ELEC 202 Cheatsheet with LaTeX

Remark. Conventions:

- UPPER CASE symbols (I, V, Z) are in phasor domain.
- Yes, $\imath$ is imaginary and i is current. imaginary $\imath$ are usually at MSB end of the equations.
- all good looking images are from [1]
- turn off independent source is replace voltage source with $v = 0$, which is a short circuit, and current source with open source.

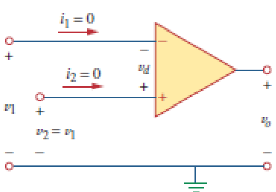
Basic Circuit

Terminology

- Resistance: R = $\Re(Z)\ \Omega$
- Reactance: X = $\Im(Z)\ \Omega$
- Impedance: Z = $R + \imath X\ \Omega$
- Admittance: Y = $\frac{1}{Z} = G + \imath B\ S$
- Conductance: G = $\Re(\frac{1}{Z})\ S$
- Susceptance: B = $\Im(\frac{1}{Z})\ S$

Operational Amplifier

Ideal Op Amp:



Ideal Opamp

- $i_1 = i_2 = 0$
- $v_1 = v_2$

Inverting Op Amp:

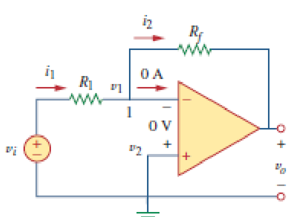


Figure 1: Inverting opamp

- $v_0 = -\frac{R_f}{R_1} v_i$

AC steady state: frequency and amplitude are not varying and all current, voltage are at the same frequency(for phasor). If not, use circuit's superposition.
DC steady state: i and v of the circuit stopped changing.

Analysis and Transformation

Laplace Transformation

Definition.
$s = \sigma + \imath\omega$
Laplace: $\mathcal{L}$[f(t)] = F(s) = $\int_{0^-}^{\infty} f(t)e^{-st}dt$
Inverse Laplace: $\mathcal{L}^{-1}$[F(s)] = f(t) = $\frac{1}{2\pi\imath}\int_{\sigma-\imath\infty}^{\sigma+\imath\infty} F(s)e^{st}ds$

See figure 24 for some transformation and proprieties.
Solve inverse laplace: For transfer function or general F(s), use partial fraction to simplify each terms and look up it's inverse from table 24. What am I kidding just use HP Prime.

Laplace transform of circuit

- $Z(s) = R$
- $Z(s) = sL$
- $Z(s) = \frac{1}{sC}$

Don't forget $u(t) = \frac{1}{s}$, sometimes it's ignored in DC

1. transform circuit element from time to laplace, Take the initial condition into consideration!. Get the steady state(IC) by transform showed in figure 2 for capacitor and figure 3 for inductor. Note that you can use Thevenin theorem to simplify the initial condition.
2. solve for laplace, don't forget the initial conditions
3. transform back to time.

Laplace vs Frequency

(Quick and dirty): In we analysis the laplace domain, for frequency domain, we use Fourier Transform and replace
$s = \imath\omega$
Better explanations deals that Laplace is used for stability studies and Fourier is used for sinusoidal responses of systems.

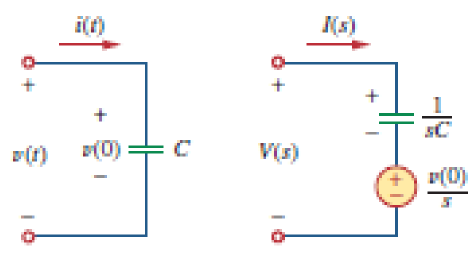


Figure 2: capacitor from time to laplace

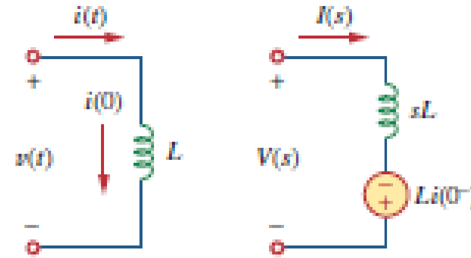


Figure 3: inductor from time to laplace

Phasor

Definition. Complex number representation of a sinusoid using amplitude and phase.
$v(t) = V_m \cos(\omega t + \phi) = V = V_m \angle\phi$

- $V_m > 0$
- $-180° < \phi < 180°$
- $\sin(x + 90) = \cos(x)$
- **leading**: In counter clockwise direction positively greater on the complex plane. Else **lagging**.

