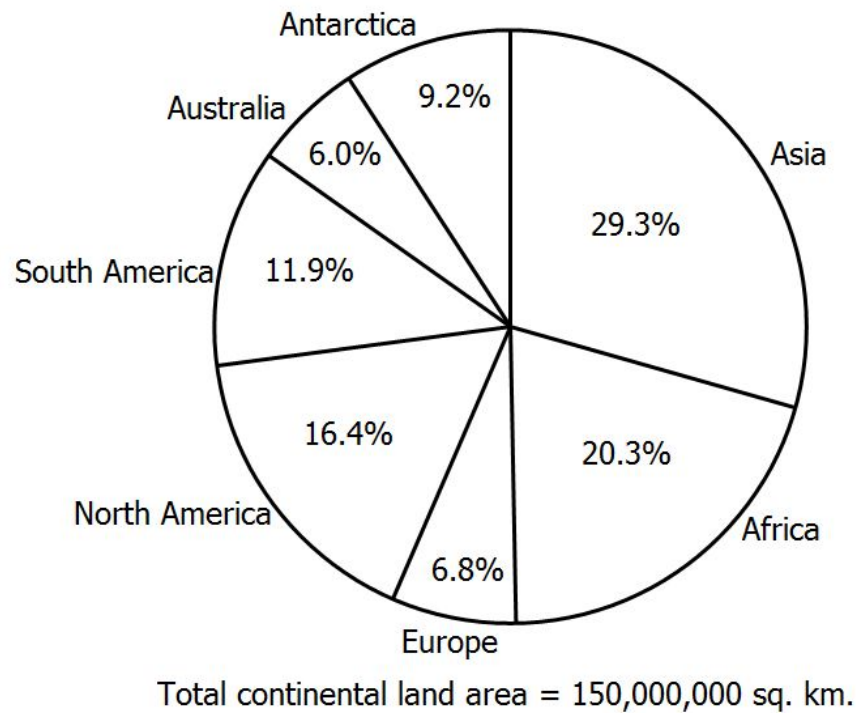
of televisions ≈ 30

population per television ≈ 25

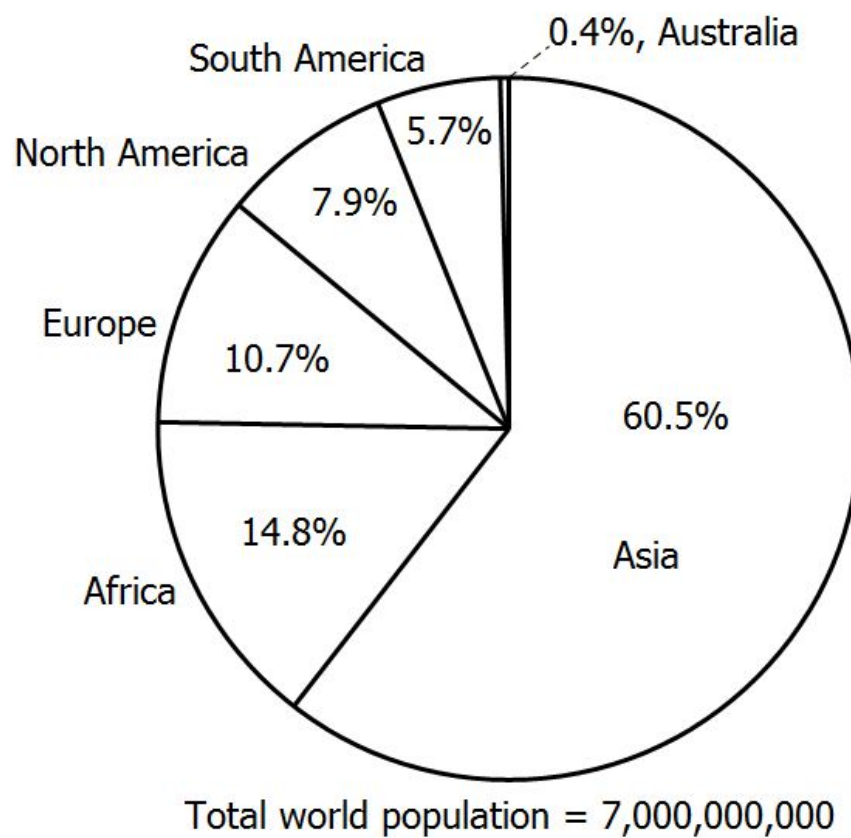
➡ population $\approx 30 \times 25 \approx 750$

九

Continents by Area



Continents by Population



1 What is the approximate area of the continent of Africa in square kilometers?

- ☐ 10,200,000
- ☐ 30,450,000
- ☐ 43,950,000
- ☐ 738,916,000
- ☐ 1,421,000,000



1) What is the approximate area of the continent of Africa?

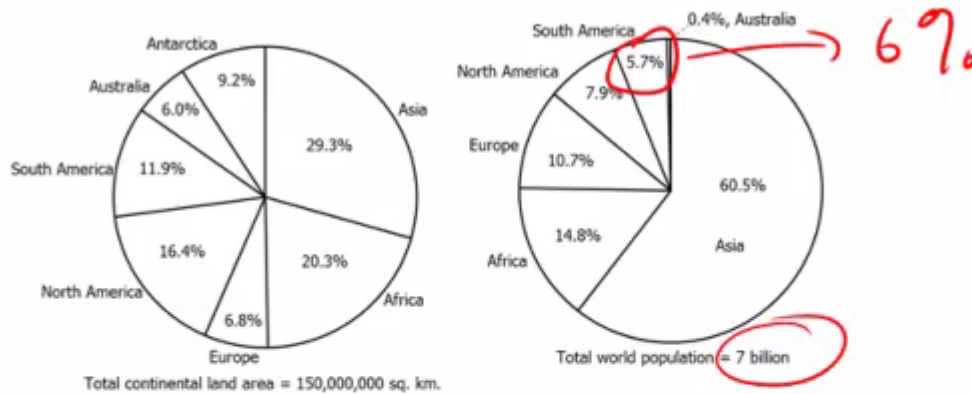
- (A) 10,200,000
- ☒ (B) 30,450,000
- (C) 43,950,000
- (D) 738,916,000
- (E) 1,421,000,000

$$20.3\% \sim 20\% \sim \frac{1}{5}$$

$$\frac{1}{5} \times 150,000,000 \sim 30,000,000$$

2 What is the approximate population of South America?

- ☐ 399,000,000
- ☐ 833,000,000
- ☐ 1,036,000,000
- ☐ 2,632,000,000
- ☐ 8,432,000,000



2) What is the approximate population of South America?

- (A) 399,000,000
 (B) 833,000,000
 (C) 1,036,000,000
 (D) 2,632,000,000
 (E) 8,432,000,000

$$0.057 \times 7b$$

$$67\% \text{ of } 7,000,000,000$$

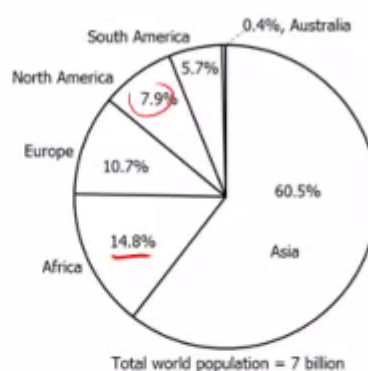
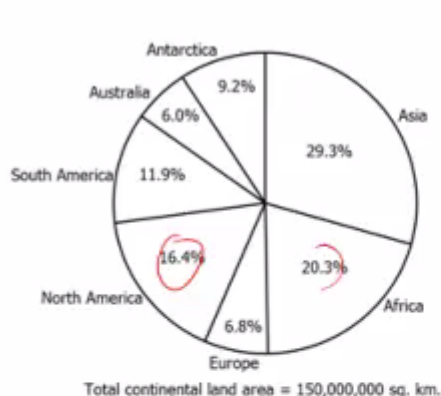
$$0.06 \times 7,000,000,000$$

$$6 \times 7,000,000,000$$

$$420,000,000$$

3 Population density is the population of a region divided by its geographic area. Of the seven continents, Asia has by far the largest population density. Which continent has the second largest?

- ☐ Africa
☐ North America
☐ South America
☐ Europe
☐ Australia



14.8% of 7b
20.3% of 150,000,000

$$A - \frac{14.8}{20.3}$$

$$NA \frac{7.9}{16.4}$$

$$SA \frac{5.7}{11.9}$$

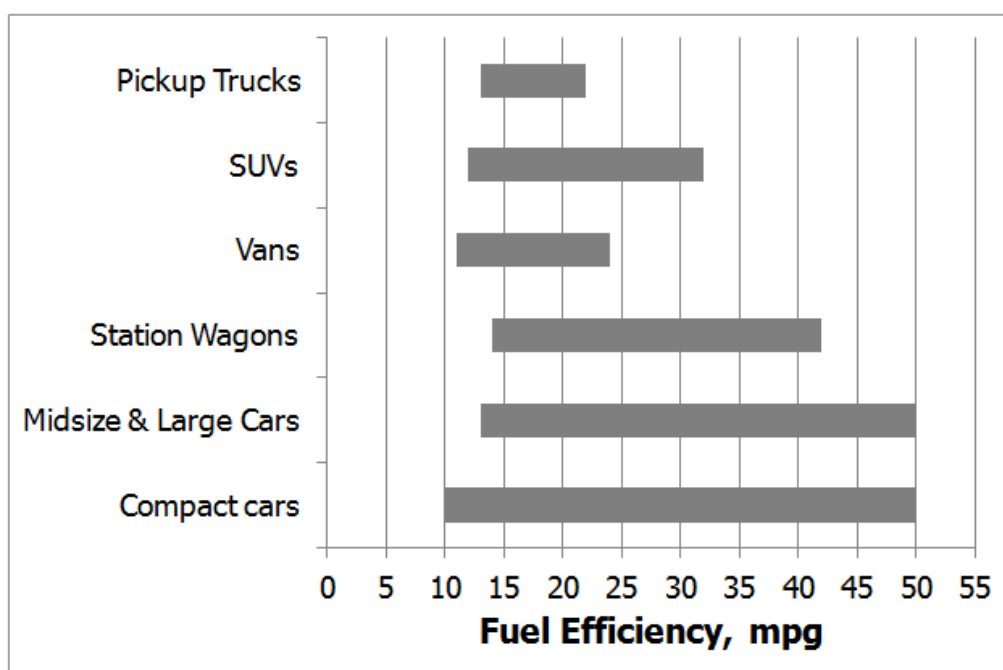
$$E \frac{10.7}{6.8}$$

$$A \frac{0.4}{6.0}$$

3) Population density is the population of a region divided by its geographic area. Of the seven continents, Asia has by far the largest population density. Which continent has the second largest?

