CS 87/187, Spring 2016

Rendering Algorithms

Subsurface Scattering

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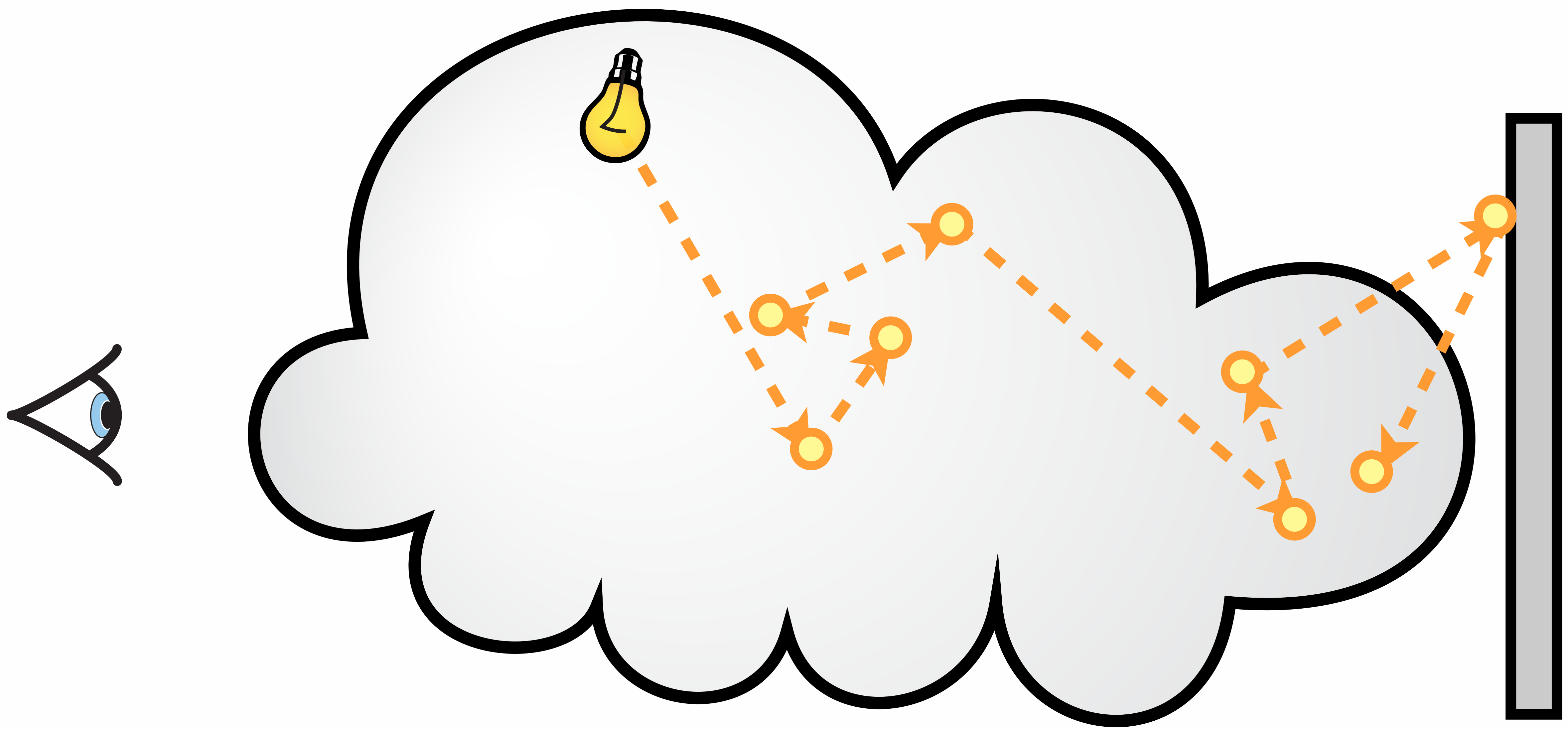
(with slide improvements by Jan Novák)

Dartmouth

Last Lecture

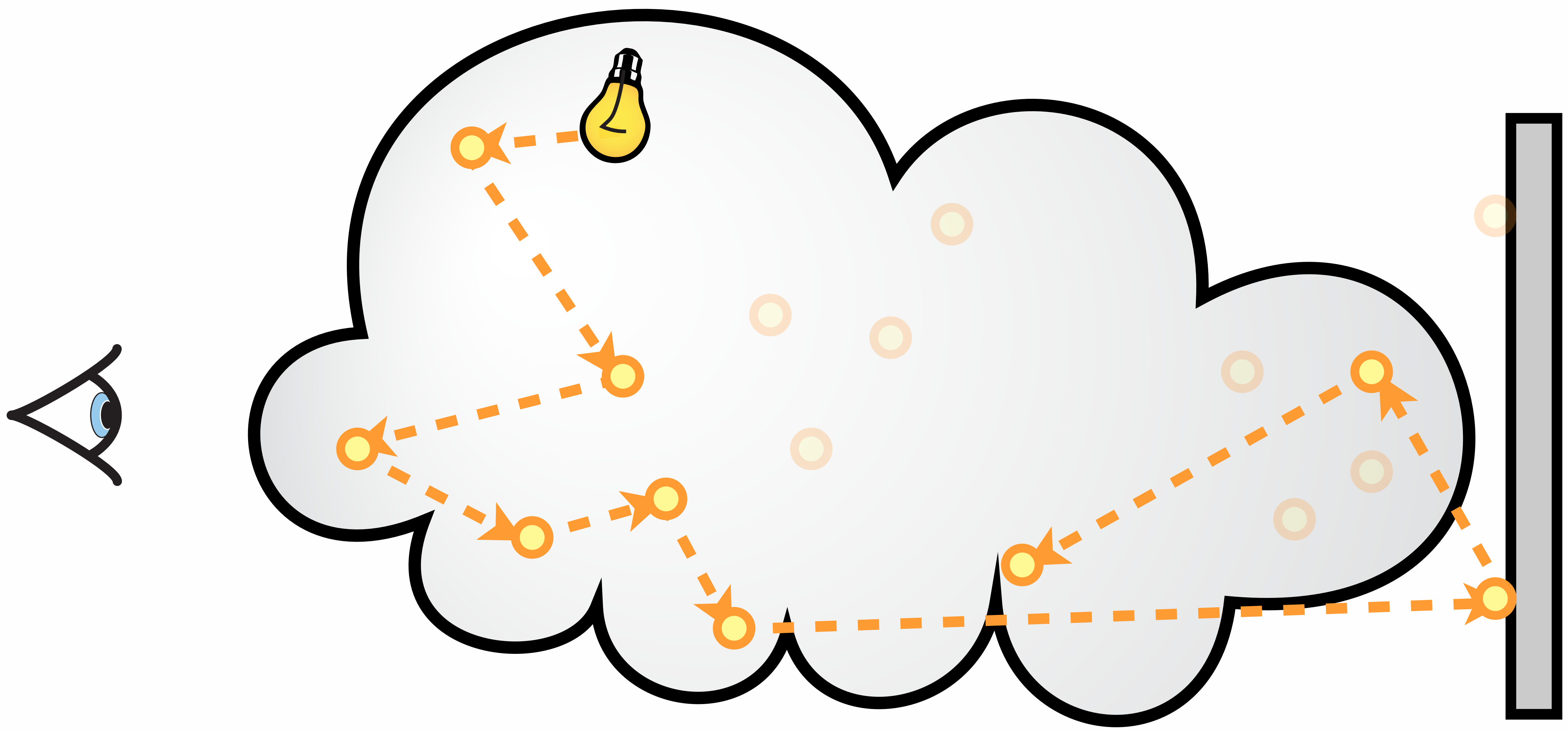
Volumetric Photon Mapping

[Jensen & Christensen 98]



Volumetric Photon Mapping

[Jensen & Christensen 98]



Volumetric Photon Mapping

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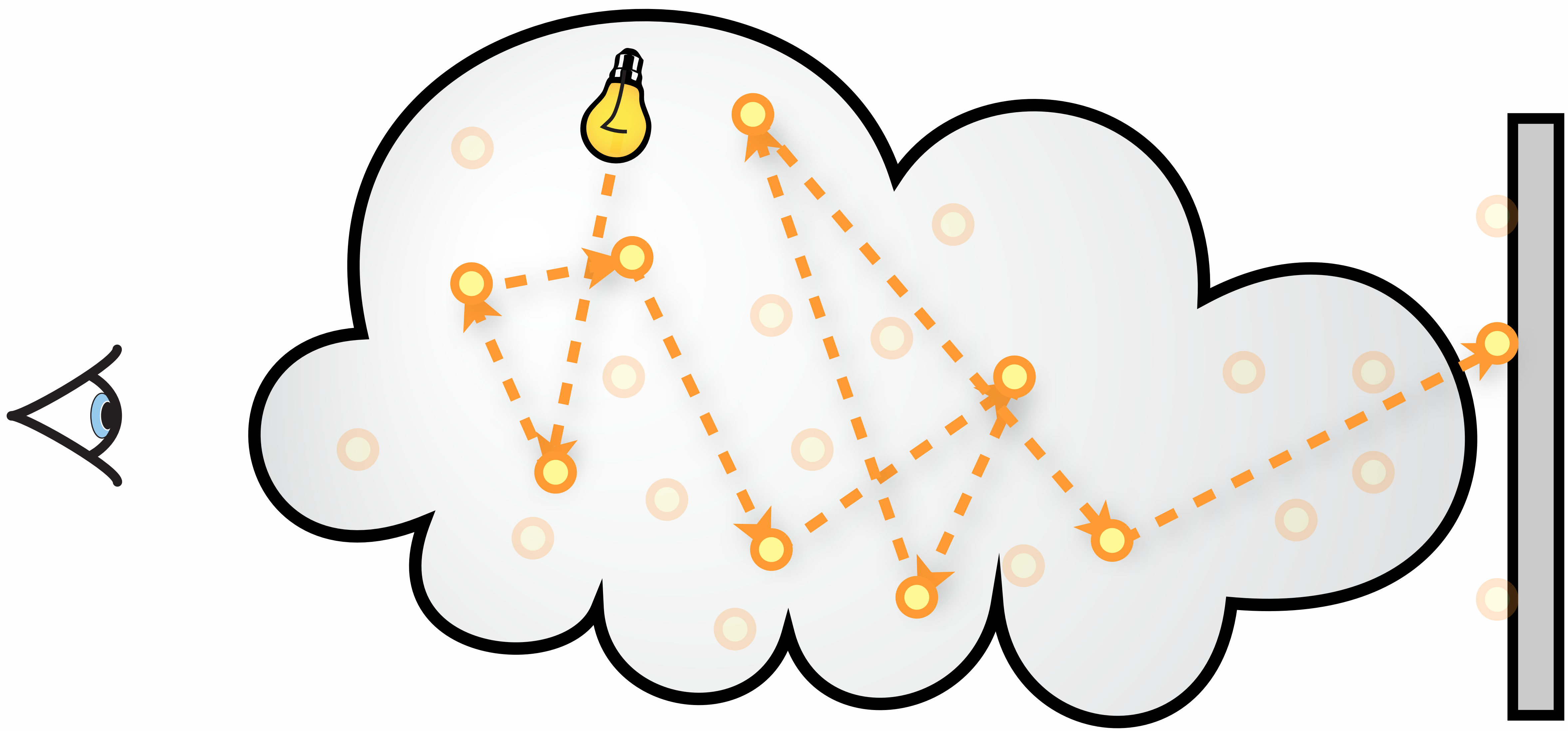

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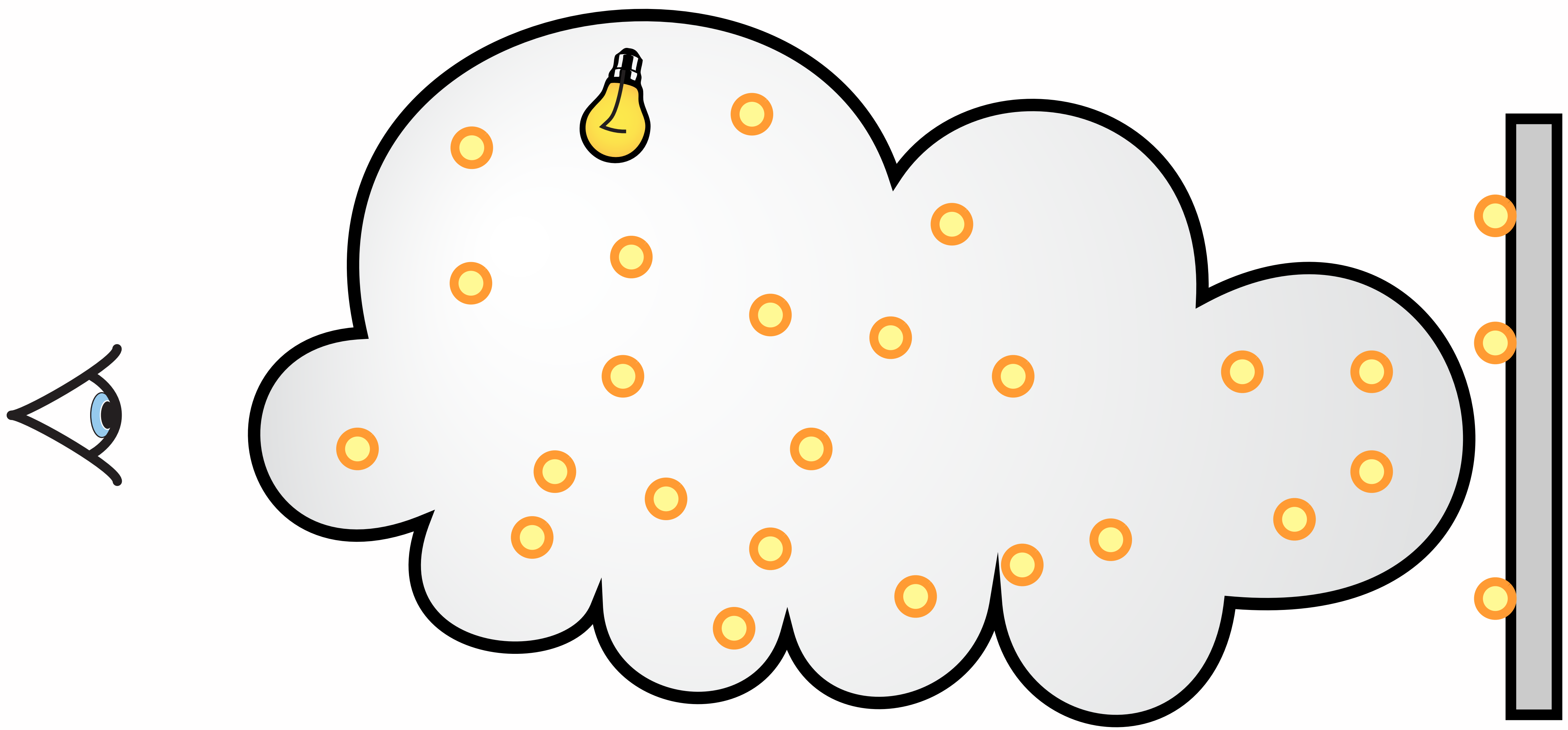

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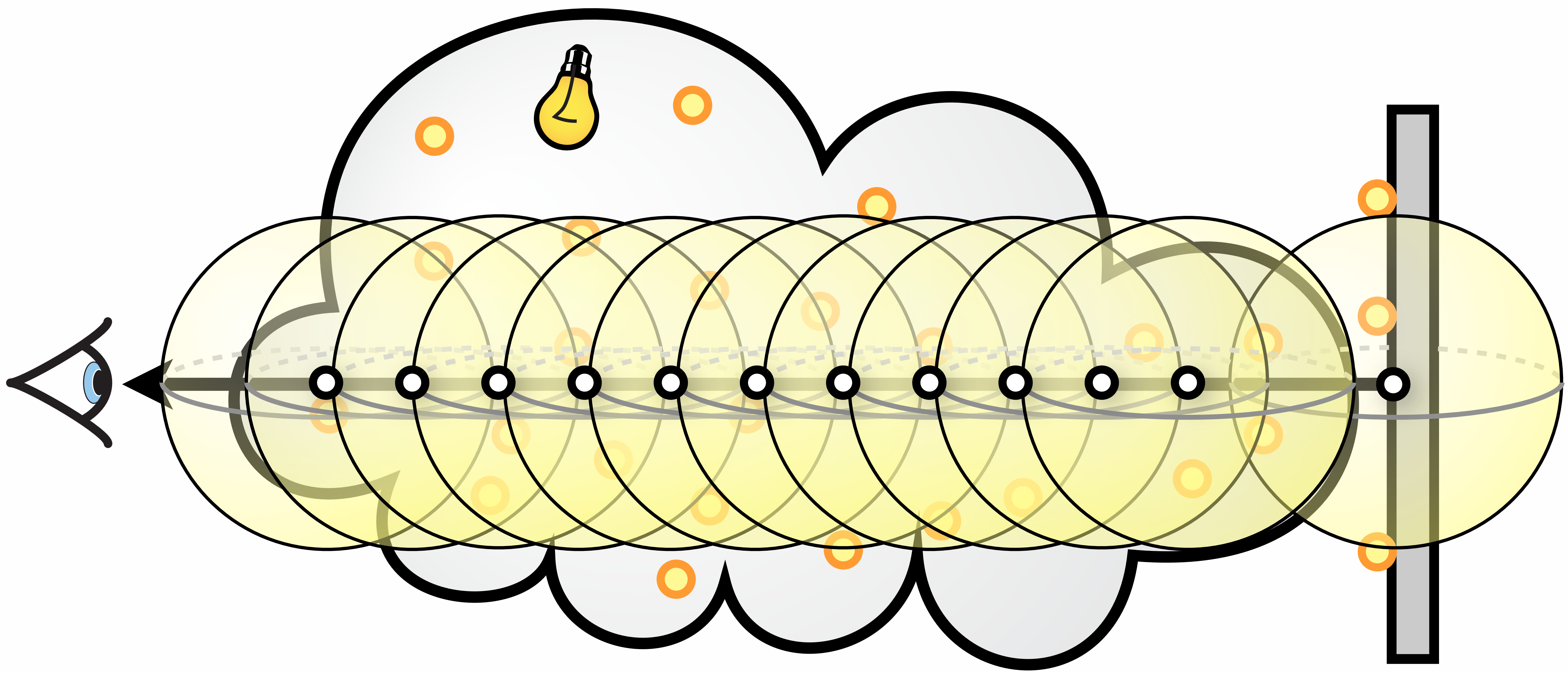

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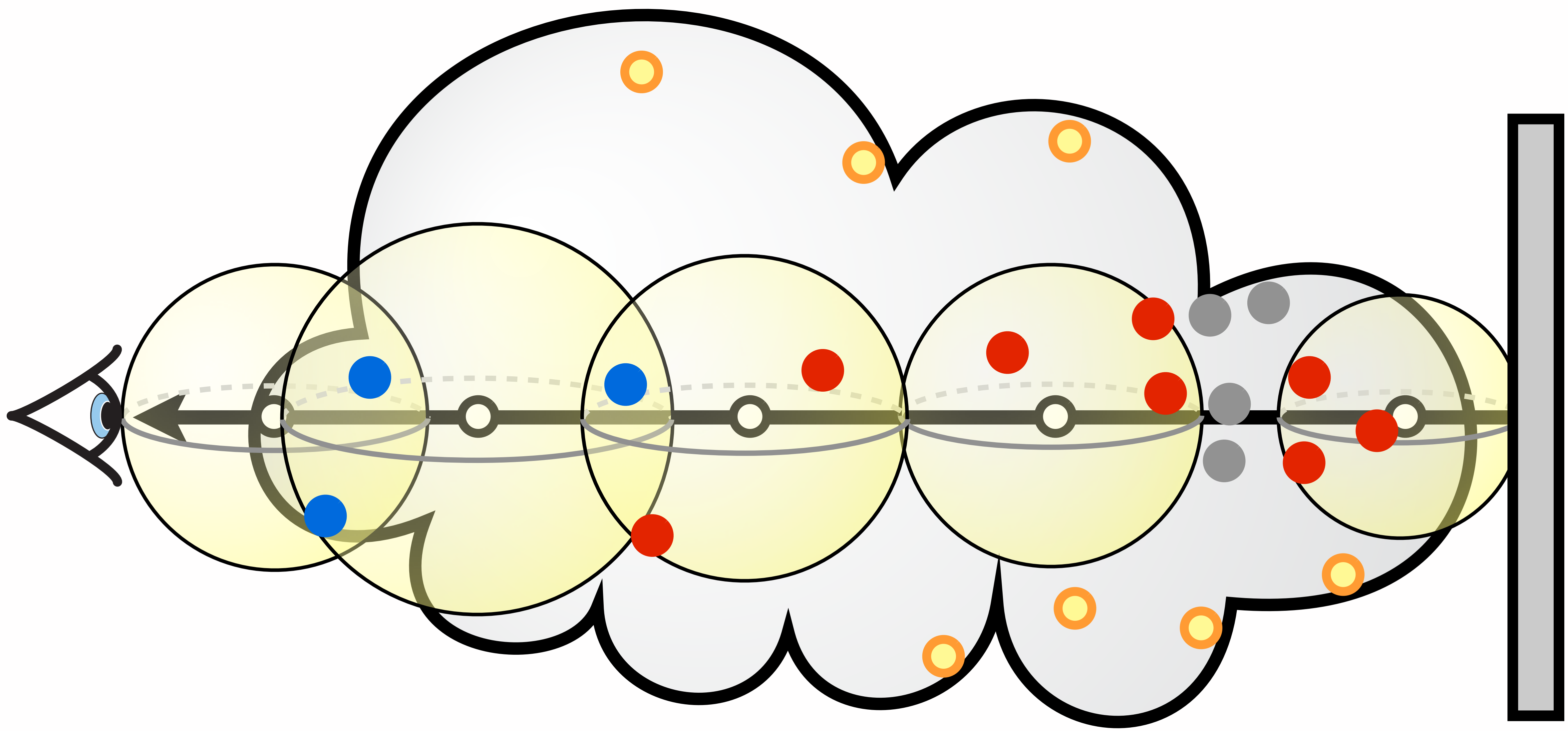
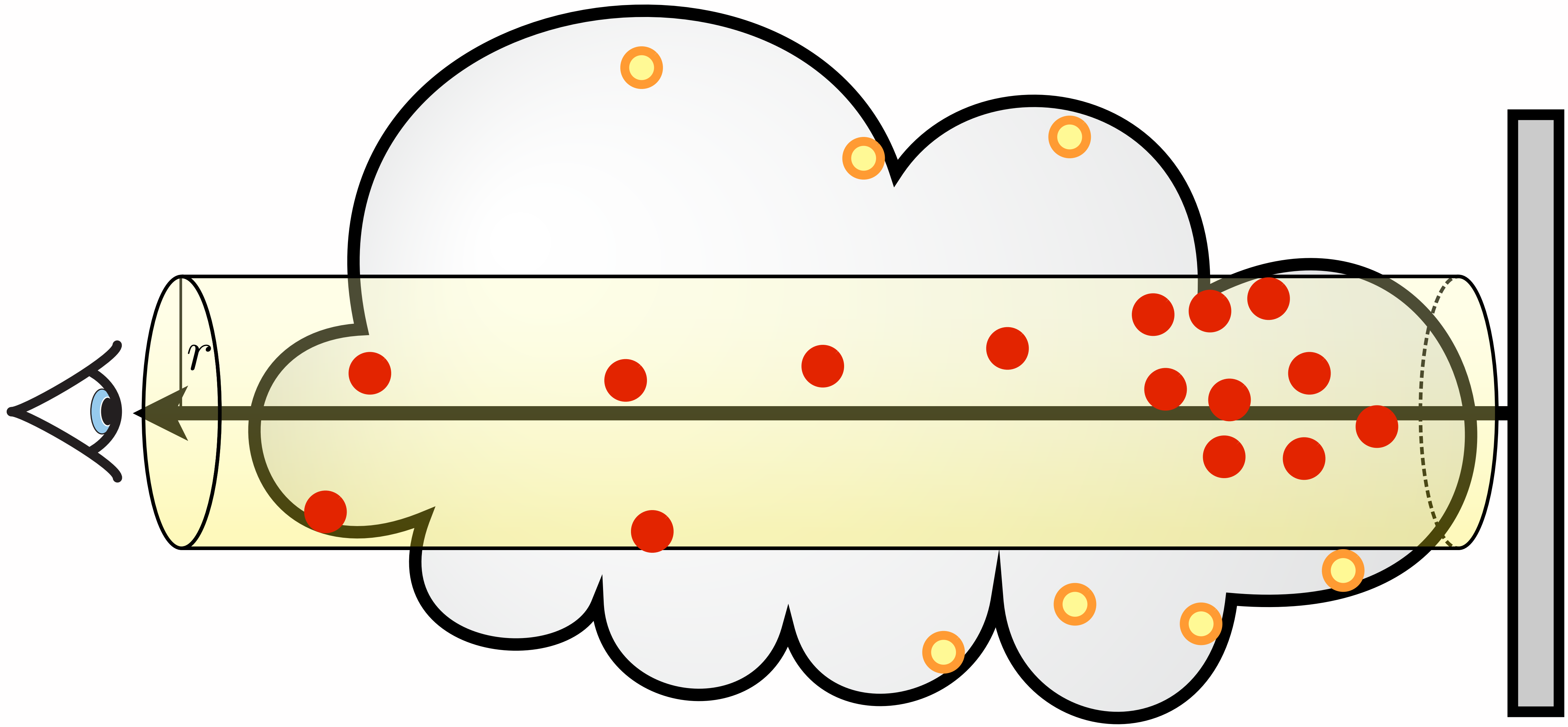


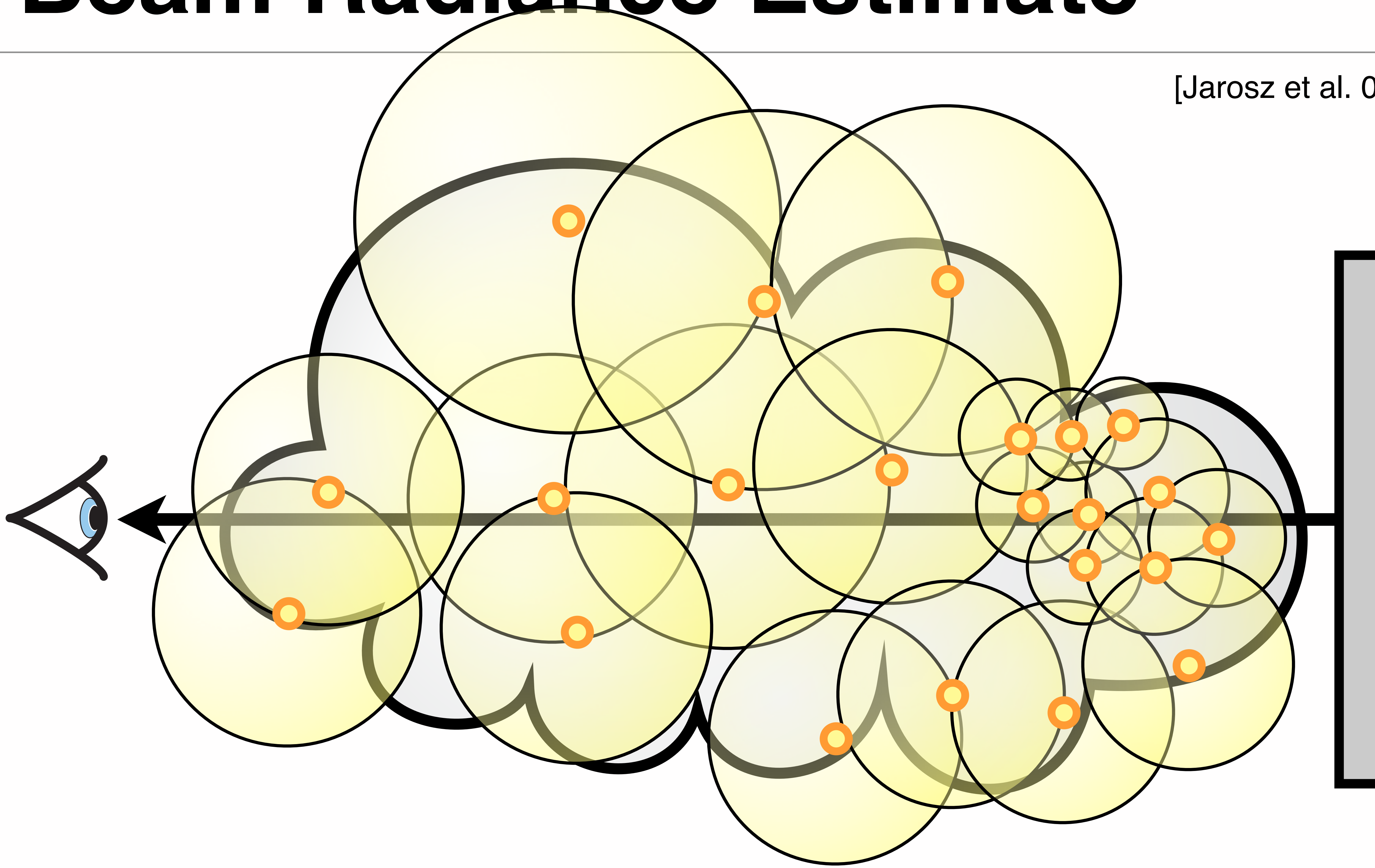
Beam Radiance Estimate

[Jarosz et al. 08]



Beam Radiance Estimate

[Jarosz et al. 08]



Beam Radiance Estimate

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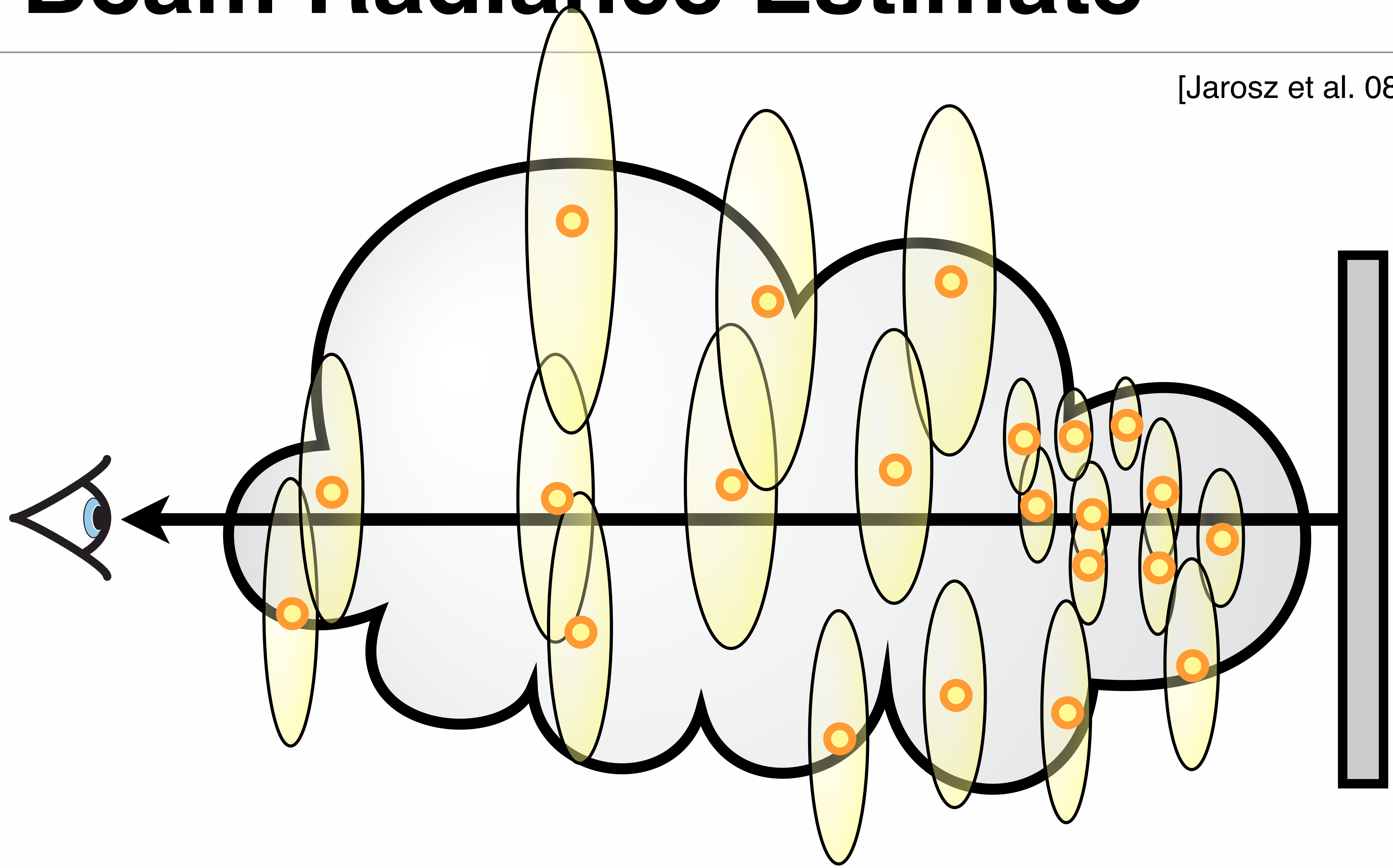

Beam Radiance Estimate

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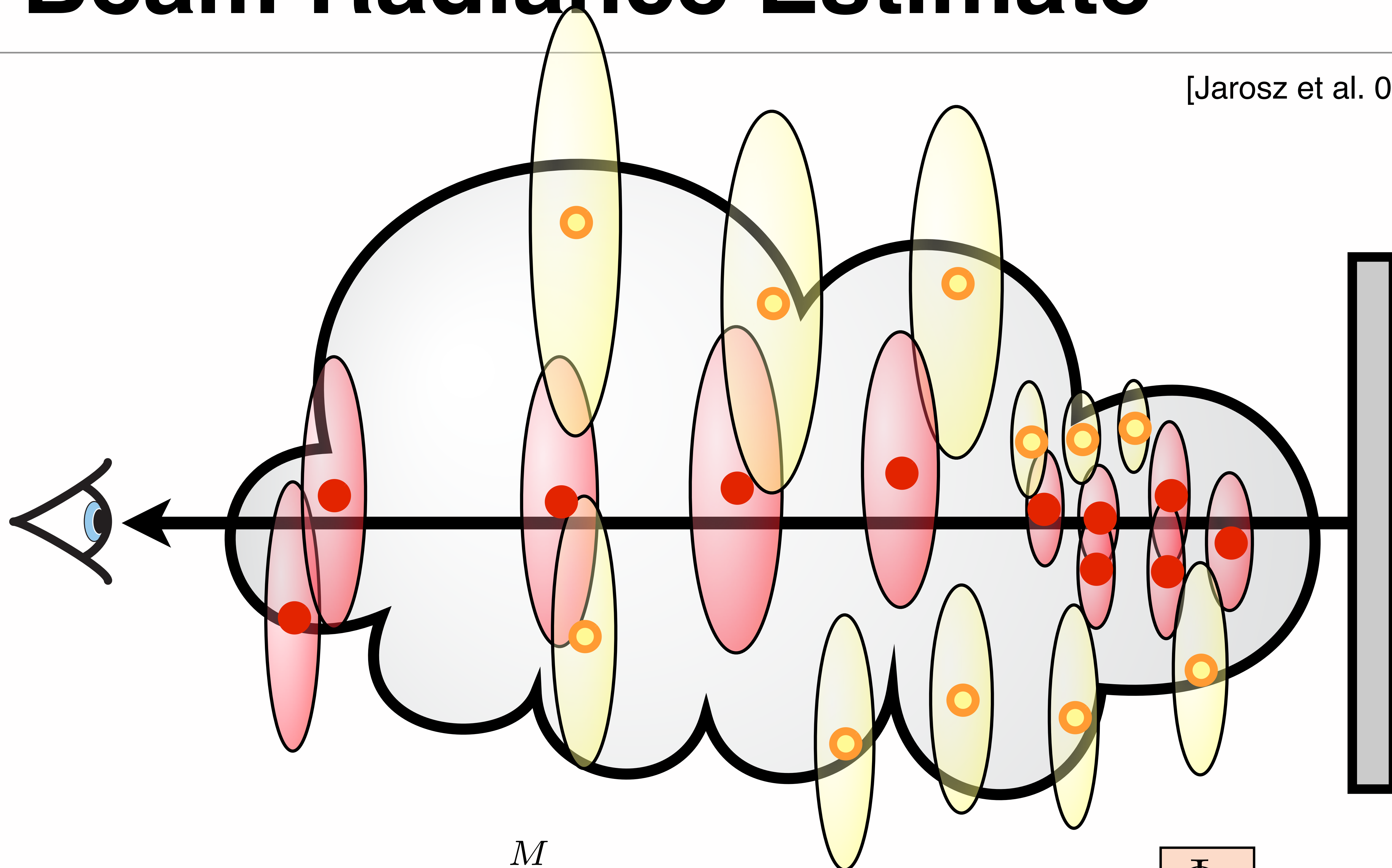
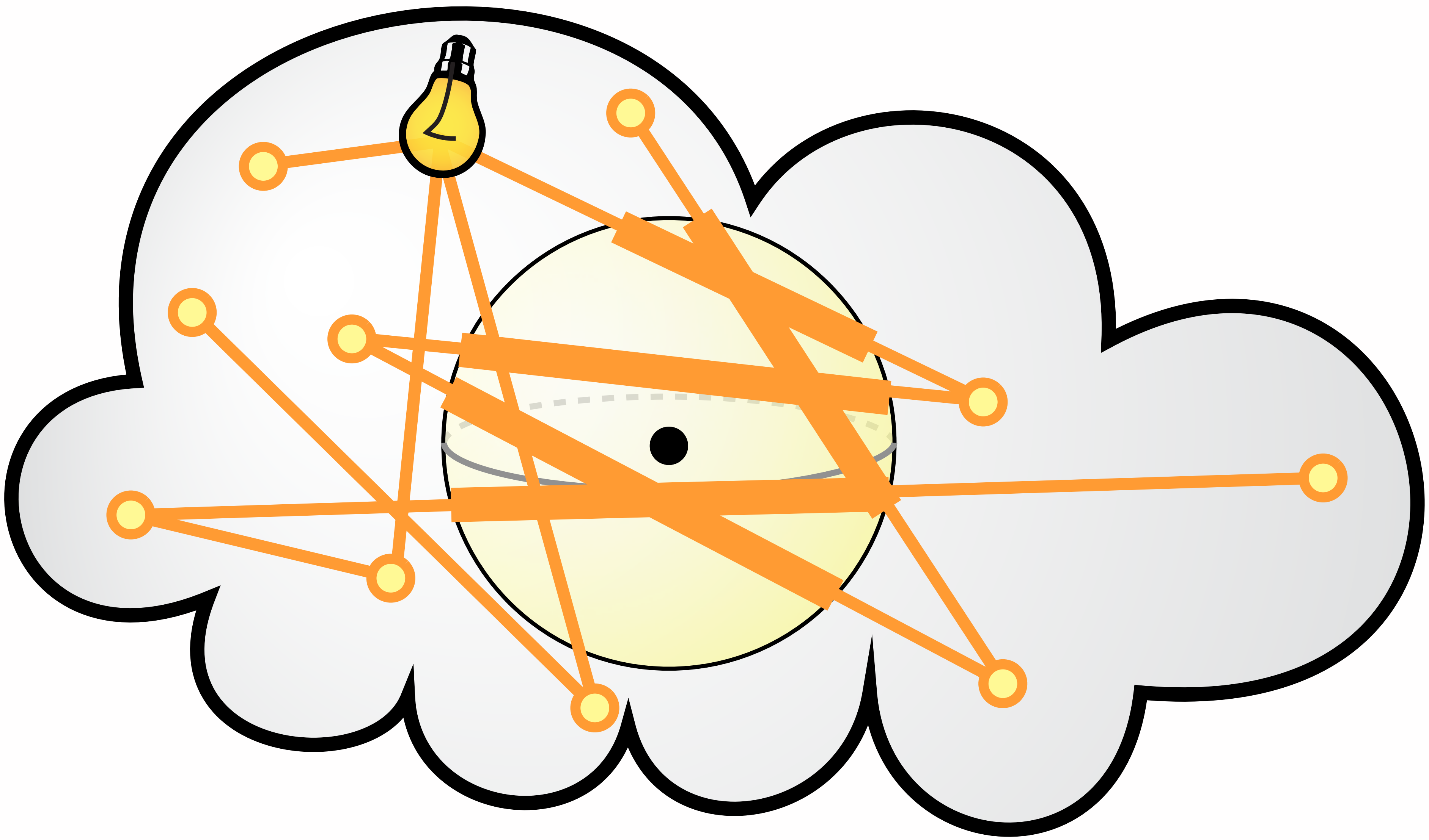


$$L_m(\mathbf{x},\vec{\omega}) \approx \sum_{i=1}^{M} T_r(\mathbf{x},\mathbf{x}_i)\sigma_s(\mathbf{x}_i) f_p(\mathbf{x}_i,\vec{\omega}_i,\vec{\omega}) \boxed{\frac{\Phi_i}{\pi r_i^2}}$$

