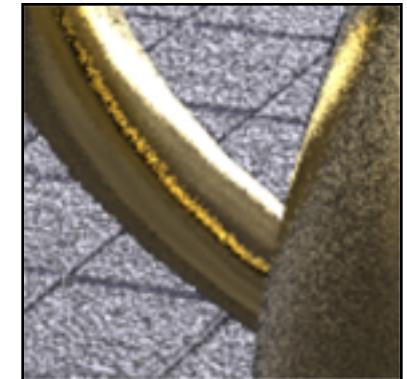
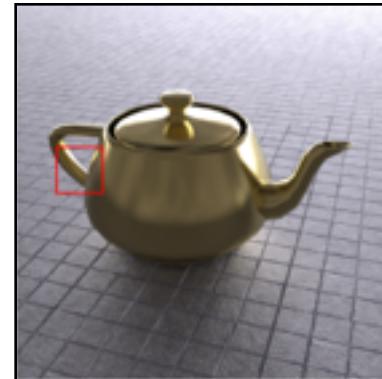


# Rendering Algorithms

## Denoising Monte Carlo Renderings

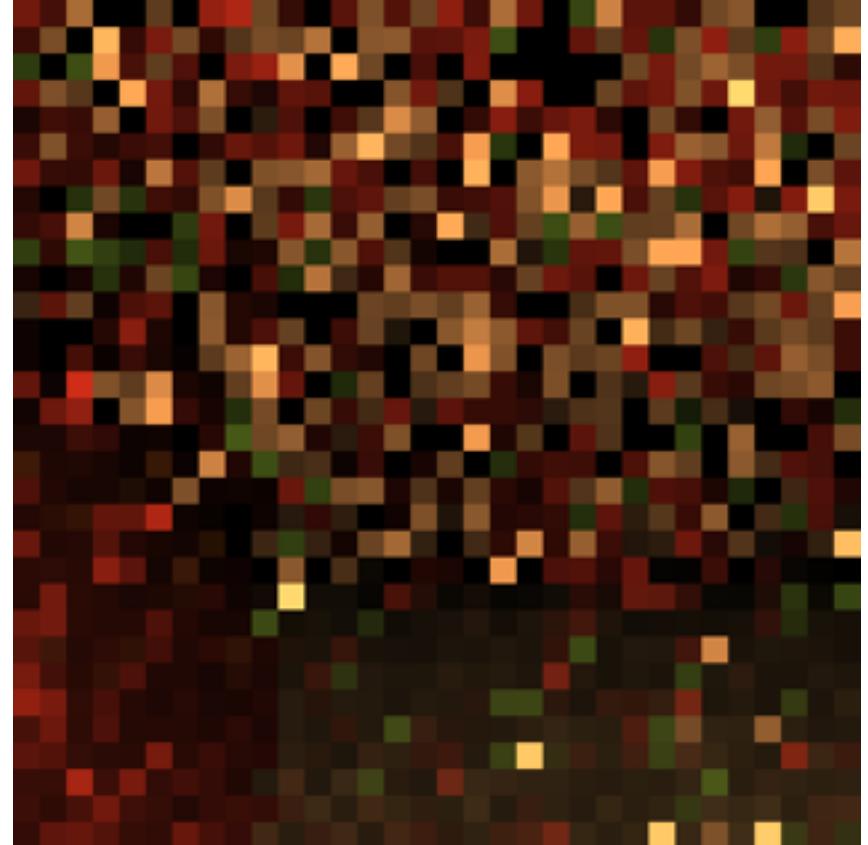
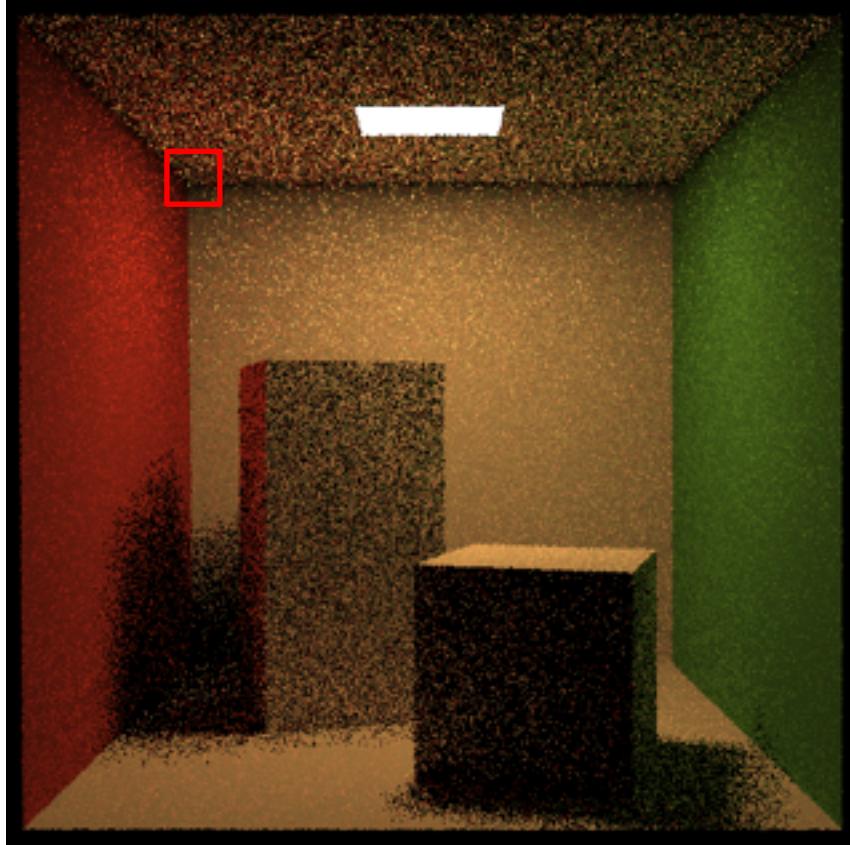


Wojciech Jarosz

(slides courtesy of Dr. Fabrice Rousselle)

# Monte Carlo Rendering

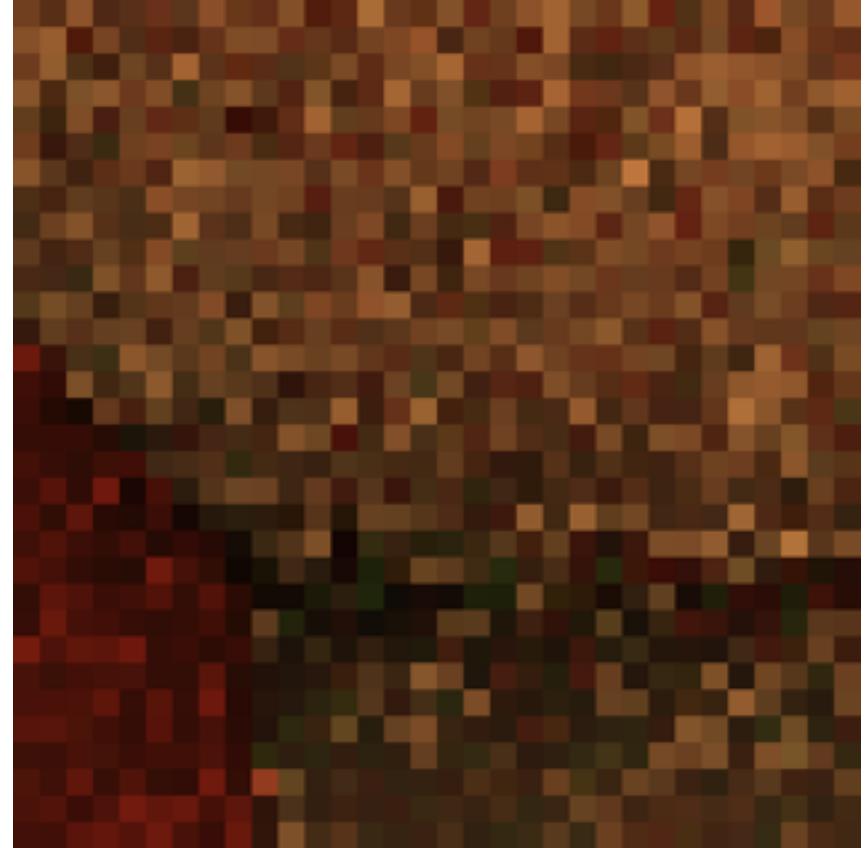
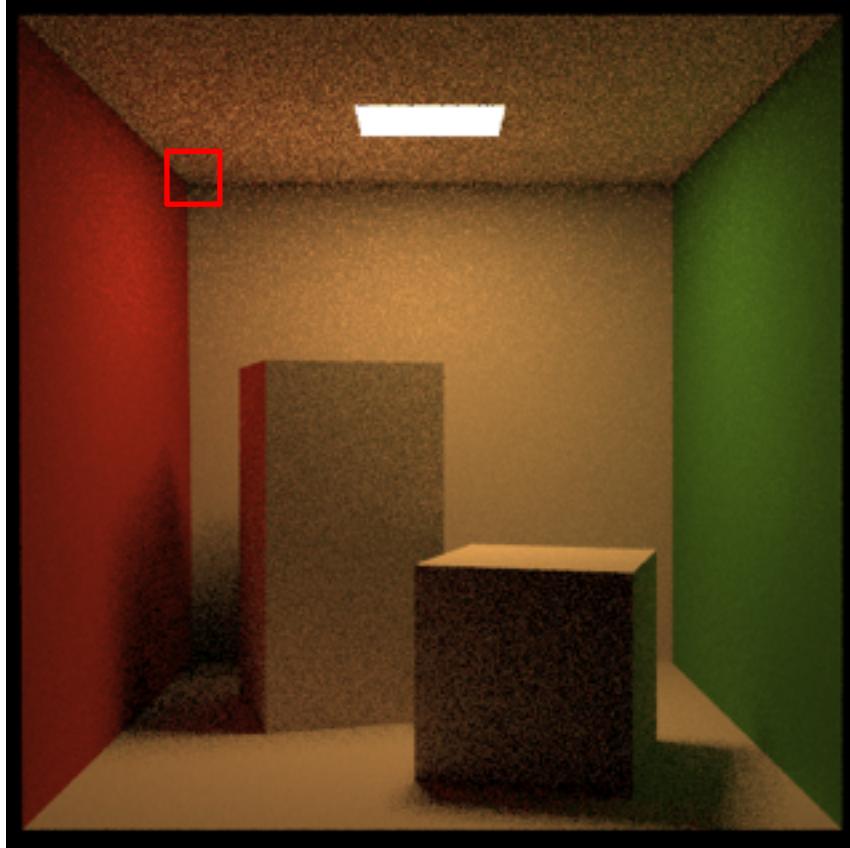
---



1 sample per pixel, 0.3s

# Monte Carlo Rendering

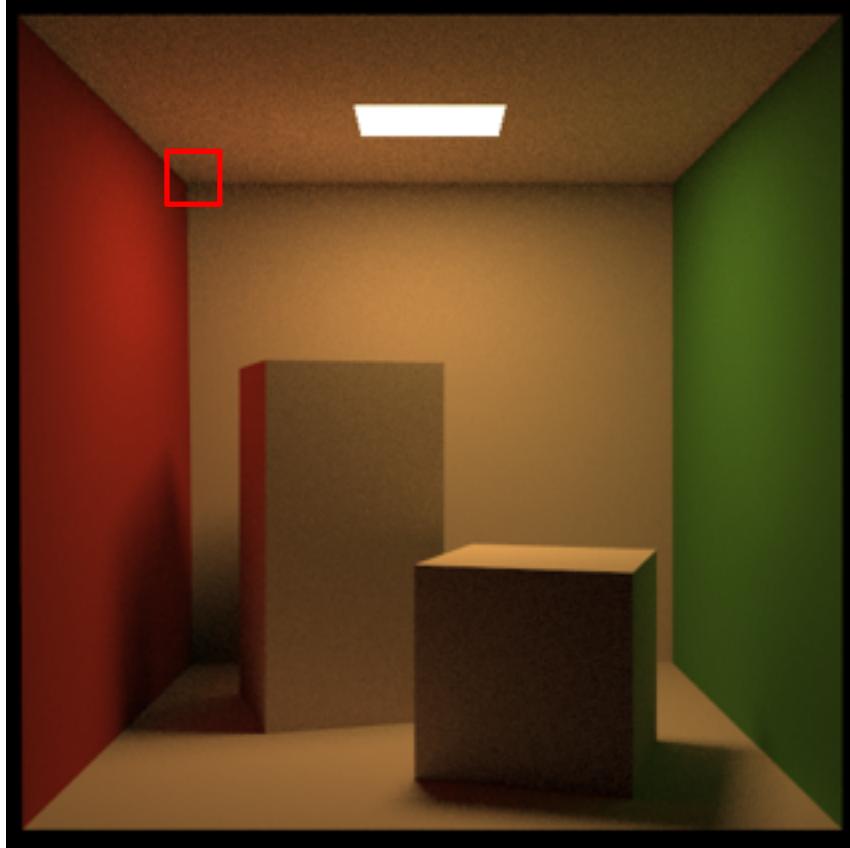
---



10 samples per pixel, 2.6s

# Monte Carlo Rendering

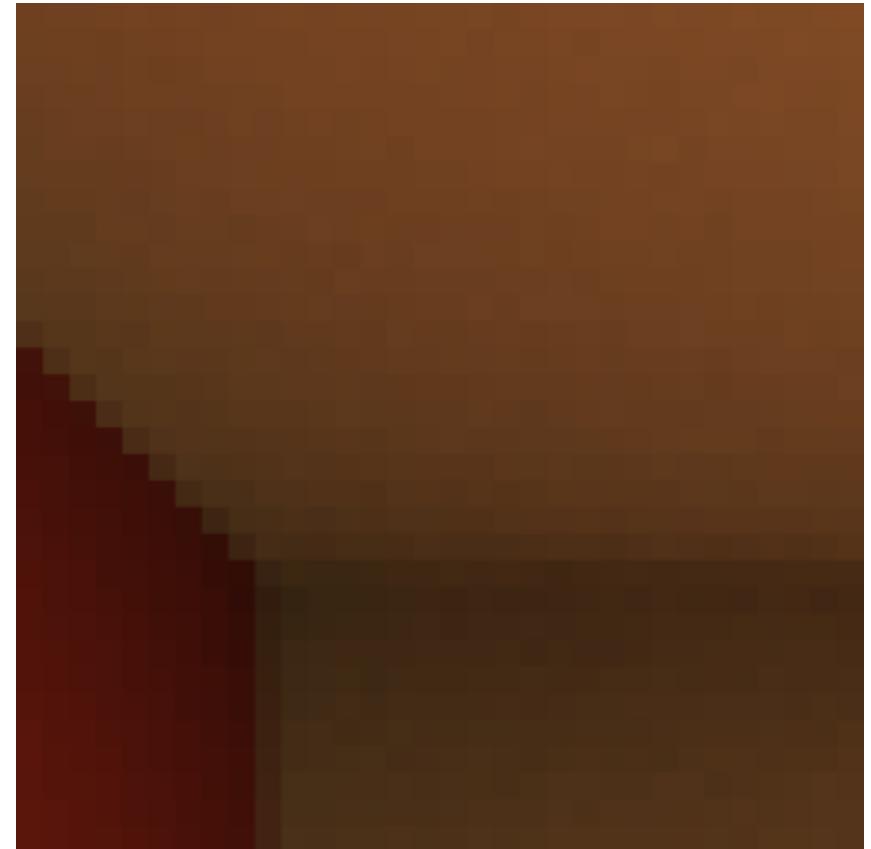
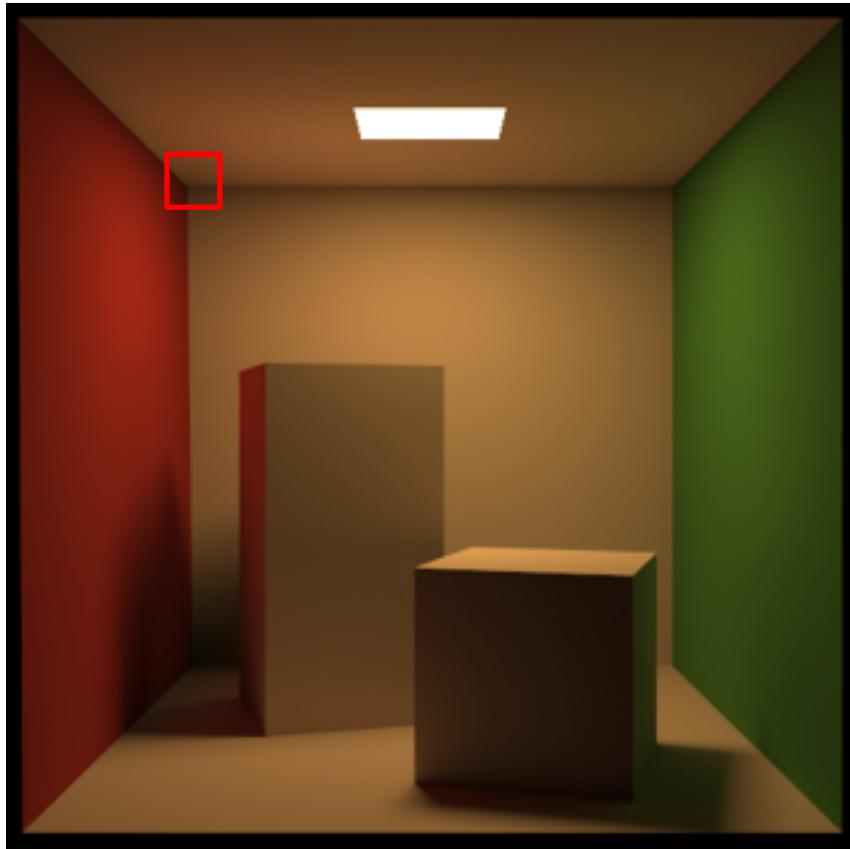
---



