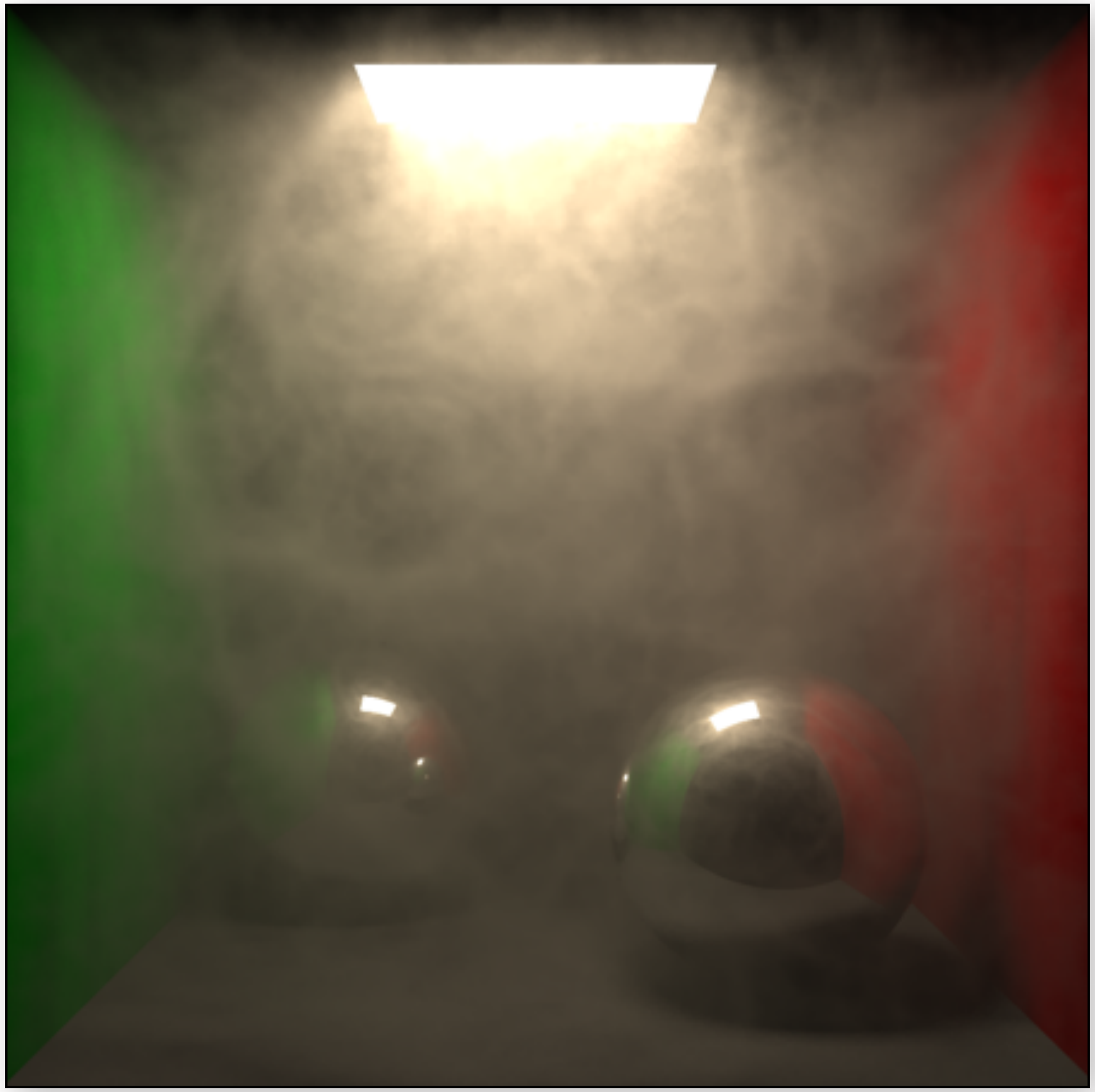
CS 87/187, Spring 2016

Rendering Algorithms

Caching Illumination



Prof. Wojciech Jarosz

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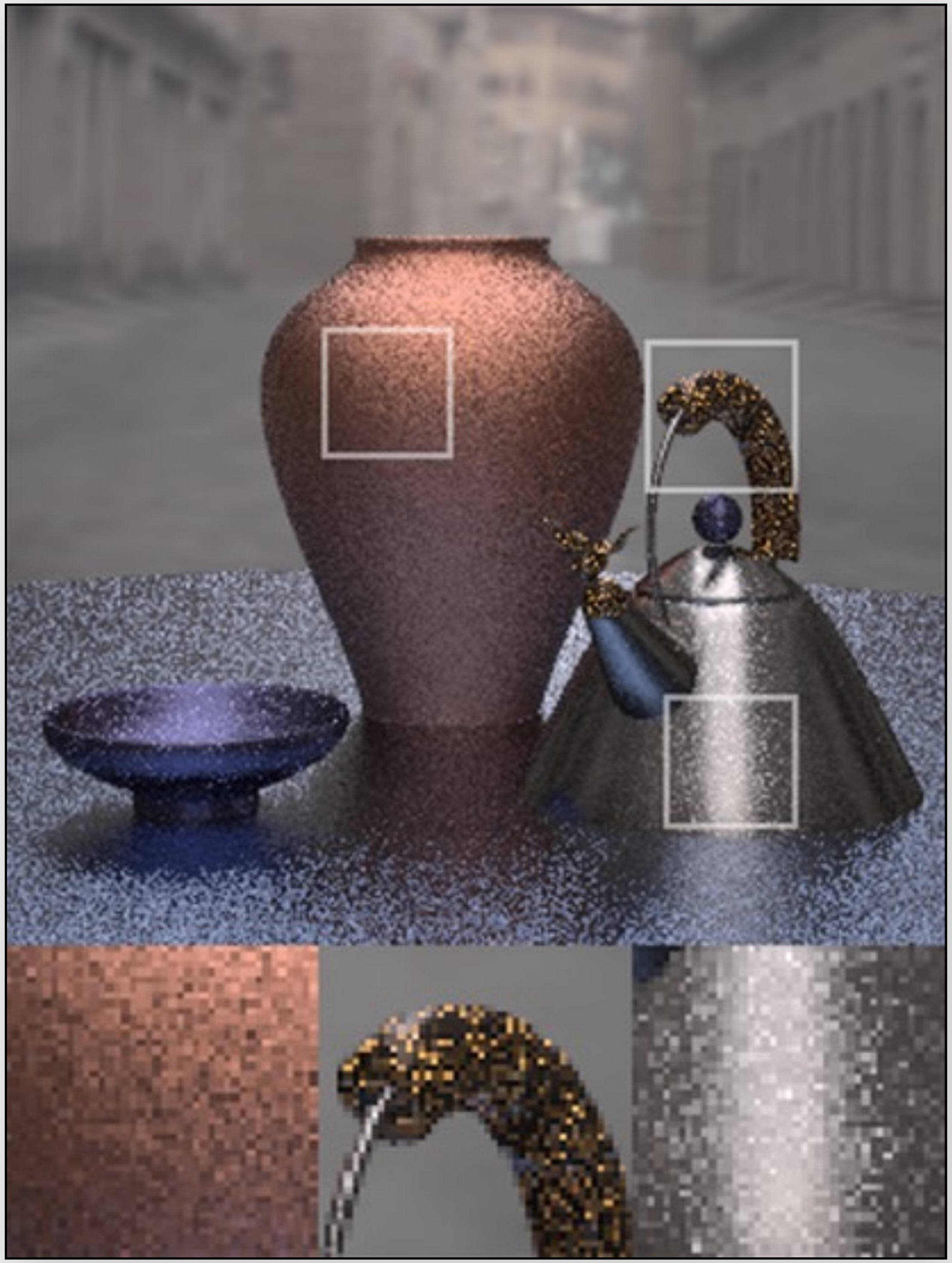
Dartmouth

Today’s menu

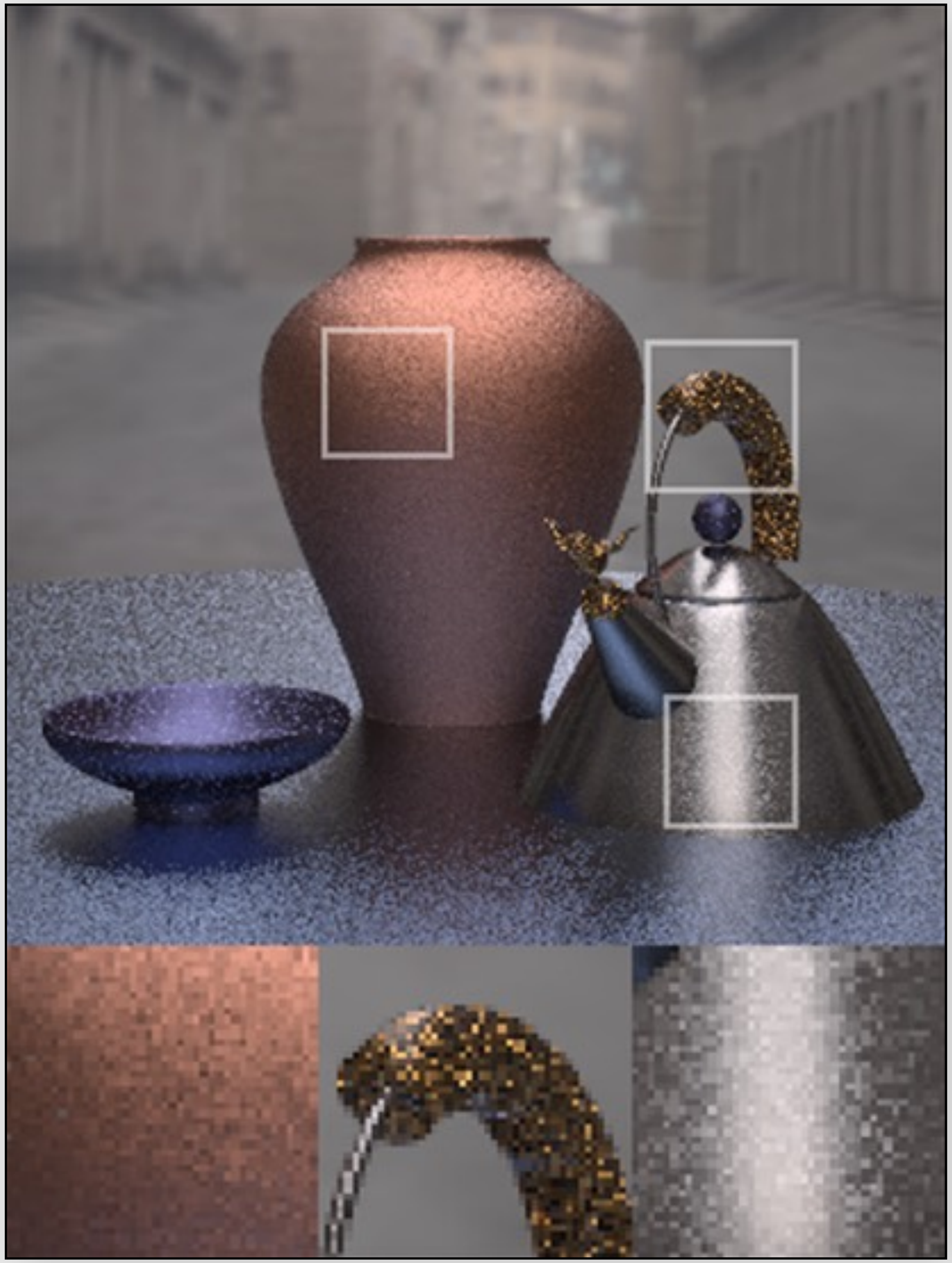
- Irradiance caching and extensions

Path tracing - 40 samples per pixel



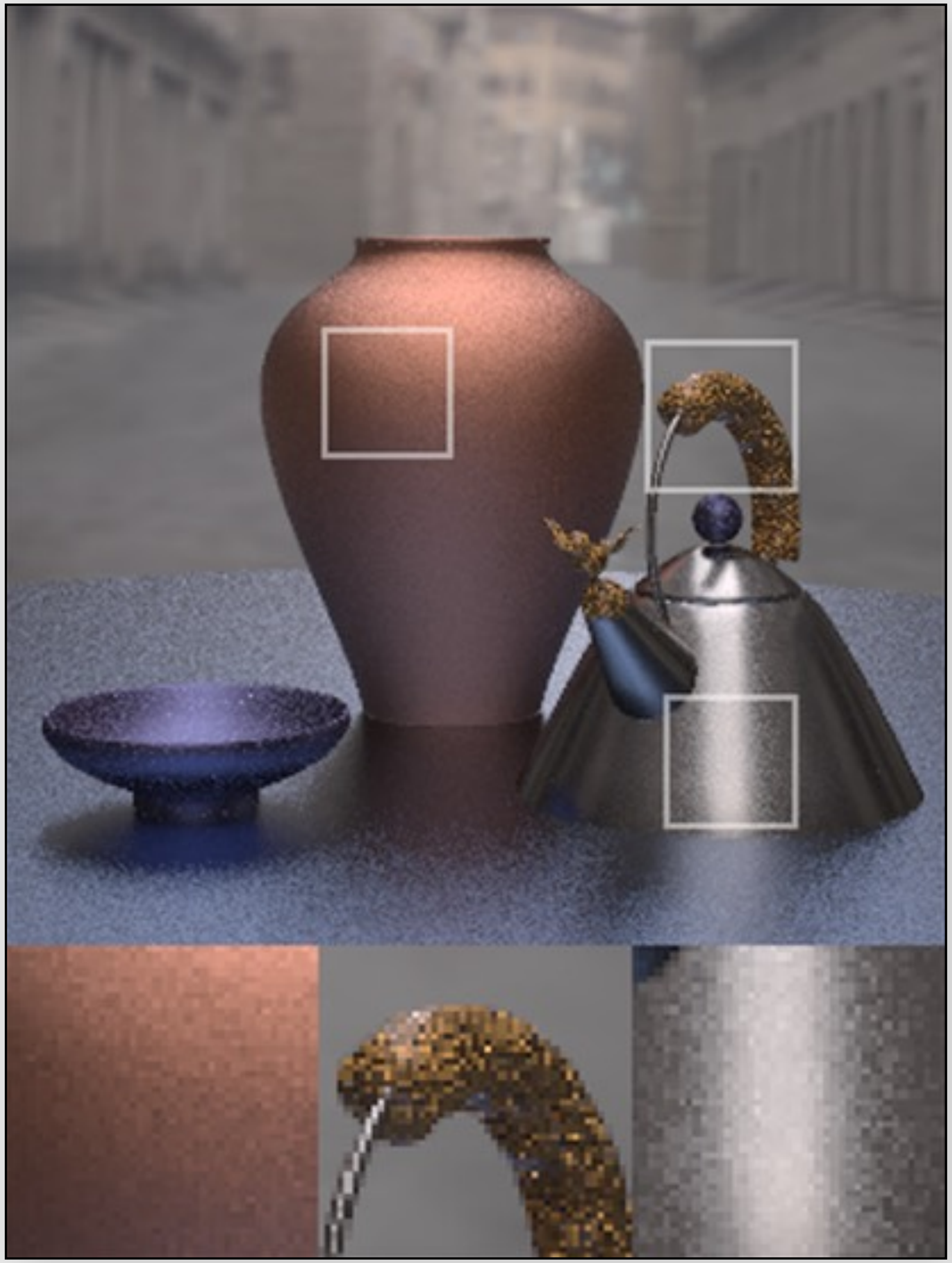
Images courtesy of **Jason Lawrence**
http://www.cs.princeton.edu/gfx/proj/brdf/

Path tracing - 100 samples per pixel



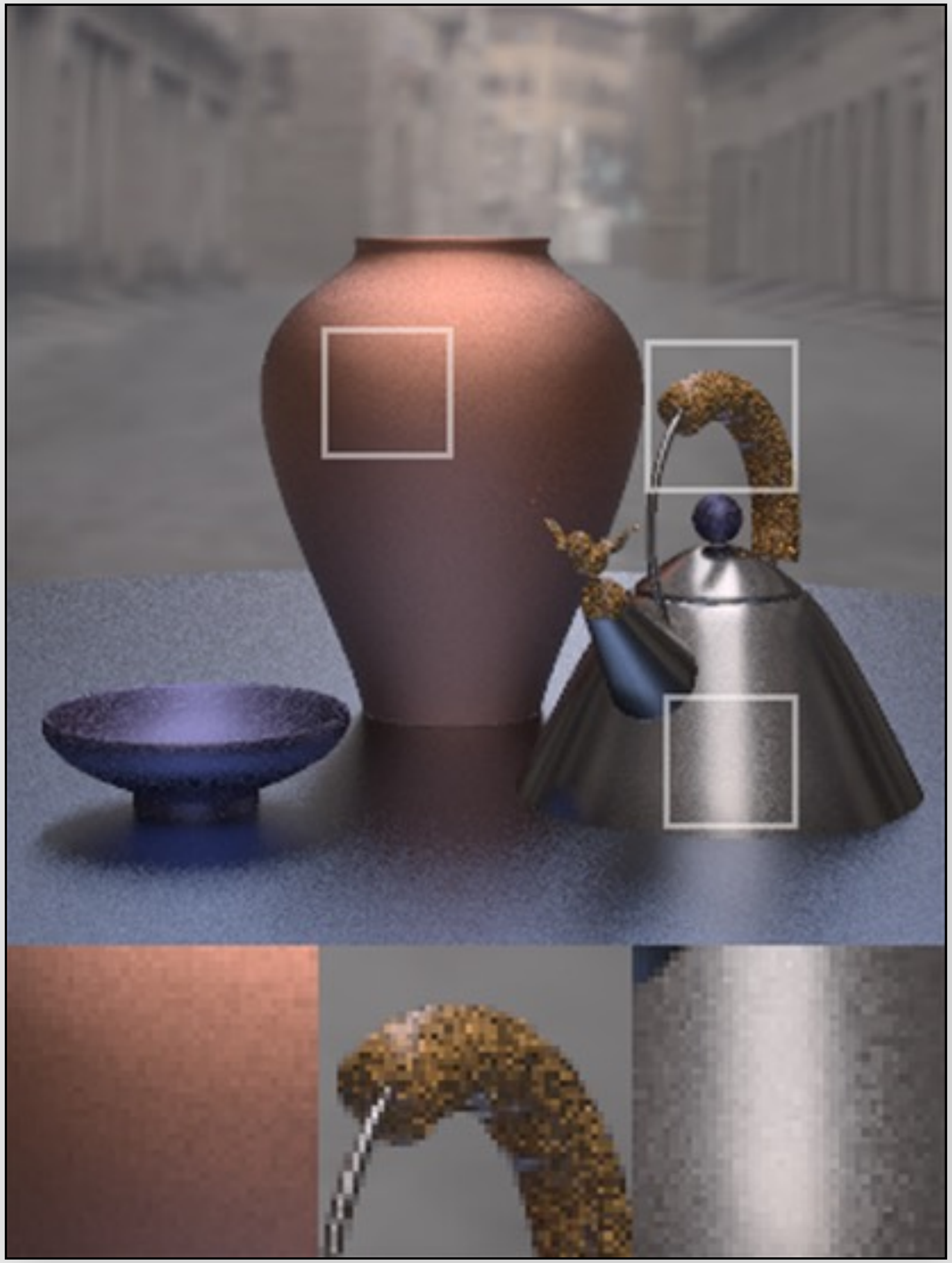
Images courtesy of **Jason Lawrence**
http://www.cs.princeton.edu/gfx/proj/brdf/

Path tracing - 300 samples per pixel



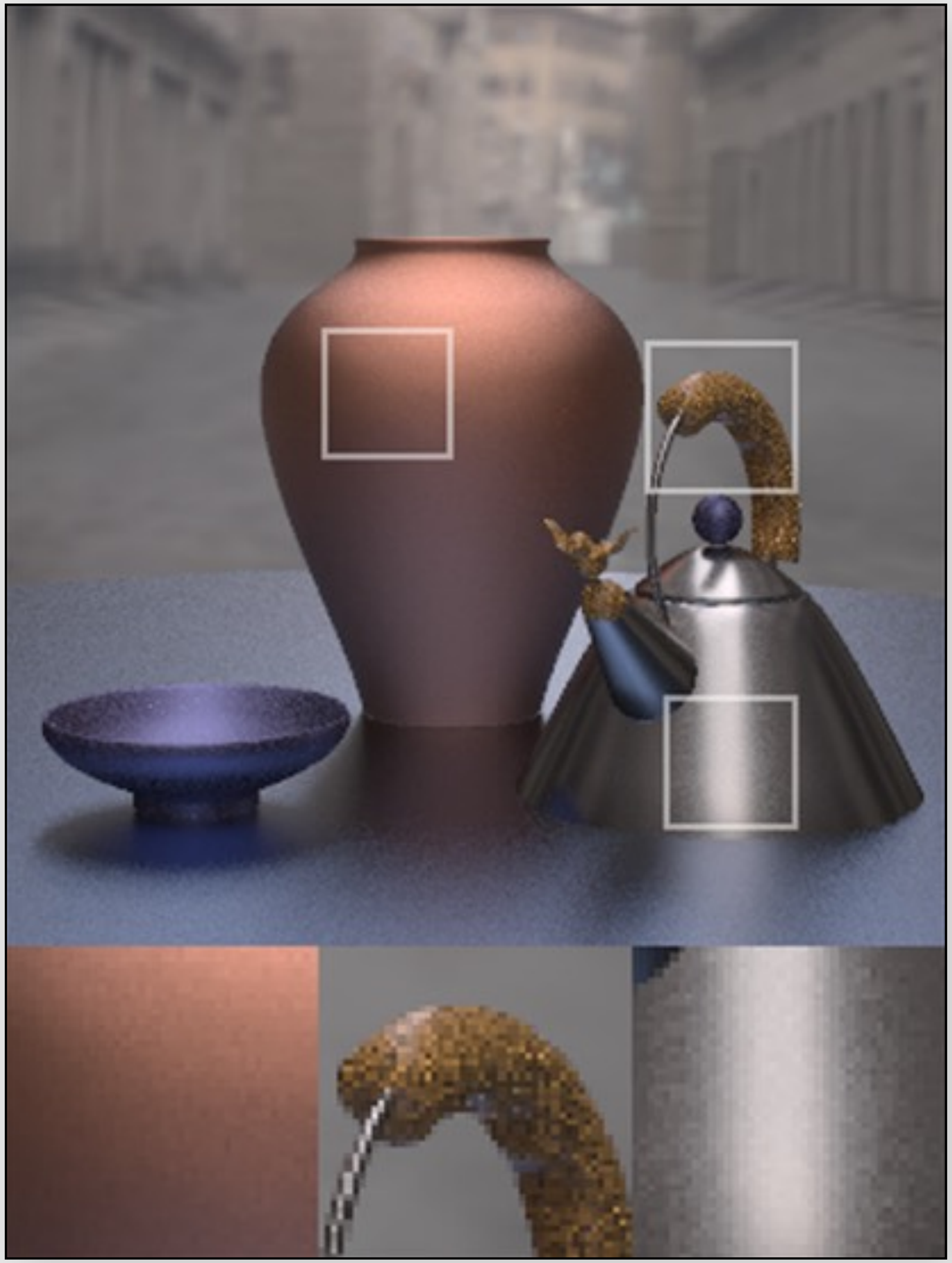
Images courtesy of **Jason Lawrence**
http://www.cs.princeton.edu/gfx/proj/brdf/

Path tracing - 600 samples per pixel



Images courtesy of **Jason Lawrence**
http://www.cs.princeton.edu/gfx/proj/brdf/

Path tracing - 1200 samples per pixel

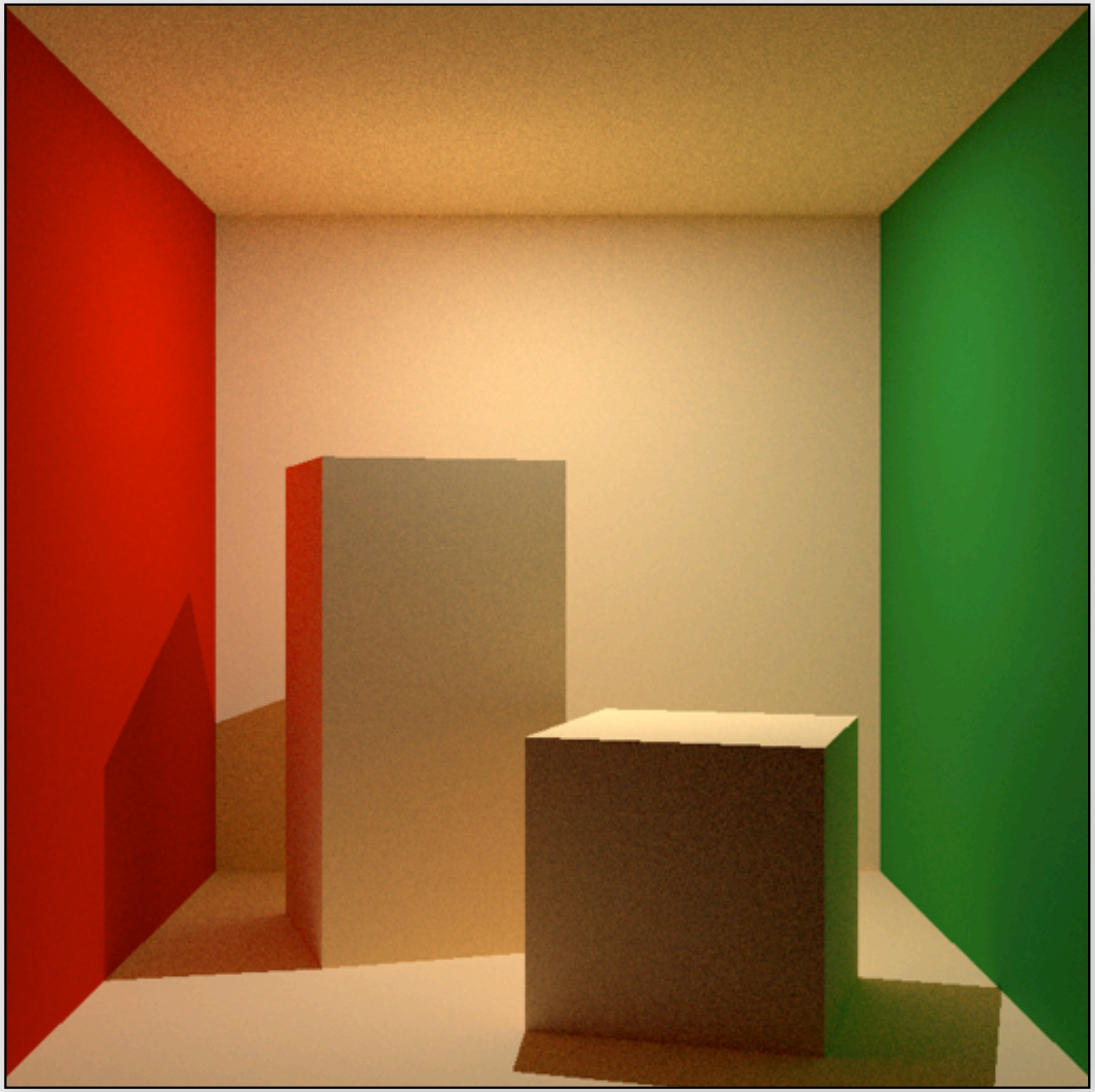


Still noisy!

Slow convergence

Images courtesy of **Jason Lawrence**
http://www.cs.princeton.edu/gfx/proj/brdf/

Path tracing - diffuse scene



128 paths/pixel

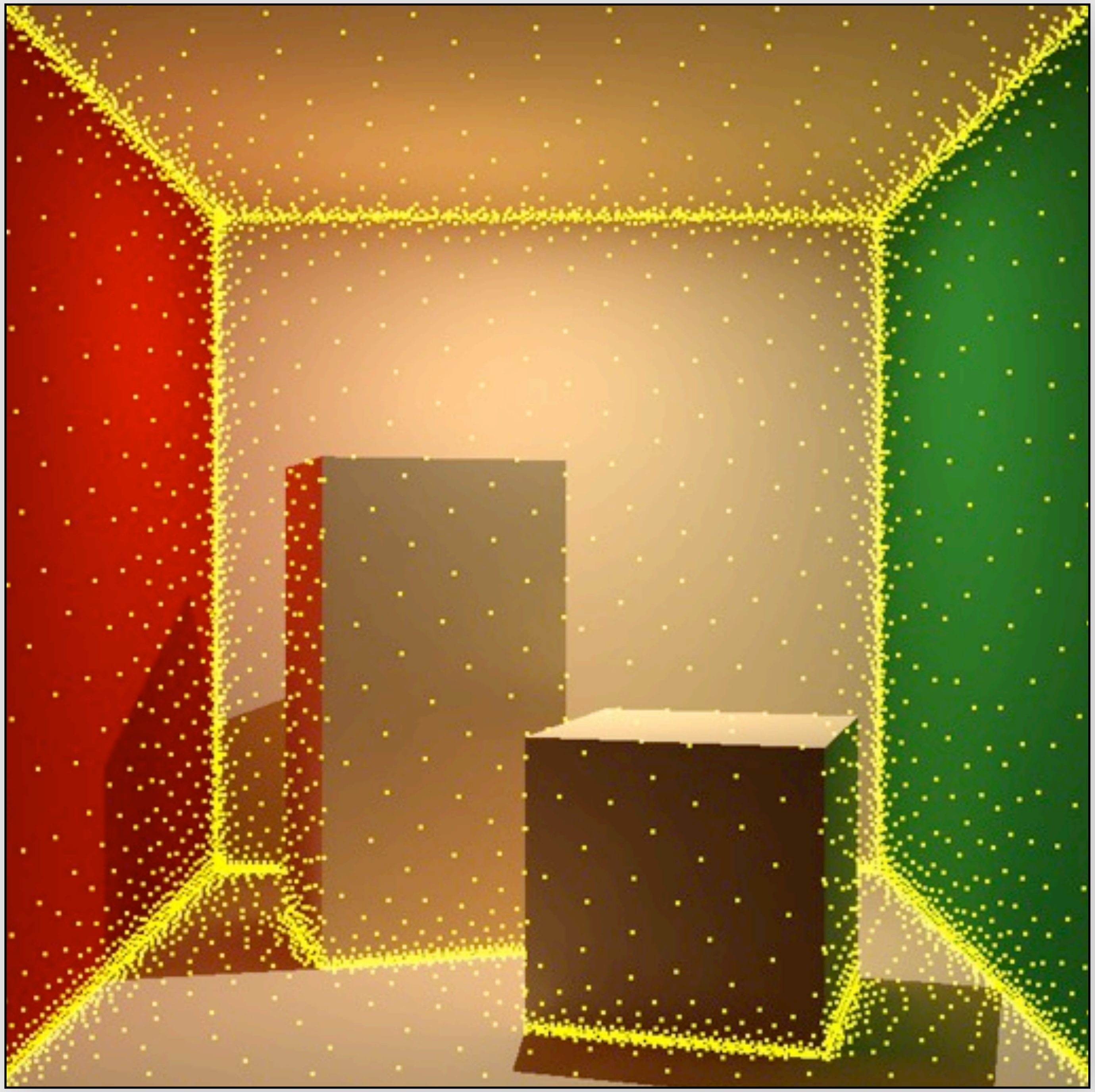
Diffuse indirect illumination is smooth

Perfect candidate for sparse sampling and interpolation

Interpolated indirect illumination

Irradiance Caching

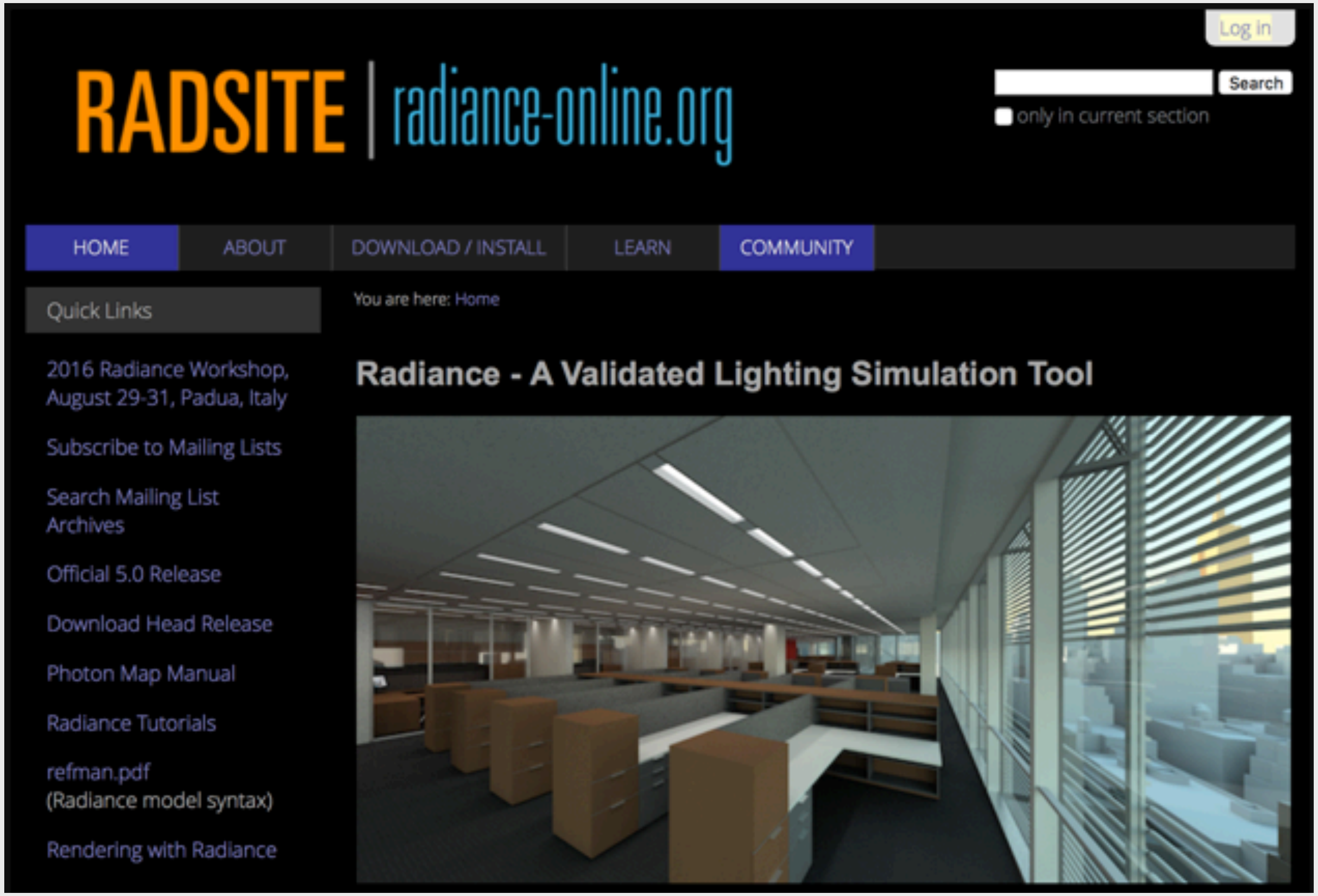
[Ward et al. 1988]



1M pixels - 4K cache points

Irradiance Caching in RADIANCE

- Originally developed by Greg Ward for RADIANCE architectural lighting simulation package



Irradiance Caching Algorithm

if at least one cached illumination value found near **x** **then**
  Interpolate illumination from the cached value(s).
else
  Compute and cache a new illumination value at **x**.

- Some questions that remain:
  - What do we cache?
  - What makes a cache point "nearby"?
  - How do we interpolate the nearby cached values?

[Ward et al. 1988]

Lambertian assumption

- Indirect illumination on a Lambertian surface:

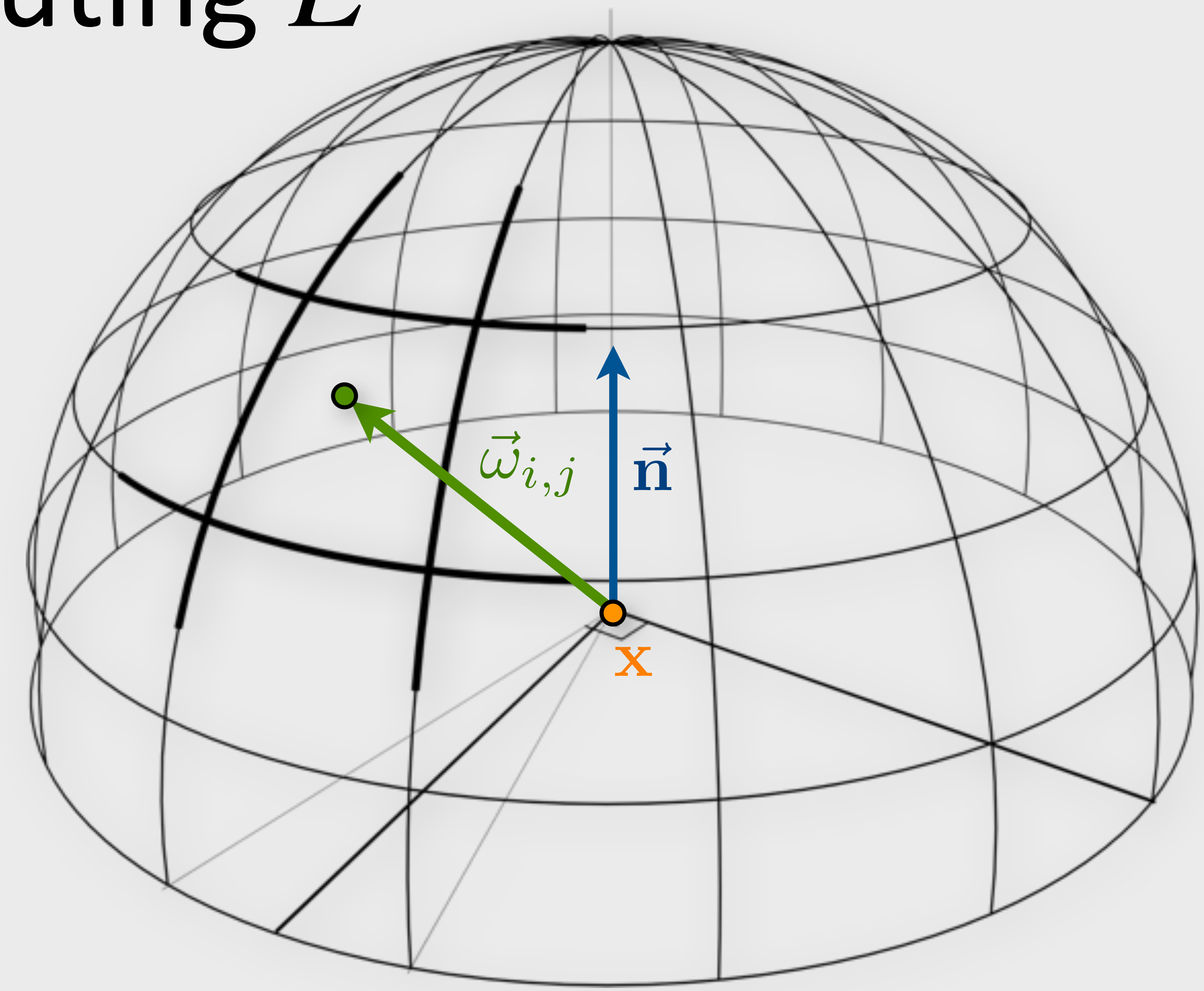
$$L_r(\mathbf{x},\vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x},\vec{\omega}_i,\vec{\omega}_r) L_i(\mathbf{x},\vec{\omega}_i)(\vec{\mathbf{n}}\cdot\vec{\omega}_i)\,\mathrm{d}\vec{\omega}_i$$

$$= \frac{\rho}{\pi} \underbrace{\int_{H^2} L_i(\mathbf{x},\vec{\omega}_i)(\vec{\mathbf{n}}\cdot\vec{\omega}_i)\,\mathrm{d}\vec{\omega}_i}_{E(\mathbf{x},\vec{\mathbf{n}}) \rightarrow \text{Irradiance}}$$

$$E(\mathbf{x},\vec{\mathbf{n}}) \approx \frac{\pi}{N}\sum_{j=1}^{N} L_i(\mathbf{x},\vec{\omega}_{i,j})$$

[Ward et al. 1988]

Computing E

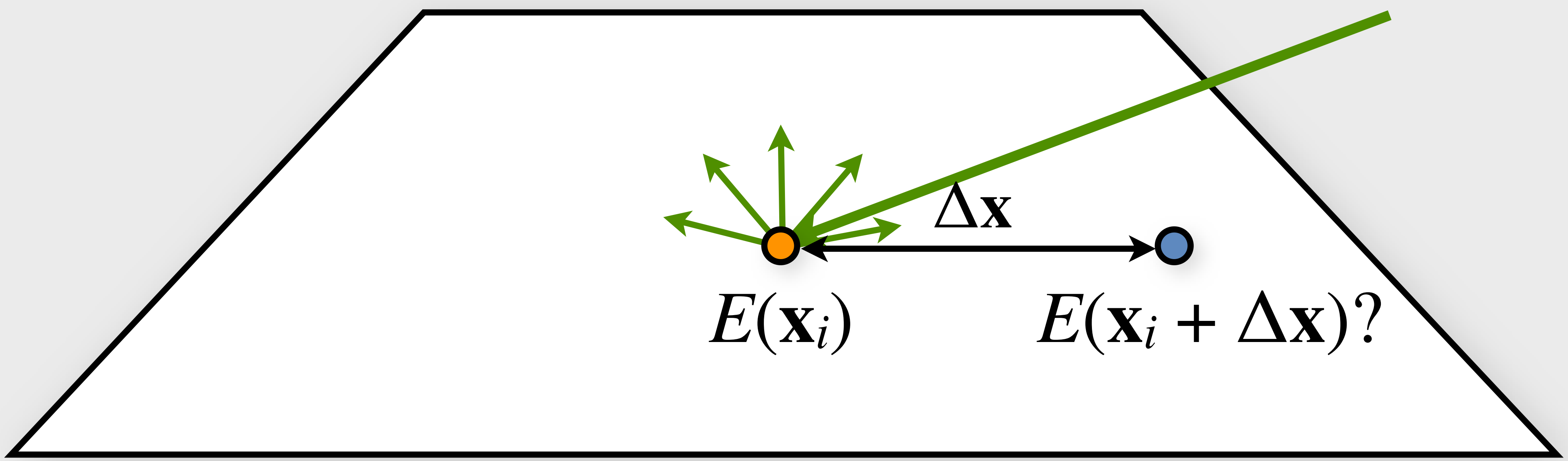


$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\pi}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} L_i(\mathbf{x}, \vec{\omega}_{i,j})$$

[Ward et al. 1988]

Interpolating Irradiance

- Irradiance computation costly, reuse whenever possible
- How far away can we reuse a cached value?

