The Impact of PRDE Parameters Changes on Bristol Stock Exchange Simulation Trading

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***Abstract*—This paper reports on differential evolution (DE) in the PRDE strategy on the Bristol Stock Exchange (BSE) and the modification of critical parameters for different simulated trades. Two key parameters k and F in PRDE are introduced in Dr Dave Cliff's paper, and this paper will be an extension of that to make assumptions and design experiments on these two parameters. It is hypothesised that different k and F will impact Profit Per Second (PPS), and several experimental groups with different k and F are designed for comparative analysis. The results will be presented as statistical plots, and the differences between each plot will be analysed to draw the appropriate conclusions or hypotheses. Finally, some problems with the experiment and subsequent considerations will be described.**

***Index Terms*—Bristol Stock Exchange, buyers, equilibrium, differential evolution, profit per second, p-value, sellers, simple moving average**

I. Introduction

A. BSE Brief Intro

The financial markets are evolving with the development of technology. Artificial securities trading is being replaced by machines that can operate trading systems automatically. These systems can adjust trading strategies to financial market trends to achieve better returns. Moreover, automated system traders are being studied to make them more human-like and flexible.

The Bristol Stock Exchange (BSE) [1], the subject of this study, is a typical analogue automated trading system that conducts continuous double auctions via a limit order book (LOB). BSE has various market&trader agents such as ZIC, ZIP, and 'house strategies' called GVWY and SHVR.

B. New Trader Types

In recent years, the BSE has added new automated traders called PRZI, PRSH and PRDE. PRZI is controlled by the trader's strategy value, denoted as $s \in [-1, +1] \in R$. For any PRZI trader, the value of s is particular, preventing financial trading from adapting strategies over time. Therefore, based on PRZI, adaptation is then added to produce PRSH. PRSH applies a loop to create a set of k, including many s. The loop selects the largest of these s values, called s_0, and then selects the remaining $k-1$ s values as the 'mutation' of s_0. They are continued as a new set in the loop.

Another strategy for optimising PRZI has since emerged, called PRDE, which uses differential evolution (DE), a similar method to PRSH, to maintain and update a population of candidate solutions. It is described in a Dr Dave Cliff's paper [2] as having a parameter NP equivalent to k in PRSH, representing the number of different candidate solutions. NP of PRDE is also denoted by k, which is convenient for outlining the analytical background of PRDE and PRSH in the next section.

Each PRDE trader has a private DE system with a particular population size, k. For instance, the set of k for trader i is $s_{i,1}$, $s_{i,2}$, ..., $s_{i,k}$. Trader i's DE will disorder the set of k and randomly pick a $s_{i,A}$ which is $k[0]$ as a possible replacement candidate in the next iteration. $s_{i,B}$, $s_{i,C}$, $s_{i,D}$ ($k[1]$, $k[2]$, $k[3]$) are then taken out and used to create new strategies, possibly one of which will replace $s_{i,A}$. The following code (line1209-23 of BSE.py in GitHub [1]) is an example:

```
# pick four individual strategies at random, but
    they must be distinct
# create sequential list of strategy-numbers
stratlist = list(range(0, self.k))
random.shuffle(stratlist) # shuffle the list

# s0 is next iteration's candidate for possible
    replacement
self.diffevol['s0_index'] = stratlist[0]

# s1, s2, s3 used in DE to create new strategy,
    potential replacement for s0
s1_index = stratlist[1]
s2_index = stratlist[2]
s3_index = stratlist[3]

# unpack the actual strategy values
s1_stratval = self.strats[s1_index]['stratval']
s2_stratval = self.strats[s2_index]['stratval']
s3_stratval = self.strats[s3_index]['stratval']
```

Another critical parameter of PRDE is the differential weight, denoted by F, which is used as an adaptive step of DE to create a new individual. Following the example above, trader i creates a new individual using $s_{i,B}$, $s_{i,C}$, $s_{i,D}$ and F_i, a new candidate strategy $s_{i,new} = max(min(s_{i,B} + F_i(s_{i,C} - s_{i,D}), +1), -1)$. Such as the following code (line1226 of BSE.py in GitHub [1]):

```python
# this is the differential evolution "adaptive step
    ": create a new individual
new_stratval = s1_stratval + self.diffevol['F'] * (
    s2_stratval - s3_stratval)
```

After that, $s_{i,new}$ is evaluated for fitness, and if its profit per second (PPS) is greater than or equal to $s_{i,A}$'s PPS, $s_{i,new}$ will replace $s_{i,A}$, and vice versa is discarded. Then the evaluation of the next strategy continues. The relevant code (line1202-04 of BSE.py [1]) shows:

```python
if fit_new >= fit_0:
    # new strat did better than old strat0, so
    overwrite new into strat0
    self.strats[i_0]['stratval'] = self.strats[i_new
    ]['stratval']
```

The literature on the PRDE of the BSE uses specific values for k and F (usually $k = 4$ and $F = 0.8$). k and F may be changed to produce better results for simulated trading. Therefore, this research aims to observe the effect of k and F on simulated trading by reasonably varying them.