100 samples per pixel, 26s

# Monte Carlo Rendering

---



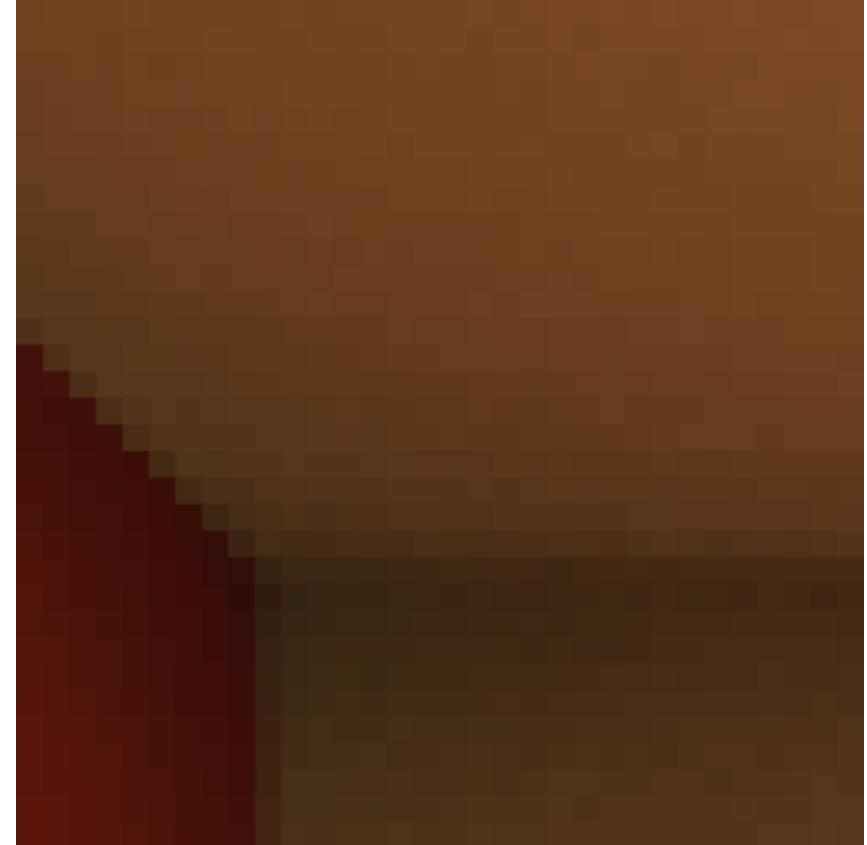
10 000 samples per pixel, 51mn

# Monte Carlo Rendering

---



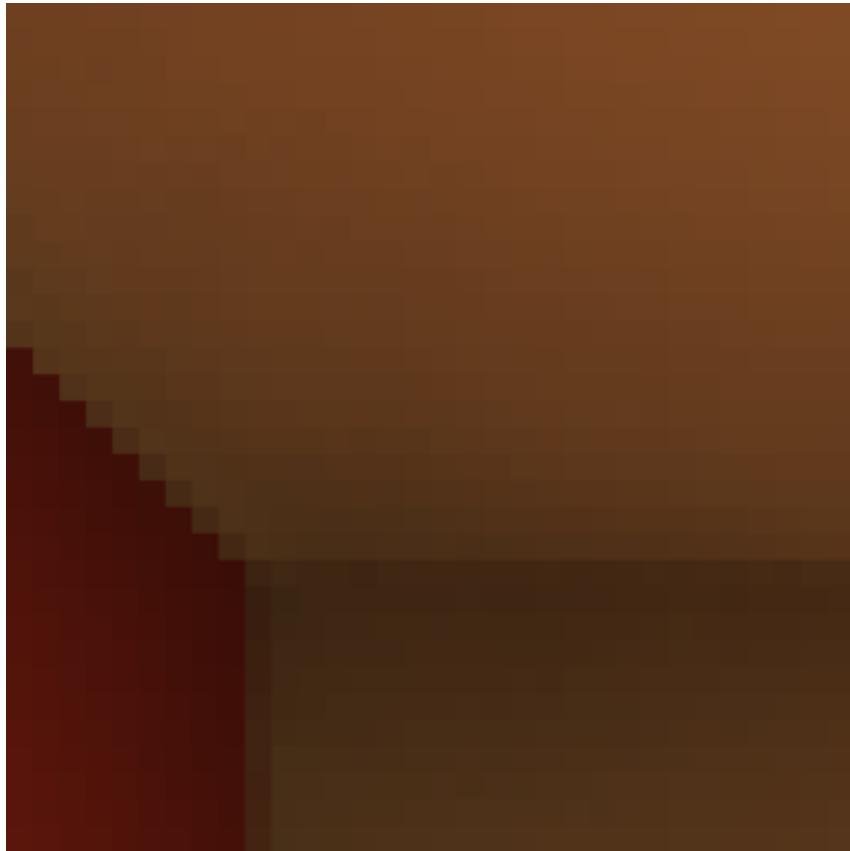
100 samples per pixel, 26s



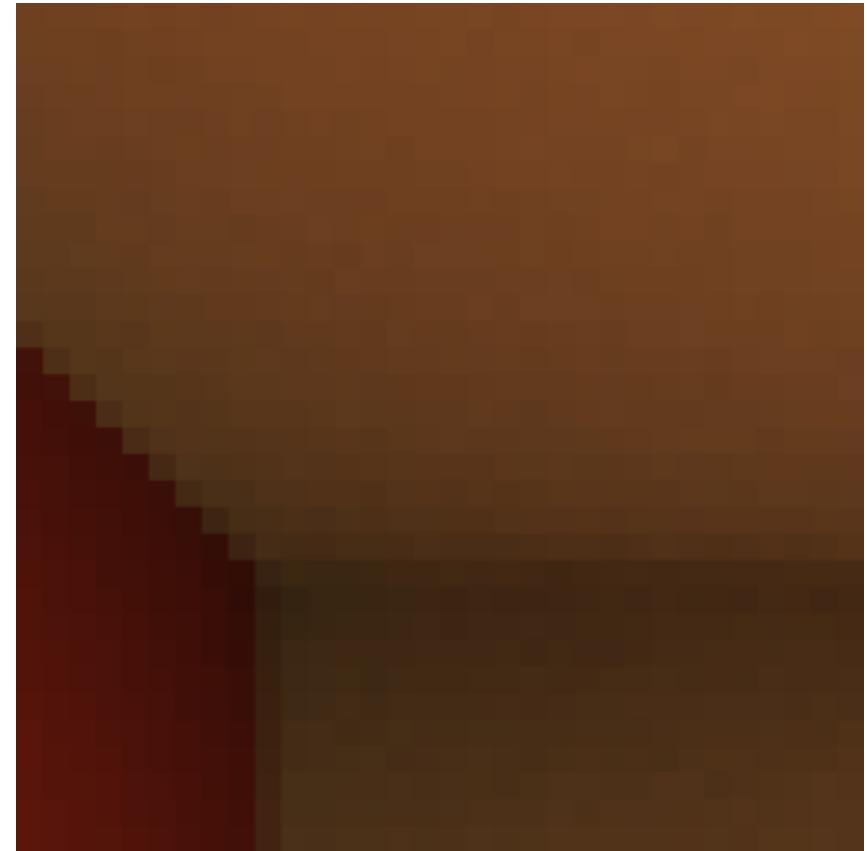
Ground truth (10 000 spp)

# Monte Carlo Rendering

---



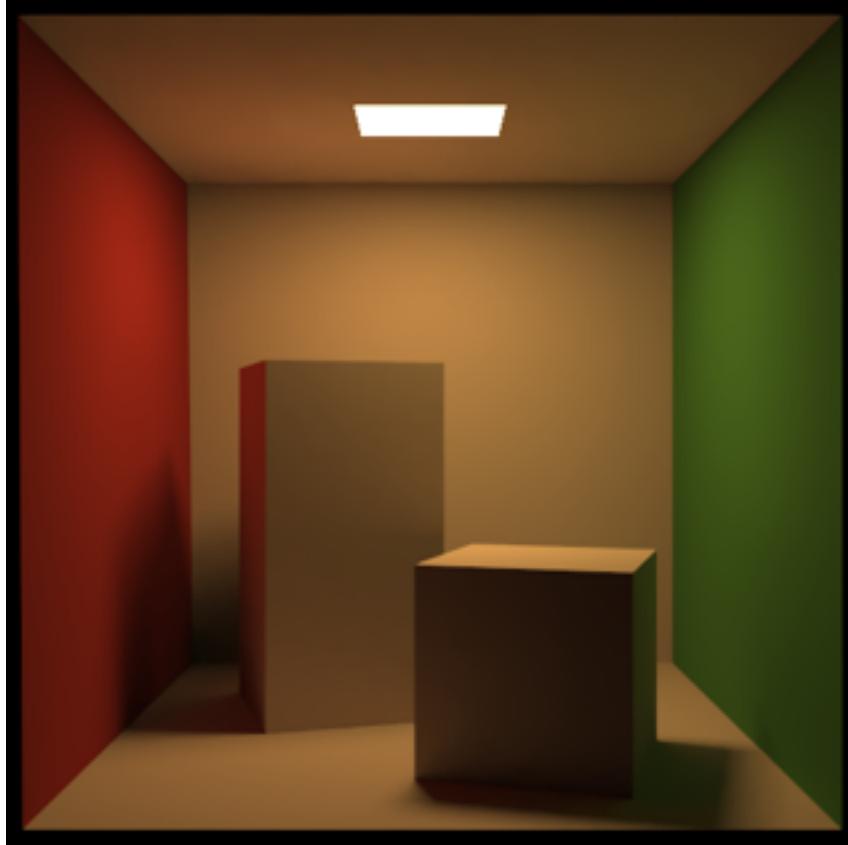
100 samples per pixel, 26s  
+ denoising, 15s (matlab)



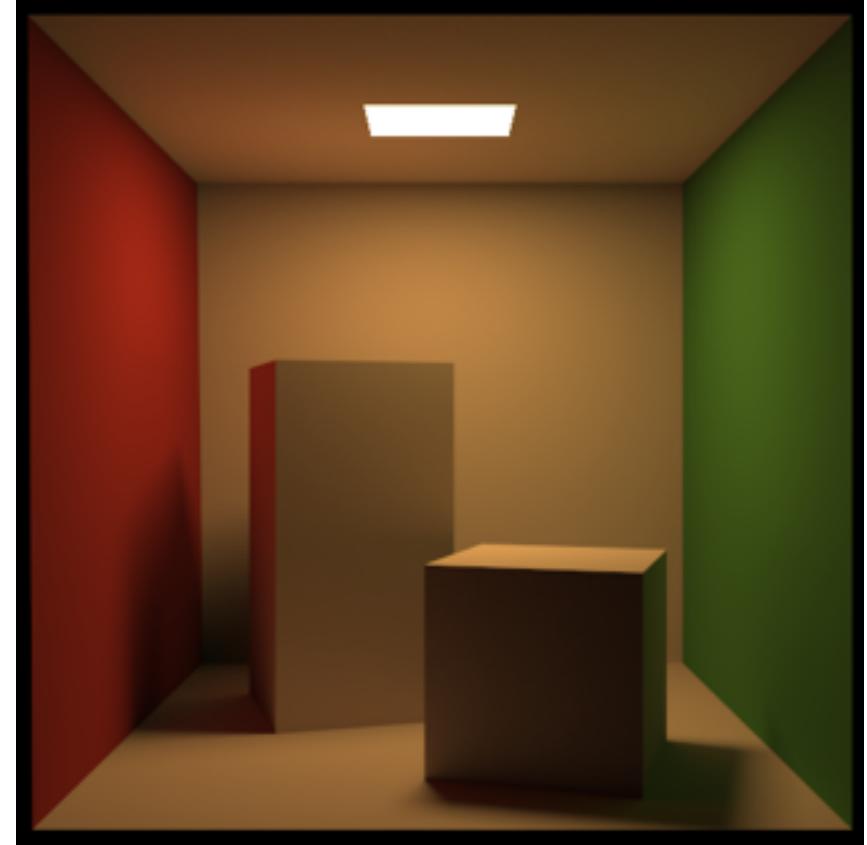
Ground truth (10 000 spp)

# Monte Carlo Rendering

---



100 spp, denoised

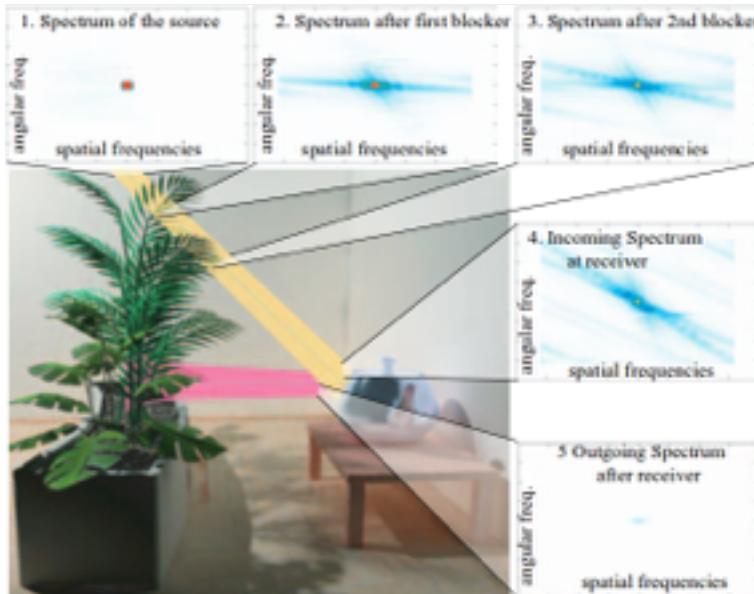


Ground truth (10 000 spp)

