ELEC 202 Cheatsheet with LaTeX

Remark. Conventions:

- UPPER CASE symbols (I, V, Z) are in phasor domain.
- Yes, $\imath$ is imaginary and i is current. imaginary $\imath$ are usually at MSB end of the equations.
- all good looking images are from [1]
- turn off independent source is replace voltage source with $v = 0$, which is a short circuit, and current source with open source.
-

Basic Circuit

- Resistance: R = $\Re(Z)$ Ω
- Reactance: X = $\Im(Z)$ Ω
- Impedance: Z = $R + \imath X$ Ω
- Admittance: Y = $\frac{1}{Z} = G + \imath B$ S
- Conductance: G = $\Re(\frac{1}{Z})$ S
- Susceptance: B = $\Im(\frac{1}{Z})$ S

AC steady state: frequency and amplitude are not varying and all current, voltage are at the same frequency(for phasor). If not, use circuit's superposition.
DC steady state: i and v of the circuit stopped changing.

Analysis and Transformation

Laplace Transformation

Definition.
$s = \sigma + \imath\omega$
Laplace: $\mathcal{L}$[f(t)] = F(s) = $\int_{0^-}^{\infty} f(t)e^{-st}dt$
Inverse Laplace: $\mathcal{L}^{-1}$[F(s)] = f(t) = $\frac{1}{2\pi\imath}\int_{\sigma-\imath\infty}^{\sigma+\imath\infty} F(s)e^{st}ds$

See figure 18 and 19 for some transformation and proprieties.
Solve inverse laplace: For transfer function or general F(s), use partial fraction to simplify each terms and look up it's inverse from table 18. What am I kidding just use HP Prime.

Laplace transform of circuit

- $Z(s) = R$
- $Z(s) = sL$
- $Z(s) = \frac{1}{sC}$

Don't forget $u(t) = \frac{1}{s}$, sometimes it's ignored in DC

1. transform circuit element from time to laplace, Take the initial condition into consideration!. Get the steady state(IC) by transform showed in figure 1 for capacitor and figure 2 for inductor. Note that you can use Thevenin theorem to simplify the initial condition.
2. solve for laplace, don't forget the initial conditions
3. transform back to time.

Laplace vs Frequency

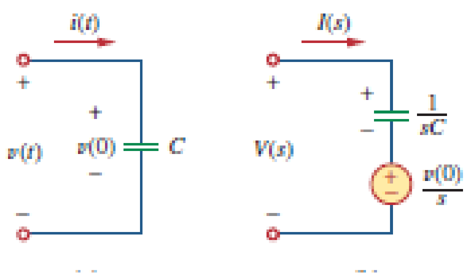

Figure 1

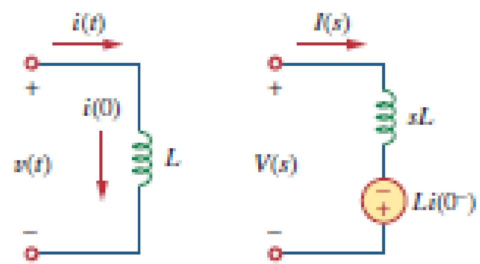

Figure 2

Phasor

Definition. Complex number representation of a sinusoid using amplitude and phase.
$v(t) = V_m \cos(\omega t + \phi) = V = V_m\angle\phi$

- $V_m > 0$
- $-180° < \phi < 180°$
- $\sin(x + 90) = \cos(x)$
- **leading**: In counter clockwise direction positively greater on the complex plane. Else **lagging**.

Operations

$A\angle\Phi_1 \cdot B\angle\Phi_2 = (AB)\angle(\Phi_1 + \Phi_2)$

Transfer Function

Definition. H(s), or transfer function, is the ratio of output response Y(s) to the input excitation X(s), assume the initial condition is zero.

Bode Plot Elements

Definition. Unit of decibel(dB), For the power gain: $G_{dB} = 10\log_1 0(\frac{P_2}{P_1})$

Definition. decade: an interval between two frequency with a ratio of 10.

Definition. Bode plot: 2D plot of Magnitude H (dB) and frequency (decade) as z and x axis. Although there is the third y axis from σ.

