Physikalisches Anfängerpraktikum
Sommersemester 2023

Versuch 15 Tutor: Pamela Ochoa Parra

Inclined Plane

1 Introduction

1.1 The purpose of the experiment

In this experiment we will determine the acceleration on an inclined plane for two rolling bodies (solid cylinder and hollow cylinder) and study the law of conservation of Energy using light barriers so that the velocity of the body can be determined at different time points. The reasons for the difference in accelerations are determined using the moment of inertia theory.

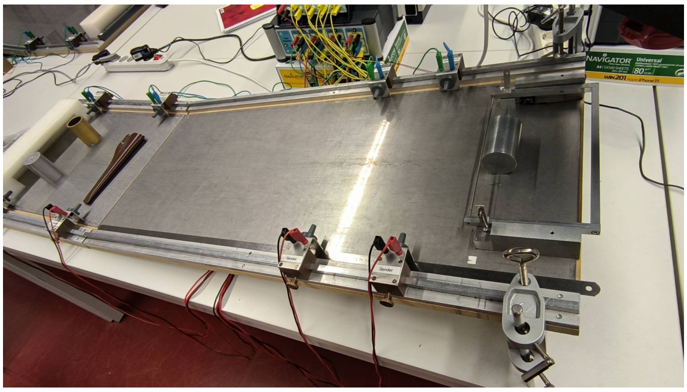


Abbildung 1: Experimental setup

1.2 Physical Principles

In this part all the necessary basic physics knowledge and instructions for using the experimental setup will be provided, as well as answers to the preparatory questions.

1.2.1 Inclined plane

The inclined plane is one of the six classical simple machines defined by Renaissance scientists. The experimental use of an inclined plane is widely employed across various scientific disciplines to investigate the principles of mechanical physics such as demonstration of energy conservation, moment of inertia.

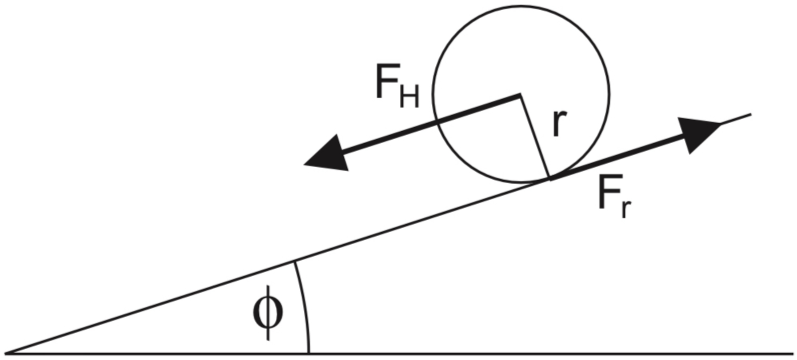


Abbildung 2: Derivation of the acceleration on the inclined plane

There are several forces that act on a body on an inclined plane: The gravitational force which can be separated into the downhill force F_H and the normal force F_N. With m describing the mass of the body, g the gravitational acceleration and ϕ the angle by which the plane is inclined, the two forces are given by:

$$F_H = mg \sin\phi \tag{1}$$
$$F_N = mg \cos\phi \tag{2}$$

1.2.2 The Moment of Inertia

Officially, the expression of the moment of inertia in theoretical physics is formulated as:

$$\Theta_{jk} = \sum_{i=1}^{N} m_i [\vec{x_i}^2 \delta_{jk} - x_{i,j} x_{i,k}] \tag{3}$$

Thus, theta is revealed as a second-order tensor (the moment of inertia tensor of a rigid body with N mass points), and sigma represents the Kronecker delta.

When the body is continuous, and the density is a function of the position vector, integration isperformed:

$$\Theta_{jk} = \int\int\int dx_1\, dx_2\, dx_3 \rho_{(\vec{x})} [\vec{x}^2 \delta_{jk} - x_j x_k] \tag{4}$$

The moment of inertia I of a homogenous body with the density ρ and volume V regarding to the rotational axis is given by:

$$I = \rho \int_V r_\perp^2 \mathrm{d}V \tag{5}$$

$r_\perp$ is the distance between the mass element and the axis of rotation.

Now we can solve it for a solid cylinder with radius r height h, for a hollow cylinder with inner radius r_i exterior radius r and for a ball with radius r. The three bodies have the same mass m:

$$I_s = \int_0^r hdr \cdot 2\pi r \rho r^2 = 2\pi h \rho \cdot \frac{1}{4} r^4 = \frac{1}{2} m r^2 \qquad m = \rho \pi r^2 h \tag{6}$$
$$I_h = \int_{r_i}^r hdr \cdot 2\pi r \rho r^2 = 2\pi h \rho \cdot \frac{1}{4} \cdot (r^4 - r_i^4) = \frac{1}{2} m d^2 \tag{7}$$
$$m = \rho \pi (r^2 - r_i^2) h, \; d^2 = r_i^2 + r^2 \geqslant r^2 \tag{8}$$
$$\tag{9}$$

We observe that when the inner radius approaches infinity relative to the external radius: $I_h \to mr^2$

Determination of the moment of inertia of the ball:

$$I_k = \int_0^r \pi \rho (r^2)^2 dx = \pi \rho \int_0^r (r^4 - 2r^2 x^2 + x^4) dx = \frac{8}{15} \cdot \pi \rho r^5 = \frac{2}{5} m r^2 \tag{10}$$
$$m = \frac{4}{3} \pi r^3 \tag{11}$$

1.2.3 Difference of acceleration

- A "frictionless" cuboid, a solid cylinder, a hollow cylinder and a solid ball, with the same radius and mass, slide or roll down an inclined plane. We compare the time that they reaches the bottom:

The cuboid has the acceleration $a_c = g \cdot cos\phi$

For the bodies which will also rotate during the translation. The downhill force accelerates the body downhill, and the normal force results in a frictional force F_r that opposes F_H. The frictional force generates a torque M, which is expressed by the following equation:

$$M = rF_r = I \frac{\mathrm{d}\omega}{\mathrm{d}t} = I \frac{\mathrm{d}(v/r)}{\mathrm{d}t} = \frac{I}{r} a_s \;\Leftrightarrow\; F_r = \frac{I}{r^2} a_s \tag{12}$$

where a_s is the acceleration of the center of mass.