# Denoising – Two Approaches

---

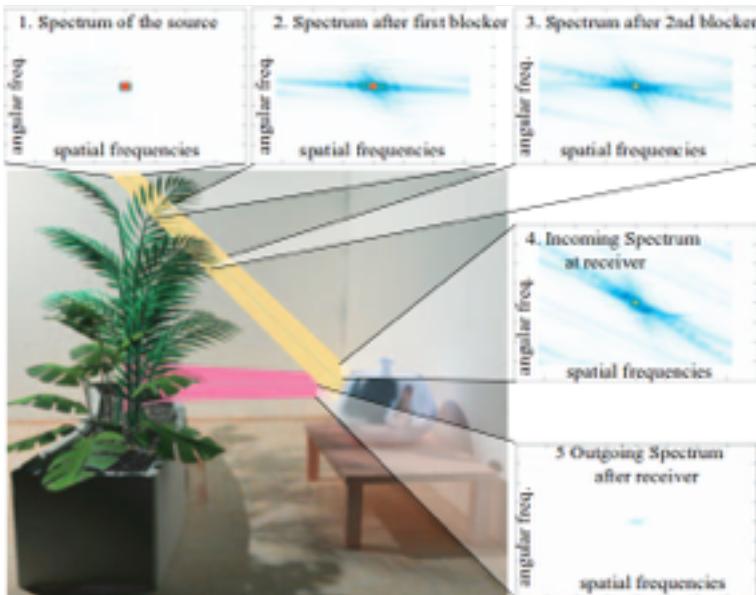
- *A priori* methods  
Analytical



Durand et al. 2005

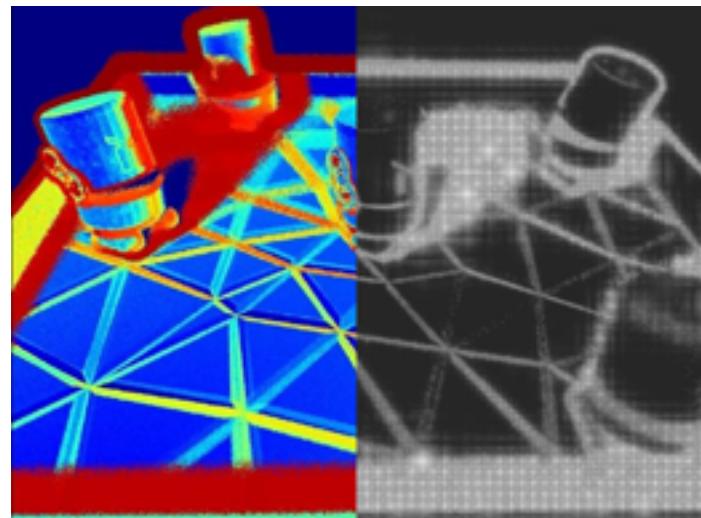
# MC Noise – Two Approaches

- *A priori* methods  
Analytical



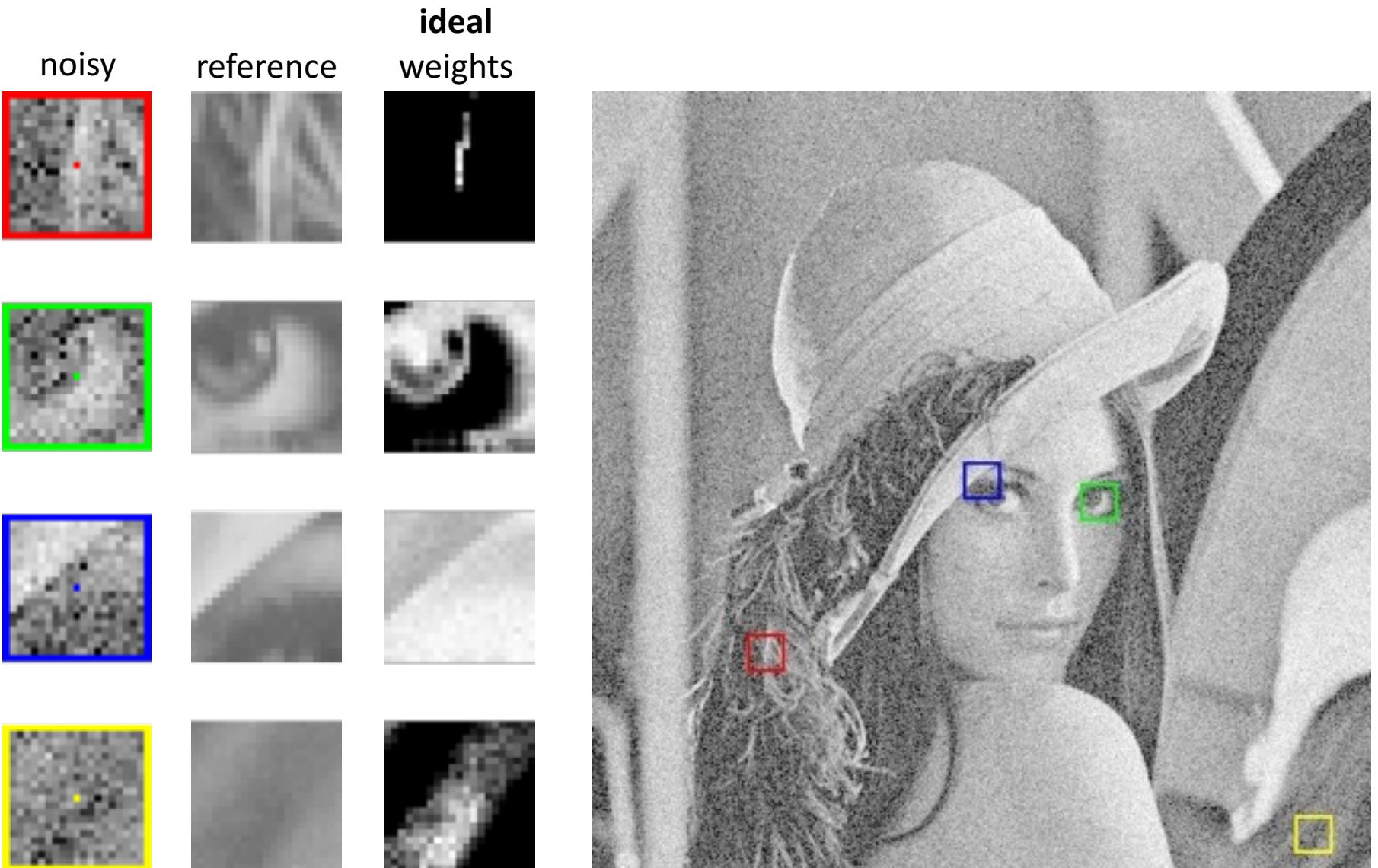
**A frequency analysis of light transport**  
Durand et al., SIGGRAPH 2005

- *A posteriori* methods  
Empirical



**Adaptive Wavelet Rendering**  
Overbeck et al., SIGGRAPH Asia 2009

# Image-Based Denoising



# Image-Based Denoising

---

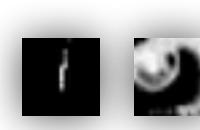
- Use a **flexible** and **robust** filter

- Flexible: arbitrary filter support
- Robust: few denoising artifacts

Noisy data



Bilateral Filter



Non-Local  
Means Filter



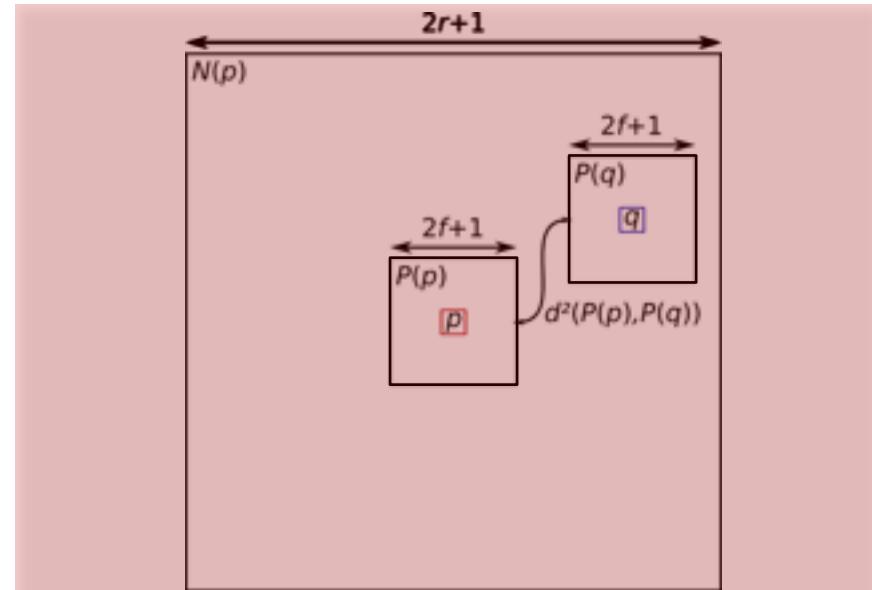
Buades et al. 2006

# The Non-Local Means Filter

- Per pixel weight  $w(p, q)$
- Generalized distance  $d$ 
  - spatial (in pixels, exact)
  - range (difference in value  $u$ , noisy)

$$d^2(p, q) = \frac{(u(p) - u(q))^2 + 2\sigma^2}{\epsilon + k^2 2\sigma^2}$$

- Weights
  - decrease exponentially with the distance



Variance  $\sigma^2$

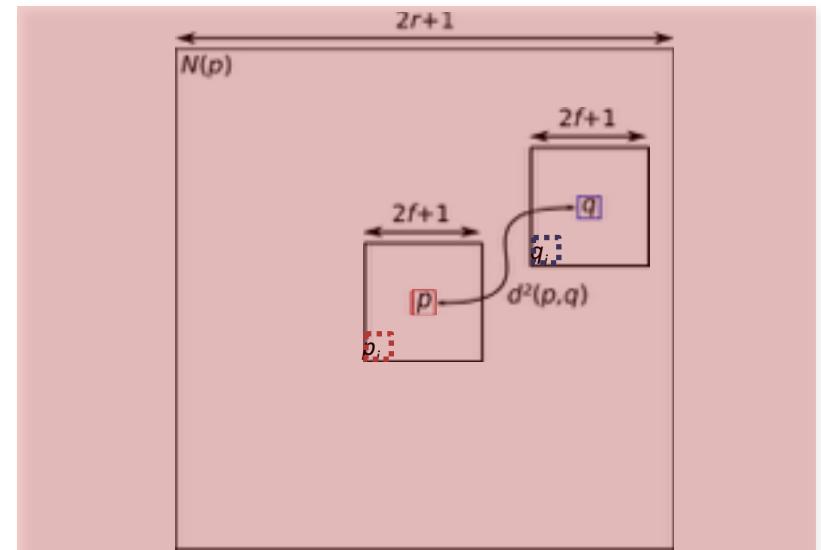


$ff=3$

bilateral filter

# NL-Means – Naïve Implementation

```
INPUT: dat, datvar  
INPUT: r(10), f(3), k(0.45)  
flt = wgtsum = 0 // 1. Initialize output  
for each pixel p // 2. Loop over pixels  
    for each pixel q in N(p) // 3. Loop over neighbors ( $r^2$ )  
        d2patch = 0 // 4. Loop over pairs  
        for each offset i in P() // (P(p), P(q)) to get  
            d2patch += d2( $p_i, q_i, k$ ) // mean distance over  
        d2patch /= area(P()) // patch
```



**Complexity:**  $O(r^2 \times f^2)$

# NL-Means – Naïve Implementation

---

```
INPUT: dat, datvar
INPUT: r(10), f(3), k(0.45)
flt = wgtsum = 0                                // 1. Initialize output
for each pixel p                               // 2. Loop over pixels
    for each pixel q in N(p)                   // 3. Loop over neighbors
        d2patch = 0                            // 4. Loop over pairs
        for each offset i in P()               //      (P(p), P(q)) to get
            d2patch += d2(pi, qi, k)      //      mean distance over
        d2patch /= area(P())                  //      patch
        wgt = exp(-max(0, d2patch))          //      Weight of P(q)
        for each offset i in P()               // 5. Store weighted
            wgtsum(pi) += wgt           //      contribution of P(q)
            flt(pi) += wgt * dat(qi)  //
    flt /= wgtsum                                // 6. Normalize output
    //
```

**Complexity:**  $O(r^2 \times f^2)$

# NL-Means – Fast Implementation

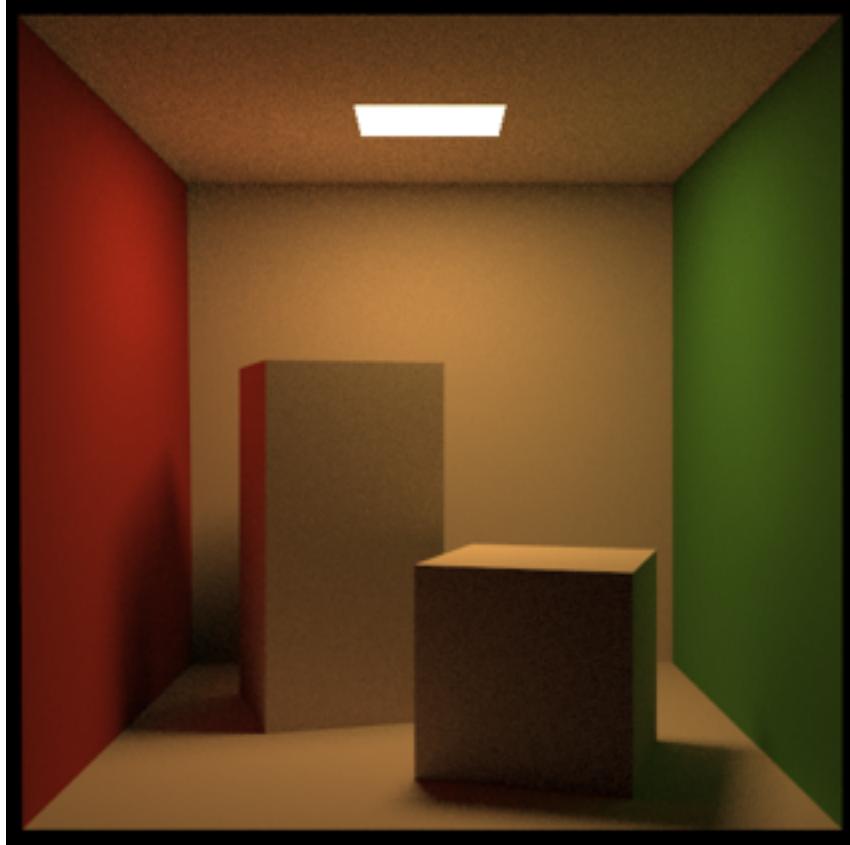
---

```
INPUT: dat, datvar
INPUT: r(10), f(3), k(0.45)
flt = wgtsum = 0                                // 1. Initialize output
for each offset (dx,dy) in N()                // 2. Loop over neighbors
    ngb = shift(dat,dx,dy)                      // 3. Compute distance to
    d2pixel = d2(ngb,dat,k)                     //      neighbor (dx,dy)
    d2patch = conv(d2pixel,boxf)              // 4. Apply box filter for
    wgt = exp(-max(0,d2patch))                  //      patch distance+weight
    wgt = conv(wgt,boxf-1)                   // 5. Box filter weights for
    flt += wgt * ngb                           //      patch contribution
    wgtsum += wgt
    flt /= wgtsum                               // 6. Normalize
```

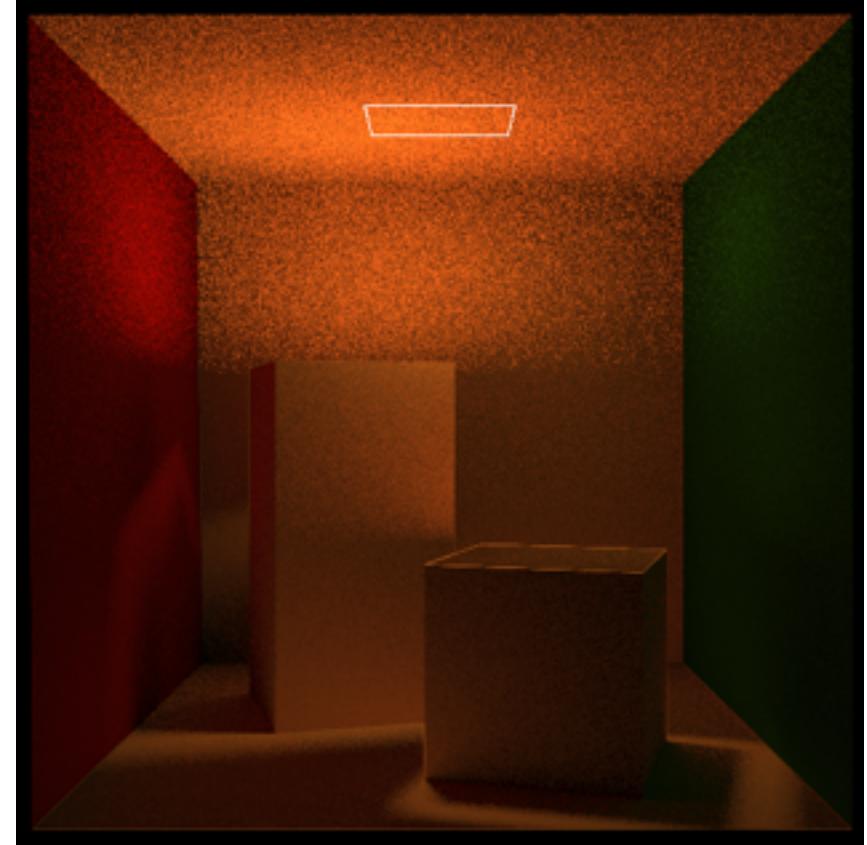
**Complexity:**  $O(r^2 \times f)$

# NL-Means – Application

---



100 samples per pixel



Variance of pixel mean (x1000)