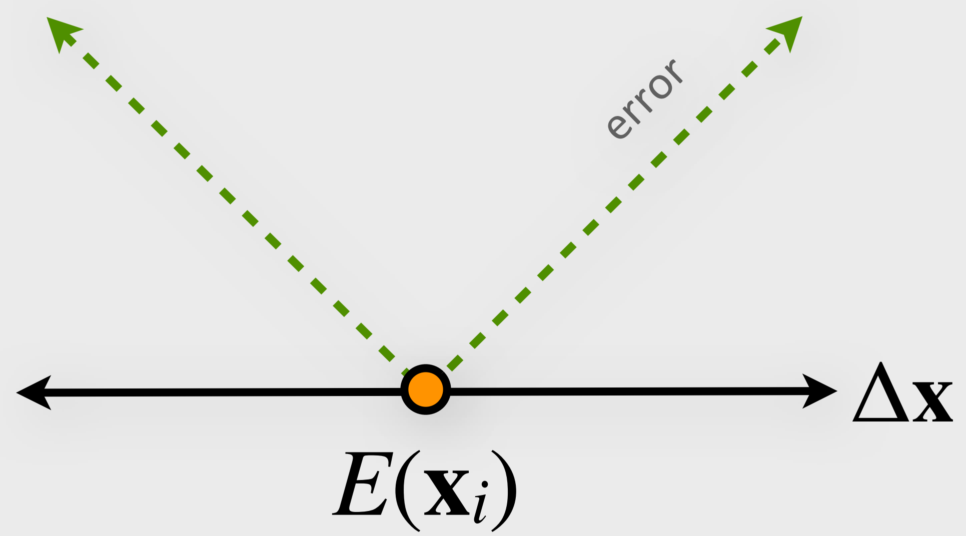


$$E(\mathbf{x}) \approx E(\mathbf{x}_i) + \left(\frac{\partial E}{\partial \mathbf{x}}\Delta\mathbf{x} + \frac{\partial E}{\partial \vec{\mathbf{n}}}\Delta\vec{\mathbf{n}}\right)$$

error

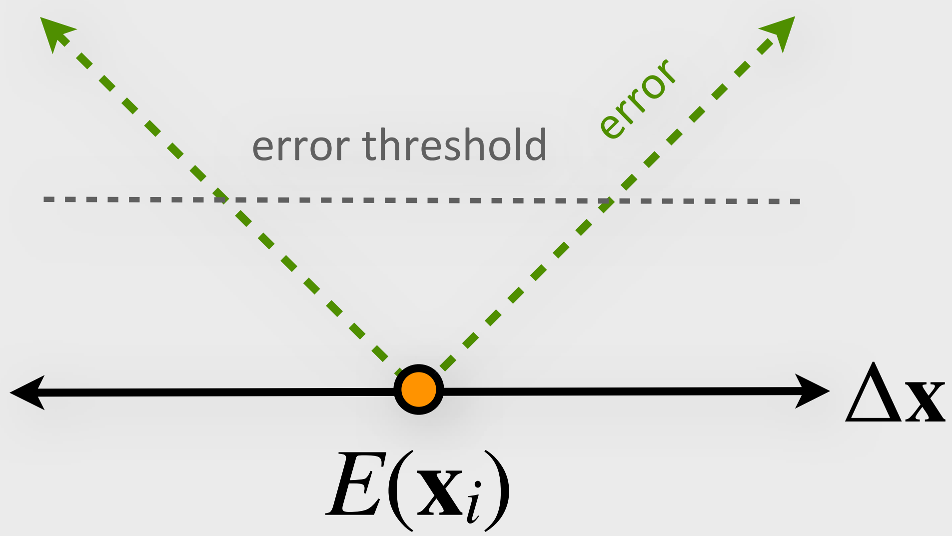
[Ward et al. 1988]

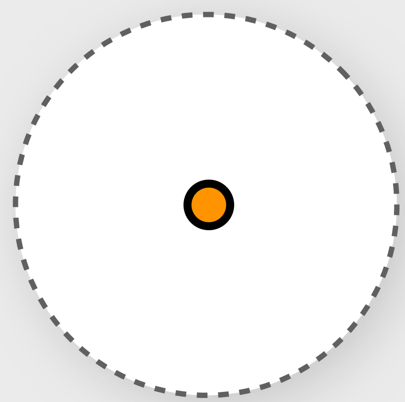
Interpolating Irradiance



[Ward et al. 1988]

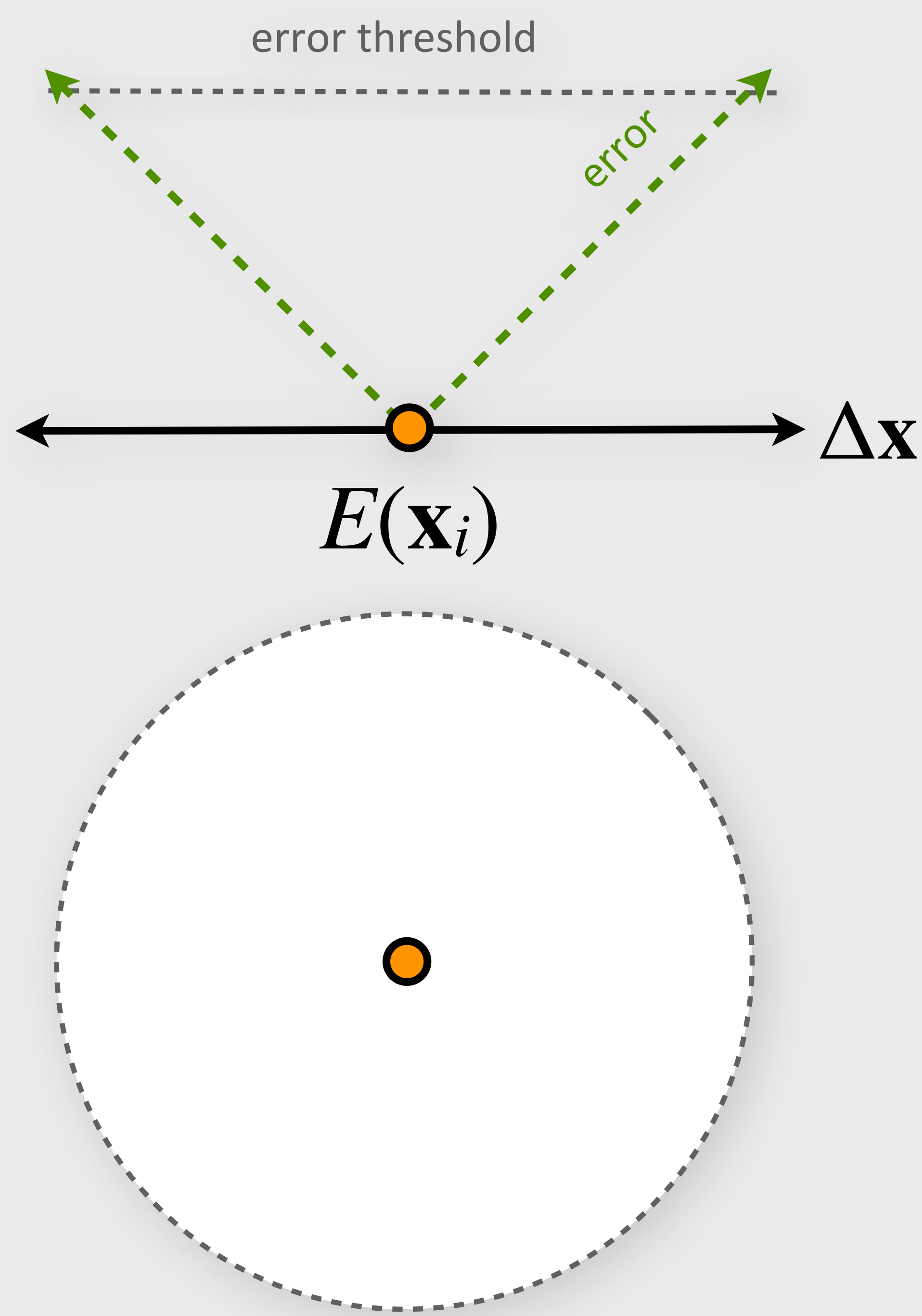
Interpolating Irradiance





[Ward et al. 1988]

Interpolating Irradiance



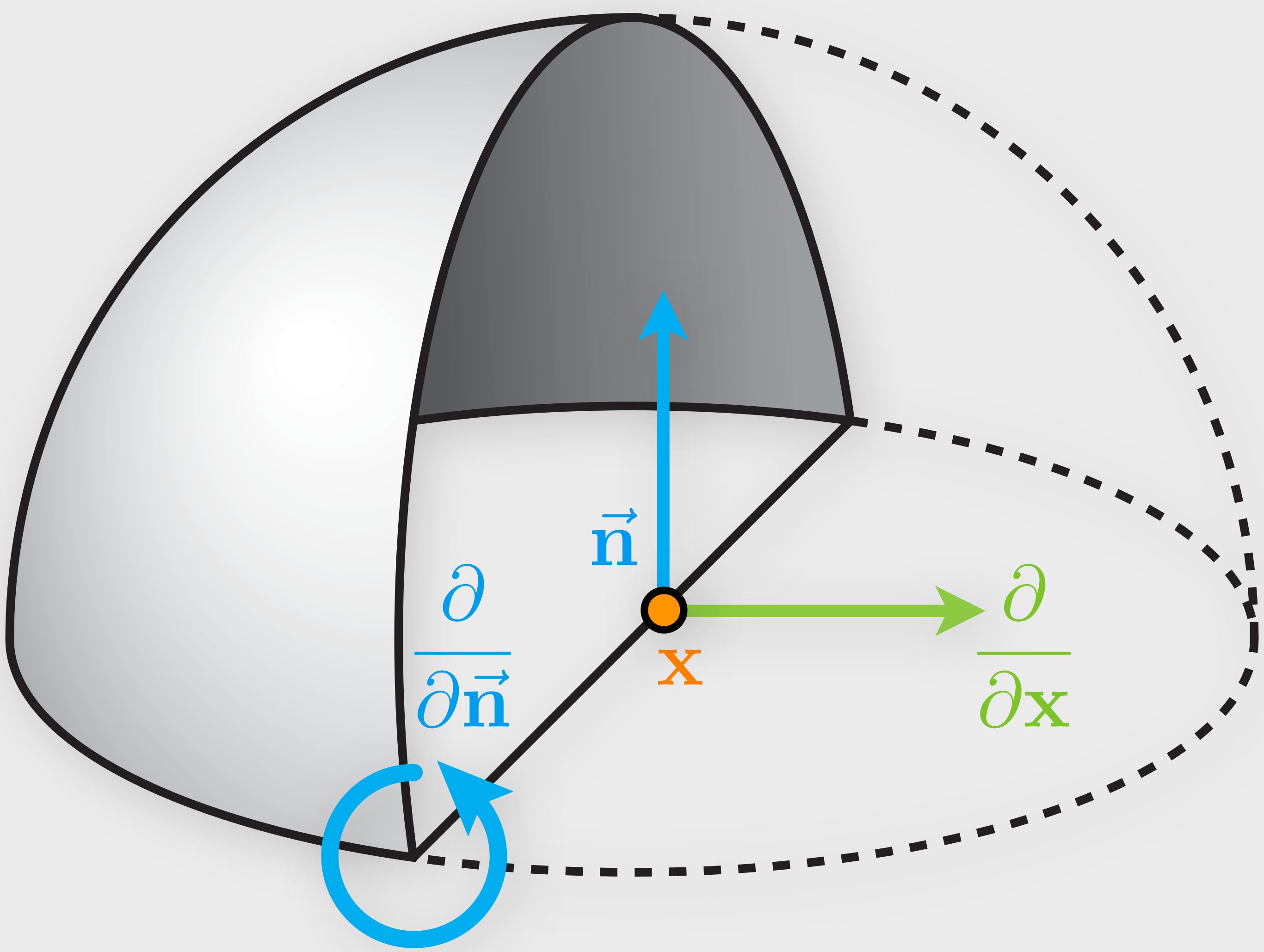
[Ward et al. 1988]

The "Split-Sphere" Heuristic

- To compute valid region, need to estimate change in irradiance

$$\varepsilon_i \lesssim \left| \frac{\partial E}{\partial \mathbf{x}} \Delta \mathbf{x} + \frac{\partial E}{\partial \vec{\mathbf{n}}} \Delta \vec{\mathbf{n}} \right|$$

- Consider hypothetical, worst-case scene: the "**Split-Sphere**"



[Ward et al. 1988]

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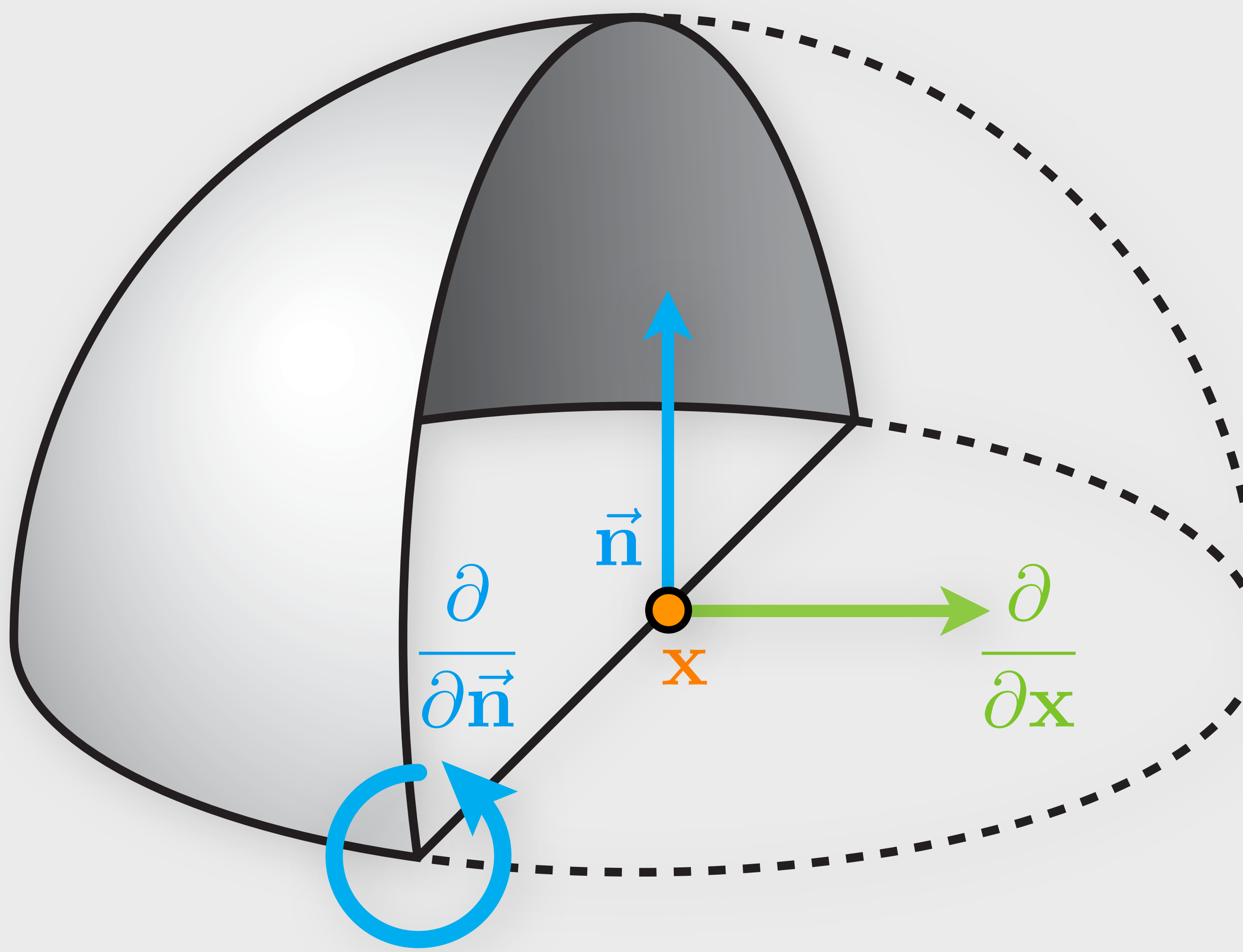

[Ward et al. 1988]

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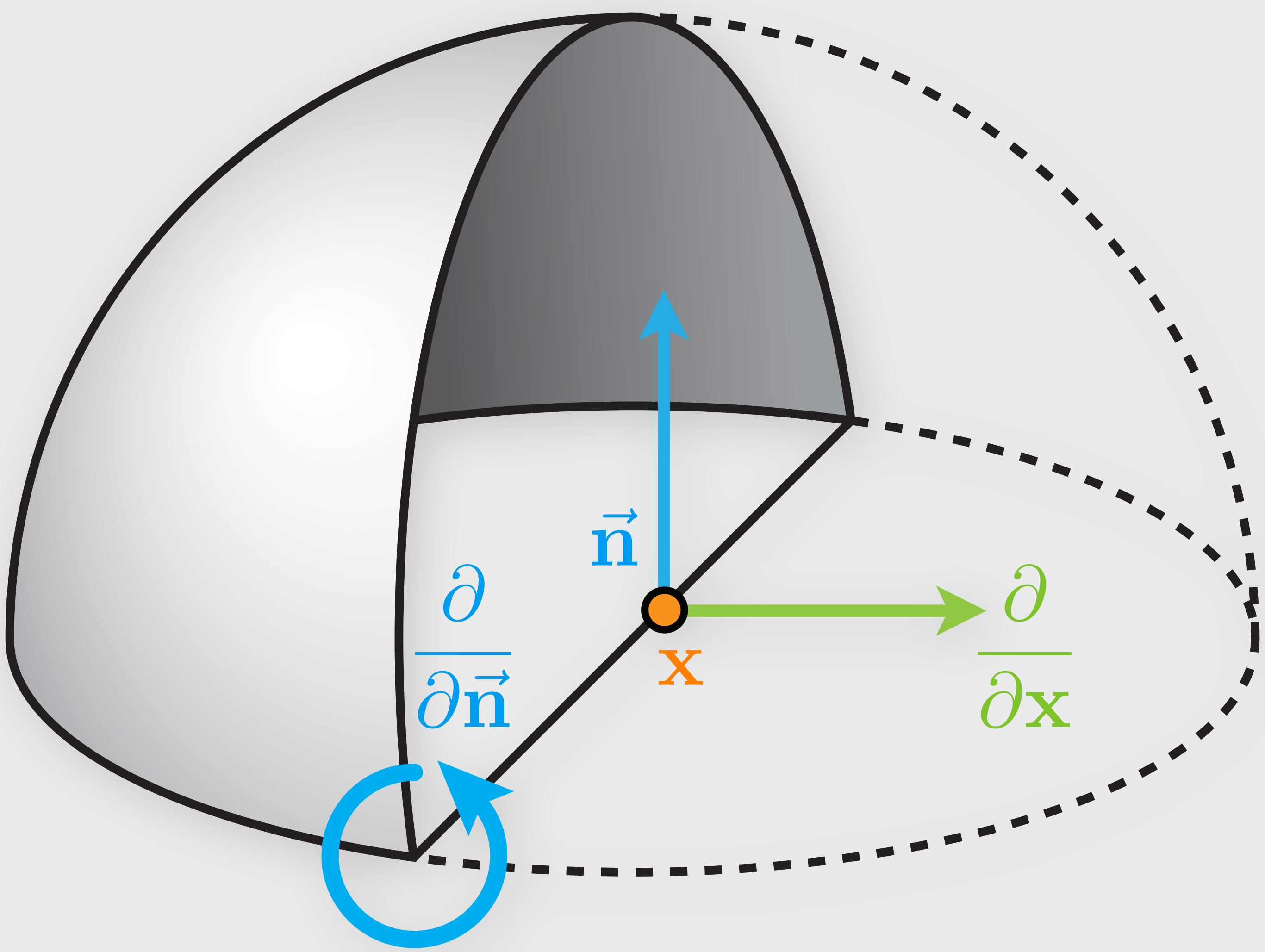
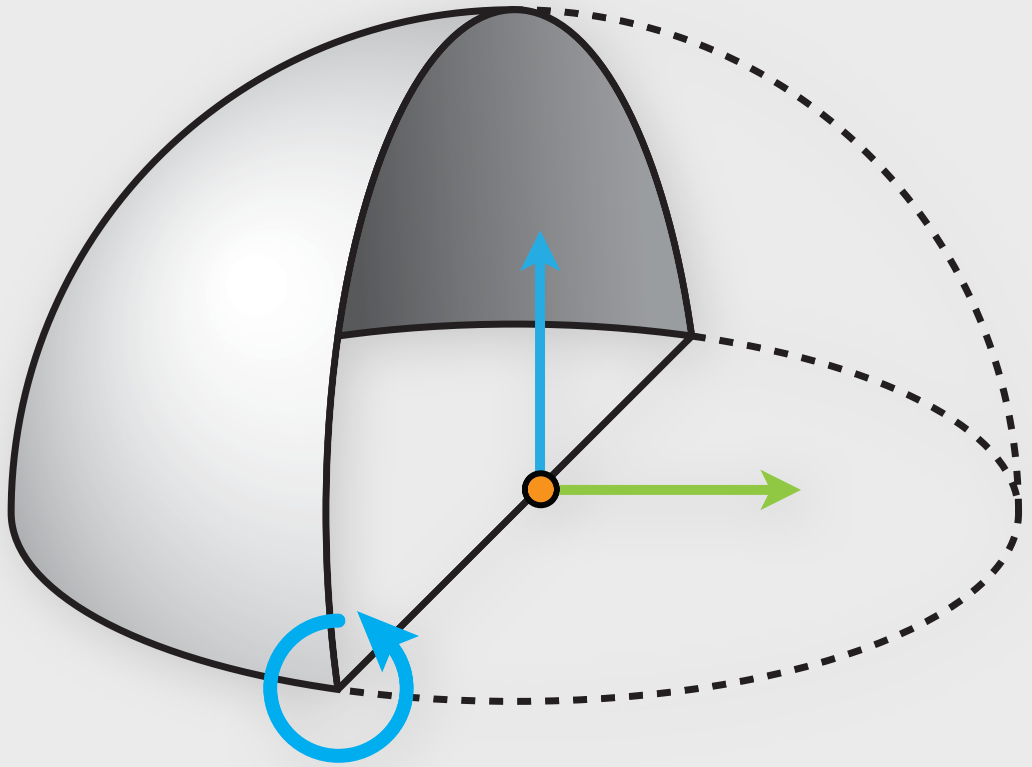


[Ward et al. 1988]

The "Split-Sphere" Heuristic

- In the "Split-Sphere" environment the error becomes:

$$\varepsilon_i \lesssim \left| \frac{\partial E}{\partial \mathbf{x}} \Delta \mathbf{x} + \frac{\partial E}{\partial \vec{\mathbf{n}}} \Delta \vec{\mathbf{n}} \right|$$



[Ward et al. 1988]

The “Split-Sphere” Heuristic

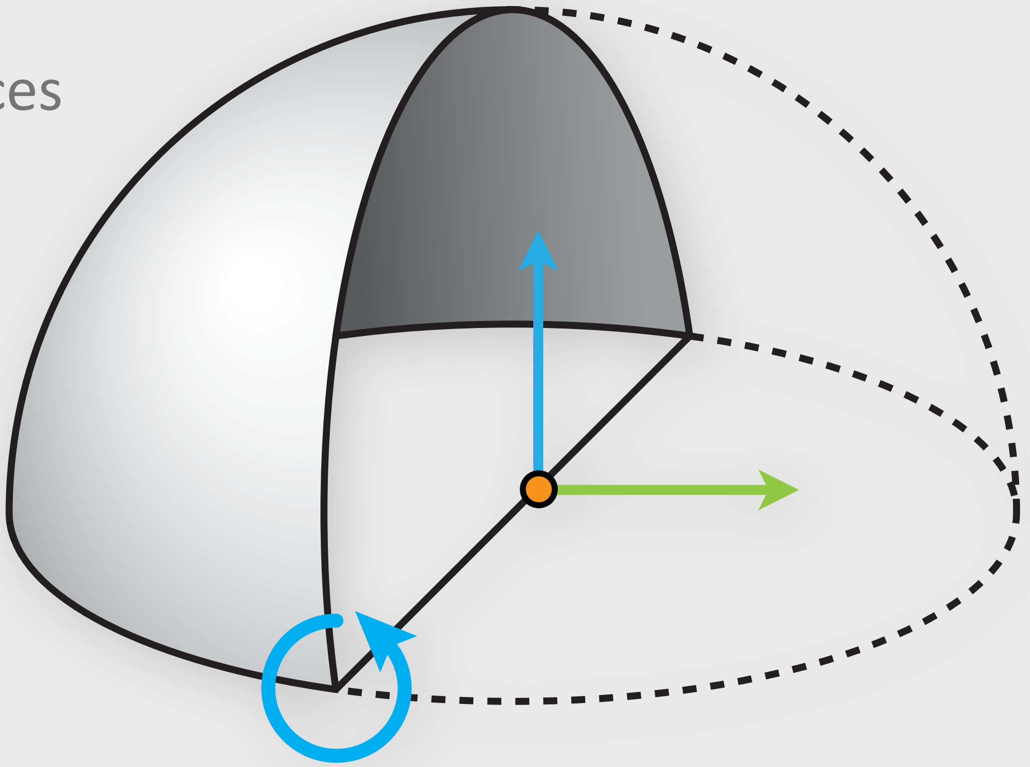
- In the “Split-Sphere” environment the error becomes:

position difference, relative to radius of sphere

orientation difference

$$\varepsilon_i \lesssim E_i \left(\frac{4}{\pi} \frac{\|\mathbf{x} - \mathbf{x}_i\|}{R_i} + \sqrt{1 - (\vec{\mathbf{n}} \cdot \vec{\mathbf{n}}_i)} \right)$$

“average” distance to visible surfaces

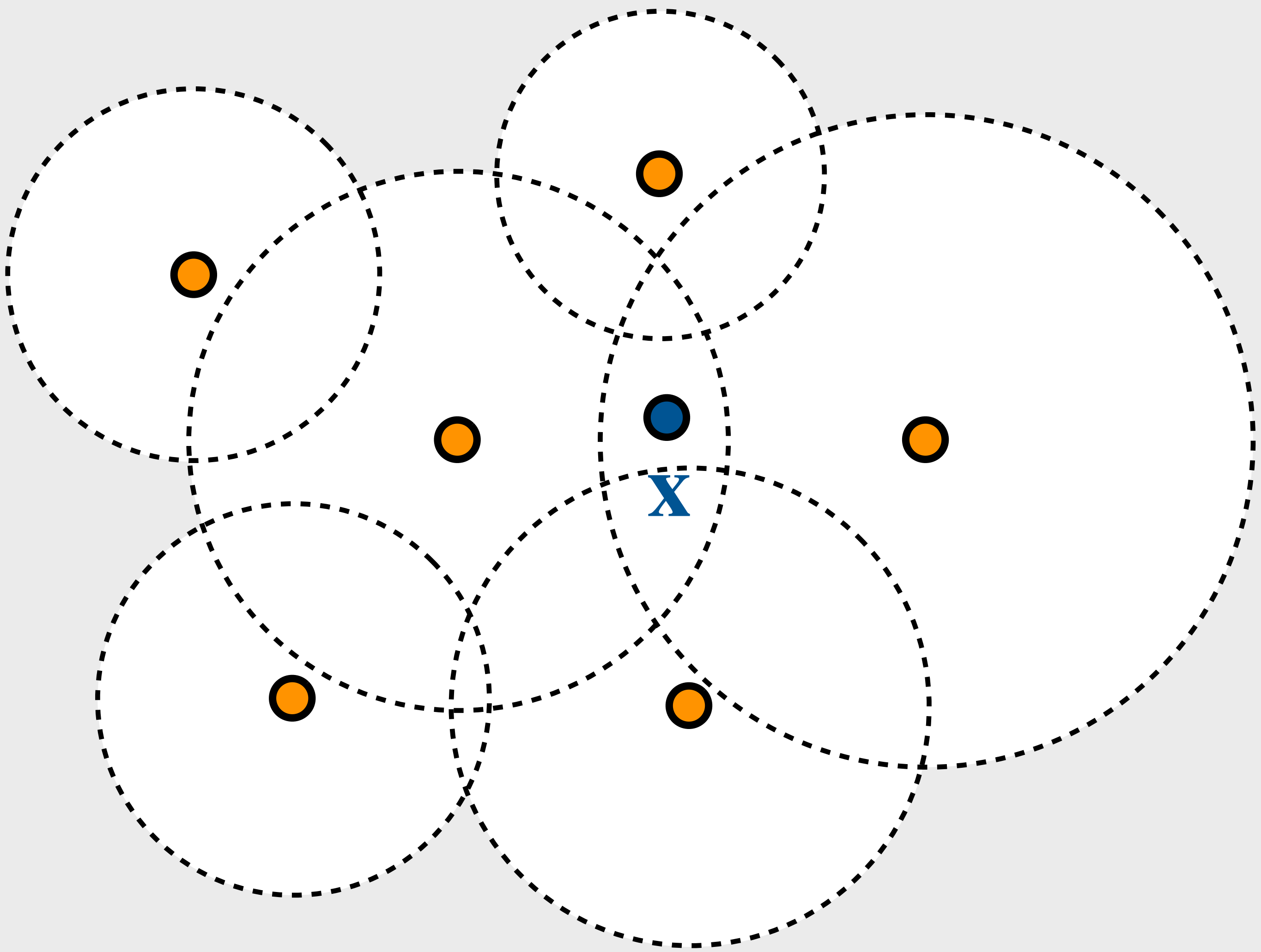


[Ward et al. 1988]

Interpolating Irradiance

- At each shading location, perform a weighted average of all cached values which have an error below some threshold.

$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})\, E_i}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})}$$



[Ward et al. 1988]

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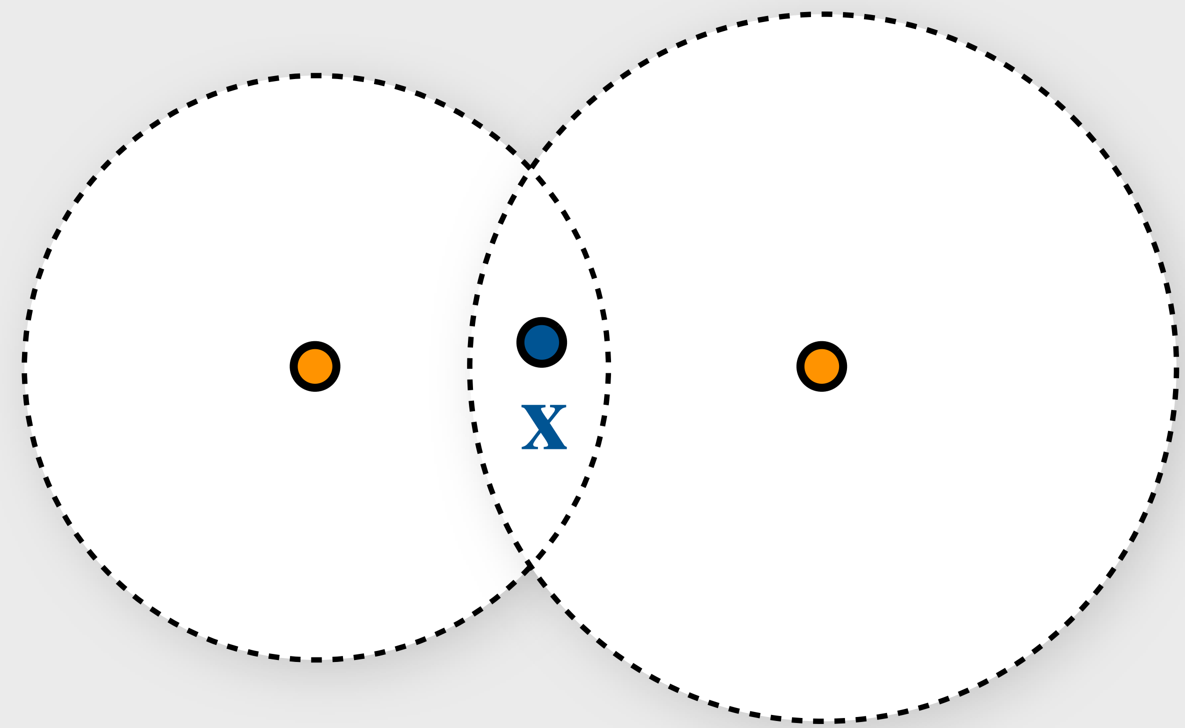
where:

$$S = \{i : \epsilon_i(\mathbf{x}, \vec{\mathbf{n}}) < a\}$$



[Ward et al. 1988]

Interpolating Irradiance

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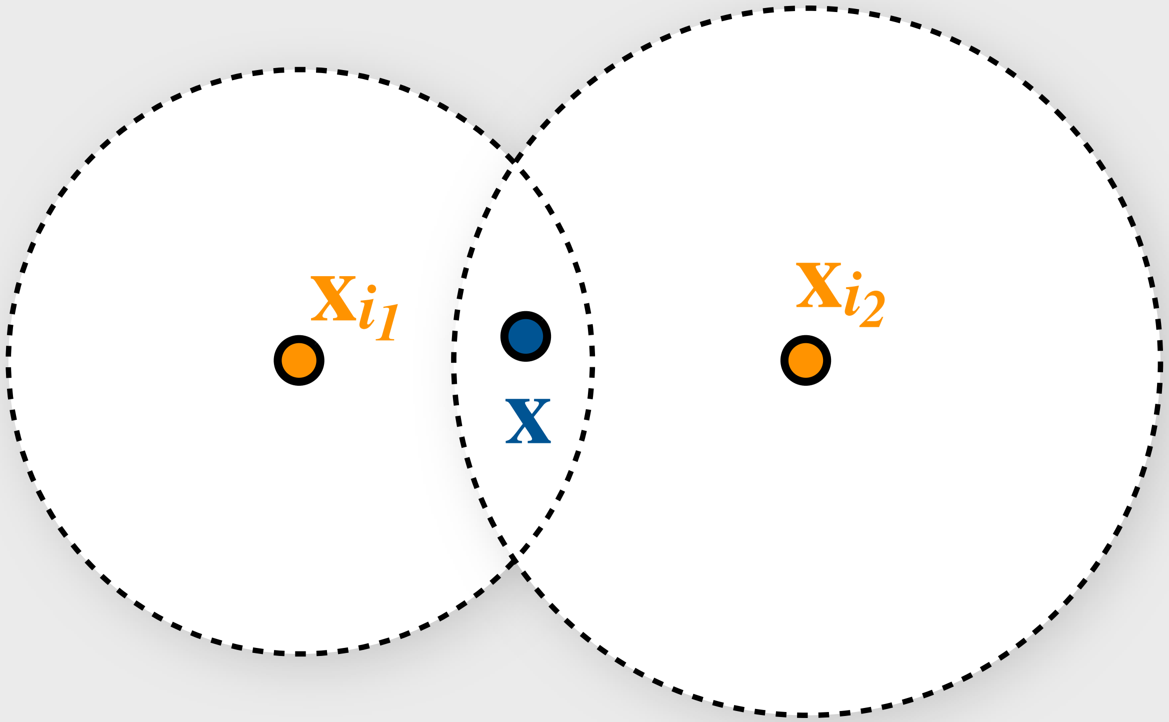
$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}}) \boxed{E_i}}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})}$$

where:

$$S = \{i : \epsilon_i(\mathbf{x}, \vec{\mathbf{n}}) < a\}$$



Interpolating Irradiance

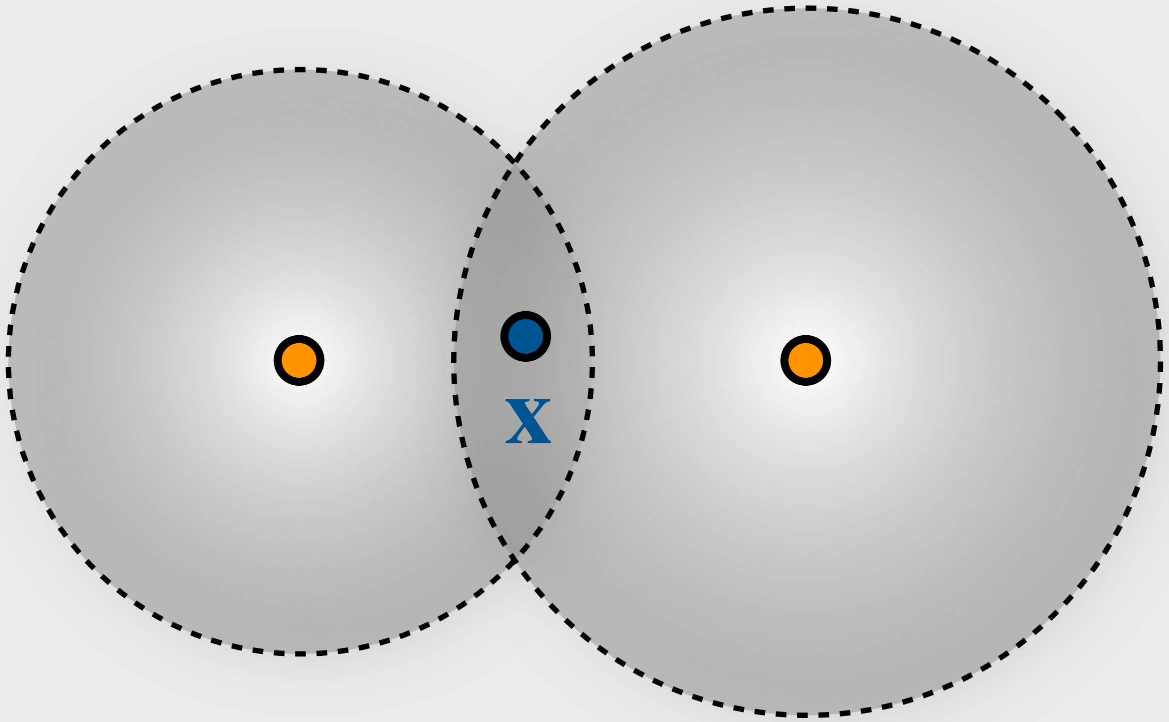
- At each shading location, perform a weighted average of all cached values which have an error below some threshold.
- Reciprocal of the error is used as the weight

