CS 87/187, Spring 2016

Rendering Algorithms

Direct Illumination I

Prof. Wojciech Jarosz

wojciech.k.jarosz@dartmouth.edu

(with many slides adapted from Dr. Jan Novák)

Dartmouth

Last time

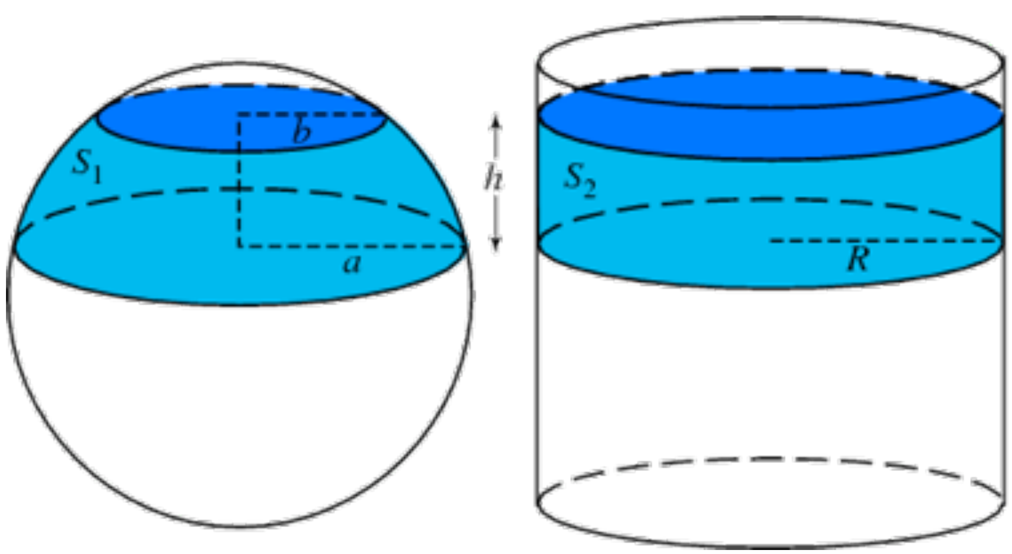
- Goal: evaluate integral $\int_a^b f(x)dx$
- Random variable $X_i \sim p(x)$
- Monte Carlo Estimator $F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(X_i)}{p(X_i)}$
- Expectation $E[F_N] = \int_a^b f(x)dx$

Last time

- Sampling:
  - uniform disk
  - uniform sphere
  - uniform hemisphere
  - cosine-weighted hemisphere
- The inversion method
- Conditional and marginal distributions

Archimedes' Hat-Box Theorem

- Uniformly sampling a hemisphere
- The surface area of a sphere between any two horizontal planes is equal to the corresponding area on the circumscribing cylinder

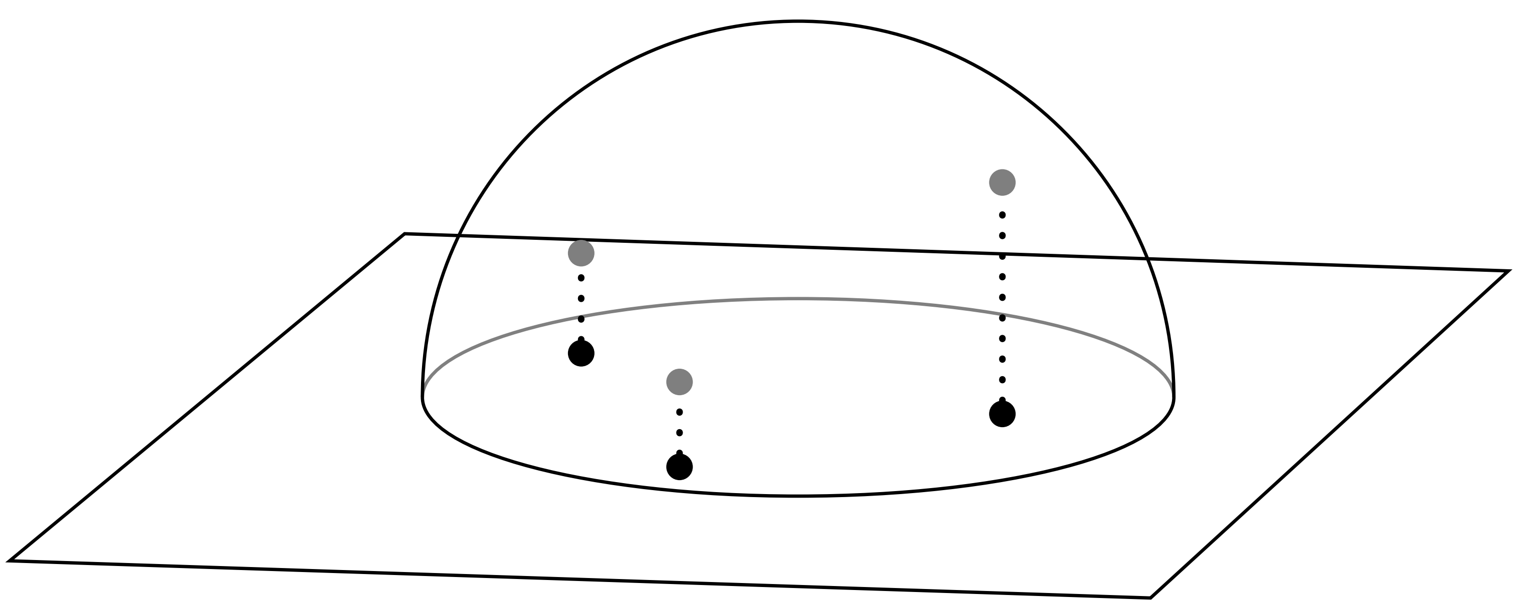


a hat-box!

Weisstein, Eric W. "Archimedes' Hat-Box Theorem." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/ArchimedesHat-BoxTheorem.html

Cosine-weighted Hemispherical Sampling

- Generate points uniformly on the disc, and then project these points to the surface of the hemisphere
  - Called the "Nusselt Analog"



Today’s Menu

- *Direct* vs. *indirect* illumination
- *Hemispherical* vs. *area* formulation of the reflection equation
- Light source models
- Importance sampling
- Multiple importance sampling

Direct vs. Indirect Illumination

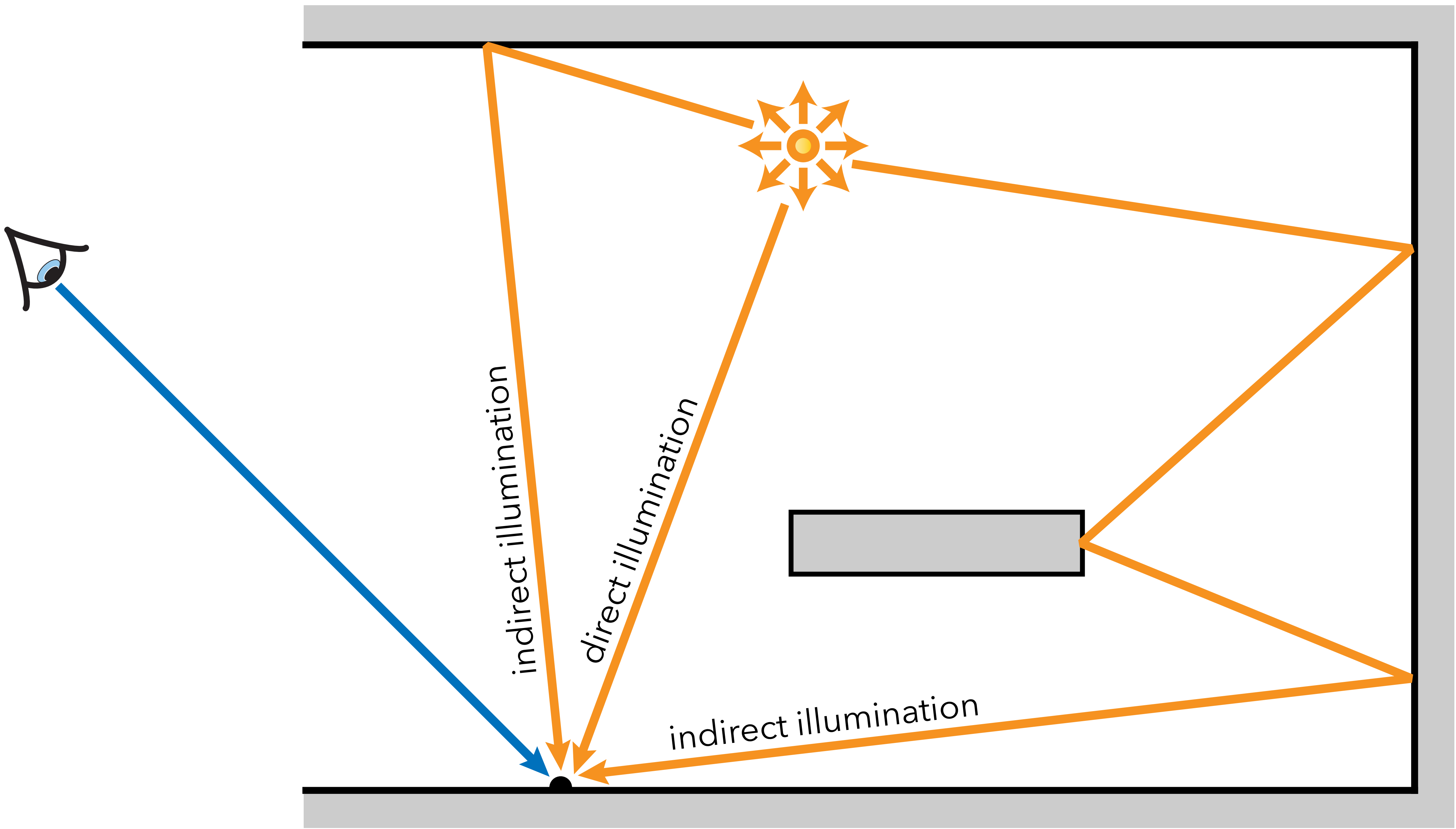
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- Where does L_i "come from"?
  - if directly from an emitter, we refer to it as *direct* illumination

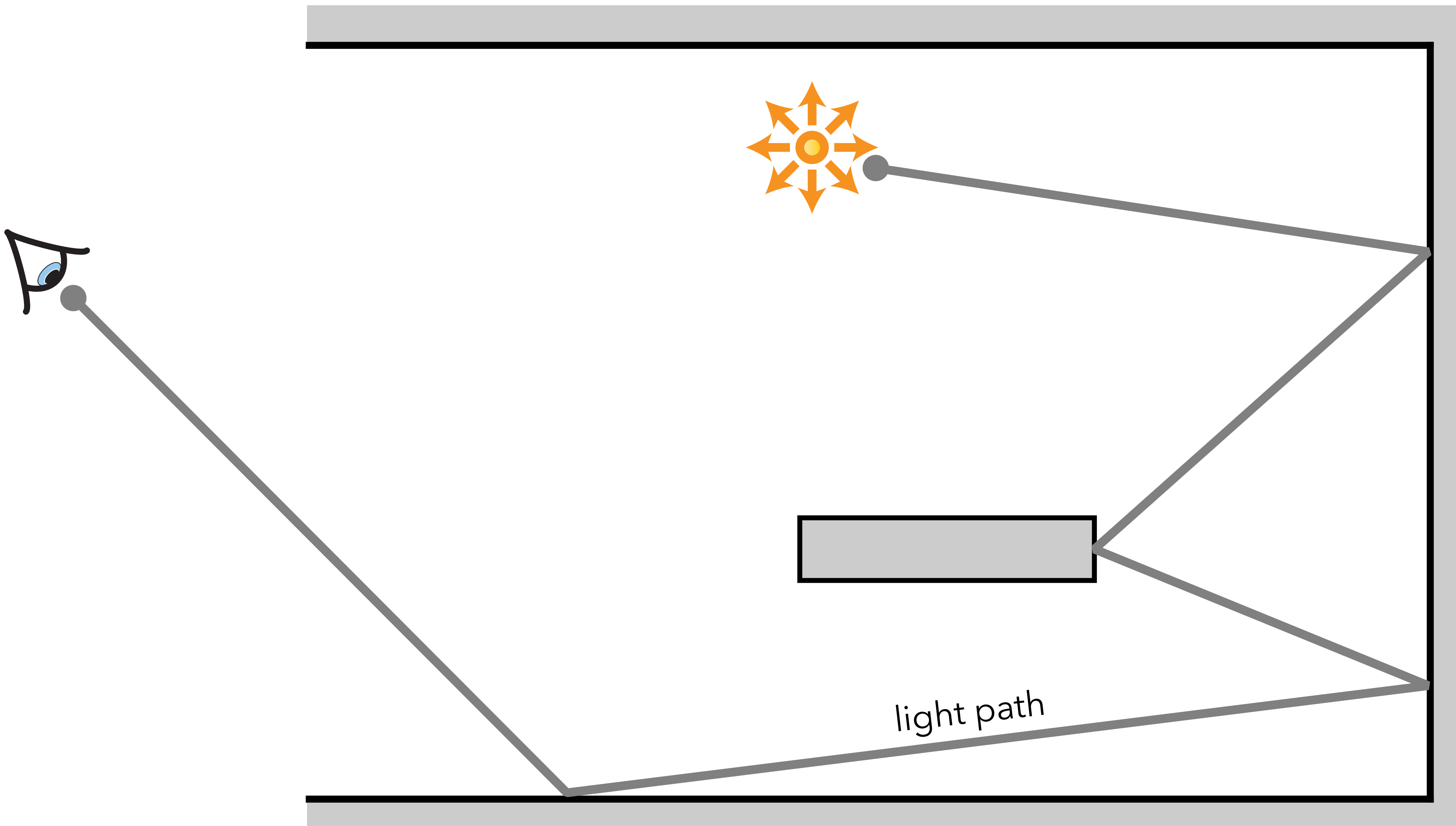
$$L_i(\mathbf{x}, \vec{\omega}) = L_e(r(\mathbf{x}, \vec{\omega}), -\vec{\omega})$$

  - if indirectly, i.e. by bouncing off a scattering surface, we call it *indirect* illumination

Direct vs. Indirect Illumination



Light Path

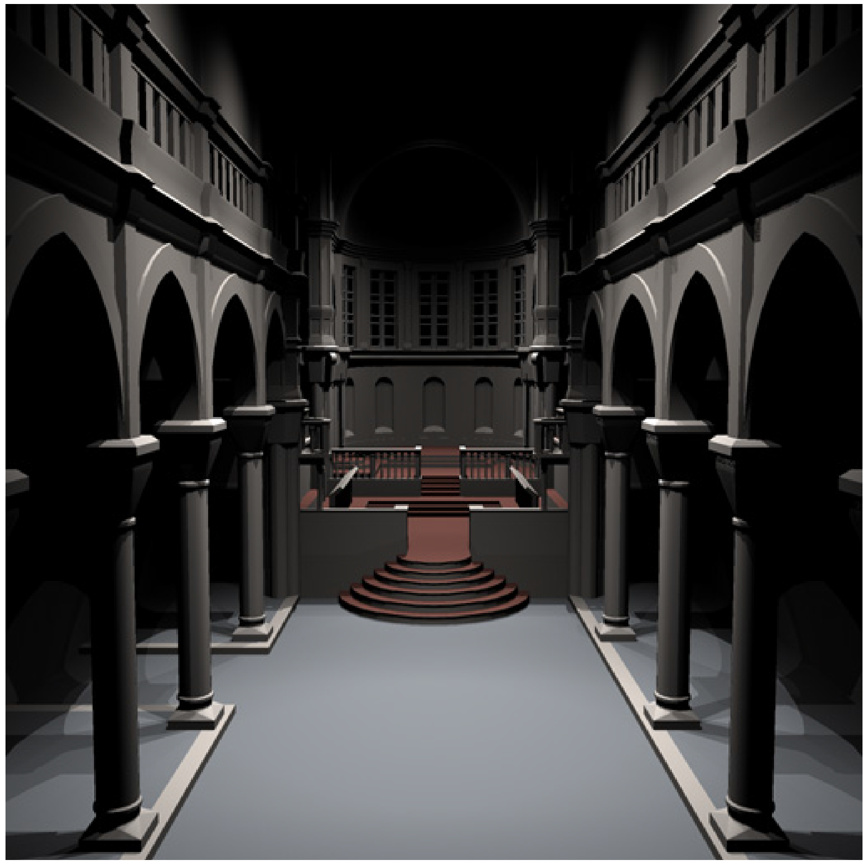


Light Path

- Connects a light source to a sensor
- Constructed by tracing from:
  - light source…*light tracing*
  - from sensor… *path tracing*
  - or from both… *bidirectional path tracing*
- Length of light path:
  - = 2 segments… *direct illumination, direct lighting*
  - > 2 segments… *indirect illumination, indirect lighting*

Direct vs. Indirect Illumination

Direct illumination

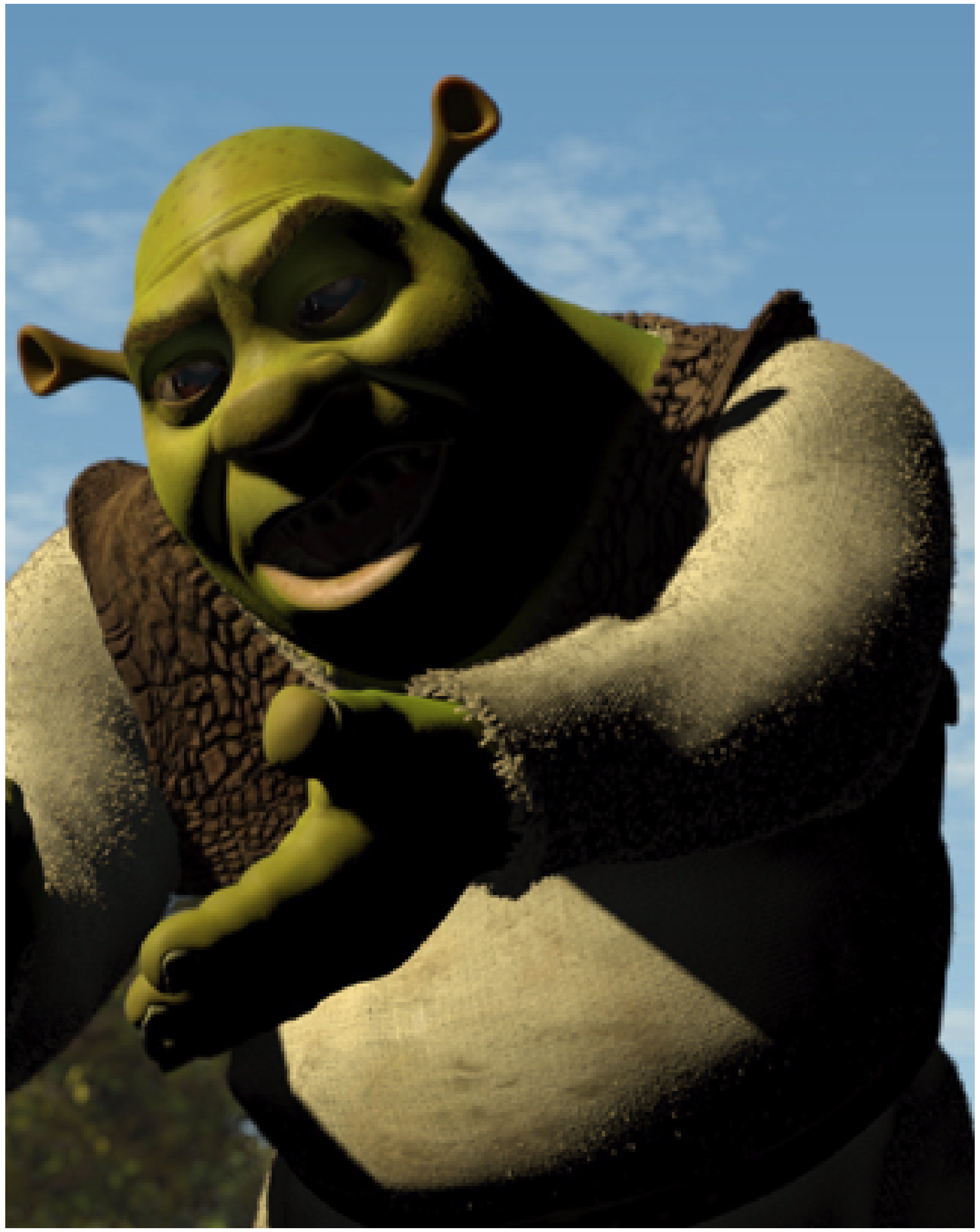


Indirect illumination

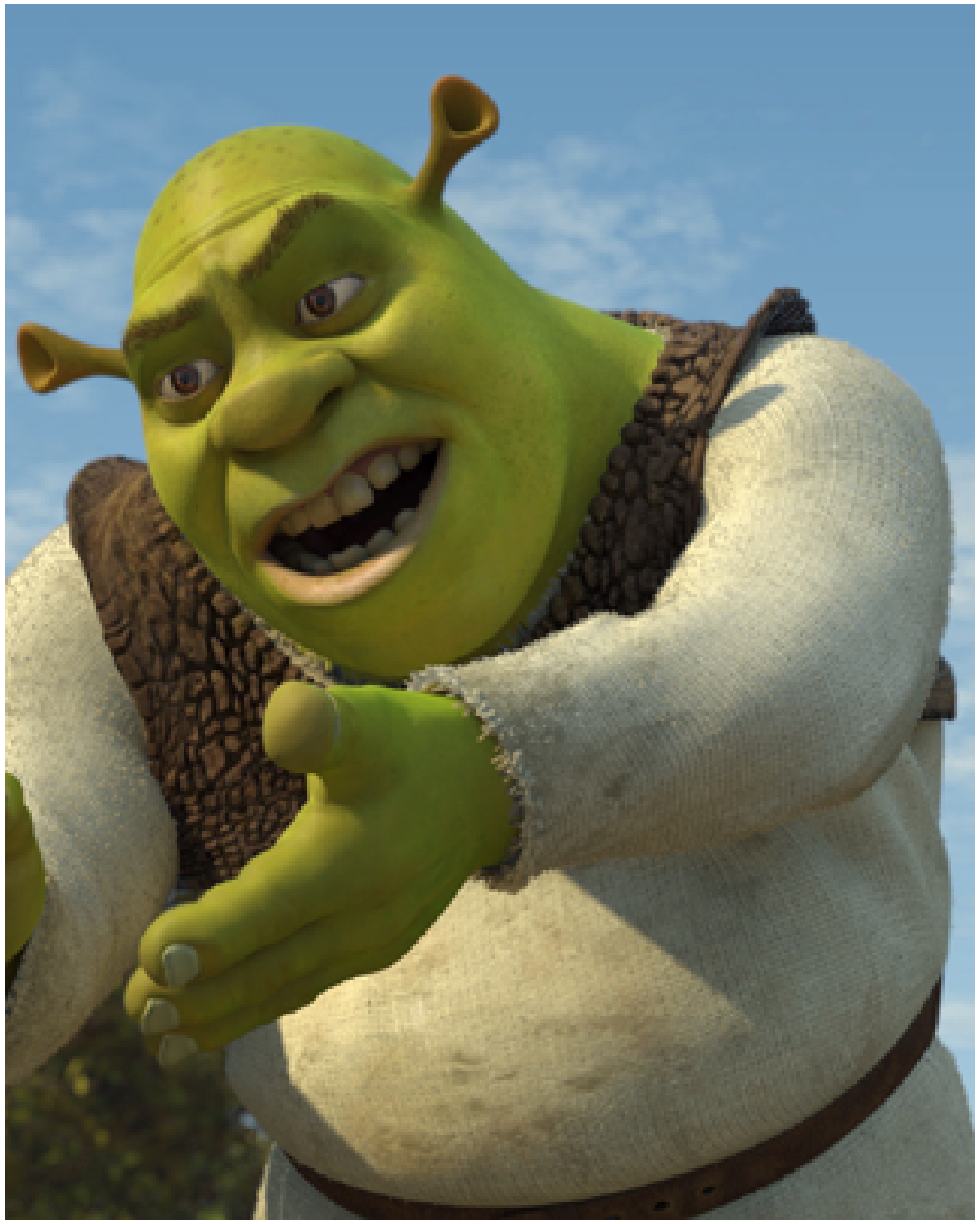
Direct + indirect illumination

Direct vs. Indirect Illumination

Direct illumination only



Direct + Indirect illumination



Images courtesy of PDI/DreamWorks

Direct Illumination

(indirect illumination will be covered later)

Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

The general incident radiance L_i
can be replaced by emission elsewhere:

$$L_i(\mathbf{x}, \vec{\omega}_i) = L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i)$$

Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

How can we estimate the integral?

Direct Illumination

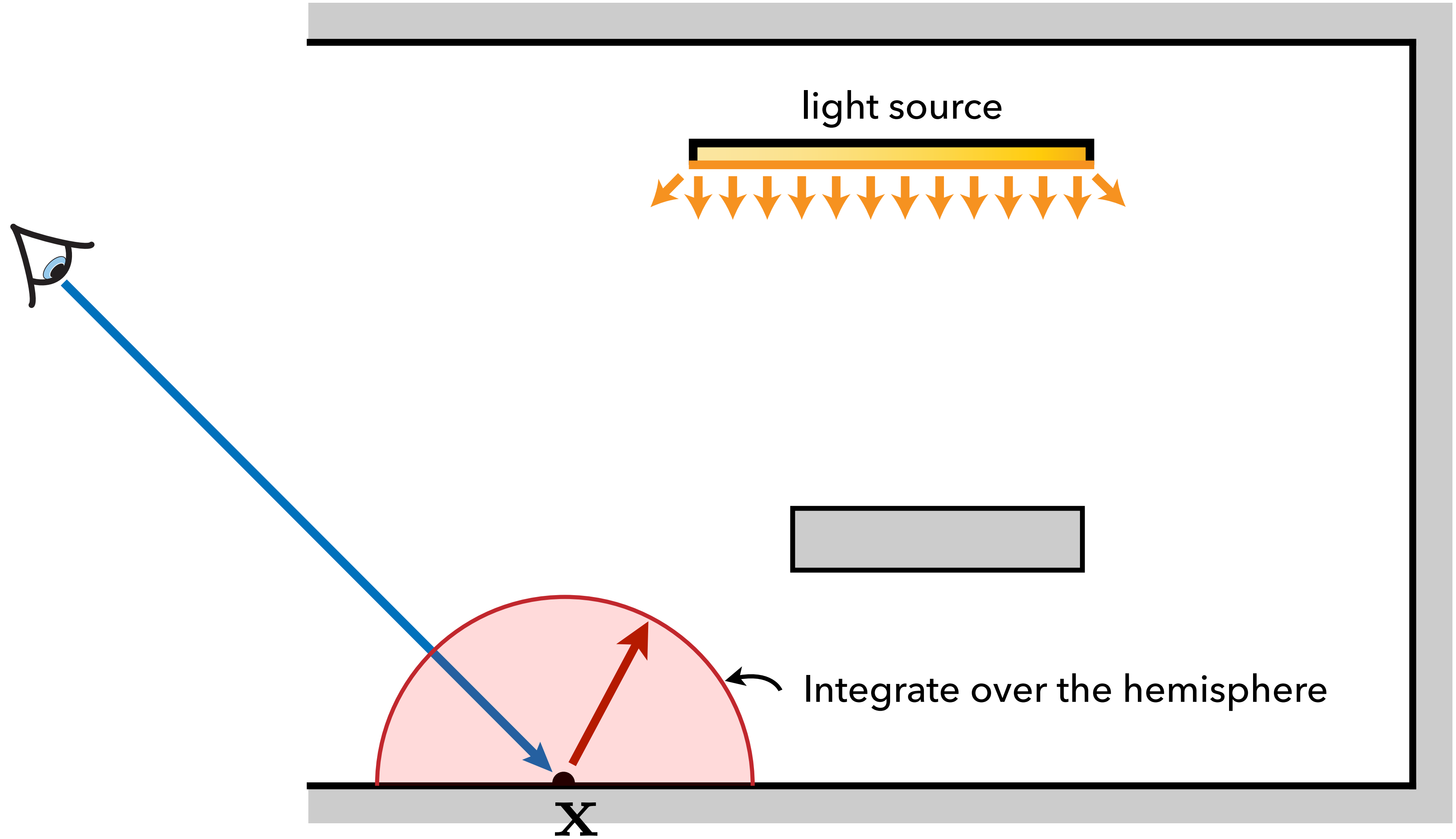
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

How can we estimate the integral?

$$\langle L_r(\mathbf{x}, \vec{\omega}_r)^N \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_{i,k}, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_{i,k}), -\vec{\omega}_{i,k}) \cos\theta_{i,k} \, d\vec{\omega}_{i,k}}{p_\Omega(\vec{\omega}_{i,k})}$$

