

CS 87/187, Spring 2016

# RENDERING ALGORITHMS

## Direct Illumination II



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(with some slides adapted from Dr. Jan Novák and Dr. Paul Debevec)



Dartmouth

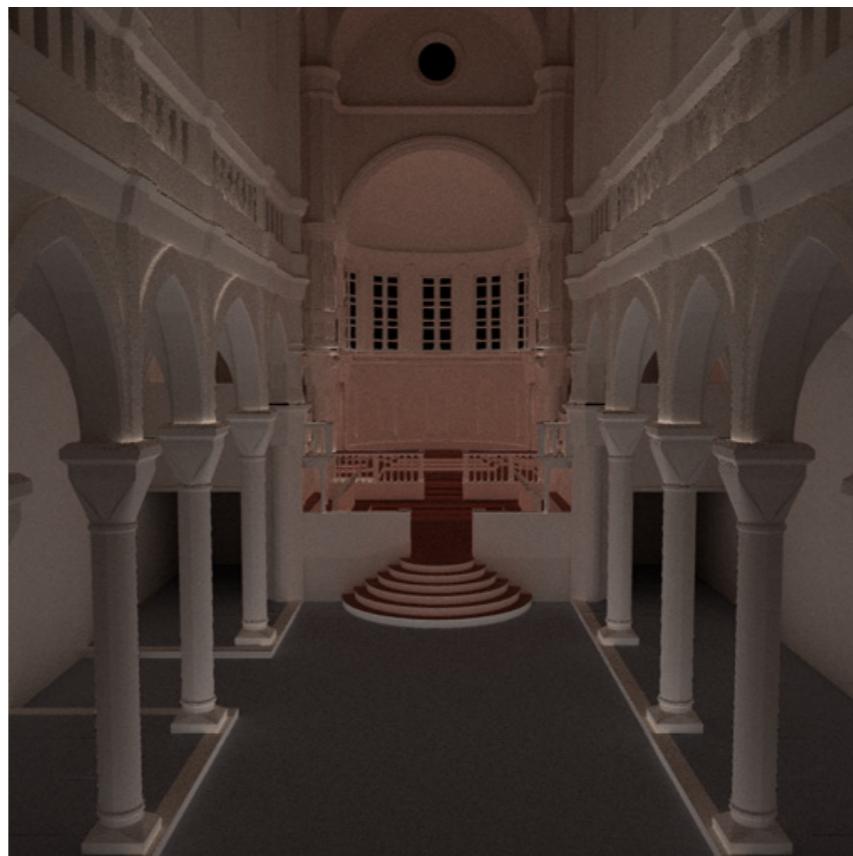
VCE

# Direct vs. Indirect Illumination

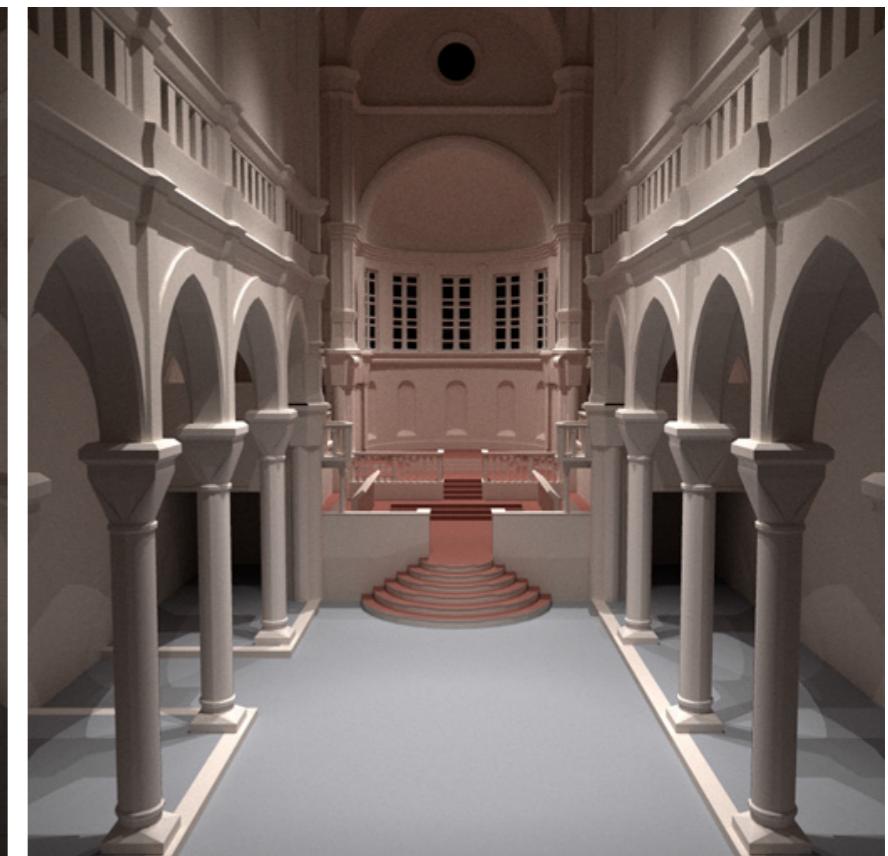
Direct illumination



Indirect illumination

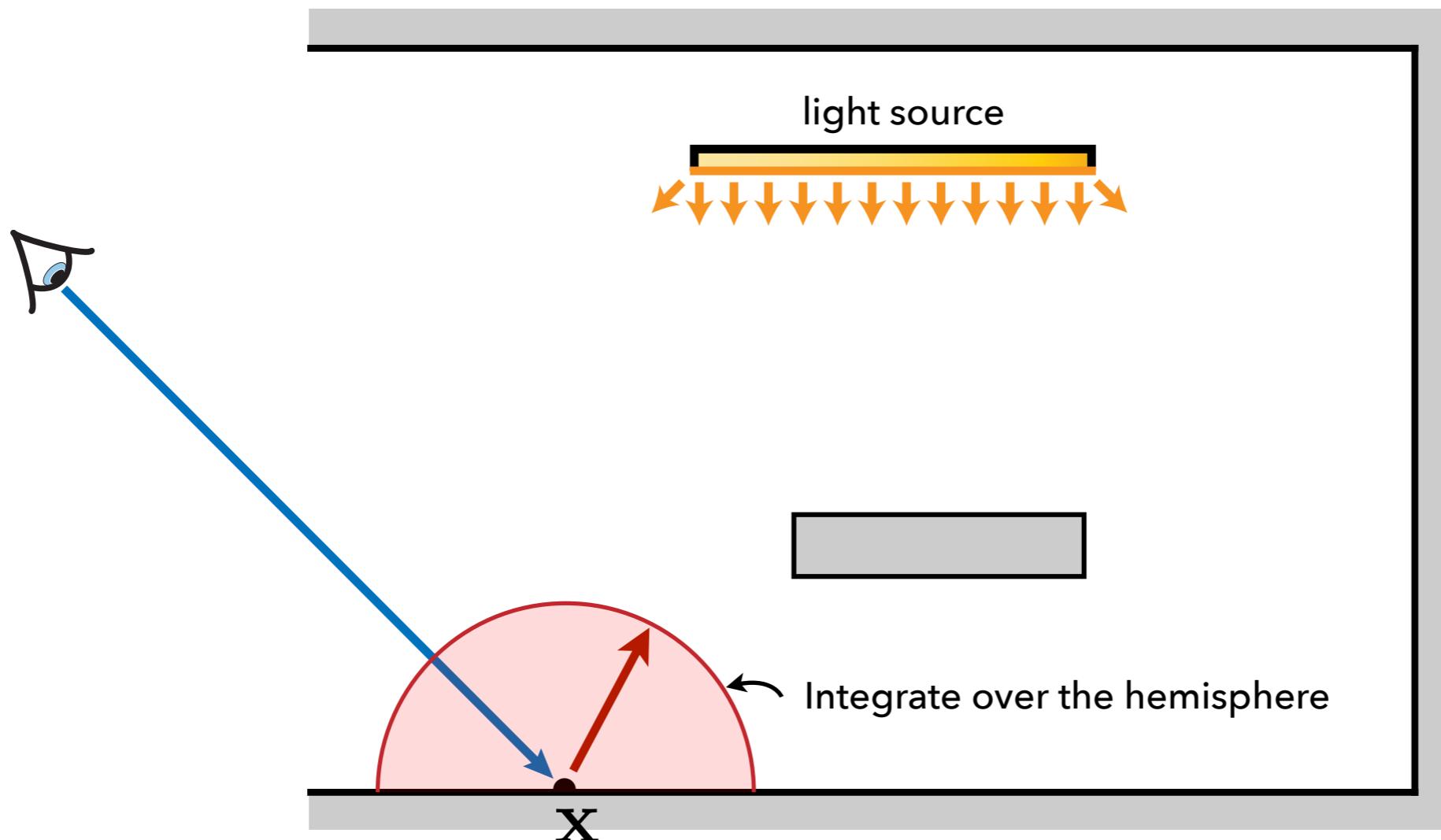


Direct + indirect  
illumination



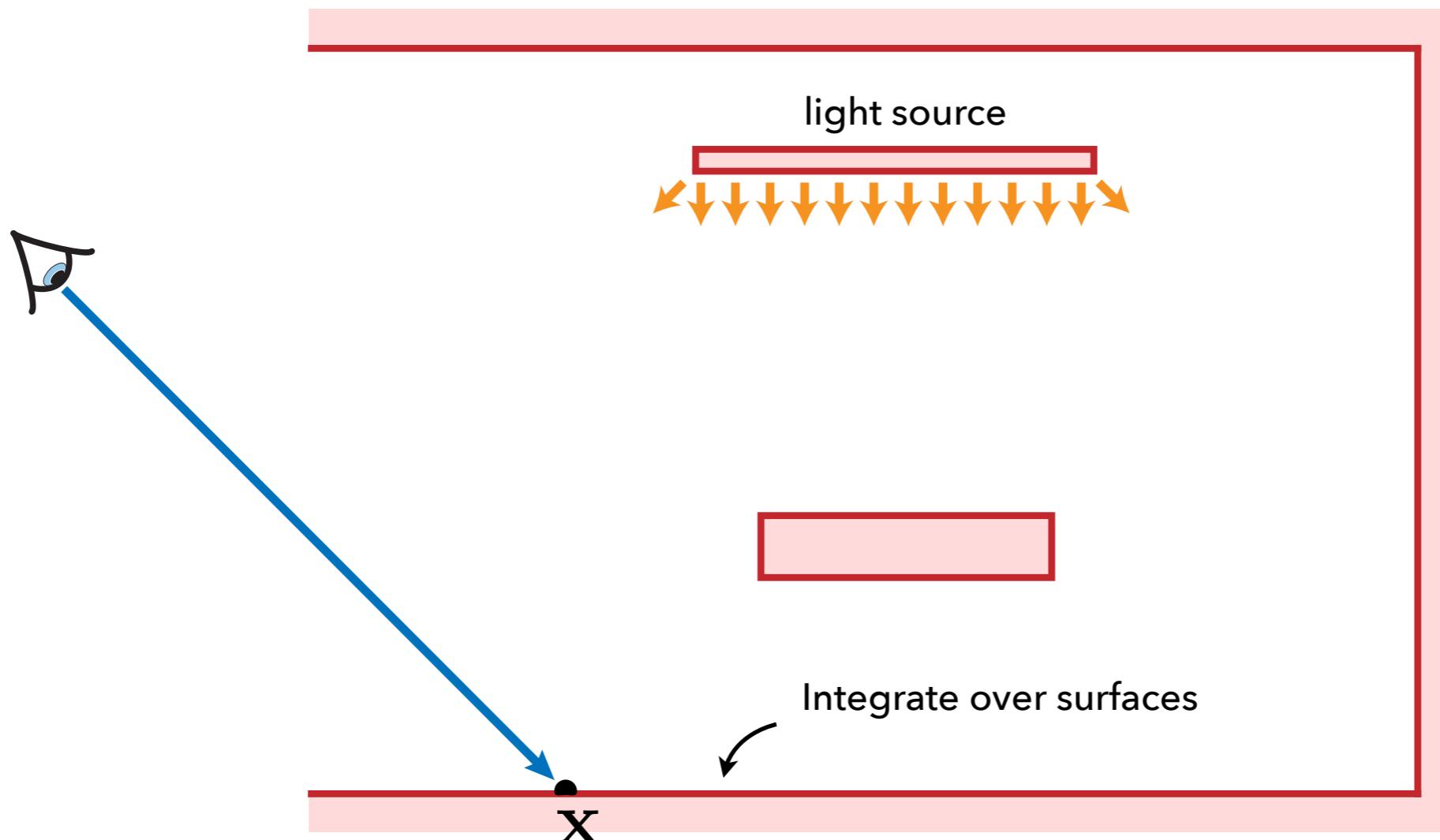
# RE - Hemispherical Form

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$



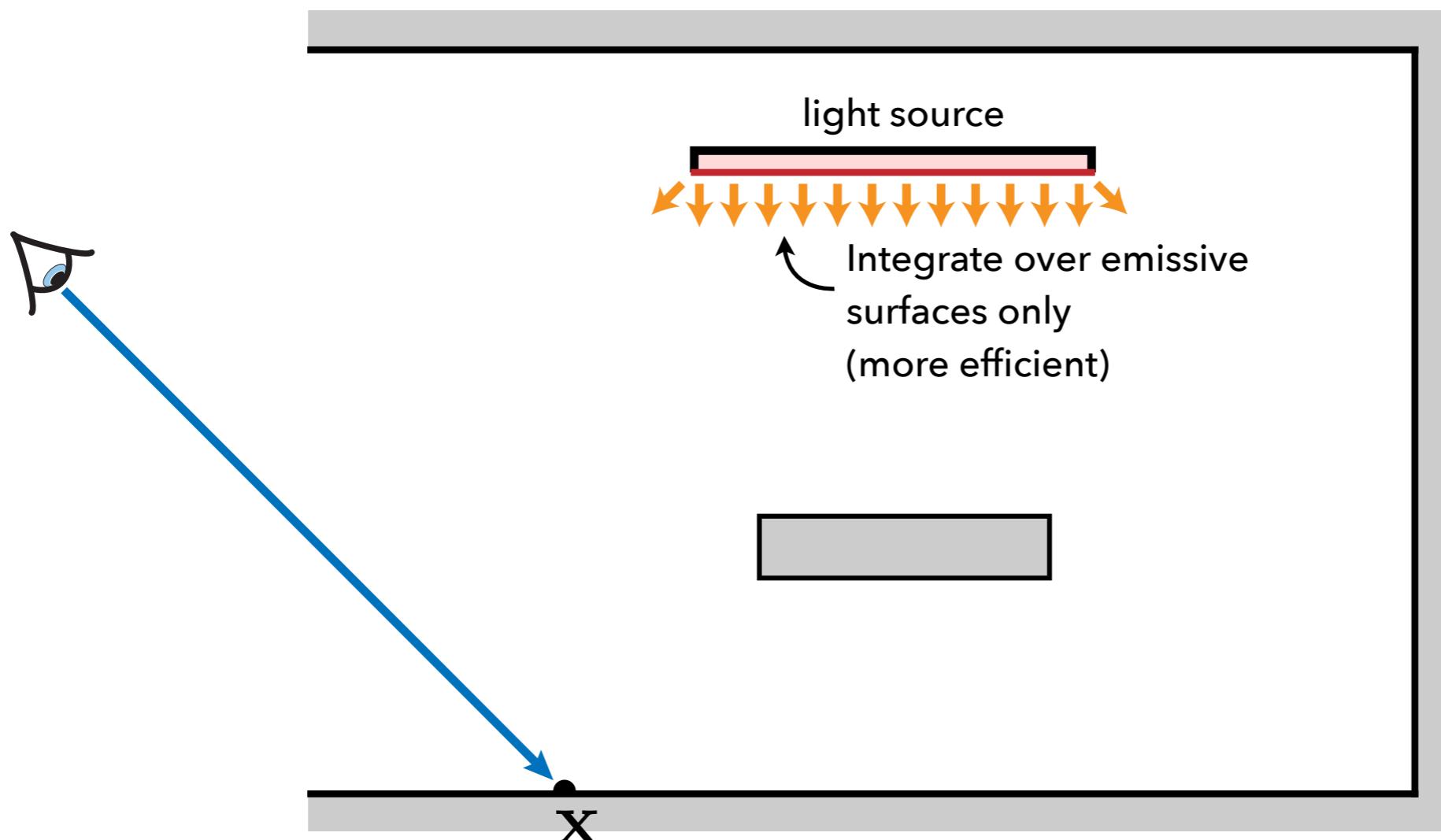
# RE - Surface Area Form

$$L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_e(\mathbf{y}, \mathbf{x}) V(\mathbf{x}, \mathbf{y}) \frac{|\cos \theta_i| |\cos \theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA(\mathbf{y})$$



