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PHYSICS 3081, 4051

MEASUREMENT OF THE CHARGE OF THE ELECTRON

References for Millikan Oil Drop Experiment

1. Harnwell and Livingood. Experimental Atomic Physics, (1933), p. 91 and pp. 98–101.
2. **Hong and Korff, Electron and Nuclear Physics (1948), pp. 1–6.
3. *Millikan, The Electron (1917), especially Chapter 5.
4. Semat, Introduction to Atomic and Nuclear Physics, 4th Ed., 1962; pp. 61–65.
5. *Wohr and Richards, Physics of the Atom, (1960), pp. 37–41.
6. Hoag, pp. 1–6, Identical with Ref. 2.

Method

In the Millikan apparatus, oil spray from an atomizer passes through a small hole into the space between two plane parallel metal condenser plates, P_1 and P_2.

The droplets of oil are so small that the force due to gravity–and the electrical force due to a few hundred volts between the plates–are of the same order of magnitude. They move downward under gravity and may be caused to reverse their direction by a sufficiently large electrical field.

Because the droplets are moving through a viscous medium (air), they move with constant velocity rather than constant acceleration. (See theory section below.) The drops are visible by scattered light from a strong source S focused at the center of the condenser as shown in figure 1. They are seen using a long focus microscope and a video camera. Line drawn on the surface of the monitor screen are used as a distance scale. They may be calibrated by focusing on a standard ruler. Then when the microscope has been focused on a droplet as carefully as possible, its actual motion from position d_1 to d_2 between the plates can be obtained from the scale readings. Thus if the droplet is timed with a stop watch, and it takes t_g seconds to fall from d_1 to d_2 under gravity,

$$v_g = \frac{d_2 - d_1}{t_g} = \frac{L}{t_g}, \quad L = d_2 - d_1$$

With an appropriate electric field E, the droplet drifts upward, with velocity V_E. The temperature, voltage, t_g and t_E are the data that must be obtained for the calculation of e. A thermometer is provided for measuring the temperature.

Procedure

The preliminary adjustments such as focusing the microscope and lining up the light source will be clear when the apparatus is seen. The instructor will show how to open up the plates and will give necessary directions and precautions. Students are requested not to try to make adjustments until the instructor has been consulted.

The velocity of a drop will fluctuate statistically due in part to impacts with air molecules. To obtain values of time worth using, several observations of time under gravity alone, and with gravity plus electric field, are necessary, the more the better to make the average significant. If a droplet changes speed suddenly under the field, try to keep from losing it to a condenser plate. It will be equivalent to two drops for which e can be calculated. It is interesting to try balancing the drops, and takes less time. For a balanced drop, give some thought to the length of time needed to prove that the drop was balanced.

Drops that pass over three division on the screen ($\approx 3/64$ inch) at some medium speed say 30 seconds will give the best values for e. Very slow drops are apt to be erratic and, in addition, do not satisfy the assumptions used in theory as well as faster drops. A very fast drop, on the other hand, cannot be timed with as good percentage precision. Furthermore, a fast drop is a large one, and is apt to carry so many charges that the number cannot be found for certain. These comments do not mean that only drops taking 30 seconds are to be considered. It is only that these facts about the speed should be kept in mind.

Many droplets will be in the field just after the atomizer is used. A particular drop may be isolated by use of the electric field to keep it moving up and down between the plates. Drops not in synchronization with the field changes will disappear into the plates. It is necessary to observe the drop for several traversals and measure the corresponding time intervals.

Theory of the Millikan Oil Drop Experiment

The small droplet moving through the air is acted upon by a retarding force due to the viscosity of the air (references 3 and 5). If the air can be considered homogeneous, that is, if the radius of the droplet is larger than the irregularities in the air, this retarding force is proportional to the velocity of the drop and can be calculated. When a droplet first starts to move in the viscous medium, under any force, say that of gravity, it picks up speed. As the speed increases, the retarding force increases in proportion and soon balances the force causing the motion. With zero force on it, the droplet is not accelerated and from then on moves with a constant velocity, the so called terminal velocity. Such motion is well known in fluid flow. Stokes derived an expression, called Stokes' Law, for the retarding force, F_r,

$$F_r = kv \text{ where } k = 6\pi\eta a_1 \tag{1.1}$$

η is the viscosity of the medium, and a_1 is the radius of the drop.

