

# Introduction to Relational Databases

- Bachelor CS, Lille 1 University
- Oct 14th, 2015 (lecture 7/12)
- Topic: Formal query languages
  - from SQL a step back to
  - Tuple-oriented Relational Calculus - TRC

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## Rendu du TP4

- Prolongation: deux semaines
  - Date: lundi 26 octobre
- Sur **Moodle**
- Ré-inscription nécessaire avec clé pour votre groupe
  - **L3S5\_15\_bdd\_Gi**
  - Où **i** est votre numéro de groupe

## Précisions pour les notes de CC

Formule habituelle de la L3 (voir sur portail)

**Note de TP (compte 25% de la note totale):**

- 1 pt par séance sur 8 seances et fait d'avoir rendu. Max 8pt.
- Qualité des rendus, sur 2 ou 3 rendus aléatoires par étudiants: max 6 pt.
- Controle de TP (derniere seance): max 6 pt.

**Note de TD (“règle du max” avec DS):**

- Présence et participation: max 10 pt
- Deux interros surprise à max 5 points

## Relational calculus

One of two major formal languages:

TRC, **T**uple **R**elational **C**alculus

DRC, Domain Relational Calculus

- Two versions of TRC:
  - with range-restricted tuples
  - with arbitrary tuples

We will study TRC with arbitrary tuples

## TRC is declarative

Express *what* we want in the result, but not *how* to obtain it.

Quite different from relational algebra

Declarativeness is a typical feature of relational languages. It holds for TRC and SQL!

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## TRC: definition

- Standard form:  $\{ t \mid p(t) \}$   
 $p(t)$  is a **formula**, built with atoms
- Definition of **atoms**:
  - $t \in R$
  - $\text{expr comp expr}$ 
    - $\text{comp}$  is comparison operator:  $=, <, >, \geq, \leq, \leq$
    - $\text{expr}$  is an expression using **constants** and  $t[A]$
    - $t[A]$  is a restriction of a tuple  $t$  on its attribute(s)  $A$
- Example:  $\{ t \mid t \in R \}$

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## TRC: rules to construct a valid formula

- an atom is a formula
- if  $p$  is a formula,  $\neg p$  (negation) and  $(p)$  are also valid formulas
- if  $p_1$  and  $p_2$  are valid formulas, then  
 $p_1 \wedge p_2$ ,  $p_1 \vee p_2$ ,  $p_1 \Rightarrow p_2$  are valid formulas  
(**conjunction**  $\wedge$ , **disjunction**  $\vee$ , **implication**  $\Rightarrow$ )
- if  $p$  is a formula, with variable  $s$ , then the following are valid formulas:
  - $\exists s \in R (p(s))$  existential quantification
  - $\forall s \in R (p(s))$  universal quantification

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## RA can be expressed with TRC

It is sufficient to show that the fundamental operators of RA can be expressed in TRC. Let  $R(A,B,C)$  be a relation.

Selection,  $\sigma_{A=1} R$ :

$$\{ t \mid \exists t \in R (t[A]=1) \}$$

Projection,  $\Pi_{AC} R$ :

$$\{ t \mid \exists t_1 \in R (t[A,C]=t_1[A,C]) \}$$

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## RA can be expressed with TRC

Cartesian Product,  $R(A,B,C) \times S(D,E,F)$ :

$$\{ t \mid \exists t1 \in R, \exists t2 \in S \\ (t[A,B,C]=t1[A,B,C] \wedge \\ t[D,E,F]=t2[D,E,F]) \}$$

Join (on common attributes A and B),

$R(A,C) * S(B,D)$ :

$$\{ t \mid \exists t1 \in R, \exists t2 \in S \\ (t[A,C] = t1[A,C] \wedge \\ t[B,D] = t2[B,D] \wedge \\ t[A] = t[B]) \}$$

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Other direction:

## TRC can be expressed with RA

However, the proof is ways more complex

Need to exclude unsafe and domain-dependent expressions

Under this hypothesis, TRC and AR have the same expressive power

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## RA can be expressed with TRC

Union,  $R \cup S$ :

$$\{ t \mid \exists t1 \in R, \exists t2 \in S \\ (t = t1 \vee t = t2) \}$$

Difference,  $R - S$ :

$$\{ t \mid \exists t \in R (t \notin S) \}$$

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## Equivalence laws for TRC

- De Morgan's Laws
$$p1 \wedge p2 \equiv \neg (\neg p1 \vee \neg p2)$$
- Correspondence between quantifiers
$$\forall t \in R (p(t)) \equiv \neg \exists t \in R (\neg p(t))$$
- Implication, defined as:
$$p1 \Rightarrow p2 \equiv \neg p1 \vee p2$$

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## Normal forms

- The above three laws imply that it is possible to write all kinds of expressions without implication:
  - Only one quantifier
  - Only one binary operator
- The most known normal form (similar to SQL) uses the existential quantifier and conjunction

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## TRC: examples

- Names of students with grade A in mathematics
- $\{ t \mid \exists t1 \in \text{STUDENT},$   
     $\exists t2 \in \text{EXAM},$   
     $\exists t3 \in \text{CLASS}$   
     $( t[\text{name}] = t1[\text{name}] \wedge$   
     $t1[\text{sid}] = t2[\text{sid}] \wedge$   
     $t2[\text{cid}] = t3[\text{cid}] \wedge$   
     $t2[\text{grade}] = A \wedge$   
     $t3[\text{title}] = \text{'mathematics'})$   
     $\}$

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## TRC: examples

Ids of the students who have passed the exam for “mathematics” but not for “databases”

$$\{ t \mid \exists t1 \in \text{EXAM}, \exists t2 \in \text{CLASS}$$
$$(t[\text{sid}] = t1[\text{sid}] \wedge$$
$$t1[\text{cid}] = t2[\text{cid}] \wedge$$
$$t2[\text{title}] = \text{'mathematics'}) \wedge$$
$$\neg ( \exists t3 \in \text{EXAM}, \exists t4 \in \text{CLASS}$$
$$(t[\text{sid}] = t3[\text{sid}] \wedge$$
$$t3[\text{cid}] = t4[\text{cid}] \wedge$$
$$t4[\text{title}] = \text{'databases'}) ) \}$$

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## Correctness

We avoid to use ‘unsafe’ formulas:

$\{ t \mid t \notin R \}$  returns an infinite result!

- Only formulas that are independent of the domain are correct
  - the solution does not depend of the domain of the attributes, but solely on the database instances

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