# RE - Surface Area Form

$$L_r(\mathbf{x}, \mathbf{z}) = \int_{A_e} f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_e(\mathbf{y}, \mathbf{x}) V(\mathbf{x}, \mathbf{y}) \frac{|\cos \theta_i| |\cos \theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA(\mathbf{y})$$



# Light Sources

Point  
light



Spot  
light



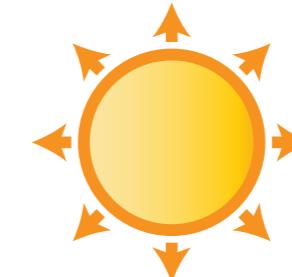
Directional  
light



Quad  
light



Sphere  
light



Mesh  
light



Delta lights

(create hard shadows)

Area/Shape lights

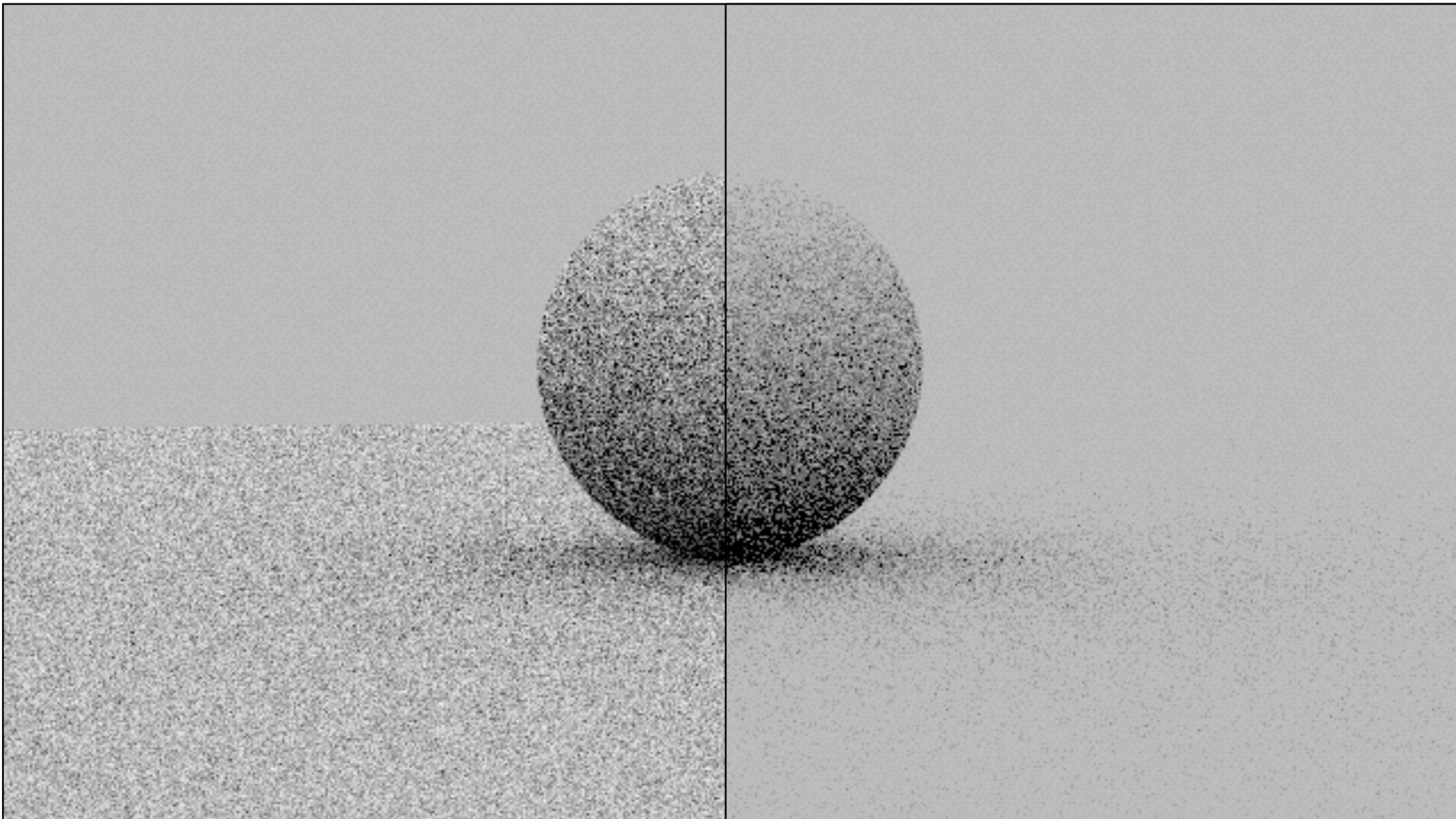
(create soft shadows)