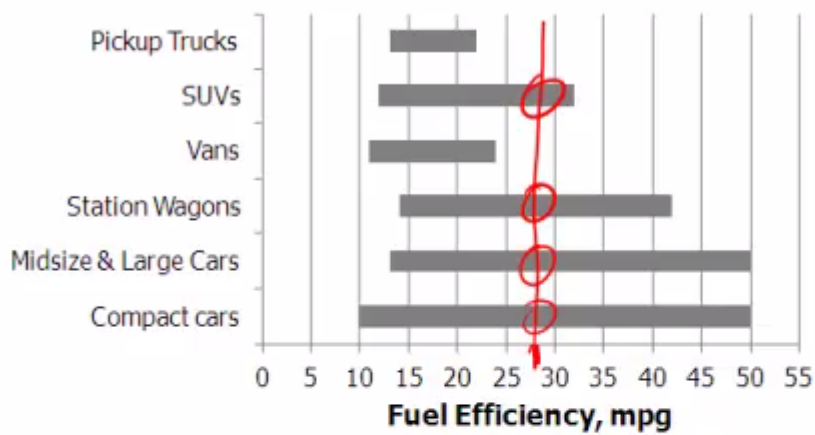
- (A) Africa
- (B) North America
- (C) South America
- ☒ (D) Europe
- (E) Australia

+



Note: mpg = miles per gallon

1 In how many different categories is possible to select a vehicle with a fuel efficiency of 28 mpg?

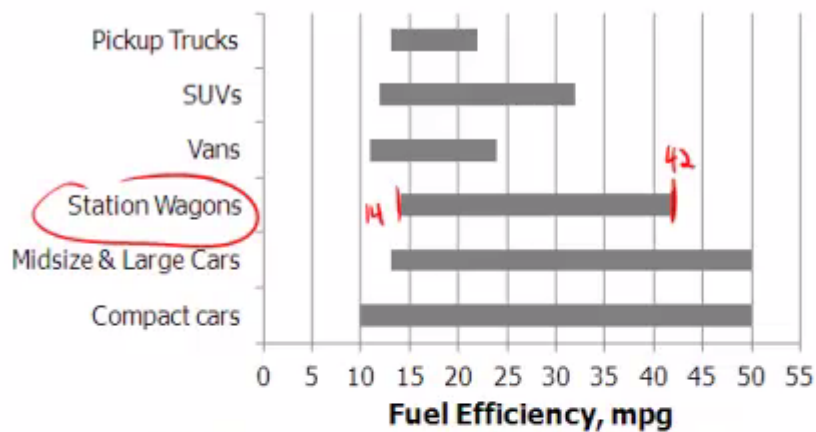


2) In how many different categories is possible to select a vehicle with a fuel efficiency of 28 mpg?

4 categories

2 The range from the least fuel efficient Station Wagon to the most fuel efficient Station Wagon is what?

- ☐ 20 mpg
- ☐ 28 mpg
- ☐ 36 mpg
- ☐ 42 mpg
- ☐ 50 mpg

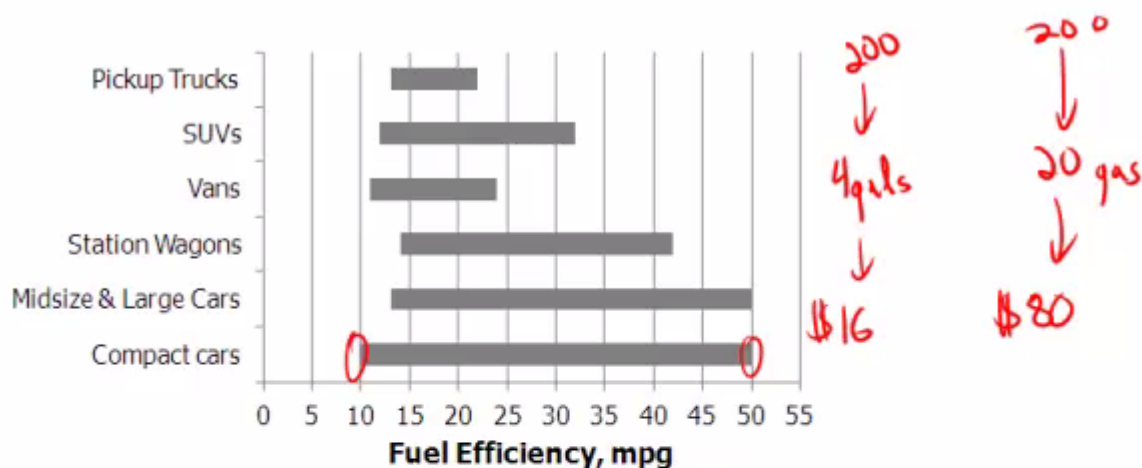


1) The range from the least fuel efficient Station Wagon to the most fuel efficient is what?

- (A) 20 mpg
- (B) 28 mpg
- (C) 36 mpg
- (D) 42 mpg
- (E) 50 mpg

3 If gas costs \$4/gallon, and one is going to drive a compact car on a 200 mile trip, what is the difference in fuel costs required for this trip between the most fuel efficient and least fuel efficient compact car?

- ☐ \$64
- ☐ \$80
- ☐ \$160
- ☐ \$200
- ☐ \$800



3) If gas costs \$4/gallon, and one is going to drive a compact car on a 200 mile trip, what is the difference in fuel costs required for this trip between the most fuel efficient and least fuel efficient compact car?

(A) \$64

(B) \$80

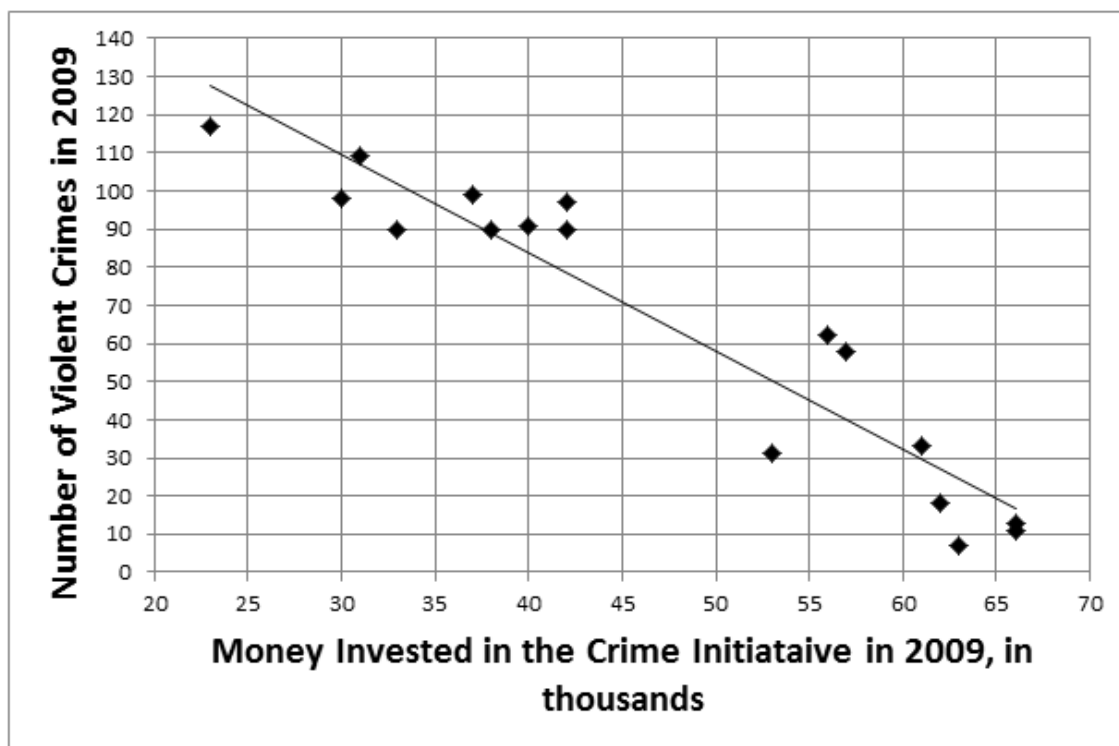
(C) \$160

(D) \$200

(E) \$800



In the final months of 2008, the state legislature of a certain state announced the availability of a new special statewide Crime Initiative, designed to fight violent crimes in the state. Towns and cities throughout the state could decide to invest in this new crime initiative in 2009, at any level up to \$70,000. The graph below shows, for 17 similarly sized towns throughout the state, the money they invested in the Crime Initiative in 2009, and the number of violent crimes that year in that town. A trend line is already displayed on the graph.



