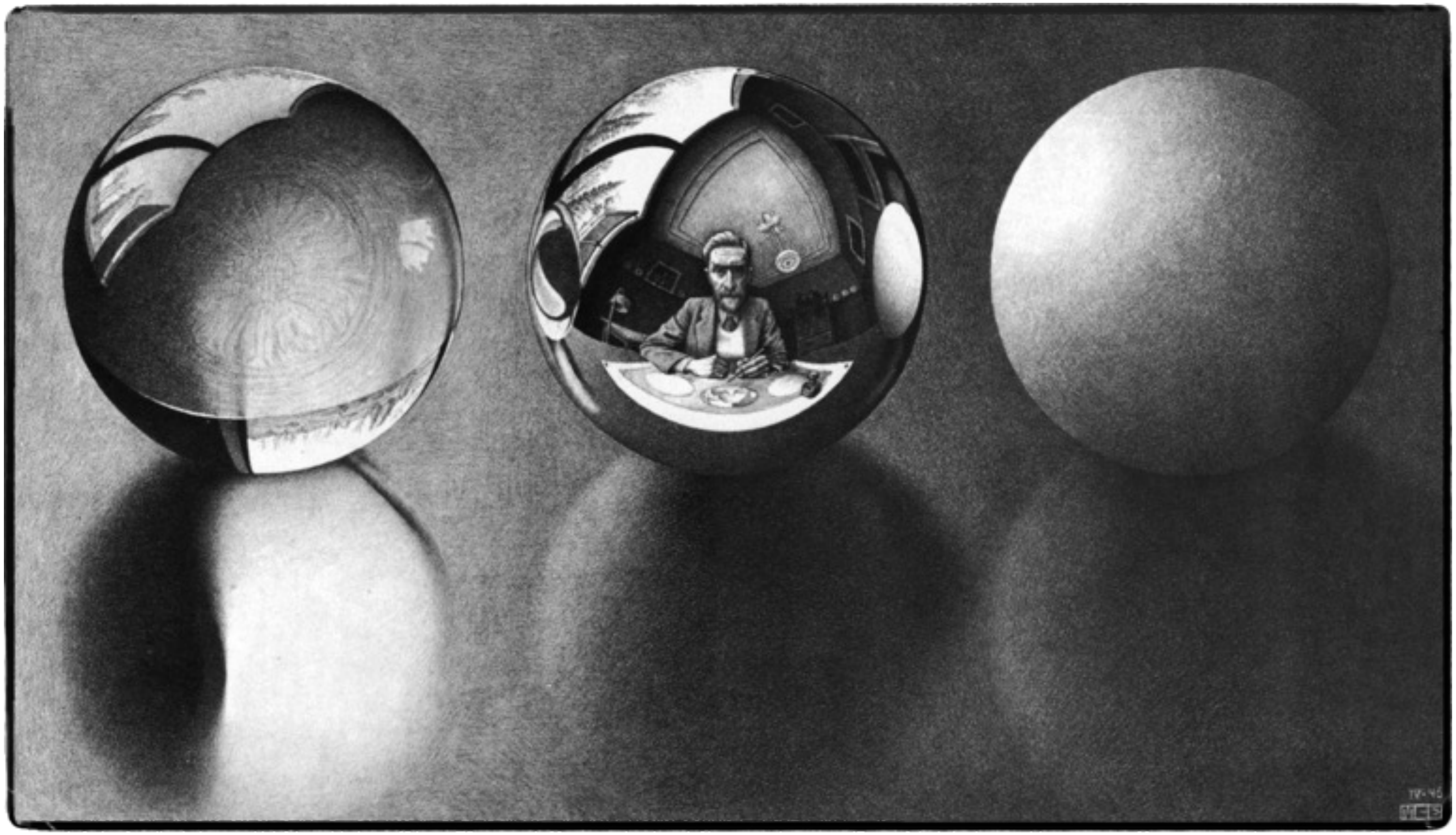
CS 87/187, Spring 2016

# RENDERING ALGORITHMS

## Appearance Modeling: Idealized BRDFs

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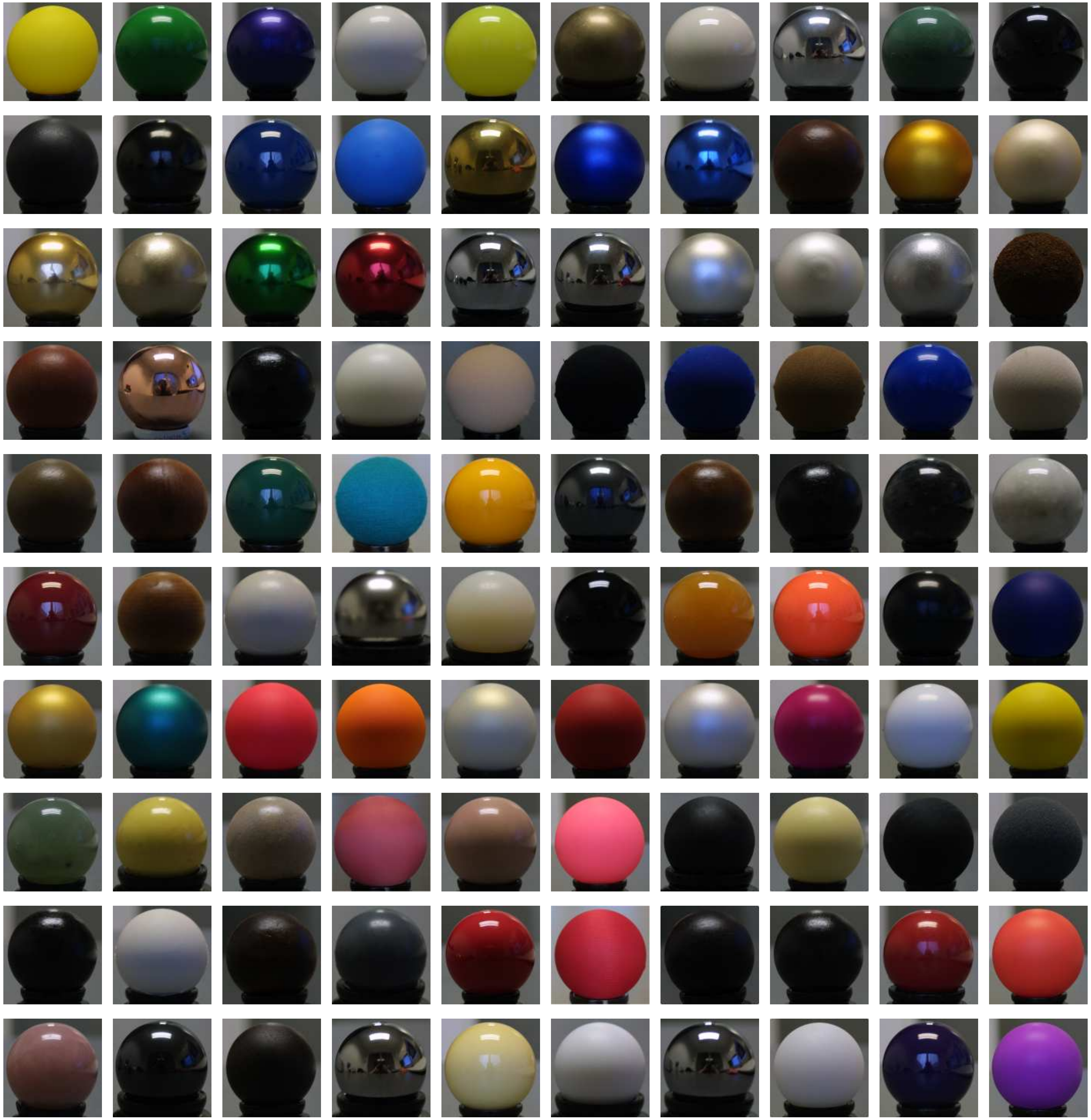
(with slide improvements from Jan Novák)

Dartmouth

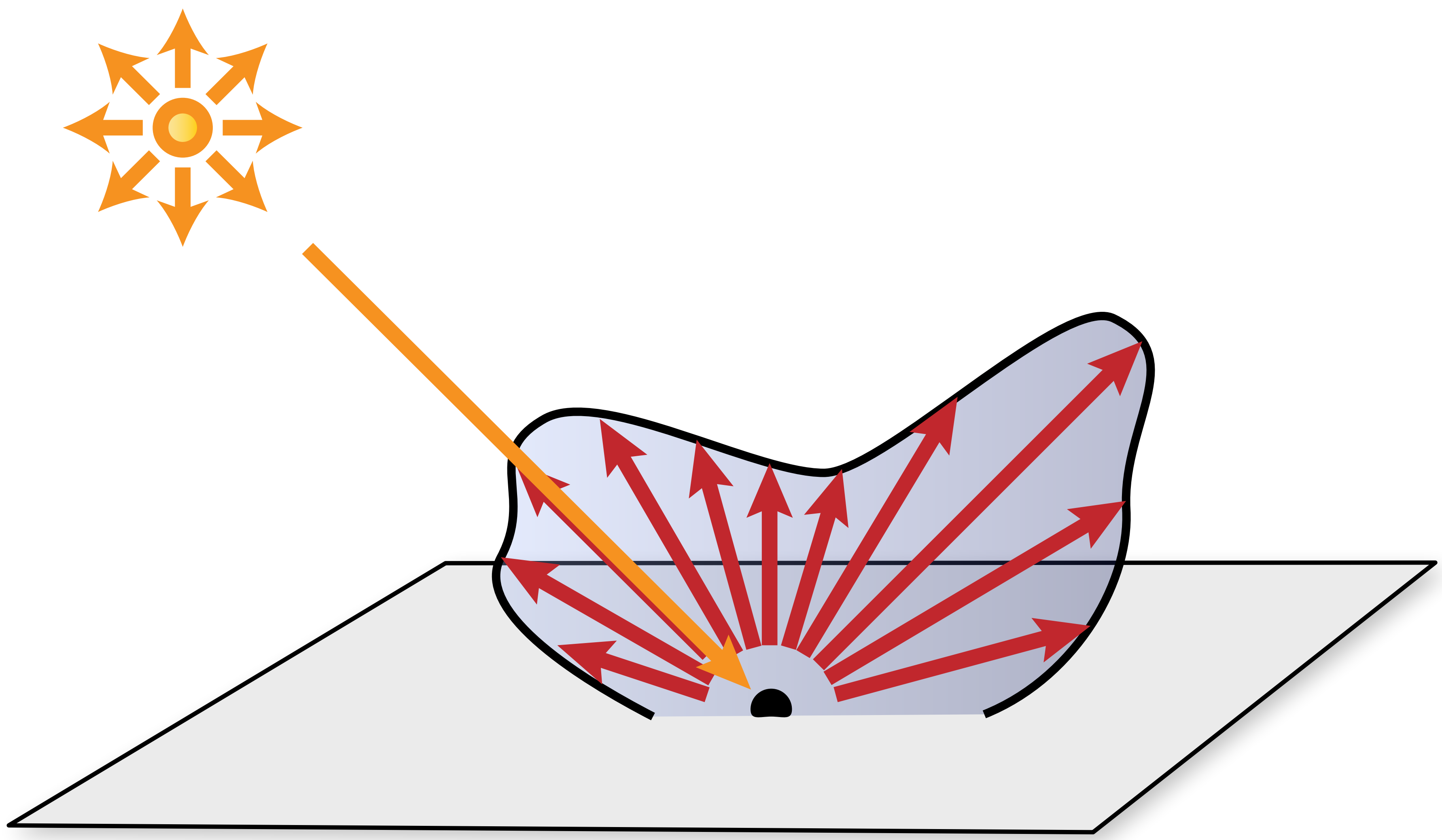
# Today's Menu

- The BRDF: how can we characterize how a material interacts with light?
- The reflection equation
- Ideal diffuse materials
- Ideal specular materials