Photon Beams

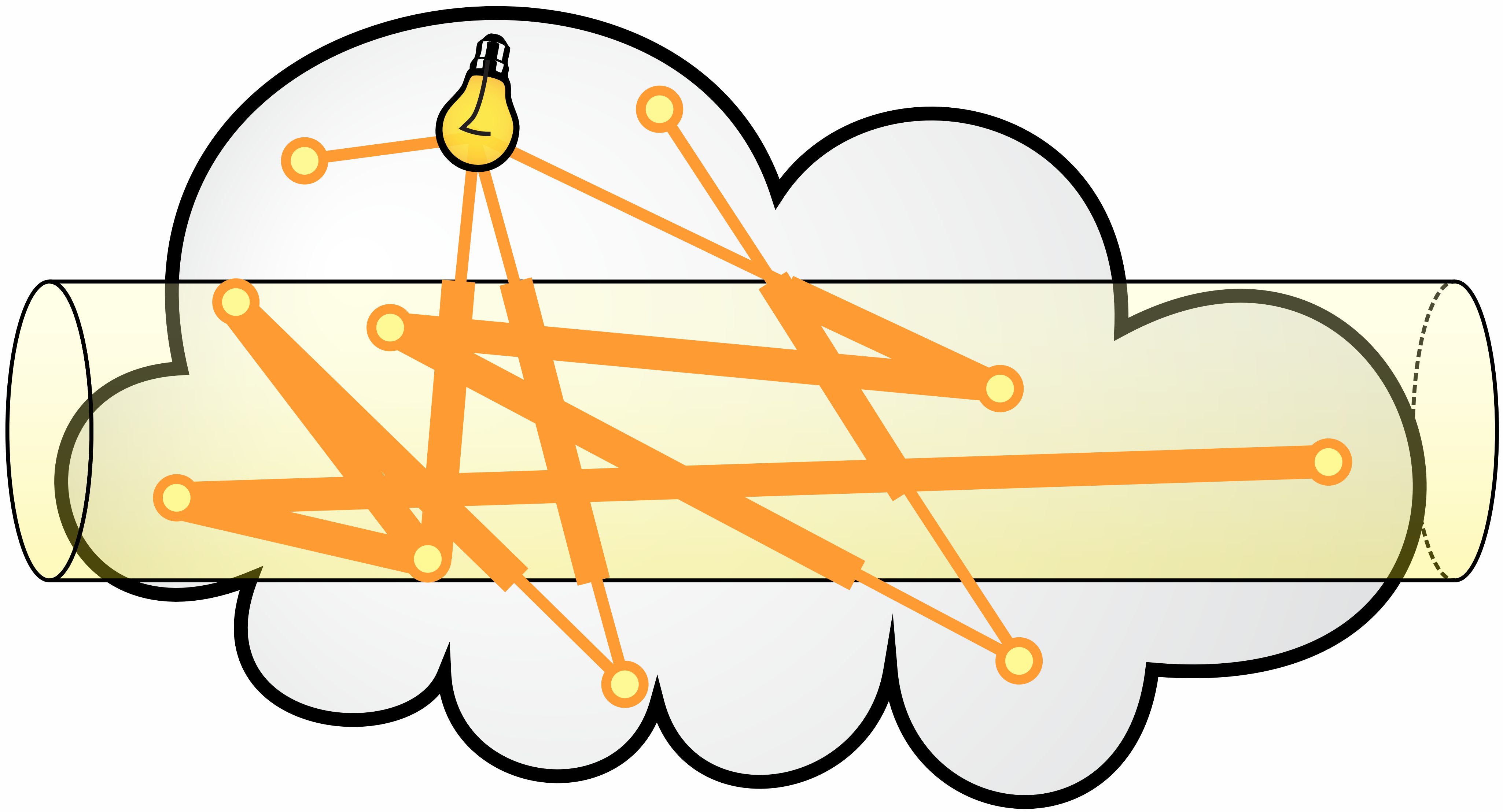
[Jarosz et al. 11]



Photon Beams **x** Point Query

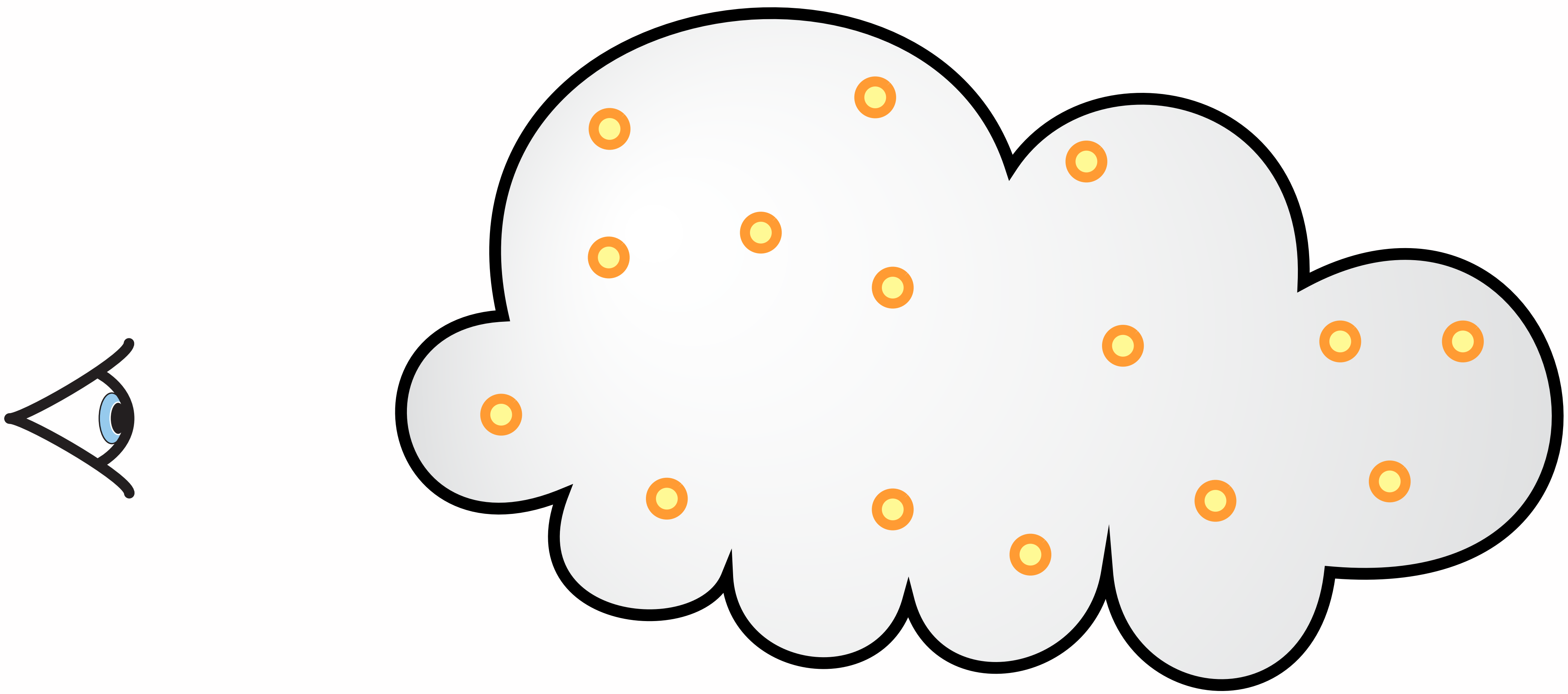
Photon Beams

[Jarosz et al. 11]

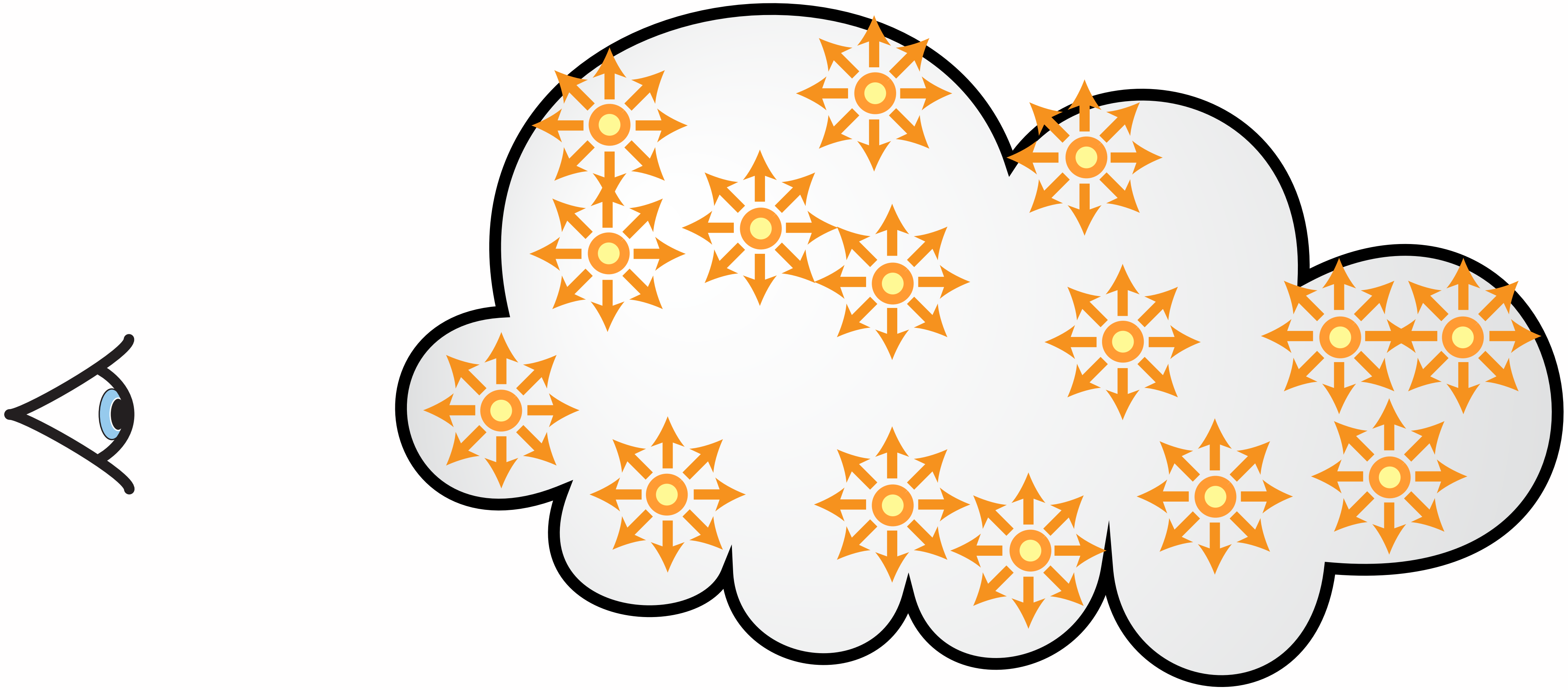


Photon Beams **x** Beam Query

Volumetric VPLs

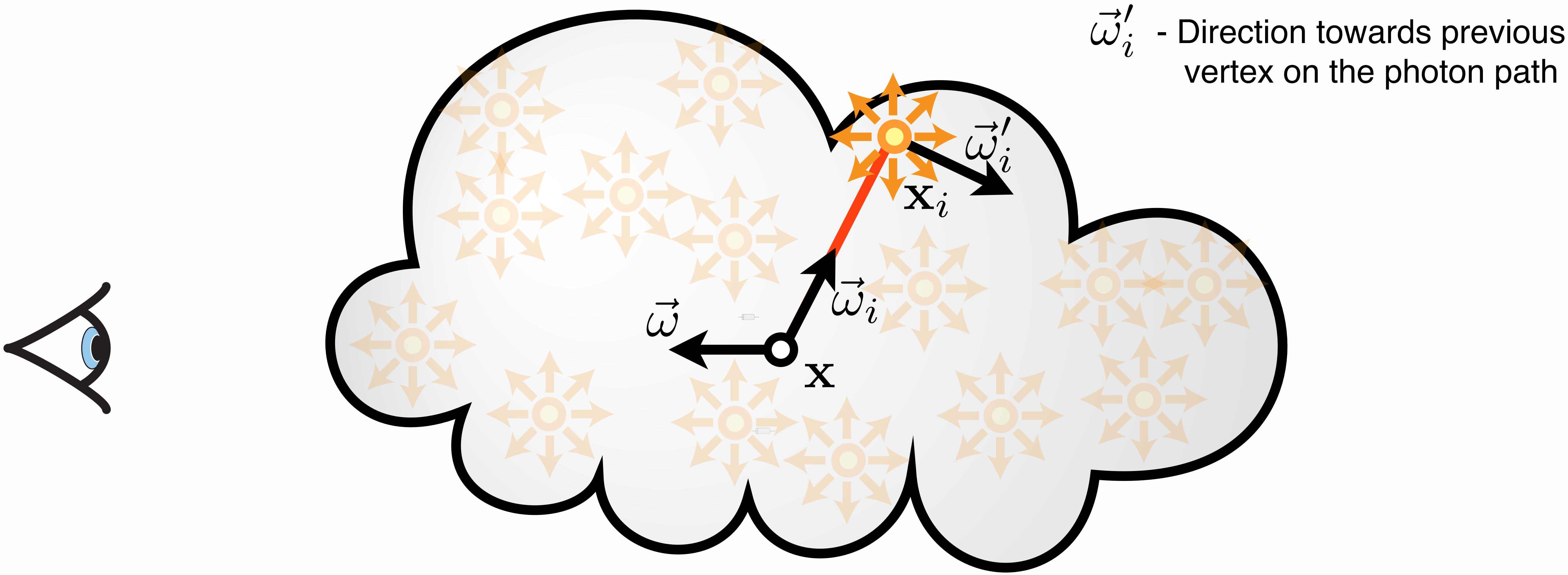


Volumetric VPLs



Turn photons into volumetric virtual point lights

Volumetric VPLs



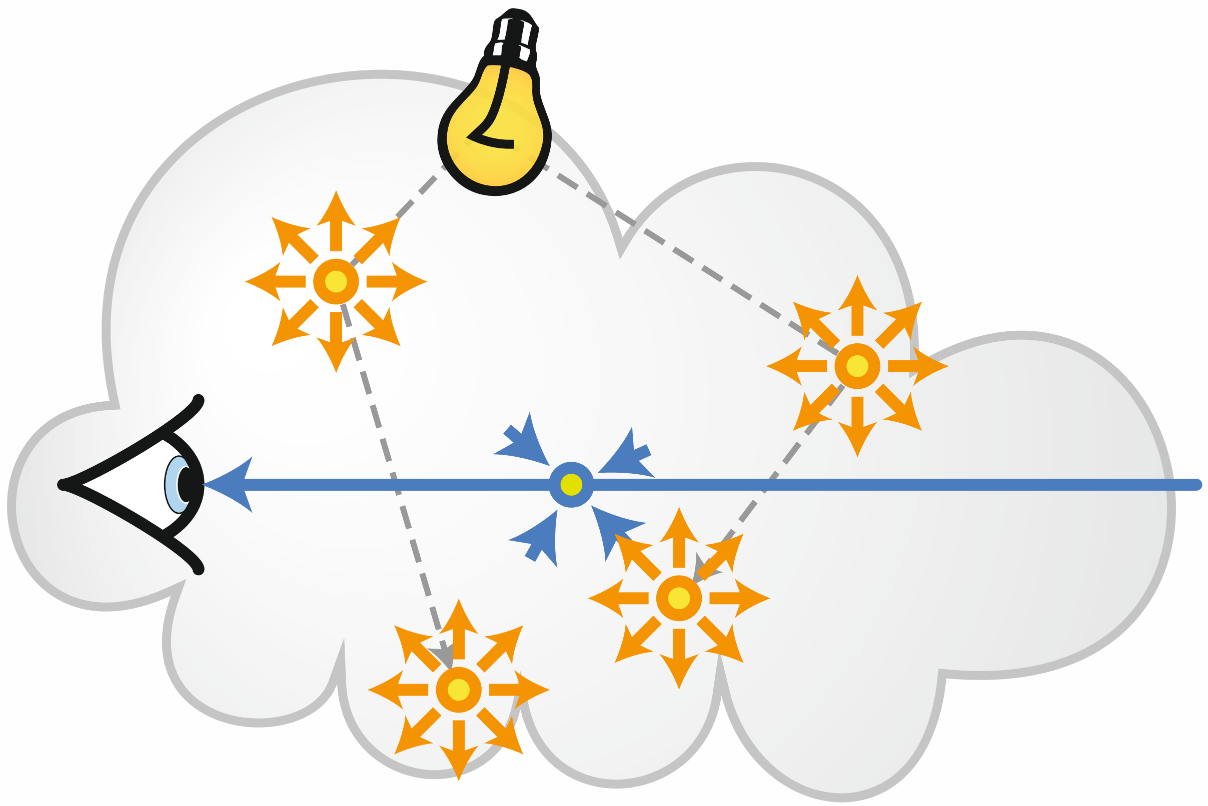
$$L_s(\mathbf{x}, \vec{\omega}) \approx \sum_{i=1}^{N} f_p(\mathbf{x}, \vec{\omega}_i, \vec{\omega}) T_r(\mathbf{x}, \mathbf{x}_i) G(\mathbf{x}, \mathbf{x}_i) f_p(\mathbf{x}_i, \vec{\omega}_i', -\vec{\omega}_i) \alpha(\mathbf{x}_i) \Phi_i$$

$$G(\mathbf{x}, \mathbf{x}_i) = V(\mathbf{x}, \mathbf{x}_i) \frac{1}{\|\mathbf{x} - \mathbf{x}_i\|^2}$$

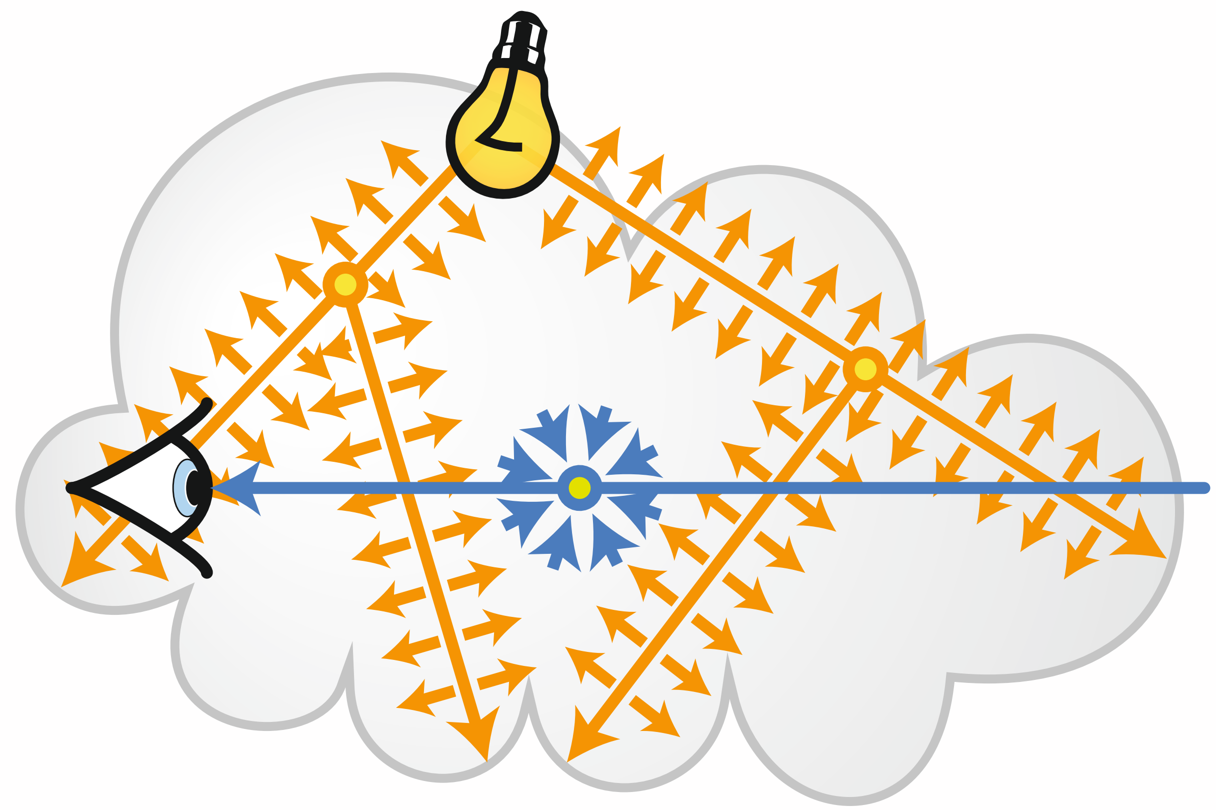
Virtual Ray Lights

[Novák et al. 12a]

- “Spread” emission over segments of photon path
  - Similar concept as in photon beams
  - Reduces the degree of the singularity



Virtual Point Lights

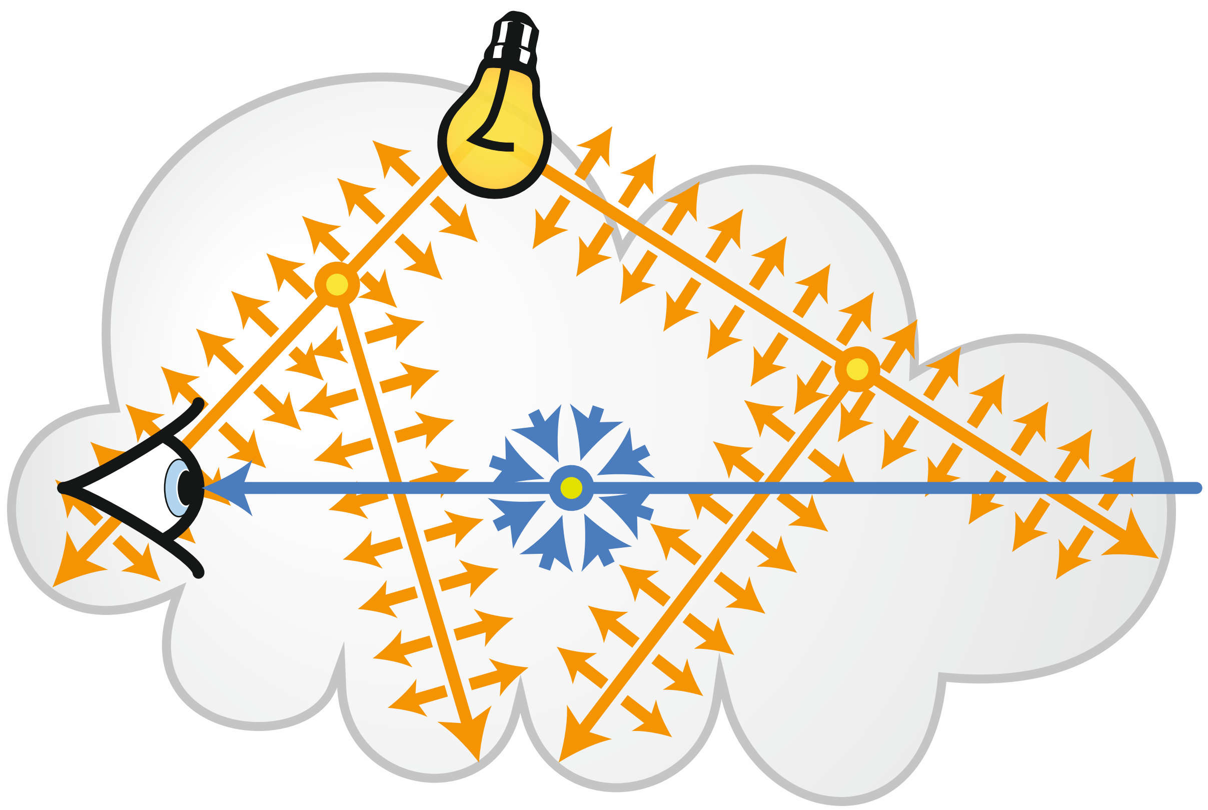


Virtual Ray Lights

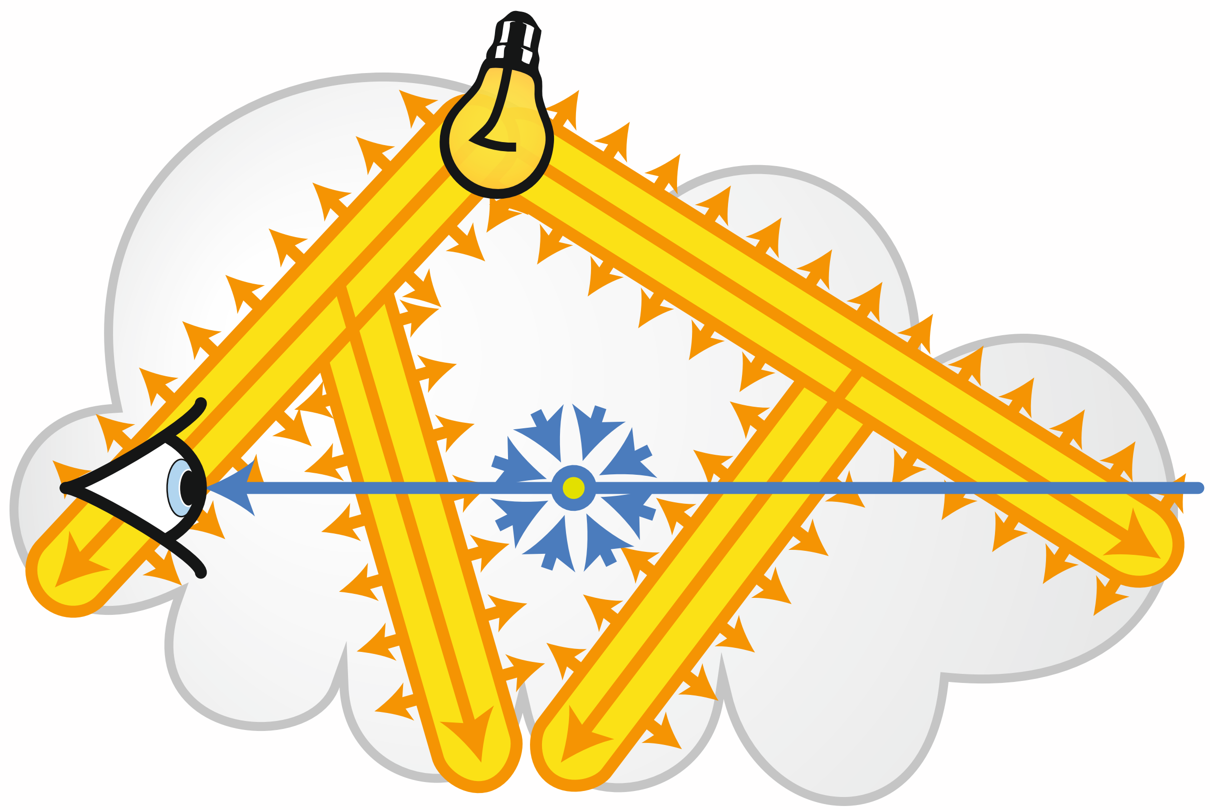
Virtual Beam Lights

[Novák et al. 12b]

- “Spread” emission over volume of a beam beam
  - Finite-thickness beam instead of infinitesimal ray

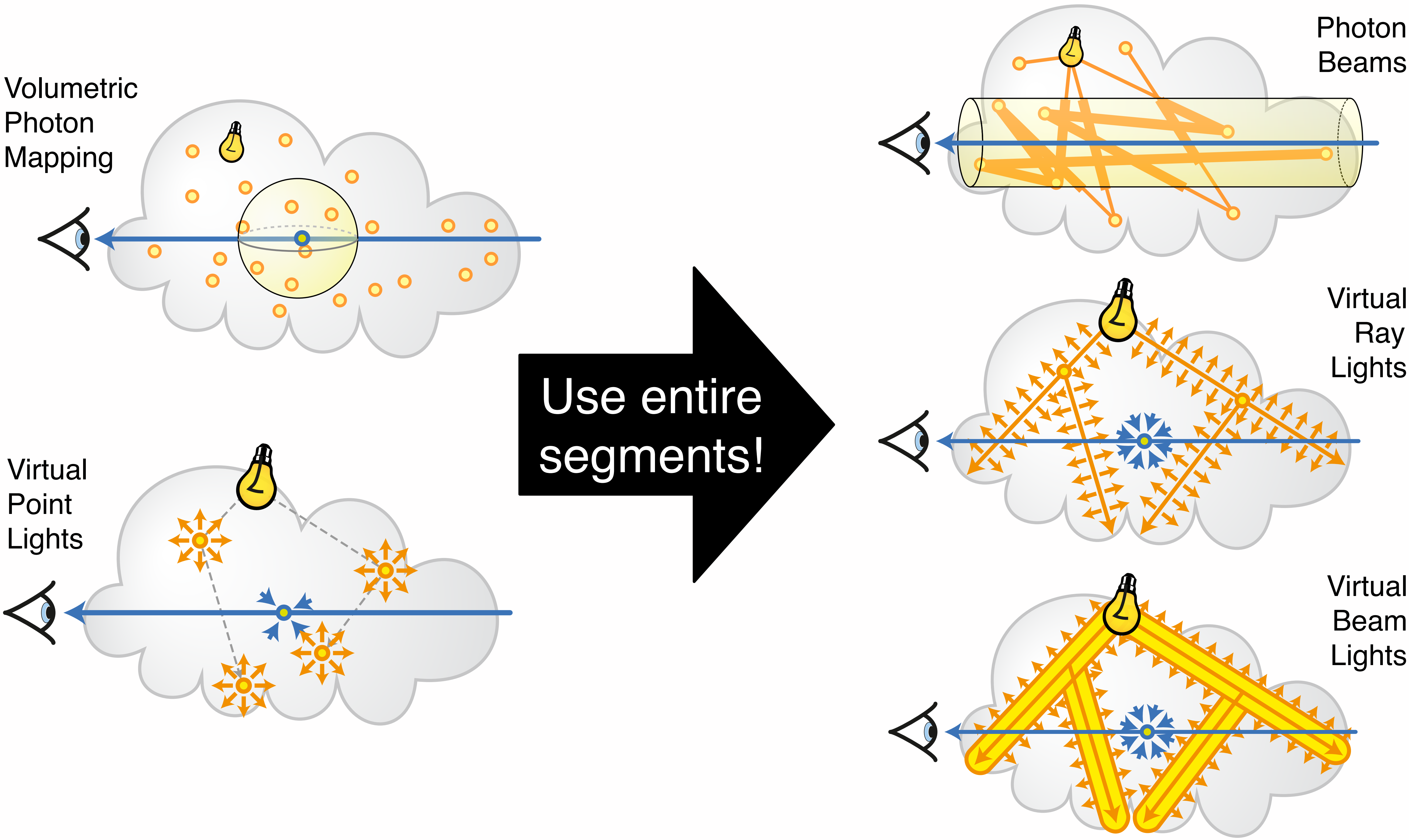


Virtual Ray Lights



Virtual Beam Lights

Care about efficiency?

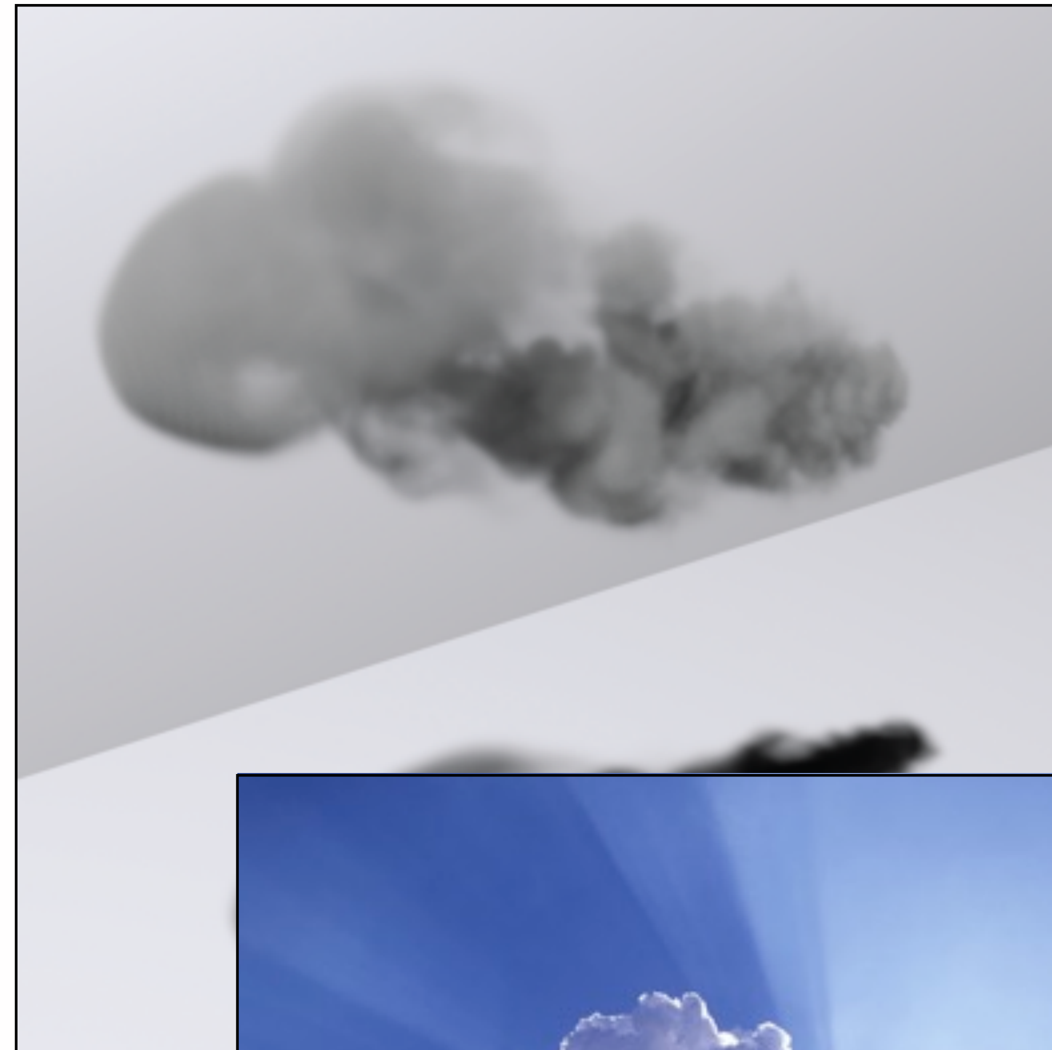


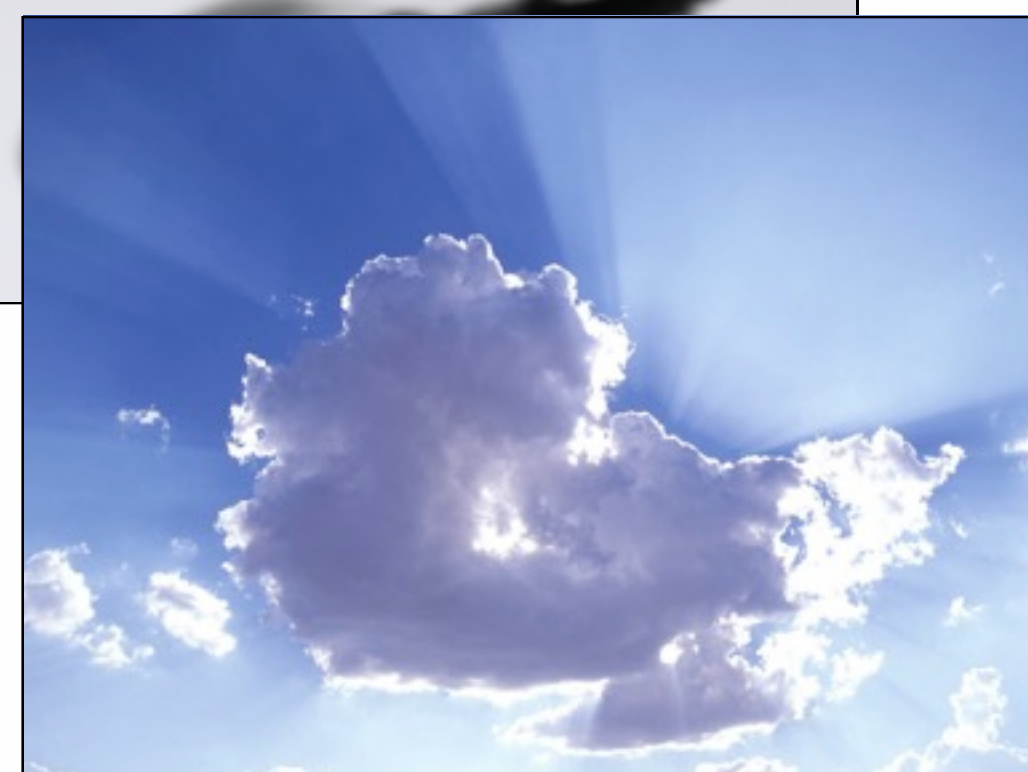
Today's Menu

- What is subsurface scattering?
- The BSSRDF
  - SSS as participating media rendering
- Diffusion approximation
- Searchlight problem
- Method of Images
  - Hierarchical evaluation
- The multipole
- Quantized diffusion
- Photon beam diffusion

Surfaces vs Volumes

source: luxology.com





Surface or Volume?


