Camera Calibration and 3D Reconstruction
摄像机标定和3D重建

Modules 模块

Fisheye camera model 鱼眼相机模型

Detailed Description 详细描述

The functions in this section use a so-called pinhole camera model. The view of a scene is obtained by projecting a scene's 3D point P_w into the image plane using a perspective transformation which forms the corresponding pixel p. Both P_w and p are represented in homogeneous coordinates, i.e. as 3D and 2D homogeneous vector respectively. You will find a brief introduction to projective geometry, homogeneous vectors and homogeneous transformations at the end of this section's introduction. For more succinct notation, we often drop the 'homogeneous' and say vector instead of homogeneous vector.

本节中的函数使用所谓的针孔相机模型。通过使用形成对应像素 p 的透视变换将场景的3D点 P_w 投影到图像平面中来获得场景的视图。 P_w 和 p 都以齐次坐标表示，即分别表示为3D和2D齐次向量。在这一节的介绍的最后，你会发现一个关于射影几何、齐次向量和齐次变换的简要介绍。为了更简洁的表示法，我们经常去掉“齐次”，而用向量代替齐次向量。

The distortion-free projective transformation given by a pinhole camera model is shown below.

针孔相机模型给出的无失真投影变换如下所示。

$$s\ p = A[R|t]P_w,$$

where P_w is a 3D point expressed with respect to the world coordinate system, p is a 2D pixel in the image plane, A is the camera intrinsic matrix, R and t are the rotation and translation that describe the change of coordinates from world to camera coordinate systems (or camera frame) and s is the projective transformation's arbitrary scaling and not part of the camera model.

其中 P_w 是相对于世界坐标系表示的3D点， p 是图像平面中的2D像素， A 是相机固有矩阵， R 和 t 是描述坐标从世界坐标系到相机坐标系的变化的旋转和平移 s 是投影变换的任意缩放，而不是相机模型的一部分。

The camera intrinsic matrix A (notation used as in [314] and also generally notated as K) projects 3D points given in the camera coordinate system to 2D pixel coordinates, i.e.

摄像机固有矩阵 A （如[314]中所使用的符号，并且通常也表示为 K ）将摄像机坐标系中给出的3D点投影到2D像素坐标，即，

$$p = AP_c.$$

The camera intrinsic matrix A is composed of the focal lengths f_x and f_y, which are expressed in pixel units, and the principal point (c_x, c_y), that is usually close to the image center:

照相机固有矩阵 A 由以像素为单位表示的焦距 f_x 和 f_y 以及通常靠近图像中心的主点 (c_x, c_y) 组成:

$$A = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix},$$

and thus 并且因此

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}.$$

The matrix of intrinsic parameters does not depend on the scene viewed. So, once estimated, it can be re-used as long as the focal length is fixed (in case of a zoom lens). Thus, if an image from the camera is scaled by a factor, all of these parameters need to be scaled (multiplied/divided, respectively) by the same factor.

内参数矩阵不依赖于所观察的场景。因此，一旦估计，只要焦距固定，就可以重新使用它（在变焦透镜的情况下）。因此，如果来自相机的图像按因子缩放，则所有这些参数都需要按相同因子缩放（分别乘以/除以）。

The joint rotation-translation matrix $[R|t]$ is the matrix product of a projective transformation and a homogeneous transformation. The 3-by-4 projective transformation maps 3D points represented in camera coordinates to 2D points in the image plane and represented in normalized camera coordinates $x' = X_c/Z_c$ and $y' = Y_c/Z_c$:

联合旋转-平移矩阵 $[R|t]$ 是投影变换和齐次变换的矩阵积。3 × 4投影变换将以照相机坐标表示的3D点映射到图像平面中的以标准化照相机坐标 $x' = X_c/Z_c$ 和 $y' = Y_c/Z_c$ 表示的2D点:

$$Z_c \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}.$$

The homogeneous transformation is encoded by the extrinsic parameters R and t and represents the change of basis from world coordinate system w to the camera coordinate sytem c. Thus, given the representation of the point P in world coordinates, P_w, we obtain P's representation in the camera coordinate system, P_c, by

齐次变换由外部参数 R 和 t 编码，并且表示从世界坐标系 w 到相机坐标系 c 的基的变化。因此，给定点 P 在世界坐标系 P_w 中的表示，我们通过下式获得 P 在相机坐标系 P_c 中的表示：

$$P_c = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} P_w,$$

This homogeneous transformation is composed out of R, a 3-by-3 rotation matrix, and t, a 3-by-1 translation vector:

