1. For each of the trees in the following Figure 1 and Figure 2, please tell whether it is an AVL tree, and if not, indicate which nodes are unbalanced. **(10 points)**

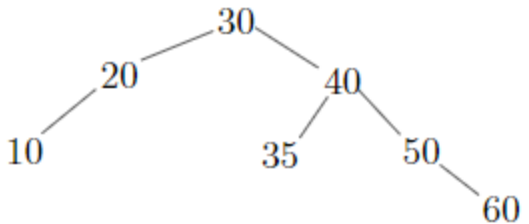


Figure 1:

Balanced.

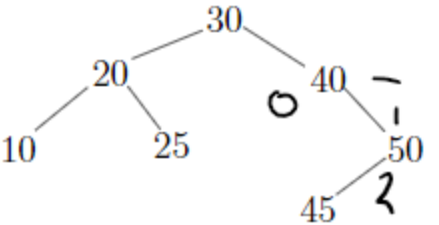


Figure 2:

Not Balanced

40 is unbalanced

(a) Consider the AVL tree in Figure 3. Suppose we want to insert 28. Draw the new tree after the insertion.

(b) Consider the AVL tree in Figure 4. Suppose we want to insert 55. Draw the new tree after the insertion.

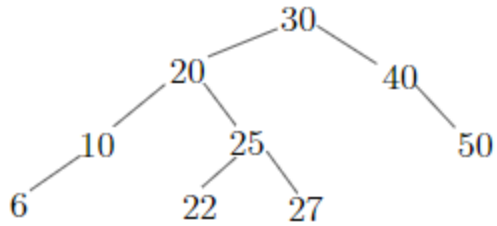


Figure 3:

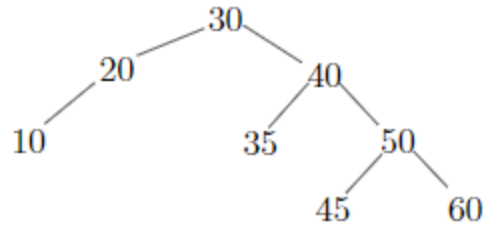
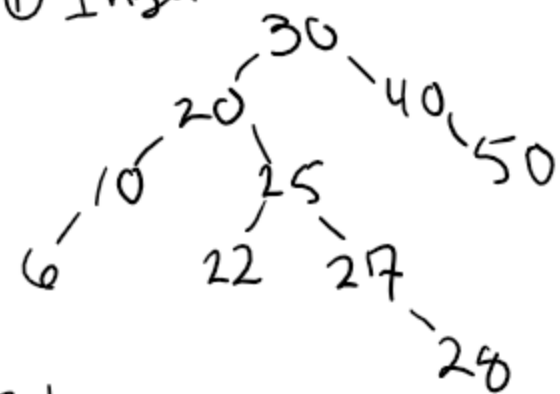


Figure 4:

Figure 3:

① Insert

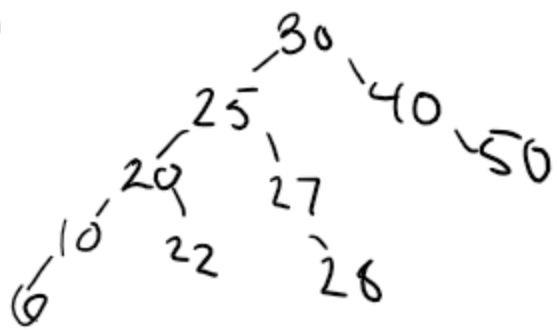


② Check balance.

Not-Balanced.

Balance

1)



→

2)

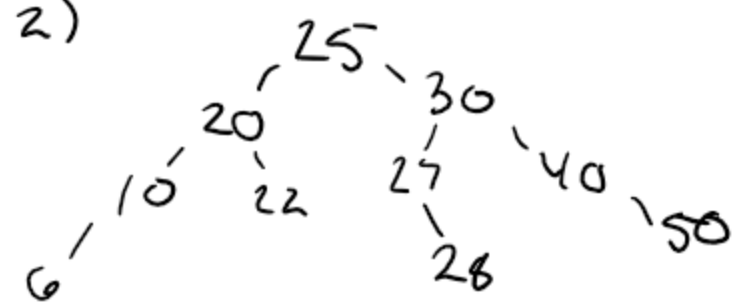
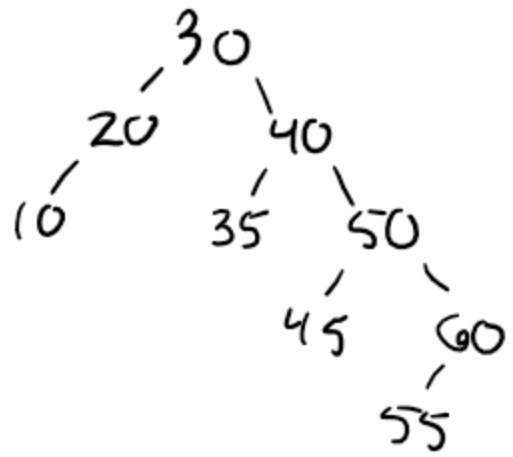


Figure 4:

① Insert

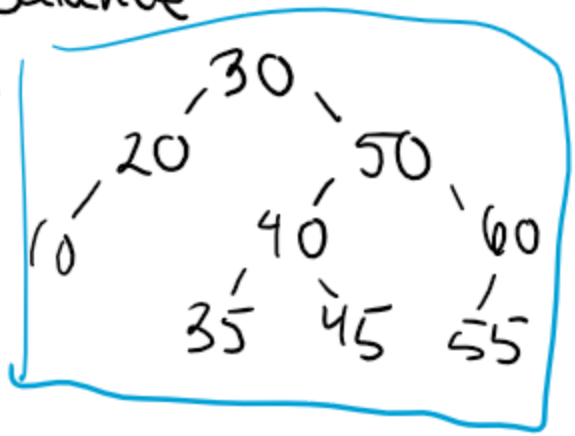


② Check balance.

