Phenomenological Renormalization Group (RG) approach to neural data

Serena Di Santo 04/15/2020

The Renormalization Group

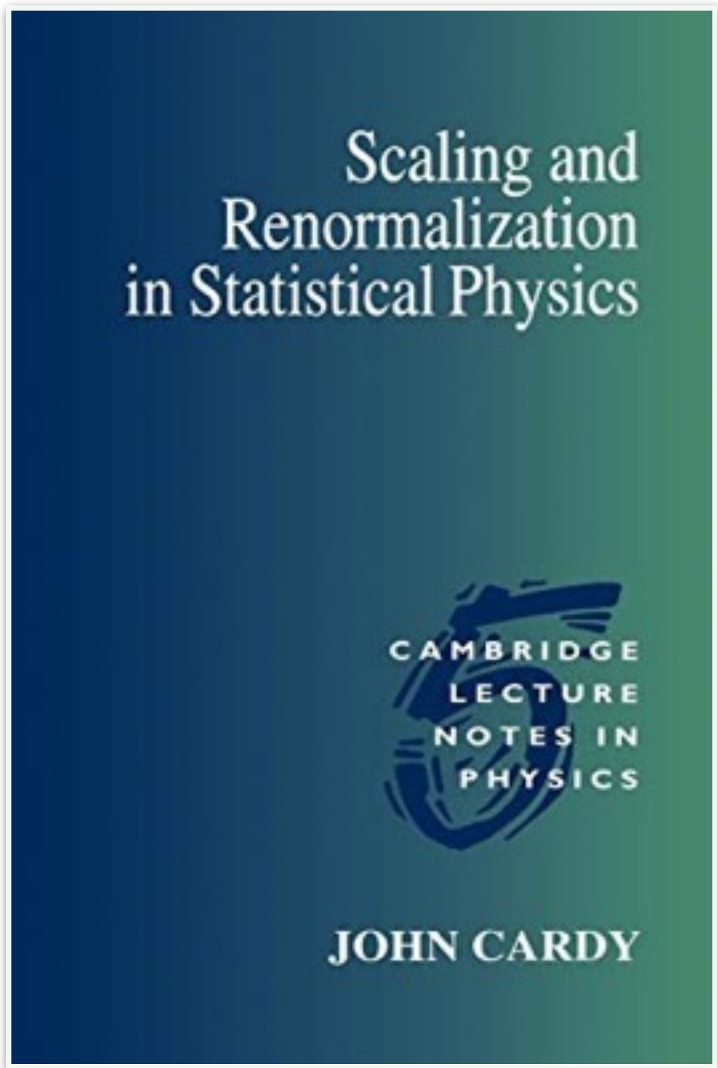
describes the evolution through the space of possible models,
towards filtering out all irrelevant features/operators
(coarse– graining)

The Renormalization ~~Group~~

The Renormalization Group

The Renormalization Group

the sources



Coarse–graining and hints of scaling in a population of 1000+ neurons

Leenoy Meshulam,[1,2,3] Jeffrey L. Gauthier,[1] Carlos D. Brody,[1,4,5] David W. Tank,[1,2,4] and William Bialek[2,3,6]
[1]Princeton Neuroscience Institute, [2]Joseph Henry Laboratories of Physics,
[3]Lewis–Sigler Institute for Integrative Genomics, [4]Department of Molecular Biology,
and [5]Howard Hughes Medical Institute, Princeton University, Princeton, NJ 08544
[6]Initiative for the Theoretical Sciences, The Graduate Center,
City University of New York, 365 Fifth Ave., New York, NY 10016
(Dated: January 1, 2019)

In many systems we can describe emergent macroscopic behaviors, quantitatively, using models that are much simpler than the underlying microscopic interactions; we understand the success of this simplification through the renormalization group. Could similar simplifications succeed in complex biological systems? We develop explicit coarse–graining procedures that we apply to experimental data on the electrical activity in large populations of neurons in the mouse hippocampus. Probability distributions of coarse–grained variables seem to approach a fixed non–Gaussian form, and we see evidence of power–law dependencies in both static and dynamic quantities as we vary the coarse–graining scale over two decades. Taken together, these results suggest that the collective behavior of the network is described by a non–trivial fixed point.

Testing the critical brain hypothesis using a phenomelogical renormalization group

Giorgio Nicoletti[1], Samir Suweis[1,2], Amos Maritan[1]
[1]Dipartimento di Fisica "G. Galilei", Università di Padova
Via Marzolo 8, 35131 Padova, Italy
[2]Padova Neuroscience Center
University of Padova, Padova, Italy
(Dated: January 14, 2020)

We present a systematic study to test a recently introduced phenomenological renormalization group, proposed to coarse-grain data of neural activity from their correlation matrix. The approach allows, at least in principle, to establish whether the collective behavior of the network of spiking neurons is described by a non-Gaussian critical fixed point. We test this renormalization procedure in a variety of models focusing in particular on the contact process, which displays an absorbing phase transition at $\lambda = \lambda_c$ between a silent and an active state. We find that the results of the coarse-graining do not depend on the presence of long-range interactions, but some scaling features persist in the super-critical system up to a distance of 10% from λ_c. Our results provide insights on the possible subtleties that one needs to consider when applying such phenomenological approaches directly to data to infer signatures of criticality.

RG in *real space*

$s_i = \pm 1$
$i = 1..N$

$s'_{i'} = \pm 1$
$i' = 1..N'$

define blocks of 3*3

$$s'_{i'} = +1 \text{ if } \sum_{i \in \text{block}} s_i > 0$$

$$s'_{i'} = -1 \text{ if } \sum_{i \in \text{block}} s_i < 0$$

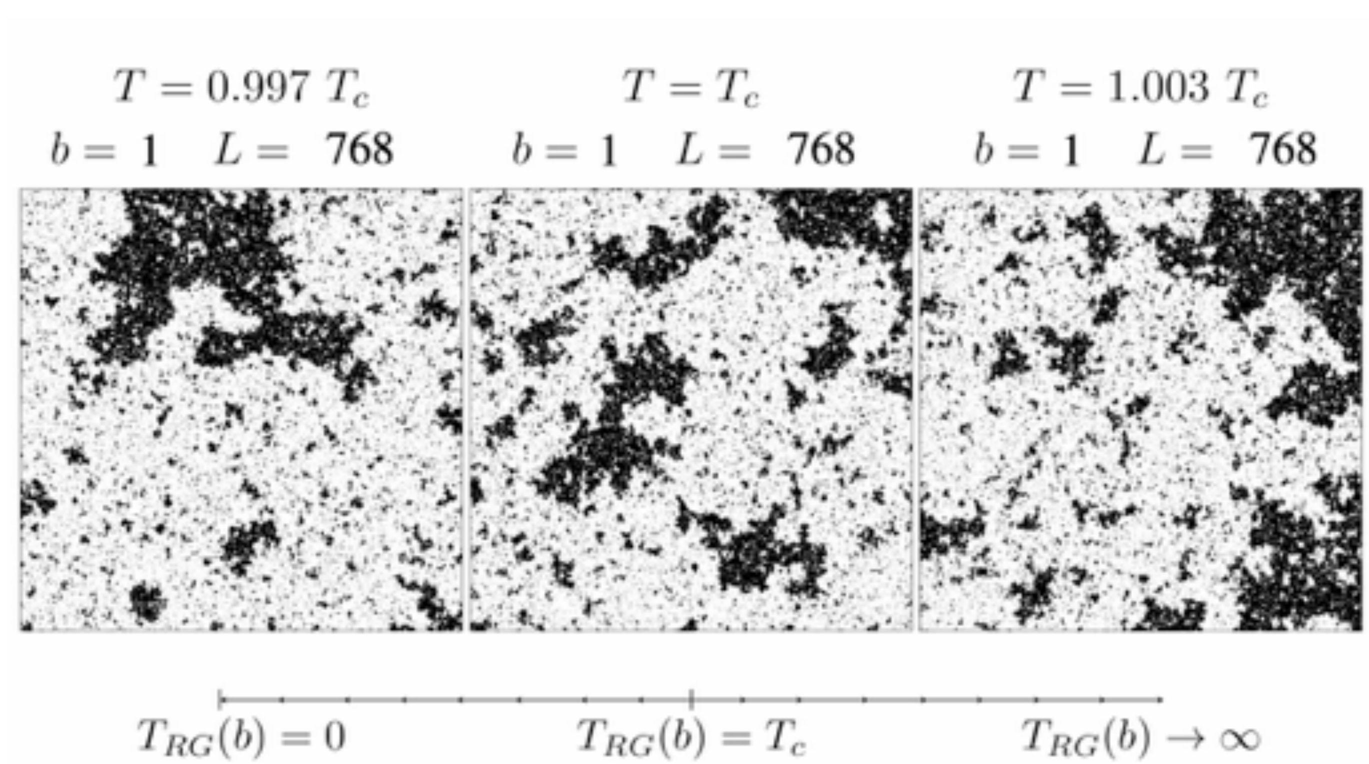
imagine that by zooming out
you can sample a larger system
so that $N' = N$