# Light-Material Interactions



# Light-Material Interactions



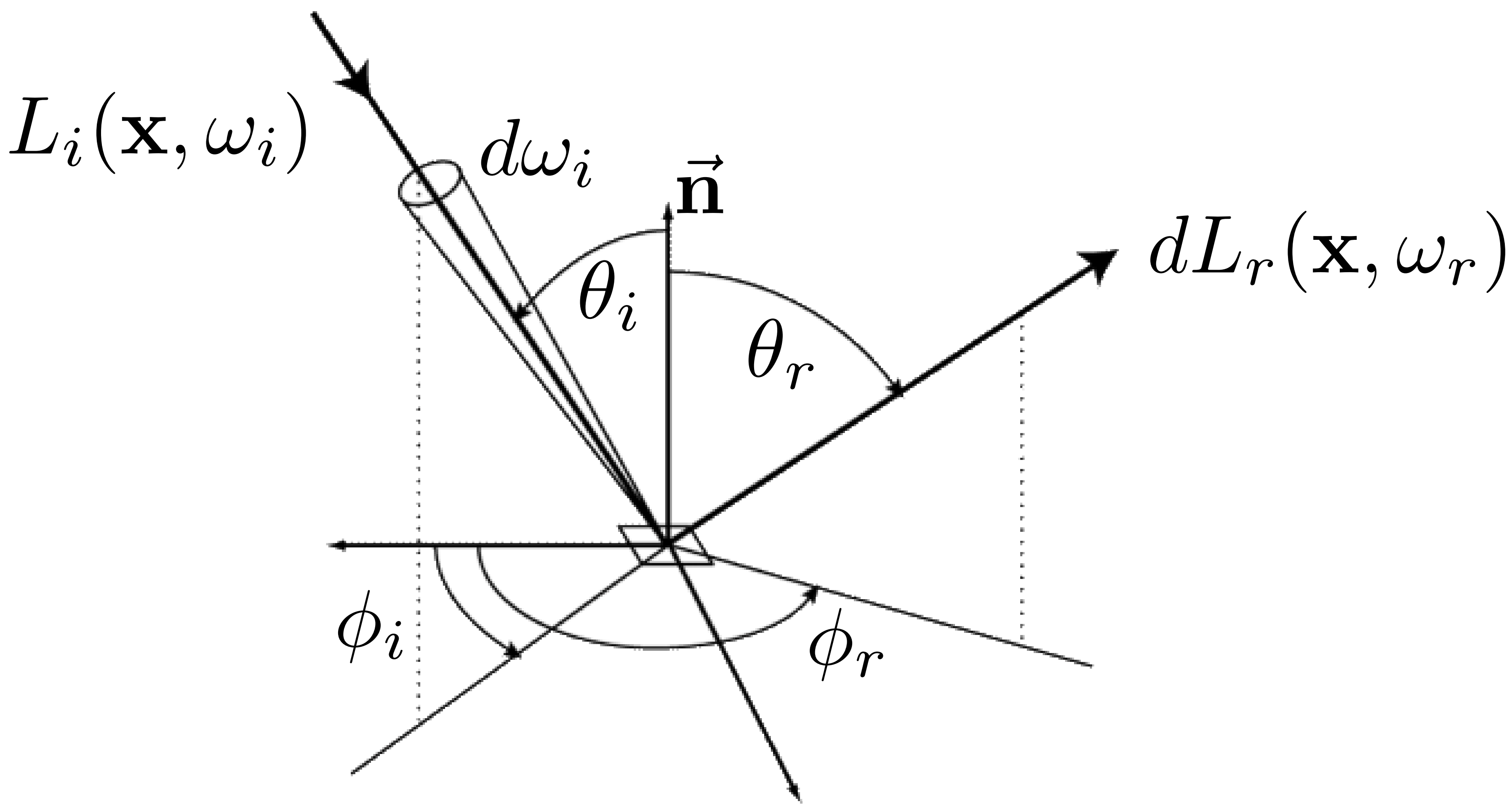
# BRDF

- **B**idirectional **R**eflectance **D**istribution **F**unction
  - ratio of differential reflected radiance
    to differential incident irradiance

# BRDF

- **B**idirectional **R**eflectance **D**istribution **F**unction

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{dE_i(\mathbf{x}, \vec{\omega}_i)} = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i} \quad [1/sr]$$



# BRDF

- **B**idirectional **R**eflectance **D**istribution **F**unction

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i}$$

- From this we can derive the **Reflection Equation**:

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{d\vec{\omega}_i}$$

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i = L_r(\mathbf{x}, \vec{\omega}_r)$$

# Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

# Reflection Equation

- The Reflection Equation describes a *local illumination* model
  - reflected radiance due to incident illumination from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

# BRDFs Properties

- Real/physically-plausible BRDFs obey:
  - Helmholtz reciprocity

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$
$$f_r(\mathbf{x}, \vec{\omega}_r \leftrightarrow \vec{\omega}_i)$$

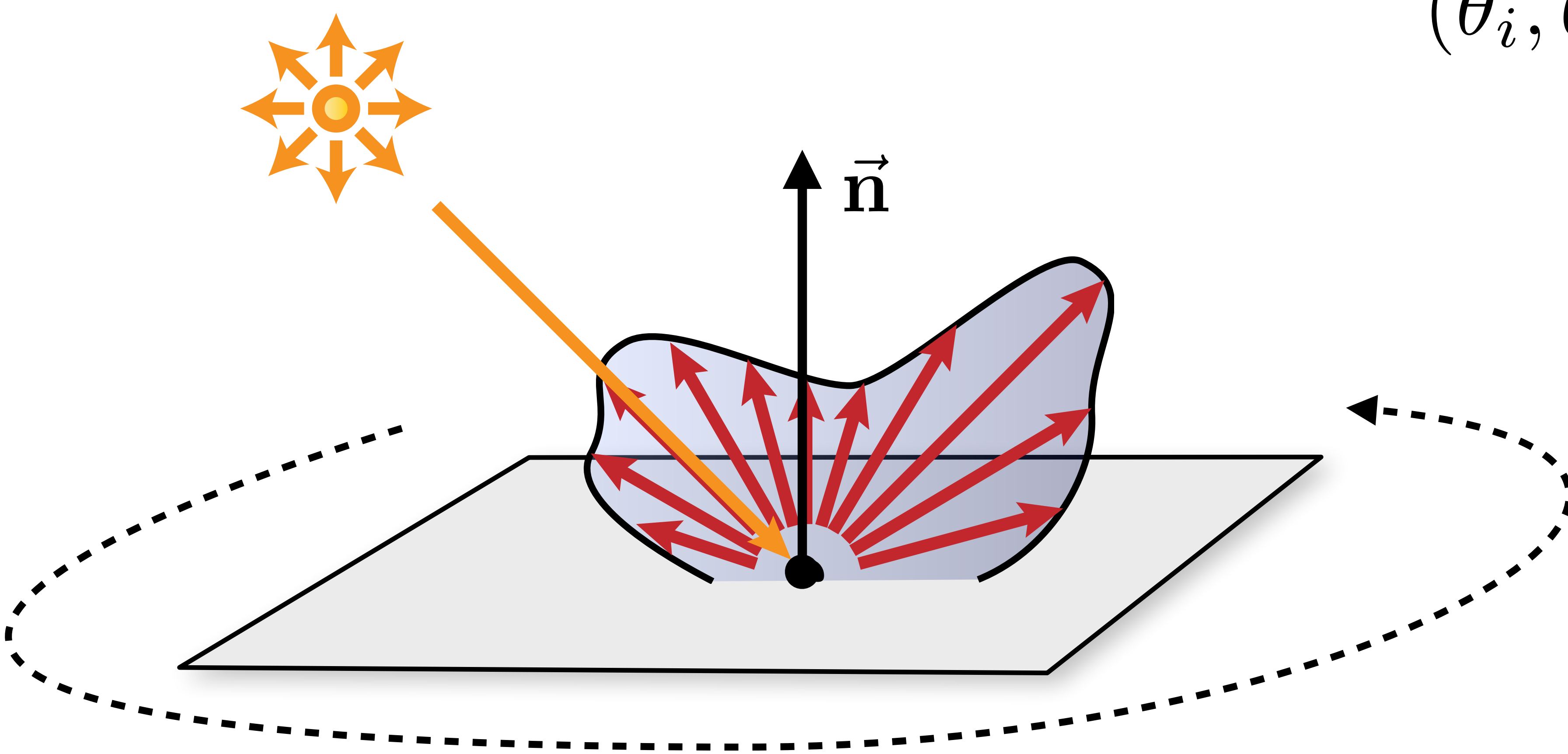
  - Energy conservation

$$\int_{H^2} f_r(\vec{\omega}_i, \vec{\omega}_r) \cos\theta_i \, d\omega_i \leq 1, \quad \forall\, \vec{\omega}_r$$

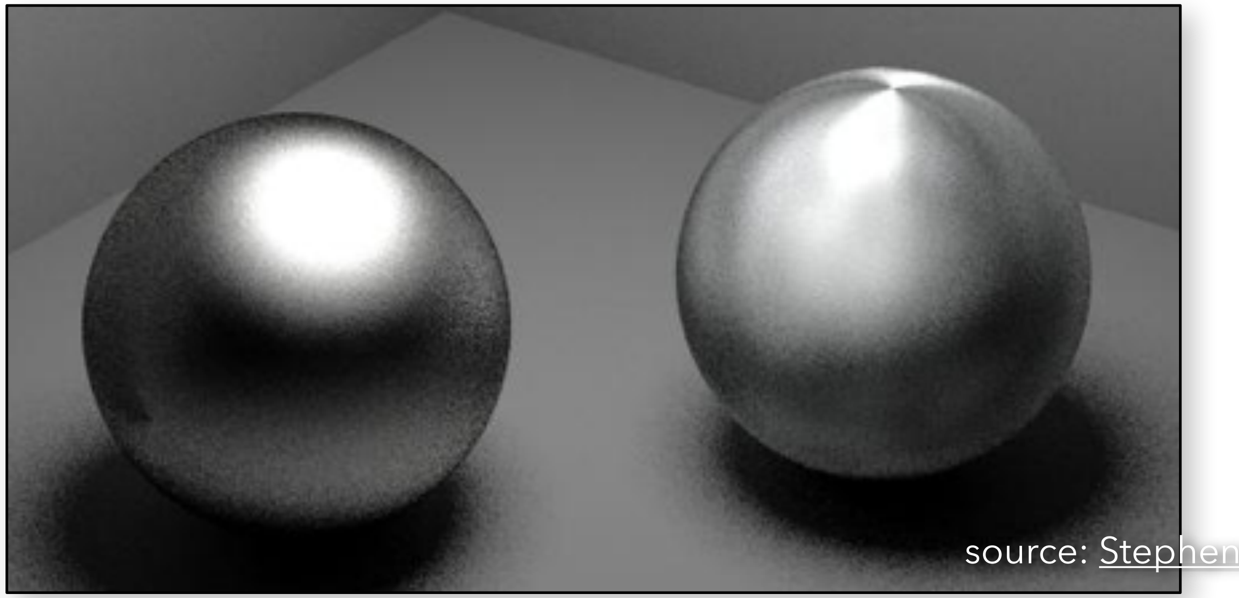
# BRDFs Properties

- If the BRDF is unchanged as the material is rotated around the normal, then it is ***isotropic***, otherwise it is ***anisotropic***.
- Isotropic BRDFs are functions of just 3 variables