Take transfer function of the form:

$$H(w) = \frac{K_1 s^{\pm 1}(s+z_0)(s+z_1)^1 ...(s+z_{n_1})^{n_1}(s+z_z)(s+\overline{z_z})...}{K_2(s+p_0)(s+p_1)^1...(s+p_{n_2})^{n_2}(s+p_z)(s+\overline{p_z})...}$$

- real constant: zero of $\frac{K_1}{K_2}$ or pole of $\frac{K_2}{K_1}$
- real simple: zero of s or pole of $\frac{1}{s}$.
- real n order: zero of $(s + z_{n_1})$ or pole of $(s + p_{n_2})^{n_2}$
- complex: let $\zeta = \cos\Omega$, where $\Omega = 180° - \theta$. Then $p_z = a + \imath b = \omega_0 e^{\imath\Omega}$ $(s + p_z)(s + \overline{p_z})|_{p_z = a+\imath b} = (s^2 + 2\zeta\omega_0 s + \omega_0^2)|_{s=jw} = (\omega_0^2 - \omega^2) + \imath 2\zeta\omega\omega_0$, which $\approx 40\log_1 0\omega_0$ when $\omega \ll \omega_0$ and $\approx 40\log_1 0\omega$ when $\omega \gg \omega_0$. At around ω_0, $H_{dB} \approx 20\log_{10}(\sqrt{2}) \approx 3$

Construct and Analysis of Bode Plot

zeros create a positive gain, where constant is a positive vertical shift of $20\log_{10} K$, real simple is a ramp with $20NdB/decade$ gain that intersect ω axis at 1 and real n order is a unit step ramp with $20NdB/decade$ gain that starts at ω, and complex zero has with $40NdB/decade$ gain instead.
poles has negative gain instead of positive.

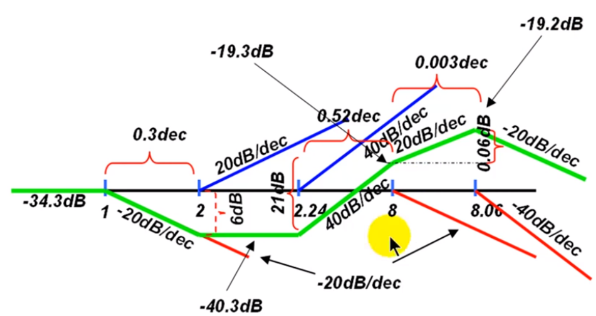

Figure 3

Procedure:

- FInish this

Construct and Analysis of Phase Plot

Definition. Phase plot has degree/dec as y/x axis.

zeros create a -45 degree/dec decline, starting from $0.1\omega_0$ to $10\omega_0$, result in a 90 degree decline. no order zero has -45 N degree/dec decline, result in 90 N degree decline. Complex zero is also centered at ω_0, differ in having $\omega_1 = \frac{\omega_0}{10^\zeta}$ and $\omega_1 = 10^\zeta \omega_0$, adn slope of $-\frac{90}{\zeta}$ degree/dec.

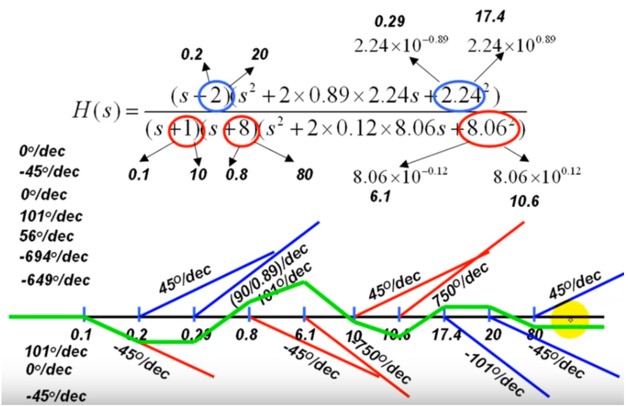

Figure 4

Procedure:

- find the knees of each pole and zero.
- FInish this

Filters

Definition.
In an RLC AC circuit:

- **resonance frequency**: $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/s. At ω_0, result in purely resisitve circuit with no power loss. Resonance circuit is designed to operate at or near resonant frequency.
- **Half power frequency**: $\omega_{1,2} = \pm\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$, and $\omega_0 = \sqrt{\omega_1\omega_2}$
- **Bandwidth** $B = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$
- **Quality Factor**: ration of resonant frequency to its bandwidth.
  $Q = 2\pi\frac{\text{Peak energy stored}}{\text{Energy dissipated in one cycle}}\frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$
- for high-Q circuit with $Q \geq 10$, $\omega_{1,2} \approx \omega_0 \pm \frac{B}{2}$

active vs passive:

type	adv & disadv
active	can amp the input not affected by loads dont need inductor
	less stable not good at high freq
passive	stable can have high freq
	Might have large C, L need tp check cut off freq

Passive Filter

- High Pass
- Low Pass
- Band Pass
- Band Stop

Active Filter

- First Order Low Pass

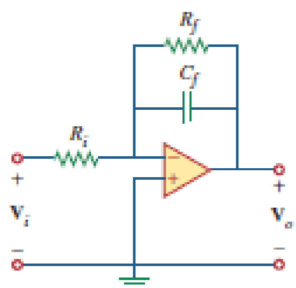

Figure 14.42
Active first-order lowpass filter.

Figure 5

- First Order High Pass

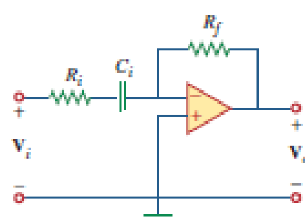

Figure 14.43
Active first-order highpass filter.

Figure 6

- Band Pass

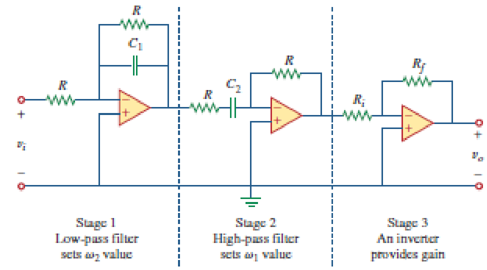

Figure 14.45
Active bandpass filter.

Figure 7

- Band reject

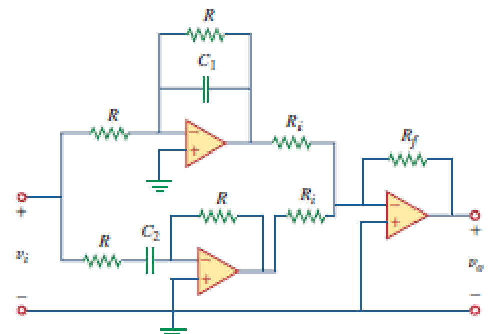

Figure 14.47
Active bandreject filter.

Figure 8

Chebyshev filter

Sallen Key configuration

Butterworth low pass filter

Two Ports network

Definition. Two port network is an electrical network with two separate ports for input and output.

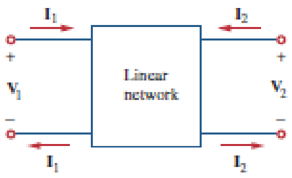

Figure 9

Most basic form:

$$\begin{cases} V_1 = z_{11}I_1 + z_{12}I_2 = \frac{V_1}{I_1}I_1 + \frac{V_2}{I_1}I_2 \\ V_2 = z_{21}I_1 + z_{22}I_2 = \frac{V_1}{I_2}I_1 + \frac{V_2}{I_2}I_2 \end{cases}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Calculate impedance parameter (z)

Method 1: add an voltage source to input of the network to get z_{11}, z_{12}, and do the same for the output.
Method 2: KCL, KVL.

Conversion of two port parameters

z: impedence parameter
y: admittence parameter
t: transmission
Finish this after doing some practices

Power

Definition. Rate of absorbing or releasing energy. As a convention, absorbing is positive and supplying is negative.

Instantaneous power(W):

$$p(t) = \frac{1}{2}V_mI_m\cos(\theta_v - \theta_i) + \frac{1}{2}V_mI_m\cos(2\omega t + \theta_v + \theta_i)$$

Average power(W):

$$P = \frac{1}{2}V_mI_m\cos(\theta_v - \theta_i)$$

Complex power(VA):

$$\begin{aligned} \boldsymbol{S} &= \frac{1}{2}\boldsymbol{VI}^* \\ &= \boldsymbol{V_{rms}I_{rms}}^* \\ &= V_{rms}I_{rms}\angle(\theta_v - \theta_i) \\ &= P + \imath Q \end{aligned}$$

Apparent power(VA):

$$\begin{aligned} S &= |\boldsymbol{S}| \\ &= V_{rms}I_{rms} = \frac{1}{2}V_mI_m \\ &= \sqrt{P^2 + Q^2} \end{aligned}$$

Power factor:

pf lag: I lags or V leads I
pf lead: I leads or V lags I

$$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

Real power(W)

$$P = \Re(\boldsymbol{S}) = S\cos(\theta_v - \theta_i)$$

Reactive power(VAR)

$$Q = \Im(\boldsymbol{S}) = S\sin(\theta_v - \theta_i)$$

Three Phase System

Definition. A system produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.