1 Of the four towns shown each with fewer than twenty violent crimes in 2009, the average amount they invested in the statewide Crime Initiative in 2009 is

- ☐ \$29,250
- ☐ \$41,000
- ☐ \$49,500
- ☐ \$58,750
- ☐ \$64,250

Do NOT solve this question with an exact calculation. That would be a huge strategic mistake, even if your mathematical calculation is correct. This question is crying out for quick shortcut. All four of the highest values invested are above \$60,000, so their average *has to be* above \$60,000. The average of any four numbers can't possibly be lower than all four of the numbers! Therefore, the only possible answer is (E).

2 Among these 17 towns, the median amount invested in the statewide Crime Initiative is

- ☐ \$37,000
- ☐ \$42,000
- ☐ \$53,000
- ☐ \$62,000
- ☐ \$90,000

There are 17 numbers, so the median will be the middle number when the numbers are in order. $17 = 8 + 1 + 8$, which means the median will have eight numbers above it and eight numbers below it, making it the ninth number on the list. How do we find the ninth number?

Well, moving across horizontally, the points are arranged in increasing order by money invested in the initiative. If we start counting from the left, we find the 8th & 9th points are horizontally at the same place --- over the same value, \$42,000. One of those two points is the median (it doesn't matter which one we call the median). Either way, the median value is \$42,000. Answer = **(B)**.

3 The town that spent \$61,000 on the Crime Initiative had a total number of violent crimes that was approximately what percentage of the total number of violent crime of the town that spent \$40,000 on the Crime Initiative?

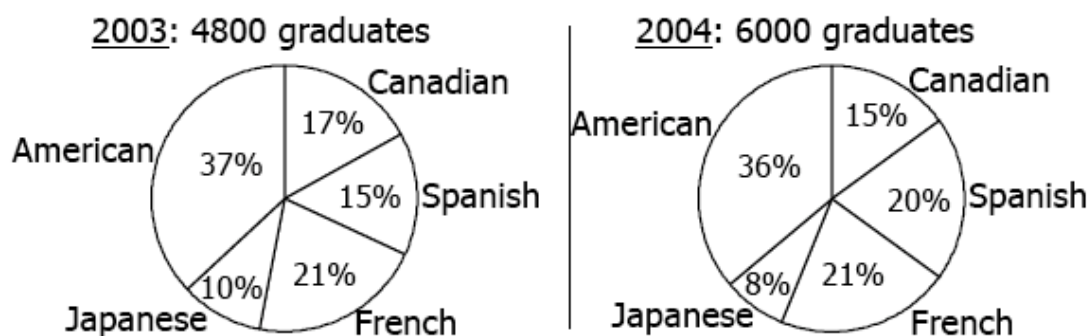
- ☐ 6.4%
- ☐ 27.3%
- ☐ 34.4%
- ☐ 41.3%
- ☐ 58.7%

The town that spent \$61,000 on the Crime Initiative had 32 violent crimes. The town that spent \$40,000 on the Crime Initiative had 91 violent crimes. The question is really asking: 32 is what percent of 91?

Because the answer choices are spread so far apart, this question is begging us to solve it via approximation. I will estimate the fraction $32/91$ as $30/90 = 1/3 = 33\%$. The only answer close to this is **(C)**.

十二

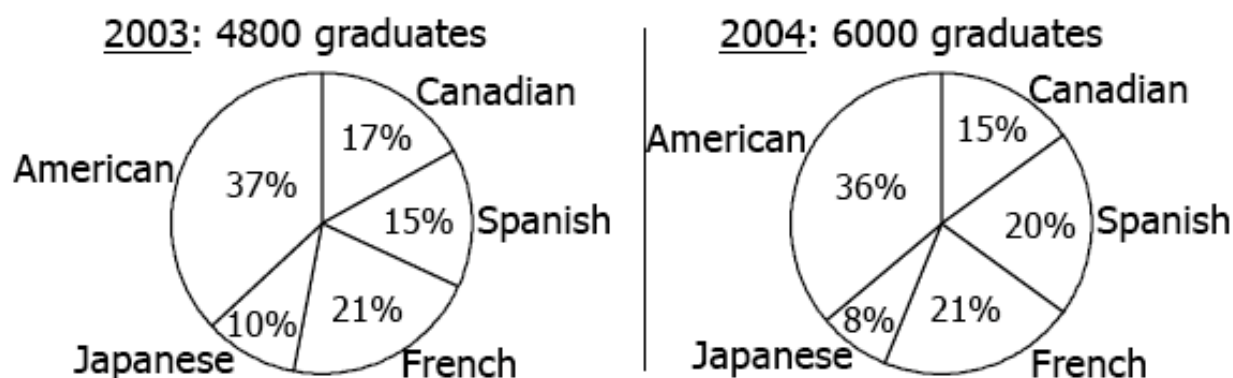
DISTRIBUTION OF GRADUATES FROM ABC COLLEGE
BY NATIONALITY IN 2003 AND 2004



1 How many more French students graduated in 2004 than in 2003?

- ☐ 0
- ☐ 120
- ☐ 202
- ☐ 222
- ☐ 252

**DISTRIBUTION OF GRADUATES FROM ABC COLLEGE
BY NATIONALITY IN 2003 AND 2004**



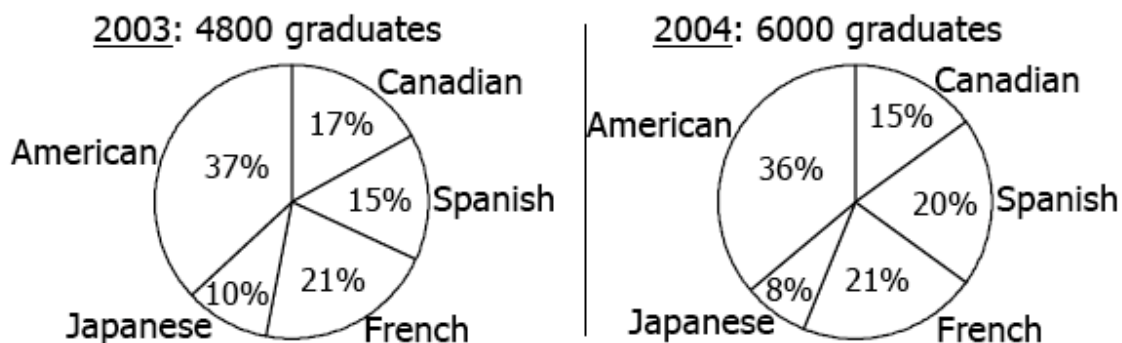
How many more French students graduated in 2004 than in 2003?

- | | | | |
|----------------|---------------------------|---|---------------------------|
| (A) 0 | <u>#of French in 2004</u> | – | <u>#of French in 2003</u> |
| (B) 120 | 21% of 6000 | – | 21% of 4800 |
| (C) 202 | 0.21 × 6000 | – | 0.21 × 4800 |
| (D) 222 | 0.21(6000 – 4800) | | |
| (E) 252 | 0.21(1200) | | |
| | 252 | | |

2 What was the percent decrease in the number of Japanese graduates from 2003 to 2004?

- ☐ 0
- ☐ 0.2
- ☐ 2
- ☐ 20
- ☐ 25

DISTRIBUTION OF GRADUATES FROM ABC COLLEGE BY NATIONALITY IN 2003 AND 2004



What was the percent decrease in the number of Japanese graduates from 2003 to 2004?

- (A) 0
(B) 0.2
(C) 2
(D) 20
(E) 25

#of Japanese in 2003
10% of 4800
480

#of Japanese in 2004
8% of 6000
480

十三

In the downtown of a certain city, there are 8,000 apartments for rent. Here is their breakdown by total area:

under 500 sq. ft.	5%
500 – 750 sq. ft.	18%
750 – 1000 sq. ft.	23%
1000 – 1250 sq. ft.	26%
1250 – 1500 sq. ft.	19%
1500 – 1750 sq. ft.	3%
1750 – 2000 sq. ft.	5%
over 2000 sq. ft.	1%

1 What is the total number of apartments with area between 500 and 1000 sq. ft.?

- 1440
○ 1840
○ 2080
○ 3280
○ 3920

In the downtown of a certain city, there are 8000 apartments for rent. Here is their breakdown by total area.

under 500 sq. ft.	5%
500 – 750 sq. ft.	18%
750 – 1000 sq. ft.	23%
1000 – 1250 sq. ft.	26%
1250 – 1500 sq. ft.	19%
1500 – 1750 sq. ft.	3%
1750 – 2000 sq. ft.	5%
over 2000 sq. ft.	1%

41%

10% 800
40% 3200

2) What is the total number of apartments with area between 500 and 1000 sq. ft.?

(A) 1440

(B) 1840

(C) 2080

(D) 3280

(E) 3920

2 The median area falls into what group?

☐ 500 – 750 sq. ft.

☐ 750 – 1000 sq. ft.

☐ 1000 – 1250 sq. ft.

☐ 1250 – 1500 sq. ft.

☐ 1500 – 1750 sq. ft.

In the downtown of a certain city, there are 8000 apartments for rent. Here is their breakdown by total area.

under 500 sq. ft.	5%
500 – 750 sq. ft.	18%
750 – 1000 sq. ft.	23%
1000 – 1250 sq. ft.	26%
1250 – 1500 sq. ft.	19%
1500 – 1750 sq. ft.	3%
1750 – 2000 sq. ft.	5%
over 2000 sq. ft.	1%

46%

median = 50 percentile

21%

1) The median area falls into what group?

(A) 500 – 750 sq. ft.

(B) 750 – 1000 sq. ft.