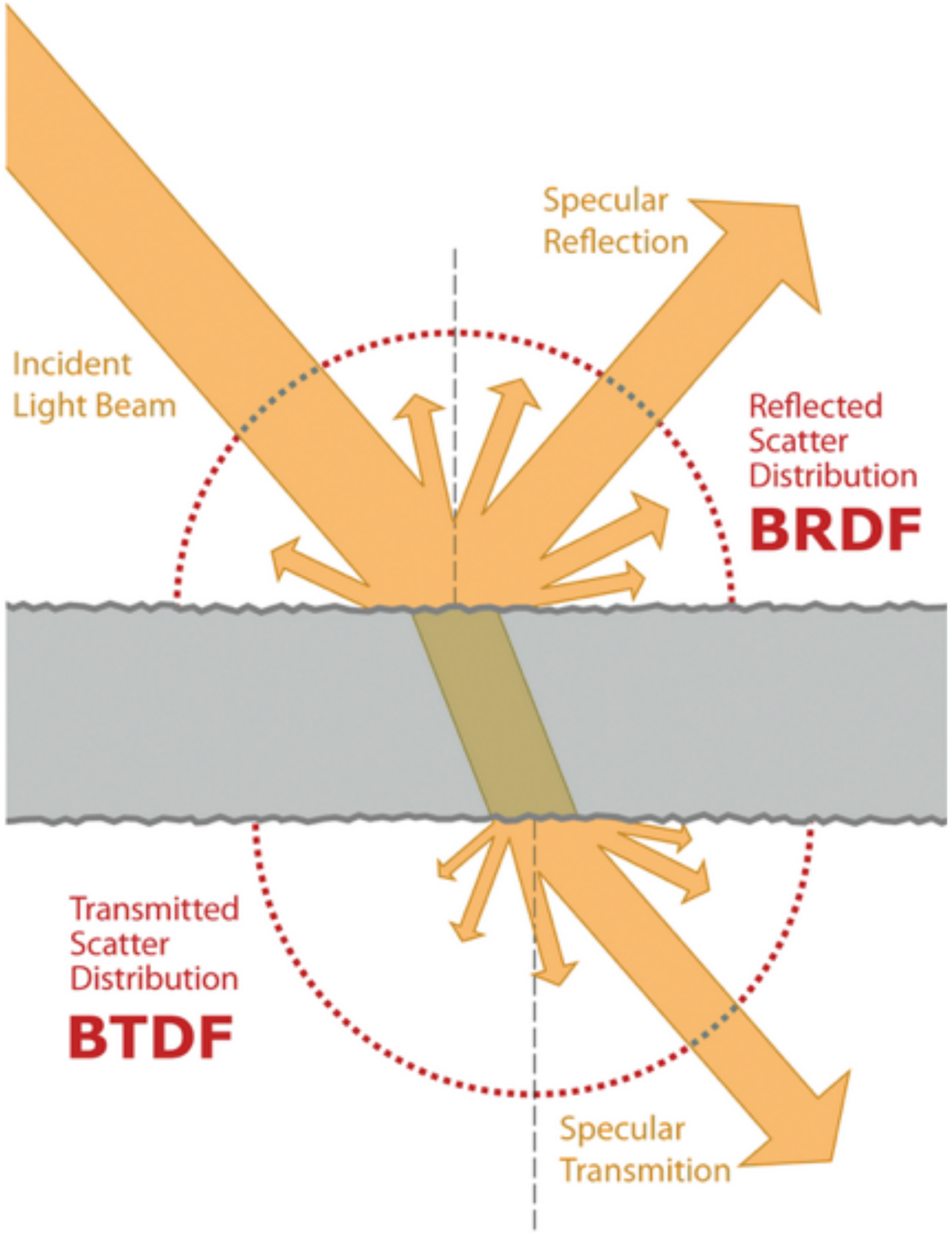
$$(\theta_i, \theta_r, \Delta\phi)$$



# Isotropic vs Anisotropic Reflection

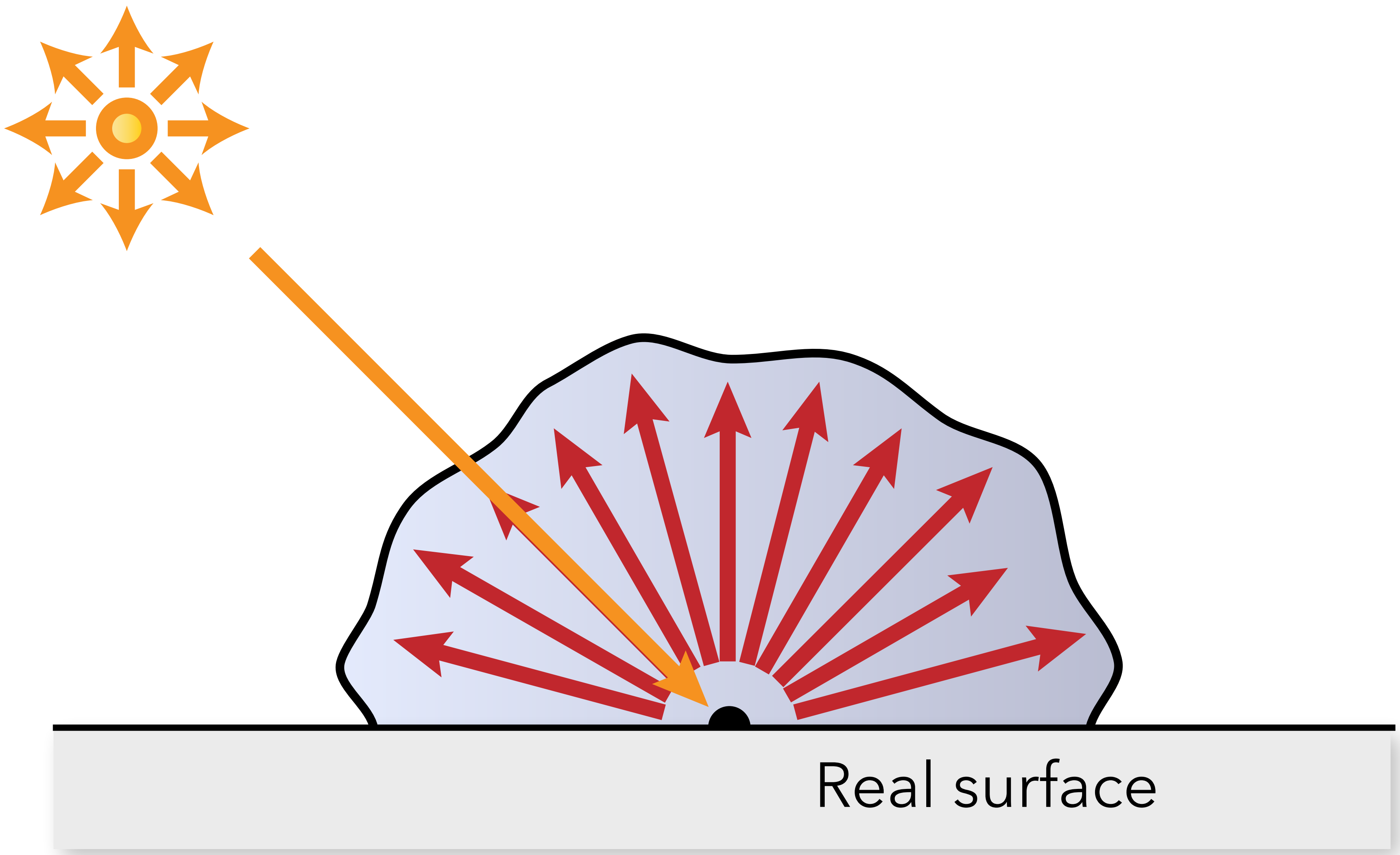




# Reflection vs. Refraction

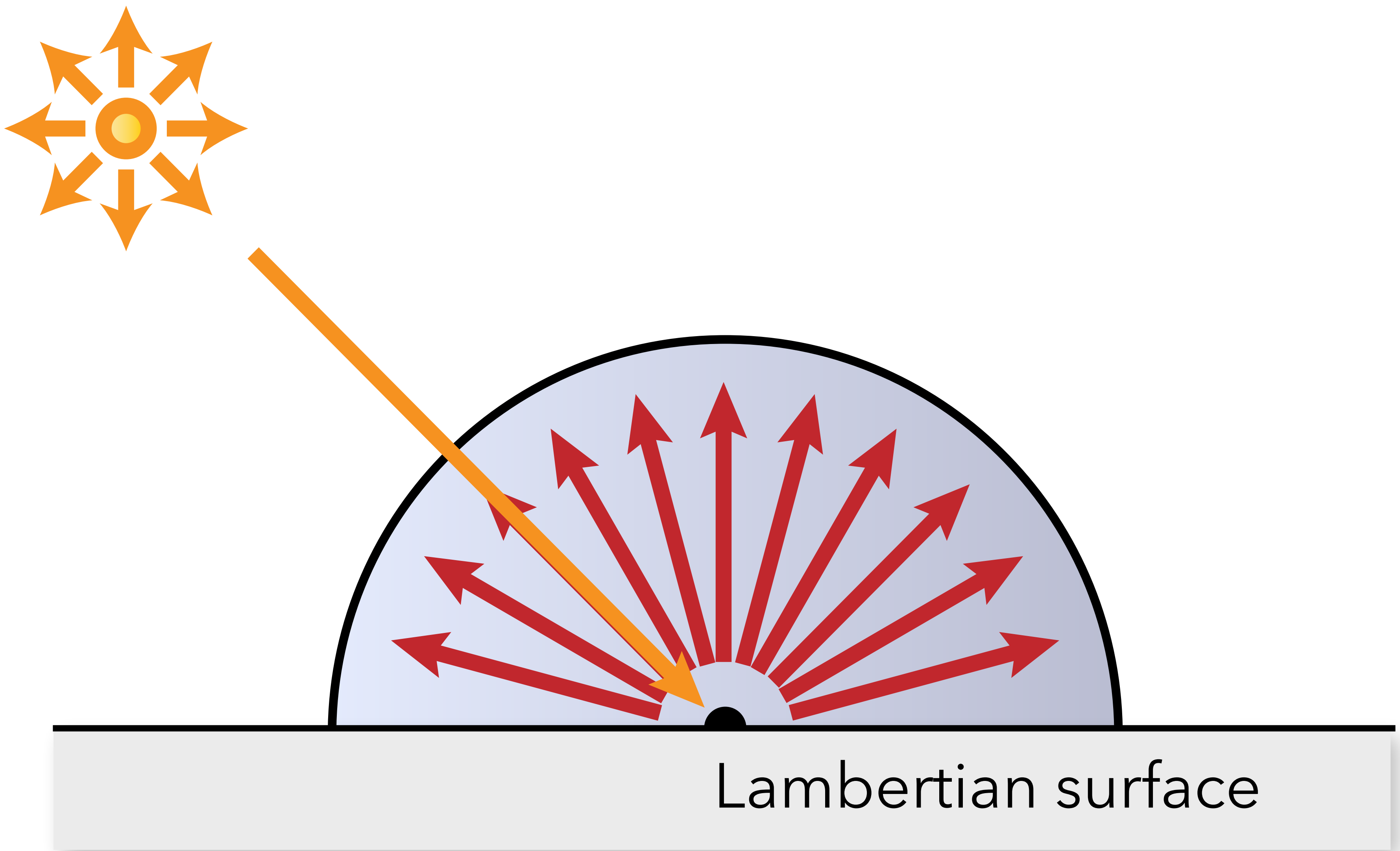


# Idealized BRDF Models (diffuse, perfectly specular)

# Diffuse Reflection



# Lambertian Reflection




Also called ideal diffuse reflection

# Ideal Diffuse BRDF

- For Lambertian reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r \, E(\mathbf{x})$$

- If *all* incoming light is reflected:

$$E(\mathbf{x}) = B(\mathbf{x})$$

$$E(\mathbf{x}) = \int_{H^2} L_r(\mathbf{x}) \cos\theta \, d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \int_{H^2} \cos\theta \, d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \, \pi$$

$$f_r = \frac{1}{\pi}$$

# Diffuse BRDF

- For Lambertian reflection, the BRDF is a constant:

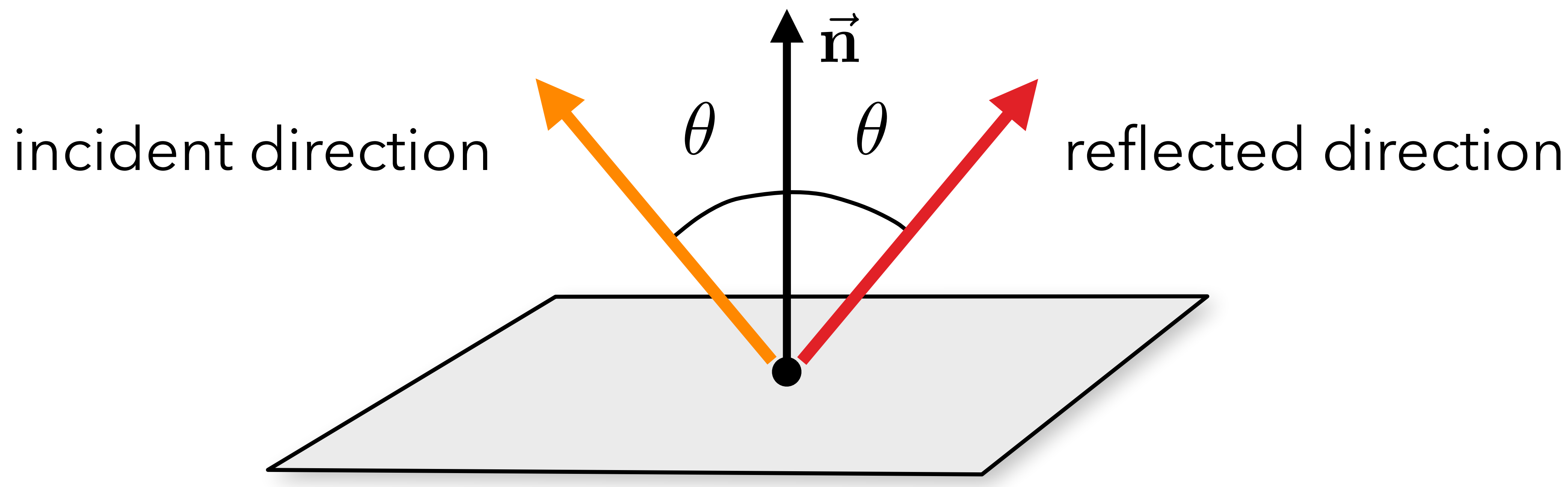
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- $\rho$: Diffuse reflectance (albedo) [0..1)

