special communications

Simple, accurate equations for human blood O_2 dissociation computations

JOHN W. SEVERINGHAUS
Cardiovascular Research Institute and Department of Anesthesia,
University of California Medical School, San Francisco, California 94143

SEVERINGHAUS, JOHN W. *Simple, accurate equations for human blood O_2 dissociation computations.* J. Appl. Physiol.: Respirat. Environ. Exercise Physiol. 46(3): 599–602, 1979.—Hill's equation can be slightly modified to fit the standard human blood O_2 dissociation curve to within ±0.0055 fractional saturation (S) from $0 < S < 1$. Other modifications of Hill's equation may be used to compute Po_2 (Torr) from S (*Eq. 2*), and the temperature coefficient of Po_2 (*Eq. 3*). Variations of the Bohr coefficient with Po_2 are given by *Eq. 4*.

$$S = (((Po_2{}^3 + 150\ Po_2)^{-1} \times 23{,}400) + 1)^{-1} \qquad (1)$$

$$\ln Po_2 = 0.385 \ln (S^{-1} - 1)^{-1} + 3.32 - (72\ S)^{-1} - 0.17(S^6) \qquad (2)$$

$$\Delta\ln Po_2/\Delta T = 0.058((0.243 \times Po_2/100)^{3.88} + 1)^{-1} + 0.013 \qquad (3)$$

$$\Delta\ln Po_2/\Delta pH = (Po_2/26.6)^{0.184} - 2.2 \qquad (4)$$

Procedures are described to determine Po_2 and S of blood iteratively after extraction or addition of a defined amount of O_2, and to compute P_{50} of blood from a single sample after measuring Po_2, pH, and S.

blood Po_2; Bohr effect; temperature effect; (a-v)O_2 content computation; O_2 saturation equation; pH effect on Po_2 calculation

---

THEORETIC ANALYSES OF O_2 EXCHANGES between gas, blood, and tissue, and the effects of temperature and pH, require reasonably accurate expressions of the O_2 dissociation curve and temperature coefficient. The Adair equation (6) although theoretically sound, could not be made to conform to whole blood O_2 dissociation to better than about ±1% saturation by Dr. E. DeLand, using a Rand Corporation computer curve-fitting program (6). Both the Adair and several empiric power functions (1, 4, 7, 9) are cumbersome multiconstant equations that may unacceptably delay small desk top calculators when iterative solutions are needed, as in computing arteriovenous Po_2 differences. The following equations were developed primarily to facilitate investigation of the determinants of transcutaneous Po_2 under heated skin surface electrodes.

Symbols and units of measurement are defined.

Po_2 partial pressure of O_2 (Torr at 37°C)

S   fractional O_2 saturation (% saturation/100)
P_{50} Po_2 of a whole blood sample at 37°C (S = 0.5, pH = 7.4)
C O_2 content (milliliters O_2 STPD per milliliters blood)
BE   base excess of whole blood (meq/l)
$'$ and $''$ before and after gas exchange, i.e., arterial venous but valid for both increase and decrease of O_2
$*$ a trial computation of a variable
T   temperature (°C)
ln   natural logarithm, used for all computations unless noted "$\log_{10}$"

I. O_2 Saturation from Po_2

The Hill equation approximates the human blood O_2 dissociation curve reasonably well for $S > 0.3$ but is unacceptably low at the bottom of the curve. The following modification of Hill's equation corrects most of this misfit, and matches the standard human blood O_2 dissociation curve at pH = 7.4, T = 37°C (6). Table 1 includes values obtained by extrapolation between the points in Table 2, columns 1 and 2 of Ref. 6. Figure 1 displays the error, as a function of O_2 saturation, of the equation

$$S = (((Po_2{}^3 + 150\ Po_2)^{-1} \times 23{,}400) + 1)^{-1} \qquad (1)$$

The greatest error is +0.55% at 98.77% Sat. The mean absolute error (with equal weight to each increment of saturation) is 0.26% saturation, less than half that of the Adair equation (6). P_{50} with *Eq. 1* is 26.86, compared with 26.67 in the standard curve.

II. Po_2 from O_2 Saturation

Equation 1 cannot be directly solved for Po_2. Another empiric modification of Hill's equation may be used to compute Po_2 from S for $S < 0.965$

$$\ln Po_2 = 0.385 \ln (S^{-1} - 1)^{-1} \qquad (2)$$
$$+ 3.32 - (72\ S)^{-1} - (S^6)/6$$

Figure 2 depicts the error of *Eq. 2*, the mean absolute error being 0.16 Torr for the range $0 < S < 0.965$. The maximum error is 0.6 Torr. At S = 0.99, *Eq. 2* is 37 Torr

599

below the standard Po_2 of 174 Torr. *Equation 1* is similarly low in this range, although for both, the error is less than 0.006 S.

