08.09.2023 Messprotokoll von Yuting Shi

Physikalisches Anfängerpraktikum
Sommersemester 2023

Versuch 13 Tutor: Lorenzo Sanfilippo

<u>Resonance</u>

In this Experiment we will determine the oscillation period T_0 of a continuous free rotating pendulum. Damped and stimulated oscillations are also investigated using an eddy current brake and a stepper motor based on Pohl's wheel. In consider of the reduction of the oscillation amplitude with time or the width and height of the resonance curve, the decay constant for different values of the current passing through the brake will be determined.

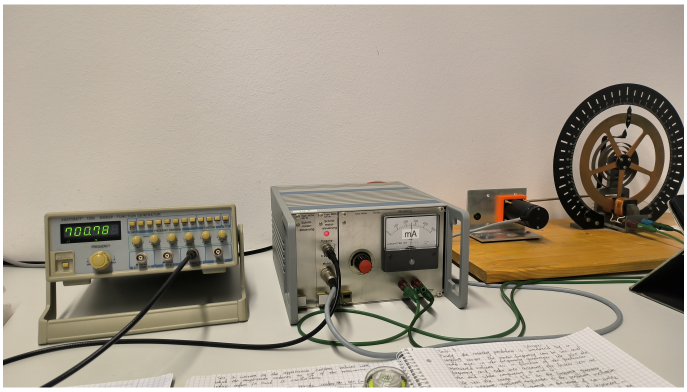


Abbildung 1: Experimental setup

1 Introduction

1.1 Physical Principles

We will first explain all the important experimental apparatus and control questions before proceeding with the experiment.

- Principle of Pohl's Wheel:

A Pohl's wheel is a rotational pendulum equipped with a Bearing block (3), variable eddy current elektromagnetic brake (4), which allows the system to extract energy by generating eddy currents, and an exciter offers stimulation (1) coupled to a spiral spring (5), which enables the addition of external energy to the oscillation.

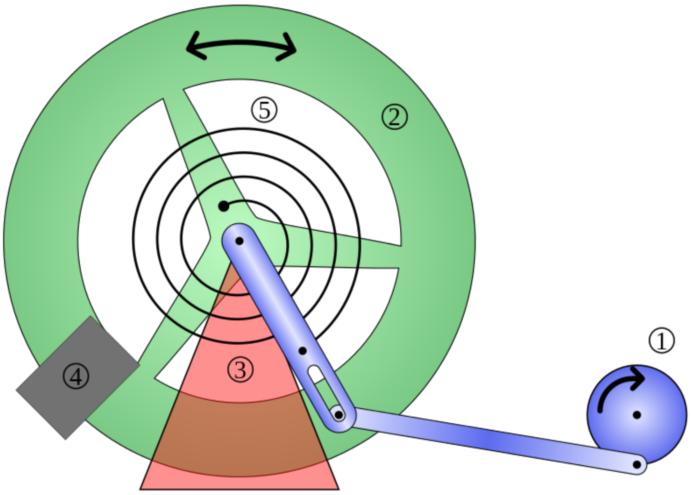


Abbildung 2: Pohl's wheel with a stepper motor (right)

- descriptionof an oscillation with differential equations:

An undamped oscillation with the moment of inertia tensor Θ and the torque D is one that continues indefinitely without any external damping force. The motion follows Hooke's Law with differential equation:

$$\Theta\ddot{\varphi} + D\varphi = 0 \qquad \varphi = Acos(\omega_0 t + \phi) \tag{1}$$

A: amplitude of the oscillation, $\omega_0 = \sqrt{\frac{D}{\Theta}}$: angular frequency, ϕ: phase angle

Damped oscillations are the osciallation, which that its amplitude attenuates with time and the pendulum comes gradually to the rest. The amplitude decay appears as a result of the action of a force, which counteracts with the evolution direction of the system (e.g. friction force). Damped oscillations are described by the following differential equation:

$$\Theta\ddot{\varphi} + 2\gamma\dot{\varphi} + D\varphi = 0 \tag{2}$$

The additional term describes a damping torque $M = -2\gamma\dot{\varphi}$

The differential equation (2) can be solved using an integrating factor $\dot{\varphi}$ or the quation $\dot{\varphi} = C\exp(\mu t)$, C and $\mu \in \mathbb{C}$:

$$\varphi(t) = A_0 \exp(-\gamma t)(cos(\omega' t) + \frac{\gamma}{\omega'} sin(\omega' t)), \gamma \leq \omega_0, strongdamping$$
$$\varphi(t) = A_0 \exp(-\gamma t)(cosh(kt) + \frac{\gamma}{\omega} sinh(kt)), \gamma \geq \omega_0, weakdamping$$
$$\varphi(t) = A_0 \exp(-\gamma t)(1 + \gamma t), aperiodiclimitcase$$
$$\omega' = \sqrt{w_0^2 - \gamma^2} \qquad k = \sqrt{\gamma^2 - \omega_0^2}$$

Here, A_0 represents the initial amplitude, and $\omega' = \sqrt{\omega_0^2 - \gamma^2}$ represents the angular frequency of the damped, free oscillating oscillator. The Amplitude is time-dependent: $A(t) = A_0 \exp(-\gamma t)$. Das Pendel schwingt also harmonisch mit einer exponentiell mit der Zeit abklingenden Amplitude.

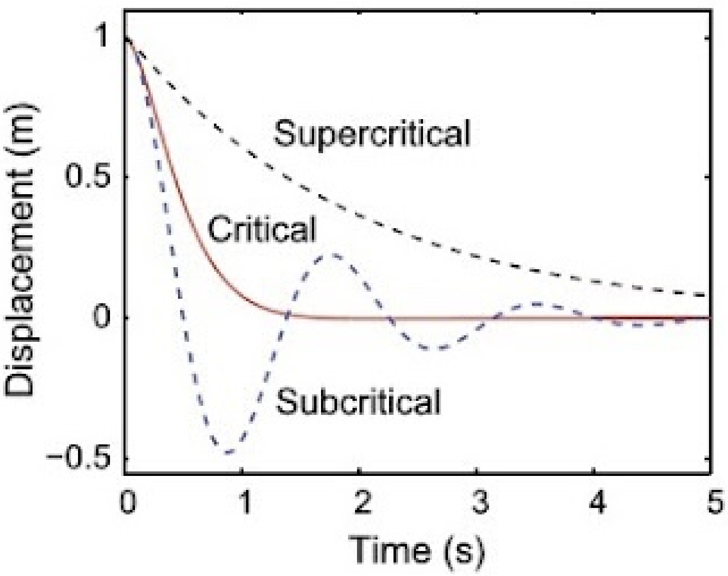


Abbildung 3: Oszillations with different Damp

- Differential equation of the oszillatin with Stimulation and Damper:

If the pendulum is now externally driven by a periodic torque $M = c \cdot cos(\omega t)$, equation (2) can be expanded as follows:

$$\Theta\ddot{\varphi} + 2\gamma\dot{\varphi} + D\varphi = c \cdot cos(\omega t) \tag{3}$$

We solve the equation in the complex plane:

$$\Theta\ddot{z} + 2\gamma\dot{z} + Dz = c\exp(wt) \tag{4}$$

We already know the general homogeneous solution from (2):

$$z_h(t) = e^{-\lambda t}e^{\pm i\bar{\omega}t}, \lambda \leq \omega_0$$
$$with\ \lambda = \frac{\gamma}{\Theta} \qquad \bar{\omega} = \sqrt{\omega_0^2 - \lambda^2}$$

To find the general solution of this inhomogeneous, ordinary, linear second-order differential equation, we need a particular solution of the inhomogeneous equation:

$$z_p(t) = z_0 e^{i\omega t}$$

We substitute z_0 into equation (4) and obtain:

$$z_0 = \frac{c/m}{(\omega_0^2 - \omega^2) + 2i\omega\lambda} = \frac{c/m[(\omega_0^2 - \omega^2) - 2i\omega\lambda]}{(\omega_0^2 - \omega^2)^2 + 4\omega^2\lambda^2}$$
$$z_p(t) = z_0 e^{i\omega t} = Ae^{-i\phi}e^{i\omega t} = Ae^{i(\omega t - \phi)}$$

With

$$A(\omega) = \frac{c/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\lambda^2}} \qquad tan\phi = \frac{2\omega\lambda}{\omega_0^2 - \omega^2} \tag{5}$$

Here, ω_0 represents the natural frequency of the undamped oscillator, and A represents the amplitude of the excitation.

A particular solution of the inhomogeneous equation is therefore:

$$\varphi_p(t) = \mathfrak{R}_{Z_p}(t) = Acos(\omega t - \phi) \tag{6}$$

The general solution is given by:

$$\varphi(t) = \varphi_p(t) + \varphi_h(t) = Acos(\omega t - \phi) + \bar{A}e^{-\lambda t}cos(\bar{\omega}t - \bar{\phi}) \tag{7}$$

A and ϕ are already determined, while $\bar{A}$ and $\bar{\phi}$ are available to satisfy the initial conditions.

The pendulum's displacement $\varphi(t)$ is composed of an exponentially decaying oscillation with the frequency $\bar{\omega}$ of the free damped oscillation and an undamped oscillation with the excitation frequency ω. $\bar{\phi}$ describes a phase dependent on the initial state of the system, and ϕ describes the phase shift between the excitation and the forced oscillation. The amplitude $A(\omega)$ depends on the excitation frequency.

The first term describes an exponentially decaying oscillation with time, having the natural frequency. If a sufficient amount of time has passed $t \gg 1/\lambda$, the first term becomes negligibly small due to exponential damping. and the second term dominates. After a transient period, the oscillator then oscillates at the same frequency as the external force but shifted by the phase ϕ. For $\omega = \omega_0$, $\phi = \pi/2$. The time it takes for this behavior to occur is also known as the transient phase or settling time.

- Resonance and half-width:

The amplitude A from equation (6) exhibits typical resonance behavior. It reaches a maximum at:

$$\frac{d}{d\omega}((\omega_0^2 - \omega^2)^2 + 4\omega^2\lambda^2) = 4\omega(-\omega_0^2 + \omega^2 + 2\lambda^2) = 0$$

$$\Rightarrow \omega_{max} = \sqrt{\omega_0^2 - 2\lambda^2} \approx \omega_0(1 - \frac{\lambda^2}{\omega_0^2} + O(\frac{\lambda^3}{\omega_0^3})) \tag{8}$$

In this case, we refer to it as resonance, and the frequency ω_{max} is called the resonance frequency. The phase shift ε between the excitation frequency and the oscillation frequency is then precisely $\phi = \frac{\pi}{2}$.

The second relevant quantity is the resonance amplification. This indicates how much the amplitude of the oscillation is amplified by the excitation. Assuming $\omega_0 \approx \omega_{max}$, the resonance amplification is given by:

$$\frac{b_{(max)}}{b_{(\omega\to 0)}} \approx \frac{\omega_0}{2\lambda} \tag{9}$$

Where, in the Taylor series, we assume weak damping $\lambda \ll \omega_0$. The maximum reaches a height of:

$$A_{max} = \frac{c/m}{\sqrt{(2\lambda^2)^2 + 4(\omega_0^2 - 2\lambda^2)\lambda^2}} = \frac{c/m}{2\lambda\sqrt{\omega_0^2 - \lambda^2}} \tag{10}$$

The half-width of the resonance is defined by the condition:

$$A^2(\omega_{1,2}) = \frac{1}{2} A_{max}^2$$

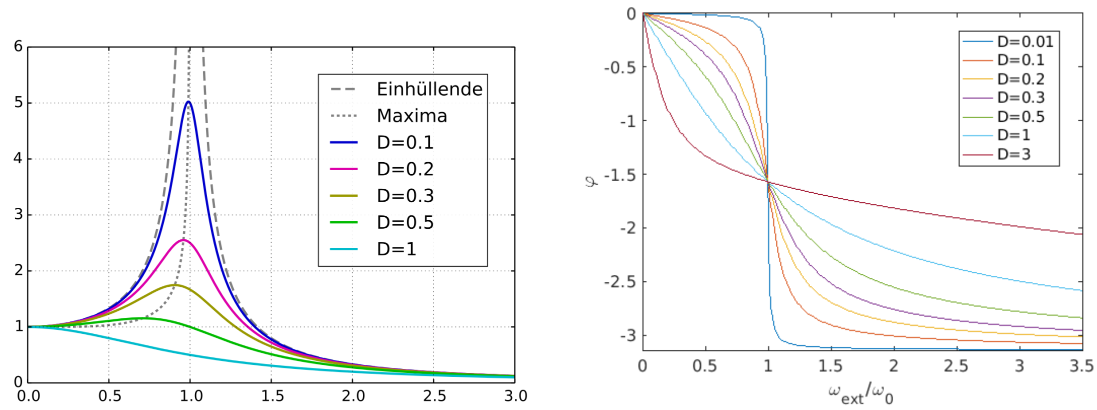
Strictly speaking, this is the half-width for an intensity that depends quadratically on the amplitude. This leads to the condition:

$$\omega_{1,2} \approx \omega_0 \pm \lambda \qquad \qquad \Gamma = |\omega_1 - \omega_2| = 2\lambda \tag{11}$$

Γ is characterized by being the width at half-height when plotting the square of the amplitude against frequency.

With an increase in damping, the half-width increases, the maximum amplitude decreases, and the local maximum shifts to the left as the damping coefficient γ becomes larger.