Using that the frictional force acts in the opposite direction than the downhill force, the resulting force that accelerates the body is given by:

$$F = ma_s = mg\sin\phi - F_r \tag{13}$$

We substitute F_r into equation (8) and solve the equation for a_s:

$$a_s = \frac{mg\sin\phi}{m + \frac{I}{r^2}} \tag{14}$$

The solution reveals that rotating bodies with the moment of inertia I always have an acceleration smaller than $gsin\phi$. Moreover, the larger the moment of inertia, the slower the body's acceleration. ($I_k \leqslant I_s \leqslant I_h$). Therefore, the times required to reach the bottom are as follows:

$$t_c \leqslant t_k \leqslant t_s \leqslant t_h \tag{15}$$

When objects roll down an inclined plane, they can have both translational (linear) motion and rotational motion. The energy is shared between these two forms of motion. An object with a lower moment of inertia can accelerate its r otation more easily because it has a smaller resistance to changes in angular velocity. They have their mass distributed close to their axis of rotation, making it easier for them to start and maintain rolling motion. so they can convert potential energy into kinetic energy efficiently. This results in a quicker descent down the inclined plane.

- A "frictional" cuboid should be trimmed by surface treatment, that its acceleration on an inclined plane (10° inclination) is the same that the acceleration of a solid cylinder. We determine now the sliding coefficient:

The acceleration of cuboid is described as:

$$a_{s_c} = \frac{F_H - F_R}{m} = (sin\phi - \mu cos\phi)g \tag{16}$$

Equation (14) with the moment of inertia of solid cylinder is now equal to (16), thus we can calculate the coefficient:

$$\mu = tan\phi - \frac{mtan\phi}{m + \frac{I_s}{r^2}} = \frac{1}{3}tan\phi \tag{17}$$

1.3 Relevant Formulas

The initial potential energy of the rolling body is converted into kinetic energy. The kinetic Energy can be divided into a rotation and a translation part, so that the total energy is given by the following equation:

$$W_{ges} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh \quad E_{kin} = \frac{1}{2}mv^2 E_{rot} = \frac{1}{2}I\omega^2 \quad E_{pot} = mgh \tag{18}$$

where v is the translation velocity, I the moment of inertia and ω the angular velocity of the rolling body. $E_{kin} = \frac{1}{2}mv^2$ refers to translational energy, $E_{rot} = \frac{1}{2}I\omega^2$ to rotational energy and $E_{pot} = mgh$ the potential energy.

We will also need equations that describe the movement of a body that moves with a constant acceleration a. v descibes the body's velocity, s describes the body's position as functions of the time t. In the experiment, we will set the starting position zero at $t = 0$ and the bodies start from rest. Thus, the relevant equations are:

$$v(t) = at \tag{19}$$

$$s(t) = \frac{a}{2}t^2 \tag{20}$$

The rest of the relevant formulas are already in **1.2**.

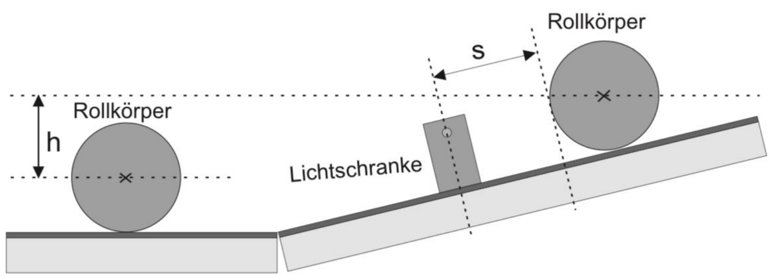


Abbildung 3: Conversion of energy

2 Implementation

2.1 Measurement Record

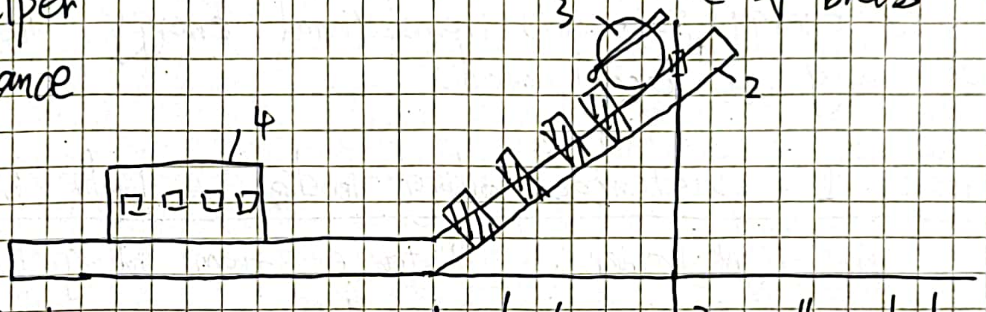
For **Measurement Setup and Record**, see following pages.