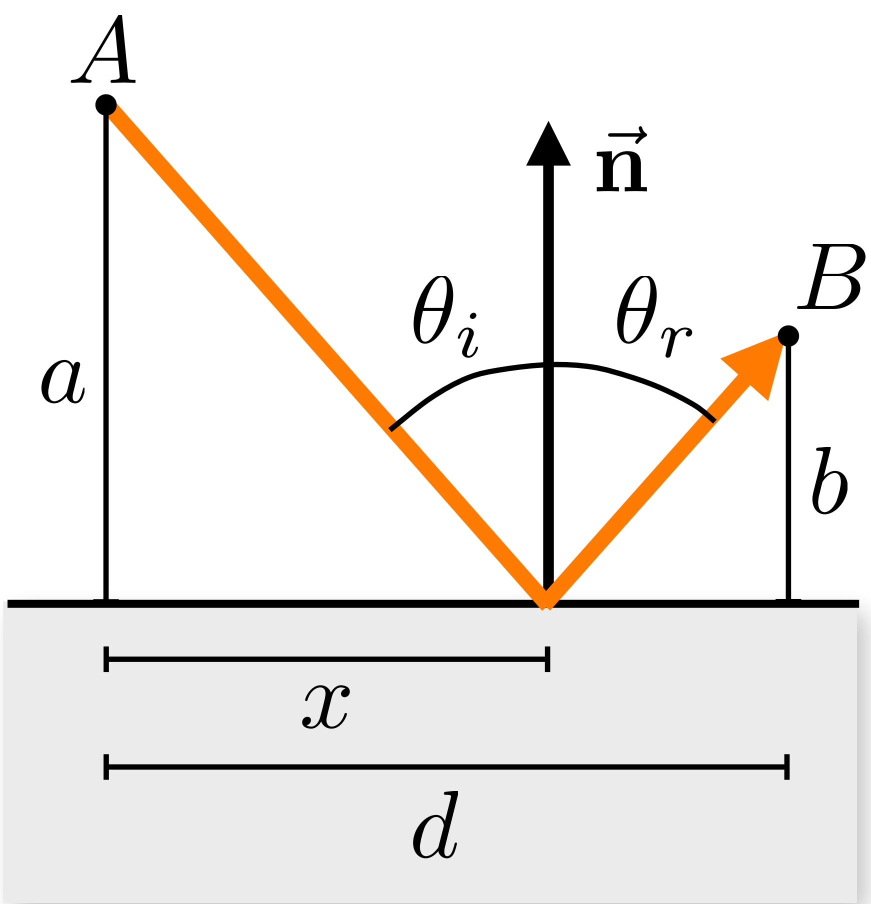
# Lambertian Reflection

# Ideal Specular Reflection



# Fermat's Principle for Reflection

- Light follows the path of least time.




Minimize the path length between A and B:

Path length:

$$l = \sqrt{a^2 + x^2} + \sqrt{b^2 + (d - x)^2}$$

Set derivative to zero and express angles:

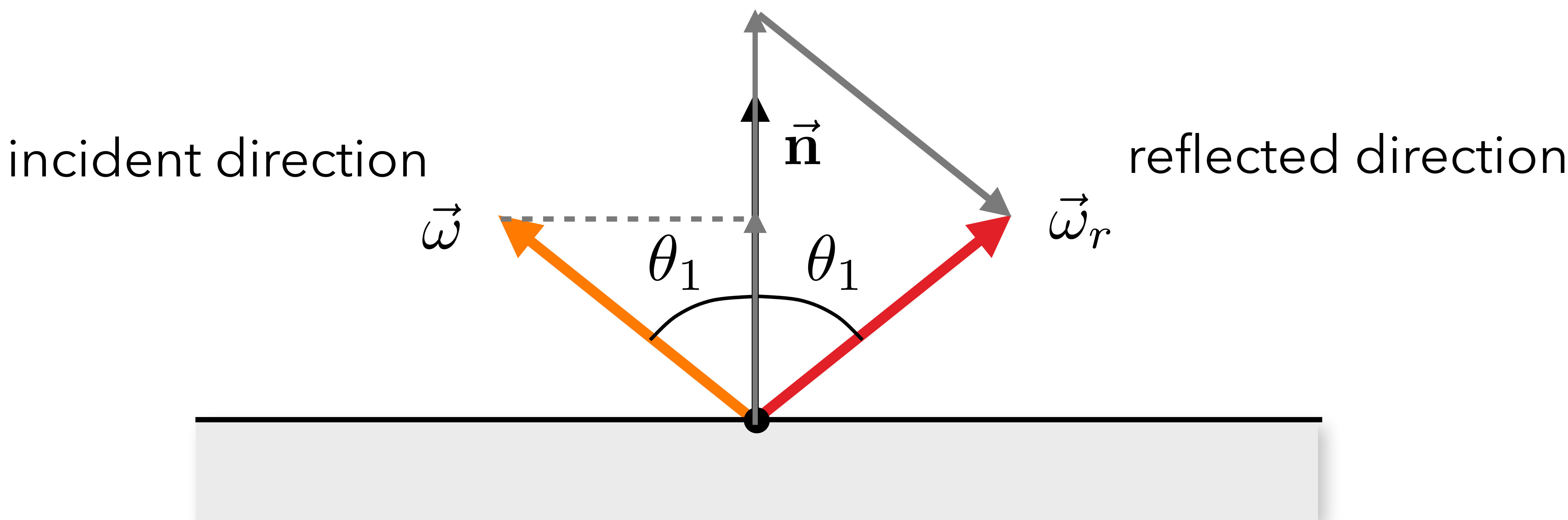
$$\frac{dl}{dx} = \frac{1}{2}\frac{2x}{\sqrt{a^2 + x^2}} + \frac{1}{2}\frac{2(d-x)(-1)}{\sqrt{b^2 + (d-x)^2}} = 0$$

$$\frac{x}{\sqrt{a^2 + x^2}} = \frac{d - x}{\sqrt{b^2 + (d-x)^2}}$$

$$\sin\theta_i = \sin\theta_r$$

$$\theta_i = \theta_r$$

# Reflected Direction

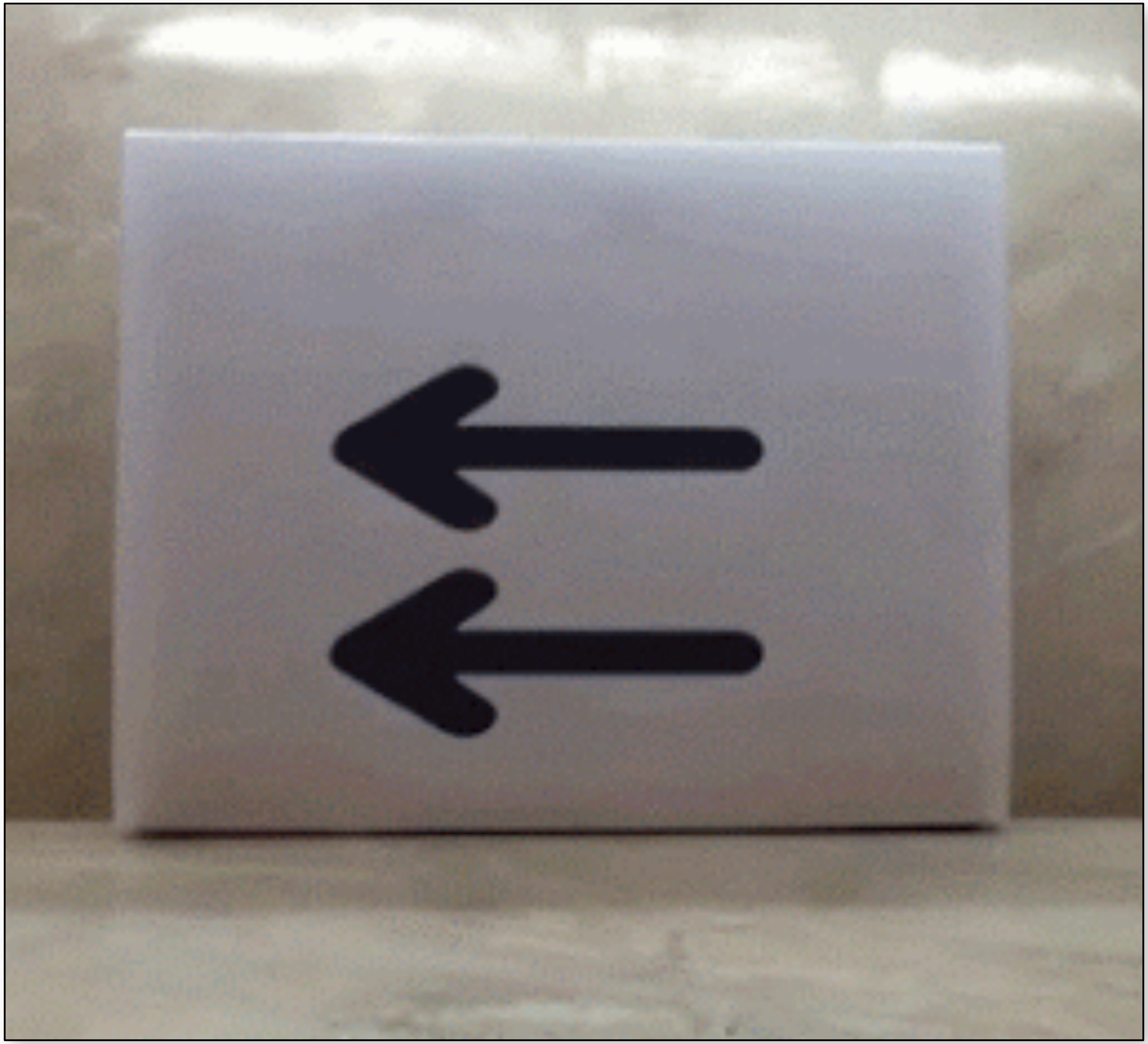



$$\vec{\omega}_r = 2\vec{\mathbf{n}}\cos\theta - \vec{\omega} = 2(\vec{\mathbf{n}}\cdot\vec{\omega})\vec{\mathbf{n}} - \vec{\omega}$$

# Ideal Specular Reflection

# Refraction

# Refraction



# Index of Refraction

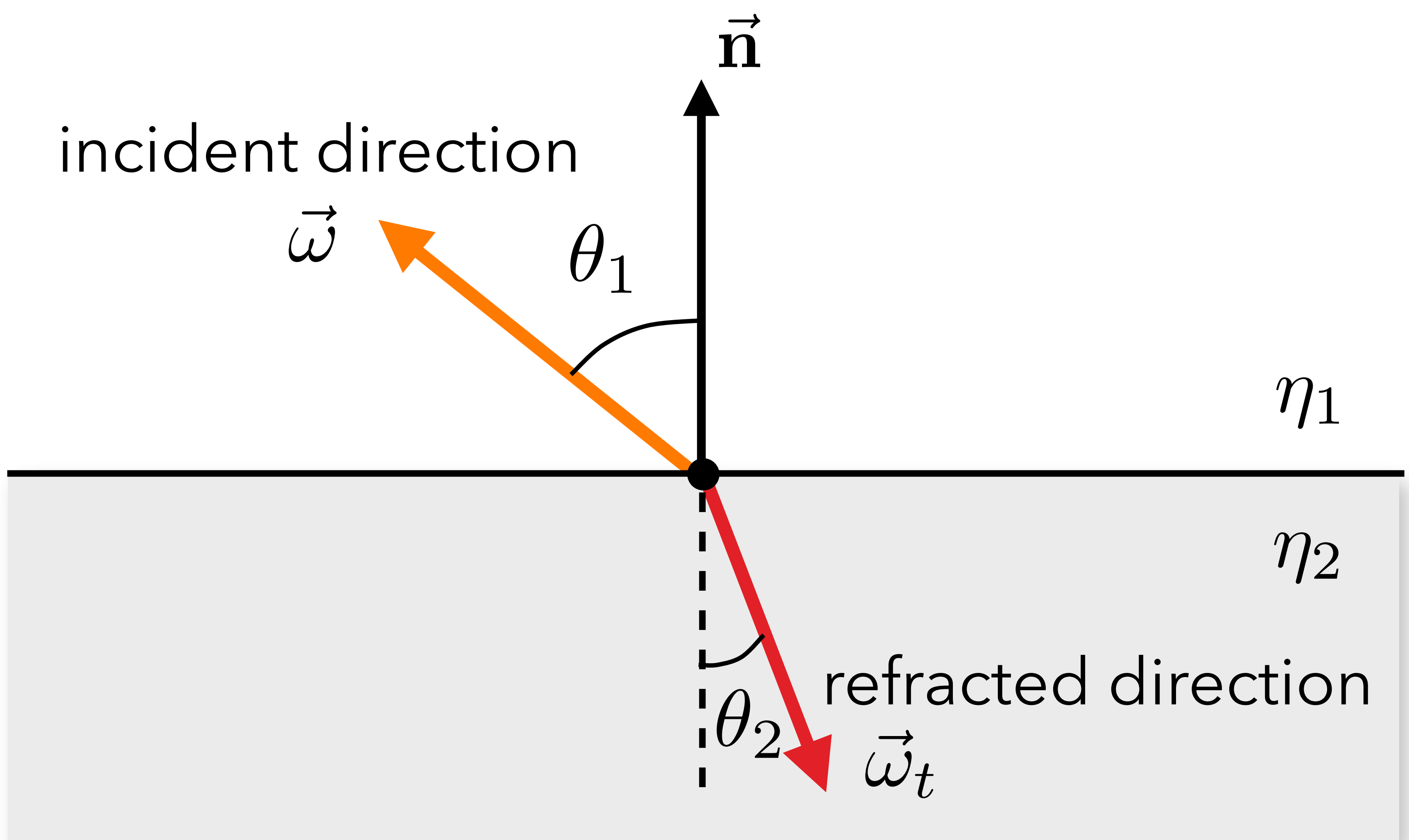
- speed of light in vacuum / speed of light in medium

| Some values of $\eta$ | |
|---|---|
| Vacuum | 1 |
| Air at STP | 1.00029 |
| Ice | 1.31 |
| Water | 1.33 |
| Crown glass | 1.52 - 1.65 |
| Diamond | 2.417 |

- These are actually wavelength dependent!