☒ (C) 1000 – 1250 sq. ft.

(D) 1250 – 1500 sq. ft.

(E) 1500 – 1750 sq. ft.

3 A developer proposes converting a gigantic old warehouse complex into apartments. The proposed new building would add 250 economy apartments (area = 625 sq. ft.), 200 regular apartments (area = 925 sq. ft.), and 50 luxury apartments (area = 1800 sq. ft.). If these apartments are added, then apartments with an area of 750 – 1000 sq. ft. will constitute what percent of the total number of apartments downtown.

- ☐ 16%
- ☐ 24%
- ☐ 35%
- ☐ 50%
- ☐ 72%

In the downtown of a certain city, there are 8000 apartments for rent. Here is their breakdown by total area.

under 500 sq. ft.	5%
500 – 750 sq. ft.	18%
750 – 1000 sq. ft.	23%
1000 – 1250 sq. ft.	26%
1250 – 1500 sq. ft.	19%
1500 – 1750 sq. ft.	3%
1750 – 2000 sq. ft.	5%
over 2000 sq. ft.	1%

- 3) A developer proposes converting a gigantic old warehouse complex into apartments. The proposed new building would add 250 economy apartments (area = 625 sq. ft.), 200 regular apartments (area = 925 sq. ft.), and 50 luxury apartments (area = 1800 sq. ft.). If these apartments are added, then apartments with an area of 750 – 1000 sq. ft. will constitute what percent of the total number of apartments downtown.

(A) 16%

(B) 24%

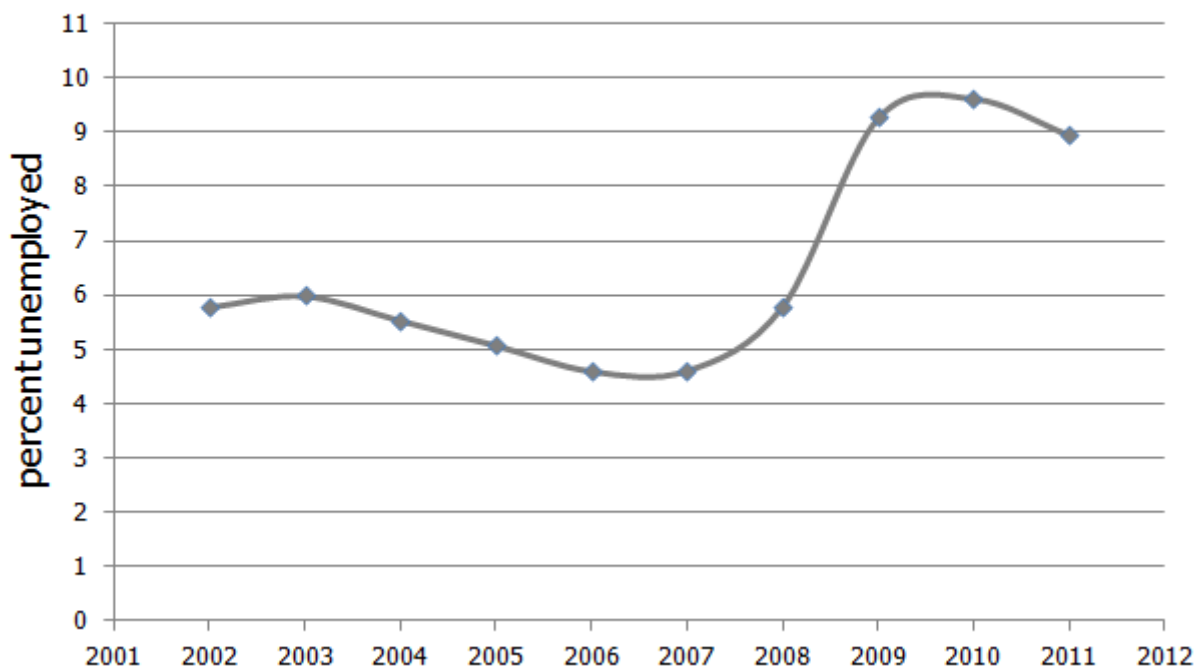
(C) 35%

(D) 50%

(E) 72%

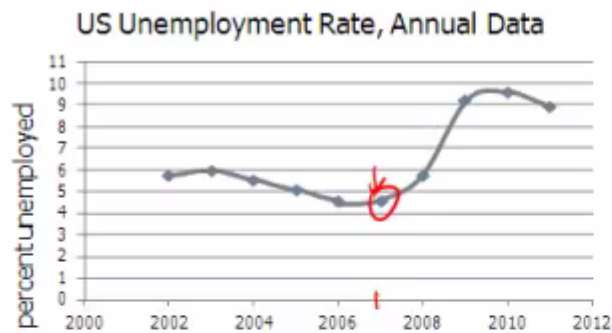
十四

US Unemployment Rate, Annual Data



1 The US unemployment rate in 2007 was approximately

- ☐ 3.5%
- ☐ 4.6%
- ☐ 5.2%
- ☐ 5.8%
- ☐ 7.2%

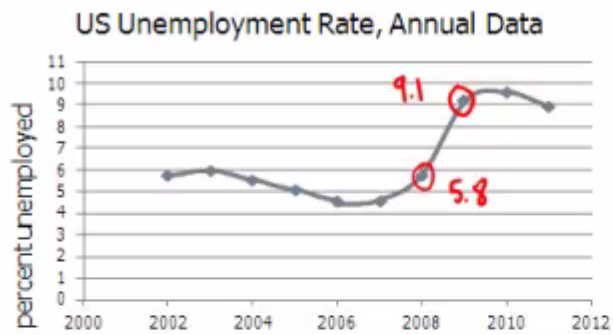


1) The US unemployment rate in 2007 was approximately

- (A) 3.5%
- (B) 4.6%
- (C) 5.2%
- (D) 5.8%
- (E) 7.2%

2 The percent increase in unemployment rate from 2008 to 2009 is approximately

- ☐ 3.5%
- ☐ 12.6%
- ☐ 23.7%
- ☐ 37.5%
- ☐ 59.9%



6 → 9
50% increase

<6 → 9<
more than 50%

2) The percent increase in unemployment rate from 2008 to 2009 is approximately

(A) 3.5%

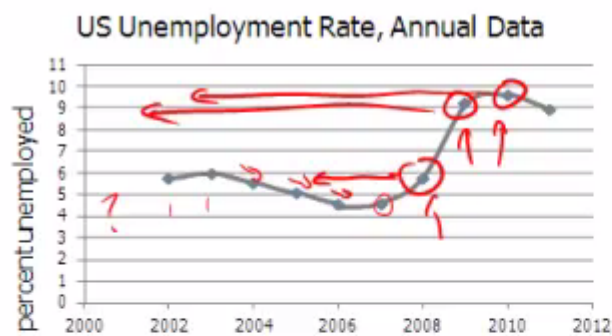
(B) 12.6%

(C) 23.7%

(D) 37.5%

(E) 59.9%

3 For years from 2004 onward, for how many years shown on the chart was the unemployment rate higher than it was in each of the previous two years?



- 3) For years from 2004 onward, for *how many* years shown on the chart was the unemployment rate higher than it was in each of the previous two years?

3 years

十五

ANIMAL DISTRIBUTION AT THE ZOO

Animal	Percent
Lions	32%
Leopards	16%
Ocelots	20%
Tigers	8%
Bobcats	24%

- 1 If there are 44 leopards at the zoo, what is the zoo's total animal population?

- ☐ 225
- ☐ 275
- ☐ 325
- ☐ 350
- ☐ 375

If there are 44 leopards at the zoo, what is the zoo's total animal population?

(A) 225

(B) 275

(C) 325

(D) 350

(E) 375

Let x = total zoo population

$$16\% \text{ of } x = 44$$

$$\frac{16}{100} x = 44$$

$$16x = 4400$$

$$x = 275$$

ANIMAL DISTRIBUTION
AT THE ZOO

Animal	Percent
Lions	32%
Leopards	16%
Ocelots	20%
Tigers	8%
Bobcats	24%

2 If 8 tigers were added to the zoo, the new ratio of lions to tigers would be 4 to 3. How many bobcats are at the zoo?

- ☐ 4
- ☐ 8
- ☐ 12
- ☐ 24
- ☐ 48

If 8 tigers were added to the zoo, the new ratio of lions to tigers would be 4 to 3. How many bobcats are at the zoo?

- (A) 4
(B) 8
(C) 12
(D) 24
(E) 48

Let T = Current # of tigers

Let L = Current # of lions

$$\frac{L}{T} = \frac{32}{8} \Rightarrow 32T = 8L \Rightarrow 4T = L$$

$$\frac{L}{T+8} = \frac{4}{3} \Rightarrow 4(T+8) = 3L$$

$$4(T+8) = 3(4T)$$

$$4T + 32 = 12T$$

$$32 = 8T$$

$$4 = T$$

ANIMAL DISTRIBUTION
AT THE ZOO

Animal	Percent
Lions	32%
Leopards	16%
Ocelots	20%
Tigers	8% 4
Bobcats	24% 12

3 If a pie graph were drawn to scale to represent the animal distribution at the zoo, what would be the measure (in degrees) of the central angle of the sector representing bobcats?

- 43.2
○ 48
○ 86.4
○ 93.6
○ 96

If a pie graph were drawn to scale to represent the animal distribution at the zoo, what would be the measure (in degrees) of the central angle of the sector representing bobcats?