II. Background

With the rapid development of machine trading, many of the newer types of machine traders are not well known to the general public. They may have potential risks, such as inadequate legal policies for the machine trader.

In Dr Dave Cliff's paper 'Metapopulation Differential Co-Evolution of Trading Strategies in a Model Financial Market' [2], he described the concept of a new type of trader, PRDE, and its advantages.

Dr Dave Cliff conducted an experiment to simulate BSE by comparing PRSH with PRDE. He first set up 30 buyers and 30 sellers (60 PRSH traders in total) with $k = 4$ per trader. Secondly, all buyers paid no more than \$140 per item, and sellers paid no less than \$60 per item. He had these 60 PRSH auto-traders simulate 300 days of trading and calculated the profit per second (PPS) for all buyers and all sellers. The data revealed an adaptation phase for the first 25 days, followed by a more consistent total of roughly 90 PPS for the sum of all buyers and sellers on day 50 and beyond.

He then set up another 60 PRDE traders with the same configuration as those 60 PRSH traders. After 300 days of simulated trading, he found that PRDE's total PPS (the sum of all sellers' and buyers' PPS) was about twice that of PRSH (plot shows in fig 1).

The only difference between PRDE and PRSH is the use of DE in PRDE and stochastic hill-climbing in PRSH, and this performance improvement can be attributed to DE.

This paper [2] by Dr Dave Cliff demonstrates that if each machine trader uses DE for automated trading, the resulting profit can be significantly higher.

His experiment was conducted with $k = 4$ (the minimum feasible value). He believes that it is a future direction to observe market results if key parameters are changed, e.g. whether a uniform increase in the value of k affects PPS. So the current research topic is an extension of his paper. The implications of changing k and F in PRDE will be discussed in various scenarios.

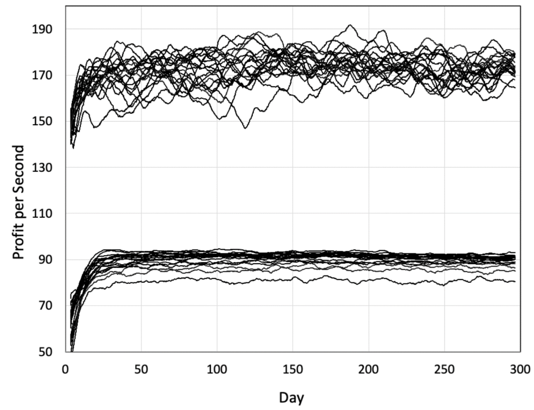

Fig. 1. Comparison of PRSH and PRDE of total PPS [2]

III. Hypothesis

Dr Dave Cliff's paper [2] calculates the PPS of all traders, which is a compelling simulation of trading data. Changing the key parameters k and F may directly impact PPS, so it is assumed that increasing the value of k or F will increase the value of PPS.

IV. Methods

When setting up an experiment, some conditions are always constant:

- The total number of traders is 60 (Buyers 30, Sellers 30).
- For each trader, $s \in [-1, +1] \in R$.
- The range of supply is $(60, 200)$, and the range of demand is $(1, 140)$.
- The stepmode of supply and demand (the stepmode is how orders are assigned to traders) is *fixed*. BSE gives orders with equal intervals, and there is a constant difference between successive prices in the supply and demand curves.
- The start time of the transaction is 0 s, and the end time is 15 days ($60 * 60 * 24 * 15$ seconds). The BSE uses a time step of $\Delta t = 1/number\ of\ traders = 1/60$ seconds to simulate continuous time, and k strategies take $7200k$ seconds to evaluate, which consumes much computational time. The performance of the laptops used for the experiments did not allow for 300 days of simulated trading as in Dr Dave Cliff's paper, so 15 days of trading was chosen.
- Order interval (the number of seconds it takes for an order to cycle through all traders and be available to them) is 30 s.
- Order timemode is *periodic* (all traders are offered new orders at the beginning of each interval).

$k = 4$ and $F = 0.8$ were set as the initial conditions (IC) and applied to experimental group A) to investigate the effects of changing the k and F conditions. The following experimental ideas were designed:

1) Experimental group A) keeps the initial conditions of k and F for all traders.

Experimental group B) increases the k of each trader to a constant $k = 7$ before the start of the trade, and $F = 0.8$ keeps constant. Observe how the total PPS differs between A) and B).