# NL-Means – Non-Uniform Variance

---

- Uniform variance

$$d^2(p, q) = \frac{(u(p) - u(q))^2 - 2\sigma^2}{\epsilon + k^2 2\sigma^2}$$

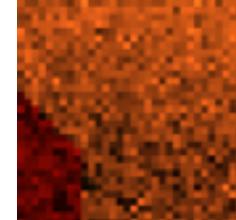
- Non-uniform variance

$$d^2(p, q) = \frac{(u(p) - u(q))^2 - (\text{Var}[p] + \min(\text{Var}[q], \text{Var}[p])))}{\epsilon + k^2(\text{Var}[p] + \text{Var}[q])}$$

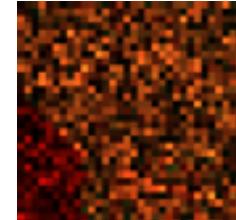
# Variance Estimation

---

- For independent samples, use sample mean variance

$$\text{Var}[p] = \left( \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \right) / n$$
( $\times 1000$ )

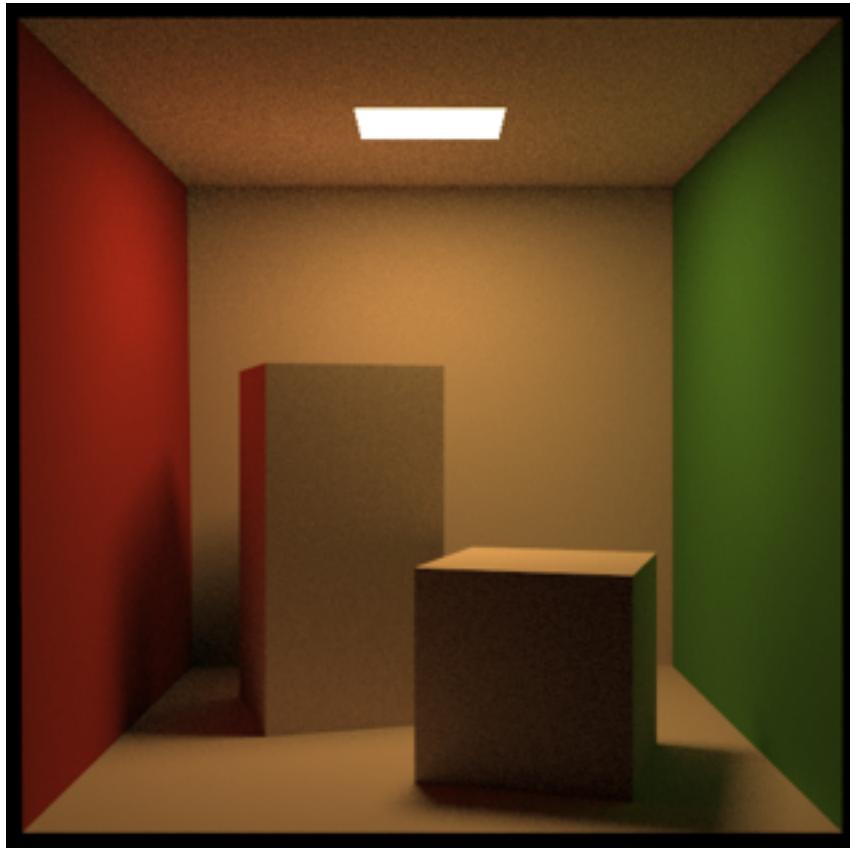
- For correlated samples, use buffer difference
  - Stratified sampling, low-discrepancy sampling, MLT, etc.
  - Generate two images with independent seeds
  - Compute the variance of the mean of the two images

$$\left( \begin{img alt="A small square image showing a brown gradient with a red triangle at the bottom left." data-bbox="228 755 375 908} - \begin{img alt="A small square image showing a brown gradient with a green triangle at the bottom left." data-bbox="405 755 532 908} \right)^2 / 4 =$$
( $\times 1000$ )

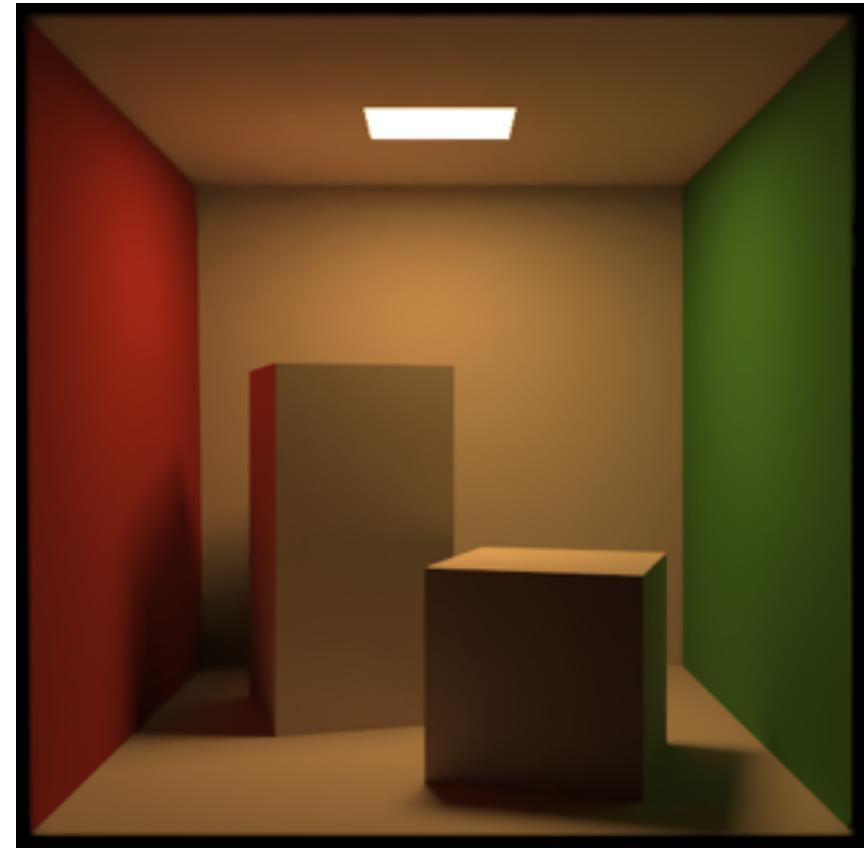
# NL-Means – Application

---

Independent samples



100 samples per pixel



Denoised

# NL-Means – Application

---

Independent samples



100 samples per pixel  
relative MSE: 1.573E-3



Denoised  
relative MSE: 0.198E-3

# NL-Means – Application

---

## Low-discrepancy samples



2 x 50 LD samples per pixel  
relative MSE: 0.405E-3

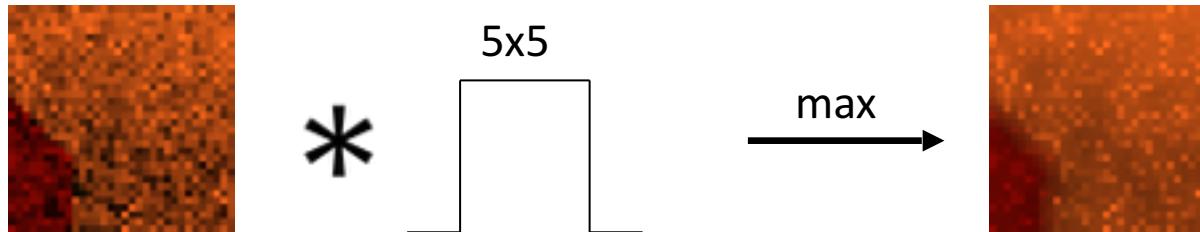


Denoised  
relative MSE: 0.257E-3

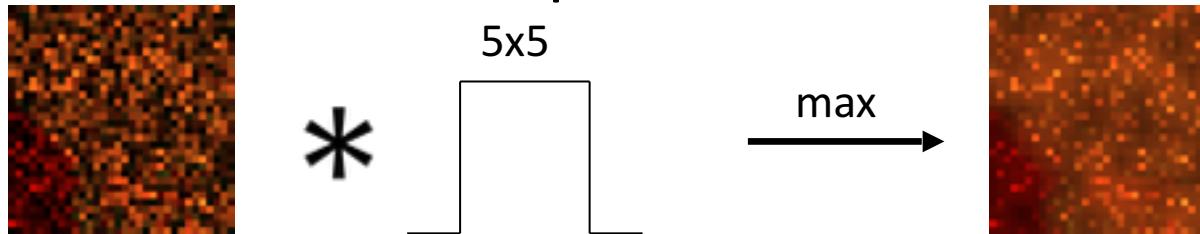
# Variance Estimation

---

- For independent samples, use sample mean variance



- For correlated samples, use buffer difference



# NL-Means – Fast Implementation

---

```
INPUT: dat, datvar  
datvar = max(datvar, conv(datvar,box2)  
flt = wgtsum = 0  
for each offset (dx,dy) in N()  
    ngb = shift(dat,dx,dy)  
    d2pixel = d2(ngb,dat)  
    d2patch = conv(d2pixel,boxf)  
    wgt = exp(-max(0,d2patch))  
    wgt = conv(wgt,boxf-1)  
    flt += wgt * ngb  
    wgtsum += wgt  
flt /= wgtsum
```

# NL-Means – Application

---

## Low-discrepancy samples



2 x 50 LD samples per pixel  
relative MSE: 0.405E-3



Denoised  
relative MSE: 0.257E-3

# NL-Means – Application

---

Low-discrepancy samples



2 x 50 LD samples per pixel  
relative MSE: 0.405E-3

With filtered variance



Denoised  
relative MSE: 0.038E-3

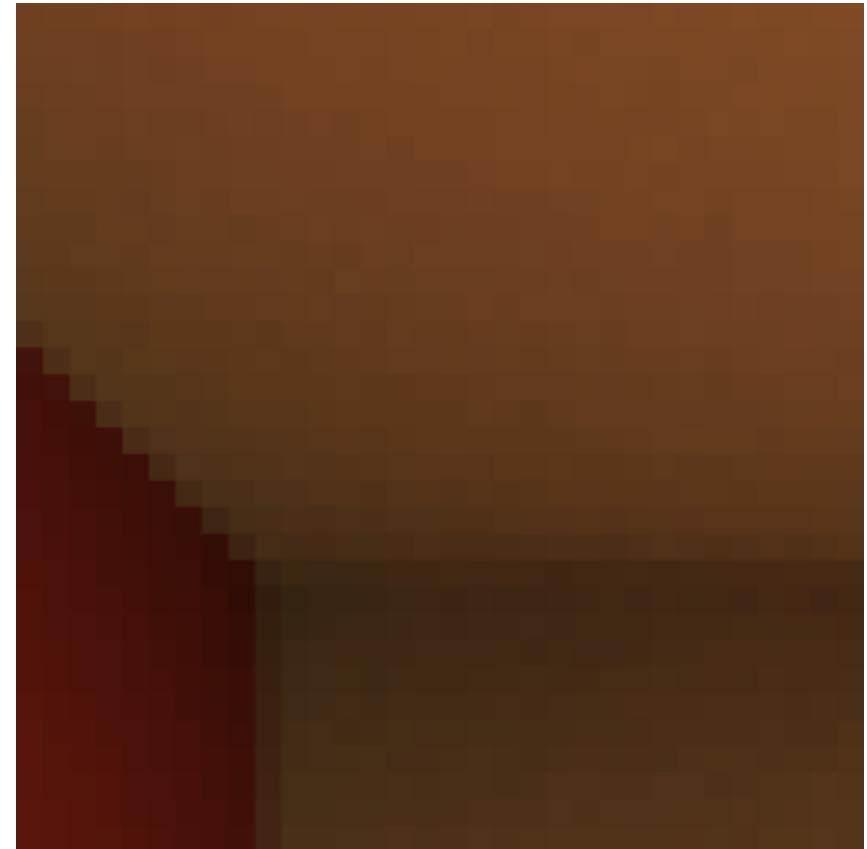
# NL-Means – Application

---

## Low-discrepancy samples



2 x 50 LD samples per pixel  
relative MSE: 0.405E-3



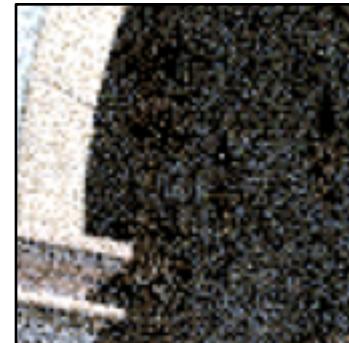
Ground truth (10 000 spp)