- (A) 43.2
- (B) 48
- (C) 86.4
- (D) 93.6
- (E) 96



ANIMAL DISTRIBUTION
AT THE ZOO

Animal	Percent
Lions	32%
Leopards	16%
Ocelots	20%
Tigers	8%
Bobcats	24%

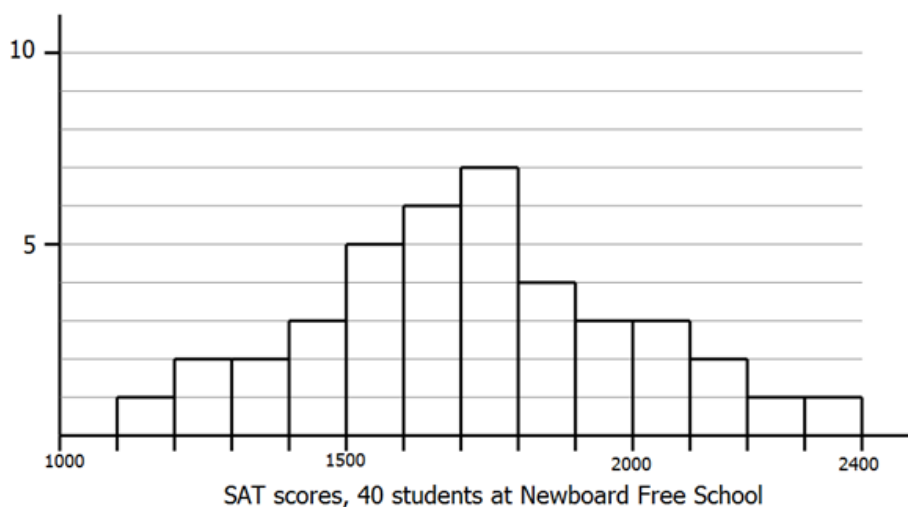
$$24\% \text{ of } 360 = 90$$

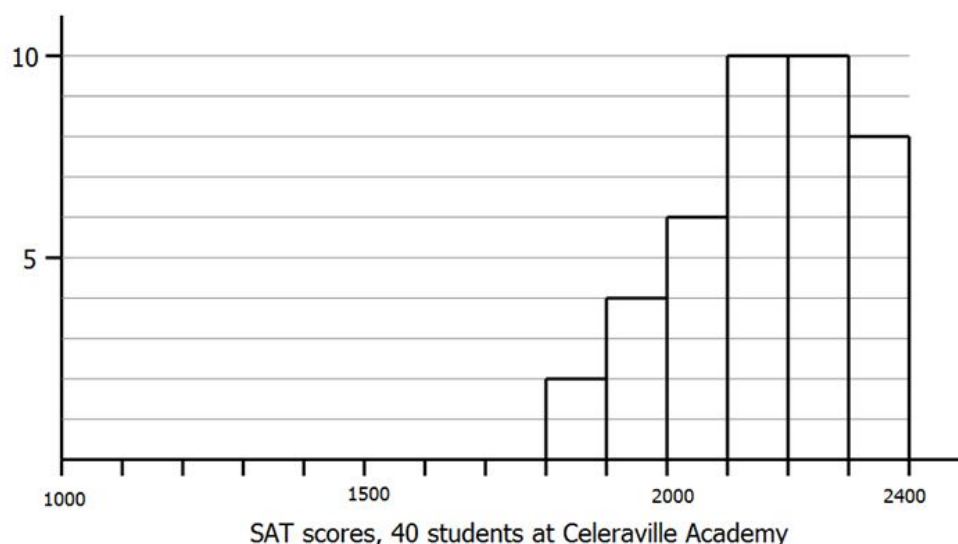
$$24\% \text{ of } 360 = 90^\circ$$

十六

The following two histograms show the distribution of SAT scores of all forty students at each of two schools. The Newboard Free School is a mixed population school, with forty students of a variety of ability levels. The Celeraville Academy is an elite college prep school for forty gifted students.

A note on rounding: in the histograms below, a score divisible by 100 would be included in the column below that score: thus, for example, a score of exactly 1900 would be included as part of the column between 1800 and 1900.





1 A score of 1800 would be zeroth percentile among the students at Celeraville Academy. What would its percentile rank be at among the students the Newboard Free School? (Assume that no one at Newboard Free School scored exactly 1800.)

- ☐ 21st percentile
- ☐ 35th percentile
- ☐ 56th percentile
- ☐ 61st percentile
- ☐ 65th percentile

Do NOT solve this question with an exact calculation. That would be a huge strategic mistake, even if your mathematical calculation is correct. This question is crying out for quick shortcut. All four of the highest values invested are above \$60,000, so their average *has to be* above \$60,000. The average of any four numbers can't possibly be lower than all four of the numbers! Therefore, the only possible answer is **(E)**.

2 In how many of the columns (2300-2400, 2200-2300, 2100-2200, etc.) is the number of students with a score in that category from the Celeraville Academy is greater than the number of students with a score in that category from the Newboard Free School?

First, the four columns on the far right of the Celeraville are all above 5 students, and the corresponding columns at Newboard Free School are below 5 students, so those four are definitely higher at Celeraville. The next column, 1900 – 2000, has 4 students at Celeraville and 3 students at Newboard Free School, so this is a fifth column that is higher at Celeraville. For the last column, the far left column on the Celeraville chart, there are only two students, and there are four students in the corresponding column at Newboard Free School, so this column does not meet the condition. Five columns meet the condition. Answer = 5.

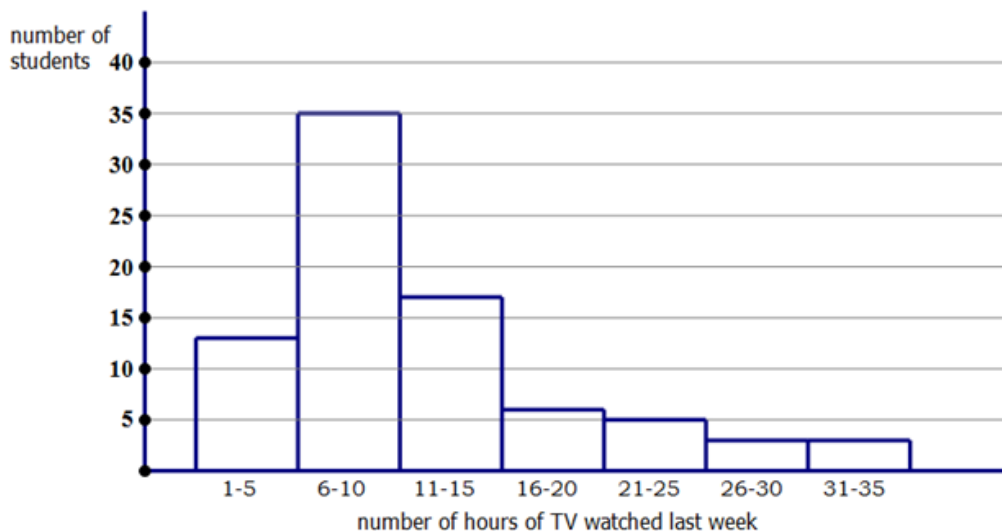
3 The first quartile SAT score among the forty scores at the Newboard Free School is in which score range?

- ☐ 1300 – 1400
- ☐ 1400 – 1500
- ☐ 1500 – 1600
- ☐ 1600 – 1700
- ☐ 1700 – 1800

There are 40 students, that is, 40 numbers on the list. The median would be the average of the 20th and 21st numbers, and that divides the whole list into a "lower list" (1st through 20th values) and an "upper list" (21st through 40th). The first quartile is the median of the "lower list," so it would be between the 10th and 11th scores. In what column do we know we are above the 10 lowest scores? Well, exactly at 1500, there are eight students below that line: we need two more. The next column, 1500-1600, has 3 members, so the first quartile would be between the first two and third member of that column. We don't know the exact value of the first quartile, but we know it must be inside that column, 1500-1600. Answer = (C).

十七

In a survey, 82 high school students were randomly selected and asked how many hours of television they had watched in the previous week. The histogram below displays their answers.



1 Which of the following gives the range of the median number of hours of TV watched last week?

- ☐ 1-5
- ☐ 6-10

- ☐ 11-15
- ☐ 16-20
- ☐ 41-45

The median of a list is the middle value, when all the values in the list are put in order from smallest to biggest. Here, we are concerned with the 82 high school students --- for each, we have a number: the number of hours of TV they watched last week. Since 82 is an even number, the median would be the average of the middle two numbers, the average of 41st and 42nd numbers.

1st number

2nd number

3rd number

.

.

.

40th number

41st number

42nd number

43rd number

} median

.

.

.

80th number

81st number

82nd number

Here, we don't have any of the individual numbers, only their groupings, but we can figure out where the 41st and 42nd numbers would be. Clearly, the first column is 13 units high, we means it contains the first 13 numbers on the list. The second column is 35 units high --- $13 + 35 = 48$, so that's the first 48 numbers on the list. If we count all the way down to the 48th number on the list, that means we have already passed the 41st and 42nd numbers, which means the median must be included in the numbers in the second column. Answer = **(B)**.

2 Which of the following could be the third quartile value for number of hours of TV watched last week?

- ☐ 11
- ☐ 17
- ☐ 21
- ☐ 23
- ☐ 26

The third quartile is the 75th percentile, dividing the lower 3/4 from the upper 1/4. If we think of the median as dividing the whole list into an upper half and lower half, the third quartile is the median of that upper list.

This whole list has 82 numbers. The median would be the average of the 41st and 42nd numbers. The "upper list" would consist of 41 numbers, starting at the 42nd number and ending at the 82 number. The number 41 can be expressed as $41 = 20 + 1 + 20$ --the median of the upper list, the third quartile of the whole list, will have 20 number of the upper list above it and 20 below it.