The dependence of the amplitude and phase shift on the excitation frequency is depicted in the following two diagrams:



(a) Amplitude vs. ω/ω_0 (b) Phase vs. ω/ω_0

Abbildung 4: The amplitude and phase of the forced oscillation as a function of the excitation frequency

From an energy consideration, it becomes clear that (8) is valid: The amplitude is maximal when the excitation force acts precisely in the direction in which the oscillator is currently moving, meaning that the velocity (i.e., the first derivative of angle) perfectly aligns with the oscillator's position.

- Q-Factor of the resonator:

The Q-factor is defined as the ratio of the energy stored in the resonator to the energy dissipated per cycle. Mathematically, it is expressed as:

$$Q = \frac{Energie\ stored}{Energy\ Dissipated\ per\ Cycle} \tag{12}$$

1.2 Relevant formulas

Here, we will write down some important formulas that are particularly relevant to the experiment:

- If these quantities are proportional to velocity (which is often the case), the time dependence of the motion is described by the differential equation:

$$a(t) = a_0 e^{-\delta t} sin\omega_f t \tag{13}$$

Here, ω_f represents the angular frequency of the damped, freely oscillating oscillator, a_0 is the initial amplitude, and δ is the damping constant. If we consider the time dependence of the amplitude only at one of the turning points:

$$a(t) = a_0 e^{-\delta t} \qquad y := lna(t) = lna_0 - \delta t \tag{14}$$

If $t_{1/2}$ is the time at which the amplitude has dropped to half of the initial amplitude, then it follows:

$$a(t_{1/2}) = \frac{a_0}{2} = a_0 e^{-\delta t_{1/2}} \Rightarrow \delta = \frac{ln2}{t_{1/2}} \tag{15}$$

- There is a relationship between the angular frequency ωf of the damped oscillation and the angular frequency ω0 of the undamped oscillation:

$$\omega_f = \sqrt{\omega_0^2 - \delta^2} \tag{16}$$

- Furthermore, the amplitude of the oscillation depends on the frequency of the driving generator. The corresponding curve is shown in Figure 4 and described by the following equation:

$$b(\omega) = \frac{A\omega_0^2}{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2} \tag{17}$$

At which the amplitude $b(\omega)$ is maximal: $\omega' = \sqrt{\omega_0^2 - 2\delta^2}$

With Half-width:

$$H = (\omega_2 - \omega_1) = 2\delta \tag{18}$$

- The resonance ratio is defined via the following quotient:

$$\frac{b(\omega')}{b(\omega \to 0)} = \frac{\omega_0}{2\delta} \tag{19}$$

2 Execution of this experiment

Experimental setup, procedure, and measurement protocol: see the following pages.

Messprotokoll PAP 1 10 08.09.2023

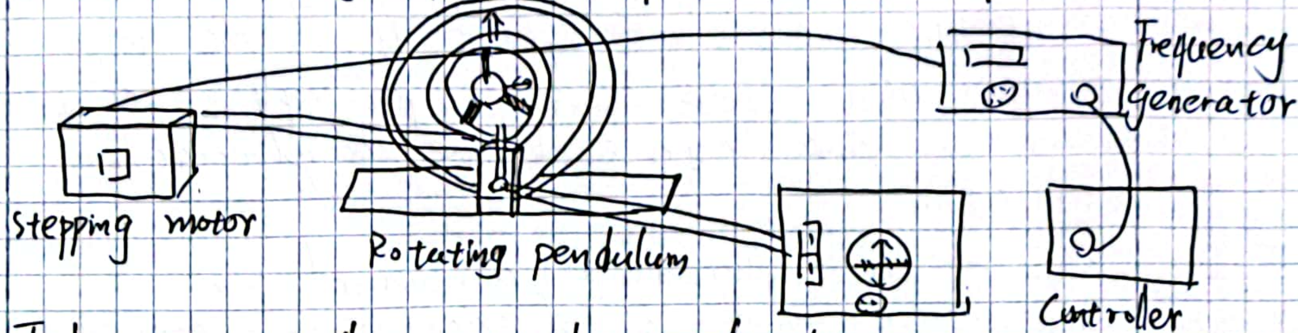
Yuting Shi, Yulai Shi 9:00 ~ 12:00

Versuch 13 — Resonance

Measuring equipment:

- Rotating pendulum driven by a stepping motor with the eccentric
- Controller of the stepping motor with a power supply
- Frequency generator
- Power supply for regulation of the oscillation damping

Task 1: Drawing of the experimental setup



Task 2: The oscillation period T_0 of the continuous free rotating pendulum

We have the pendulum without the generator and measure the duration of 20 oscillations, for 3 times repeated.

Table 1: Duration of the free undamped Pendulum

Nr.	Duration for 20 oscillations [s]
1	39.48
2	39.53
3	39.54

error $\Delta T = 0.2s$

Task 3: Determination of the duration until 5% of the output amplitude

We turn the oscillation damping on, with damping and observe qualitatively its effect on the oscillation amplitude at different currents passing through the damper magnetic coil.

Set 2 written on the apparatus current values with which the amplitude reduces to 5% of the maximum value after 10 and 15 oscillations.

$\Delta I = 10mA$ Table 2: Duration for 10.15 periods under 280, 360 [mA] damping

$\Delta a = 0.2$ Skt gilt für Tabellen 2, 3

	15 periods 280 mA	10 periods 360 mA
Duration [s]	29.56	19.75

4. Task 4: Determination of the temporal decrease in amplitude during attenuation

We measure the oscillation period T_f of the pendulum for both current values chosen in paragraph 3 and record the time course of the amplitude attenuation.

We start the oscillation from a reversal point at $t=0$ and note the amplitude of each complete oscillation.

$\hat{s}_1$: Measure 1

$\hat{s}_2$: Measure 2

Table 3: Amplitude in reversal point after x oscillations under 360, 280 mA in [skt]

360 mA

n.	0 (max)	1	2	3	4	5	6	7	8	9	10
$\hat{s}_1$	20.0	14.4	10.9	8.0	5.9	4.4	3.3	2.4	1.9	1.4	1.1
$\hat{s}_2$	20.0	14.4	10.5	7.8	6.0	4.5	3.2	2.5	1.9	1.4	1

280 mA

n.	0 (max)	1	2	3	4	5	6	7	8	9
$\hat{s}_1$	20.0	17.0	14.3	12.2	10.2	8.6	7.3	6.3	5.2	4.4
$\hat{s}_2$	20.0	17.0	14.4	12.2	10.2	8.7	7.4	6.2	5.3	4.4

280 mA

n.	10	11	12	13	14	15
$\hat{s}_1$	3.8	3.2	2.7	2.3	1.9	1.7
$\hat{s}_2$	3.8	3.2	2.6	2.3	2.0	1.7

扫描全能王 创建

Task 5:

Firstly, the rotating pendulum is (driven) simulated by a stepping motor. The motor frequency can be set and read out in the frequency generator. We plot the measured values as a function of the generator frequency and take into account the factor 2500 at the end while computing δ and $\bar{\omega}'$

We set the range "1k" on the frequency generator, then, measure stationary amplitude of the pendulum oscillation as a function of the frequency in the range of 300-2100Hz. We firstly make ~~the~~ a measurements with a increment ~ 200Hz, then with an increment of ±150Hz, and finally with an increment of 50Hz around the resonance frequency.