# Importance Sampling the Cosine

Uniform hemispherical sampling

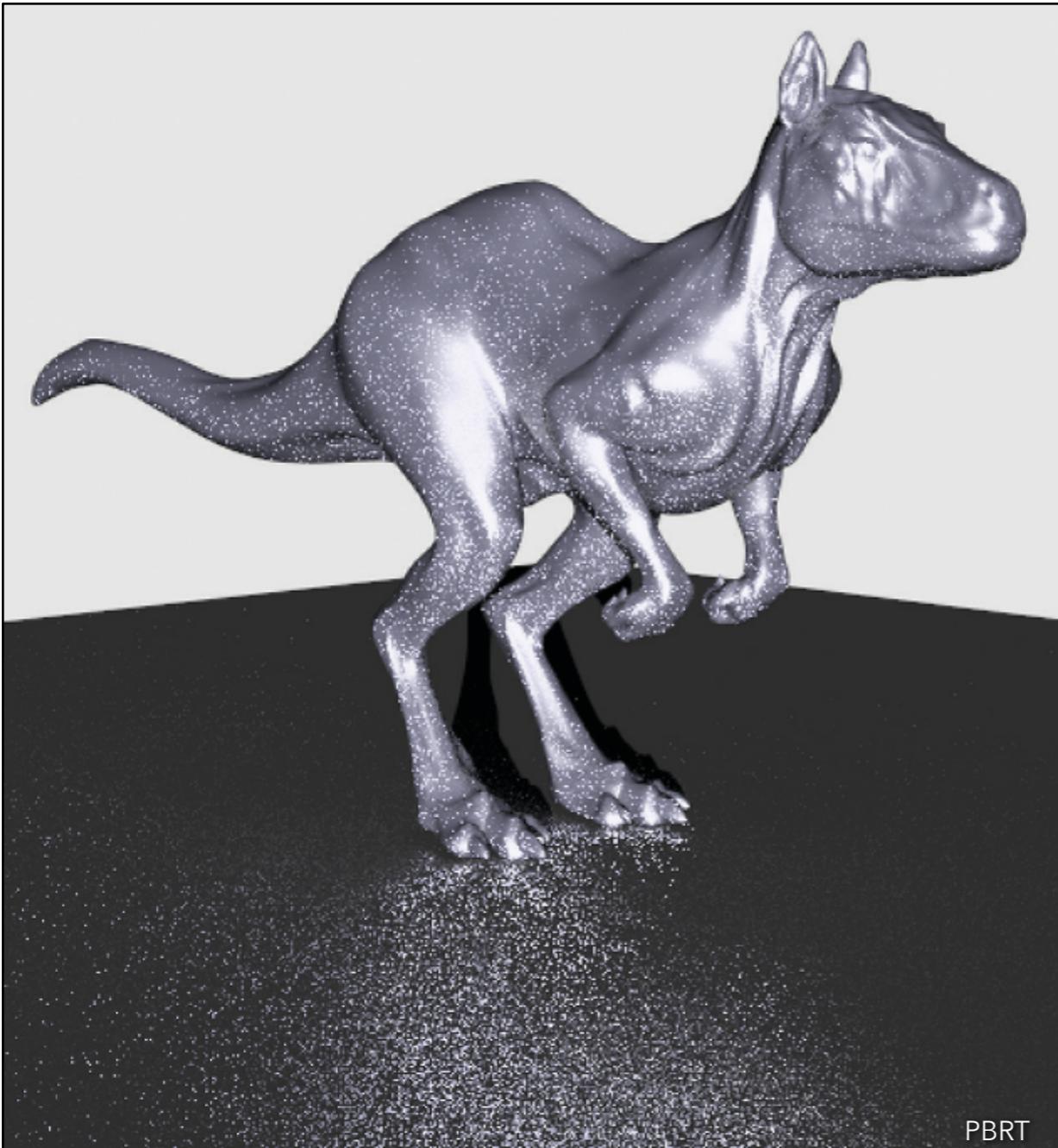
Cosine-weighted importance sampling

4 samples/pixel

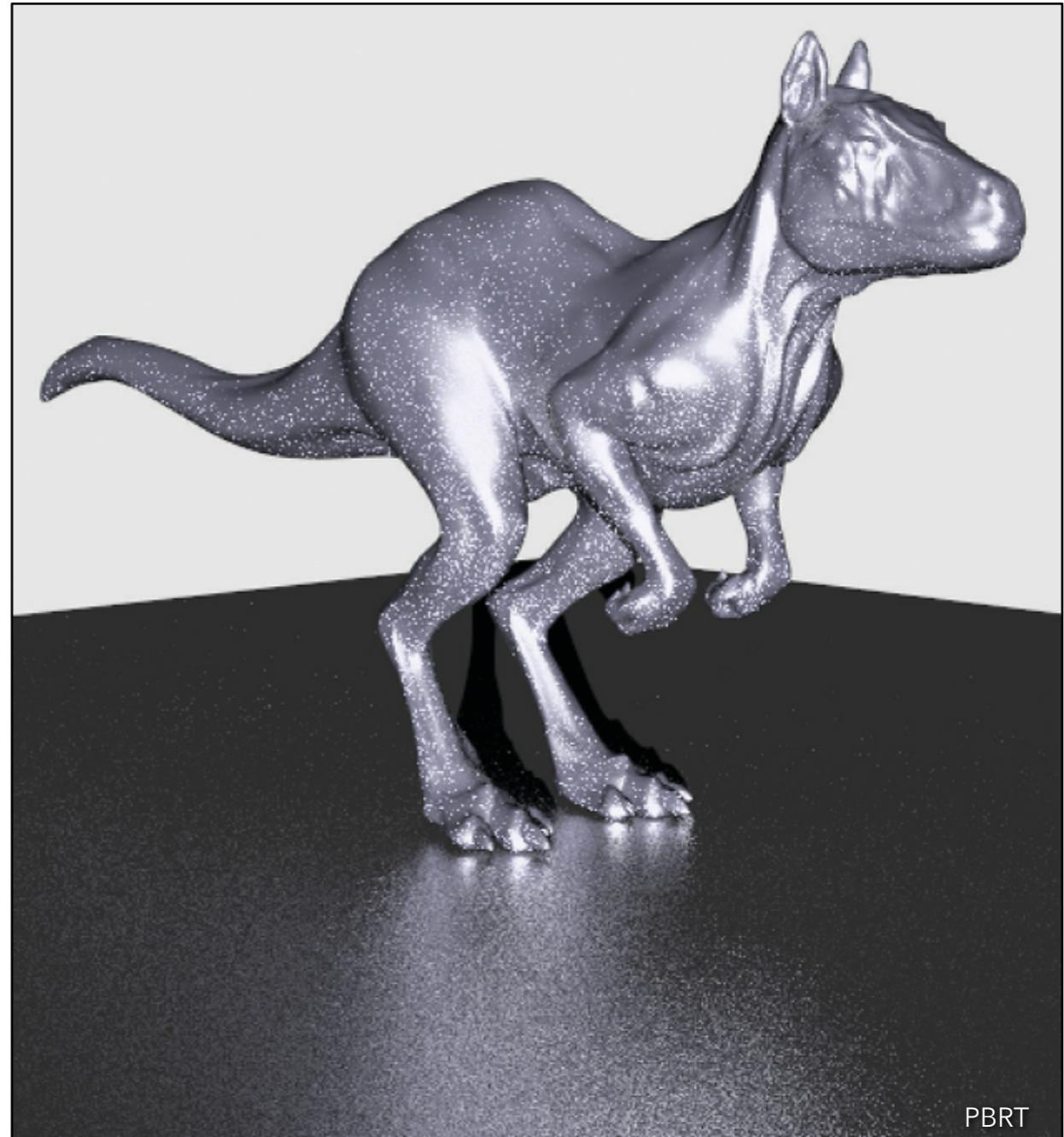


# Importance Sampling the BRDF

Uniform hemispherical sampling



BRDF importance sampling



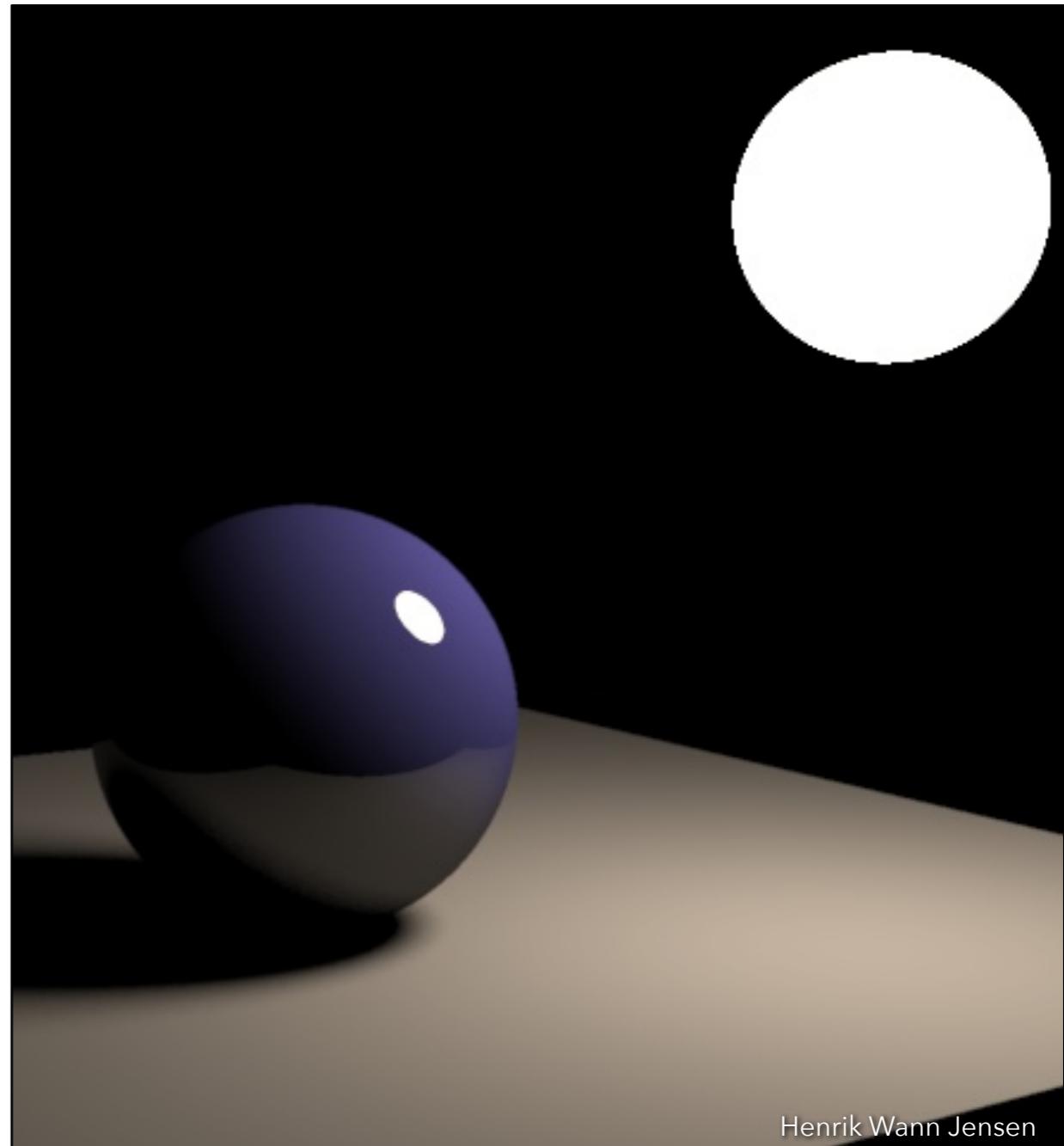
# Importance Sampling the Light

Uniform hemispherical sampling



Henrik Wann Jensen

Uniform sampling of subtended solid angle



Henrik Wann Jensen

# Multiple Importance Sampling

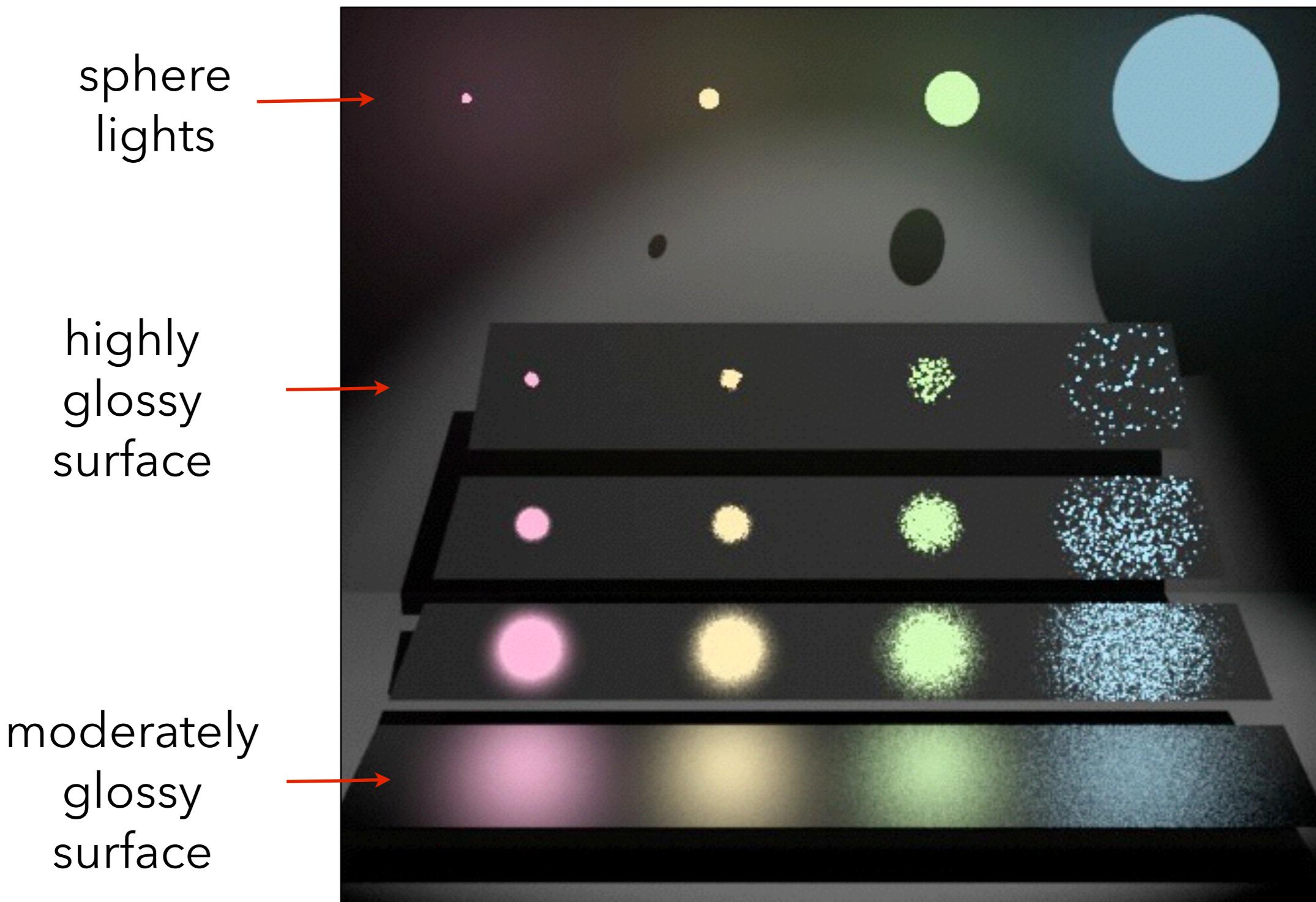
- *Multi-sample* model

$$\langle F^{\sum N_s} \rangle = \sum_{s=1}^M \frac{1}{N_s} \sum_{i=1}^{N_s} w_s(x_i) \frac{f(x_i)}{p_s(x_i)}$$

- *Balance* heuristic:

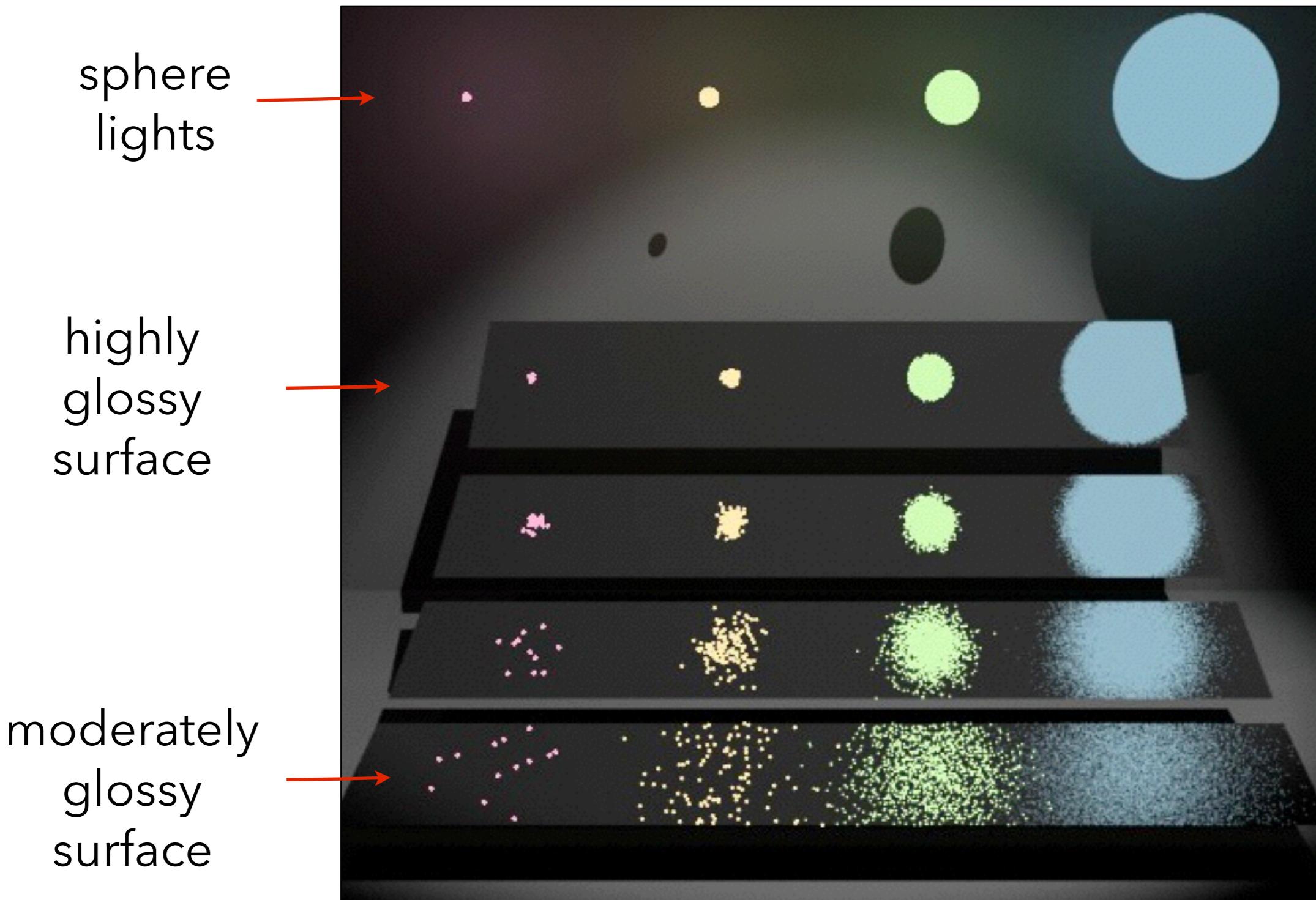
$$w_s(x) = \frac{N_s p_s(x)}{\sum_j N_j p_j(x)}$$

# Sampling the Light



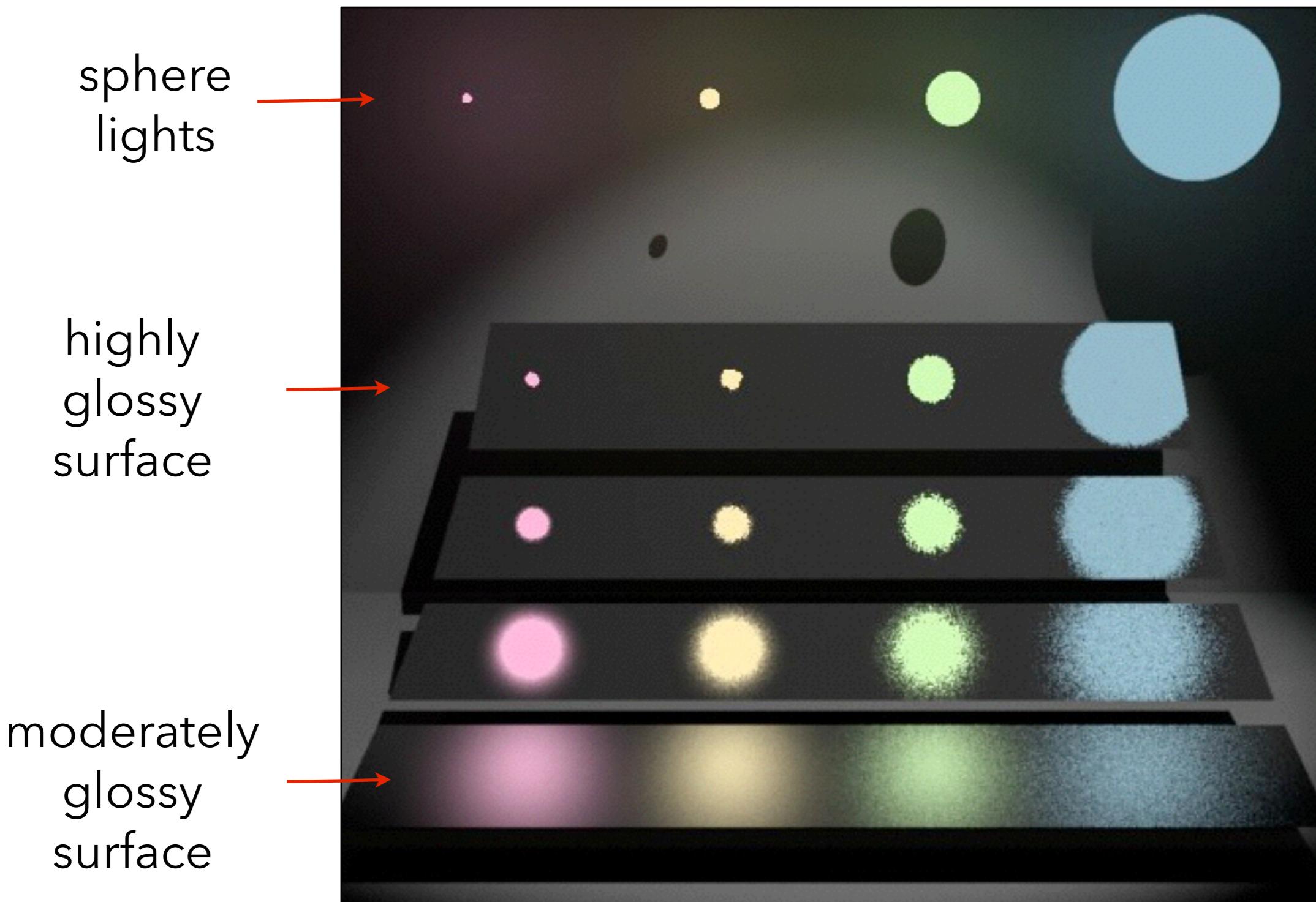
Eric Veach and Leonidas J. Guibas 1995.

# Sampling the BRDF



Eric Veach and Leonidas J. Guibas 1995.

# Multiple Importance Sampling



Eric Veach and Leonidas J. Guibas 1995.

# Today's Menu

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- Environment lighting (image-based lighting)
- Marginal/conditional inversion sampling
- Hierarchical sample warping
- Product sampling

# Environment Lighting

# Related techniques

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- Reflection mapping
- Environment mapping/lighting
- Image-based lighting (IBL)

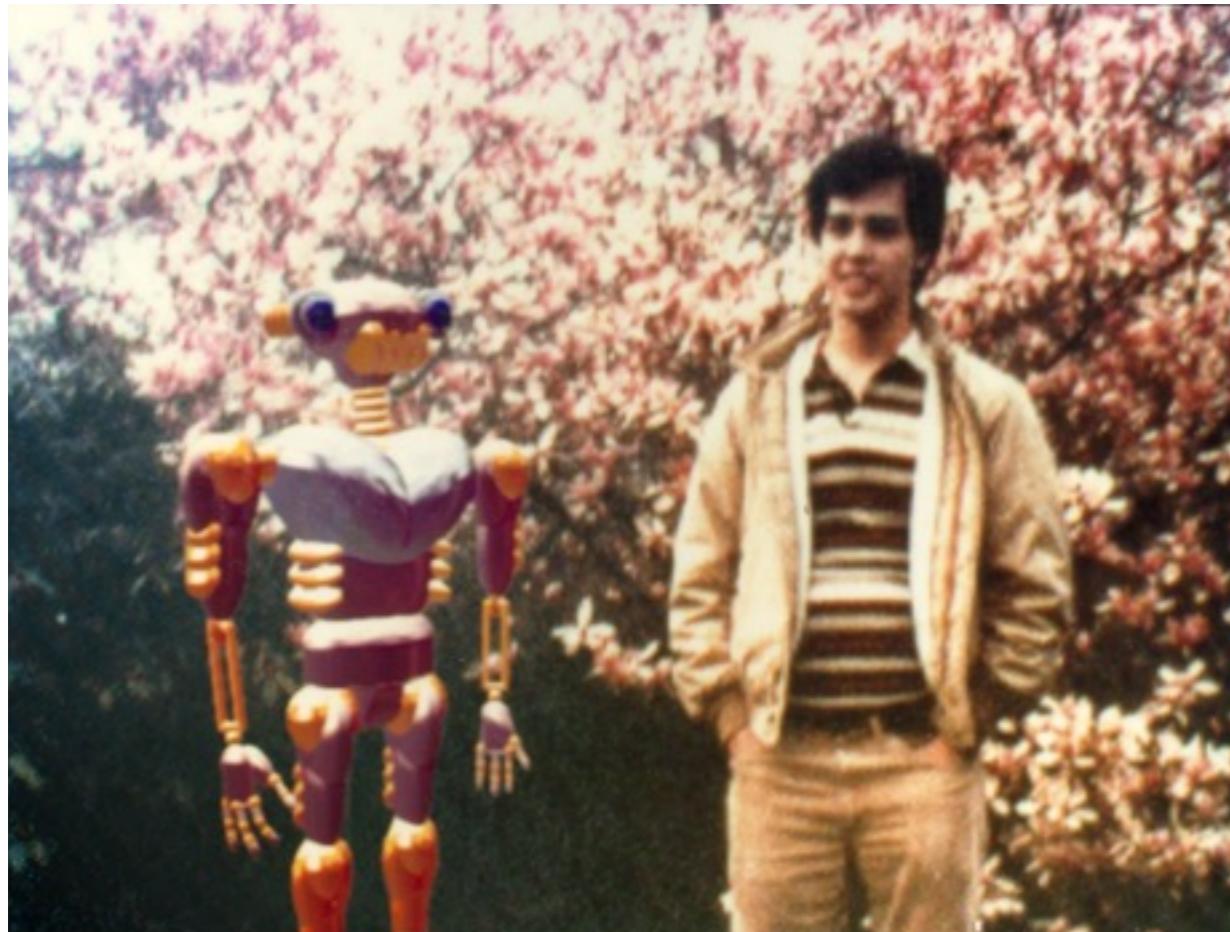
# Environment Lighting

- Sidesteps tedious modeling of the environment by representing it using one or more *images*
- The image “wraps” around the virtual scene, serving as a *distant* source of illumination



source: [3delight](#)