On that upper list, the first 20 numbers go from the 42nd number to the 61st number; the 62nd would be the median; and the last 20 numbers would go from the 63rd number to the 82nd number.

Where is the 62nd number on this list? Well, we could count up 62 places from the left side, or we could just count down 20 from the right side.

The rightmost column has 3 numbers. The two rightmost columns have 6 numbers. The three rightmost columns have 11 numbers. The four rightmost columns have 17 numbers--still not quite enough. The five rightmost columns have 28 numbers-- these columns contain the third quartile. It must be in the 11-15 column. The only number on the list in that column is 11, so Answer = **(A)**.

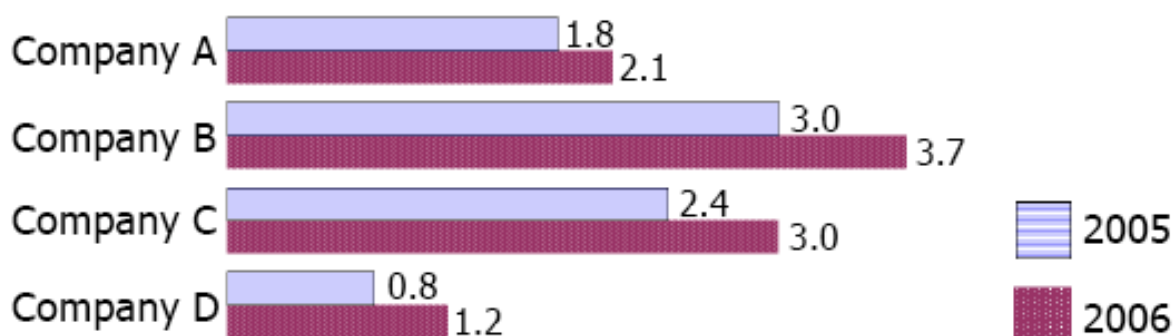
3 Suppose all students surveyed answered in integer number of hours only. Suppose, of 82 surveyed, only one respondent answered "16 hours." Within this group, the approximate percentile of this person would be:

- ☐ 32nd percentile
- ☐ 51st percentile
- ☐ 67th percentile
- ☐ 75th percentile
- ☐ 80th percentile

If all students answered in integers only, and if only one answered "16 hours", that one would be higher than the first three columns --- higher than $13 + 35 + 17 = 65$ of the people. In other words, she is higher than $65/82$. Approximate this as $64/80 = 8/10 = 80\%$. Answer = **(E)**.

In a more precise calculation, $65/82 = 0.7926$, and with percentiles, we round up to the next nearest integer --- the 80th percentile. Answer = **(E)**.

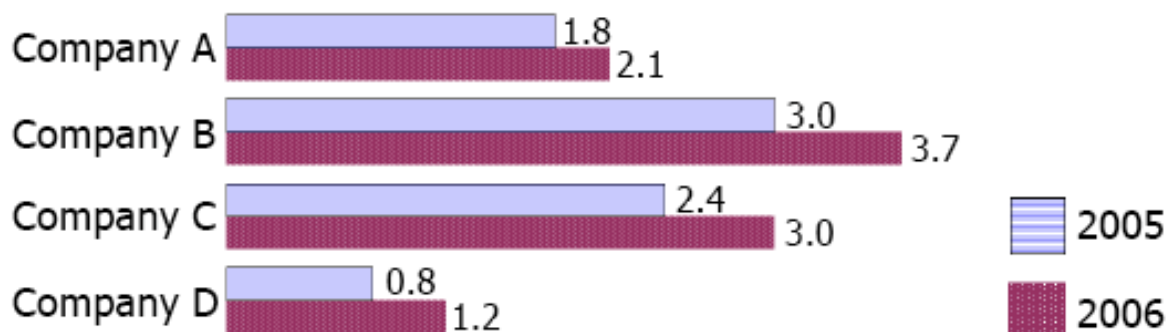
NUMBER OF WIDGETS SOLD BY SELECTED COMPANIES IN 2005 AND 2006 (in millions)



1 In 2005, Company C sold what percent of the widgets sold by the four companies listed?

- ☐ 24
- ☐ 25
- ☐ 30
- ☐ 37.5
- ☐ 42.9

NUMBER OF WIDGETS SOLD BY SELECTED COMPANIES IN 2005 AND 2006 (in millions)



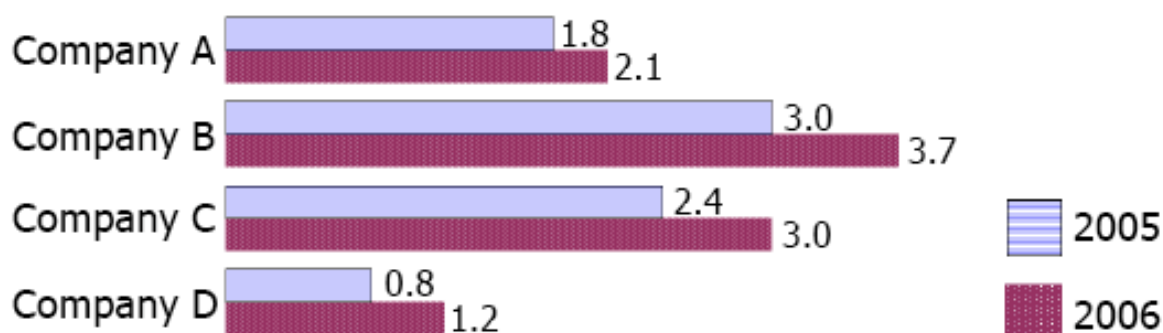
In 2005, Company C sold what percent of the widgets sold by the four companies listed?

<p>(A) 24</p> <p>(B) 25</p> <p>(C) 30</p> <p>(D) 37.5</p> <p>(E) 42.9</p>	$\frac{\text{\# sold by Company C}}{\text{Total \# sold}} = \frac{2.4}{1.8 + 3.0 + 2.4 + 0.8}$ $= \frac{2.4}{8}$ $= \frac{0.3}{1} = 0.3 = 30\%$
--	---

2 In 2006, the ratio of the number of widgets sold by Company C, Company E (not shown) and Company D was 5 to 8 to 2, respectively. How many widgets did Company E sell in 2006?

- ☐ 300,000
- ☐ 600,000
- ☐ 2,400,000
- ☐ 4,800,000
- ☐ 6,000,000

NUMBER OF WIDGETS SOLD BY SELECTED COMPANIES
IN 2005 AND 2006 (in millions)



In 2006, the ratio of the number of widgets sold by Company C, Company E and Company D (not shown) was 5 to 8 to 2, respectively. How many widgets did Company E sell in 2006?

(A) 300,000

(B) 600,000

(C) 2,400,000

☒ (D) 4,800,000

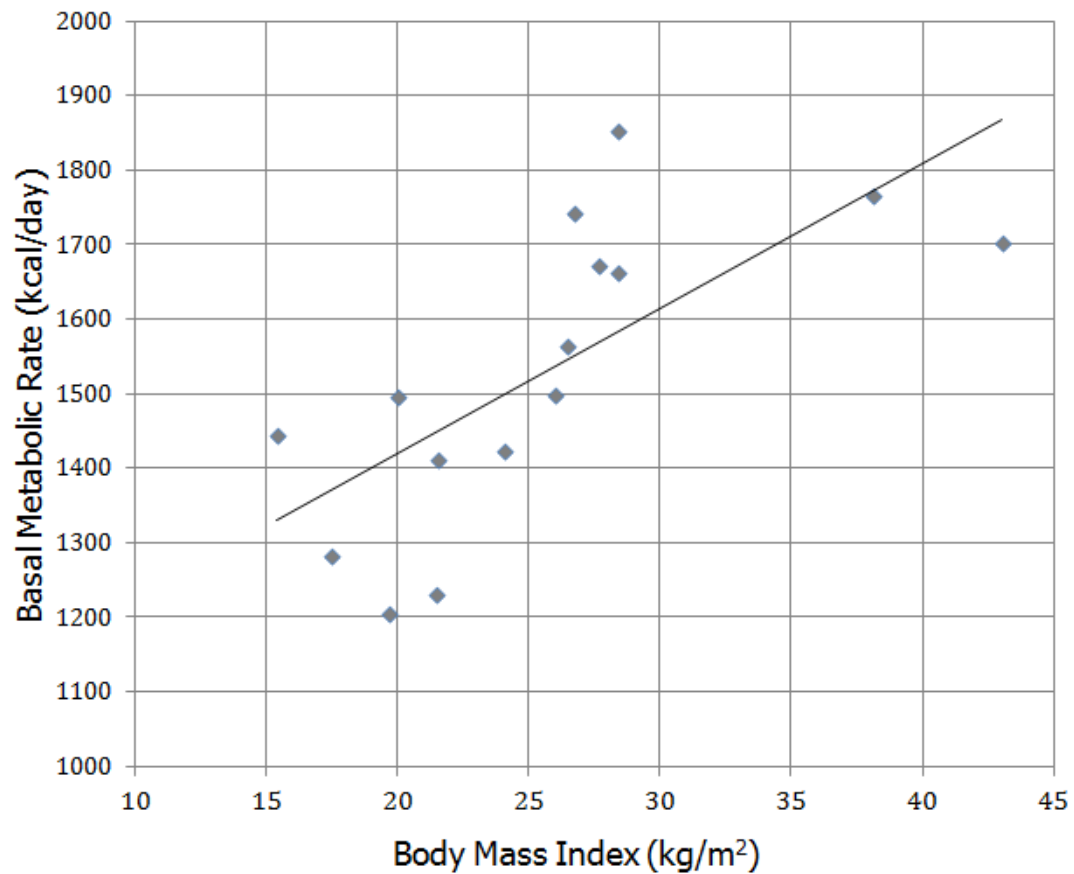
(E) 6,000,000

$$C : E : D = 5 : 8 : 2 \Rightarrow E : D = 8 : 2 \Rightarrow E : D = 4 : 1$$

$$\frac{\text{Company E}}{\text{Company D}} : \frac{x}{1.2} = \frac{4}{1} \Rightarrow x = (1.2)(4) = 4.8$$

十九

The graph below shows the Body Mass Index (BMI) and Basal Metabolic Rate (BMR) of fifteen males between the ages of 43 and 65.

