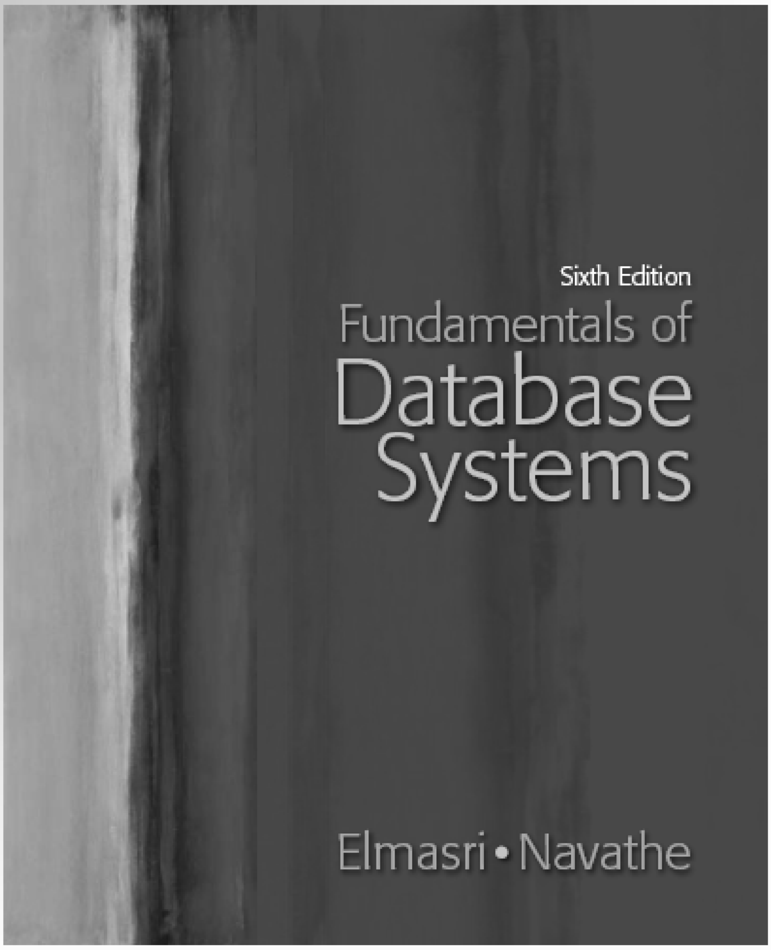
Université de Lille 1 Sciences et Technologies

L3 Informatique

BDD, Cours 2

Relational Algebra



Outline

- Unary Relational Operations: SELECT and PROJECT
- Relational Algebra Operations from Set Theory
- Binary Relational Operations: JOIN and DIVISION
- Additional Relational Operations

Outline (cont'd.)

- Examples of Queries in Relational Algebra
- The Tuple Relational Calculus
- [The Domain Relational Calculus]

The Relational Algebra and Relational Calculus

- **Relational algebra**
  - Basic set of operations for the relational model
- **Relational algebra expression**
  - Sequence of relational algebra operations
- **Relational calculus**
  - Higher-level declarative language for specifying relational queries

Example database

- A company and its employees
- For each employees we know
  - the name
  - The social security number
  - Birth date, gender
  - His personal address, salary
  - the company's department where he works
  - his superior

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date

DEPT_LOCATIONS

Dnumber	Dlocation

PROJECT

Pname	Pnumber	Plocation	Dnum

WORKS_ON

Essn	Pno	Hours

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship

Figure 3.5
Schema diagram for the COMPANY relational database schema.

Figure 3.6
One possible database state for the COMPANY relational database schema.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

Continued next page...

Unary Relational Operations: SELECT and PROJECT

- The SELECT Operation
  - Subset of the tuples from a relation that satisfies a selection condition:

$$\sigma_{<\text{selection condition}>}(R)$$

    - Boolean expression contains clauses of the form <attribute name> <comparison op> <constant value>

      or
    - <attribute name> <comparison op> <attribute name>

Unary Relational Operations: SELECT and PROJECT (cont'd.)

- Example:

$$\sigma_{(\text{Dno}=4 \textbf{ AND } \text{Salary}>25000) \textbf{ OR } (\text{Dno}=5 \textbf{ AND } \text{Salary}>30000)}(\text{EMPLOYEE})$$

- <selection condition> applied independently to each individual tuple *t* in *R*
  - If condition evaluates to TRUE, tuple selected
- Boolean conditions **AND**, **OR**, and **NOT**
- **Unary**
  - Applied to a single relation

Unary Relational Operations: SELECT and PROJECT (cont'd.)