Not balanced

Balance

1)



3. **(15 points)**

(a) Consider the AVL tree in Figure 5. Suppose we want to remove 30 (use the smallest key in the right subtree of 30 to replace it). Draw the new tree after the removal.

(b) Consider the AVL tree in Figure 6. Suppose we want to remove 20. Draw the new tree after the removal.

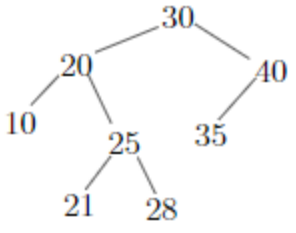


Figure 5:

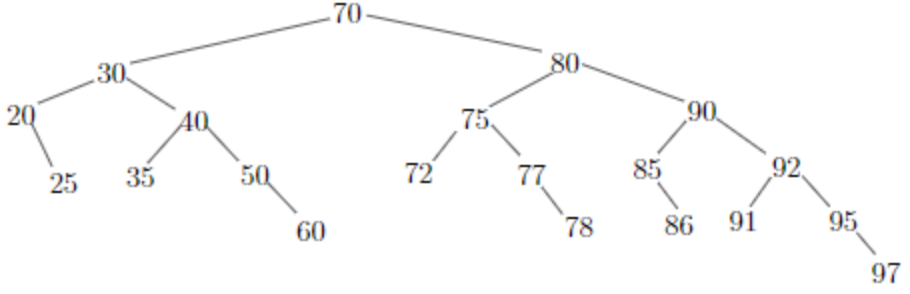
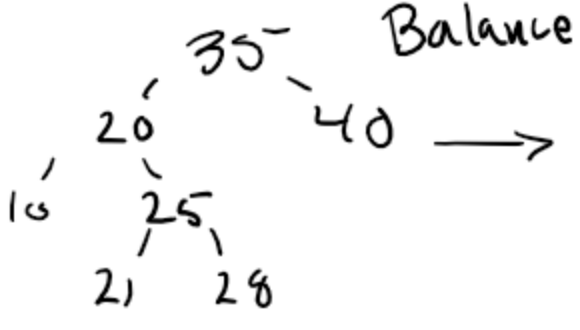


Figure 6:

Figure 5:

① remove



②

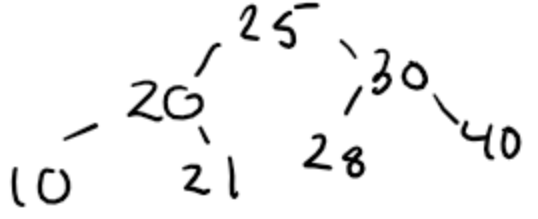
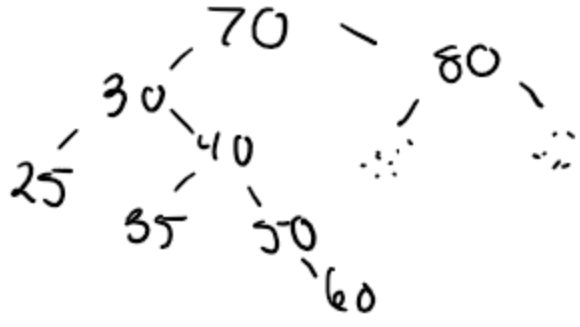


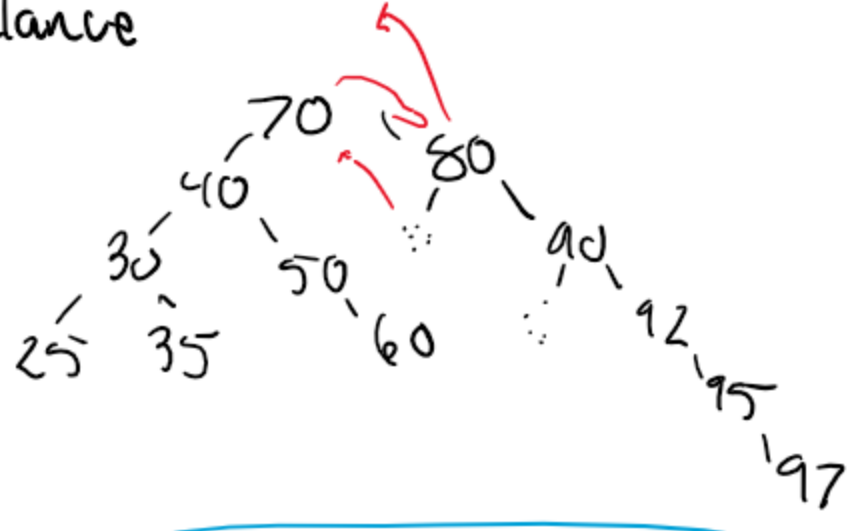
Figure 6:

① remove



② Balance

1)



2)