Messprotokoll PAP 1 13.09. 2023
Yuting Shi, Yulai Shi 9:00 ~ 12:00

Experiment 15 — Inclined plane

1. Experimental Setup:

- Height-adjustable runway
- Light barrier with control unit
- Spirit level
- Ruler
- Rolling bodies:
  - Solid cylinder (aluminium $\rho = 2.70 g/cm^3$)
  - Hollow cylinder (brass, $\rho = 8.44 g/cm^3$)
  - Composite cylinder: sheath of aluminium, core of brass
- calliper
- Balance



1 – Light sensor 2 – inclined plane 3 – rolling body
4 – control unit

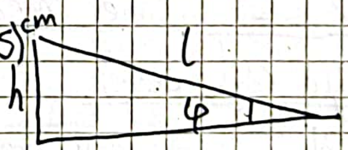
2. Measuring the rolling bodies and inclined plane:

First we will measure the diameter d of the rolling bodies: $d = 5{,}000$ cm $\Delta d = 0.005$ cm

and the adjustable plane:

$L = 89{,}56$ cm $L = (89.56 \pm 0.05)$ cm

$h = 13{,}25$ cm



To calculate the angle we will use: $\sin\varphi = \frac{h}{L}$

$\Rightarrow \varphi = \arcsin\left(\frac{h}{L}\right) = 8{,}51°$

3. Examination of the movement type ~~of~~ at the inclined plane

The bodies need different time to roll down the inclined plane because of their different movement of inertia. Before rolling down the plane, they all have the same potential energy <u>per mass</u>. The potential energy goes into kinetic energy which can be divided into rotational and translational energy.

The rotational energy is directly proportional to the moment of inertia; thus the different moments of inertia result in different ratios of translational to rotational kinetic energies and the more of the kinetic energy goes into translational energy, the more the body is accelerated.

Table 1: Distances rolling bodies - light barrier

The error of length measurements is 0,05cm

Nr. Light barrier	Distance from rolling body [cm]
1	9,00
2	25,00
3	49,00
4	81,00

Table 2: Time measured at the light barries (Solid cylinder)

$\Delta t = 0{,}001s$

No. light barrier	time [s]	time^2 [s^2]
1	0,485	0,2352
2	0,774	0,5990
3	1,106	1,2232
4	1,401	1,9628
1	0,484	0,2343
2	0,774	0,5991
3	1,105	1,2210
4	1,399	1,9572

No. Light barriers	time [s]	time² [s²]
1	0,467	0.2181
2	0,756	0.5715
3	1,087	1.1816
4	1,395	1.9460
1	0,470	0.2209
2	0,758	0.5746
3	1,089	1.1859
4	1,384	1.9155
1	0,467	0.2181
2	0,754	0.5885
3	1,087	1.1816
4	1,396	1.9488

Errors:

$\Delta t = 0.001 s$

$\Delta(t^2) = 2t\Delta t$

Table 3: Time measured at the light barriers (hollow cylinder)

No. Light barriers	time [s]	time² [s²]
1	0,524	0.2746
2	0,846	0.7157
3	1,216	1.2679
4	1,563	2.4430
1	0,515	0.2652
2	0,838	0.7022
3	1,207	1.4568
4	1,552	2.4087
1	0,533	0.2841
2	0,855	0.7310
3	1,223	1,4957
4	1,568	2,4586
1	0,517	0,2673
2	0,838	0,7022
3	1,206	1,4544
4	1,551	2,4056
1	0,524	0,2746
2	0,844	0,7123
3	1,213	1,4714
4	1,557	2,4243

5. Studies on the law of energy

To calculate the translational speed we will measure the time between sensor 1 and sensor 2. and the distance:

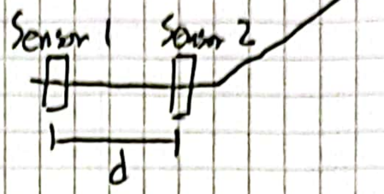


Table 4: Time and distance measurements at the end of the plane

d=30,00 cm

Type of cylinder	No.	Sensor 1 [s]	Sensor 2 [s]	difference [s]
hollow	1	1,689	1,983	0.294
	2	1,686	1,981	0,295
	3	1,703	1,997	0.294
	4	1,706	1,998	0.292
	5	1,703	1,996	0.293
solid	1	1,526	1,789	0.263
	2	1,523	1,786	0.263
	3	1,529	1,793	0,264
	4	1,530	1,793	0,263
	5	1,526	1,789	0,263
composite	1	1,424	1,673	0,249
	2	1,427	1,674	0,247
	3	1,426	1,676	0,250
	4	1,421	1,671	0,250
	5	1,431	1,678	0,247

We then calculate the mean values of the times measured at the light barriers from table 2 & 3 as well as their errors due to the calculation of the mean.

	light barrier	time t_{mean} [s]	Δt_{mean} [s]
solid cylinder	1	0,4746	0,0041
	2	0,7632	0,0048
	3	1,0948	0,0044
	4	1,3950	0,0030
Hollow cylinder	1	0.5226	0.0032
	2	0.8442	0.0043
	3	1.2130	0.0031
	4	1.5582	0.0032

We add the error due to limited measuring accuracy Δt according to:

$$\Delta t_{total} = \sqrt{\Delta t_{mean}^2 + \Delta t^2} \qquad \Delta t = 0.001\,s$$

Using error propagation, we find that the error $\Delta(t^2)$ is given by: $\Delta(t_{mean}^2) = 2t_{mean}\Delta t_{total} =: \Delta(t^2)$

The results of the calculations can be seen in the table below:

Light barrier	$[t_{mean}^2 \pm \Delta(t^2)]$ $[s^2]$ (Solid)	$[t_{mean}^2 \pm \Delta(t^2)]$ $[s^2]$ (hollow)
1	0,2253 ± 0,0040	0.2731 ± 0.0035
2	0,5725 ± 0,0072	0.7127 ± 0.0075
3	1,1986 ± 0,0099	1.4714 ± 0.0079
4	1,9460 ± 0,0088	
		2.4280 ± 0.0104

Nr.	[g] Mass	Error [g]
Solid Cylinder	444,8	0,1
Hollow Cylinder	443,8	
Composite Cylinder	443,5	

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3 Evaluation

There are two measurement series in this experiment, one with a solid cylinder and one with a hollow cylinder. In order to better distinguish each other, $s_s,\ t_s$ are used to describe the solid cylinder and $s_h,\ t_h$ are used to describe the hollow cylinder.

