Optimal feedback control as a theory of motor coordination

E. Todorov and M.I. Jordan

Nature Neuroscience 2002

State of the art before this publication: signal dependent noise for motor control

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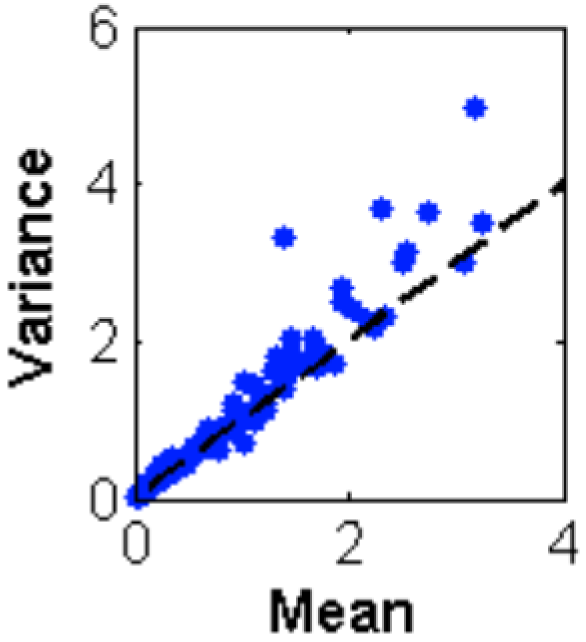
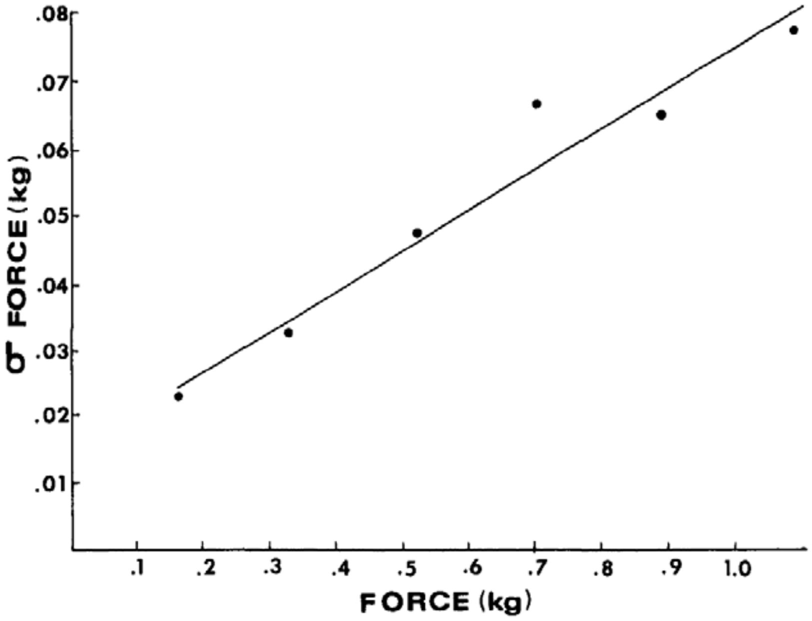
1. Physiology

Increase of spike count variability with mean in motor cortex neurons
Zacksenhouse, 2007



Standard deviation of muscle force grows linearly with its mean
Schmidt, 1979



State of the art before this publication: signal dependent noise for motor control

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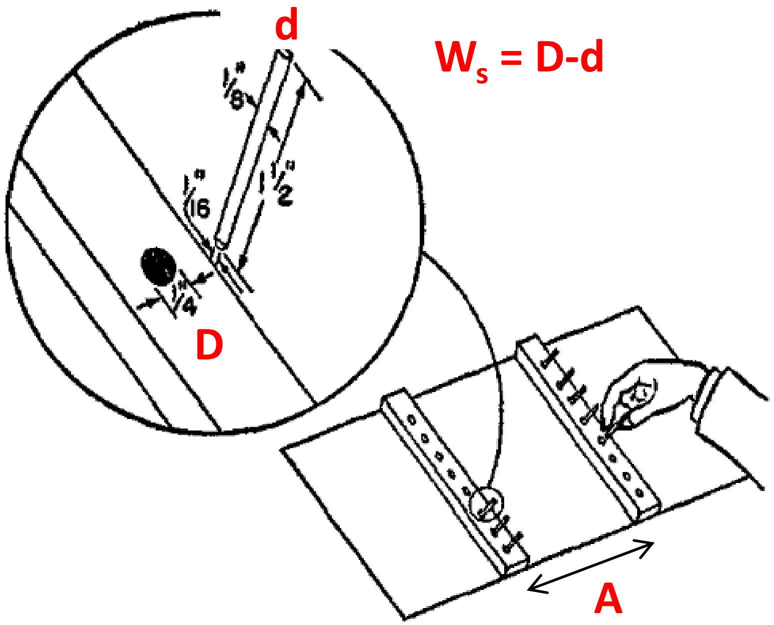
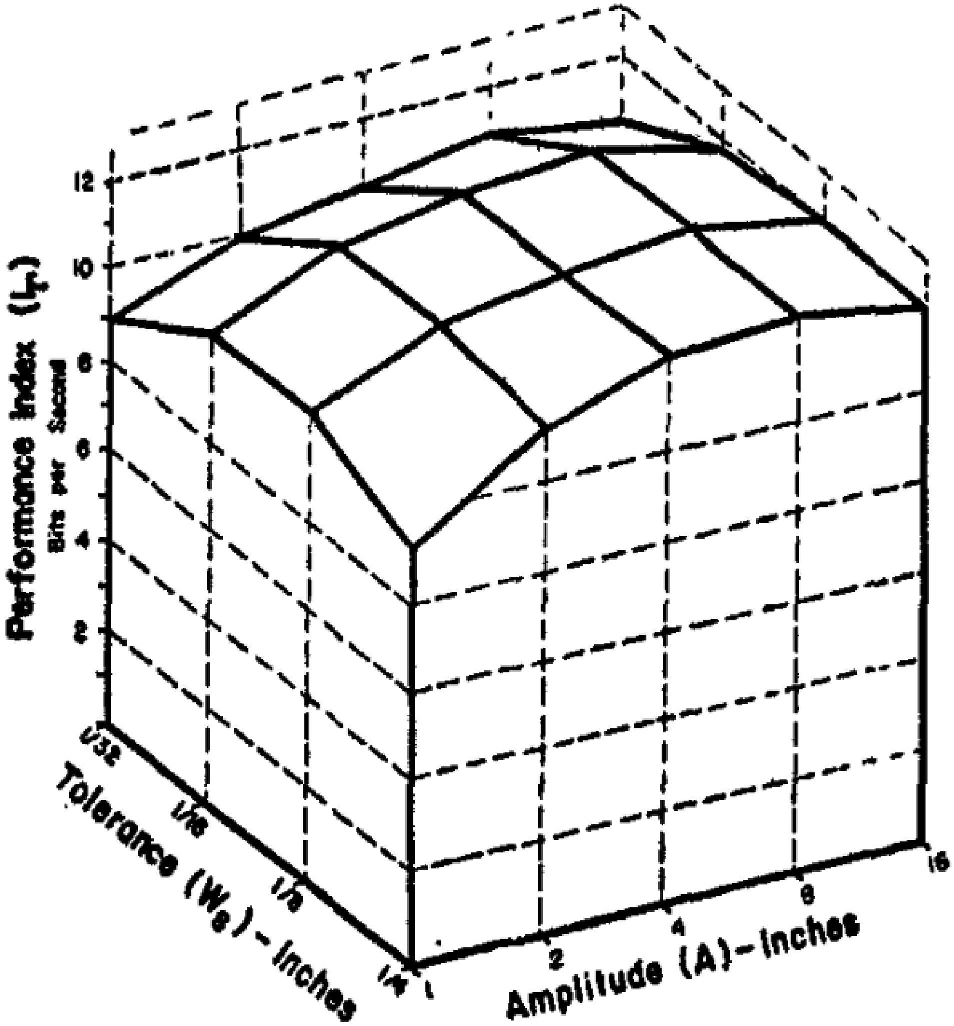