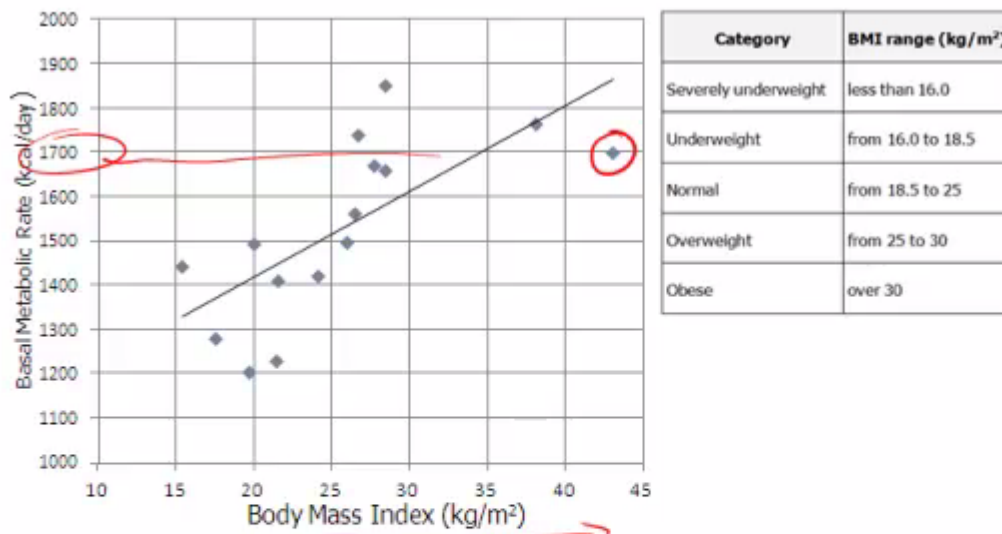


Category	BMI range (kg /m ²)
Severely underweight	less than 16.0
Underweight	from 16.0 to 18.5
Normal	from 18.5 to 25
Overweight	from 25 to 30
Obese	over 30

1 The individual on this chart with the highest BMI has a BMR of approximately

- ☐ 1204
- ☐ 1444

- 1563
○ 1702
○ 1853

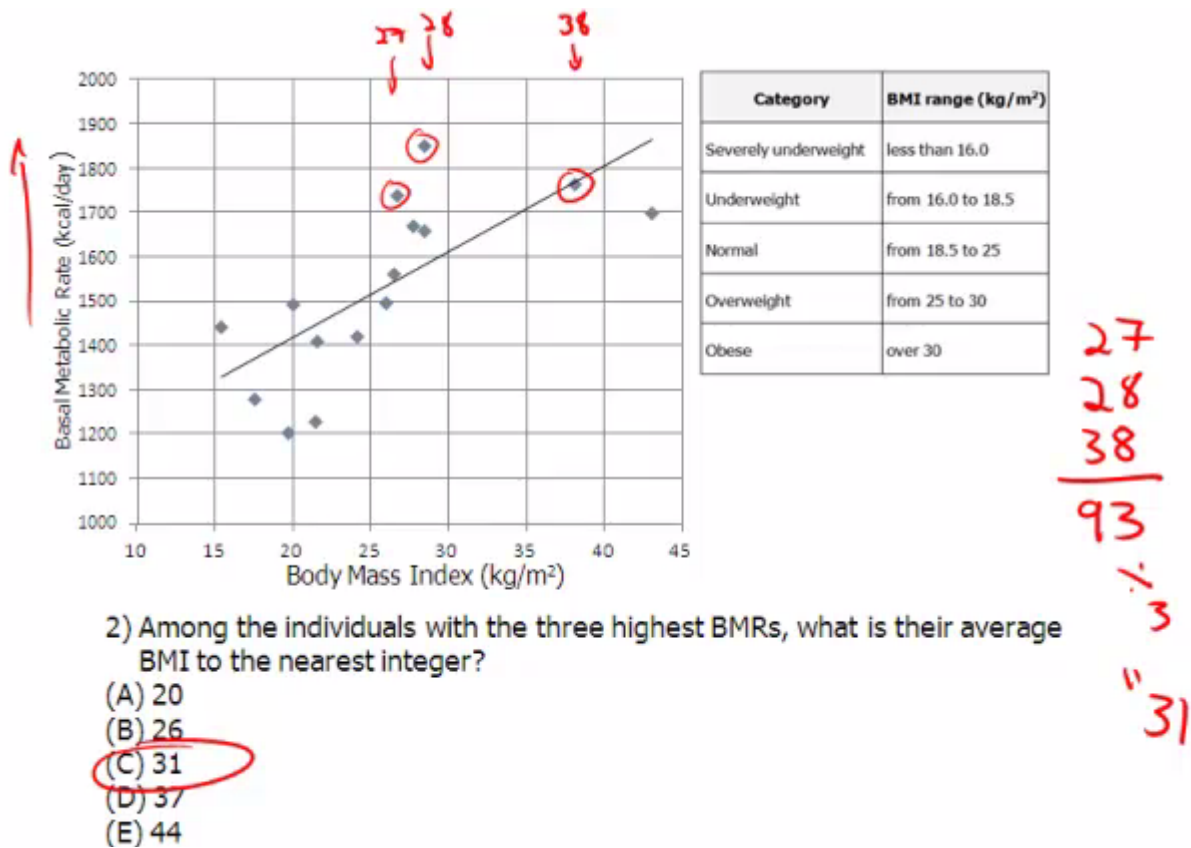


1) The individual on this chart with the highest BMI has a BMR of approximately

- (A) 1204
(B) 1444
(C) 1563
(D) 1702
(E) 1853

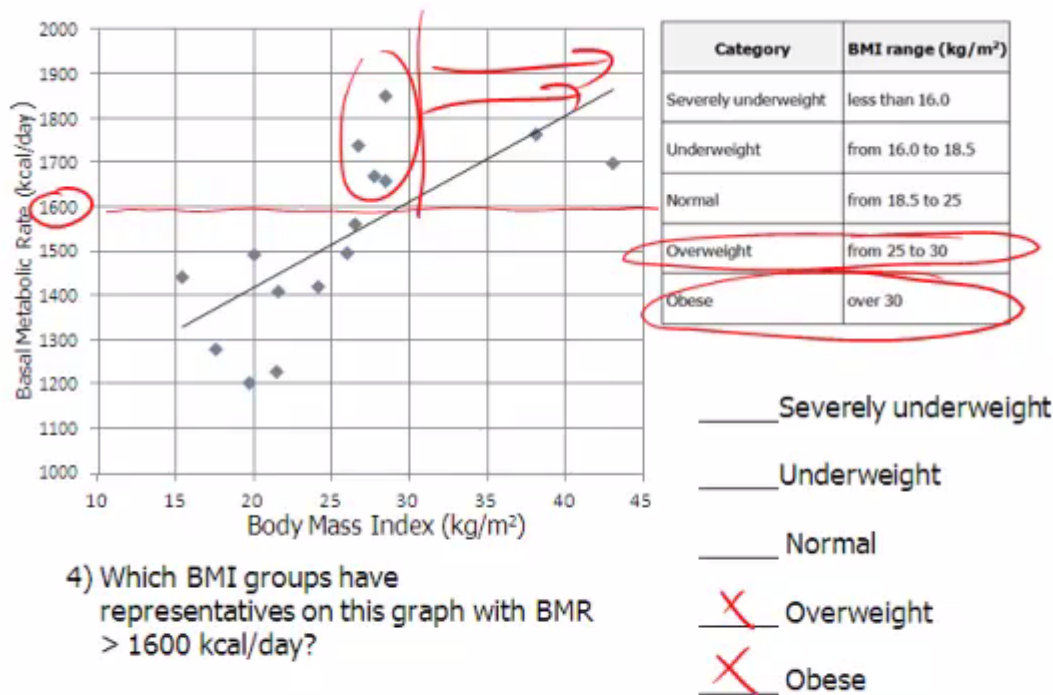
2 Among the individuals with the three highest BMRs, what is their average BMI to the nearest integer?

- 20
○ 26
○ 31
○ 37
○ 44



3 Which BMI groups have representatives on this graph with BMR > 1600 kcal/day?

- ☐ Severely underweight
- ☐ Underweight
- ☐ Normal
- ☐ Overweight
- ☐ Obese



三十

Melpomene High School has 400 students, and Thalia High School has 700 students. The following table shows the percentage breakdown for various groups in each school.

	percentage in Melpomene H.S.	percentage in Thalia H.S.
in band only	11%	8%
on an athletic team only	12%	42%
in honor society only	16%	2%
in band & athletic team only	14%	10%
in honor society & band only	26%	15%
in honor society & athletic team only	10%	8%
in band & athletic team & honor society	4%	2%
in none of these three groups	7%	13%

1 The total number of people in honor society at Melpomene High School, regardless of other activities, is approximately what percent higher than the total number of people in honor society at Thalia High School, regardless of other activities?

- ☐ 2%
- ☐ 8%
- ☐ 19%
- ☐ 29%
- ☐ 56%

In this question, we need to find the two numbers, and then find how much bigger one is than the other.