该齐次变换由3 × 3旋转矩阵 R 和3 × 1平移向量 t 组成：

$$\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and therefore 并且因此

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}.$$

Combining the projective transformation and the homogeneous transformation, we obtain the projective transformation that maps 3D points in world coordinates into 2D points in the image plane and in normalized camera coordinates:

结合投影变换和齐次变换，我们获得将世界坐标中的3D点映射到图像平面中和归一化相机坐标中的2D点的投影变换：

$$Z_c \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = [R|t] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix},$$

with $x' = X_c/Z_c$ and $y' = Y_c/Z_c$. Putting the equations for instrincs and extrinsics together, we can write out $s\ p = A[R|t]P_w$ as

$x' = X_c/Z_c$ 和 $y' = Y_c/Z_c$ 。把双反函数和外反函数的方程放在一起，我们可以把 $s\ p = A[R|t]P_w$ 写成：

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}.$$

If $Z_c \neq 0$, the transformation above is equivalent to the following,

如果 $Z_c \neq 0$ ，则上面的变换等价于下面的变换，

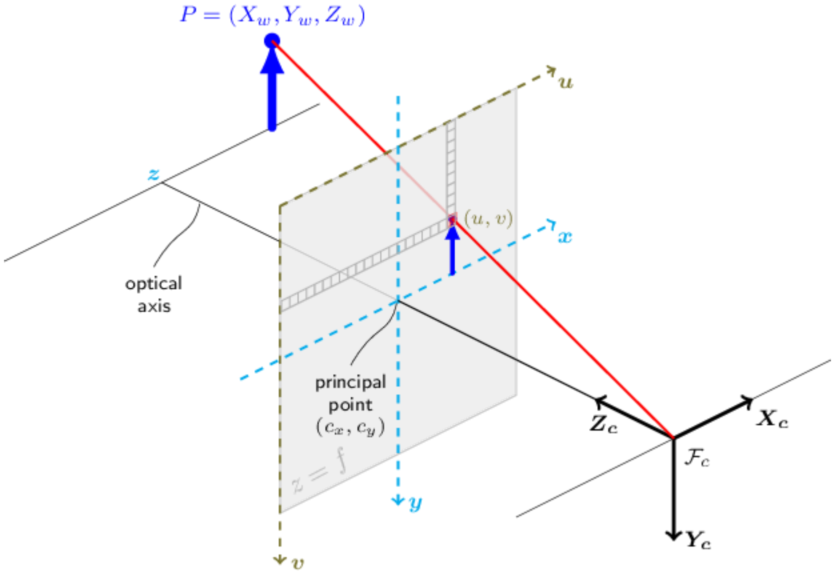
$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f_x X_c/Z_c + c_x \\ f_y Y_c/Z_c + c_y \end{bmatrix}$$

with 与

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = [R|t] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}.$$

The following figure illustrates the pinhole camera model.

下图说明了针孔相机模型。



Pinhole camera model 针孔相机模型

Real lenses usually have some distortion, mostly radial distortion, and slight tangential distortion. So, the above model is extended as:

真实的透镜通常有一些畸变，主要是径向畸变，和轻微的切向畸变。因此，上述模型扩展为：

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f_x x'' + c_x \\ f_y y'' + c_y \end{bmatrix}$$

where 哪里

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x' \frac{1+k_1 r^2 + k_2 r^4 + k_3 r^6}{1 + k_4 r^2 + k_5 r^4 + k_6 r^6} + 2 p_1 x' y' + p_2(r^2 + 2 x'^2) + s_1 r^2 + s_2 r^4 \\ y' \frac{1+k_1 r^2 + k_2 r^4 + k_3 r^6}{1 + k_4 r^2 + k_5 r^4 + k_6 r^6} + p_1 (r^2 + 2 y'^2) + 2 p_2 x' y' + s_3 r^2 + s_4 r^4 \end{bmatrix}$$

with 与

$$r^2 = x'^2 + y'^2$$

and

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} X_c/Z_c \\ Y_c/Z_c \end{bmatrix},$$

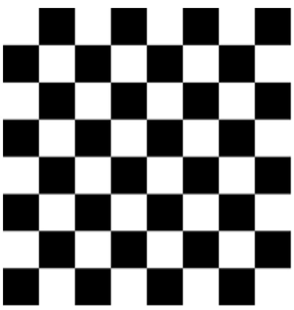
if $Z_c \ne 0$. 如果 $Z_c \ne 0$ 。

The distortion parameters are the radial coefficients k_1, k_2, k_3, k_4, k_5, and k_6 , p_1 and p_2 are the tangential distortion coefficients, and s_1, s_2, s_3, and s_4, are the thin prism distortion coefficients. Higher-order coefficients are not considered in OpenCV.

