Rosenbrock函数

多元 Rosenbrock 函数的一种形式如下：

$$f(x) = \sum_{i=1}^{n-1} \left[(1-x_i)^2 + 100(x_{i+1} - x_i^2)^2\right], x \in \mathbb{R}^n.$$

其梯度为：

$$\begin{aligned}
\frac{\partial f}{\partial x_1} &= \frac{\partial}{\partial x_1}\left[(1-x_1)^2 + 100(x_2 - x_1^2)^2\right] \\
&= 2(x_1 - 1) + 400x_1(x_1^2 - x_2); \\
\frac{\partial f}{\partial x_i} &= \frac{\partial}{\partial x_i}\left[(1-x_i)^2 + 100(x_i - x_{i-1}^2)^2 + 100(x_{i+1} - x_i^2)^2\right] \\
&= 2(x_i - 1) + 400x_i(x_i^2 - x_{i+1}) + 200(x_i - x_{i-1}^2), \\
&\quad i = 2, 3, \cdots, n-1; \\
\frac{\partial f}{\partial x_n} &= 200(x_n - x_{n-1}^2).
\end{aligned}$$

其 Hessian 为：

$$\begin{aligned}
\frac{\partial^2 f}{\partial x_1^2} &= 1200x_1^2 - 400x_2 + 2; \\
\frac{\partial^2 f}{\partial x_1 \partial x_2} &= -400x_1; \\
\frac{\partial^2 f}{\partial x_i \partial x_{i-1}} &= -400x_{i-1}; \\
\frac{\partial^2 f}{\partial x_i^2} &= 1200x_i^2 - 400x_{i+1} + 202; \\
\frac{\partial^2 f}{\partial x_i \partial x_{i+1}} &= -400x_i, \\
& i = 2, 3, \cdots, n-1; \\
\frac{\partial^2 f}{\partial x_n \partial x_{n-1}} &= -400x_{n-1}; \\
\frac{\partial^2 f}{\partial x_n^2} &= 200;
\end{aligned}$$