# Leveraging Scene Information

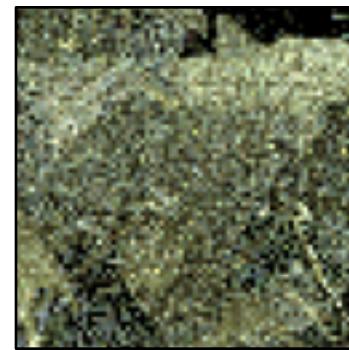
---



32 samples per pixel



normal

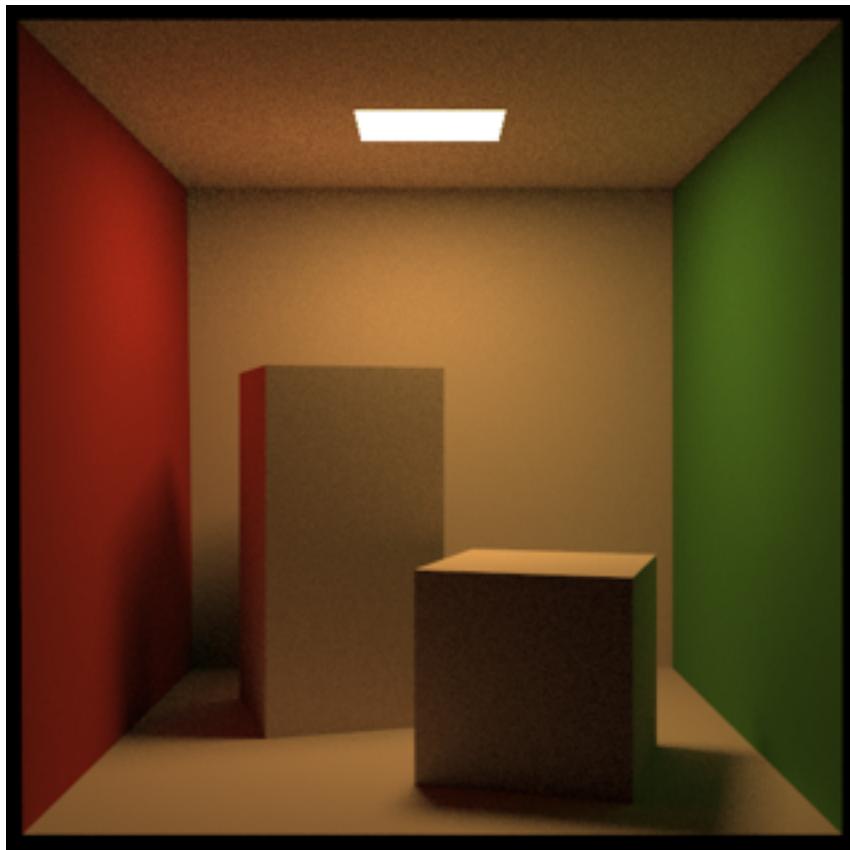


texture

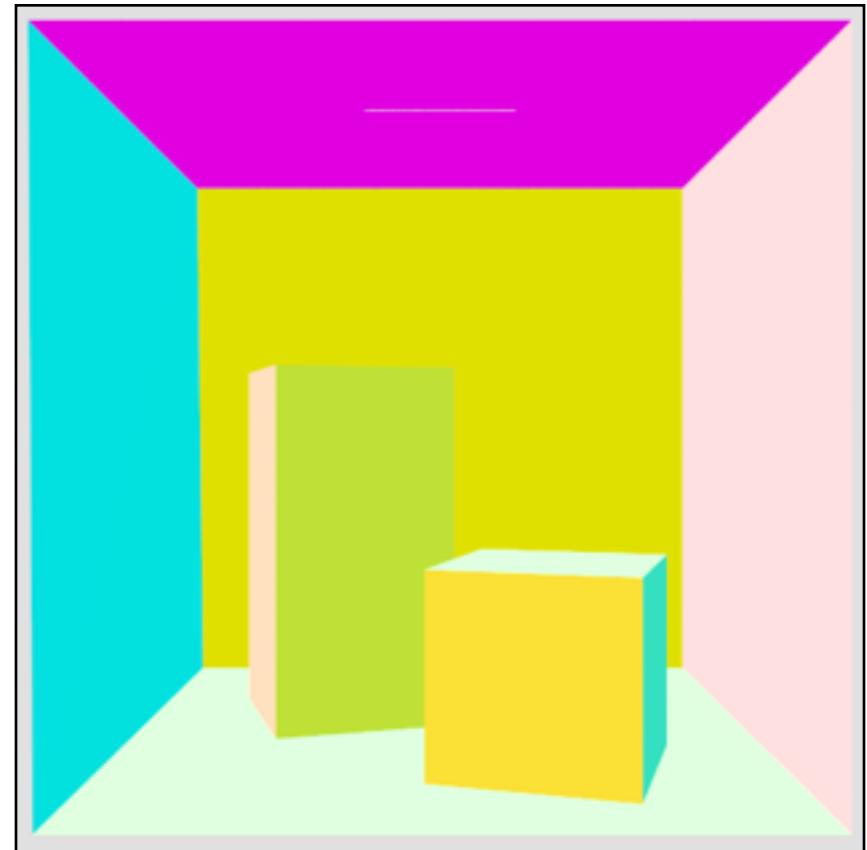


# Joint NL-Means Filter

---



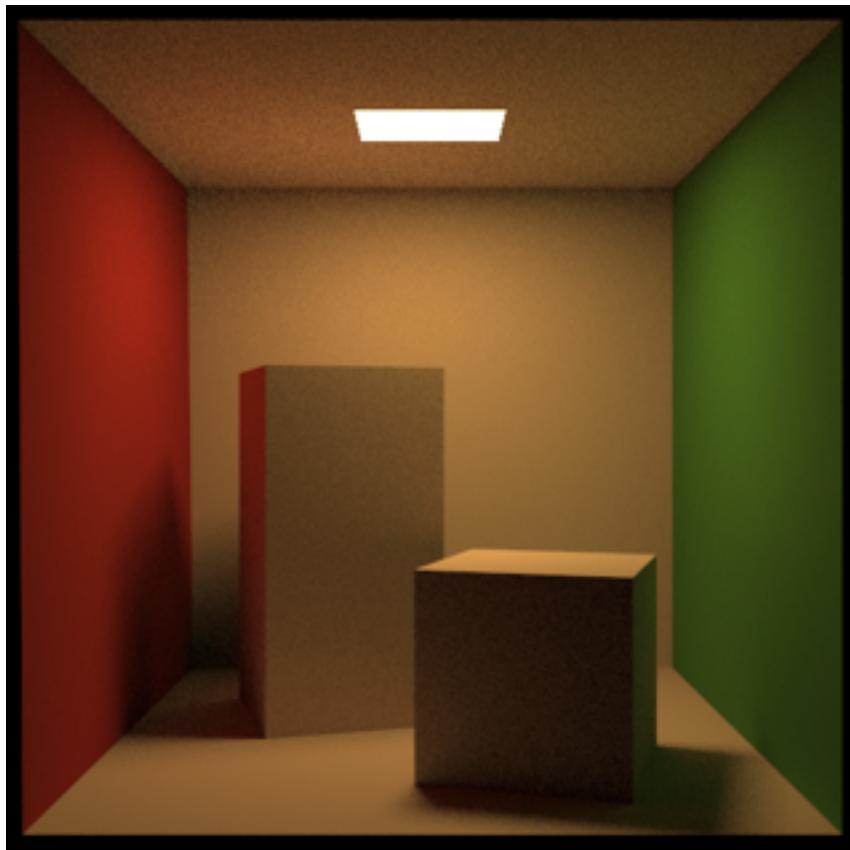
100 samples per pixel, 26s



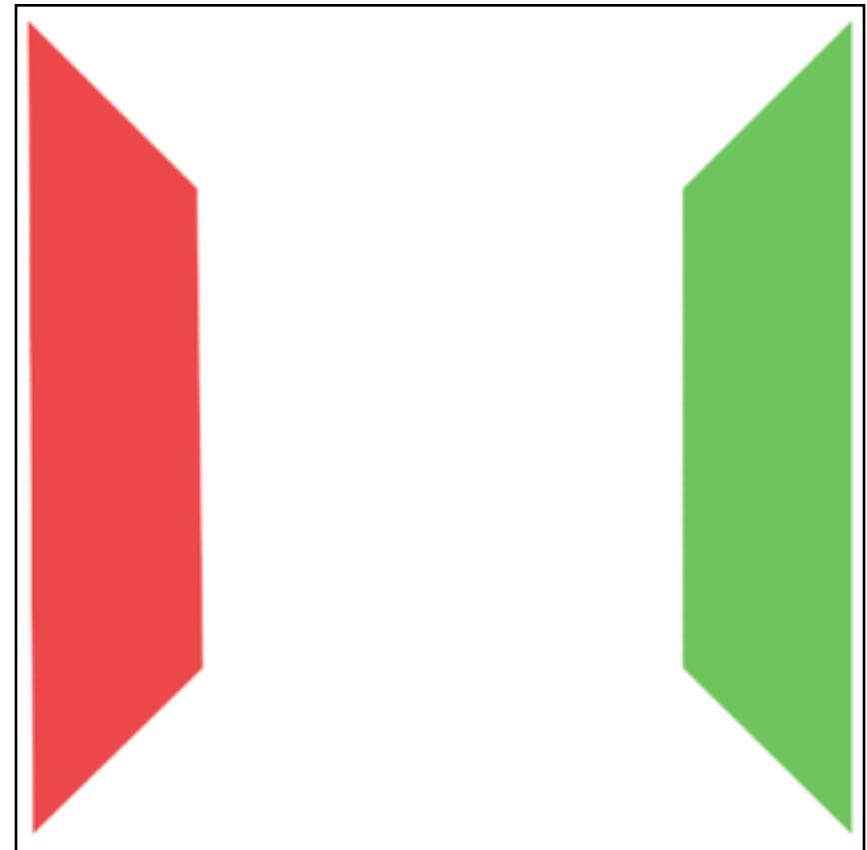
Normal, remapped to [0,1]

# Joint NL-Means Filter

---



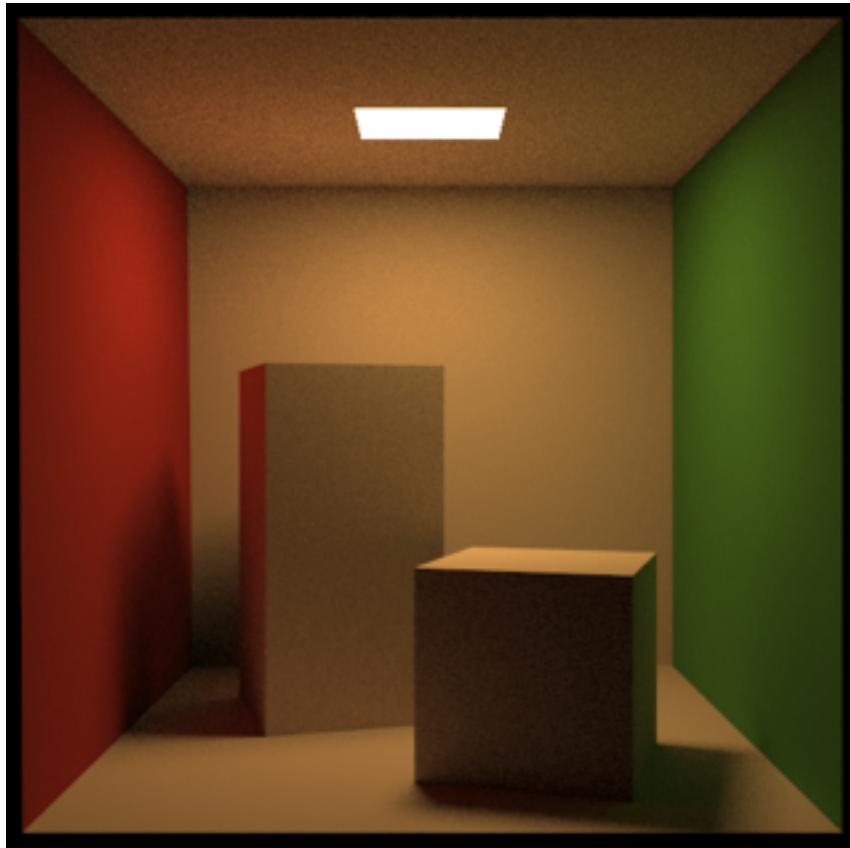
100 samples per pixel, 26s



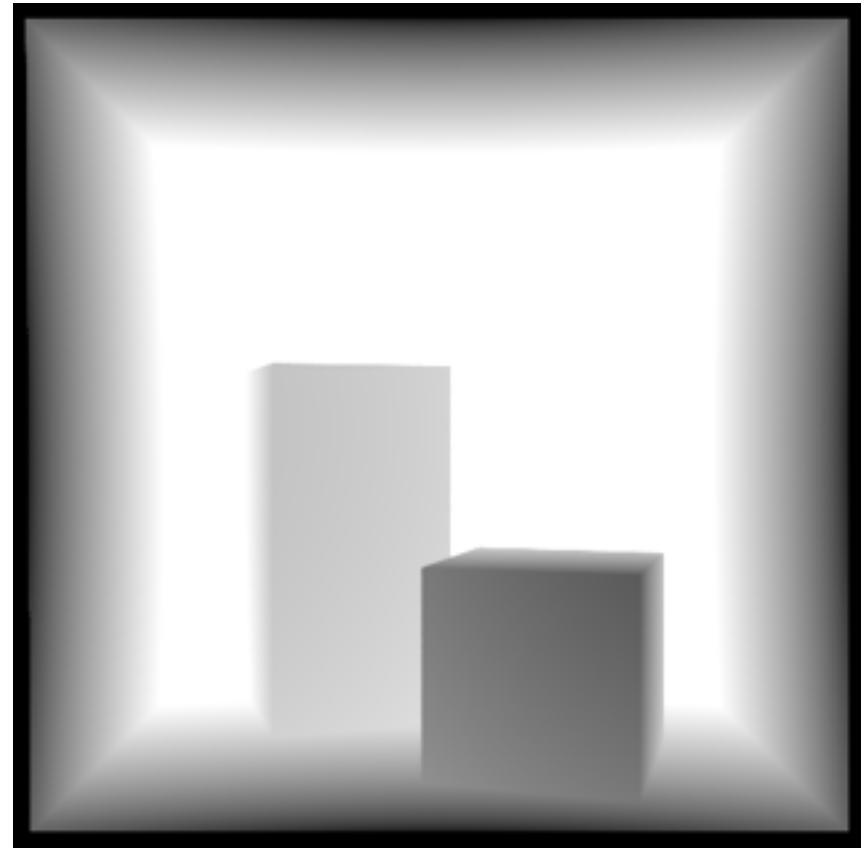
Albedo

# Joint NL-Means Filter

---



100 samples per pixel, 26s



Depth, remapped to  $[0,1]$

# Joint NL-Means – Feature Weights

---

- Feature Distance
  - Assume noise-free features

$$\Phi_j^2(p, q) = \frac{(f_j(p) - f_j(q))^2}{k_f^2 \max(\tau, \|\text{Grad}_j[p]\|^2)} \quad k_f = 0.60$$
$$\tau = 1e-3$$

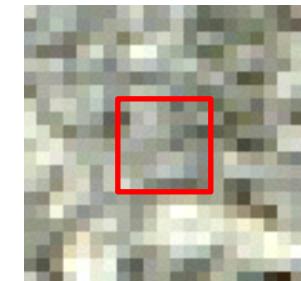
$$d_f^2(p, q) = \underset{j \in [1 \dots M]}{\operatorname{argmax}} \Phi_j^2(p, q)$$

- Pixel-based distance – ***not*** patch-based

$$w_f(p, q) = \exp^{-d_f^2(p, q)}$$

- Joint weight

$$w(p, q) = \min(w_c(p, q), w_f(p, q))$$



# Features Squared Gradient

---

```
INPUT: features (n-dimensional array)
gL = (features - shift(features, [0 -1]))/2
gR = (features - shift(features, [0 +1]))/2
gU = (features - shift(features, [+1 0]))/2
gD = (features - shift(features, [-1 0]))/2
sqrggrad = min(gL2, gR2) + min(gU2, gD2)
```

# NL-Means Filter

---

## NL-Means Denoising



100 samples per pixel  
relative MSE: 0.089E-3



Ground truth (10 000 spp)

# Joint NL-Means Filter

---

Joint NL-Means denoising

**Without** feature gradients



100 samples per pixel  
relative MSE: 0.081E-3



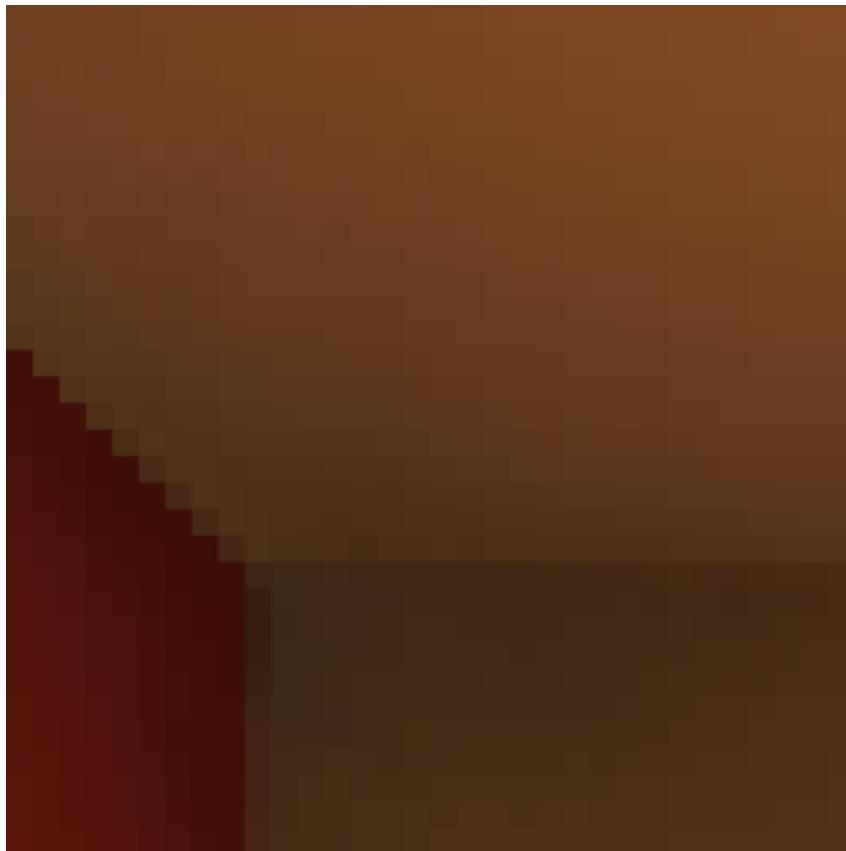
Ground truth (10 000 spp)

# Joint NL-Means Filter

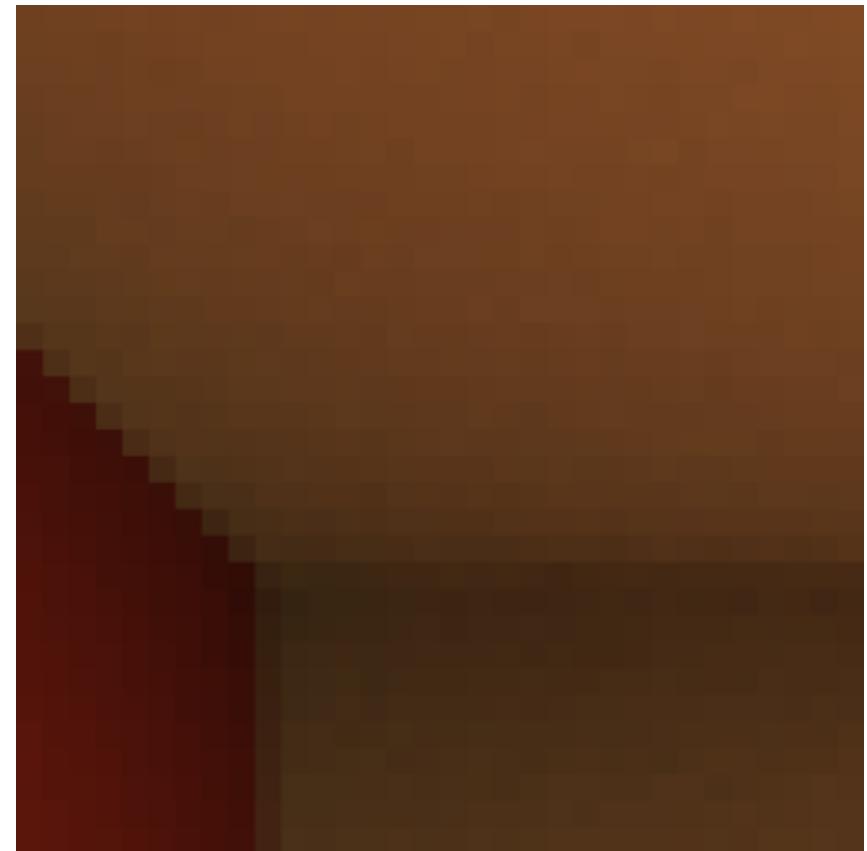
---

Joint NL-Means denoising

**With** feature gradients



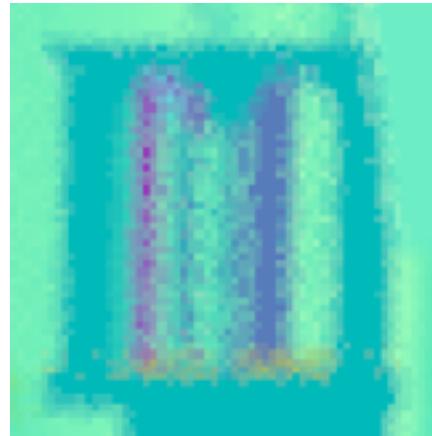
100 samples per pixel  
relative MSE: 0.071E-3