Operations

$A\angle\Phi_1 \cdot B\angle\Phi_2 = (AB)\angle(\Phi_1 + \Phi_2)$

Question.

- Basic complex number calculations.
- Use phasor to solve AC circuits: given operating frequency, ask for the value of components.
- Find Thevenin equivalent circuit
- Superposition of inputs in different frequency. ASN 3

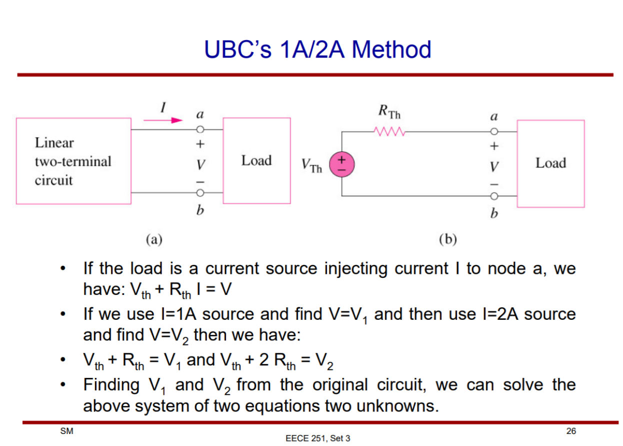


Figure 4: [2]

Method. **1A/2A method:**

- Replace load with 1A/2A current source.
- Find V_{1A}, V_{2A}.
- Solve V_{th}, R_{th} with $\begin{cases} V_{th}+R_{th}=V_{1A} \\ V_{th}+2R_{th}=V_{2A} \end{cases}$

Transfer Function

Definition. H(s), or transfer function, is the ratio of output response Y(s) to the input excitation X(s), assume the initial condition is zero.
Stability: If the transfer function has no pole in the right half-plane, it's stable.

Bode Plot Elements

Definition. Unit of decibel(dB), For the power gain: $G_{dB}=10\log_{10}(\frac{P_2}{P_1})$. For voltage and current, $G_{dB}=20\log_{10}(\frac{V_2}{V_1})$
decade: an interval between two frequency with a ratio of 10.
Bode plot: 2D plot of Magnitude H (dB) and frequency (decade) as z and x axis. Although there is the third y axis from σ.

Construct and Analysis of Bode Plot

General Transfer Function

$$H(w)=\frac{K_1 s^{\pm 1}(s+z_0)(s+z_1)^1...(s+z_{n_1})^{n_1}(s+z_z)(s+\overline{z_z})...}{K_2(s+p_0)(s+p_1)^1...(s+p_{n_2})^{n_2}(s+p_z)(s+\overline{p_z})...}$$

- real constant: zero of $\frac{K_1}{K_2}$ or pole of $\frac{K_2}{K_1}$
- real simple: zero of s or pole of $\frac{1}{s}$.
- real n order: zero of $(s+z_{n_1})$ or pole of $(s+p_{n_2})^{n_2}$
- complex: let $\zeta=\cos\Omega$, where $\Omega=180°-\theta$. Then $p_z=a+\imath b=\omega_0 e^{\imath\Omega}$. Keep in mind that nit all s^2+as+b are complex. $(s+p_z)(s+\overline{p_z})|_{p_z=a+\imath b}=(s^2+2\zeta\omega_0 s+\omega_0^2)|_{s=jw}=(\omega_0^2-\omega^2)+\imath 2\zeta\omega\omega_0$, which $\approx 40\log_1 0\omega_0$ when $\omega\ll\omega_0$ and $\approx 40\log_1 0\omega$ when $\omega\gg\omega_0$. At around ω_0, $H_{dB}\approx 20\log_{10}(\sqrt{2})\approx 3$