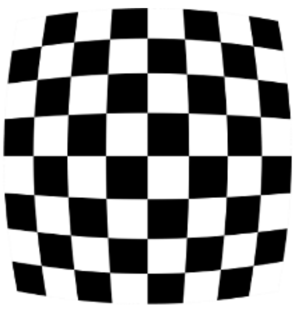
畸变参数是径向系数 k_1 、 k_2 、 k_3 、 k_4 、 k_5 和 k_6 ， p_1 和 p_2 是切向畸变系数， s_1 、 s_2 、 s_3 和 s_4 是薄棱镜畸变系数。OpenCV中不考虑高阶系数。

The next figures show two common types of radial distortion: barrel distortion ($1 + k_1 r^2 + k_2 r^4 + k_3 r^6$ monotonically decreasing) and pincushion distortion ($1 + k_1 r^2 + k_2 r^4 + k_3 r^6$ monotonically increasing). Radial distortion is always monotonic for real lenses, and if the estimator produces a non-monotonic result, this should be considered a calibration failure. More generally, radial distortion must be monotonic and the distortion function must be bijective. A failed estimation result may look deceptively good near the image center but will work poorly in e.g. AR/SFM applications. The optimization method used in OpenCV camera calibration does not include these constraints as the framework does not support the required integer programming and polynomial inequalities. See issue #15992 for additional information.

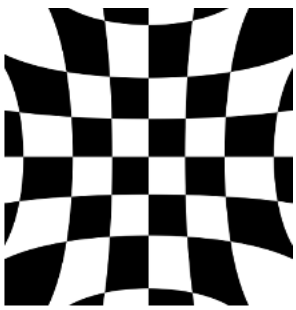
下图显示了两种常见的径向失真：桶形失真（ $1 + k_1 r^2 + k_2 r^4 + k_3 r^6$ 单调递减）和枕形失真（ $1 + k_1 r^2 + k_2 r^4 + k_3 r^6$ 单调递增）。径向畸变对于真实的镜片总是单调的，如果估计器产生非单调结果，则应将其视为校准失败。更一般地，径向失真必须是单调的，并且失真函数必须是双射的。失败的估计结果可能在图像中心附近看起来很好，但在例如AR/SFM应用中效果很差。OpenCV相机校准中使用的优化方法不包括这些约束，因为该框架不支持所需的整数规划和多项式不等式。更多信息请参见问题#15992。



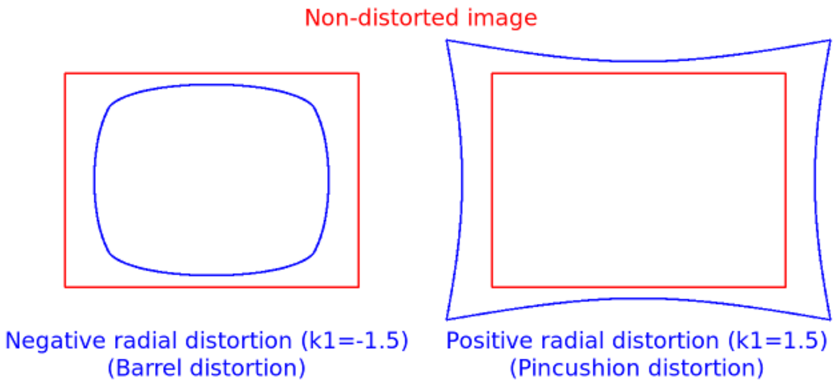
No distortion



Negative radial distortion
(Barrel distortion)



Positive radial distortion
(Pincushion distortion)



In some cases, the image sensor may be tilted in order to focus an oblique plane in front of the camera (Scheimpflug principle). This can be useful for particle image velocimetry (PIV) or triangulation with a laser fan. The tilt causes a perspective distortion of x'' and y''. This distortion can be modeled in the following way, see e.g. [167].