source: onebigphoto.com


TexasArrowheads.com

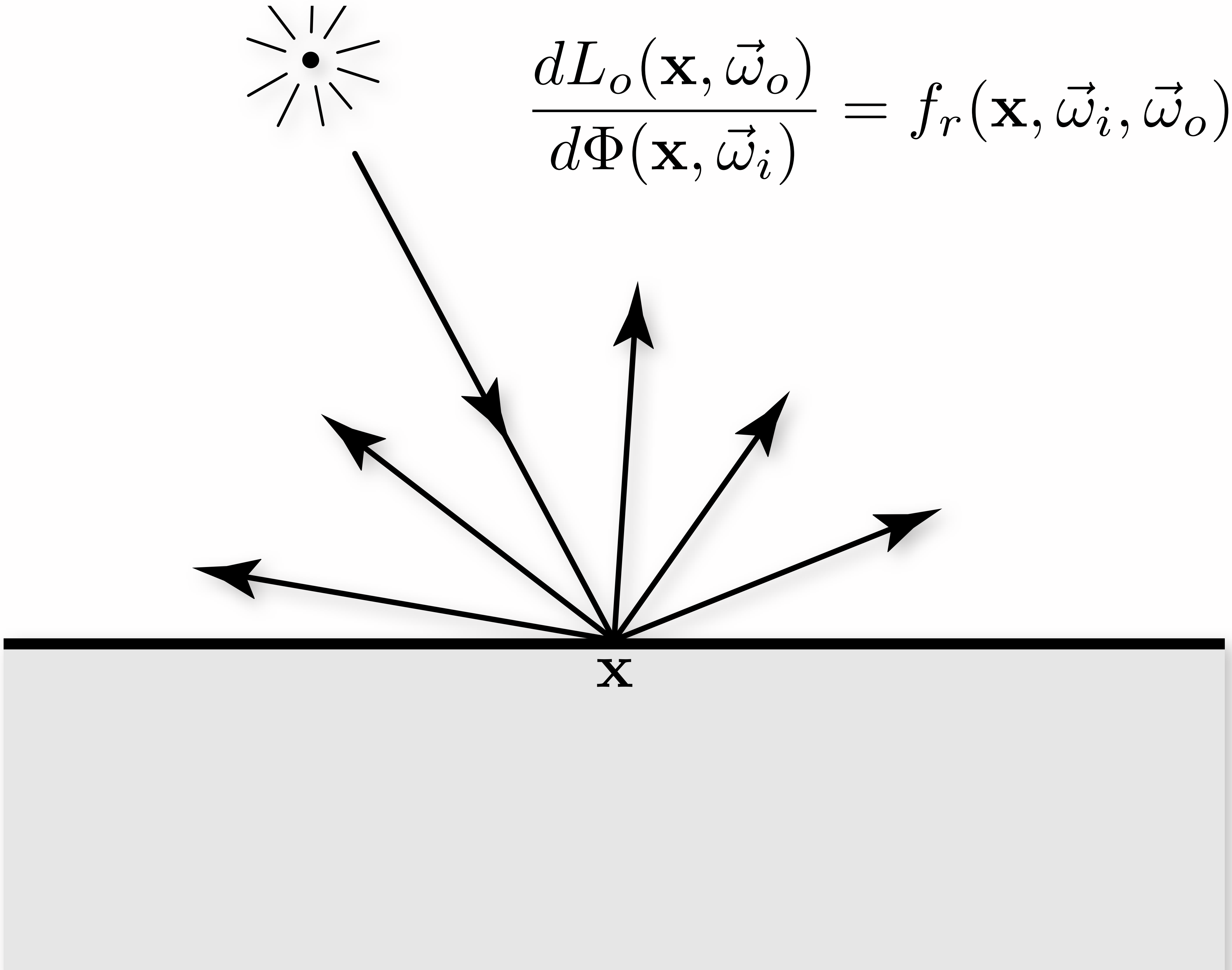

TexasArrowheads.com


source: showfoodchef.com


source: freshlocalandbest.blogspot.com


source: euglossine.wordpress.com

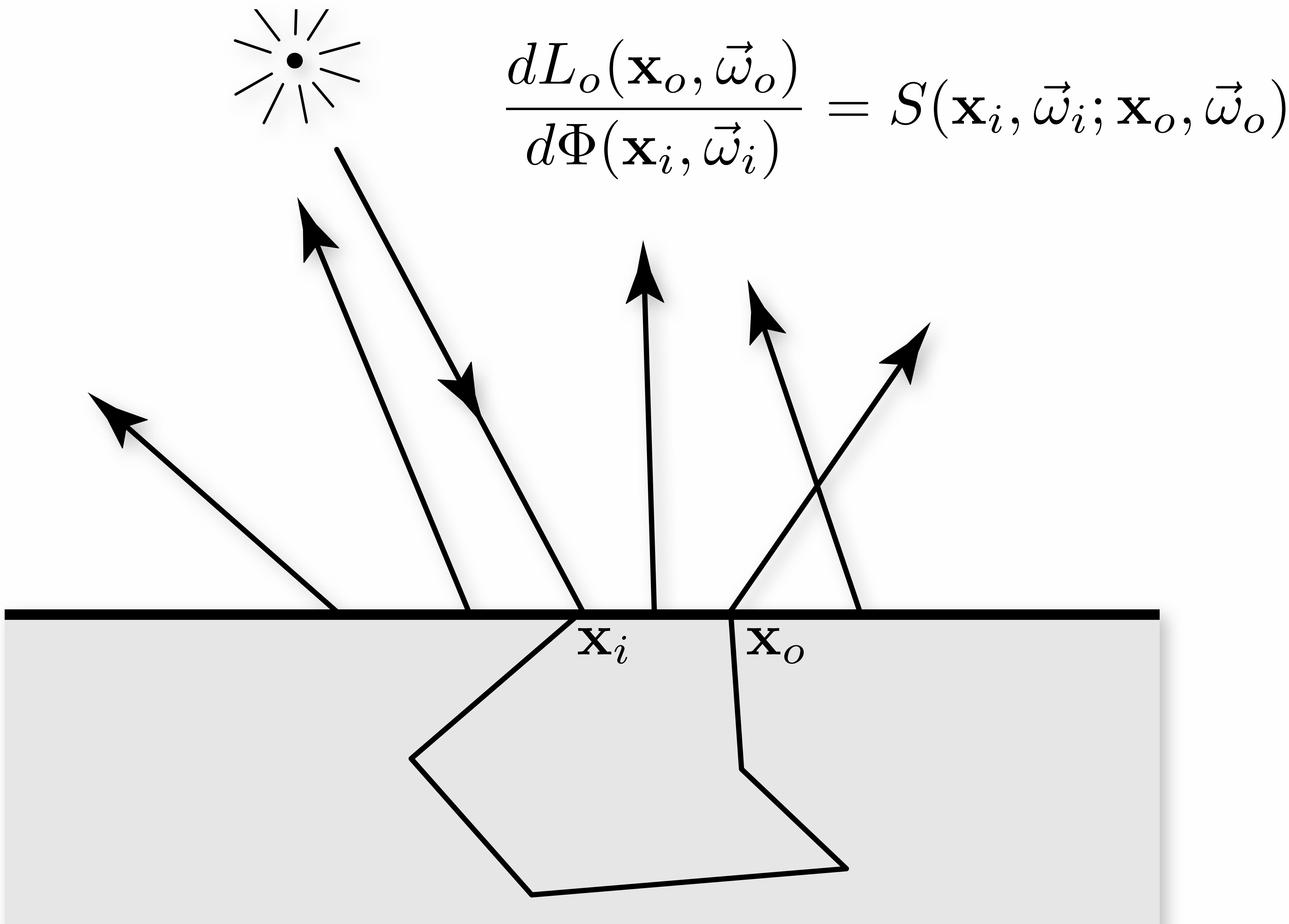
Surface Scattering (BRDF)



$$\frac{dL_o(\mathbf{x}, \vec{\omega}_o)}{d\Phi(\mathbf{x}, \vec{\omega}_i)} = f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o)$$

source: Jensen et al. 2001

Subsurface Scattering (BSSRDF)

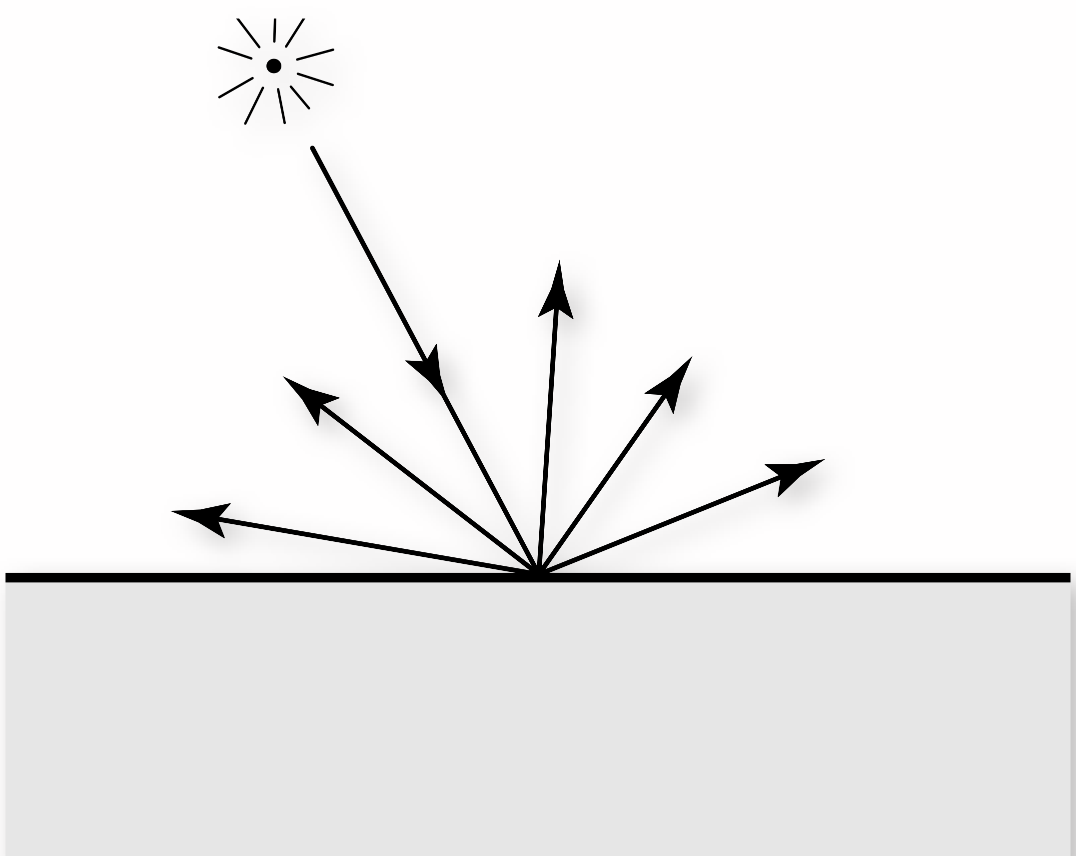


source: Jensen et al. 2001

BRDF vs BSSRDF

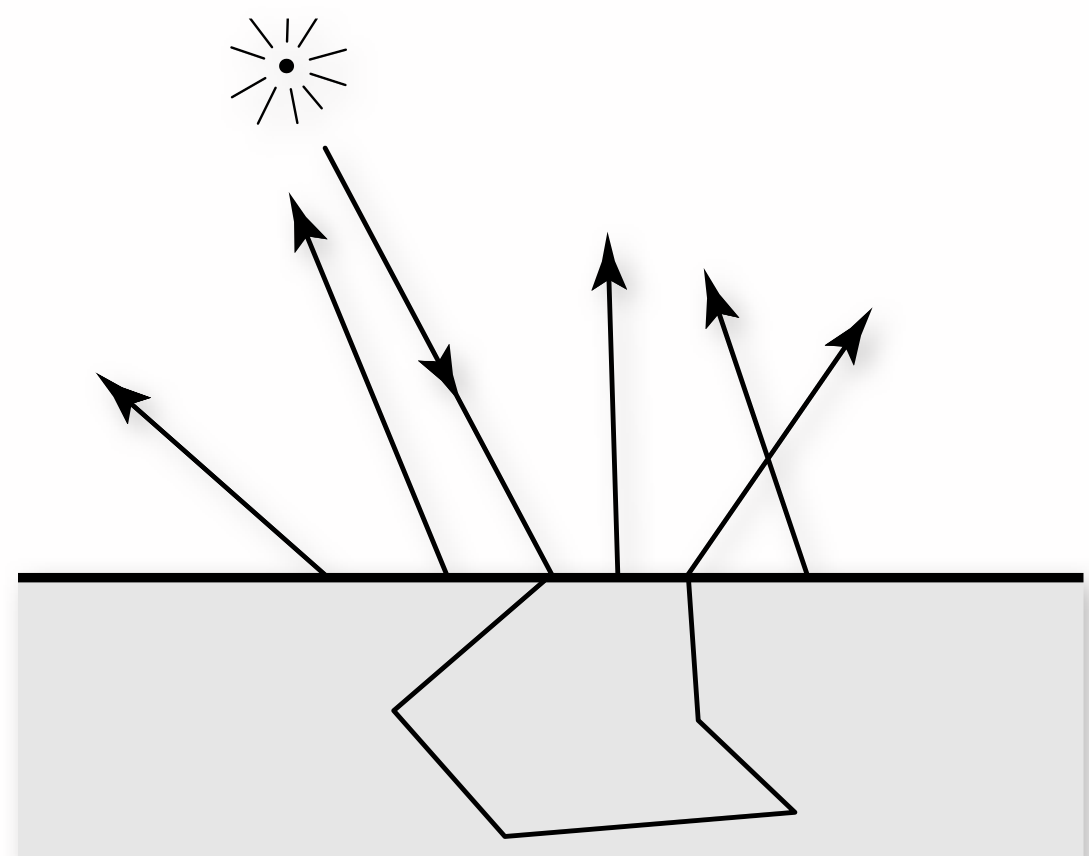
The BRDF

$$\frac{dL_o(\mathbf{x},\vec{\omega}_o)}{d\Phi(\mathbf{x},\vec{\omega}_i)} = f_r(\mathbf{x},\vec{\omega}_i,\vec{\omega}_o)$$



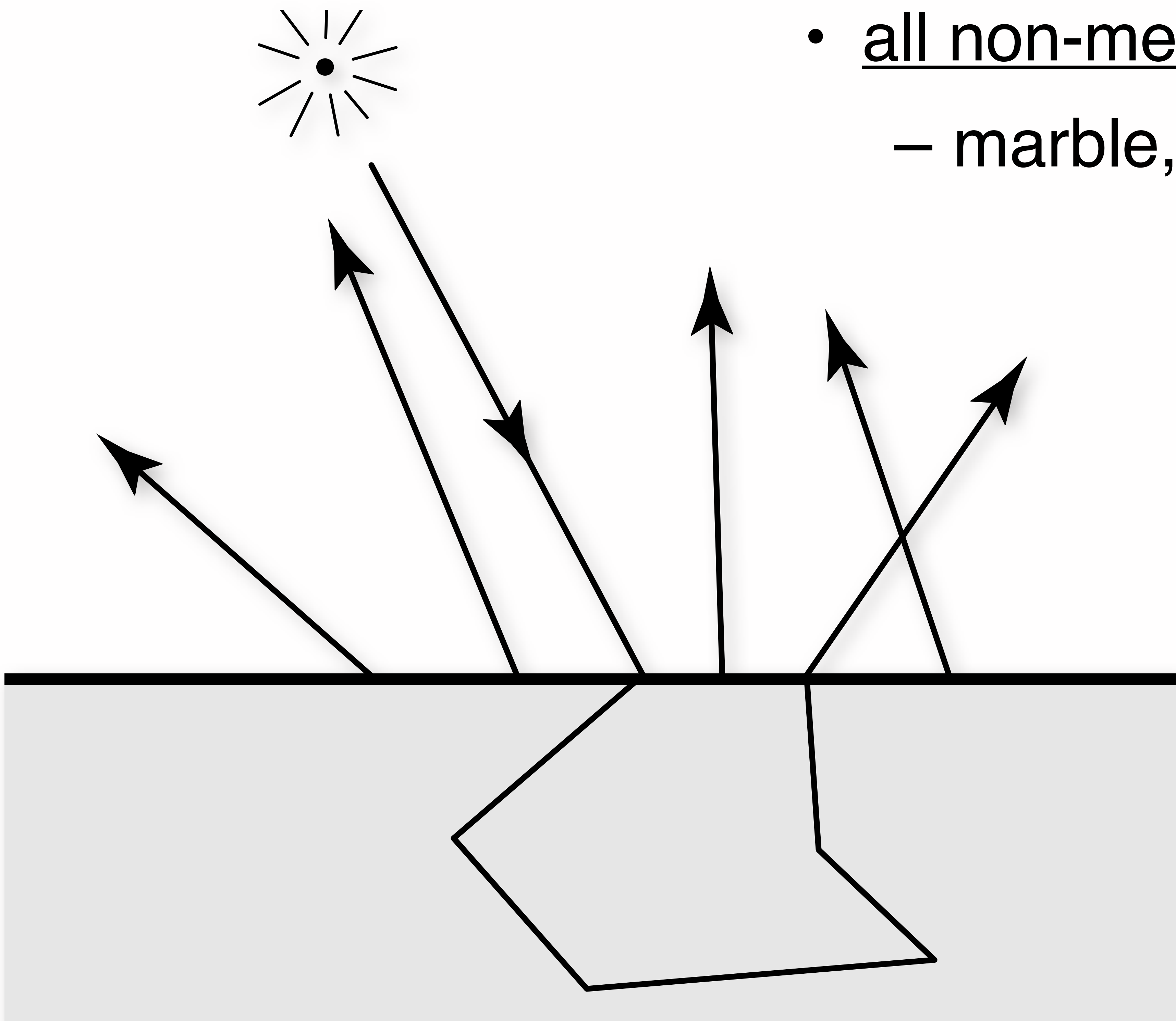
The BSSRDF

$$\frac{dL_o(\mathbf{x}_o,\vec{\omega}_o)}{d\Phi(\mathbf{x}_i,\vec{\omega}_i)} = S(\mathbf{x}_i,\vec{\omega}_i;\mathbf{x}_o,\vec{\omega}_o)$$



Subsurface Scattering

- all non-metals
  - marble, skin, milk, food, etc



Dielectric Boundary + Absorbing Medium

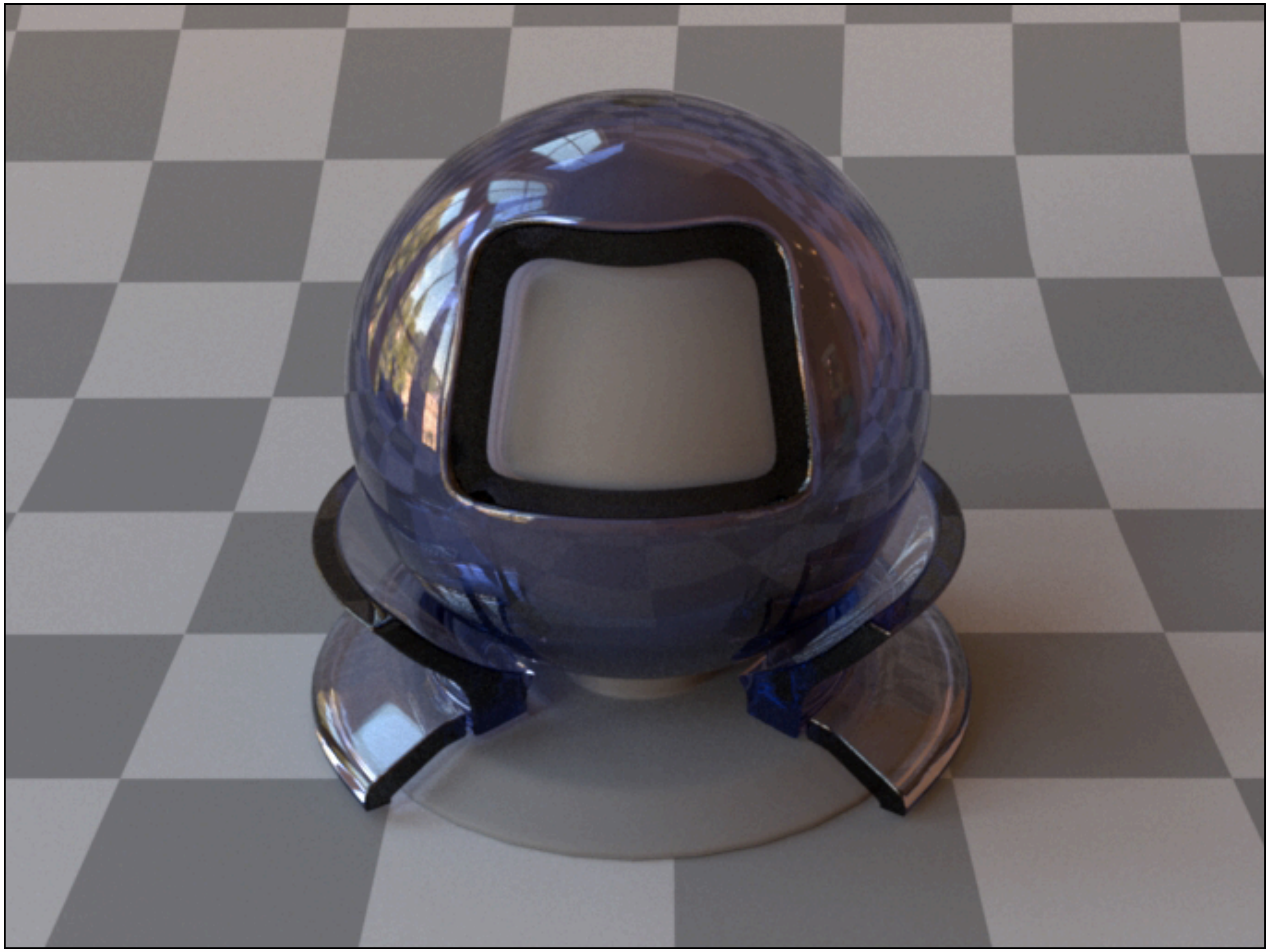


Image courtesy of Wenzel Jakob

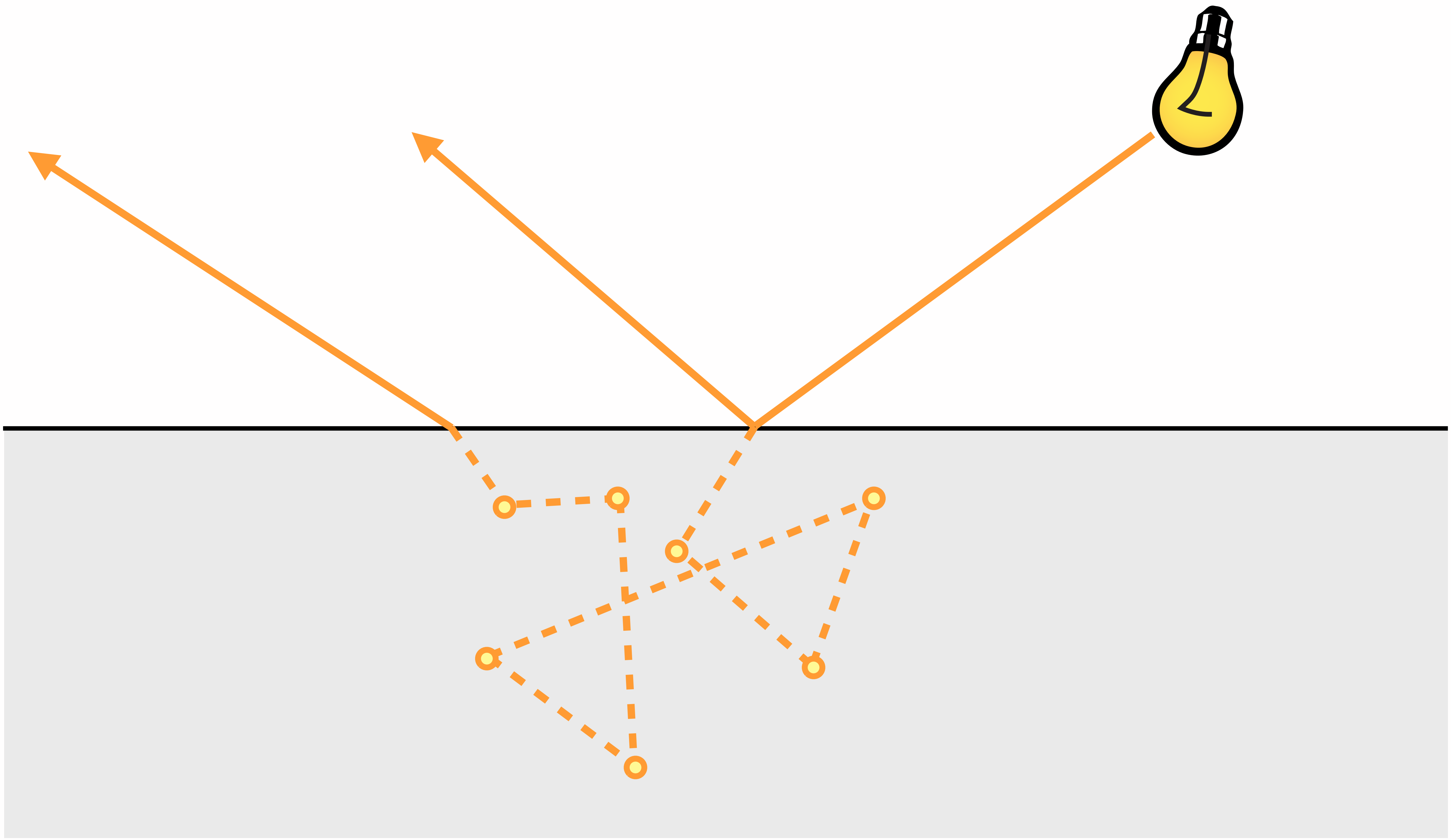




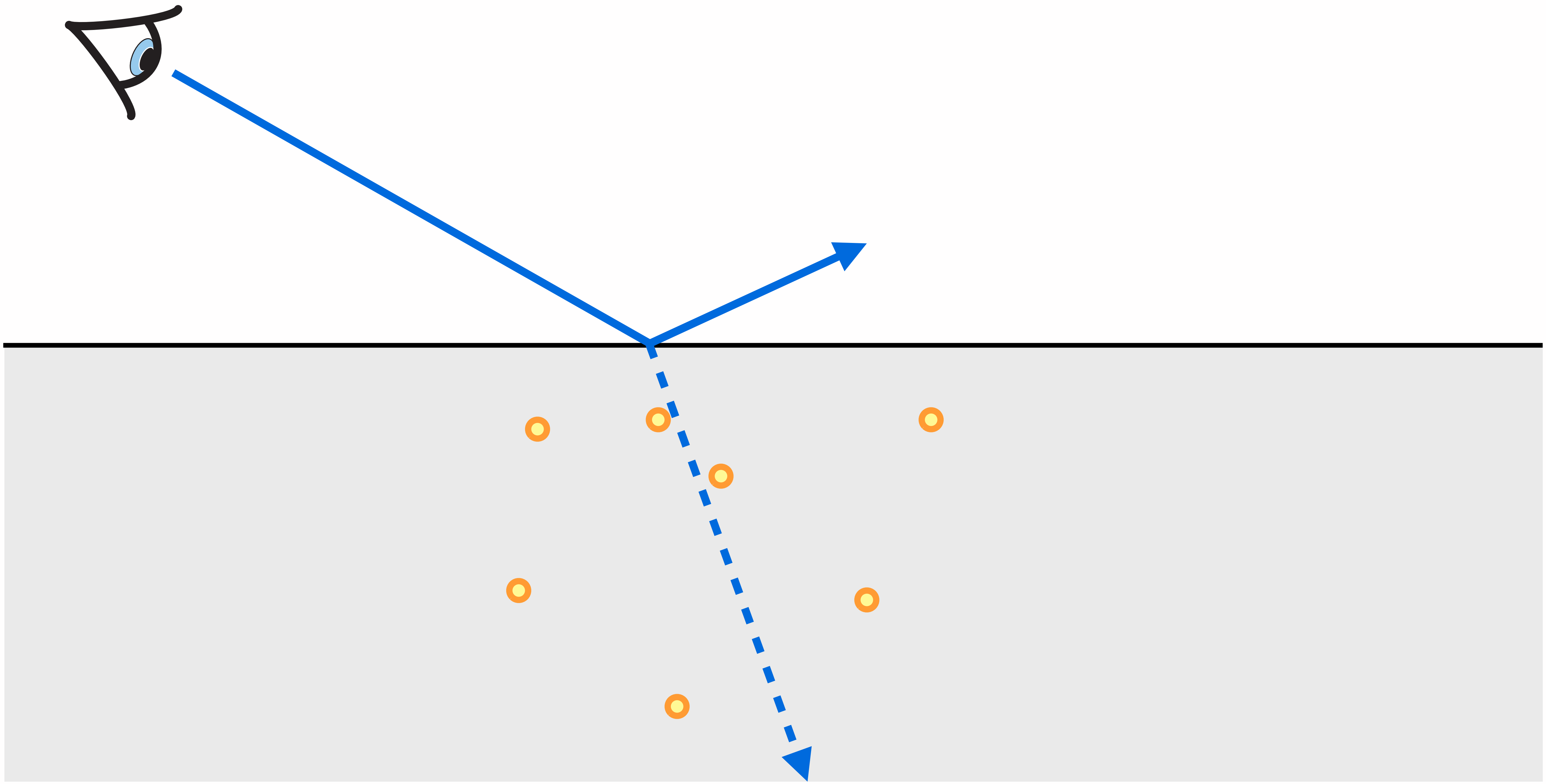
Subsurface Scattering

- Can use standard participating media rendering techniques
  - volumetric path tracing/photon mapping, etc
  - very general!

Photon Tracing



Radiance Estimation



MC Photon Mapping

BRDF

Marble

BRDF

render time: seconds

Photon Mapping

render time: minutes-hours

Subsurface Scattering

- Need to solve a generalized version of the reflection equation:

$$L_o(\mathbf{x}, \vec{\omega}_o) = \int_\Omega f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o) L_i(\mathbf{x}, \vec{\omega}_i)(\vec{n} \cdot \vec{\omega}_i)\, d\vec{\omega}_i$$

$$\downarrow$$

$$L_o(\mathbf{x}_o, \vec{\omega}_o) = \int_A \int_\Omega S(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o) L_i(\mathbf{x}_i, \vec{\omega}_i)(\vec{n} \cdot \vec{\omega}_i)\, d\vec{\omega}_i dA(\mathbf{x}_i)$$

- MC approaches will be very costly and often unnecessarily general

Highly-Scattering Materials

- Scattering albedo greatly influences random-walk techniques (path tracing, photon mapping)
- Milk albedo: 0.9987
  - 87.8% energy after 100 scattering events
  - 52% after 500 scattering events
  - 27% after 1000!
- Need to simulate thousands of bounces for each light path! Too expensive!
- Many materials are highly scattering

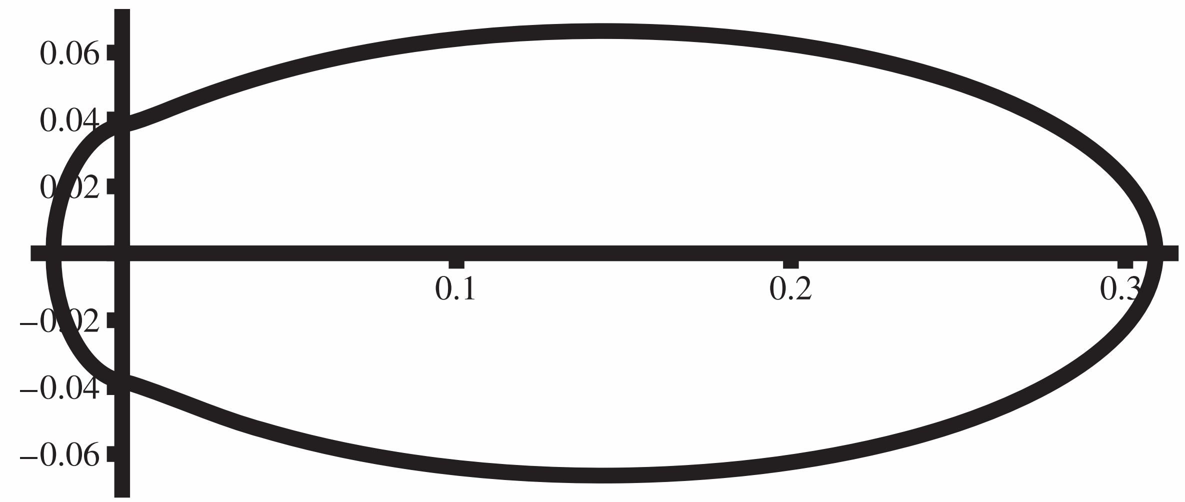
Highly-Scattering Materials

- In highly scattering media, the distribution of light approaches uniformity

Highly-Scattering Materials

- In highly scattering media, the distribution of light approaches uniformity

after 10 bounces

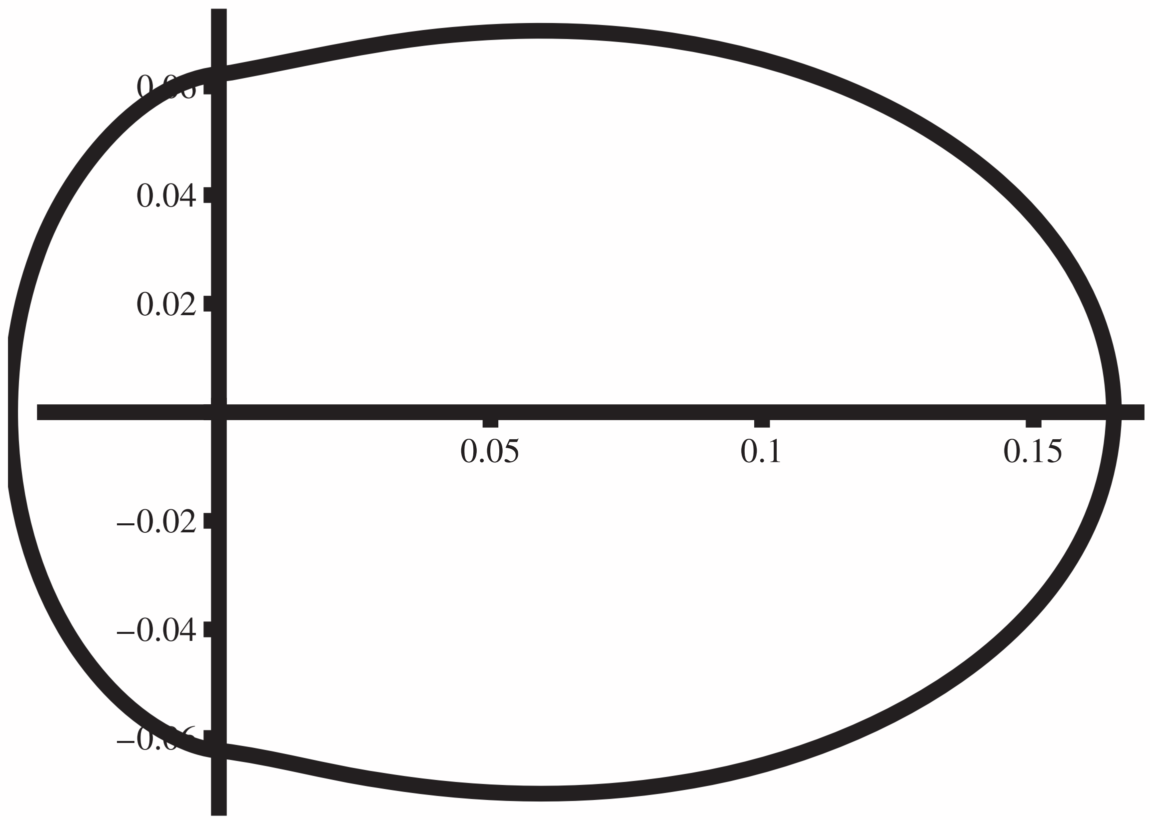


g = 0.9

Highly-Scattering Materials

- In highly scattering media, the distribution of light approaches uniformity

after 100
bounces



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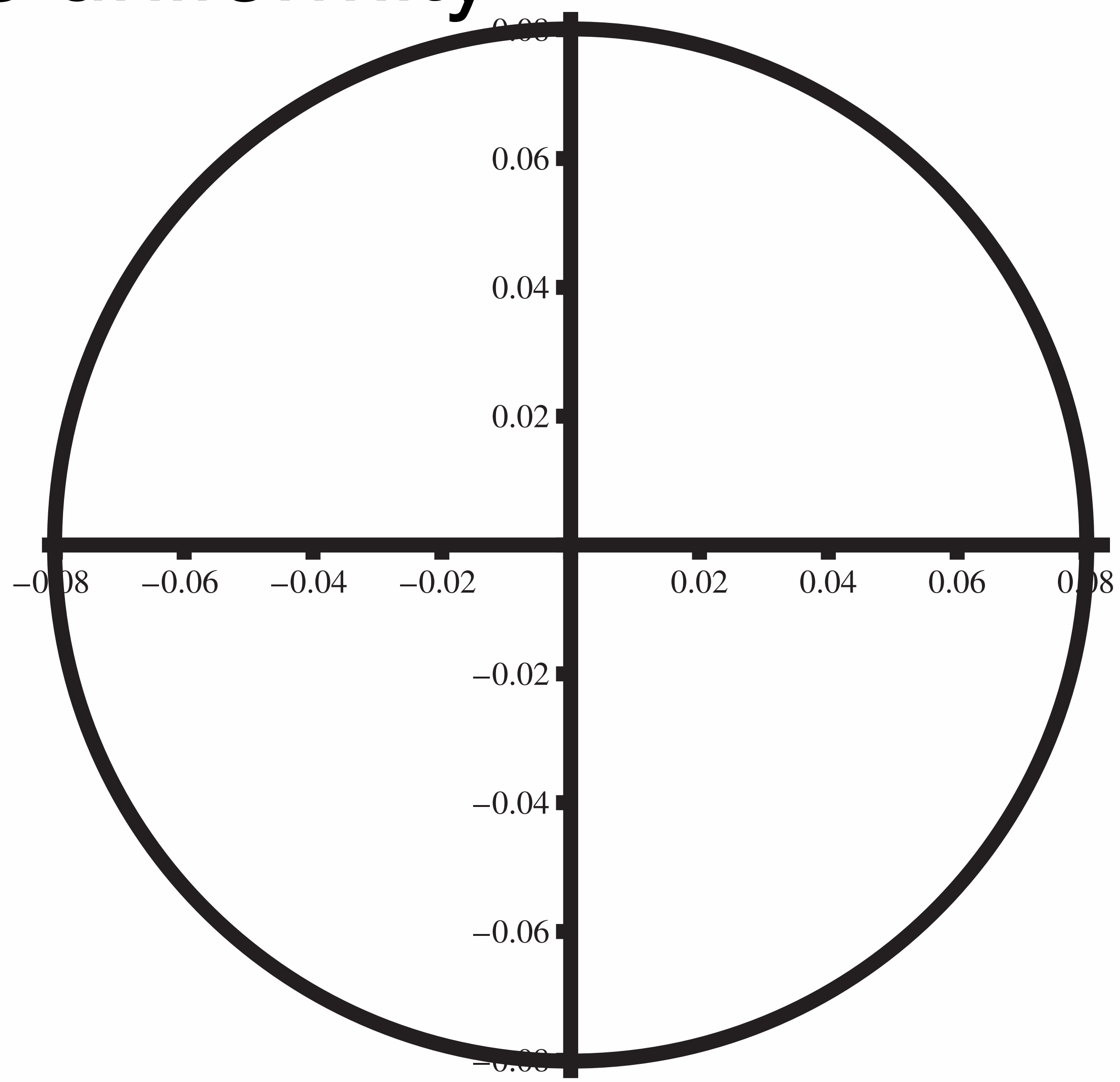
Highly-Scattering Materials

- In highly scattering media, the distribution of light approaches uniformity

after 1000 bounces



g = 0.9

Principle of Similarity

- We can approximate *anisotropic* scattering ($|g| > 0$) as *isotropic* ($g = 0$) with modified parameters
- Reduced scattering coefficient: $\sigma_s' = (1 - g)\sigma_s$
- Reduced extinction coefficient: $\sigma_t' = \sigma_a + \sigma_s'$
- Intuition:
  - when $g \to 1$, light continues in same direction more often, propagates in forward direction faster (like less scattering)
  - when $g \to -1$, light reverses direction often, has trouble making forward progress (like more scattering)

