Examples of control theory in neuroscience

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What is control theory?

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- Study of how internal variables and outputs of a system respond to their inputs and initial conditions, and how they can be inferred from input/output measurements.

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- Study of how internal variables and outputs of a system respond to their inputs and initial conditions, and how they can be inferred from input/output measurements.
- Linear models are particularly successful:
  - they describe small perturbations, which most control theory is aimed at regulating
  - they are more tractable and allow developing systematic and detailed control approaches
  - engineered systems often made up of linear modules with any nonlinearities introduced in carefully selected locations and forms

Network control theory

- TC Kao, G Hennequin, Neuroscience out of control: control-theoretic perspectives on neural circuit dynamics, Current Opinion in Neurobiology, 2019.
- E Tang, DS Bassett, Colloquium: Control of dynamics in brain networks, Reviews of modern physics, 2018.

Key goal in neuroscience: How do recurrent neural network lead to behaviour

Typical approaches:

- direct implementation of physiological features
- low-dimensional latent feature dynamical models fitted to exp. recordings
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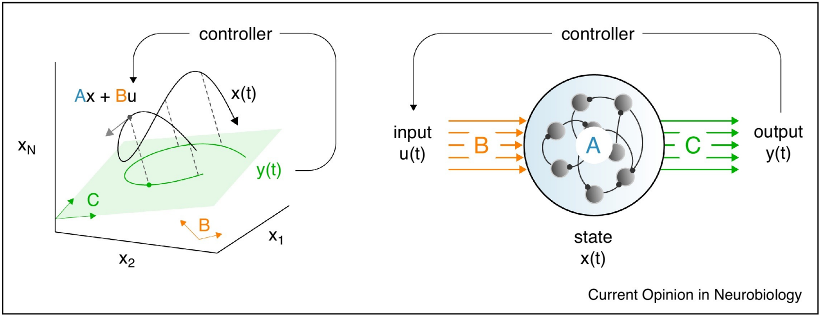
$\Rightarrow$ missing/developing "algebra of dynamics" a.k.a. Control theory

Control of linear state-space models

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \text{noise}$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \text{noise}$$



Observability

How well can we infer the network state given the input?

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Given $\mathbf{u}(t)$ and $\mathbf{y}(t)$ for $0 \leq t < T$:

$$\begin{pmatrix} \mathbf{y}(0) \\ \mathbf{y}(1) \\ \dots \\ \mathbf{y}(T-1) \end{pmatrix} = \begin{pmatrix} \mathbf{C} \\ \mathbf{CA} \\ \dots \\ \mathbf{CA}^{T-1} \end{pmatrix} \mathbf{x}(0) + \begin{pmatrix} 0 & 0 & 0 & 0 \\ \mathbf{CB} & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ \mathbf{CA}^{T-2}\mathbf{B} & \mathbf{CA}^{T-3}\mathbf{B} & \dots & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u}(0) \\ \mathbf{u}(1) \\ \dots \\ \mathbf{u}(T-1) \end{pmatrix}$$

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$$\bar{\mathbf{y}} = \begin{pmatrix} \mathbf{C} \\ \mathbf{CA} \\ \dots \\ \mathbf{CA}^{T-1} \end{pmatrix} \mathbf{x}(0) = \mathcal{O}_T \mathbf{x}(0)$$

with *observability matrix* $\mathcal{O}_T$.

A state is unobservable if it cannot be distinguished from the zero state. The dynamic system is called unobservable if it has an unobservable state, otherwise it is called observable.

Observability

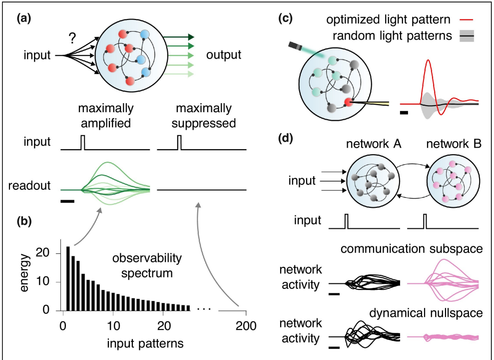
Observability Gramian:

$$\begin{aligned}\mathbf{Q} &= \mathcal{O}_T^T \mathcal{O}_T \\ &\rightarrow \int_0^\infty e^{t\mathbf{A}^T} \mathbf{C}^T \mathbf{C} e^{t\mathbf{A}} dt\end{aligned}$$

characterizes how well initial state $\mathbf{x}(0)$ can be inferred from $\mathbf{y}(t)$ and $\mathbf{u}(t)$.
If $\mathcal{E}(\mathbf{v}) = \mathbf{v}^T \mathbf{Q} \mathbf{v} > 0$ system is observable.

Geometry of input sensitivity in neural circuits

Observability Gramian provides information about a network's sensitivity to specific input patterns.
Think about $\mathcal{E}(\mathbf{v})$ as the energy in $\mathbf{y}(t)$ evoked by an input pulse along direction $\mathbf{v}$.



Current Opinion in Neurobiology

Controllability

Can network state $\mathbf{x}(t)$ be steered along desired trajectory using control inputs $\mathbf{u}(t)$?