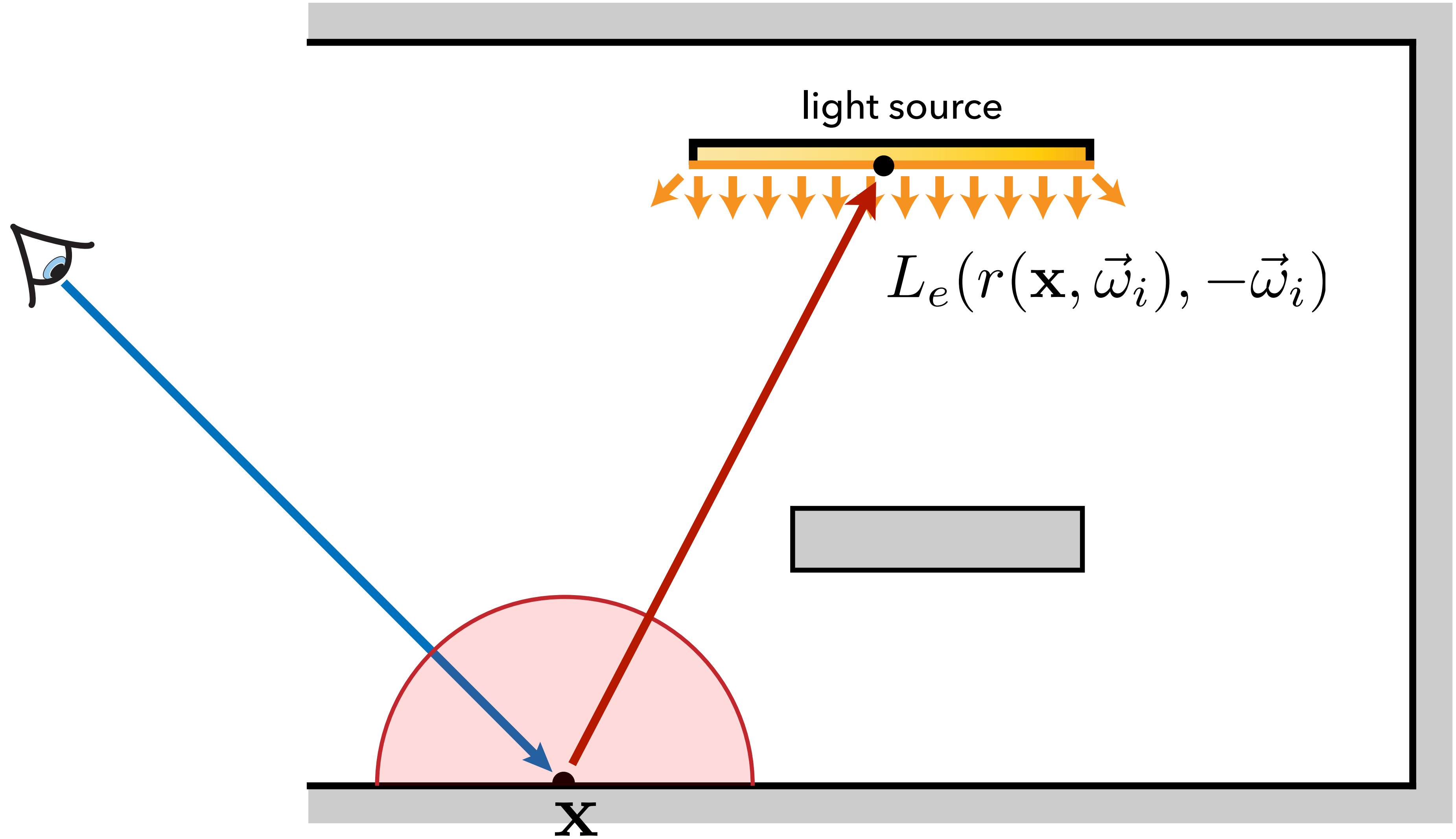
Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \boxed{\int_{H^2}} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



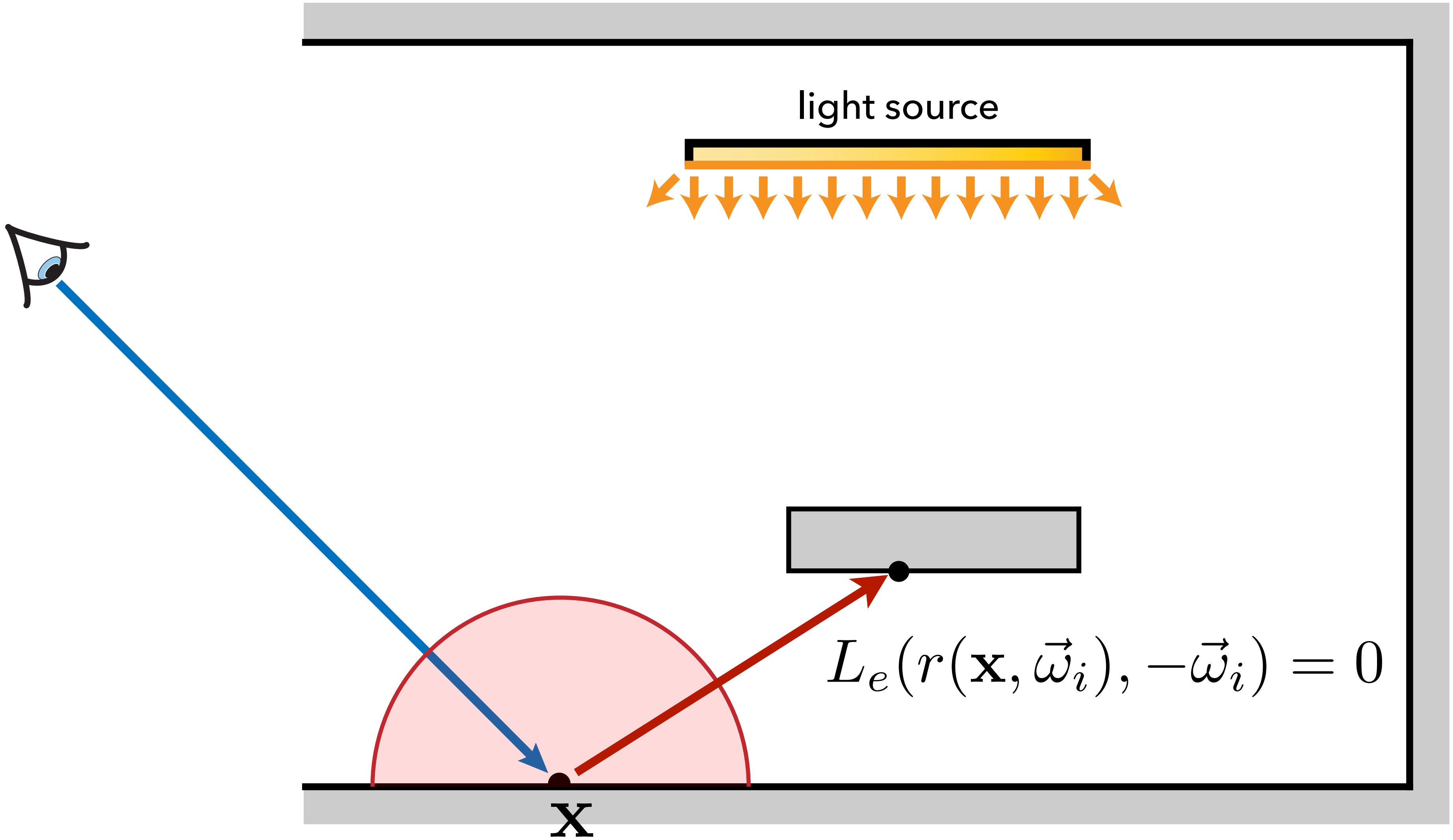
Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \boxed{L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i)} \cos\theta_i \, d\vec{\omega}_i$$



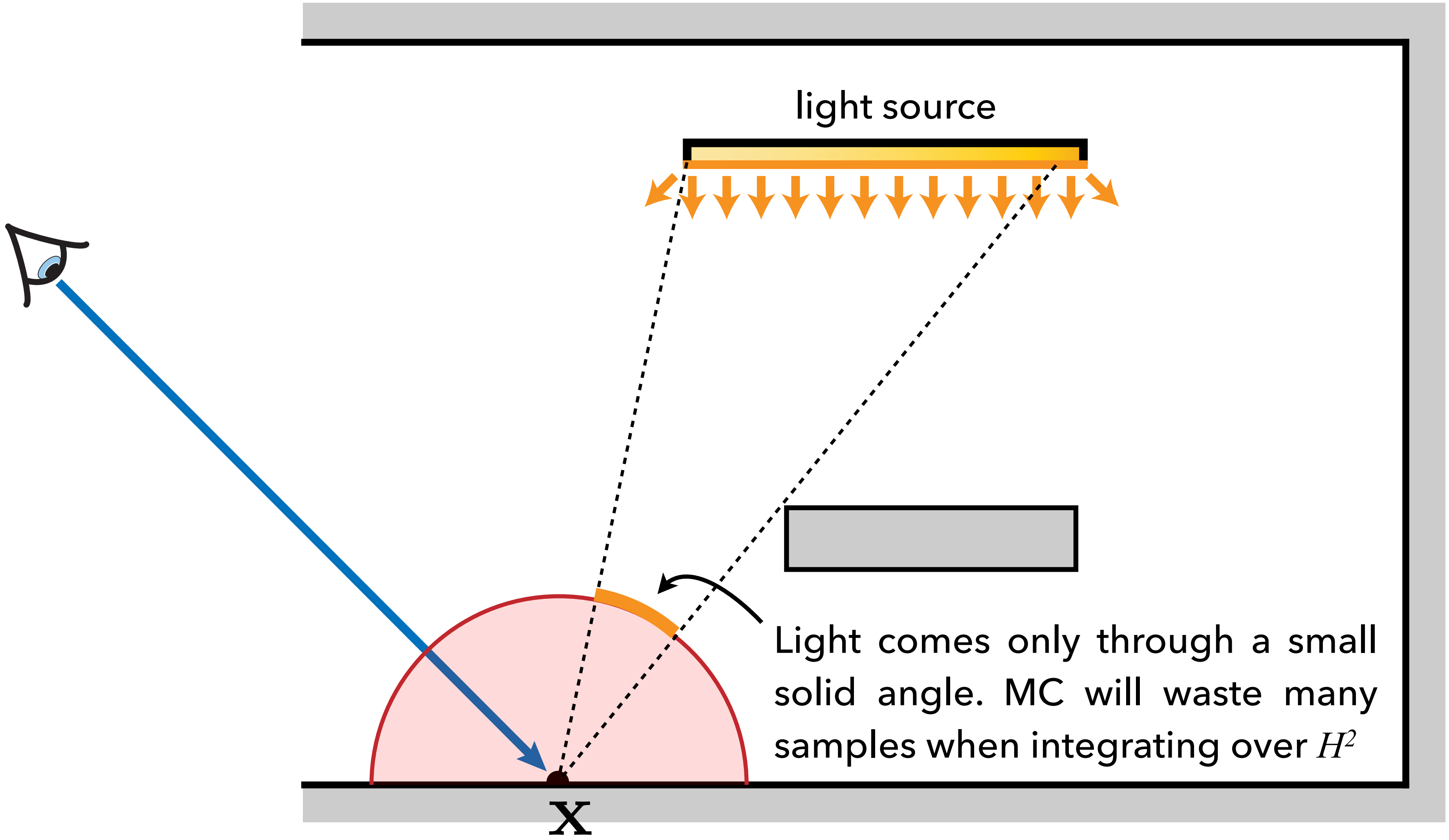
Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \boxed{L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i)} \cos\theta_i \, d\vec{\omega}_i$$



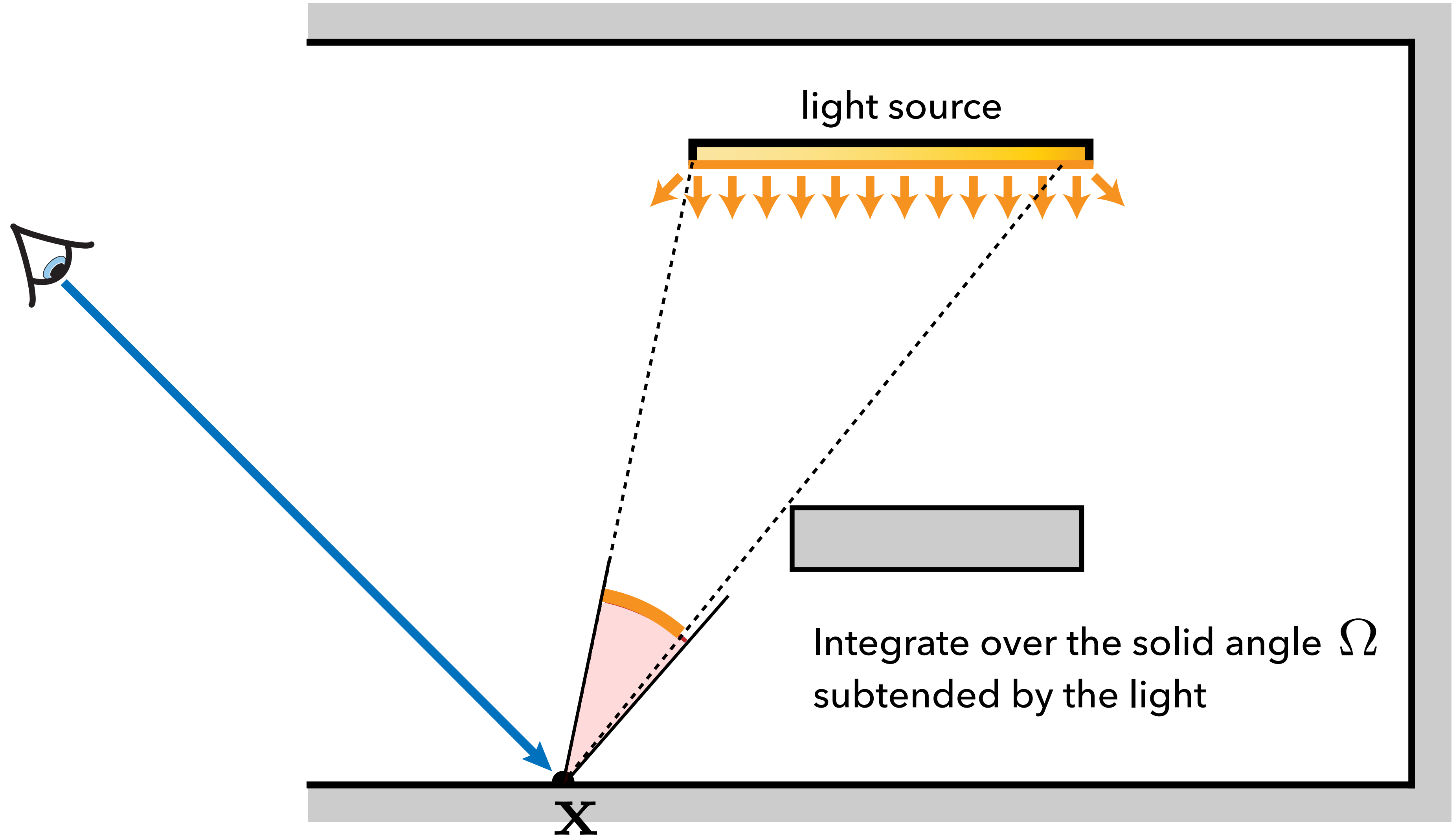
Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{\boxed{\Omega}} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) |\cos\theta_i| d\vec{\omega}_i$$

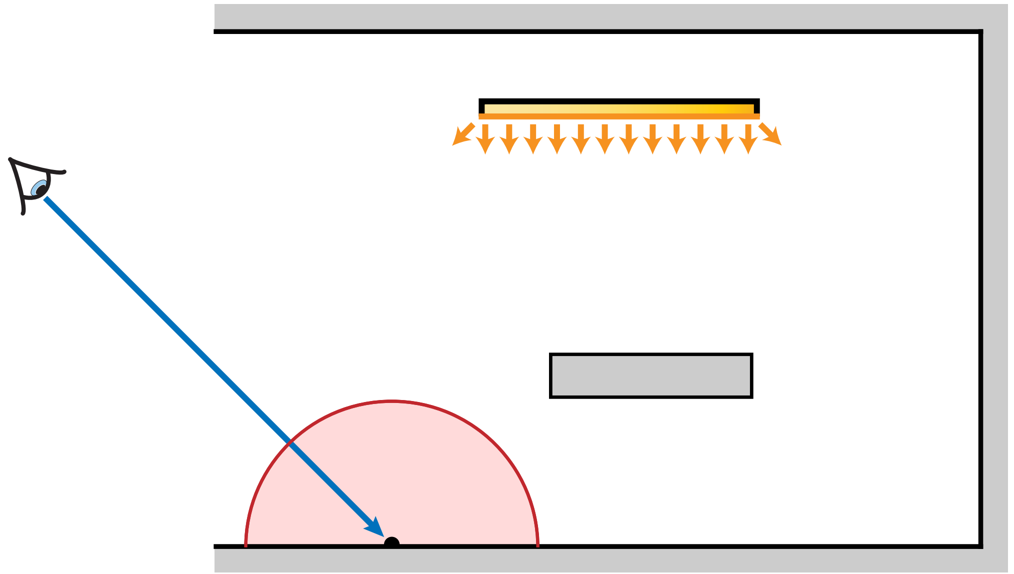


Integration over Solid Angle

- Can significantly improve efficiency of MC estimation
- Difficult for anything but spheres and disks
- Can we do better for light sources of arbitrary shapes?

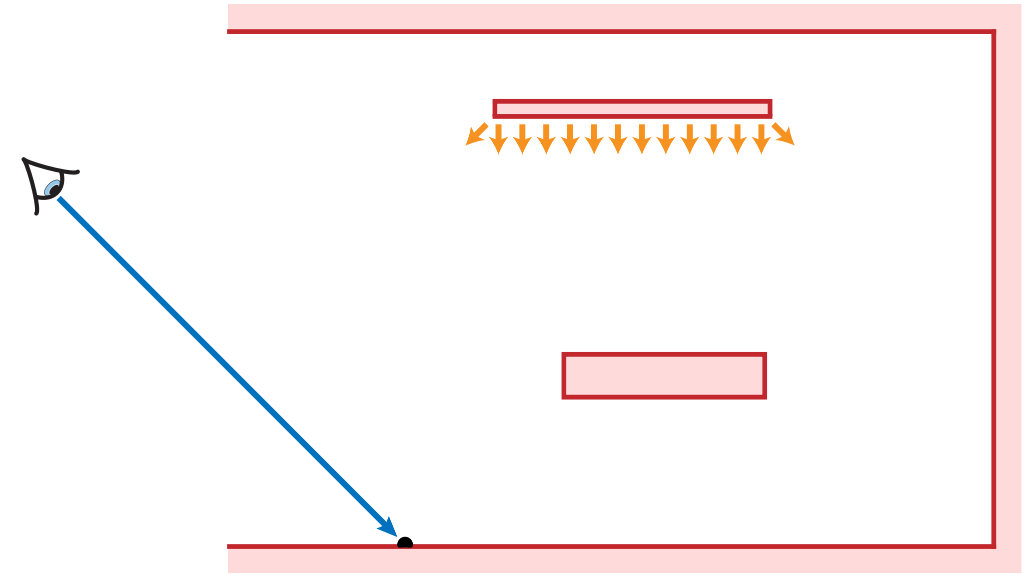
Forms of Reflection Equation

Hemispherical integration



$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Surface Area integration

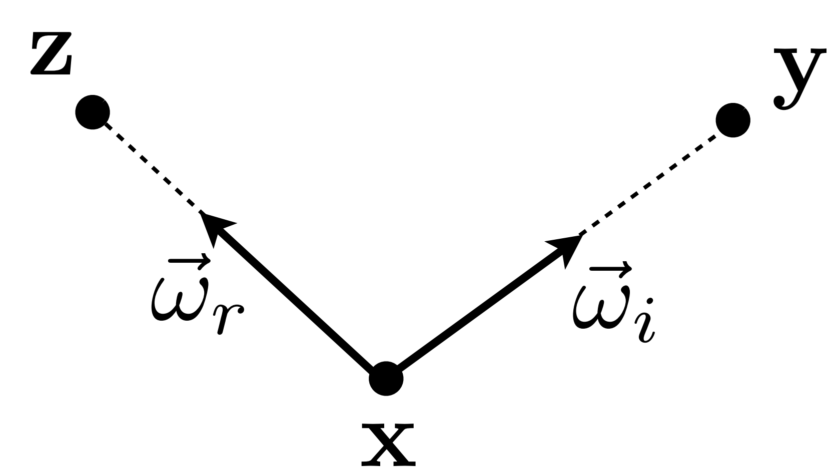


$$L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) G(\mathbf{x}, \mathbf{y}) \, dA(\mathbf{y})$$

Forms of Reflection Equation

- Change in notation:

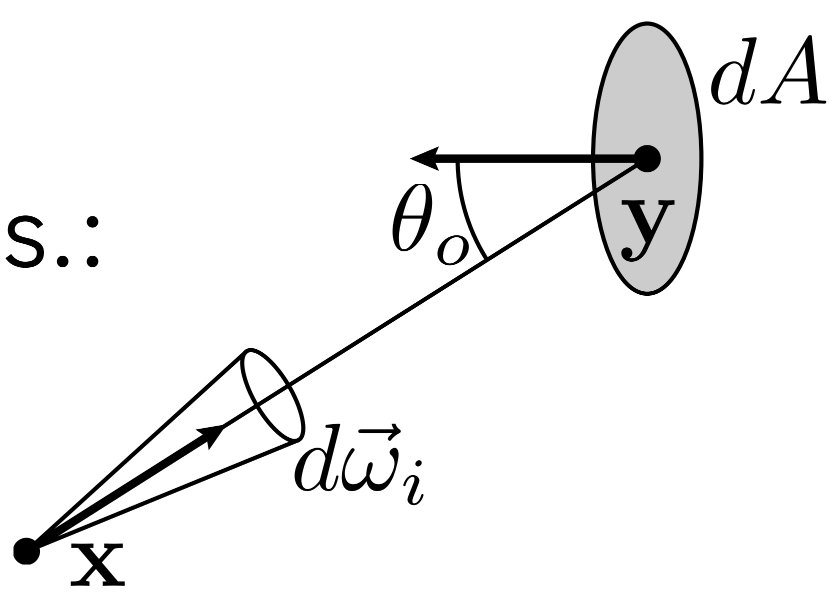
$$L_i(\mathbf{x}, \vec{\omega}_i) = L_i(\mathbf{x}, \mathbf{y})$$
$$L_r(\mathbf{x}, \vec{\omega}_r) = L_r(\mathbf{x}, \mathbf{z})$$
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \mathbf{y}, \mathbf{z})$$



- Transform integral over directions into integral over surface area.

- Jacobian determinant of the trans.:

$$d\vec{\omega}_i = \frac{|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA$$



Forms of Reflection Equation

$$L_i(\mathbf{x}, \vec{\omega}_i) = L_i(\mathbf{x}, \mathbf{y})$$

$$L_r(\mathbf{x}, \vec{\omega}_r) = L_r(\mathbf{x}, \mathbf{z})$$

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \mathbf{y}, \mathbf{z})$$

$$d\vec{\omega}_i = \frac{|\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA$$

Hemispherical form:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Surface area form:

$$L_r(\mathbf{x}, \mathbf{z})$$

Forms of Reflection Equation

$$L_i(\mathbf{x}, \vec{\omega}_i) = L_i(\mathbf{x}, \mathbf{y})$$

$$L_r(\mathbf{x}, \vec{\omega}_r) = L_r(\mathbf{x}, \mathbf{z})$$

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \mathbf{y}, \mathbf{z})$$

$$d\vec{\omega}_i = \frac{|\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA$$

Hemispherical form:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Surface area form:

$$L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) G(\mathbf{x}, \mathbf{y}) \, dA(\mathbf{y})$$

Area Form of the Reflection Eq.

$$L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) \boxed{G(\mathbf{x}, \mathbf{y})} \, dA(\mathbf{y})$$

Geometry term:

$$G(\mathbf{x}, \mathbf{y}) = \boxed{V(\mathbf{x}, \mathbf{y})} \, \frac{|\cos\theta_i| \, |\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2}$$

Visibility term:

$$V(\mathbf{x}, \mathbf{y}) = \begin{cases} 1: & \text{visible} \\ 0: & \text{not visible} \end{cases}$$

Area Form of the Reflection Eq.

$$L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) \boxed{G(\mathbf{x}, \mathbf{y})}\, dA(\mathbf{y})$$

Original foreshortening term

Geometry term:

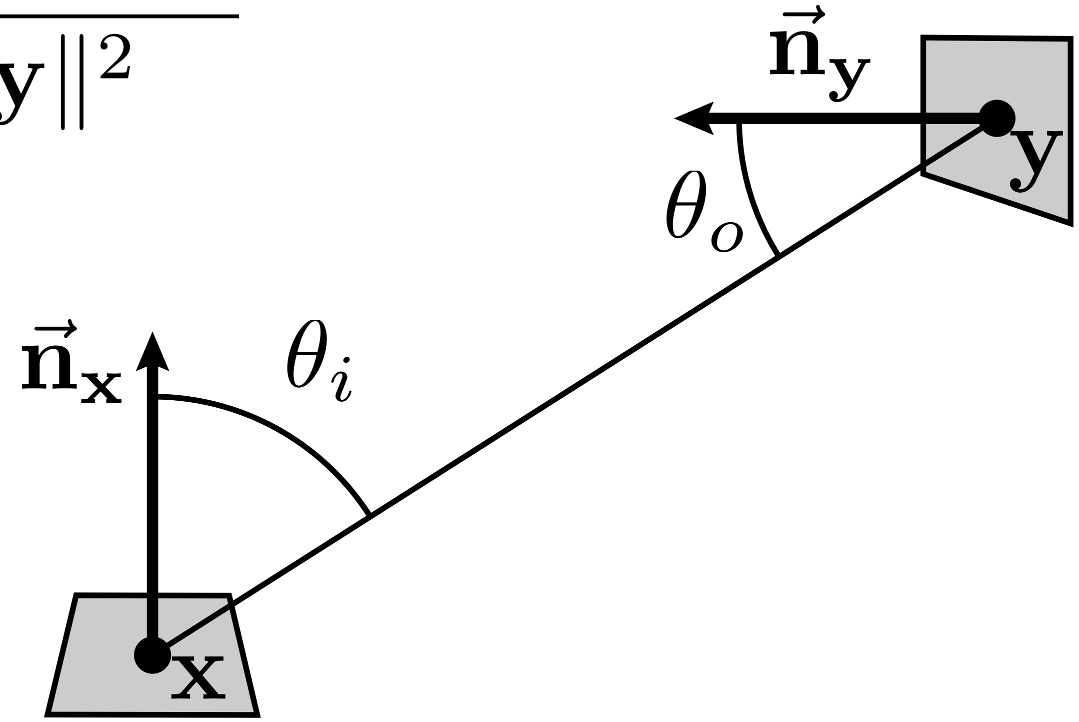
$$G(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \mathbf{y}) \frac{\boxed{|\cos\theta_i|}\ \boxed{|\cos\theta_o|}}{\boxed{\|\mathbf{x} - \mathbf{y}\|^2}}$$

Jacobian determinant
of the transform

$$d\vec{\omega}_i = \frac{|\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA$$

Area Form of the Reflection Eq.

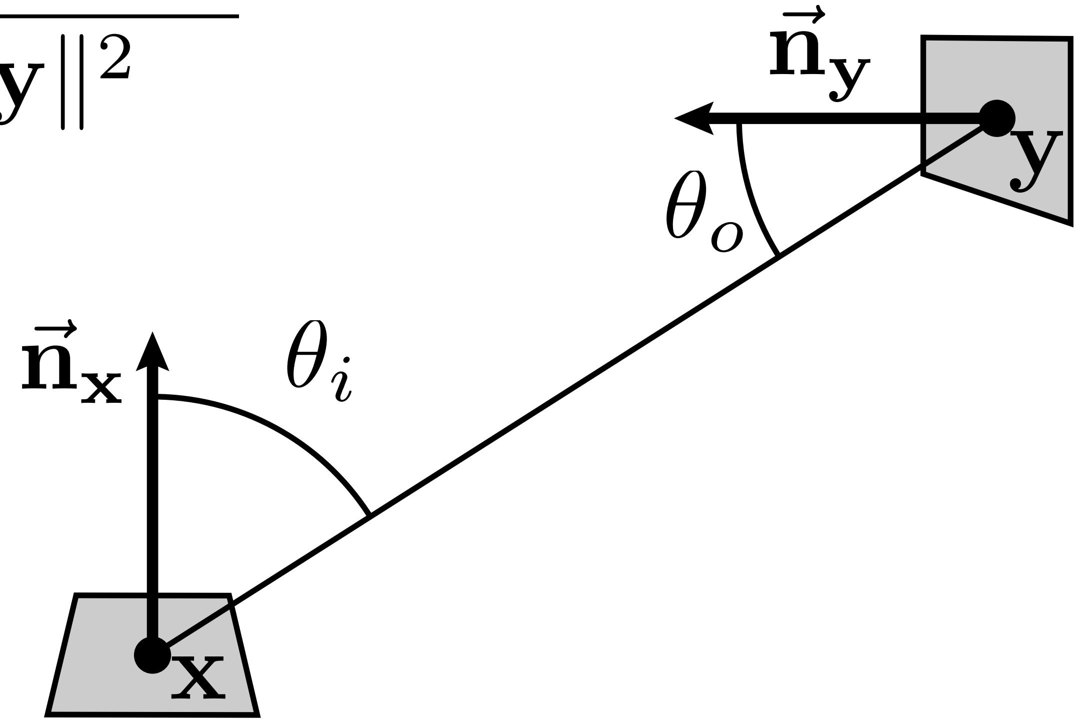
- Interpreting $$\frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2}$$



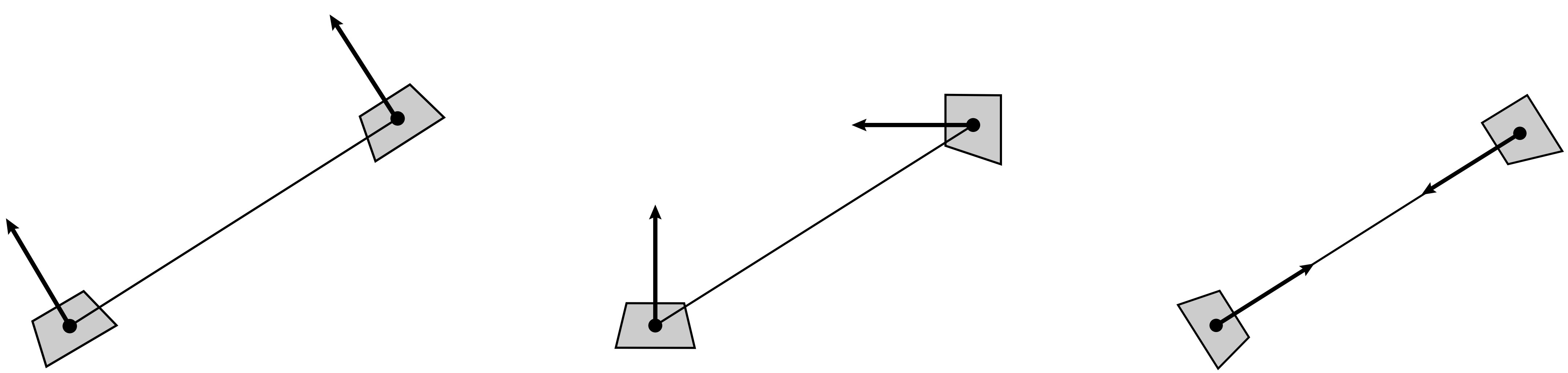
- The chance that a photon emitted from a differential patch will hit another diff patch decreases as:
  - the patches face away from each other (numerator)
  - the patches move away from each other (denominator)

Area Form of the Reflection Eq.