$$\{K\} \equiv (K_1, K_2, ...) = (1, 0, 0, ...)$$

$$\{K'\} \equiv (K'_1, K'_2, ...)$$

$$\{K'\} = \mathcal{R}(\{K\})$$

…iterate



$$\{K^*\} = \mathcal{R}(\{K^*\})$$

(Youtube video by Douglas Ashton)

what is it good for??

$\{K'\} = \mathcal{R}(\{K\})$ fixed point $\{K\} = \{K^*\}$

linearize around K*

$$K'_a - K^*_a \sim \sum_b T_{ab}(K_b - K^*_b) \qquad T_{ab} = \left(\frac{\partial K'_a}{\partial K_b}\right)_{K=K^*}$$

diagonalize T

$$\sum_a e^i_a T_{ab} = \lambda^i e^i_b$$

define “RG eigenvalues” y: $\lambda^i = b^{y_i}$

b is the length rescaling factor

$y_i > 0$ relevant eigenvalue

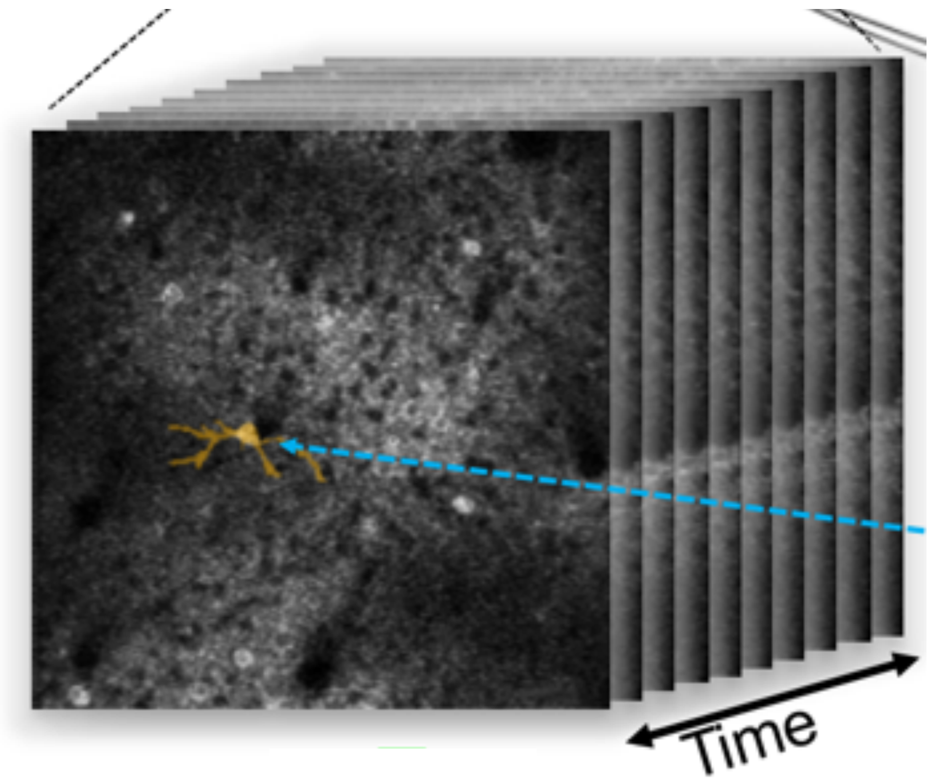
$y_i < 0$ irrelevant eigenvalue

$y_i = 0$ marginal eigenvalue

RG tell us the relevant operators to describe the coarse grained system in each of the fixed points

It also allows to characterise criticality (measure the critical exponents, recognise universality class)

is RG of any use in neuroscience??



most of the times we do not have neighbouring information.
how would we coarse grain?

Zhou et al, biorxiv 2020

RG in *momentum space*: relation with PCA

Dimensionality reduction: look for a projection of data on lower dimensional space

PCA: diagonalize the covariance matrix: the PC are the modes that capture most of the variance, other modes are discarded. But what if the spectrum is continuous?

RG (momentum space) is the analysis of what happens when cutoff on short wavelengths is shifted (short scales are averaged out).

RG in *momentum space*: relation with PCA

RG

$$s(\mathbf{k}) = \mathcal{F}\left[\sigma(\mathbf{x})\right] = \frac{1}{\sqrt{N}} \sum_i e^{-i\mathbf{k}\cdot x_i} \sigma(x_i)$$

$$\sigma(\mathbf{x}) = \mathcal{F}^{-1}\left[s(\mathbf{k})\right] \to \tilde{\sigma}(\mathbf{x}) = \frac{z_\Lambda}{\sqrt{N}} \sum_{|\mathbf{k}|<\Lambda} e^{i\mathbf{k}\cdot\mathbf{x}} s(\mathbf{k})$$

PCA: $C_{\mathrm{ij}} = \langle \sigma_{\mathrm{i}} \sigma_{\mathrm{j}} \rangle - \langle \sigma_{\mathrm{i}} \rangle \langle \sigma_{\mathrm{j}} \rangle$

project sigma onto the subspace
of first K eigenvectors

note: if translation invariant, C is diagonal in Fourier space!
—>first k eigenvectors are the first k Fourier modes!
—>the PC are the Fourier components

$$C_{\mathrm{ij}} = \frac{1}{N} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot(\mathbf{x}_{\mathrm{i}} - \mathbf{x}_{\mathrm{j}})} G(\mathbf{k})$$

coarse grained variables *are* the projections of the original variables on the PC