(only measure, when the amplitude reaches the steady level.)

Table 4: Amplitude by excitation in [skt]

$\hat{S}_{280}$: Measurement with frequency 280Hz

$\hat{S}_{360}$: ~360Hz

f [Hz]	300	500	700	900	1050	1150	1200	1250
$\hat{S}_{280}$	0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.9
$\hat{S}_{360}$	0.5	0.5	0.6		0.7		0.7	

f [Hz]	1300	1350	1400	1600	1800	2100	2150	2200
$\hat{S}_{280}$	0.9	0.9	1.0	1.3	2.2	4.2	3.0	2.2
$\hat{S}_{360}$	0.8	0.9	1.0	1.3	2.1	3.4	2.7	2.1

$\Delta f = 1\,Hz \quad \Delta \hat{S} = 0.2$

	2300	2400
$\hat{S}_{280}$	1.5	1.1
$\hat{S}_{360}$	1.5	1.1

because of the ~~vibrant~~ drastic change between 1400 ~ 2400 HZ we proceed the preciser Experiment:

	1450	1650	1700	1750	1850	1900	1950	2000	2050	2250	2350
$\hat{S}_{280}$	1.0	1.4	1.6	1.9	2.8	3.7	5.3	7.3	6.3	1.8	1.3
$\hat{S}_{360}$	1.0	1.4	1.6	1.8	2.6	3.2	3.8	4.4	4.2	1.7	1.2

Diagramm 1: Auslenkung als Funktion der Schwingung
(δstark, 360mA)

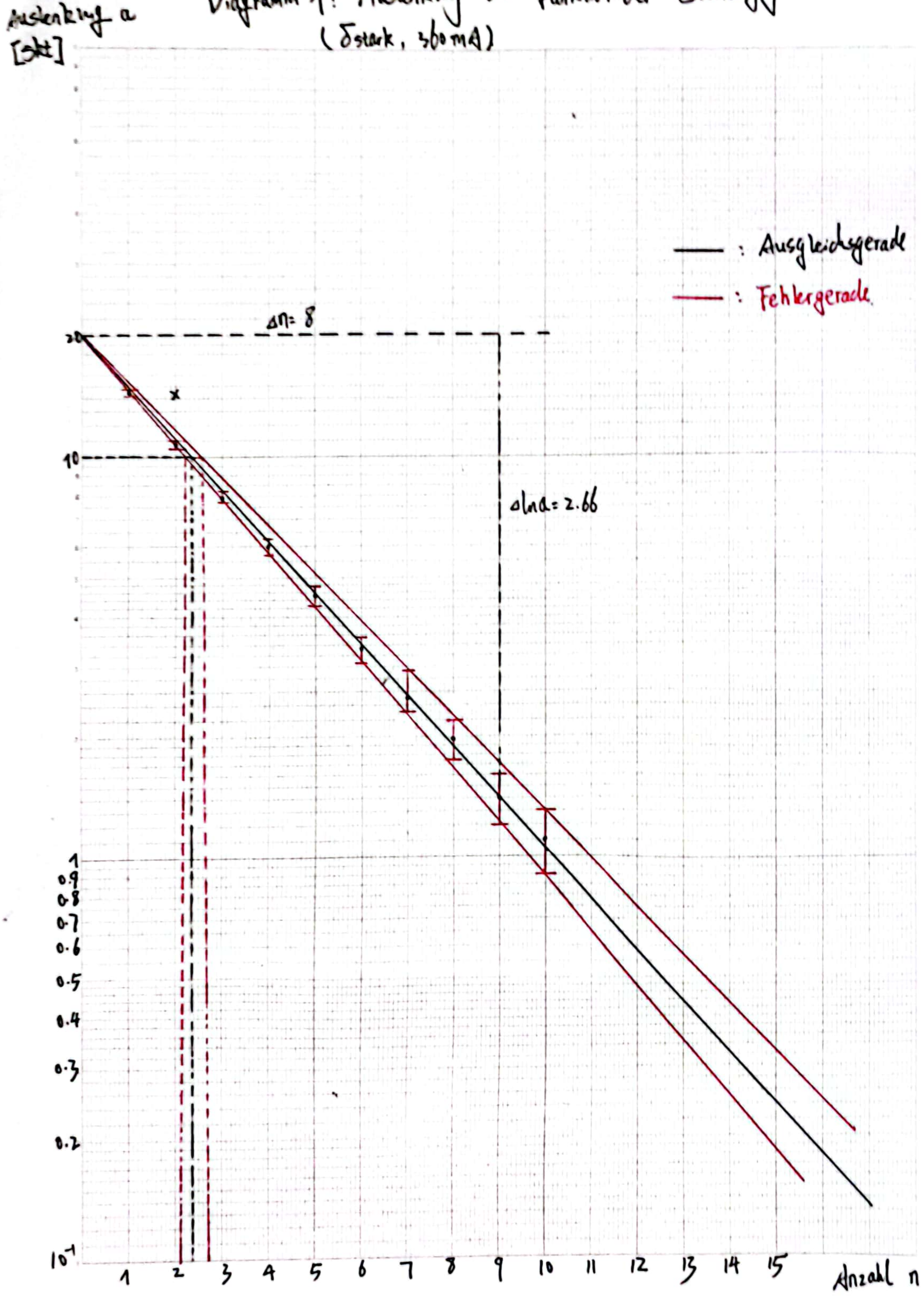
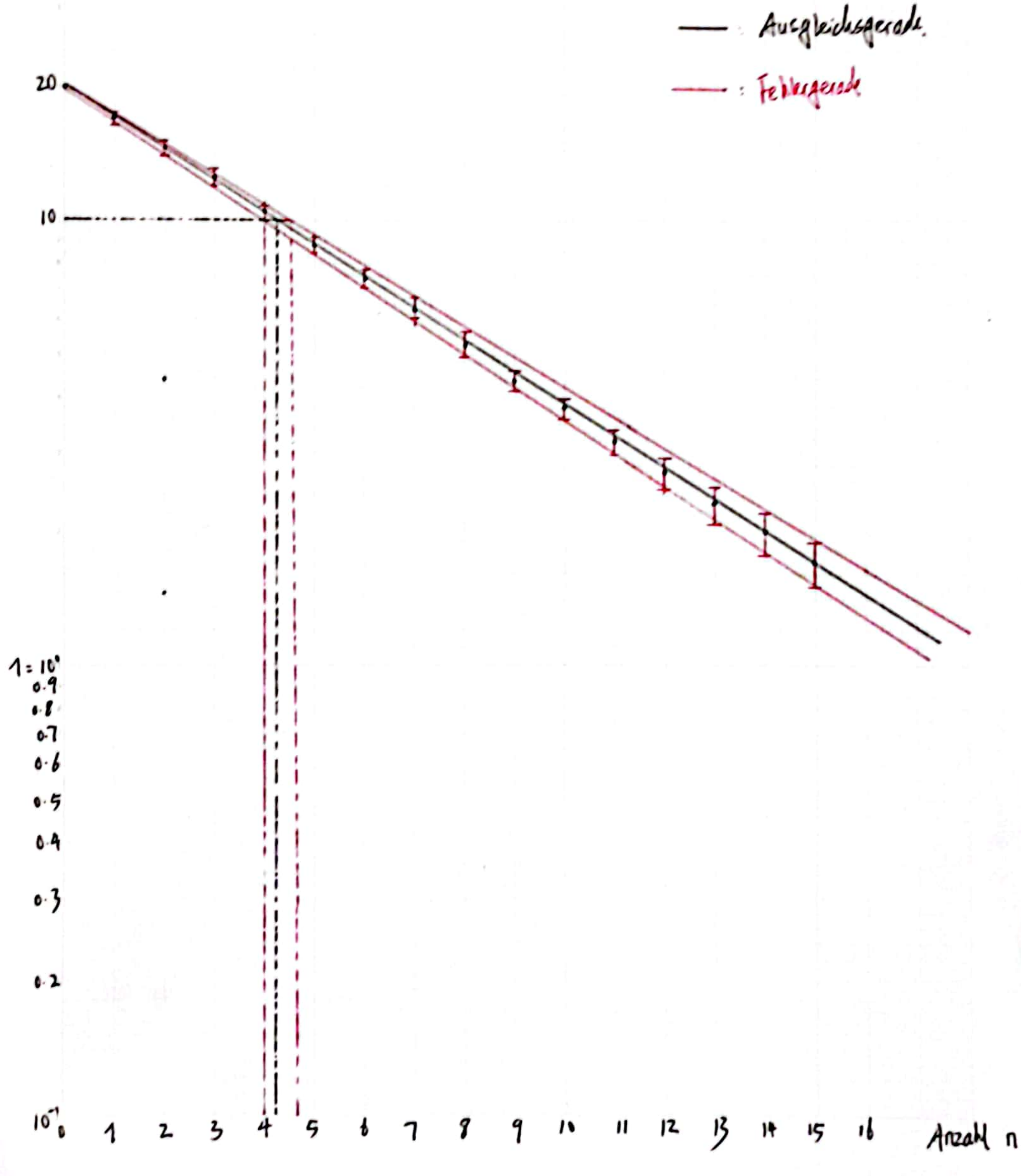