$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \boxed{\mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})} E_i}{\boxed{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})}}$$

where:

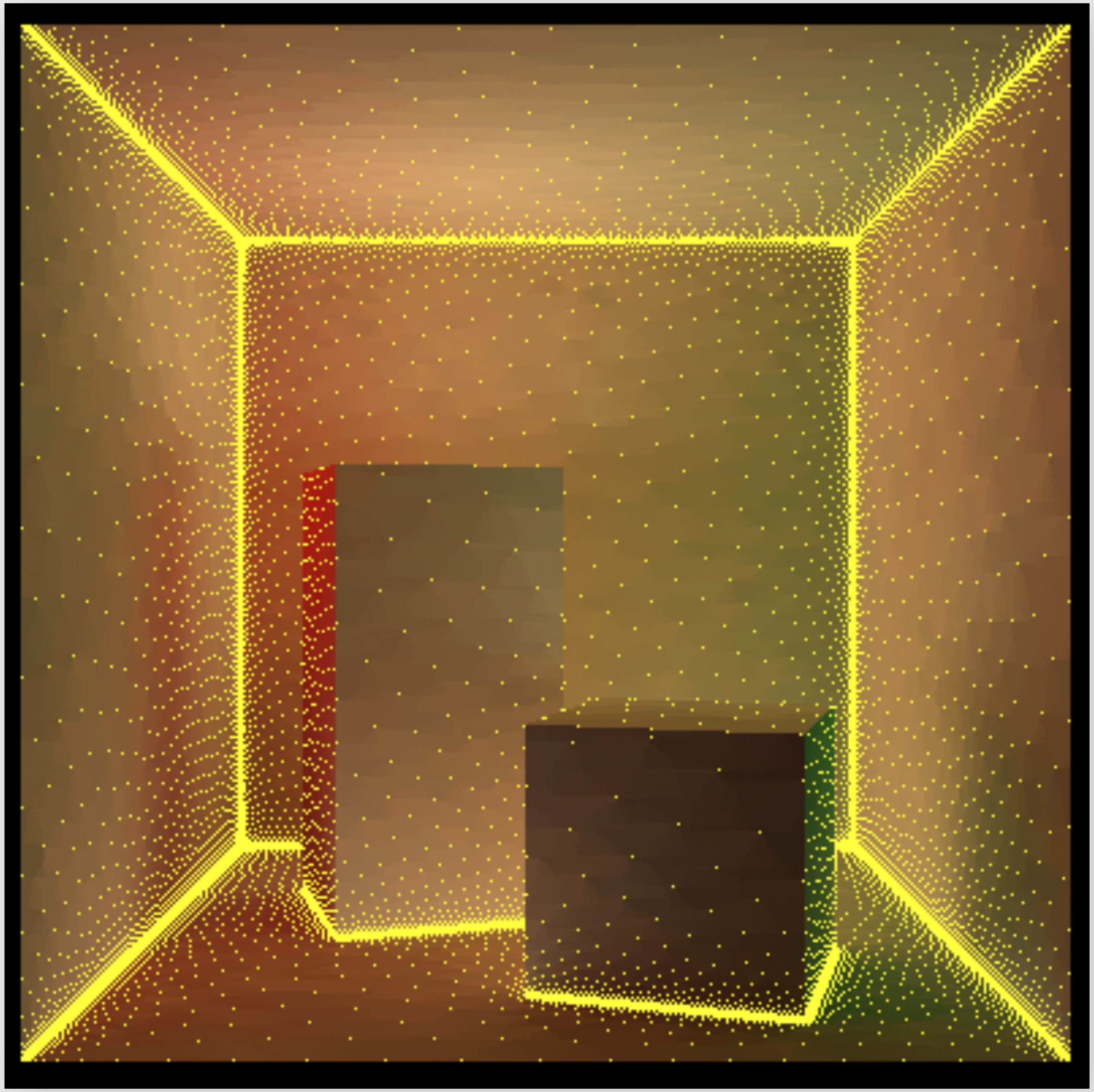
$$S = \{i : \epsilon_i(\mathbf{x}, \vec{\mathbf{n}}) < a\}$$

$$\mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}}) = \frac{1}{\epsilon_i(\mathbf{x}, \vec{\mathbf{n}})} - \frac{1}{a}$$



Practical Considerations

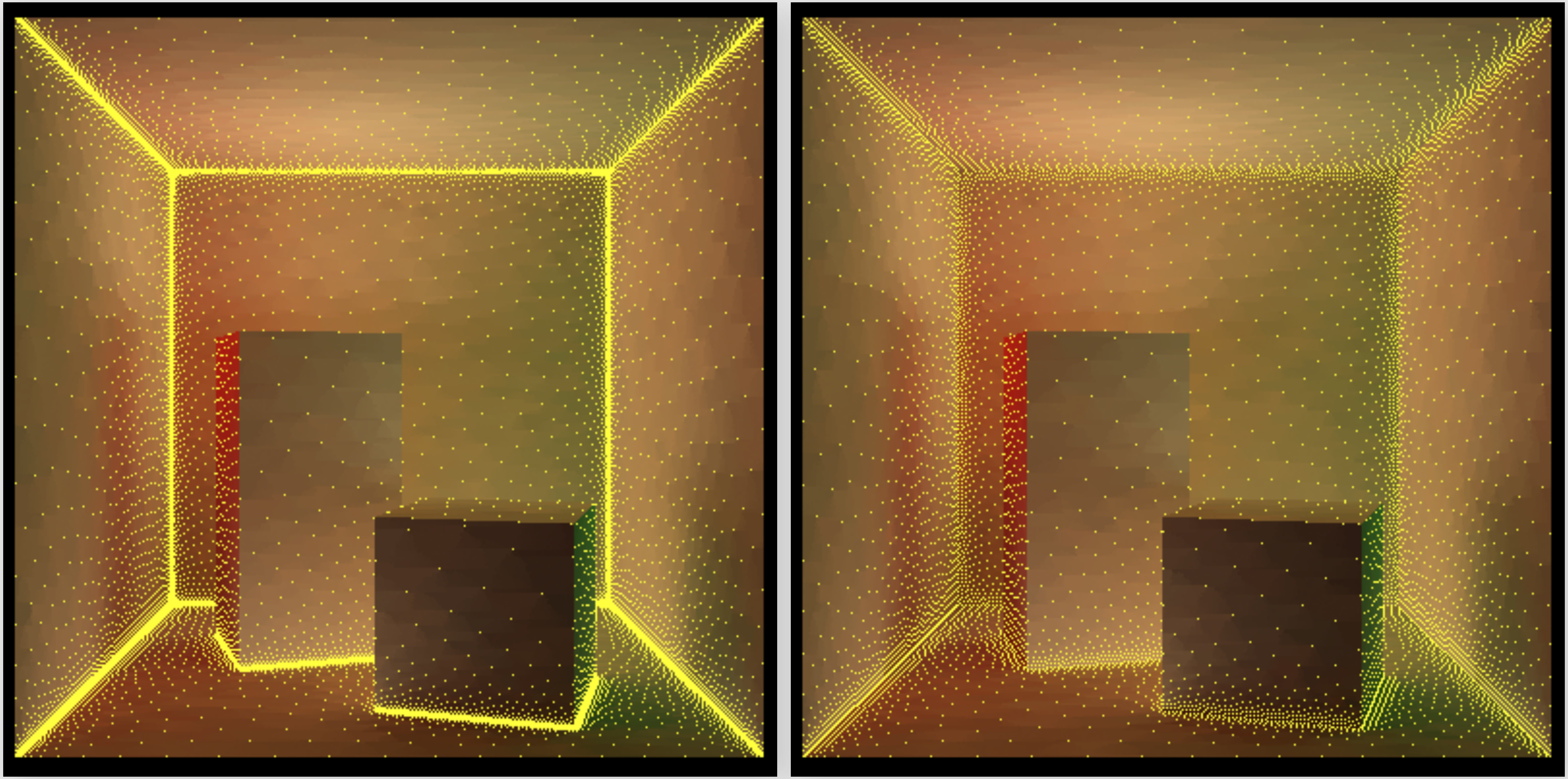
Over-Sampling/Clumping



split-sphere metric

[Ward et al. 1988]

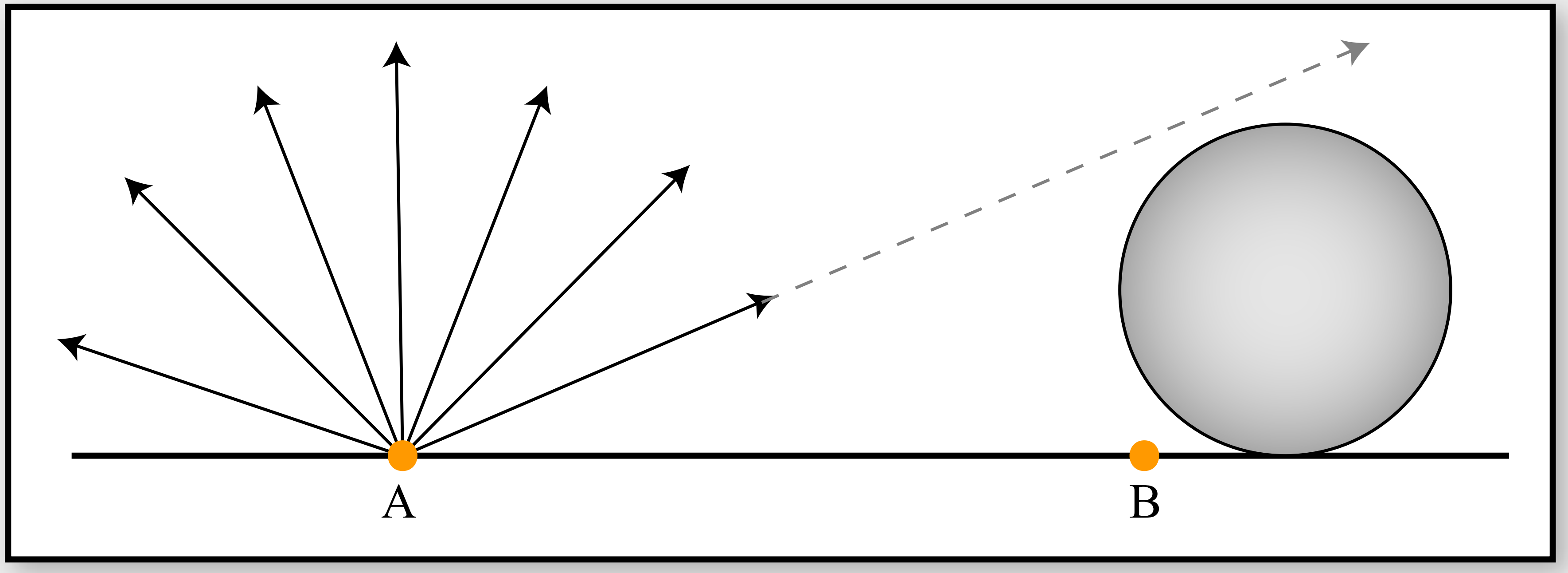
Over-Sampling/Clumping



split-sphere metric　　　　with minimum radius clamping

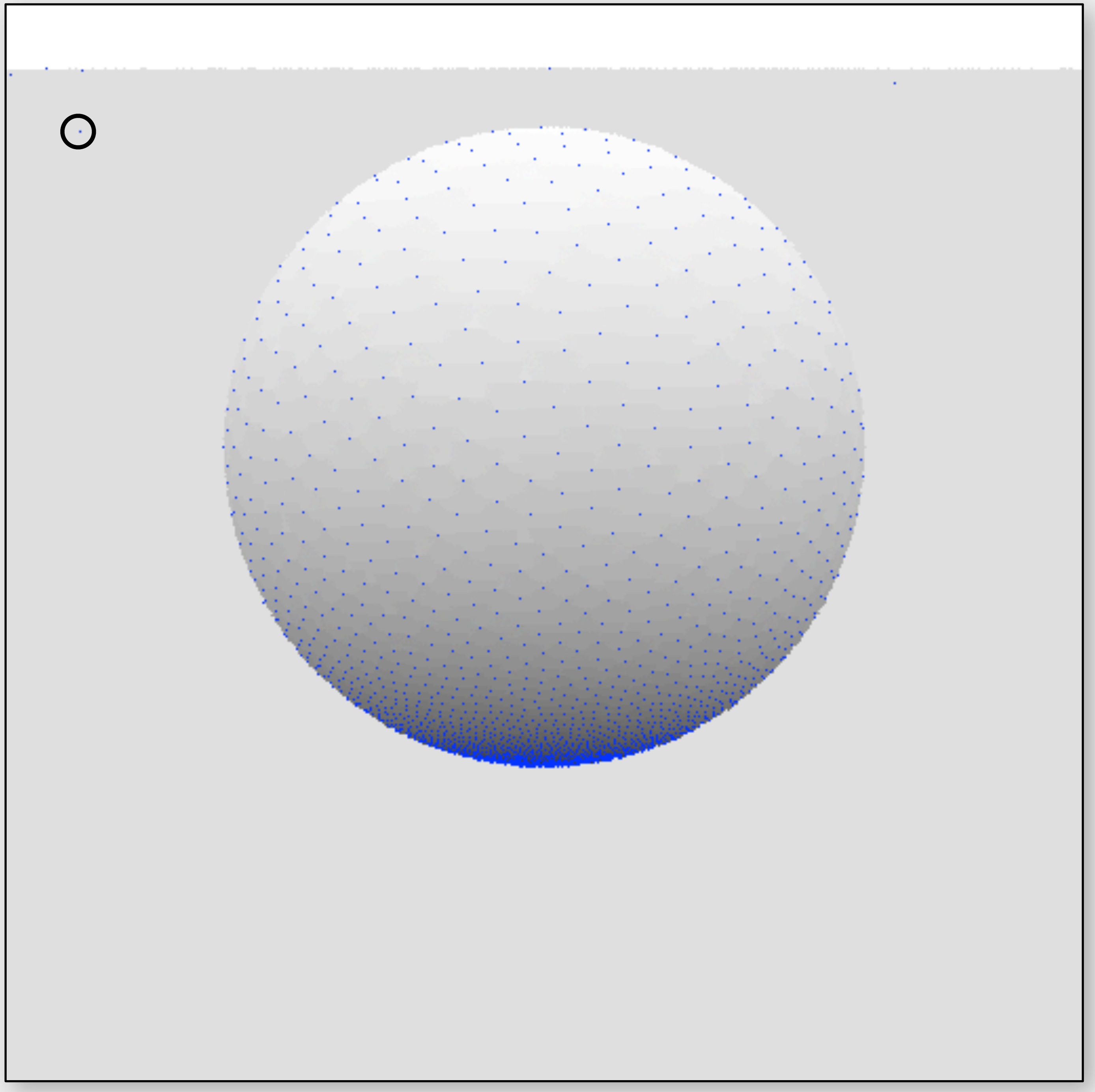
[Ward et al. 1988]

Under-Sampling



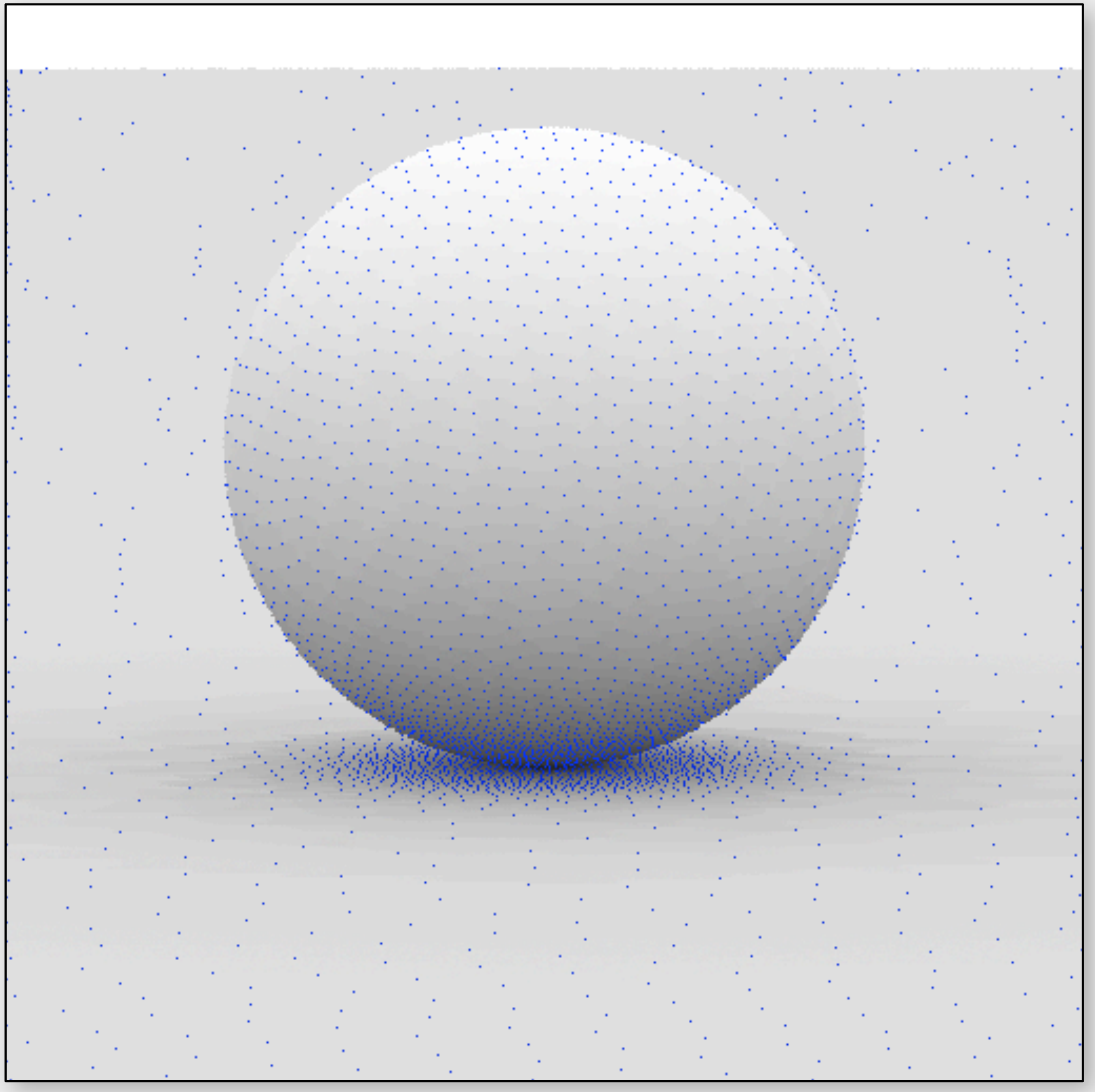
[Ward et al. 1988]

Under-Sampling



[Ward et al. 1988]

Under-Sampling



[Ward et al. 1988]

Split-Sphere Heuristic

- In practice, must also specify:
  - Min/max screen-space & world-space radii

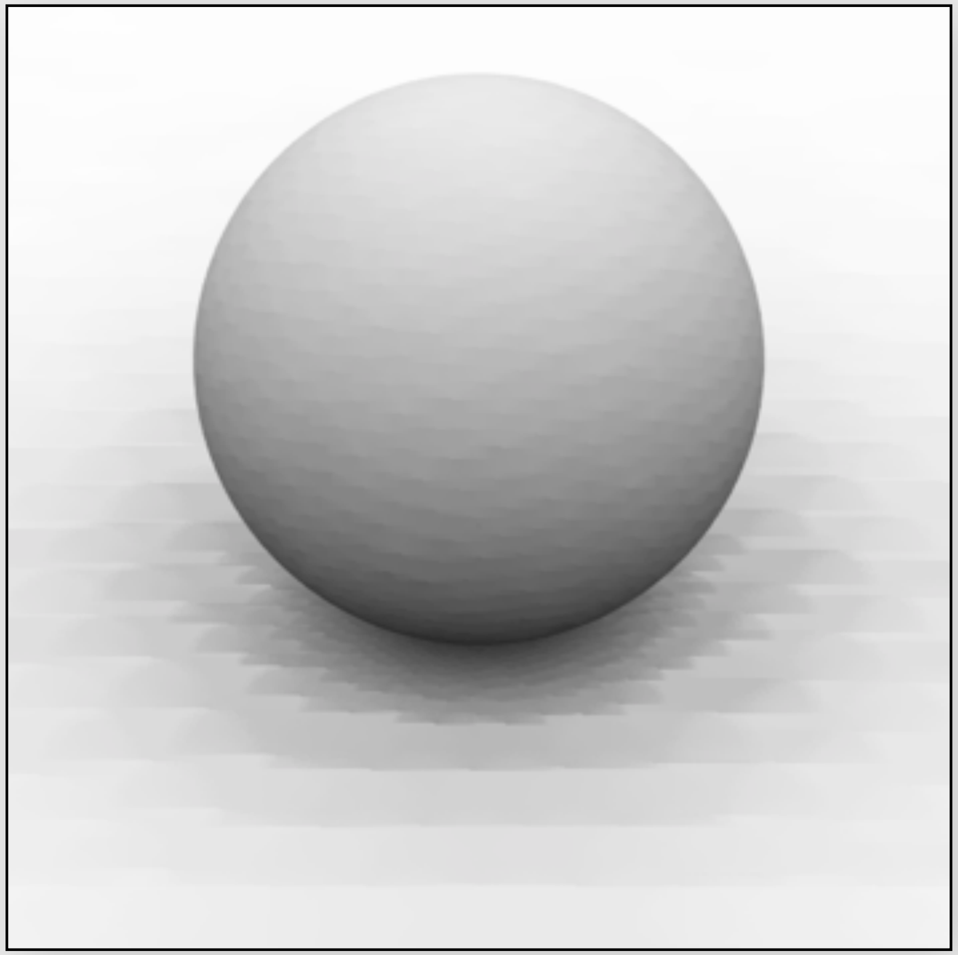
Irradiance Caching

- Pros:
  - Independent of resolution.
  - Computation amortized across many pixels
  - Concentrates computation in visible regions were illumination changes rapidly

[Ward et al. 1988]

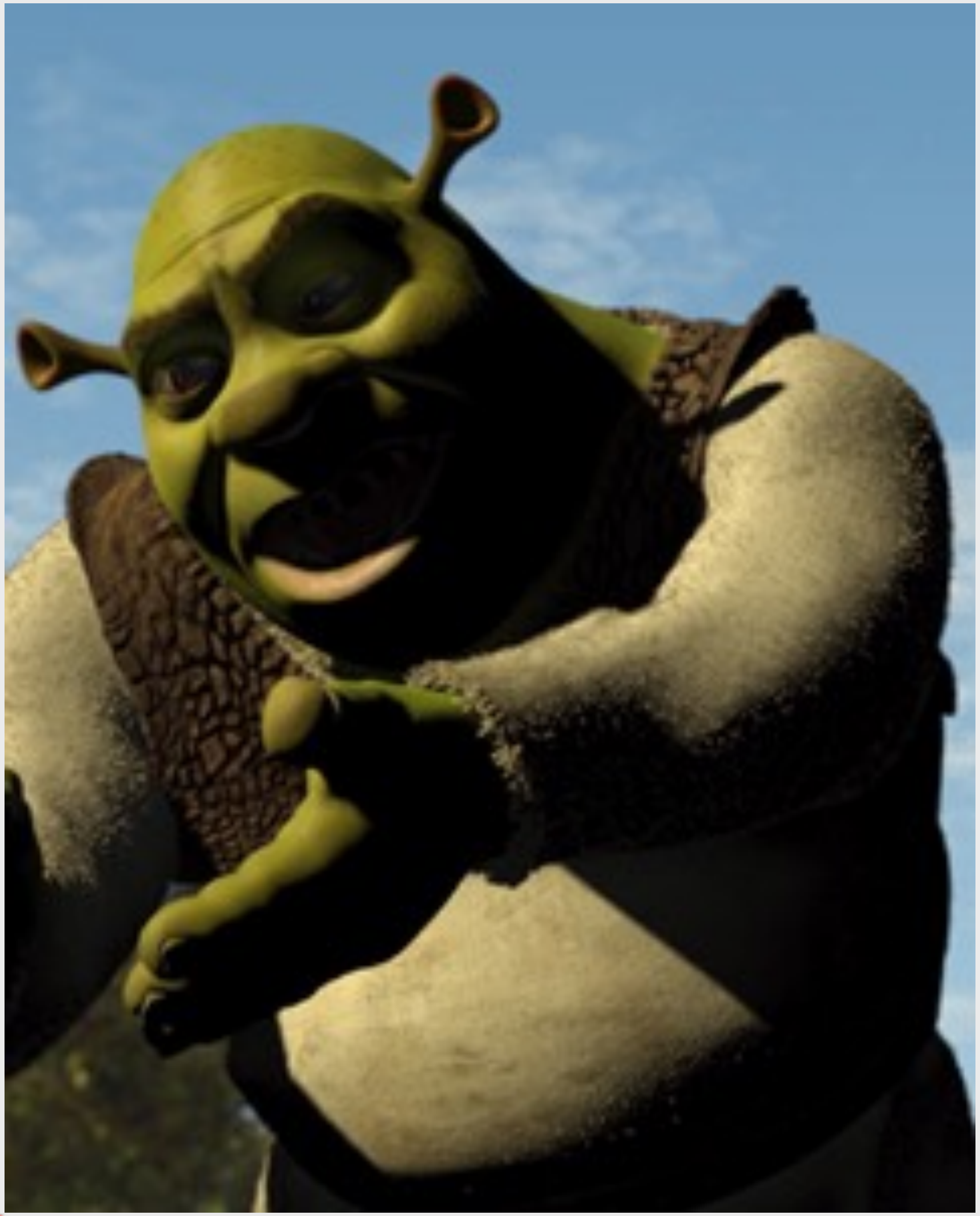
Irradiance Caching

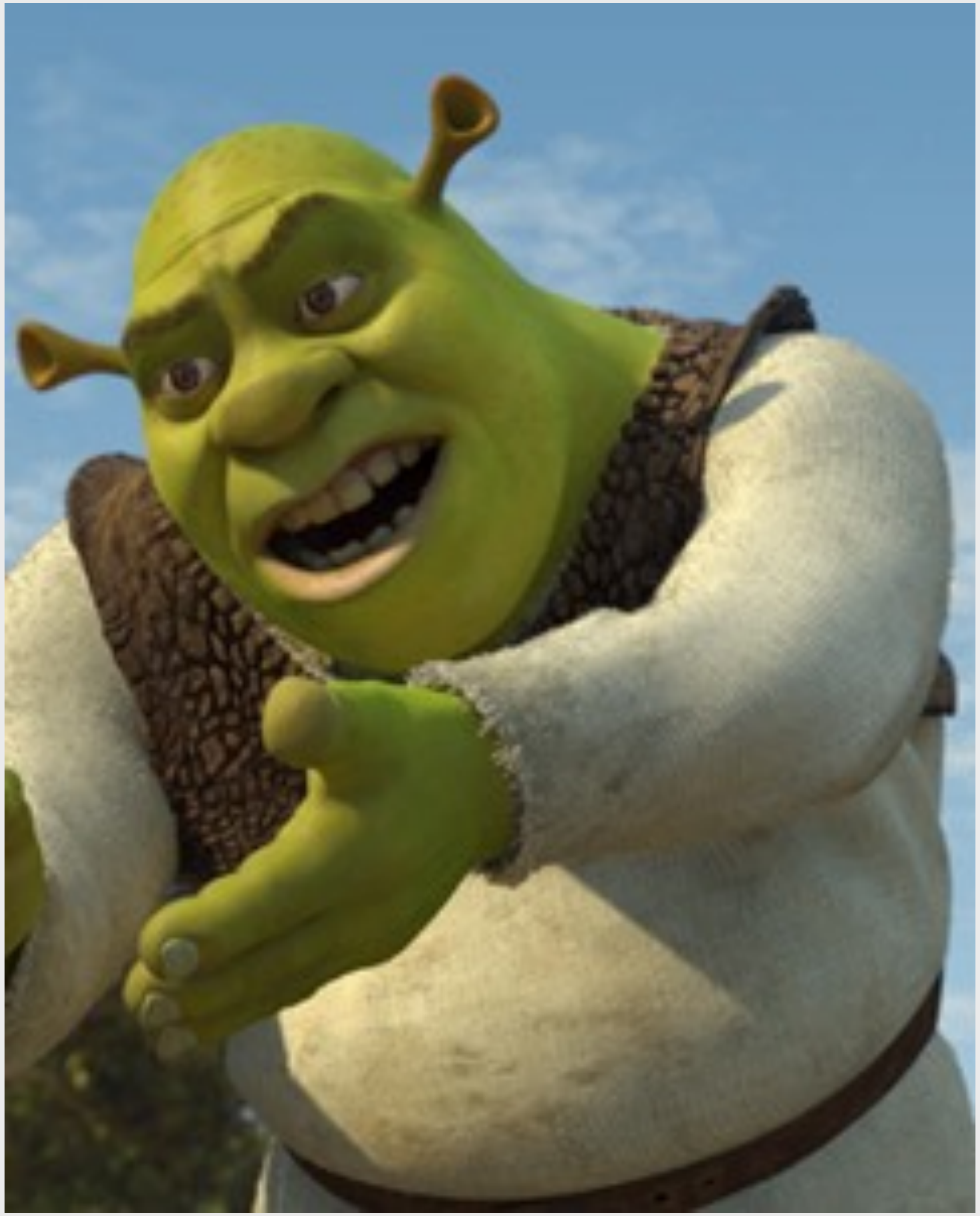
- Cons:
  - Interpolation/extrapolation can introduce visible artifacts
  - Valid radius metric not always robust
  - Limited to Lambertian (matte) surfaces



[Ward et al. 1988]

Use in the film industry





[Tabellion and Lamorlette 2004]

Improvements/Extensions

- Many extensions:
  - Ward and Heckbert ‘92 - better interpolation
  - Křivánek et al. ‘05a, ‘05b - glossy surfaces
  - Jarosz et al. ’08 - participating media
  - Jarosz et al. ‘12 - irradiance Hessians
  - Schwarzhaupt et al. ‘12 - better error control
  - ...

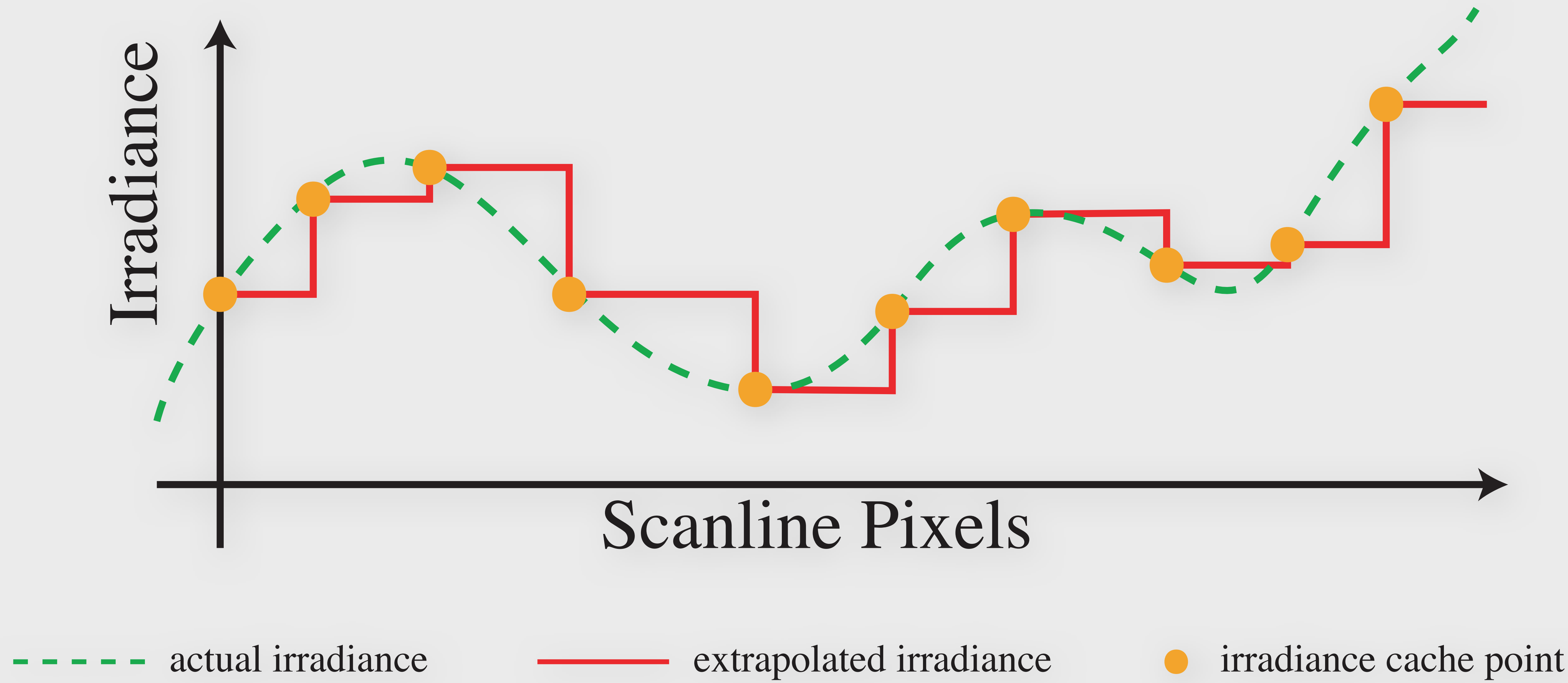
Irradiance Caching Artifacts

Irradiance gradients

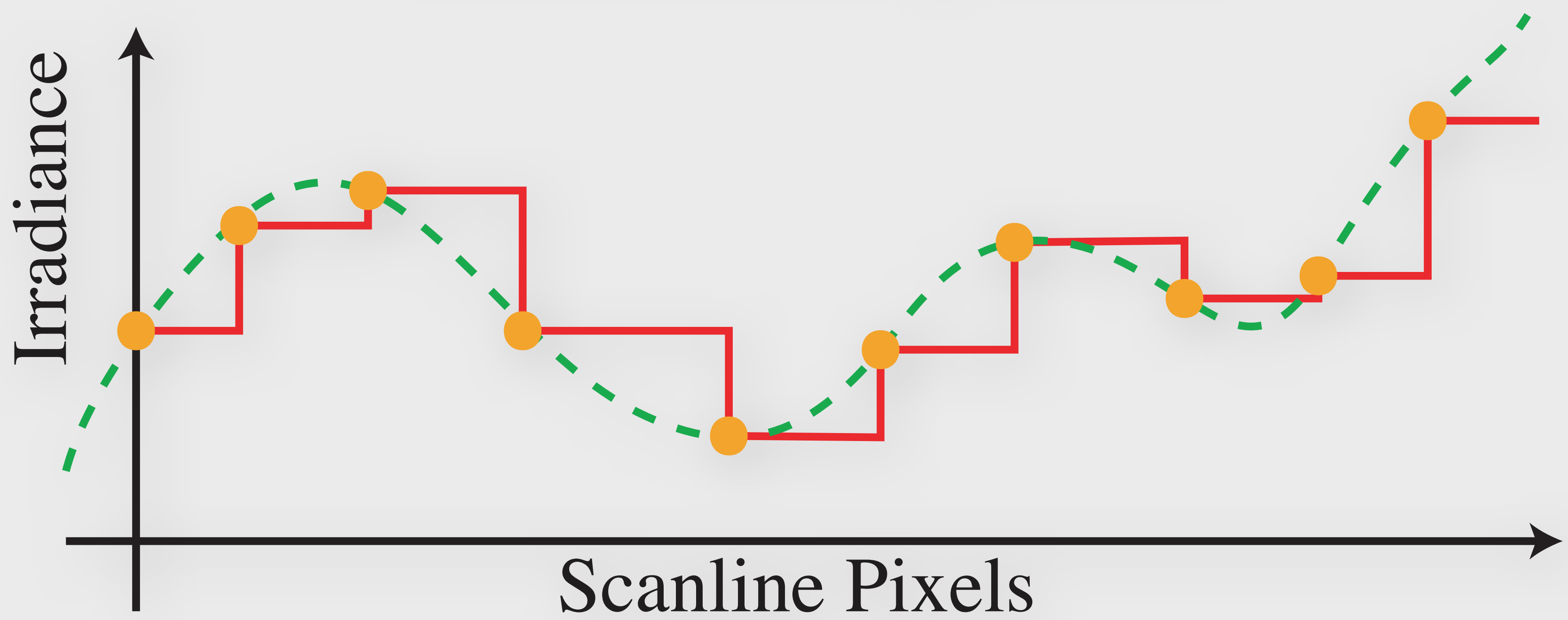
- Improve interpolation/extrapolation quality using gradients
- Irradiance Gradients [Ward and Heckbert 1992]
  - Estimate an actual derivative to the irradiance
  - Apply this derivative to the weighted average

[Ward and Heckbert 1992]