TABLE 1. *Values for standard human blood O_2 dissociation curve at 37°C, pH = 7.4, extrapolated between points from Ref. 6*

Po_2	%S	Po_2	%S	Po_2	%S
1	0.60	34	65.16	80	95.84
2	1.19	36	68.63	85	96.42
4	2.56	38	71.94	90	96.88
6	4.37	40	74.69	95	97.25
8	6.68	42	77.29	100	97.49
10	9.58	44	79.55	110	97.91
12	12.96	46	81.71	120	98.21
14	16.89	48	83.52	130	98.44
16	21.40	50	85.08	140	98.62
18	26.50	52	86.59	150	98.77
20	32.12	54	87.70	175	99.03
22	37.60	56	88.93	200	99.20
24	43.14	58	89.95	225	99.32
26	48.27	60	90.85	250	99.41
28	53.16	65	92.73	300	99.53
30	57.54	70	94.06	400	99.65
32	61.69	75	95.10	500	99.72

III. Temperature Coefficient of Po_2 in Blood

The effect of changing temperature on blood Po_2 varies from 7.4%/°C at low saturation, to 1.3%/°C at high Po_2. This factor depends on the slope of the O_2 dissociation curve at the Po_2 of the blood. A modification of Hill's equation expresses slope, and may be used to compute the temperature coefficient at any given Po_2. This equation is

$$\Delta\ln Po_2/\Delta T = 0.058((0.243 \times Po_2/100)^{3.88} + 1)^{-1} + 0.013 \tag{3}$$

In Fig. 3, *Eq. 3* is plotted as a function of both Po_2 and saturation. This equation applies to Po_2 at 37°C. To begin with some other temperature, one may estimate a trial 37°C Po_2 using the factors 6%/°C if $Po_2 < 100$, and 6 Torr/°C above 100 Torr, and proceed iteratively in *Eq. 3*.

Little experimental data is available to verify the temperature correction. Nunn et al. (5) obtained a few points in good agreement with the values computed from the dissociation curve by the present author (8). *Equation 3* yields values in close accord with the complex procedure of Thomas (9), and that of Ruiz et al. (7).
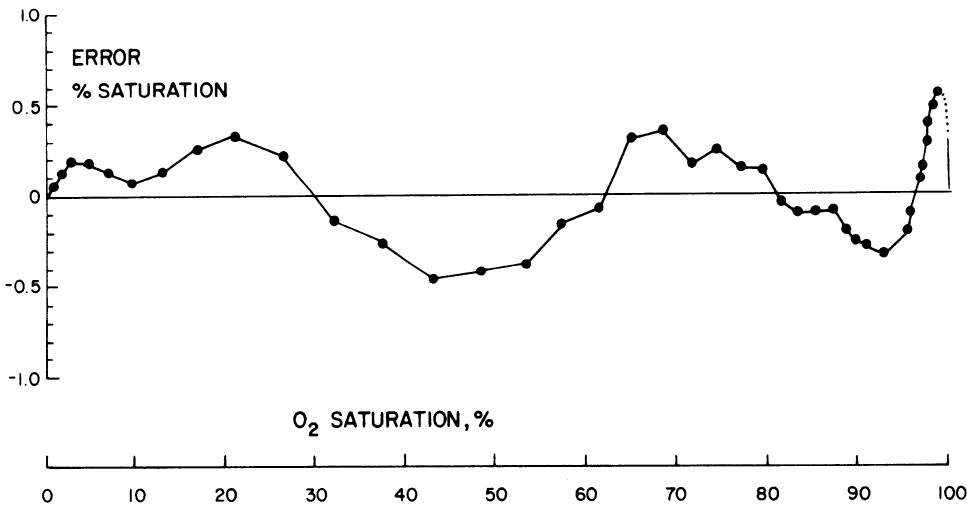