Fitts' Law for performance:

$$I_p = -\frac{1}{t}\log_2\frac{W_s}{2A}\ \text{bits/sec.}$$

movement time



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1. Physiology of motor neuronal firing: Clamann, 1969
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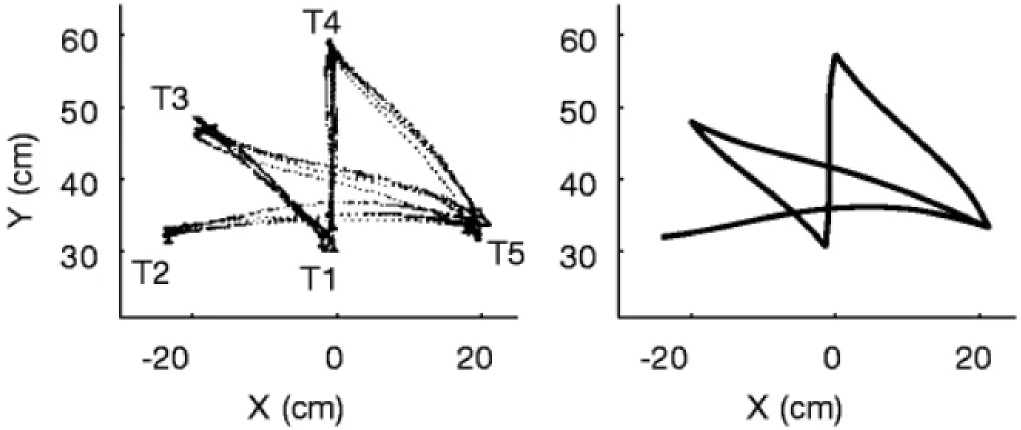
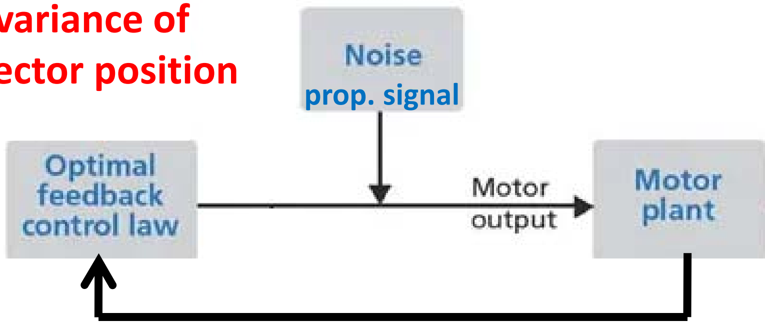
3. Behavior: Harris and Wolpert, 1998

   Predicts average eye and hand trajectories in goal-reaching tasks:

Data **Model**



Minimize variance of effector position



Adapted from Scott, Nat. Neuro. 2002

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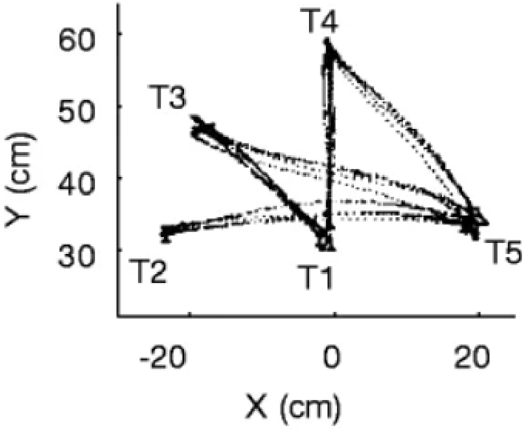
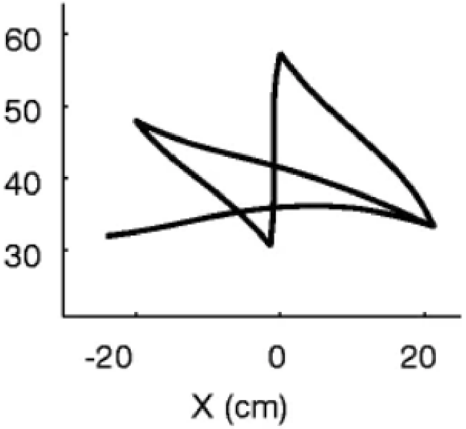