Controllability Gramian:

$$\mathbf{P} = \int_0^\infty e^{t\mathbf{A}}\mathbf{B}\mathbf{B}^T e^{t\mathbf{A}^T} dt$$

$\mathbf{v}^T\mathbf{P}^{-1}\mathbf{v}$ is proportional to minimal energy required to move $\mathbf{x}(t)$ along direction $\mathbf{v}$.

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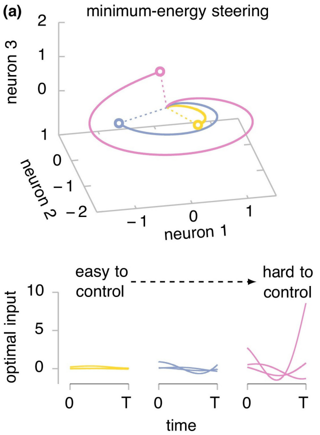
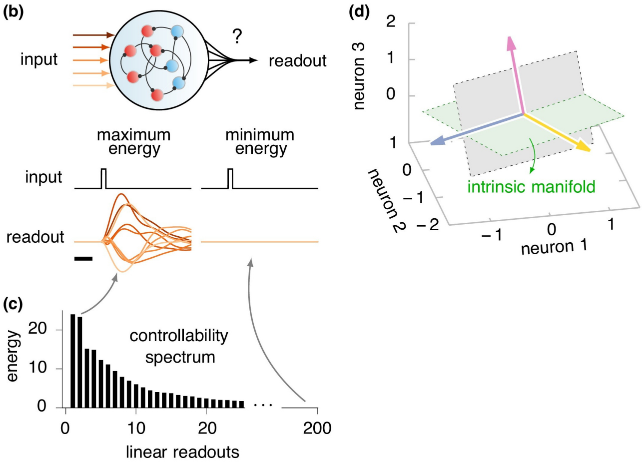
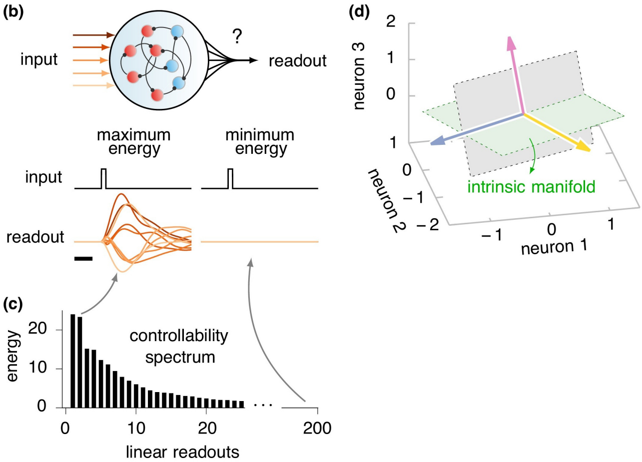


Intrinsic manifolds and control costs



Intrinsic manifolds and control costs



Modulation of network activity within intrinsic manifold is cheap whereas control of activity outside intrinsic manifolds requires larger inputs.

Model reduction

- Observability and controllability $\Rightarrow$ model reduction
- *Balanced truncation*

Stability and homeostasis

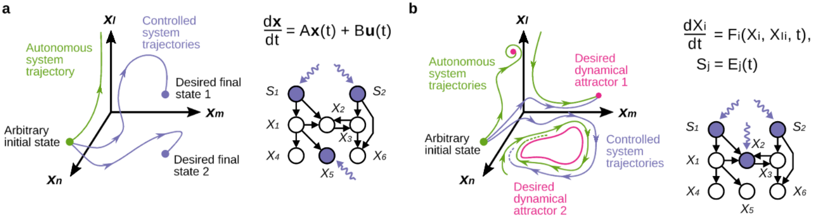
How can complex, inherently unstable dynamical processes be controlled and stabilized?

- Positive feedback due to recurrent excitatory connections
- Positive feedback in Hebbian learning

Seizure suppression in epilepsy

- Treatment: medication, surgery or neurostimulation
- Problem: different brain areas can play diverse roles in production and propagation of epileptiform dynamics between individuals.
- Potential applications of control theory:
  - Transcranial electric stimulation using a closed-loop system
  - Model electrode stimuli to direct/halt propagation of pathological activity
  - Identify suitable lesion points to prevent synchronous activity

Non-linear dynamics - feedback vertex sets



- consider nonlinear dynamics:
  $\frac{dx_i(t)}{dt} = F_i(x_i, x_{\text{pre},i}, t), \frac{dS_j(t)}{dt} = G_j(t)$
- identify minimal set of nodes that controls network (Zanudo et al 2016)
- switch network dynamics from one attractor to another one by overriding source and feedback vertex set nodes

Time-dependent control

- Without structural information
- Limited time-series data describing evolution of spreading process
- Determine uncertainty set containing all networks consistent with observed data (Han et al 2015)
- May be applicable to control of seizures

Exploiting system properties for network control

- Compensatory perturbations: iteratively identify perturbations by considering admissible perturbations and by nonlinear optimization of these perturbations (Cornelius et al 2013)
- Network topology: source and sink nodes give lower bound on number of nodes that need to be controlled (Ruths & Ruths 2014)

Conclusions

- Control theory offers simple geometric descriptions of the dynamic input/output behaviour of neural circuits.
- Most methods apply primarily to linear state space models.
- Development of new control methods for more realistic networks potentially applicable in neuroscience

Thank you for your attention!