Diagramm 2: Auslenkung als Funktion der Schwingungen ($\delta_{schwach}$, 280 mA)



3 Evaluation and Analysis for the Experiment

In the following, the damping constant δ is determined for weak and strong damping in three ways. To distinguish them better, δ represents the damping constant determined from the observation of the amplitude decrease, $\bar{\delta}$ represents the one determined from the half-life, and $\tilde{\delta}$ represents the damping constant calculated from the resonance peak.

3.1 Determination of the period duration T_0

Measurment	T [s]	T_0 [s]	$deltaT$ [s]	
1	39,48	1,97	0,2	
2	39,53	1,98	0,2	
3	39,54	1,98	0,2	

We do not turn on the alternating current and the generator first. The period duration T_0 of the undamped, free oscillation of Pohl's wheel is calculated as follows:

$$T_0 = \frac{\bar{T}}{n} = \frac{T_1 + T_2 + T_3}{3 \cdot n} = \frac{39,48 + 39,53 + 39,54}{3 \cdot 20} \approx 1,98s \tag{20}$$

Here, $\bar{T}$ denotes the average of the three measured durations for n =20 oscillations.

We calculate the errors of T_0:

The error of the main value:

$$\sigma_{\bar{T}_0} = \frac{\sigma_{T_0}}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{n} \left(T_i - \bar{T}_0\right)^2}{n(n-1)}} = \sqrt{\frac{0,04^2 + 0,01^2 + 0,02^2}{3 \cdot 2}} \approx 0,0187s \tag{21}$$

Incorporating quadratic addition, we also account for the reaction time error that occurs in each individual measurement: $\Delta T = 0.20s$

$$\Longrightarrow \Delta_r T_0 = \sqrt{3(\Delta T)^2} = 0.35s$$

We add the error from the averaging process $\sigma_{\bar{T}_0}$ and the error from the reading inaccuracy $\Delta_r T_0$:

$$\Delta T_0 = \sqrt{(\sigma_{\bar{T}_0})^2 + (\Delta_r T_0)^2} \approx 0,35s \qquad \Rightarrow \underline{\underline{T_0 = (1,98 \pm 0,35)s}} \tag{22}$$

3.2 Determination of the damping of the amplitude

We compile the measurements of the damped oscillations, that is, the amplitudes of both dampings over the number of oscillations on a logarithmic scale.

From Diagram 1, we read that half of the amplitude of the Strong damping (360mA) is reached after $n_{1/2} = 2,34 \pm 0,24$ periods. Therefore, for the half-life time $t_{1/2}$, we have:

$$t_{1/2} = n_{1/2} \cdot T_0 = 2,34 \cdot 1,98s = 4,63s \tag{23}$$

With Equation (??), it follows that:

$$\delta_{strong} = \frac{ln(2)}{t_{1/2}} = 0,149\ s^{-1} \tag{24}$$

We estimate the error $\Delta\delta_{strong}$ using the Gaussian error propagation law:

$$\Delta t_{1/2} = \sqrt{(T_0 \cdot \Delta n_{1/2})^2 + (n_{1/2} \cdot \Delta T_0)^2} = 1,15s$$
$$\Rightarrow t_{1/2} = (4,63 \pm 1,15)s$$

From this $\Delta t_{1/2}$, it follows that:

$$\Delta\delta_{strong} = \frac{d\delta_{strong}}{dt_{1/2}} = \frac{ln(2)}{(t_{1/2})^2} \cdot \Delta t_{1/2} = 0,037\ s^{-1} \tag{25}$$

$$\Rightarrow \underline{\underline{\delta_{strong} = (0,149 \pm 0,037)\ s^{-1}}} \tag{26}$$

We now perform the same calculation with the measurement taken at a damping of 280mA in Diagram 2:

$n'_{1/2} = 4,23 \pm 0,23$ periods. Therefore, for the half-life time $t'_{1/2}$, we have:

$$t'_{1/2} = n'_{1/2} \cdot T_0 = 4,23 \cdot 1,98s = 8,38s \tag{27}$$

With Equation (??), it follows that:

$$\delta_{weak} = \frac{ln(2)}{t'_{1/2}} = 0,083\ s^{-1} \tag{28}$$

We estimate the error $\Delta\delta_{strong}$ using the Gaussian error propagation law:

$$\Delta t'_{1/2} = \sqrt{(T_0 \cdot \Delta n'_{1/2})^2 + (n'_{1/2} \cdot \Delta T_0)^2} = 2,65s$$
$$\Rightarrow t'_{1/2} = (8,38 \pm 2,65)s$$