Droplet falling under gravity

For a falling drop the forces are

1. The weight, $W = mg$
2. the buoyant force, W_b, equal to the weight of air displaced
3. the retarding force, kv_g, due to the viscosity of the air, as given in equation (1.1). At terminal velocity these forces have zero resultant, figure 2.

$$W - W_b - kv_g = 0 \tag{1.2}$$

W and W_b are given by:

$$W = mg = \frac{4}{3}\pi a_1^3 \rho g \tag{1.3}$$

$$W_b = m_a g = \frac{4}{3}\pi a_1^3 \rho_a g \tag{1.4}$$

where ρ is the density of the oil and ρ_a that of air. Substituting equations (1.1), (1.3) and (1.4) into (1.2) we can solve for the radius of the droplet a_1:

$$a_1 = \sqrt{\frac{9\eta v_g}{2(\rho - \rho_a)g}} \tag{1.5}$$

Thus, measuring the free fall velocity of the droplet allows the determination of its size. We also have

$$k = 18\pi\sqrt{\frac{\eta^3 v_g}{2(\rho - \rho_a)g}} \tag{1.6}$$

Droplet rising under an electric field plus gravity

1. kv_E is the Stokes force now in the opposite direction because the velocity has reversed
2. neE is the force in the drop due to its total charge of n electronic charges. E, the electric field, is V/D where D is the distance between the plates.

Since there is no net force on the drop we have, figure 3, $W - W_b = neE - kv_E$ and, using equation (1.2):

$$kv_g = neE - kv_E \quad \text{or} \quad ne = k\frac{v_g + v_E}{E} \tag{1.7}$$

Finally, substituting the value of k from equation (1.6), we have an expression giving ne in terms of the velocity of fall under gravity, the velocity under an electric field, the voltage applied, and various constants of the apparatus.

$$ne = \frac{18\pi D}{V}(v_E + v_g)\sqrt{\frac{\eta^3 v_g}{2(\rho - \rho_a)g}} \tag{1.8}$$

This formula does not however give the correct value of e. Millikan found that values of e calculated from equation (1.8) grew progressively larger the slower (smaller) the drop. This he showed to be due to the failure of Stokes' law for droplets whose radii were no longer large compared with the distances between the air molecules. In these empty spaces the drop falls free, resulting in a larger terminal velocity v_g than that given by equation (1.2). and leading in Equation (1.8) thus values of ne which are too large.

By analysis of his data and by reasonable assumptions, he was able to devise a corrective term by which the value of v_g used in calculating the radius would be less than that given by equation (1.5). How this was done is described in "The Electron", p 98, ff, resulting in:

$$a^2 = \frac{9\eta v_g}{2(\rho - \rho_a)g} \times \frac{1}{1 + b/pa} \tag{1.9}$$

Here p is the air pressure in cm of mercury, and b is a constant. Note that the unknown value of a appears in the correction. This is handled by using first the value of a_1, calculated from by equation (1.5), (the uncorrected form of Stokes' Law) to find a better value a_2.