Balanced three phase

Balanced voltage: source voltage are equal in magnitude.
Balanced load: phase impedance are equal in magnitude and phase.
Line voltage: voltage from line x to line y.
Phase voltage: voltage from line to neutral line
Line current: current in the line, from source(usually left) to right.
Phase current: current of each phase or source or load.
Connection types

- $Y - Y$:
  Provided V_p:

  $$\text{Phase voltage: } \begin{cases} V_{an} = V_p\angle 0° \\ V_{bn} = V_p\angle -120° \\ V_{cn} = V_p\angle 120° \end{cases}$$

  $$\text{Line voltage: } \begin{cases} V_{ab} = V_{an} - V_{bn} = \sqrt{3}V_p\angle 30° \\ V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_p\angle -90° \\ V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_p\angle 150° \end{cases}$$

  $$\text{Line current: } \begin{cases} I_a = \frac{V_{an}}{Z_Y} \\ I_b = I_a\angle -120° \\ I_c = I_a\angle 120° \end{cases}$$

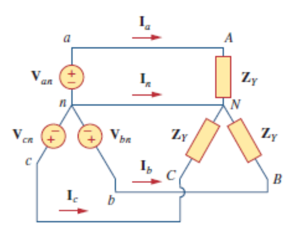

Figure 10

- $\Delta - \Delta$
  Since each load and source are parallel, the phase current can be calculated, and then line current.

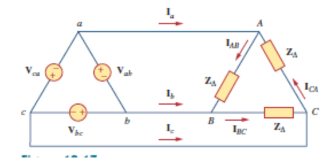

Figure 11

- $Y - \Delta$
  Change the Δ configuration to Y by :
  $\begin{cases} Z_\Delta = 3Z_Y \\ Z_Y = \frac{Z_\Delta}{3} \end{cases}$

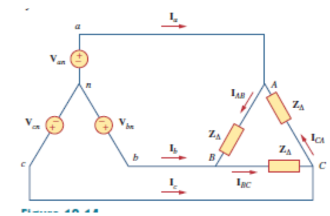

Figure 12

- $\Delta - Y$:

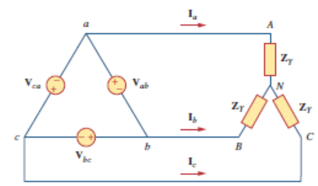

Figure 13

Change Δ to Y: First find V_L with the phase voltage of Δ, then get the line voltage for Y configuration.

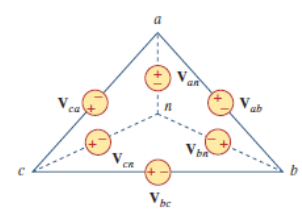

Figure 14

Phase voltage: $\begin{cases} V_{ab} = V_L \angle 0^\circ \\ V_{bc} = V_L \angle -120^\circ \\ V_{ca} = V_L \angle 120^\circ \end{cases}$

Line voltage: $\begin{cases} V_{an} = \frac{1}{\sqrt{3}} V_L \angle -30^\circ \\ V_{bn} = \frac{1}{\sqrt{3}} V_L \angle -150^\circ \\ V_{cn} = \frac{1}{\sqrt{3}} V_L \angle 90^\circ \end{cases}$

Power of Three Phase:

For Y configuration: $\begin{cases} I_p = I_L \\ \sqrt{3} V_p = V_L \end{cases}$

For Δ configuration: $\begin{cases} \sqrt{3} I_p = I_L \\ V_p = V_L \end{cases}$

- Average Power: $3P = 3V_p I_P \cos(\theta_v - \theta_i) = \sqrt{3} V_L I_L \cos(\theta_v - \theta_i)$
- Reactive Power: $3Q = 3V_p I_P \sin(\theta_v - \theta_i) = \sqrt{3} V_L I_L \sin(\theta_v - \theta_i)$
- Apparent Power: $3S = 3V_p I_P = \sqrt{3} V_L I_L$
- Complex Power: $3\boldsymbol{S} = 3V_p I_p \angle\theta \sqrt{3} V_L I_L \angle\theta$

power loss

Unbalanced three phase

Since I_n is no longer 0, take that into account.