Principle of Similarity

- Reduces noise in MC approaches (VPT, VPM,...) but these are still too expensive
- We don't need high accuracy of MC below the surface, we need something fast
- Multiple scattering in isotropic media with high albedo can be approximated well by a simple, analytic expression... the *diffusion equation*

Diffusion Approximation

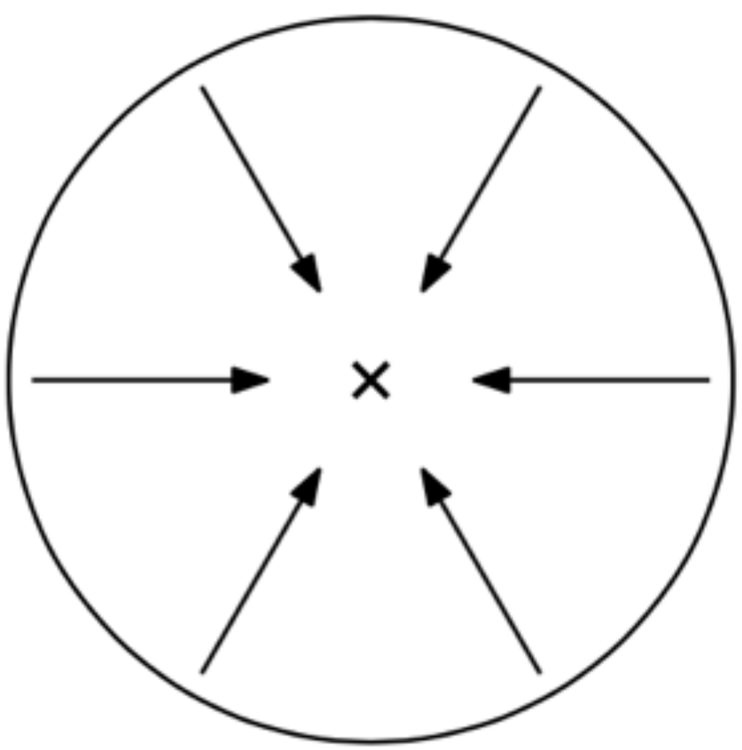
Diffusion Approximation

- Transition from individual interactions to statistical averages
- Expresses equilibrium (steady-state) probability density of a particle, which is subject to random movement, being at some location in space
- Observation:
  - As the number of scattering events increases, high-frequency directional effects disappear
  - Radiance can be approximated well using only its *first two angular moments*

Fluence

- Zero-th angular moment of radiance
  - Amount of light reaching a point

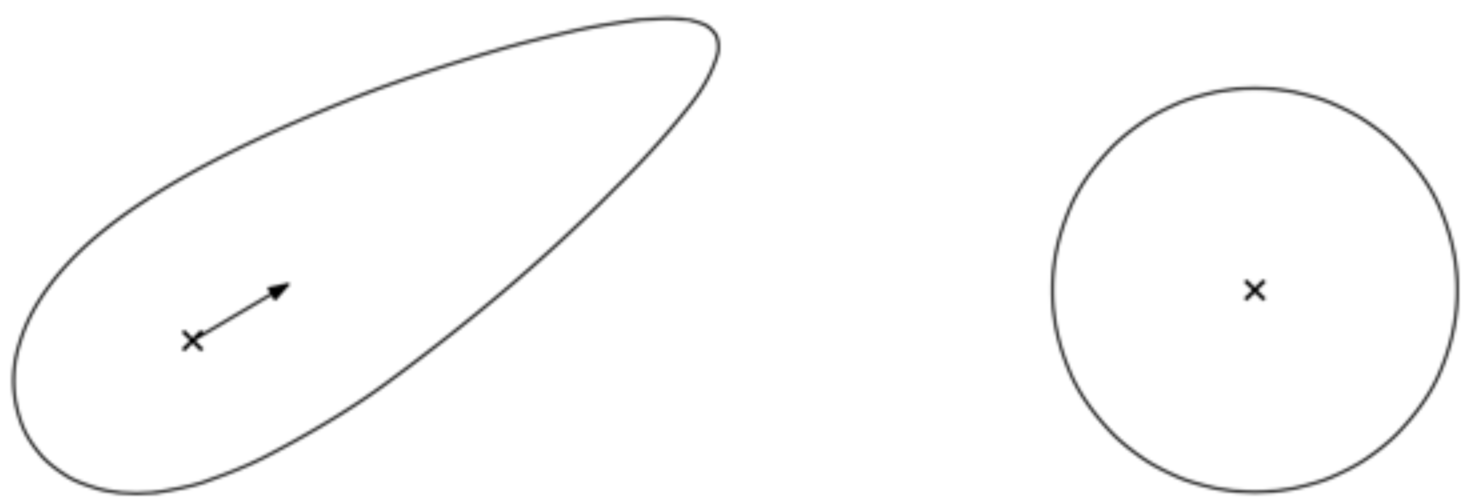
$$\phi(\mathbf{x}) = \mu_0[L(\mathbf{x}, \cdot)] = \int_{S^2} L(\mathbf{x}, \vec{\omega}) d\vec{\omega}$$



Vector Irradiance

- First angular moment of radiance
  - Weighted mean direction of radiance

$$\vec{E}(\mathbf{x}) = \mu_1[L(\mathbf{x}, \cdot)] = \int_{S^2} \vec{\omega} L(\mathbf{x}, \vec{\omega}) d\vec{\omega}$$



First Order Approximation of $L(\mathbf{x}, \vec{\omega})$

$$L(\mathbf{x}, \vec{\omega}) \approx \frac{1}{4\pi}\phi(\mathbf{x}) + \frac{3}{4\pi}\vec{\omega} \cdot \vec{E}(\mathbf{x})$$

Fluence

Vector irradiance

Derivation of DA from RTE

- Recipe:
  1. Start from the Radiative Transfer Equation (RTE)
  2. Approximate radiance RTE using the first-order approximation of radiance
  3. Derive the diffusion approximation by requiring LHS = RHS for the first two moments

Derivation of DA from RTE

1. Radiative Transfer Equation:

$$
\begin{aligned}
(\vec{\omega} \cdot \nabla) L(\mathbf{x}, \vec{\omega}) = & - \sigma_t L(\mathbf{x}, \vec{\omega}) && \text{Extinction} \\
& + \sigma_s \int_{S^2} f_p(\mathbf{x}, \vec{\omega}', \vec{\omega}) L(\mathbf{x}, \vec{\omega}') d\vec{\omega}' && \text{Scattering} \\
& + Q(\mathbf{x}, \vec{\omega}) && \text{Emission}
\end{aligned}
$$

2. First-order approximation:

$$L(\mathbf{x}, \vec{\omega}) \approx \frac{1}{4\pi} \phi(\mathbf{x}) + \frac{3}{4\pi} \vec{\omega} \cdot \vec{E}(\mathbf{x})$$

3. By require identity of the first two moments derive the following diffusion approximation:

$$-D\nabla^2 \phi(\mathbf{x}) + \sigma_a \phi(\mathbf{x}) = Q(\mathbf{x}) \qquad D = \frac{1}{3\sigma_t'}$$

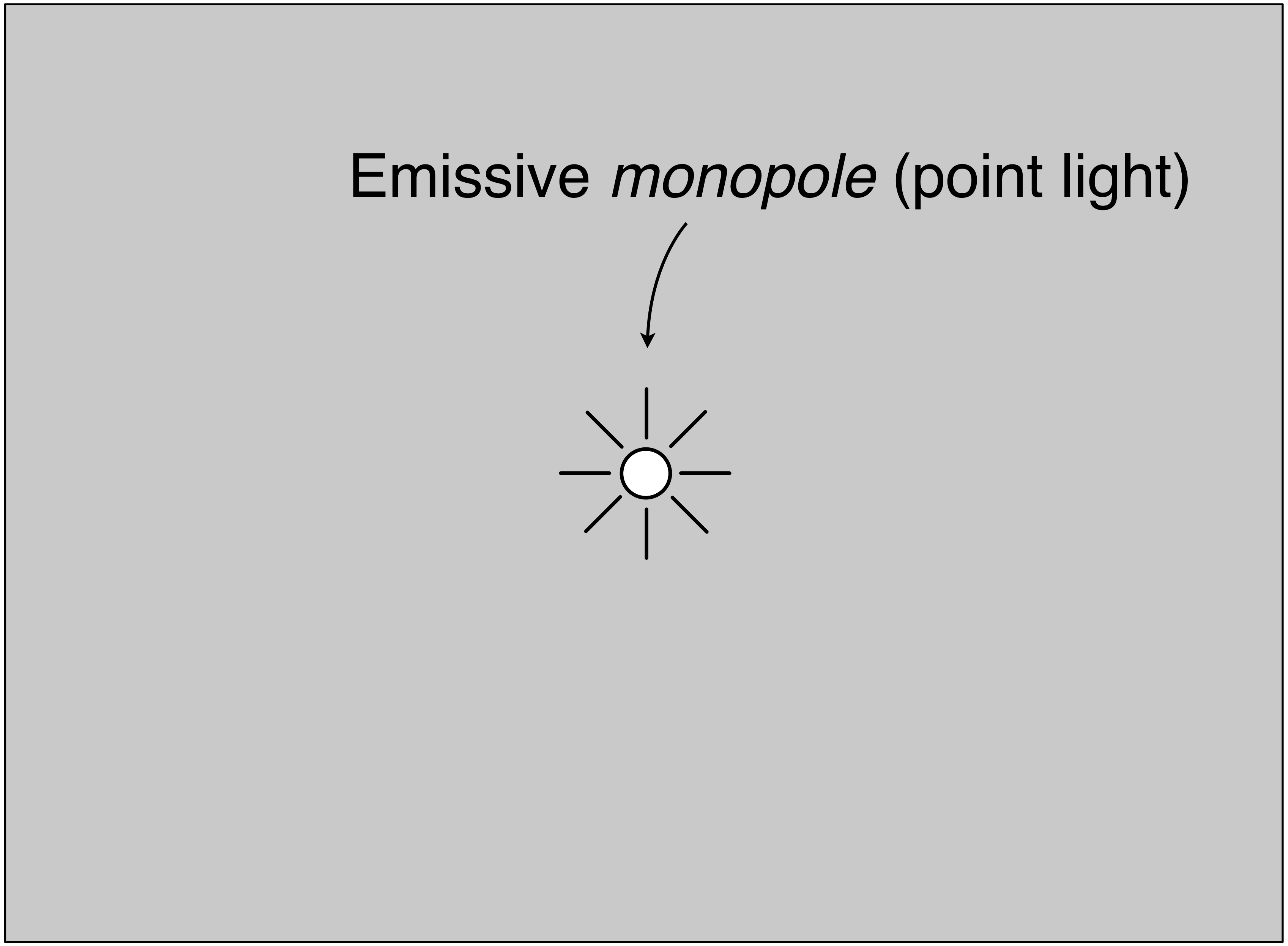
Diffusion Equation

- From the previous expression we can derive the *Classical Diffusion Equation* (aka Green's function) for an isotropic point emitter:

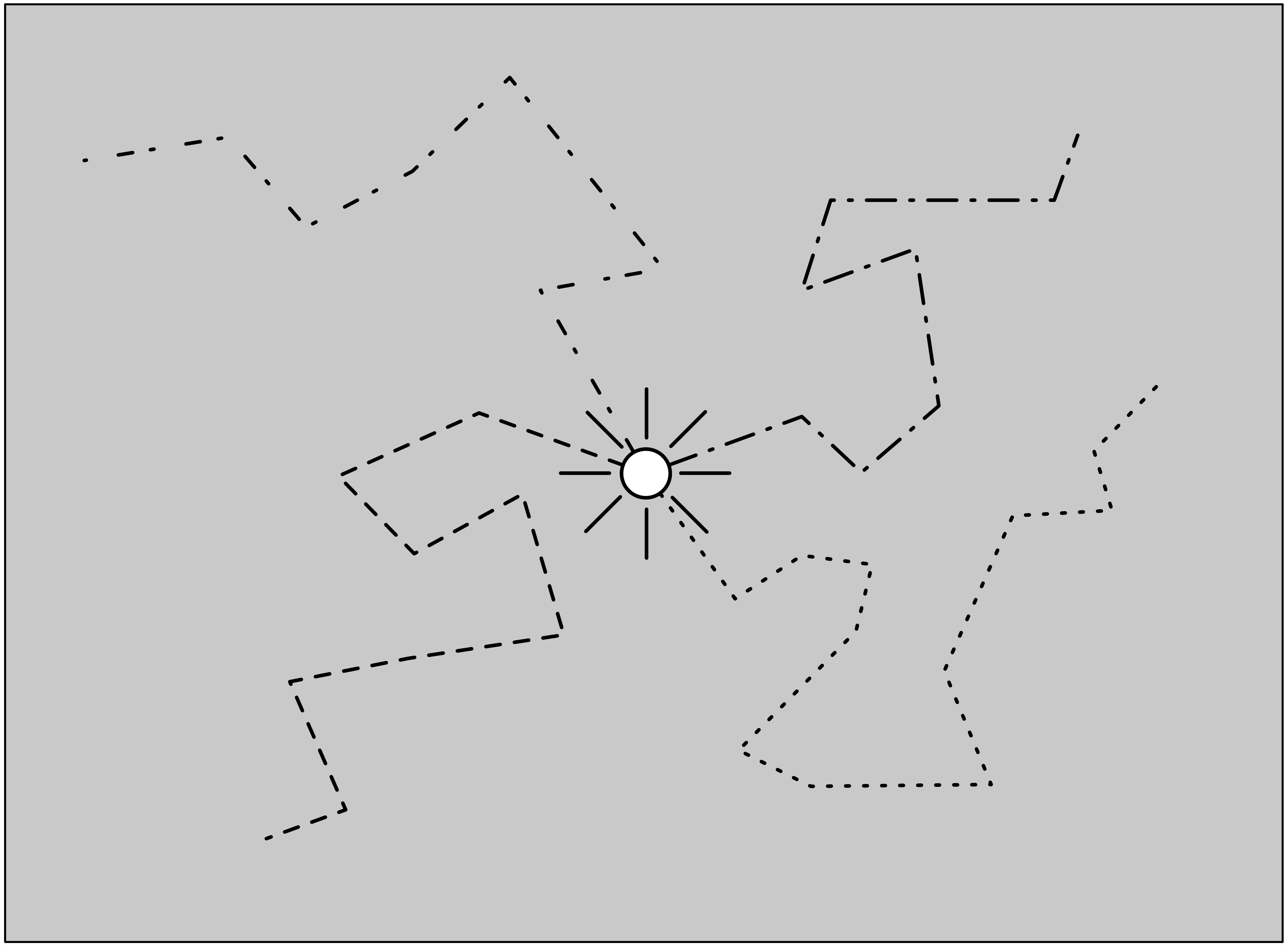
$$\phi(\mathbf{x}) = \frac{\Phi}{4\pi D}\frac{e^{-\sigma_{tr} d(\mathbf{x})}}{d(\mathbf{x})} \qquad \sigma_{tr} = \sqrt{3\sigma_t'\sigma_a}$$

Distance to the point emitter

Infinite Medium

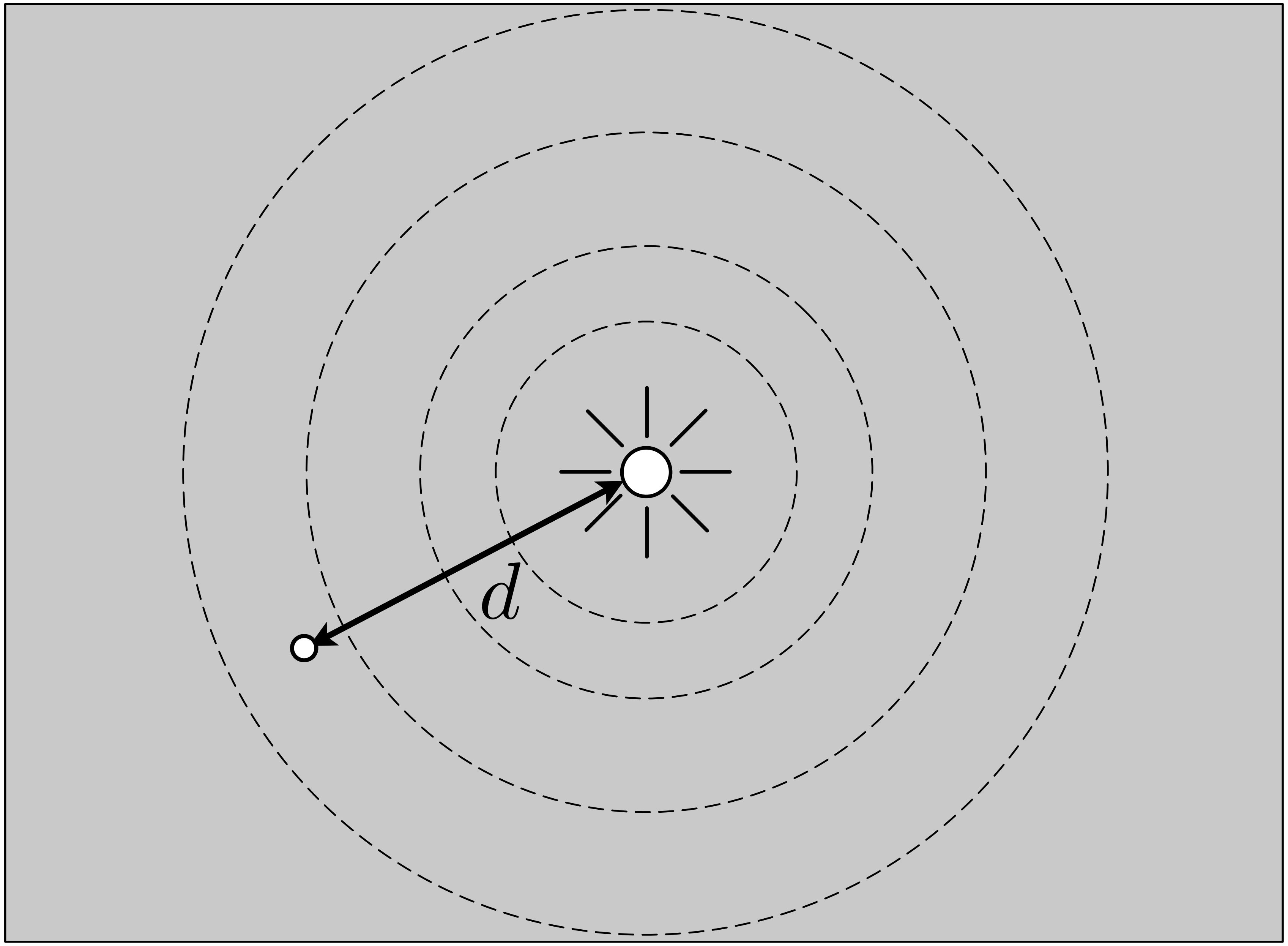


Radiative Transport Equation



$$(\vec{\omega} \cdot \nabla) L(\mathbf{x}, \vec{\omega}) = -\sigma_t L(\mathbf{x}, \vec{\omega}) + \sigma_s \int_{S^2} f_p(\mathbf{x}, \vec{\omega}', \vec{\omega}) L(\mathbf{x}, \vec{\omega}') d\vec{\omega}' + Q(\mathbf{x})$$

Diffusion Approximation



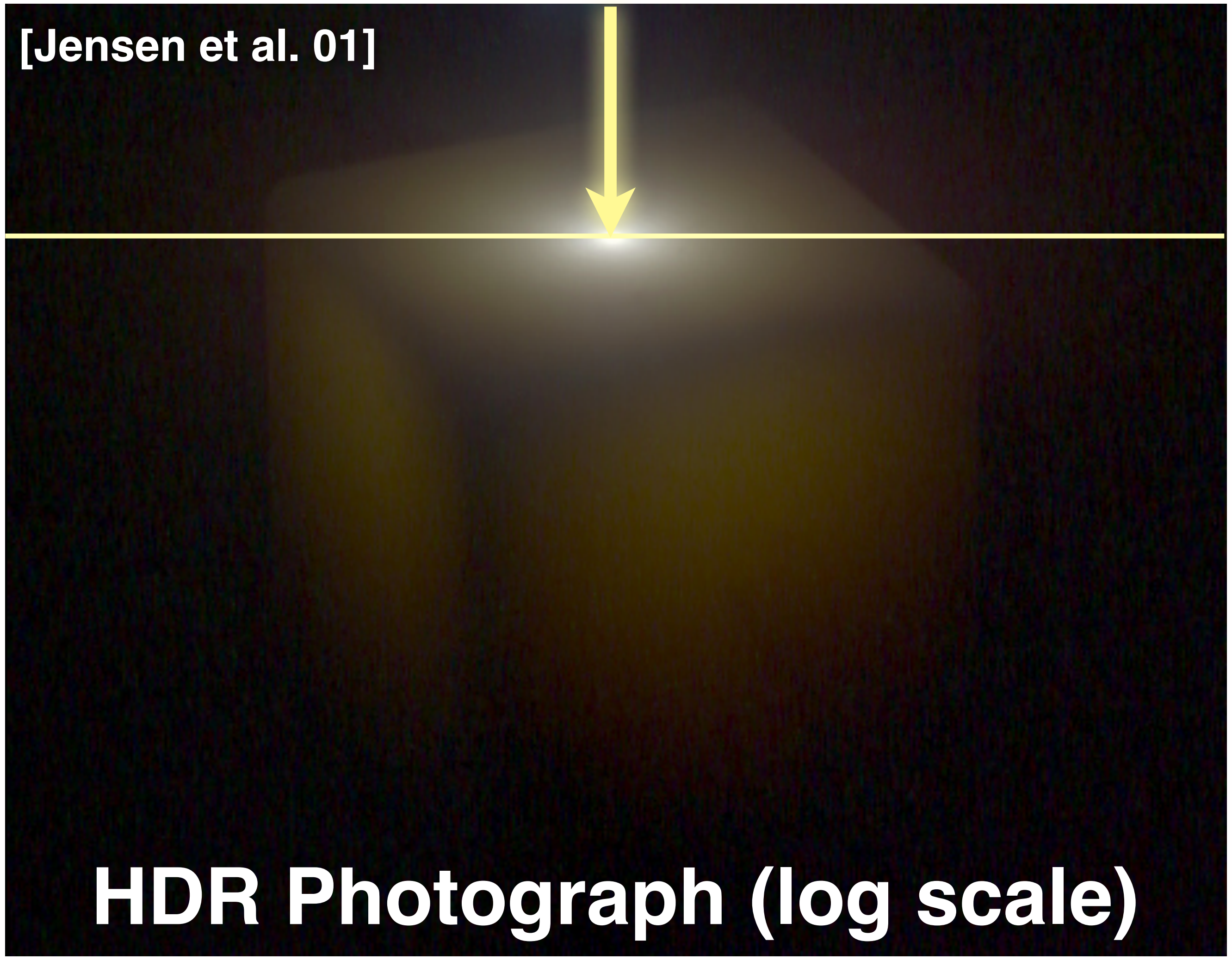
$$\phi(d) = \frac{\Phi}{4\pi D}\frac{e^{-\sigma_{tr} d}}{d}$$

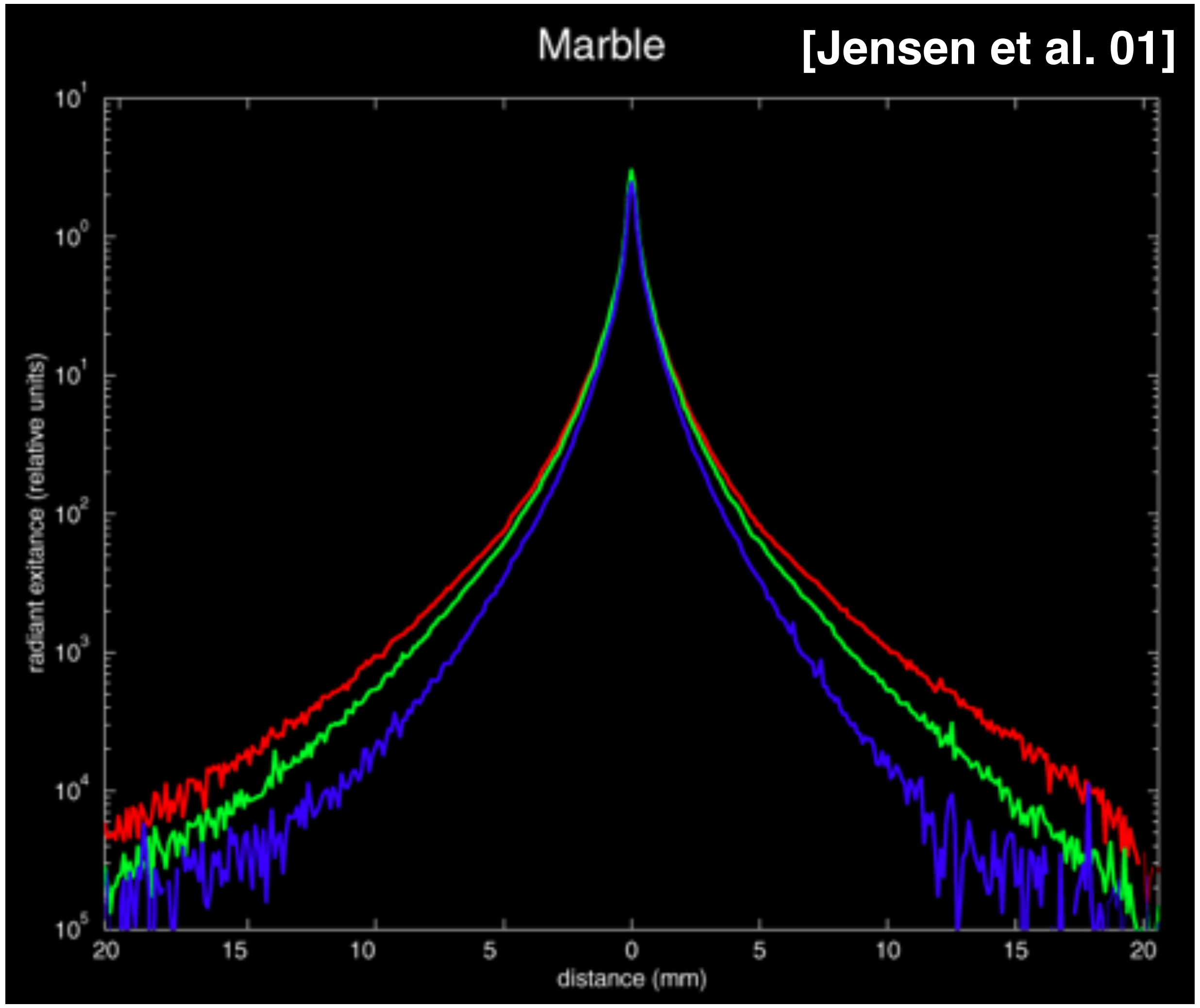
Diffusion Approximation

- So far, we can express fluence due to a point emitter in an infinite medium
- How do we apply this to rendering subsurface scattering?

Searchlight Problem (motivation)

Marble Block

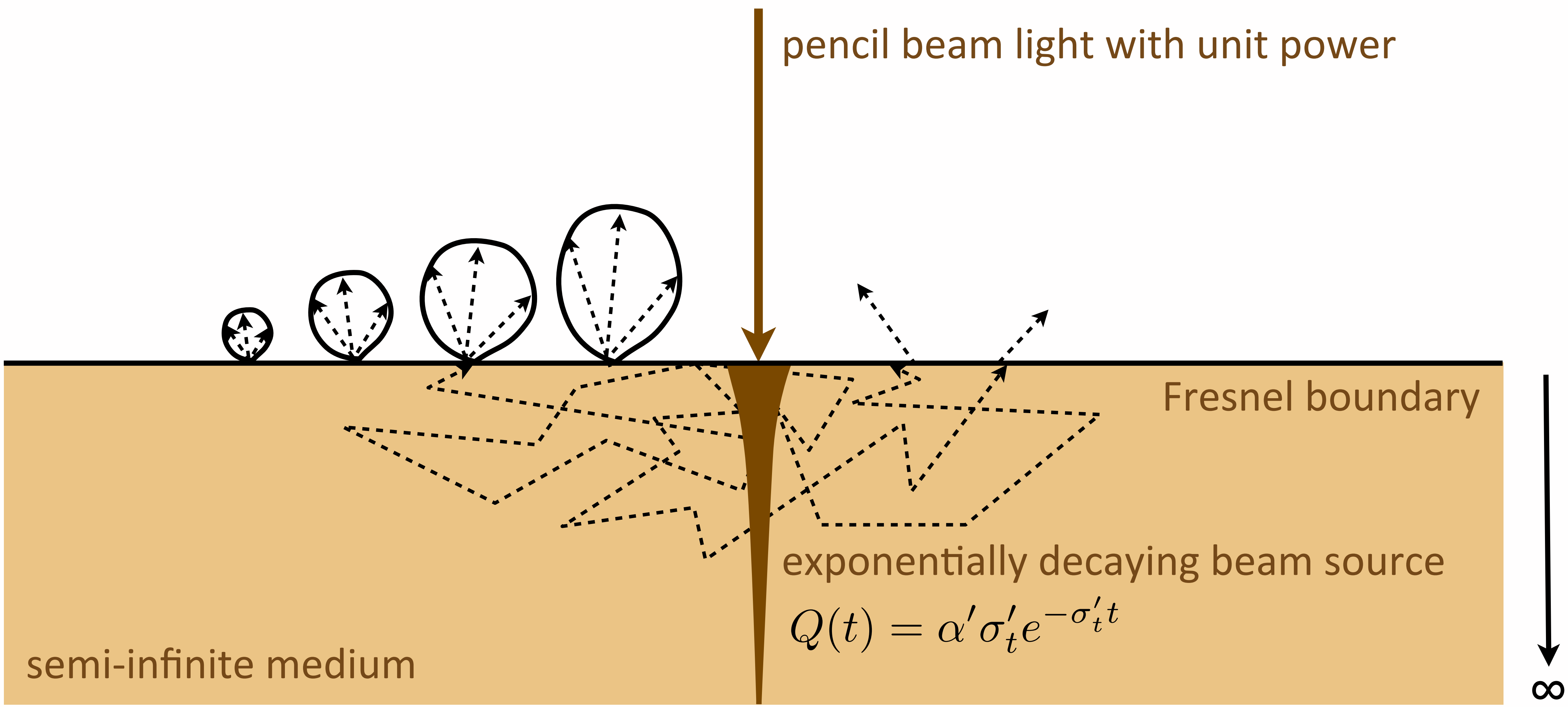




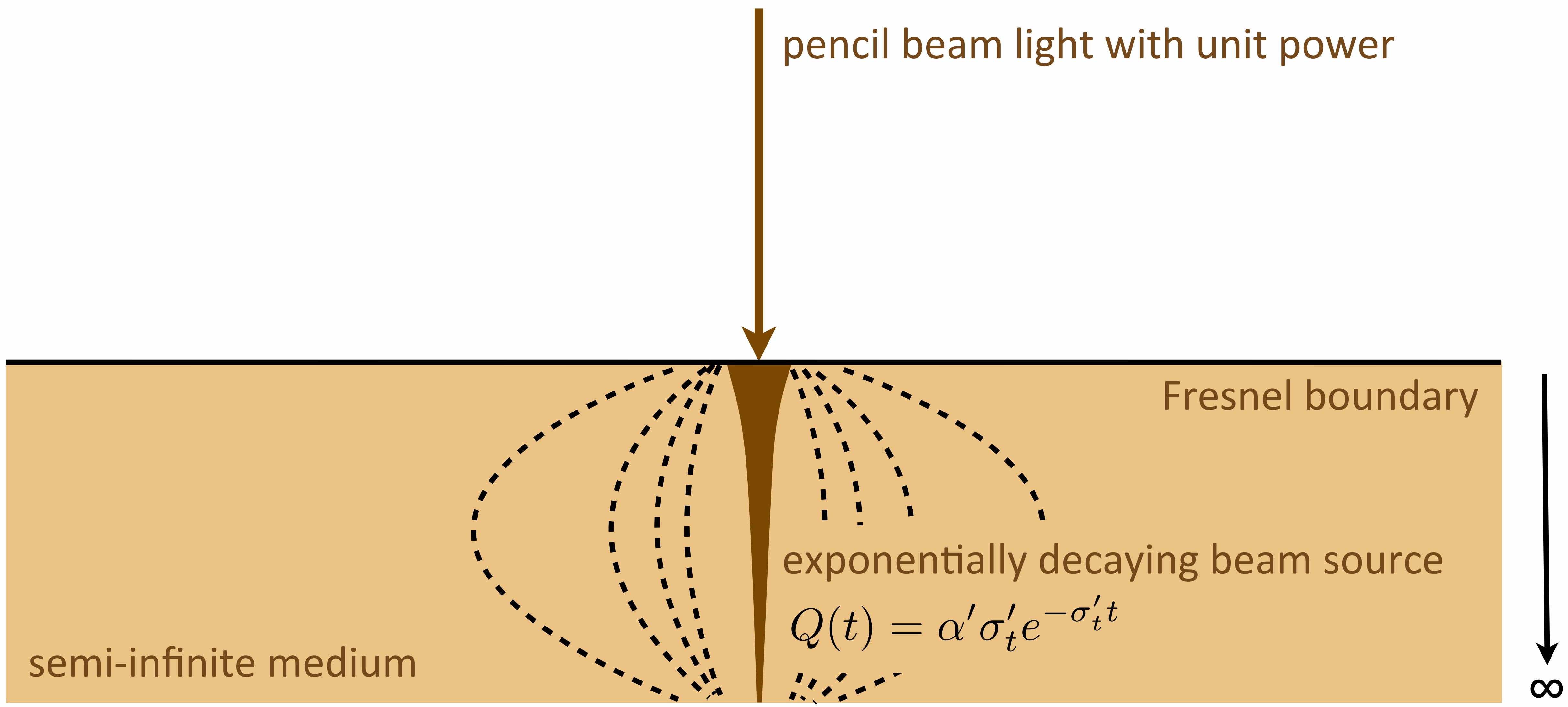
Searchlight Problem

- Goal: analytic solution for BSSRDF
- Assumptions:
  - Semi-infinite homogeneous medium
  - Focused pencil beam light orthogonal to medium
  - Dielectric boundary
- Result: exitant radiance at all positions
  - Symmetries make the solution radially symmetric

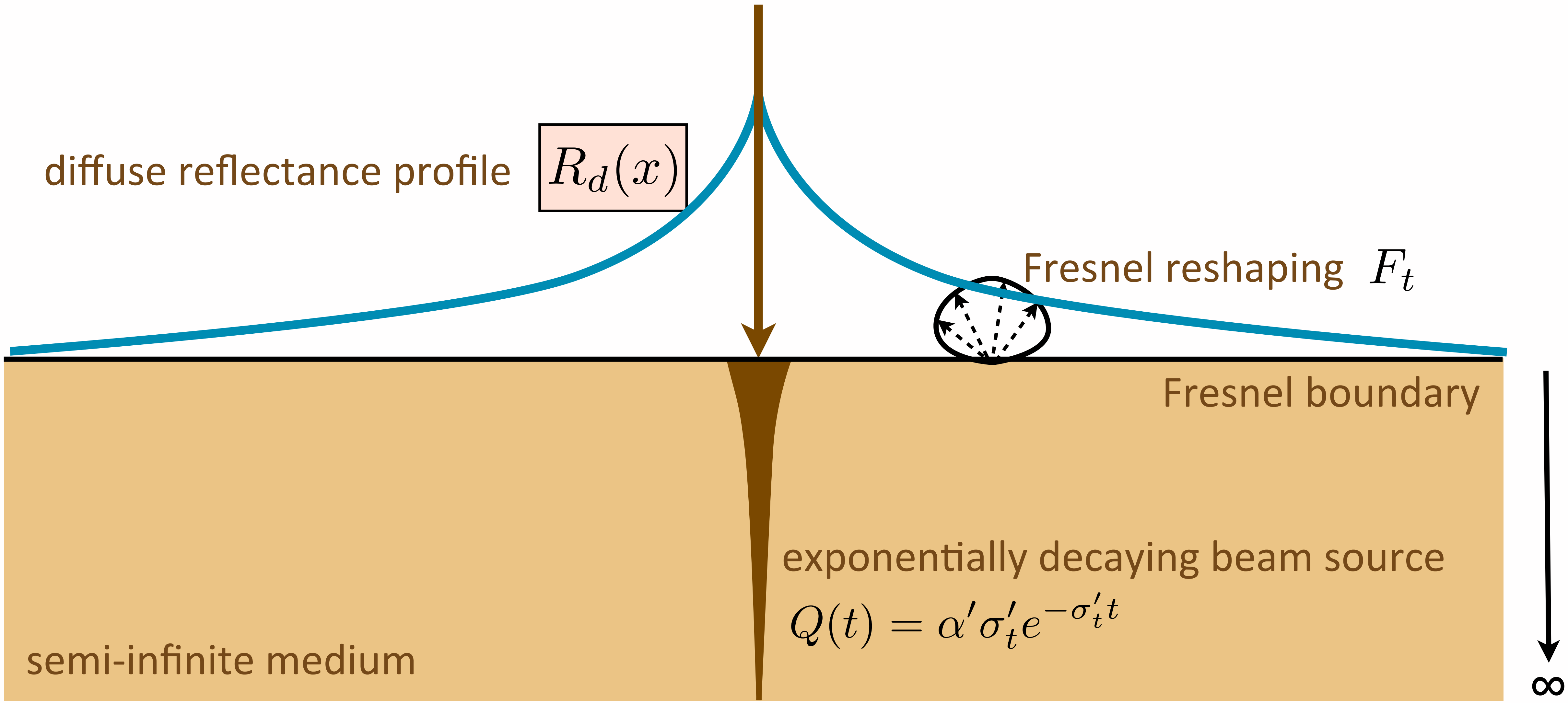
Searchlight Problem



Searchlight Problem - Diffusion



BSSRDF - Diffusion



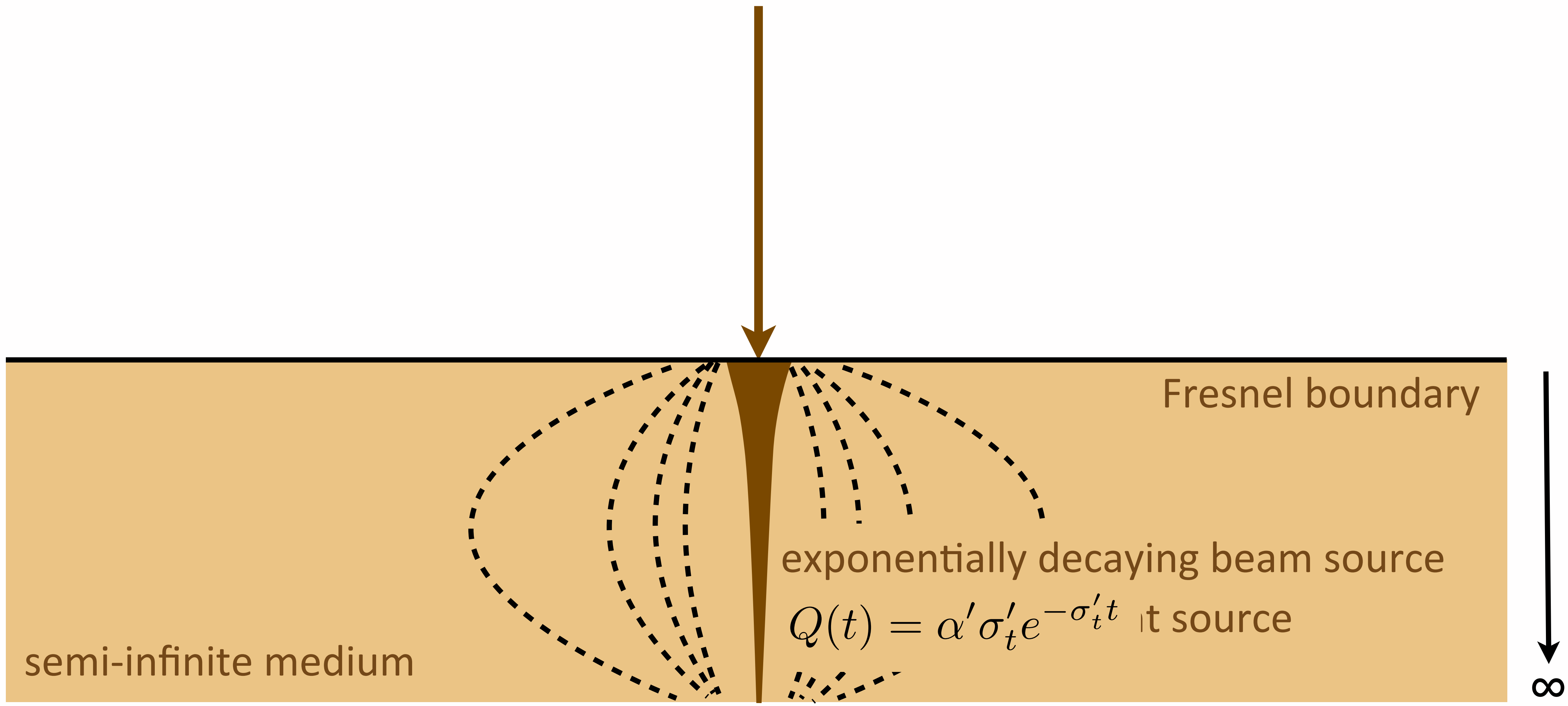
$$S_d(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) = \frac{1}{\pi} F_t(x_i, \vec{\omega}_i) \boxed{R_d(x_o - x_i)} \frac{F_t(x_o, \vec{\omega}_o)}{4C_\phi(\frac{1}{\eta})}$$