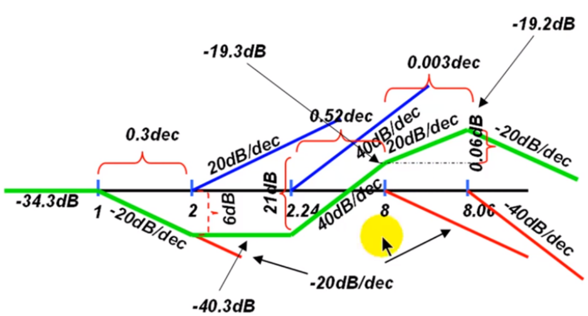


Figure 5

zeros create a positive gain, where constant is a positive vertical shift of $20\log_{10}K$, real simple is a ramp with $20NdB/decade$ gain that intersect ω axis at 1 and real n order is a unit step ramp with $20NdB/decade$ gain that starts at ω, and complex zero has with $40NdB/decade$ gain instead. poles has negative gain instead of positive.

Phase Plot

Phase plot has degree/dec as y/x axis. Zeros create a -45 degree/dec decline, starting from $0.1\omega_0$ to $10\omega_0$, result in a 90 degree decline. no order zero has -45 N degree/dec decline, result in 90 N degree decline. Complex zero is also centered at ω_0, differ in having $\omega_1=\frac{\omega_0}{10^\zeta}$ and $\omega_1=10^\zeta\omega_0$, adn slope of $-\frac{90}{\zeta}$ degree/dec.

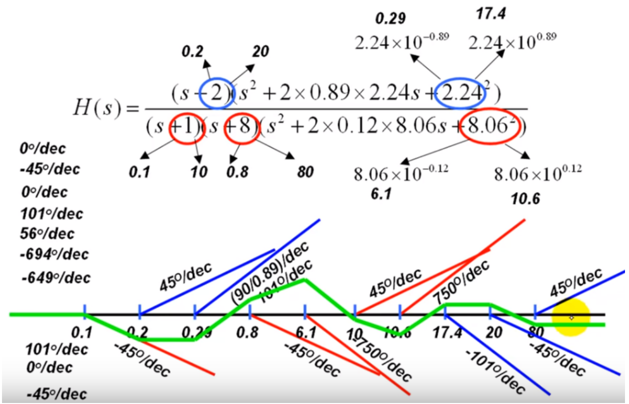


Figure 6

Plot Bode plot on HP Prime

Question.
ASN4, 5
Transfer function

- Construct Transfer function from circuit.
- Given Transfer function, ask for the value of each components.

Bode Plot

- Approx vs Actual: use Bode plot to approx, use transfer function to calculate the actual values.
- Given a transfer function, plot the Bode plot and Phase plot.

Filters

Definition.
A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

Terminologies: In an RLC AC circuit,

- **Resonance Frequency**:
$$\omega_0=\frac{1}{\sqrt{LC}}\ rad/s$$
At ω_0, result in purely resisitve circuit with no power loss. Resonance circuit is designed to operate at or near resonant frequency.
- **Half power frequency**:
$$\omega_{1,2}=\pm\frac{R}{2L}+\sqrt{(\frac{R}{2L})^2+\frac{1}{LC}}$$
, and $\omega_0=\sqrt{\omega_1\omega_2}$
- **Bandwidth** $B=\omega_2-\omega_1=\frac{\omega_0}{Q}$
- **Quality Factor**: ration of resonant frequency to its bandwidth.
$$Q=2\pi\frac{\text{Peak energy stored}}{\text{Energy dissipated in one cycle}}$$
Q is different for series and parallel circuit: Series: $Q=\frac{\omega_0 L}{R}$, Parallel: $Q=\frac{R}{\omega_0 L}$
- for **high-Q** circuit with $Q\geq 10$, $\omega_{1,2}\approx\omega_0\pm\frac{B}{2}$
- **Error**: at cutoff frequency, if it's a simple pole/zero, the error is 3dB.