- Interpreting $$\frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2}$$

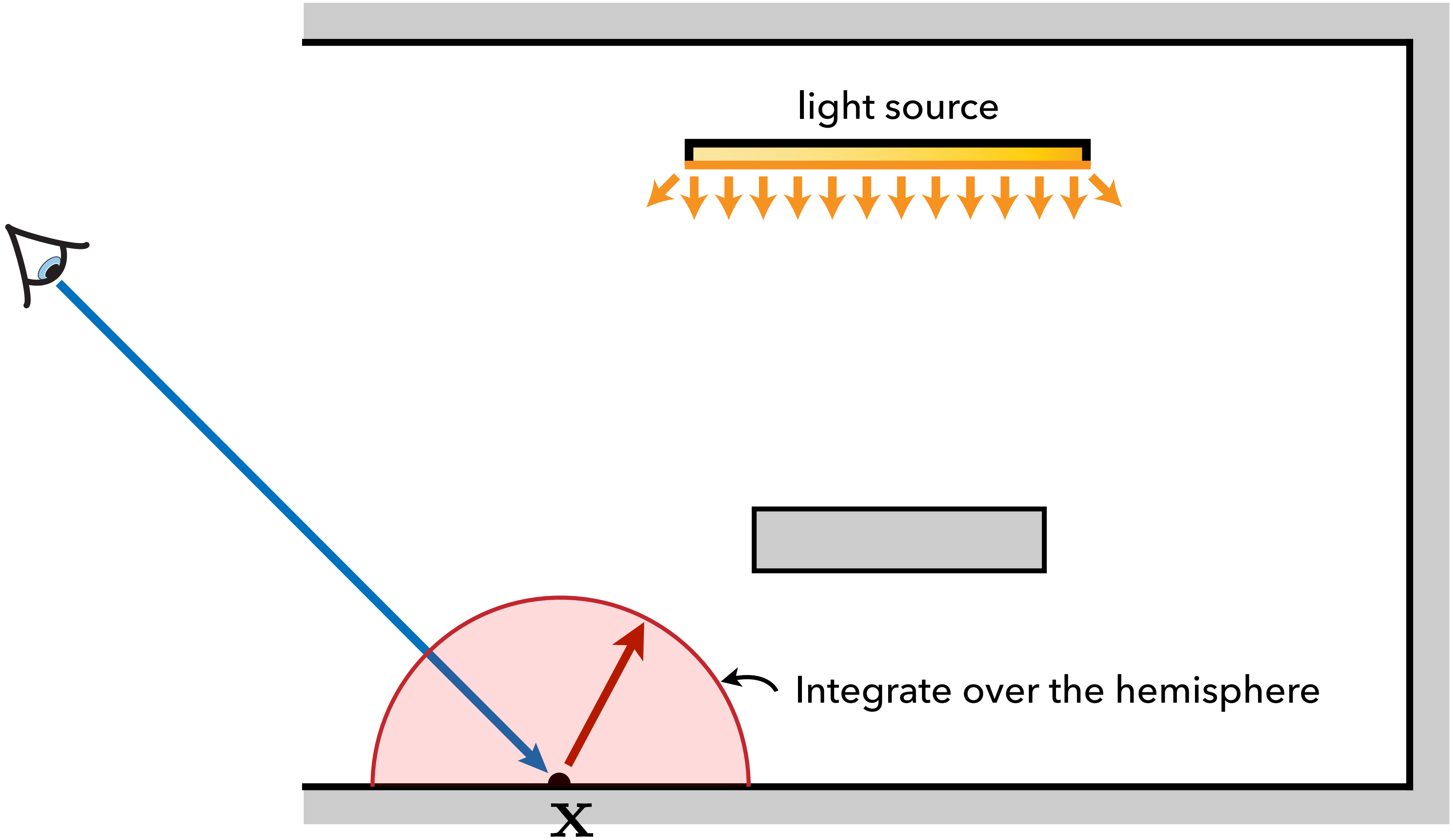


numerator = 0 | 0 < numerator < 1 | numerator = 1



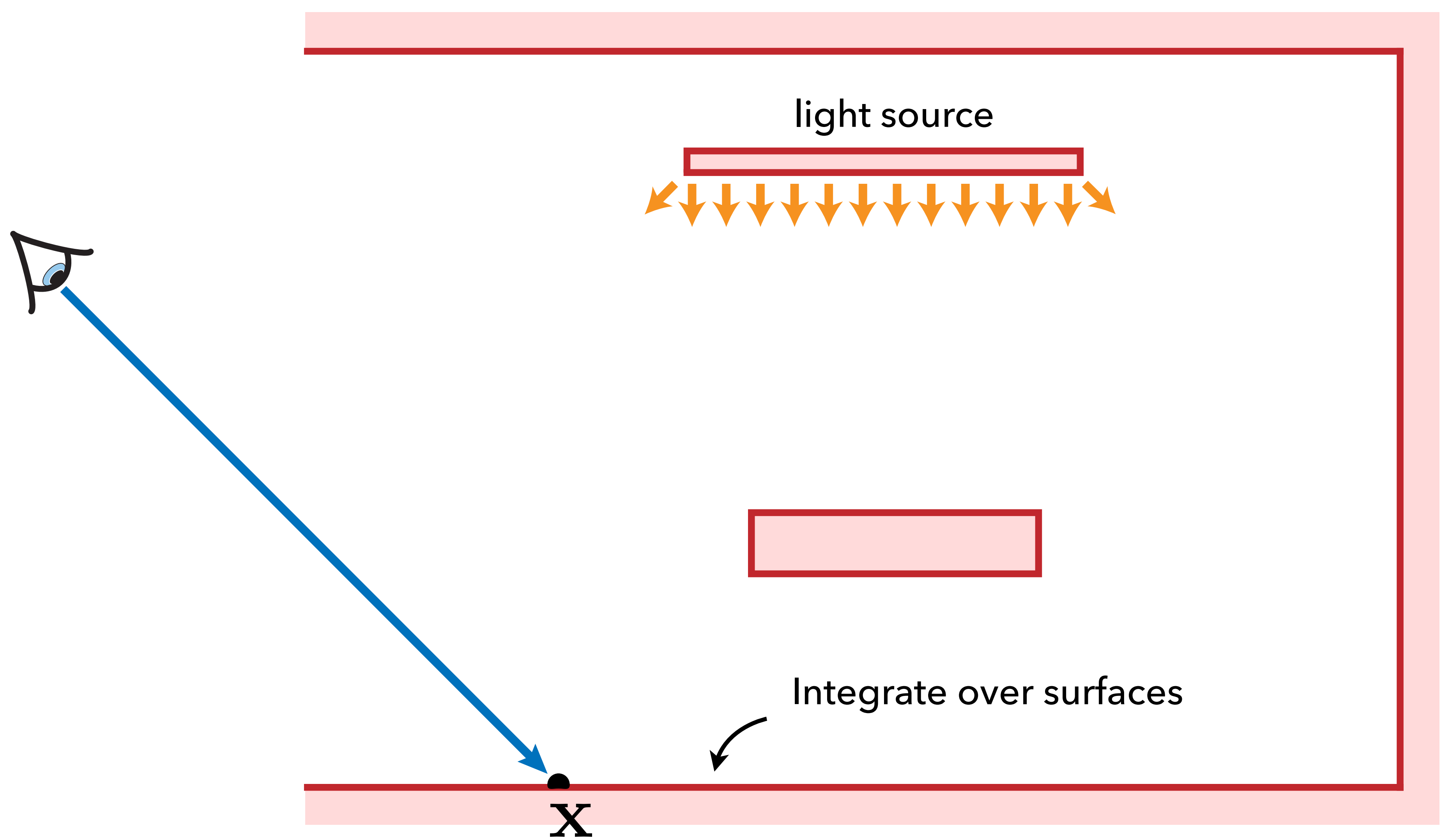
Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \boxed{\int_{H^2}} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



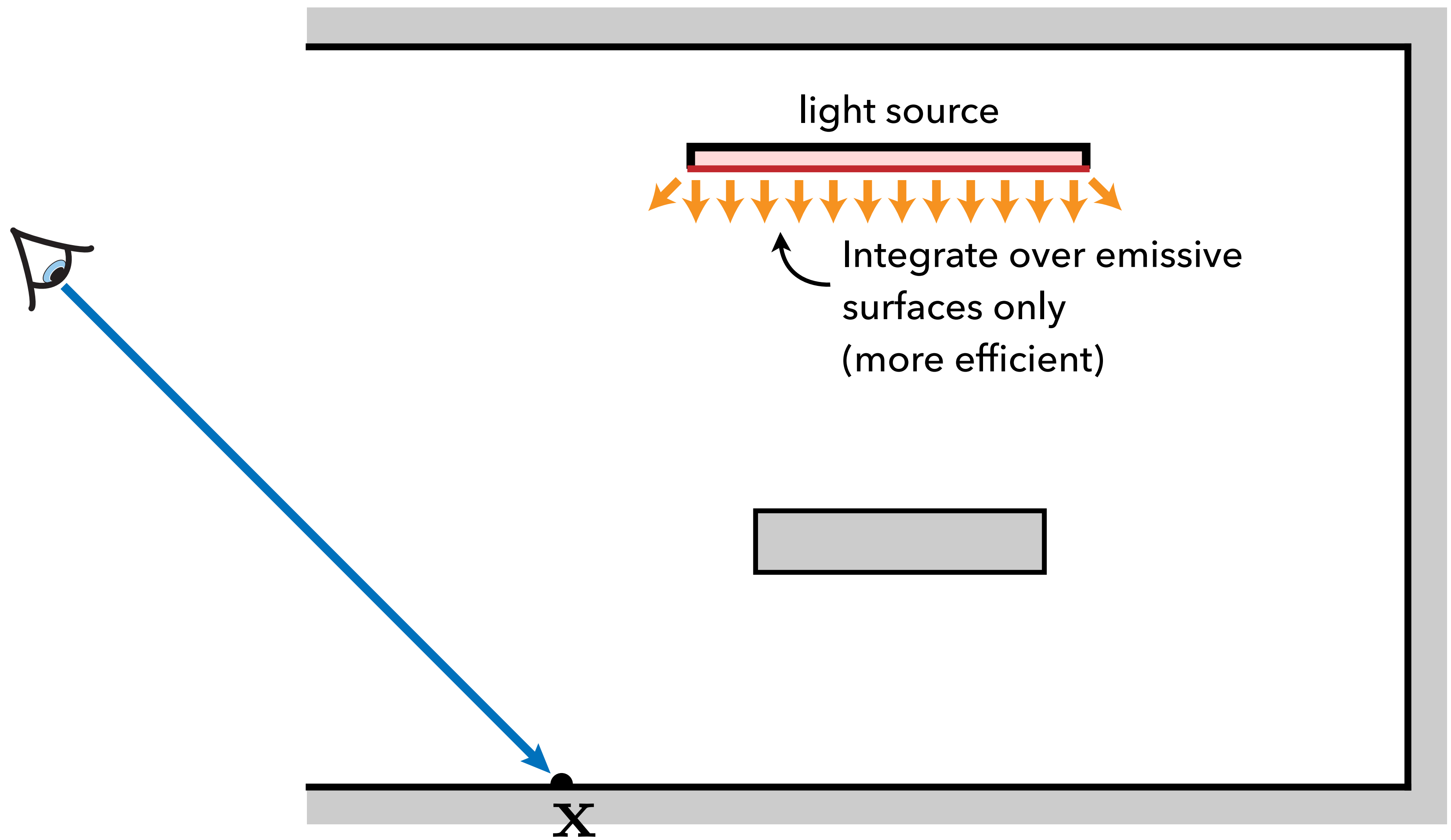
Direct Illumination

$$L_r(\mathbf{x},\mathbf{z}) = \boxed{\int_A} f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_e(\mathbf{y},\mathbf{x}) V(\mathbf{x},\mathbf{y}) \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA(\mathbf{y})$$



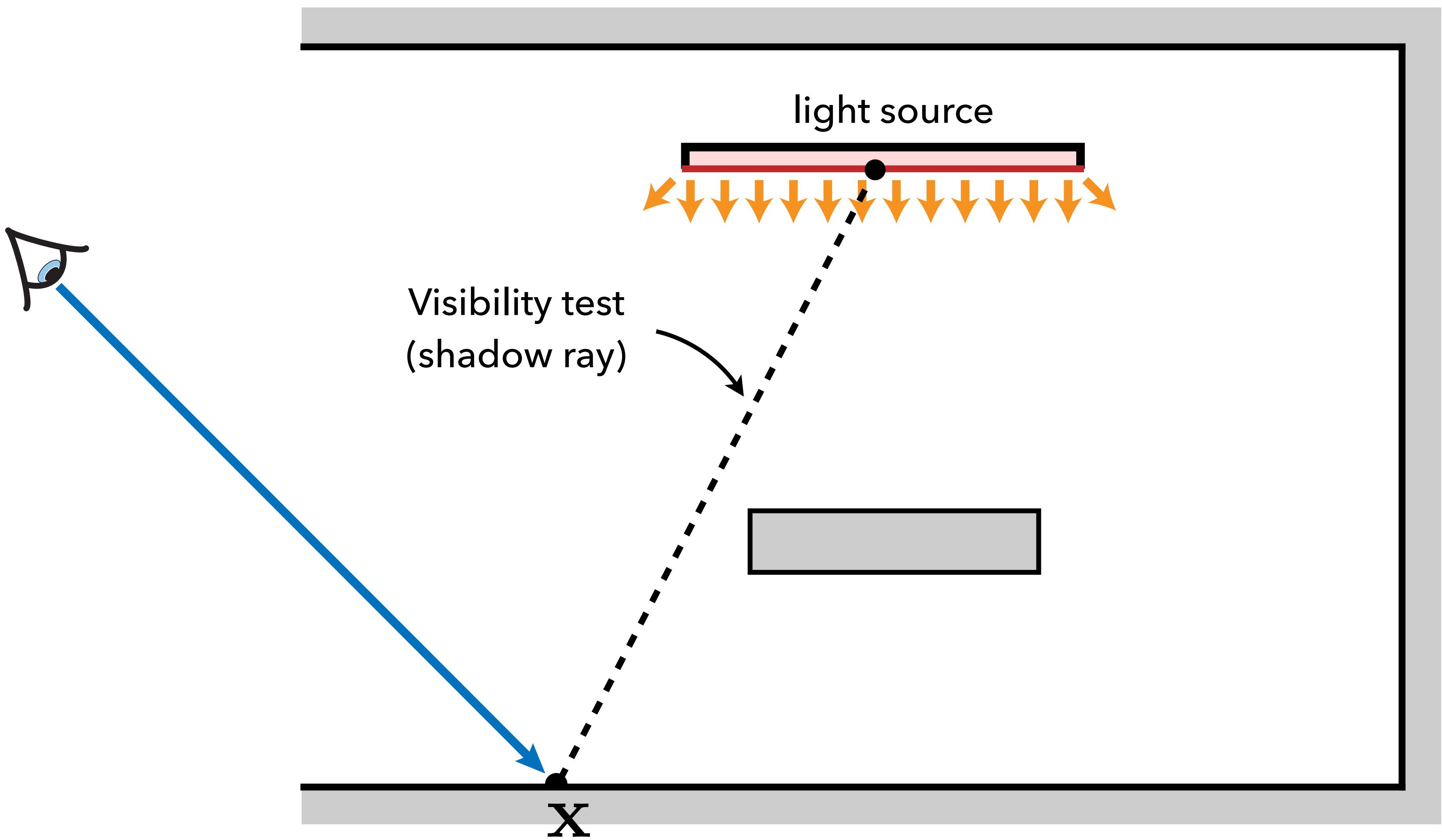
Direct Illumination

$$L_r(\mathbf{x},\mathbf{z}) = \boxed{\int_{A_e}} f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_e(\mathbf{y},\mathbf{x}) V(\mathbf{x},\mathbf{y}) \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA(\mathbf{y})$$



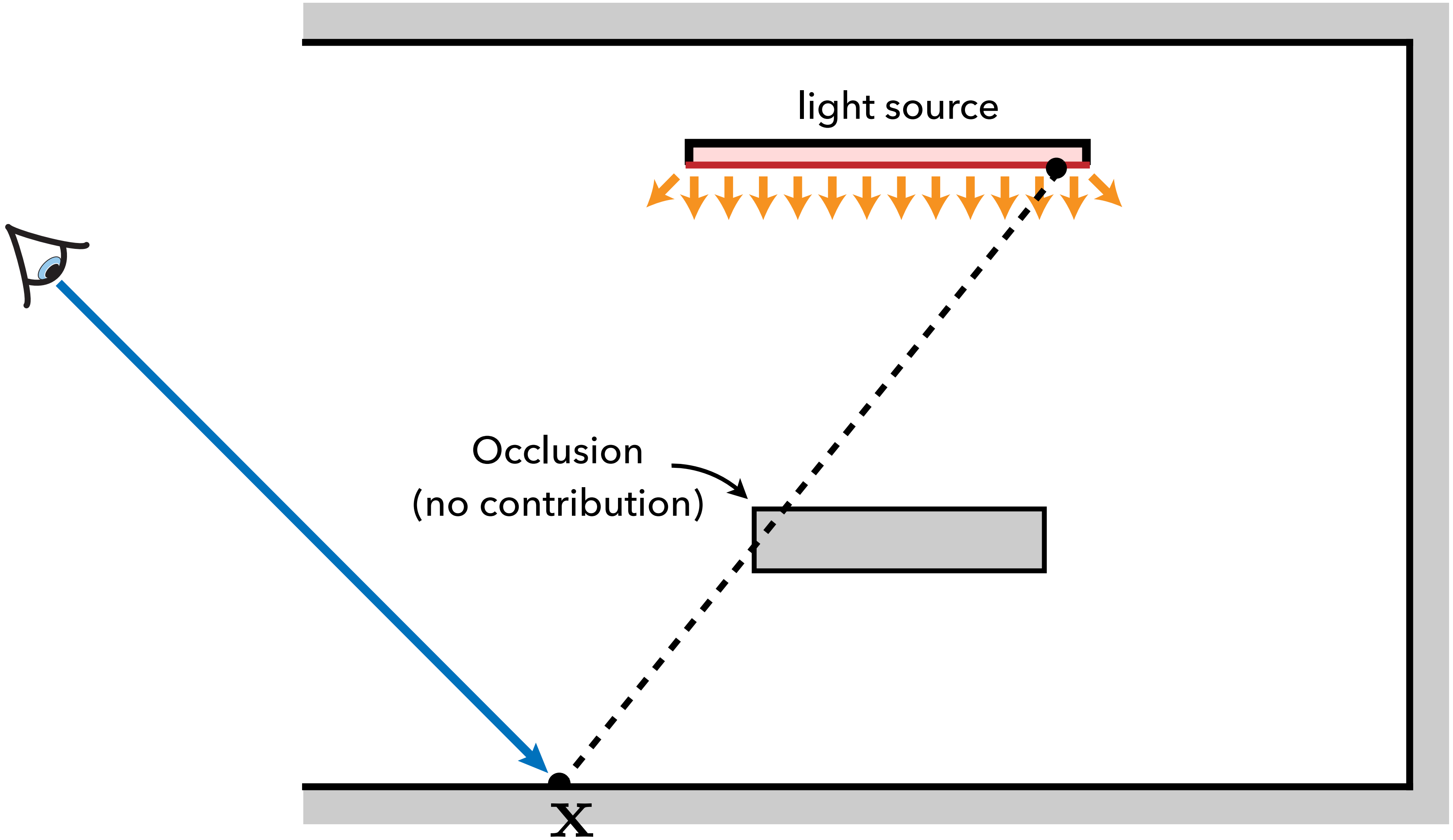
Direct Illumination

$$L_r(\mathbf{x},\mathbf{z}) = \int_{A_e} f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_e(\mathbf{y},\mathbf{x}) \boxed{V(\mathbf{x},\mathbf{y})} \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA(\mathbf{y})$$

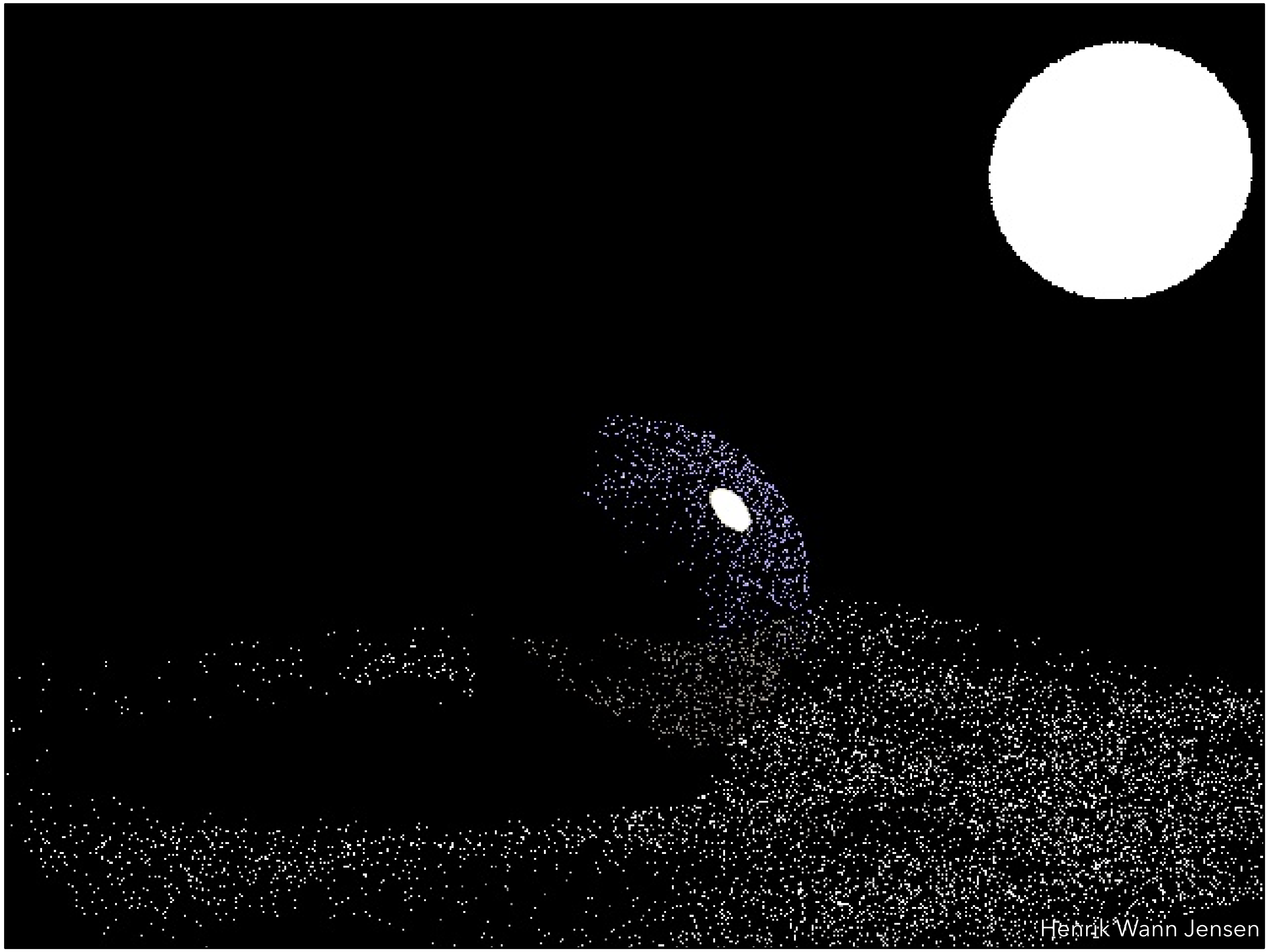


Direct Illumination

$$L_r(\mathbf{x},\mathbf{z}) = \int_{A_e} f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_e(\mathbf{y},\mathbf{x}) \boxed{V(\mathbf{x},\mathbf{y})} \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA(\mathbf{y})$$

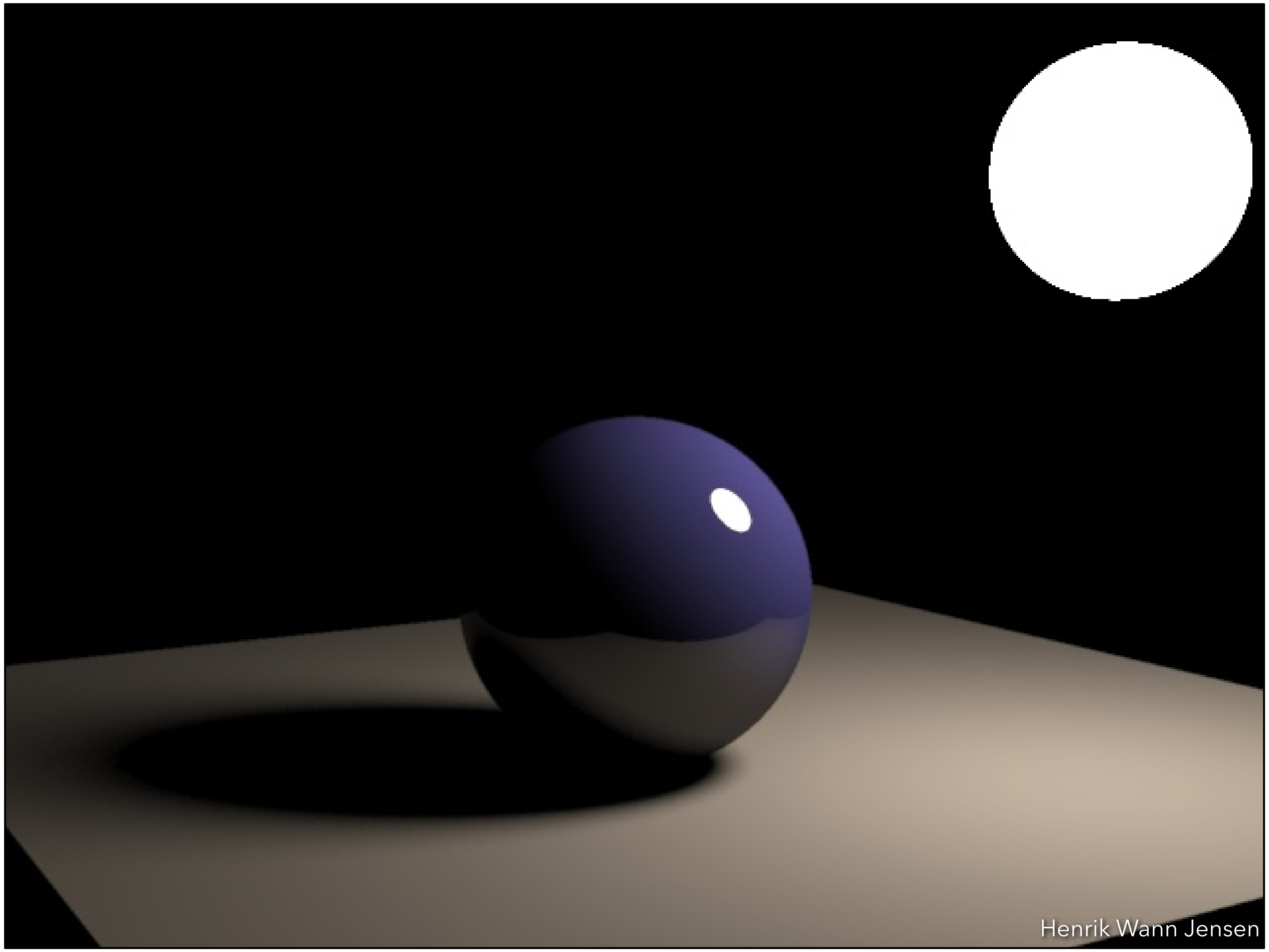


Direct Illumination



Sampling the hemisphere

Direct Illumination



Sampling the area of the light

Visual Break

Visual Break

Light Sources

Light Sources

Point light | Spot light | Directional light | Quad light | Sphere light | Mesh light