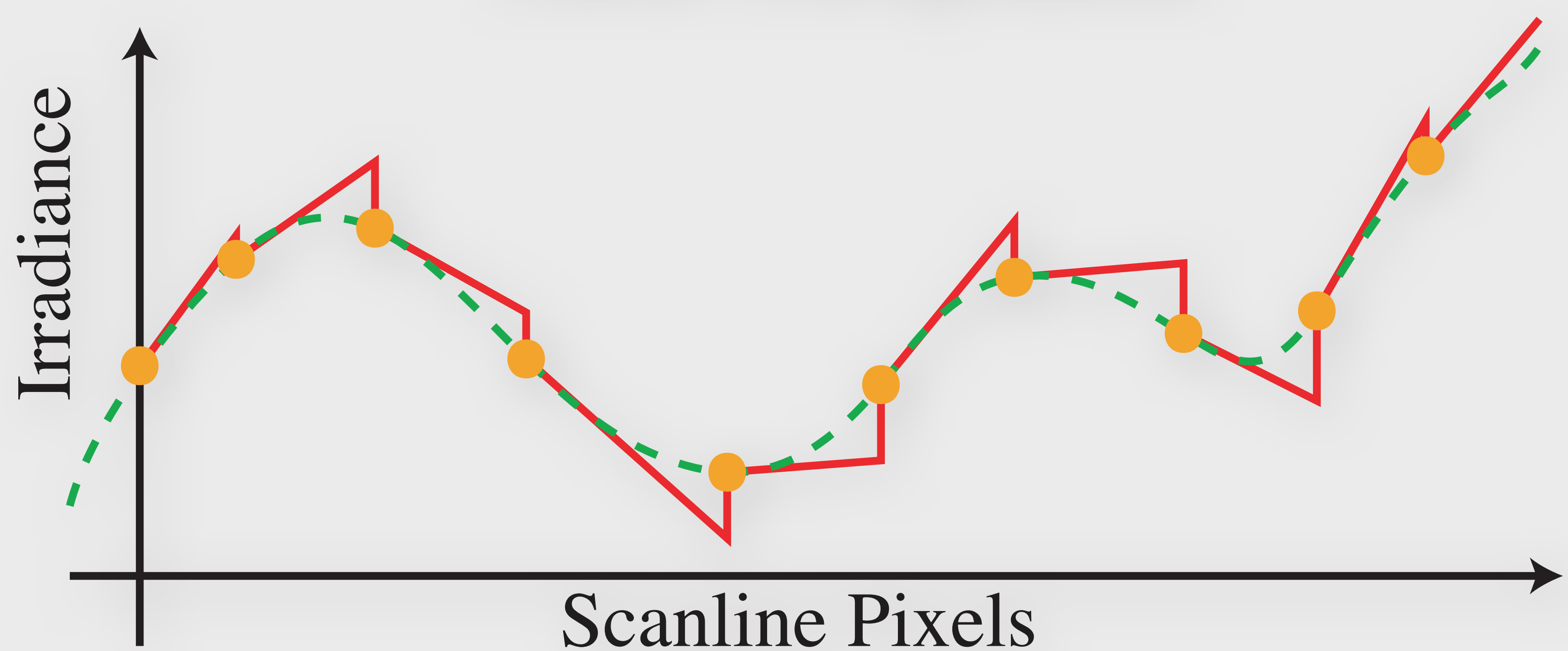
Constant Extrapolation



Constant Extrapolation

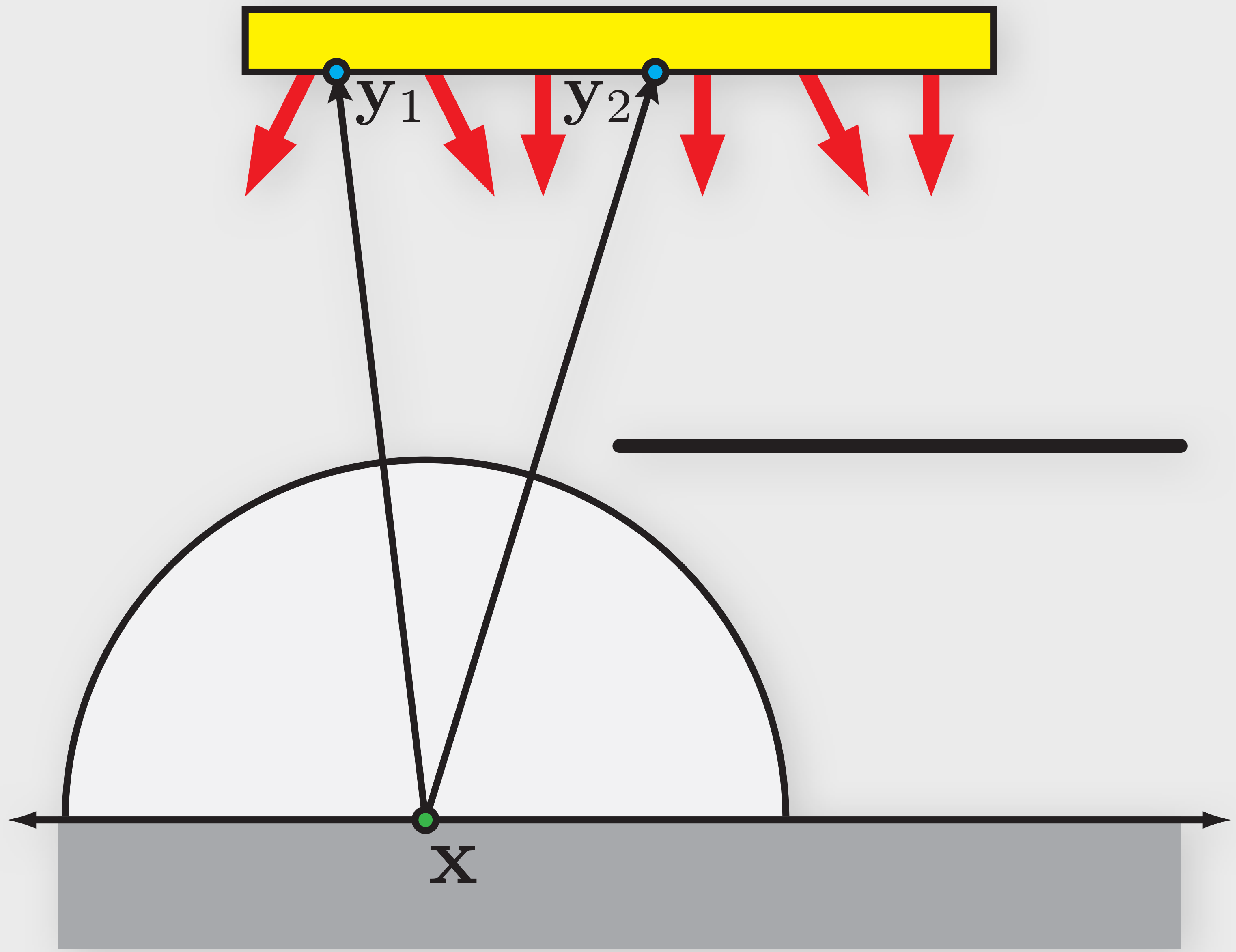


Linear Extrapolation



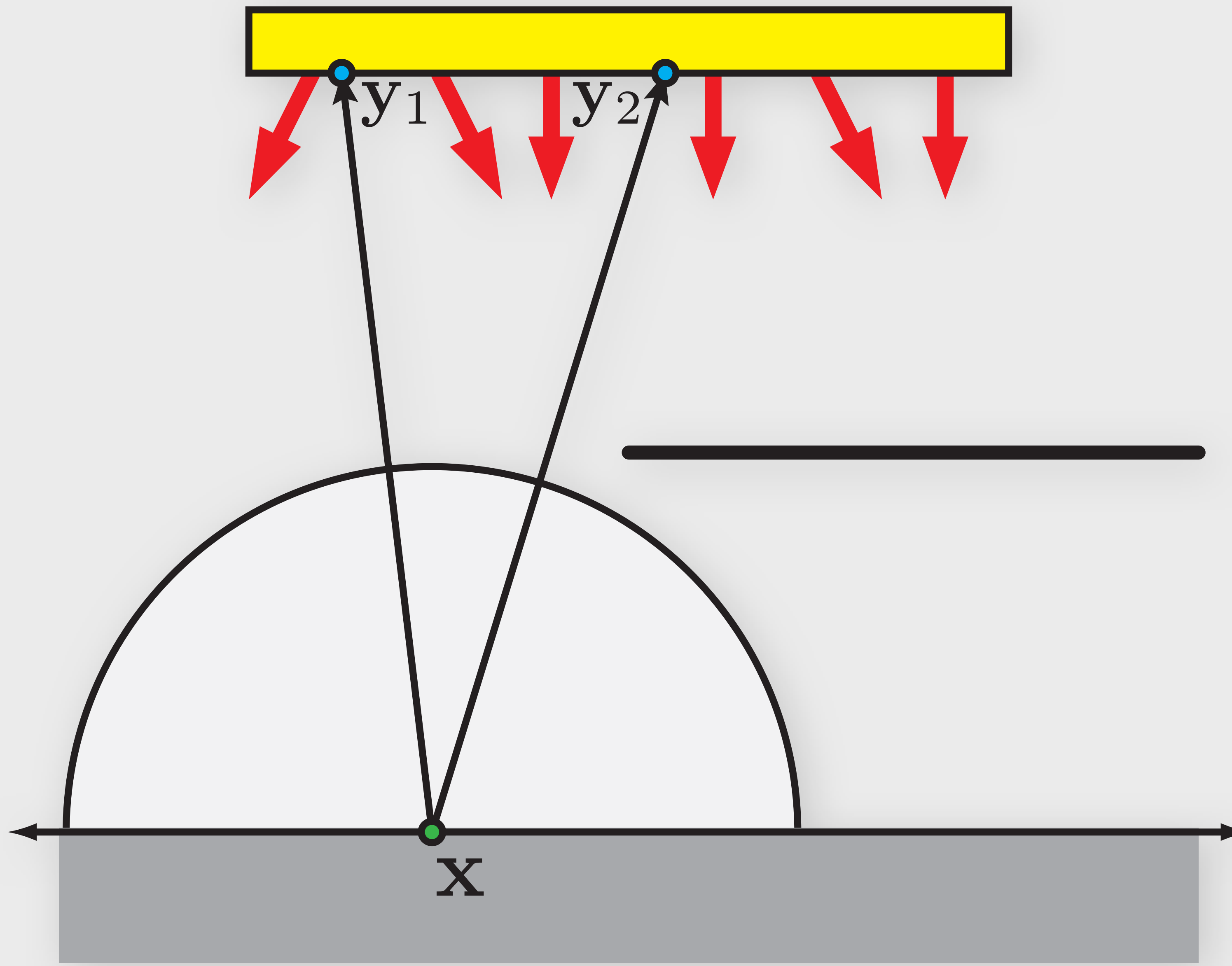
actual irradiance   extrapolated irradiance   irradiance cache point

Gradients (surface-area formulation)



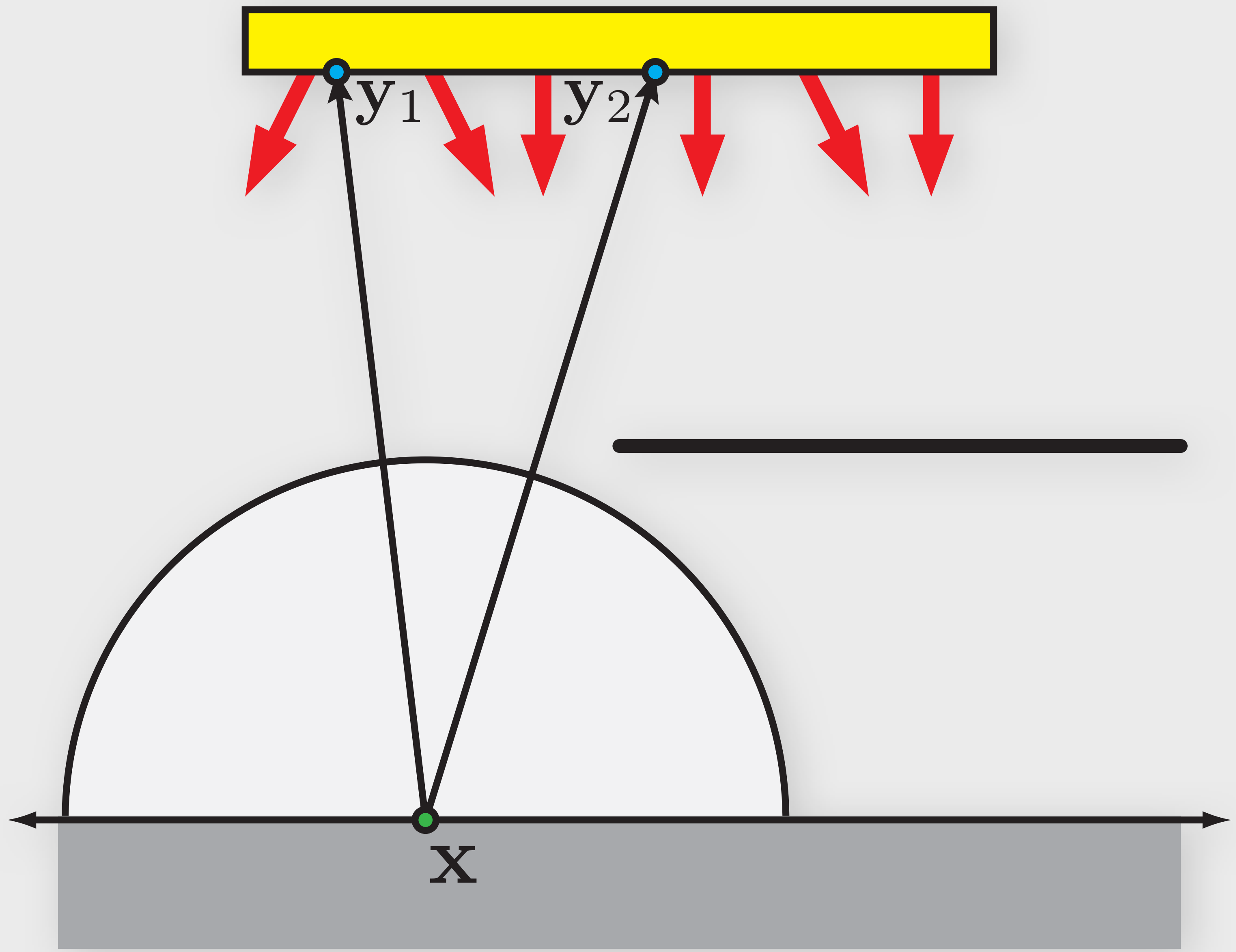
$$\nabla E(\mathbf{x}) = \nabla \int_A L(\mathbf{x} \leftarrow \mathbf{y})\, V(\mathbf{x}, \mathbf{y})\, G(\mathbf{x}, \mathbf{y})\; \mathrm{d}\mathbf{y}$$

Gradients (surface-area formulation)



$$\nabla E(\mathbf{x}) = \int_A \cancel{\boxed{\nabla L\, V\, G}} + \cancel{\boxed{L\, \nabla V\, G}} + \boxed{L\, V\, \nabla G}\, \mathrm{d}\mathbf{y}$$

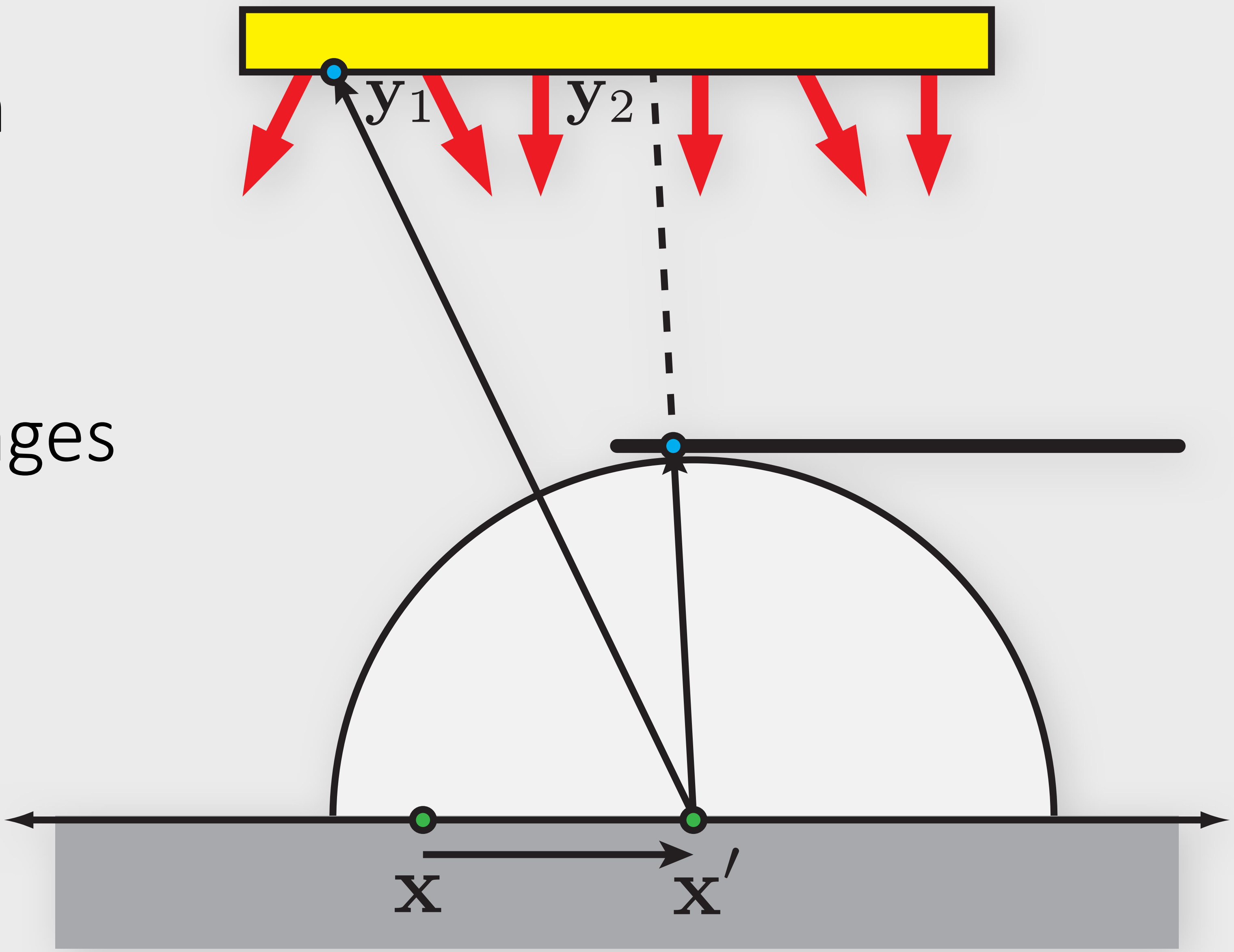
Gradients (surface-area formulation)



$$\nabla E(\mathbf{x}) \approx \int_A \boxed{L(\mathbf{x} \leftarrow \mathbf{y})\, V(\mathbf{x}, \mathbf{y})\, \nabla G(\mathbf{x}, \mathbf{y})}\, \mathrm{d}\mathbf{y}$$

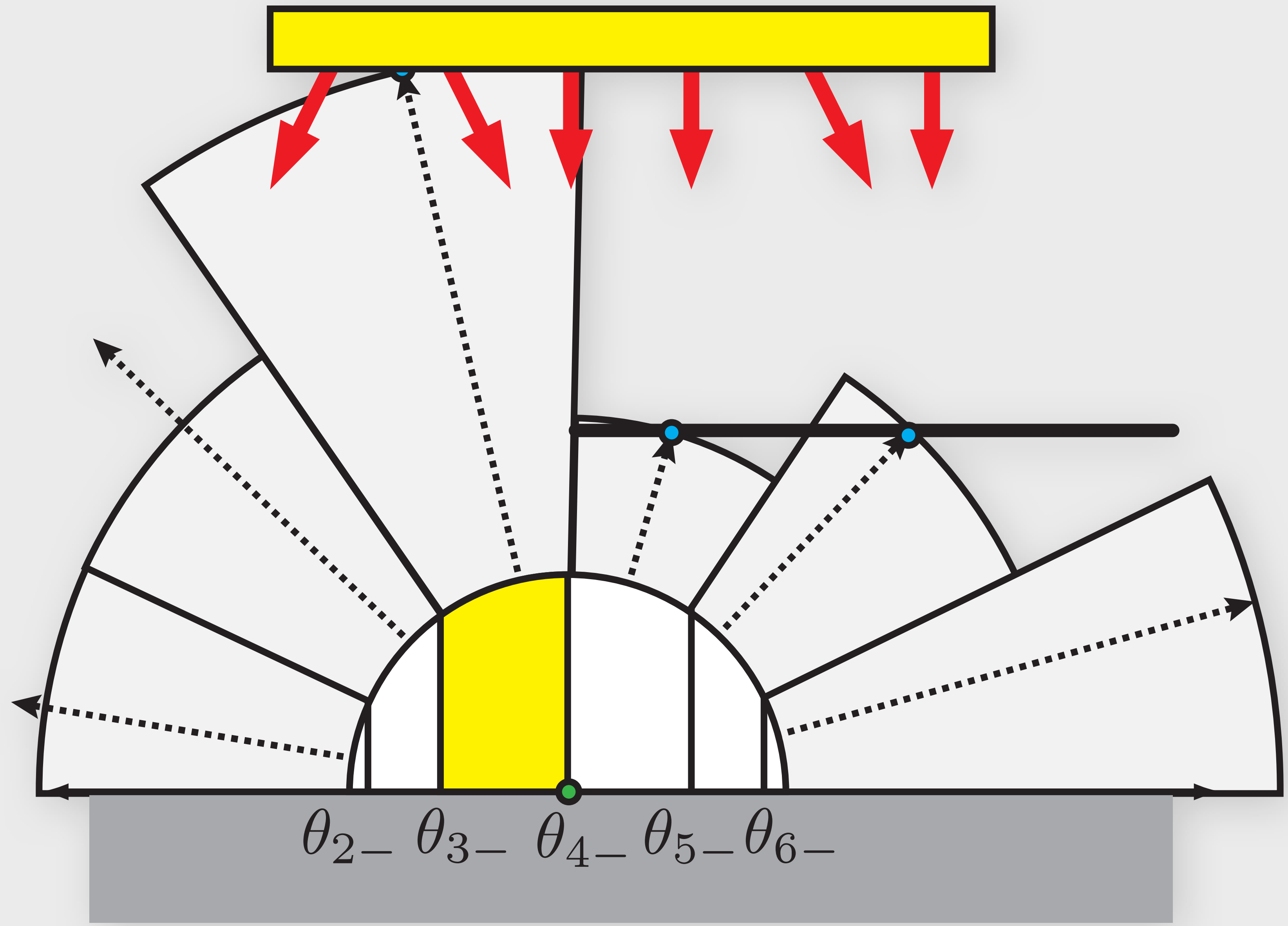
Gradients (surface-area formulation)

- Accounts for change in geometric relationship between x & y
- Ignores occlusion changes



$$\nabla E(\mathbf{x}) \approx \int_A \boxed{L(\mathbf{x}\leftarrow\mathbf{y})\, V(\mathbf{x},\mathbf{y})\, \nabla G(\mathbf{x},\mathbf{y})}\, \mathrm{d}\mathbf{y}$$

Gradients (stratified formulation)

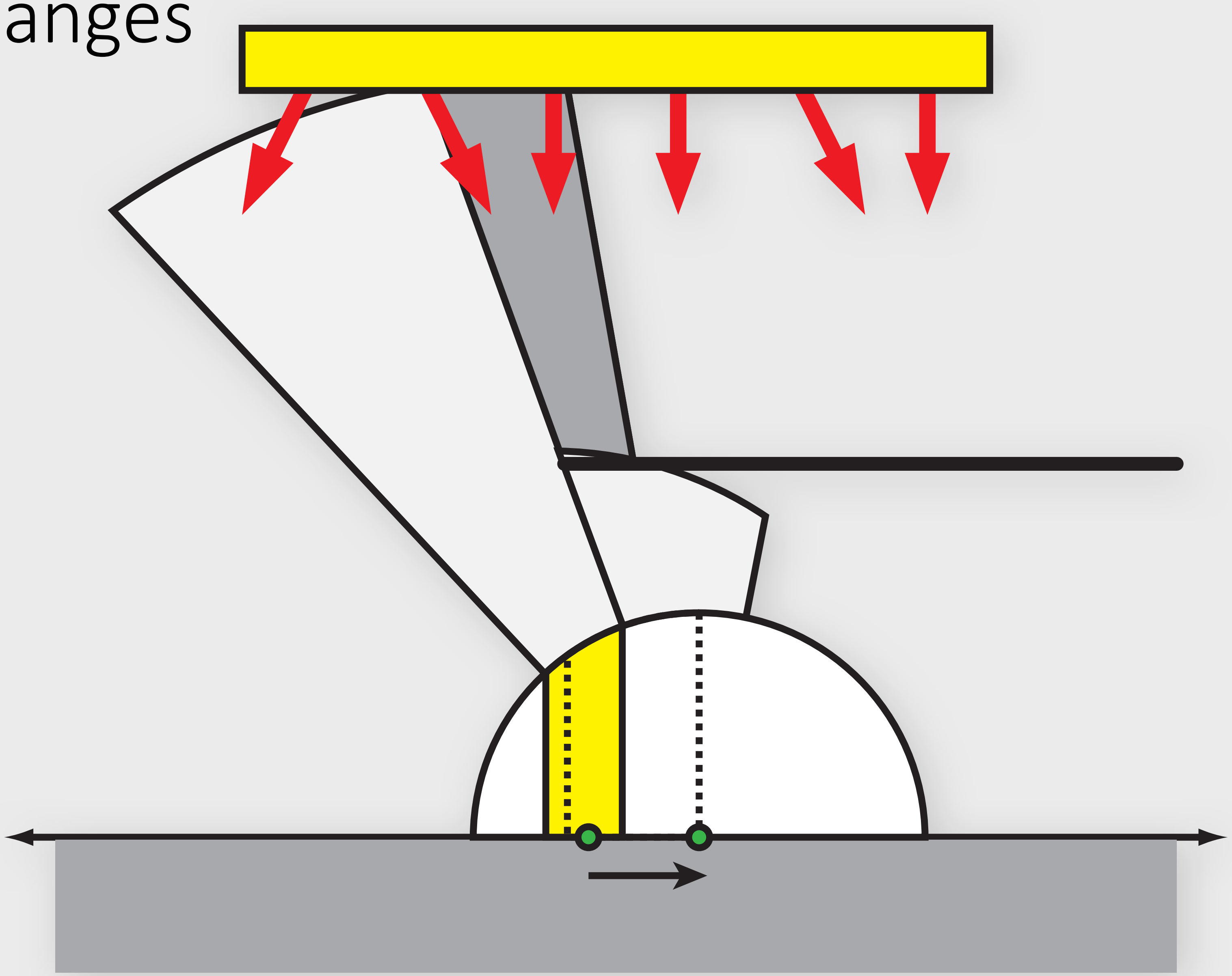


[Ward and Heckbert 1992]

Gradients (stratified formulation)

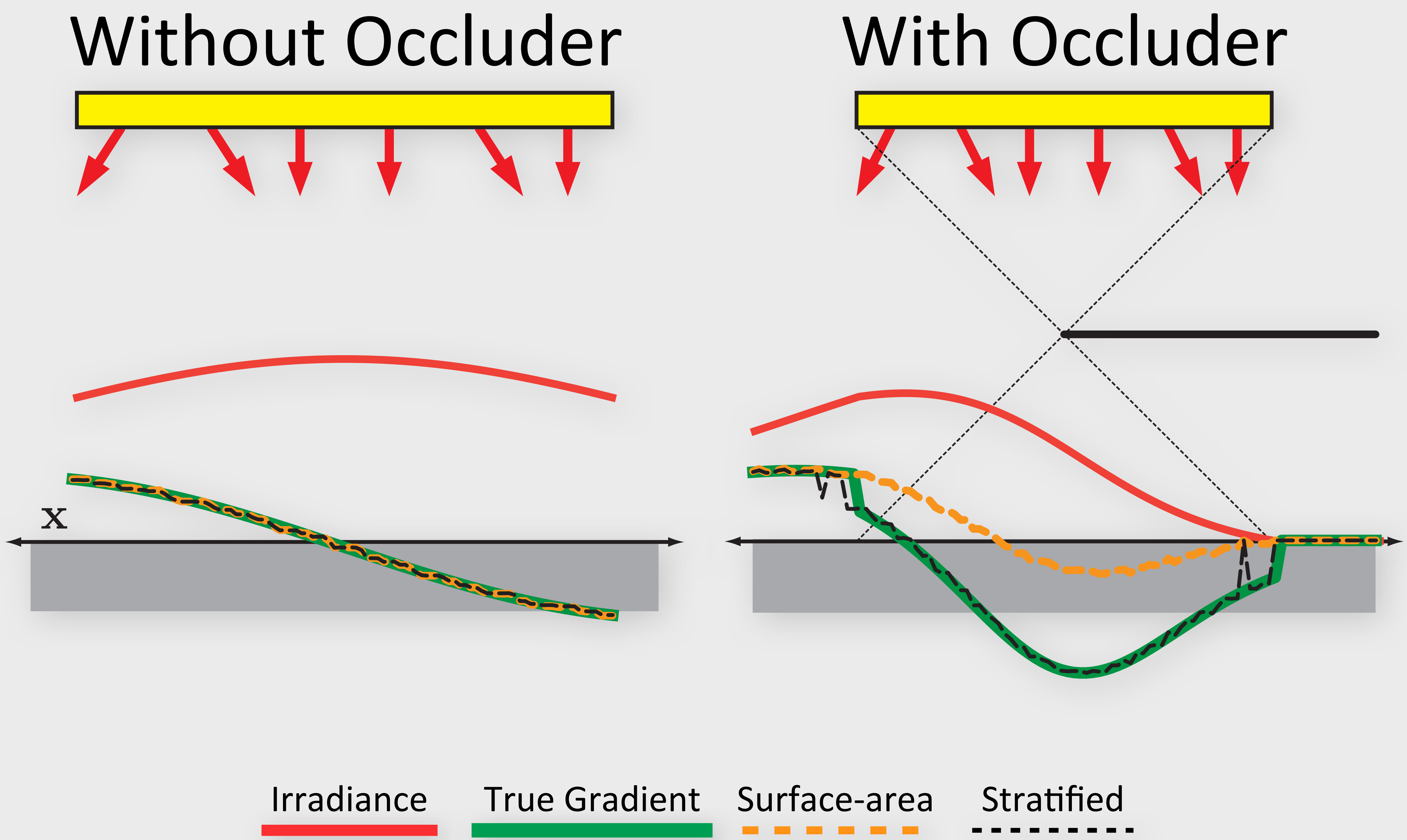
- Considers occlusion changes

Very Important!



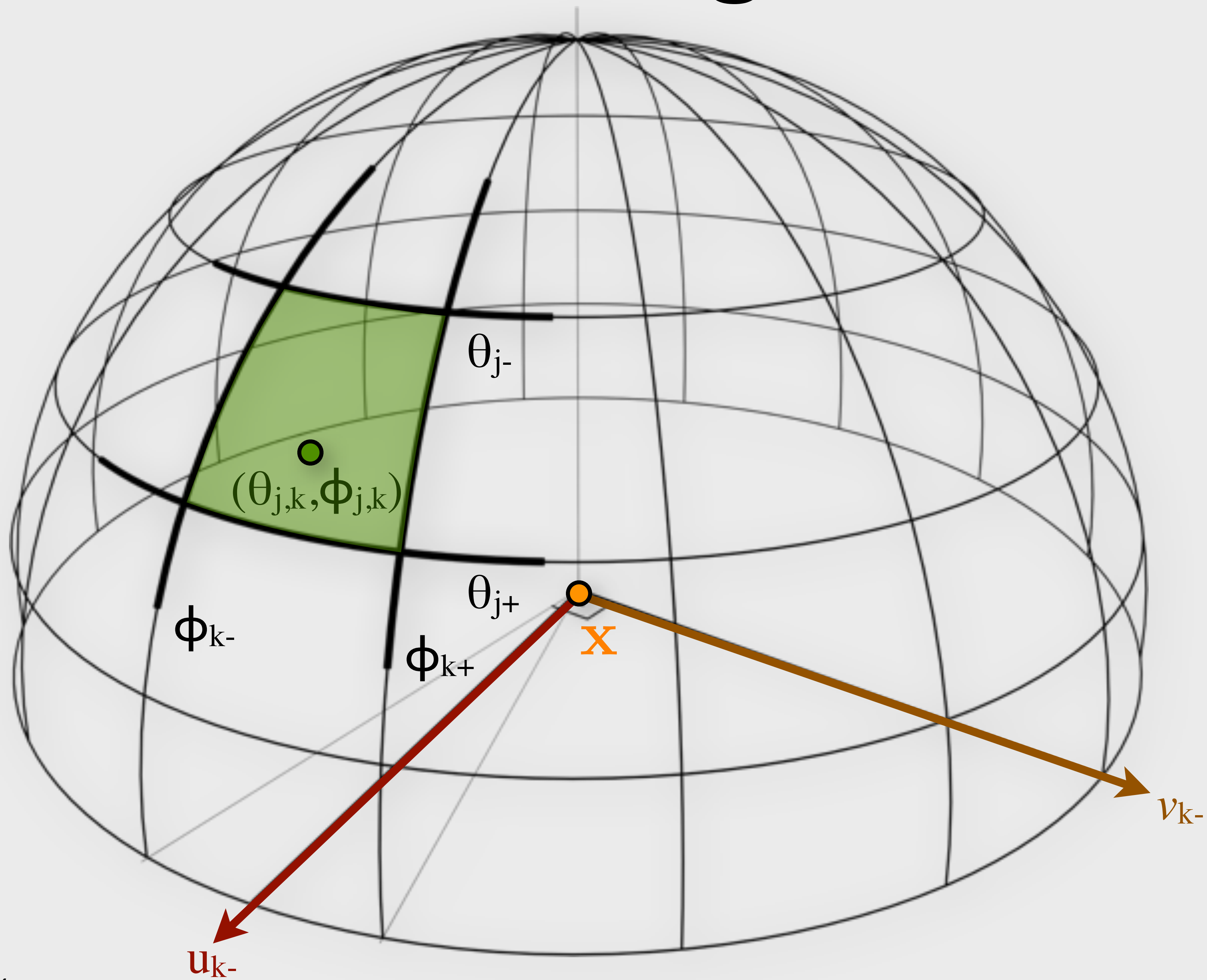
[Ward and Heckbert 1992]

Irradiance gradients comparison



[Jarosz et al. 2012]

Stratified irradiance gradient



$$\nabla_t E(\mathbf{x}) = \sum_{k=1}^{N_1} \left(\hat{u}_k \sum_{j=2}^{N_2} \nabla_{\hat{u}_k} A_{j_-,k} (L_{j,k} - L_{j\text{-}1,k}) \cos\theta_{j_-} + \hat{v}_{k_-} \sum_{j=1}^{N_2} \nabla_{\hat{v}_{k_-}} A_{j,k_-} (L_{j,k} - L_{j,k\text{-}1}) \cos\theta_j \right)$$

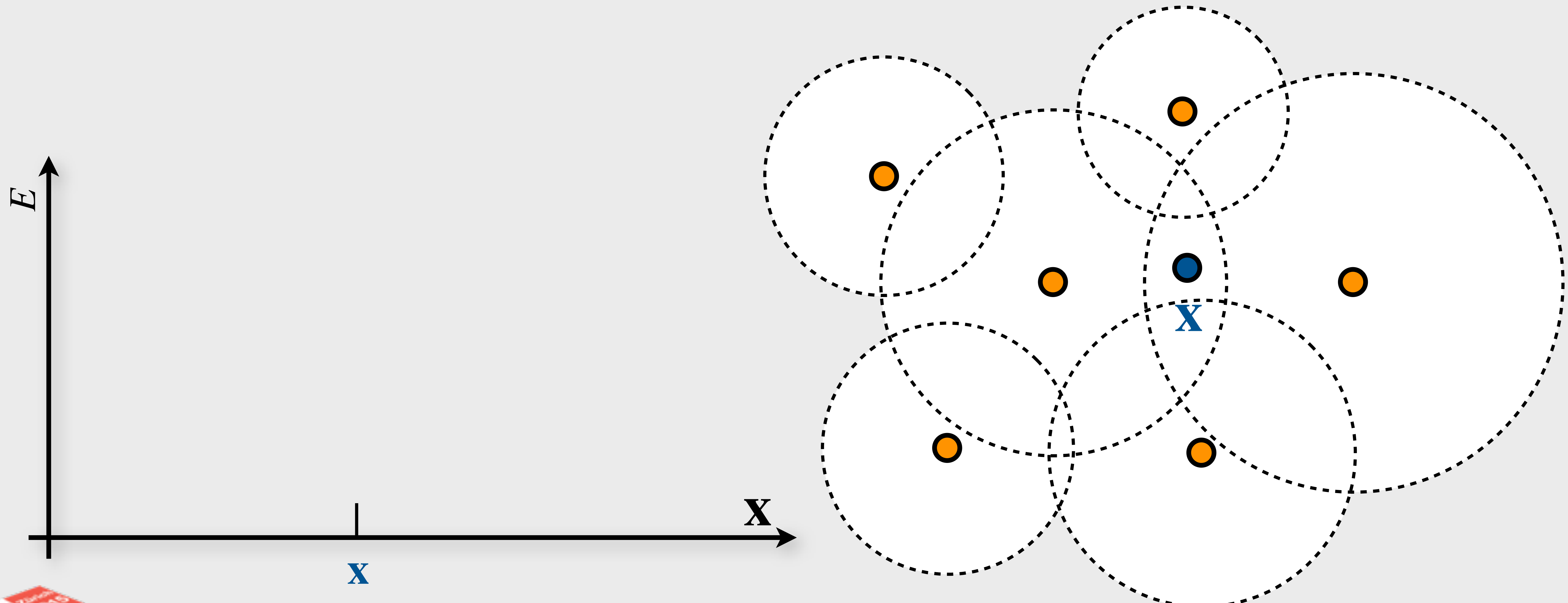
[Ward and Heckbert 1992]

Interpolating with gradients

$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\left(E_i + (\vec{n}_i \times \vec{\mathbf{n}}) \cdot (\vec{\nabla}_r E_i) + (\mathbf{x} - \mathbf{x}_i) \cdot (\vec{\nabla}_t E_i)\right)}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