# Reflection Mapping (1982)

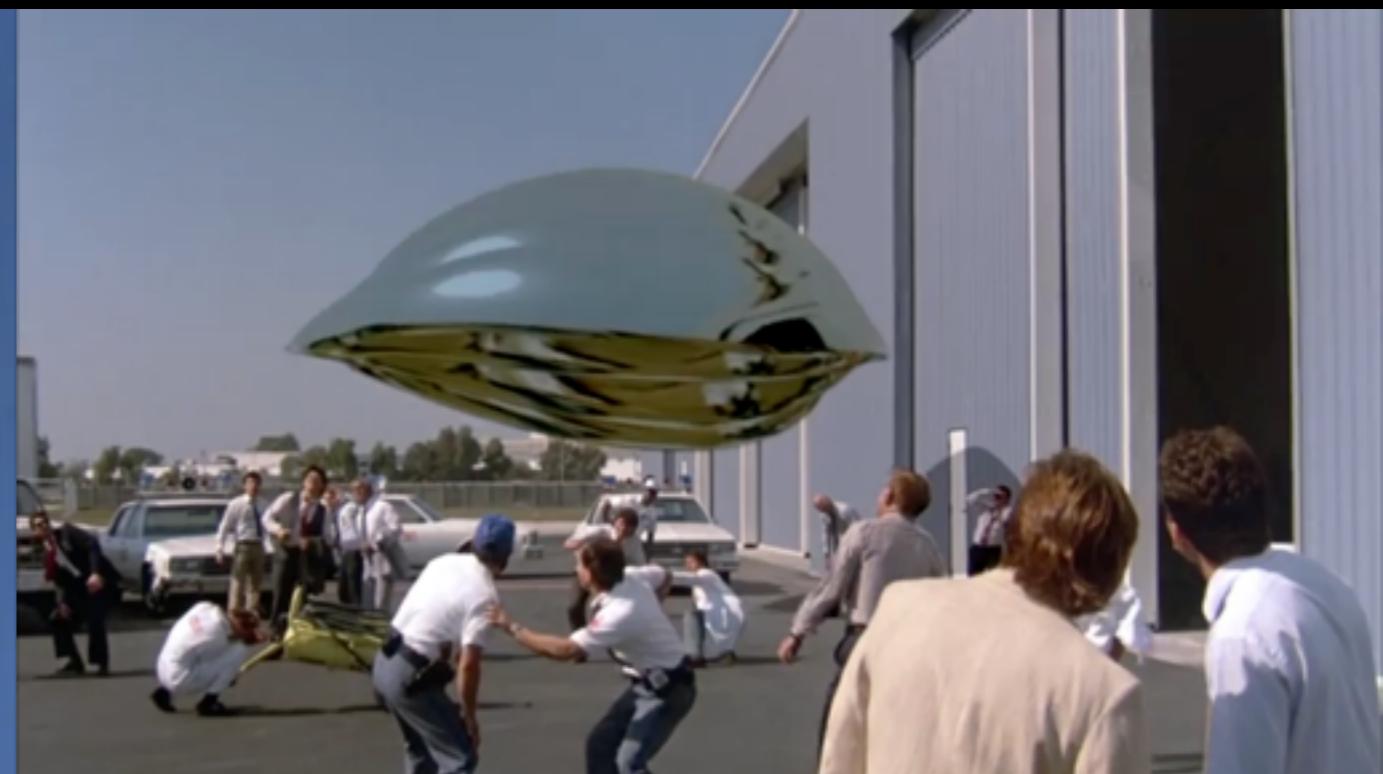


Mike Chou and Lance Williams



Gene Miller and Ken Perlin

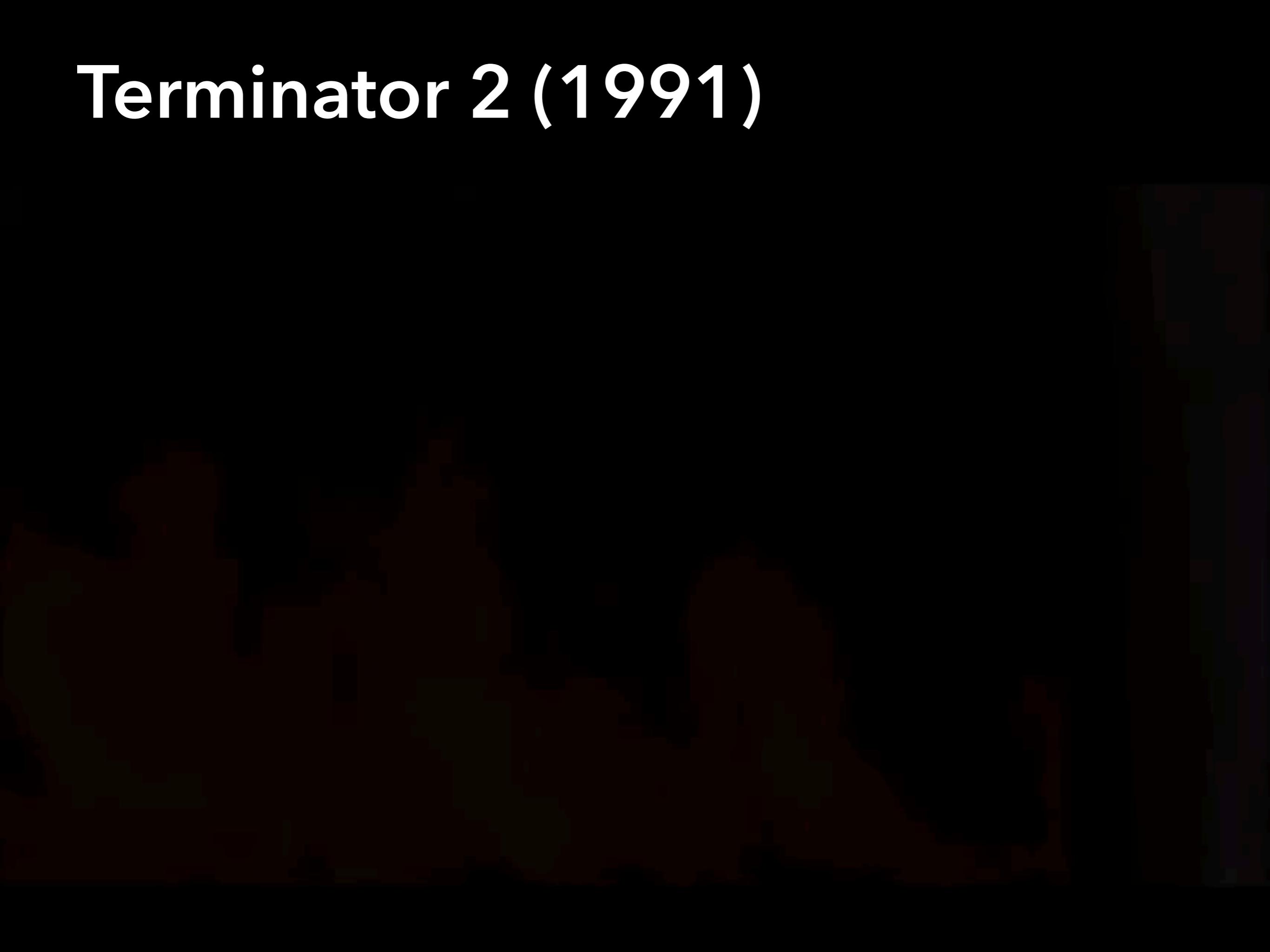
# Flight of the Navigator (1986)



# The Abyss (1989)



# Terminator 2 (1991)



# Fiat Lux

SIGGRAPH 99 Electronic Theater

# More details

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- More details on Fiat Lux
  - <http://www.pauldebevec.com/FiatLux/>
- History of reflection mapping
  - <http://www.pauldebevec.com/ReflectionMapping/>

# Acquiring Environment Lighting

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- Mirrored ball + camera
- Fisheye lens images
- Stitching images together
- Panoramic camera

# Acquisition - Low Tech.



Maurits C. Escher



ogbog.net

Lightprobe



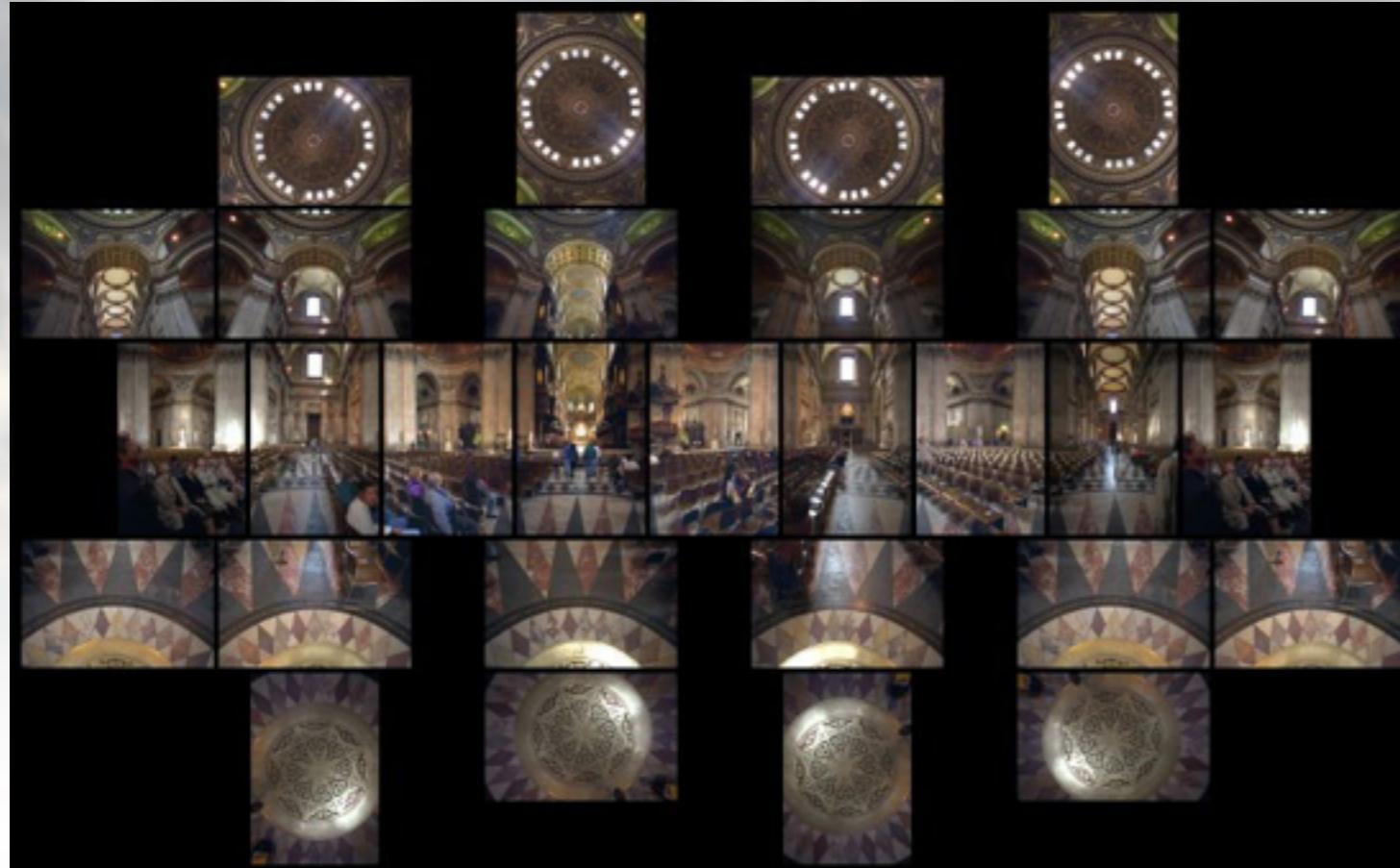
omnidirectional,  
360° panoramic,  
**HDR image**

# Acquisition - Even Lower Tech.

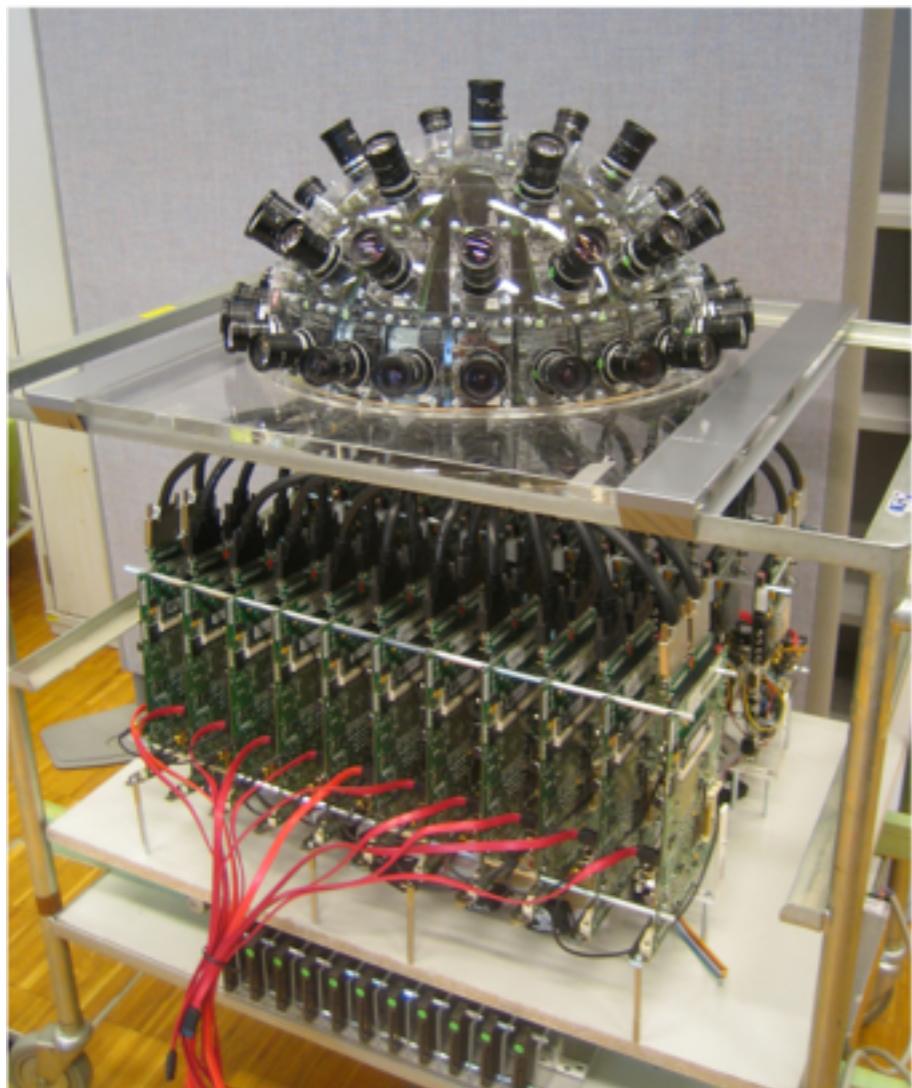


Gene Miller and Ken Perlin

# Acquisition - Stitched Panorama



# Acquisition - High Tech.



lsm.epfl.ch



[immersivemedia.com](http://immersivemedia.com)

# Storing Environment Maps

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- Various ways to represent/parametrize environment maps
- Related to cartography
  - Projecting the earth (sphere) onto a plane