Delta lights
(create hard shadows)

Area/Shape lights
(create soft shadows)

Point Light

- Typically defined using a point $\mathbf{p}$ and emitted power Φ (or radiant intensity)
- Omnidirectional emission from a single point
  - delta function

$$L_r(\mathbf{x},\mathbf{z}) = \int_{A_e} f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_e(\mathbf{y},\mathbf{x}) V(\mathbf{x},\mathbf{y}) \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA(\mathbf{y})$$

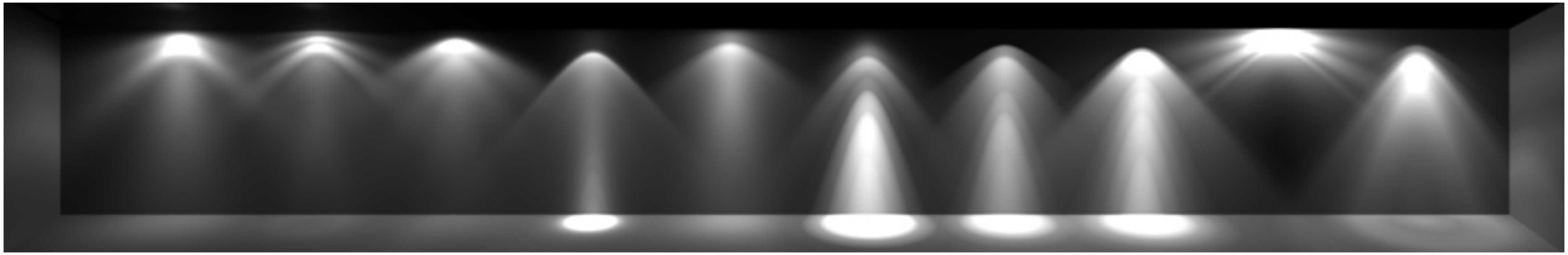
$$L_r(\mathbf{x},\mathbf{z}) = \frac{\Phi}{4\pi} f_r(\mathbf{x},\mathbf{p},\mathbf{z}) V(\mathbf{x},\mathbf{p}) \frac{|\cos\theta_i|}{\|\mathbf{x}-\mathbf{p}\|^2}$$

Spot Light

- Typically defined using a point $\mathbf{p}$ and a directionally dependent radiant intensity I

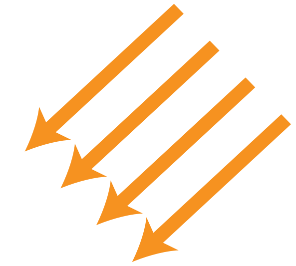
$$L_r(\mathbf{x}, \mathbf{z}) = I(\mathbf{p}, \mathbf{x}) f_r(\mathbf{x}, \mathbf{p}, \mathbf{z}) V(\mathbf{x}, \mathbf{p}) \frac{|\cos \theta_i|}{\|\mathbf{x} - \mathbf{p}\|^2}$$

- The intensity can be defined using IES profiles:



https://www.vrayschool.com

Directional Light

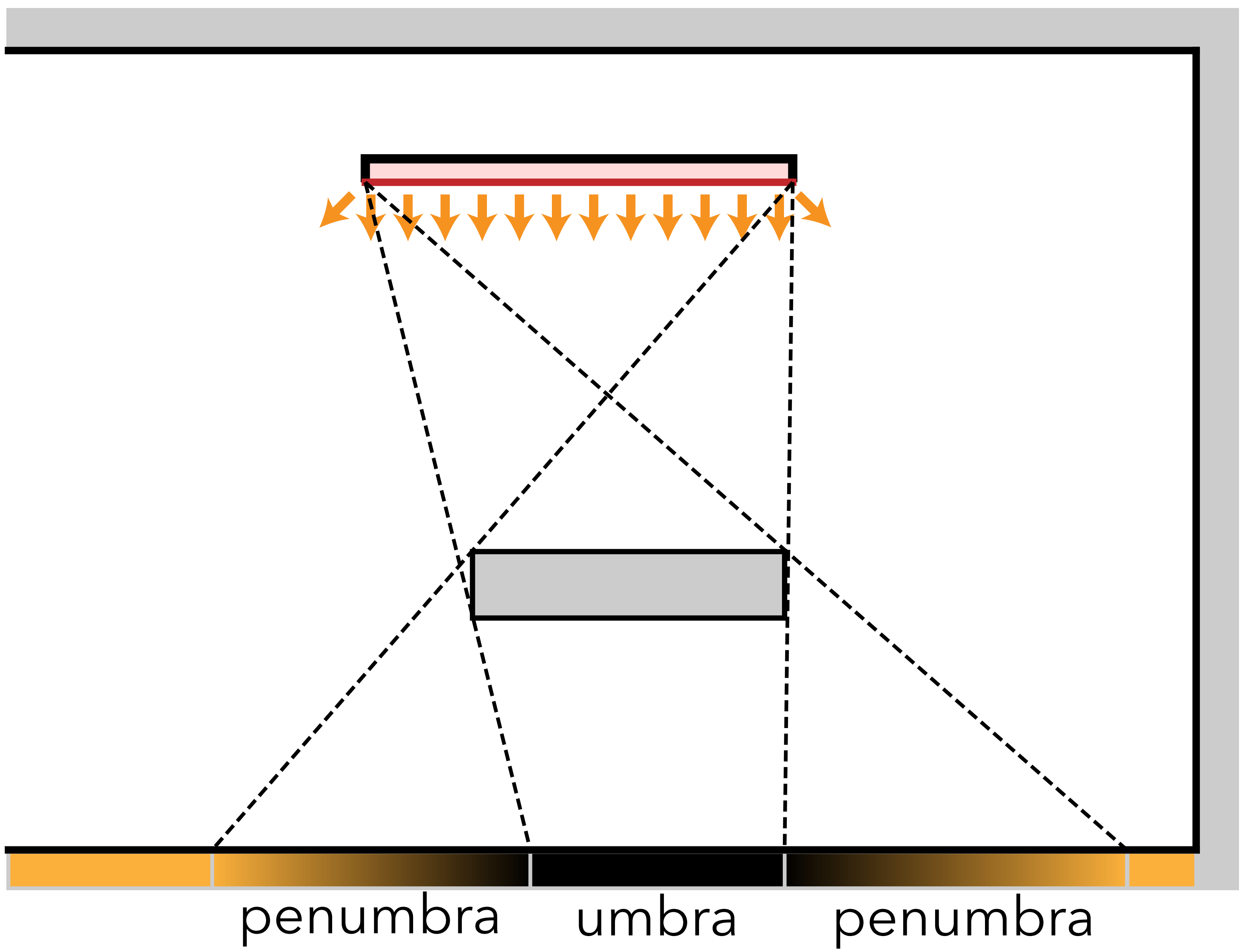


- Typically defined using direction $\vec{\omega}$ and radiance $L_d(\vec{\omega})$ coming *from* direction $\vec{\omega}$

$$L_r(\mathbf{x}, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}, \vec{\omega}_r) V(\mathbf{x}, \vec{\omega}) L_d(\vec{\omega}) \cos\theta$$

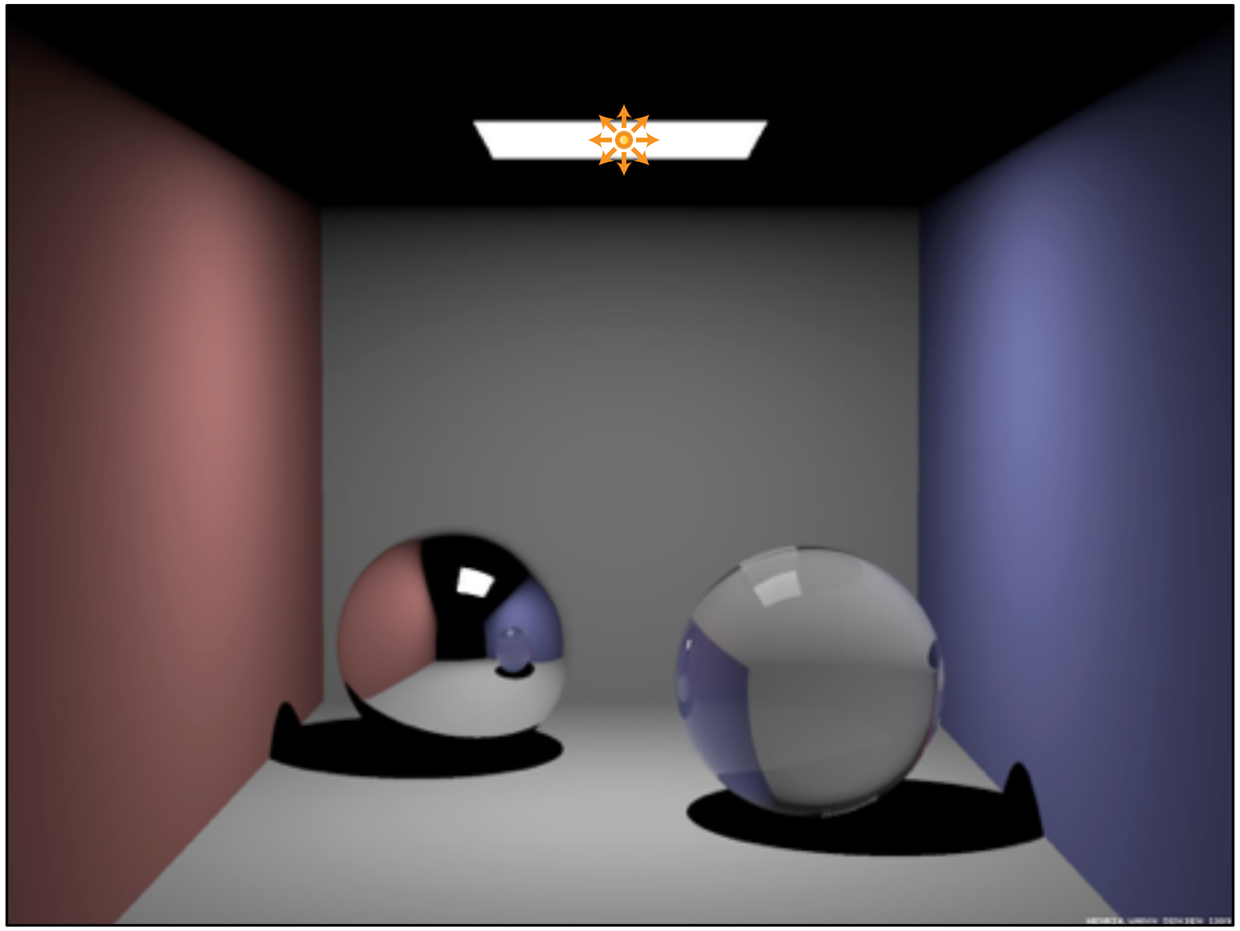
Quad Light

- Has finite area... creates soft shadows

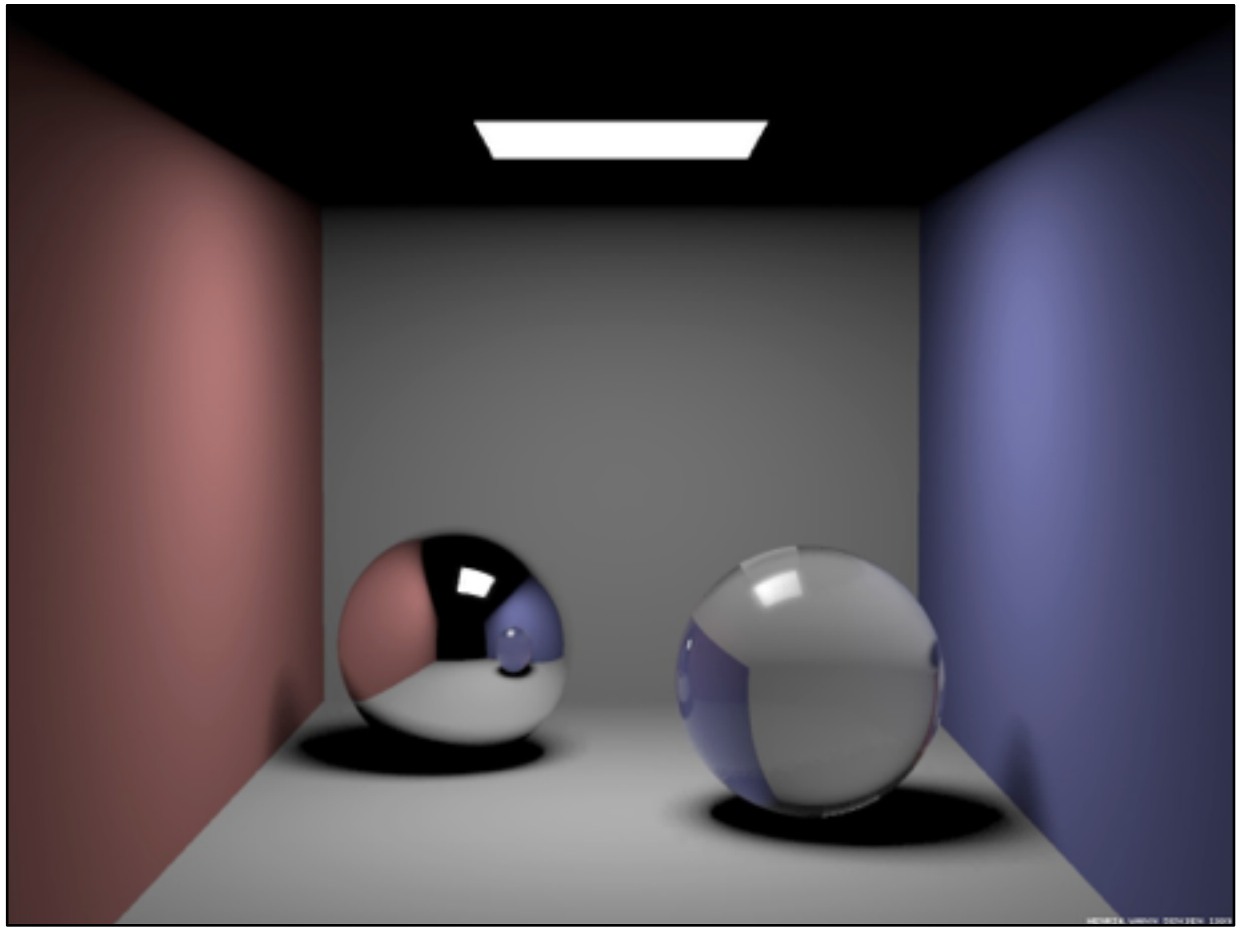


Quad Light

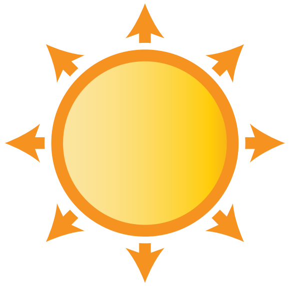
Point light



Quad light



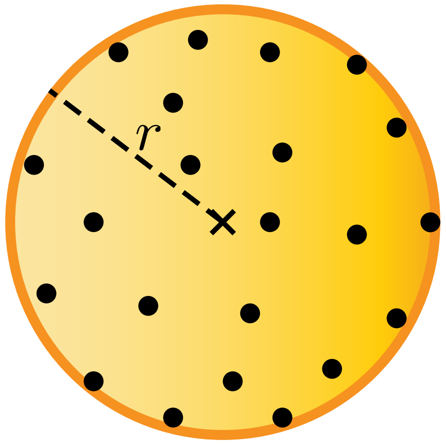
Sphere Light



- Typically defined using a point $\mathbf{p}$, radius r and emitted power Φ (or emitted radiance L_e)
- Has finite area $4\pi r^2$

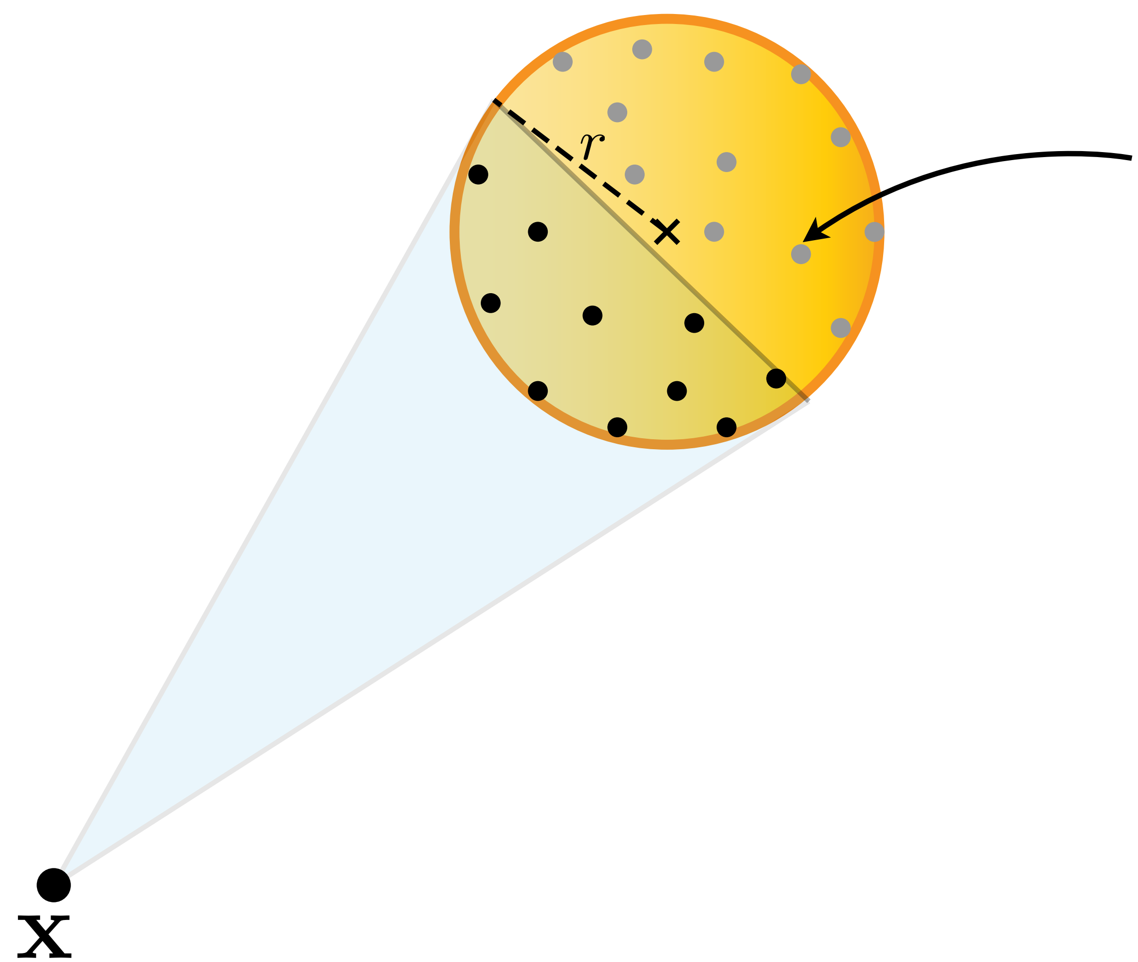
Sphere Light

- How to sample points on the sphere light?
- **Approach 1**: uniformly sample *sphere* *<u>area</u>*



Sphere Light

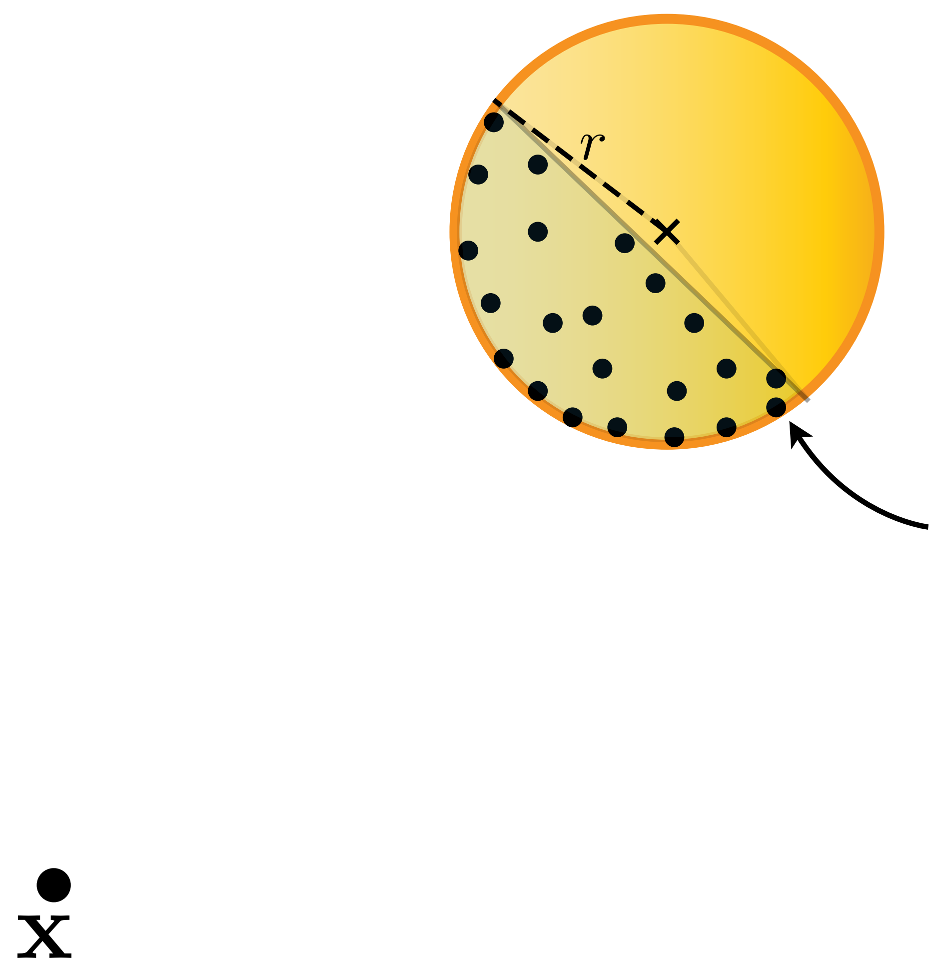
- How to sample points on the sphere light?
- **Approach 1**: uniformly sample *sphere* *area*



Many samples are not visible from the shading point!

Sphere Light

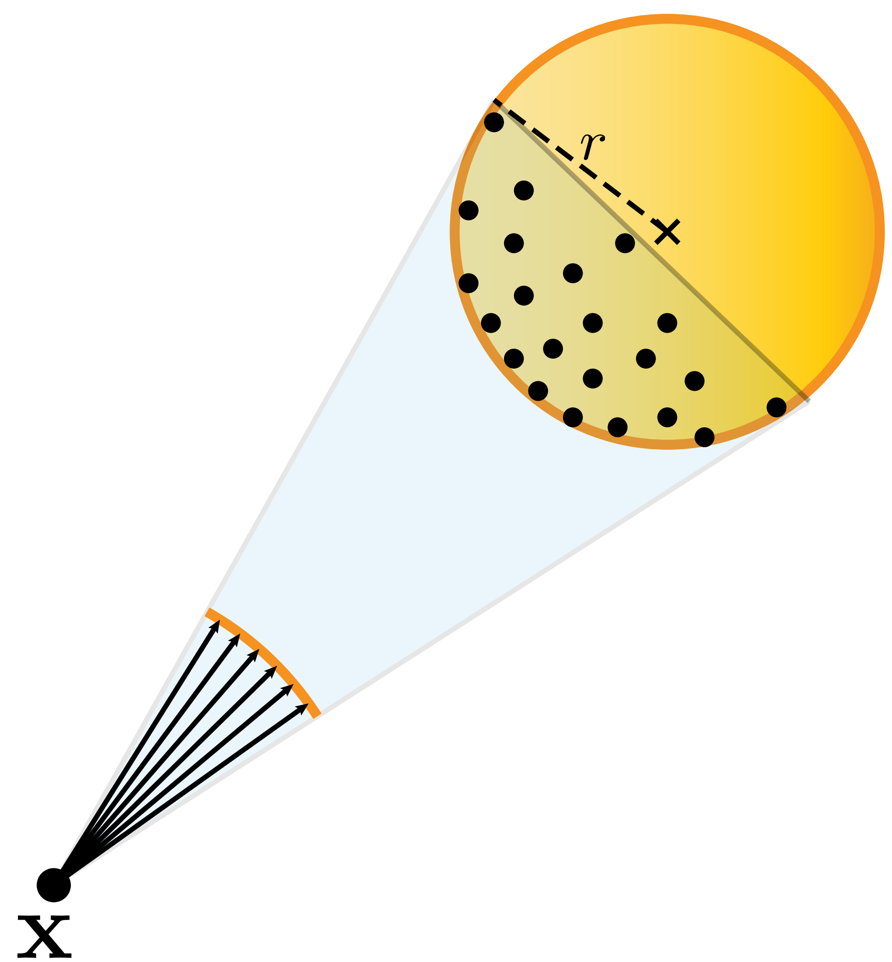
- How to sample points on the sphere light?
- **Approach 2** (better): uniformly sample *area* of the *visible spherical cap*



Uniform area-density is not ideal as emitted radiance is weighted by the cosine term
(recall the form factor in the G term)

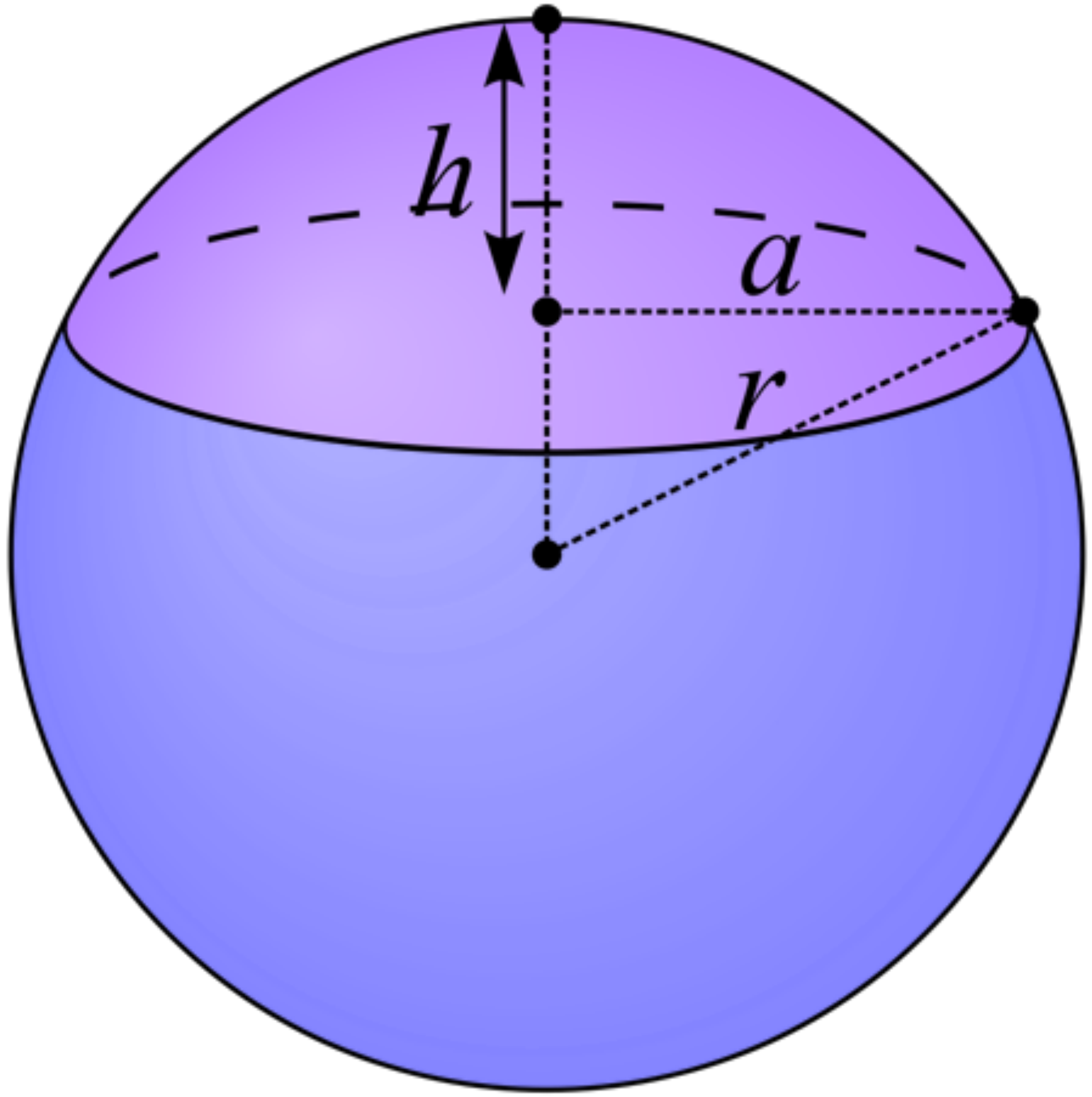
Sphere Light

- How to sample points on the sphere light?
- **Approach 3** (best): uniformly sample *solid angle subtended* by the sphere