Question. Given a three phase system, provides it's complex power and ask for line current, voltage. Ask for ways to raise power factor to unity.

Transformer

There are in total 4 configuration types for parallel coil, two for series. See figure 15, $L_{total} = L_1 + L2 + 2M$. In the other case of series, $L_{total} = L_1 + L2 - 2M$

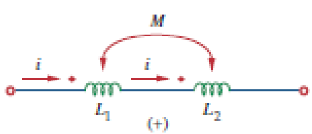

Figure 15

Dot Convention:
if a current enters the doted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

Question. Given circuit components of a transformer circuit, ask for phasor current or impedance of one side, or any missing circuit components.

Method. See figure 16 to simply the circuit, then use KCL to solve. Be careful with the direction of current!

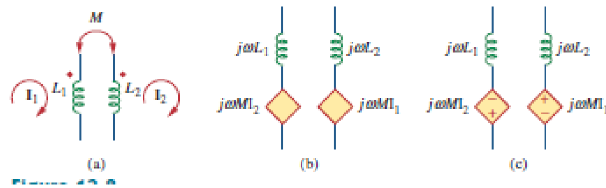

Figure 16

Energy In Transformer

Definition. Instantaneous energy stored:
$w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 \pm M i_1 i_2$, where the $\pm$ is determined by if both current enter or leave dot.
Coupling coefficient: $k = \frac{M}{\sqrt{L_1 L_2}}$ $0 \leq k \leq 1$, measure of magnetic coupling between two coils, if $k > 0.5$, they are tightly coupled, else loosely.

Question. Calculate energy in circuit at given time of a time domain voltage input.

Method. Just plug the left side in there.

Ideal Transformer

Definition. Transformer circuit with the following assumption:

- large reactants: $L_1, L_2, M \to \infty$
- coupling coefficient equals to unity, $k = 1$
- both oils are lossless ($R_1 = R_2 = 0$)

With these assumptions, we can say use turn ratio $n = \frac{N_2}{N1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$.

Question. Given circuit components of a transformer circuit, ask for phasor current or impedance of one side, or any missing circuit components.

Method.

1. See figure 17 to simply the circuit, then use KCL to solve.
2. Assign current and voltage to each coil, use turn ratio to complete KVL/KCL to solve the circuit.

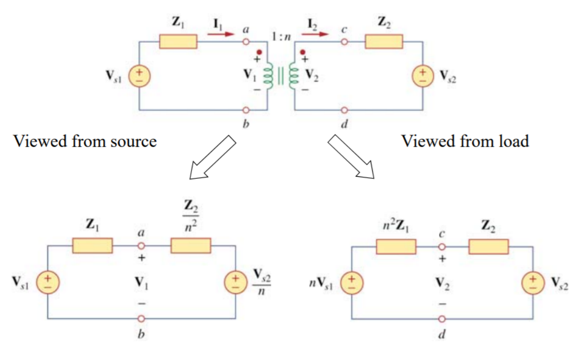

Figure 17

Second Order Circuit

Definition. Any circuits that can be simplify to a second order ODE.

Source-Free

Definition. Any Second order circuits without any independent source. It's steady state will include a steady state inductor and/or capacitor.

Solve

damping factor: $\alpha = \frac{\mathrm{L}}{2\mathrm{R}}$
resonance frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$
After transform a general source free RLC circuit into laplace, solve the characteristic function with $s = -\alpha \pm \sqrt{\alpha^2 + \omega_0^2}$ based on $\Delta = \sqrt{4\alpha^2 - 4w_0^2}$. The eigenvalue gives:

- overdamped: $\alpha > \omega_0$
- critically damped: $\alpha = \omega_0$
- underdamped: $\alpha < \omega_0$

Question. Given a RLC circuit and I_0, V_0 of inductor and capacitor,

Step-Response

Definition. RLC circuit was introduced instantaneously by a source. Solve for $v(t)$ in RLC series cricuit, $i(t)$ in RLC parallel circuit.