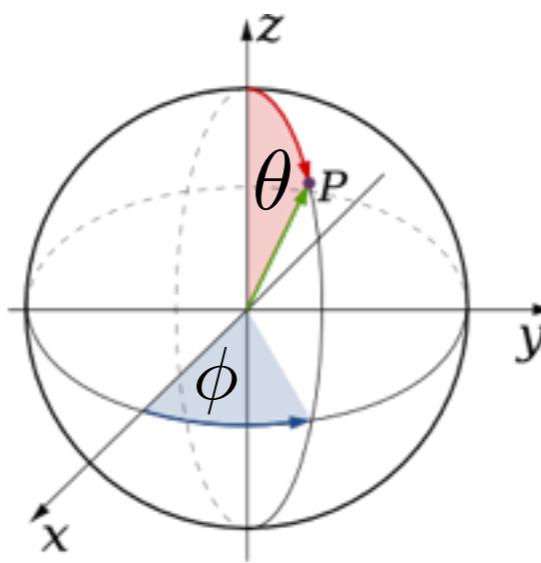
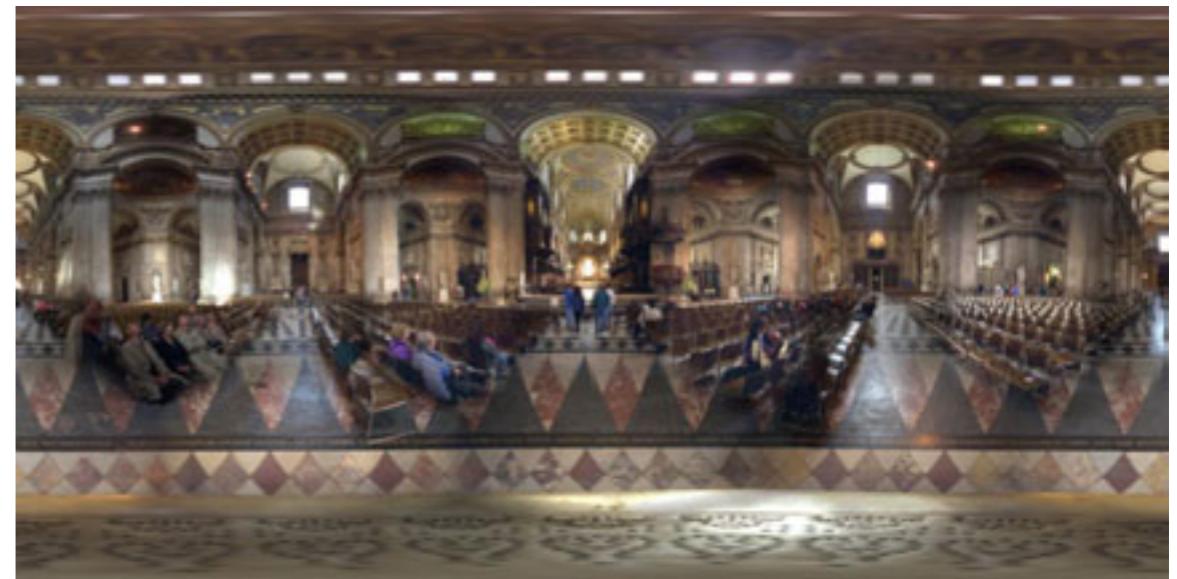
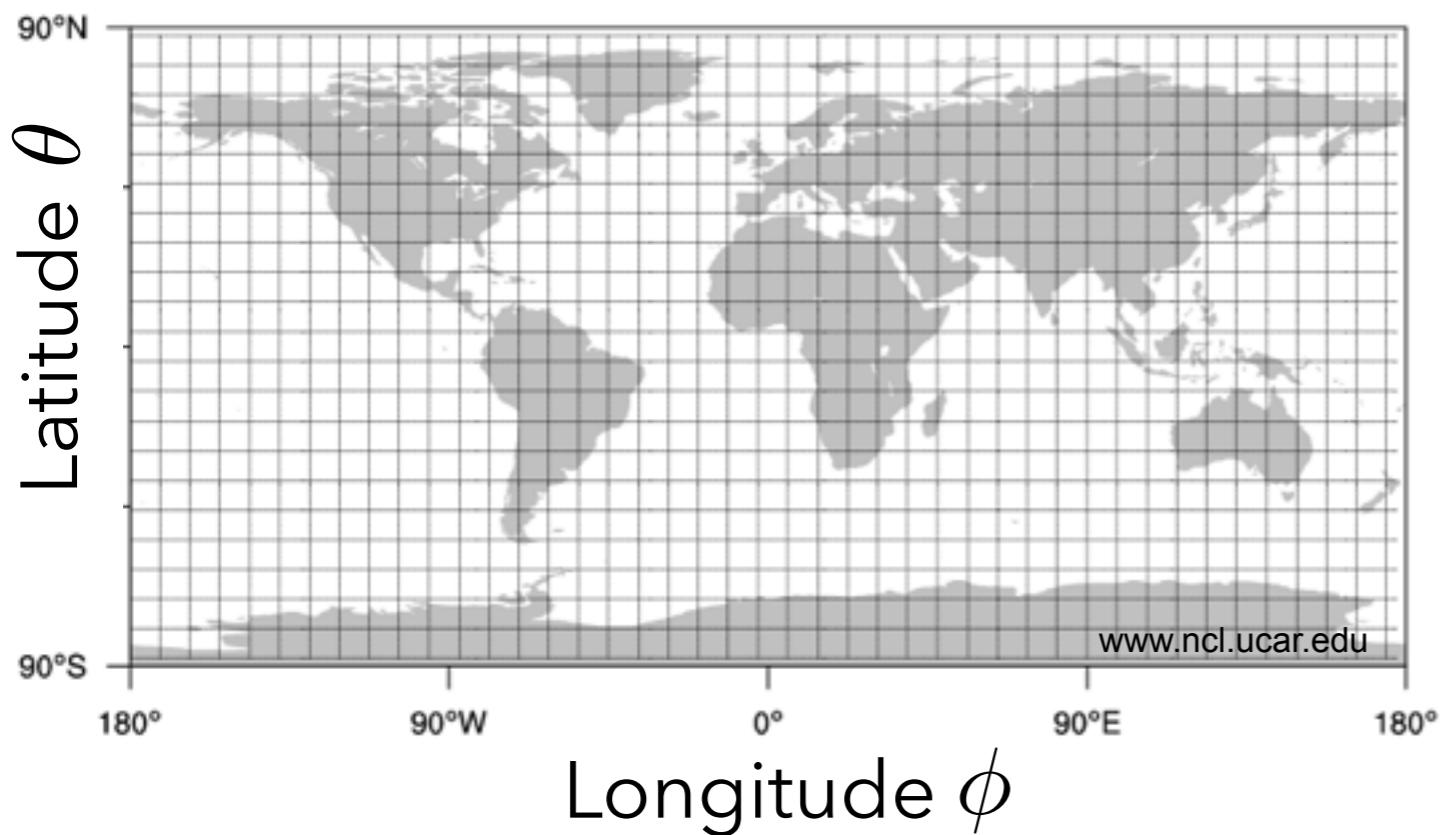
# Mirror Ball



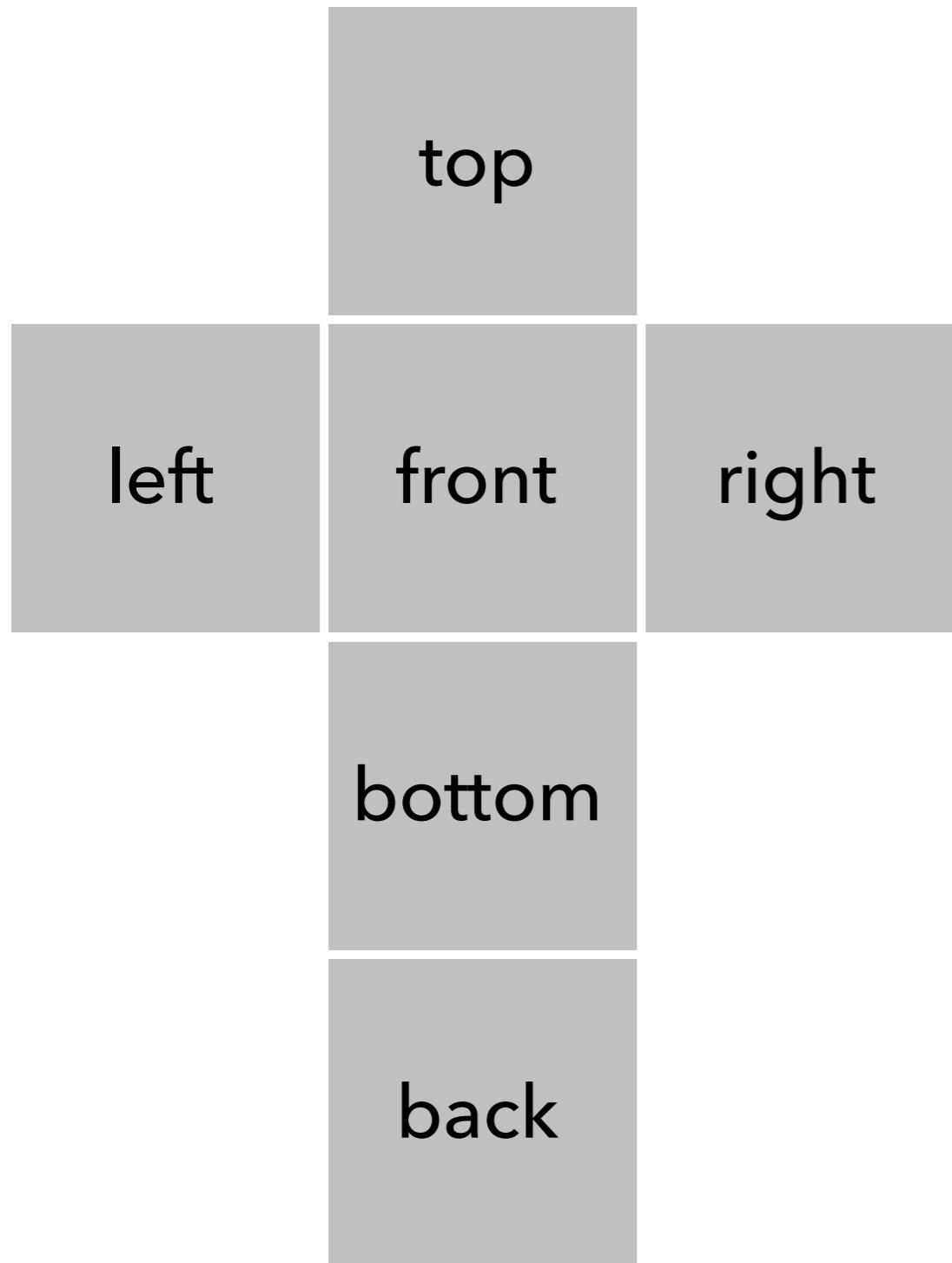
# Angular Map



# Latitude/Longitude Map

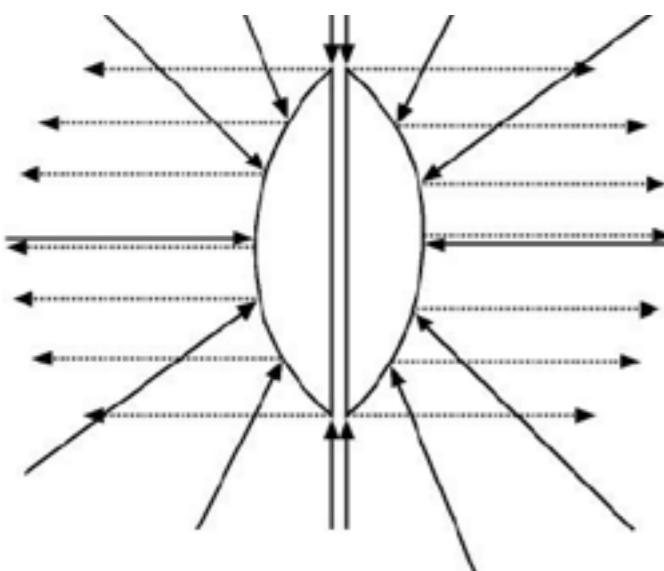
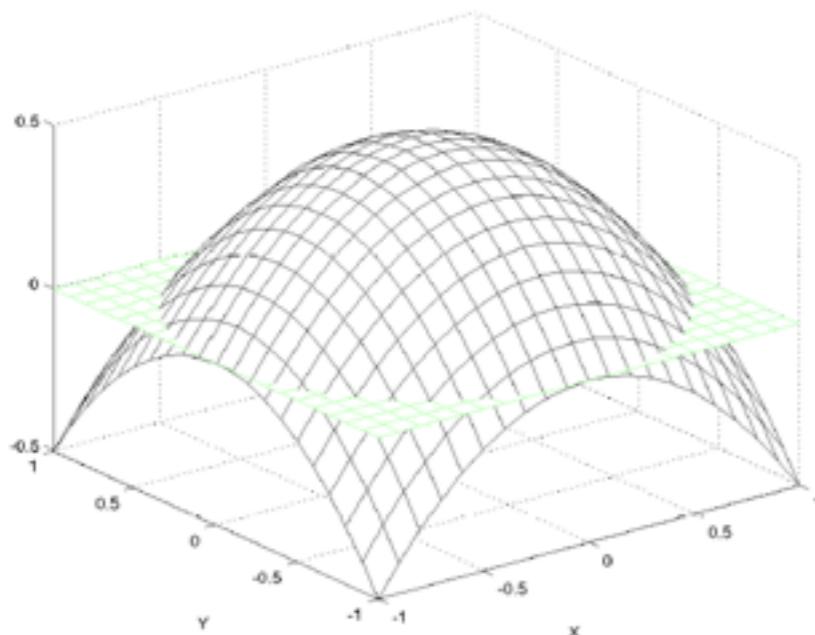


# Cube Map (Skybox)



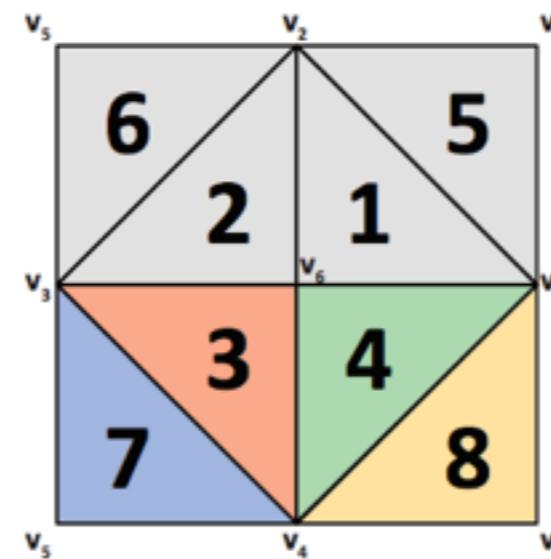
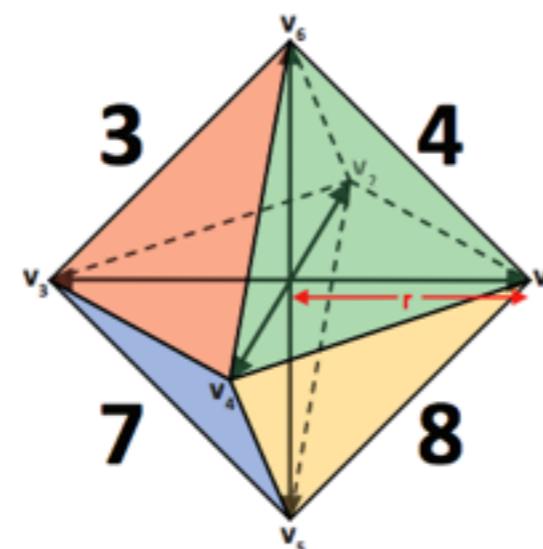
# Other Parameterizations