From this $\Delta t'_{1/2}$, it follows that:

$$\Delta\delta_{weak} = \frac{d\delta_{weak}}{dt'_{1/2}} = \frac{ln(2)}{(t'_{1/2})^2} \cdot \Delta t'_{1/2} = 0,026\ s^{-1} \tag{29}$$

$$\Rightarrow \underline{\underline{\delta_{weak} = (0,083 \pm 0,026)\ s^{-1}}} \tag{30}$$

Diagramm 3: Amplitude als Funktion der Generatorfrequenz (schwach, 280 mA)

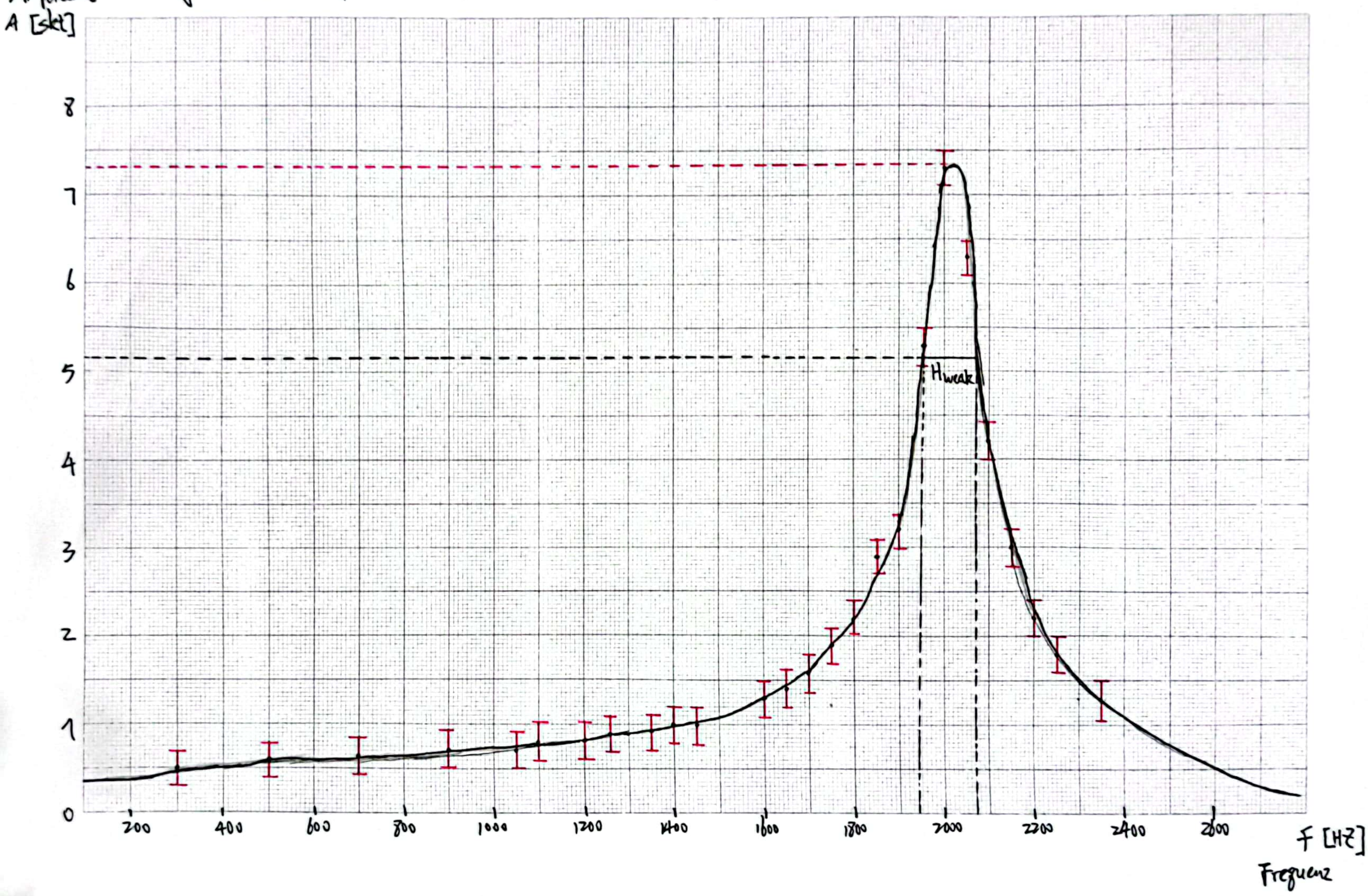
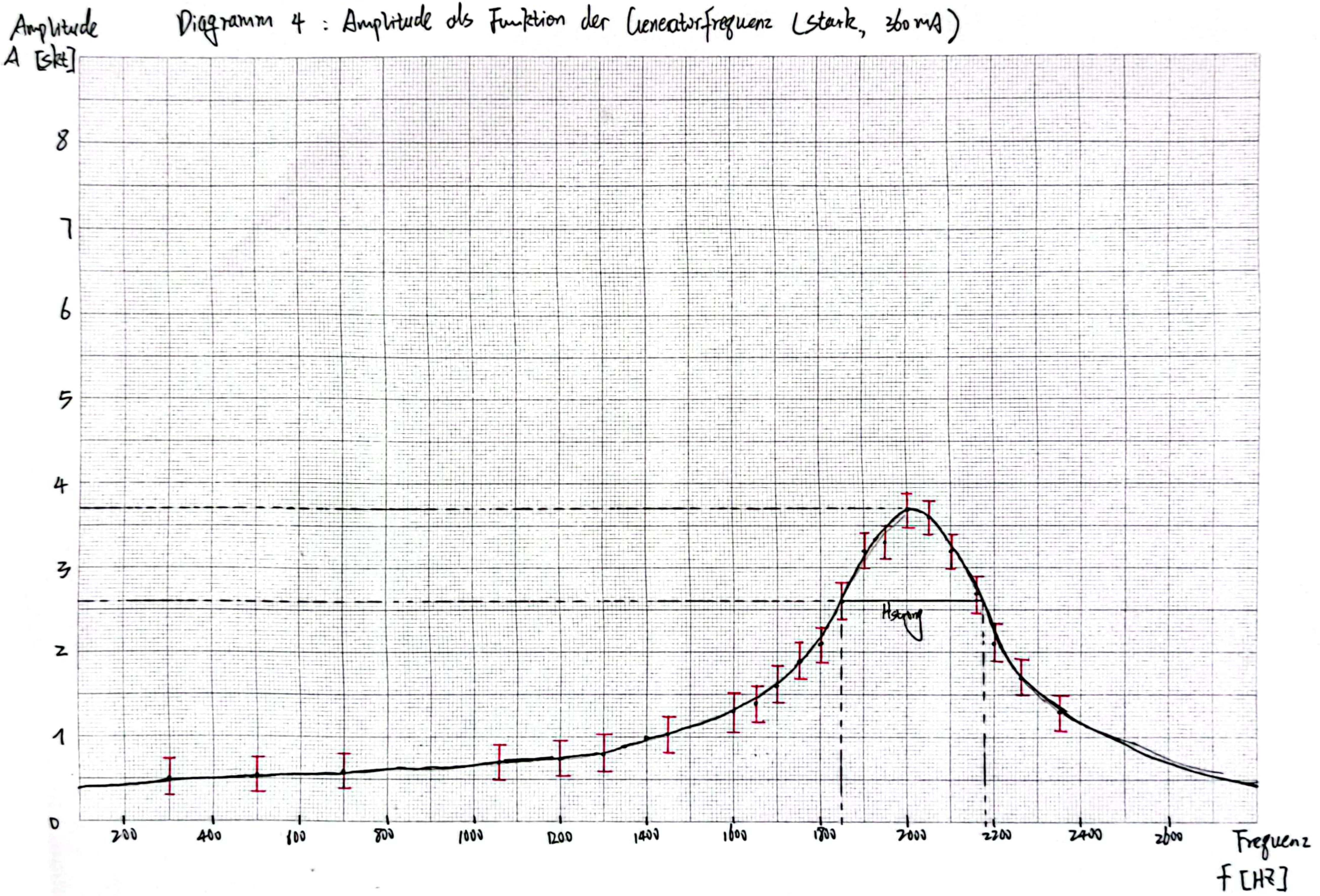