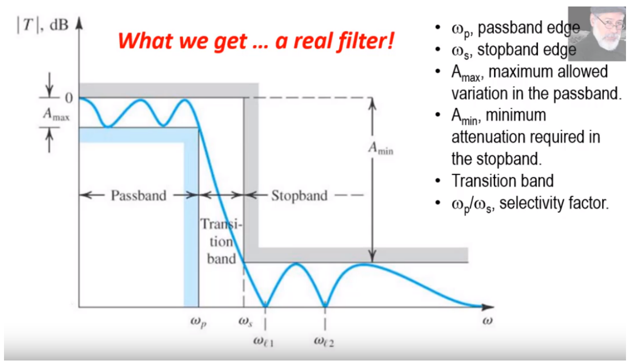


Figure 7: Elements of real filter

Active vs Passive Filters:

type	adv & disadv
active	can amp the input not affected by loads dont need inductor
	less stable not good at high freq
passive	stable can have high freq
	Might have large C, L need tp check cut off freq

Passive Filter

- High and Low Pass: cutoff frequency(half power, corner) $w_c = \frac{1}{RC}$, or w_c such that $|H(w_c)| = \frac{1}{\sqrt{2}}$
- Band Pass
- Band Stop

Question.

- Given components, find the cutoff/corner frequency, or the band corners. ASN6
- Determine type of filter.
- given frequency and band, find the value of components.

Active Filter

- First Order Low Pass

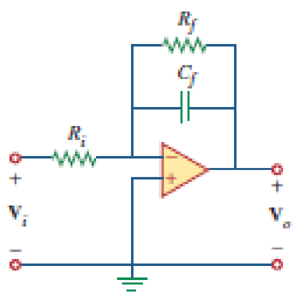


Figure 14.42 Active first-order lowpass filter.

Figure 8

- First Order High Pass

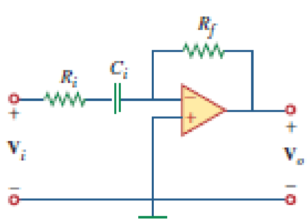


Figure 14.43 Active first-order highpass filter.

Figure 9

- Band Pass Lower Pass $\rightarrow$ High Pass $\rightarrow$ Inverter

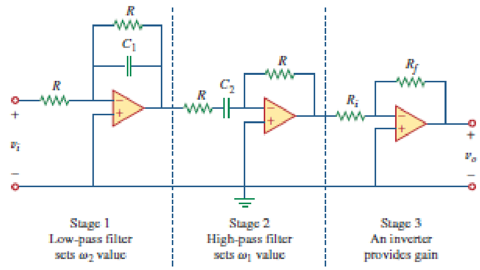


Figure 14.45 Active bandpass filter.

Figure 10

- Band reject Lower Pass and Highpass $\rightarrow$ Summing opamp.

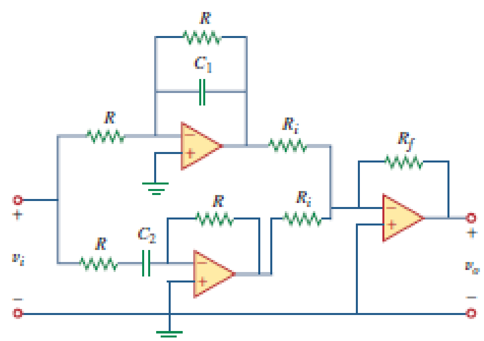


Figure 14.47 Active bandreject filter.

Figure 11

Question.

- Design a (type of function) filter with pass frequency between x and y Hz, with K(gain) of z
- Given a filter configuration, ask for the value of some components in order to archive certain affects.
- Midterm 3: given a filter network, as for passband, frequencies, component values.

Scaling

Definition. Obtain the desired value of an unknown filter from a know filter (usually taking the elementary value of 1Ω, $1H$, $1F$).

Magnitude Scaling:
Process of increasing all impedance in a network by a factor, the frequency response remaining unchanged
Impedance Scaling:
Process of shifting the frequency response of a network up or down the frequency axis while leaving the impedance the same.