Then a_2 is put into the correction term, and a still better value, a_3 obtained. This is called the method of successive approximations. It turns out that two approximations are enough, and the correction factor in terms of a_1, is

$$k_1 = 1 + \frac{b}{pa_1} + \frac{1}{2}\Big(\frac{b}{pa_1}\Big)^2 \tag{1.10}$$

Millikan states that v_g comes into ne as $v_g^{3/2}$, and uses the 3/2 power of equation (1.10) as the final correction factor. This is not quite right, but it is always used. We have then,

$$ne = \frac{18\pi D(v_E + v_g)}{V\Big[1 + \frac{b}{pa_1} + \frac{1}{2}\big(\frac{b}{pa_1}\big)^2\Big]^{3/2}}\sqrt{\frac{\eta^3 v_g}{2(\rho - \rho_a)g}} \tag{1.11}$$

$K_1 = k_1^{3/2}$ has been computed for p = normal pressure (good enough in a correction term), and for t_g in intervals of 5 seconds. This is given below in a table of K_1 against v_g. Defining:

$$K_2 = 18\pi D\sqrt{\frac{\eta^3}{2(\rho - \rho_a)g}} \tag{1.12}$$

we may write

$$ne = \frac{K_2(v_E + v_g)\sqrt{v_g}}{VK_1} \tag{1.13}$$

Numerical Constants

1. $g = 980.66\ \mathrm{cm\cdot s^{-2}}$
2. The oil is Apiezon E (Shell Oil Co.), with density $\rho = 0.8724\ \mathrm{g\cdot cm^{-3}}$ at 10 °C and $0.8606\ \mathrm{g\cdot cm^{-3}}$ at 30°C. The density is linear with the temperature.
3. $L \approx 3/64$ inch for 3 divisions on screen-calibration required.
4. $d = 4.97 \pm 0.02$ mm.
5. $\eta = (1.81920\pm0.00006)\times10^{-5}\ \mathrm{kg\cdot m^{-1}\cdot s^{-1}}$ at 20 °C. η has a temperature coefficient of 0.0494×10^{-6}/°C.
6. $\rho_a = .001205\mathrm{gm/cm^3}$ at 20°C, 1 Atm.
7. $b = .000617$ with p in cm of Hg (76 = 1 Atm) and a in cm