# Ideal Specular Refraction

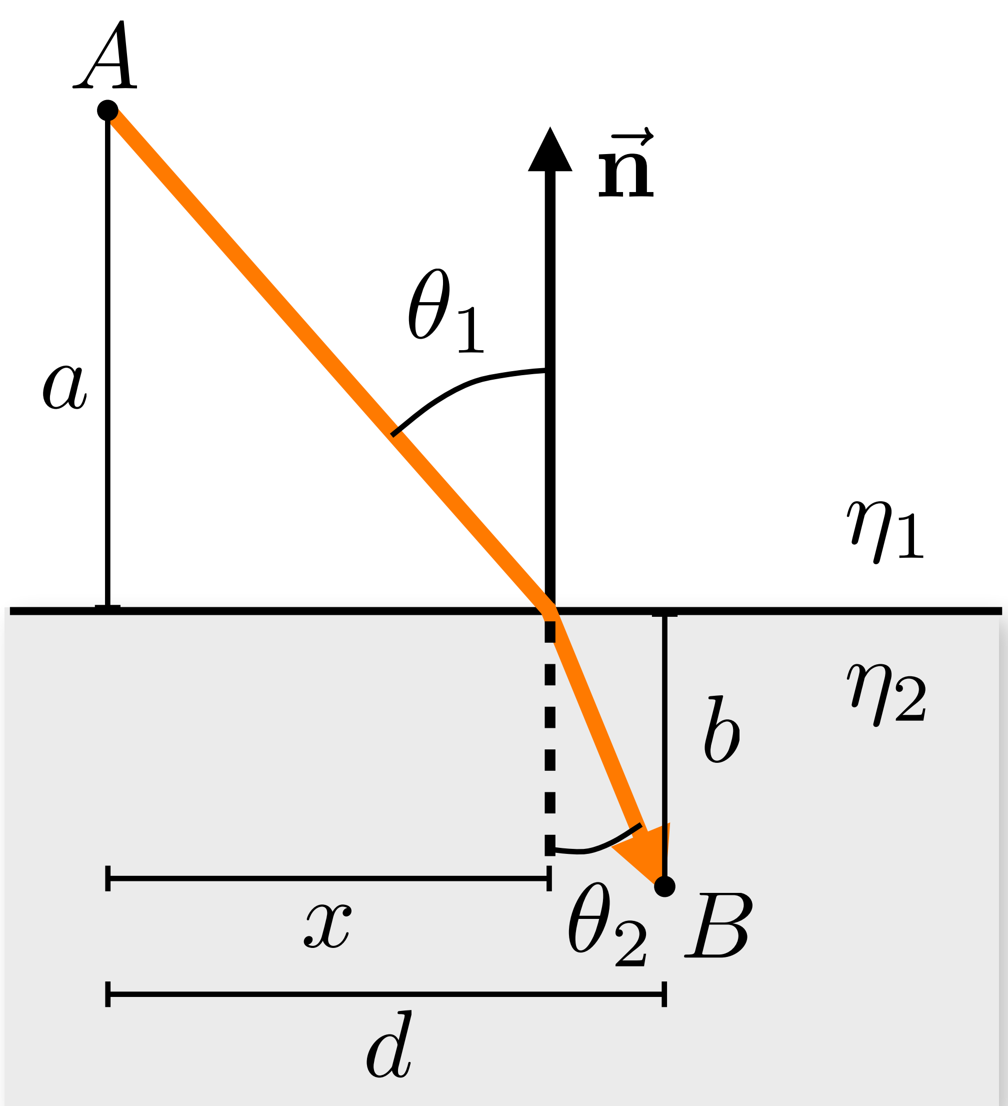
- Snell's law




$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

# Fermat's Principle for Refraction

- Light follows the path of least time.




Minimize the travel time between A and B:

Travel time:

$$t = \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{\sqrt{b^2 + (d-x)^2}}{v_2}$$

Set derivative to zero and express angles:

$$\frac{dt}{dx} = \frac{x}{v_1\sqrt{a^2+x^2}} - \frac{d-x}{v_2\sqrt{b^2+(d-x)^2}} = 0$$

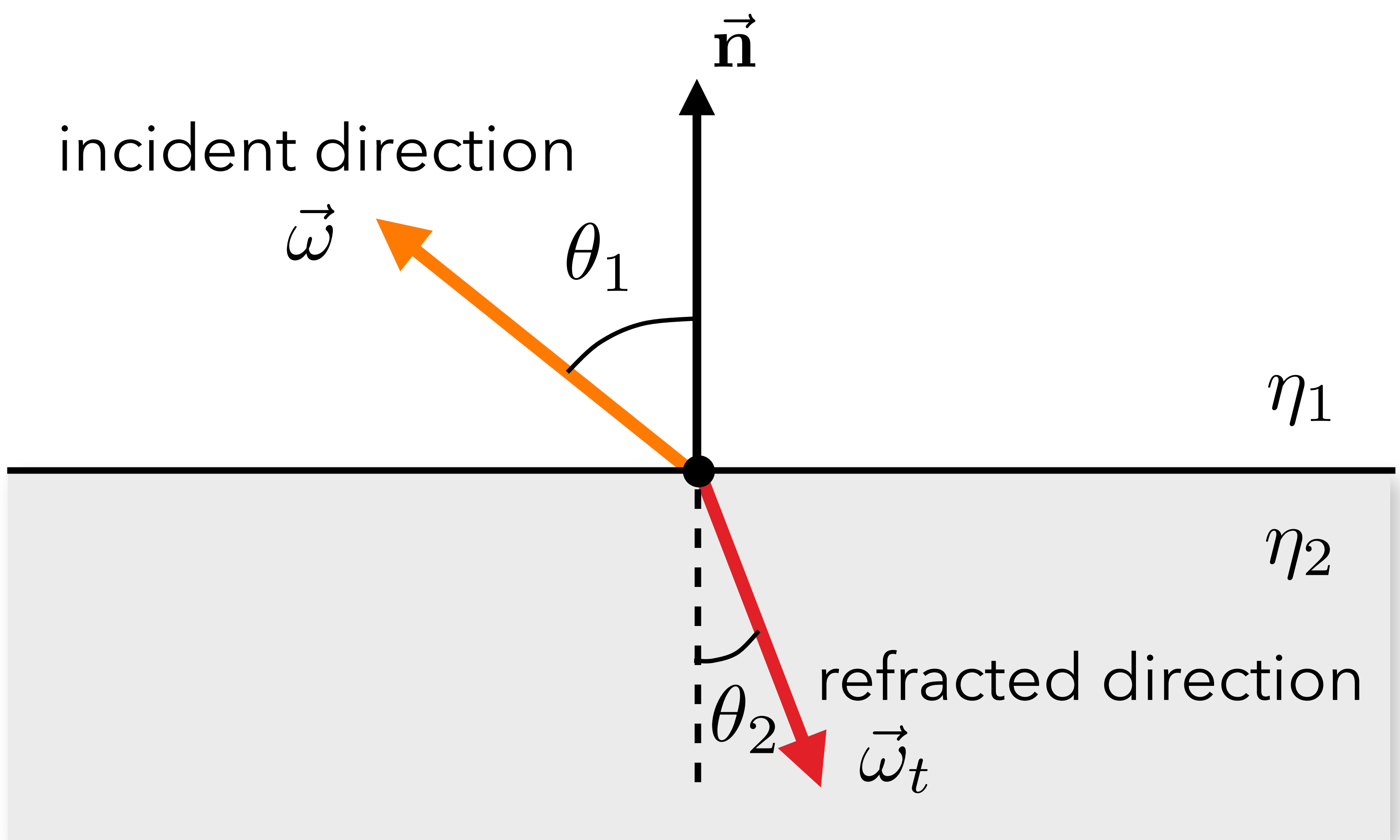
$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

$$\frac{\eta_1}{\eta_2} = \frac{\sin\theta_2}{\sin\theta_1}$$

Snell's law

# Ideal Specular Refraction

- Snell's law

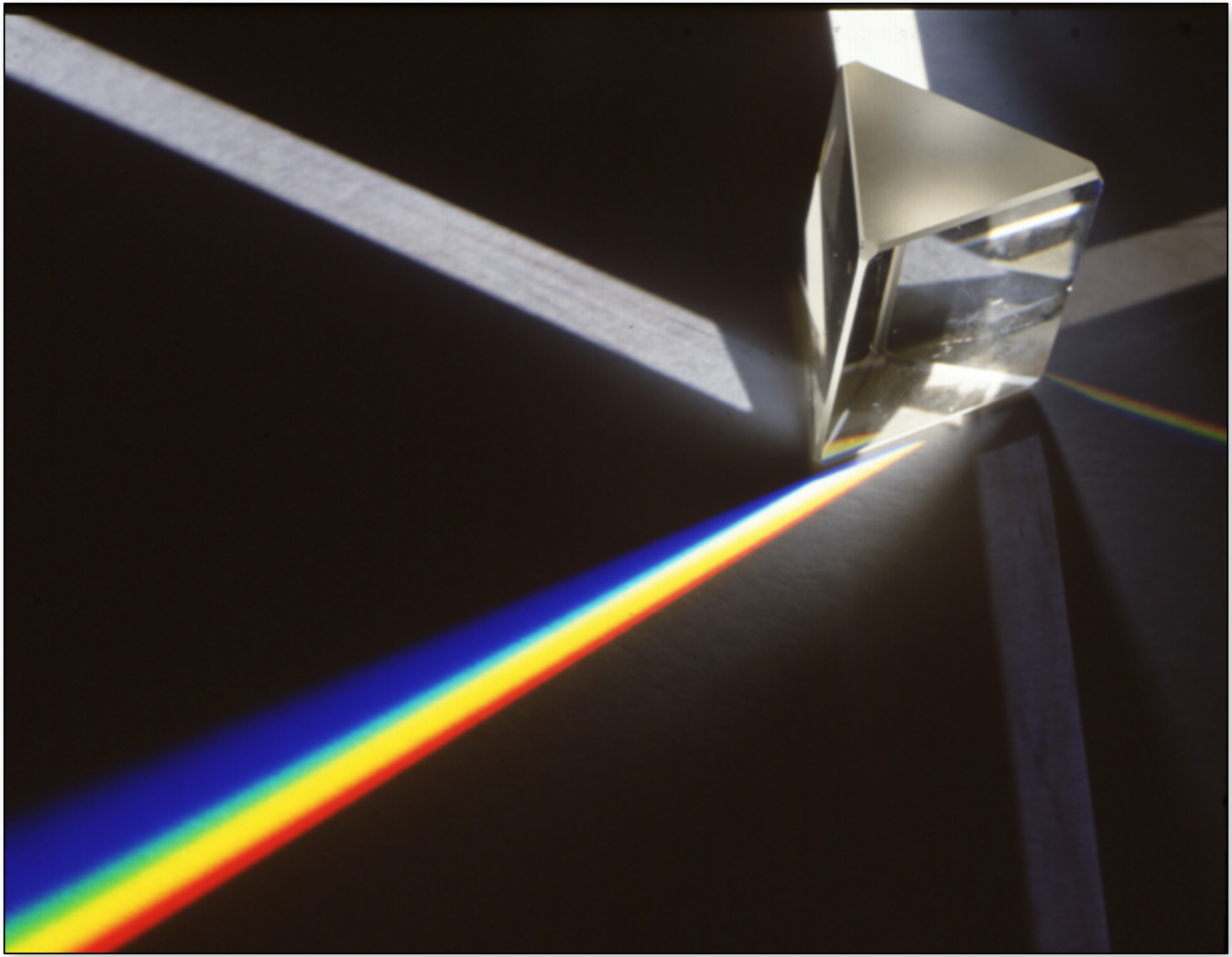



$$\vec{\omega}_t = -\frac{\eta_1}{\eta_2}\left(\vec{\omega} - (\vec{\omega}\cdot\vec{\mathbf{n}})\,\vec{\mathbf{n}}\right) - \vec{\mathbf{n}}\sqrt{1 - \left(\frac{\eta_1}{\eta_2}\right)^2\left(1 - (\vec{\omega}\cdot\vec{\mathbf{n}})^2\right)}$$