At Melpomene, the four categories that include the honor society add up to $16\% + 26\% + 10\% + 4\% = 56\%$, and 56% of $400 = (0.56)*400 = 224$ students.

At Thalia, the four categories that include the honor society add up to $2\% + 15\% + 8\% + 2\% = 27\%$, and 27% of $700 = (0.27)*700 = 189$ students.

The question really is: 224 is what percent higher than 189? Estimate --- 10% of 189 is approximately 19. $189 + 19 = 208$, a 10% increase. $208 + 19 = 227$, a 20% increase, and that's just over 224, so 224 should be very close to, maybe slight less than, a 20% increase. This leads us to the answer of **(C)**.

A more precise calculation:

$$\text{percentage change} = \frac{\text{end value} - \text{start value}}{\text{start value}} \times 100\%$$

$$\text{percentage change} = \frac{224 - 189}{189} \times 100\% = 18.5185\% = 19\%$$

Answer = **(C)**.

2 How many non-band members at Melpomene, regardless of other activities, would have to join the band so that they had the same number of band members as does Thalia High School?

- ☐ 12
- ☐ 25
- ☐ 38
- ☐ 46
- ☐ 65

This question is just asking us to figure out the number in the band at Thalia and at Melpomene, and then subtract them.

At Melpomene, the four categories that include the band add up to $11\% + 14\% + 26\% + 4\% = 55\%$, and 55% of $400 = (0.55)*400 = 220$ students.

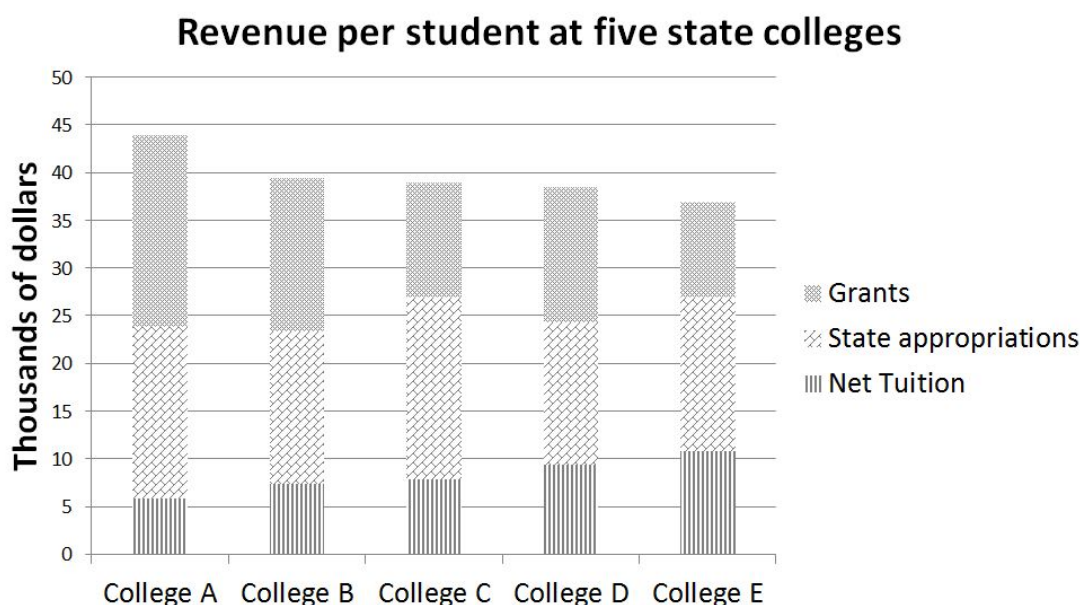
At Thalia, the four categories that include the band add up to $8\% + 10\% + 15\% + 2\% = 35\%$, and 35% of $700 = (0.35) \cdot 700 = 245$ students.

The difference is 25 students. Answer = **(B)**.

3 The total number of people at Melpomene High School who are involved in band and at least one other group is _____.

At Melpomene, the three categories that include the band and one other activity add up to $14\% + 26\% + 4\% = 44\%$, and 44% of $400 = (0.44) \cdot 400 = 176$ students.

二十一



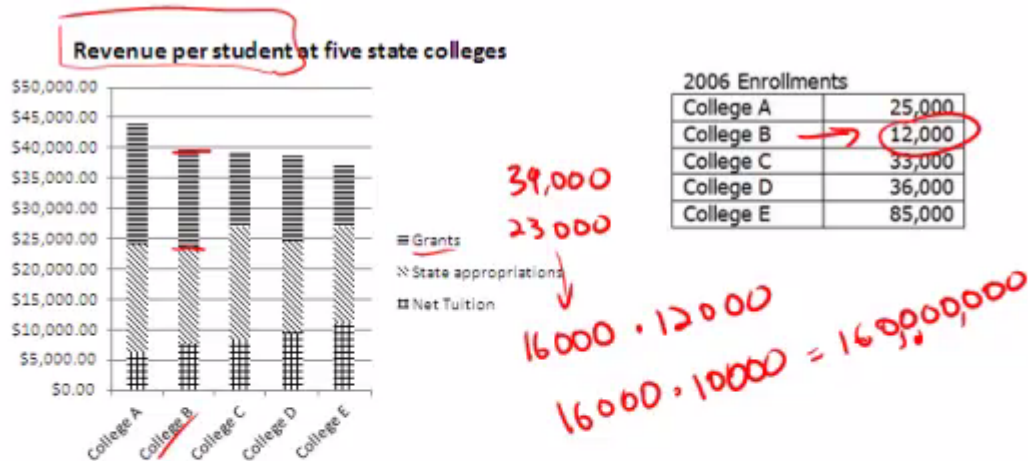
2006 Enrollments

College A	25,000
College B	12,000
College C	33,000
College D	36,000
College E	85,000

1 What is the total dollar amount that College B received in grants?

- ☐ \$16,000,000
- ☐ \$48,000,000

- ☐ \$96,000,000
- ☐ \$160,000,000
- ☐ \$192,000,000



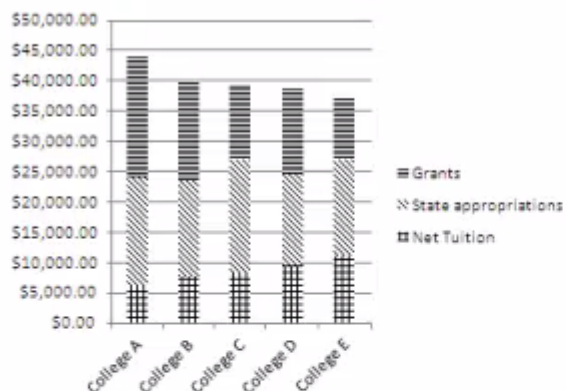
2) What is the total dollar amount that College B received in grants?

- (A) \$16,000,000
- (B) \$48,000,000
- (C) \$96,000,000
- (D) \$160,000,000
- ☒ (E) \$192,000,000

2 Suppose in the next year, 2007, College D's expenses remain about the same, but in addition to their current revenues, they receive an additional \$50,000,000 grant. This would allow them to reduce average tuition by how much?

- ☐ \$1388.89
- ☐ \$3571.43
- ☐ \$5555.56
- ☐ \$9500.00
- ☐ \$25888.89

Revenue per student at five state colleges



2006 Enrollments

College A	25,000
College B	12,000
College C	33,000
College D	36,000
College E	85,000

$$\frac{50,000,000}{36,000} = \frac{50}{36} \cdot 1000$$

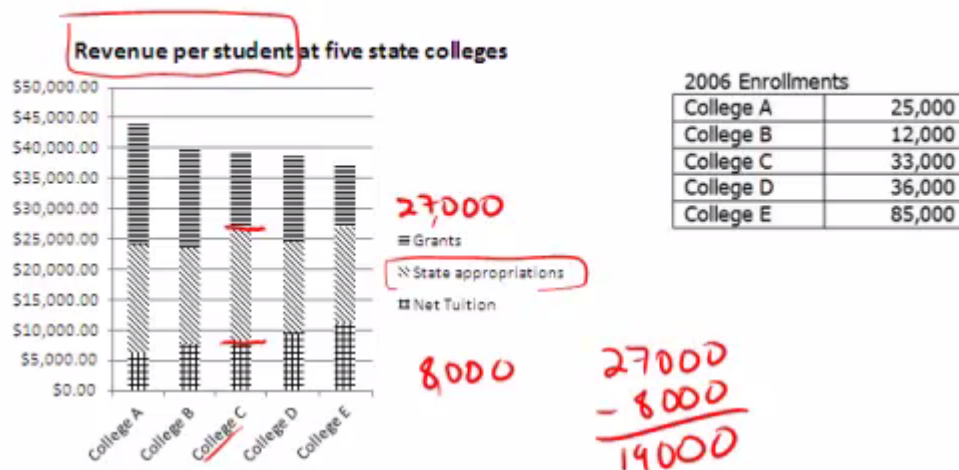
$$2 > \frac{50}{36} > 1$$

3) Suppose in the next year, 2007, College D's expenses remain about the same, but in addition to their current revenues, they receive an additional \$50,000,000 grant. This would allow them to reduce average tuition by how much?

- (A) \$1388.89
 (B) \$3571.43
 (C) \$5555.56
 (D) \$9500.00
 (E) \$25888.89

3 College C received approximately how much in state appropriations per student?

- ☐ \$9,000
☐ \$12,000
☐ \$19,000
☐ \$27,000
☐ \$39,000



1) College C received approximately how much in state appropriations per student?

- (A) \$9,000
(B) \$12,000
(C) \$19,000
(D) \$27,000
(E) \$39,000