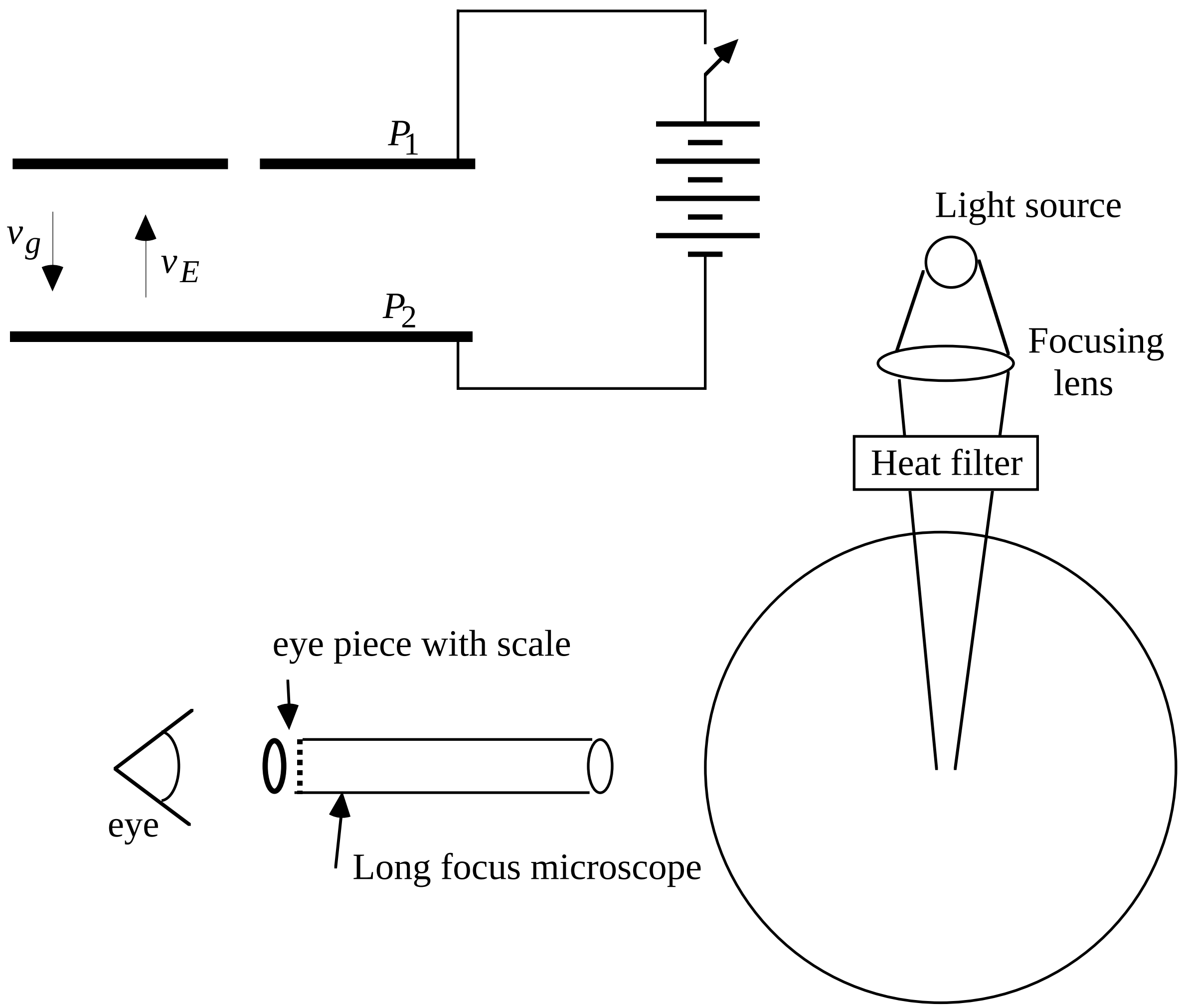
A check of your calculation

For V=300 volts, t_g=43.41 seconds, t_e=26.46 seconds T=20°C, L=3/64 inch, the droplet had very close to a single electron charge ($ne \approx 1.03$).

The following table is for L=2.559±.0003 mm. Interpolate for actual value of L used for measurement.

Table 1. Value of K_1

t_g (s)	K_1	t_g (s)	K_1
5	1.058	65	1.221
10	1.082	70	1.230
15	1.102	75	1.239
20	1.118	80	1.249
25	1.132	85	1.258
30	1.146	90	1.267
35	1.158	95	1.275
40	1.170	100	1.283
45	1.181	105	1.291
50	1.192	110	1.298
55	1.202	115	1.305
60	1.212	120	1.312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v_g
v_E
P_2
Light source
Focusing lens
Heat filter
eye piece with scale
eye
Long focus microscope

Figure 1

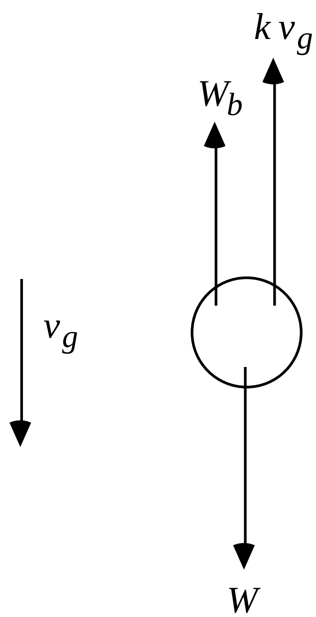

Figure 2

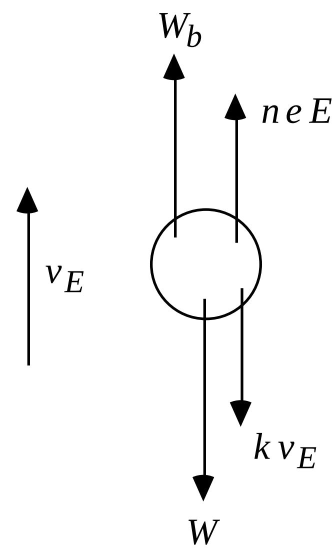

Figure 3