Interpolating with gradients

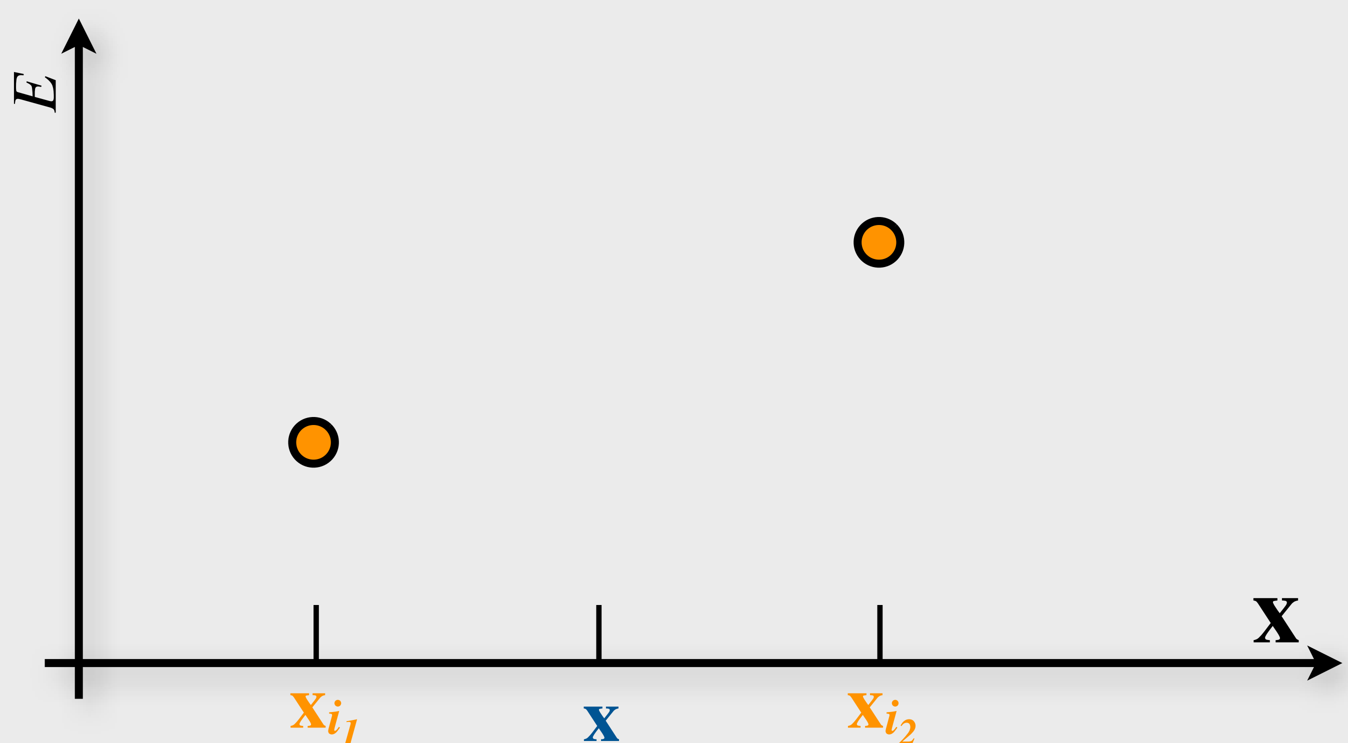
$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}}) \big(E_i + (\vec{n}_i \times \vec{\mathbf{n}}) \cdot (\vec{\nabla}_r E_i) + (\mathbf{x} - \mathbf{x}_i) \cdot (\vec{\nabla}_t E_i) \big)}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})}$$

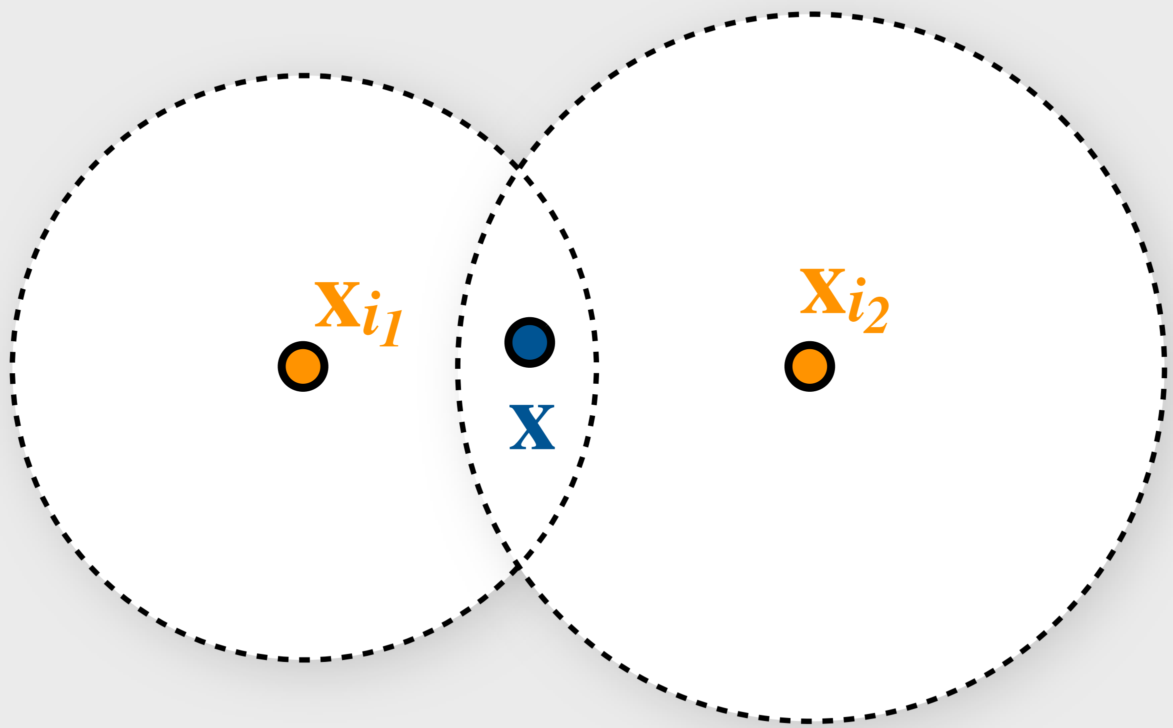


Interpolating with gradients

$$E(\mathbf{x},\vec{\mathbf{n}}) \approx \frac{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\Big(E_i+(\vec{n}_i\times\vec{\mathbf{n}})\cdot(\vec{\nabla}_r E_i)+(\mathbf{x}-\mathbf{x}_i)\cdot(\vec{\nabla}_t E_i)\Big)}{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

Find overlapping cache records

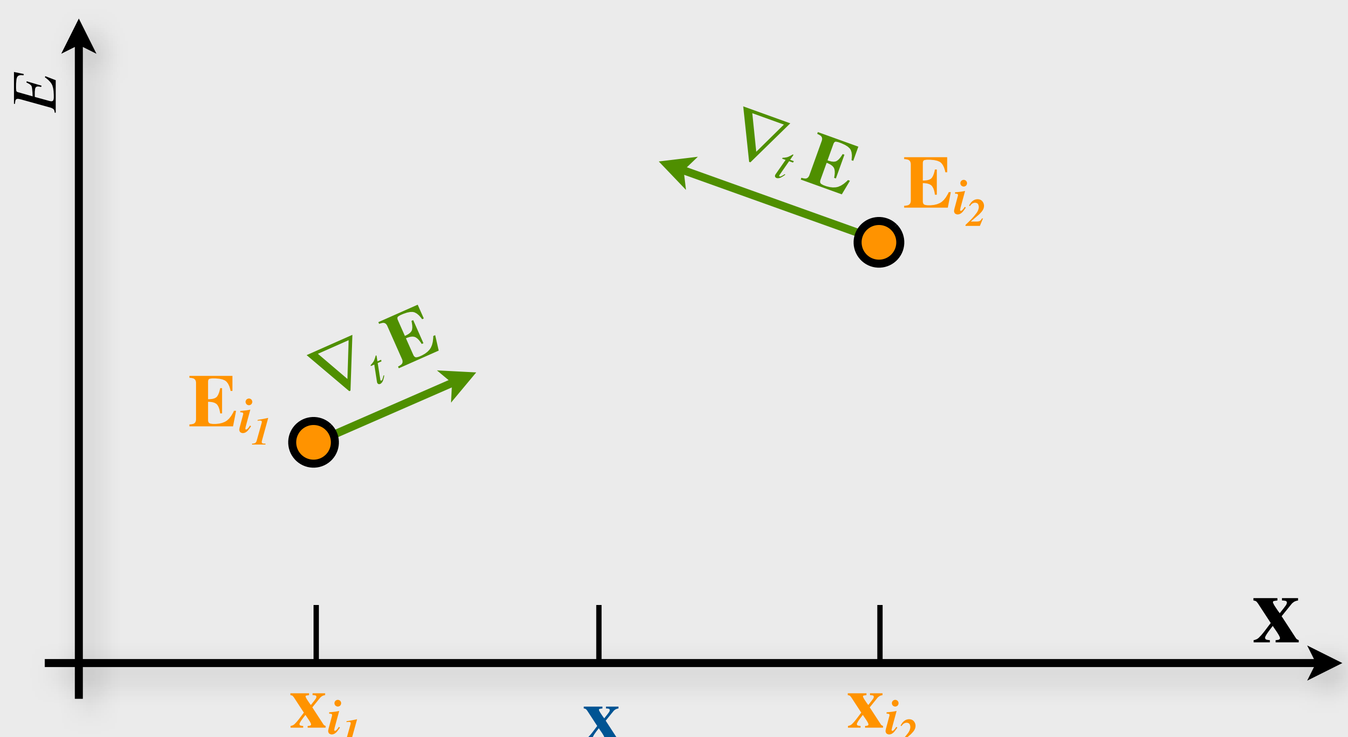


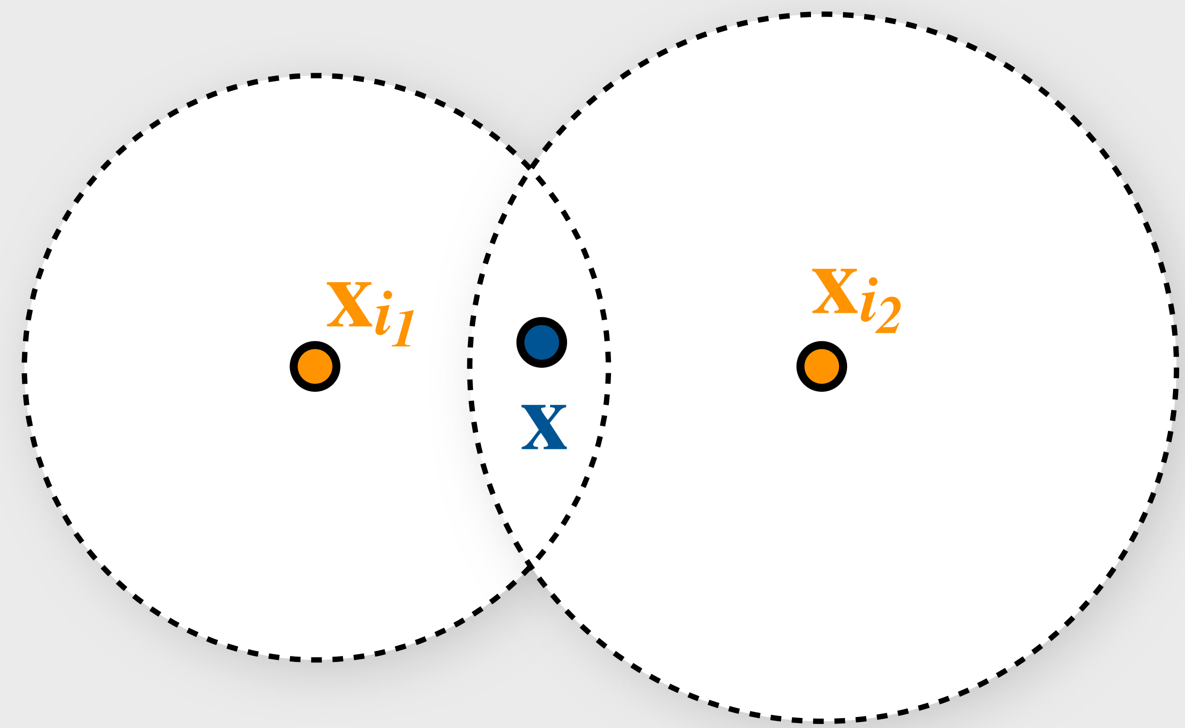


Interpolating with gradients

$$E(\mathbf{x},\vec{\mathbf{n}}) \approx \frac{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\left(\boxed{E_i+(\vec{n}_i\times\vec{\mathbf{n}})\cdot(\vec{\nabla}_r E_i)+(\mathbf{x}-\mathbf{x}_i)\cdot(\vec{\nabla}_t E_i)}\right)}{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

Extrapolate along gradients

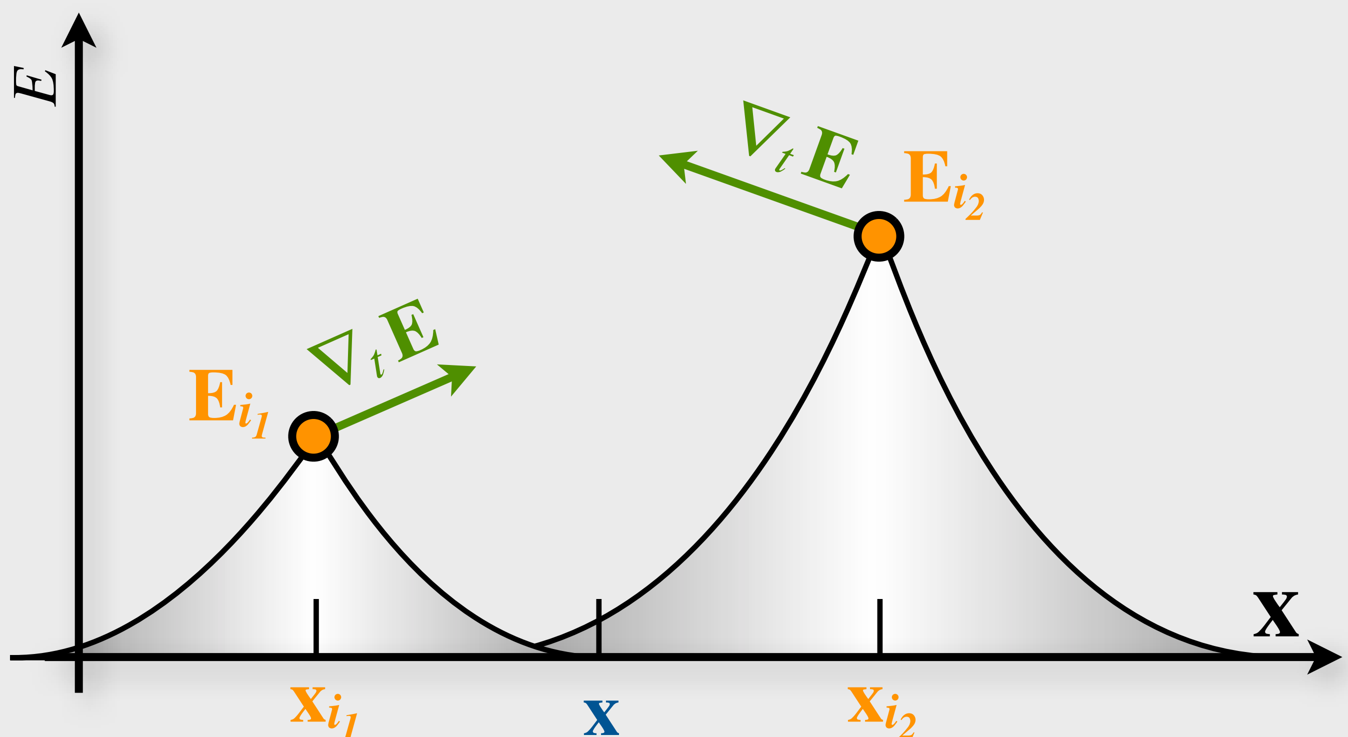


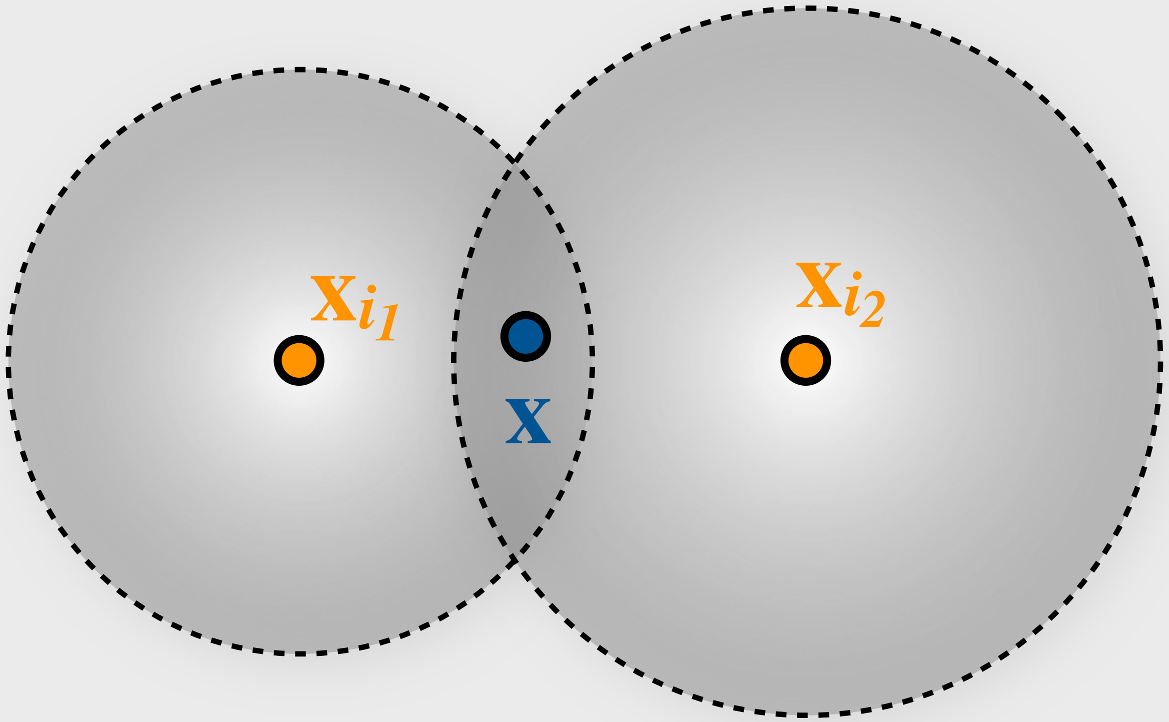


Interpolating with gradients

$$E(\mathbf{x},\vec{\mathbf{n}}) \approx \frac{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\left(E_i+(\vec{n}_i\times\vec{\mathbf{n}})\cdot(\vec{\nabla}_r E_i)+(\mathbf{x}-\mathbf{x}_i)\cdot(\vec{\nabla}_t E_i)\right)}{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

Weight contributions

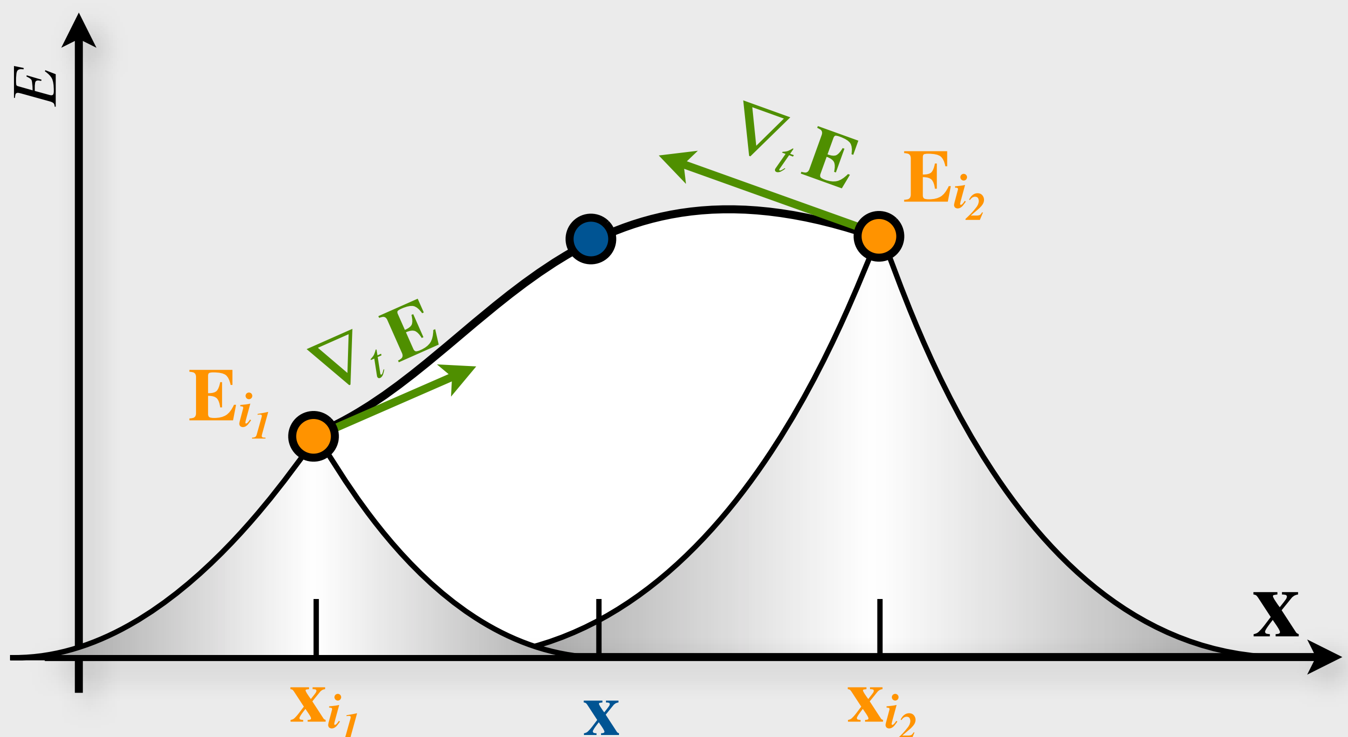


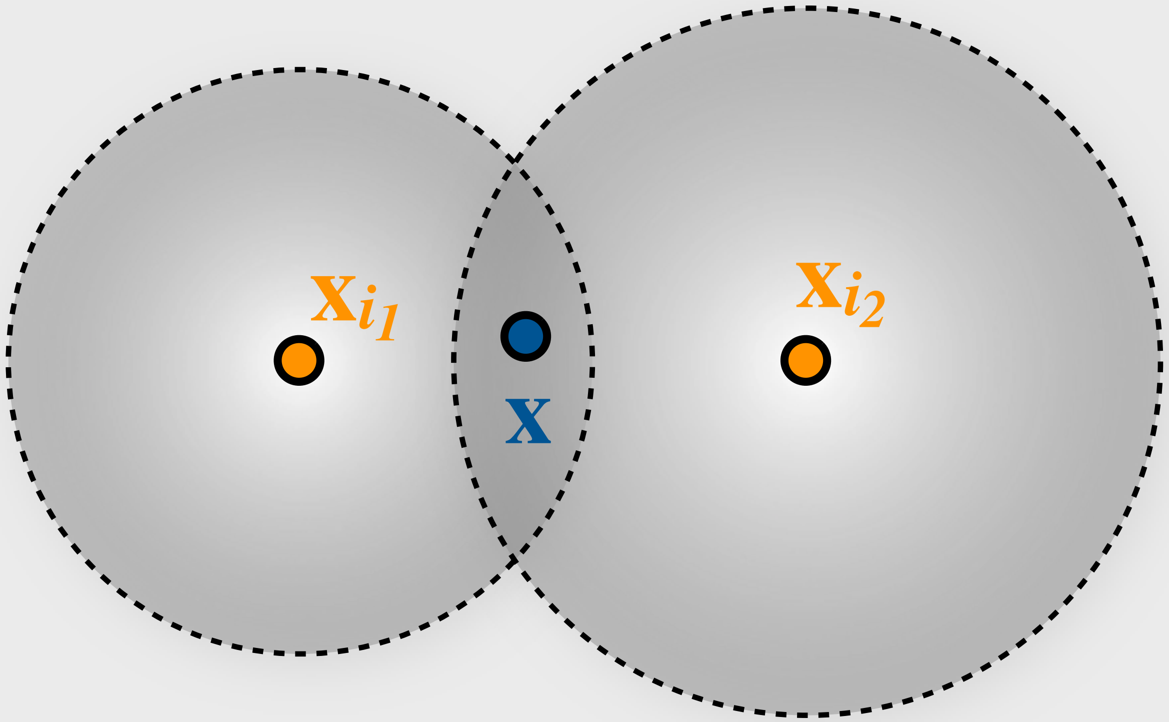


Interpolating with gradients

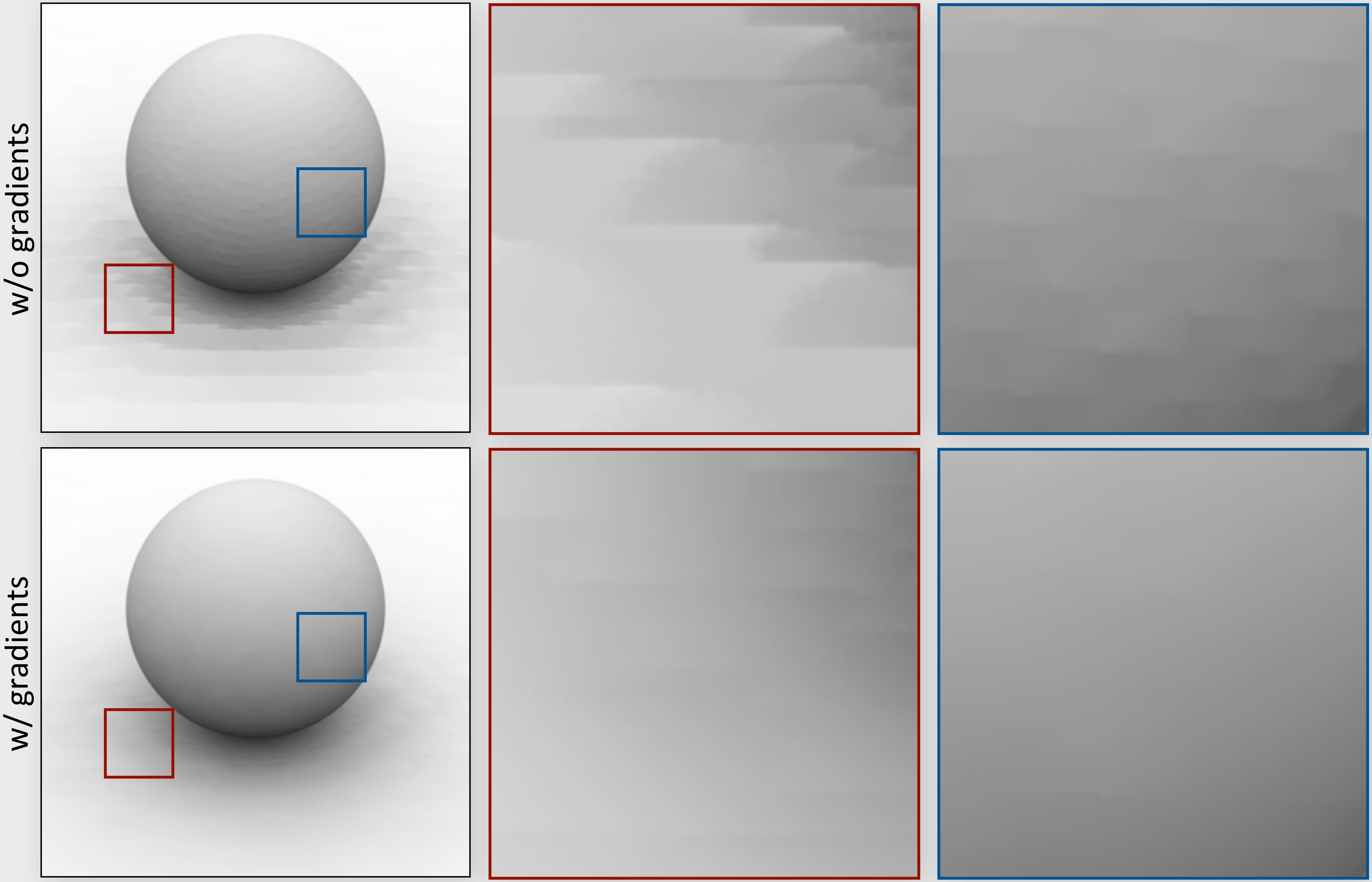
$$E(\mathbf{x},\vec{\mathbf{n}}) \approx \frac{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\Big(E_i+(\vec{n}_i\times\vec{\mathbf{n}})\cdot(\vec{\nabla}_r E_i)+(\mathbf{x}-\mathbf{x}_i)\cdot(\vec{\nabla}_t E_i)\Big)}{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

Sum extrapolated values

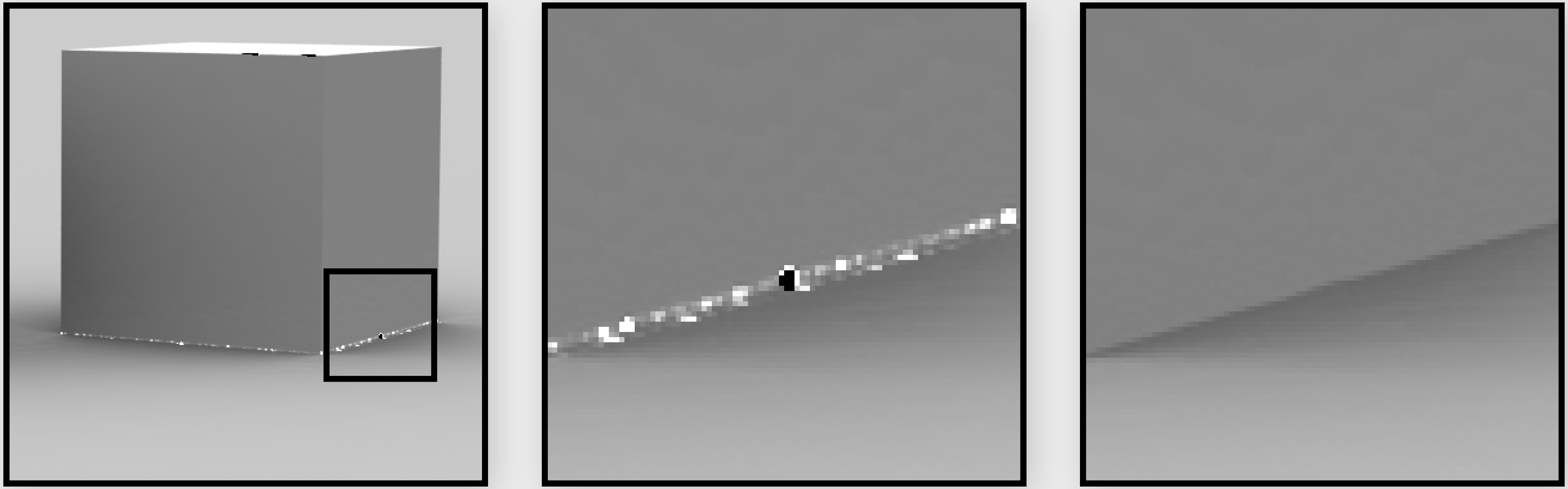




Irradiance Gradients



Practical issue: over-shooting



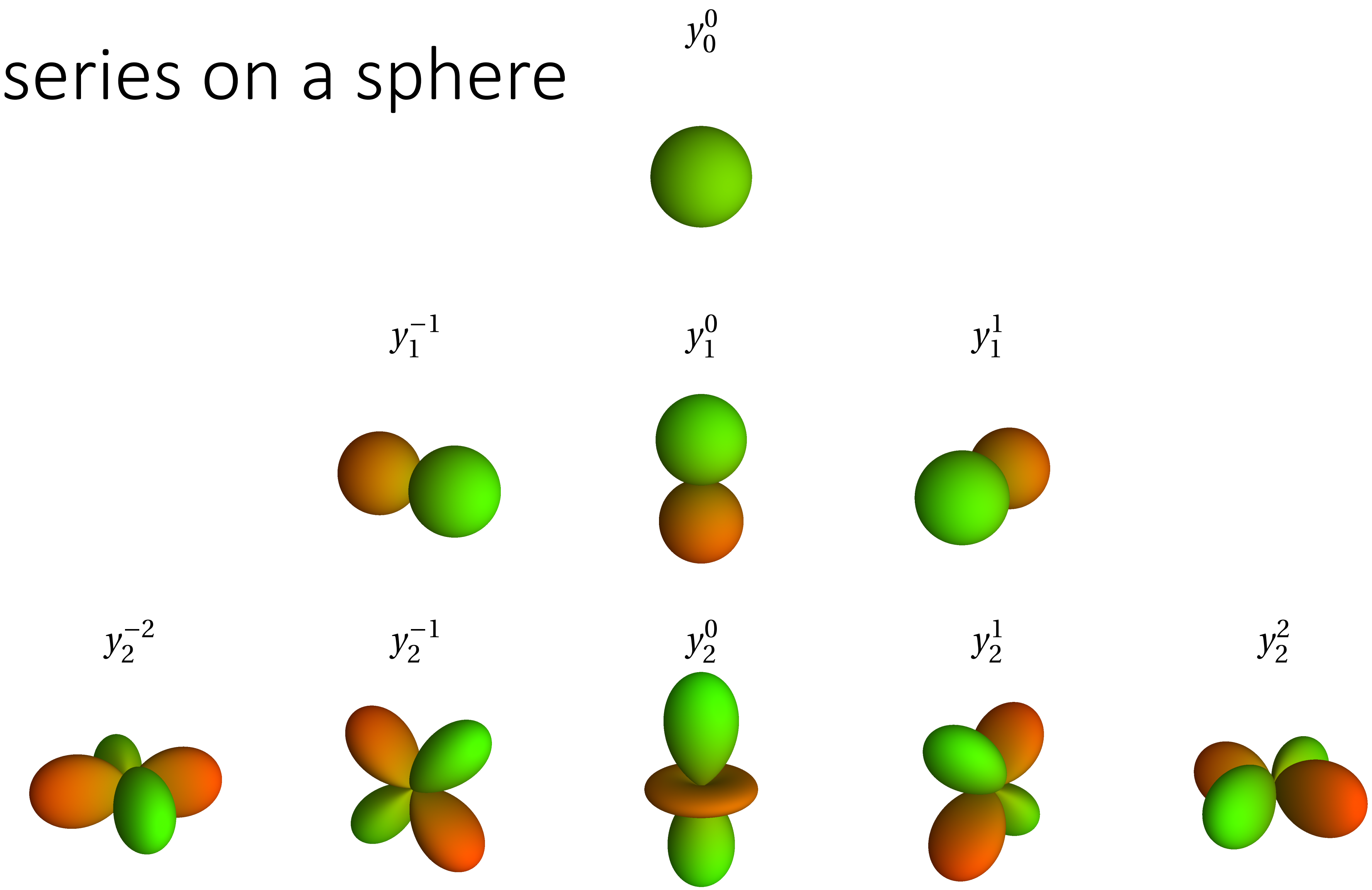
Beyond Lambertian surfaces

- Generalization to glossy surfaces
- **Radiance Caching** [Křivánek et al. 2005a,2005b]
  - Can no longer cache just the irradiance value
  - Cache full hemispherical *radiance* field at sparse locations

[Křivánek et al. 2005a,2005b]

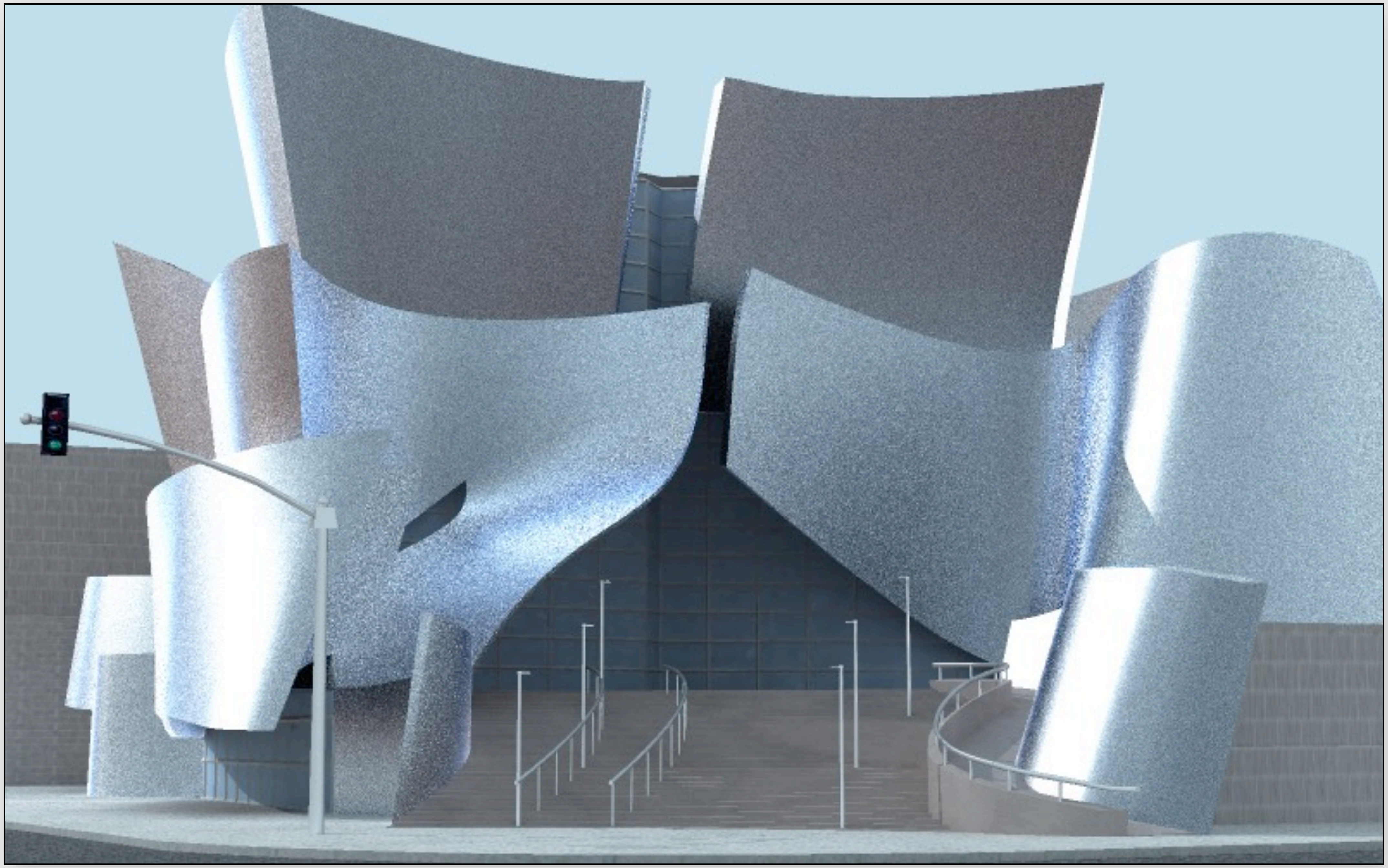
Radiance Storage

- Use spherical or hemispherical harmonics
- Approximates smooth functions with a few coefficients
- Fourier series on a sphere