Diagramm 4 : Amplitude als Funktion der Generatorfrequenz (stark, 360 mA)



3.3 Determination of the damping from the resonance curve

In the previous diagrams, the amplitudes of the torsional pendulum are plotted against the generator frequency. The measured frequencies, which are recorded in the diagram, are converted into angular frequencies as follows:

$$\omega = 2\pi f \tag{31}$$

To calculate the frequencies, the formulas provided below, as well as the determined T_0 values in section 3.2, were used. The indices signify: Generator (Gen), Exciter (Err), and Eigen (Eig).

$$\omega_{Err} = \frac{\omega_{Gen}}{2500} \tag{32}$$

The natural frequency ω_0 can also be determined using the formula:

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{1,98s} = 3.17\ s^{-1} \tag{33}$$

We once again use Gaussian error propagation:

$$\Delta\omega_0 = \sqrt{(\frac{\partial\omega_0}{\partial T_0} \cdot \Delta T_0)^2} = \frac{2\pi}{T_0^2} \cdot \Delta T_0 = 0,56\ s^{-1} \tag{34}$$

So we have the natural frequency: $\underline{\underline{\omega_0 = (3,17 \pm 0,56)\ s^{-1}}}$

3.3.1 280mA weak Damping

We read the maximum $a_{max} = (7,3 \pm 0,2)\ [Skt]$ at $f_{Gen,max} = 2020Hz$. The error can be limited to 80 Hz.

$\Rightarrow \omega_{Gen,max} = 2\pi f_{Gen,max} \approx 12686\ s^{-1} \qquad \Delta\omega_{Gen,max} = 2\pi\Delta f_{Gen,max} \approx 502\ s^{-1}$

The frequency of the exciter can be calculated als:

$$\omega_{Err,max} = \frac{\omega_{Gen,max}}{2500} = (5,08 \pm 0,20)\ s^{-1} \tag{35}$$

For reading the half-width, the difference between the resonance frequency and the frequency at which the amplitude $a_{HW} = \frac{1}{\sqrt{2}}a_{max}$. For this, we can calculate an error:

$$\Delta a_{HW} = \frac{1}{\sqrt{2}} \Delta a_{max} \approx 0,14 \tag{36}$$

The half-width for weak damping can be readed as:

$\Delta f_{weak} = \frac{(124 \pm 20)\ Hz}{2500} = (0,046 \pm 0,008)\ Hz$

$$H_{weak} = 2\pi \cdot \Delta f_{weak} = (0,289 \pm 0,050)\ s^{-1} \tag{37}$$

3.3.2 360mA strong Damping

We read the maximum $a'_{max} = (3,7 \pm 0,2)\ [Skt]$ at $f'_{Gen,max} = 2015 Hz$. The error can be limited to 80 Hz.

$\Rightarrow \omega'_{Gen,max} = 2\pi f'_{Gen,max} \approx 12654\ s^{-1} \qquad \Delta\omega'_{Gen,max} = 2\pi \Delta f'_{Gen,max} \approx 502\ s^{-1}$

The frequency of the exciter can be calculated als:

$$\omega'_{Err,max} = \frac{\omega'_{Gen,max}}{2500} = (5,06 \pm 0,20)\ s^{-1} \tag{38}$$

For reading the half-width, the difference between the resonance frequency and the frequency at which the amplitude $a'_{HW} = \frac{1}{\sqrt{2}} a'_{max}$. For this, we can calculate an error:

$$\Delta a'_{HW} = \frac{1}{\sqrt{2}} \Delta a'_{max} \approx 0,14 \tag{39}$$

The half-width for weak damping can be readed as:

$\Delta f_{strong} = \frac{(332 \pm 20)\ Hz}{2500} = (0,133 \pm 0,008)\ Hz$

$$H_{strong} = 2\pi \cdot \Delta f_{strong} = (0,289 \pm 0,050)\ s^{-1} \tag{40}$$

We now summarize everything and obtain this table:

Tabelle 6: Calculating damping using the resonance curve

	$\omega_{Gen,max}\ [s^{-1}]$	$\omega_{Err,max}\ [s^{-1}]$	$\omega_{Eig} = \omega_0\ [s^{-1}]$	Half-width $[s^{-1}]$	$a_{max}[Skt]$
$\delta_{schwach}$	$(12{,}686 \pm 0{,}502) \cdot 10^3$	$5{,}08 \pm 0{,}20$	$3,17 \pm 0,56$	$0{,}289 \pm 0{,}050$	$7{,}3 \pm 0{,}2$
δ_{stark}	$(12{,}654 \pm 0{,}502) \cdot 10^3$	$5{,}06 \pm 0{,}20$	$3,17 \pm 0,56$	$0{,}835 \pm 0{,}050$	$53{,}7 \pm 0{,}2$

From the graphically determined half-widths $H = 2\bar{\delta}$, we determine the damping constants and their errors:

$$\bar{\delta}_{weak} = \frac{H_{weak}}{2} = (0,1445 \pm 0,025)\ s^{-1} \tag{41}$$

$$\bar{\delta}_{strong} = \frac{H_{strong}}{2} = (0,4175 \pm 0,025)\ s^{-1} \tag{42}$$

3.4 Determination of the damping from the resonance peak

Finally, we determine the damping constant using the resonance peak. From the diagrams, we read: $b(\omega \to 0)_{weak} = 0.3 \pm 0.2$; $b(\omega \to 0)_{strong} = 0.3 \pm 0.2$.