Solve

Let $v_t(t)$ be the transient response, $v_{ss}(t)$ be the steady state response. $v_t(t)$ is the homogeneous solution to RLC circuit, where $v_{ss}(t)$ is the partical solution. $v(t) = v_t(t) + v_{ss}(t)$

General Second Order Circuit

- determine $x(0), \frac{\mathrm{d}x}{\mathrm{d}t}, x(\infty)$
- find $x_t(t)$ by turning off independent source, plug in initial condition $x(0), \frac{\mathrm{d}x}{\mathrm{d}t}$.
- $x(t) = x_t(t) + x_{ss}(t)$, where $x_{ss}(t)$ is $x(\infty)$.

Question. Given a circuit, solve for $v(t), i(t)$

Miscs

References

[1] Charles Alexander and Matthew Sadiku. *Fundamentals of Electric Circuits*. McGraw-Hill, Inc., New York, NY, USA, 3 edition, 2007.

[2] Patrick ONeill. Laplace transform table. https://sites.google.com/site/patrickoneill1416/laplace-transform. [Online; accessed 19-April-2019].

HP Primer a Primer

HUGE TABLE INCOMING

Command	Parameter	Function
laplace	(f(t), t, s)	
ilaplace	(F(s), s, t)	inverse laplace
partfrac	(f(x))	partial fraction
evalf	(f(x))	
proot		
normal		
solve		

Function	Time domain $f(t) = \mathcal{L}^{-1}\{F(s)\}$	Laplace s-domain $F(s) = \mathcal{L}\{f(t)\}$	Region of convergence	Reference
unit impulse	$\delta(t)$	1	all s	inspection
delayed impulse	$\delta(t-\tau)$	$e^{-\tau s}$		time shift of unit impulse
unit step	$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} > 0$	integrate unit impulse
delayed unit step	$u(t-\tau)$	$\frac{e^{-\tau s}}{s}$	$\mathrm{Re}\{s\} > 0$	time shift of unit step
ramp	$t \cdot u(t)$	$\frac{1}{s^2}$	$\mathrm{Re}\{s\} > 0$	integrate unit impulse twice
delayed nth power with frequency shift	$\frac{(t-\tau)^n}{n!} e^{-\alpha(t-\tau)} \cdot u(t-\tau)$	$\frac{e^{-\tau s}}{(s+\alpha)^{n+1}}$	$\mathrm{Re}\{s\} > -\alpha$	Integrate unit step, apply frequency shift, apply time shift
nth power (for integer n)	$\frac{t^n}{n!} \cdot u(t)$	$\frac{1}{s^{n+1}}$	$\mathrm{Re}\{s\} > 0$ $(n > -1)$	Integrate unit step n times
qth power (for complex q)	$\frac{t^q}{\Gamma(q+1)} \cdot u(t)$	$\frac{1}{s^{q+1}}$	$\mathrm{Re}\{s\} > 0$ $(\mathrm{Re}\{q\} > -1)$	ref?
nth power with frequency shift	$\frac{t^n}{n!} e^{-\alpha t} \cdot u(t)$	$\frac{1}{(s+\alpha)^{n+1}}$	$\mathrm{Re}\{s\} > -\alpha$	Integrate unit step, apply frequency shift
exponential decay	$e^{-\alpha t} \cdot u(t)$	$\frac{1}{s+\alpha}$	$\mathrm{Re}\{s\} > -\alpha$	Frequency shift of unit step
two-sided exponential decay	$e^{-\alpha\lvert t\rvert}$	$\frac{2\alpha}{\alpha^2 - s^2}$	$-\alpha < \mathrm{Re}\{s\} < \alpha$	Frequency shift of unit step
exponential approach	$(1-e^{-\alpha t}) \cdot u(t)$	$\frac{\alpha}{s(s+\alpha)}$	$\mathrm{Re}\{s\} > 0$	Unit step minus exponential decay
sine	$\sin(\omega t) \cdot u(t)$	$\frac{\omega}{s^2+\omega^2}$	$\mathrm{Re}\{s\} > 0$	ref?
cosine	$\cos(\omega t) \cdot u(t)$	$\frac{s}{s^2+\omega^2}$	$\mathrm{Re}\{s\} > 0$	ref?
hyperbolic sine	$\sinh(\alpha t) \cdot u(t)$	$\frac{\alpha}{s^2-\alpha^2}$	$\mathrm{Re}\{s\} > \lvert\alpha\rvert$	ref?
hyperbolic cosine	$\cosh(\alpha t) \cdot u(t)$	$\frac{s}{s^2-\alpha^2}$	$\mathrm{Re}\{s\} > \lvert\alpha\rvert$	ref?
Exponentially-decaying sine wave	$e^{-\alpha t} \sin(\omega t) \cdot u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$	$\mathrm{Re}\{s\} > -\alpha$	ref?
Exponentially-decaying cosine wave	$e^{-\alpha t} \cos(\omega t) \cdot u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$	$\mathrm{Re}\{s\} > -\alpha$	ref?
nth root	$\sqrt[n]{t} \cdot u(t)$	$s^{-(n+1)/n} \cdot \Gamma\left(1+\frac{1}{n}\right)$	$\mathrm{Re}\{s\} > 0$	ref?
natural logarithm	$\ln\left(\frac{t}{t_0}\right) \cdot u(t)$	$-\frac{t_0}{s}\left[\ln(t_0 s) + \gamma\right]$	$\mathrm{Re}\{s\} > 0$	ref?
Bessel function of the first kind, of order n	$J_n(\omega t) \cdot u(t)$	$\frac{\left(\sqrt{s^2+\omega^2}-s\right)^n}{\omega^n \sqrt{s^2+\omega^2}}$	$\mathrm{Re}\{s\} > 0$ $(n > -1)$	ref?
Error function	$\mathrm{erf}(t) \cdot u(t)$	$\frac{e^{s^2/4}\left(1-\mathrm{erf}(s/2)\right)}{s}$	$\mathrm{Re}\{s\} > 0$	ref?