We have first measured the length l and height h so we can calculate the inclination angle of the plane durch trigonometric:

$$\phi = arcsin(\frac{h}{l}) \approx 8,51^\circ = 0,15 \qquad \Delta l = \Delta h = 0,05cm \tag{21}$$

The error of the inclination angle ϕ is given by

$$\begin{aligned}
\Delta\phi =& \sqrt{\left(\frac{\partial\phi}{\partial h}\Delta h\right)^2 + \left(\frac{\partial\phi}{\partial l}\Delta l\right)^2} && (22)\\
=& \sqrt{\left(\frac{1}{l}\frac{1}{\sqrt{1-\left(\frac{h}{l}\right)^2}}\Delta h\right)^2 + \left(\frac{-h}{l^2}\frac{1}{\sqrt{1-\left(\frac{h}{l}\right)^2}}\Delta l\right)^2} && (23)\\
\approx& (5,7\cdot 10^{-4})^\circ \approx 1\cdot 10^{-5} && (24)
\end{aligned}$$

where the last value is ϕ and $\Delta\phi$ in radiants.

3.1 Calculation of the acceleration

After conducting the experiment, we first calculated the means and the mean standard errors of the mean for the time it takes for the bodies to reach the bottom, as shown in the measurement protocol.

Then we squared each mean time in light barriers and calculated the error of $\Delta(t^2)$ using the equation:

$$\Delta(t^2) = 2t\Delta t \tag{25}$$

Here, Δt refers to the mean standard error of the mean that we just discussed.

We insert the values in the t^2-s-diagram (diagram 1, 2). Then we calculate the slopes of the balance lines. The error Δk can be read from the diagram. It is the difference of the slope of the balance line and the error line. Compared to the length reading error, the errors $\Delta(t^2) \leq 0,010s^2$ are too small to be seen in the diagram. Therefore, we only drew the error bars taking into account the reading errors:

The slopes of both bodies are as follows:

$$k_x = \frac{\Delta s_x}{\Delta t_x^2} \tag{26}$$

$$k_s = \frac{0,4900m}{1,1986s^2} \approx 0,4088m/s^2 \qquad k_s' = \frac{0,7210m}{1,7207s^2} \approx 0,4190m/s^2 \tag{27}$$

$$k_h = \frac{0,0,4000m}{1,1983s^2} \approx 0,3338m/s^2 \qquad k_h' = \frac{0,3210m}{0,9566s^2} \approx 0,3356m/s^2 \tag{28}$$

$$\Longrightarrow \underline{\underline{k_s = (0,4088 \pm 0,0102)m/s^2}} \qquad \underline{\underline{k_h = (0,3338 \pm 0,0018)m/s^2}} \tag{29}$$

The acceleration of the bodies can be derived from the equations describing the accelerated motion of the bodies when they are at rest initially and the starting position s_0 is zero:

$$s = \frac{a}{2}t^2 \iff a = 2\frac{s}{t^2} = 2k_x \tag{30}$$

$$\Rightarrow \Delta a = 2\Delta k_x \tag{31}$$

$$\Rightarrow \underline{\underline{a_s = (0,8176 \pm 0,0204)\frac{m}{s}}} \tag{32}$$

$$\Rightarrow \underline{\underline{a_h = (0,6676 \pm 0,0036)\frac{m}{s}}} \tag{33}$$