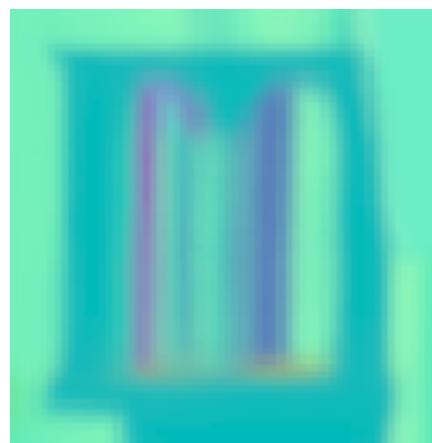
Ground truth (10 000 spp)

# Joint NL-Means – Noisy Features

---



Normal

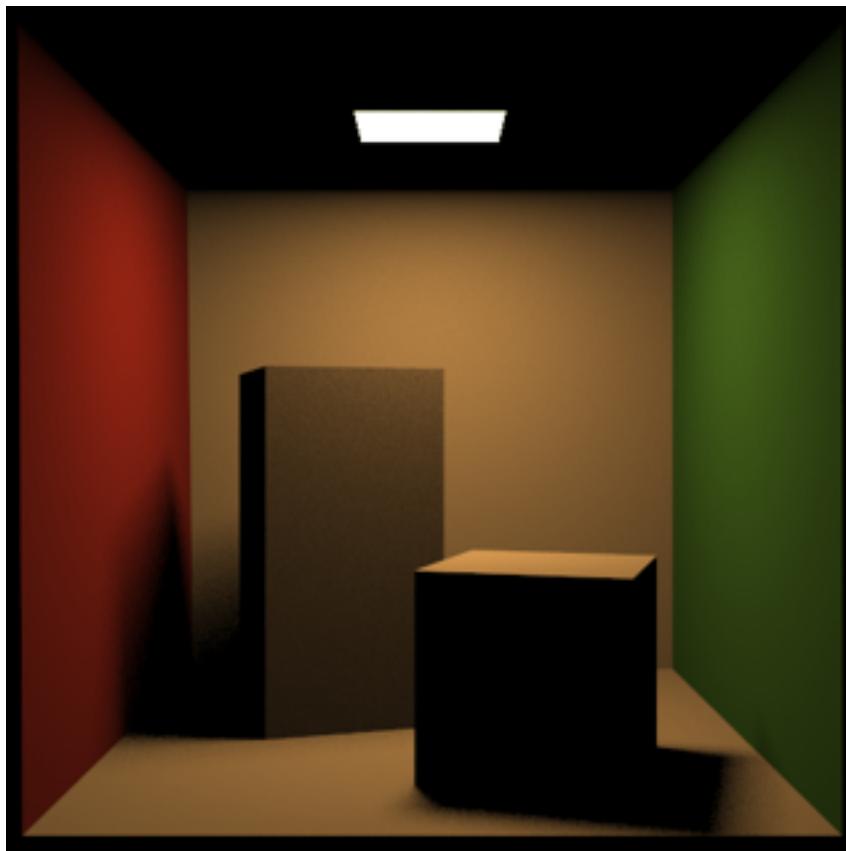


Filtered normal

# Separate Direct/Indirect Illumination

---

Direct Illumination



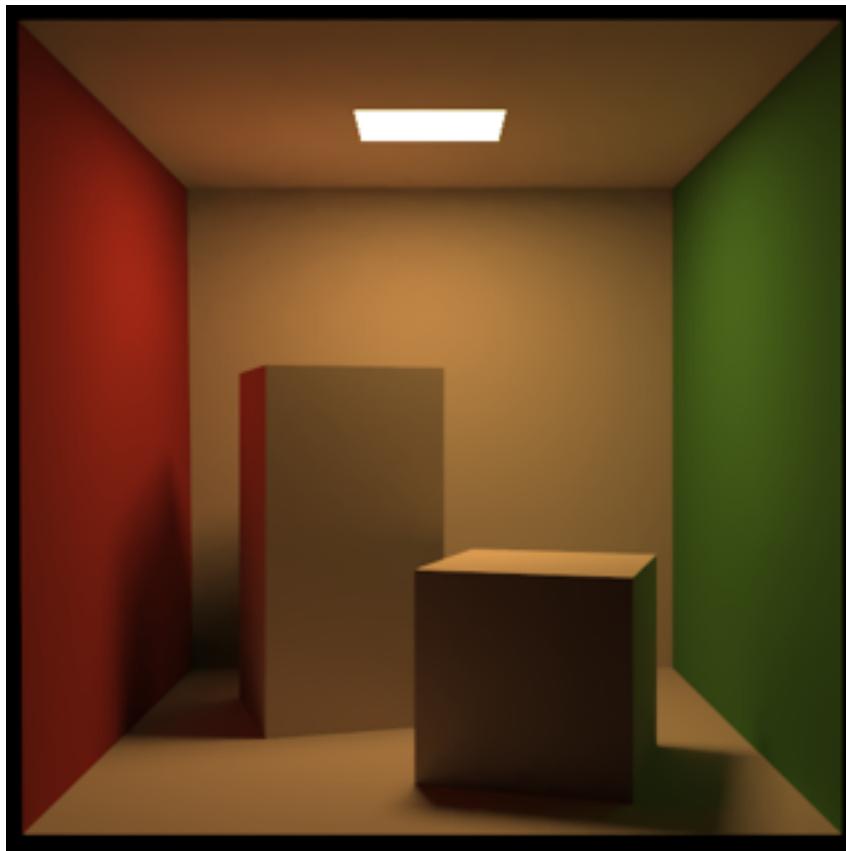
Indirect Illumination



100 samples per pixel  
relative MSE: 1.573E-3

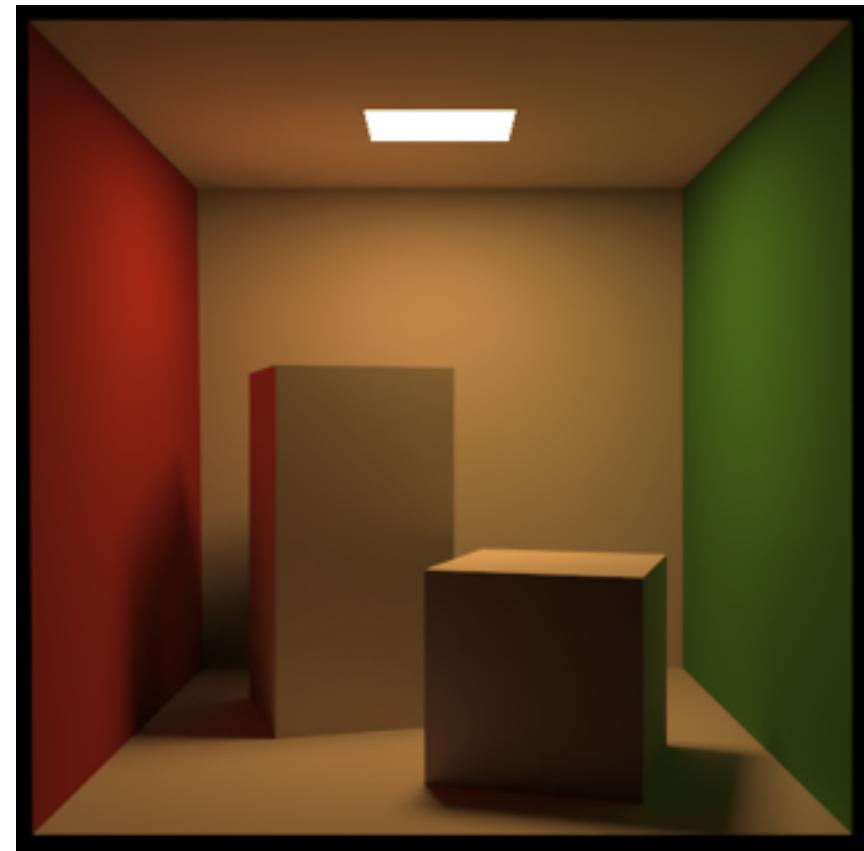
# Separate Direct/Indirect Illumination

Final color denoising



100 samples per pixel  
relative MSE: 0.071E-3

Separate  
direct/indirect denoising

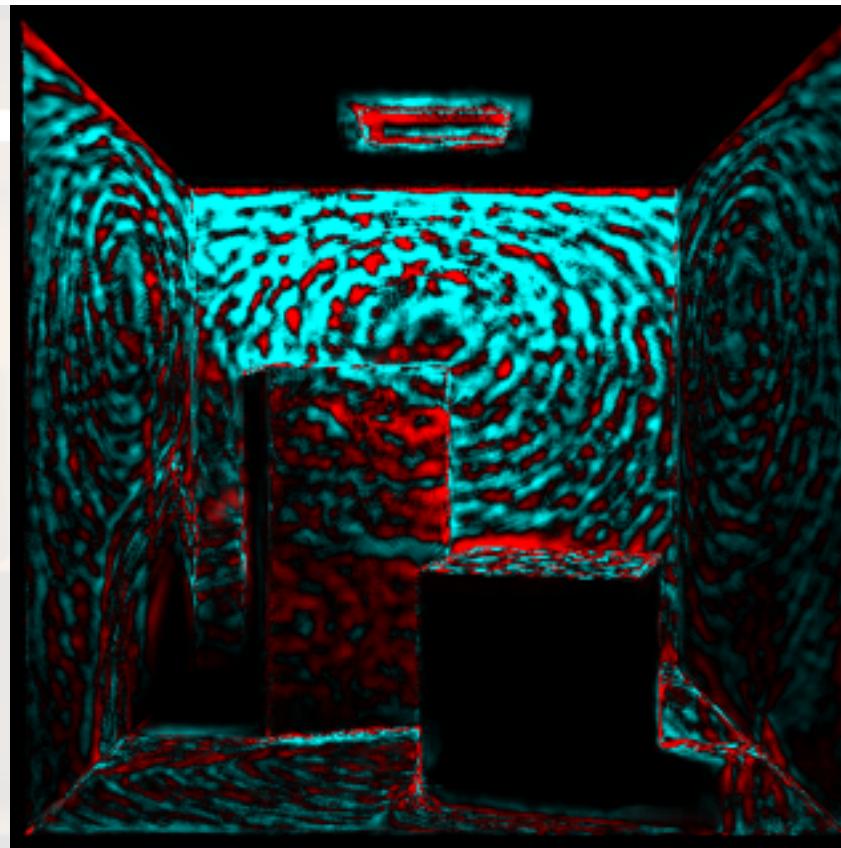


100 samples per pixel  
relative MSE: 0.059E-3

# Separate Direct/Indirect Illumination

---

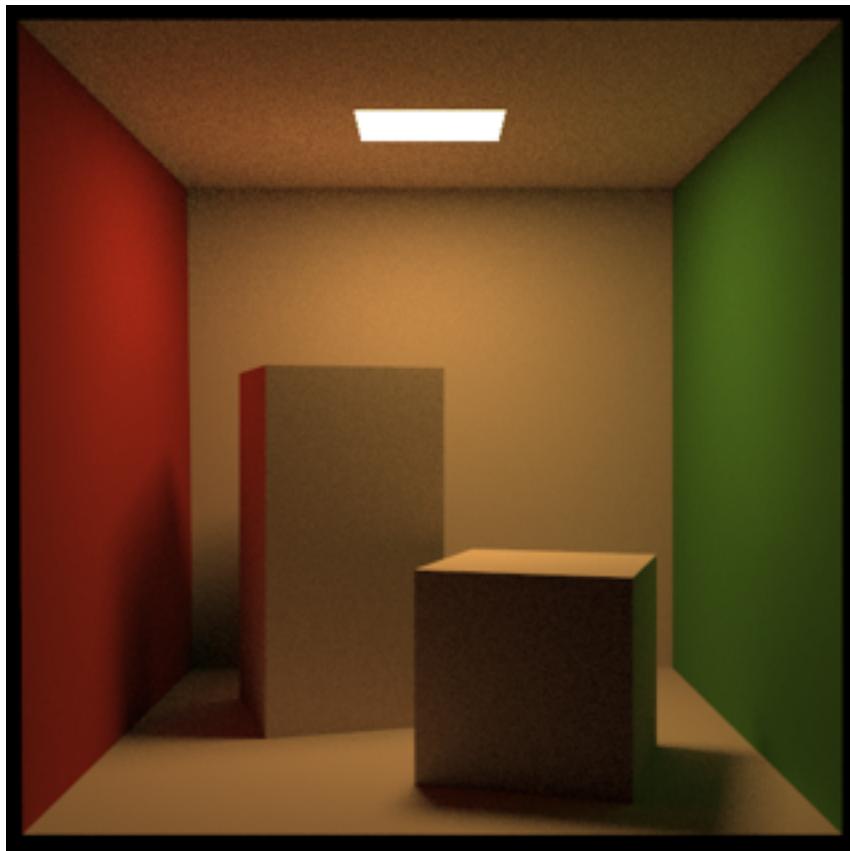