Scaling Calculation

Let K_m be the magnitude scaling, K_f be the frequency scaling.

$$R' = K_m R,\ L' = \frac{K_m}{K_f} L$$

$$C' = \frac{C}{K_m K_f},\ \omega' = K_f \omega$$

On webwork, all filters related to scaling are in its **canonical form**, where $\omega = 1$ rad/s

Advanced Filter Configurations

Chebyshev filter

Definition. It's an analog or digital filters having a steeper roll-off and more passband ripple (type I) or stopband ripple (type II) than Butterworth filters. Chebyshev filters have the property that they minimize the error between the idealized and the actual filter characteristic over the range of the filter but with ripples in the passband.

Sallen Key configuration

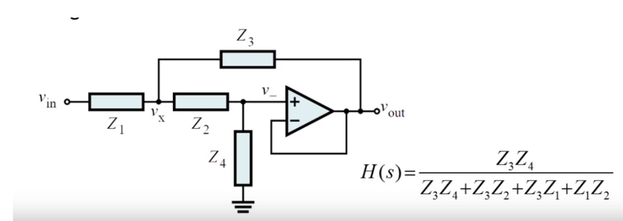


Figure 12

Butterworth low pass filter

Definition. A type of signal processing filter designed to have a frequency response as flat as possible in the passband.

Two Ports network

Definition. Two port network is an electrical network with two separate ports for input and output.

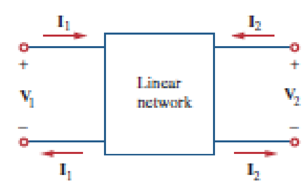


Figure 13

Most basic form:

$$\begin{cases} V_1 = z_{11}I_1 + z_{12}I_2 = \frac{V_1}{I_1}I_1 + \frac{V_2}{I_1}I_2 \\ V_2 = z_{21}I_1 + z_{22}I_2 = \frac{V_1}{I_2}I_1 + \frac{V_2}{I_2}I_2 \end{cases}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Calculate impedance parameter (z)

Method 1: add an voltage source to input of the network to get z_{11}, z_{12}, and do the same for the output.
Method 2: KCL, KVL.

Conversion of two port parameters

z: impedence parameter
y: admittence parameter
t: transmission
Search up the conversion from figure 25

Power

Definition. Rate of absorbing or releasing energy. As a convention, absorbing is positive and supplying is negative.

Instantaneous power(W):

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Average power(W):

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Complex power(VA):

$$\begin{aligned} \boldsymbol{S} &= \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^* \\ &= \boldsymbol{V_{rms}}\boldsymbol{I_{rms}}^* \\ &= V_{rms}I_{rms}\angle(\theta_v - \theta_i) \\ &= P + \imath Q \end{aligned}$$

Apparent power(VA):

$$\begin{aligned} S &= |\boldsymbol{S}| \\ &= V_{rms}I_{rms} = \frac{1}{2}V_m I_m \\ &= \sqrt{P^2 + Q^2} \end{aligned}$$

Power factor:
pf lag: I lags or V leads I
pf lead: I leads or V lags I

$$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

Real power(W)

$$P = \Re(\boldsymbol{S}) = S\cos(\theta_v - \theta_i)$$

Reactive power(VAR)

$$Q = \Im(\boldsymbol{S}) = S\sin(\theta_v - \theta_i)$$

Three Phase System

Definition. A system produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.

Balanced three phase

Balanced voltage: source voltage are equal in magnitude.
Balanced load: phase impedance are equal in magnitude and phase.
Line voltage: voltage from line x to line y.
Phase voltage: voltage from line to neutral line
Line current: current in the line, from source(usually left) to right.
Phase current: current of each phase or source or load.
Connection types

- $Y - Y$:
  Provided V_p:

  Phase voltage: $\begin{cases} V_{an} = V_p\angle 0° \\ V_{bn} = V_p\angle -120° \\ V_{cn} = V_p\angle 120° \end{cases}$

  Line voltage: $\begin{cases} V_{ab} = V_{an} - V_{bn} = \sqrt{3}V_p\angle 30° \\ V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_p\angle -90° \\ V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_p\angle 150° \end{cases}$

  Line current: $\begin{cases} I_a = \frac{V_{an}}{Z_Y} \\ I_b = I_a\angle -120° \\ I_c = I_a\angle 120° \end{cases}$

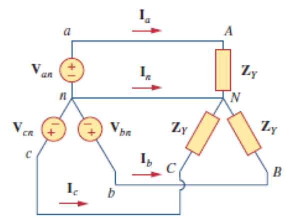


Figure 14

- $\Delta - \Delta$
  Since each load and source are parallel, the phase current can be calculated, and then line current.