Single-depth Approximation

- Assume all scattering events happen at a single point at a depth of *one mean free path*
- Turns pencil beam into a point light source
  - Rather strong assumption
  - But we can work with the point source solution

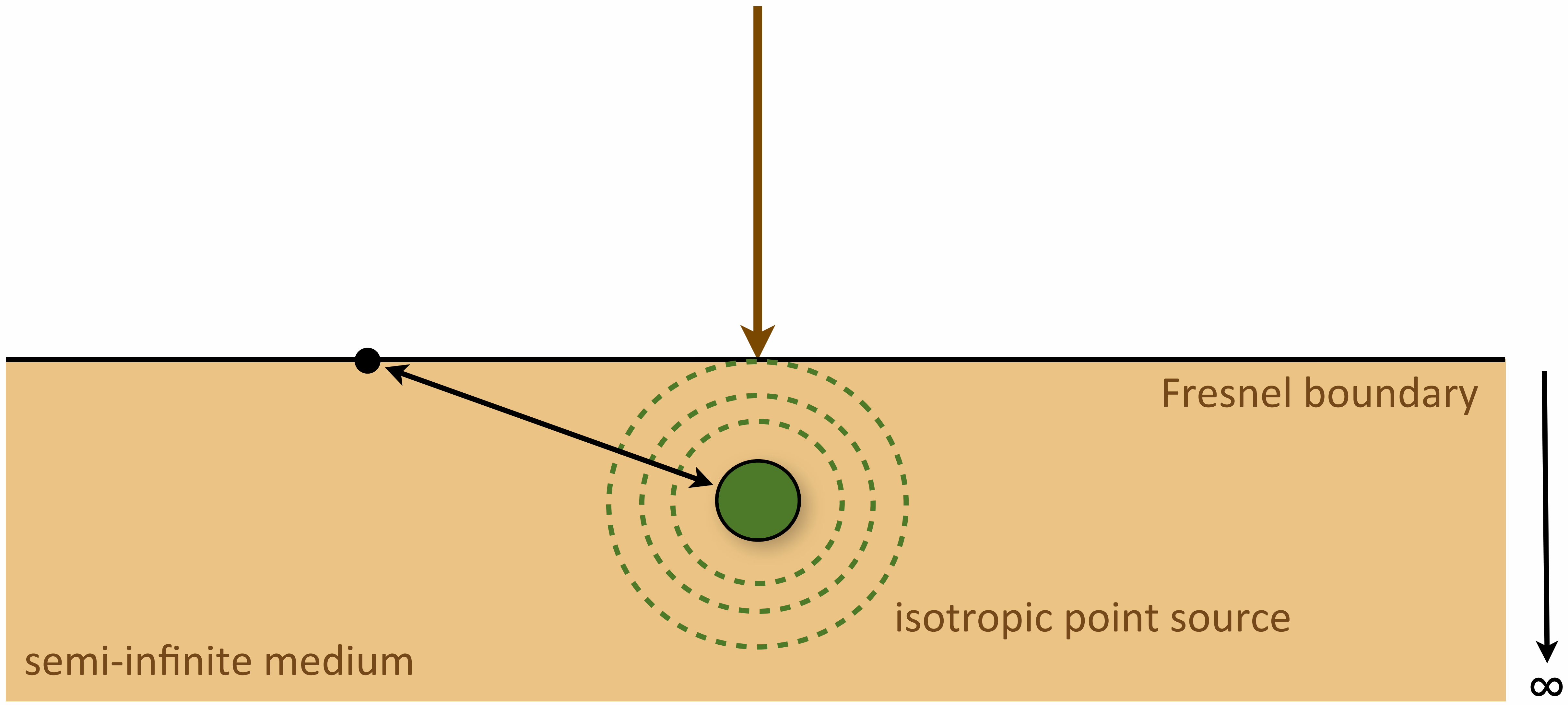
Monopole Approximation

[Farrell 92, Jensen 01]

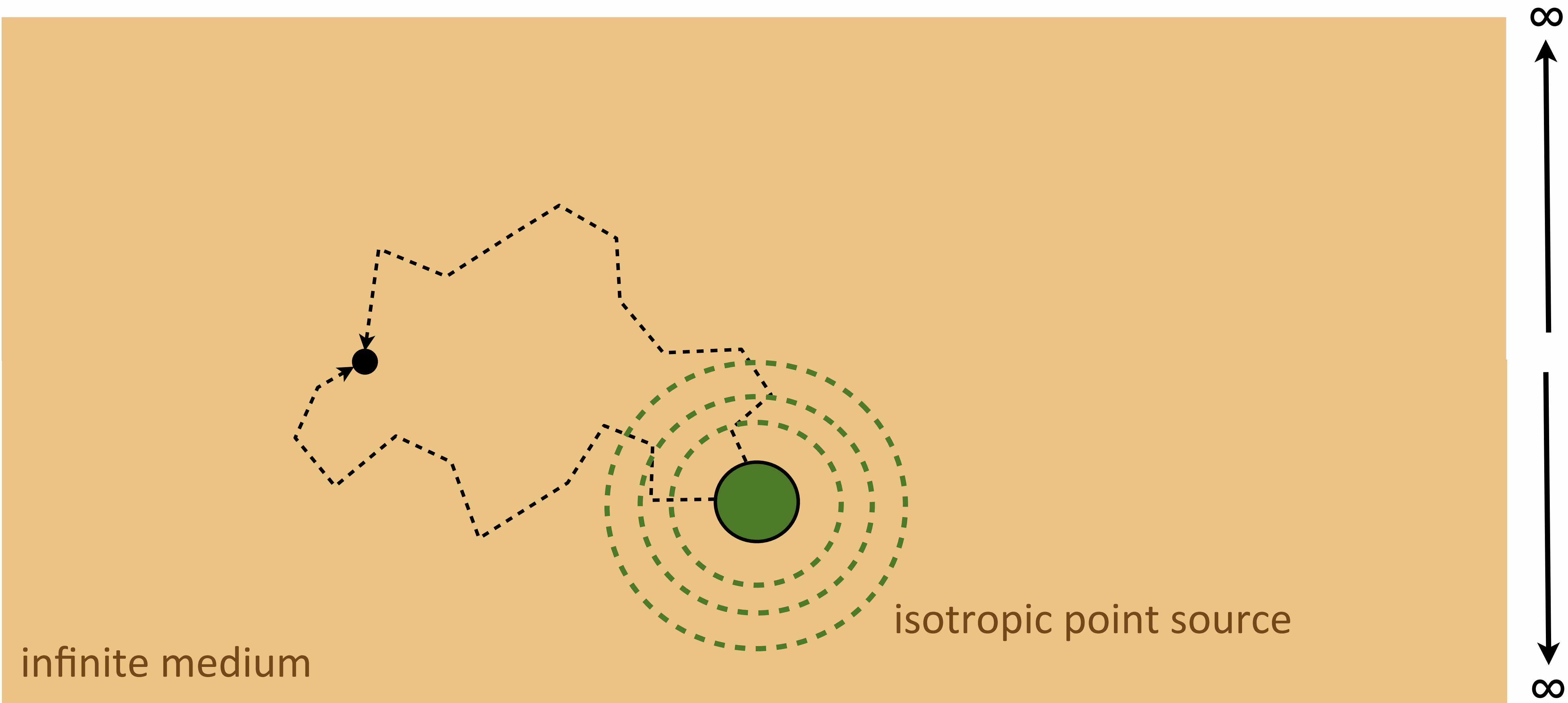


Monopole Approximation

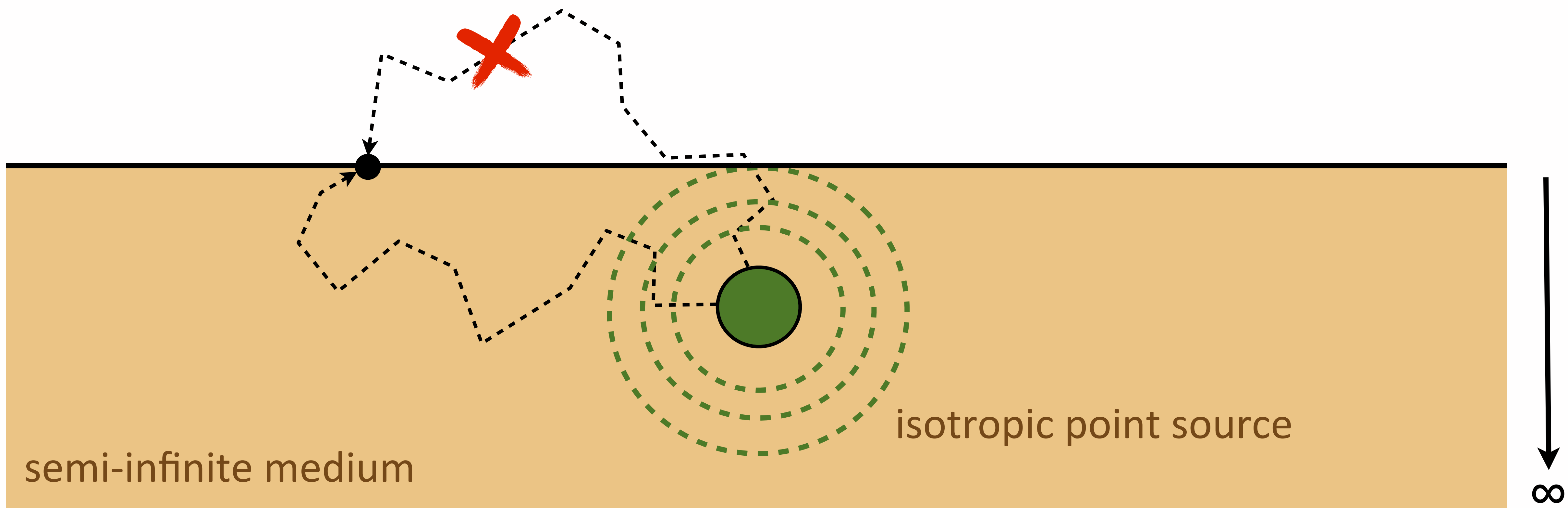
[Farrell 92, Jensen 01]



Monopole Approximation



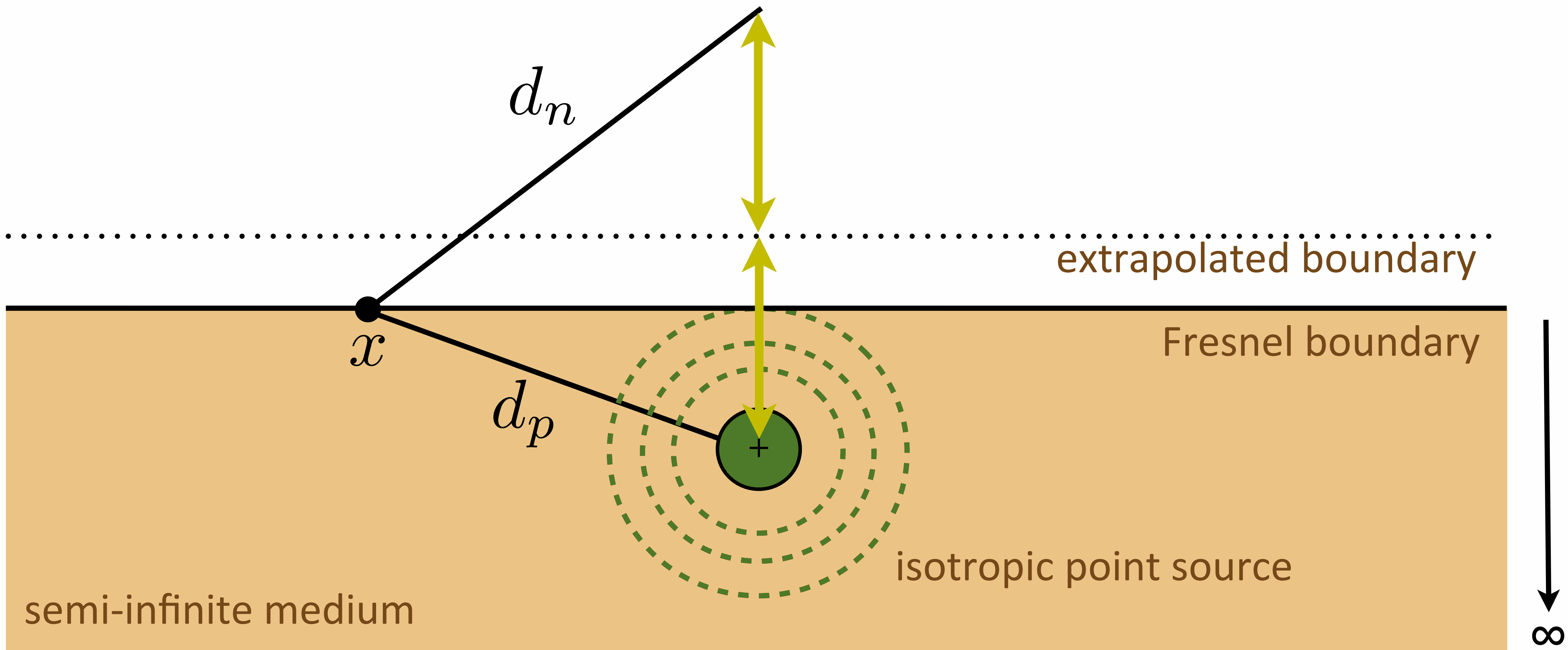
Monopole Approximation



Dipole Diffusion

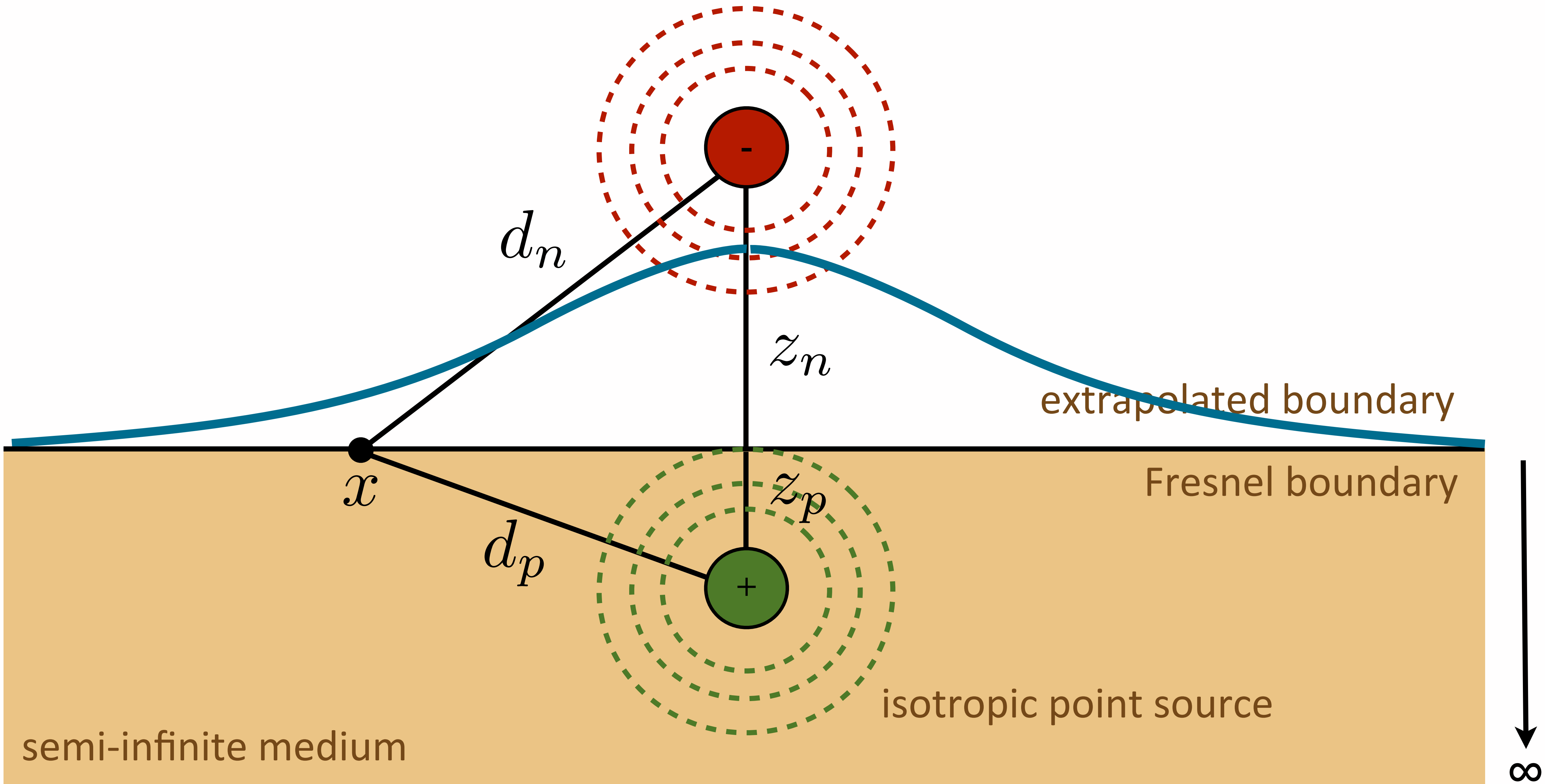
- Boundary not considered yet!
- Place *negative* source by mirroring source at an extrapolated boundary
- Negative source "subtracts" away light that exits through the boundary and never returns
  - Fluence is 0 at extrapolated boundary
  - Called "Method of images" in other fields

Method of Images - Dipole



$$\phi(x) = \frac{\Phi}{4\pi D}\left(\frac{e^{-\sigma_{tr} d_p}}{d_p} - \frac{e^{-\sigma_{tr} d_n}}{d_n}\right)$$

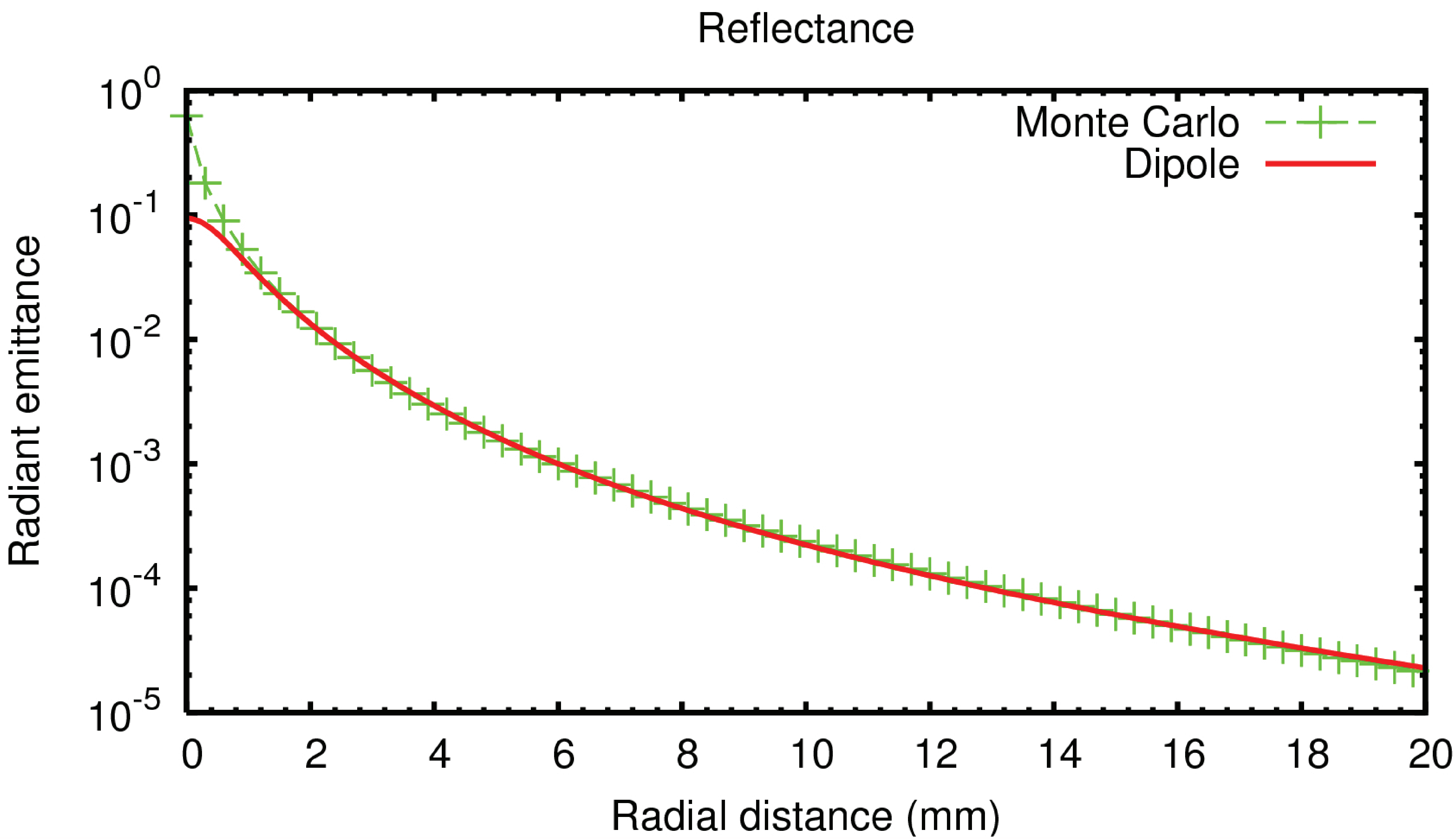
Method of Images - Dipole



$$R_d(x) = \frac{\alpha'}{4\pi}\left(\frac{z_p(1+\sigma_{tr}d_p)e^{-\sigma_{tr}d_p}}{d_p^3} + \frac{z_n(1+\sigma_{tr}d_n)e^{-\sigma_{tr}d_n}}{d_n^3}\right)$$

see [Farrell 92, Jensen 01] for detail

Dipole Diffusion Profile



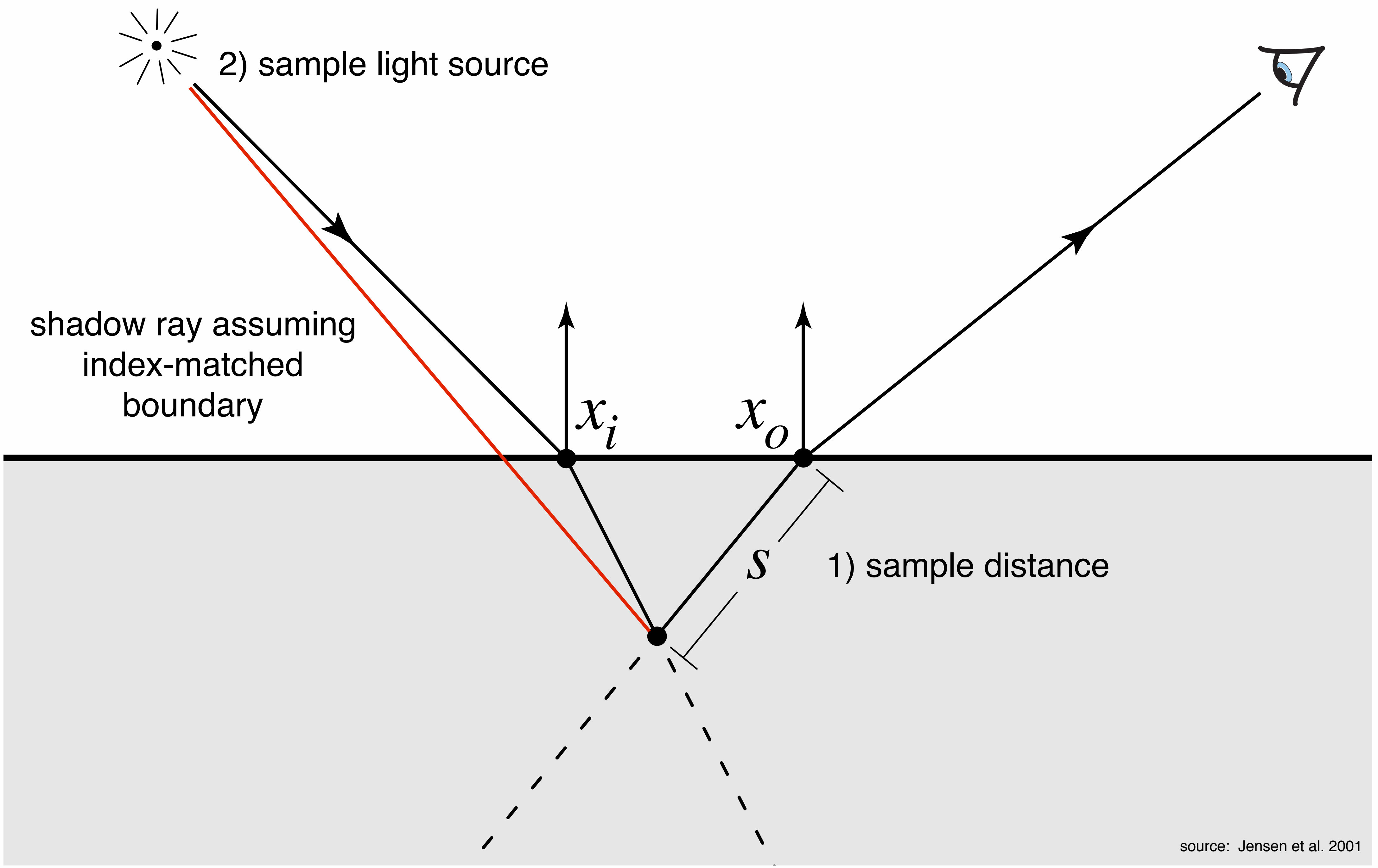
The Diffusion Dipole BSSRDF

- Only considering multiply-scattered light so far:

$$S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) = S_d(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) + S^{(1)}(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)$$

$$S_d(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) = \frac{1}{\pi} F_t(x_i, \vec{\omega_i}) R_d(x_o - x_i) \frac{F_t(x_o, \vec{\omega_o})}{4C_\phi(\frac{1}{\eta})}$$

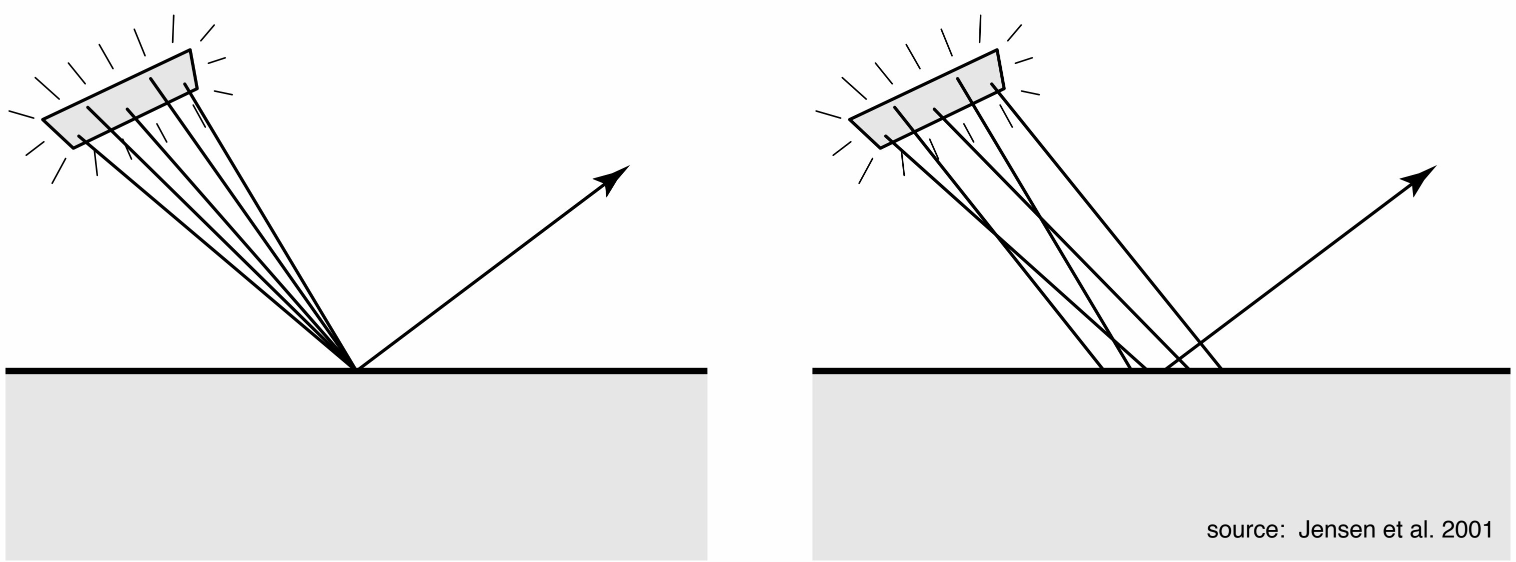
Single-Scattering



Rendering with the Diffusion Dipole

$$L_o(\mathbf{x}_o, \vec{\omega}_o) = \int_A \int_\Omega S(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o) L_i(\mathbf{x}_i, \vec{\omega}_i)(\vec{n} \cdot \vec{\omega}_i)\, d\vec{\omega}_i dA(\mathbf{x}_i)$$

- Integration over both hemisphere and area!
- Integrate $S^{(1)}$ by ray marching or random sampling refracted ray into surface
- Integrate S_d by randomly sampling a $\boldsymbol{x}_i$ around $\boldsymbol{x}_o$ with exponential density. Compute shade/shadow at each $\boldsymbol{x}_i$



source: Jensen et al. 2001

BSSRDF (Fresnel term)

source: Jensen et al. 2001

BSSRDF (single-scattering term)

source: Jensen et al. 2001

BSSRDF (diffusion term)

source: Jensen et al. 2001

BSSRDF

source: Jensen et al. 2001

Comparison

BSSRDF

render time: seconds-minutes

Photon Mapping

render time: minutes-hours

source: Jensen et al. 2001

Comparison

BRDF

BSSRDF

Photon Mapping

source: Jensen et al. 2001

BRDF

source: Jensen et al. 2001

BSSRDF

source: Jensen et al. 2001

Milk (BRDF)

source: Jensen et al. 2001

Whole Milk (BSSRDF)

source: Jensen et al. 2001

Skim Milk (BSSRDF)

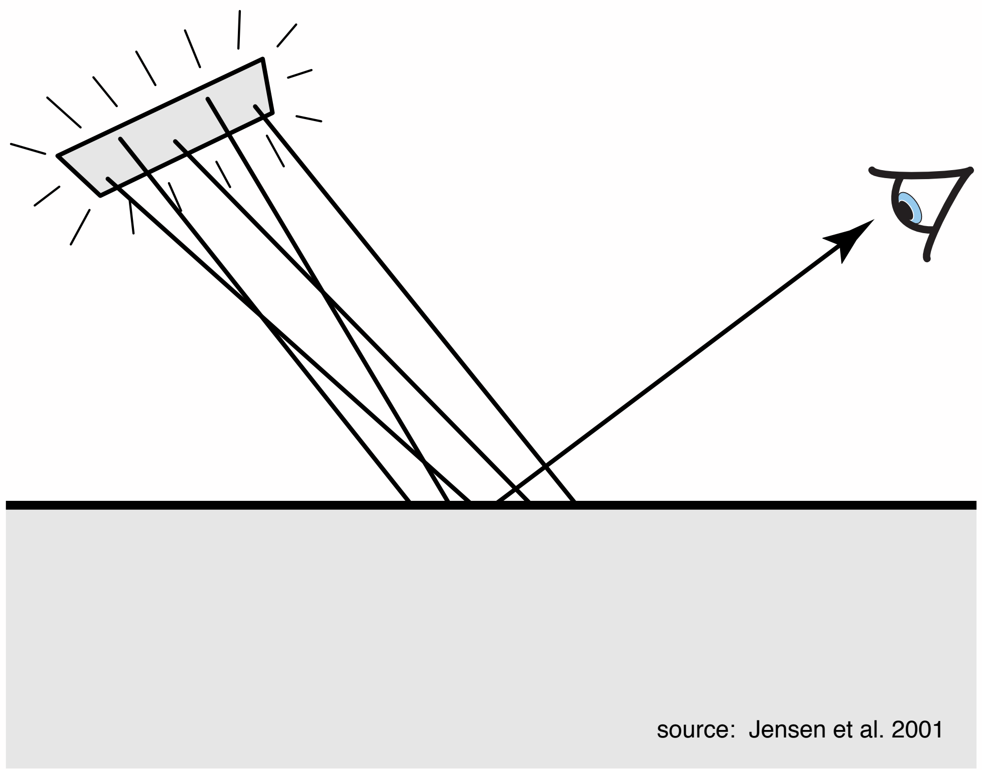
source: Jensen et al. 2001

Milk Comparison

source: Jensen et al. 2001

Practical Concerns

$$L_o(\mathbf{x}_o, \vec{\omega}_o) = \int_A \int_\Omega S(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o) L_i(\mathbf{x}_i, \vec{\omega}_i)(\vec{n} \cdot \vec{\omega}_i)\, d\vec{\omega}_i dA(\mathbf{x}_i)$$

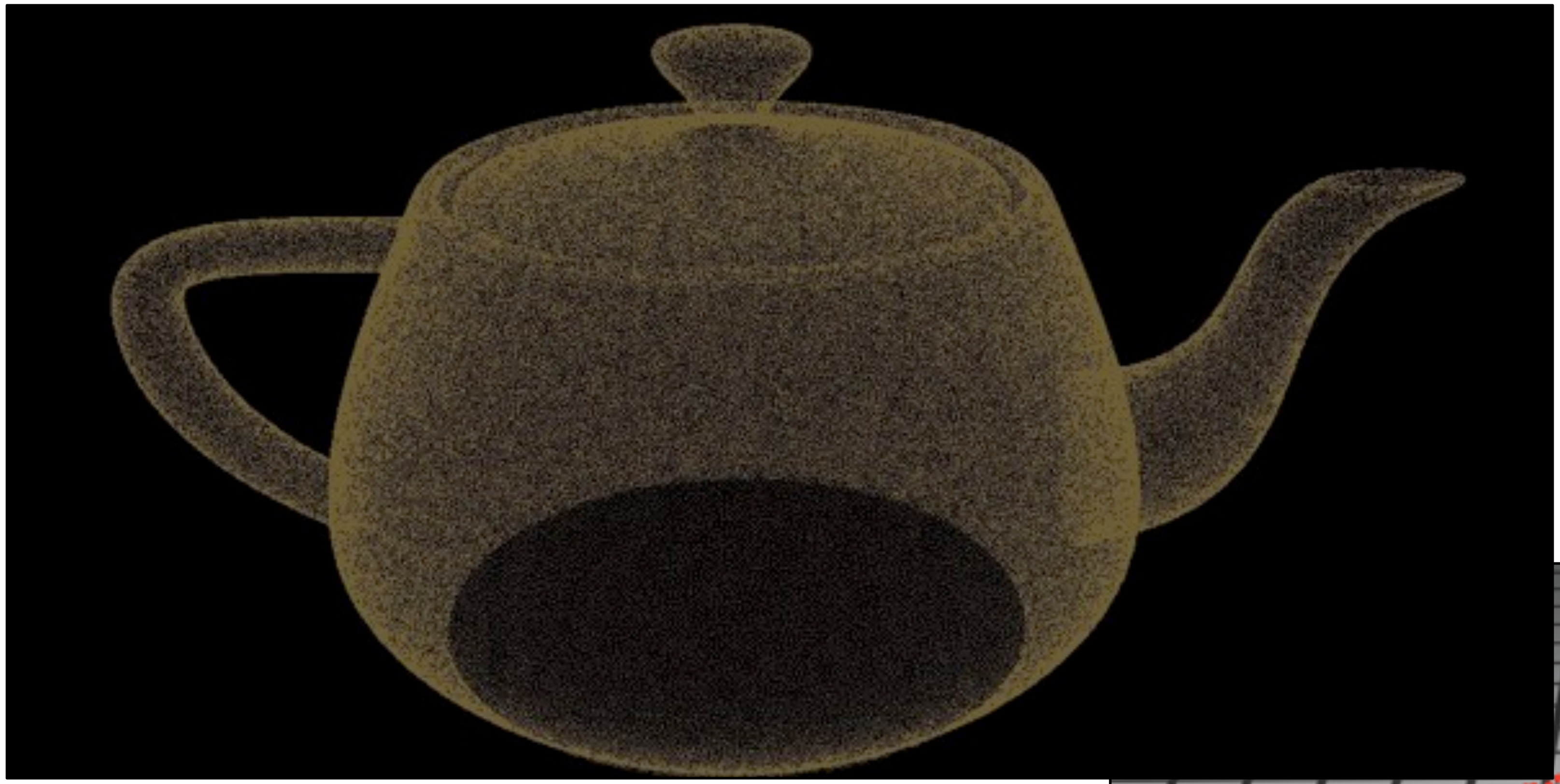


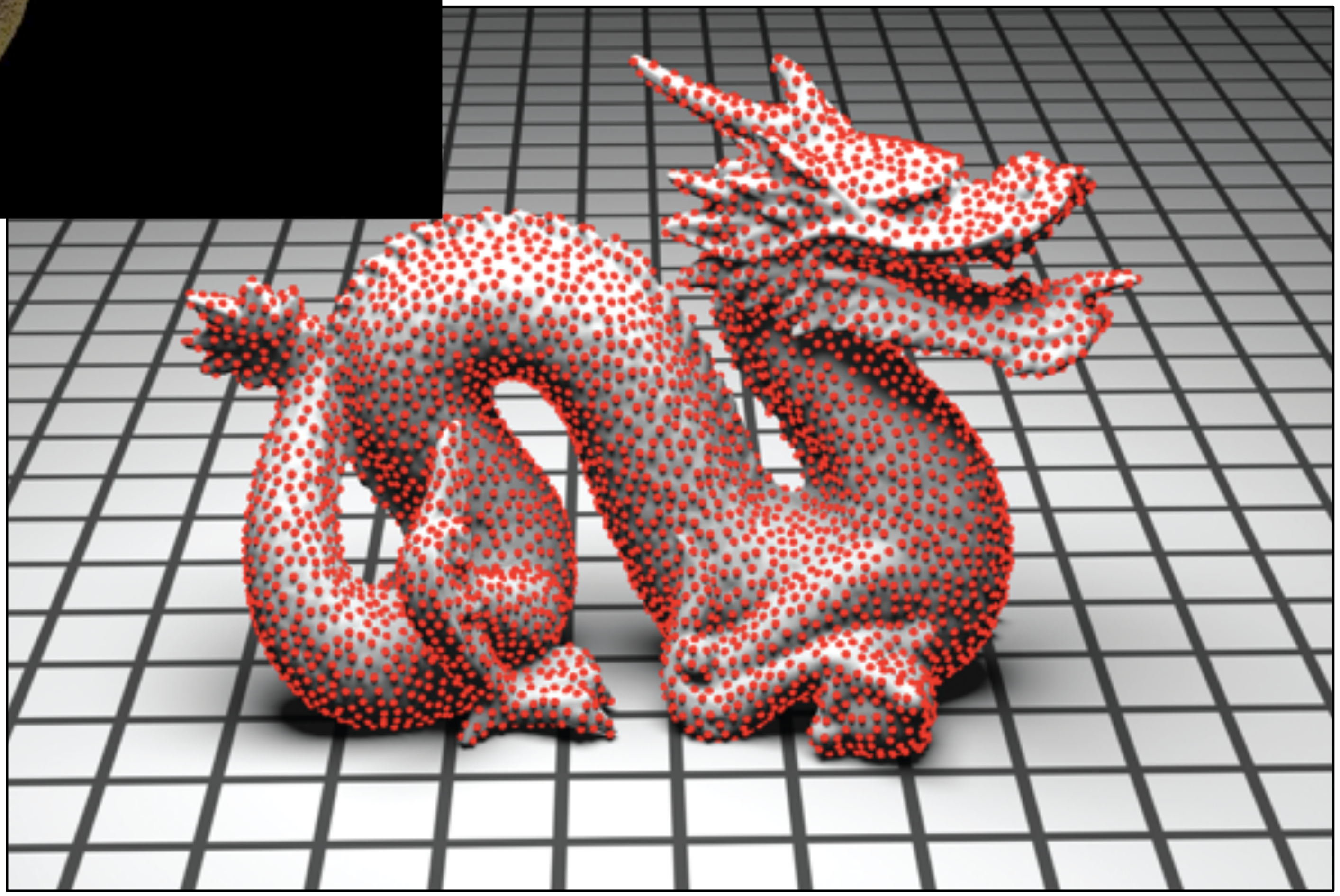
- Need to integrate/evaluate incident lighting at many x_i's when evaluating BSSRDF at x_o