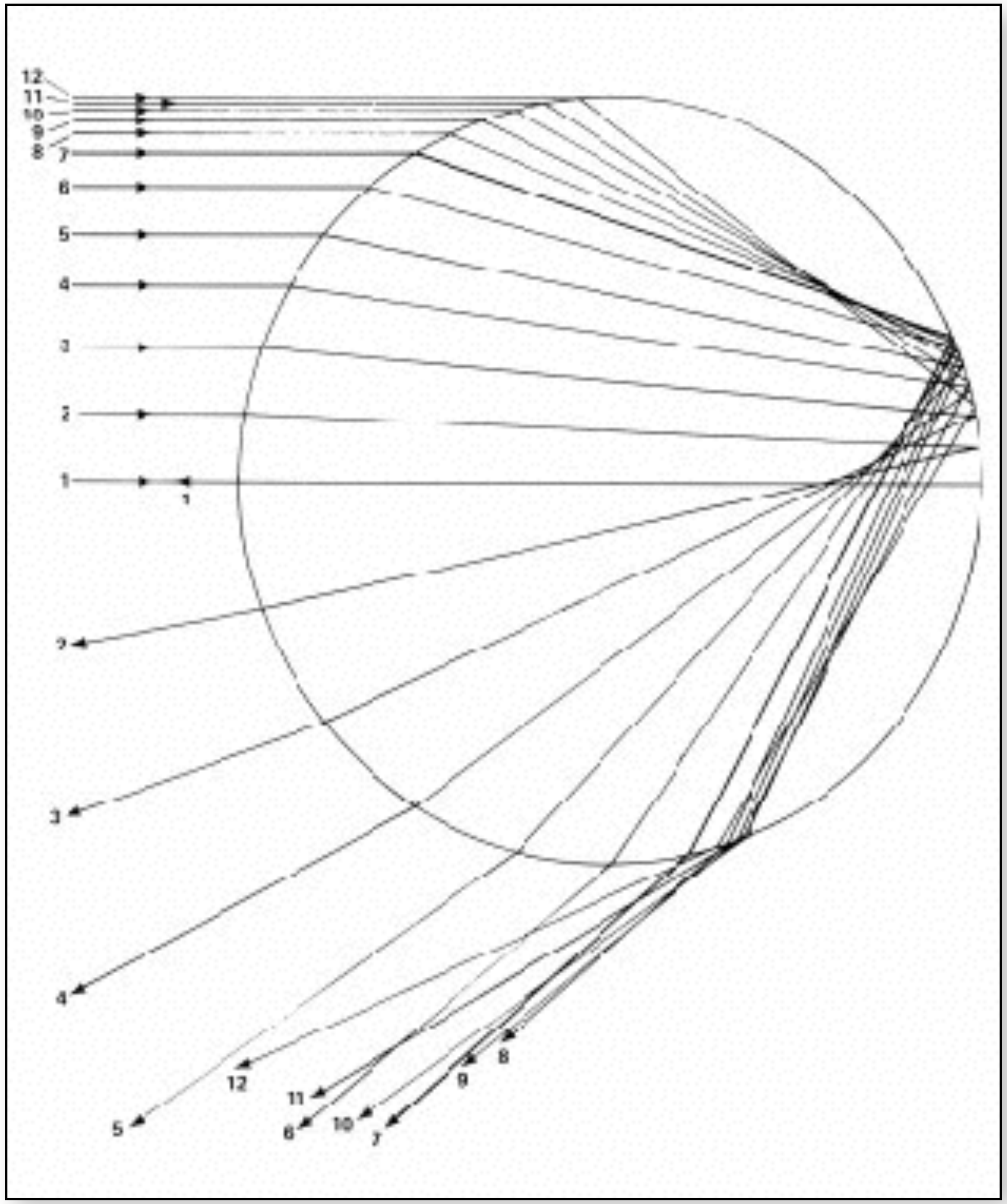
# Index of Refraction

- The index of refraction *depends on the wavelength* and the *polarization* state of the light.
- This dependency gives rise to a number of important phenomena.

# Dispersion Examples



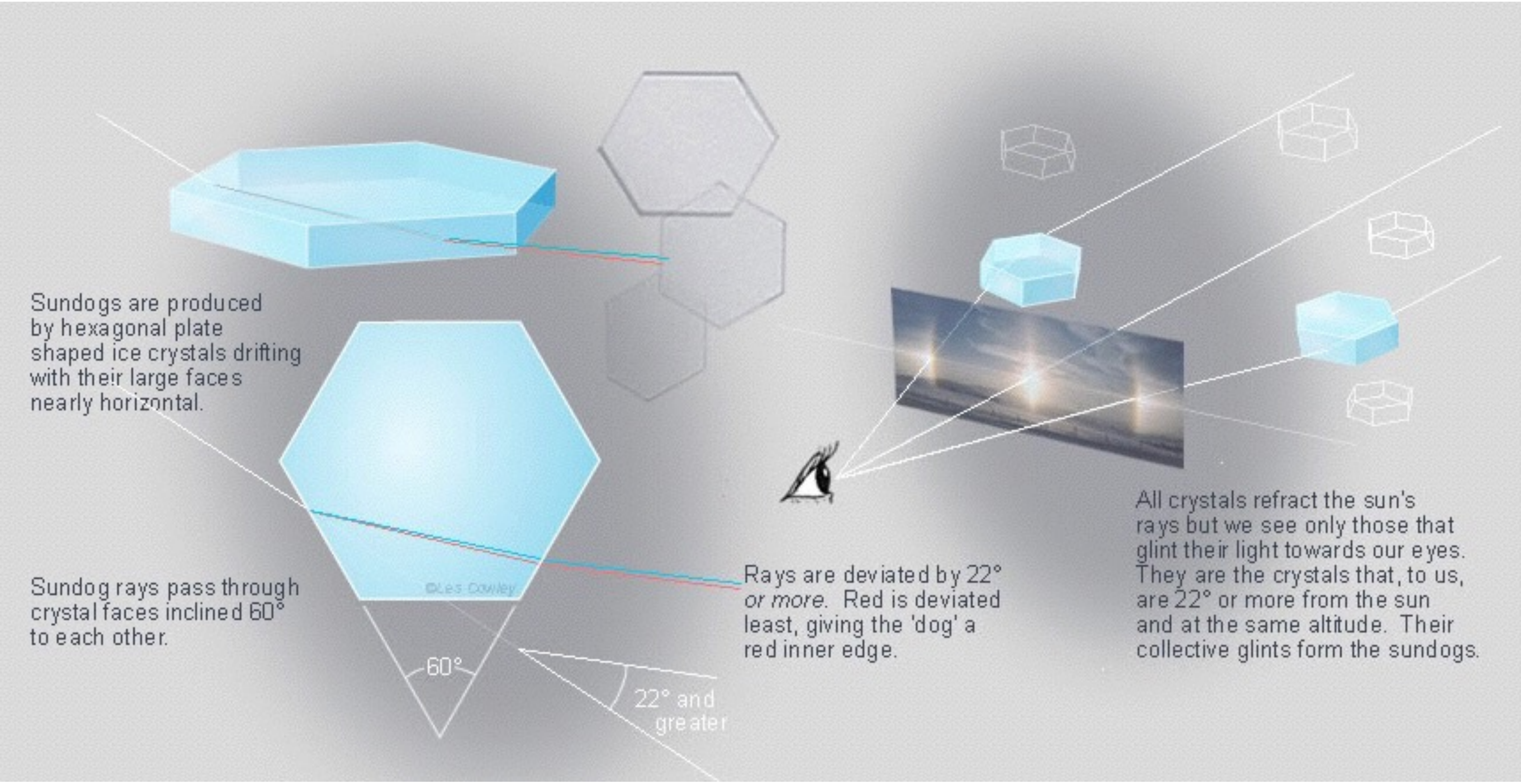
# Refraction in a Waterdrop



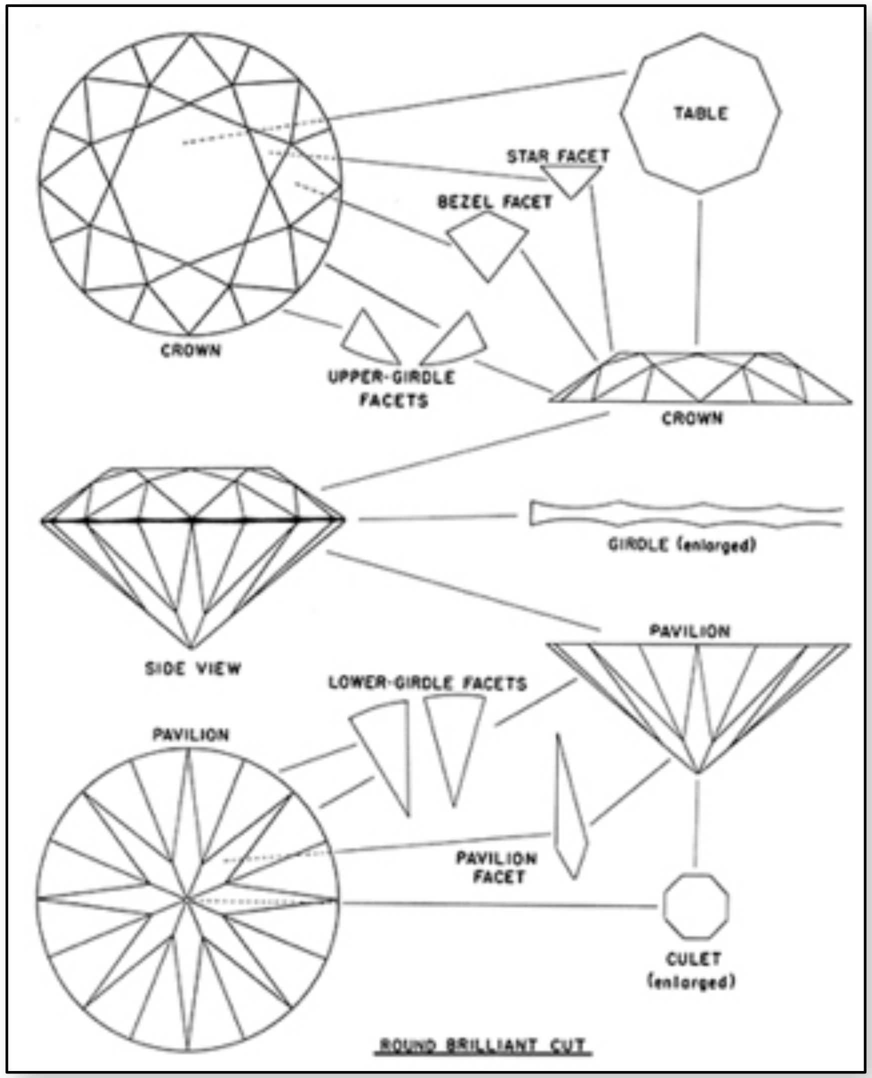
# Double rainbow all the way across the sky!