Then it holds that:

$$\frac{b(\omega_{max})}{b(\omega \to 0)} \approx \frac{\omega_0}{2\tilde{\delta}} \Rightarrow \tilde{\delta} = \left(\frac{b(\omega_{max})}{b(\omega \to 0)}\right)^{-1} \frac{\omega_0}{2} \tag{43}$$

$$\Rightarrow \tilde{\delta}_{schwach} \approx 0,065 \frac{1}{s} \tag{44}$$

$$\Rightarrow \tilde{\delta}_{schwach} \approx 0,123 \frac{1}{s} \tag{45}$$

We calculate the error using the error propagation law:

$$\Delta\tilde{\delta}_x = \sqrt{(\frac{\partial\tilde{\delta}_x}{\partial\omega_0}\Delta\omega_0)^2 + (\frac{\partial\tilde{\delta}_x}{\partial b(\omega_{max})}\Delta b(\omega_{max}))^2 + (\frac{\partial\tilde{\delta}_x}{\partial b(\omega_{\omega\to 0})}\Delta b(\omega_{\omega\to 0}))^2} \tag{46}$$

$$= \sqrt{(\frac{1}{2}(\frac{b(\omega_{max})}{b(\omega \to 0)})^{-1}\Delta\omega_0)^2 + (\frac{\omega_0}{2}\frac{b(\omega \to 0)}{(b(\omega_{max}))^2}\Delta b(\omega_{max}))^2 + (\frac{\omega_0}{2b(\omega_{max})}\Delta b(\omega_{\omega\to 0}))^2} \tag{47}$$

$$\Rightarrow \tilde{\delta}_{schwach} \approx 0,013 \frac{1}{s} \tag{48}$$

$$\Rightarrow \tilde{\delta}_{stark} \approx 0,030 \frac{1}{s} \tag{49}$$

We summarize:

Tabelle 7: Summary of the values for the damping constants

	$\delta_x[s^{-1}]$	$\bar{\delta}_x[s^{-1}]$	$\tilde{\delta}_x[s^{-1}]$
Weak Damping	$0,083 \pm 0,026$	$0,1445 \pm 0,025$	$0,065 \pm 0,013$
Strong Damping	$0,149 \pm 0,037$	$0,4175 \pm 0,025$	$0,123 \pm 0,030$

We calculate whether the deviations between any two of these values are significant as follows:

$$y := \frac{|\bar{\delta}_x - \delta_x|}{\sqrt{(\Delta\bar{\delta}_x)^2 - (\Delta\delta_x)^2}} \tag{50}$$

We can also replace one of the two expressions with $\tilde{\delta}_x$. Values above 1 indicate significant deviations. The results of the calculation are shown in the following table:

Tabelle 8: Deviation of the calculated values for weak damping

•	$\delta_{schwach}$	$\bar{\delta}_{schwach}$	$\tilde{\delta}_{schwach}$
$\delta_{schwach}$	0	2,31	0,69
$\bar{\delta}_{schwach}$	2,31	0	3,16
$\tilde{\delta}_{schwach}$	0,69	3,16	0

Tabelle 9: Deviation of the calculated values for strong damping

•	δ_{stark}	$\bar{\delta}_{stark}$	$\tilde{\delta}_{stark}$
δ_{stark}	0	10,18	0,69
$\bar{\delta}_{stark}$	10,18	0	11,37
$\tilde{\delta}_{stark}$	0,69	11,37	0

The deviations between 1 and 3 are not significant, but there is a significant discrepancy for 2, indicating a substantial error.

4 Discussion

In this experiment, we determined the damping constant of the Pohl's pendulum in three different ways. We started with free, undamped oscillation and determined the pendulum's natural frequency based on the period. We then activated the eddy current brake and a stepper motor, using two different current intensities. To obtain more accurate amplitude readings, we recorded the process during the experiment and subsequently measured the amplitude.

In the first part of the experiment, we assumed that the rotary pendulum oscillates without damping when not braked by the eddy current brake. However, this was not the case, as it lost energy due to air resistance and mechanical friction of the components. It is possible that this friction was so low that, compared to the energy loss caused by the eddy current brake for 15 or 10 oscillations, it was negligible. Nevertheless, this could have been experimentally verified, for instance, by observing the decrease in deflection of the rotary pendulum relative to the number of oscillations and then subtracting the measurement taken with the current applied to the eddy current brake. The amplitude of the damped oscillations (for both currents) was then plotted on a logarithmic scale as a function of the number of oscillations, and we determined the first damping coefficient from the graph.

We obtained the second damping coefficients using resonance curves, which, however, significantly deviated from our initial measurements. The width of the curve at half-height $b(\omega)/\sqrt{2}$ is described by the following formula when damping is not too strong: $H = 2\delta$. After obtaining the half-width, we could theoretically determine the damping constant. Signs of this significant deviation from theory were already observed during the creation of Table 6: According to resonance theory, the maximum amplitude should be reached when the excitation frequency is nearly equal to the natural frequency of the Pohl's pendulum. However, we obtained $\omega_{Err} = (5,08 \pm 0,20)\ s^{-1}$ compared to $\omega_0 = 3,17 \pm 0,56$, indicating a significant error. After reviewing the instrument photos, we realized that during the measurement of amplitude decay in the second part of the experiment, we accidentally set the eddy current brake to the wrong current scale (we initially did not consider a step size of 20 mA), resulting in actual values of 280 mA and 360 mA, but we measured them as 260 mA and 340 mA. This led to a significantly lower damping coefficient in the first part.

Interestingly, it is noticeable that the error of the value determined by the first method is slightly larger than the error of the values resulting from the resonance curve. This is surprising because larger errors are typically expected to result from inaccuracies, such as when reading . We were conservative in error estimation for the resonance curve and, to improve accuracy, we deliberately performed 11 additional measurements near the resonance peak during the experiment to better plot the curve.

A relatively large source of error in the first part was reading the deflections and oscillation durations. These errors could have been minimized if readings were not taken manually but with the help of a camera that records the oscillation in slow motion and, if necessary, multiple times in succession. With this method, a more precise scale division would have made sense. Even more accurate readings of deflection (perhaps to half-scale divisions) would have been possible and would have further reduced the overall error.

Out of curiosity, we also observed the times required for the pendulum to return to ëquilibrium"(resonance) after each change in generator frequency. It was noticed that when we changed the frequency near the maximum corresponding frequency, the pendulum oscillated vigorously and required more time to reach resonance. Furthermore, with a damping of 360 mA, the pendulum took significantly more time to return to equilibrium. This can already be inferred from Formula (5) $tan\phi = \frac{2\omega\lambda}{\omega_0^2-\omega^2}$: The larger the damping coefficient and the closer the frequency is to the natural frequency, the greater the phase shift and, consequently, the required time.