Model



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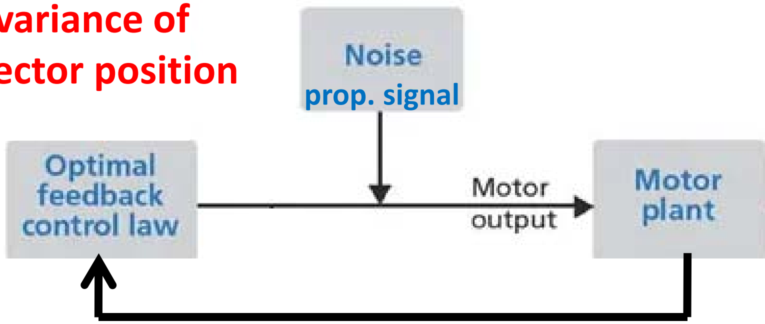
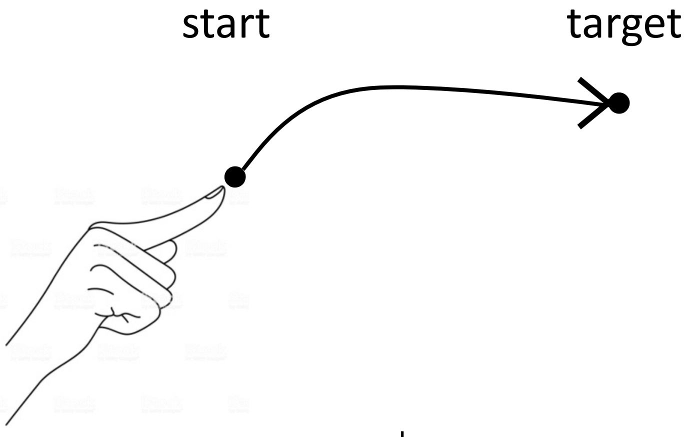


Adapted from Scott, Nat. Neuro. 2002

How to better capture movement variability?
How to rule out alternative models for motor control?

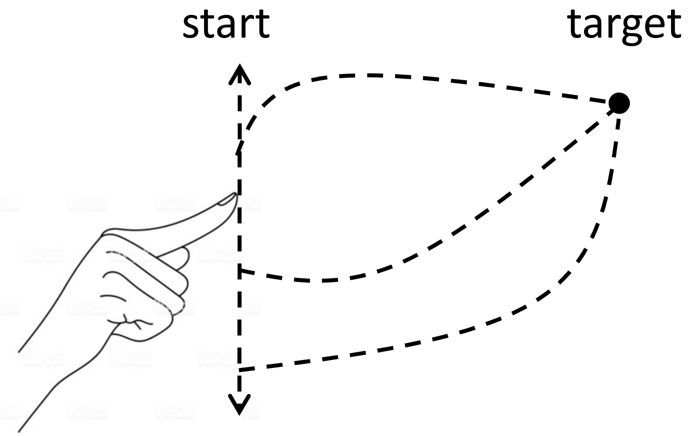
Two strategies for motor control

Impose a pre-planned trajectory



Little variability

Just try to fulfill a goal



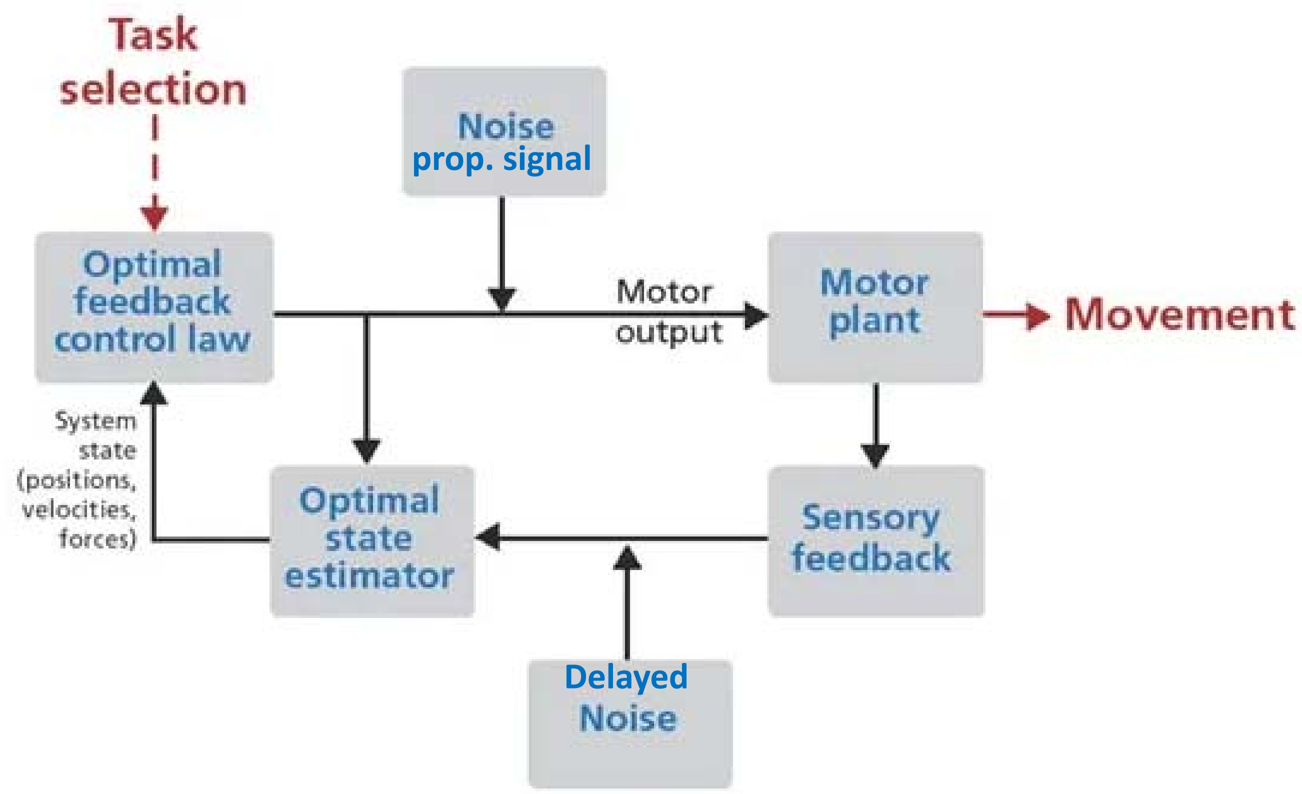
Variability allowed as long as no functional cost

Optimal feedback control for motor control

Hypothesis:

movement trajectory is mostly a consequence of optimal steering of effector towards target, rather than being pre-planned

Maximize performance index
+
Minimize effort penalty



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Linear–quadratic–Gaussian control and redundancy exploitation