# Dispersion: “Halos” and “Sun dogs”

# Halos and Sundogs



# Dispersion Examples

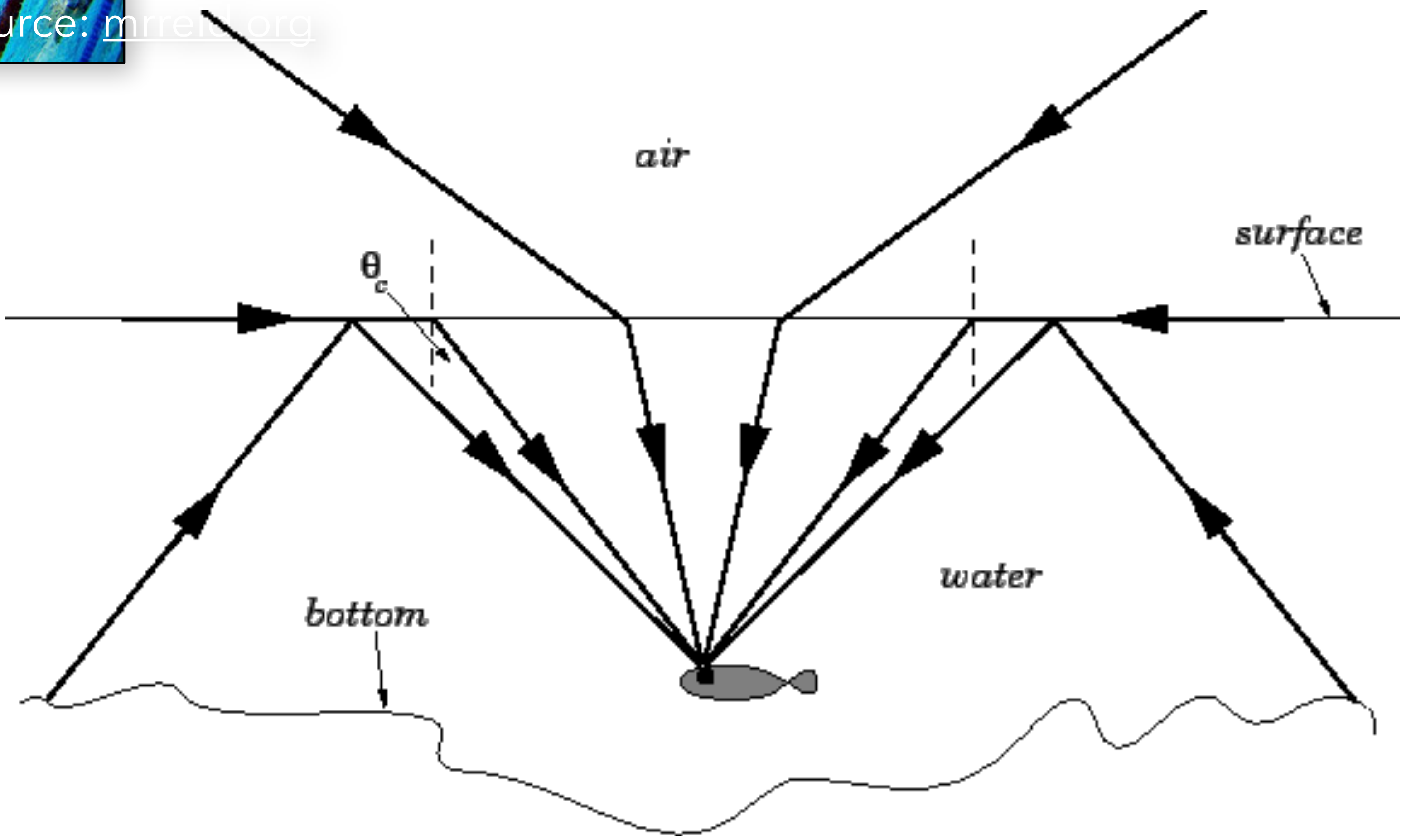
# What is this dark circle?

source: mrre

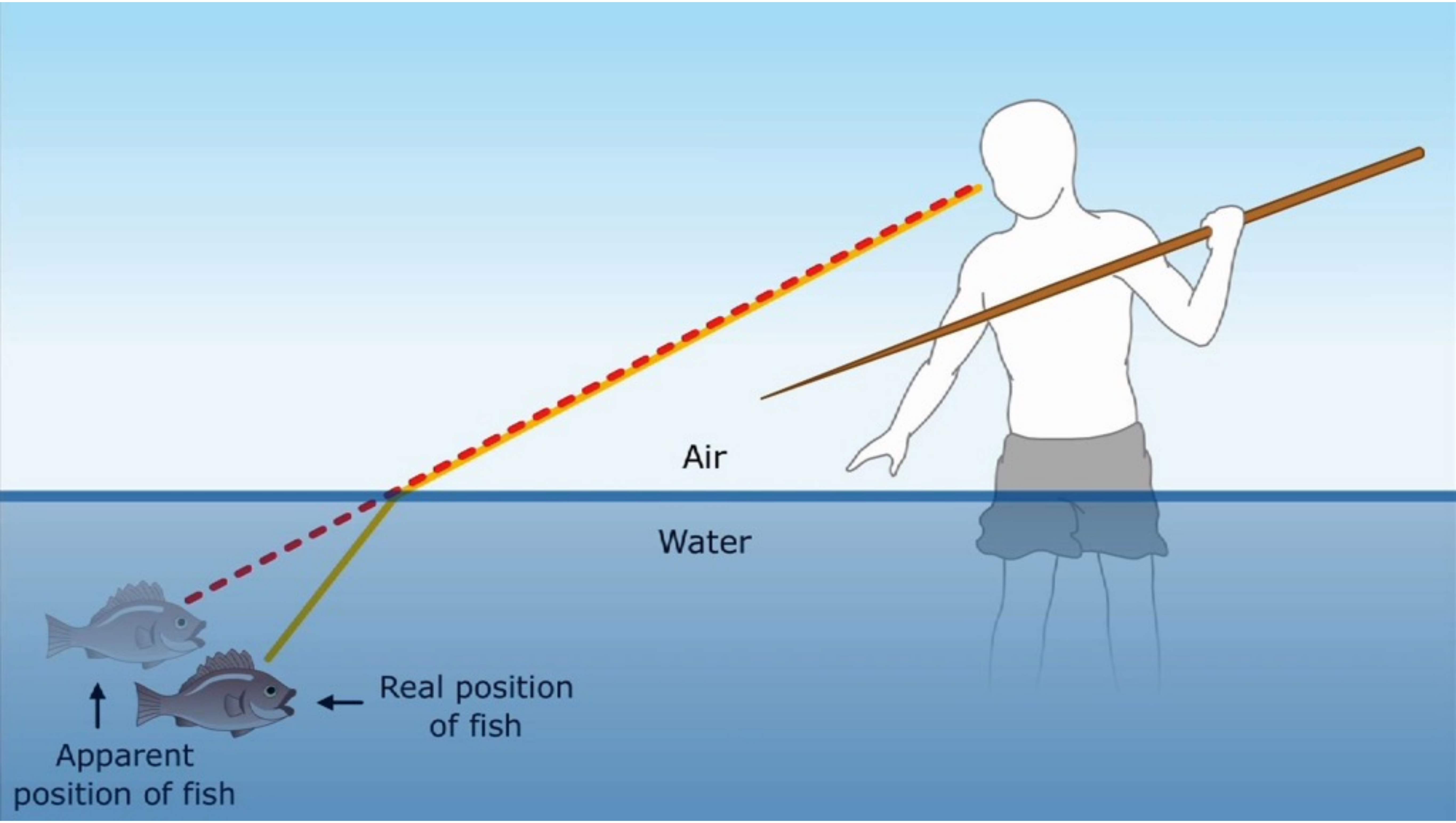
# What is this dark circle?


Called
"Snell's window" or
"optical man-hole"

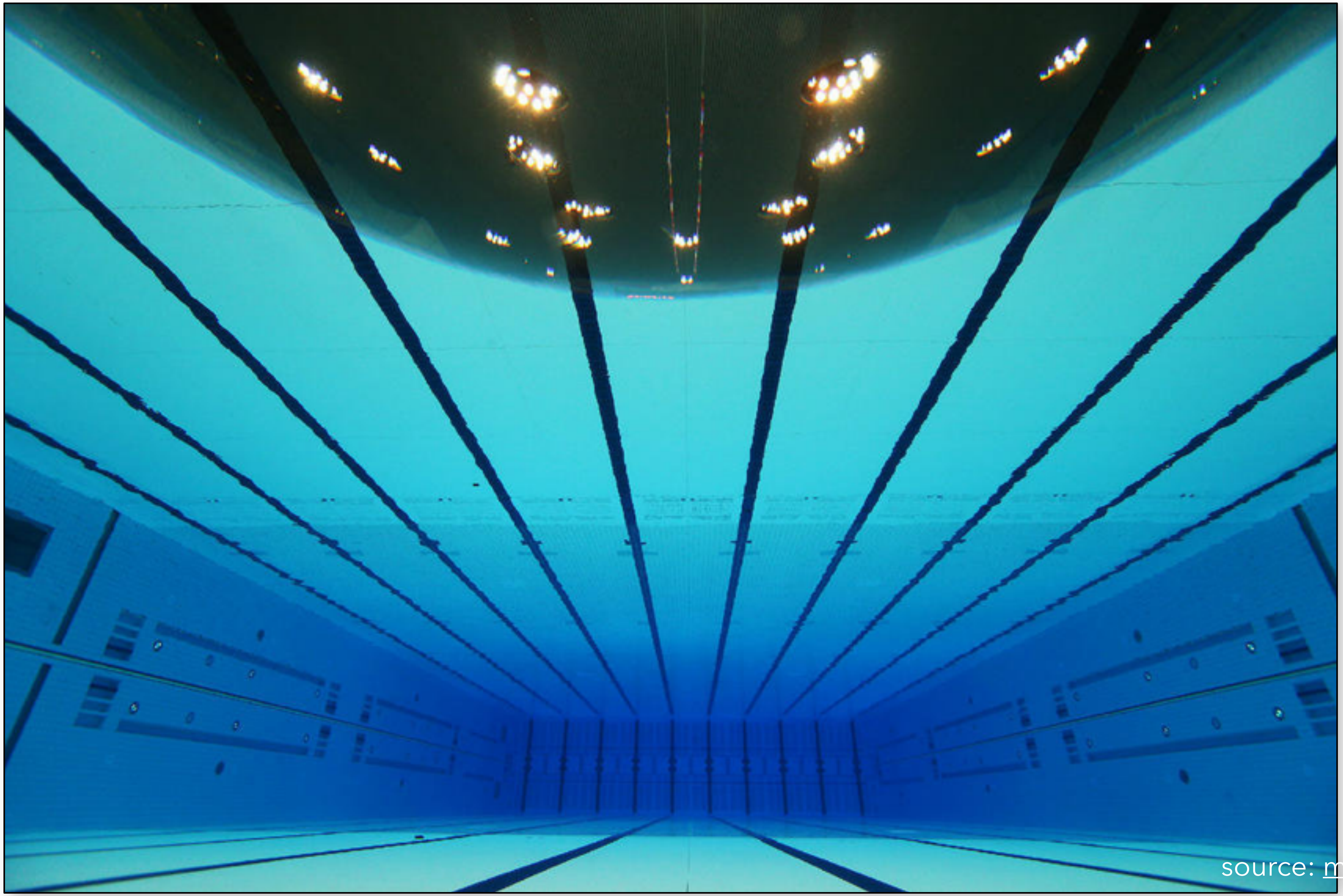
Caused by total
internal reflection



# Practical consequences of refraction



# Total Internal Reflection



source: mrreid.org

# Total Internal Reflection

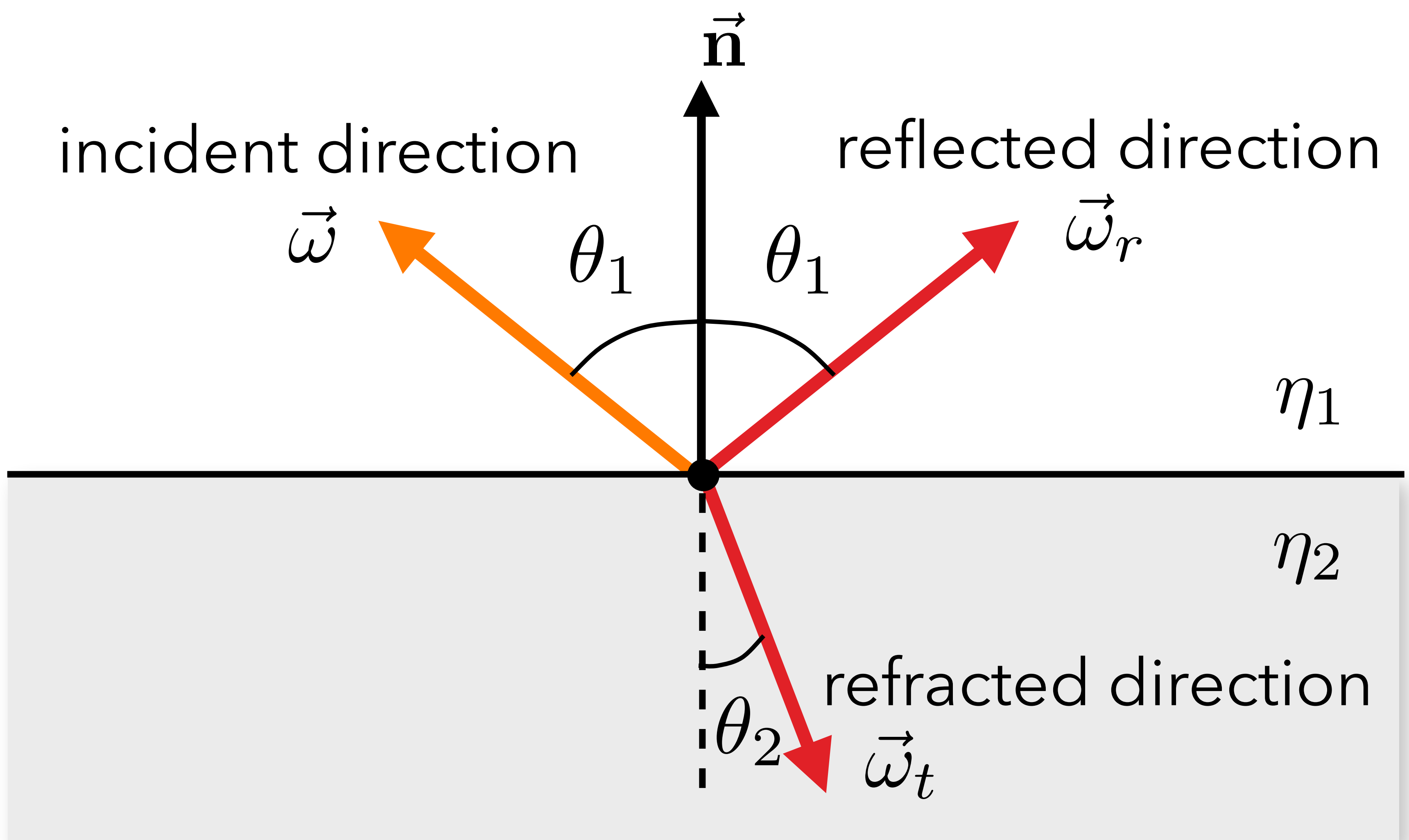
# Total Internal Reflection

- Can only happen when light passes from high index of refraction to low index of refraction

# Total Internal Reflection

# Reflection vs. Refraction

- How much light is reflected vs. refracted?