Difference x1000



# Final Result: 26x reduction of rMSE

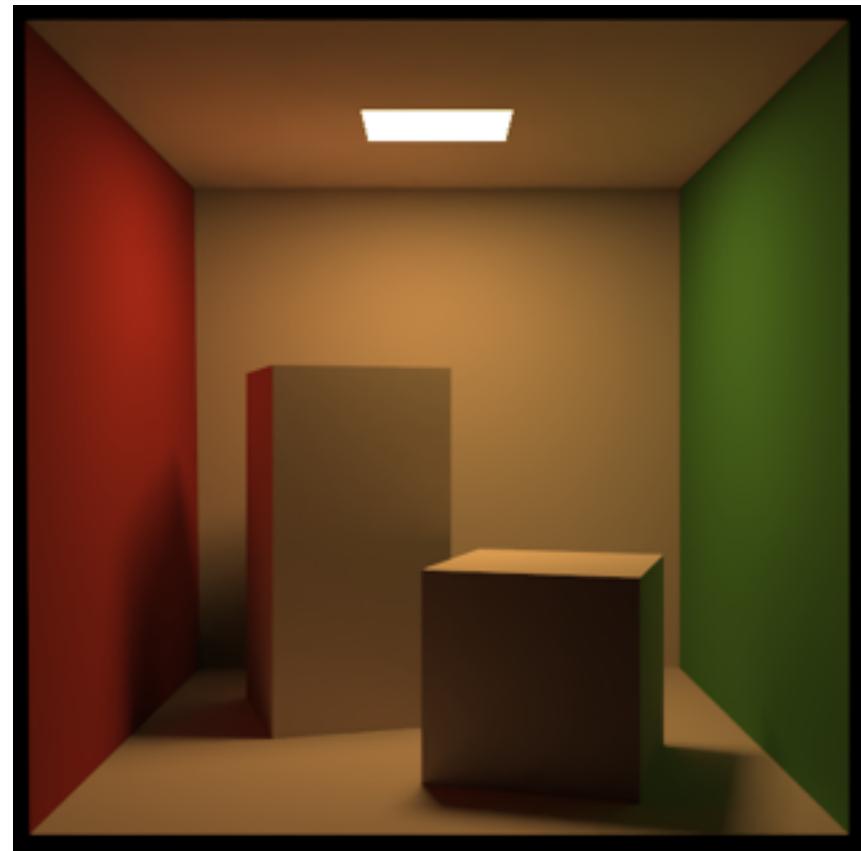
---

Input



100 samples per pixel  
relative MSE: 1.573E-3

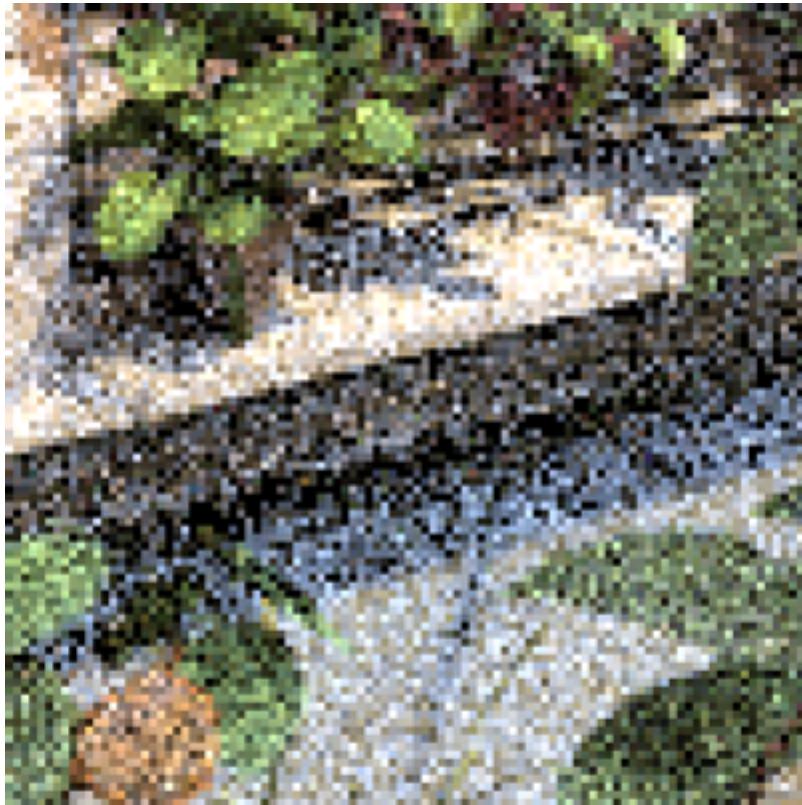
Denoised



100 samples per pixel  
relative MSE: 0.059E-3

# Denoising Results

---

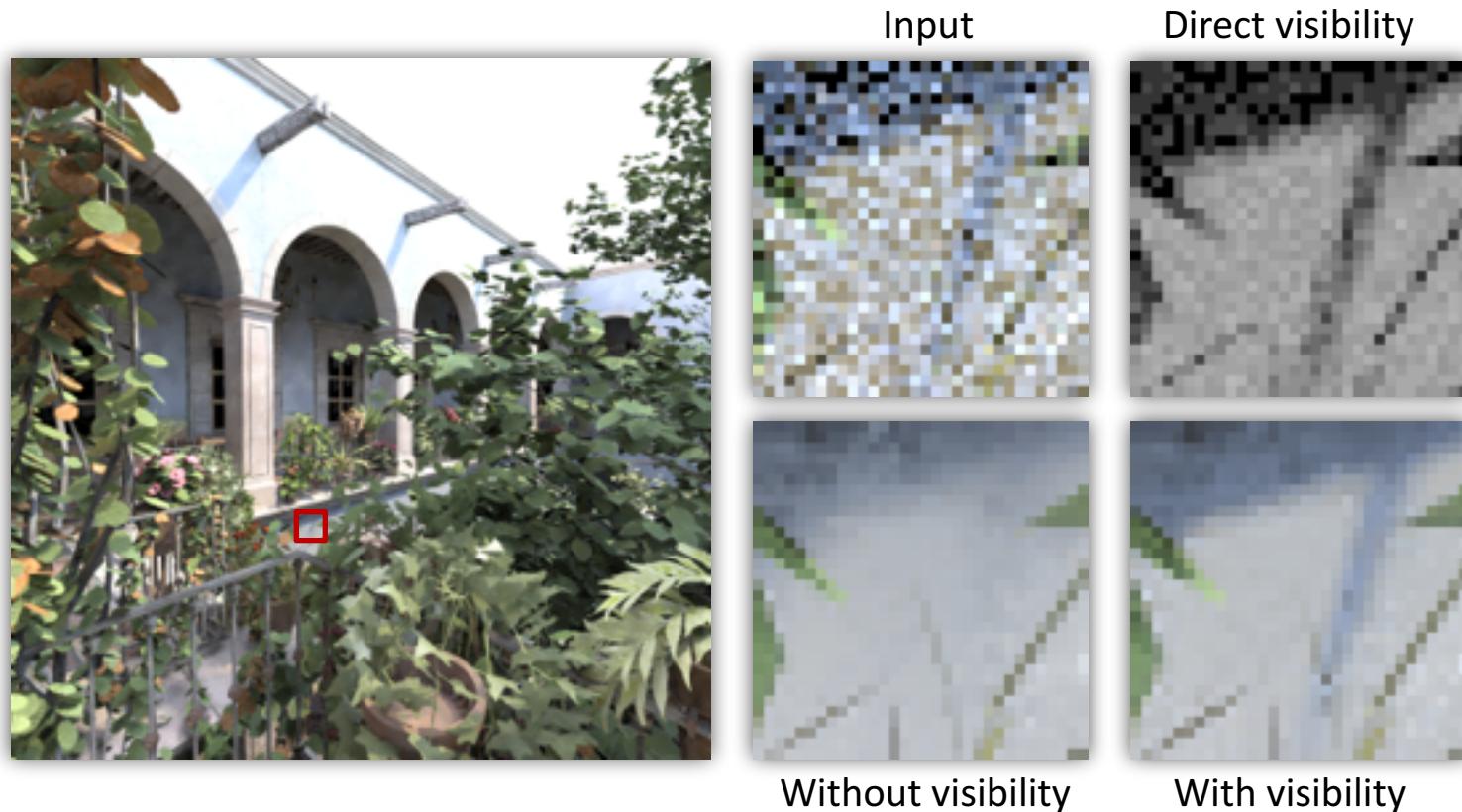


Environment lighting + indirect illumination  
Complex geometry

# Robustness – new features

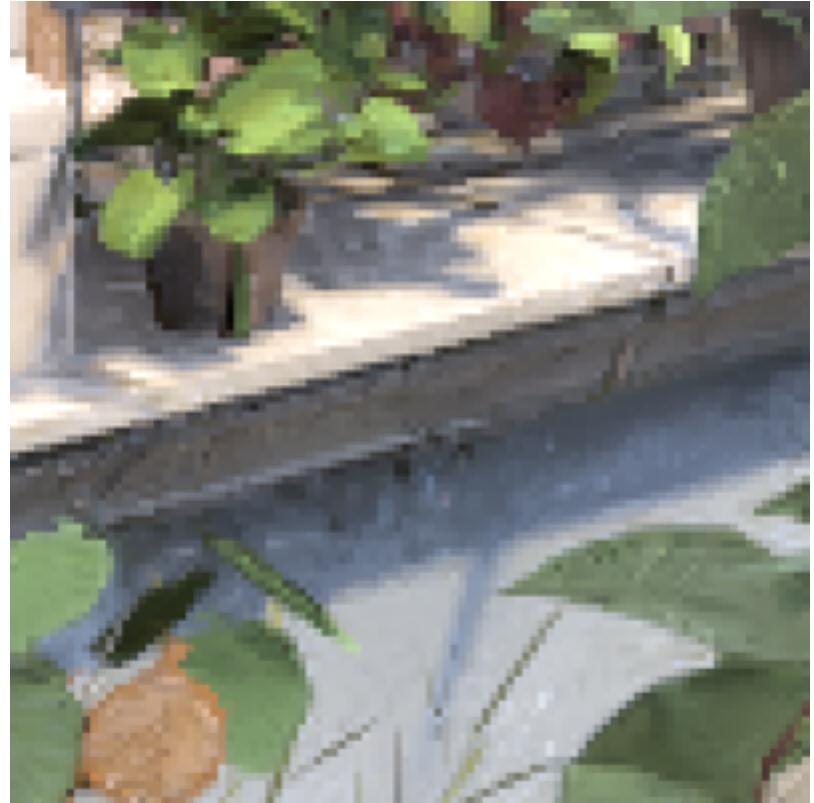
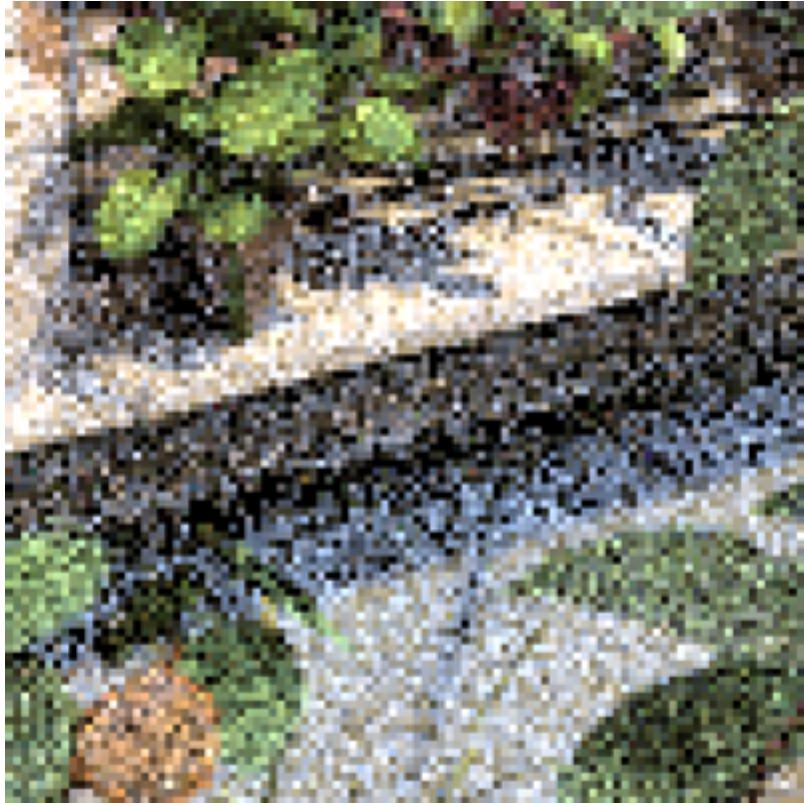
## Robust Denoising using Features and Color Information

Rousselle et al., Pacific Graphics 2013



# Denoising Results

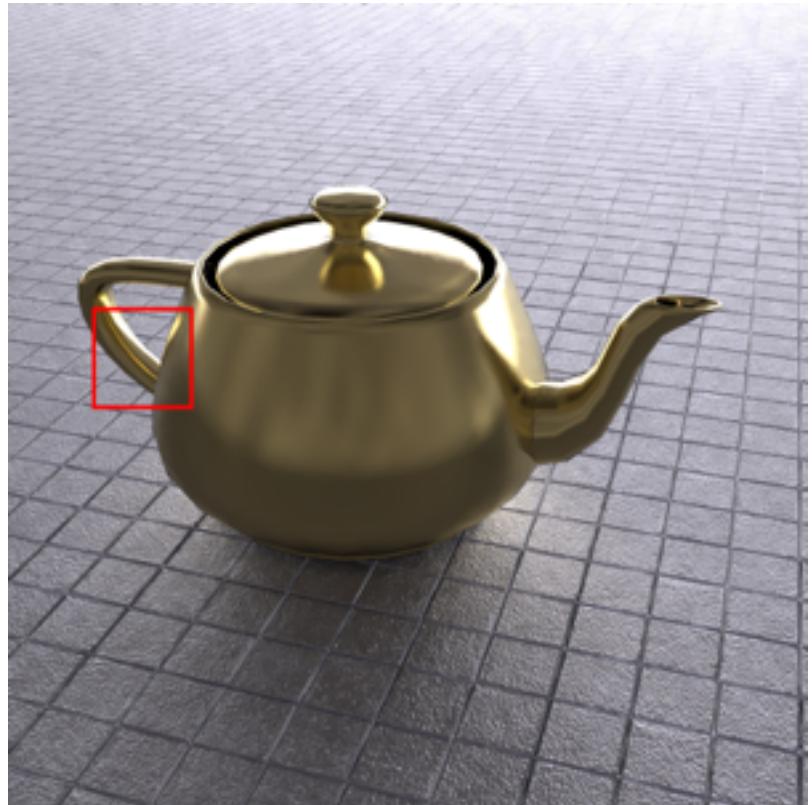
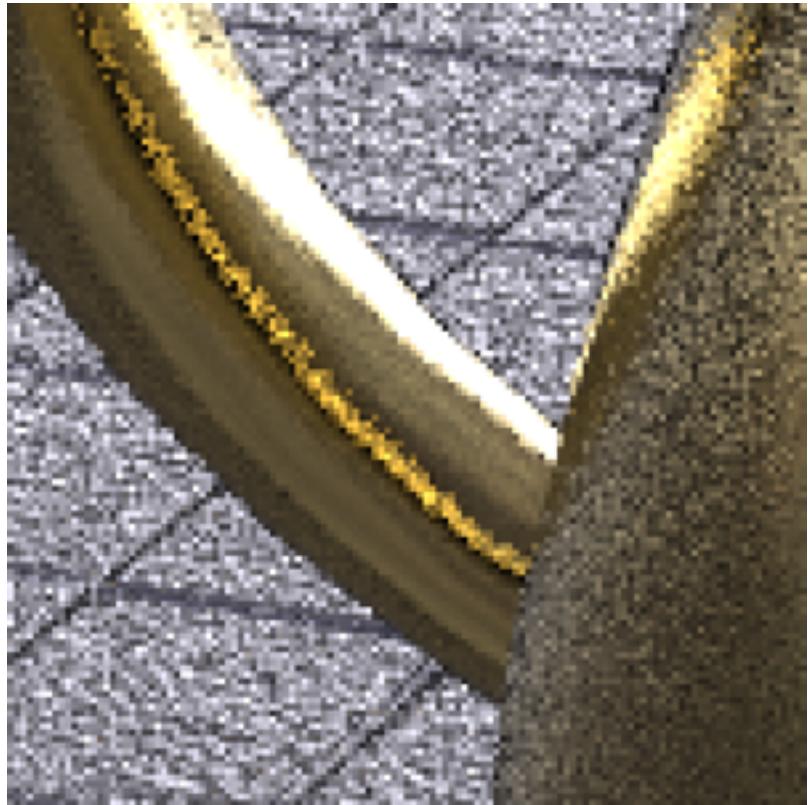
---