One-step dynamics: $x_i^{final} = ax_i + u_i(1 + \sigma\varepsilon_i);\ i \in \{1,2\}$

Control signal

Independent noise, variance 1

Linear–quadratic–Gaussian control and redundancy exploitation

One-step dynamics:

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Control signal

Independent noise, variance 1

Target sum

Cost function:

$$E_\varepsilon(x_1^{final} + x_2^{final} - X^*)^2 + r(u_1^2 + u_2^2)$$

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Optimal control: minimize $(r + \sigma^2)(u_1^2 + u_2^2)$ subject to $u_1 + u_2 = -Err$

$Err \triangleq a(x_1 + x_2) - X^*$ is the expected task error if $u_1 = u_2 = 0$

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Optimal feedback control law: $u_1 = u_2 = -Err/2$

→ depends on the sum $x_1 + x_2$, not the individual values x_1 and x_2

→ creates variability along task-irrelevant direction $x_1 - x_2$ (no control in this direction)

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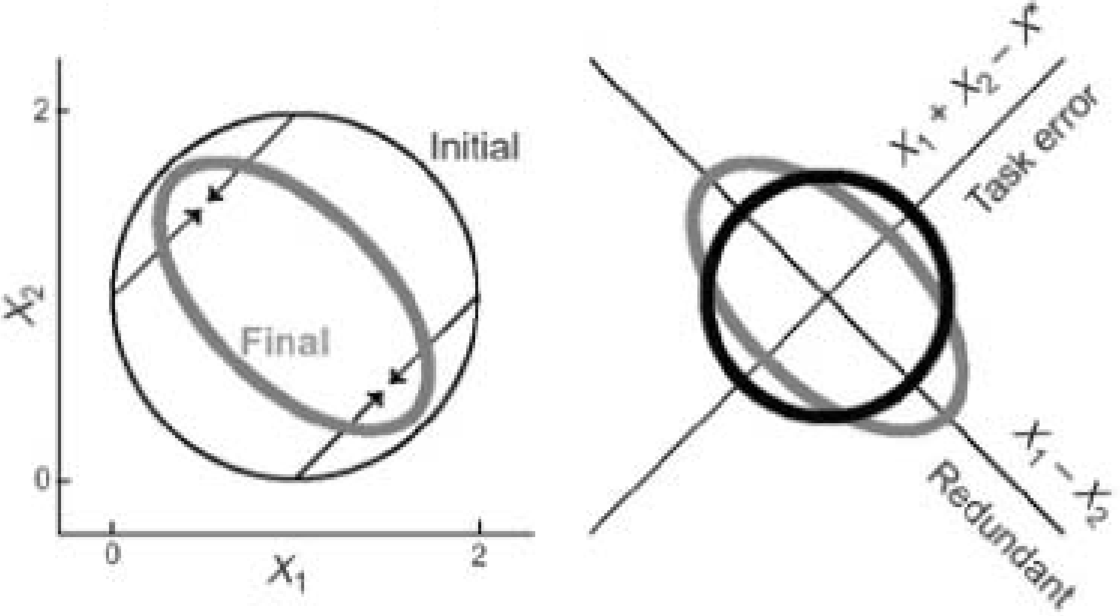
Suboptimal average law: $x_1^{final} = x_2^{final} = X^*/2 \longrightarrow u_i = X^*/2 - ax_i;\ i \in \{1,2\}$

→ no correlations along task-irrelevant directions allowed because no redundancy

Linear–quadratic–Gaussian control and redundancy exploitation

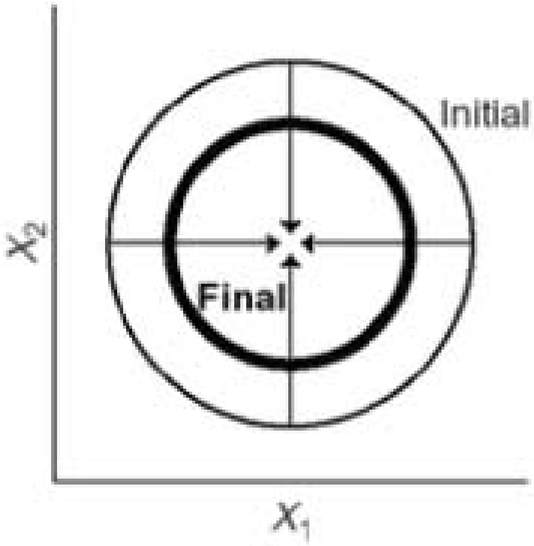
Optimal control:
takes advantage of redundancy



Smaller, synergistic and coupled control signals

« Principle of minimal intervention»

Suboptimal control:
eliminates redundancy



Optimal control: final state variability when intercepting and aiming with 2D point masses

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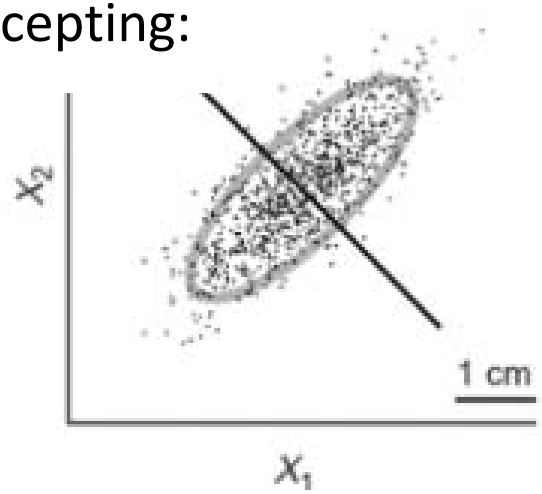
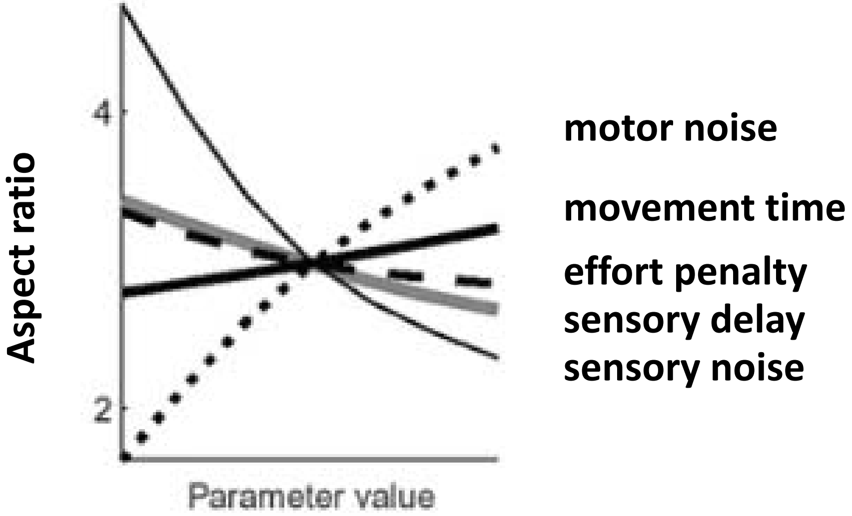
Model of intercepting:

$X_2 = X_1$





Optimal control: final state variability when intercepting and aiming with 2D point masses

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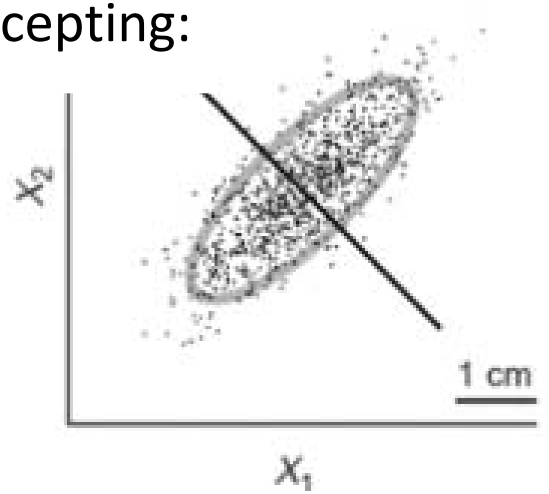
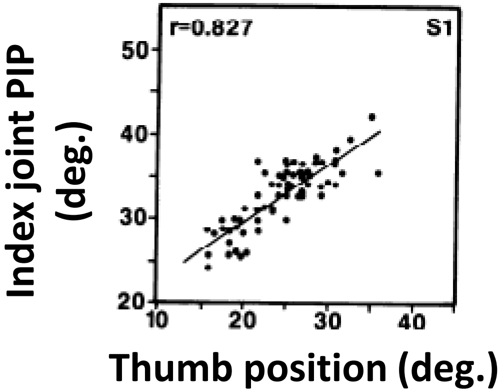


Data: 3 fingers grasp

(Cole and Abbs J. Neurophysiol. 1986)



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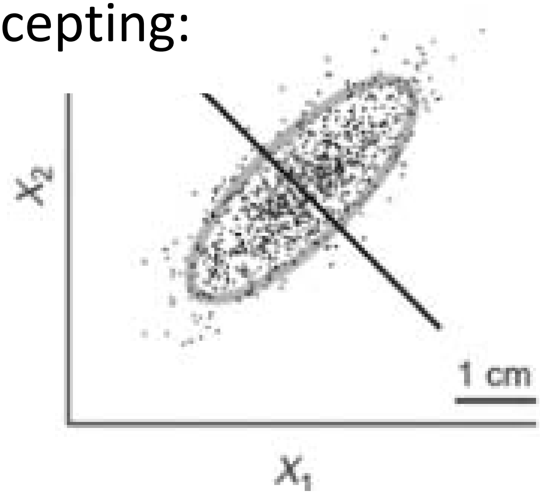

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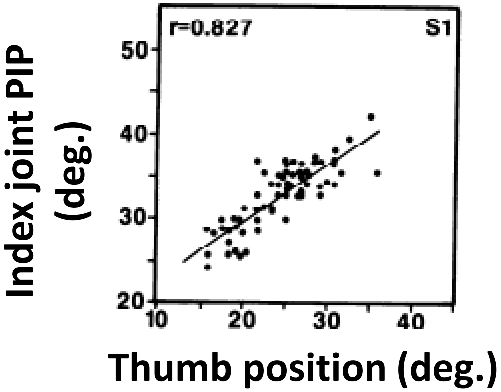
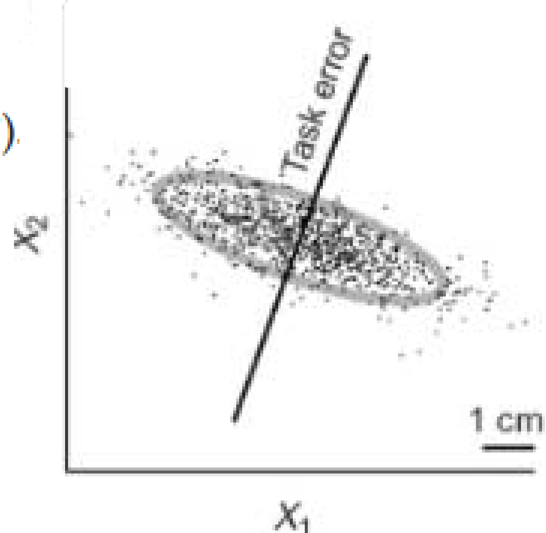


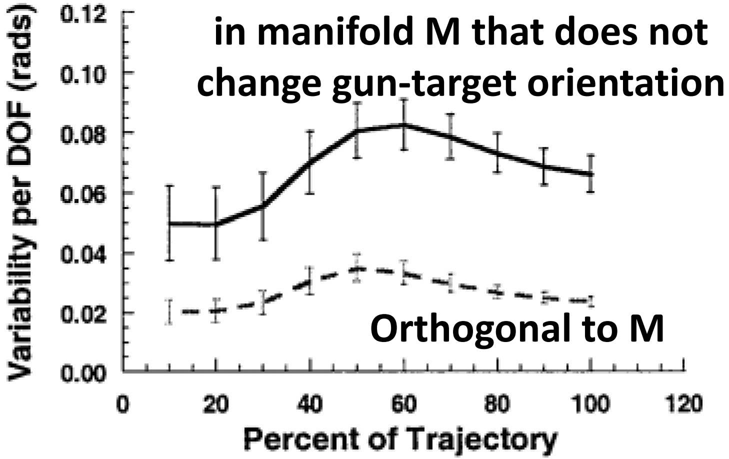
Model of aiming:

$X_2 = X_1 \tan(-20°)$

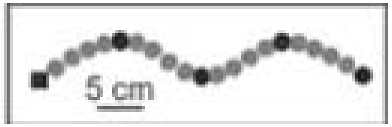


Data: gun aiming

(Scholz et al., Exp. Brain Res., 2000)