# Fresnel Equations

- *Reflection* and *refraction* from smooth *dielectric* (e.g. glass) surfaces
- *Reflection* from *conducting* (e.g. metal) surfaces
- Derived from Maxwell equations
- Involves polarization of the wave

# Fresnel Equations for Dielectrics

- Reflection of light polarized parallel and perpendicular to the plane of refraction

$$\rho_{||} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

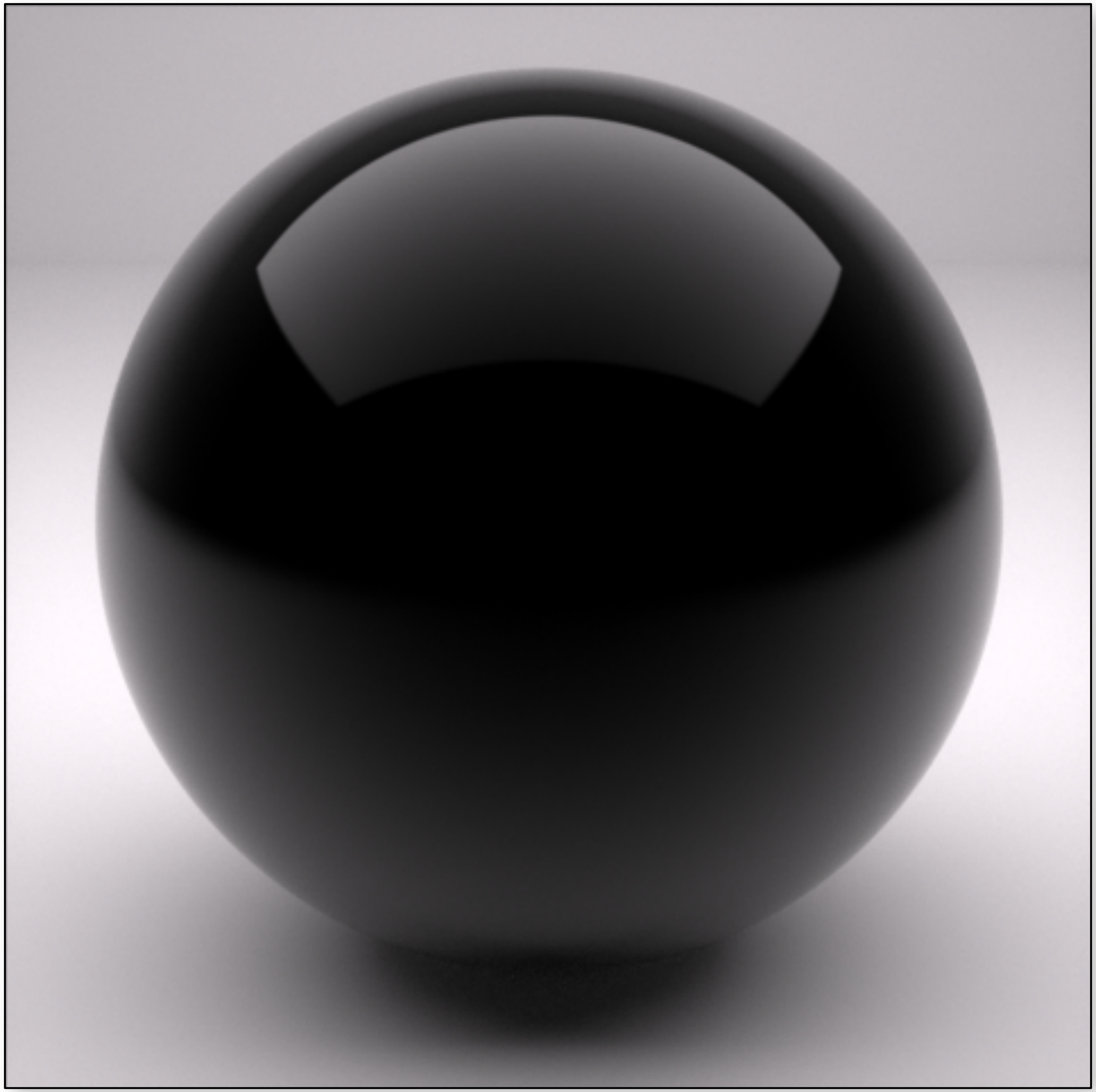
$$\rho_{\perp} = \frac{\eta_1 \cos\theta_1 - \eta_2 \cos\theta_2}{\eta_1 \cos\theta_1 + \eta_2 \cos\theta_2}$$

reflected: $F_r = \frac{1}{2}\left(\rho_{||}^2 + \rho_{\perp}^2\right)$

refracted: $F_t = 1 - F_r$

  - faster approximations possible (Schlick), but sacrifice in accuracy no longer worthwhile

# Fresnel Reflection



# Fresnel Reflection



# Recall the reflection equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- What is the BRDF for specular reflection/refraction?

# BRDF of Ideal Specular Reflection

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- What is the BRDF for specular reflection?

Dirac delta

Fresnel reflection

Reflection function (flips about normal)

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = F_r(\vec{\omega}_i) \frac{\delta(\vec{\omega}_r - R(\vec{\omega}_i, \vec{\mathbf{n}}))}{\cos\theta_i}$$

to cancel the cosine term in the reflection equation (Fresnel eqs. account for it)

# BTDF of Ideal Specular Refraction

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- What is the BTDF for specular refraction?

Dirac delta

Fresnel reflection

Transmission function

$$f_t(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{\eta_1^2}{\eta_2^2}(1 - F_r(\vec{\omega}_i)) \frac{\delta(\vec{\omega}_r - T(\vec{\omega}_i, \vec{\mathbf{n}}))}{\cos\theta_i}$$

to cancel the cosine term
in the reflection equation
(Fresnel eqs. account for it)

# Rendering with Delta BRDFs

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- Specular BRDFs contain delta functions
- Integral drops out, specular interactions are handled explicitly

# What's happening in this photo?

# Polarizing Filter

# Polarization

Without Polarizer

With Polarizing Filter

source: photog

# Polarization

Without Polarizer

With Polarizing Filter

source: wiki

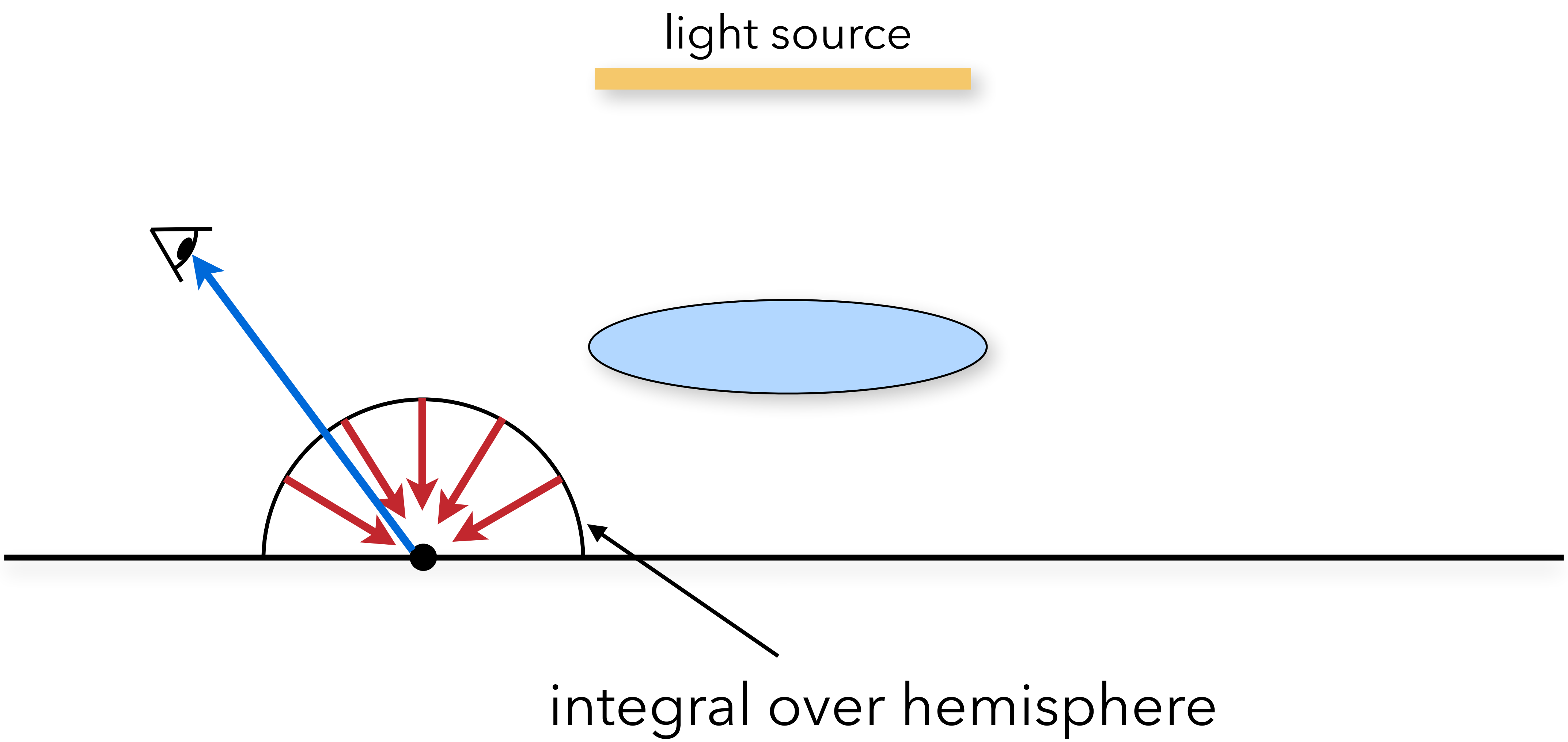
# Effect of Polarization

# Effect of Polarization

# Direct Lighting

- Area lights

$$L_r(\mathbf{x},\vec{\omega}_r) = \boxed{\int_{H^2}} f_r(\mathbf{x},\vec{\omega}_i,\vec{\omega}_r) L_i(\mathbf{x},\vec{\omega}_i)\cos\theta_i \, d\vec{\omega}_i$$



# Change of Variable

- Convert integral *over directions* into integral *over surface area*.

- Jacobian of transformation:

$$d\vec{\omega} = \frac{dA \cos\theta''}{\|\mathbf{x}'' - \mathbf{x}'\|^2}$$

# The Reflection Equation

- Surface integral form

$$L_r(\mathbf{x}', \mathbf{x}) = \int_A f_r(\mathbf{x}'', \mathbf{x}', \mathbf{x}) L_i(\mathbf{x}'', \mathbf{x}') G(\mathbf{x}'', \mathbf{x}')\, dA(\mathbf{x}'')$$

integrate over all surfaces

geometry term

$$G(\mathbf{x}'', \mathbf{x}') = V(\mathbf{x}'', \mathbf{x}') \frac{\cos\theta'' \cos\theta'}{\|\mathbf{x}'' - \mathbf{x}'\|^2}$$

visibility term

$$V(\mathbf{x}'', \mathbf{x}') = \begin{cases} 1: & \text{visible} \\ 0: & \text{not visible} \end{cases}$$

# Direct Lighting

- Area lights

$$L_r(\mathbf{x}', \mathbf{x}) = \boxed{\int_A} f_r(\mathbf{x}'', \mathbf{x}', \mathbf{x}) L_i(\mathbf{x}'', \mathbf{x}') \boxed{V(\mathbf{x}', \mathbf{x}'')} \frac{\boxed{\cos\theta''}\,\boxed{\cos\theta'}}{\|\mathbf{x}'' - \mathbf{x}'\|^2} \, dA(\mathbf{x}'')$$

integral over surface area

visibility term