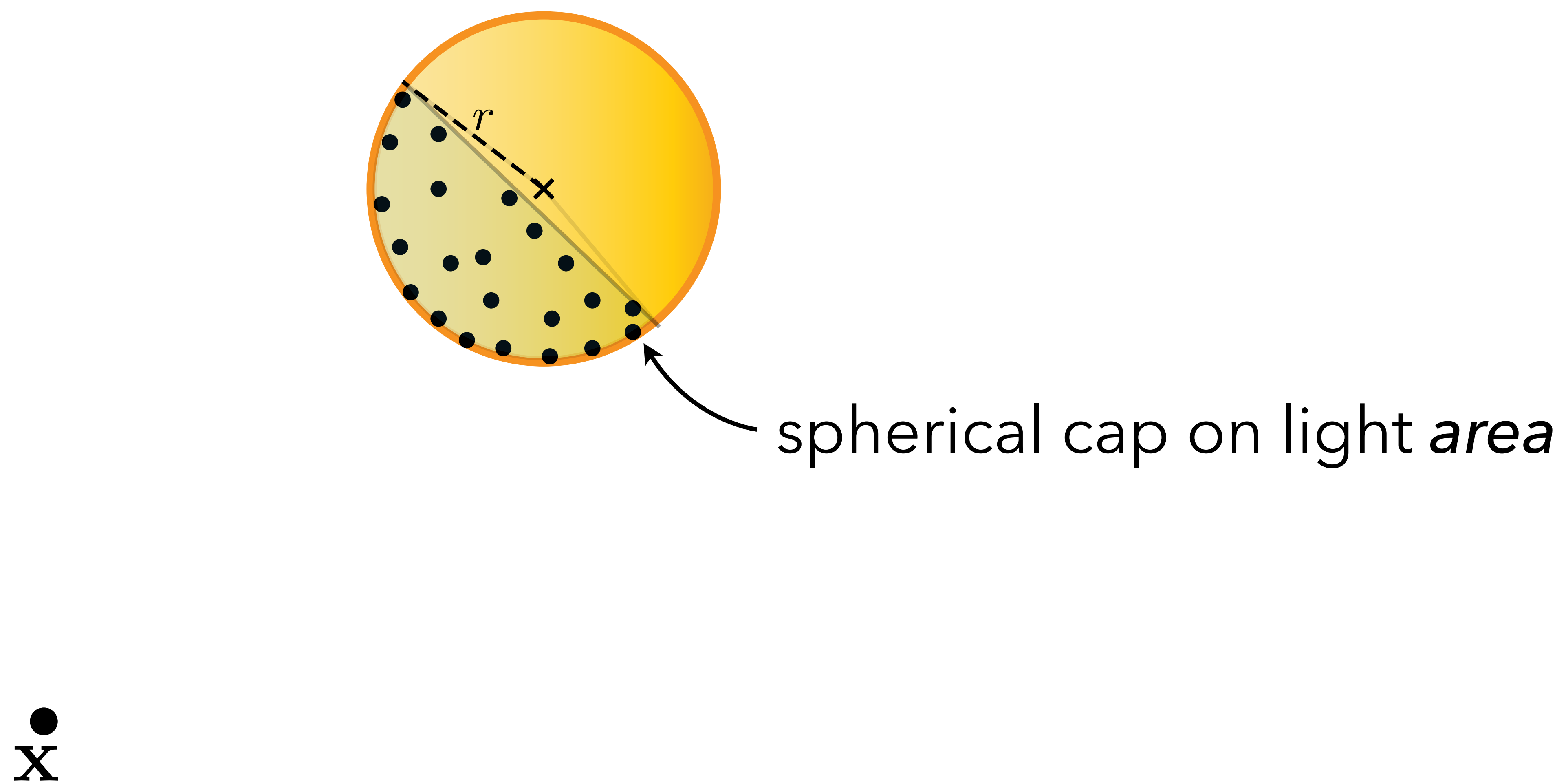
Sampling a Spherical Cap

- Sample using Hat-Box theorem with restricted z-range



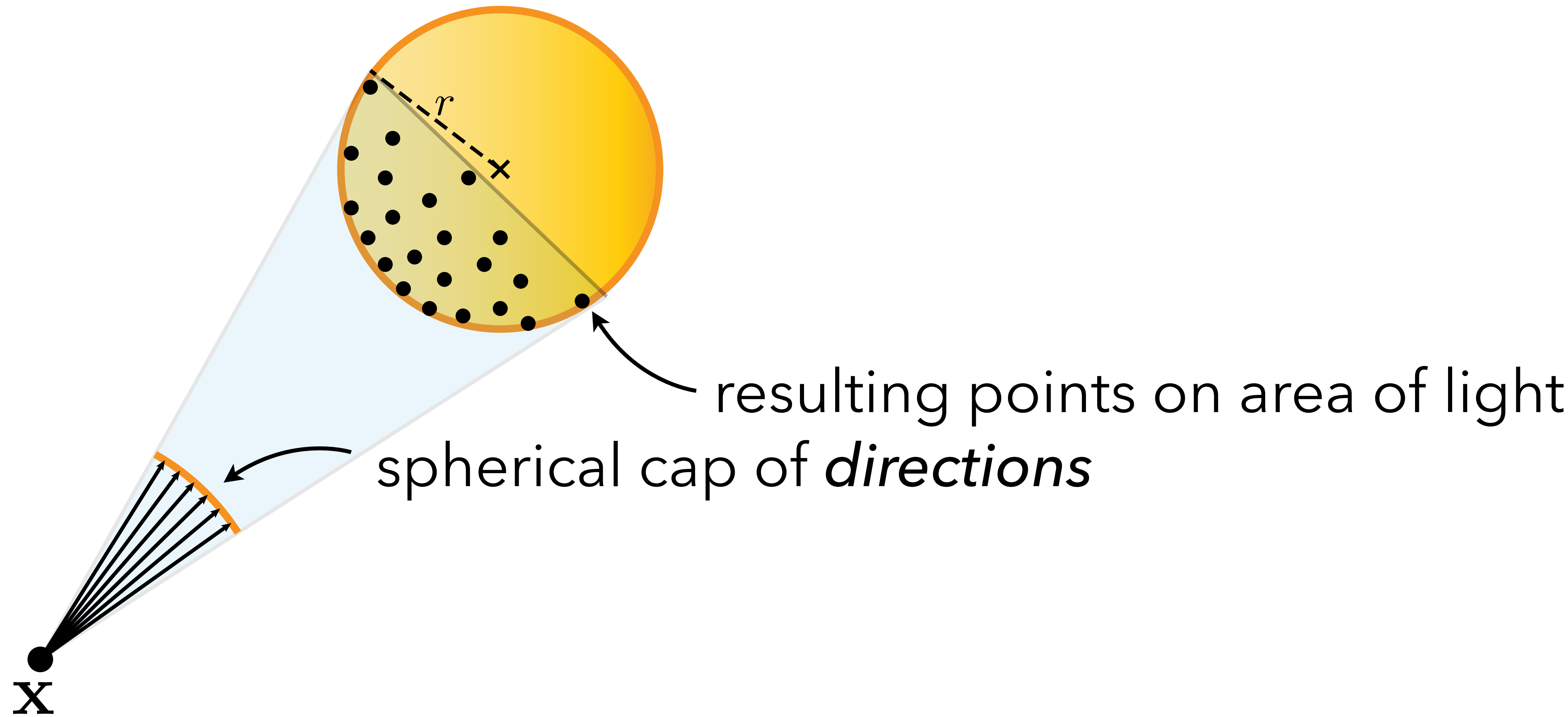
Sphere Light

- How to sample points on the sphere light?
- **Approach 2** (better): uniformly sample <u>*area*</u> of the *visible spherical cap*



Sphere Light

- How to sample points on the sphere light?
- **Approach 3** (best): uniformly sample <u>*solid angle*</u> *subtended* by the sphere

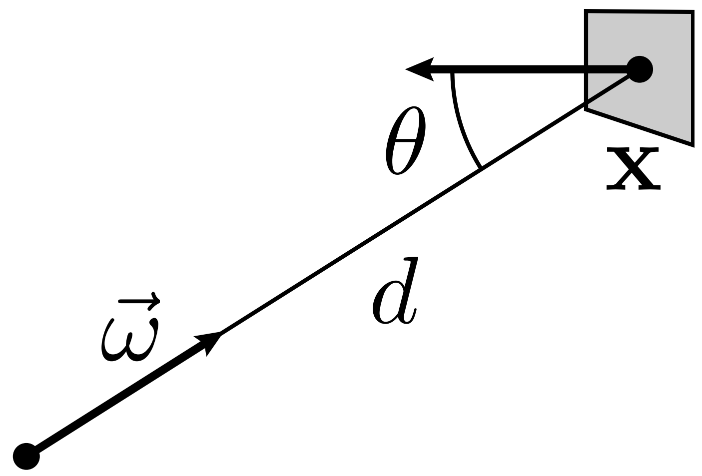


Sphere Light

- How to sample points on the sphere light?
- **Caution!**
  - Individual approaches use PDFs defined w.r.t. different measures
  - Make sure to convert the PDF into the measure of the integral!

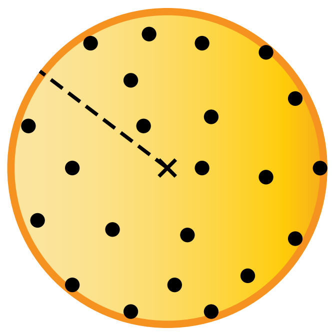
$$p_A(\mathbf{x}) = \frac{\cos\theta}{d^2} p_\Omega(\vec{\omega})$$

$$p_\Omega(\vec{\omega}) = \frac{d^2}{\cos\theta} p_A(\mathbf{x})$$



Sphere Light

- How to sample points on the sphere light?
- **Caution!**
  - Each approach uses a different measure
  - Make sure to convert the PDF into the measure of the integral
  - Example: using approach 1 for MC integration of the hemispherical formulation of the reflection eq.



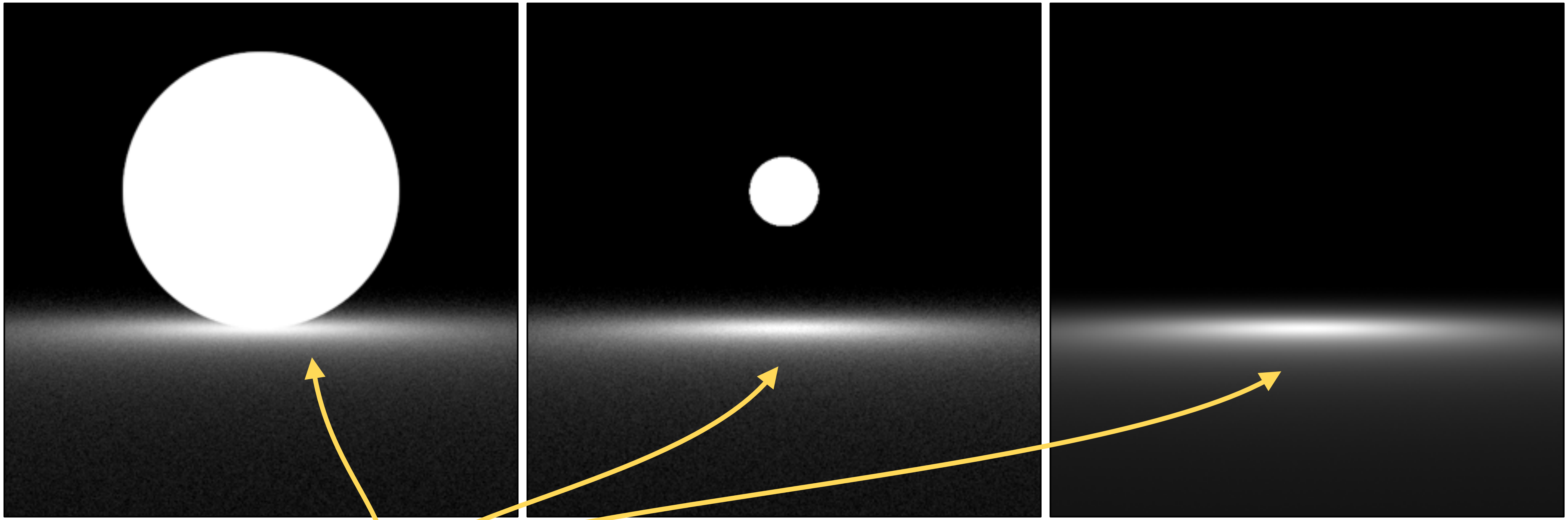
$$\langle L_r(\mathbf{x}, \vec{\omega}_r) \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_{i,k}, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k}}{p_\Omega(\vec{\omega}_{i,k})}$$

$$p_A(\mathbf{y}) = \frac{1}{4\pi r^2} \qquad p_\Omega(\vec{\omega}_i) = \frac{\|\mathbf{x} - \mathbf{y}\|^2}{|-\omega_i \cdot \mathbf{n}_\mathbf{y}|\, 4\pi r^2}$$

Sphere Light

- **Validation**: irradiance is independent of radius (assuming it emits always the same power & no occluders)

A sphere light | A smaller sphere light | A point light

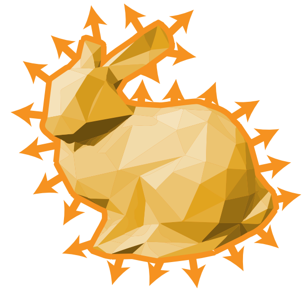


Identical irradiance profiles

Mesh Light

- An emissive mesh where every surface point emits given radiance L_e
- Total area: $\sum A(k)$

Mesh Light



- How to importance sample?
- **Preprocess**:
  1. build a discrete PDF p_Δ for choosing polygons (triangles) *proportional to their area:*

$$p_\Delta(i) = \frac{A(i)}{\sum A(k)}$$

- **Run-time**:
  2. sample a polygon i and a point $\mathbf{x}$ on i
  3. compute the PDF of choosing the point $\mathbf{x}$

$$p_A(\mathbf{x}) = p_\Delta(i)p_A(\mathbf{x}|i) = \frac{1}{\sum A(k)}$$

Light Sources

Point light | Spot light | Directional light | Quad light | Sphere light | Mesh light

Delta lights
(create hard shadows)

Area/Shape lights
(create soft shadows)

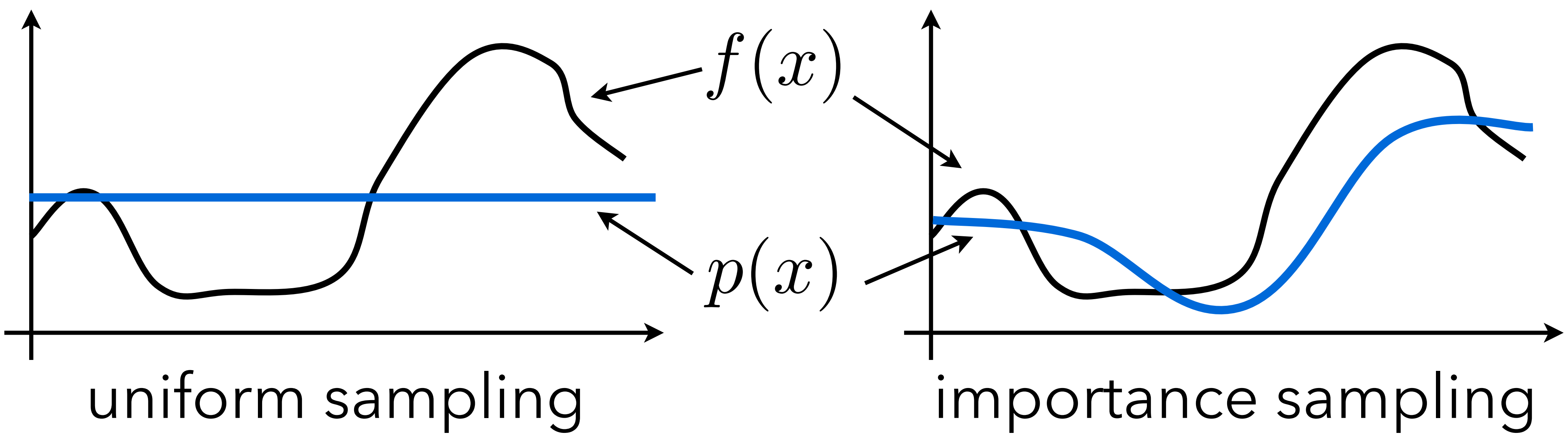
Importance Sampling

(recap)

Importance Sampling

- Placing samples intelligently reduces variance

$$\langle L_r(\mathbf{x}, \vec{\omega}_r)^N \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_{i,k}, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k} \, d\vec{\omega}_{i,k}}{p_\Omega(\vec{\omega}_{i,k})}$$



Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- What terms can we importance sample?
  - BRDF
  - incident radiance
  - cosine term

Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \boxed{\cos\theta_i} \, d\vec{\omega}_i$$

- What terms can we importance sample?
  - BRDF
  - incident radiance
  - **cosine term**

Sampling the Cosine Term

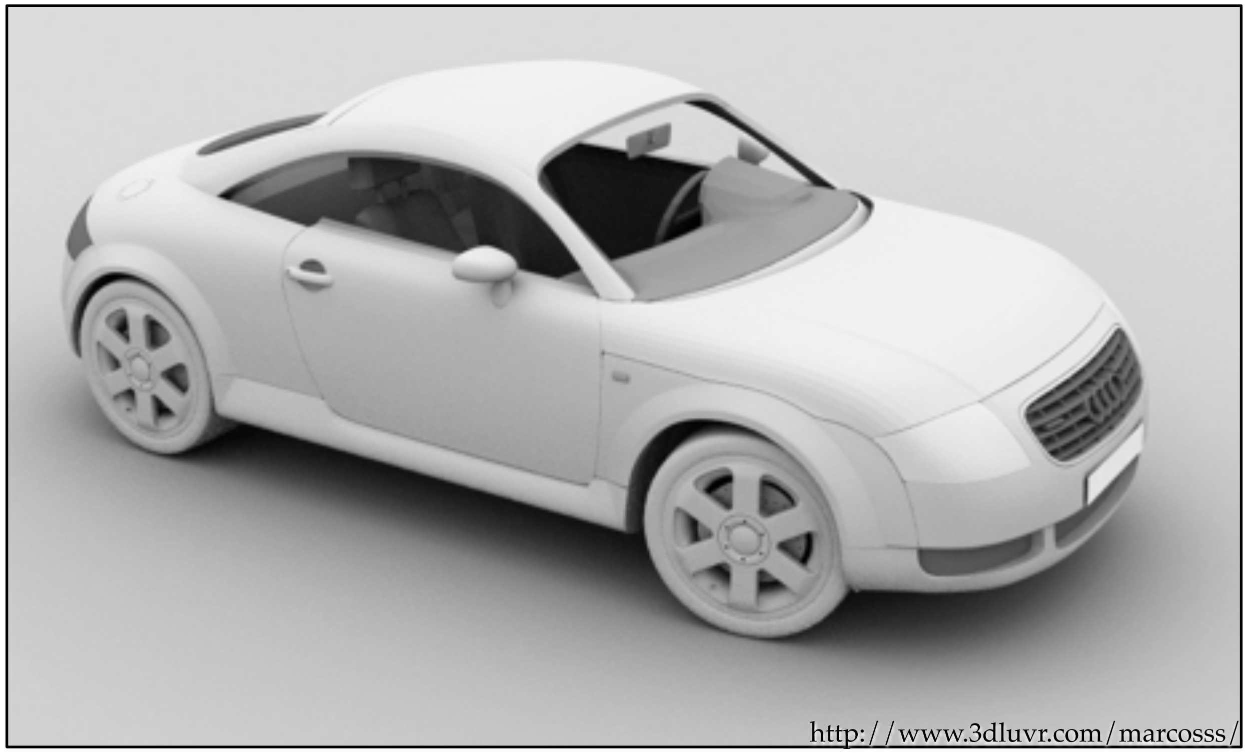
- Let's consider a simplified setup: diffuse objects illuminated by an ambient white sky

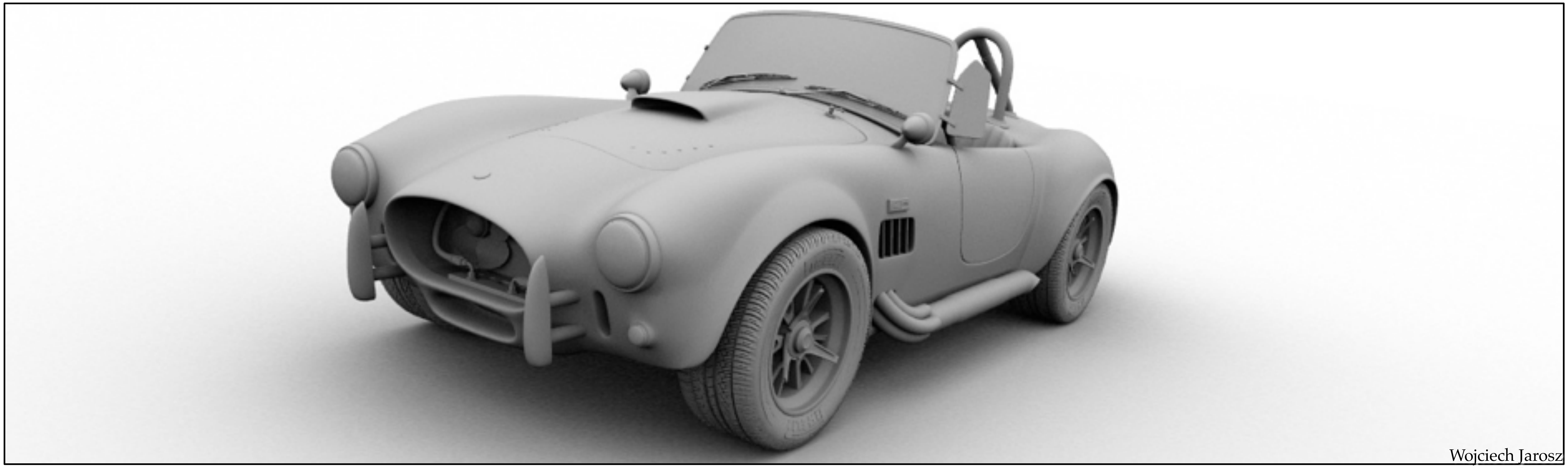
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- a.k.a. *ambient occlusion*

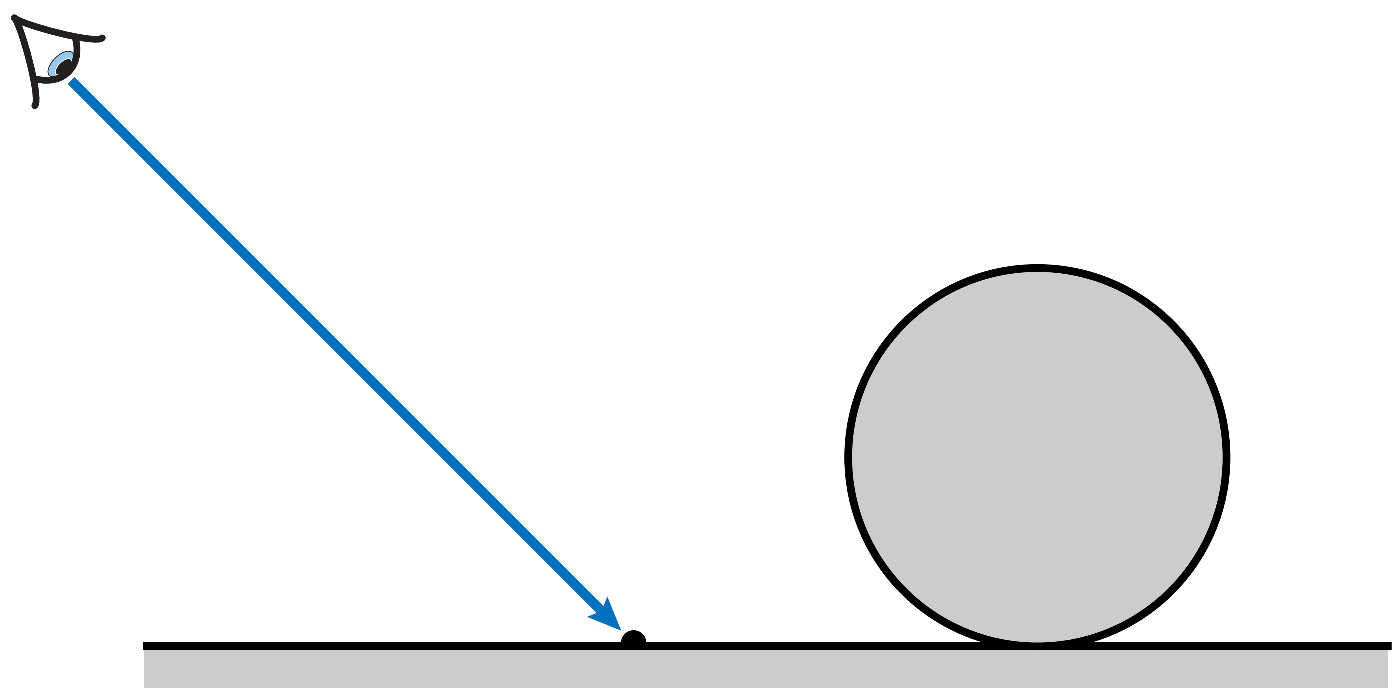
Ambient Occlusion





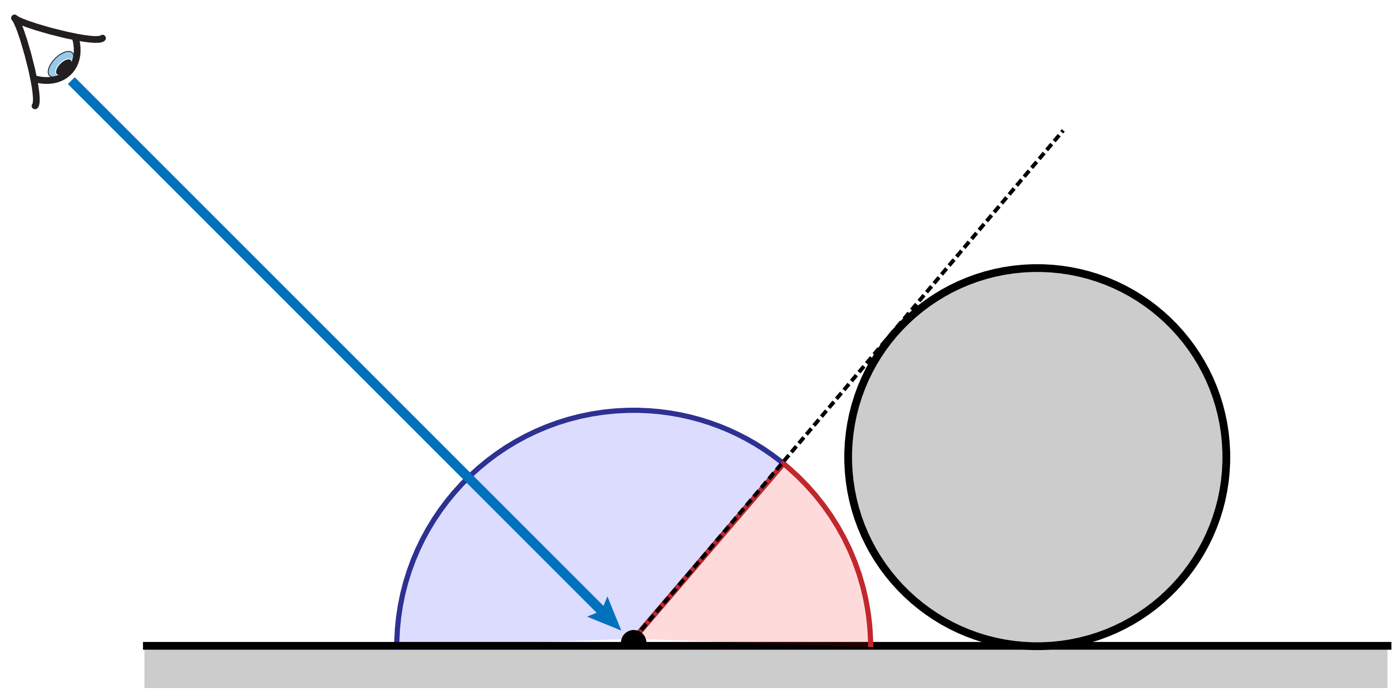
Ambient Occlusion

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



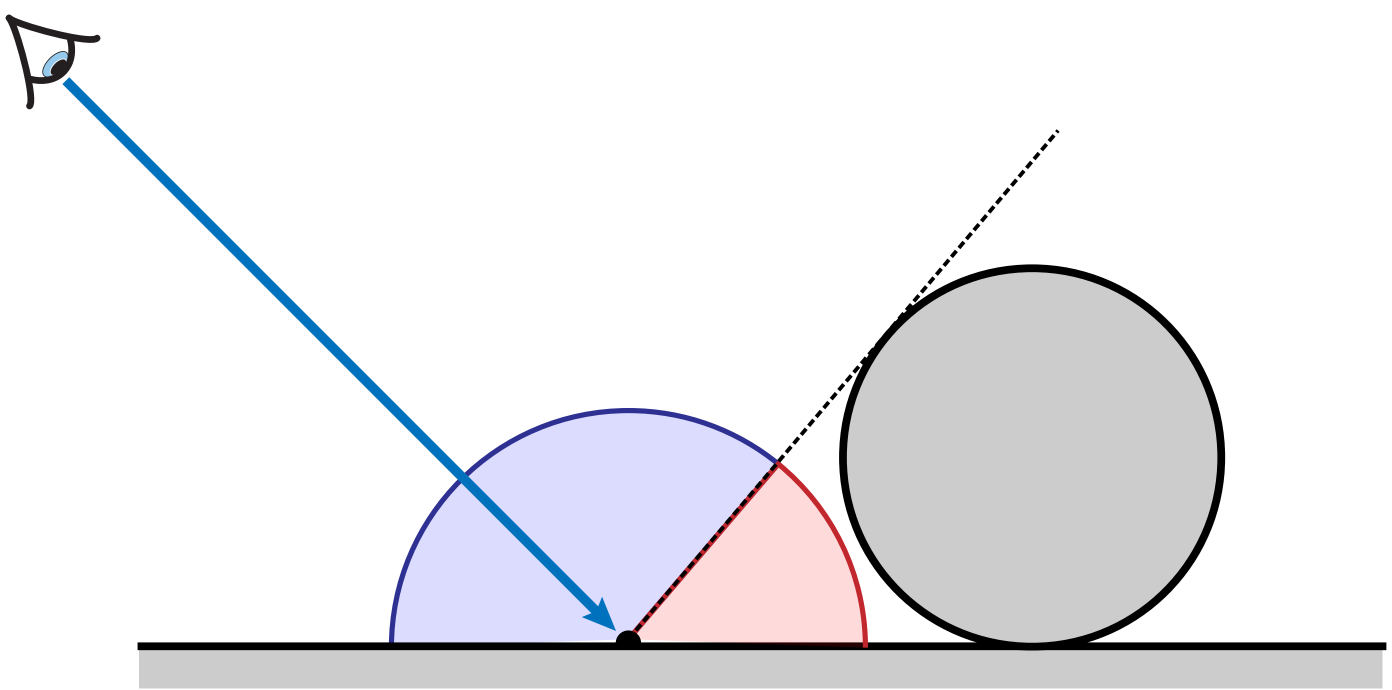
Ambient Occlusion

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



Ambient Occlusion

$$L_r(\mathbf{x}) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k}}{p(\vec{\omega}_{i,k})}$$