Practical Concerns

- Independent evaluation at each x_o is wasteful
  - Lots of potential for computation re-use
- [Jensen and Buhler 2002]
  - 2 pass approach
  - hierarchical BSSRDF evaluation

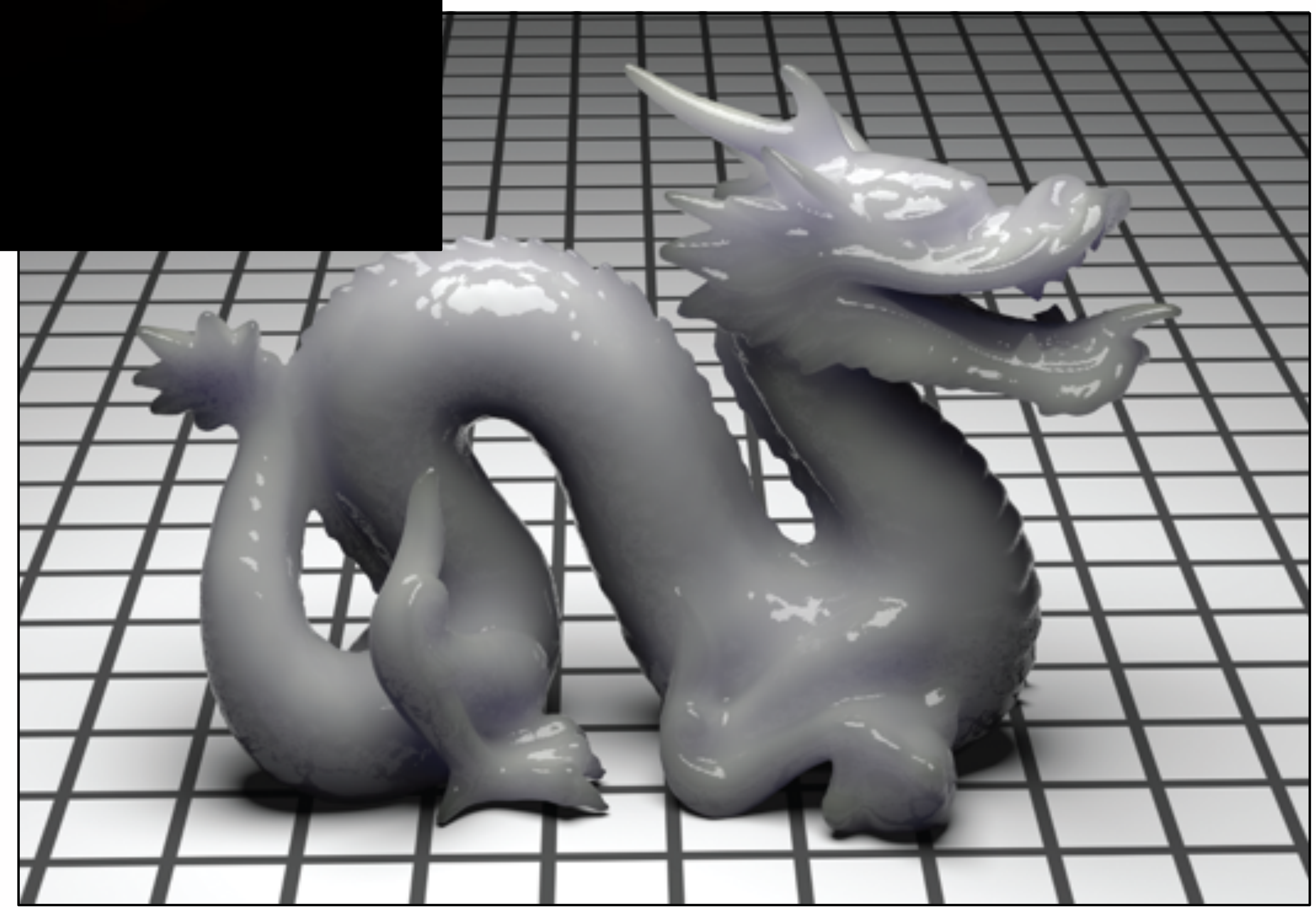
Pass 1: Sampling Irradiance





- Distribute points randomly on surface
- Compute irradiance at each point

Pass 2: Evaluating Diffusion Dipole



- integrate over all cached irradiance values, weight by diffusion profile
- hierarchical evaluation
  - distant points can be clustered

Original BSSRDF

render time: 18 minutes

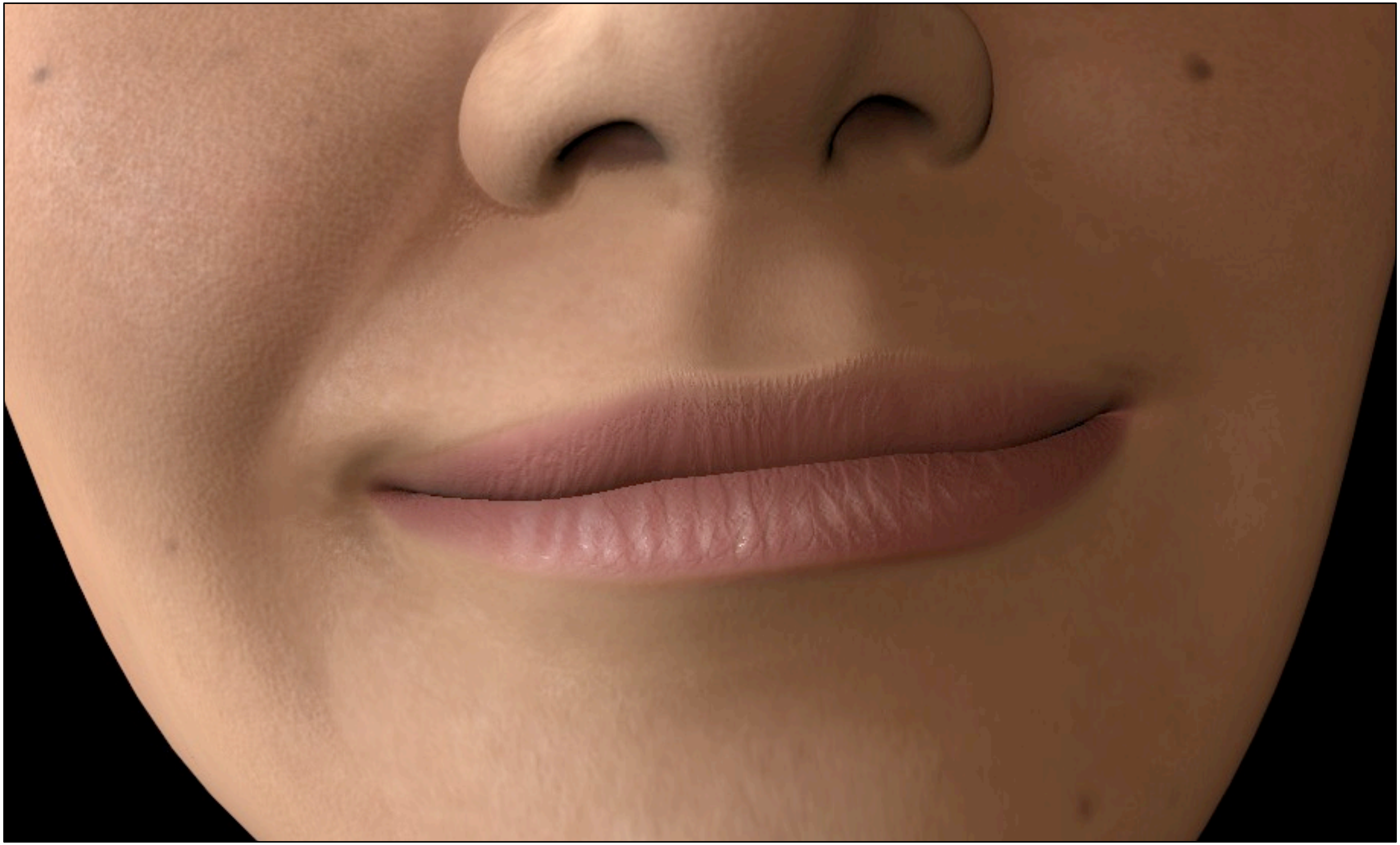
source: Jensen and Buhler 2002

Hierarchical BSSRDF

render time: 7 seconds

source: Jensen and Buhler 2002

Skin BRDF



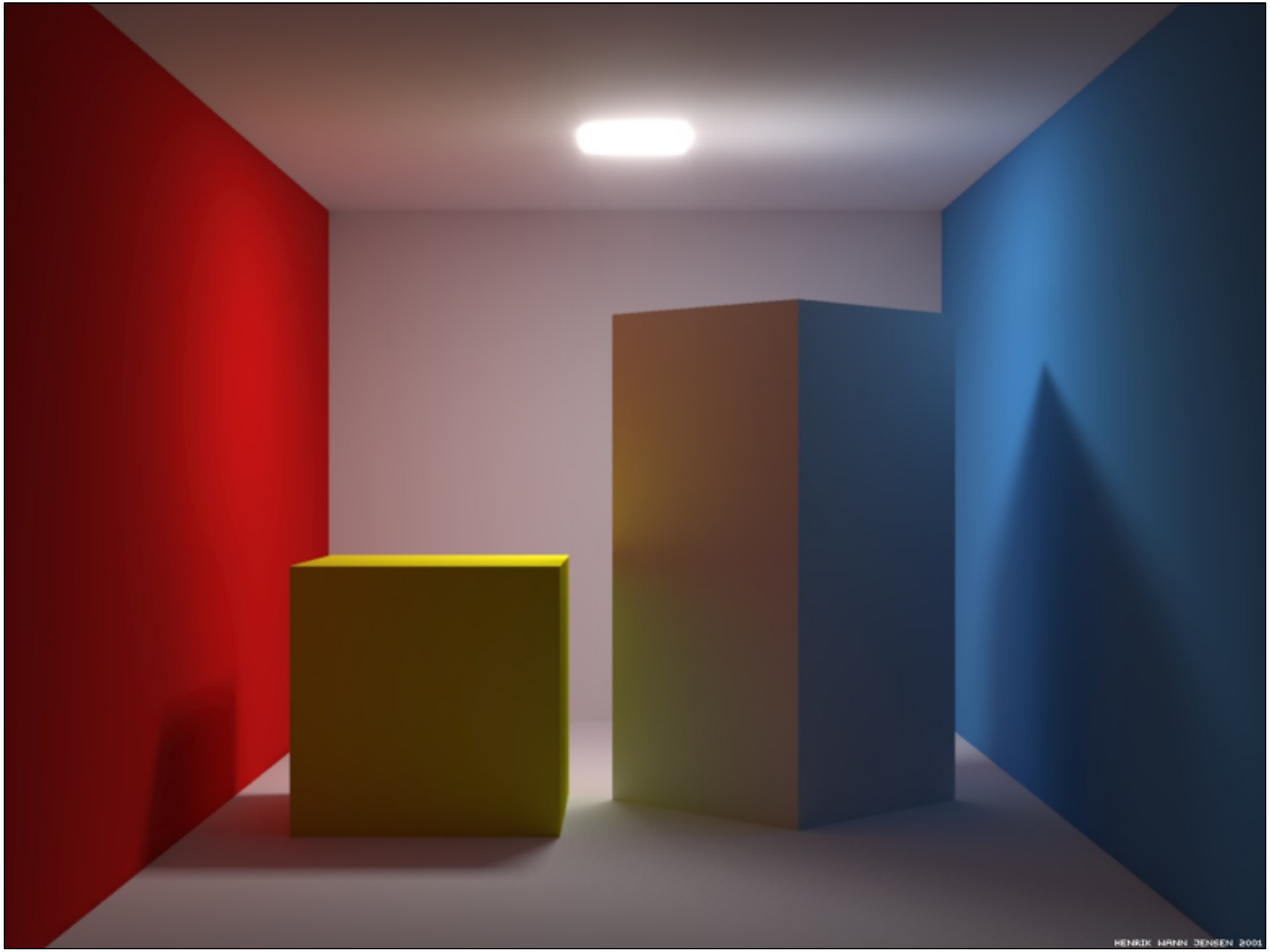
source: Jensen and Buhler 2002

Skin BSSRDF

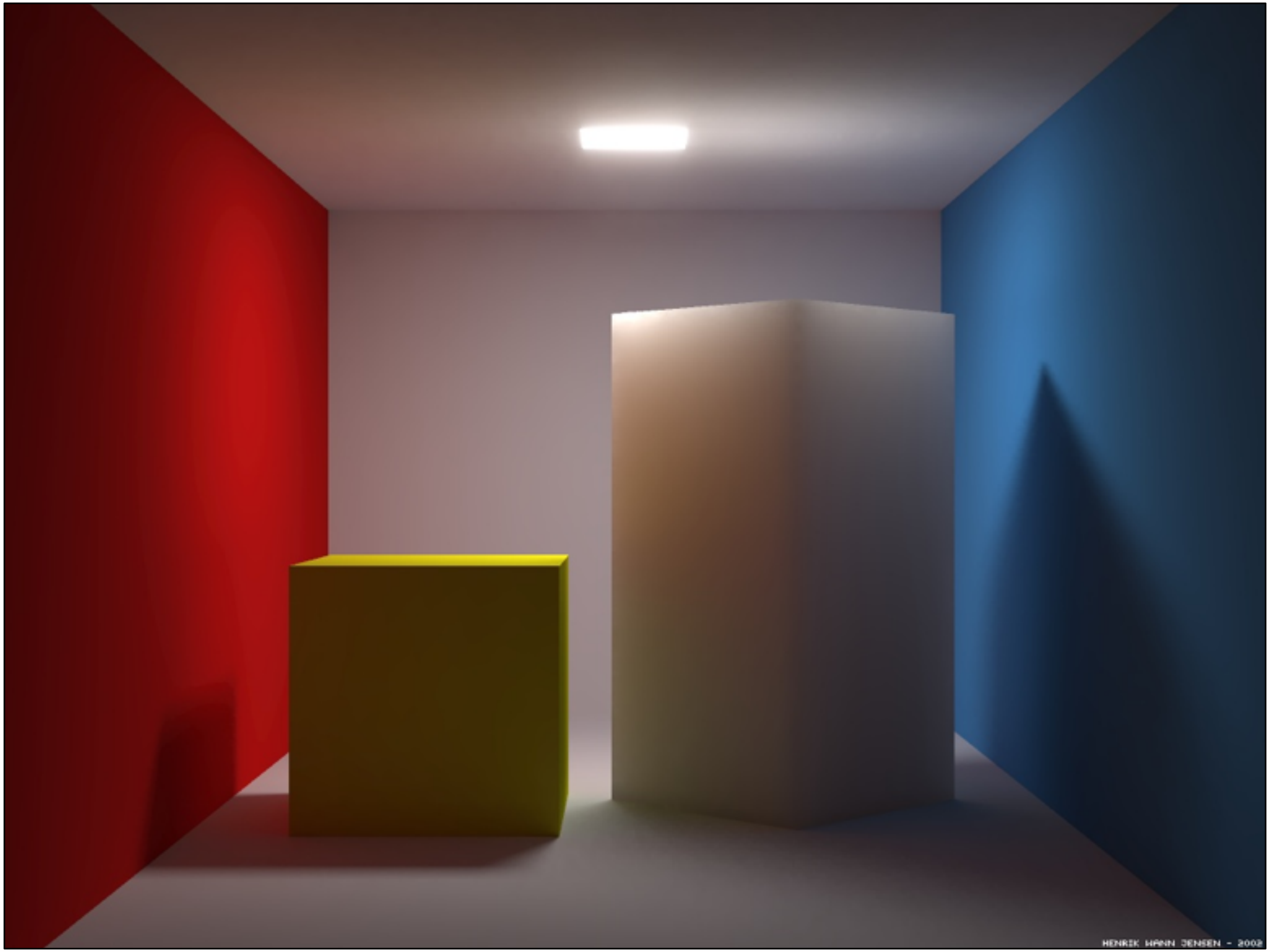
source: Jensen and Buhler 2002

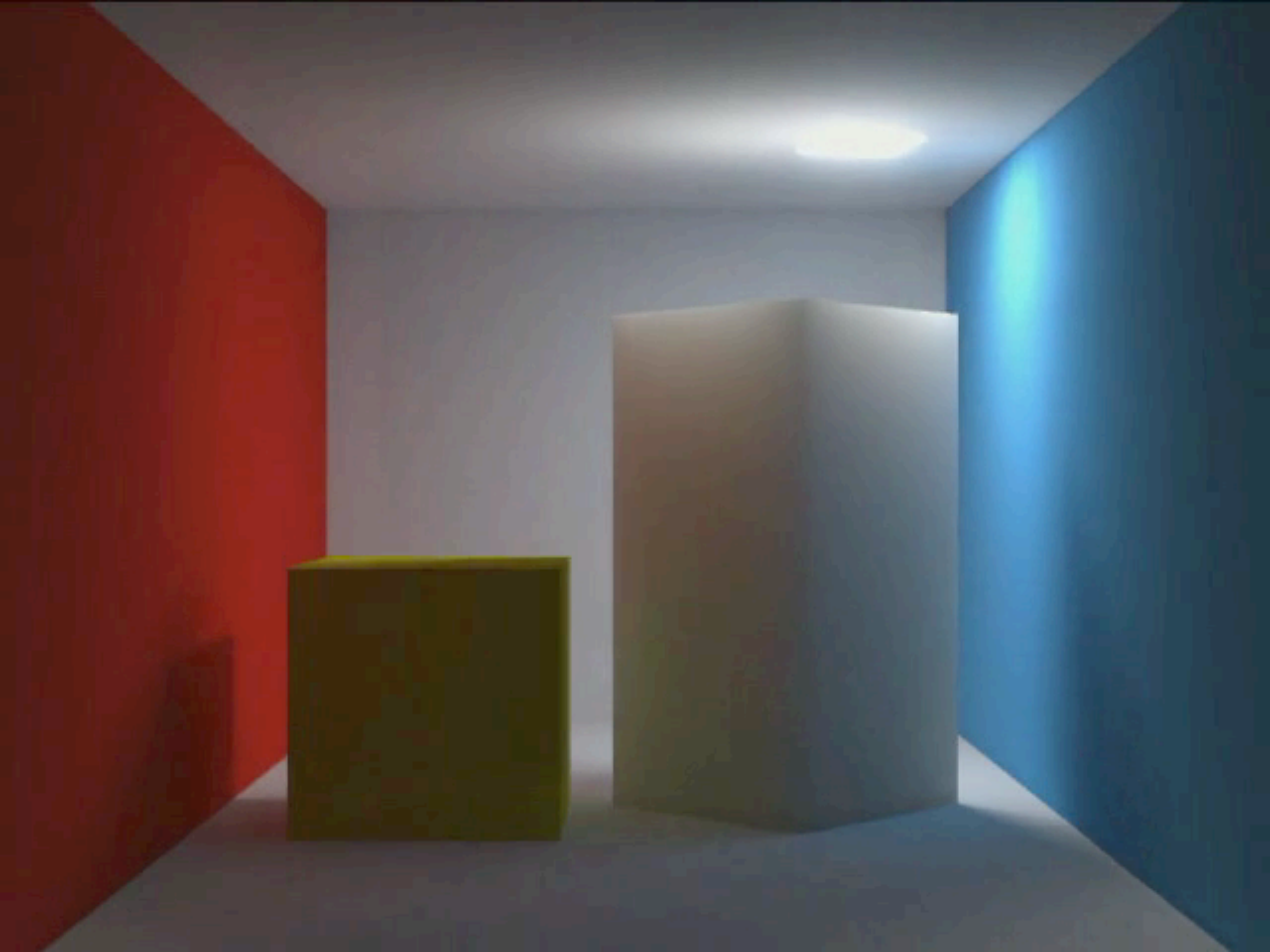
Green Skin (Shrek)

Cornell Box (BRDF)



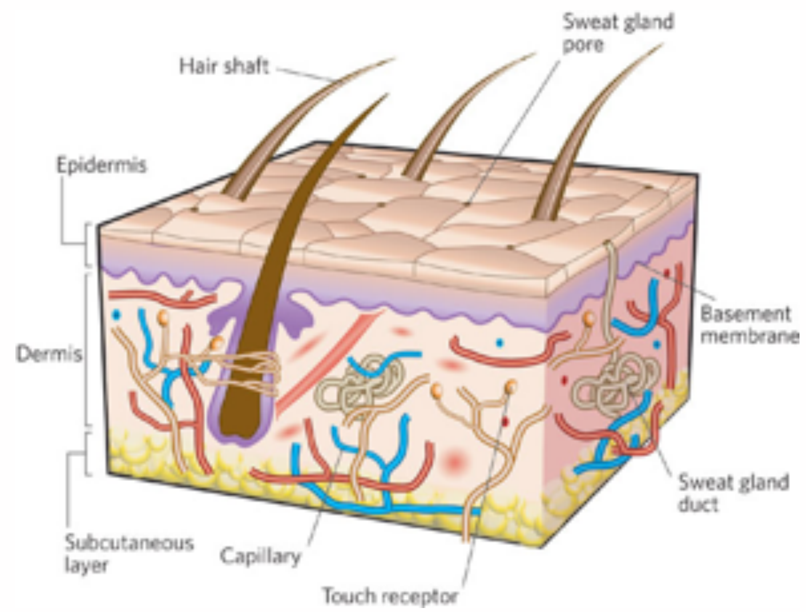
Cornell Box (BSSRDF)





Dipole Limitations

- Many materials are not actually semi-infinite
  - thin materials?
  - layered materials?
  - heterogeneous materials?

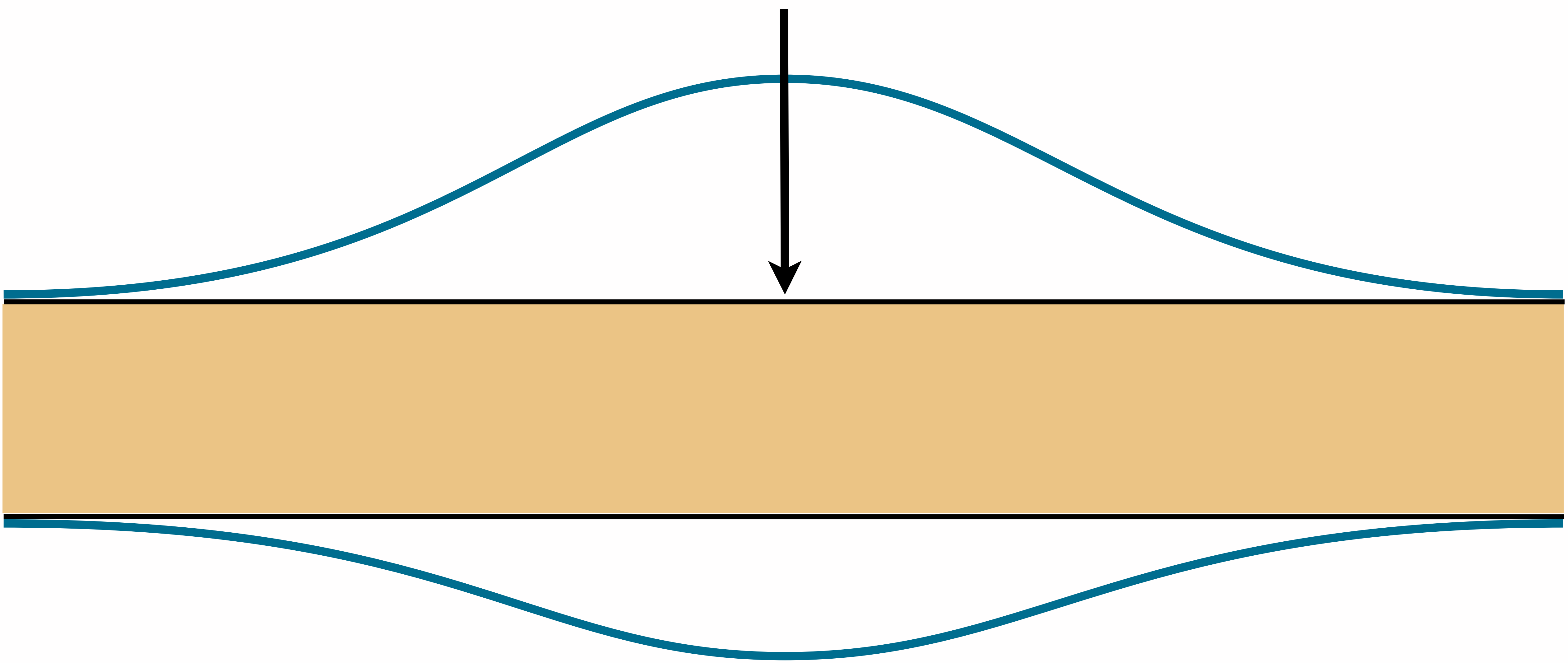


The Multipole

- [Donner and Jensen 2005]
  - Extend single semi-infinite slab into several finite slabs
  - More complex boundary condition

The Multipole

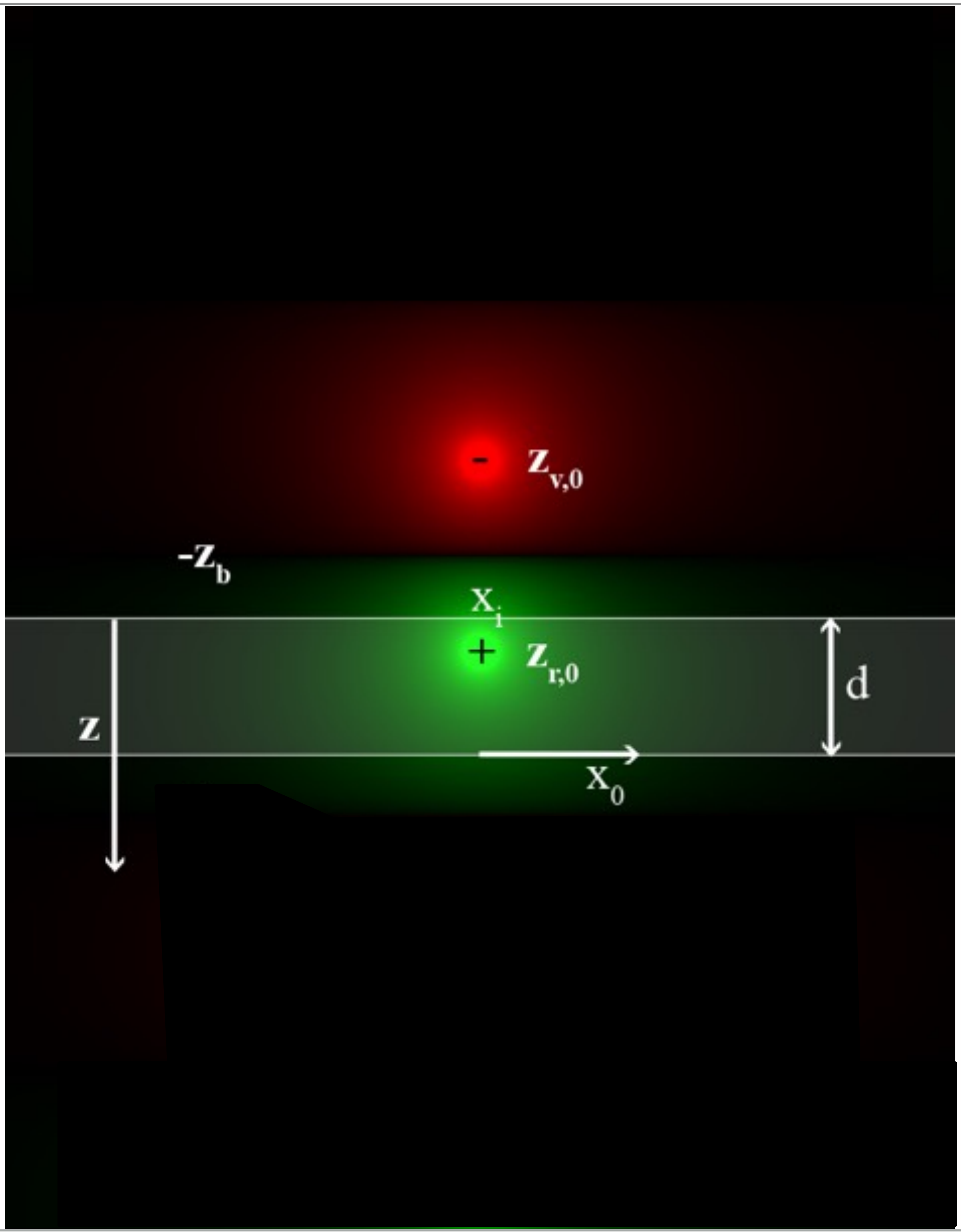
- Extended model: finite slab
- Reflectance *and* transmittance profiles



The Multipole

- Create negative dipole source by mirroring on upper extended boundary
- Mirror source on lower boundary to match conditions there
  - But now upper boundary is violated?
- Mirror the mirrored sources on respective other boundary again, etc...
- Infinite series matches to *both* boundary conditions