- **Selectivity**
  - Fraction of tuples selected by a selection condition
- SELECT operation commutative
- **Cascade** SELECT operations into a single operation with **AND** condition

The PROJECT Operation

- Selects columns from table and discards the other columns:

$$\pi_{<\text{attribute list}>}(R)$$

- **Degree**
  - Number of attributes in <attribute list>
- **Duplicate elimination**
  - Result of PROJECT operation is a set of distinct tuples

Sequences of Operations and the RENAME Operation

- **In-line** expression:

$$\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno}=5}(\text{EMPLOYEE}))$$

- Sequence of operations:

$$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{Dno}=5}(\text{EMPLOYEE})$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Salary}}(\text{DEP5\_EMPS})$$

- **Rename** attributes in intermediate results
  - RENAME operation

$$\rho_{S(B1,\, B2,\, \ldots,\, Bn)}(R) \quad \text{or} \quad \rho_{S}(R) \quad \text{or} \quad \rho_{(B1,\, B2,\, \ldots,\, Bn)}(R)$$

Relational Algebra Operations from Set Theory

- **UNION**, **INTERSECTION**, and **MINUS**
  - Merge the elements of two sets in various ways
  - Binary operations
  - Relations must have the same type of tuples
- UNION
  - $R \cup S$
  - Includes all tuples that are either in *R* or in *S* or in both *R* and *S*
  - Duplicate tuples eliminated

Relational Algebra Operations from Set Theory (cont'd.)

- INTERSECTION
  - $R \cap S$
  - Includes all tuples that are in both *R* and *S*
- SET DIFFERENCE (or MINUS)
  - $R – S$
  - Includes all tuples that are in *R* but not in *S*

The CARTESIAN PRODUCT (CROSS PRODUCT) Operation

- **CARTESIAN PRODUCT**
  - **CROSS PRODUCT** or **CROSS JOIN**
  - Denoted by ×
  - Binary set operation
  - Relations do not have to be union compatible
  - Useful when followed by a selection that matches values of attributes

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Binary Relational Operations: JOIN and DIVISION

- The **JOIN** Operation
  - Denoted by $\bowtie$
  - Combine related tuples from two relations into single “longer” tuples
  - General join condition of the form <condition> **AND** <condition> **AND**...**AND** <condition>
  - Example:

$$\text{DEPT\_MGR} \leftarrow \text{DEPARTMENT} \bowtie_{\text{Mgr\_ssn=Ssn}} \text{EMPLOYEE}$$
$$\text{RESULT} \leftarrow \pi_{\text{Dname, Lname, Fname}}(\text{DEPT\_MGR})$$

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Binary Relational Operations: JOIN and DIVISION (cont'd.)

- **THETA** JOIN
  - Each <condition> of the form $A_i \theta B_j$
  - A_i is an attribute of R
  - B_j is an attribute of S
  - A_i and B_j have the same domain
  - θ (theta) is one of the comparison operators:
    - $\{=, <, \leq, >, \geq, \neq\}$

Variations of JOIN: The EQUIJOIN and NATURAL JOIN

- **EQUIJOIN**
  - Only = comparison operator used
  - Always have one or more pairs of attributes that have identical values in every tuple
- **NATURAL JOIN**
  - Denoted by *
  - Removes second (superfluous) attribute in an EQUIJOIN condition

Variations of JOIN: The EQUIJOIN and NATURAL JOIN (cont'd.)

- **Join selectivity**
  - Expected size of join result divided by the maximum size $n_R * n_S$
- **Inner joins**
  - Type of match and combine operation
  - Defined formally as a combination of CARTESIAN PRODUCT and SELECTION

A Complete Set of Relational Algebra Operations

- Set of relational algebra operations {σ, π, ∪, ρ, –, ×} is a **complete set**
  - Any relational algebra operation can be expressed as a sequence of operations from this set

The DIVISION Operation

- Denoted by ÷
- Example: retrieve the names of employees who work on all the projects that 'John Smith' works on
- Apply to relations *R*(*Z*) ÷ *S*(*X*)
  - Attributes of *R* are a subset of the attributes of *S*

Operations of Relational Algebra

Table 6.1 Operations of Relational Algebra

OPERATION	PURPOSE	NOTATION
SELECT	Selects all tuples that satisfy the selection condition from a relation *R*.	$\sigma_{\text{<selection condition>}}(R)$
PROJECT	Produces a new relation with only some of the attributes of *R*, and removes duplicate tuples.	$\pi_{\text{<attribute list>}}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\text{<join condition>}} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\text{<join condition>}} R_2$, OR $R_1 \bowtie_{(\text{<join attributes 1>}),(\text{<join attributes 2>})} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\text{<join condition>}} R_2$, OR $R_1 *_{(\text{<join attributes 1>}),(\text{<join attributes 2>})} R_2$ OR $R_1 * R_2$

Operations of Relational Algebra (cont'd.)