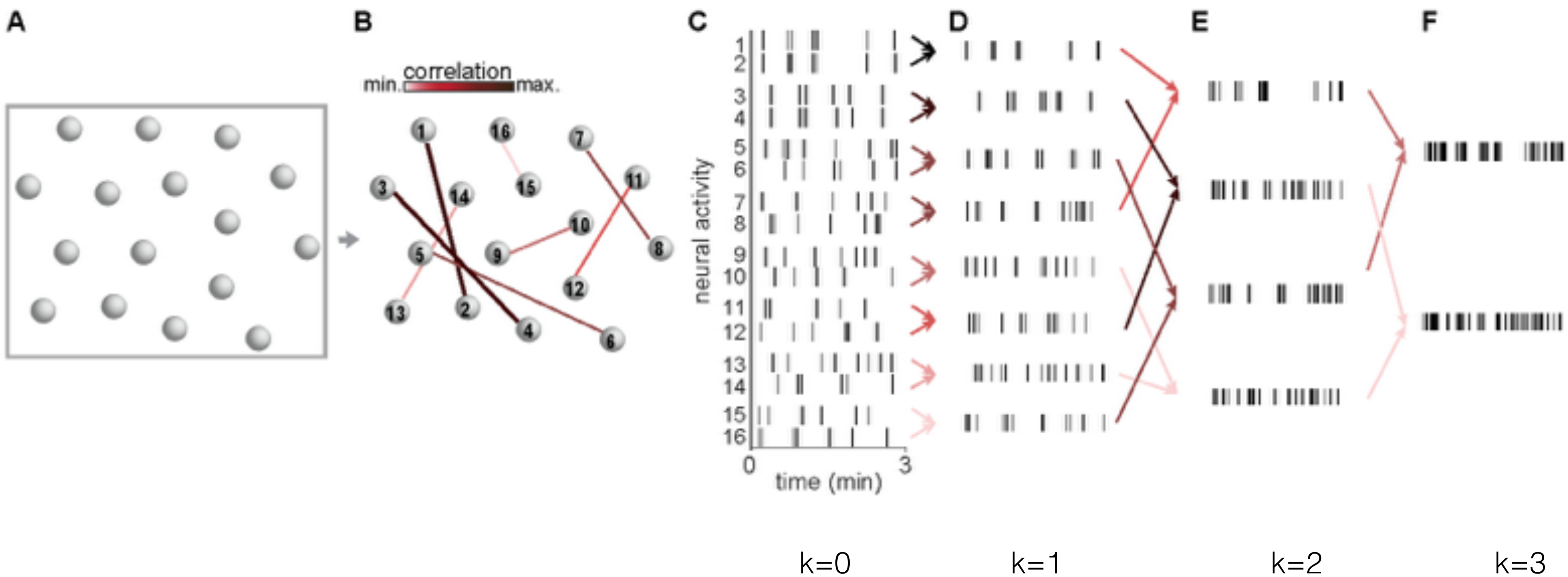
1RG step is equivalent to PCA with $K \sim \Lambda^d$

RG focus: $P(\{\sigma_{\mathrm{i}}\}) \to \tilde{P}(\{\tilde{\sigma}_{\mathrm{i}}\})$ as we move $\mathbf{\Lambda}$

if correlations are weak—>
distribution of coarse grained
variables is Gaussian!

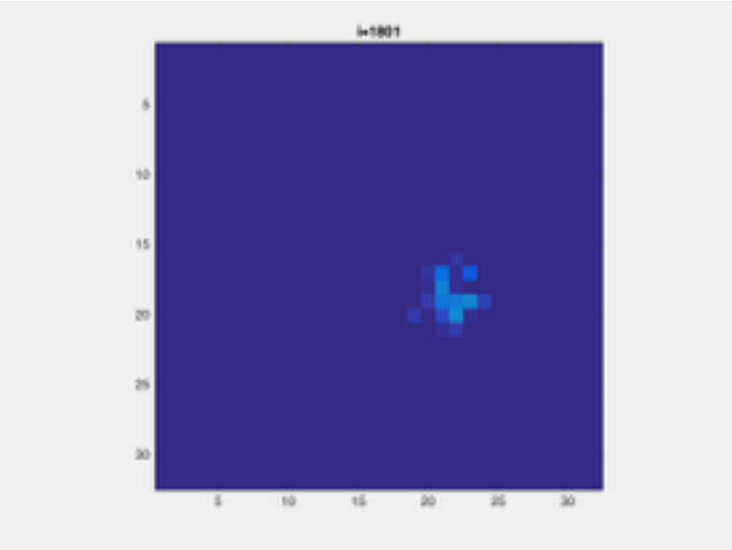
in RG dimensionality reduction is not the source of simplification:
it is a search for simplification in the space of models (rather than variables).

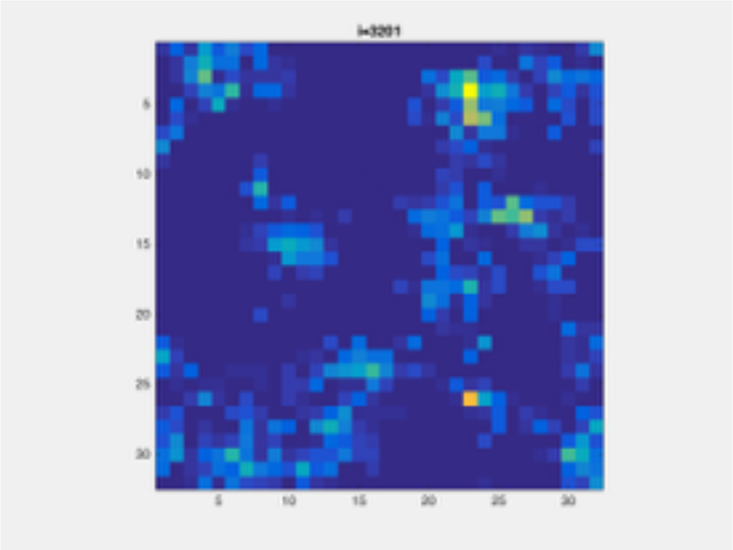
Phenomenological RG: correlation as a proxy for neighbourhood

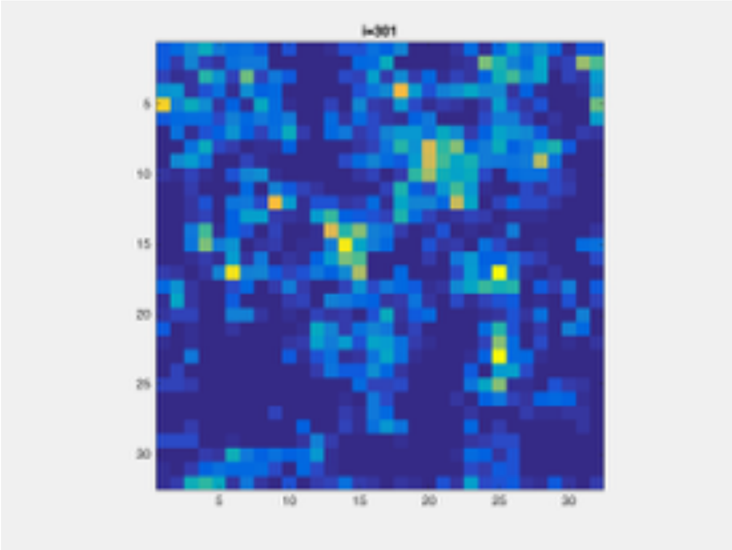


$$N_K = \lfloor N/K \rfloor \text{ clusters of size } K = 2^k$$

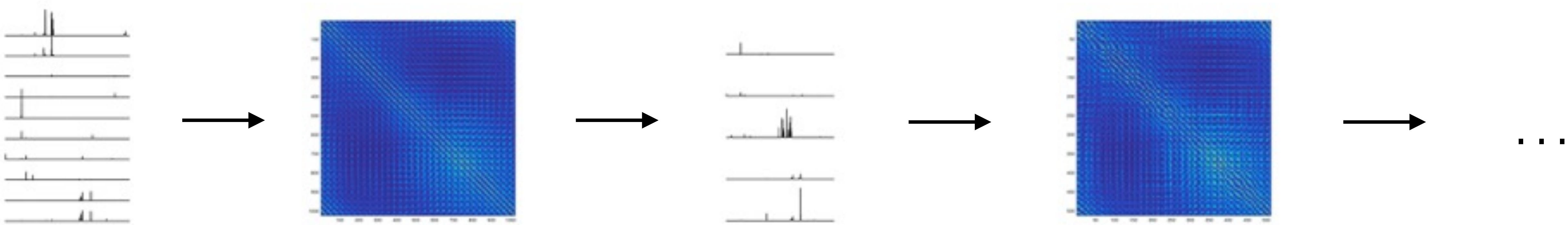
test on a control case: Contact Process on a 2D lattice