Figure 18: Table of Laplace [2]

Properties of the unilateral Laplace transform

	Time domain	's' domain	Comment
Linearity	$af(t)+bg(t)$	$aF(s)+bG(s)$	Can be proved using basic rules of integration.
Frequency differentiation	$tf(t)$	$-F'(s)$	F' is the first derivative of F.
Frequency differentiation	$t^n f(t)$	$(-1)^n F^{(n)}(s)$	More general form, n^{th} derivative of F(s).
Differentiation	$f'(t)$	$sF(s)-f(0)$	f is assumed to be a differentiable function, and its derivative is assumed to be of exponential type. This can then be obtained by integration by parts
Second Differentiation	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$	f is assumed twice differentiable and the second derivative to be of exponential type. Follows by applying the Differentiation property to $f'(t)$.
General Differentiation	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$	f is assumed to be n-times differentiable, with n^{th} derivative of exponential type. Follow by mathematical induction.
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma)\,d\sigma$	
Integration	$\int_0^t f(\tau)\,d\tau=(u*f)(t)$	$\frac{1}{s}F(s)$	$u(t)$ is the Heaviside step function. Note $(u*f)(t)$ is the convolution of $u(t)$ and $f(t)$.
Time scaling	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$	
Frequency shifting	$e^{at}f(t)$	$F(s-a)$	
Time shifting	$f(t-a)u(t-a)$	$e^{-as}F(s)$	$u(t)$ is the Heaviside step function
Multiplication	$f(t)g(t)$	$\frac{1}{2\pi i}\lim_{T\to\infty}\int_{c-iT}^{c+iT}F(\sigma)G(s-\sigma)\,d\sigma$	the integration is done along the vertical line $Re(\sigma)=c$ that lies entirely within the region of convergence of F.[12]
Convolution	$(f*g)(t)=\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)\cdot G(s)$	$f(t)$ and $g(t)$ are extended by zero for $t<0$ in the definition of the convolution.
Complex conjugation	$f^*(t)$	$F^*(s^*)$	
Cross-correlation	$f(t)\star g(t)$	$F^*(-s^*)\cdot G(s)$	
Periodic Function	$f(t)$	$\frac{1}{1-e^{-Ts}}\int_0^T e^{-st}f(t)\,dt$	$f(t)$ is a periodic function of period T so that $f(t)=f(t+T),\ \forall t\geq 0$. This is the result of the time shifting property and the geometric series.

Figure 19: Operation of Laplace [2]