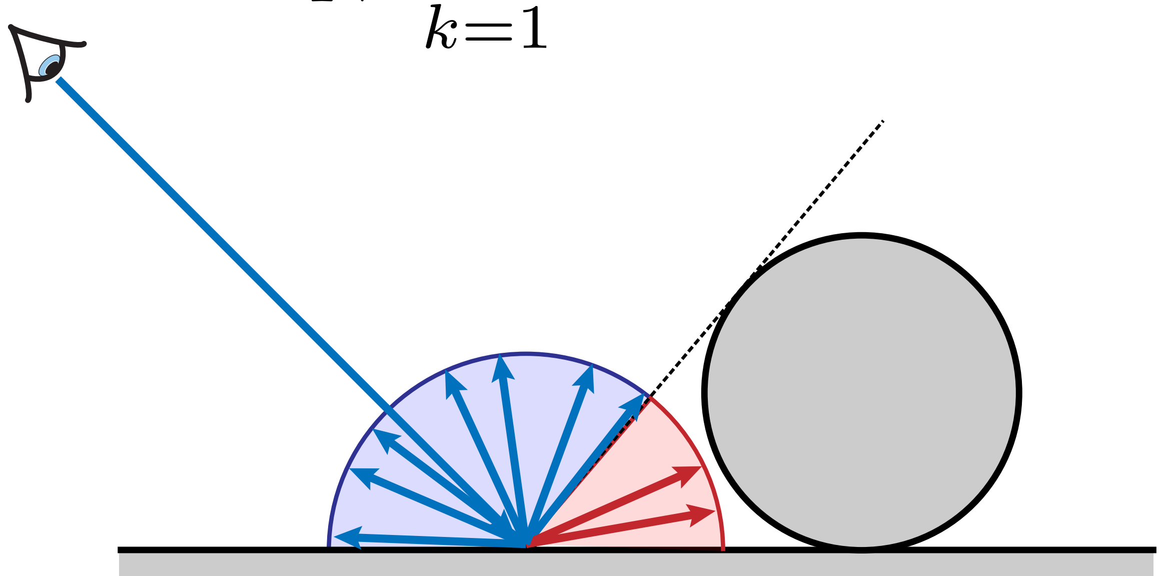
Ambient Occlusion

$$L_r(\mathbf{x}) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k}}{p(\vec{\omega}_{i,k})}$$

Uniform hemispherical sampling

$$p(\vec{\omega}_{i,k}) = 1/2\pi$$

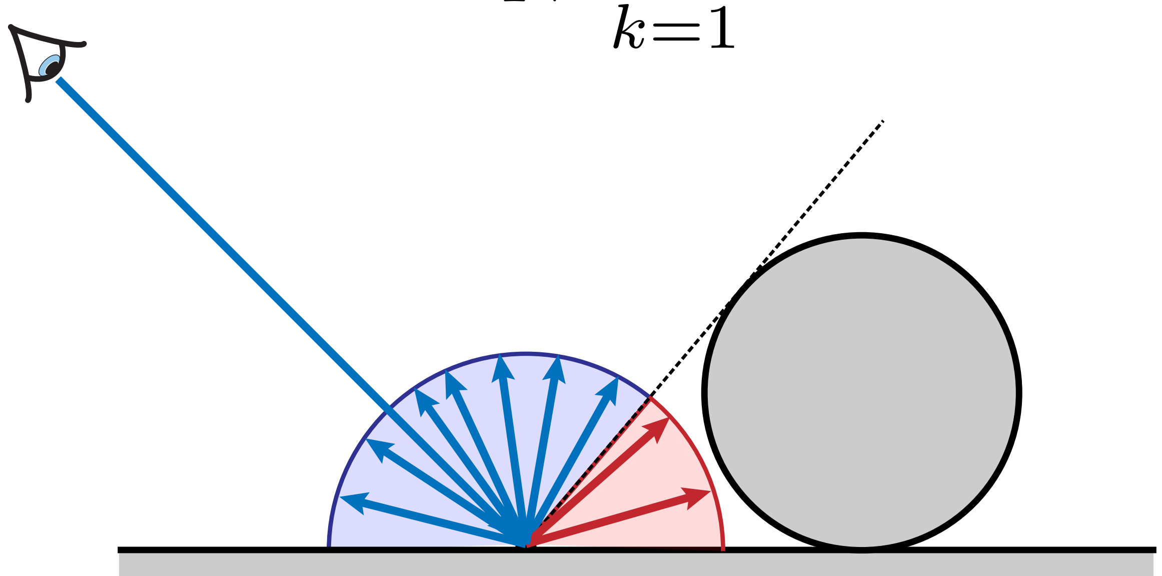
$$L_r(\mathbf{x}) \approx \frac{2\rho}{N} \sum_{k=1}^{N} V(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k}$$



Cosine-weighted importance sampling

$$p(\vec{\omega}_{i,k}) = \cos\theta_{i,k}/\pi$$

$$L_r(\mathbf{x}) \approx \frac{\rho}{N} \sum_{k=1}^{N} V(\mathbf{x}, \vec{\omega}_{i,k})$$

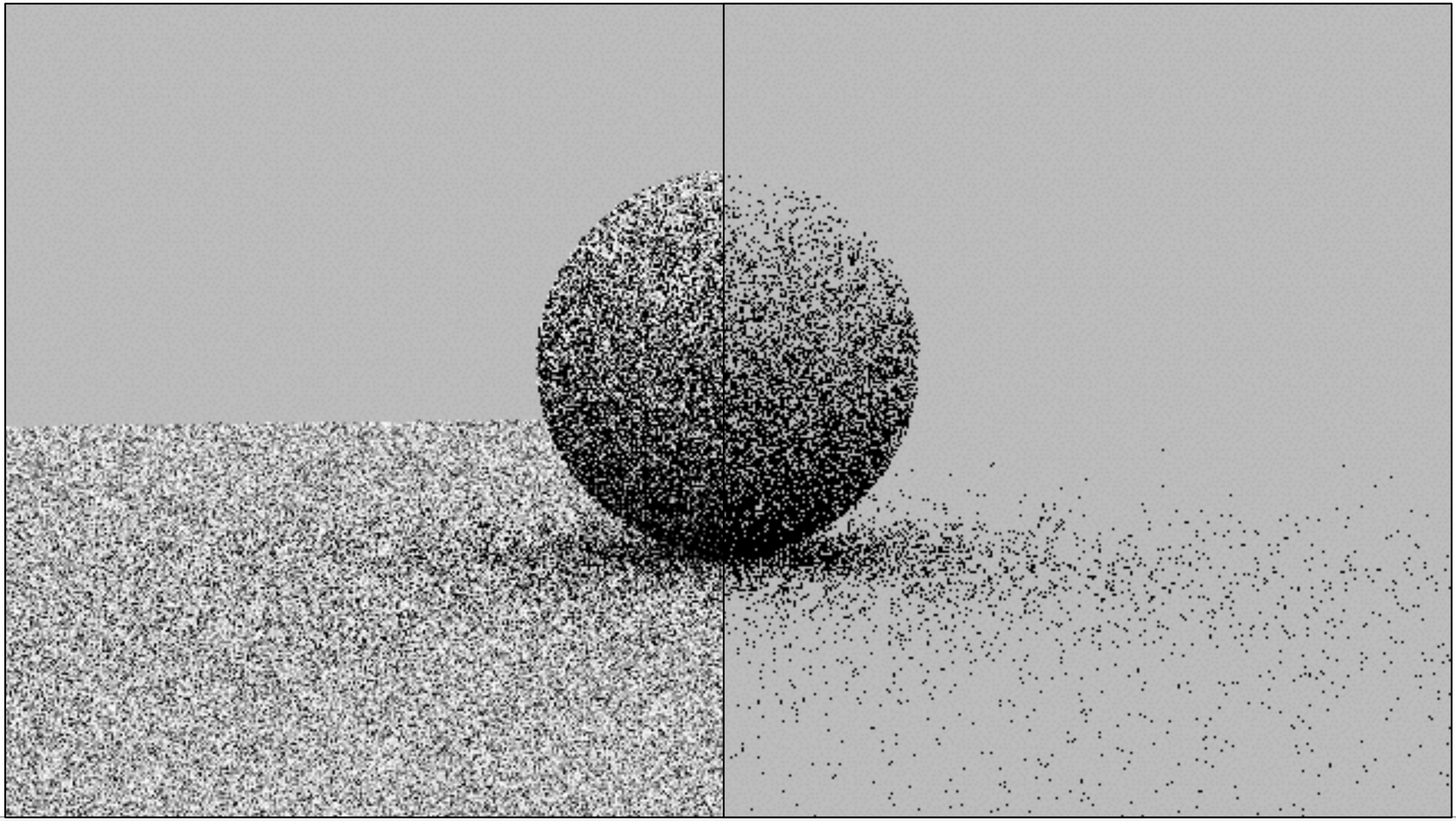


Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

1 sample/pixel

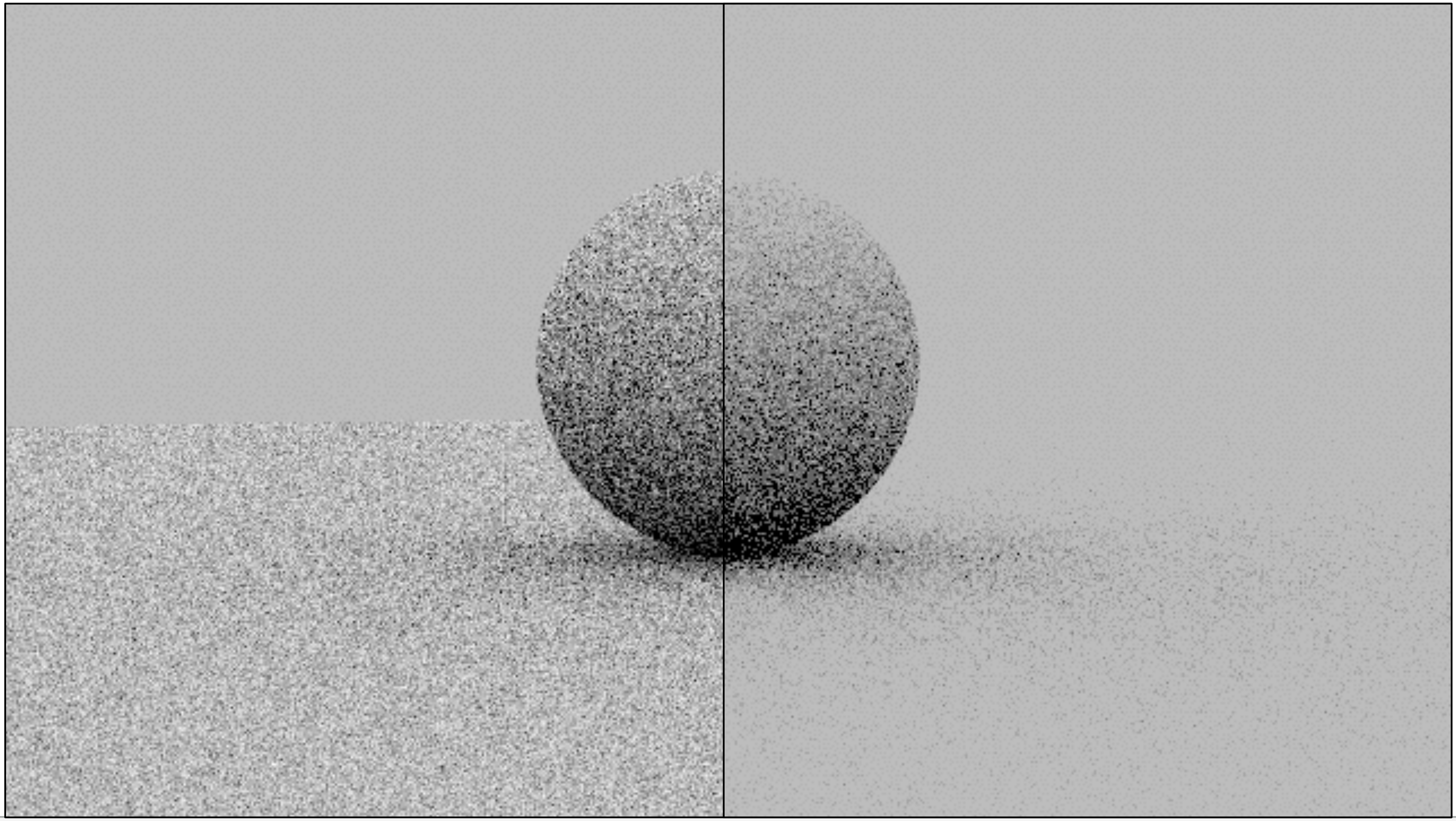


Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

4 samples/pixel

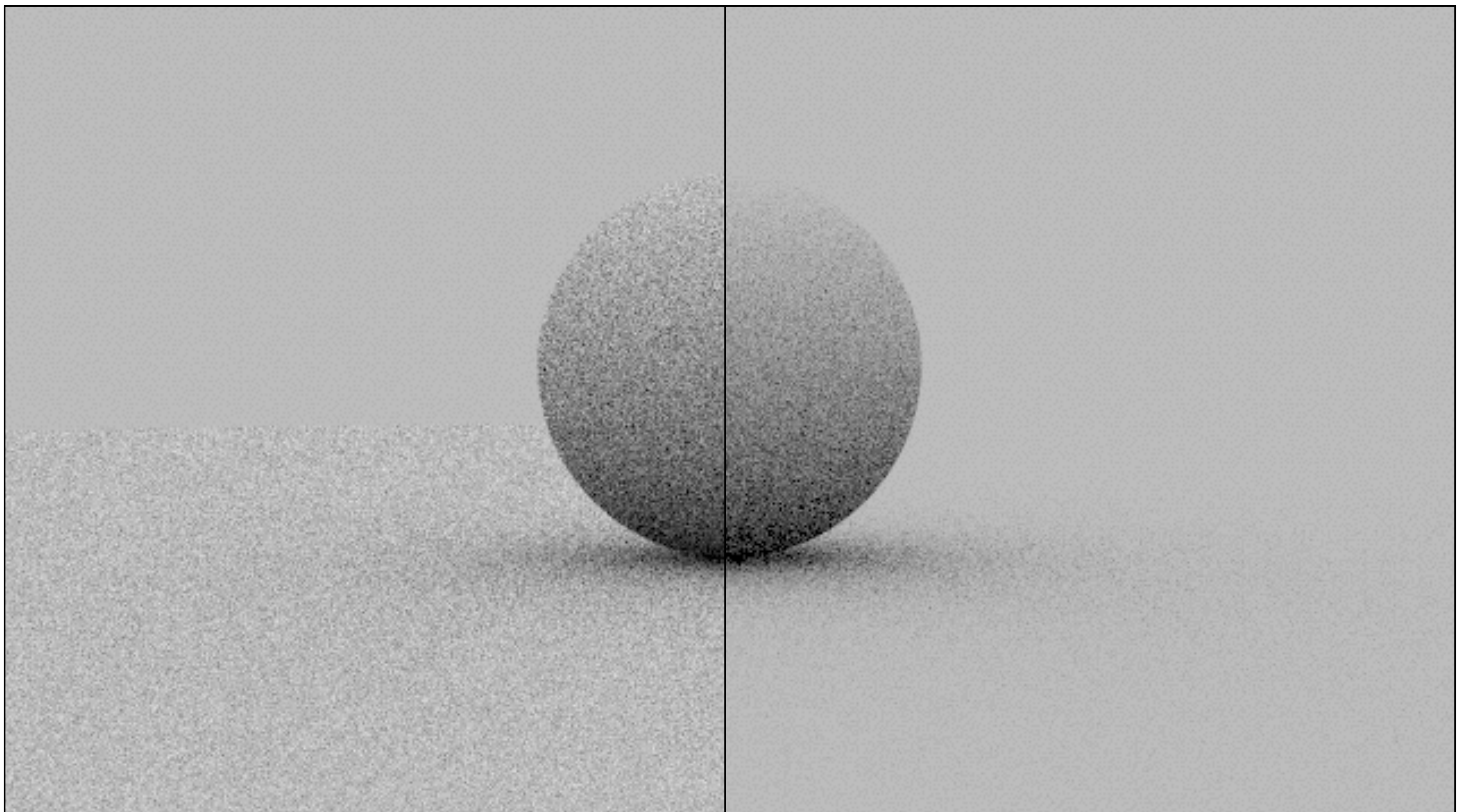


Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

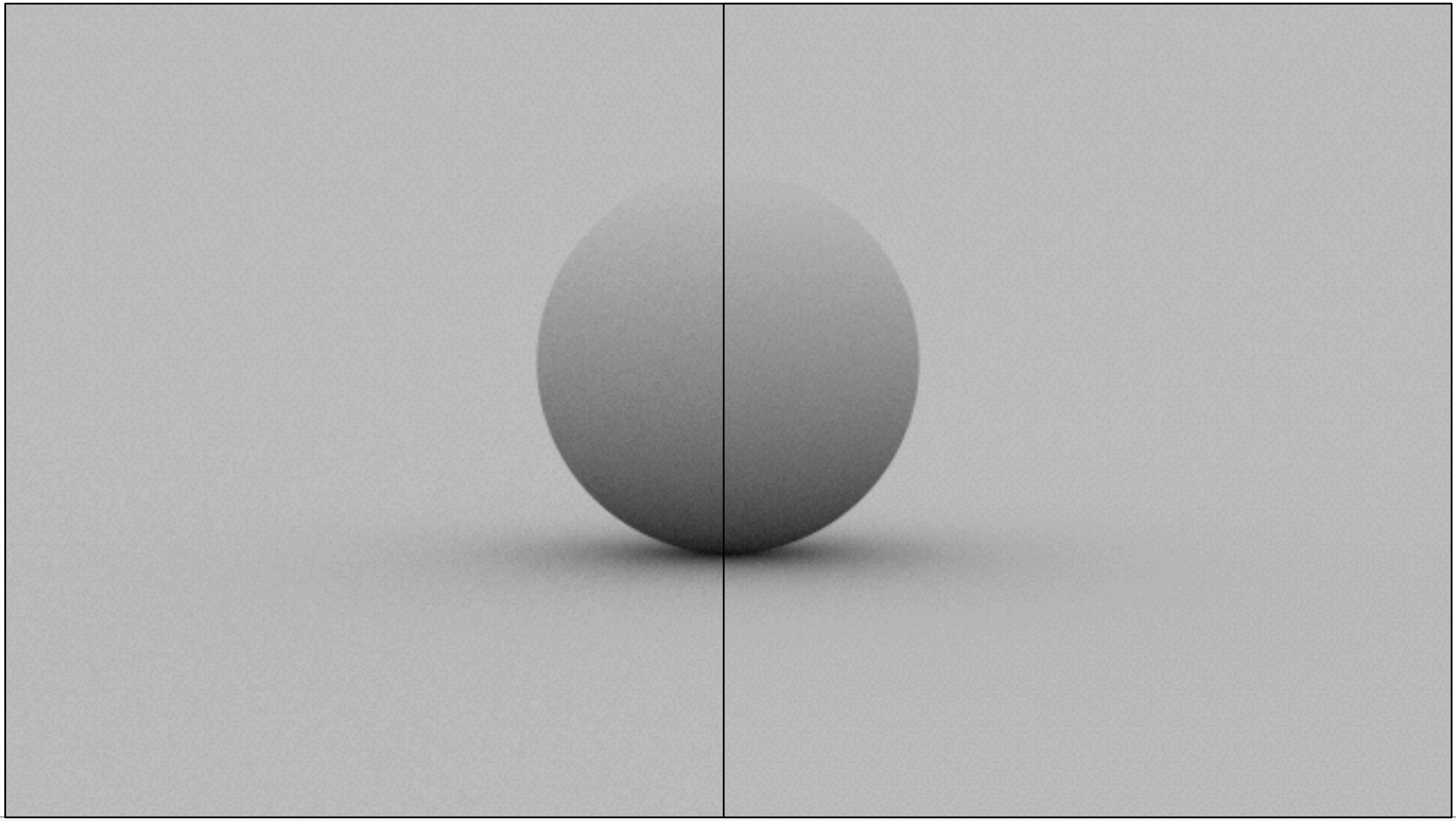
16 samples/pixel



Ambient Occlusion

Uniform hemispherical sampling | **Cosine-weighted importance sampling**

1024 samples/pixel



Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \boxed{\cos\theta_i} d\vec{\omega}_i$$

- What terms can we importance sample?
  - BRDF
  - incident radiance
  - **cosine term**

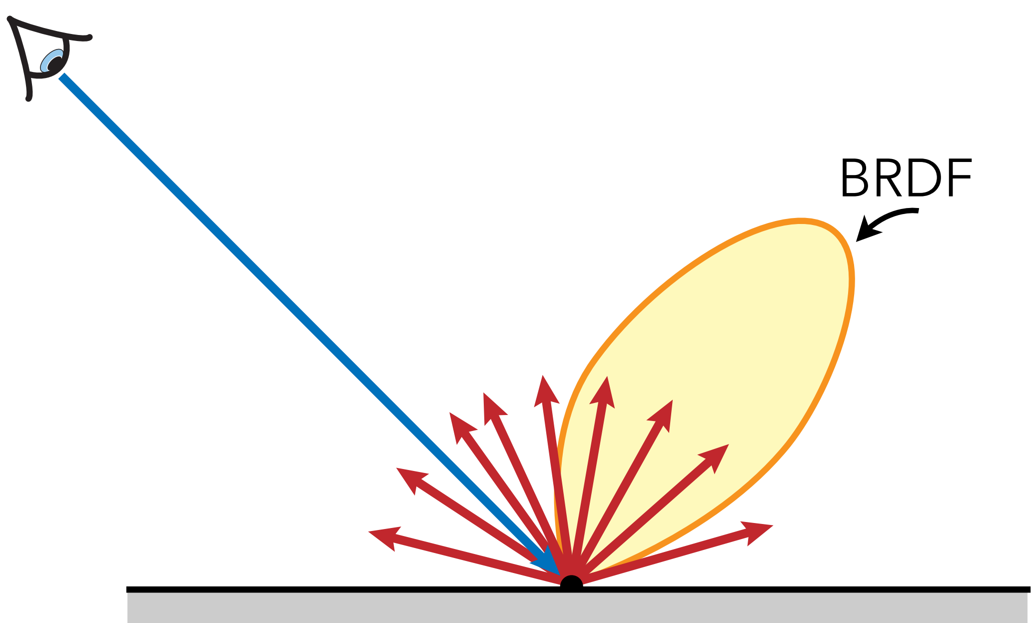
Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} \boxed{f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

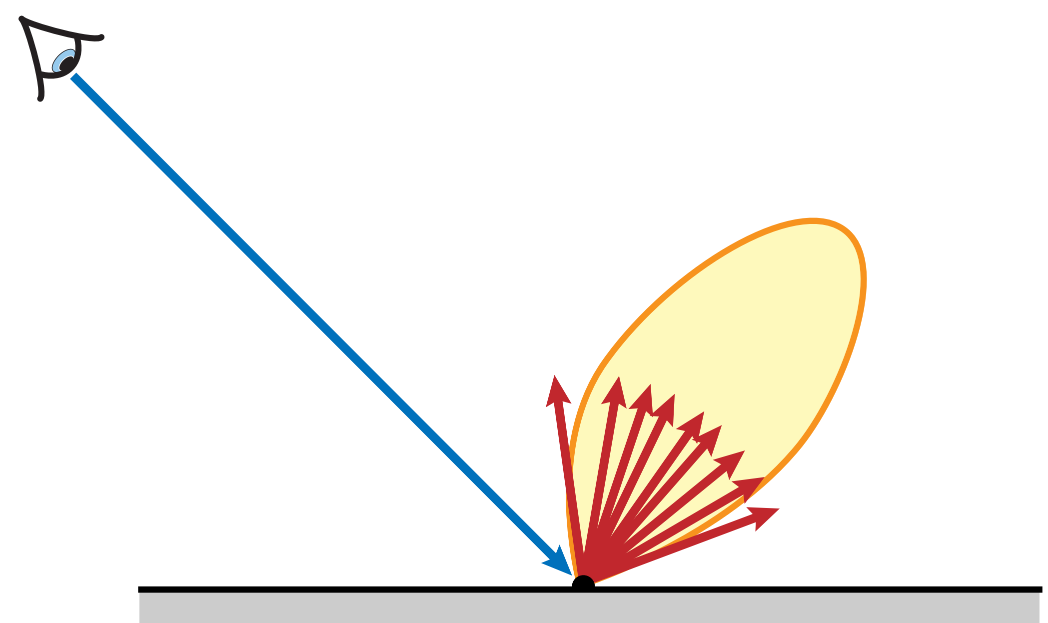
- What terms can we importance sample?
  - **BRDF**
  - incident radiance
  - cosine term

Importance Sampling the BRDF

Cosine-weighted importance sampling

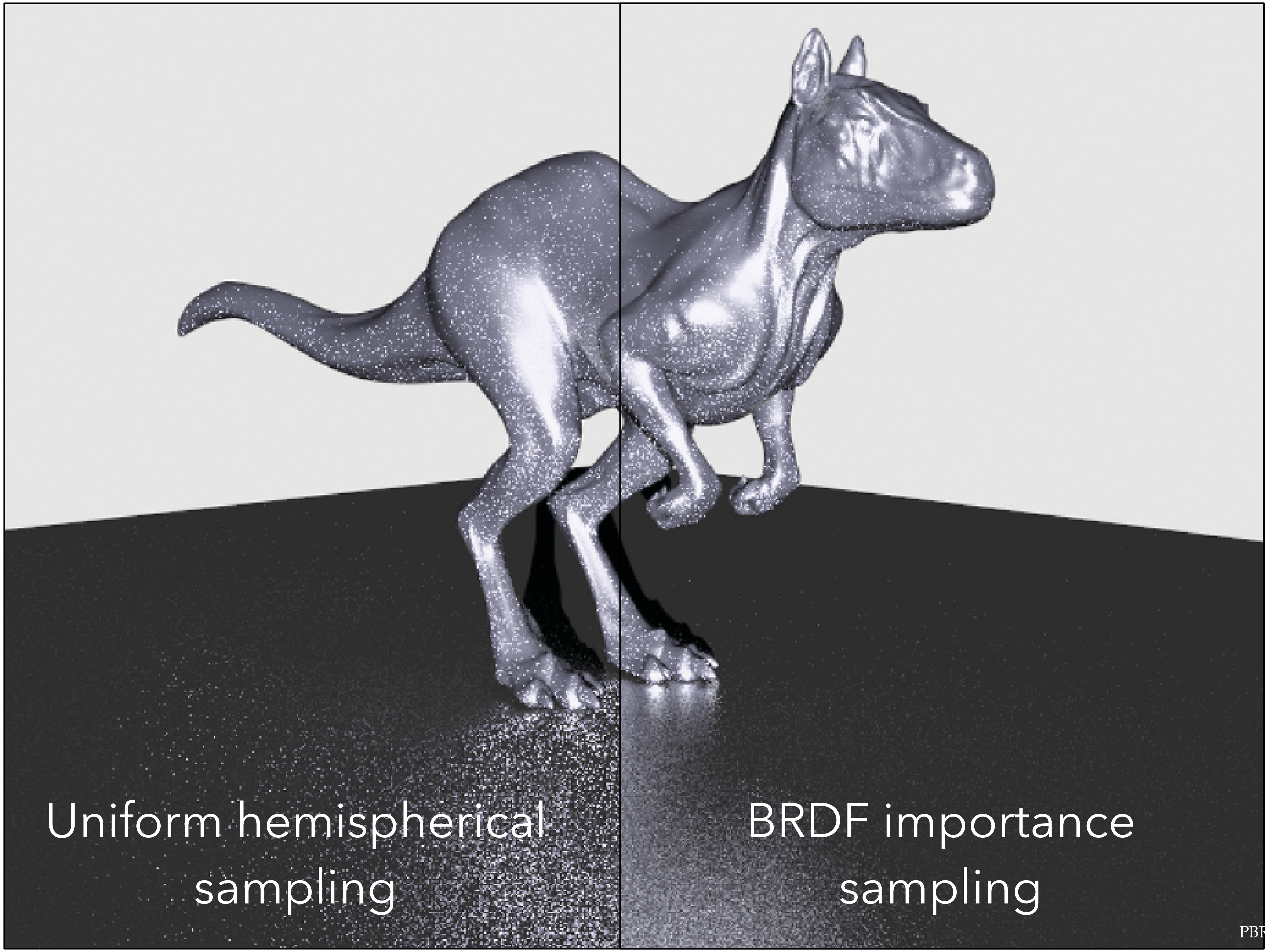


BRDF importance sampling



$$p(\vec{\omega}_i) \propto f(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)$$