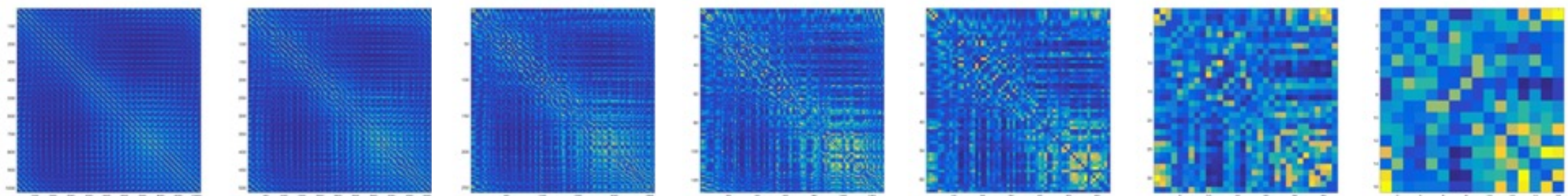
correlation matrix of coarse grained variables, PRG



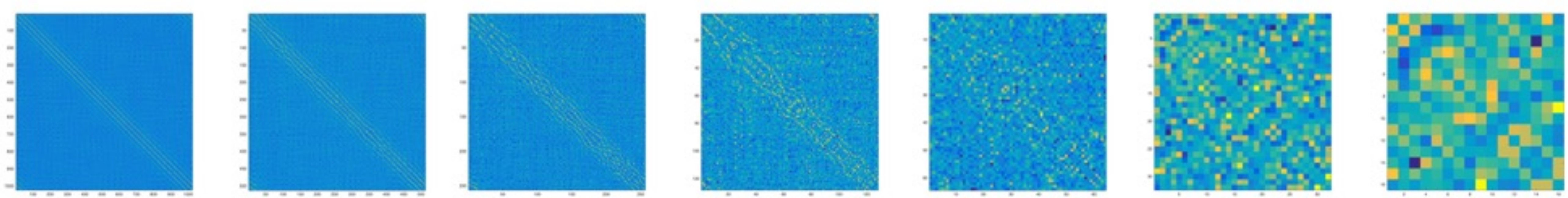
N=1024 variables:
Nk=1024 clusters
of K=1 variables

Nk=512 clusters
of K=2 variables

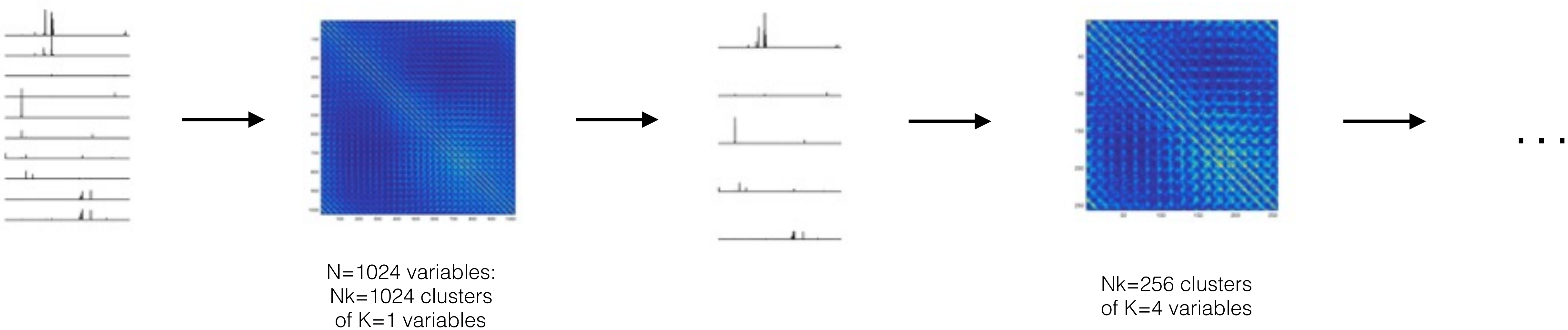
critical



supercritical



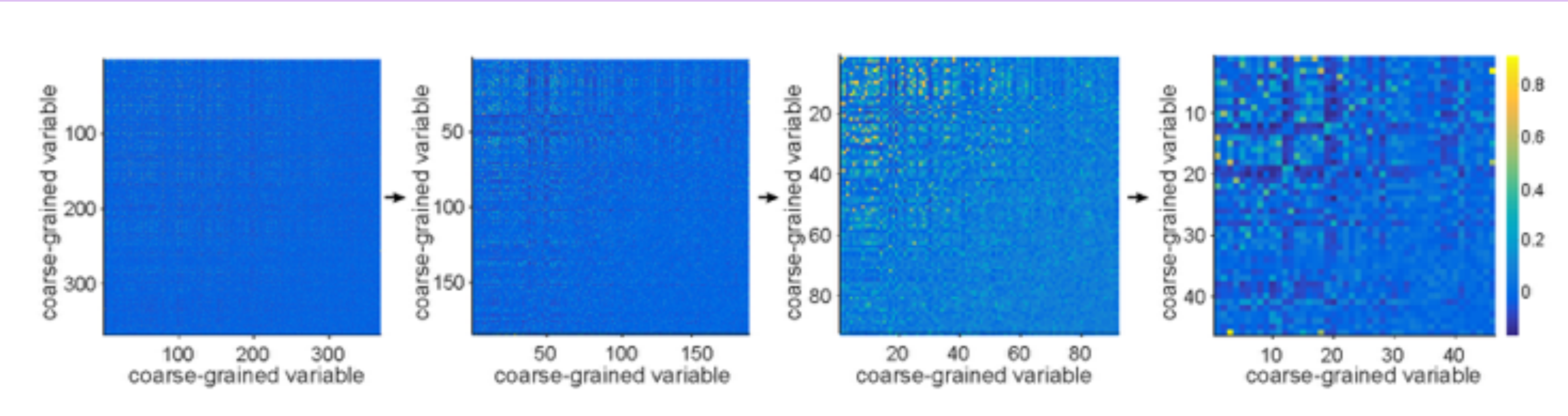
correlation matrix of coarse grained variables, *block* RG