Diagramm 1: Strecke als Funktion der t^2 für Vollzylinder

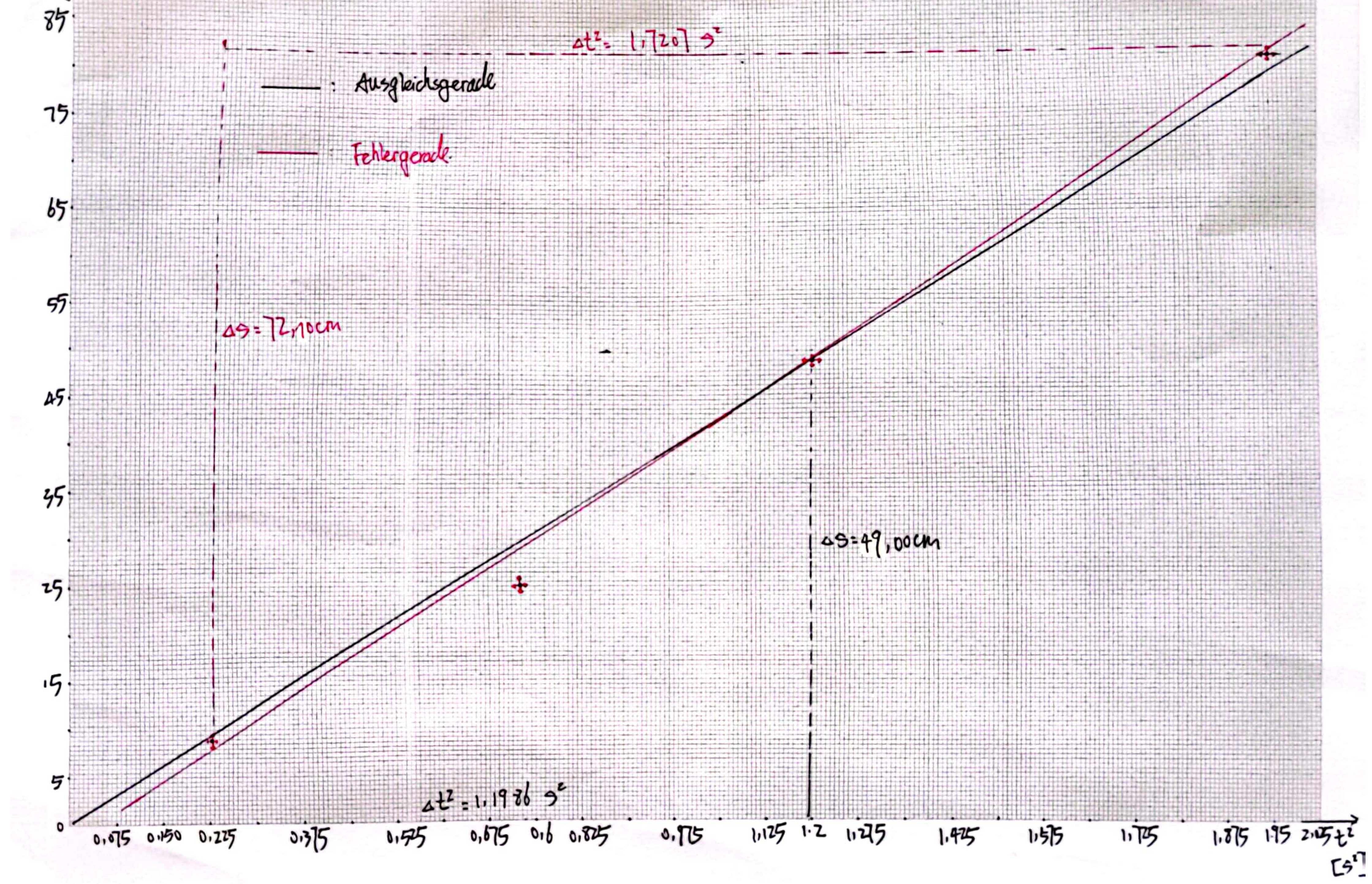
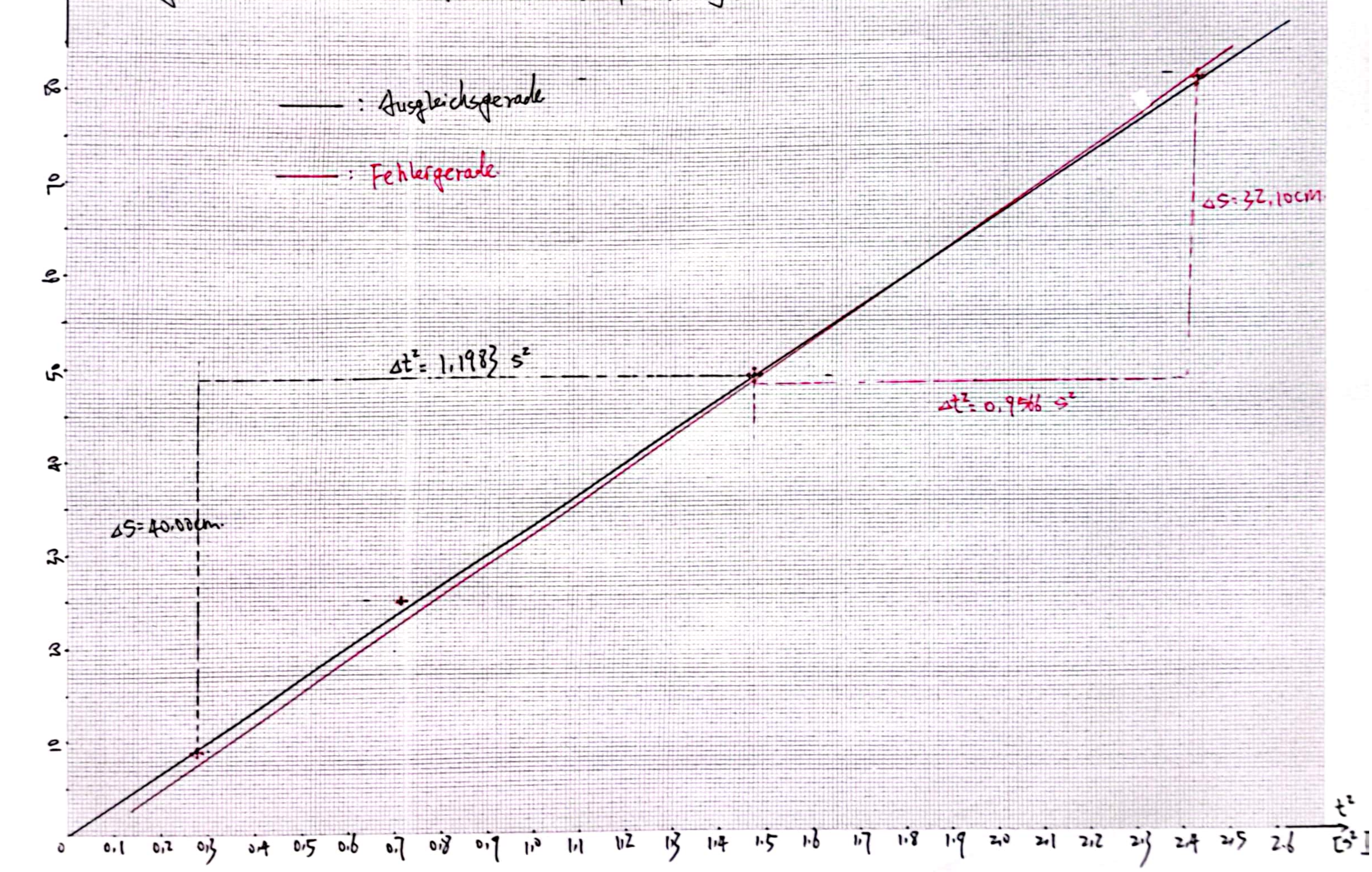


Diagramm 2: Strecke als Funktion der t^2 für Hohlzylinder.