FIG. 1. Error, in % saturation, of *Eq. 1* as a function of saturation from 0 to 100%, from "standard blood O_2 dissociation curve" data of Table 1.
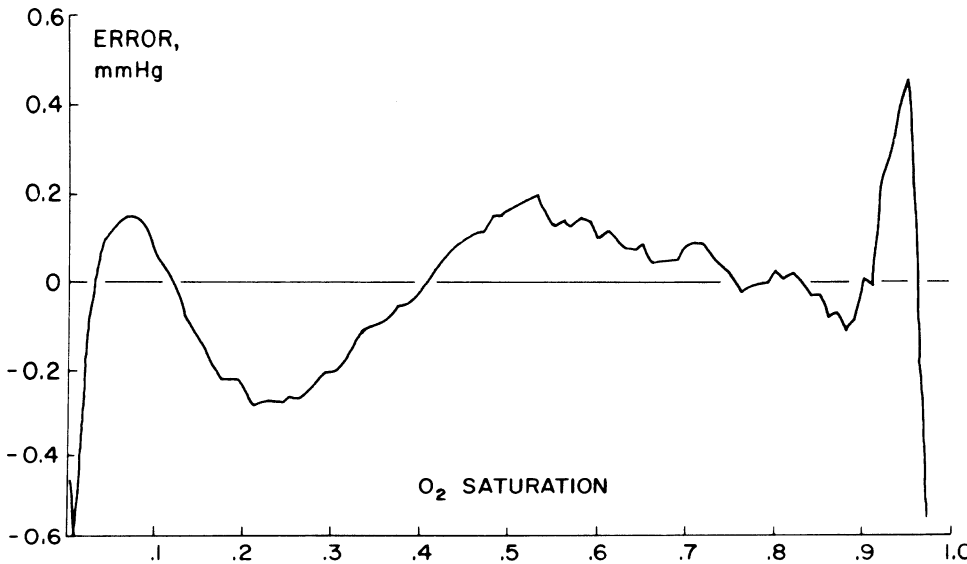

FIG. 2. Error, in Torr, of *Eq. 2* as a function of saturation fraction S, also compared with data of Table 1. Above S = 0.965, errors which are about +0.005 S, become very large as Po_2 errors.
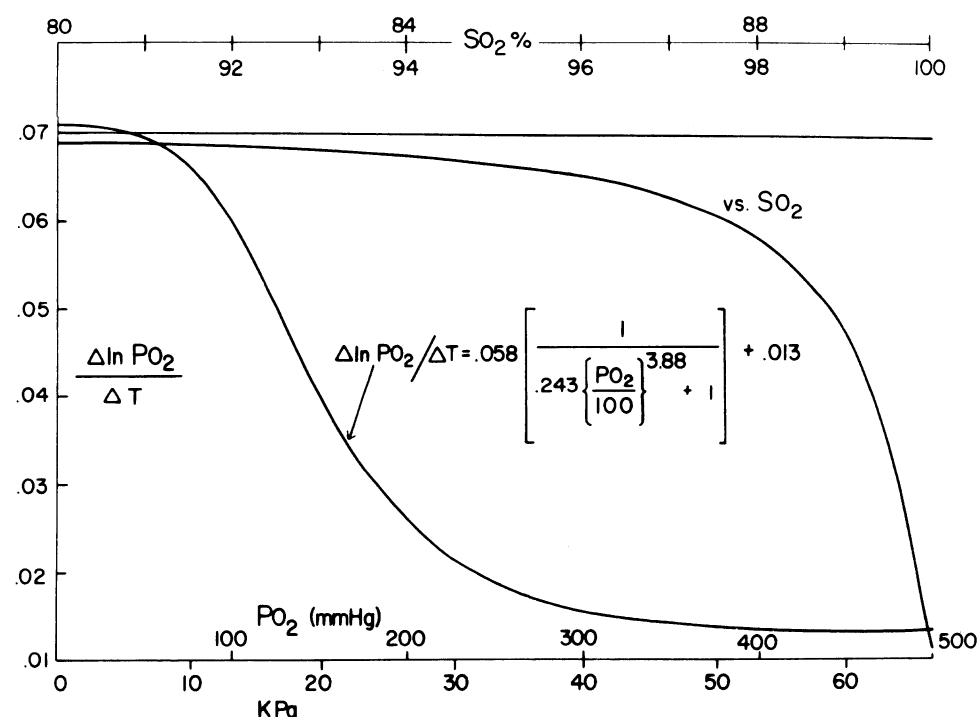
FIG. 3. Relationship of anaerobic temperature coefficient of whole blood P_{O_2} to the level of P_{O_2} (*lower curve* and *lower abscissa*, in both Torr and in kPa), or to saturation percentage (*upper ordinates* and *curves*). Nearly horizontal line at 0.07 is value from $80 < S_{O_2}\% < 90$.
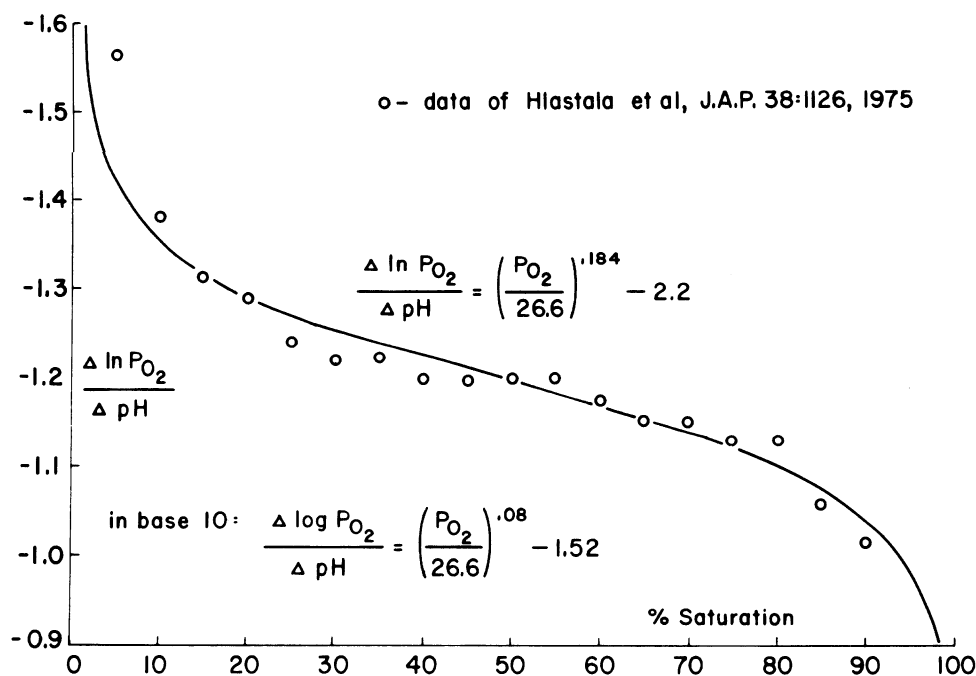
$$\frac{\Delta \ln P_{O_2}}{\Delta T} = .058 \left[\frac{1}{.243 \left\{ \frac{P_{O_2}}{100} \right\}^{3.88} + 1} \right] + .013$$