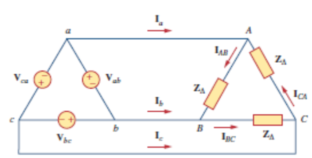


Figure 15

- $Y - \Delta$
  Change the Δ configuration to Y by :
  $\begin{cases} Z_\Delta = 3Z_Y \\ Z_Y = \frac{Z_\Delta}{3} \end{cases}$

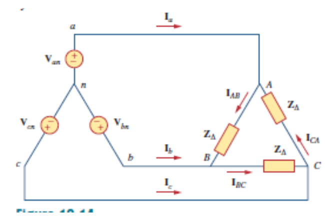


Figure 16

- $\Delta - Y$:

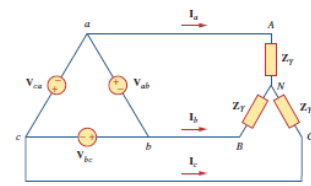


Figure 17

Change Δ to Y: First find V_L with the phase voltage of Δ, then get the line voltage for Y configuration.

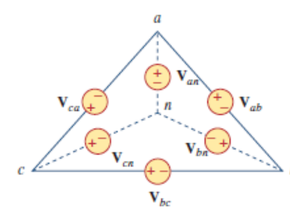


Figure 18

Phase voltage: $\begin{cases} V_{ab} = V_L\angle 0° \\ V_{bc} = V_L\angle -120° \\ V_{ca} = V_L\angle 120° \end{cases}$

Line voltage: $\begin{cases} V_{an} = \frac{1}{\sqrt{3}}V_L\angle -30° \\ V_{bn} = \frac{1}{\sqrt{3}}V_L\angle -150° \\ V_{cn} = \frac{1}{\sqrt{3}}V_L\angle 90° \end{cases}$

Power of Three Phase:
For Y configuration: $\begin{cases} I_p = I_L \\ \sqrt{3}V_p = V_L \end{cases}$

For Δ configuration: $\begin{cases}\sqrt{3}I_p = I_L\\ V_p = V_L\end{cases}$

- Average Power: $3P = 3V_pI_P\cos(\theta_v - \theta_i) = \sqrt{3}V_LI_L\cos(\theta_v - \theta_i)$
- Reactive Power: $3Q = 3V_pI_P\sin(\theta_v - \theta_i) = \sqrt{3}V_LI_L\sin(\theta_v - \theta_i)$
- Apparent Power: $3S = 3V_pI_P = \sqrt{3}V_LI_L$
- Complex Power: $3\boldsymbol{S} = 3V_pI_p\angle\theta\sqrt{3}V_LI_L\angle\theta$

power loss in three phase.

Unbalanced three phase

Since I_n is no longer 0, take that into account.

Question. Given a three phase system, provides it's complex power and ask for line current, voltage. Ask for ways to raise power factor to unity.

Transformer

There are in total 4 configuration types for parallel coil, two for series. See figure 19, $L_{total} = L_1 + L2 + 2M$. In the other case of series, $L_{total} = L_1 + L2 - 2M$

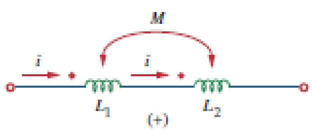


Figure 19

Dot Convention:
if a current enters the doted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

Question. Given circuit components of a transformer circuit, ask for phasor current or impedance of one side, or any missing circuit components.

Method. See figure 20 to simply the circuit, then use KCL to solve. Be careful with the direction of current!

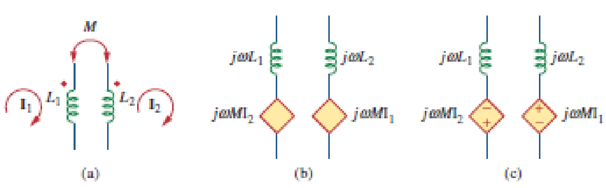


Figure 20

Energy In Transformer

Definition. Instantaneous energy stored:

$$w = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 \pm Mi_1i_2 \tag{1}$$

where the $\pm$ is determined by if both current enter or leave dot.
Coupling coefficient: $k = \frac{M}{\sqrt{L_1L_2}}$ $0 \le k \le 1$, measure of magnetic coupling between two coils, if $k > 0.5$, they are tightly coupled, else loosely.