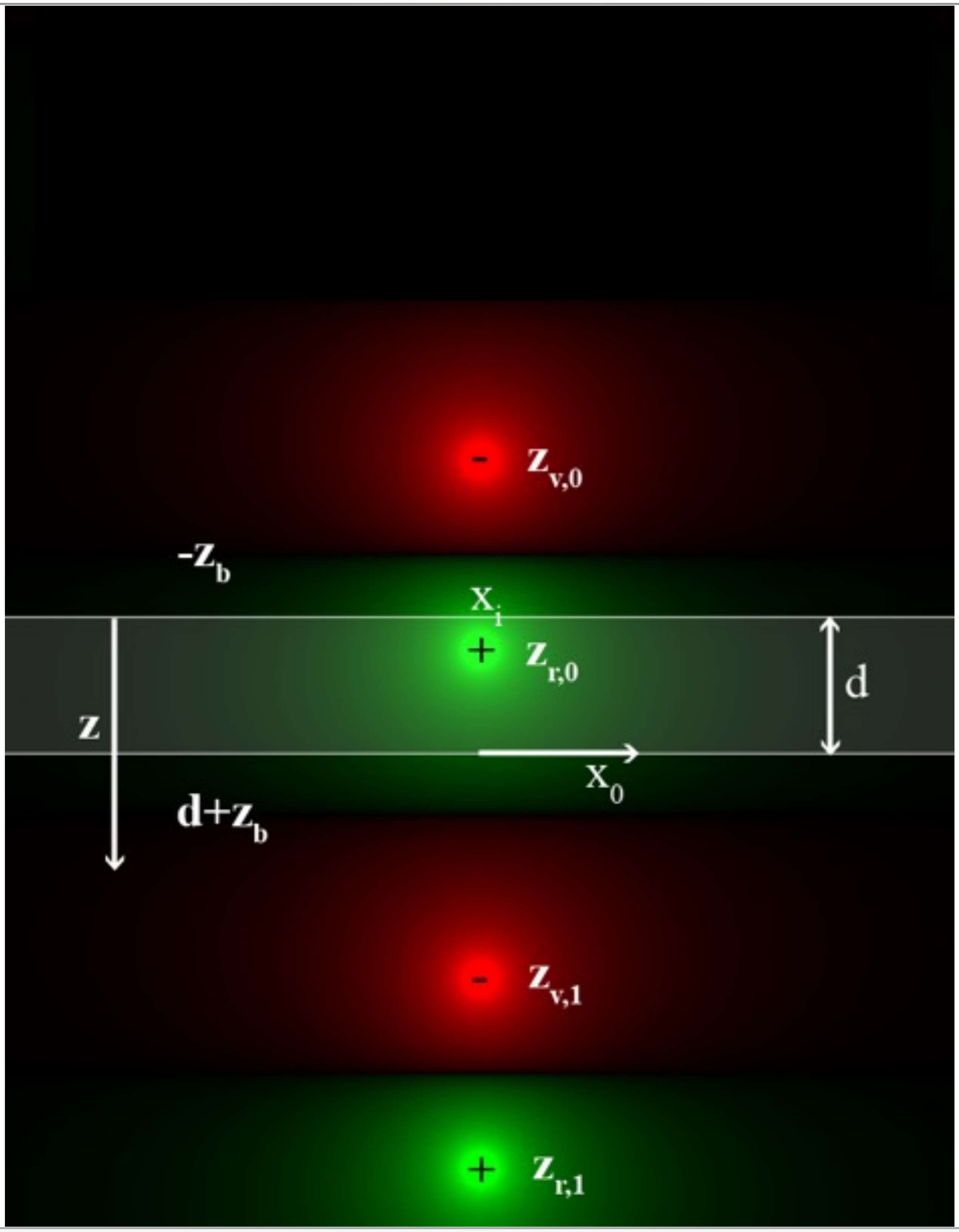
The Multipole - 1 Slab



fluence = 0

source: Habel 2007

The Multipole - 1 Slab

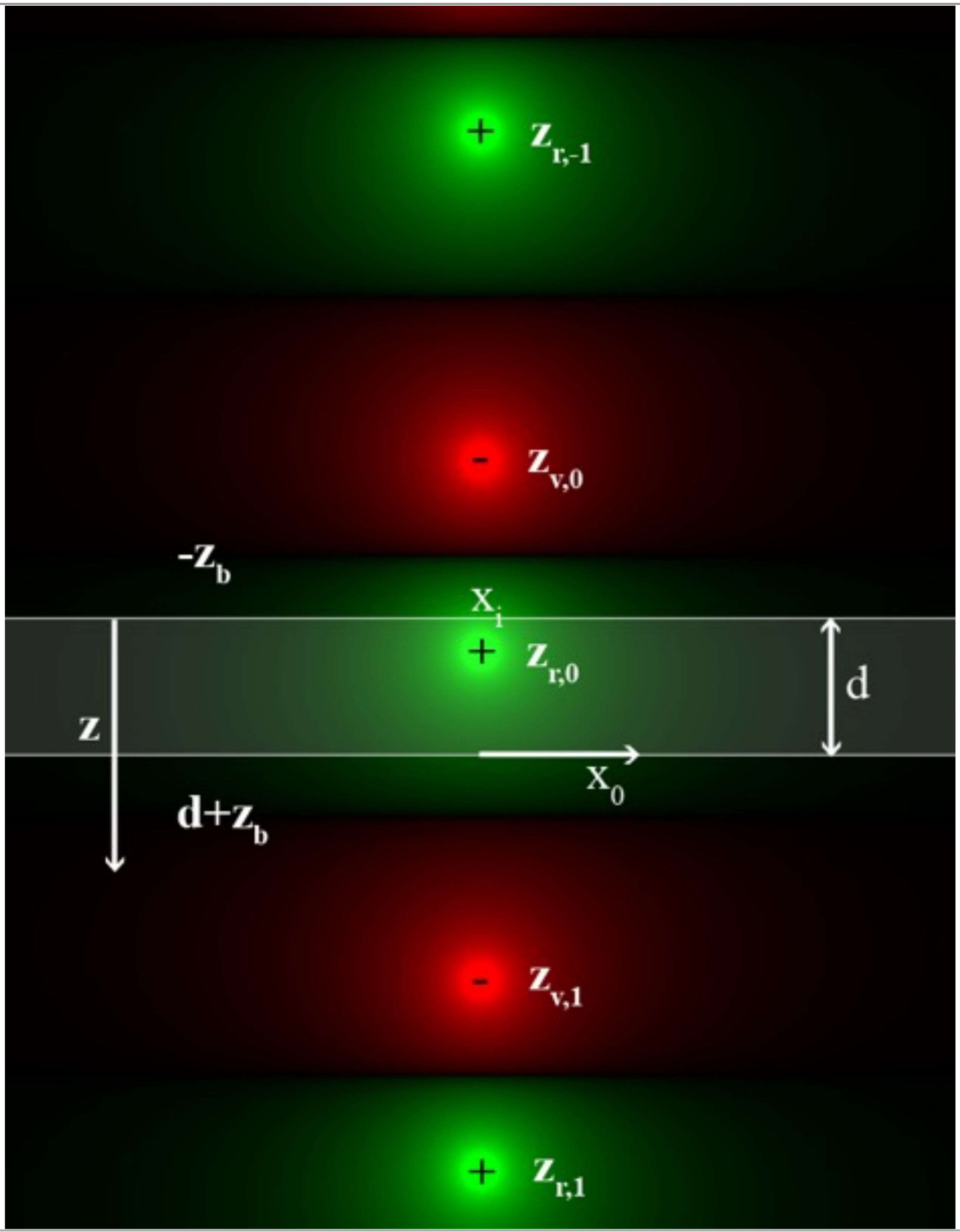


fluence != 0

fluence = 0

source: Habel 2007

The Multipole - 1 Slab



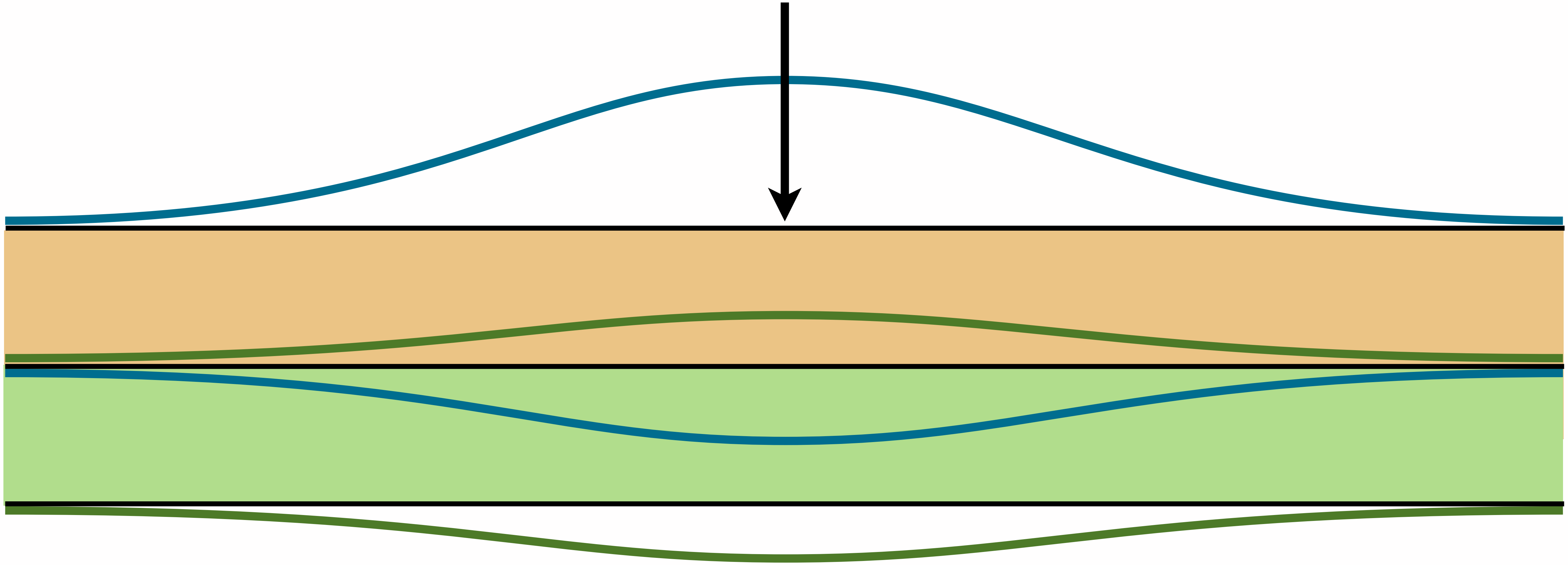
← fluence = 0

← fluence != 0

source: Habel 2007

The Multipole - 2 Slabs

- Layered media:
  - Convolute multipole profiles on boundaries



Jade

source: Donner and Jensen 2005

Jade + Paint Layer

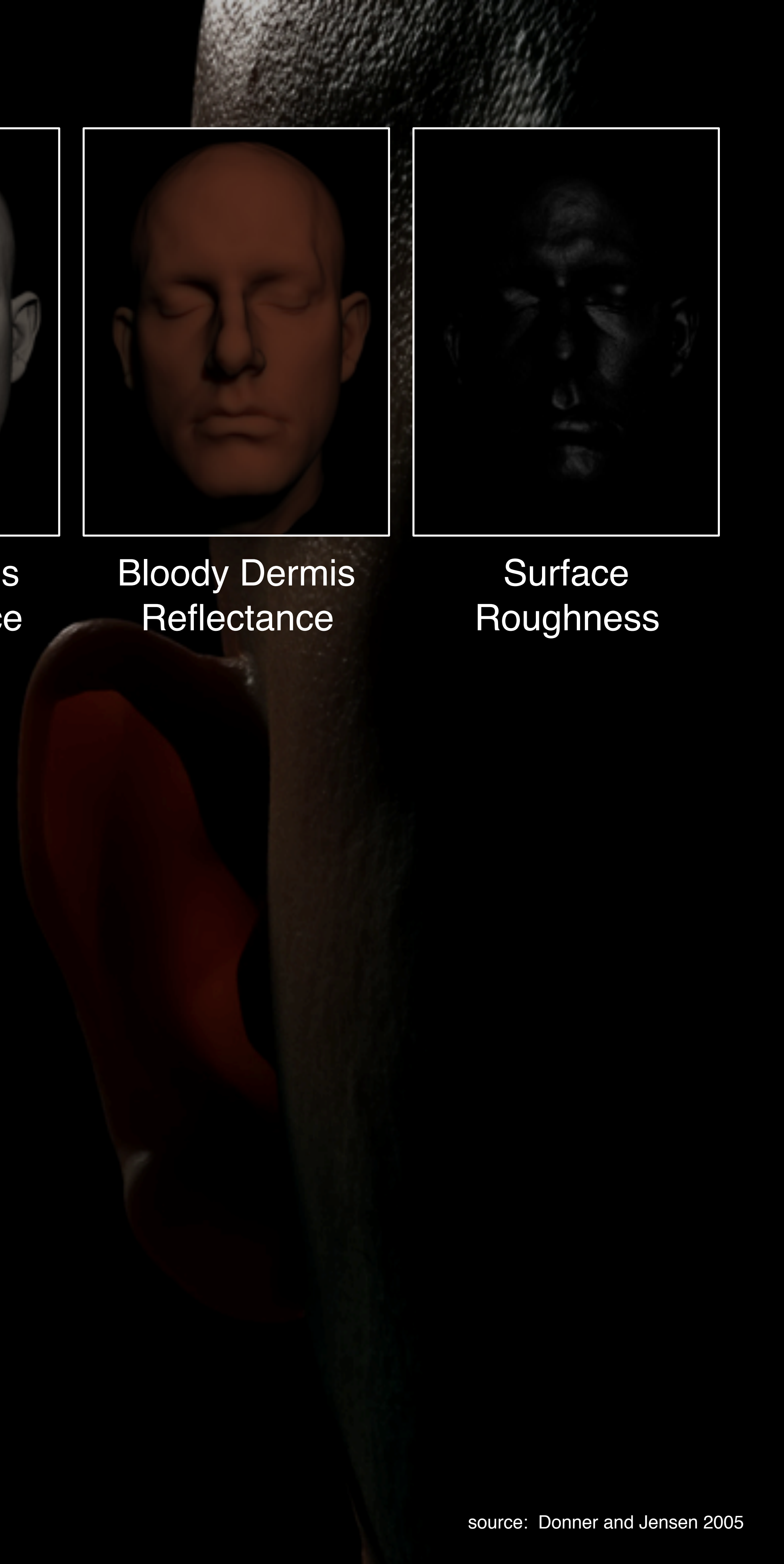
source: Donner and Jensen 2005

source: Donner and Jensen 2005

source: Donner and Jensen 2005

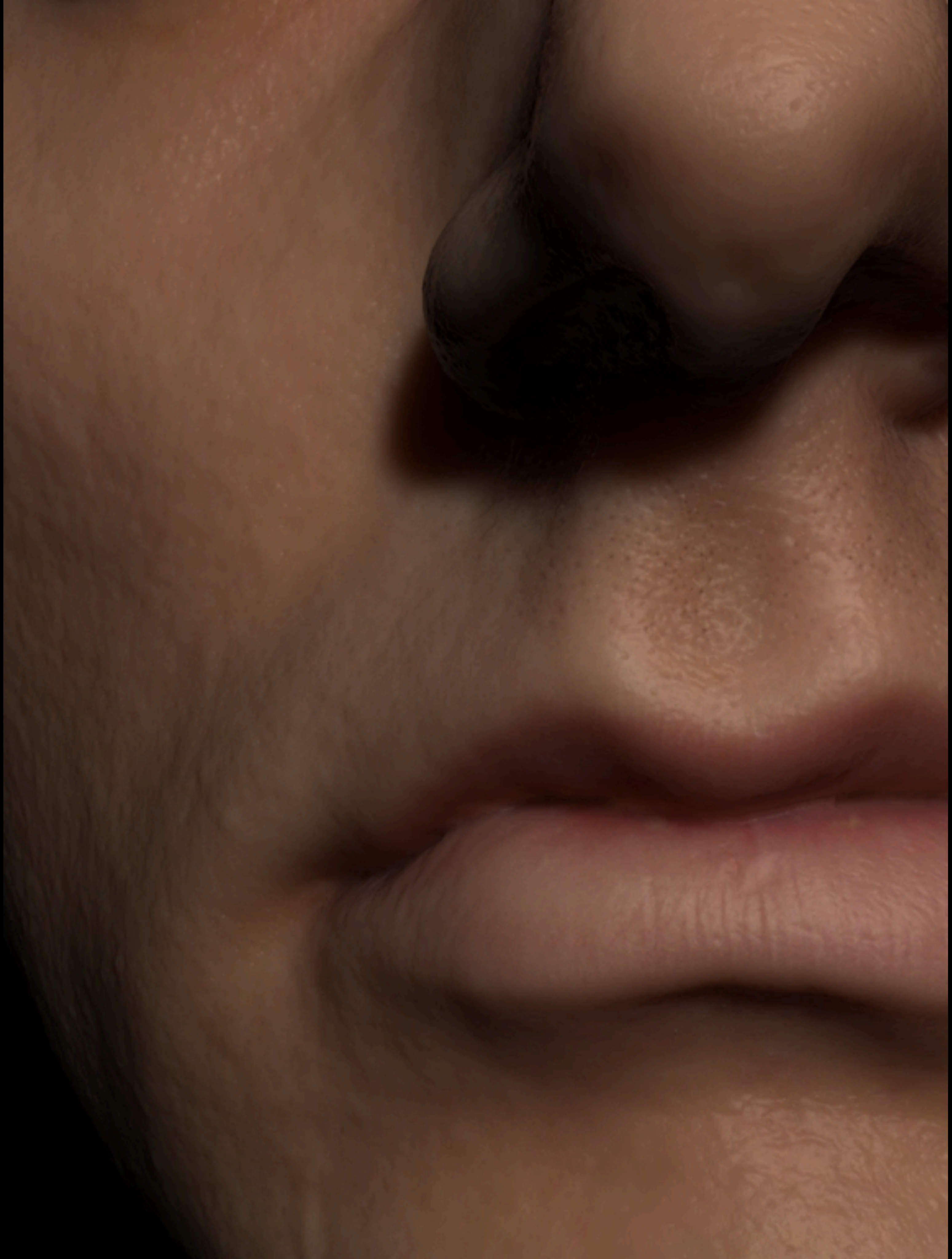
source: Donner and Jensen 2005

Epidermis Reflectance | Epidermis Transmittance | Upper Dermis Reflectance | Upper Dermis Transmittance | Bloody Dermis Reflectance | Surface Roughness



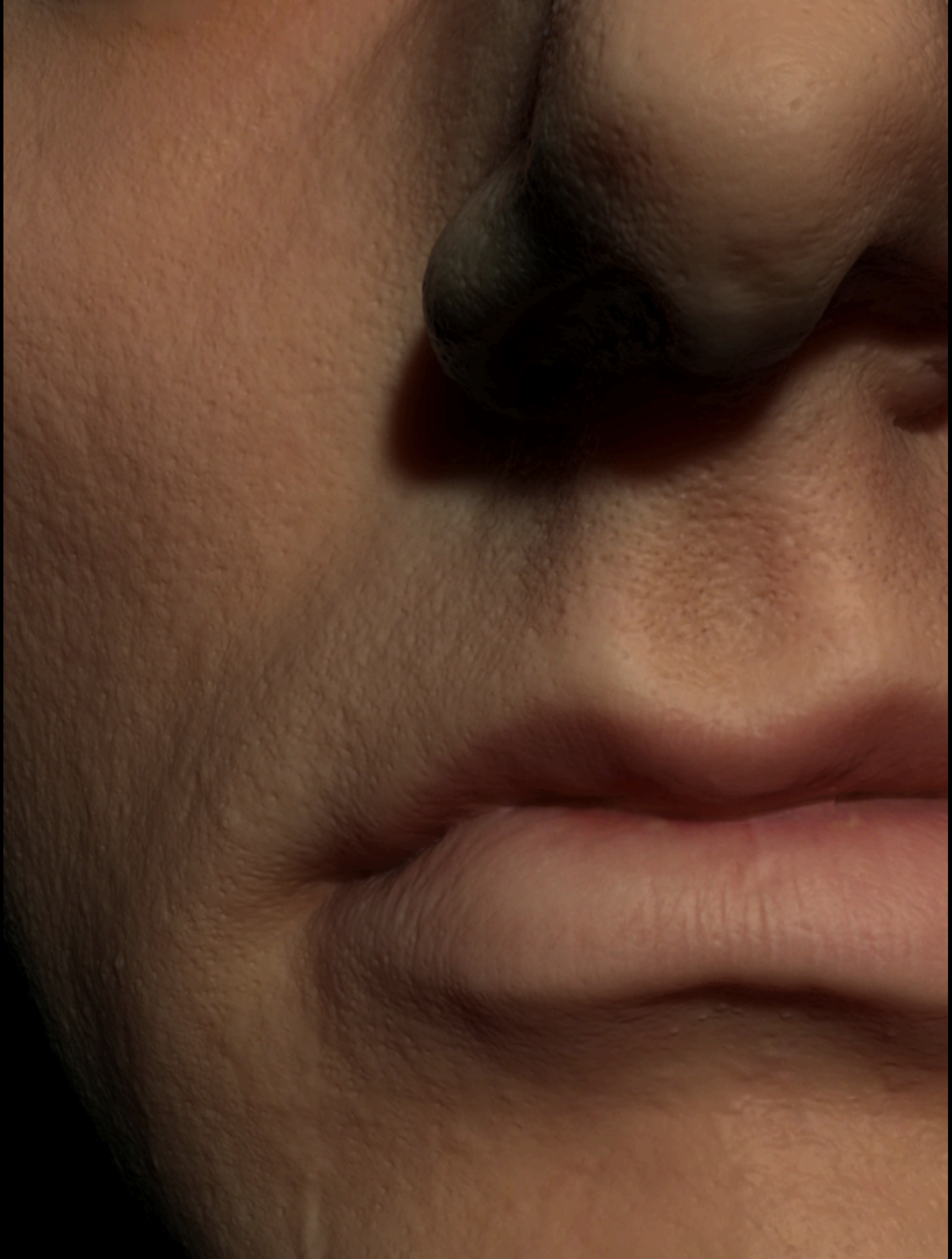
source: Donner and Jensen 2005

Dipole



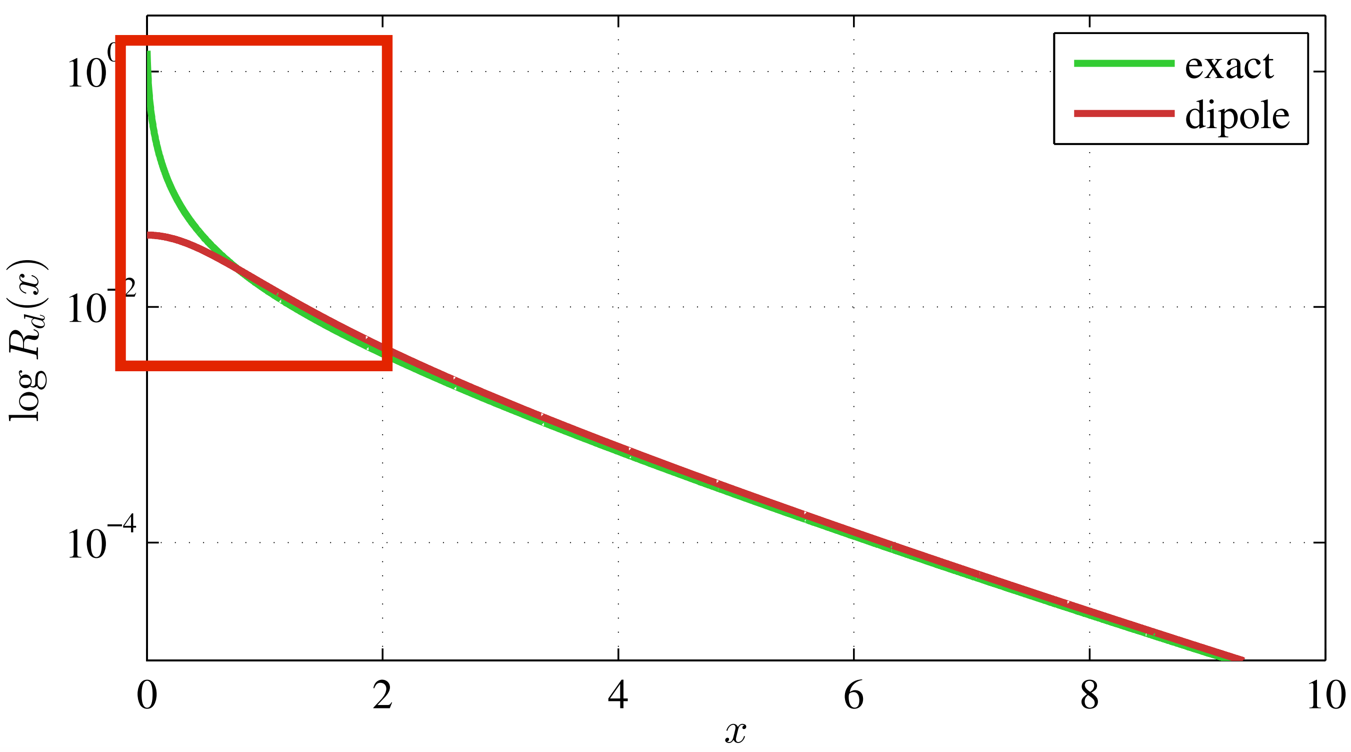
source: Donner and Jensen 2005

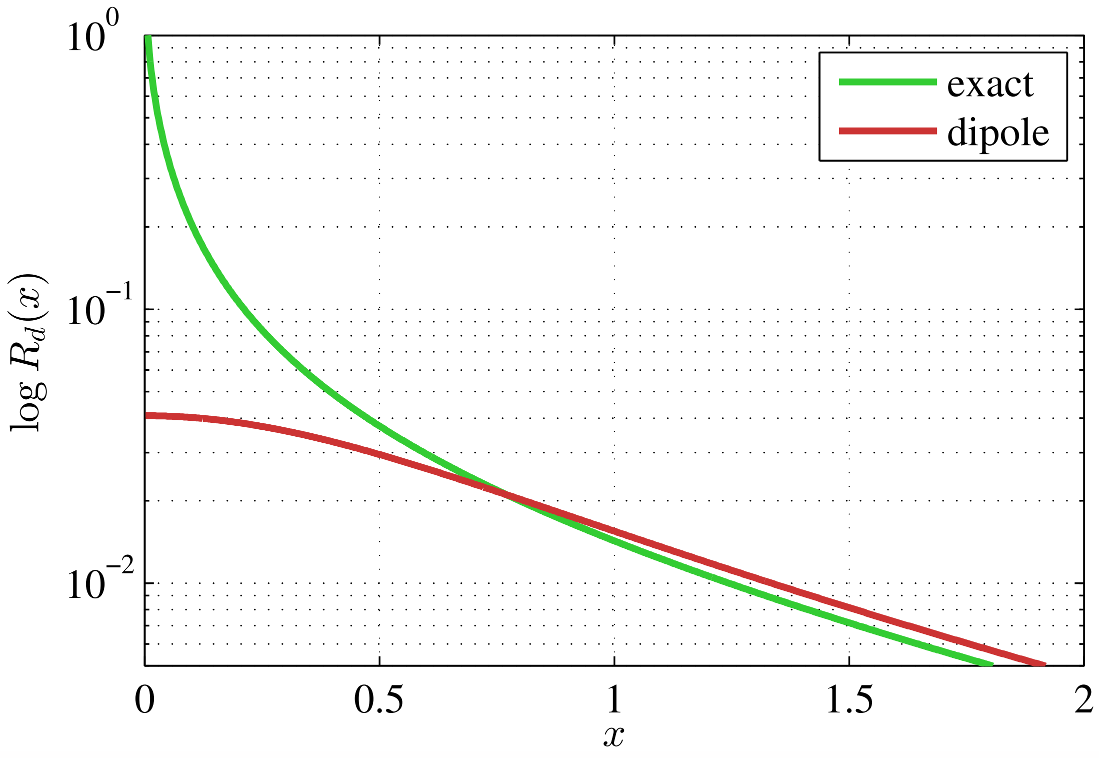
Multilayer multipole



source: Donner and Jensen 2005

Dipole - Profile





- Near incident light profile is wrong!
  - Dipole is off by a factor of 12x near incident light
  - Over-smoothing occurs

Dipole

Subsurface scattering appears too waxy

Dipole - Darkening

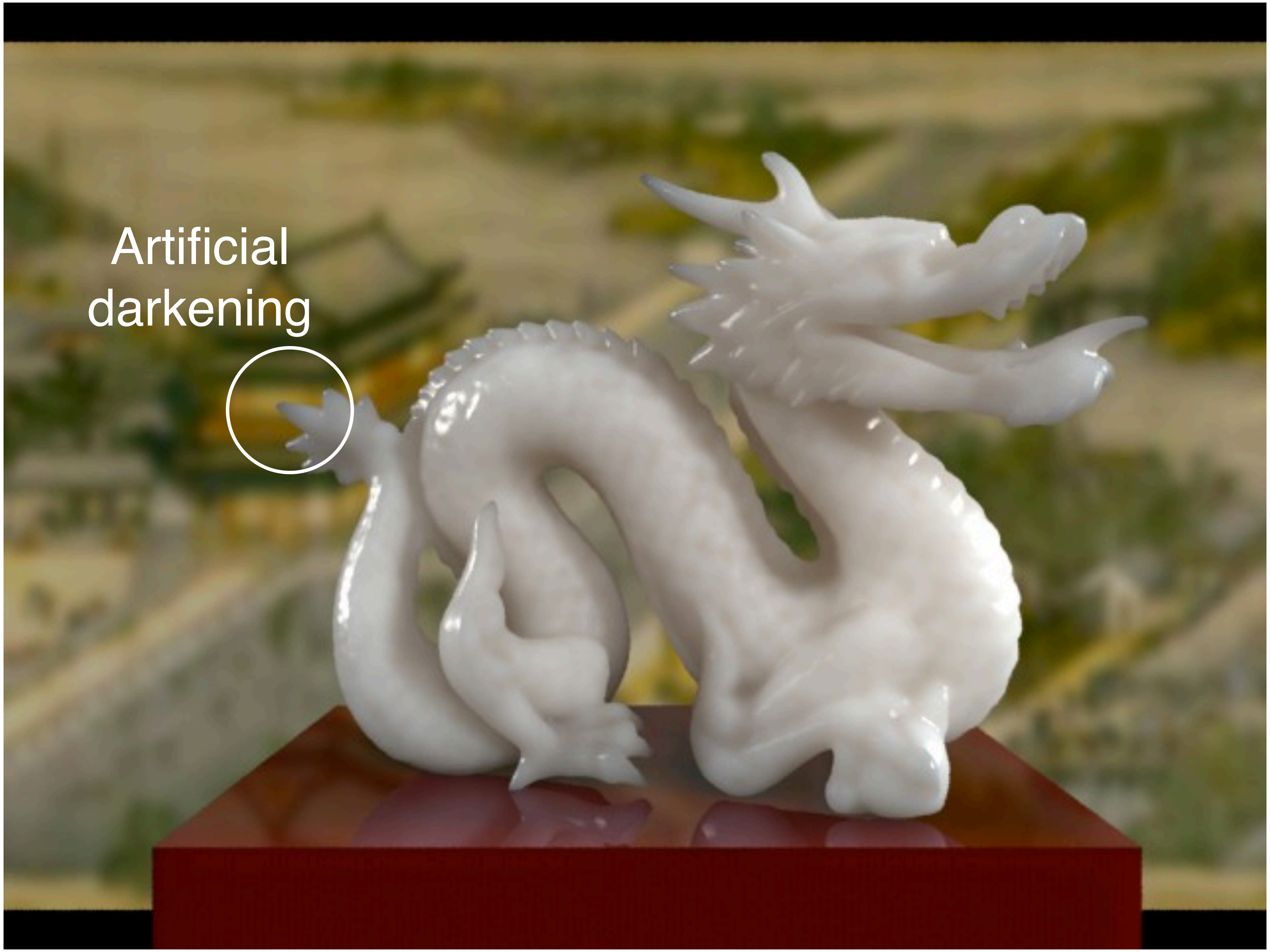
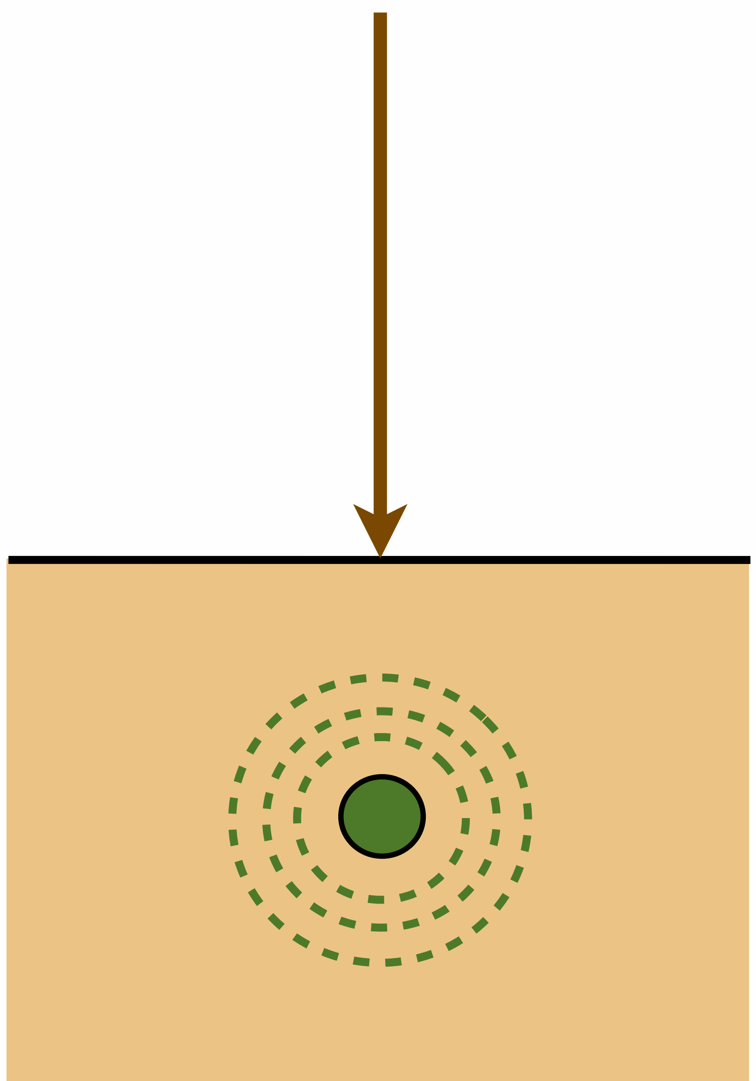


Image courtesy of Rui Wang

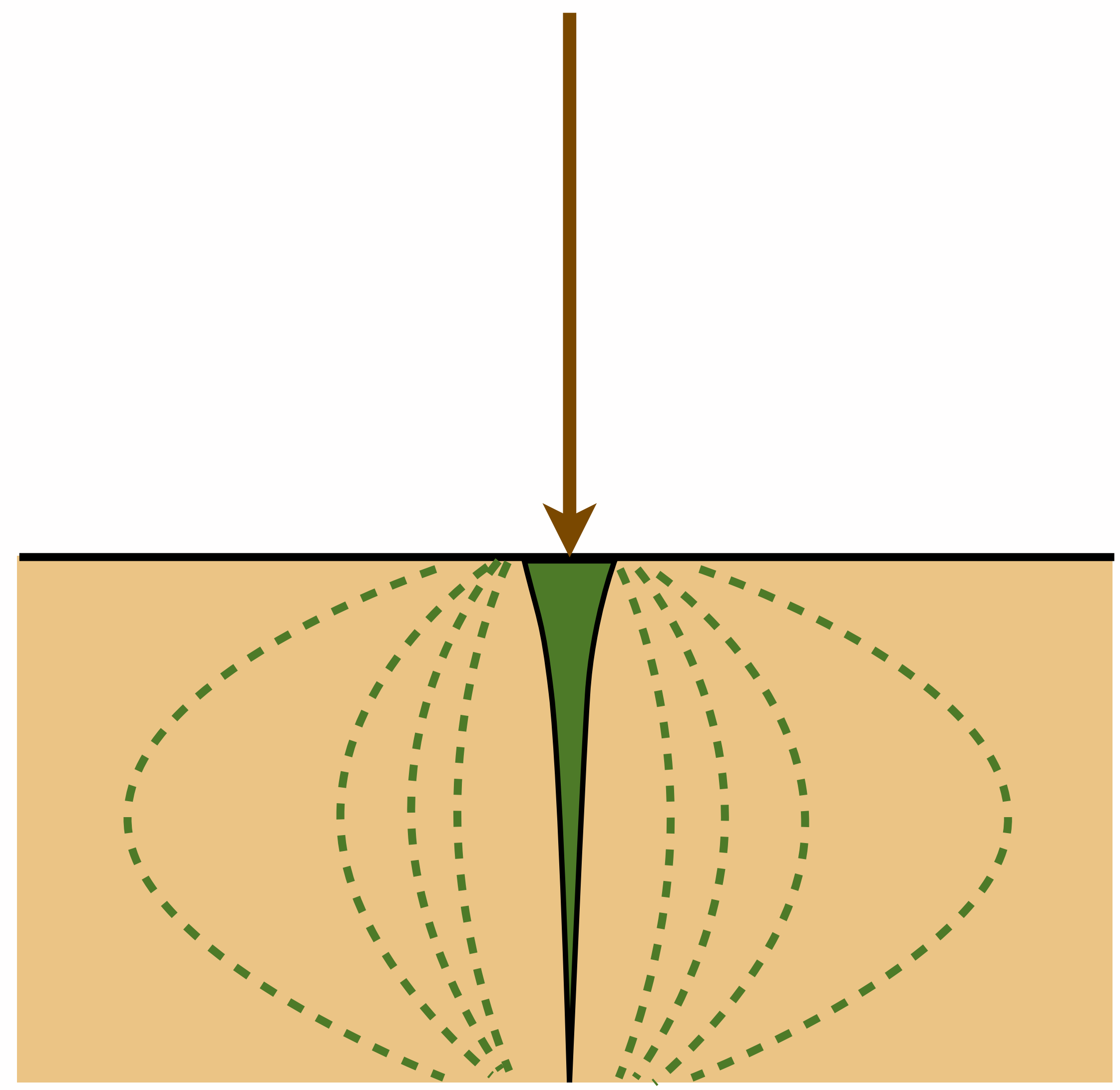
Dipole Approximation

Single depth approximation
(positive source placed at 1 mfp)



$$R_d(\vec{x}) = r(\vec{x}, \vec{x_p}, \vec{x_n})$$

Integration along the beam yields more accuracy

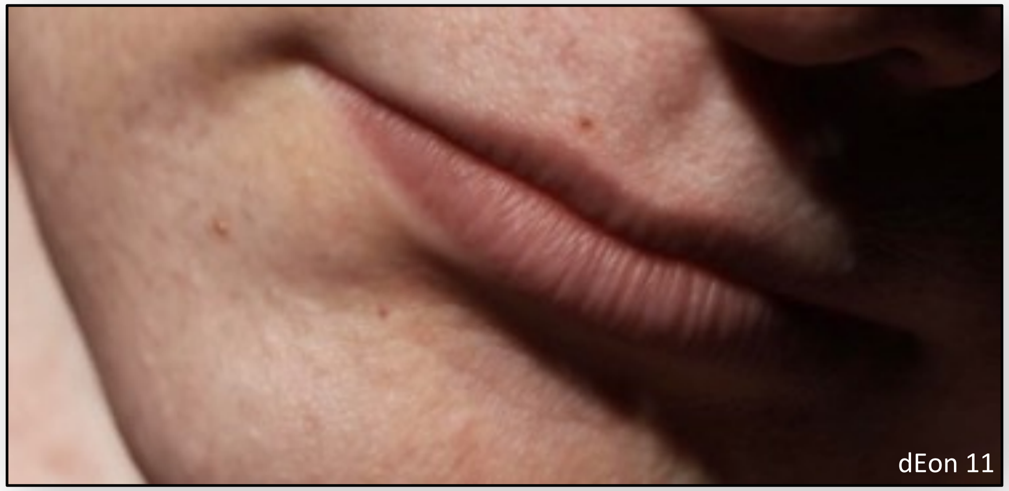


$$R_d(\vec{x}, \vec{\omega}) = \int_0^\infty r(\vec{x}, \vec{x}_p(t), \vec{x}_n(t))\, Q(t) dt$$

Quantized Diffusion

[d'Eon and Irving 11]

- Compute beam source profile as a sum of Gaussians
- Integrate time-dependent to steady-state solution
- Uses improved diffusion theory and boundary conditions
  [Grojean 58, Kienle-Patterson 97]
- Requires complex calculations
- Some (avoidable) numerical instabilities



Quantized Diffusion

[d'Eon and Irving 11]

Dipole

Quantized Diffusion

Photon Beam Diffusion

[Habel et al. 13]

- Previous diffusion work: Numerical integration of

$$R_d(\vec{x},\vec{\omega}) = \int_0^{\infty} r(\vec{x},\vec{x}_p(t),\vec{x}_n(t))\, Q(t)dt$$

  not efficient

- Not if sampling is clever enough!