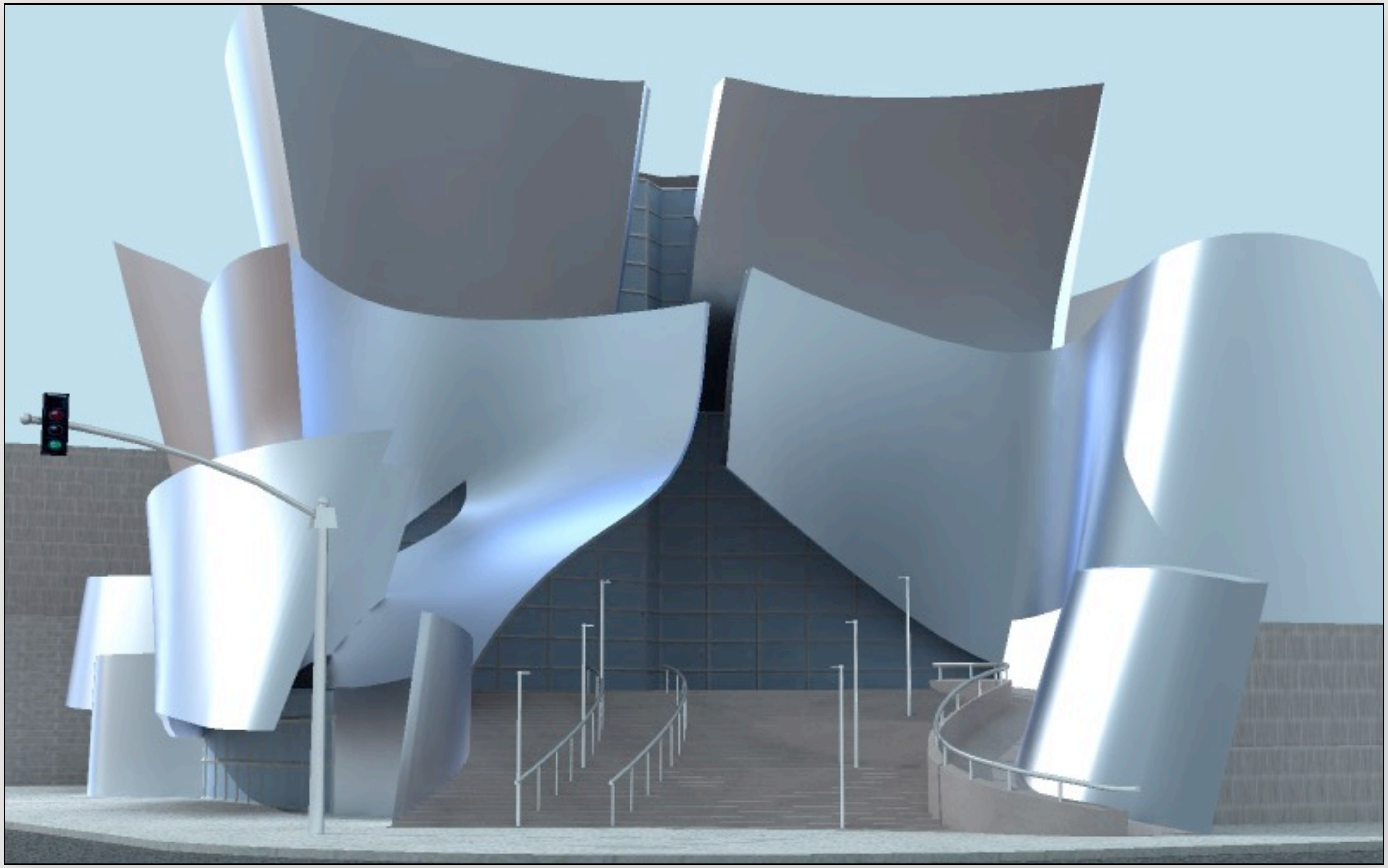
[Křivánek et al. 2005a,2005b]

Monte Carlo



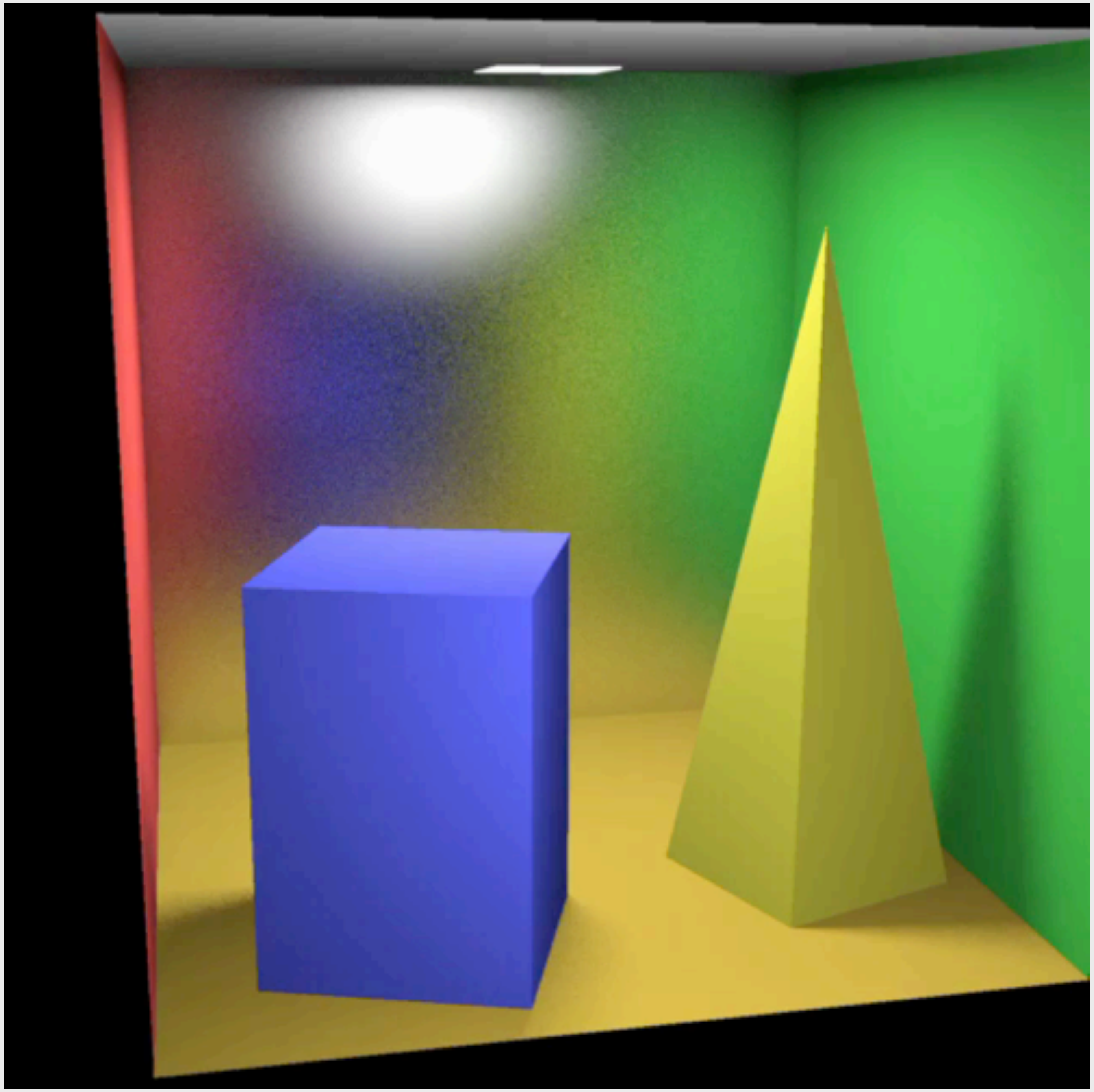
[Křivánek et al. 2005a,2005b]

Radiance Caching

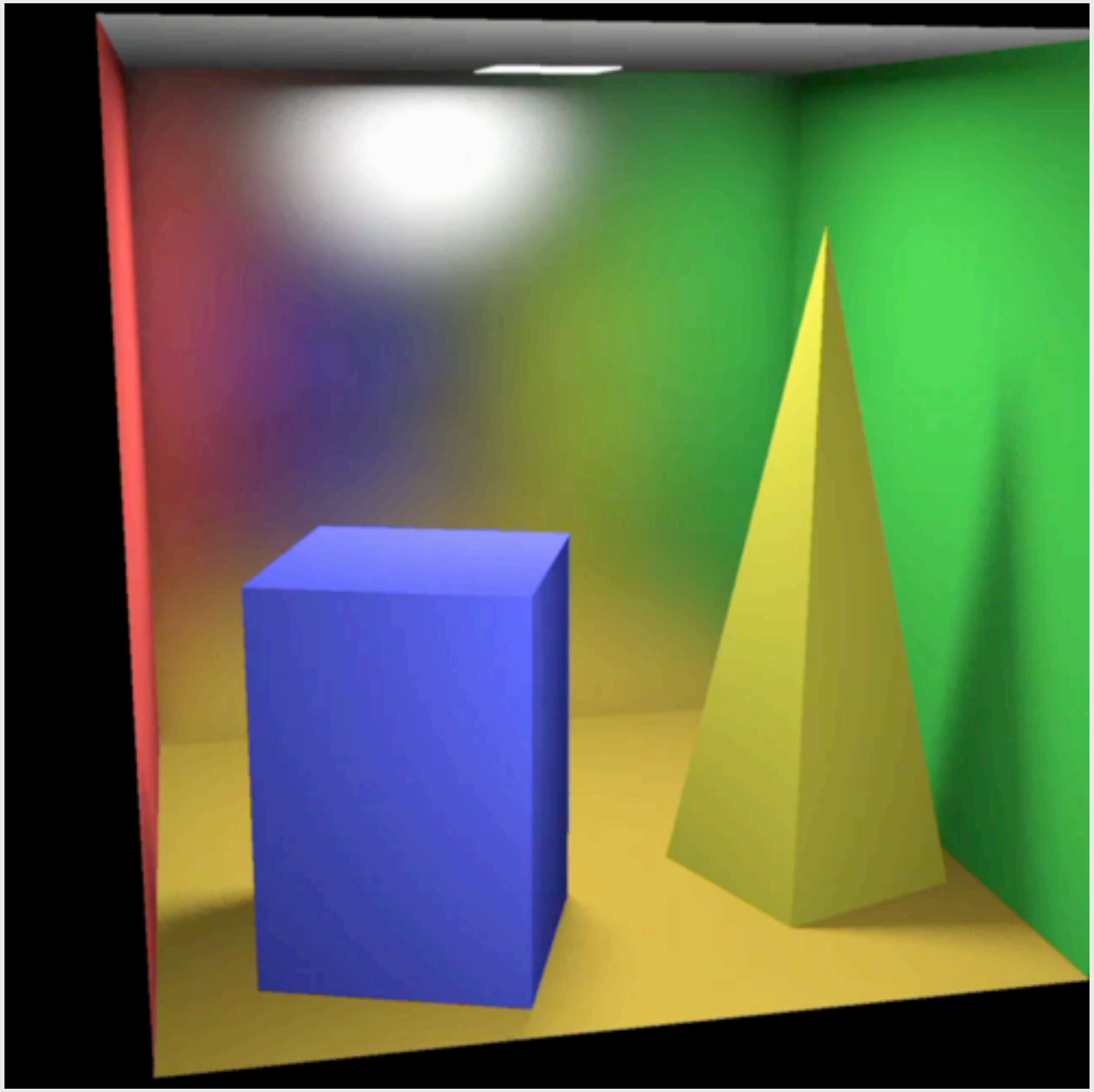


[Křivánek et al. 2005a,2005b]

Glossy Surfaces



Path Tracing



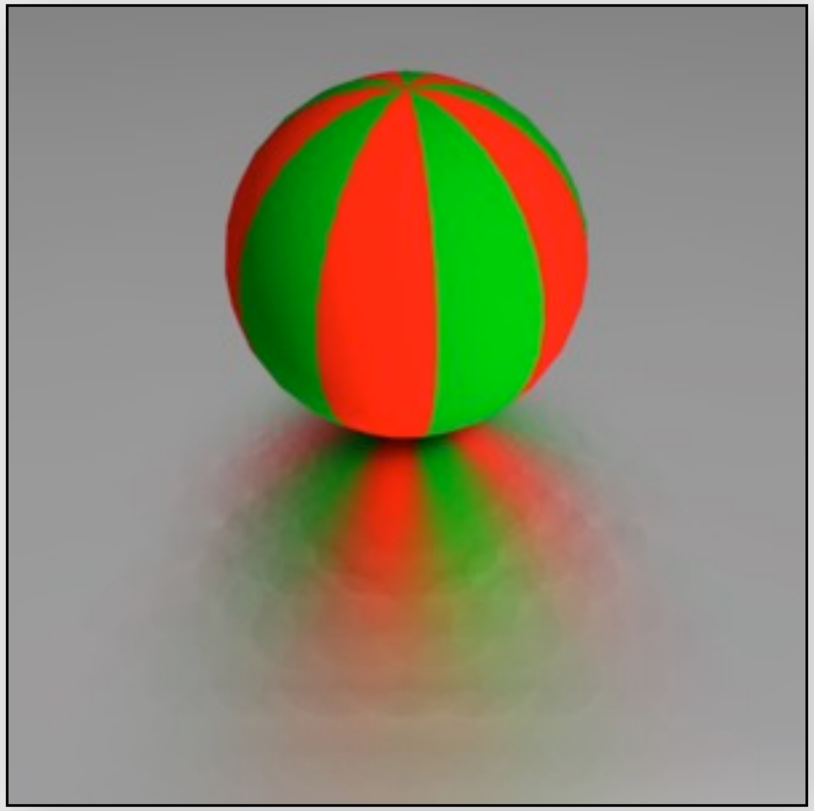
Radiance Caching

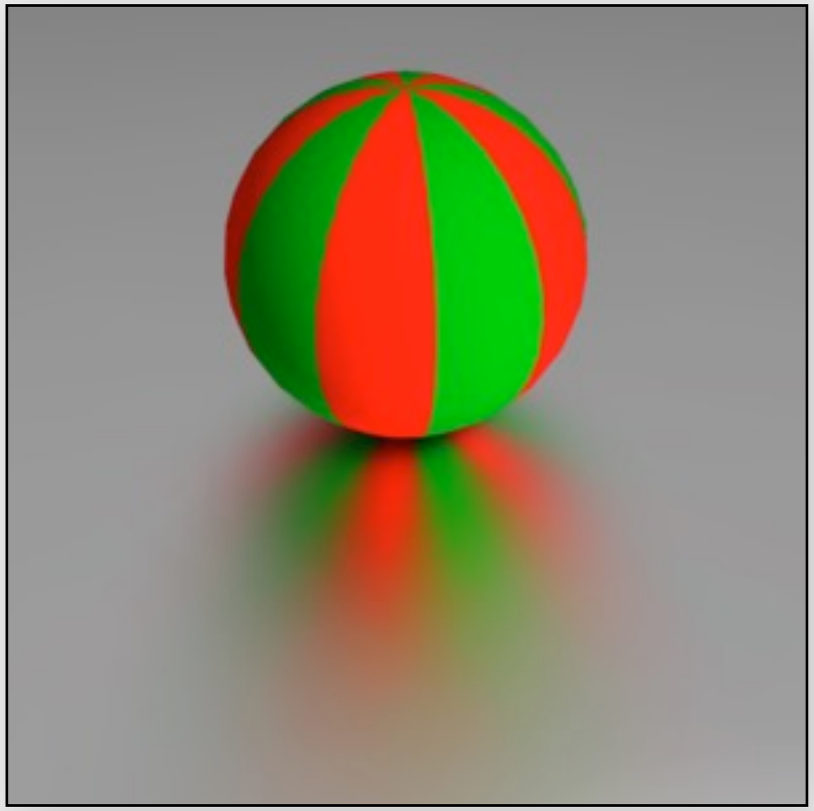
[Křivánek et al. 2005a,2005b]

Radiance Gradients

- Improve interpolation quality by storing gradient of incoming radiance field

[Křivánek et al. 2005a,2005b]





occlusion-unaware

occlusion-aware

[Křivánek et al. 2005a,2005b]

occlusion-unaware

occlusion-aware

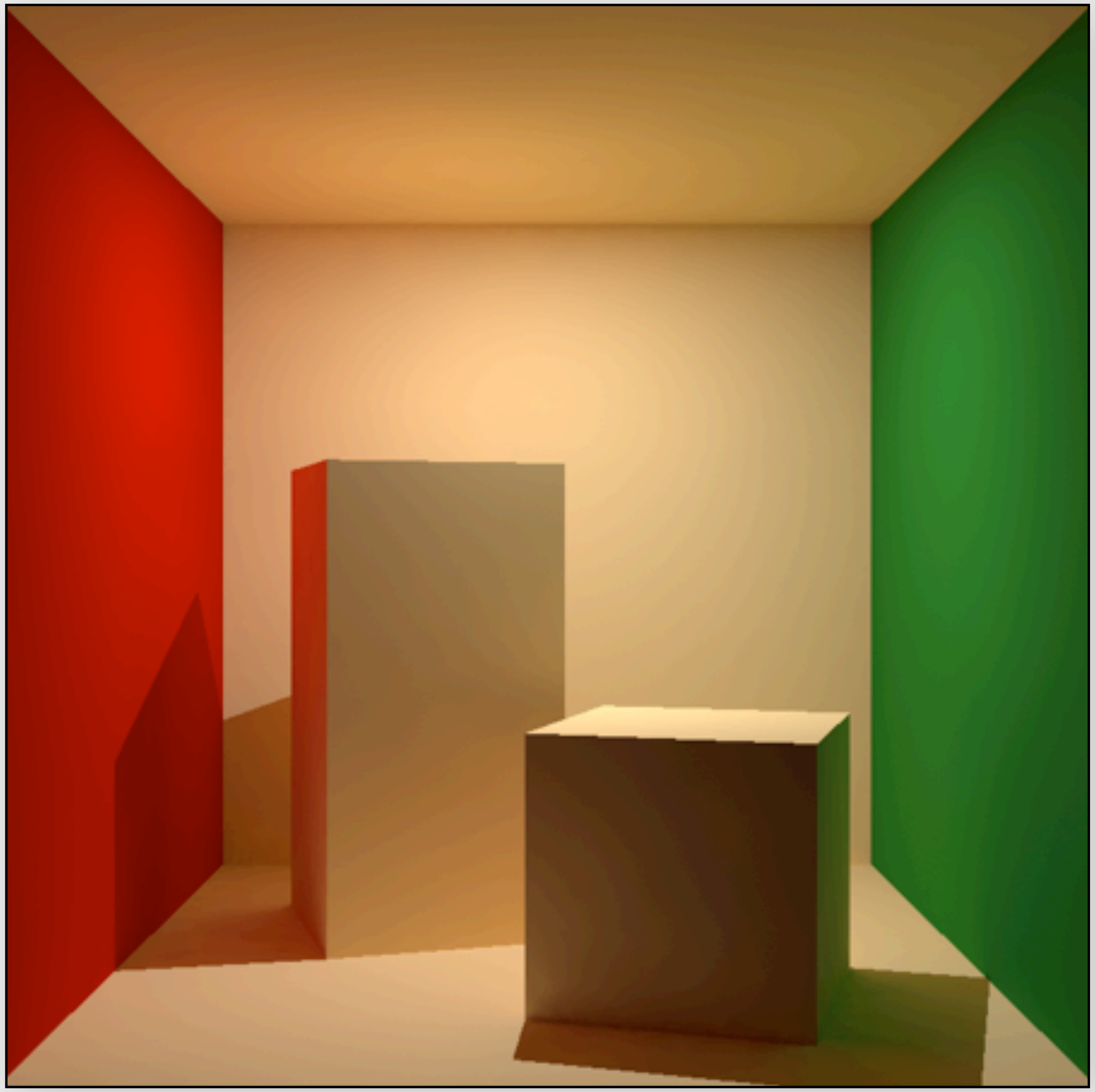
[Křivánek et al. 2005a,2005b]

Beyond surfaces

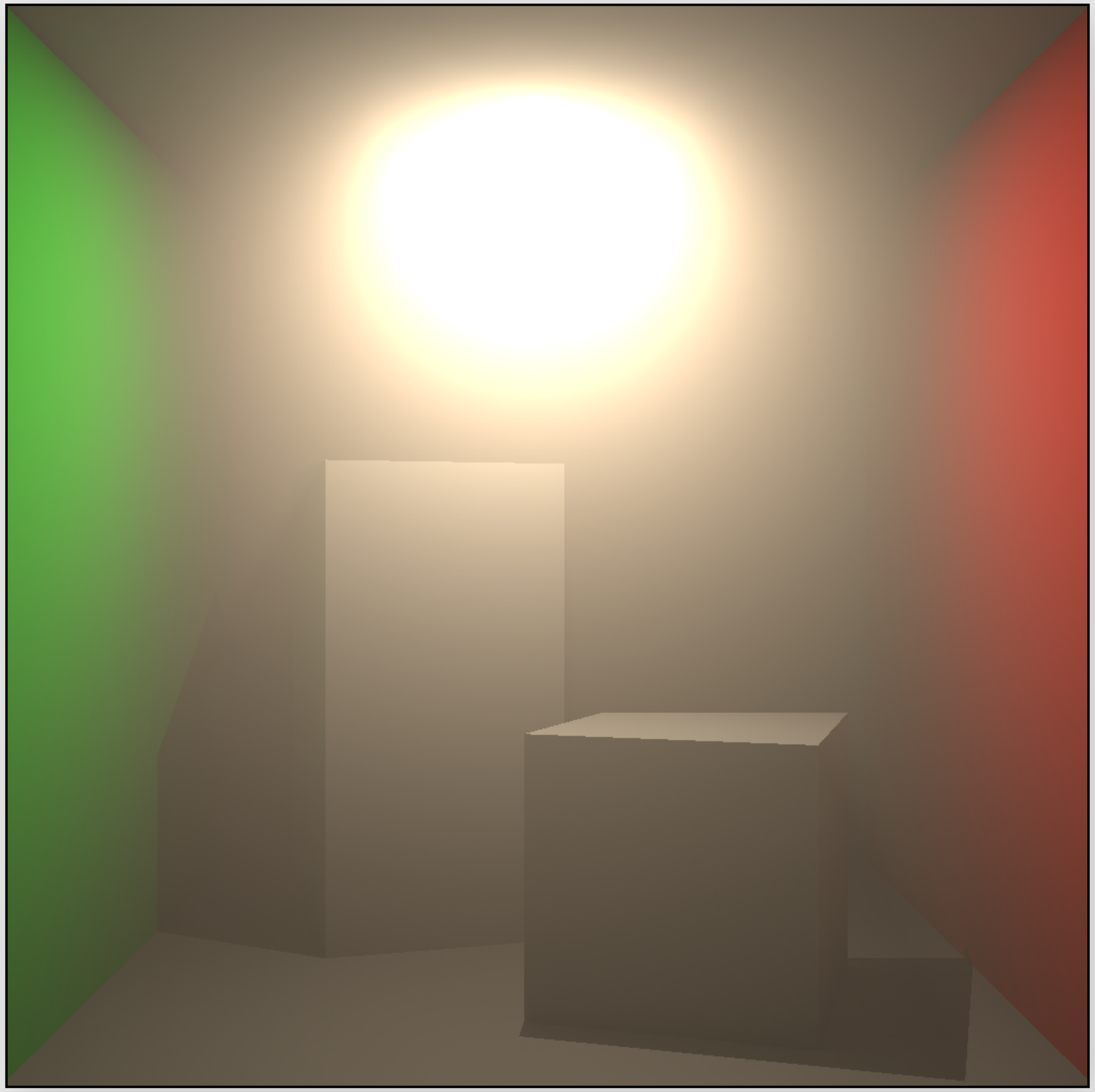
- Generalizations to participating media
- **Volumetric Radiance Caching** [Jarosz et al. 2008a, 2008b]
  - Cache radiance and gradients within volume

[Jarosz et al. 2008a, 2008b]

Participating Media



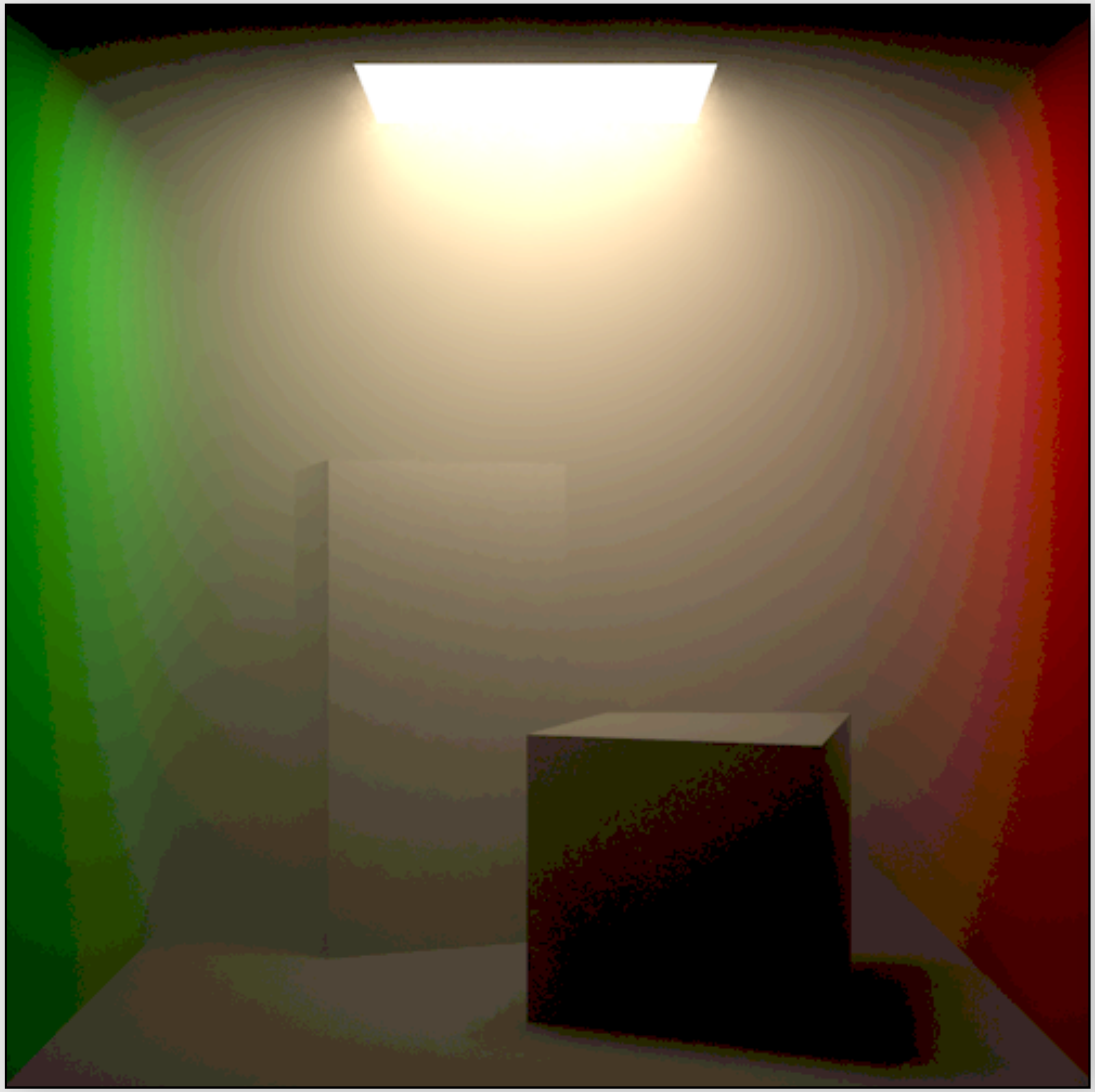
no media

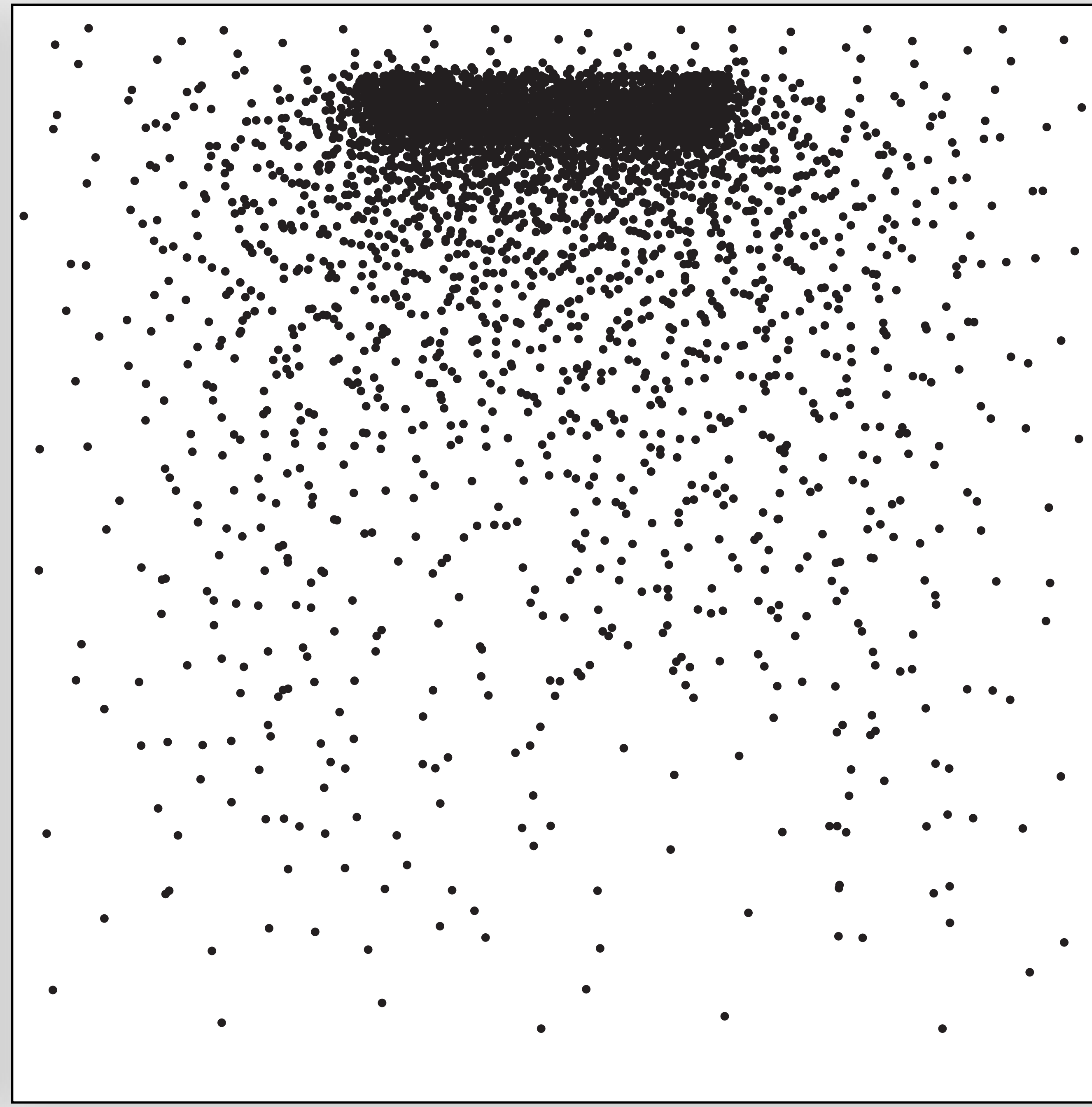


with media

[Jarosz et al. 2008a]

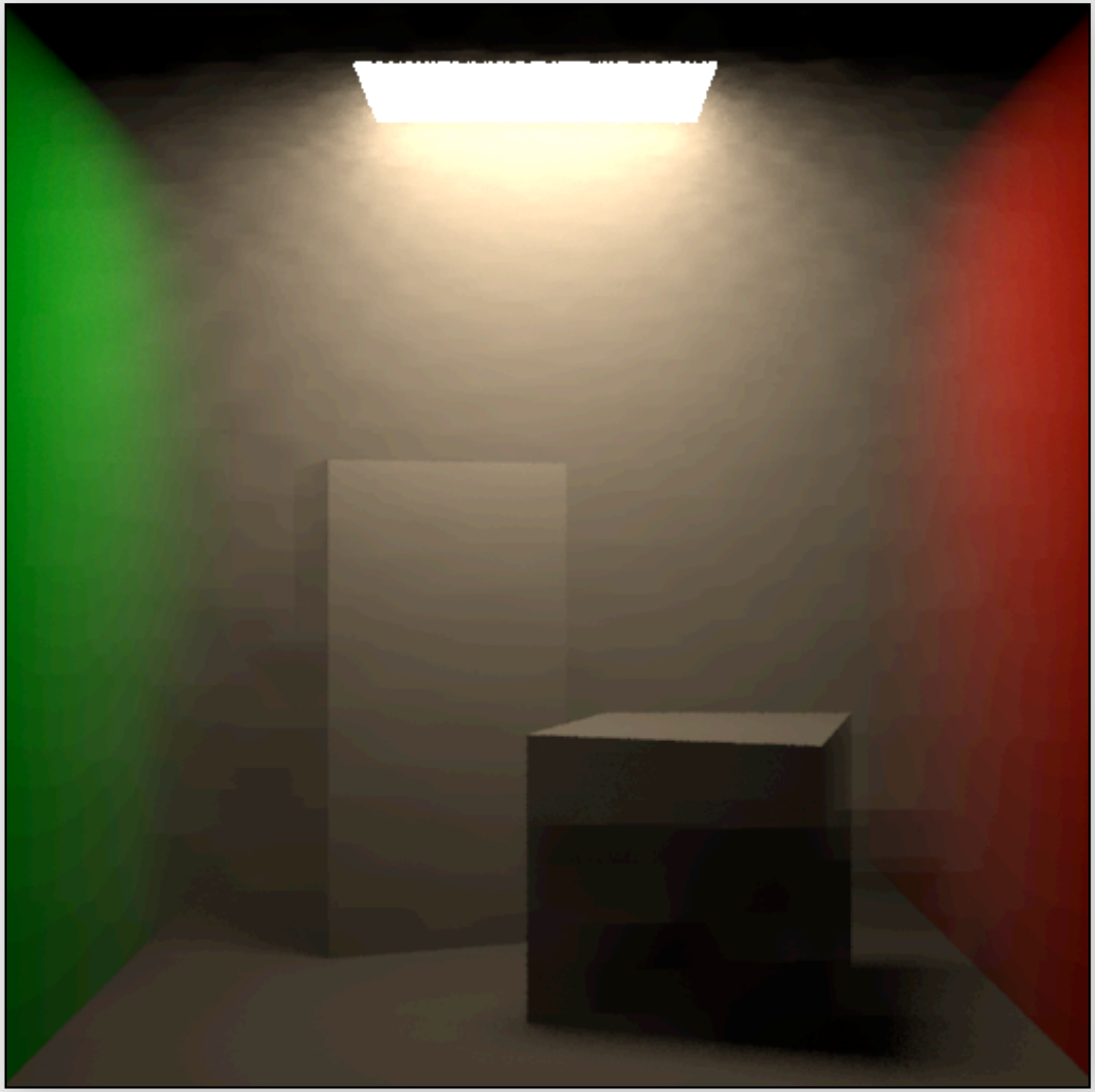
Valid Radius





[Jarosz et al. 2008a, 2008b]

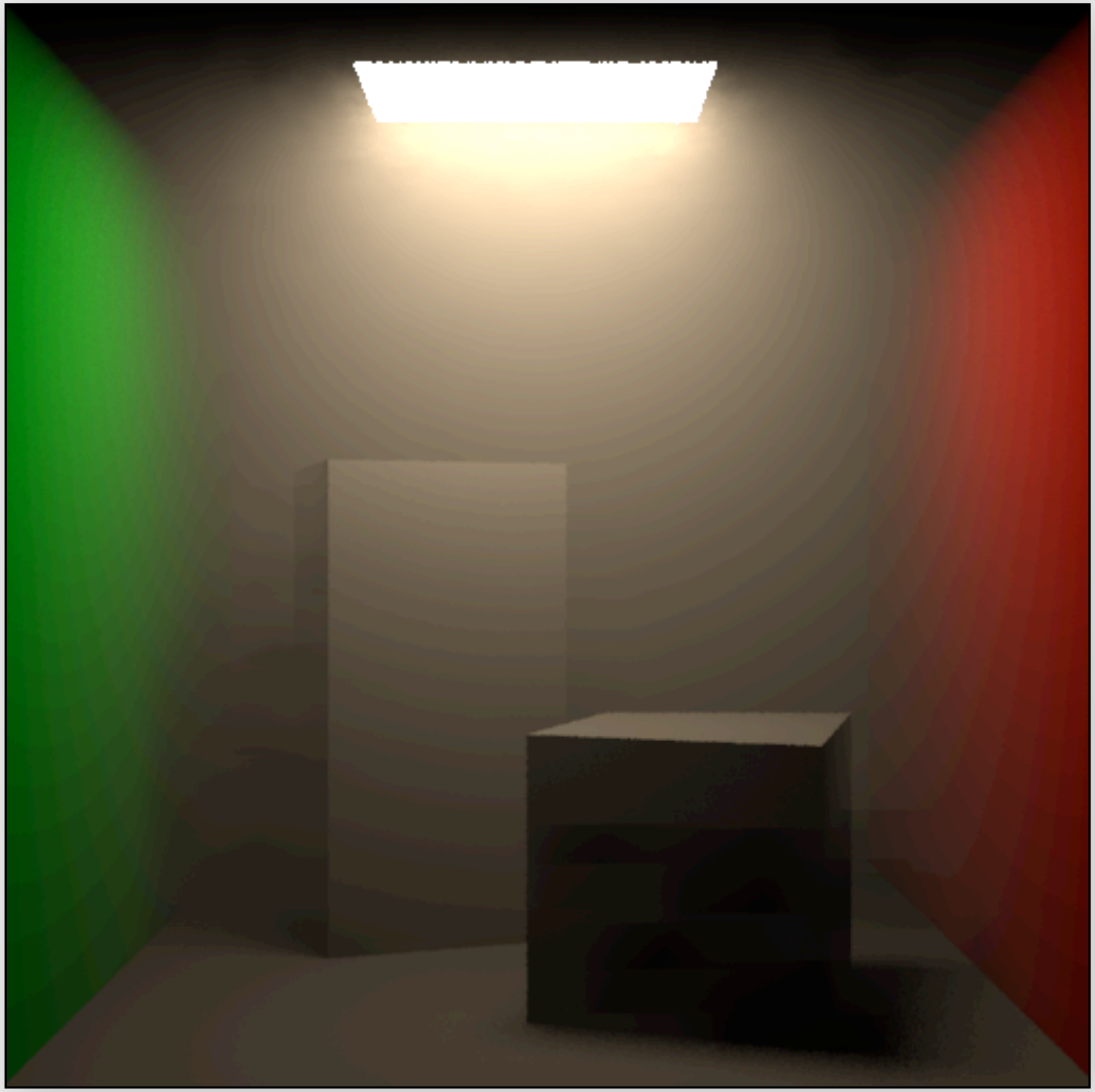
Gradients



no gradients

[Jarosz et al. 2008a]