Consequences of optimal control for trajectory variability depending on intermediate goals



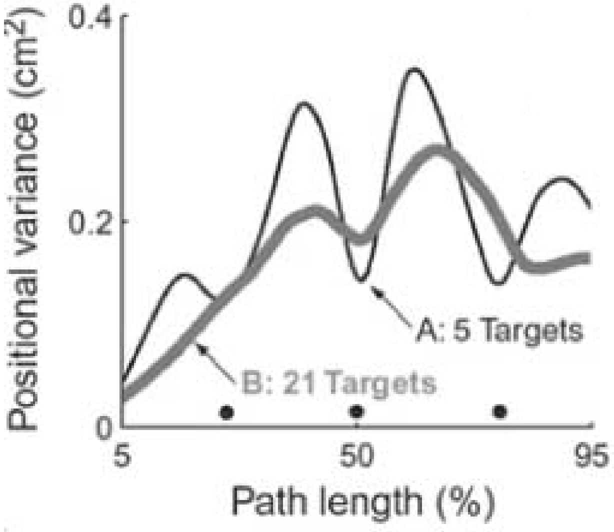
— **5 targets**

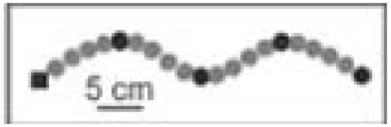
— 21 targets

Similar average trajectory

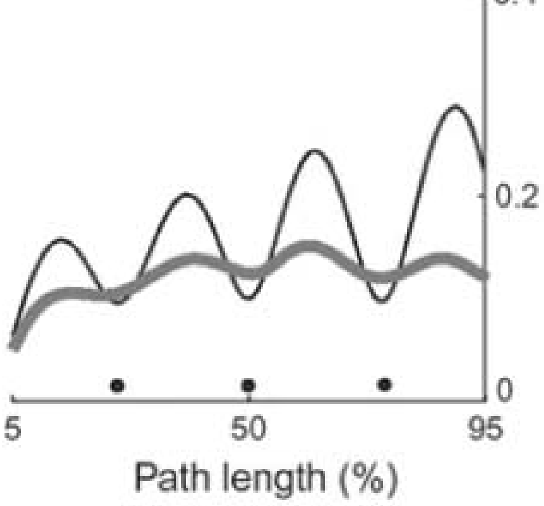
Consequences of optimal control for trajectory variability depending on intermediate goals

Data





Model



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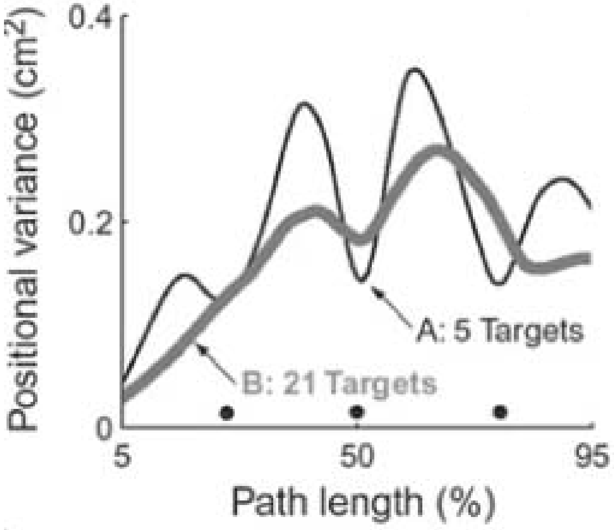

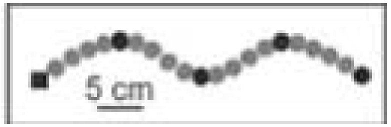

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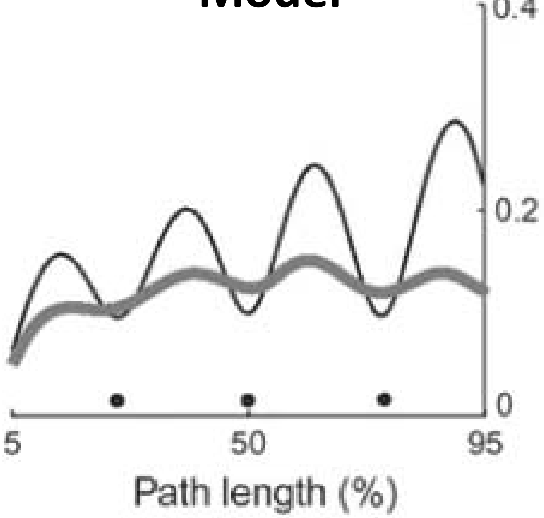
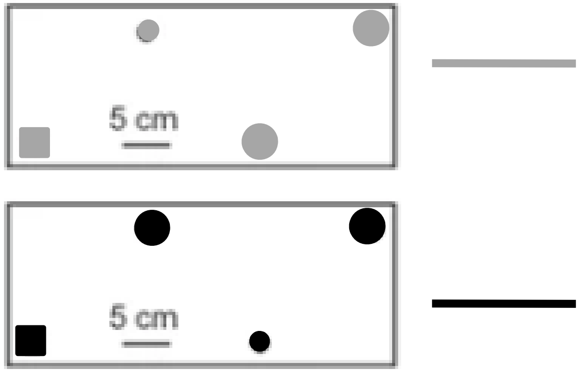




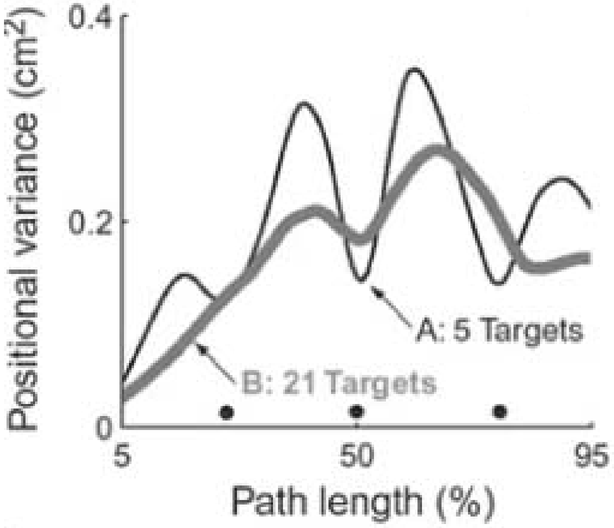
small
first target

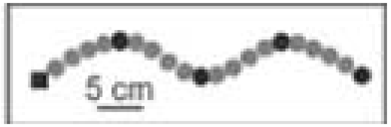
small
second target

Similar average trajectory

Consequences of optimal control for trajectory variability depending on intermediate goals