Question. Calculate energy in circuit at given time of a time domain voltage input.

Method. Just plug 1 into the circuit after all circuit components are calculated. Use $\boldsymbol{S} = \boldsymbol{VI^*}$ for power in each components.

Ideal Transformer

Definition. Transformer circuit with the following assumption:

- large reactants: $L_1, L_2, M \to \infty$
- coupling coefficient equals to unity, $k = 1$
- both oils are lossless ($R_1 = R_2 = 0$)
- $Z_{in} = \frac{Z_L}{n^2}$ to calculate how load impedance reflects to the primary side.
- With these assumptions, we can say use turn ratio $n = \frac{N_2}{N1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$.

Question. Given circuit components of a transformer circuit, ask for phasor current or impedance of one side, or any missing circuit components.

Method.

1. See figure 21 to simply the circuit, then use KCL to solve.
2. Assign current and voltage to each coil, use turn ratio to complete KVL/KCL to solve the circuit.

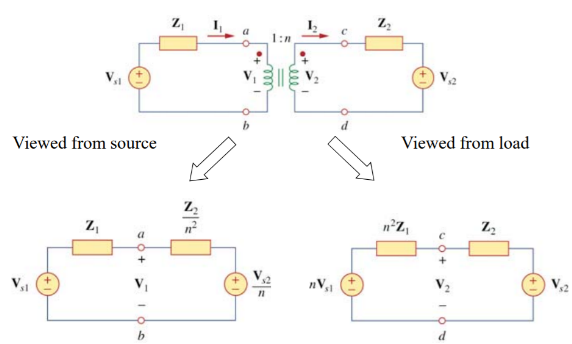


Figure 21

Second Order Circuit

Definition. Any circuits that can be simplify to a second order ODE.

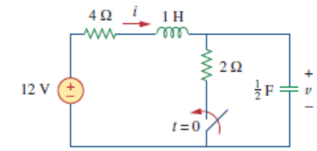


Figure 22: Example of Second Order Circuit 2

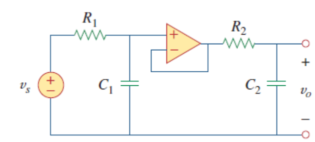


Figure 23: Example of Second Order Circuit 2

Source-Free

Definition. Any Second order circuits without any independent source. It's steady state will include a steady state inductor and/or capacitor.

Solve Source-Free:
damping factor: $\alpha = \frac{\mathrm{L}}{2\mathrm{R}}$
resonance frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$
After transform a general source free RLC circuit into laplace, solve the characteristic function with $s = -\alpha \pm \sqrt{\alpha^2 + \omega_0^2}$ based on $\Delta = \sqrt{4\alpha^2 - 4w_0^2}$. The eigenvalue gives:

- overdamped: $\alpha > \omega_0$
- critically damped: $\alpha = \omega_0$
- underdamped: $\alpha < \omega_0$

Question. Given a RLC circuit and I_0, V_0 of inductor and capacitor,

Step-Response

Definition. RLC circuit was introduced instantaneously by a source. Solve for $v(t)$ in RLC series cricuit, $i(t)$ in RLC parallel circuit.

Solve Step-Response: Let $v_t(t)$ be the transient response, $v_{ss}(t)$ be the steady state response. $v_t(t)$ is the homogeneous solution to RLC circuit, where $v_{ss}(t)$ is the partical solution.
$\boldsymbol{v(t) = v_t(t) + v_{ss}(t)}$

General Second Order Circuit

Method.

- determine $x(0^+)$, $\frac{\mathrm{d}x}{\mathrm{d}t}(0^+), x(\infty)$ and other initial conditions for the ODE.
- find $x_t(t)$ by turning off independent source, plug in initial condition and boundary values. Construct KCL/KVL using function of time, and extract ODE for a value. Note that KVL is simpler for inductor only circuit, KCL is easier for capacitor.
- $x(t) = x_t(t) + x_{ss}(t)$, where $x_{ss}(t)$ is $x(\infty)$.