Importance Sampling the BRDF



Importance Sampling the BRDF

Recipe:

1. Express the desired distribution in a convenient coordinate system
   - Requires computing the determinant of the Jacobian matrix
2. Compute marginal and conditional 1D PDFs
3. Sample 1D PDFs using the inversion method

Glossy Phong

- Normalized Phong-like $\cos^n$ lobe:

$$p(\theta, \phi) = \frac{n+2}{2\pi} \cos^n \theta$$

$$(\theta, \phi) = \left(\cos^{-1}\left((1-\xi_1)^{\frac{1}{n+2}}\right), 2\pi\xi_2\right)$$

Sampling the Microfacet BRDF

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|(\vec{\omega}_i \cdot \vec{\mathbf{n}})(\vec{\omega}_o \cdot \vec{\mathbf{n}})|}$$

- Pre-2014: "Impractical to importance sample the full model".
- Since then: *Importance Sampling Microfacet-Based BSDFs using the Distribution of Visible Normals (Heitz and D'Eon 2014)*
- Naive approach: Importance sample the microfacet distribution
- General recipe:
  - randomly generate a ω_h, with PDF proportional to D (e.g. Beckmann)
  - reflect incident direction ω_i about ω_h to obtain ω_o
  - convert PDF(ω_h) to PDF(ω_o) (change-of-variable)
- Read PBRe2 14.5

BRDFs with Multiple Lobes

- Typically, each lobe has a scaling coefficient
  - k_d diffuse coefficient
  - k_s specular/glossy coefficient
- Importance sampling:
  - Probabilistically choose a lobe, e.g. proportional to the coefficient
  - Sample a direction using the lobe

$$p_\Omega(\omega) = p_l(l) p_\Omega(\omega | l)$$

*or use multiple importance sampling (preferred)

Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} \boxed{f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- What terms can we importance sample?
  - **BRDF**
  - incident radiance
  - cosine term

Reflection Equation

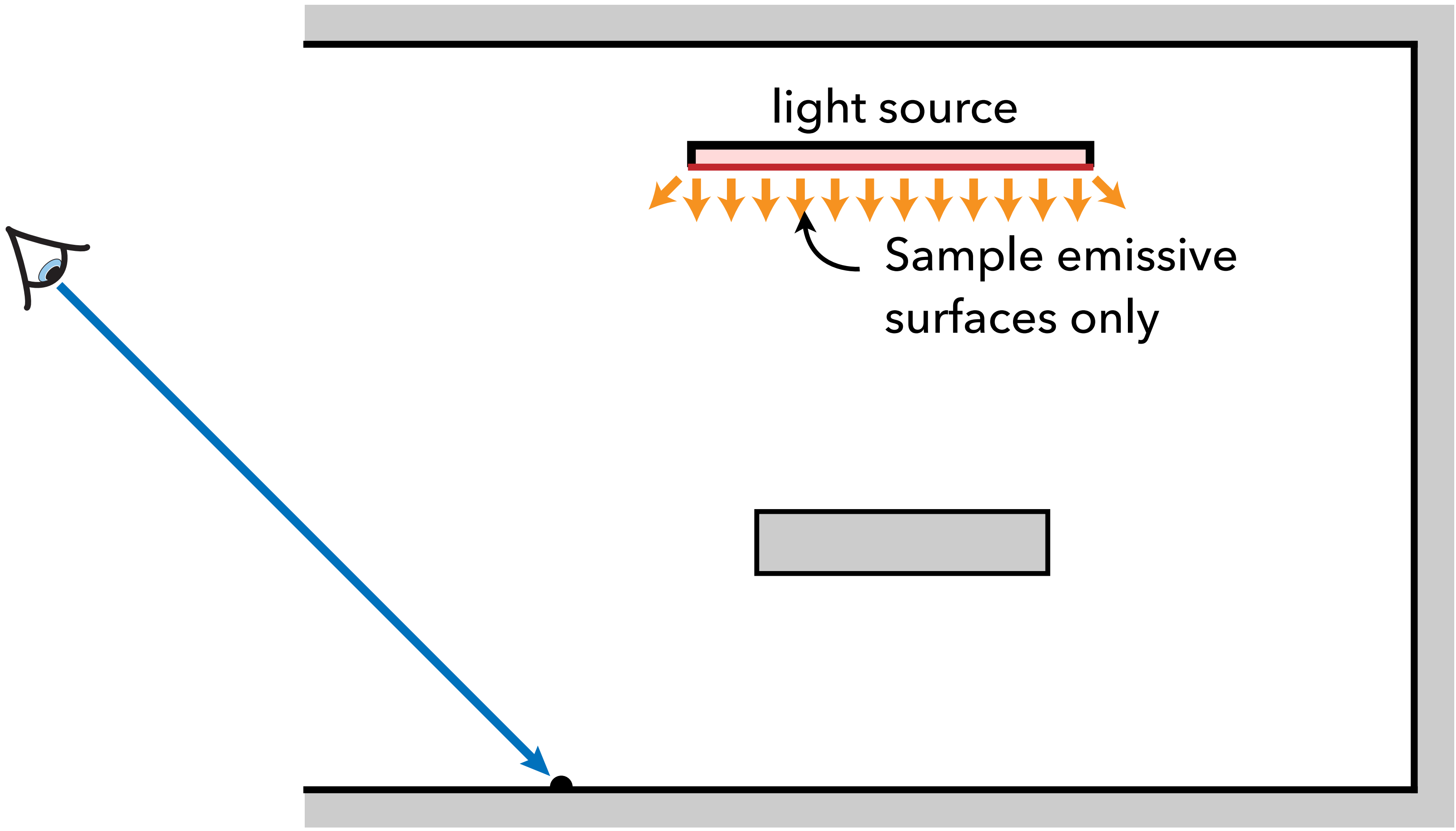
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \boxed{L_i(\mathbf{x}, \vec{\omega}_i)} \cos\theta_i \, d\vec{\omega}_i$$

- What terms can we importance sample?
  - BRDF
  - **incident radiance**
  - cosine term

Importance Sampling Incident Radiance

- Generally impossible, but…

  for direct illumination we can explicitly sample emissive surfaces



Importance Sampling Incident Radiance

- Generally impossible, but…

  for direct illumination we can explicitly sample emissive surfaces

- Use e.g. the area form of the reflection eq.:

$$L_r(\mathbf{x}, \mathbf{z}) = \int_{A_e} f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_e(\mathbf{y}, \mathbf{x}) V(\mathbf{x}, \mathbf{y}) \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA(\mathbf{y})$$

Integrate over emissive surfaces only

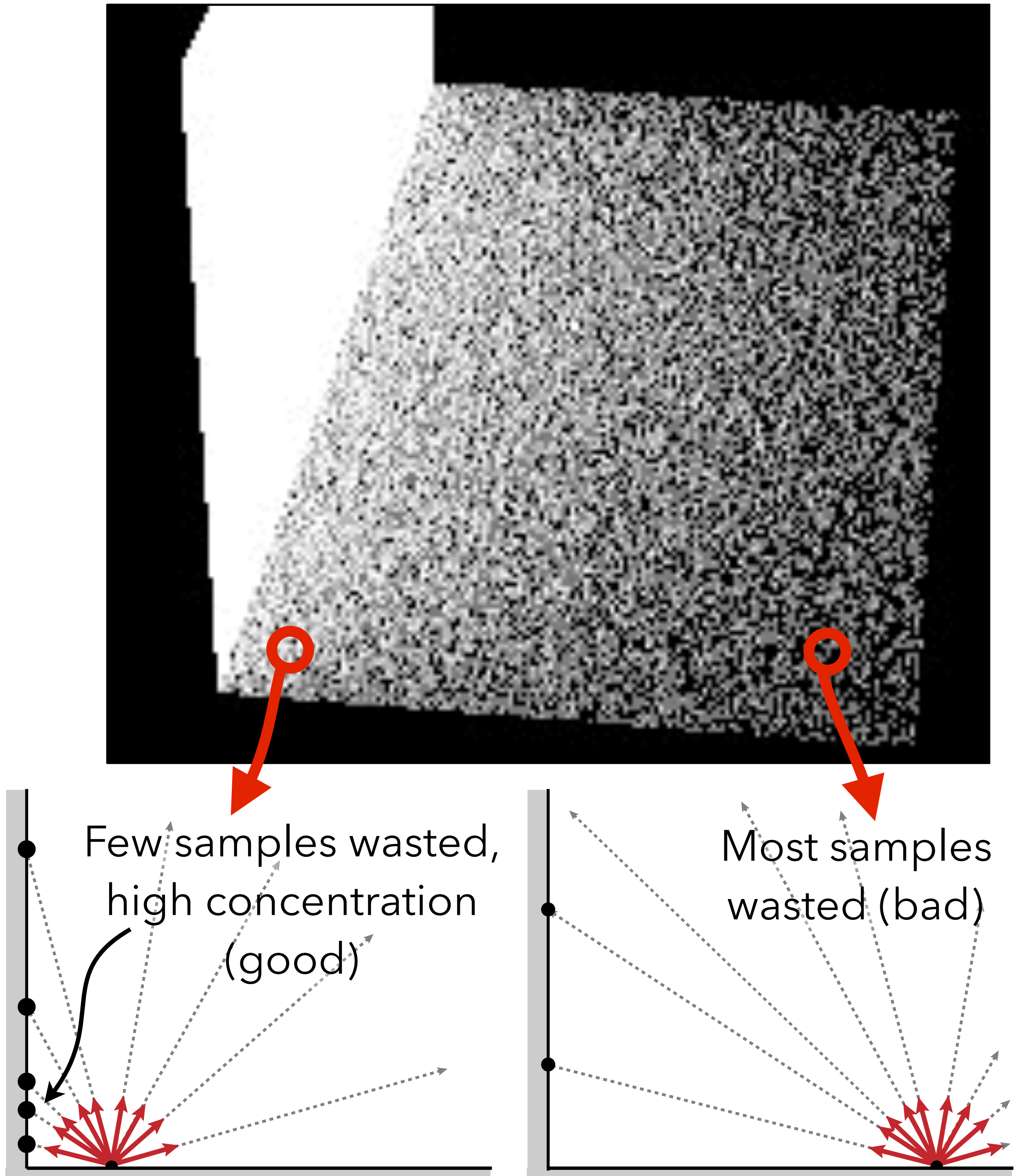
Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

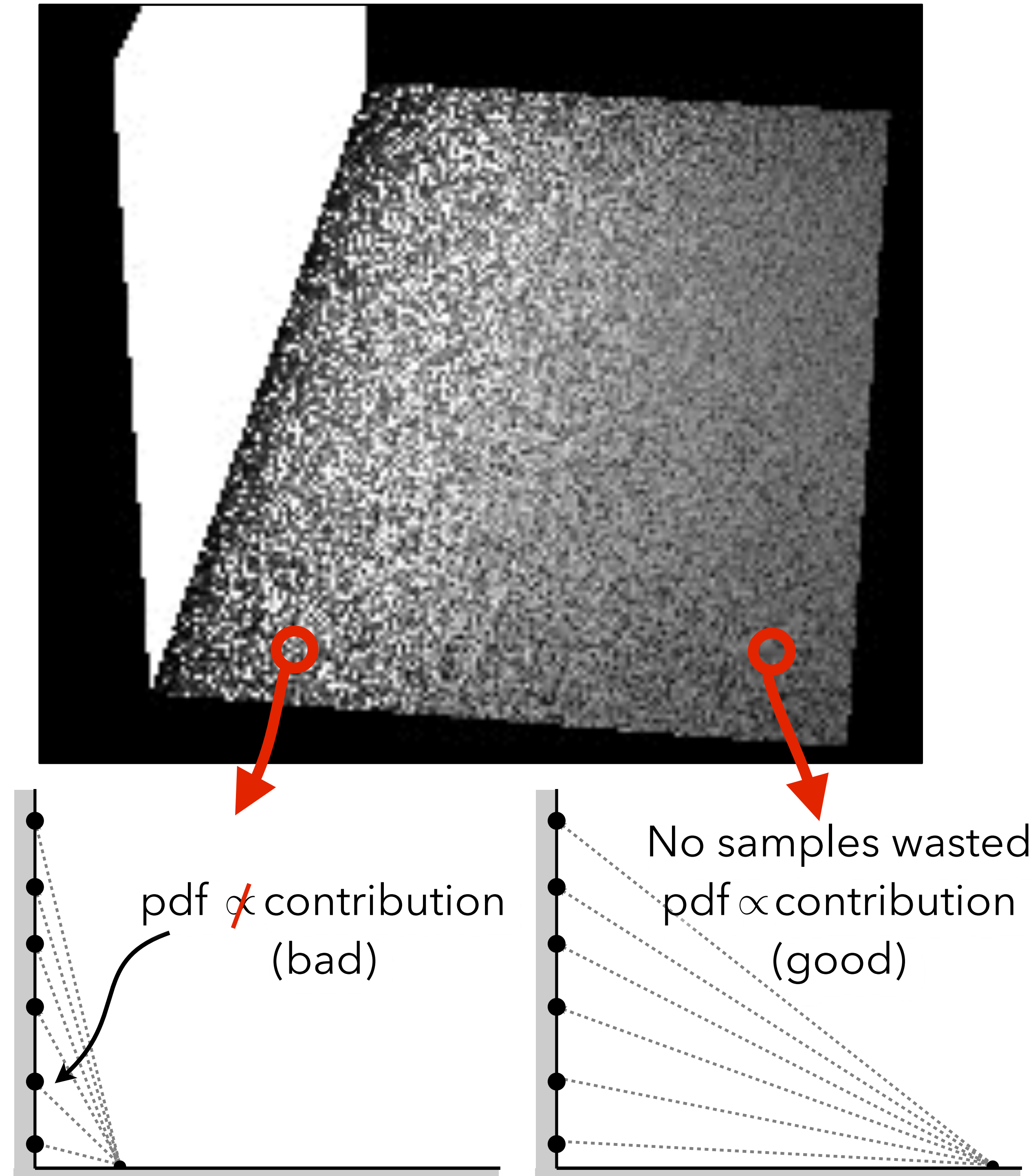
- What terms can we importance sample?
  - BRDF
  - incident radiance
  - cosine term
- What terms **should** we importance sample?
  - depends on the context, hard to make a general statement

Multiple Strategies

Cosine-weighted hemisphere

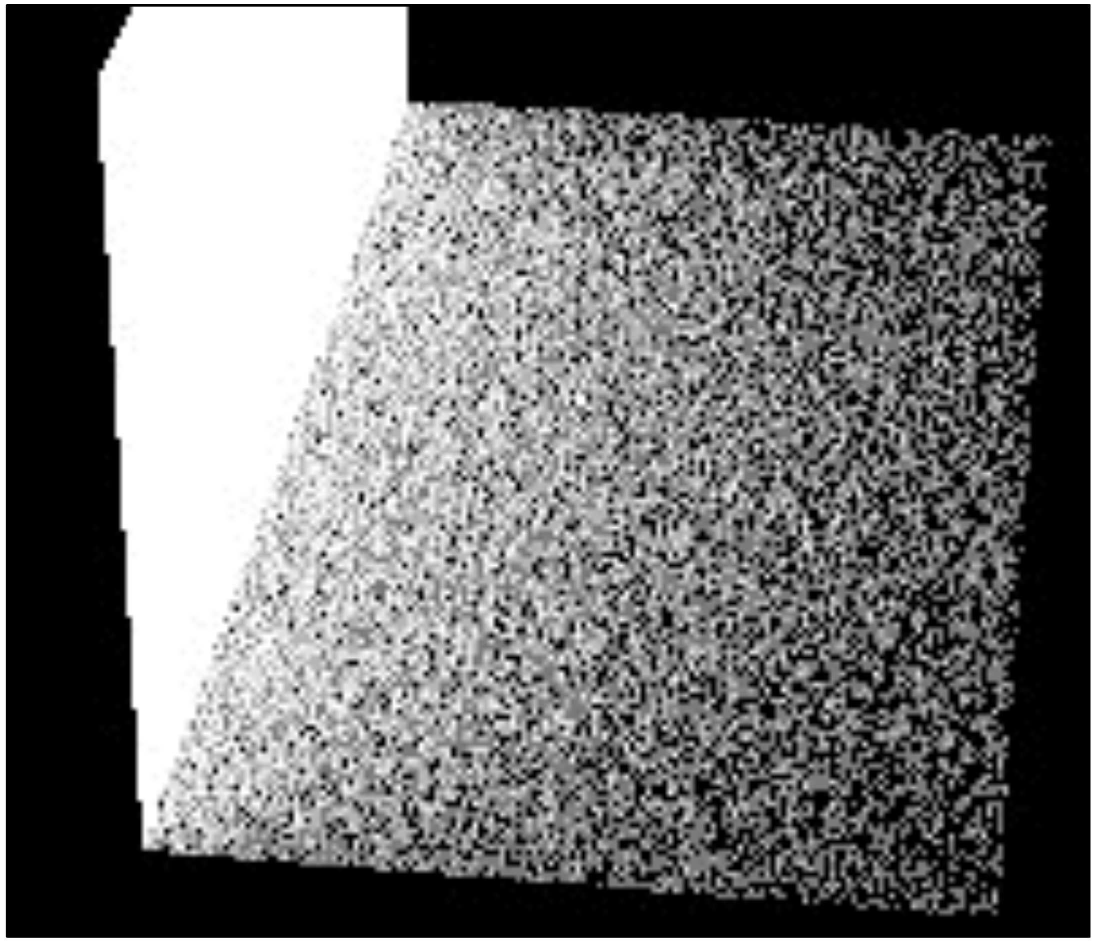


Uniform surface area



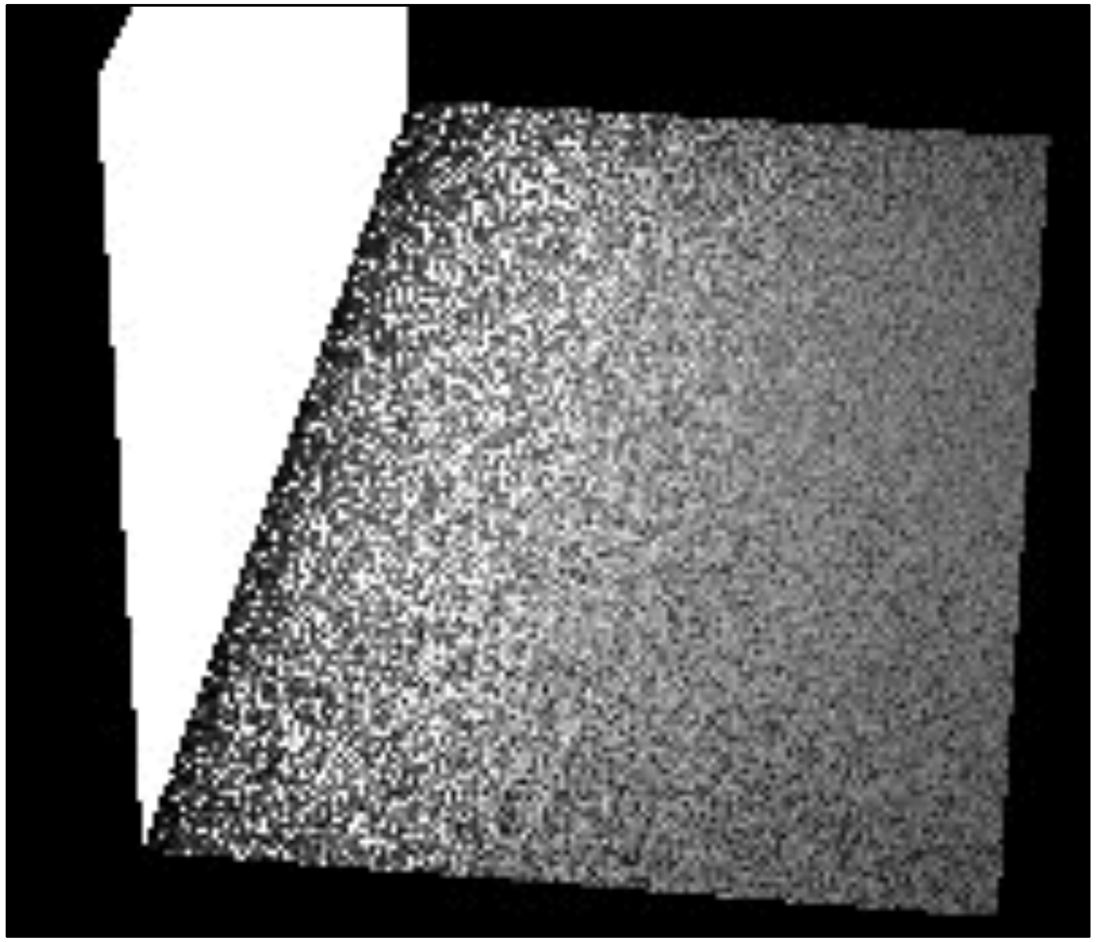
Combining Multiple Strategies

Cosine-weighted hemisphere



$$p_1(\vec{\omega}) = \frac{\cos\theta}{\pi}$$

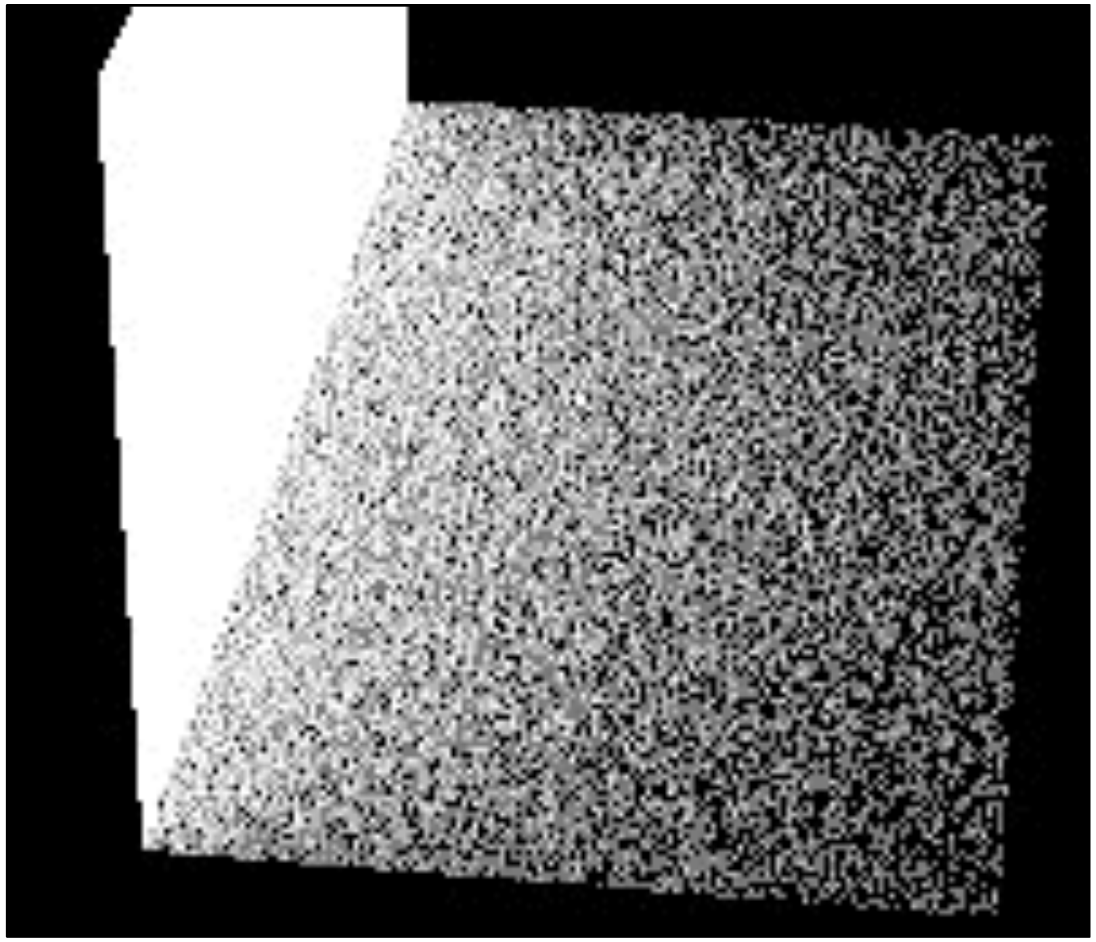
Uniform surface area



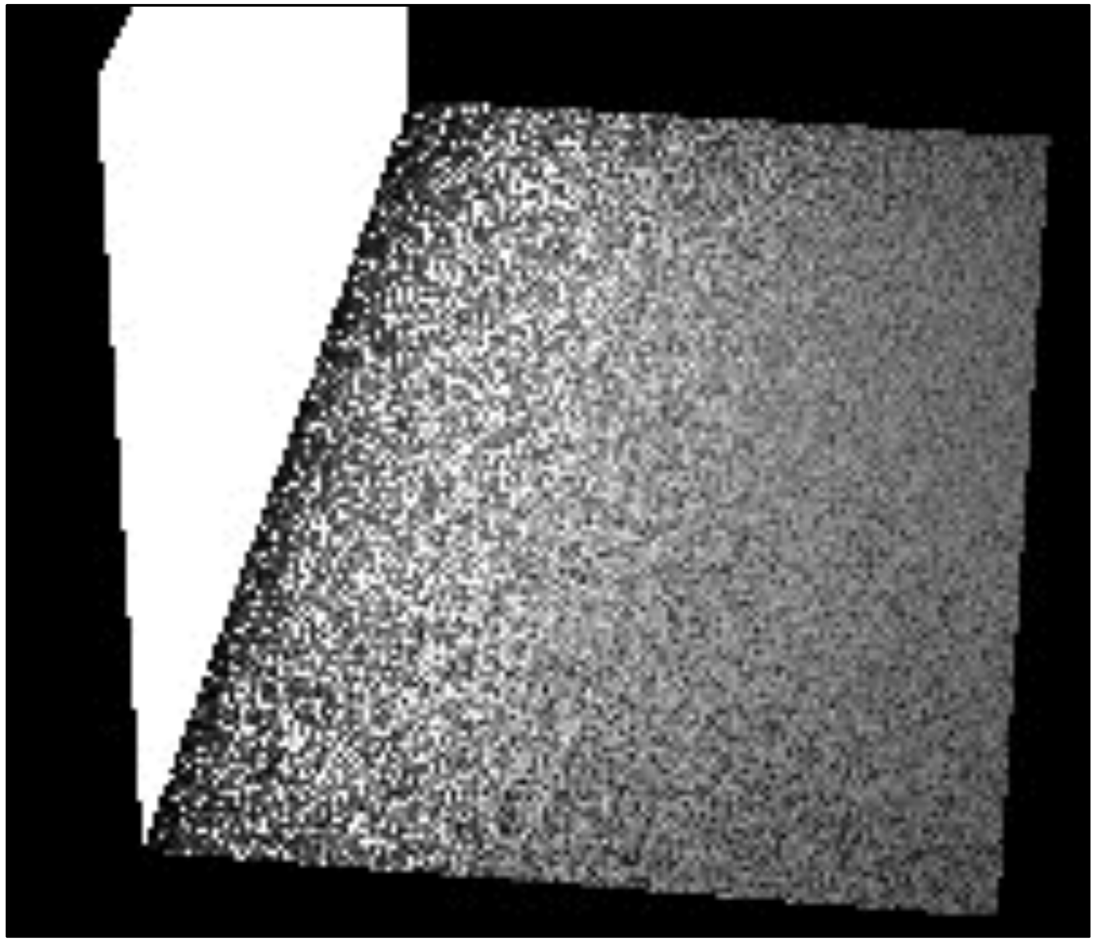
$$p_2(\mathbf{x}) = \frac{1}{A}$$

Combining Multiple Strategies

Cosine-weighted hemisphere



Uniform surface area



$$p_1(\vec{\omega}) = \frac{\cos\theta}{\pi}$$

$$p_2(\mathbf{x}) = \frac{1}{A} \qquad p_2(\vec{\omega}) = \frac{1}{A}\frac{d^2}{\cos\theta}$$