critical

supercritical

correlation matrix of coarse grained variables, PRG on data

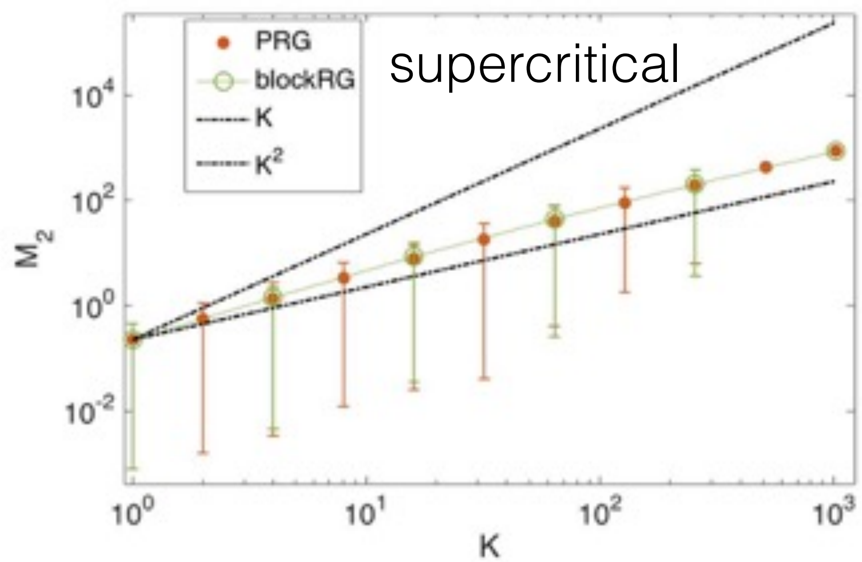


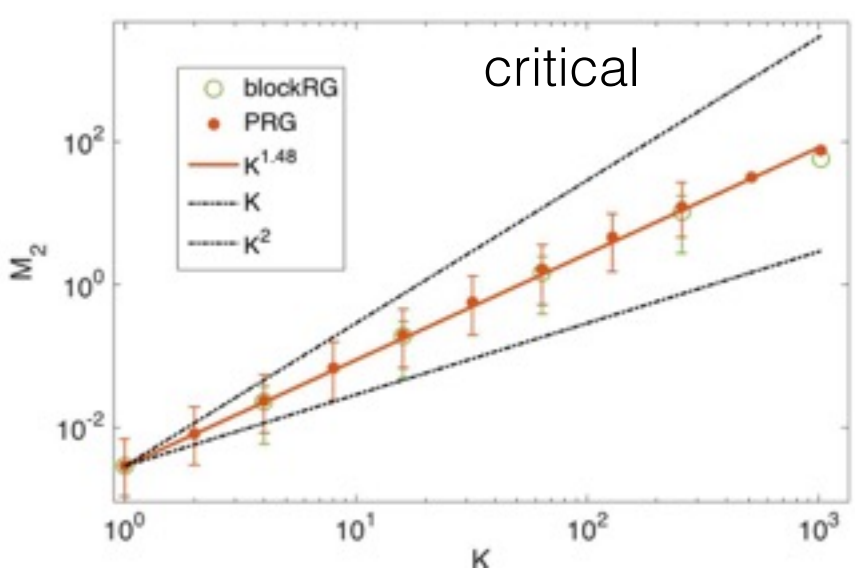
mean variance of activity

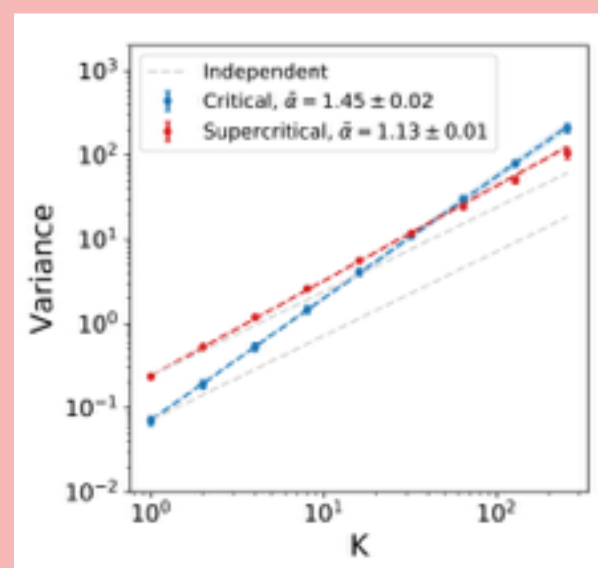
$$M_2(K) = \frac{1}{N_k} \sum_{i=1}^{N_k} \left[\left\langle \left(\sigma_i^{(k)} \right)^2 \right\rangle - \left\langle \sigma_i^{(k)} \right\rangle^2 \right]$$

$$M_2(K) \propto K^{\tilde{\alpha}} \text{ with } \tilde{\alpha} = 1$$

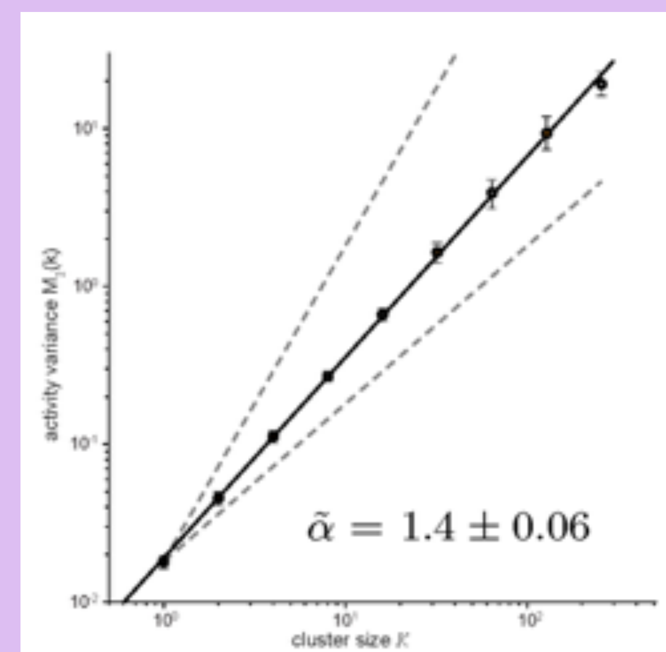
for independent variables







Nicoletti et al



Meshulam et al

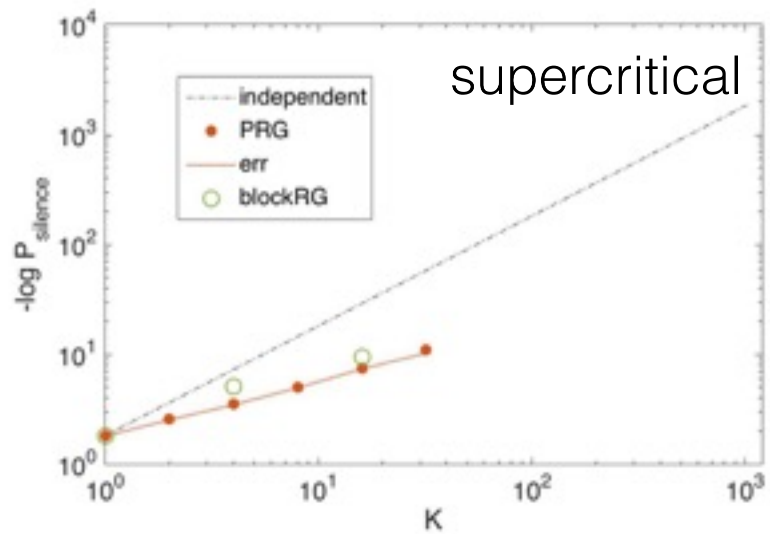
distribution of coarse grained variables: silence