Question.

- Given a circuit, solve for $v_0(t), i_0(t)$: **ASN13.10, 11**
-

References

[1] Charles Alexander and Matthew Sadiku. *Fundamentals of Electric Circuits*. McGraw-Hill, Inc., New York, NY, USA, 3 edition, 2007.

[2] Elec 251 slide. http://www.ece.ubc.ca/~shahriar/eece251_notes/eece251_set3_1up.pdf. [Online; accessed 21-April-2019].

[3] J.C. Linares. Elec 202 course website. https://courses.ece.ubc.ca/elec202/linares.html. [Online; accessed 19-April-2019].

HP Primer a Primer

HUGE TABLE INCOMING

Command	Parameter	Function
laplace	(f(t), t, s)	
ilaplace	(F(s), s, t)	inverse laplace
partfrac	(f(x))	partial fraction
evalf	(f(x))	
proot	(f(x))	get the root of polynoimal
normal		
solve	system of equations, variables	
fsolve	same as solve, sometimes	

Laplace Tables

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(t)dt$	$\frac{1}{s}F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds}F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1-e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2+\omega^2}$
$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

*Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.

Figure 24: Table of Laplace [3]

TABLE 19.1

Conversion of two-port parameters.

	z		y		h		g		T		t	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$	$\frac{d}{c}$	$\frac{1}{c}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta_g}{g_{11}}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{\Delta_t}{c}$	$\frac{a}{c}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta_g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$	$\frac{a}{b}$	$-\frac{1}{b}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$	$-\frac{1}{B}$	$\frac{A}{B}$	$-\frac{\Delta_t}{b}$	$\frac{d}{b}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta_g}$	$-\frac{g_{12}}{\Delta_g}$	$\frac{B}{D}$	$\frac{\Delta_T}{D}$	$\frac{b}{a}$	$\frac{1}{a}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{g_{21}}{\Delta_g}$	$\frac{g_{11}}{\Delta_g}$	$-\frac{1}{D}$	$\frac{C}{D}$	$\frac{\Delta_t}{a}$	$\frac{c}{a}$
g	$\frac{1}{z_{11}}$	$-\frac{z_{12}}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta_h}$	$-\frac{h_{12}}{\Delta_h}$	g_{11}	g_{12}	$\frac{C}{A}$	$-\frac{\Delta_T}{A}$	$\frac{c}{d}$	$-\frac{1}{d}$
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta_z}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$-\frac{h_{21}}{\Delta_h}$	$\frac{h_{11}}{\Delta_h}$	g_{21}	g_{22}	$\frac{1}{A}$	$\frac{B}{A}$	$\frac{\Delta_t}{d}$	$-\frac{b}{d}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$	A	B	$\frac{d}{\Delta_t}$	$\frac{b}{\Delta_t}$
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta_g}{g_{21}}$	C	D	$\frac{c}{\Delta_t}$	$\frac{a}{\Delta_t}$
t	$\frac{z_{22}}{z_{12}}$	$\frac{\Delta_z}{z_{12}}$	$-\frac{y_{11}}{y_{12}}$	$-\frac{1}{y_{12}}$	$\frac{1}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$-\frac{\Delta_g}{g_{12}}$	$-\frac{g_{22}}{g_{12}}$	$\frac{D}{\Delta_T}$	$\frac{B}{\Delta_T}$	a	b
	$\frac{1}{z_{12}}$	$\frac{z_{11}}{z_{12}}$	$-\frac{\Delta_y}{y_{12}}$	$-\frac{y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta_h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$	$-\frac{1}{g_{12}}$	$\frac{C}{\Delta_T}$	$\frac{A}{\Delta_T}$	c	d

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$, $\Delta_T = AD - BC$

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$, $\Delta_g = g_{11}g_{22} - g_{12}g_{21}$, $\Delta_t = ad - bc$

Figure 25: Two Port Network Conversion Table [3]