These are our experimentally measured accelerations. Additionally, we would like to compare them with the theoretically determined values using equation (14):

$$a_{theo} = \frac{mg\sin\phi}{(m + \frac{I}{r^2})} \tag{34}$$

Inserting the above-calculated moments of inertia $I_s = \frac{m_s}{2}r^2,\ I_h = \frac{1}{2}m_h(r_i^2 + r^2) \approx m_h r^2$ and the calculated angle $\phi = 8,51°$ ($sin\phi = 0,148$) of the inclined plane, we find:

$$\Longrightarrow a_{s,theo} = \frac{g\sin\phi}{(1+\frac{1}{2})} = \frac{9,80984 \cdot 0,148}{1,5} \approx 0,9679\frac{m}{s^2} \tag{35}$$

$$a_{h,theo} = \frac{g\sin\phi}{(1+1)} = \frac{9,80984 \cdot 0,148}{2} \approx 0,7259\frac{m}{s^2} \tag{36}$$

In order to calculate the error of the theoretically determined acceleration we pay attention to the error $\Delta(\sin\phi)$:

$$\Delta(\sin\phi) = \cos\phi\Delta\phi = \cos(8,51°) \cdot 1 \cdot 10^{-5} \approx 9,8 \cdot 10^{-6} \tag{37}$$

We can calculate the error of the theoretically determined accelerations:

$$\Delta a_{s,theo} = \sqrt{(\frac{2g\Delta sin\phi}{3})^2 + (\frac{2sin\phi\Delta g}{3})^2} \approx 6,5 \cdot 10^{-5} m/s^2 \tag{38}$$

$$\Delta a_{h,theo} = \sqrt{(\frac{g\Delta sin\phi}{2})^2 + (\frac{sin\phi\Delta g}{2})^2} \approx 4,9 \cdot 10^{-5} m/s^2 \tag{39}$$

We now summarize both values in the following table:

Tabelle 5: Measured energies of the rolling cylinders

	$a_{Exp}[m/s^2]$	$a_{Theo}[m/s^2]$
solid cylinder	0,8176 $\pm 0,0204$	0,9679 $\pm 6,5 \cdot 10^{-6}$
hollow cylinder	0,6676 $\pm 0,0036$	0,7259 $\pm 4,9 \cdot 10^{-6}$

The much smaller error is caused by the small angle error of the inclined plane.

We compare the experimentally found values to the theoretically determined ones according to:

$$y := \frac{|a_{exp} - a_{theo}|}{\sqrt{(\Delta a_{exp})^2 + (\Delta a_{theo})^2}} \tag{40}$$

$$z := \frac{|a_{exp} - a_{theo}|}{a_{theo}} \tag{41}$$

Values of $y > 1$ indicate a significant difference of the calculated values.

$$\Longrightarrow y_s = 7,368 \qquad z_s = 0,155 \tag{42}$$

$$\Longrightarrow y_h = 16,19 \qquad z_h = 0,080 \tag{43}$$

There is a perfectly normal relative deviation, but the error deviation already exceeds $\geqslant 3\sigma$, which signifies a significant deviation. However, due to our conservative error estimation in this experiment, the deviation is still acceptable.

3.2 The law of energy conservation

In order to compare the kinetic energy (translation and rotation energy) with the potential energy, we first calculate the potential energy E_{pot} as well as its error ΔE_{pot} of the bodies inserting the values:

$$E_{pot} = mgh \tag{44}$$

$$\Delta E_{pot} = E_{pot} \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta h}{h}\right)^2} \tag{45}$$

Wir insert the mass of the solid and hollow cylinders $m_s = (444,8 \pm 0,1)g$, $m_h = (443,8 \pm 0,1)g$, the gravitational acceleration in Heidelberg $g = (9,80984 \pm 2 \times 10^{-5})\ m/s^2$ and the height $h = (13,25 \pm 0,05)cm$:

$$E_{pot,s} = 0,5823J \qquad E_{pot,h} = 0,5769J \tag{46}$$

$$\Delta E_{pot,s} = 0,0001J \qquad \Delta E_{pot,h} = 0,0001J \tag{47}$$

$$\Longrightarrow \underline{\underline{E_{pot,s} = (0,5823 \pm 0,0001)J}} \qquad \underline{\underline{E_{pot,h} = (0,5769 \pm 0,0001)J}} \tag{48}$$

To calculate the rotational energy, we need to determine the moments of inertia of the bodies using equations (6) and (7). Substituting the values radius $r = (2,5000 \pm 0,0025)cm$, inner radius $r_i = (1,2500 \pm 0,0025)cm$ we obtain:

$$I_s = \frac{1}{2} m_s r^2 \approx (1,3901 \cdot 10^{-4}) kg \cdot m^2 \tag{49}$$

$$I_h = \frac{1}{2} m_h d^2 = \frac{1}{2} m_h (r_i^2 + r^2) \approx (1,7336 \cdot 10^{-4}) kg \cdot m^2 \tag{50}$$