o— data of Hlastala et al, J.A.P. 38:1126, 1975

$$\frac{\Delta \ln P_{O_2}}{\Delta pH} = \left(\frac{P_{O_2}}{26.6} \right)^{.184} - 2.2$$

in base 10: $\dfrac{\Delta \log P_{O_2}}{\Delta pH} = \left(\dfrac{P_{O_2}}{26.6} \right)^{.08} - 1.52$

FIG. 4. Bohr factor as a function of saturation. Data points from Hlastala, and one (not shown) at S = 0.99 from Ref. 6, are represented by an empiric equation. Data are for CO_2 variations of pH in whole human blood with normal BE and 2,3-DPG.

IV. Correction of P_{O_2} to pH = 7.40 (Bohr Effect)

As a first approximation, the effect of respiratory variations of pH on P_{O_2} at constant saturation is a factor independent of P_{O_2}.

$\Delta \log_{10} P_{O_2}/pH = -0.48$, or computing $-0.48(\ln 10) = 1.1$

$$P_{O_2}(7.4) = P_{O_2} \times e^{1.1(pH-7.4)} \qquad (4)$$

The Bohr effect falls off at high saturation and increases at very low saturation, as illustrated in Fig. 4, taken from the work of Hlastala and Woodson (2). An additional point Roughton and I (6) obtained at S = 0.99

is −0.78 (−0.34 in $\log_{10}$) is not shown in Fig. 4. When CO_2 is the acid variable, about 20% of the Bohr effect is due to a CO_2 effect independent of pH. For this one may add a correction of +0.003 BE (8). An empiric estimate of the Bohr effect taking these data into account is then

$$\Delta \ln P_{O_2}/\Delta pH = (P_{O_2}/26.7)^{0.184} + 0.003 \, BE - 2.2 \qquad (5)$$

Values from *Eq. 5* may be substituted in *Eq. 4* in place of (1.1).

V. Program for Computing S from P_{O_2} and pH

These equations are easily programmed on hand-held

calculators. A typical program (Novus 4515 Mathematician PR) including initial P_{O_2} correction for pH, is as follows: Start; Enter; Halt; Enter; 7.4; −; 1.1; x; e^x; x; Enter; Fx^2; 150; +; 23400; /; 1/x; 1; +; 1/x; Skip.