Environment lighting + indirect illumination  
Complex geometry

# Denoising Results

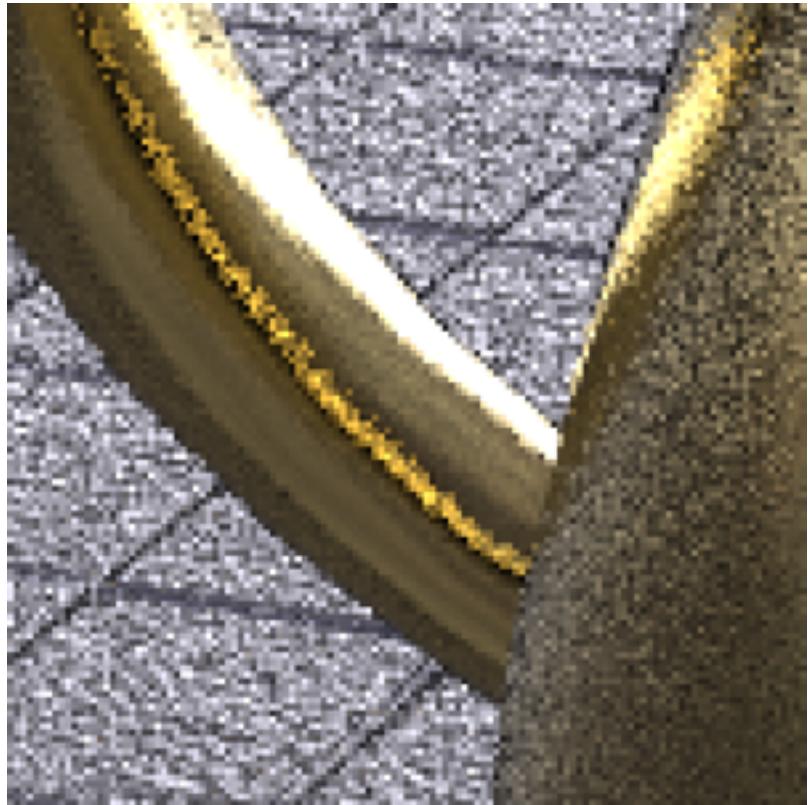
---



Environment lighting + indirect illumination  
Bump map + glossy reflections

# Denoising Results

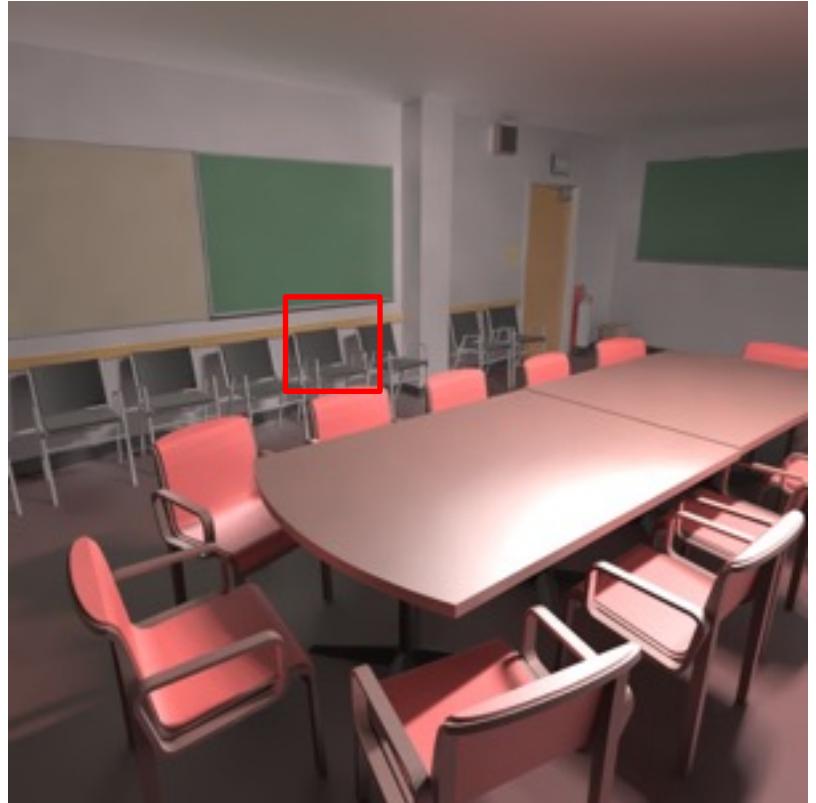
---



Environment lighting + indirect illumination  
Bump map + glossy reflections

# Denoising Results

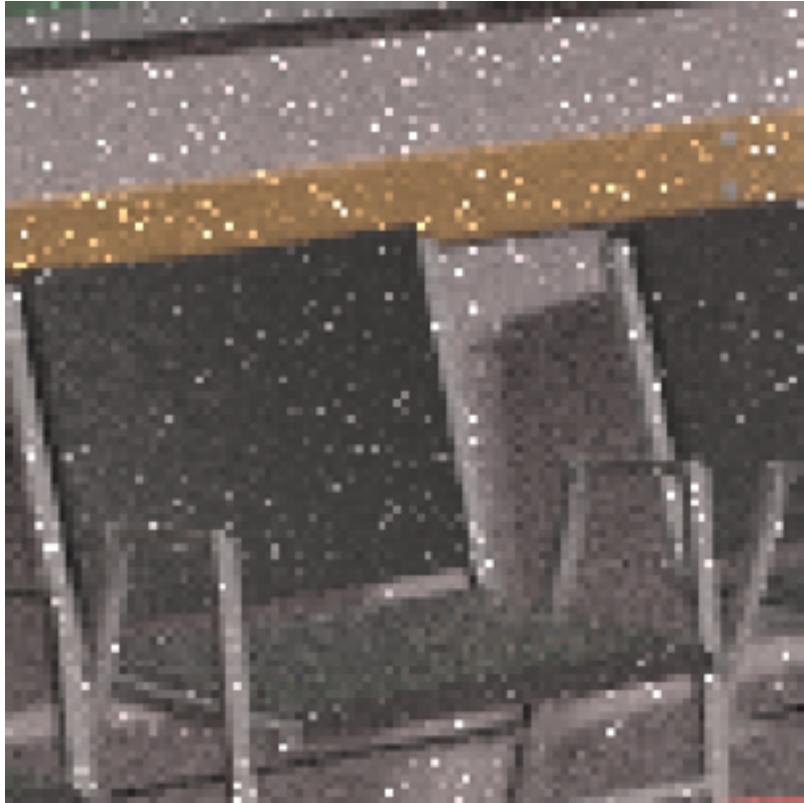
---



Area lighting + indirect illumination  
Spike noise

# Denoising Results

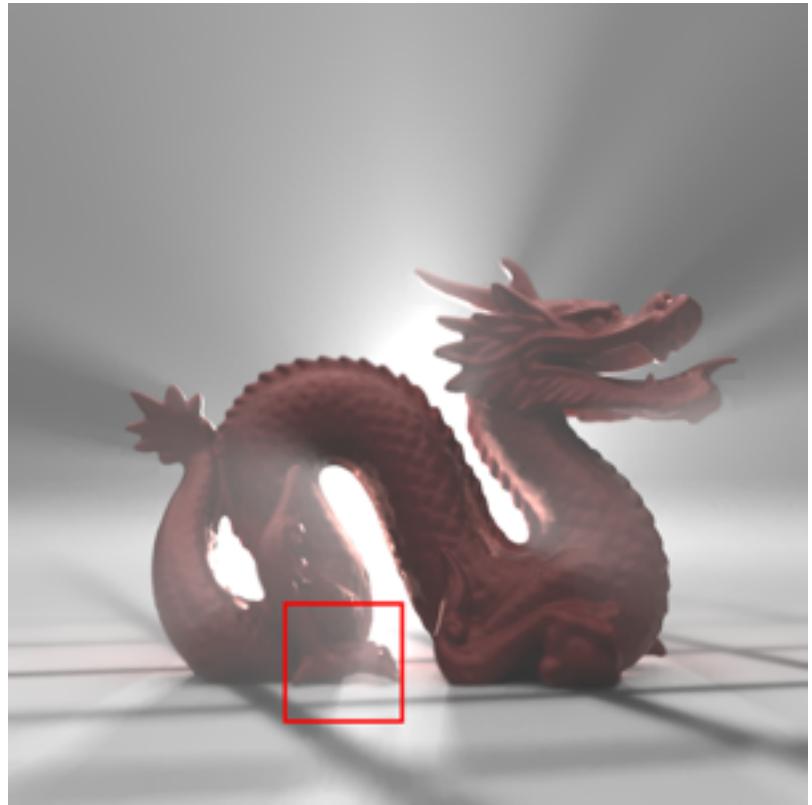
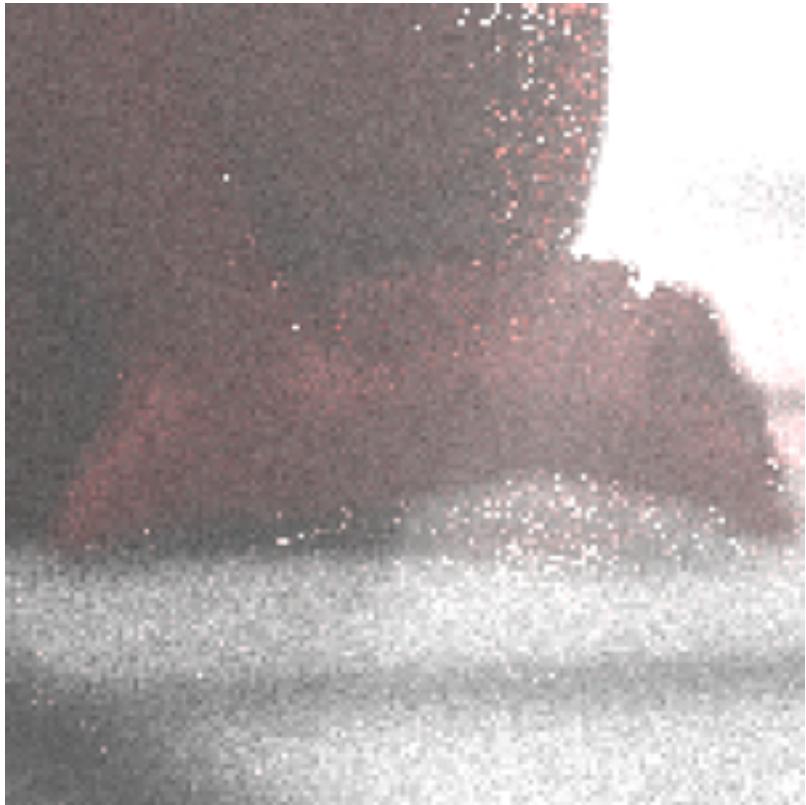
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Area lighting + indirect illumination  
Spike noise

# Denoising Results

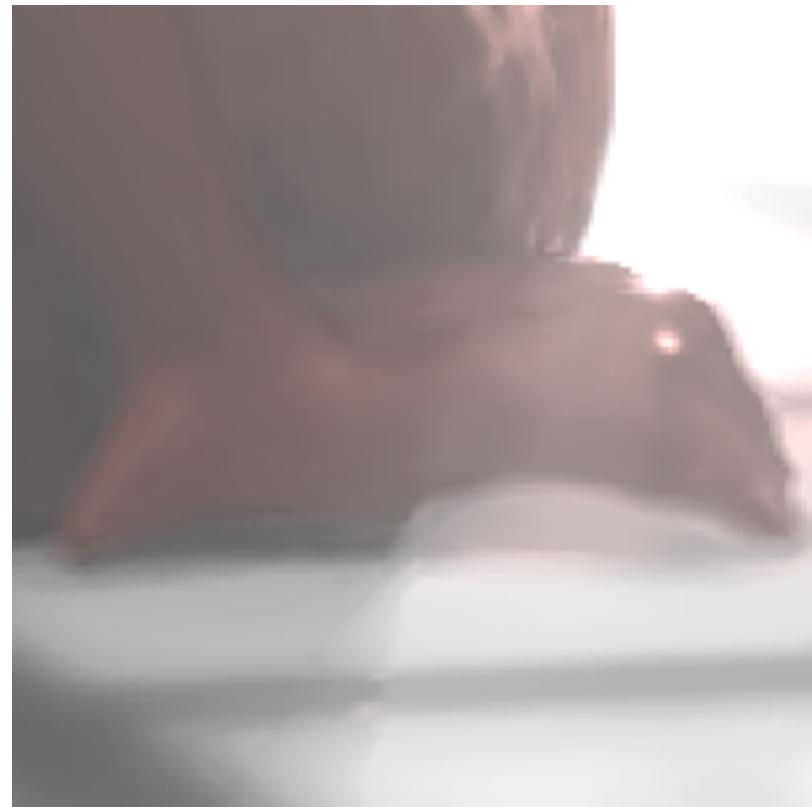
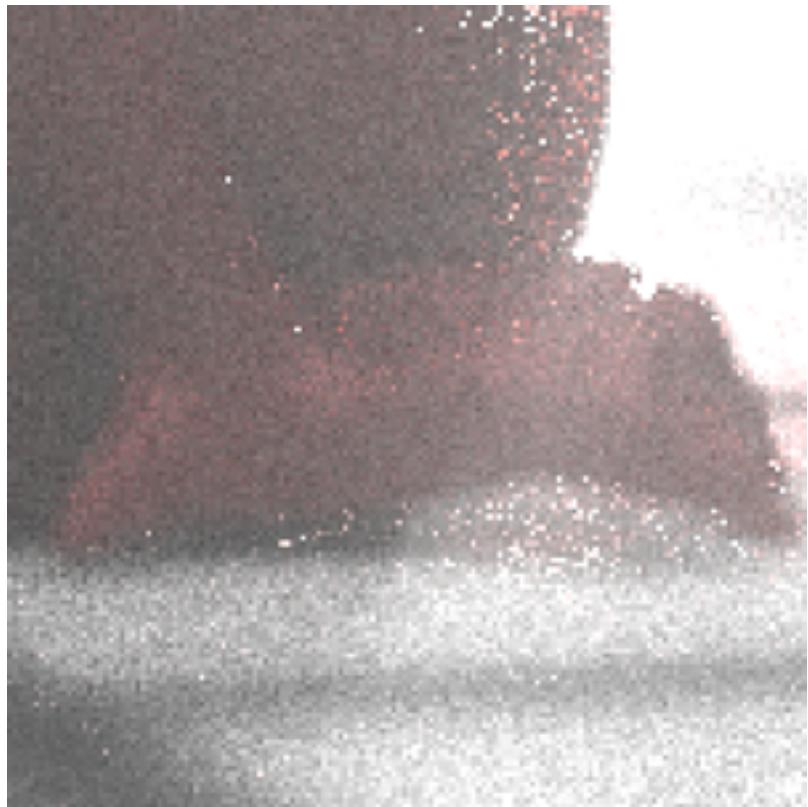
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Area lighting + indirect illumination +  
depth-of-field + participating media

# Denoising Results

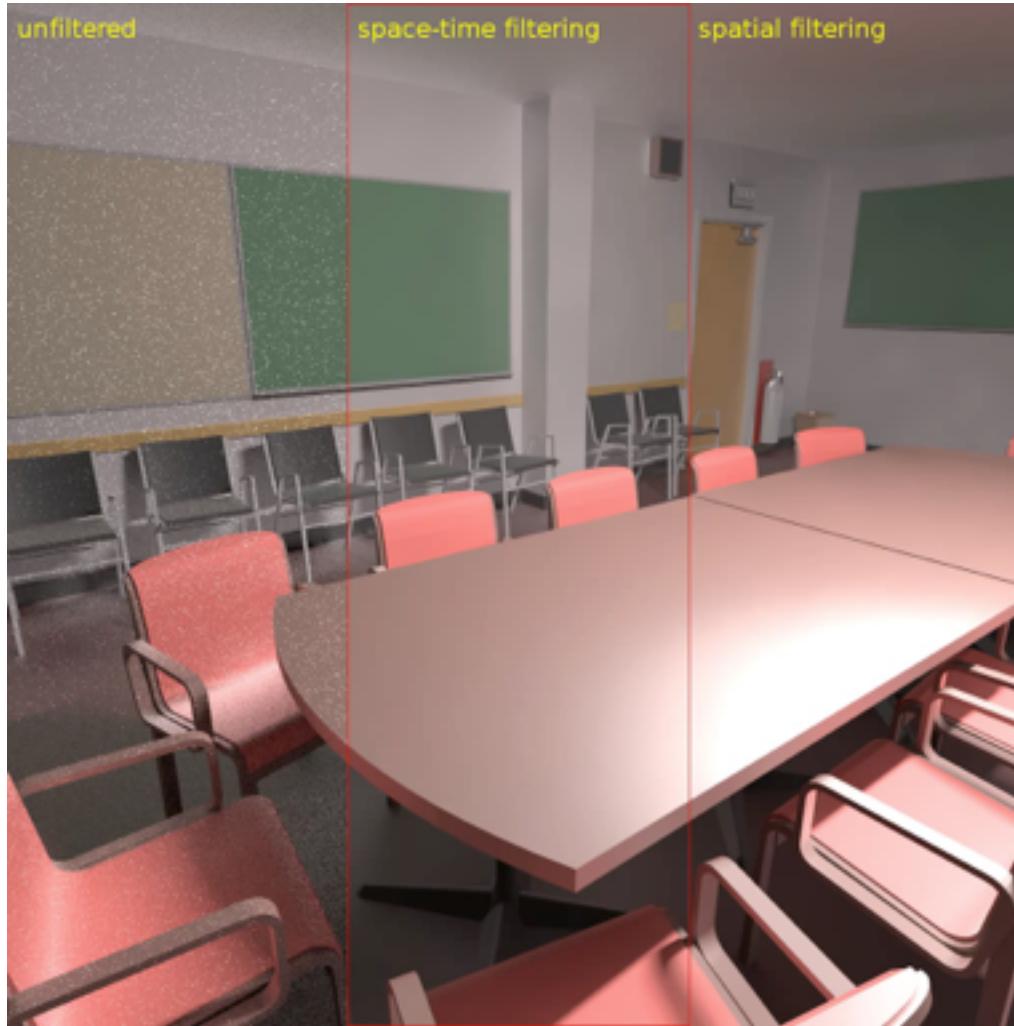
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Area lighting + indirect illumination +  
depth-of-field + participating media

# Denoising Results – Animation

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# Image-based *a posteriori* methods

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- Very effective
  - Preserve the generality of MC rendering
  - Low computational overhead
  - Moderately intrusive for the renderer
- Limitations
  - Residual low-frequency noise (use spatio-temporal denoising)
  - Not robust at low sampling rates
  - Cannot handle conflicting constraints (motion blur in front of a static object)

# NL-Means filter – production use

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