2) Group C) has $k = 4$ for all traders before starting and increases k for each trader by 3 every 7.5 days after the start. It corresponds to $k = 4$ for days 0-7.5 and $k = 7$ for days 7.5-15. $F = 0.8$ remains constant. Observe the trend in total PPS for experimental groups C) and A).
3) Group D) increases the k to 7 for 30 Sellers, F remains unchanged, and the k and F for 30 Buyers is the IC. Group E) increases the k to 7 for 30 Buyers, F is unchanged, and the k and F of 30 Sellers are IC. Observe the PPS of D), E) and A).
4) F) sets $k = 7$ for 15 Buyers, another 15 are IC, and sets $k = 7$ for 15 Sellers, another 15 are IC. Both F remains unchanged. Observe the PPS of F) and A).
5) G) has $F = 0$ for all traders before starting and increases the F of each trader by 2 for every 7.5 days after the start. It corresponds to $F = 0$ for days 0-7.5, $F = 2$ for days 7.5-15. $k = 4$ remains constant. Observe the trend in total PPS for G) and A).
6) H) has $F = 2$ for 30 Sellers and $F = 0.8$ for 30 Buyers, with both k remaining unchanged. I) has $F = 2$ for 30 Buyers and $F = 0.8$ for 30 Sellers, with both k remaining unchanged. Observe the PPS of H), I) and A).
7) J) sets $F = 2$ for 15 Sellers and 15 Buyers, and $F = 0.8$ for 15 Sellers and 15 Buyers, with both k remaining unchanged. Observe J) PPS with A).

Before starting the experiment, it is necessary to add a function to calculate the total PPS based on BSE.py [1]. Each trader's PPS is in the parameter **self.strats**, a multi-dimensional dictionary, the dimension of which is determined according to the range of k. The current PPS is the PPS of the current strategy, **self.active_strat**. Therefore, the trader's PPS is **traders.strats[traders.active_strat][‘pps’]**. It is obvious to set a for loop to get each trader's PPS:

```
for t in traders:
    if traders[t].tid[:1] == 'B': #if trader is buyer
        pps_b.append(traders[t].strats[traders[t].active_strat]['pps'])
    if traders[t].tid[:1] == 'S': #if trader is seller
        pps_s.append(traders[t].strats[traders[t].active_strat]['pps'])
```

The **pps_b** obtained is the PPS of each buyer, and **pps_s** is the PPS of each seller. So afterwards, they are combined to obtain each total PPS, like this:

```
sumB_pps.append(sum(pps_b)) #sum all buyers PPS
sumS_pps.append(sum(pps_s)) #sum all sellers PPS
```

In addition, when using ZIP or ZIC as trader types, conditional parameters such as k and F are not required, but trader types such as PRZI, PRSH and PRDE will report an error if they do not add them. Moreover, in BSE.py [1], the **unpack_params** function is used to get the trader's parameters. However, its dictionary has no F parameter because BSE sets a default value of $F = 0.8$. It is necessary to add the parameter $'F'$ if setting a particular F:

```
parameters = {'optimizer': 'PRDE', 'k': trader_params['k'], 'strat_min': trader_params['s_min'], 'strat_max': trader_params['s_max'], 'F':trader_params['F']}
```

Furthermore, each trader's condition parameter needs to be set before starting a trade.

```
trader_params_1 = {'k': 4, 's_min':-1.0, 's_max':+1.0,'F':0.8}  # IC, k = 4, F = 0.8
trader_params_2 = {'k': 7, 's_min':-1.0, 's_max':+1.0,'F':0.8} # k = 7, F = 0.8
trader_params_3 = {'k': 4, 's_min':-1.0, 's_max':+1.0,'F':0}  # k = 4, F = 0
trader_params_4 = {'k': 4, 's_min':-1.0, 's_max':+1.0,'F':2}  # k = 4, F = 2
```

A. Experiment 1: Keeping a constant k

For obtaining data for experimental groups A) and B), it is sufficient to initially change the value of k for all traders.

```
#For group A)
sellers_A= [('PRDE', 30, trader_params_1)]
buyers_A = sellers_A
traders_A = {'sellers':sellers_A, 'buyers':buyers_A}
#For group B)
sellers_B= [('PRDE', 30, trader_params_2)]
buyers_B = sellers_B
traders_B = {'sellers':sellers_B, 'buyers':buyers_B}
```

B. Experiment 2: Increasing k over time

For obtaining data for C), some changes need to be made to BSE.py [1]. In the *time* loop of the function **market_session**, the **respond** function is called for each trader and is used to update each trader's internal variables, such as evaluating k strategies. When the trading time reaches 7.5 days, each trader's value of k needs to increase. So, it is feasible to add a k value parameter to the **respond** function and set up an *if* condition that if $time >= 86400 * 7.5$ (7.5 days), $k + 3$, otherwise, it does not change. Example:

```
for t in traders:
    k = traders[t].k  #get current k value
    if time_order >= 86400*7.5 :
        traders[t].respond(time, lob, trade, respond_verbose, k+3)
    else:
        traders[t].respond(time, lob, trade, respond_verbose, k)
```