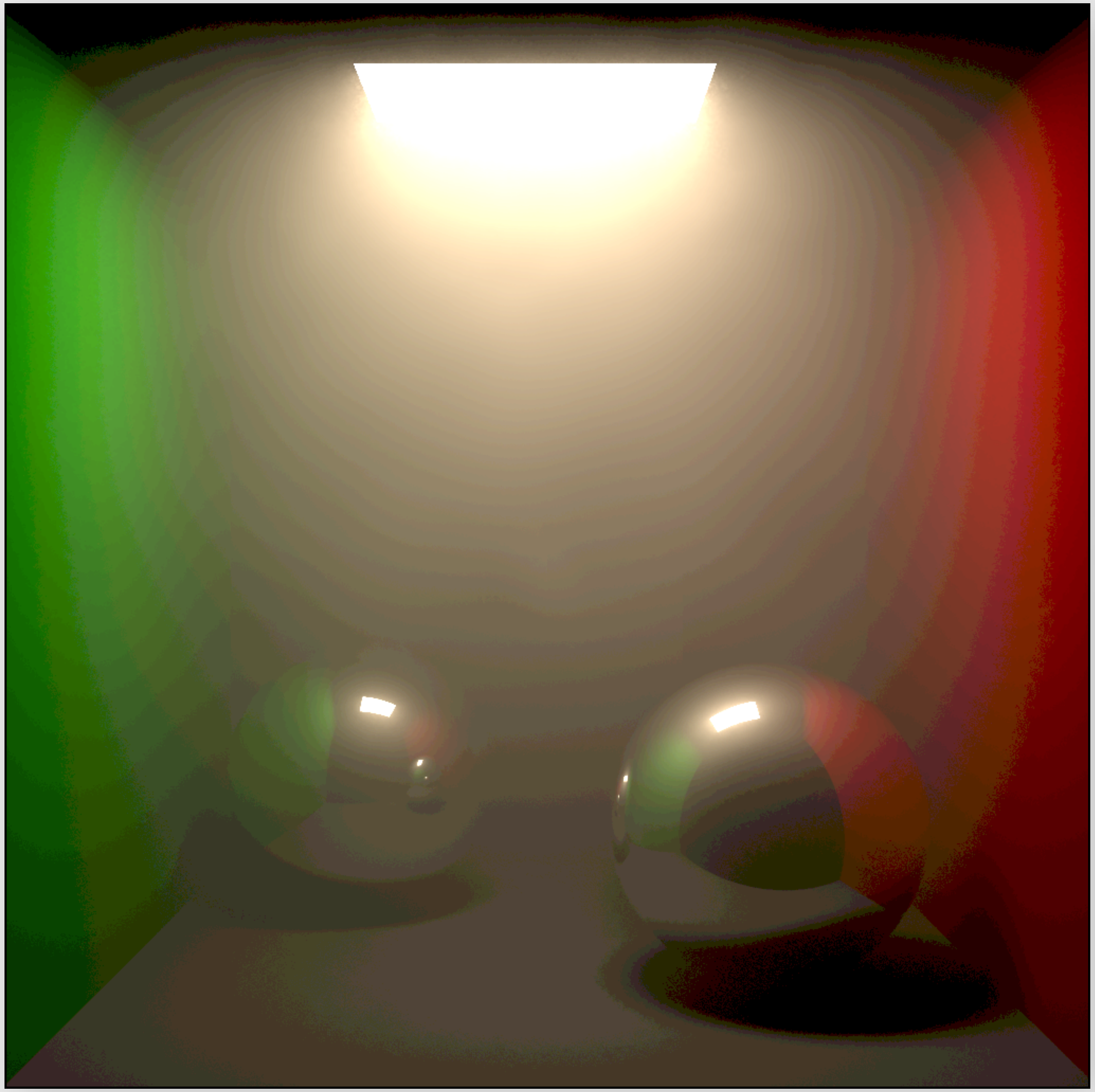
Gradients

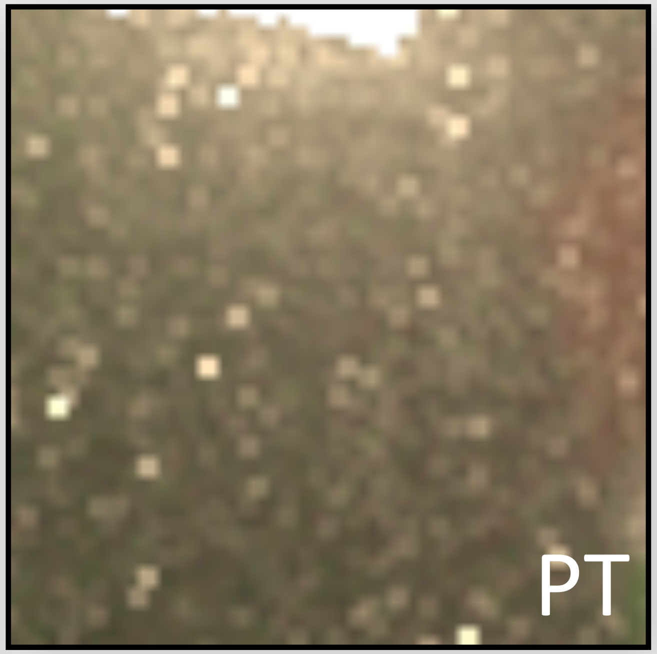


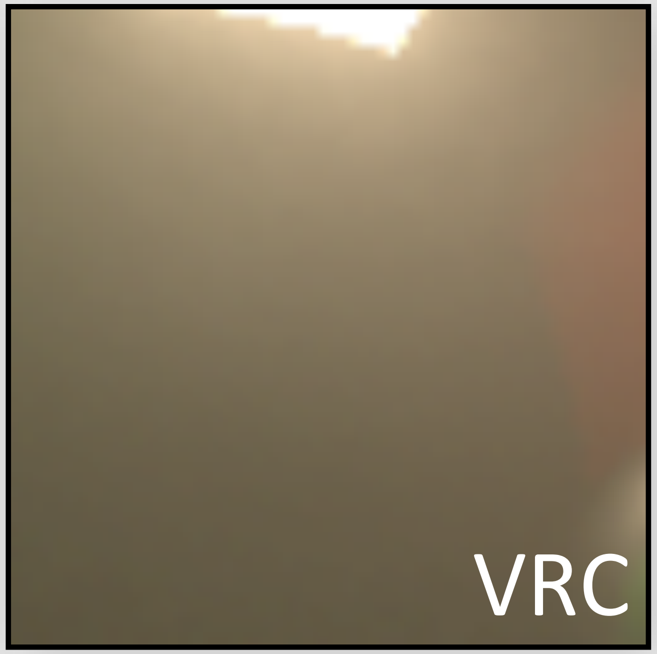
with gradients

[Jarosz et al. 2008a]

Results

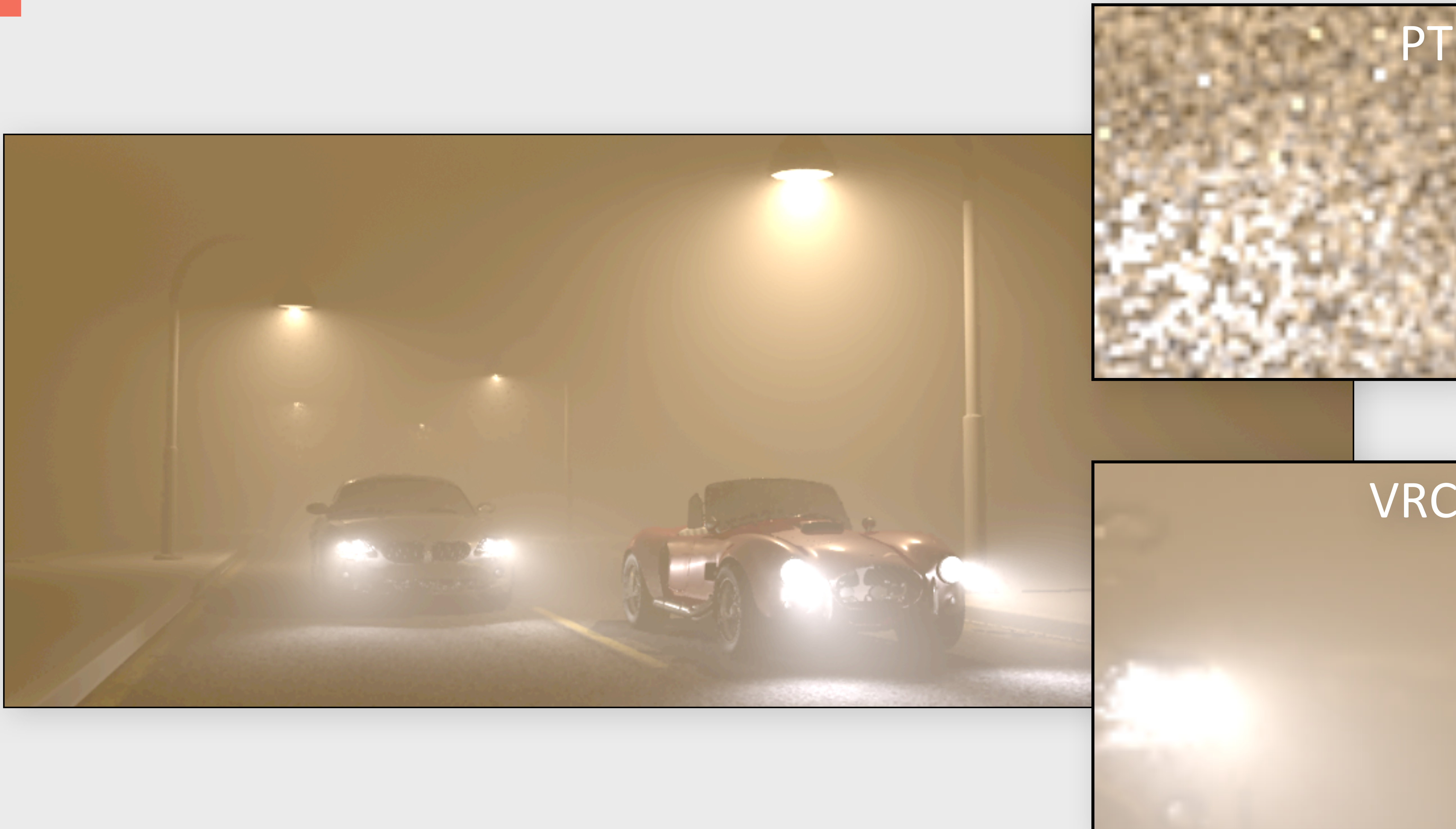






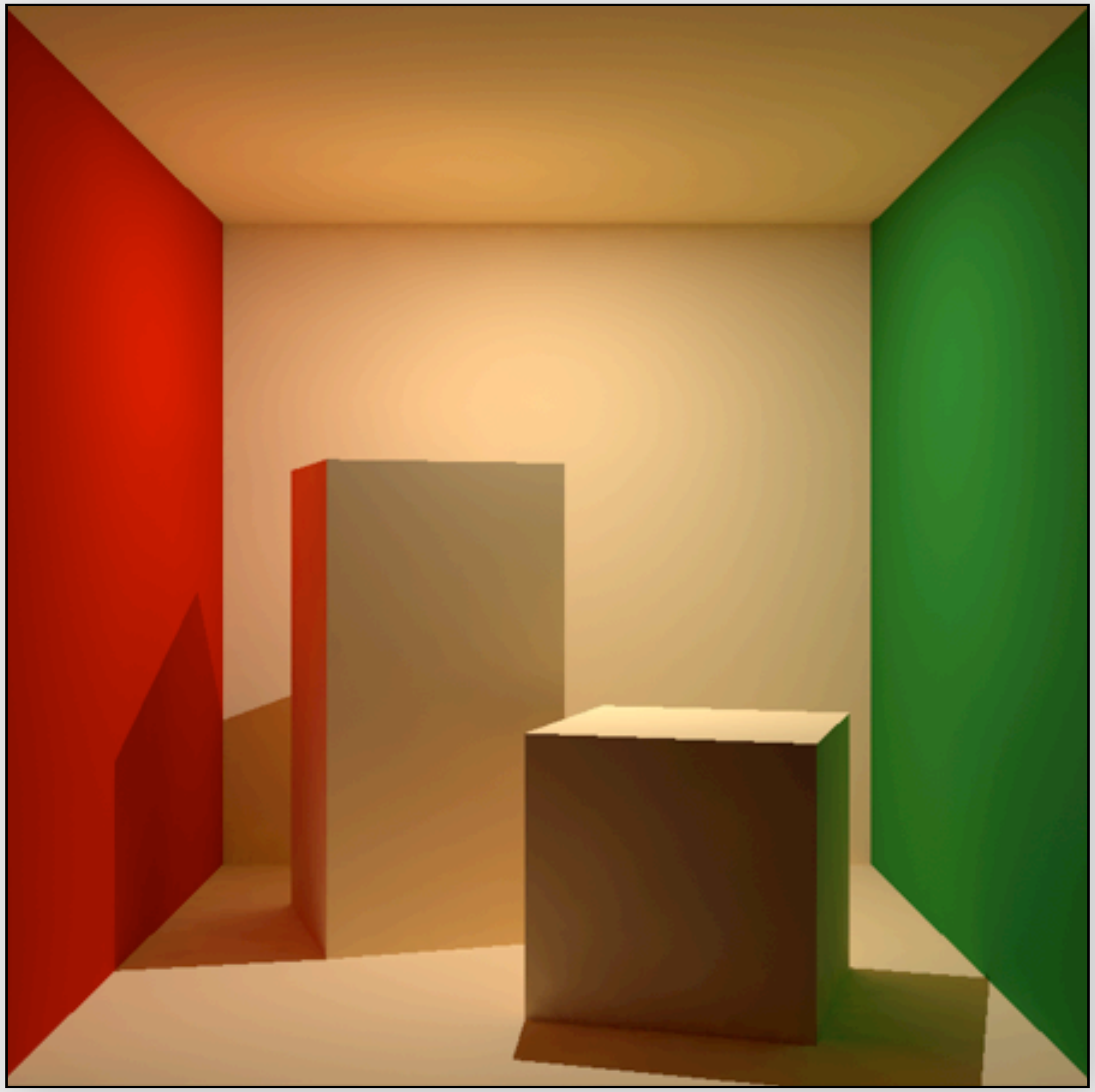
[Jarosz et al. 2008a]

Results

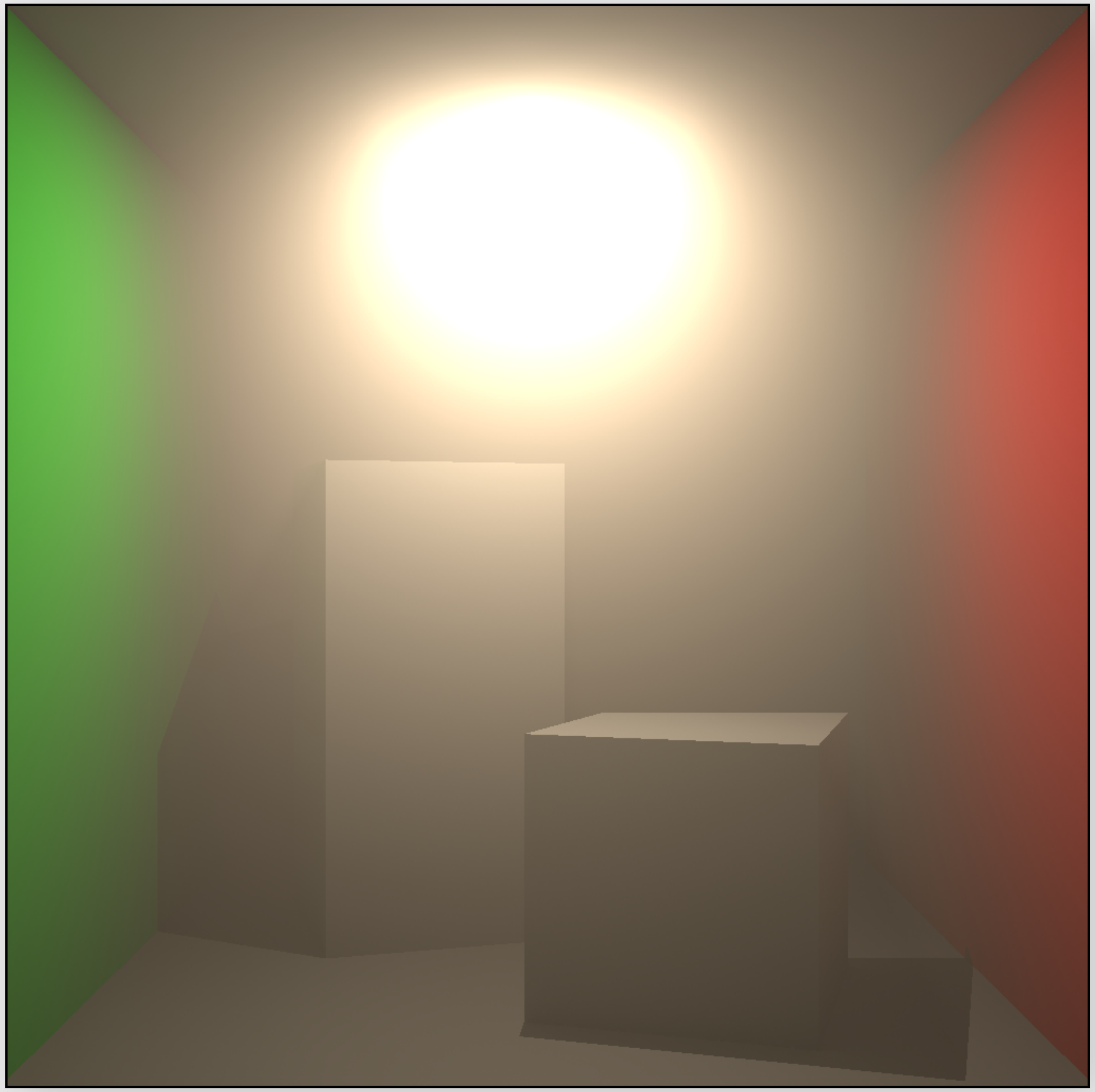


[Jarosz et al. 2008a]

Participating media



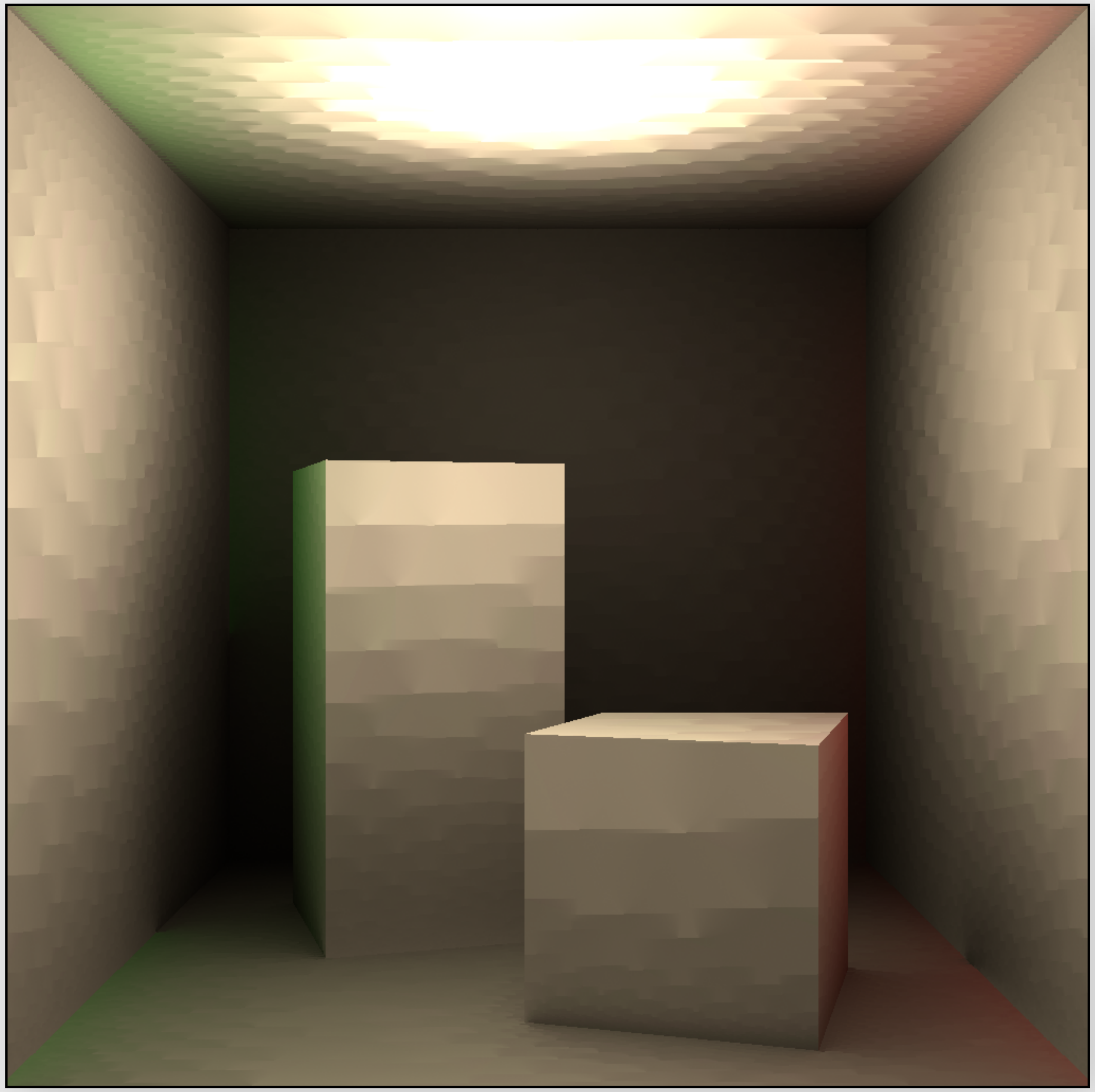
no media



with media

Surfaces in participating media

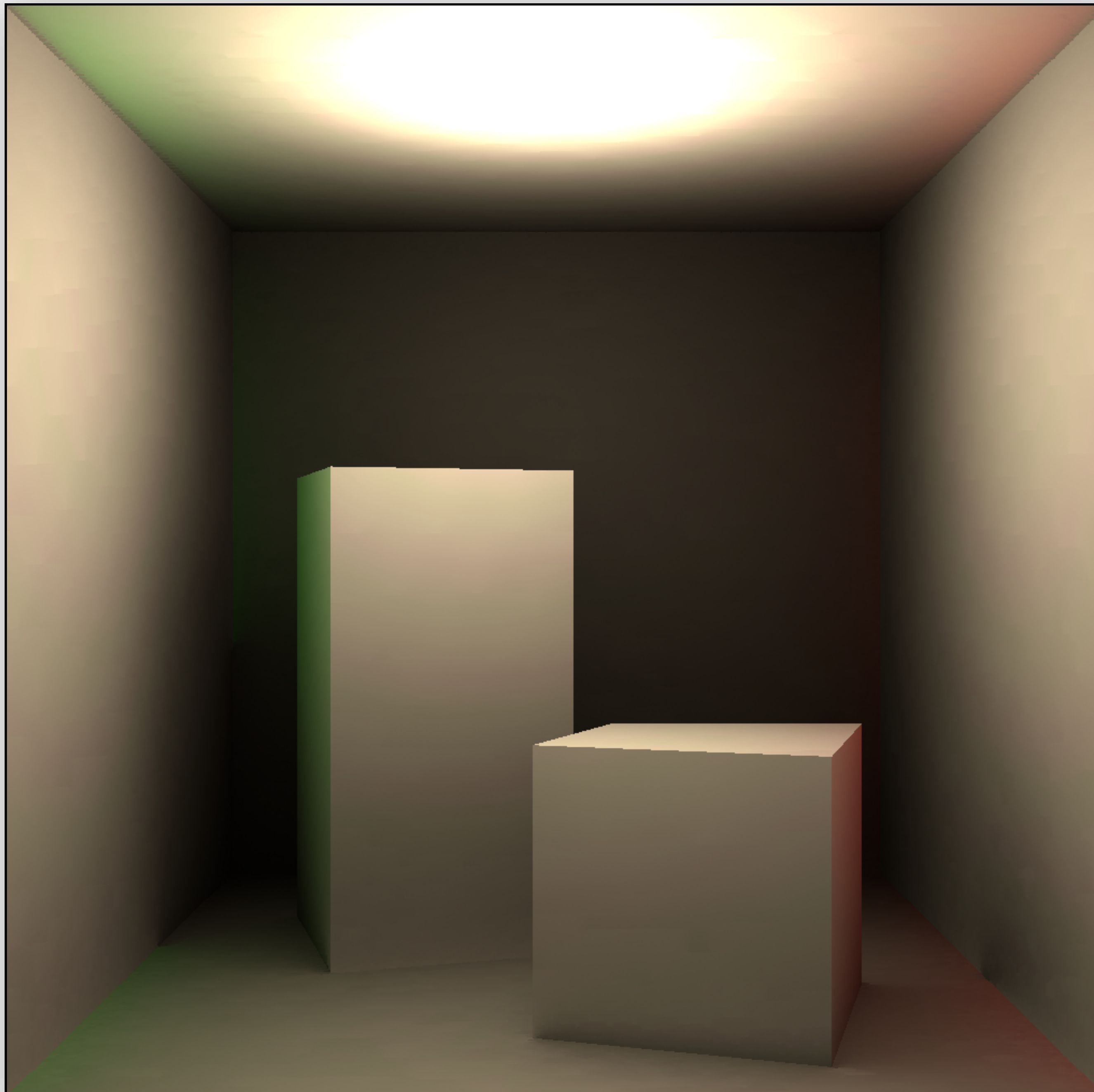
no media
(indirect irradiance)



with media
(indirect irradiance)

[Jarosz et al. 2008b]

Surfaces in participating media



Occlusion aware, but media unaware gradients

[Ward and Heckbert 1992]

Occlusion and media aware gradients

[Jarosz et al. 2008b]

[Jarosz et al. 2008b]

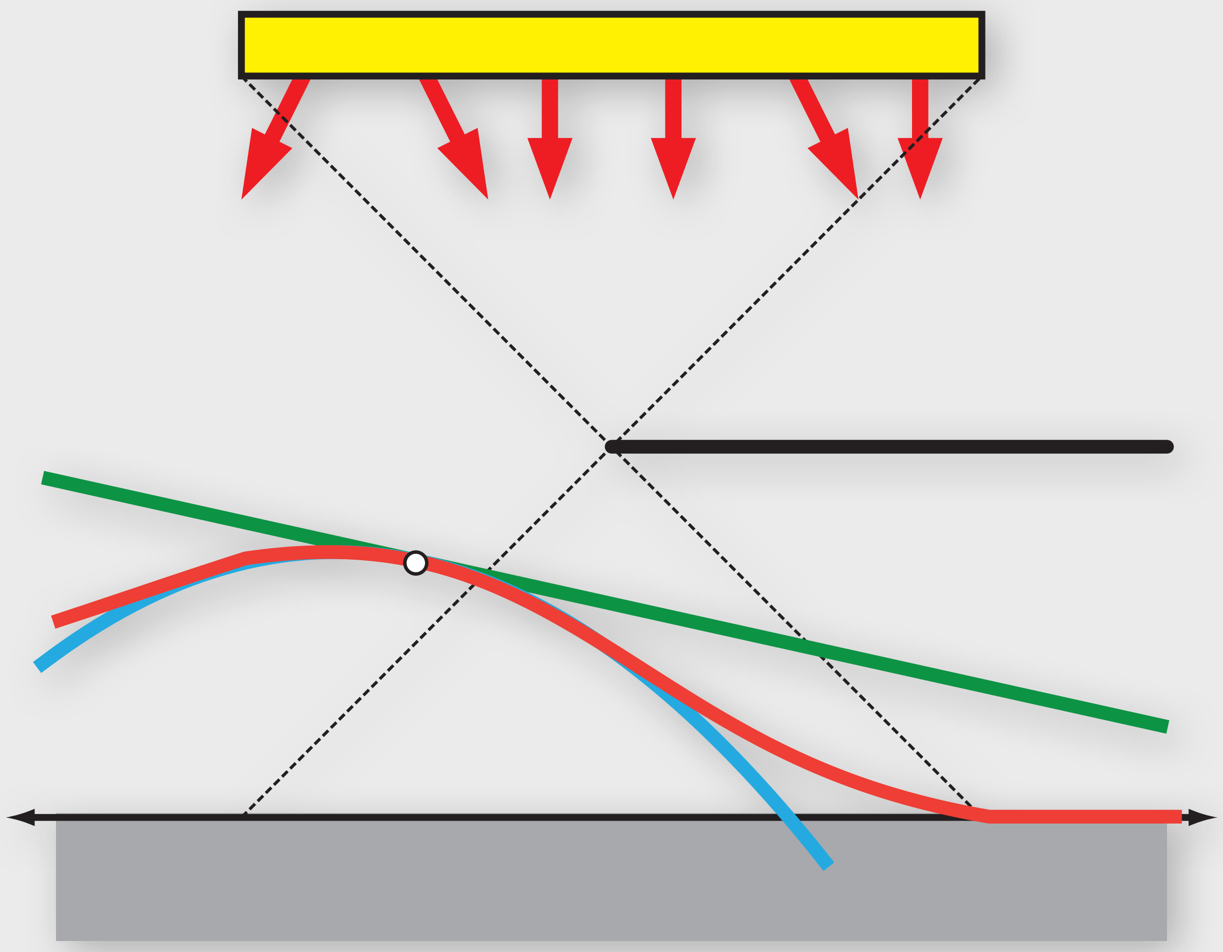
Sun beam through window

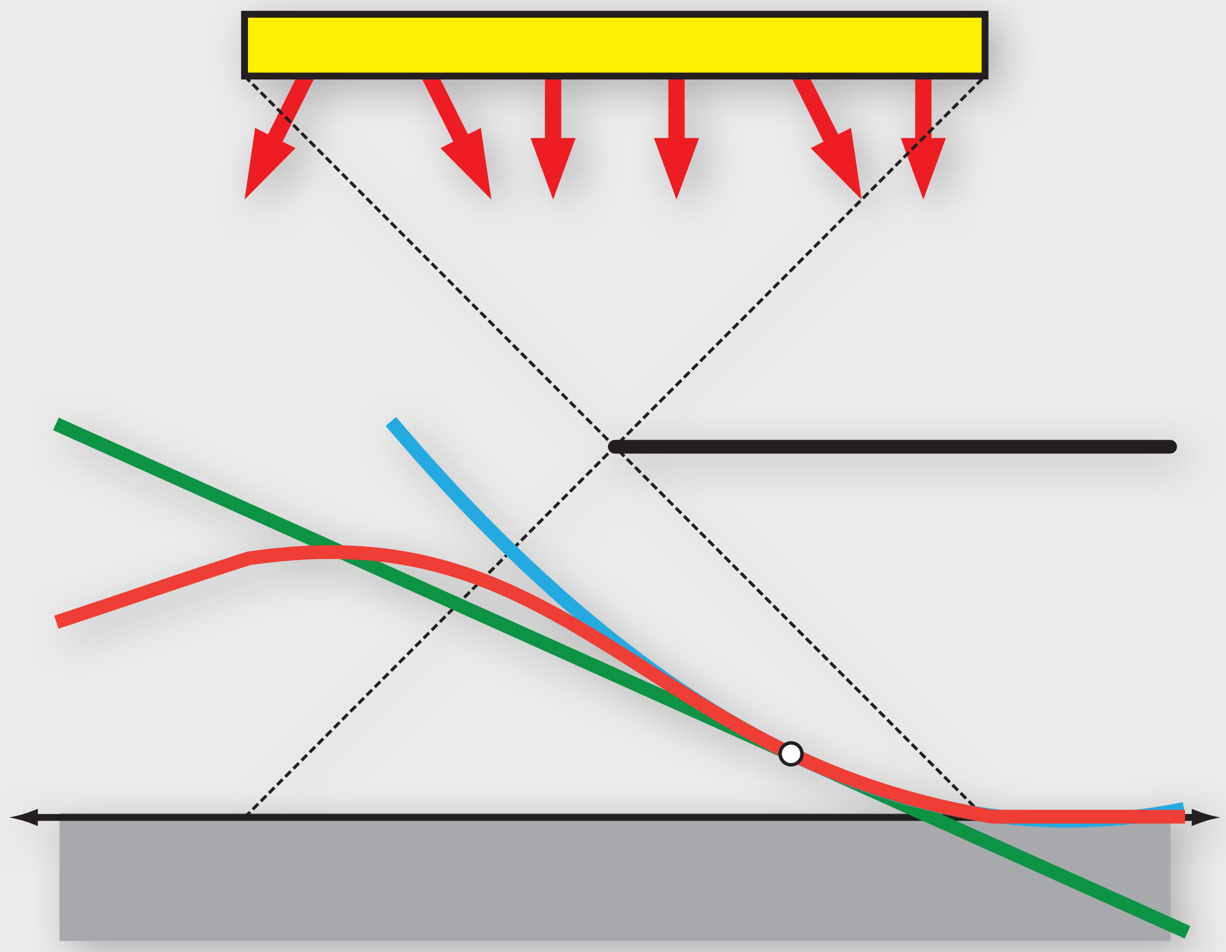
Gradients by [Ward and Heckbert 1992]
Gradients by [Jarosz et al. 2008b]

Higher-order derivatives

- Exploit higher-order derivatives for better error control
  - [Jarosz et al. 2012] - Hessians (occlusion-unaware)
  - [Schwarzhaupt et al. 2012] - occlusion-aware Hessians & practical details

1st & 2nd Order Extrapolation





Irradiance 1st order 2nd order

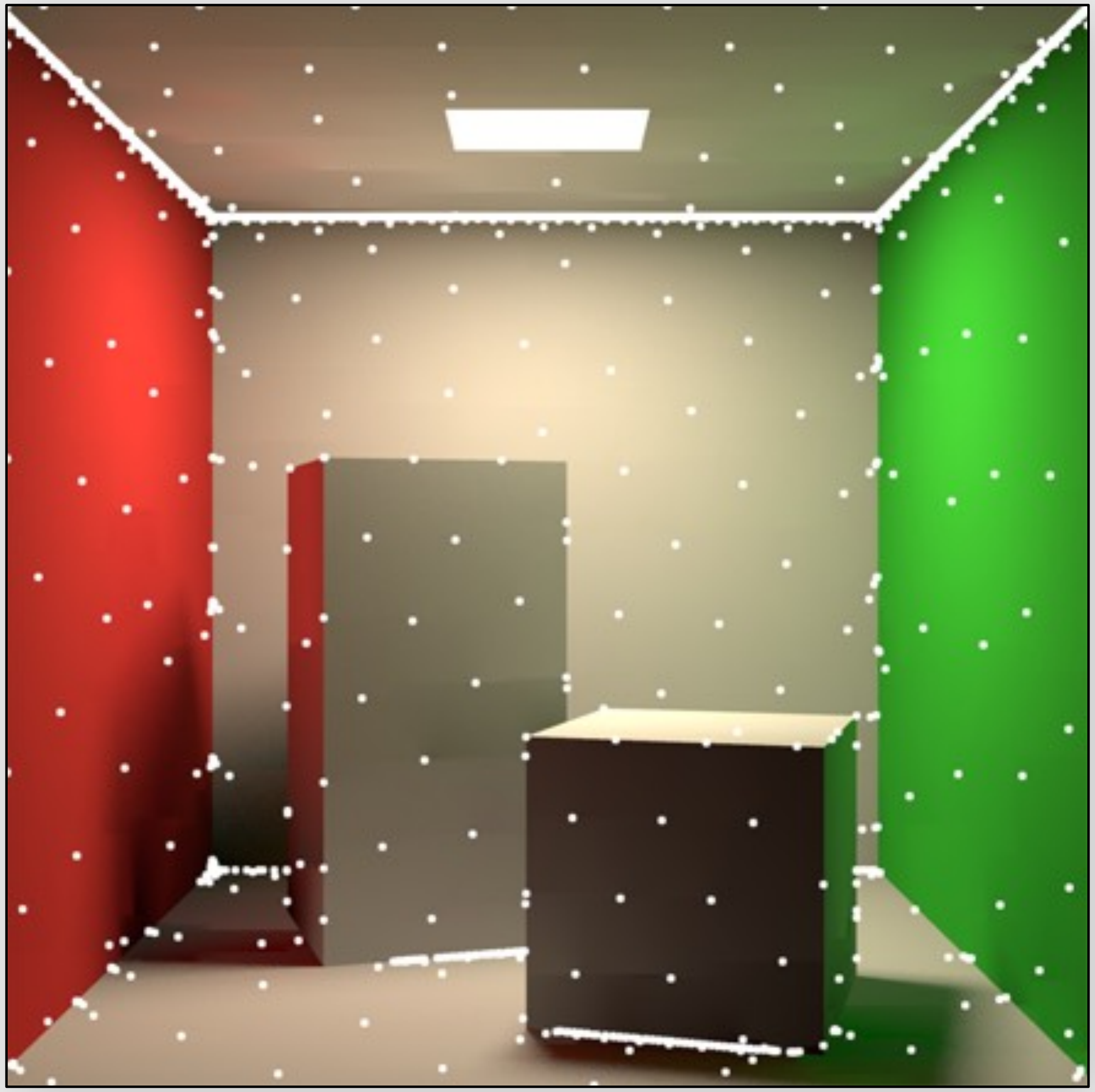
[Jarosz et al. 2012]

Split-Sphere Heuristic

- Basis for most irradiance caching algorithms for 20+ years
- Fix-ups to original metric lead to many parameters
  - error threshold
  - min/max screen-space radii
  - min/max world-space radii
  - gradient clamping
  - ...
- Hard to control!