Data





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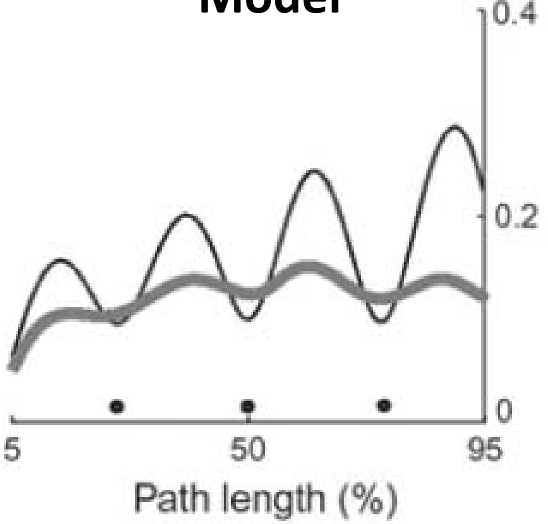
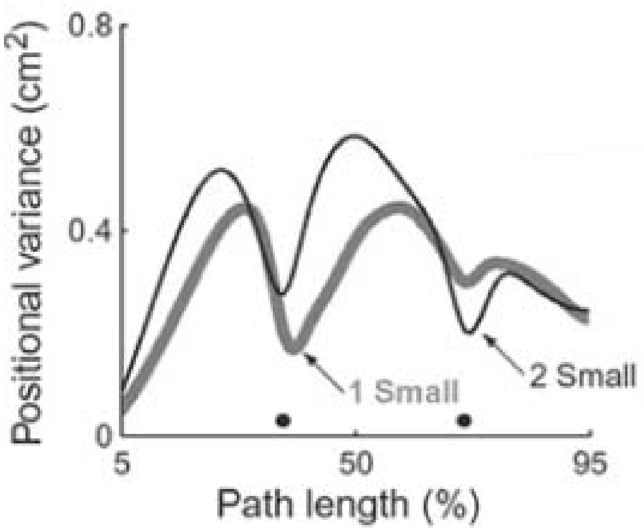
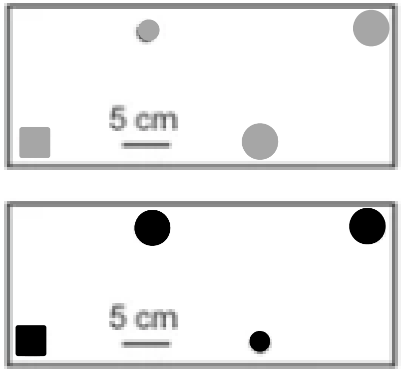
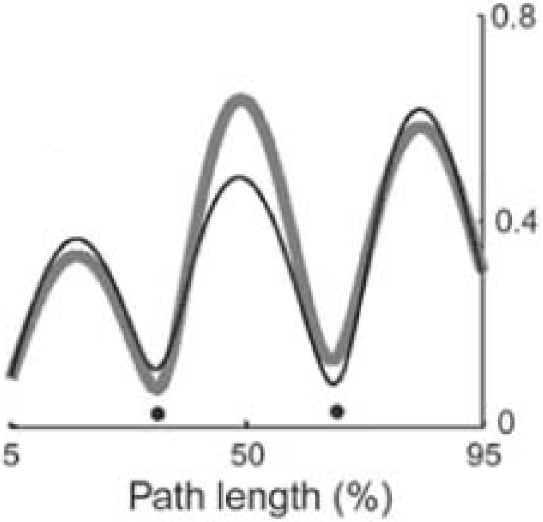