The errors of the moments of inertia are given by:

$$\Delta I_s = \sqrt{(\frac{\partial I_s}{\partial m_s} \Delta m_s)^2 + (\frac{\partial I_s}{\partial r} \Delta r)^2} = \sqrt{(\frac{1}{2} r^2 \Delta m_s)^2 + (m_s r \Delta r)^2} \approx (0,0003 \cdot 10^{-4}) kg \cdot m^2 \tag{51}$$

$$\Delta I_h = \sqrt{(\frac{\partial I_h}{\partial m_h} \Delta m_h)^2 + (\frac{\partial I_h}{\partial r} \Delta r)^2 + (\frac{\partial I_h}{\partial r_i} \Delta r_i)^2} \tag{52}$$

$$= \sqrt{(\frac{1}{2}(r_i^2 + r^2) \Delta m_h)^2 + (m_h r \Delta r)^2 + (m_h r_i \Delta r_i)^2} \approx (0,0004 \cdot 10^{-4}) kg \cdot m^2 \tag{53}$$

$$\Longrightarrow \underline{\underline{I_s = (1,3901 \pm 0,0003) \cdot 10^{-4} kg \cdot m^2}} \tag{54}$$

$$\Longrightarrow \underline{\underline{I_h = (1,7336 \pm 0,0004) \cdot 10^{-4} kg \cdot m^2}} \tag{55}$$

We calculate the velocity of the bodies at the bottom of the inclined plane so that we can get the rotational and translational energy. In this context, we considered the time difference between photocells 1 and 2 from Table 4 and used their average as the time required for the traveled distance:

$$t_h = \frac{0,294 + 0,294 + 0,295 + 0,292 + 0,293}{5} = 0,2936 s \tag{56}$$

$$t_s = \frac{0,263 + 0,263 + 0,264 + 0,263 + 0,263}{5} = 0,2632 s \tag{57}$$

The error Δt which includes the error due to calculating the mean of the measured values $\sigma_{\bar{t_s}}$ and the error $\Delta t_{read} = 0,001 s$ due to the limited reading accuracy of the meauring device according to:

$$\sigma_{\bar{t_h}} = \frac{\sigma_{t_h}}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{n}(t_{s_i} - t_s)^2}{n(n-1)}} \approx 0,0005s \tag{58}$$

$$\sigma_{\bar{t_s}} = \frac{\sigma_{t_s}}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{n}(t_{s_i} - t_s)^2}{n(n-1)}} \approx 0,0002s \tag{59}$$

$$\Longrightarrow \Delta t_s = \sqrt{(\sigma_{\bar{t_s}})^2 + (\Delta t)^2_{read}} = \sqrt{(0,0002)^2 + (0,001)^2} \approx 0,0010s \tag{60}$$

$$\Longrightarrow \Delta t_h = \sqrt{(\sigma_{\bar{t_h}})^2 + (\Delta t)^2_{read}} = \sqrt{(0,0005)^2 + (0,001)^2} \approx 0,0010s \tag{61}$$

$$\tag{62}$$

Due to the stabil measurement form uns only the accuracy of the light barriers is taken into account.

$$\Longrightarrow \underline{\underline{t_s = (0,263 \pm 0,001)s}} \qquad \underline{\underline{t_h = (0,294 \pm 0,001)s}} \tag{63}$$

The velocity v of a body that is not accelerated and travels a distance s at a time t is: $v = \frac{s}{t}$. The error Δv is given by

$$\Delta v = \sqrt{(\frac{\partial v}{\partial s}\Delta s)^2 + (\frac{\partial v}{\partial t}\Delta t)^2} = \sqrt{(\frac{\Delta s}{t})^2 + (\frac{s\Delta t}{t^2})^2} \tag{64}$$

which we can calculate assuming $\Delta s = 0,05cm$, $s = 30,00cm$ and an error Δt :

$$v_s = \frac{s}{t_s} = 1,141\ m/s \qquad \Delta v_s = \sqrt{(\frac{\Delta s}{t_s})^2 + (\frac{s\Delta t_s}{t_s^2})^2} \approx 0,002\ m/s \tag{65}$$

$$v_s = \frac{s}{t_h} = 1,020\ m/s \qquad \Delta v_h = \sqrt{(\frac{\Delta s}{t_h})^2 + (\frac{s\Delta t_h}{t_s^2})^2} \approx 0,002\ m/s \tag{66}$$

$$\Longrightarrow \underline{\underline{v_s = (1,141 \pm 0,002)\ m/s}} \qquad \underline{\underline{v_h = (1,020 \pm 0,002)\ m/s}} \tag{67}$$

Now, the rotational and the translational kinetic energies can be calculated according to (18) and using that $\omega = \frac{v}{r}$:

$$E_{rot,s} = \frac{I_s}{2}\omega_s^2 = \frac{I_s}{2}(\frac{v_s}{r})^2 \approx 0,1448J \tag{68}$$

$$E_{rot,h} = \frac{I_h}{2}\omega_s^2 = \frac{I_h}{2}(\frac{v_h}{r})^2 \approx 0,1443J \tag{69}$$

$$E_{trans,s} = \frac{m_s}{2}v_s^2 \approx 0,2916J \tag{70}$$

$$E_{trans,h} = \frac{m_h}{2}v_h^2 \approx 0,2309J \tag{71}$$

$$\tag{72}$$

In order to calculate the error of the energy we will also use the partial Derivative:

$$\Delta E_{rot,x} = \sqrt{(\frac{\partial E_{rot,x}}{\partial I_x}\Delta I_x)^2 + (\frac{\partial E_{rot,x}}{\partial v_x}\Delta v_x)^2 + (\frac{\partial E_{rot,x}}{\partial r}\Delta r)^2} \tag{73}$$

$$= \sqrt{(\frac{v_x^2 \Delta I_x}{2r^2})^2 + (\frac{I_x v_x \Delta v_x}{r^2})^2 + (\frac{I_x v_x^2}{r^3}\Delta r)^2} \tag{74}$$

$$\tag{75}$$

$$\Delta E_{trans,x} = \sqrt{(\frac{\partial E_{trans,x}}{\partial m_x}\Delta m_x)^2 + (\frac{\partial E_{tans,x}}{\partial v_x}\Delta v_x)^2} \tag{76}$$