In the **respond** function, something needs to be added as well. If the referenced variable k, called *new_k*, is greater than **self.k**, then *new_k* replaces **self.k**, and a new initial strategy is added to **self.strats**.

```
if new_k > self.k:
    self.k = new_k
    #add new strategy
    for s in range(new_k - self.k):
        self.strats.append({'stratval': self.mutate_strat(self.strats[0]['stratval'], 'uniform_bounded_range'), 'start_t': time,'profit': 0, 'pps': 0, 'lut_bid': None , 'lut_ask': None}))
```

Note that these changes are commented on when doing other experiments.

C. Experiments 3 and 4: Markets with different k distributions

For obtaining D), E) and F), change the k value of all the traders at the beginning.

```
#For group D)
sellers_D= [('PRDE', 30, trader_params_2)]
buyers_D = [('PRDE', 30, trader_params_1)]
traders_D = {'sellers':sellers_D, 'buyers':buyers_D}
#For group E)
sellers_E= [('PRDE', 30, trader_params_1)]
buyers_E = [('PRDE', 30, trader_params_2)]
traders_E = {'sellers':sellers_E, 'buyers':buyers_E}
#For group F)
sellers_F= [('PRDE', 15, trader_params_2), ('PRDE',
    15, trader_params_1)]
buyers_F = [('PRDE', 15, trader_params_1), ('PRDE',
    15, trader_params_2)]
traders_F = {'sellers':sellers_F, 'buyers':buyers_F}
```

D. Experiment 5: Increasing F over time

The value of F is located in **diffevol['F']** of all traders. Similar to experiment 2, increasing the parameter F of the **respond** function in the *time* loop is also necessary. Add an *if* condition that if $time >= 86400 * 7.5$ (7.5 days), $F + 2$, otherwise, it does not change. Example:

```
for t in traders:
    F_value = traders[t].diffevol['F'] #get current
    F value
    if time_order >= 86400*7.5 :
        traders[t].respond(time, lob, trade,
    respond_verbose, F_value + 2)
    else:
        traders[t].respond(time, lob, trade,
    respond_verbose, F_value)
```

In the **respond**, if the updated variable F, called *new_F*, is greater than **self.diffevol['F']**, then *new_F* will be the current F.

```
if new_F > self.diffevol['F']:
    self.diffevol['F'] = new_F
```

These changes will also be commented on when performing other experiments.

E. Experiments 6 and 7: Markets with different F distributions.

For obtaining H), I), and J), change the F value of all the traders at the beginning. Such as J)'s traders setup will be:

```
#For group J) choose params 1 and 4
sellers_J= [('PRDE', 15, trader_params_1), ('PRDE',
    15, trader_params_4)]
buyers_J = [('PRDE', 15, trader_params_4), ('PRDE',
    15, trader_params_1)]
traders_J = {'sellers':sellers_J, 'buyers':buyers_J}
```

After obtaining the data for these groups, it is necessary to smooth data with simple moving average (SMA) [3] to reduce noise. Moreover, it is significant to use the Shaprio-Wilk [4] method to determine whether these data are normally distributed and select the appropriate analysis method for Hypothesis testing.

V. Results

If these data are to be analysed accurately, their distribution needs to be known. The Shaprio-Wilk [4] method gives a p-value if $p >= 0.05$, which means that the data are approximately normally distributed. Because the length of each group was too large to calculate the p-value accurately, 3000 rows of data (smoothed) were randomly selected from each group. The following is a list of the p-value for each data set:

TABLE I
Shaprio-Wilk to get each group p-value

Group	p-value
A)	3.57E-34
B)	1.15E-23
C)	1.99E-28
D)	7.92E-30
E)	5.04E-44
F)	2.30E-31
G)	3.11E-34
H)	1.64E-12
I)	9.75E-41
J)	1.81E-37

The results show that all the data are not normally distributed, and then the Hypothesis testing can be analysed using Mann-Whitney [5] or Kruskal-Wallis [6].

The PPS data for Group A) is shown in the following graph:

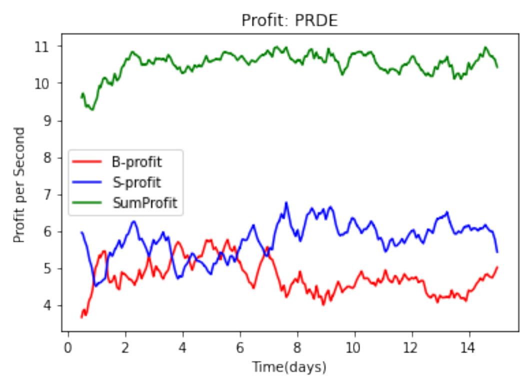

Fig. 2. Plot for Group A) of the 15-day simulated trading PPS include all PRDE traders. Horizontal axis is time, measured in days; vertical axis is simple moving average of profit per second (PPS) over the preceding 32400 seconds. B-profit is the sum of the PPS of all Buyers, S-profit is the sum of the PPS of all Sellers, Sumprofit = B-profit + S-profit.