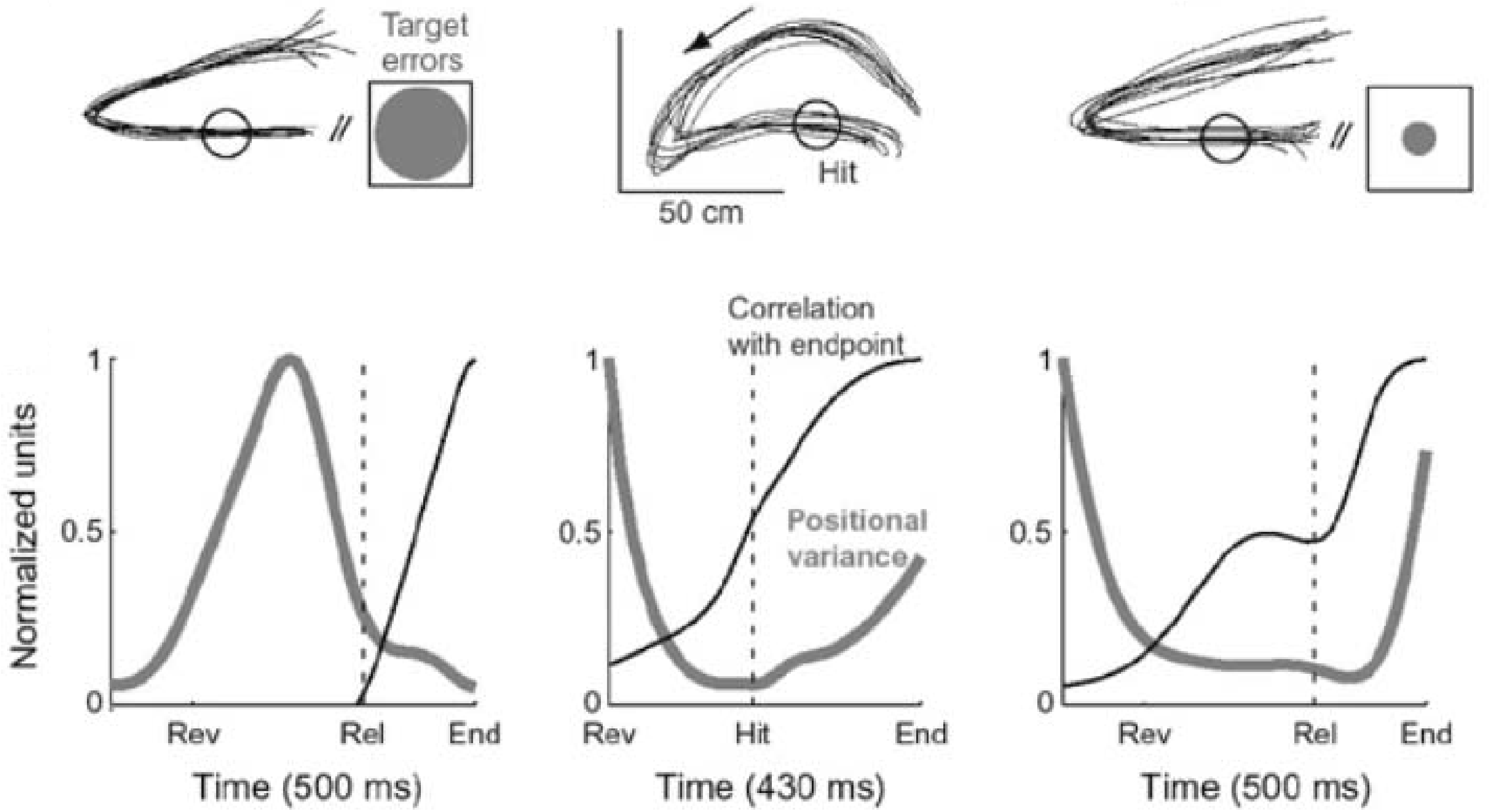






Consequences of optimal control for hitting a ball towards a target (releasing in a certain area)

Trajectory-control model **Data** **Optimal-control model**



Emergent properties of optimal feedback control: different roles assigned to different effectors

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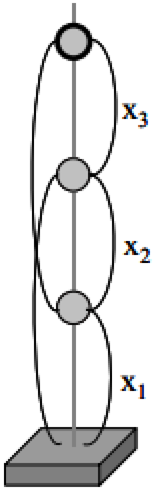
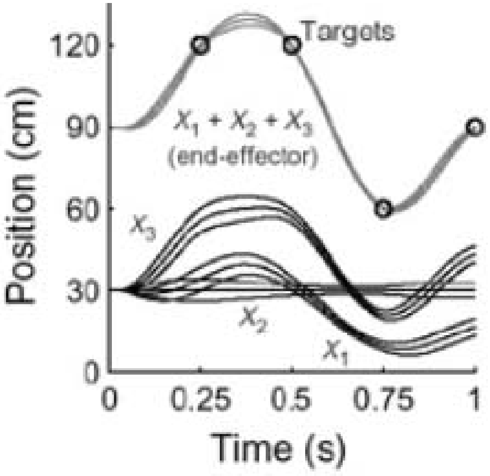
Multijoint « arm » model:



Hit four targets at particular times:



X_2 is used for variability correction in synergy with ‘mean’ actuators

Emergent properties of optimal feedback control: different roles assigned to different effectors

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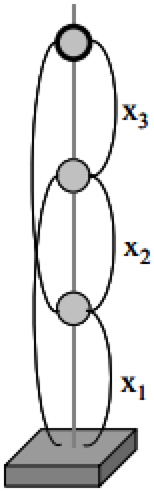

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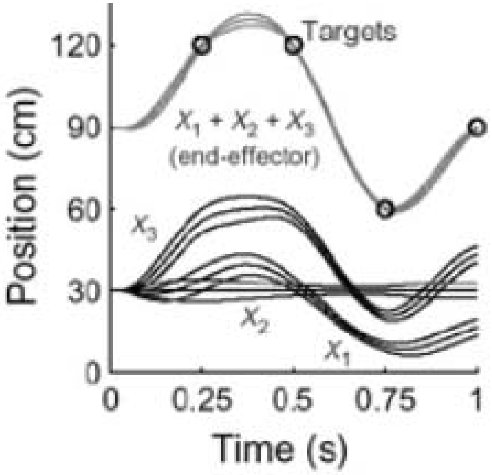
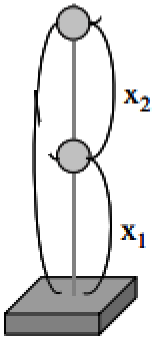
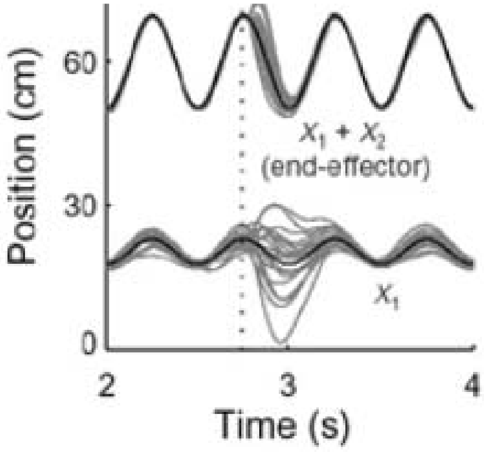
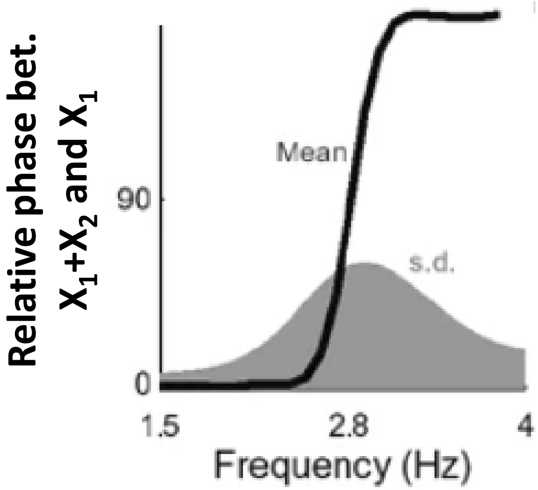


X_2 is used for variability correction in synergy with 'mean' actuators

Track sinusoid with noise injection:







Synergies depend on context: worse coupling around 2.8Hz

Conclusions

- In a wide range of tasks, very good match between data and optimal (noisy) feedback control with multiplicative motor noise
- Mainly focuses on tasks with clear visual targets to reach: different from speech production, acrobatics, dancing, when active sensing is required (Yeo et al., 2016)
- Mathematical approach only applicable to systems with fully explicit linear dynamics (e.g. point masses in simple motion)