Table 6.1 Operations of Relational Algebra

UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

Notation for Query Trees

- **Query tree**
  - Represents the input relations of query as leaf nodes of the tree
  - Represents the relational algebra operations as internal nodes

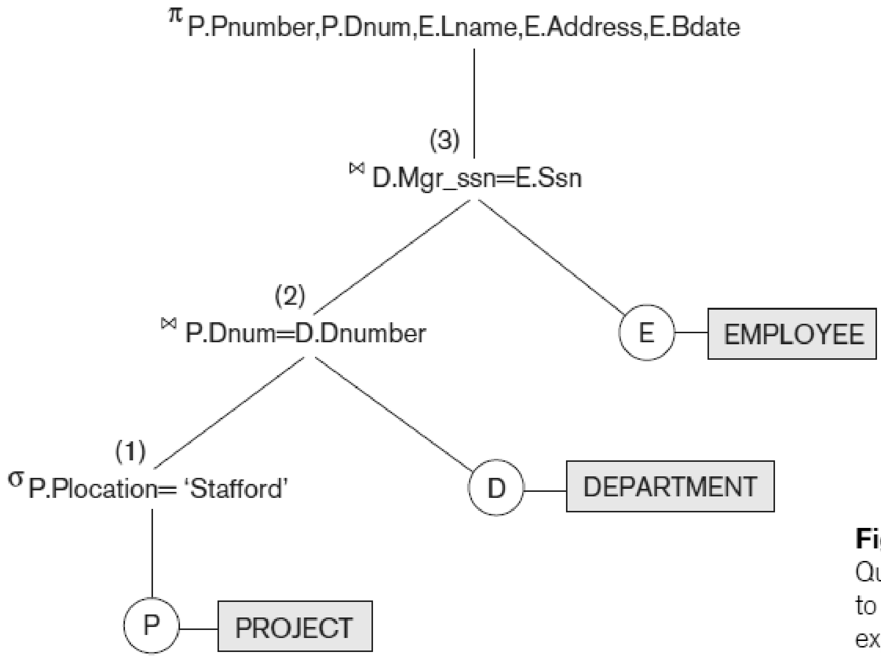

Figure 6.9
Query tree corresponding to the relational algebra expression for Q2.

Additional Relational Operations

- **Generalized projection**
  - Allows functions of attributes to be included in the projection list

    $$\pi_{F1,\ F2,\ \ldots,\ Fn}\ (R)$$

- **Aggregate functions** and **grouping**
  - Common functions applied to collections of numeric values
  - Include SUM, AVERAGE, MAXIMUM, and MINIMUM

Additional Relational Operations (cont'd.)

- Group tuples by the value of some of their attributes
  - Apply aggregate function independently to each group

$$\text{<grouping attributes>} \; \mathfrak{I} \; \text{<function list>} \; (R)$$

Figure 6.10
The aggregate function operation.

a. $\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(_{\text{Dno}}\,\mathfrak{I}_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE}))$.

b. $_{\text{Dno}}\,\mathfrak{I}_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

c. $\mathfrak{I}_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

(a) R

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

(b)

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

(c)

Count_ssn	Average_salary
8	35125

[8] Note that this is an arbitrary notation we are suggesting. There is no standard notation.

Recursive Closure Operations

- Operation applied to a **recursive relationship** between tuples of same type

$$\text{BORG\_SSN} \leftarrow \pi_{\text{Ssn}}(\sigma_{\text{Fname='James' AND Lname='Borg'}}(\text{EMPLOYEE}))$$
$$\text{SUPERVISION}(\text{Ssn1}, \text{Ssn2}) \leftarrow \pi_{\text{Ssn,Super\_ssn}}(\text{EMPLOYEE})$$
$$\text{RESULT1}(\text{Ssn}) \leftarrow \pi_{\text{Ssn1}}(\text{SUPERVISION} \bowtie_{\text{Ssn2=Ssn}} \text{BORG\_SSN})$$