Dual-paraboloid



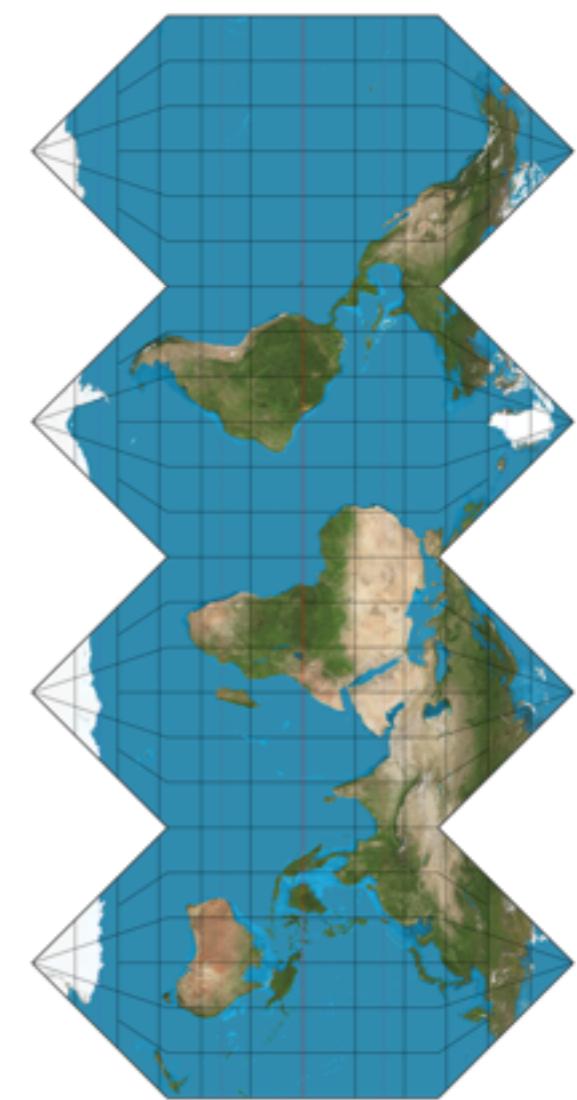
Courtesy of Brabec

Octahedron map



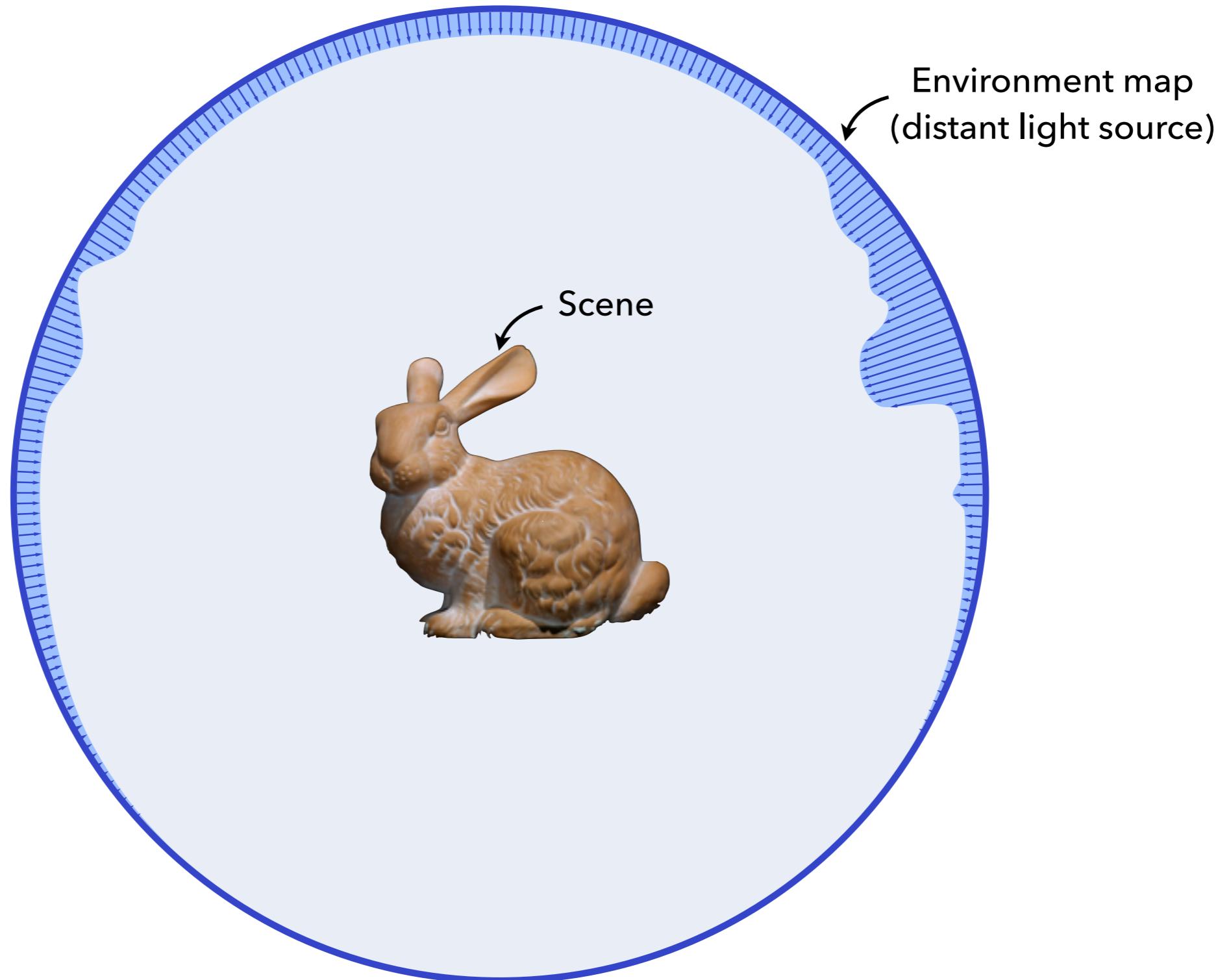
Courtesy of Engelhardt and Dachsbacher

HEALPix



Courtesy of Ryazanov

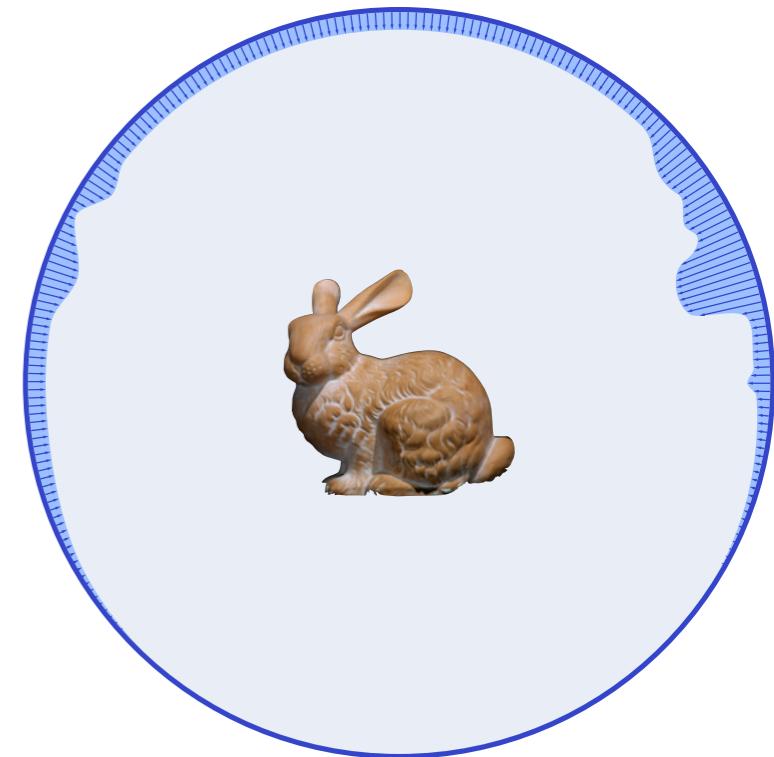
# Environment Lighting



# Environment Lighting

- The image “wraps” around the virtual scene, serving as a *distant* source of illumination

$$\begin{aligned} L_r(\mathbf{x}, \vec{\omega}_r) &= \int_{\Omega} f_r(\vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i \\ &= \int_{\Omega} f_r(\vec{\omega}_i, \vec{\omega}_r) L_{\text{env}}(\vec{\omega}_i) V(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i \end{aligned}$$



# Environment Lighting



HENRIK WANN DENSEN - 2002

# Environment Lighting



$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{\Omega} f_r(\vec{\omega}_i, \vec{\omega}_r) L_{\text{env}}(\vec{\omega}_i) V(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

# Importance Sampling $L_{\text{env}}$



$$p(\vec{\omega}_i) \propto L_{\text{env}}(\vec{\omega}_i)$$

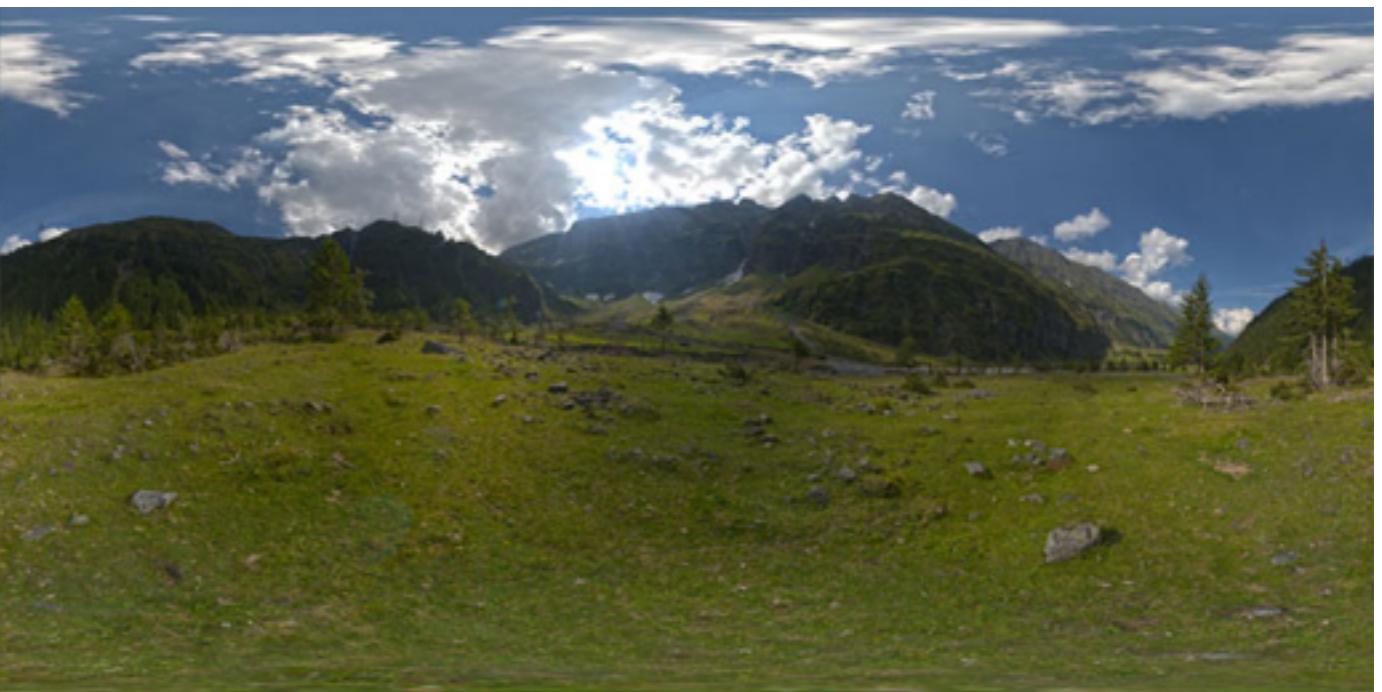
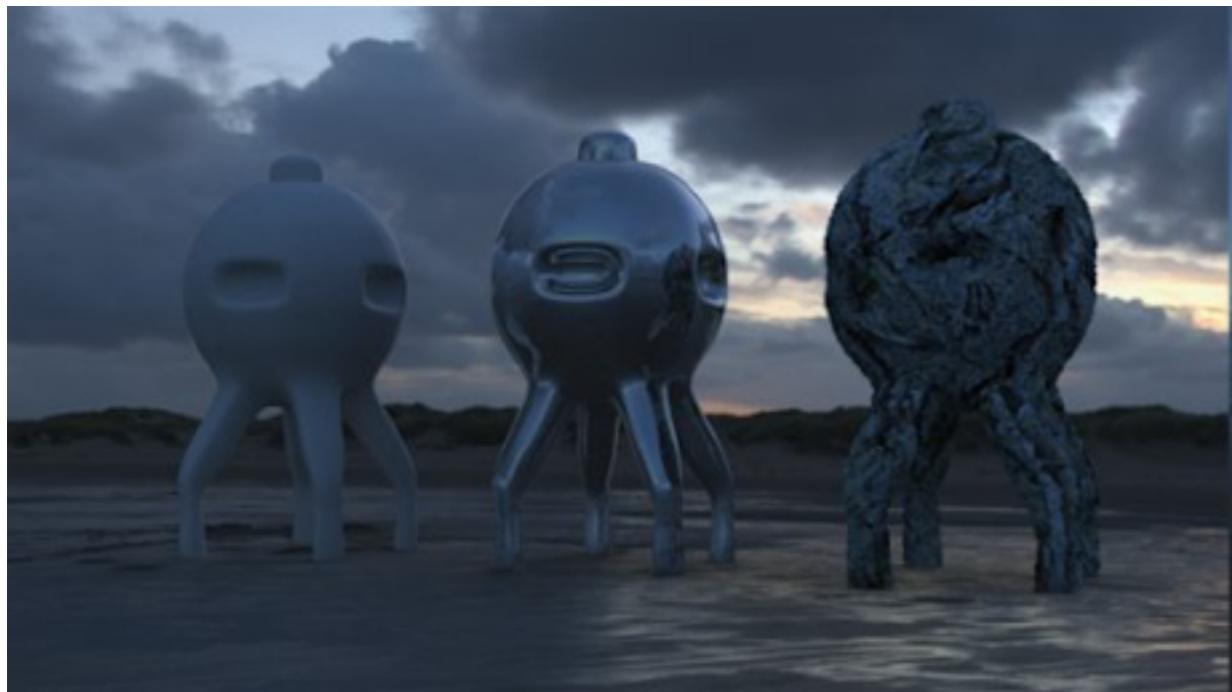
# Importance Sampling $L_{\text{env}}$

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$$p(\vec{\omega}_i) \propto L_{\text{env}}(\vec{\omega}_i)$$

- Several strategies exist
- We'll discuss:
  - Marginal/Conditional CDF method
  - Hierarchical warping method

# Visual Break



[openfootage.net](http://openfootage.net)

# Marginal/Conditional CDF

- Assume the lat/long parameterization
- Draw samples from joint  $p(\theta, \phi) \propto L_{\text{env}}(\theta, \phi) \sin \theta$

# Why the Sine?

- General case of integrating some  $f(\vec{\omega})$  over  $S^2$

$$d\vec{\omega} = \sin \theta d\theta d\phi$$

↗ Comes from the Jacobian

$$\begin{aligned}\int_{S^2} f(\vec{\omega}) d\vec{\omega} &= \int_0^{2\pi} \int_0^\pi f(\theta, \phi) \sin \theta d\theta d\phi \\ &\approx \frac{1}{N} \sum_{i=1}^N \frac{f(\theta_i, \phi_i) \sin \theta_i}{p(\theta_i, \phi_i)}\end{aligned}$$

- If we set  $p(\theta, \phi) \propto f(\theta, \phi) \sin \theta$  then the sines will effectively cancel out

# Marginal/Conditional CDF

- Assume the lat/long parameterization
- Draw samples from joint  $p(\theta, \phi) \propto L_{\text{env}}(\theta, \phi) \sin \theta$ 
  - Step 1: create scalar version  $L'(\theta, \phi)$  of  $L_{\text{env}}(\theta, \phi) \sin \theta$

- Step 2: compute marginal PDF

$$p(\theta) = \int_0^{2\pi} L'(\theta, \phi) d\phi$$

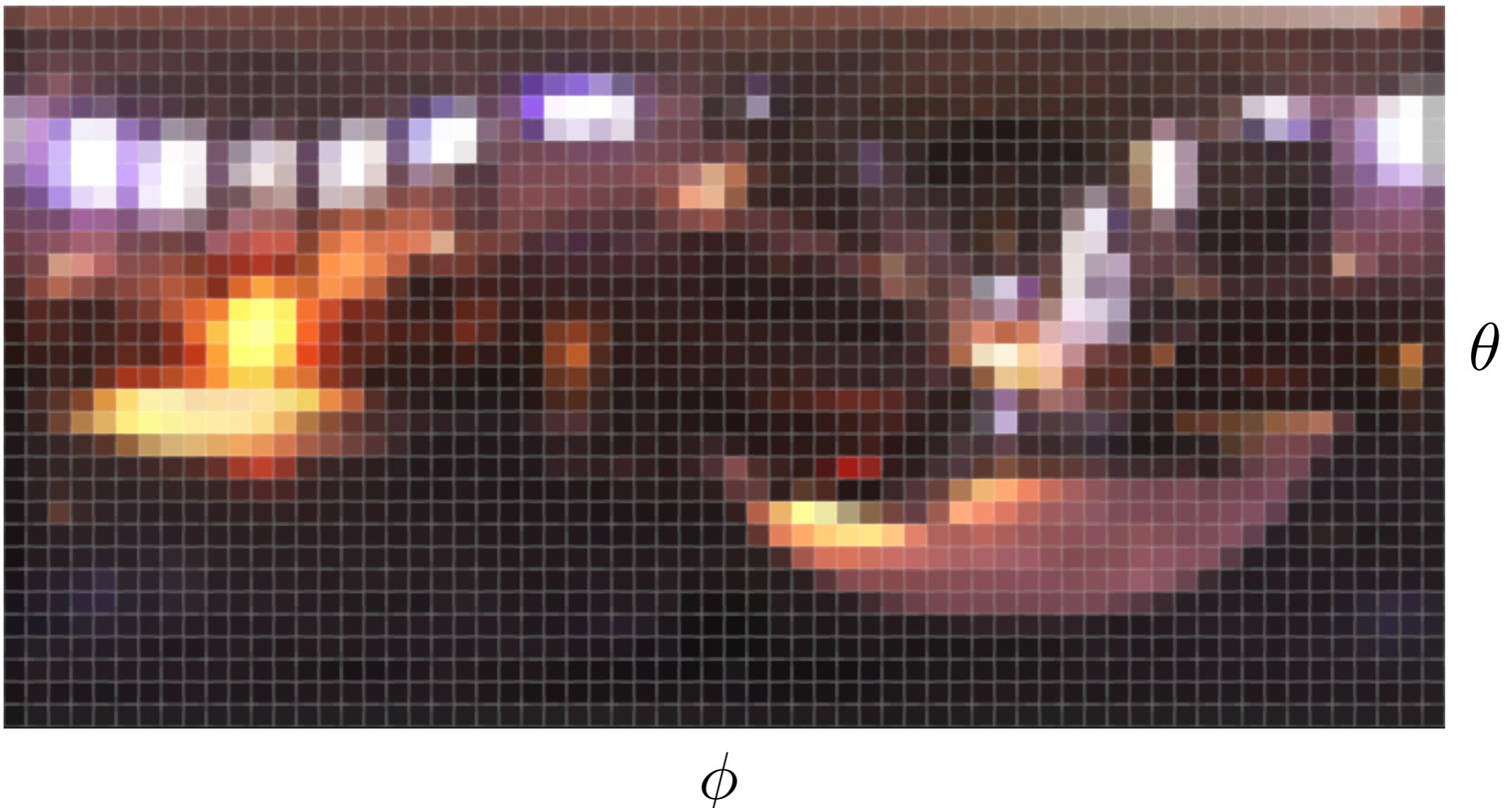
- Step 3: compute conditional PDF

$$p(\phi|\theta) = \frac{p(\theta, \phi)}{p(\theta)}$$

- Step 4: draw samples  $\theta_i \sim p(\theta)$  and  $\phi_i = p(\phi|\theta)$