The total PPS of A) remains around 10, with only slight fluctuations.

A)'s PPS of sellers and buyers is superimposed (i.e. B-profit and S-profit are close to each other and tend to be equilibrium) during the 0-6 day trading period, while after 6 days, it gradually stabilises as S-profit > B-profit.

A) as the IC will be compared with other groups.

A. Maintain constant k

Group B) vs A)

B)'s PPS data after 15 days of trading is shown in Fig3:

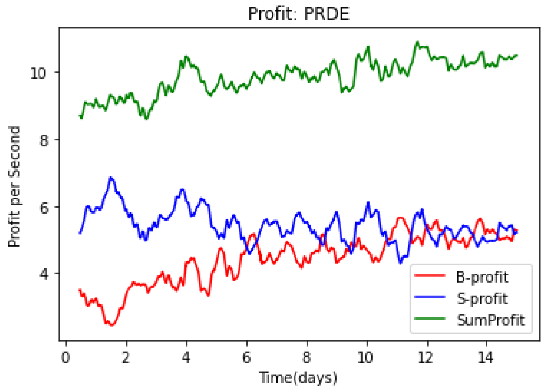

Fig. 3. Plot for Group B) of the 15-day simulated trading PPS has the same structure as fig2.

Similarities to A)

The total PPS for A) and B) remains about the same at around 10, with no significant difference.

Sellers and Buyers both partially converge to equilibrium.

S-profit $>$ B-profit is for most of the trading time.

Differences from A)

The distribution of the superposition state of B) is the opposite of A). S-profit $>$ B-profit) in days 0-6 and superposition after 6 days.

B. Increasing k over time

Group C) vs A)

Group C) for 15-day trading (Fig4):

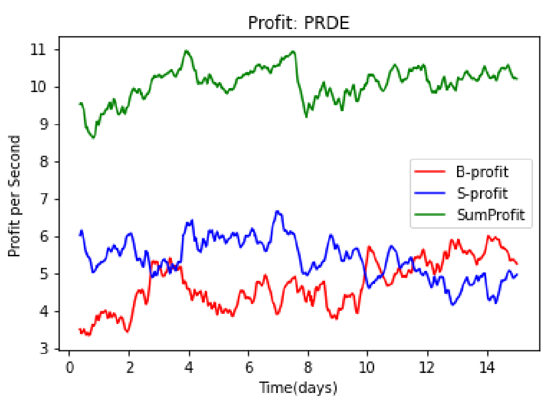

Fig. 4. Group C) for the 15-day simulated trading PPS

Similarities to A)

The total PPS for both C) and A) remains about the same at around 10, with no significant difference.

Sellers and Buyers partially converge to equilibrium.

S-profit $>$ B-profit is for most of the trading time.

Differences from A)

On day 7.5, C)'s total PPS drops, then gradually rises and plateaus.

Except for day 3, C) shows convergence towards equilibrium between days 7.5 and 15 ($k = 7$).

C. Markets with different k distribution

Group D) vs A) and E) vs A)

PPS for D) (Fig5):

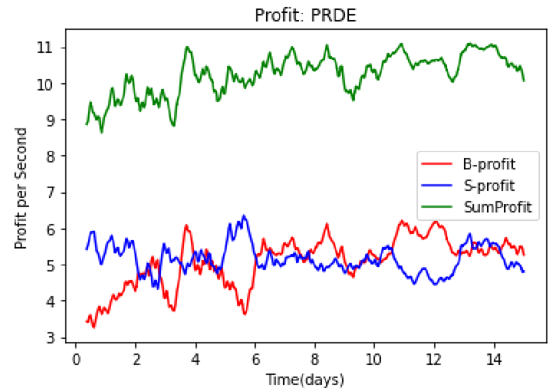

Fig. 5. Group D) for the 15-day simulated trading PPS

Similarities)

Total PPS remain around 10.

Sellers and Buyers both tend to be partially equivalent to equilibrium.

Differences)

Sellers and Buyers in D) approach equilibrium most of the time.