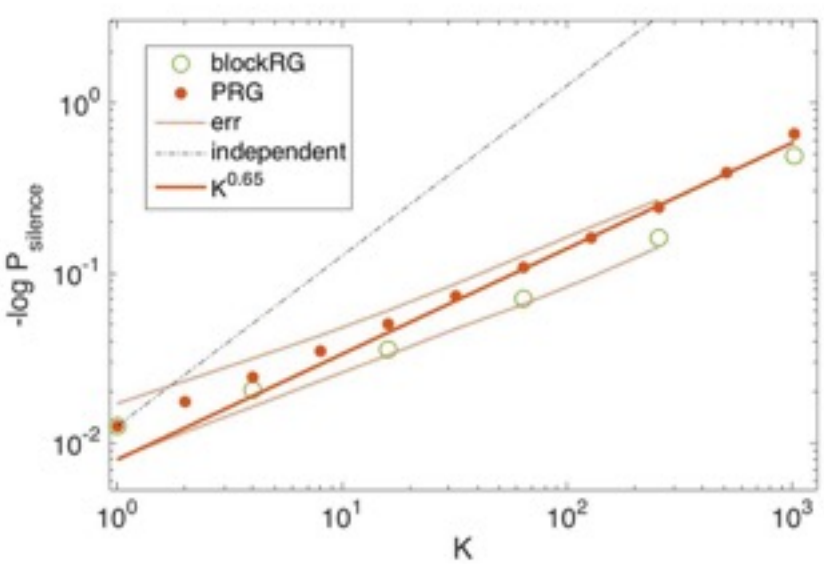
$$P(\sigma_{\mathrm{i}}^{(k)}) \;=\; P_{\text{silence}}(K)\delta\left(\sigma_{\mathrm{i}}^{(k)},0\right) + \left[1-P_{\text{silence}}(K)\right] F_K(\sigma_{\mathrm{i}}^{(k)}/K)$$

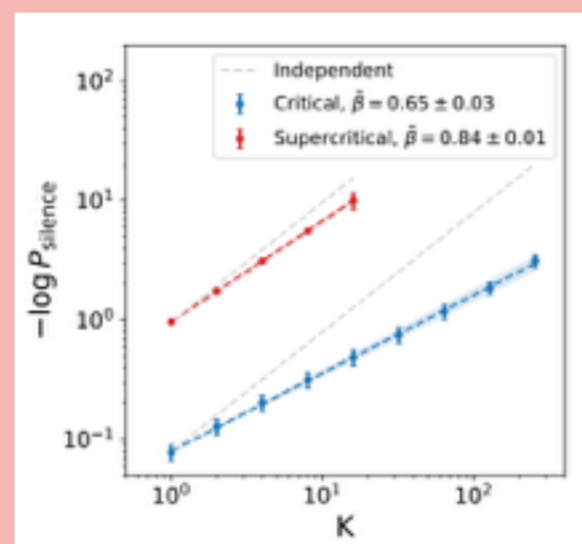
$$P_0(K) = \exp(-aK^{\tilde{\beta}}),$$

Probability that variable x at step k is 0=
Probability that all K neurons are silent

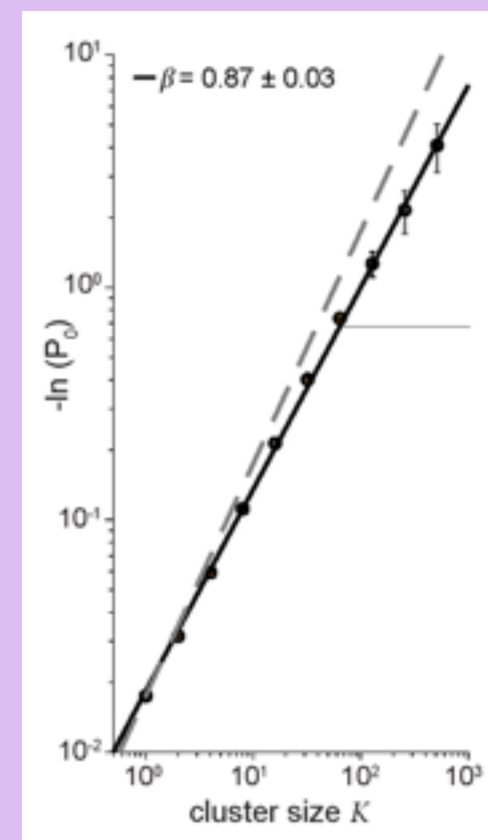
beta=1 for independent variables,
beta<1 for correlated variables