To use, key in P_{O_2}, Start, pH, Start.

VI. Arteriovenous P_{O_2} Differences from O_2 Content Differences

When blood exchanges O_2 with tissue or gas, variable amounts of the O_2 come from dissolved O_2 and HbO_2, depending on the slope of the dissociation curve over which the blood moves. O_2 content (C) is computed as

$$C = (0.0134 \times Hb \times S) + (0.000031 \tag{6}$$
$$\times P_{O_2}) \text{ (at } 37°C)$$

Assuming known values of P'_{O_2}, Hb, and the change in C (ΔC), one needs an iterative method to determine the final P''_{O_2} and S''. Proceed as follows.

With Eq. 1, compute S'.

Let $P'_{O_2} = P^*_{O_2}$ and $\Delta C/2 = \Delta C^*$. Iteration

$$P^*_{O_2}(1 - ((\Delta C - \Delta C^*)2VP^*_{O_2})) = P''_{O_2} \tag{7}$$
$$\text{(if this } P''_{O_2} < 0, \text{ set } P''_{O_2} = 10)$$

Compute S'' from new P''_{O_2} (Eq. 1)

$$0.0134 \times Hb \times (S' - S'') + 0.000031$$
$$\times (P'_{O_2} - P''_{O_2}) = \Delta C^*$$

Let $P''_{O_2} = P^*_{O_2}$ and iterate from Eq. 7 until ($\Delta C - \Delta C^*$) $< \pm 0.0001$. The empiric term $VP^*_{O_2}$ in Eq. 7 scales the error in C to approximately match the slope of the O_2 dissociation curve at $P^*_{O_2}$.

Note that Eqs. 1 and 2 are not precisely equivalent, and therefore can not be used sequentially in iterative solutions.

VII. Computation of P_{50}

Obtain a sample of the blood in question with $0.2 < S < 0.8$. Measure S spectrophotometrically or by determination of C and O_2 capacity. Measure P_{O_2} and pH of the same sample at 37°C. Use Eq. 2 to compute P_{O_2} from measured S, and term this P_{O_2}(std). Use Eqs. 4 and 5 to compute P_{O_2} at pH = 7.4 from the measured P_{O_2} and pH, and term this P_{O_2}(obs).

$$P_{50} = 26.7 \times P_{O_2}\text{(obs)}/P_{O_2}\text{(std)} \tag{8}$$

REFERENCES

1. ABERMAN, A., J. M. CAVANILLES, J. TROTTER, D. ERBECK, M. H. WEIL, AND H. SHUBIN. An equation for the oxygen hemoglobin dissociation curve. J. Appl. Physiol. 35: 570–571, 1973.
2. HLASTALA, M. P., AND R. D. WOODSON. Saturation dependency of the Bohr effect: interactions among H^+, CO_2, and DPG. J. Appl. Physiol. 38: 1126–1131, 1975.
3. KELMAN, G. R. Digital computer subroutine for the conversion of oxygen tension into saturation. J. Appl. Physiol. 21: 1375–1376, 1966.
4. LUTZ, J., H. G. SCHULZE, AND U. F. MICHAEL. Calculation of O_2 saturation and of the oxyhemoglobin dissociation curve for different species, using a new programmable pocket calculator. Pfluegers Arch. 359: 285–295, 1975.
5. NUNN, J. F., N. A. BERGMAN, A. BUNATYAN, AND A. J. COLEMAN. Temperature coefficients for P_{CO_2} and P_{O_2} of blood in vitro. J. Appl. Physiol. 20: 23–26, 1965.
6. ROUGHTON, F. J. W., AND J. W. SEVERINGHAUS. Accurate determination of O_2 dissociation curve of human blood above 98.7% saturation with data on O_2 solubility in unmodified human blood from 0° to 37°C. J. Appl. Physiol. 35: 861–869, 1973.
7. RUIZ, B. C., W. K. TUCKER, AND R. R. KIRBY. A program for calculation of intrapulmonary shunts, blood-gas and acid-base values with a programmable calculator. Anesthesiology 42: 88–95, 1975.
8. SEVERINGHAUS, J. W. Blood gas calculator. J. Appl. Physiol. 21: 1108–1116, 1966.
9. THOMAS, L. J., JR. Algorithms for selected blood acid-base and blood gas calculations. J. Appl. Physiol. 33: 154–158, 1972.