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OUTER JOIN Operations

- **Outer joins**
  - Keep all tuples in *R*, or all those in *S*, or all those in both relations regardless of whether or not they have matching tuples in the other relation
  - **Types**
    - **LEFT OUTER JOIN, RIGHT OUTER JOIN, FULL OUTER JOIN**
  - Example: $\text{TEMP} \leftarrow (\text{EMPLOYEE} ⟕_{\text{Ssn=Mgr\_ssn}} \text{DEPARTMENT})$

$$\text{RESULT} \leftarrow \pi_{\text{Fname, Minit, Lname, Dname}}(\text{TEMP})$$

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The OUTER UNION Operation

- Take union of tuples from two relations that have some common attributes
  - Not union (type) compatible
- **Partially compatible**
  - All tuples from both relations included in the result
  - Tut tuples with the same value combination will appear only once

Examples of Queries in Relational Algebra

Query 1. Retrieve the name and address of all employees who work for the 'Research' department.

$\text{RESEARCH\_DEPT} \leftarrow \sigma_{\text{Dname}=\text{'Research'}}(\text{DEPARTMENT})$
$\text{RESEARCH\_EMPS} \leftarrow (\text{RESEARCH\_DEPT} \bowtie_{\text{Dnumber}=\text{Dno}} \text{EMPLOYEE})$
$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Address}}(\text{RESEARCH\_EMPS})$

As a single in-line expression, this query becomes:

$\pi_{\text{Fname, Lname, Address}}\ (\sigma_{\text{Dname}=\text{'Research'}}(\text{DEPARTMENT} \bowtie_{\text{Dnumber}=\text{Dno}}(\text{EMPLOYEE}))$

Examples of Queries in Relational Algebra (cont'd.)

Query 2. For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

$STAFFORD\_PROJS \leftarrow \sigma_{Plocation=\text{'Stafford'}}(PROJECT)$
$CONTR\_DEPTS \leftarrow (STAFFORD\_PROJS \bowtie_{Dnum=Dnumber} DEPARTMENT)$
$PROJ\_DEPT\_MGRS \leftarrow (CONTR\_DEPTS \bowtie_{Mgr\_ssn=Ssn} EMPLOYEE)$
$RESULT \leftarrow \pi_{Pnumber,\ Dnum,\ Lname,\ Address,\ Bdate}(PROJ\_DEPT\_MGRS)$

Query 3. Find the names of employees who work on *all* the projects controlled by department number 5.

$DEPT5\_PROJS \leftarrow \rho_{(Pno)}(\pi_{Pnumber}(\sigma_{Dnum=5}(PROJECT)))$
$EMP\_PROJ \leftarrow \rho_{(Ssn,\ Pno)}(\pi_{Essn,\ Pno}(WORKS\_ON))$
$RESULT\_EMP\_SSNS \leftarrow EMP\_PROJ \div DEPT5\_PROJS$
$RESULT \leftarrow \pi_{Lname,\ Fname}(RESULT\_EMP\_SSNS * EMPLOYEE)$

Examples of Queries in Relational Algebra (cont'd.)

Query 6. Retrieve the names of employees who have no dependents.

This is an example of the type of query that uses the MINUS (SET DIFFERENCE) operation.

$ALL\_EMPS \leftarrow \pi_{Ssn}(EMPLOYEE)$
$EMPS\_WITH\_DEPS(Ssn) \leftarrow \pi_{Essn}(DEPENDENT)$
$EMPS\_WITHOUT\_DEPS \leftarrow (ALL\_EMPS - EMPS\_WITH\_DEPS)$
$RESULT \leftarrow \pi_{Lname,\ Fname}(EMPS\_WITHOUT\_DEPS * EMPLOYEE)$

Query 7. List the names of managers who have at least one dependent.

$MGRS(Ssn) \leftarrow \pi_{Mgr\_ssn}(DEPARTMENT)$
$EMPS\_WITH\_DEPS(Ssn) \leftarrow \pi_{Essn}(DEPENDENT)$
$MGRS\_WITH\_DEPS \leftarrow (MGRS \cap EMPS\_WITH\_DEPS)$
$RESULT \leftarrow \pi_{Lname,\ Fname}(MGRS\_WITH\_DEPS * EMPLOYEE)$

Summary

- Formal languages for relational model of data:
  - Relational algebra: operations, unary and binary operators
  - Some queries cannot be stated with basic relational algebra operations
    - But are important for practical use
- Relational calculus
  - Based predicate calculus