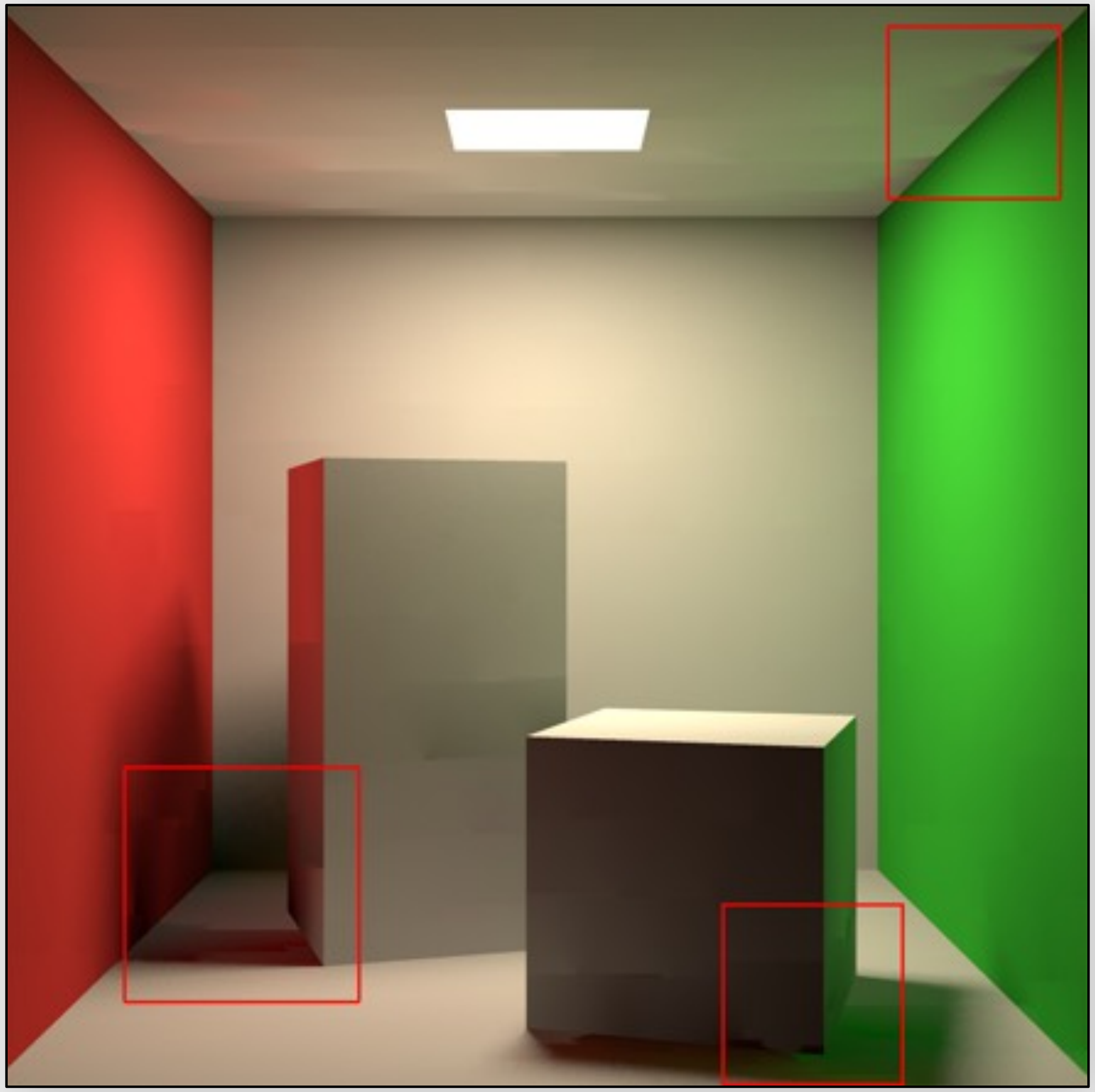
[Jarosz et al. 2012]

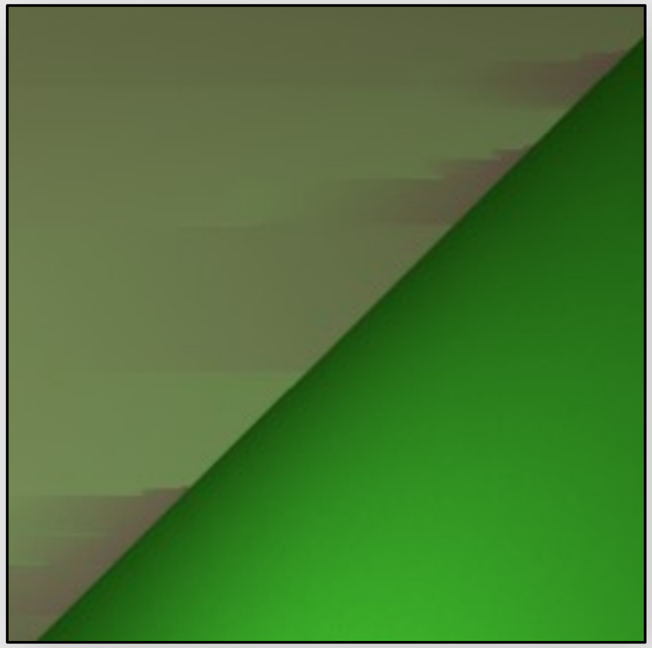
The Split Sphere

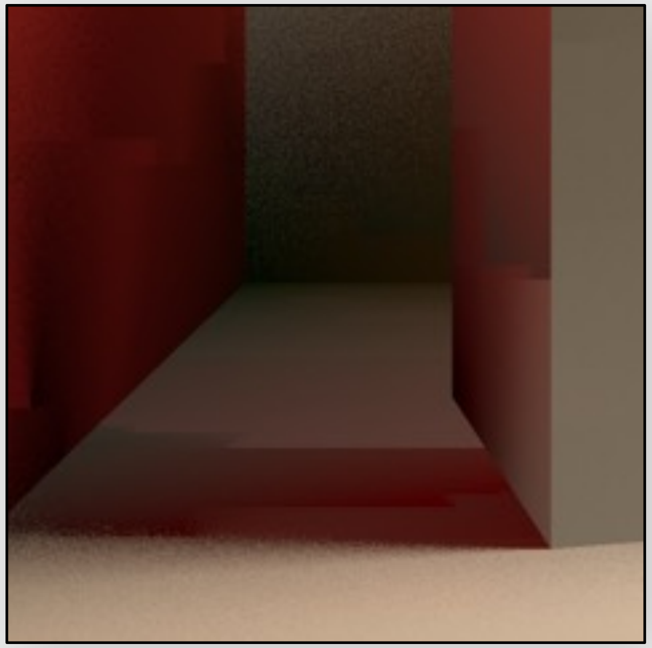


[Jarosz et al. 2012]

The Split Sphere



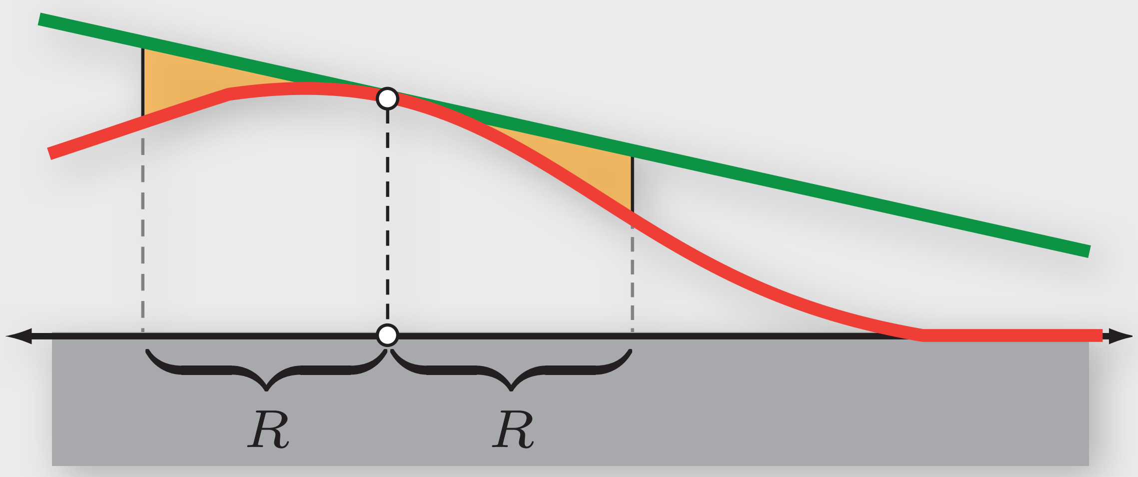




[Jarosz et al. 2012]

Better Error Control

- **total error** ϵ^t = integrated difference between extrapolated and correct irradiance

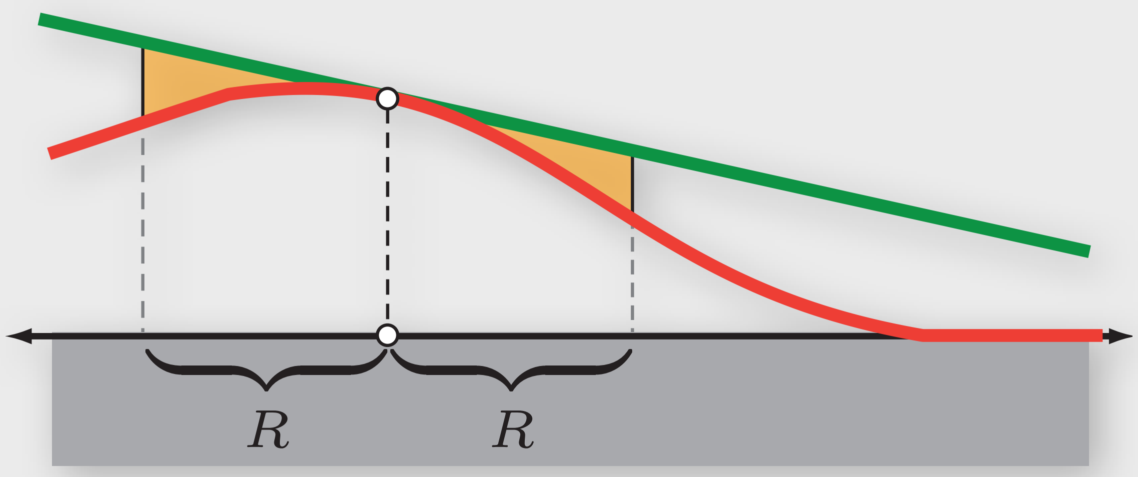


$$\epsilon^t = \int_{-R_i}^{R_i} \left| E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x) \right| dx$$

[Jarosz et al. 2012]

Better Error Control

- E' is 1st-order Taylor extrapolation
- E is unknown!

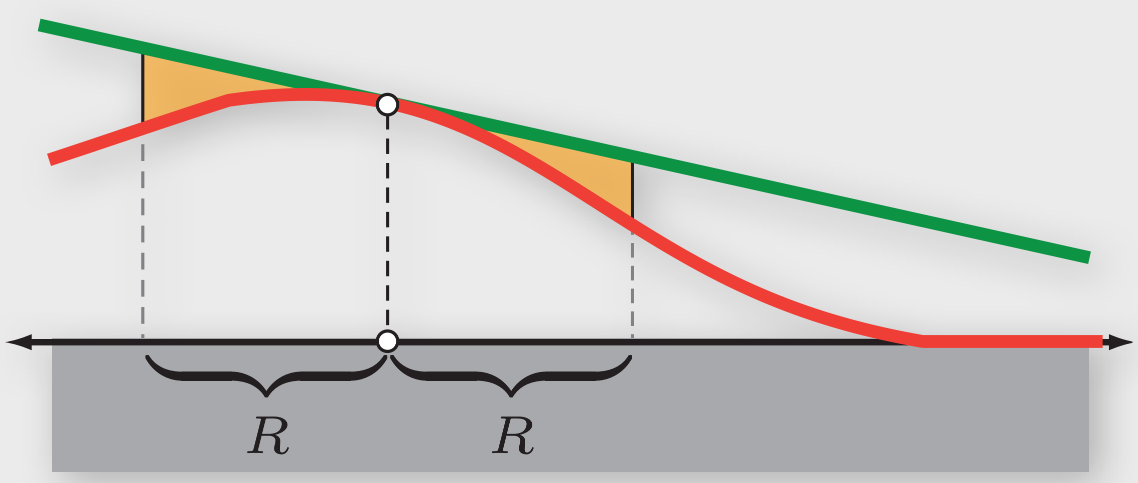


$$\epsilon^t = \int_{-R_i}^{R_i} |E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x)|\, dx$$

[Jarosz et al. 2012]

Better Error Control

- E' is 1st-order Taylor extrapolation
- E is unknown!

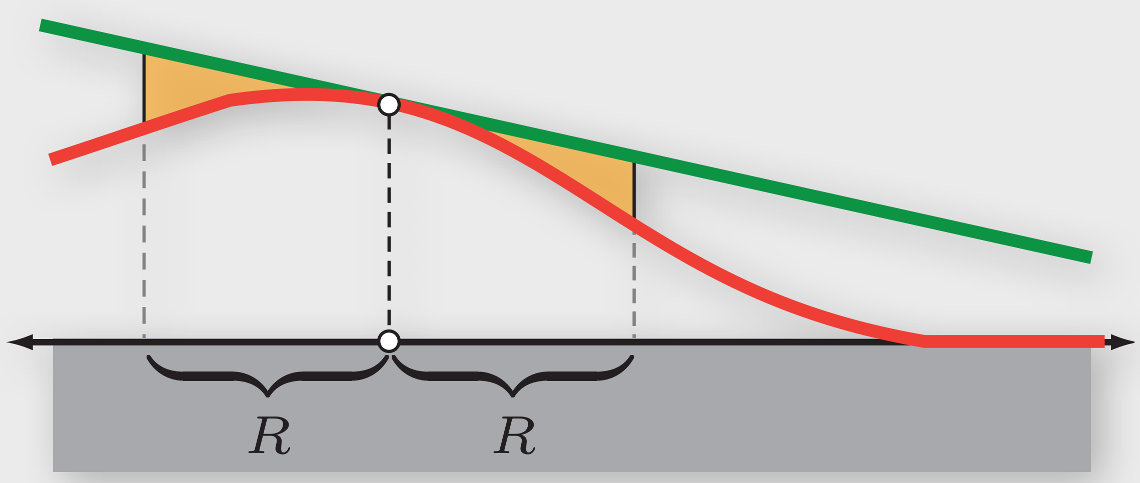


$$\epsilon^t = \int_{-R_i}^{R_i} \left| E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x) \right| \, dx$$

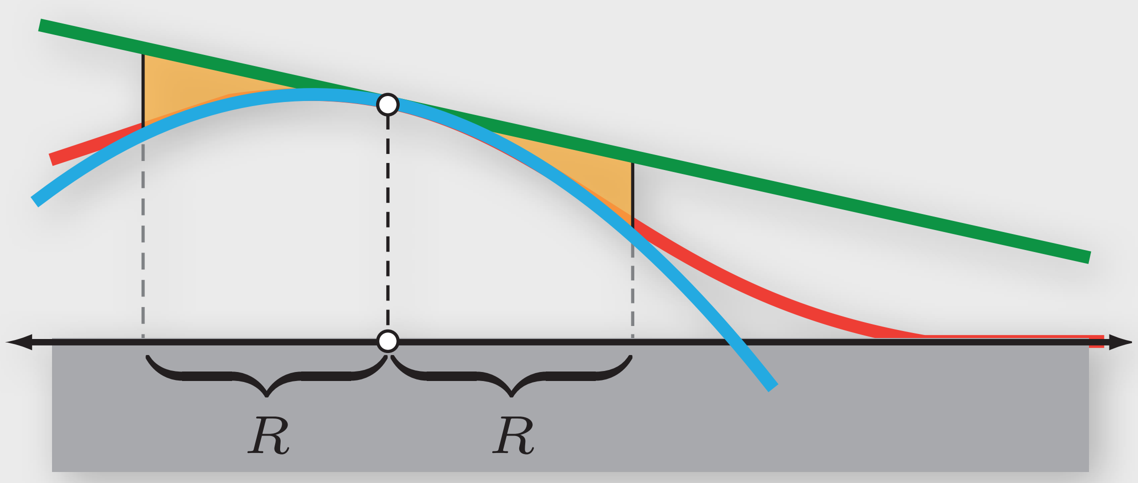
[Jarosz et al. 2012]

Better Error Control

- *E'* is 1st-order Taylor extrapolation
- *E* is unknown!



2nd-order Taylor extrapolation

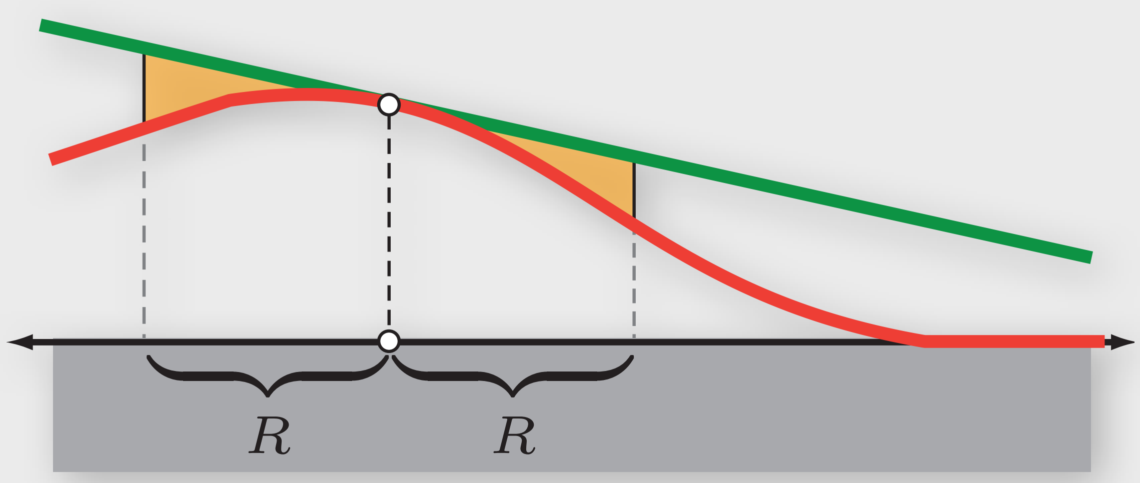


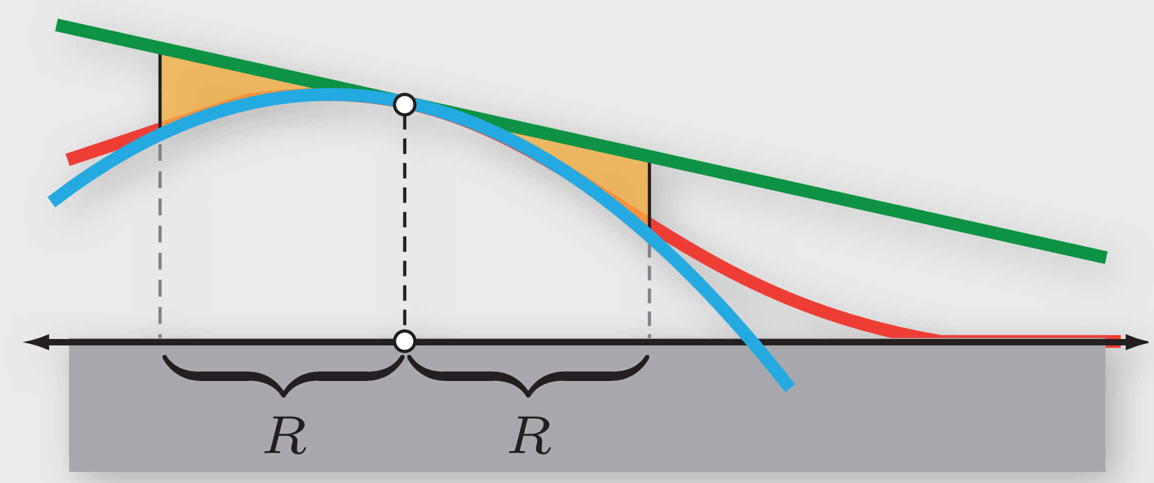
$$\epsilon^t = \int_{-R_i}^{R_i} \left| E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x) \right| dx$$

[Jarosz et al. 2012]

Hessian-based Error Control

- *E'* is 1st-order Taylor extrapolation
- 2nd-order Taylor extrapolation approximates *E*





$$\epsilon^t = \int_{-R_i}^{R_i} |E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x)| \, dx \quad \approx \quad \hat{\epsilon}^t = \frac{1}{2} \int_{-R_i}^{R_i} |x \, \mathbf{H}_\mathbf{x}(E_i) \, x| \, dx$$

[Jarosz et al. 2012]

Hessian-based Error Control

$$\hat{\epsilon}^t = \frac{1}{2}\int_{-R_i}^{R_i} |x\,\mathbf{H}_{\mathbf{x}}(E_i)\,x|\,dx = \frac{1}{3}\boxed{|h_{\mathbf{x}}(E_i)|}R_i^3$$

second derivative

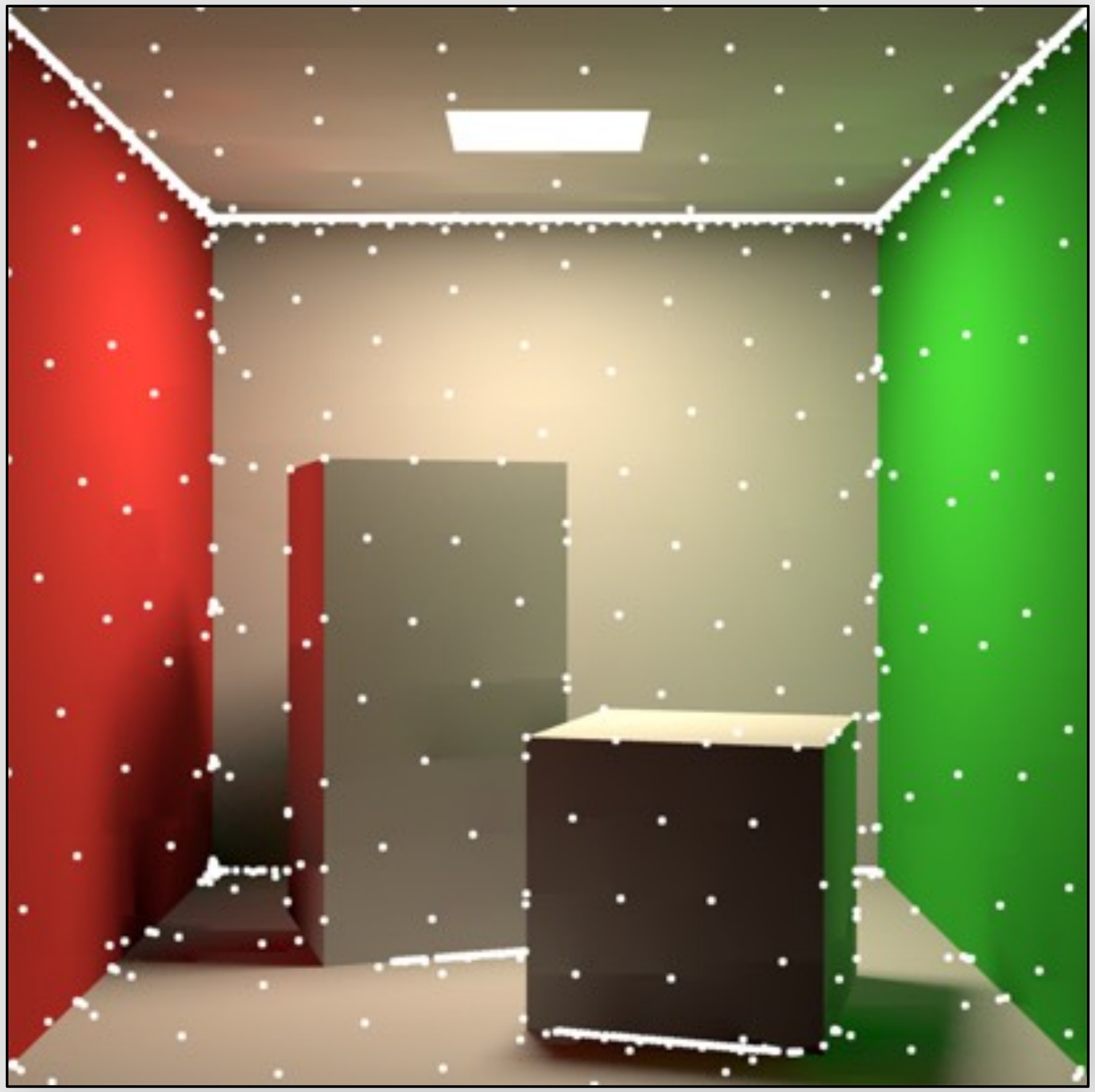
- fix ϵ^t, solve for R_i

$$\boxed{R_i = \sqrt[3]{\frac{3\hat{\epsilon}^t}{|h_{\mathbf{x}}(E_i)|}}}$$

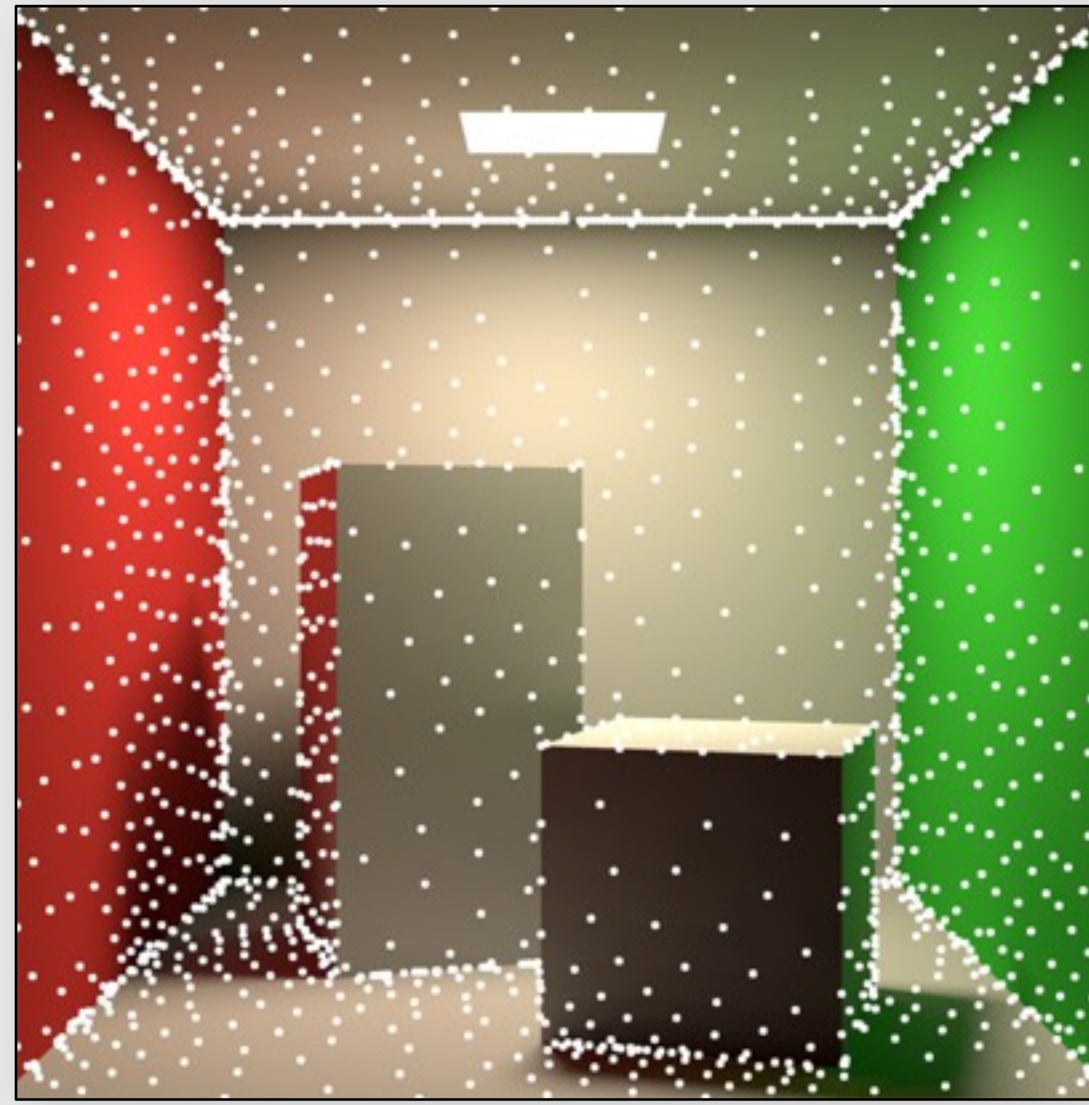
[Jarosz et al. 2012]

Beyond the Split-Sphere

~1,700 Cache Points



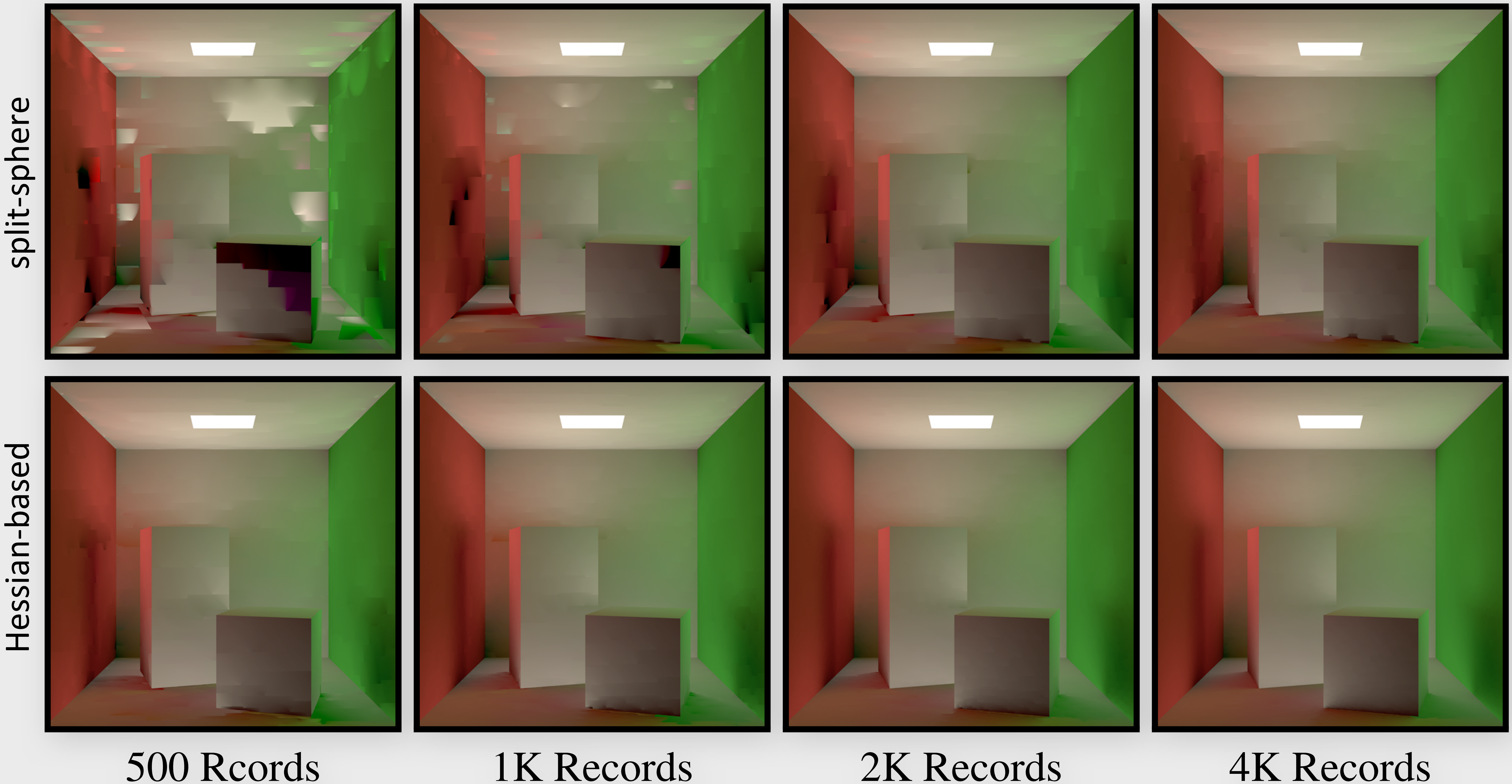
Split-Sphere



Hessian-based

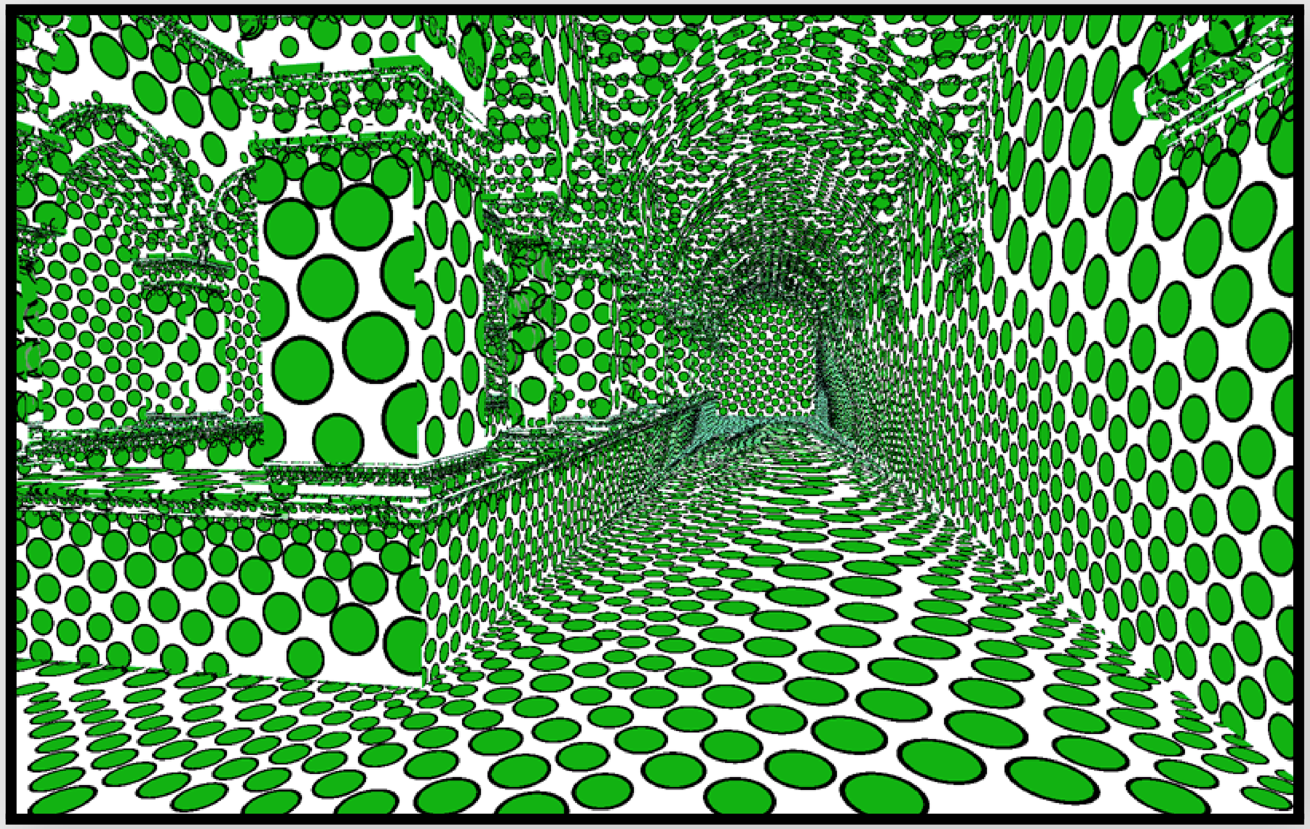
[Schwarzhaupt et al. 2012]

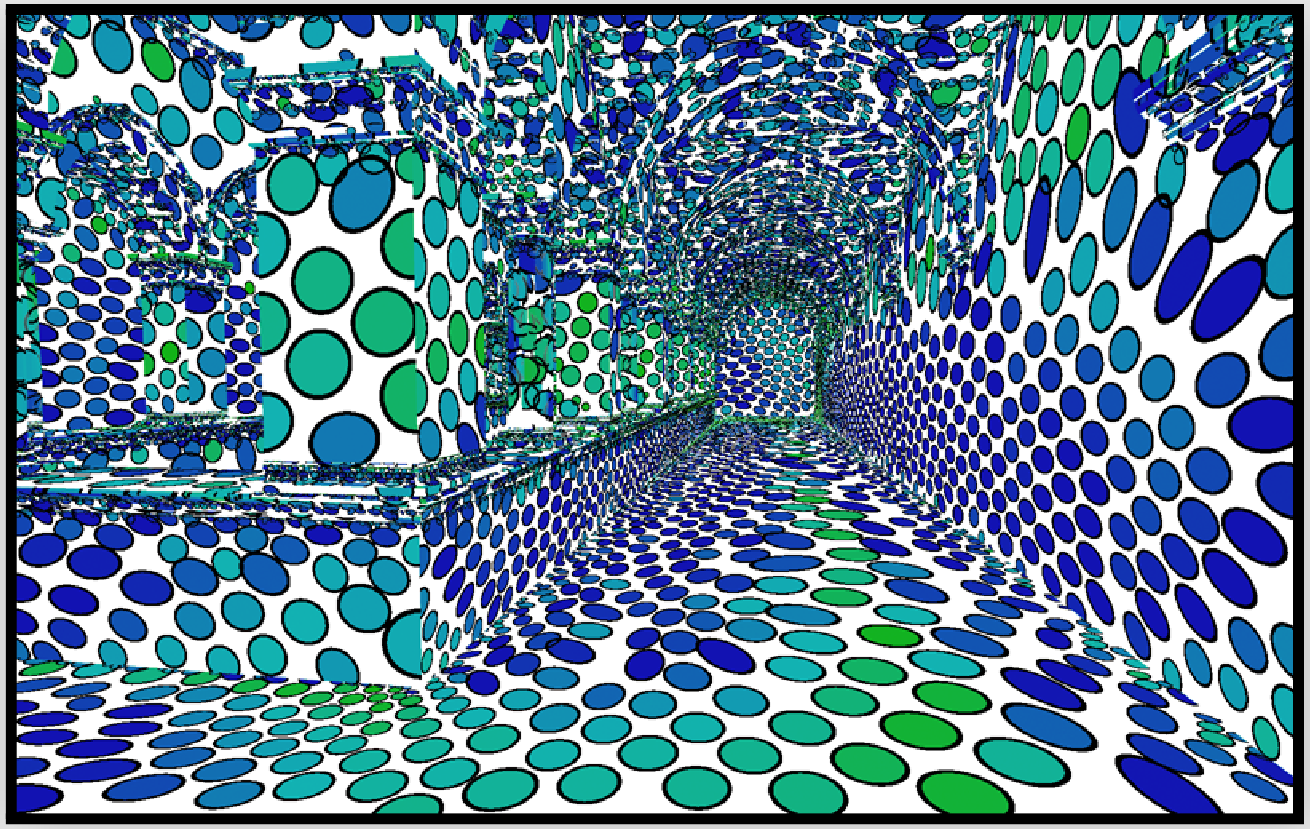
Split-Sphere vs Hessian-based



[Schwarzhaupt et al. 2012]

Anisotropic Cache Records

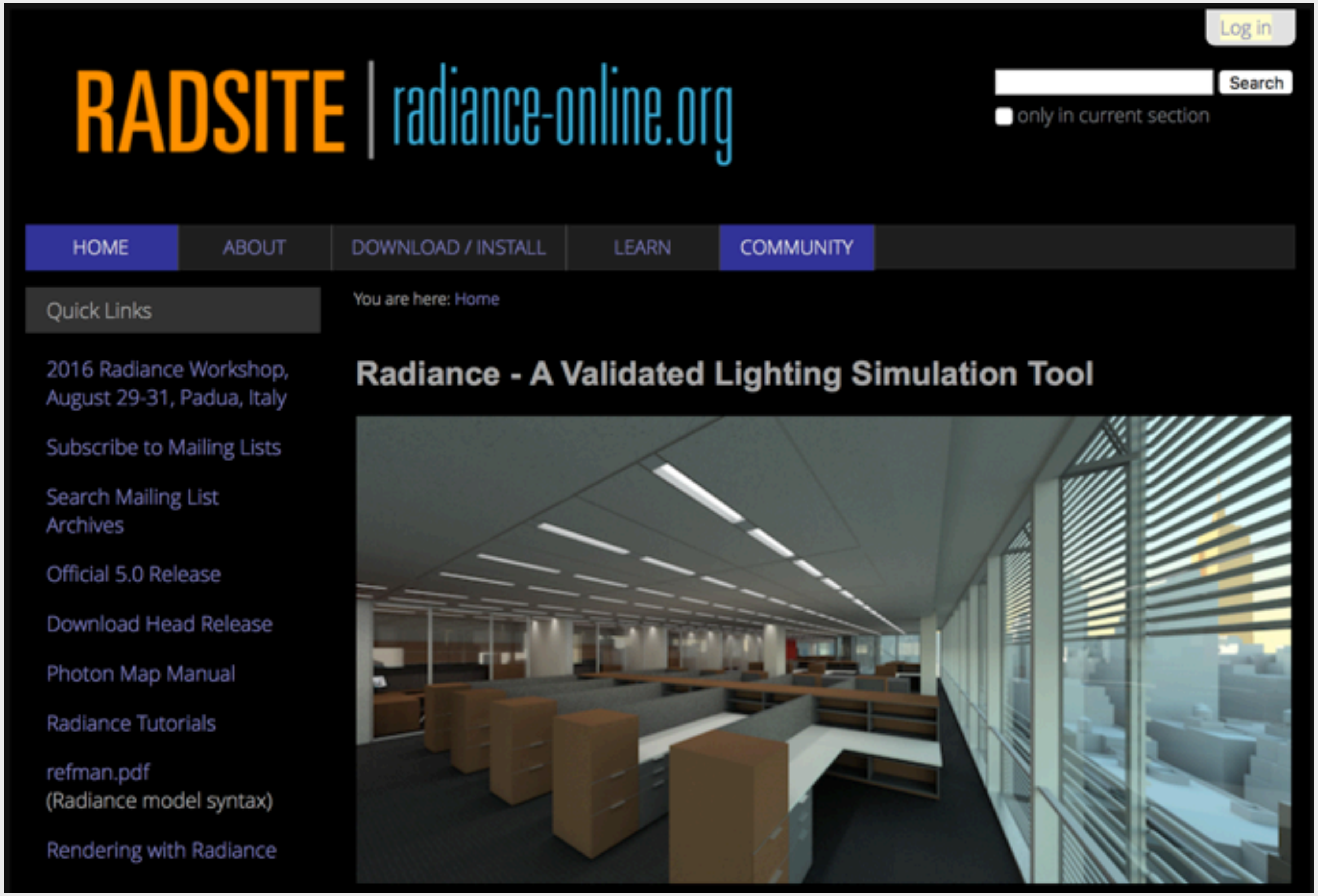




Reference Bounded Split-Sphere Occlusion Hessian

Hessians in RADIANCE

- Now the default approach used in RADIANCE



Summary

- Derivatives can estimate local function smoothness
- Amortize illumination computation across many pixels
- Accounting for occlusions is challenging but critical
- Specialized techniques only for diffuse or moderately glossy