PPS for E) (Fig6):

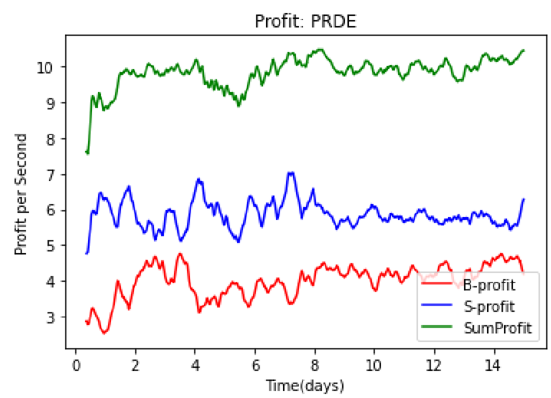

Fig. 6. Group E) for the 15-day simulated trading PPS

Similarities

The total PPS remains about the same at around 10.

Differences

The 15-day trading of E) consistently shows S-profit $>$ B-profit, and the difference is significant.

F) vs A)

PPS for F) (Fig7):

Similarities

The total PPS remains about the same at 10.

Sellers and Buyers both trend partially towards equilibrium.

Differences

B-profit $>$ S-profit is for most of the trading time, A) is the opposite.

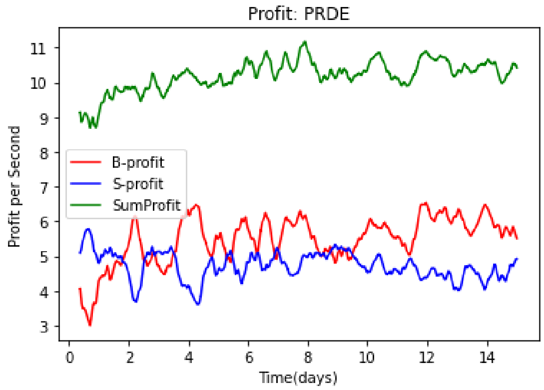

Fig. 7. Group F) for the 15-day simulated trading PPS

D. Increasing F over time

G) vs A)

G) for the 15-day trading PPS (Fig8):

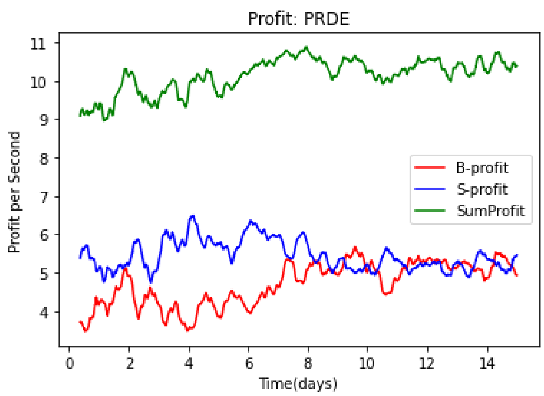

Fig. 8. Group G) for the 15-day simulated trading PPS

Similarities

Total PPS remain around 10.

Sellers and Buyers both converge partially to equilibrium.

Most of the time is S-profit > B-profit, except for the superposition state.

Differences

After 7.5 days (from $F = 0$ to after $F = 2$), there is a convergence to equilibrium.

E. Markets with different F distributions

H) vs A) and I) vs A)

PPS for H) (Fig9):

Similarities

Total PPS remain around 10 . Sellers and Buyers both approach partial equilibrium.

Most of the time is S-profit > B-profit.

Differences

H) is close to equilibrium for a small proportion of the trading time.

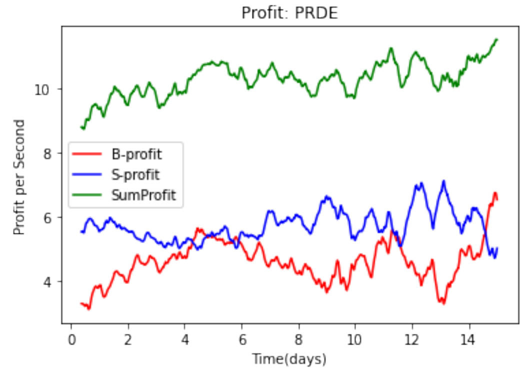

Fig. 9. Group H) for the 15-day simulated trading PPS

PPS for I) (Fig10):

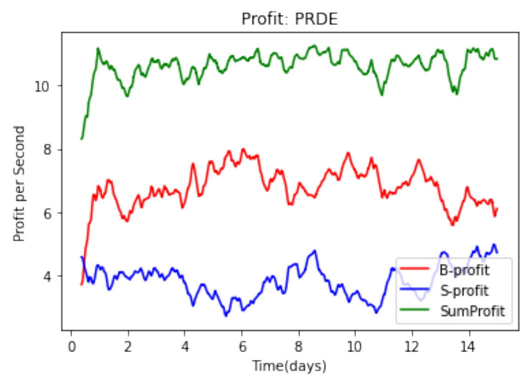

Fig. 10. Group I) for the 15-day simulated trading PPS

Similarities

Total PPS remains about at 10.

