Regarding (0.1) as made up of the two conditions

(0.2) $F(x, r, pX) \leq F(x, s, p, X)$ whenever $r \leq s$

and (0.3) $F(x, r, pX) \leq F(x, r, p, Y)$ whenever $Y \leq X$

we will give the name "degenerate ellipticity " to the second. That is, F is said to be degenerate elliptic if (0.3) holds. When (0.2) also holds (equivalently, (0.1) holds), we will say that F is proper.

The examples given in 81 will illustrate the fact that the antimonotonicity in X is indeed an "ellipticity" condition. The possibility of "degeneracies" is clearly exhibited by considering the case in which $F(x, r, p, X)$ does not depend on $X -$ it is then degenerate elliptic. The monotonicity in r, while easier to understand, is a slightly subtle selection criterion that, in particular, excludes the use of the viscosity theory for first order equations of the form $b(u)u_x = f(x)$ in $\mathbb{R}$ when b is not a constant function, since then $F(x, r, p) = b(r)p - f(x)$ is not nondecreasing in r for all choices of p (scalar conservation laws are outside of the scope of this theory).

The presentation begins with $1, which, as already mentioned, provides a list of examples. This rather long list is offered to meet several objectives. First, we seek to bring the reader to our conviction that the scope of the theory is quite broad while providing a spectrum of meaningful applications and, at the same time, generating some insight as regards the fundamental structural assumption (0.1). Finally, in the presentation of examples involving famous second order equations, the very act of writing the equations in a form compatible with the theory will induce an interesting modification of the classical viewpoint concerning them. In $2 we begin an introductory presentation of the basic facts of the theory. The style is initially leisurely and expository and technicalities are minimized, although complete discussions of various key points are given and some simple arguments inconveniently scattered in the literature are presented. Results are illustrated with simple examples making clear their general nature. Section 2 presents the basic notions of solution used in the theory, the analytical heart of which lies in comparison results. Accordingly, $3 is devoted to explaining comparison results in the simple setting of the Dirichlet problem; roughly speaking, they are proved by methods involving extensions of the maximum principle to semicontinuous functions. Once these comparison results are established, existence assertions can be established by Perron's method, a rather striking tale that is told in $4. With this background in hand, the reader will have an almost complete (sub) story and with some effort (but not too much!) should be able to absorb in an efficient way some of the more technical features of the theory that are outlined in the rest of the paper.

Other important ideas are to be found in $6, which is concerned with the issue of taking limits of viscosity solutions and applications of this and in $7 which describes the adaptation of the theory to accommodate problems with other boundary conditions and problems in which the boundary condition cannot be strictly satisfied. In the later case, the entire problem has a generalized interpretation for which there is often existence and uniqueness. While the description of these results is deferred to $7, they are fundamental and dramatic.