Nicoletti et al

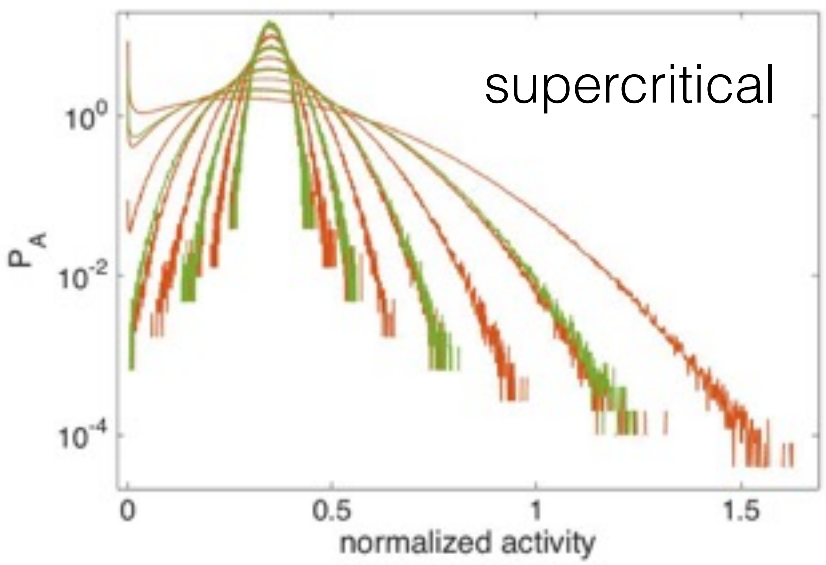


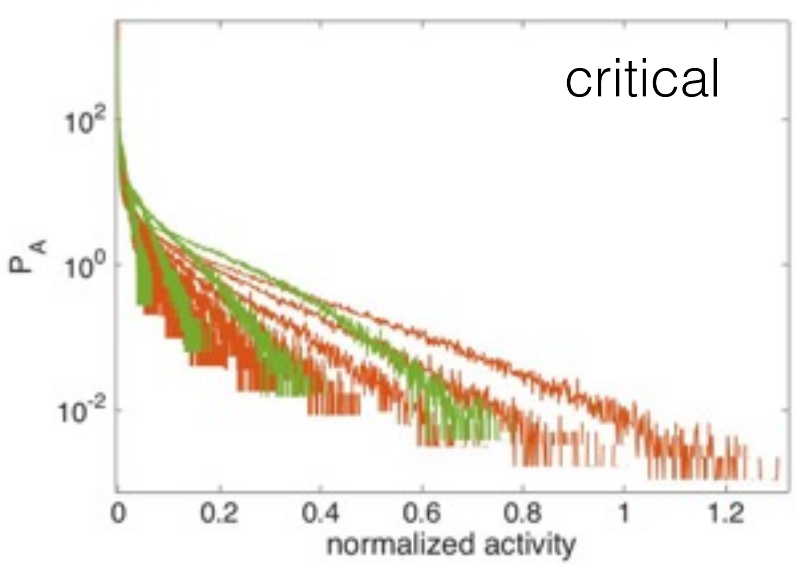
Meshulam et al

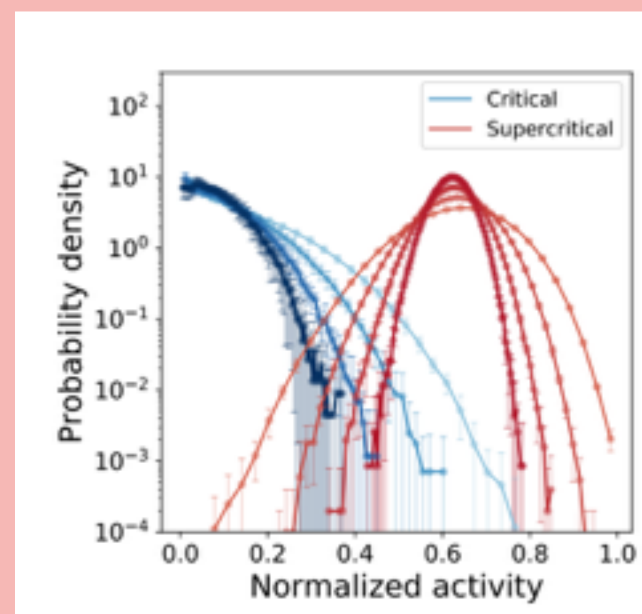
distribution of coarse grained variables: activity

distribution of active sites

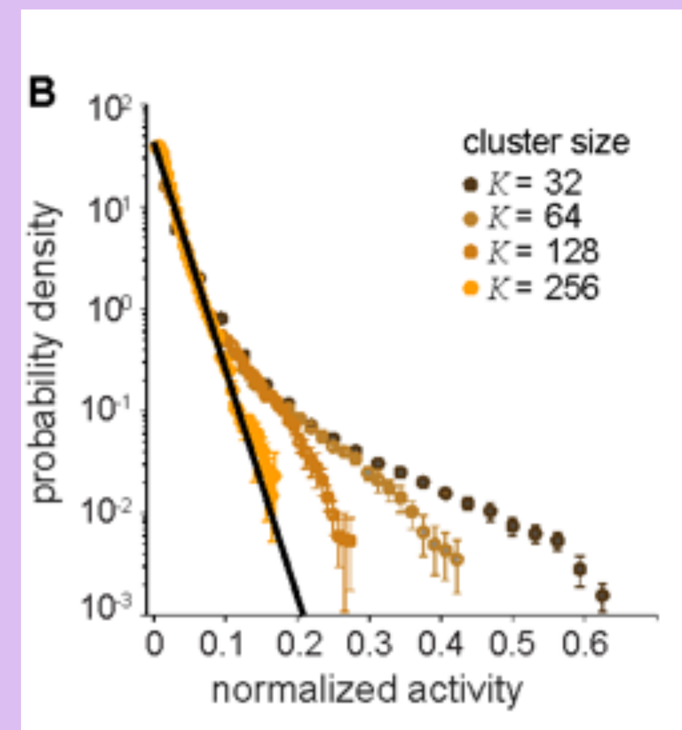
$$P(\sigma_{\rm i}^{(k)}) \;=\; P_{\rm silence}(K)\delta\left(\sigma_{\rm i}^{(k)},0\right) + \left[1-P_{\rm silence}(K)\right] F_K(\sigma_{\rm i}^{(k)}/K)$$







Nicoletti et al



Meshulam et al

spectrum of the covariance matrix

the eigenvalues of Cij (if translation invariance) are the wave vectors \bold{k}.
at a fixed point of the RG transformation these scale as a power law

$$G(\mathbf{k}) \propto \frac{1}{|\mathbf{k}|^{2-\eta}}$$

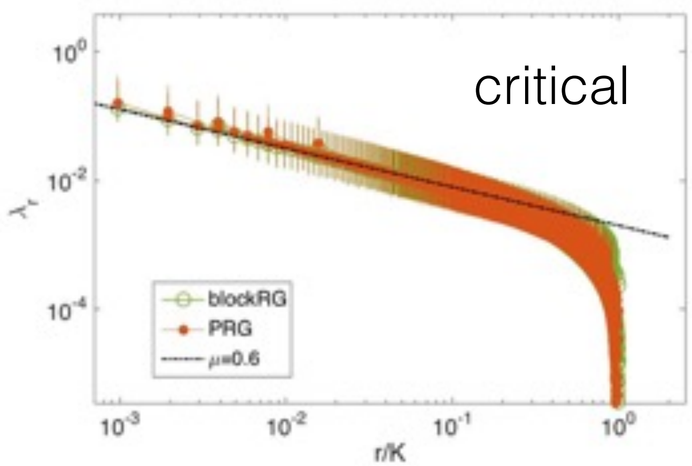
ranking $\mathrm{r} \sim |\mathbf{k}|^d$

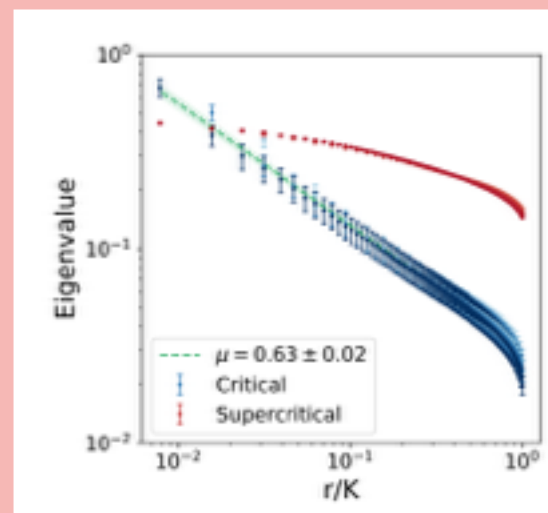
I want to see the scaling of this quantity with the number of degrees of freedom