Differences

I) has been B-profit > S-profit, except for the initial adaptation phase, and has not converged to equilibrium.

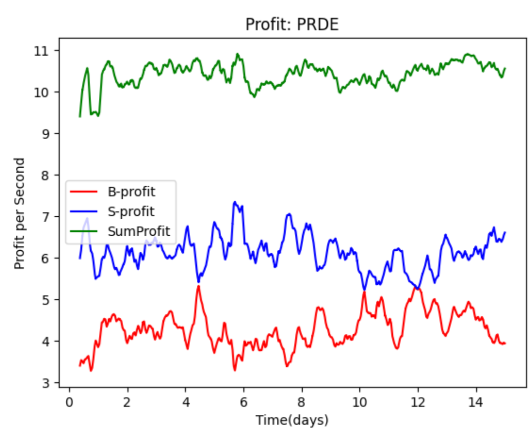

Fig. 11. Group J) for the 15-day simulated trading PPS

J) vs A)

PPS for J) (Fig11 shows above):

Similarities

Total PPS remains at around 10.

Differences

J) is always S-profit > B-profit and does not converge to equilibrium.

F. Statistical Difference in Total PPS

The total PPS for each group in the above graphs is not yet significantly different, so it is necessary to test Mann-Whitney [5] against A) separately to determine whether there is a significant difference between them. If p-value $>= 0.05$, then there is no significant difference between each group (i.e. different strategies) and A), and vice versa. The following data table is available:

TABLE II
EACH GROUP MANN-WHITNEY TEST WITH A)

Group	p-value of Mann-Whitney with A)
B)	5.15E-38
C)	5.2E-178
D)	2.9E-287
E)	8.6E-43
F)	6.7E-289
G)	1.4E-234
H)	0.00
I)	0.00
J)	3.59E-66

The result gives a p-value < 0.05 for all data. Indicating that A) differs from B) to J), i.e. changing k and F values affects PPS.

Mann-Whitney [5] only used A) with each data group and did not yet perform a multiple-group analysis. Kruskal-Wallis [6]b can be used for multiple groups by placing A) to J) in the **stats.kruskal** function. Finally, obtaining p-value = 0.0 shows a difference in the data again.

VI. DISCUSSION

With the results above, the figures and p-value for each group demonstrate that changing k and F makes a difference to PPS, but the change in total PPS is not very significant. Increasing k or F did not significantly increase PPS, so the hypothesis previously made is not valid.

TABLE III
BUYERS' AND SELLER'S PPS INFO FOR EACH GROUP

Group	Sellers Mean PPS	Buyers Mean PPS	Main Trend
A)	5.175462	4.406035	equilibrium (eq) to separation(spr)
B)	5.4192	4.419223	spr to eq
C)	5.372086	4.687787	spr to eq
D)	*5.15641*	*5.08677*	main eq
E)	5.853033	3.991782	main spr
F)	**4.714169**	**5.48574**	eq to spr
G)	5.441559	4.698158	spr to eq
H)	5.719337	4.5766	main spr
I)	**3.831057**	**6.790324**	main spr
J)	5.411586	4.49639	main spr

In addition, the variation in the data for Sellers and Buyers for each group was puzzling, so the average Sellers' and Buyers' PPS for each group is calculated and grouped into the table3.

The above table shows that in the mean PPS, F) and I) are Buyers > Sellers (**bold**), and the other groups are Sellers > Buyers. The mean PPS of D) is the closest to each other (*italic*). However, there is no potential pattern in the Main Trend.

A. Comparison with A)

Several phenomena can be identified through the similarities and differences with A):

When F is constant and k varies

- B) changes only constant k based on A), and in making $k = 7$, the trend of PPS for sellers and buyers is the opposite of A). B) gradually converges to equilibrium.
  So a larger k may make the PPS converge more to equilibrium.
- C) is the same as A) for days 0-7.5. However, at the start of day 7.5 (when k increases to 7), total PPS first plummets, then returns to its stable state. It is possible that when adding k to traders (adding strategies), the DE in PRDE has to re-evaluate each new strategy with increased evaluation range and time, and the profit and PPS of the new strategy are 0.
  Therefore, when k is added during a trade, it may cause the PPS of traders to plummet and then gradually rise and stabilise. The PPS trend for Sellers and Buyers also converges to equilibrium at 7.5-15 days, the same as the finding in B).
- D) changes only Sellers' k based on A), and D) converges to equilibrium most of the time. In table 3, the mean PPS of D) is closest to each other (Sellers profit decreases and Buyers profit increases based on A)).
  Therefore, increasing Sellers' k may reduce Sellers' profit but increase Buyers' to reach equilibrium.
- E) is the exact opposite of D), changing only the k of Buyers based on A). E) has always been a state of separation, with the Sellers' profit increasing and the Buyers' profit decreasing.
  So, increasing Buyers' k may increase Sellers' profit, but it will decrease Buyers', avoiding equilibrium.
- F) makes more changes from A). It allocates Sellers and Buyers with different k. It results in F) having B-profit > S-profit for most of the trading time, in contrast to A). The Mean PPS also illustrates this in Table 3. Therefore, it is possible to compare the results of C), D) and E). When more strategic Sellers trade with fewer strategic Buyers, their impact is more significant than fewer strategic Sellers trade with more strategic Buyers. In conclusion, the impact of increasing Sellers' k may be more significant than the impact of increasing Buyers' k.