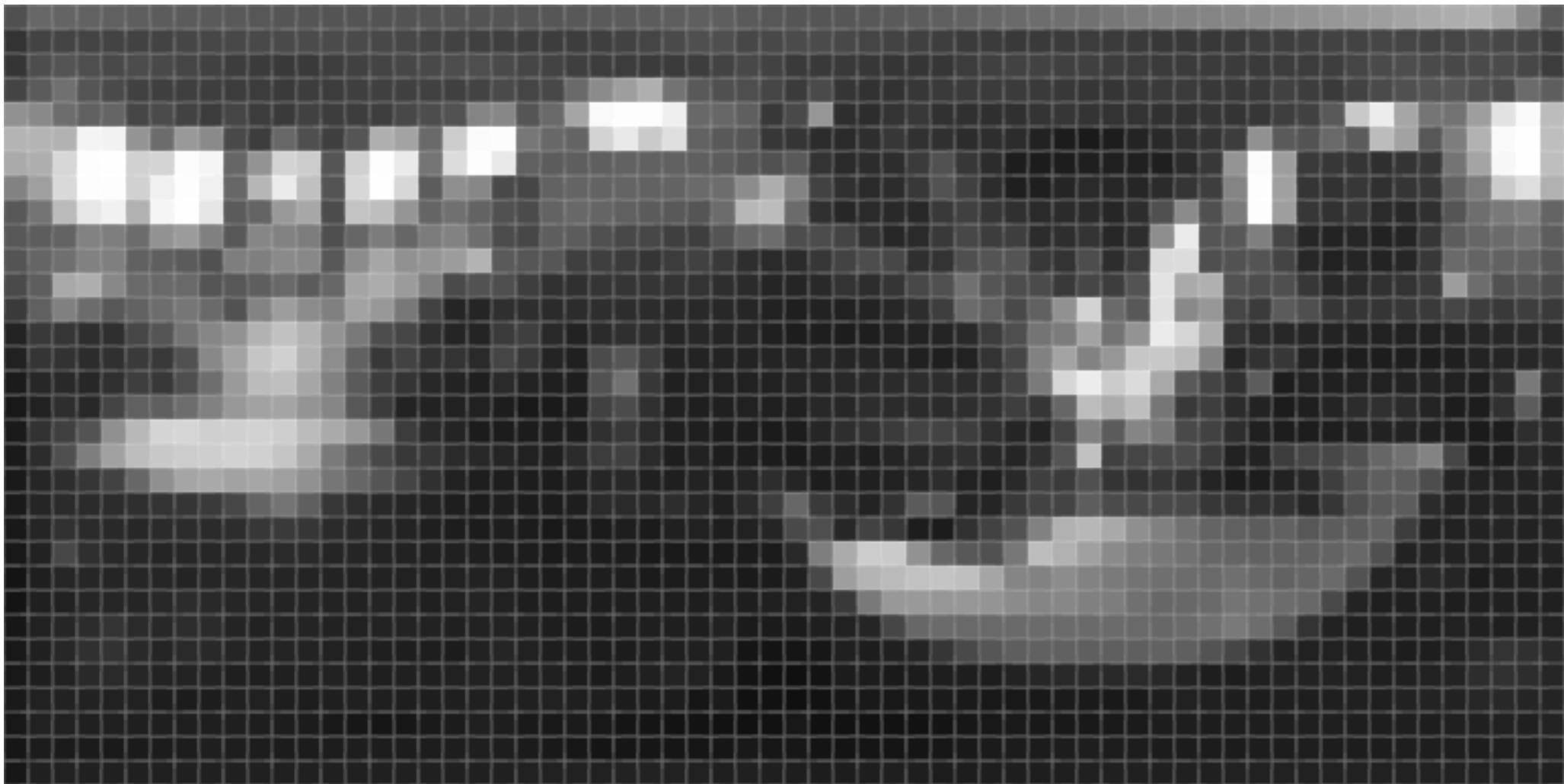
# Step 1: Scalar Importance Func.

Original environment map



# Step 1: Scalar Importance Func.

Scalar version  
(average, max, or luminance of RGB channels)