$$\lambda_r \propto \left(\frac{K}{r}\right)^{\mu}$$

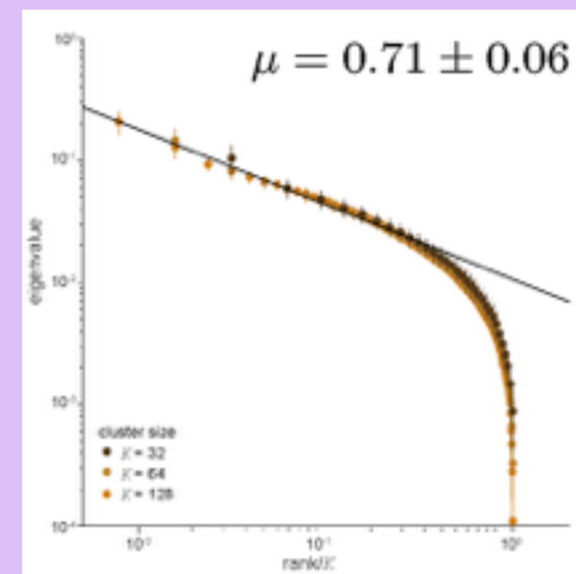
compatible with hyper scaling relation

$$\eta = d - 2 + \frac{\beta}{\nu_\perp}$$

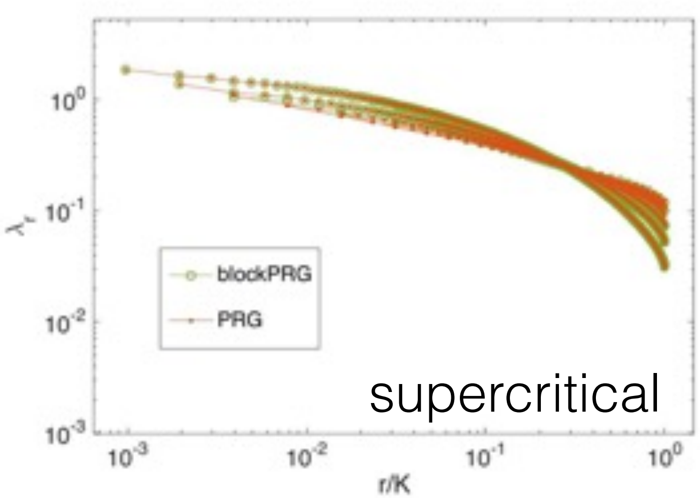


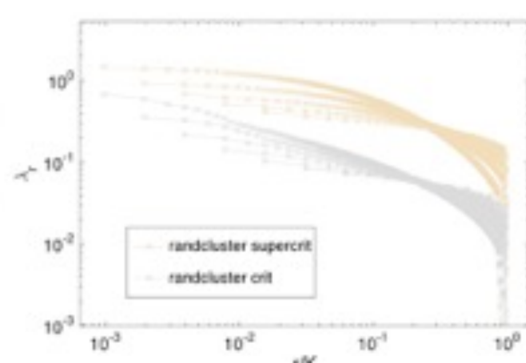


Nicoletti et al



Meshulam et al





autocorrelation function

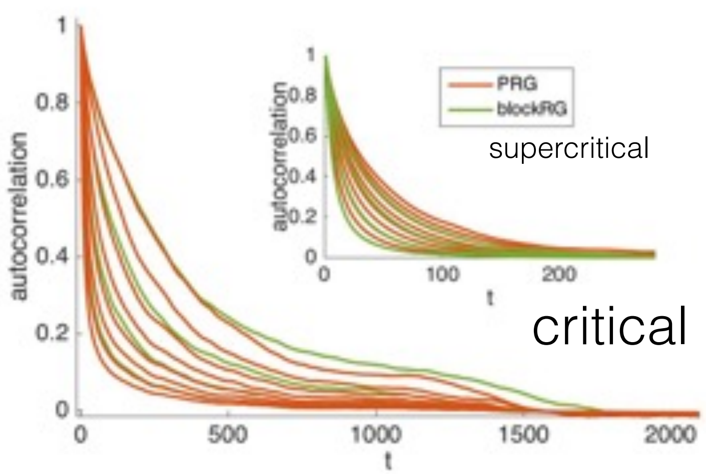
$$C^{(k)}(t) = \frac{1}{N_k} \sum_i C_i^{(k)}(t)$$

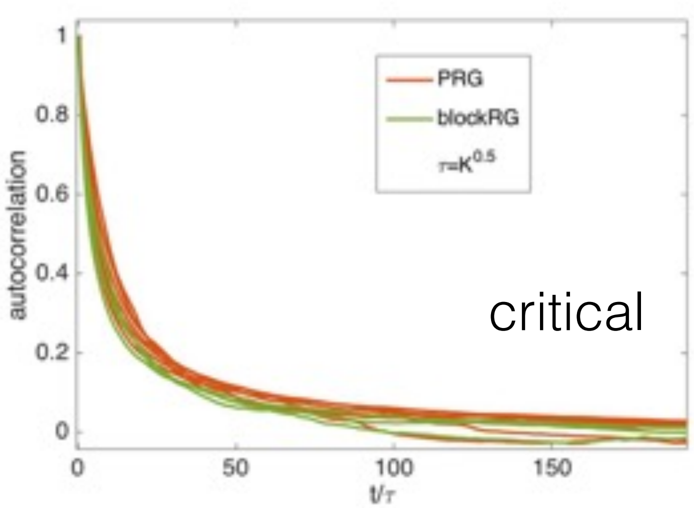
where

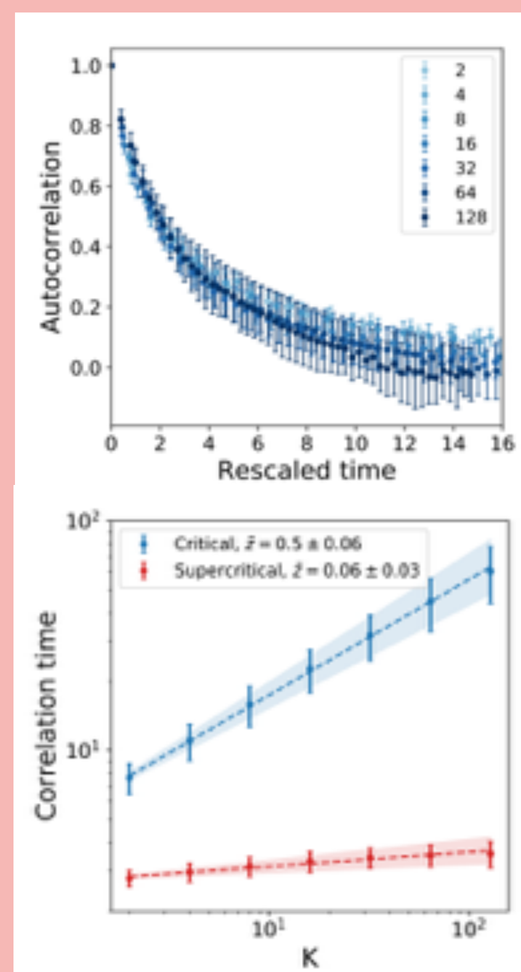
$$C_i^{(k)}(t) = \frac{\langle \sigma_i^{(k)}(t_0) \sigma_i^{(k)}(t_0 + t) \rangle - \langle \sigma_i^{(k)} \rangle^2}{\langle (\sigma_i^{(k)})^2 \rangle - \langle \sigma_i^{(k)} \rangle^2}.$$

$$C^{(k)}(t) = C[t/\tau_c(k)]$$

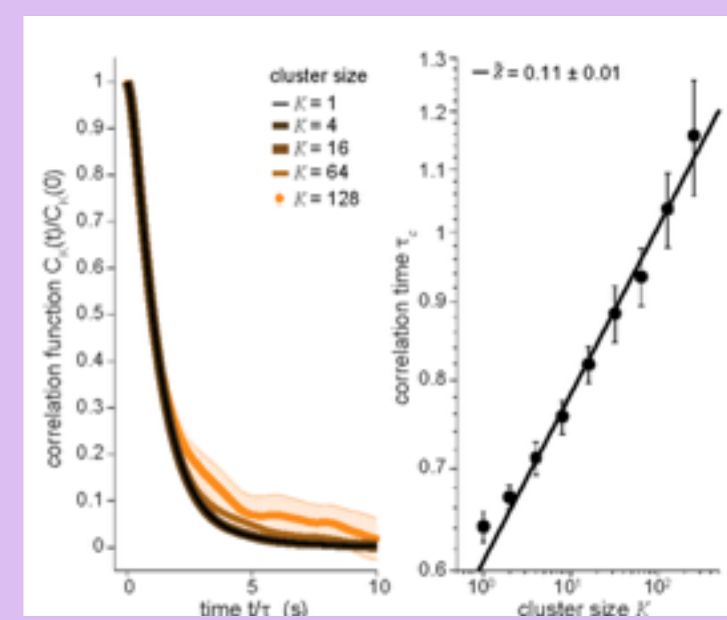
$$\tau_c(k) \propto K^{\tilde{z}}$$







Nicoletti et al



Meshulam et al

the conclusion is: a simpler model might exist.

PRG is compatible with block scheme!

inspection of a subset of observables
might be misleading