When k is constant and F varies

- G) is similar to C) in design direction. Comparing A) with $F = 0.8$, the PPS of sellers and buyers is much more dispersed in days 0-7.5 ($F = 0$) than in the first 7.5 days of A), and after 7.5 days ($F = 2$), the PPS of Sellers and Buyers converges to equilibrium.

So, increasing F may make the PPS more convergent to equilibrium and vice versa, the smaller the F, the greater the dispersion.

- H) changing only $F = 2$ for Sellers from A) results in a PPS that rarely converges to equilibrium. Because Buyers' F is equal to $F = 0.8$ for A), Buyers' mean PPS does not change much, but Sellers' mean profit increases significantly.
  It illustrates that a larger F may generate more PPS and, vice versa, a smaller profit.
- I) is parameterised by $F = 0.8$ for Sellers and $F = 2$ for Buyers, resulting in B-profit $>$ S-profit all the time, but Buyers in H) do not change as much as Sellers in I) when $F = 0.8$. Moreover, when $F = 2$, H) Sellers do not change as much as I) Buyers.
  Thus, when F increases, it may affect Buyers more than Sellers, and conversely, it may affect Sellers more than Buyers.
- The result is that the PPS is always S-profit $>$ B-profit and does not converge to equilibrium. This result is inconsistent with the conclusion of I), so the hypothesis of I) is excluded, and no other favourable information is found.

B. Problems with the Experiment

Too many parameter changes

Many of the experimental groups had too many parameter changes to allow accurate analysis of the variation between them; thus, conclusions could not be drawn. Examples include I) and J).

Too many experimental groups

Too many experimental groups will result in a large amount of runtime consumed to calculate the data and thus not much effort spent analysing the data. (Calculated as an average of 5 hours per experimental group)

Conclusions cannot be confirmed

For the Sellers and Buyers PPS distributions, due to the poor performance of the equipment, each experiment was not validated multiple times, and each simulated trade with the same experimental group may produce different results and is random. Therefore, the analysis for Sellers and Buyers can only be treated as hypotheses.

Transaction time limits

The PRDE has an evaluation period of $7200k$ seconds, and then the elite and mutants are selected and ready to move on to the next iteration of the evaluation cycle, which takes much time. However, simulated trading of just 15 days may not yield an ideal elite, so the results are not the most reasonable.

C. Other Findings

The conclusions of Sellers' and Buyers' PPS could not be proven at this time, so going in search of some other generated data to use in the analysis.

Some of the data sets had complex changes in parameters that did not allow differences to be found visually, so A) and B) were chosen to observe k, and H) and I) to observe F.

After 15 days of trading, the total profit of all traders in A) versus B):

TABLE IV
PROFIT FOR GROUP A) AND B)

Group	*k value*	*Total Profit*	*Mean Profit*
A)	4	12439292	207321.5
B)	7	**12755007**	**212583.5**

It demonstrates a significant increase in total profit for B). Therefore, an increase in the k value may lead to more profit.

Another data set, H) versus I) for all the trades that took place:

TABLE V
TRADES THAT TOOK PLACE FOR H) AND I)

Group	*Sellers F value*	*Buyers F value*	*mean price for all trades*
H)	2	0.8	100.779
I)	0.8	**2**	**93.7127**
A)	0.8	0.8	96.737

It follows that F may affect the overall transaction amount, increasing the amount in favour of Sellers when Sellers $F >$ Buyers F and decreasing the amount in favour of Buyers when Sellers $F <$ Buyers F.

VII. CONCLUSION

As a result of this experiment, it is known that changes in k and F make a difference to total PPS, but there is no significant change. Increasing k or F does not increase PPS.

By comparing other data, it is known that an increase in k may result in more profit. In addition, a change in F may affect the transaction amount.

The current experiments do not provide enough evidence to support the assumptions used in Sellers' and Buyers' PPS analyses, such as the notion that the greater the k, the closer their PPS is to equilibrium. They will need to be addressed in future experiments, provided that the experimental methods are refined by reducing the number of experimental groups and obtaining more diverse data for analysis.

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