在某些情况下，图像传感器可以倾斜，以便将倾斜平面聚焦在相机前面（Scheimpflug原理）。这对于粒子图像测速（PIV）或使用激光风扇的三角测量非常有用。倾斜导致 x'' 和 y'' 的透视失真。这种失真可以用以下方式建模，参见例如[167]。

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f_x x''' + c_x \\ f_y y''' + c_y \end{bmatrix},$$

where 哪里

$$s \begin{bmatrix} x''' \\ y''' \\ 1 \end{bmatrix} = \begin{bmatrix} R_{33}(\tau_x, \tau_y) & 0 & -R_{13}(\tau_x, \tau_y) \\ 0 & R_{33}(\tau_x, \tau_y) & -R_{23}(\tau_x, \tau_y) \\ 0 & 0 & 1 \end{bmatrix} R(\tau_x, \tau_y) \begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix}$$

and the matrix $R(\tau_x, \tau_y)$ is defined by two rotations with angular parameter τ_x and τ_y, respectively,

并且矩阵 $R(\tau_x, \tau_y)$ 由分别具有角度参数 τ_x 和 τ_y 的两个旋转定义，

$$R(\tau_x, \tau_y) = \begin{bmatrix} \cos(\tau_y) & 0 & -\sin(\tau_y) \\ 0 & 1 & 0 \\ \sin(\tau_y) & 0 & \cos(\tau_y) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\tau_x) & \sin(\tau_x) \\ 0 & -\sin(\tau_x) & \cos(\tau_x) \end{bmatrix} = \begin{bmatrix} \cos(\tau_y) & \sin(\tau_y)\sin(\tau_x) & -\sin(\tau_y)\cos(\tau_x) \\ 0 & \cos(\tau_x) & \sin(\tau_x) \\ \sin(\tau_y) & -\cos(\tau_y)\sin(\tau_x) & \cos(\tau_y)\cos(\tau_x) \end{bmatrix}.$$

In the functions below the coefficients are passed or returned as

在下面的函数中，系数被传递或返回为

$$(k_1, k_2, p_1, p_2[, k_3[, k_4, k_5, k_6[, s_1, s_2, s_3, s_4[, \tau_x, \tau_y]]]])$$

vector. That is, if the vector contains four elements, it means that $k_3 = 0$. The distortion coefficients do not depend on the scene viewed. Thus, they also belong to the intrinsic camera parameters. And they remain the same regardless of the captured image resolution. If, for example, a camera has been calibrated on images of 320 x 240 resolution, absolutely the same distortion coefficients can be used for 640 x 480 images from the same camera while f_x, f_y , c_x, and c_y need to be scaled appropriately.

vector.也就是说，如果向量包含四个元素，则意味着 $k_3 = 0$ 。失真系数不依赖于所观察的场景。因此，它们也属于相机的固有参数。无论捕获的图像分辨率如何，它们都保持相同。例如，如果相机已经在320 x 240分辨率的图像上进行了校准，则绝对相同的失真系数可以用于来自同一相机的640 x 480图像，而 f_x 、 f_y 、 c_x 和 c_y 需要适当地缩放。

The functions below use the above model to do the following:

下面的函数使用上述模型执行以下操作:

- Project 3D points to the image plane given intrinsic and extrinsic parameters.

  在给定内部和外部参数的情况下，将3D点投影到图像平面。

- Compute extrinsic parameters given intrinsic parameters, a few 3D points, and their projections.

  在给定内部参数、一些3D点及其投影的情况下计算外部参数。

- Estimate intrinsic and extrinsic camera parameters from several views of a known calibration pattern (every view is described by several 3D-2D point correspondences).

  从已知校准图案的几个视图估计内部和外部相机参数（每个视图由几个3D-2D点对应描述）。

- Estimate the relative position and orientation of the stereo camera "heads" and compute the rectification* transformation that makes the camera optical axes parallel.

  估计立体相机“头”的相对位置和方向，并计算使相机光轴平行的校正 * 变换。

Homogeneous Coordinates 齐次坐标

Homogeneous Coordinates are a system of coordinates that are used in projective geometry. Their use allows to represent points at infinity by finite coordinates and simplifies formulas when compared to the cartesian counterparts, e.g. they have the advantage that affine transformations can be expressed as linear homogeneous transformation.