$$= \sqrt{(\frac{v_x^2 \Delta m_x}{2})^2 + (m_x v_x \Delta v_x)^2} \tag{77}$$

We substitute the respective values into equations (49) through (53) and obtain the errors for the energy:

$$\Delta E_{rot,s} = \sqrt{(\frac{v_s^2 \Delta I_s}{2r^2})^2 + (\frac{I_s v_s \Delta v_s}{r^2})^2 + (\frac{I_s v_s^2}{r^3}\Delta r)^2} \approx 0,0003J \tag{78}$$

$$\Delta E_{rot,h} = \sqrt{(\frac{v_h^2 \Delta I_h}{2r^2})^2 + (\frac{I_h v_h \Delta v_h}{r^2})^2 + (\frac{I_h v_h^2}{r^3}\Delta r)^2} \approx 0,0003J \tag{79}$$

$$\Delta E_{trans,s} = \sqrt{(\frac{v_s^2 \Delta m_s}{2})^2 + (m_s v_s \Delta v_s)^2} \approx 0,0010 \tag{80}$$

$$\Delta E_{trans,h} = \sqrt{(\frac{v_h^2 \Delta m_h}{2})^2 + (m_h v_h \Delta v_h)^2} \approx 0,0009 \tag{81}$$

Next, the total kinetic energy $E_{kin} = E_{rot} + E_{trans}$ of the two bodies is calculated by adding the rotational and the translational kinetic energy and compared with the potential energy. The total error of the kinetic energy consists of the error of the rotational and the translational energy as follows:

$$\Delta E_{kin} = \sqrt{(\Delta E_{rot})^2 + (\Delta E_{trans})^2} \tag{82}$$

$$\implies \Delta E_{kin,s} \approx 0,0010J \qquad \Delta E_{kin,h} \approx 0,0009J \tag{83}$$

We add the rotational energy to the translational energy and present the kinetic energy along with its error in the following table:

Tabelle 6: Measured energies of the rolling cylinders

Body	$E_{pot} \pm \Delta E_{pot}$ $[J]$	$E_{kin} \pm \Delta E_{kin}$ $[J]$	Difference [J]
Solid	$0,5823 \pm 0,0001$	$0,4364 \pm 0,0010$	0,1459
Hollow	$0,5769 \pm 0,0001$	$0,3752 \pm 0,0009$	0,2017

Finally, we would like to calculate the relative errors as well as the error range so that we can compare the kinetic energy at the bottom of the runway with the potential energy at the beginning.

$$y := \frac{|E_{pot} - E_{kin}|}{\sqrt{(\Delta E_{pot})^2 + (\Delta E_{kin})^2}} \tag{84}$$

$$z := \frac{|E_{pot} - E_{kin}|}{E_{pot}} \tag{85}$$

Values of $y > 1$ mean a significant difference of the calculated values.

$$\implies y_s = 145.18 \qquad z_s = 0.25 \tag{86}$$

$$\implies y_h = 222.74 \qquad z_h = 0.35 \tag{87}$$

$$\tag{88}$$

These errors have definitely become too significant, with the relative errors exceeding a quarter. This probably indicates that the conservation of energy principle does not hold under a conservative force field. This is actually not correct, and we will further investigate the reasons for such a large deviation in the following discussion.

4 Discussion

In this experiment, we initially attempted to determine the acceleration of both a solid cylinder and a hollow cylinder on an inclined plane. To do this, we placed light barriers at specific intervals along the plane and graphically represented the distance between the light barrier and the initial position as a function of time squared. The slope of this graph allowed us to experimentally calculate the acceleration. Subsequently, we theoretically determined two additional accelerations by utilizing the measured incline angle and the theory of moment of inertia. We then compared these theoretical values with the experimentally measured results. It was noted that both rolling bodies exhibited a significant deviation from the expected values, despite the noticeable errors. However, due to the relatively low relative error, the agreement between the experimental and theoretical results was confirmed.

In the second part of the experiment, we examined the conservation of energy within a conservative force field (gravity). To do this, we compared the potential energy at the beginning with the kinetic energy at the end. According to the law of conservation of energy, potential energy should be converted into kinetic energy, comprising both rotational and translational energy. The hollow cylinder, due to its larger moment of inertia, acquired more rotational energy, resulting in a slower acceleration and lower translational velocity at the end. Surprisingly, a significant discrepancy between kinetic and potential energy arose, with an error exceeding 100 σ. One contributing factor to this discrepancy was our overly conservative error estimation, which led to wide error margins for most values (e.g. ϕ, I_x, v_x). Additionally, mechanical friction factors such as air resistance and friction coefficients played a significant role in energy loss. This situation could be improved by conducting the experiment on a surface with a lower friction coefficient, such as an air cushion track, or by determining the friction coefficient before the experiment to account for energy loss.

Furthermore, we considered the bending at the lower end of the plane, which resulted in further energy loss. We observed that, after repeated experiments, there was a crack at the junction point of the bevel with the horizontal track due to the impact force of the cylinder, causing an additional loss of kinetic energy. For future experiments, it would be advisable to securely connect these two components.

The angle ϕ could have been more precisely determined by measuring the length of the plane's projection on the table. However, the results yielded a very small error (approximately 10^{-5}) because we used the smallest resolution of the measuring device, which appeared negligible compared to our measuring length.

In the second part of the experiment, we used time measurements at the lower end of the plane to calculate the final velocities of the bodies and compare energies. The energy values significantly differed for both cylinders, with potential energy being greater in both cases. This aligns with our expectations, as there is also friction between the rolling bodies and the air and plane during rolling, leading to measurable energy loss. The two cylinders may have exhibited different behavior at this point due to their distinct geometries, possibly explaining the larger deviation of 145σ for the solid cylinder compared to $223s$ for the hollow one.