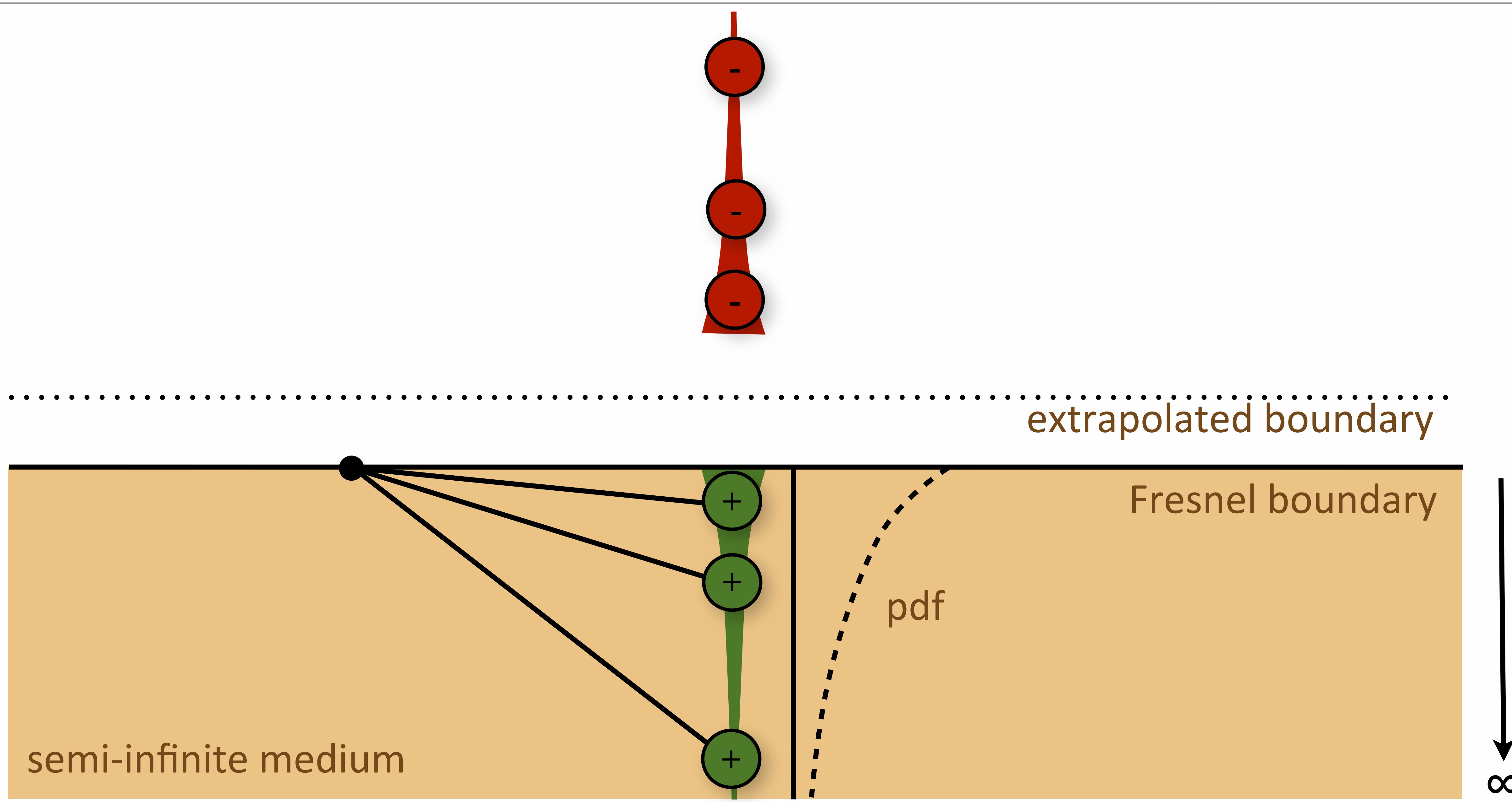
Numerically integrate beam source

- Monte Carlo integration with importance sampling:

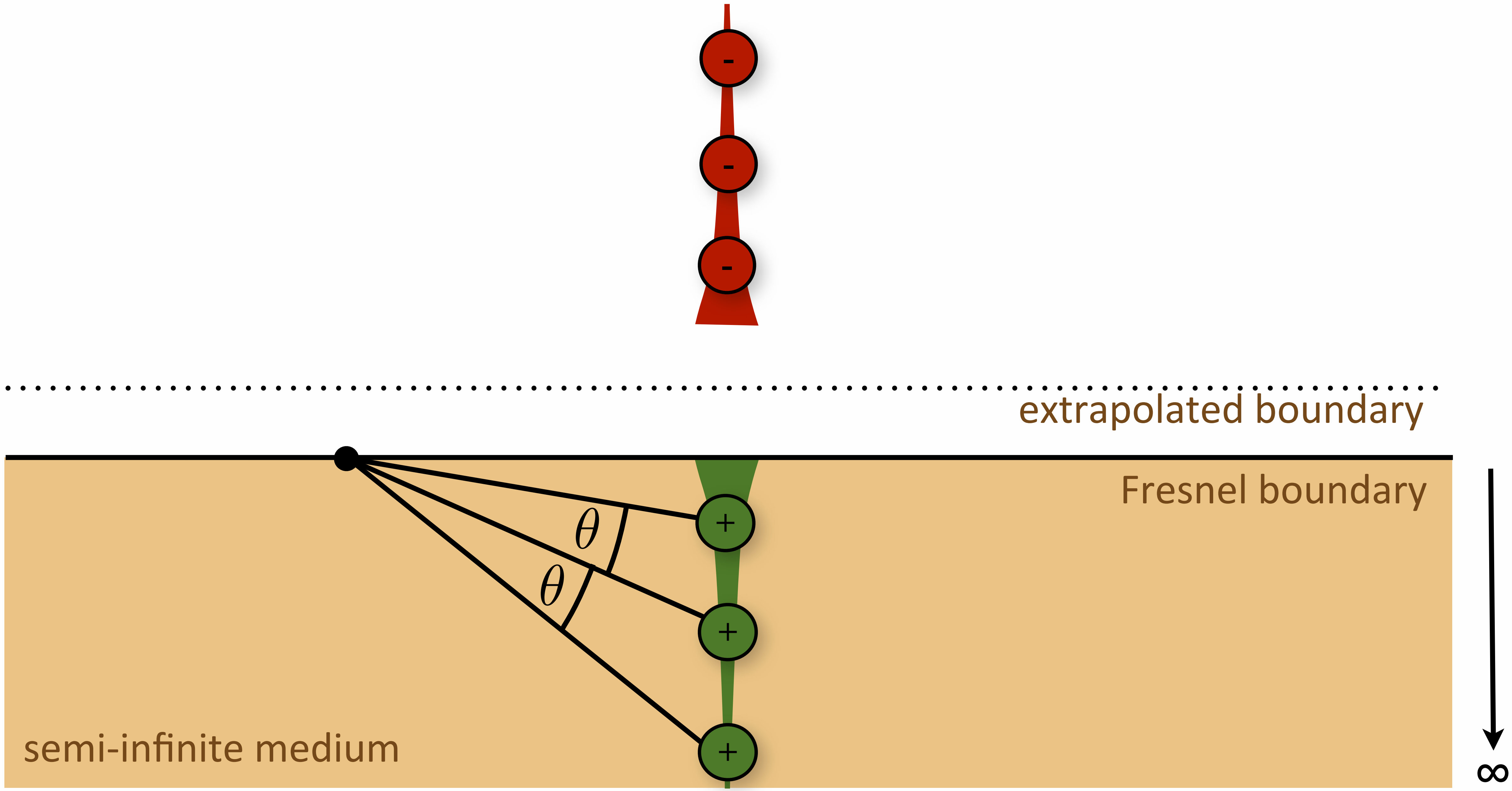
$$R_d(\vec{x},\vec{\omega}) = \int_0^\infty \underbrace{r(\vec{x},\vec{x}_p(t),\vec{x}_n(t))\,Q(t)}_{f(\vec{x},\vec{\omega},t_i)}dt \approx \frac{1}{N}\sum_{i=1}^{N}\frac{f(\vec{x},\vec{\omega},t_i)}{\boxed{\mathrm{pdf}(\vec{x},\vec{\omega},t_i)}}$$

- Find a good PDF for importance sampling

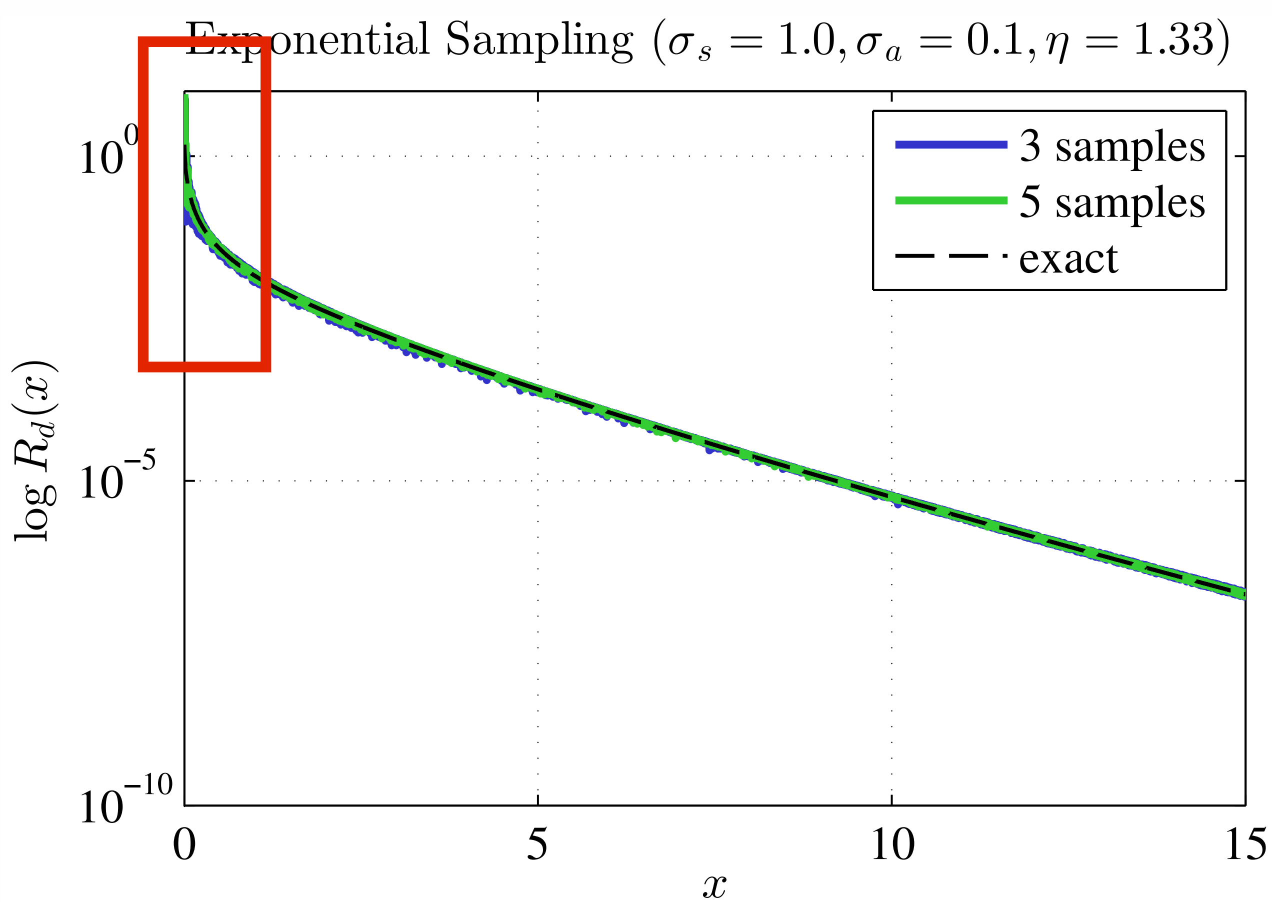
Exponential PDF

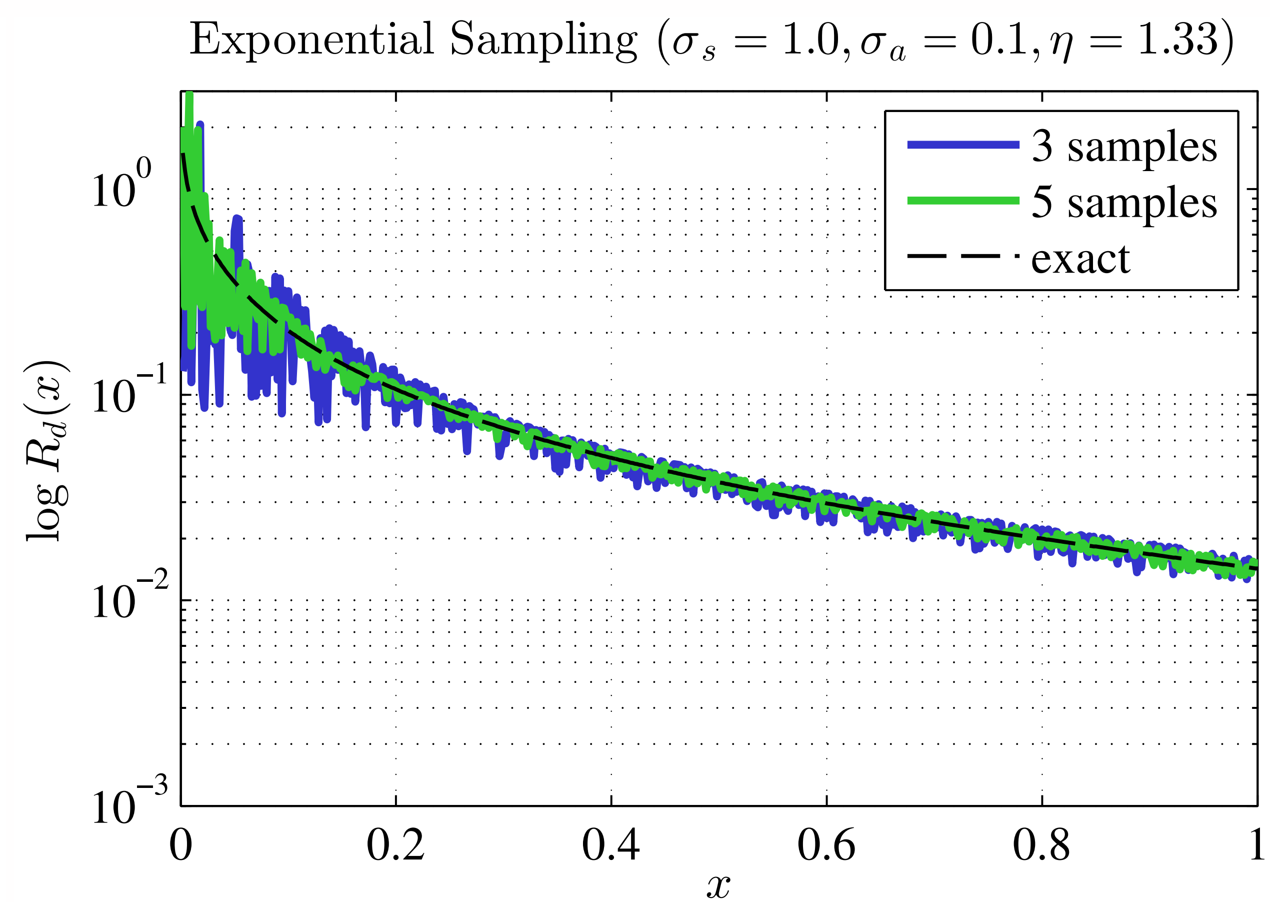


Equiangular PDF



Exponential PDF

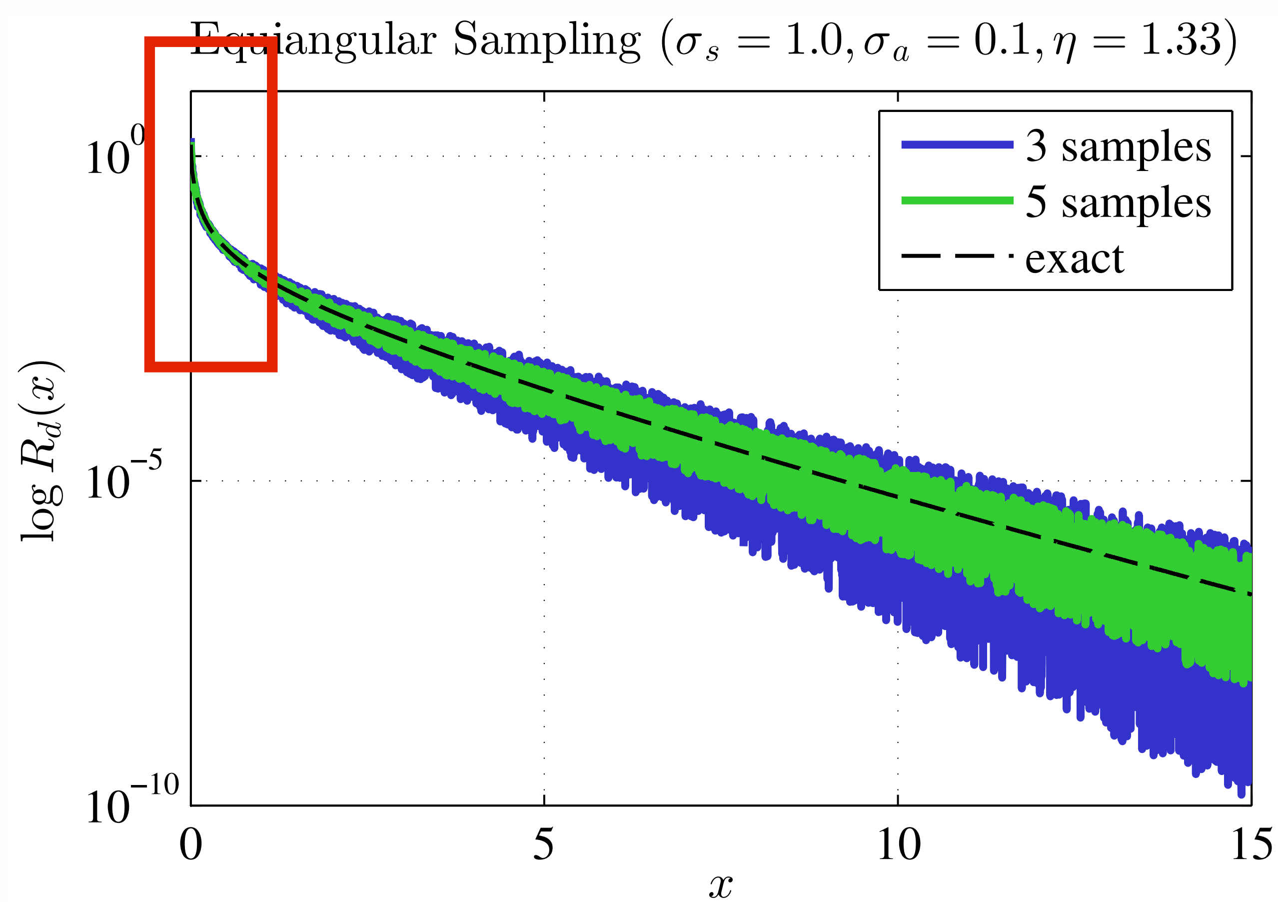


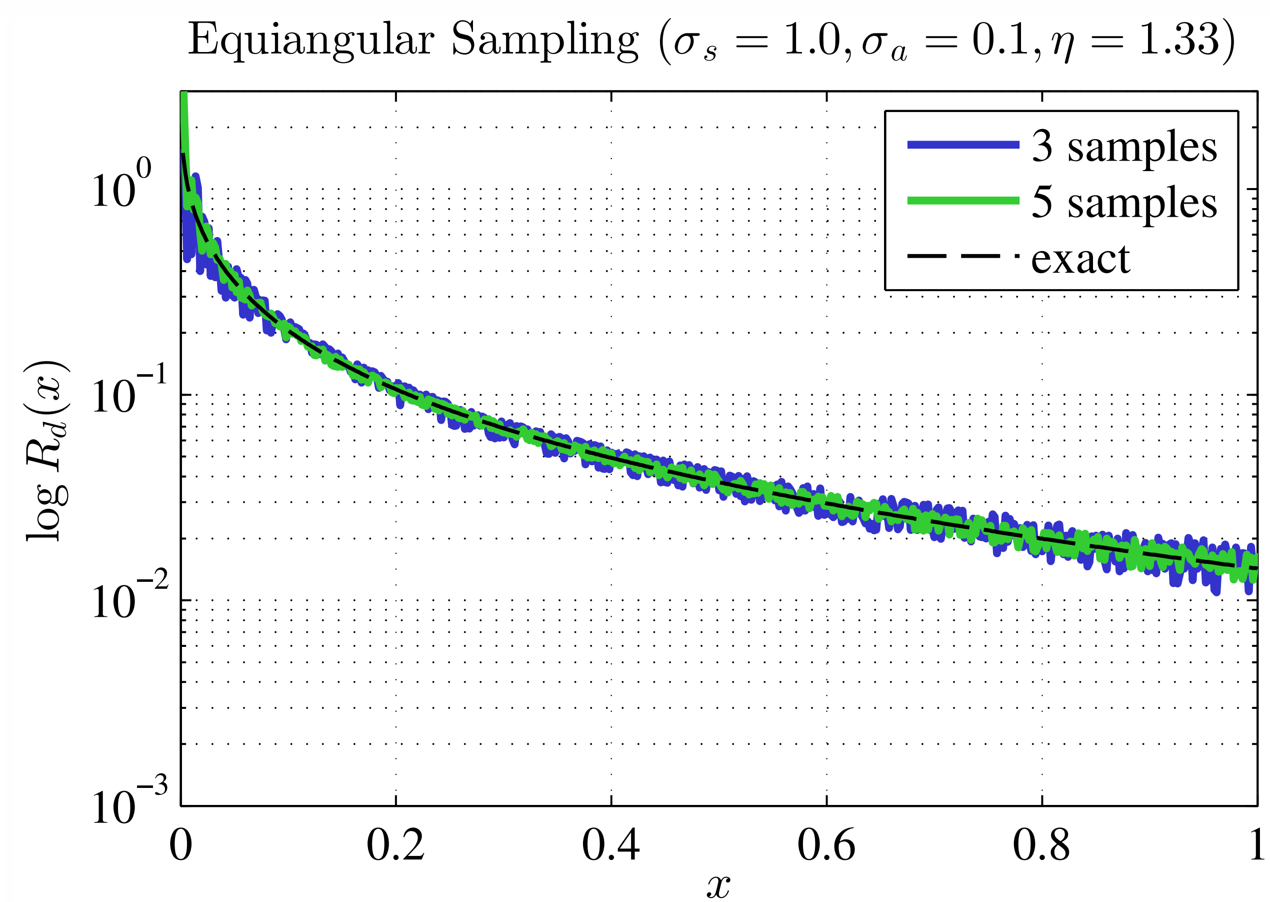


Good in far field

Lacks in near field

Equiangular PDF



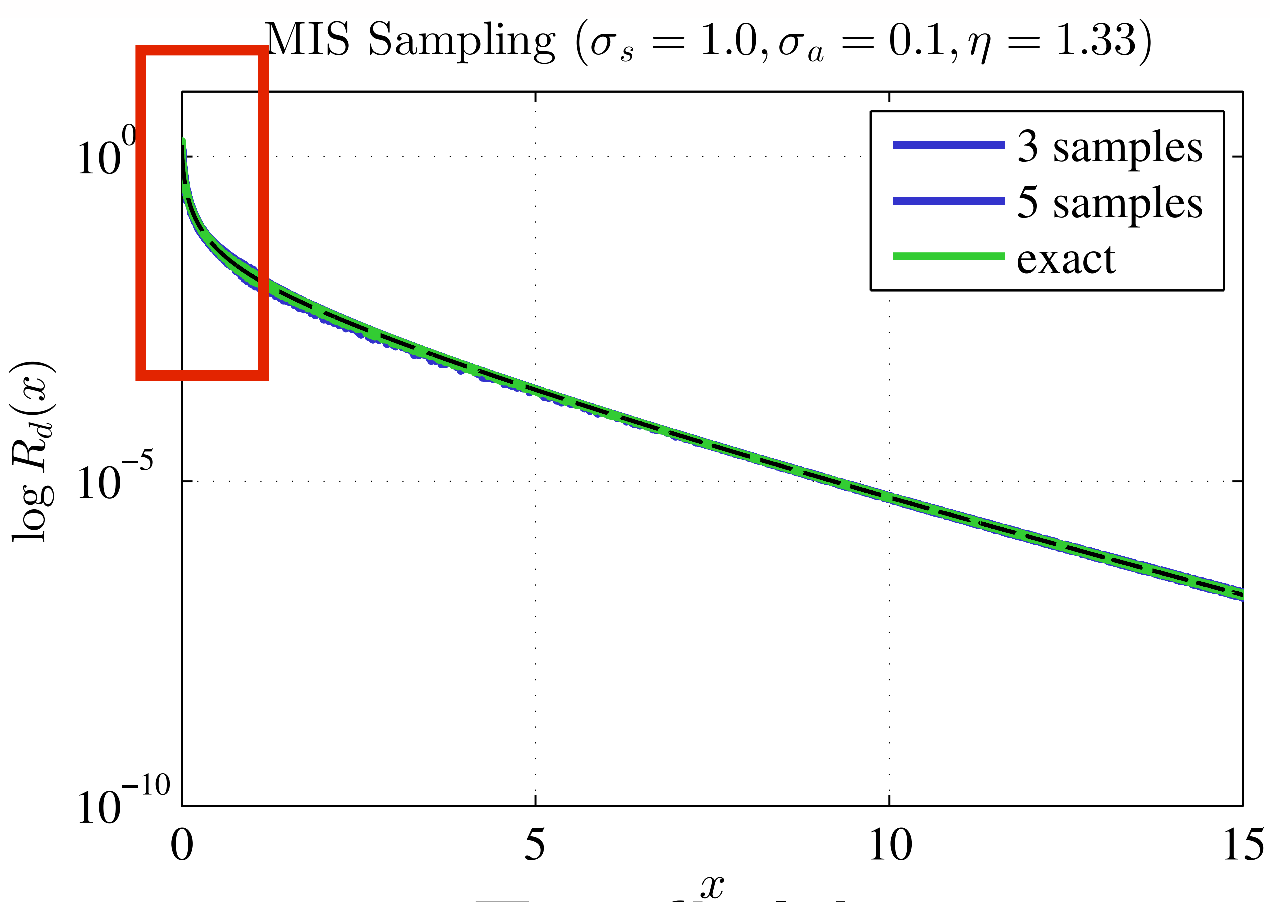


Lacks in far field

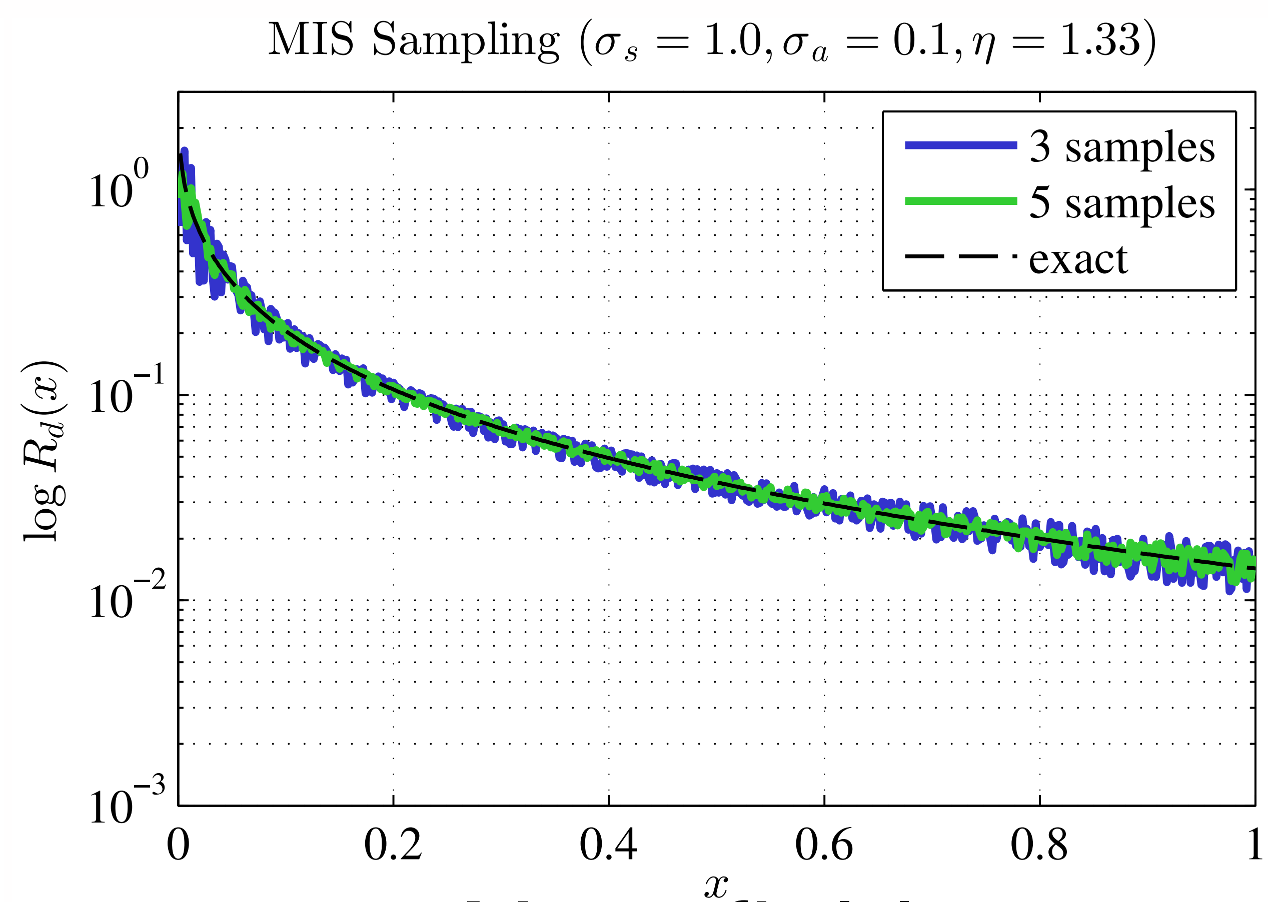
Good in near field

Combining Strategies

- Choose equiangular in near field, exponential in far field
- MIS in small blending range around mean free path



Far field



Near field

Photon Beam Diffusion

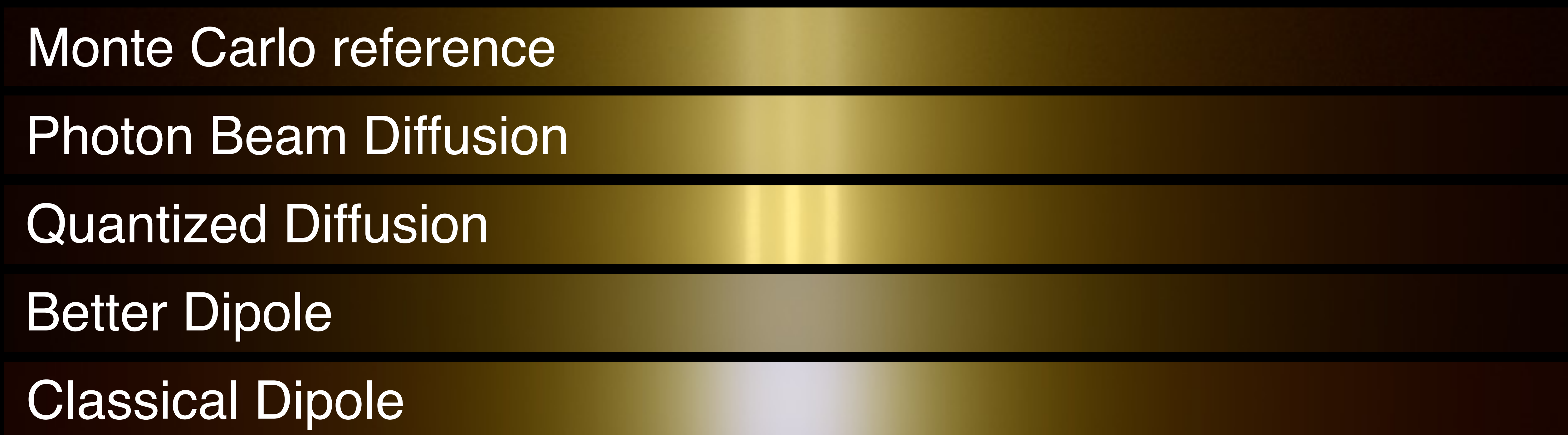
- Observation: 3-5 samples enough to get very accurate results
  - weighted sum of 3-5 dipole evaluations
  - 3-28x faster than Quantized Diffusion
- Easy to implement:

```
at distance dist
for each sample [0..1]

   compute depth and pdf_weight from exp/equi sampling;
   compute dipole R_d(depth, dist);
   R_d_beam += R_d/pdf_weight;

end
R_d_beam /= number_samples;
```

Stripe Comparison



Dipole

Photon Beam Diffusion

Dipole

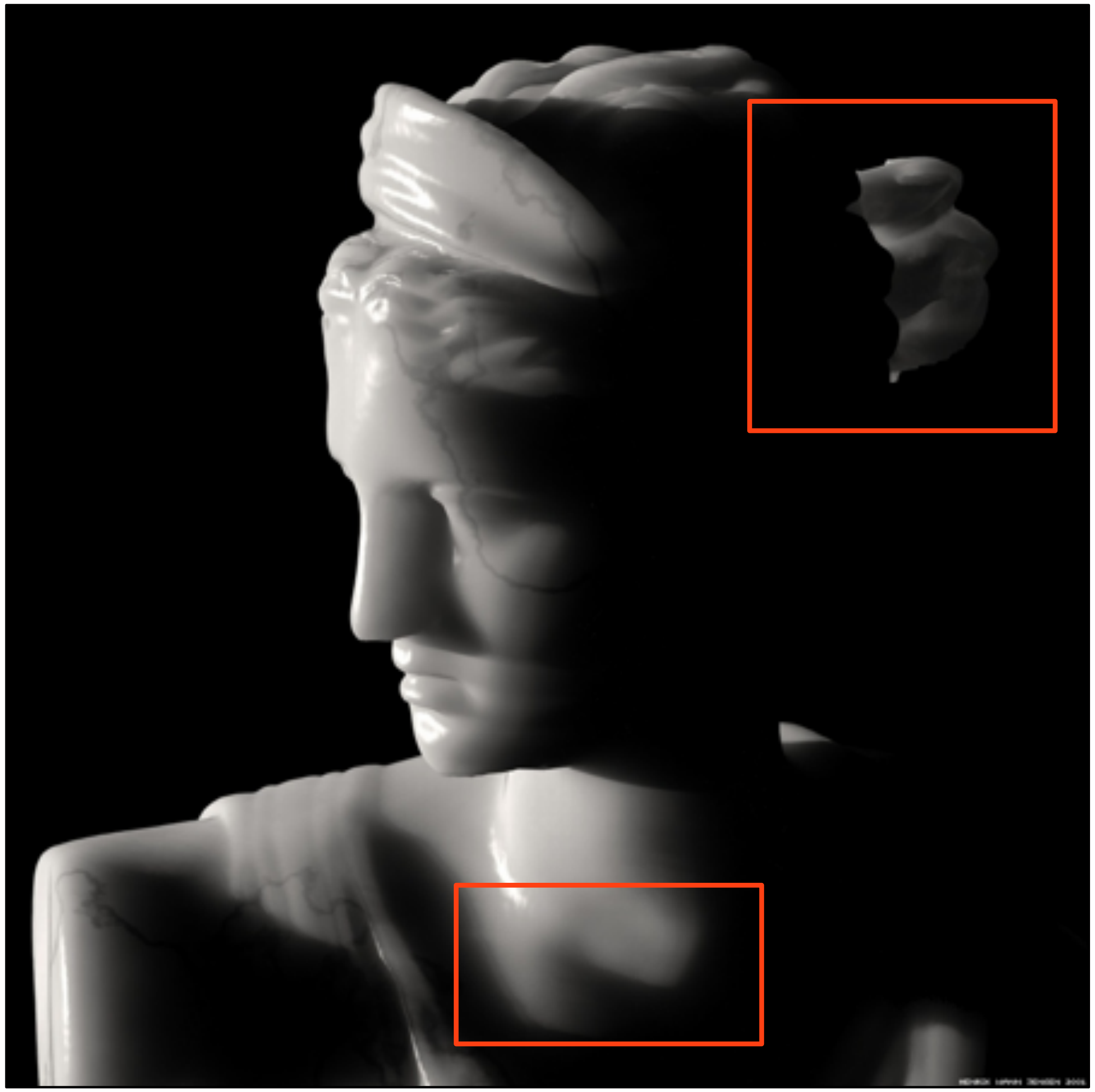
Photon Beam Diffusion

Conclusion

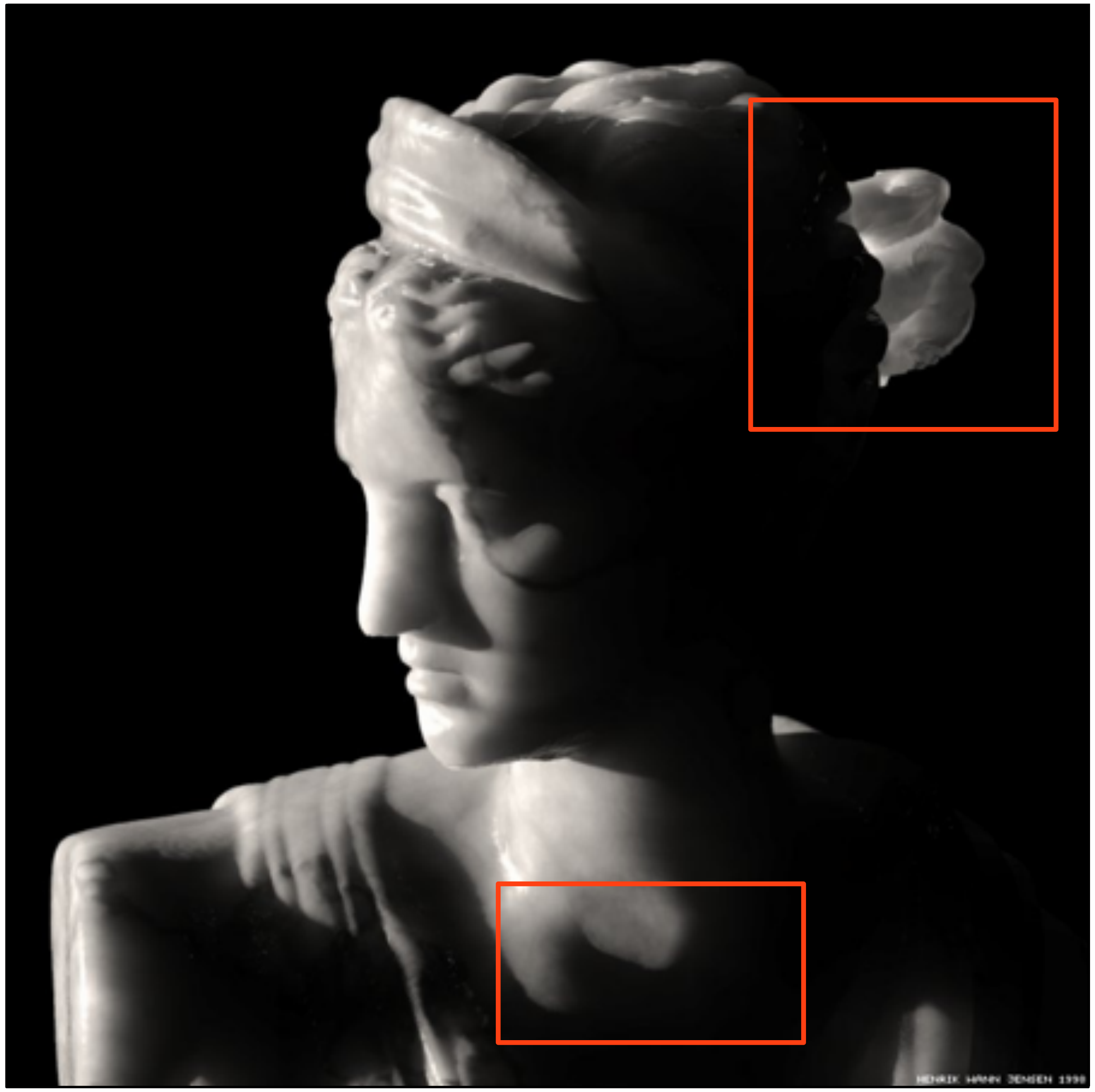
- Diffusion is approximation to light transport
  - First order can be generalized to n-th order approximations
- Semi-infinite and finite slab are very strong approximations
  - Arbitrary errors if applied to general geometry
  - Model does not know how the vicinity looks like
- We cannot expect physically accurate results
  - But it *looks* pretty close

Errors in Dipole Method

Dipole



Photon Mapping



source: Jensen et al. 2001

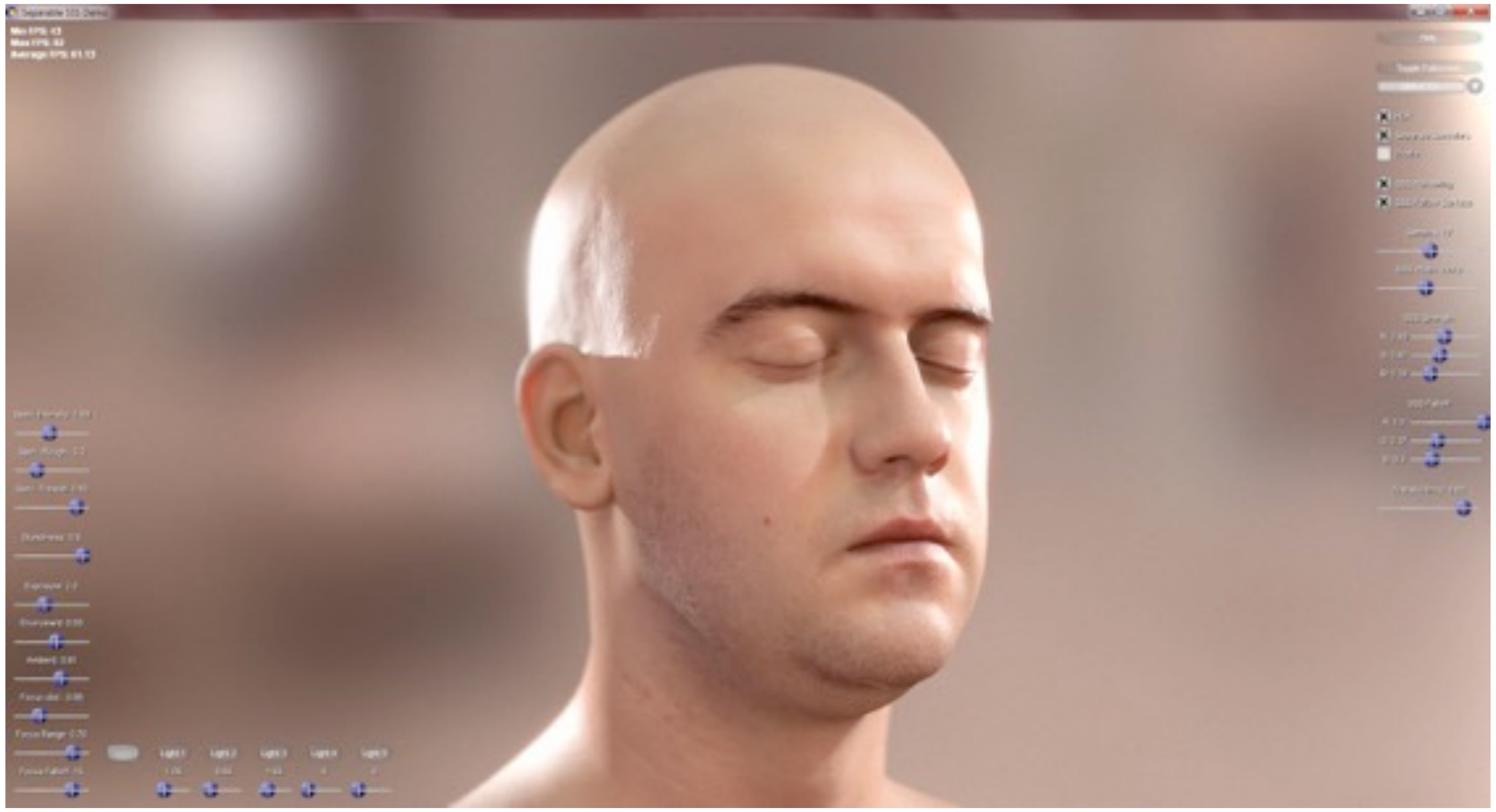
Conclusion

- Used on almost all animated and VFX characters

source: : Pixar, Industrial Light and Magic

Conclusion

- Real-time approaches:
  - Calculate irradiance in texture space or screen space
  - Blur texture according to profile
  - Or do per vertex/per texel finite element transport



End


source: onebigphoto.com