齐次坐标系是射影几何中使用的坐标系。它们的使用允许用有限坐标表示无穷远处的点，并且当与笛卡尔对应物相比时简化了公式，例如，它们具有仿射变换可以表示为线性齐次变换的优点。

One obtains the homogeneous vector P_h by appending a 1 along an n-dimensional cartesian vector P e.g. for a 3D cartesian vector the mapping $P \rightarrow P_h$ is:

通过沿着n维笛卡尔向量 P 附加1来获得齐次向量 P_h ，例如，对于3D笛卡尔向量，映射 $P \rightarrow P_h$ 为：

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}.$$

For the inverse mapping $P_h \rightarrow P$, one divides all elements of the homogeneous vector by its last element, e.g. for a 3D homogeneous vector one gets its 2D cartesian counterpart by:

对于逆映射 $P_h \rightarrow P$ ，将齐次向量的所有元素除以其最后一个元素，例如，对于3D齐次向量，通过下式得到其2D carnival对应物：

$$\begin{bmatrix} X \\ Y \\ W \end{bmatrix} \rightarrow \begin{bmatrix} X/W \\ Y/W \end{bmatrix},$$

if $W \neq 0$. 如果 $W \neq 0$ 。

Due to this mapping, all multiples kP_h, for $k \neq 0$, of a homogeneous point represent the same point P_h. An intuitive understanding of this property is that under a projective transformation, all multiples of P_h are mapped to the same point. This is the physical observation one does for pinhole cameras, as all points along a ray through the camera's pinhole are projected to the same image point, e.g. all points along the red ray in the image of the pinhole camera model above would be mapped to the same image coordinate. This property is also the source for the scale ambiguity s in the equation of the pinhole camera model.

由于该映射，齐次点的所有倍数 kP_h （对于 $k \neq 0$ ）表示相同的点 P_h 。对这个性质的直观理解是，在射影变换下， P_h 的所有倍数都映射到同一点。这是针对针孔相机所做的物理观察，因为沿着沿着穿过相机的针孔的射线的所有点被投影到相同的图像点，例如，沿着上面的针孔相机模型的图像中的红色射线的所有点沿着将被映射到相同的图像坐标。这一性质也是针孔相机模型方程中尺度模糊的根源。

As mentioned, by using homogeneous coordinates we can express any change of basis parameterized by R and t as a linear transformation, e.g. for the change of basis from coordinate system 0 to coordinate system 1 becomes:

如上所述，通过使用齐次坐标，我们可以将由 R 和 t 参数化的基的任何变化表示为线性变换，例如，对于从坐标系0到坐标系1的基的变化，变为：

$$P_1 = RP_0 + t \rightarrow P_{h_1} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} P_{h_0}.$$

Note 注意

- Many functions in this module take a camera intrinsic matrix as an input parameter. Although all functions assume the same structure of this parameter, they may name it differently. The parameter's description, however, will be clear in that a camera intrinsic matrix with the structure shown above is required.

  此模块中的许多函数都将摄像机内禀矩阵作为输入参数。尽管所有函数都假定此参数的结构相同，但它们可能会对其命名不同。然而，参数的描述将是清楚的，因为需要具有上述结构的相机固有矩阵。
- A calibration sample for 3 cameras in a horizontal position can be found at opencv_source_code/samples/cpp/3calibration.cpp

  3台摄像机水平位置的校准示例可在opencv_source_code/samples/cpp/3calibration.cpp中找到
- A calibration sample based on a sequence of images can be found at opencv_source_code/samples/cpp/calibration.cpp

  基于图像序列的校准示例可在opencv_source_code/samples/cpp/calibration. cpp中找到
- A calibration sample in order to do 3D reconstruction can be found at opencv_source_code/samples/cpp/build3dmodel.cpp

  可在opencv_source_code/samples/cpp/build3dmodel.cpp中找到用于进行3D重建的校准样本
- A calibration example on stereo calibration can be found at opencv_source_code/samples/cpp/stereo_calib.cpp

  有关立体声校准的校准示例，请访问opencv_source_code/samples/cpp/stereo_calib. cpp
- A calibration example on stereo matching can be found at opencv_source_code/samples/cpp/stereo_match.cpp

  有关立体匹配的校准示例，请访问opencv_source_code/samples/cpp/stereo_match. cpp
- (Python) A camera calibration sample can be found at opencv_source_code/samples/python/calibrate.py

  （Python）相机校准示例可以在opencv_source_code/samples/python/calibrate.py上找到

Classes 类

struct	**cv::CirclesGridFinderParameters** **cv：：CirclesGridFinder参数**
class 类	**cv::LMSolver cv：：LMSolver**
class 类	**cv::StereoBM cv：：StereoBM**