Combining Multiple Strategies

- Could just average two different estimators:

$$\frac{0.5}{N_1}\sum_{i=1}^{N_1}\frac{f(x_i)}{p_1(x_i)} + \frac{0.5}{N_2}\sum_{i=1}^{N_2}\frac{f(x_i)}{p_2(x_i)}$$

  - doesn't really help: *variance is additive*

- Instead, sample from the average PDF

$$\frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{0.5(p_1(x_i)+p_2(x_i))}$$

Sample from Average PDF

- You are given two sampling functions and their corresponding pdfs:

```
float sample1(float rnd); float pdf1(float x);
float sample2(float rnd); float pdf2(float x);
```

- Create a new function:

```
float sampleAvg(float rnd);
```

- which has the corresponding pdf:

```
float pdfAvg(float x)
{
    return 0.5 * (pdf1(x) + pdf2(x));
}
```

Sample from Average PDF

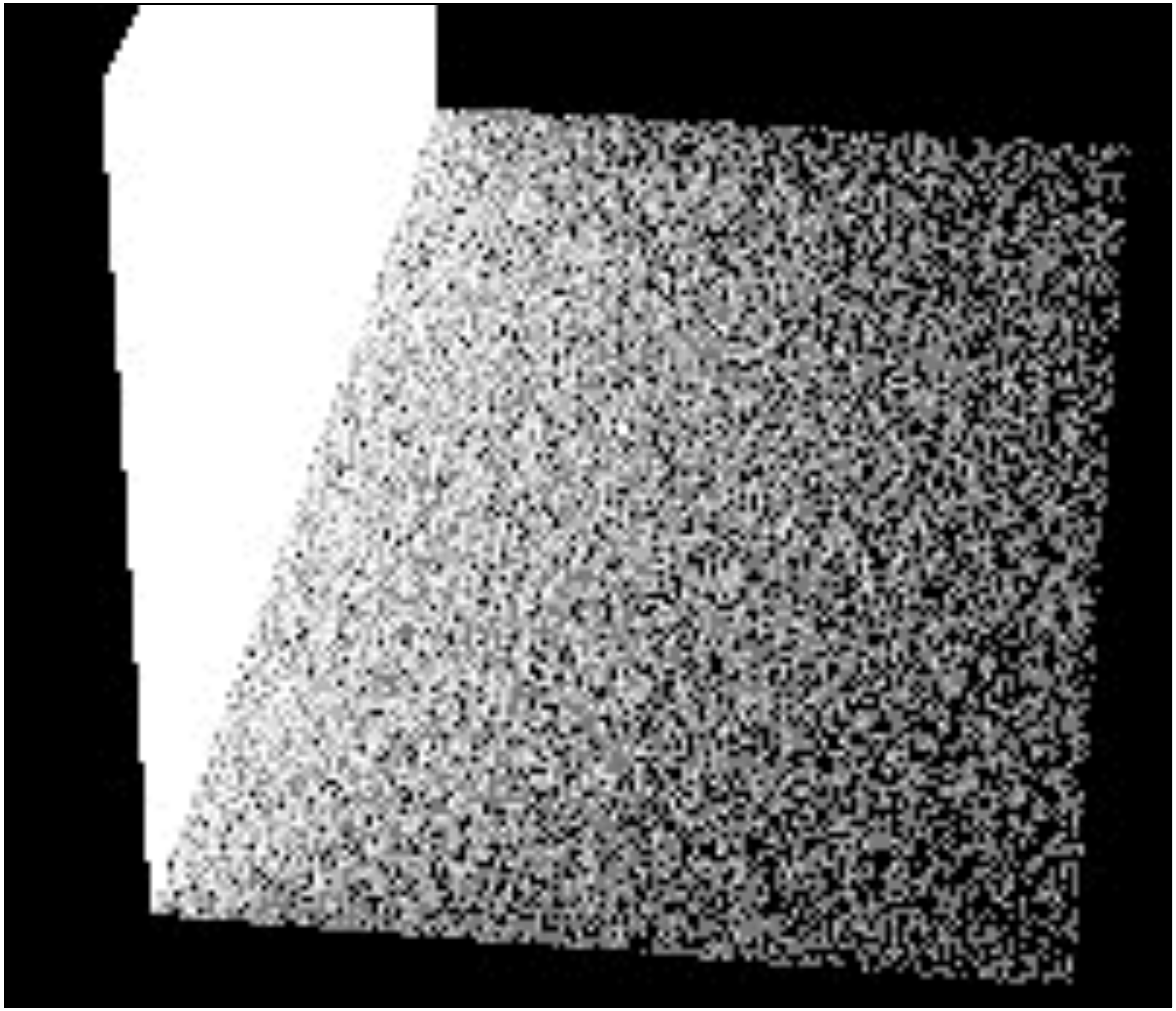
```
float sampleAvg(float rnd)
{
   if (rand.nextFloat() < 0.5)
      return sample1(rnd);
   else
      return sample2(rnd);
}
```

Requires extra random number (can be avoided)

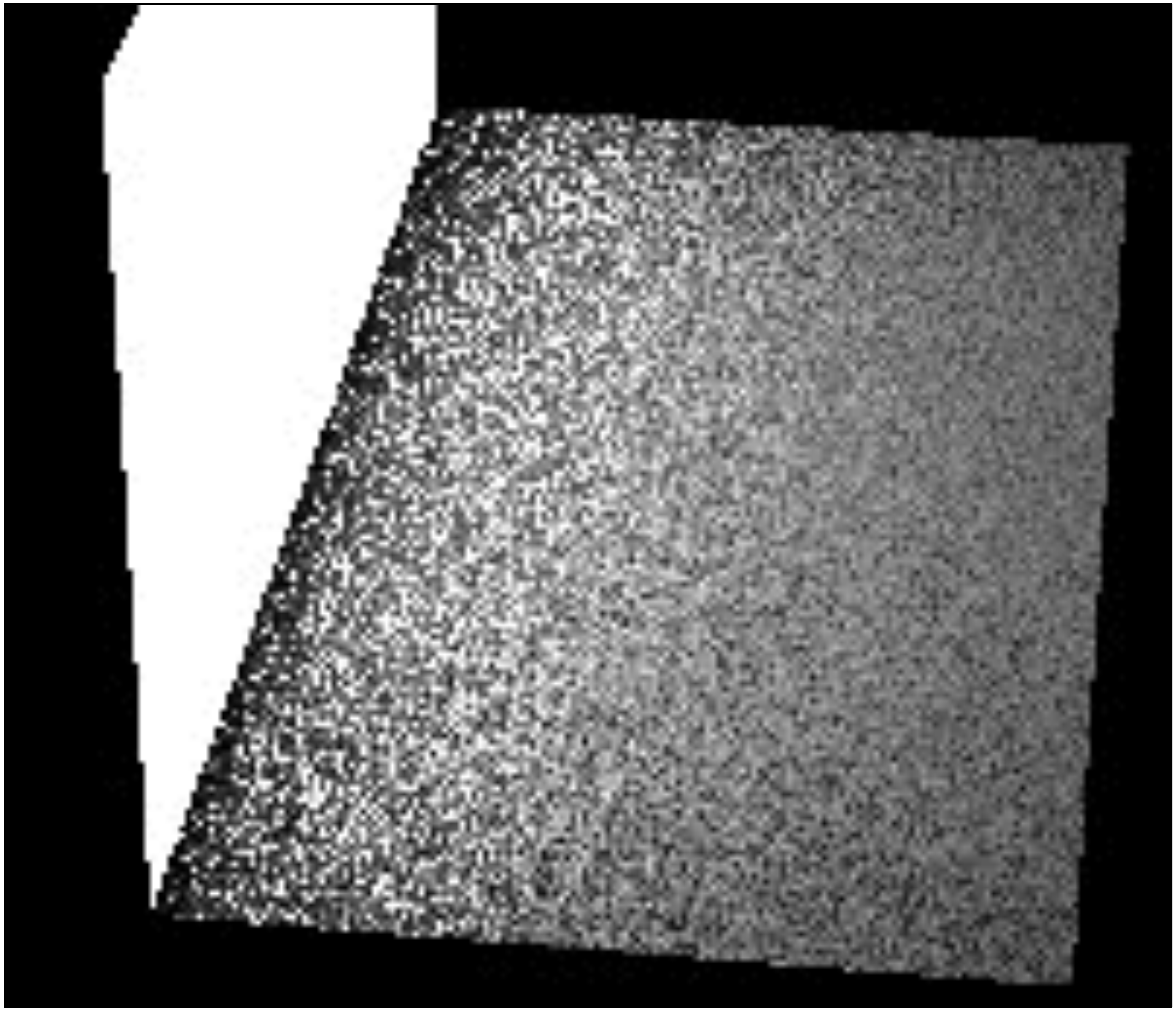
Sample from Average PDF

```
float sampleAvg(float rnd)
{
   if (rnd < 0.5)
      return sample1(2.0 * rnd);
   else
      return sample2(2.0 * rnd - 1.0);
}
```

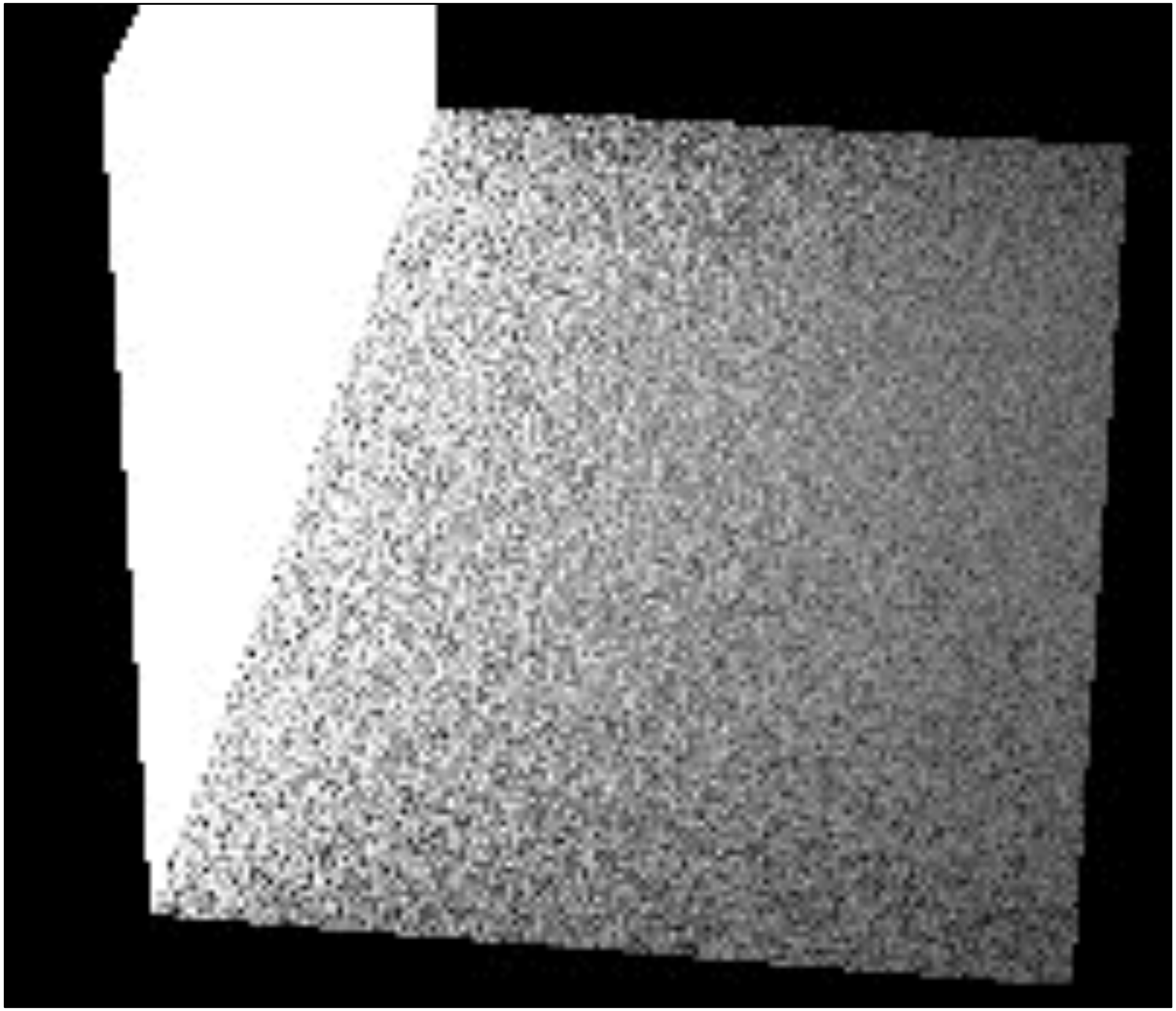
Cosine-weighted Hemisphere



Uniform Surface Area



Average PDF



Visual Break


source: onebigphoto.com

Visual Break


source: flickr

Multiple Importance Sampling (MIS)

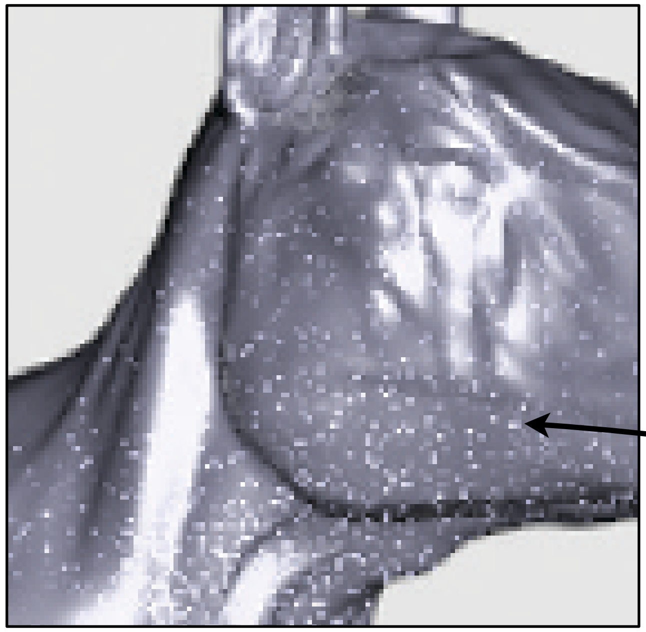
Motivation

- In MC integration, variance is high when the PDF is not proportional to the integrand
- Worst case: *rare* samples with *huge* contributions

$$\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

large value

small value



"fireflies"

Motivation

- In MC integration, variance is high when the PDF is not proportional to the integrand
- Worst case: *rare* samples with *huge* contributions

$$\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

large value

small value

- We often have multiple sampling strategies
- If at least one covers each part of the integrand well, then combining them should reduce fireflies

Multiple Importance Sampling

- Weighted combination of 2 strategies

$$\langle F^{N_1+N_2} \rangle = \frac{1}{N_1} \sum_{i=1}^{N_1} w_1(x_i) \frac{f(x_i)}{p_1(x_i)} + \frac{1}{N_2} \sum_{i=1}^{N_2} w_2(x_i) \frac{f(x_i)}{p_2(x_i)}$$

  - where:

$$w_1(x) + w_2(x) = 1$$

Multiple Importance Sampling

- Weighted combination of *M* strategies

$$\langle F^{\sum N_s} \rangle = \sum_{s=1}^{M} \frac{1}{N_s} \sum_{i=1}^{N_s} w_s(x_i) \frac{f(x_i)}{p_s(x_i)}$$

  - where:

$$\sum_{s=1}^{M} w_s(x) = 1$$

- How to choose the weights?

Multiple Importance Sampling

- Balance heuristic (provably good):

$$w_s(x) = \frac{N_s\, p_s(x)}{\sum_j N_j\, p_j(x)}$$

- Power heuristic (more aggressive, can be better):

$$w_s(x) = \frac{(N_s\, p_s(x))^\beta}{\sum_j (N_j\, p_j(x))^\beta}$$

- Other heuristics exist (e.g. cutoff heuristic, maximum heuristic,...)

Multiple Importance Sampling

- *Multi-sample* model

$$\langle F^{\sum N_s} \rangle = \sum_{s=1}^{M} \frac{1}{N_s} \sum_{i=1}^{N_s} w_s(x_i) \frac{f(x_i)}{p_s(x_i)}$$

- What if we want to draw just *one* sample?

Multiple Importance Sampling

- *One-sample* model:

$$\langle F^1 \rangle = w_s(x) \frac{f(x)}{q_s\, p_s(x)}$$

- where q_s is the probability of using strategy *s*

$$\sum_{s=1}^{N} q_s = 1$$

Interpreting the Balance Heuristic

- Balance heuristic for the one-sample model:

$$w_s(x) = \frac{q_s\, p_s(x)}{\sum_j q_j\, p_j(x)}$$

- Plugged into the one-sample model:

$$\langle F^1 \rangle = w_s(x) \frac{f(x)}{q_s\, p_s(x)} = \frac{q_s\, p_s(x)}{\sum_j q_j\, p_j(x)} \frac{f(x)}{q_s\, p_s(x)} = \frac{f(x)}{\sum_j q_j\, p_j(x)}$$

- Balance heuristic samples from average PDF

Why Does it Work?

- Using a single strategy:

$$\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

large value

small value

- Combining multiple strategies using balance h.:

$$\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{\sum_j q_j p_j(x_i)}$$

large value

relatively large value

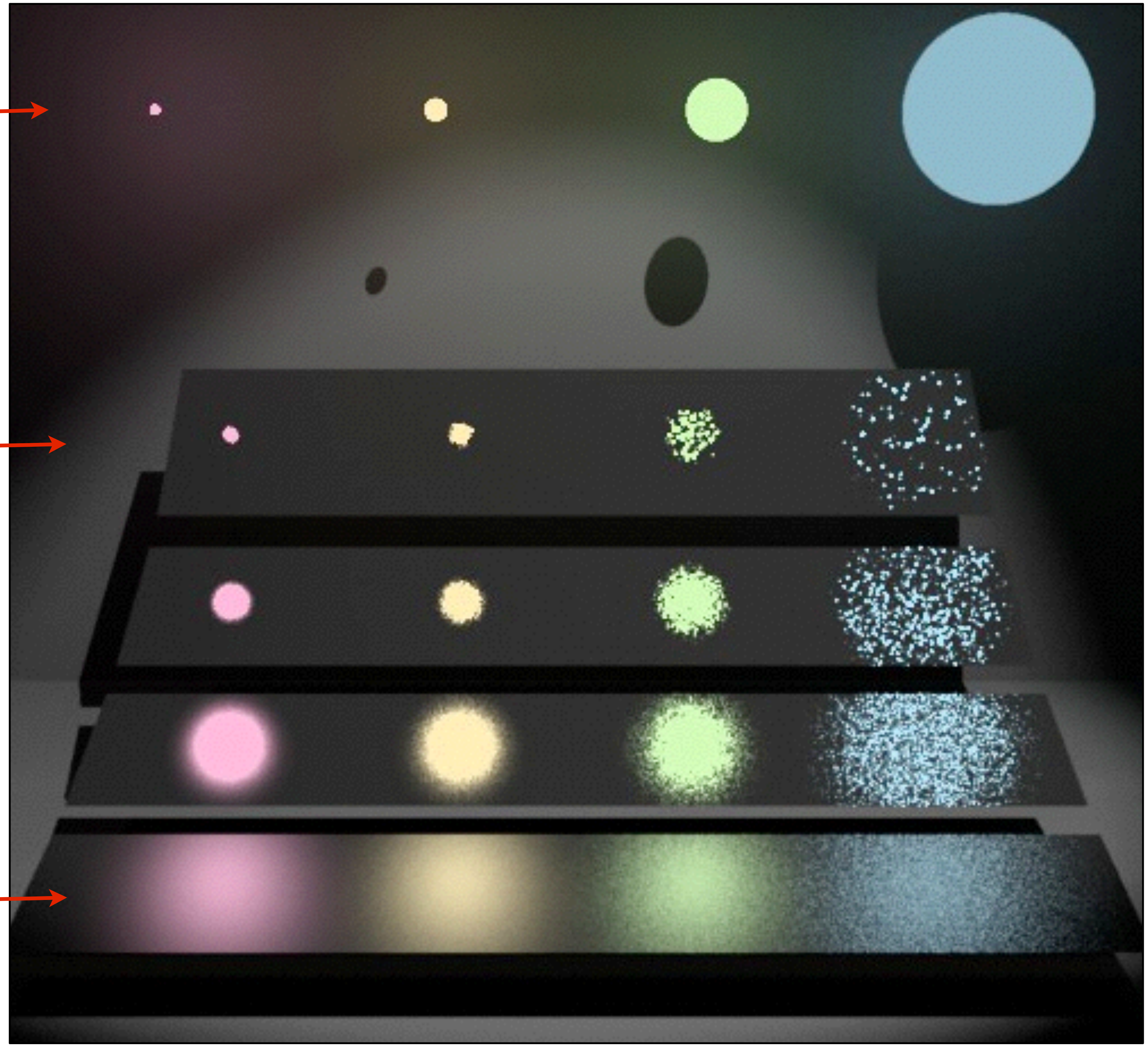
(as long as at least one PDF is large)

Sampling the Light

sphere lights

highly glossy surface

moderately glossy surface



Eric Veach and Leonidas J. Guibas 1995.

Sampling the BRDF

sphere lights

highly glossy surface

moderately glossy surface

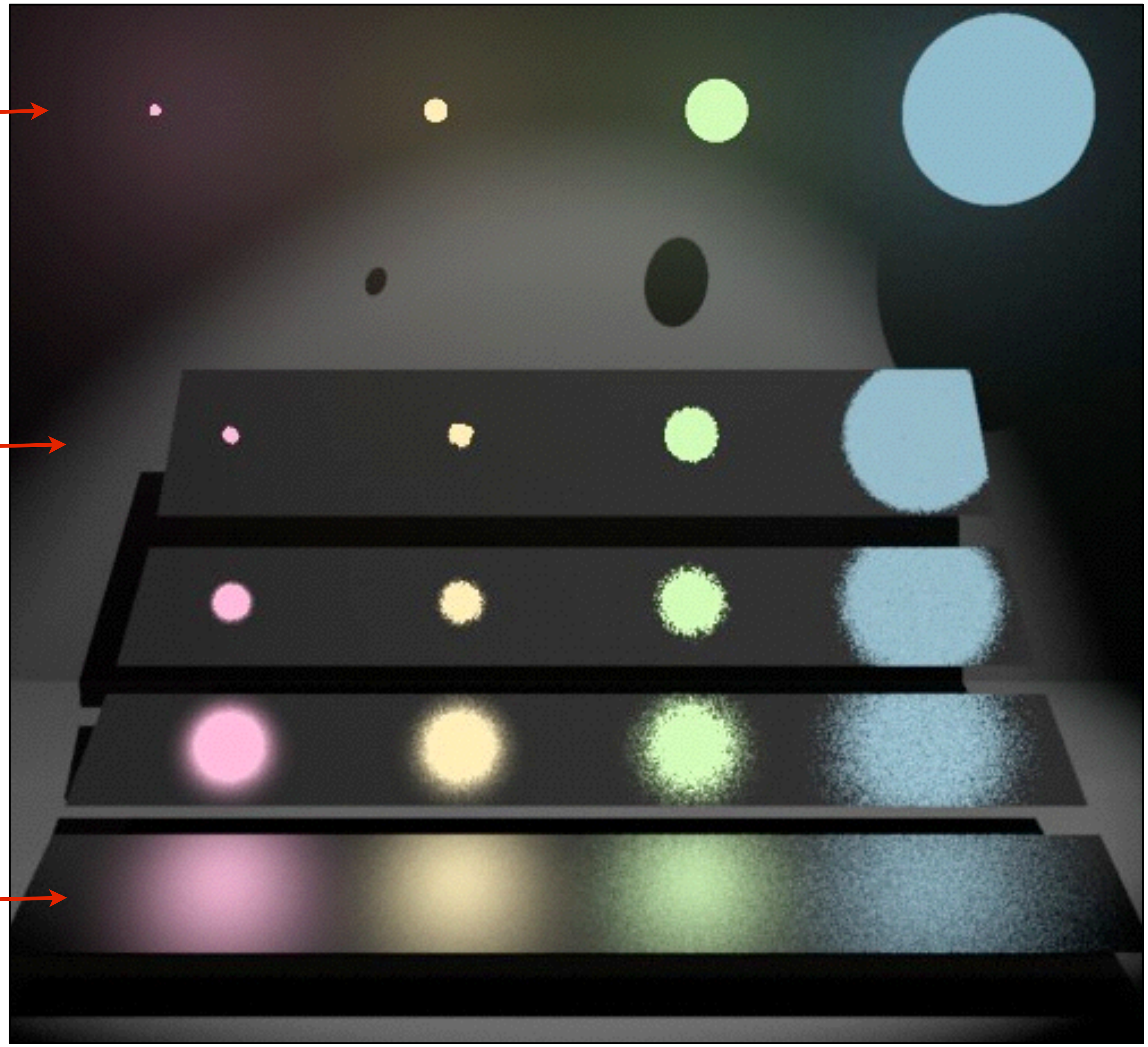
Eric Veach and Leonidas J. Guibas 1995.

Multiple Importance Sampling

sphere lights

highly glossy surface

moderately glossy surface



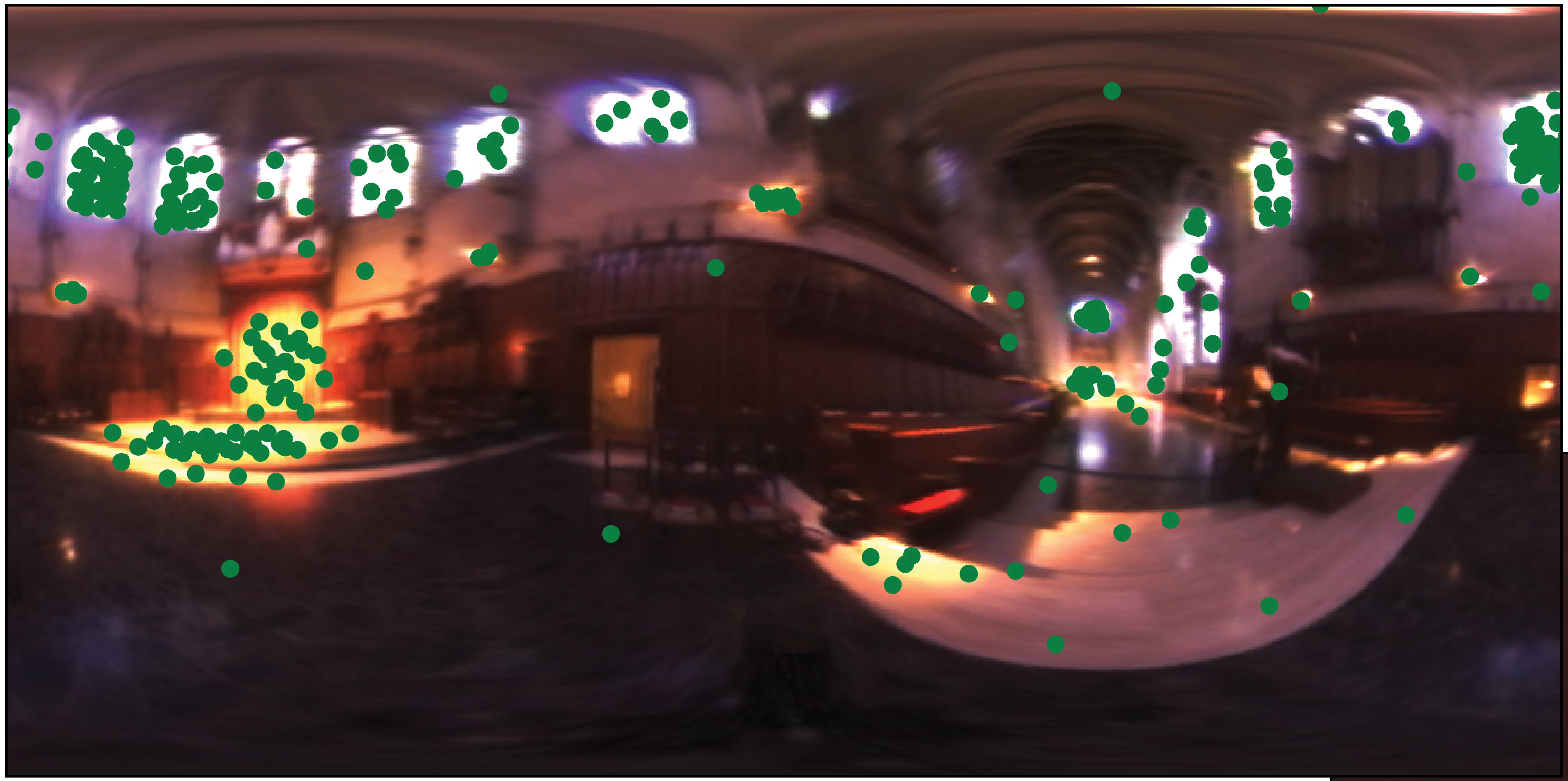
Eric Veach and Leonidas J. Guibas 1995.

Multiple Importance Sampling

- See PBRe2 14.4.1 for more details

Eric Veach and Leonidas J. Guibas 1995.

Next Lecture: more MC & direct illum.



Eric Veach and Leonidas J. Guibas 1995.