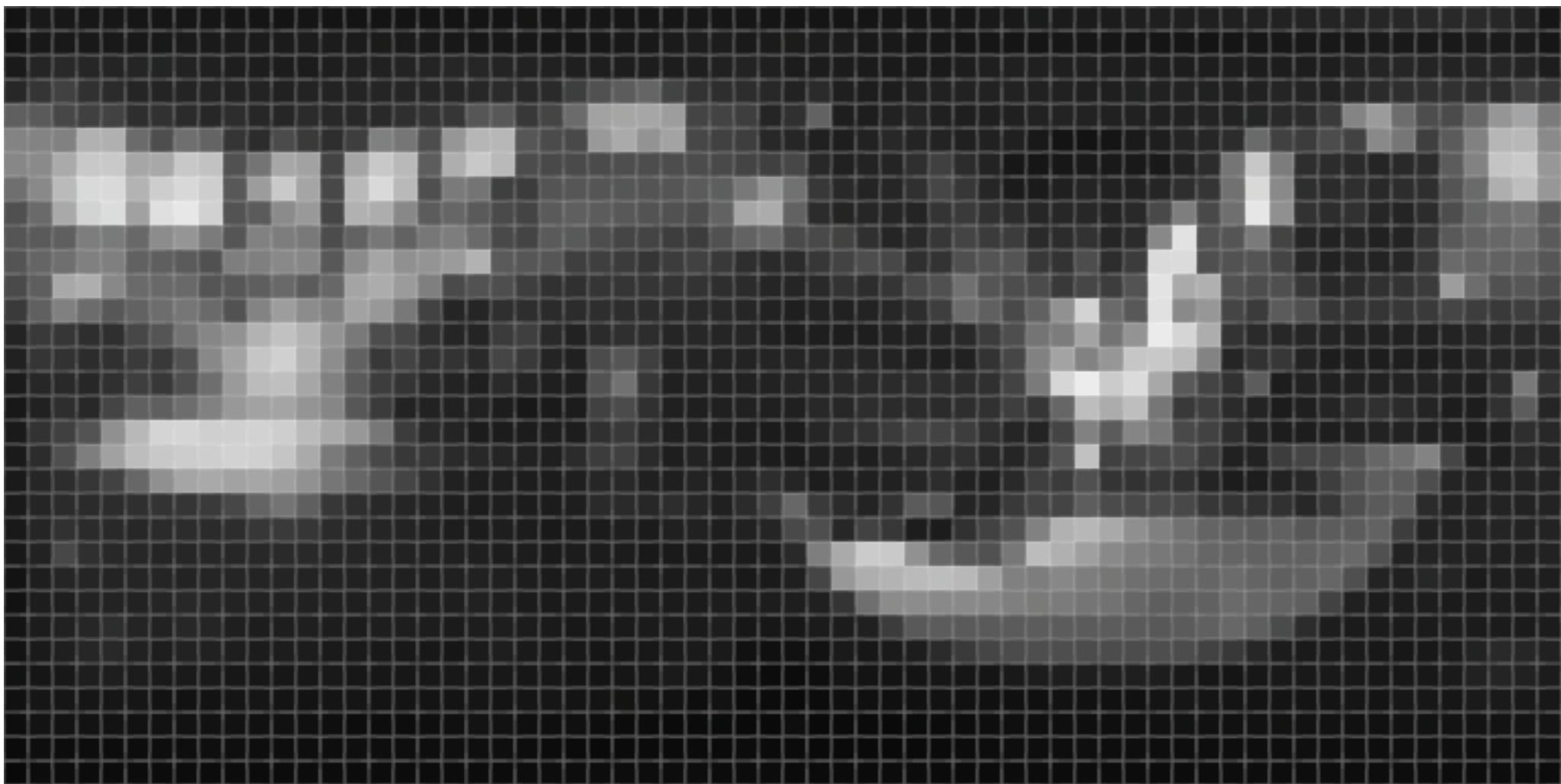


$\phi$

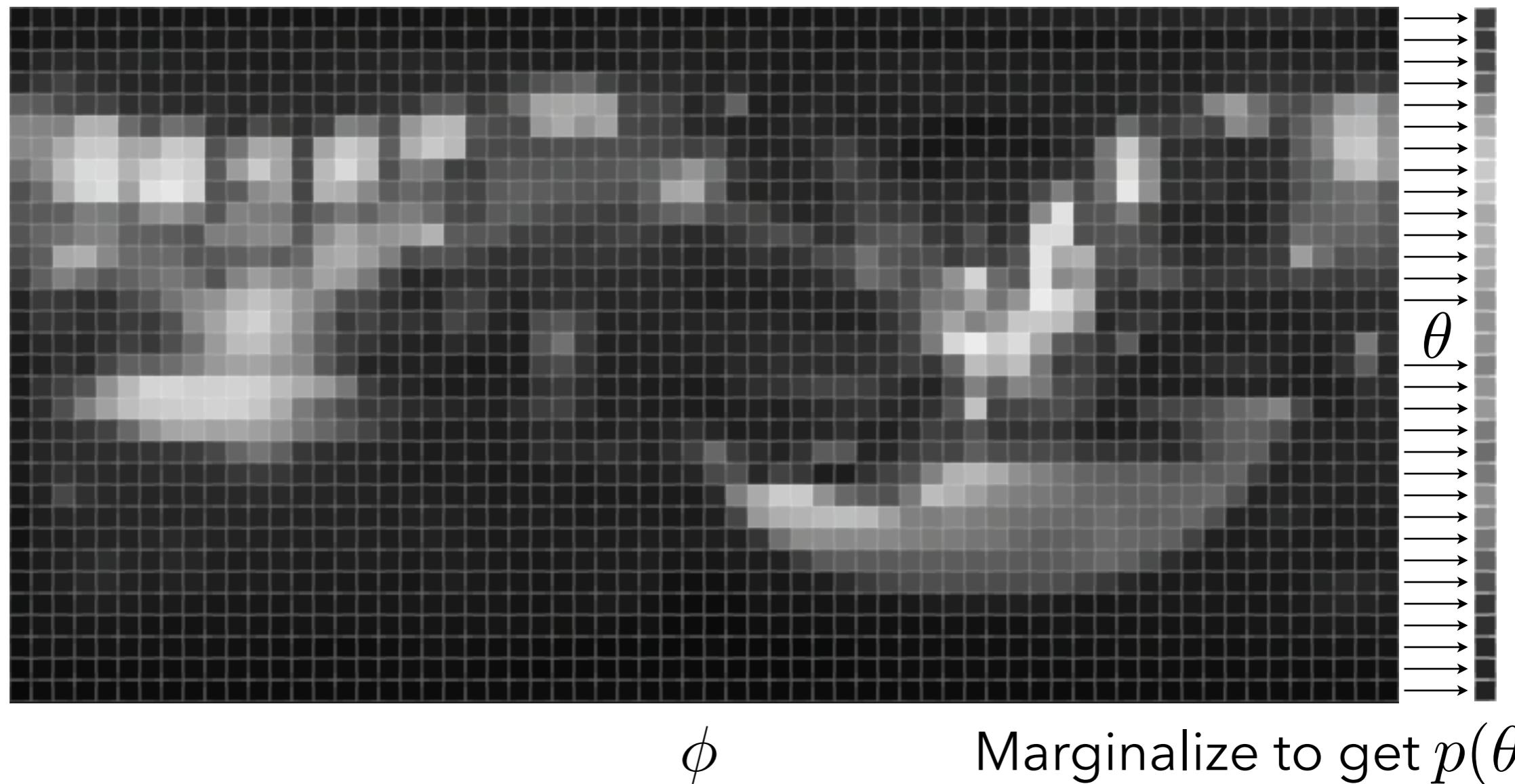
$\theta$

# Step 1: Scalar Importance Func.

Multiplied by  $\sin \theta$

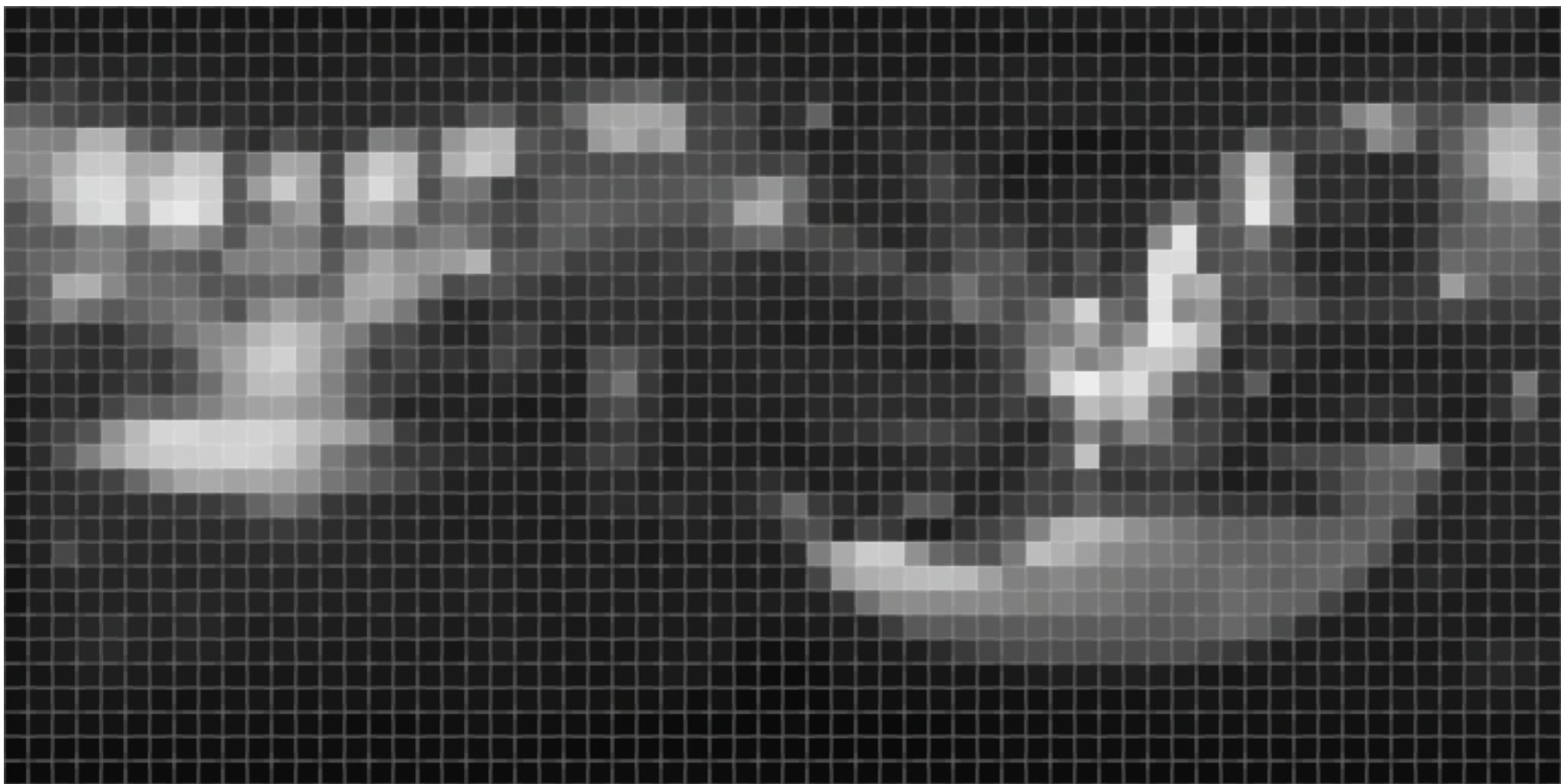


# Step 2: Marginalization

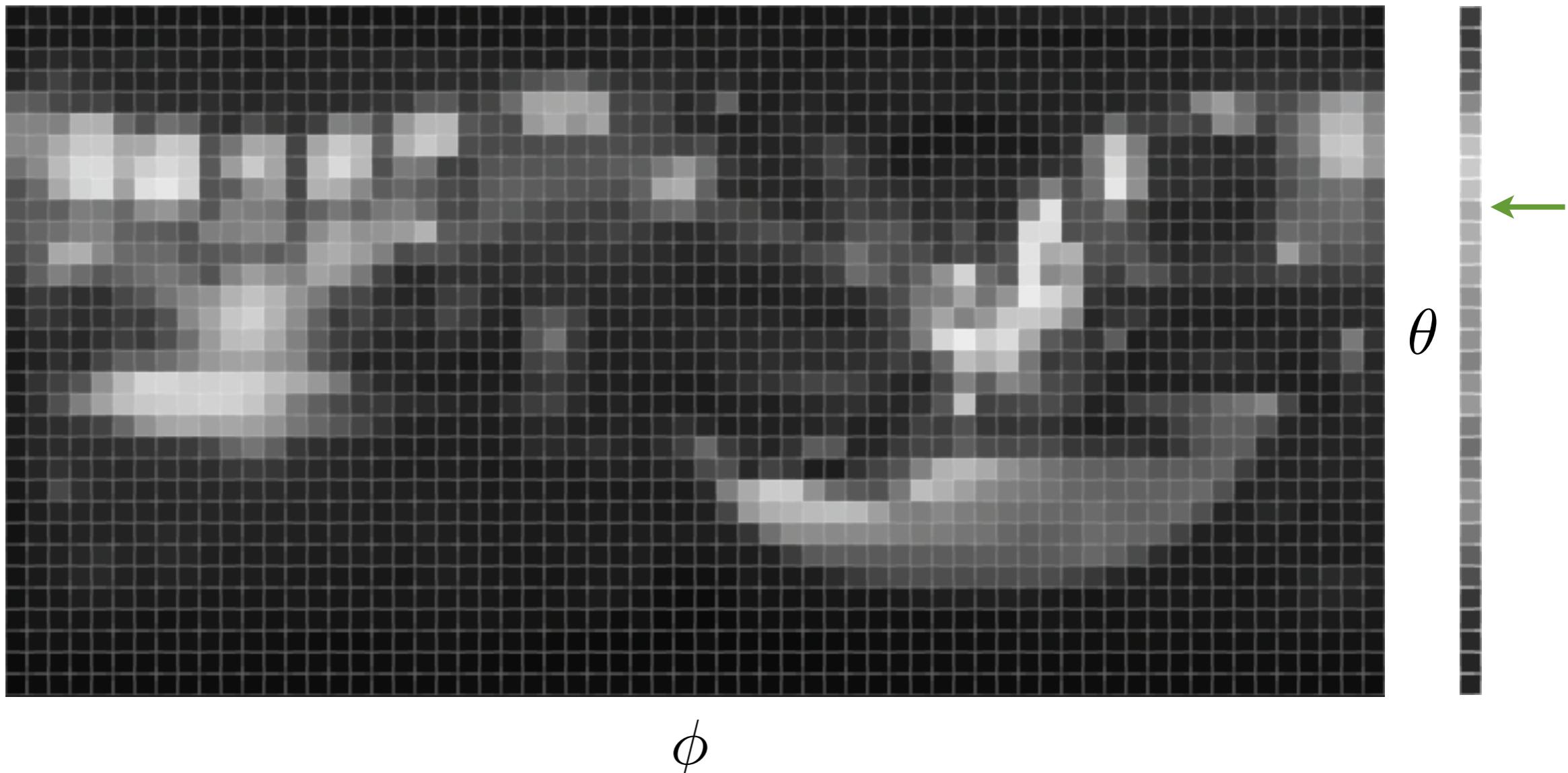


# Step 3: Conditional PDFs

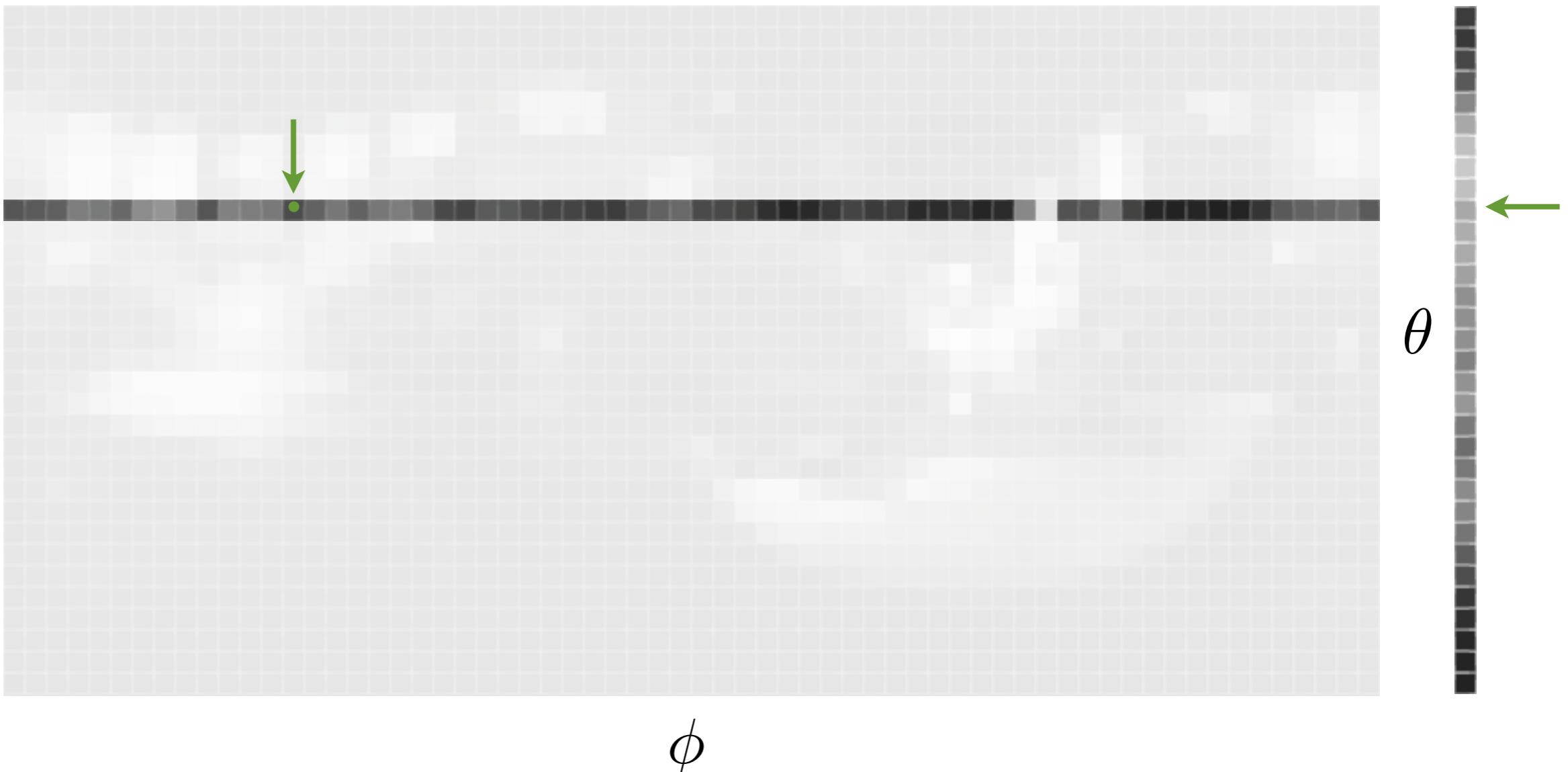
Once normalized, each row can  
serve as the conditional PDF



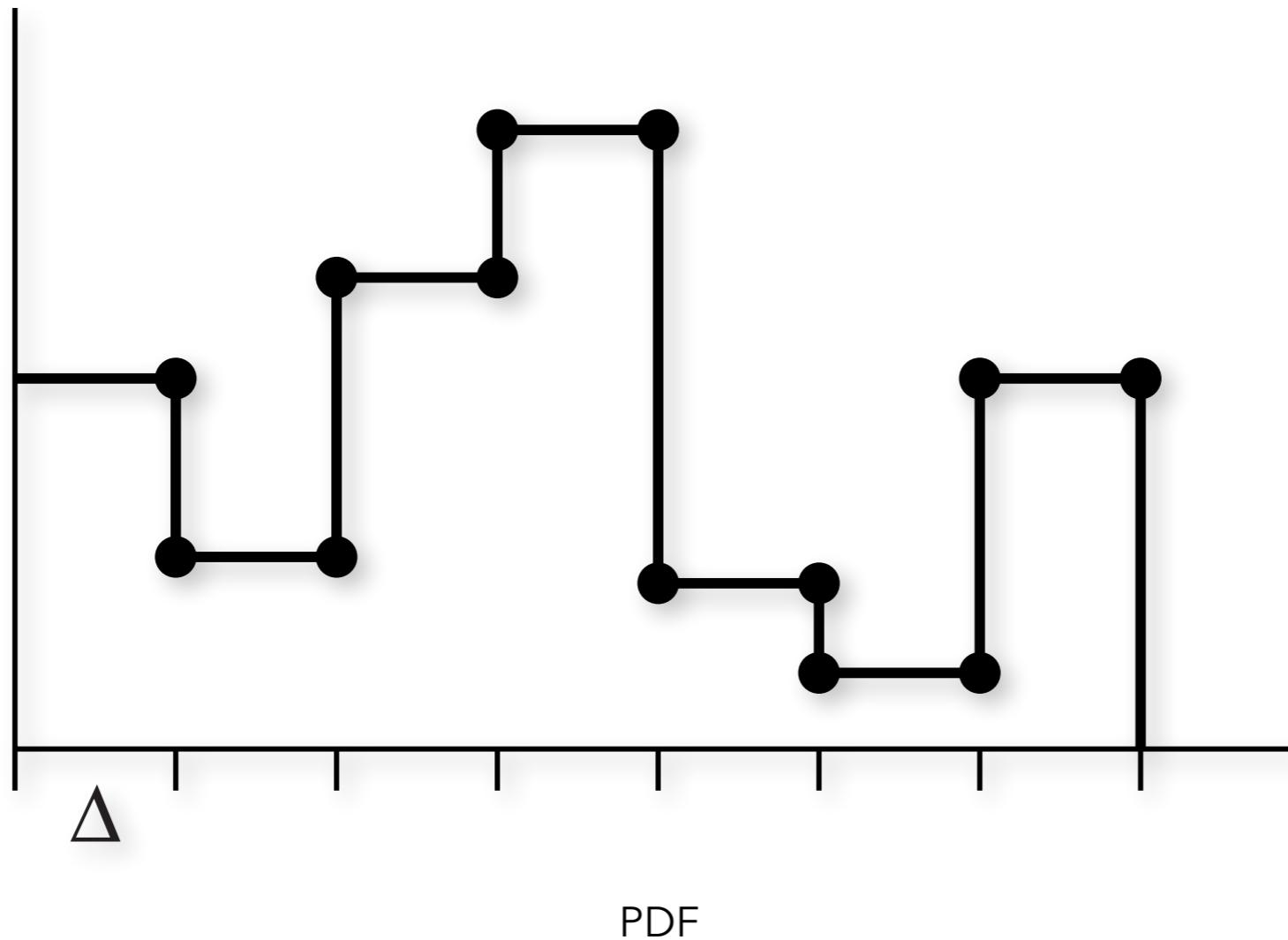
# Step 4: Sampling



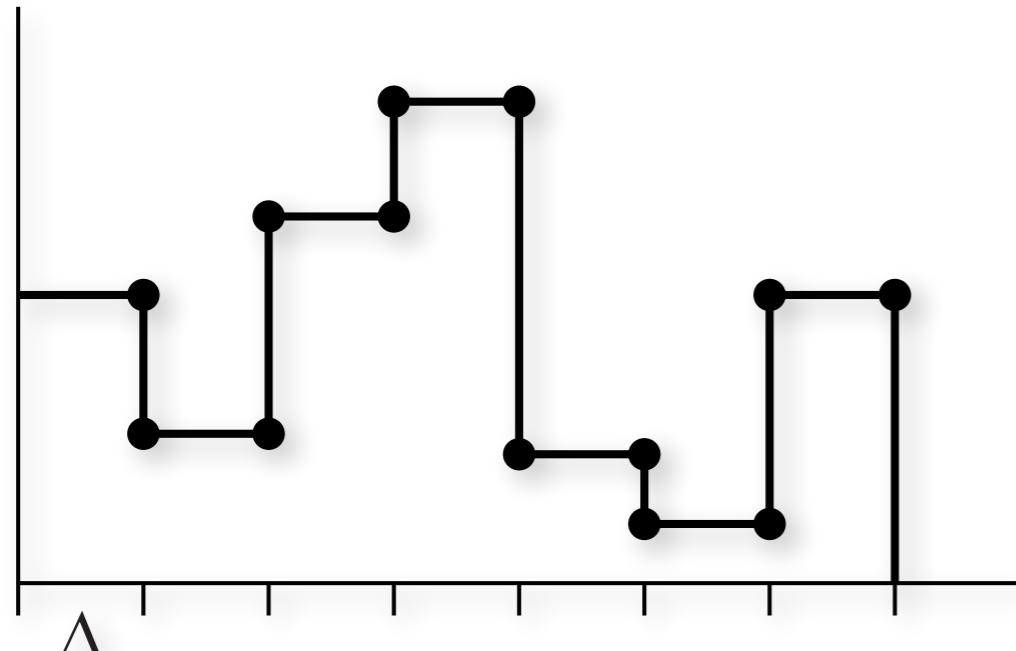
# Step 4: Sampling



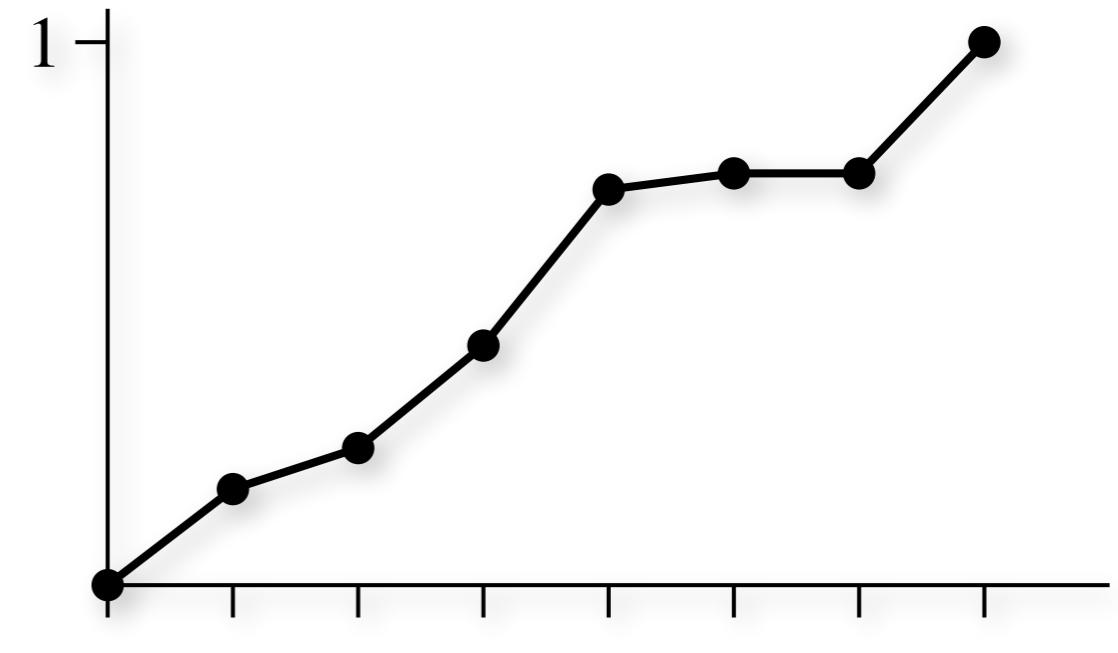
# Sampling Discrete 1D PDFs



# Sampling Discrete 1D PDFs



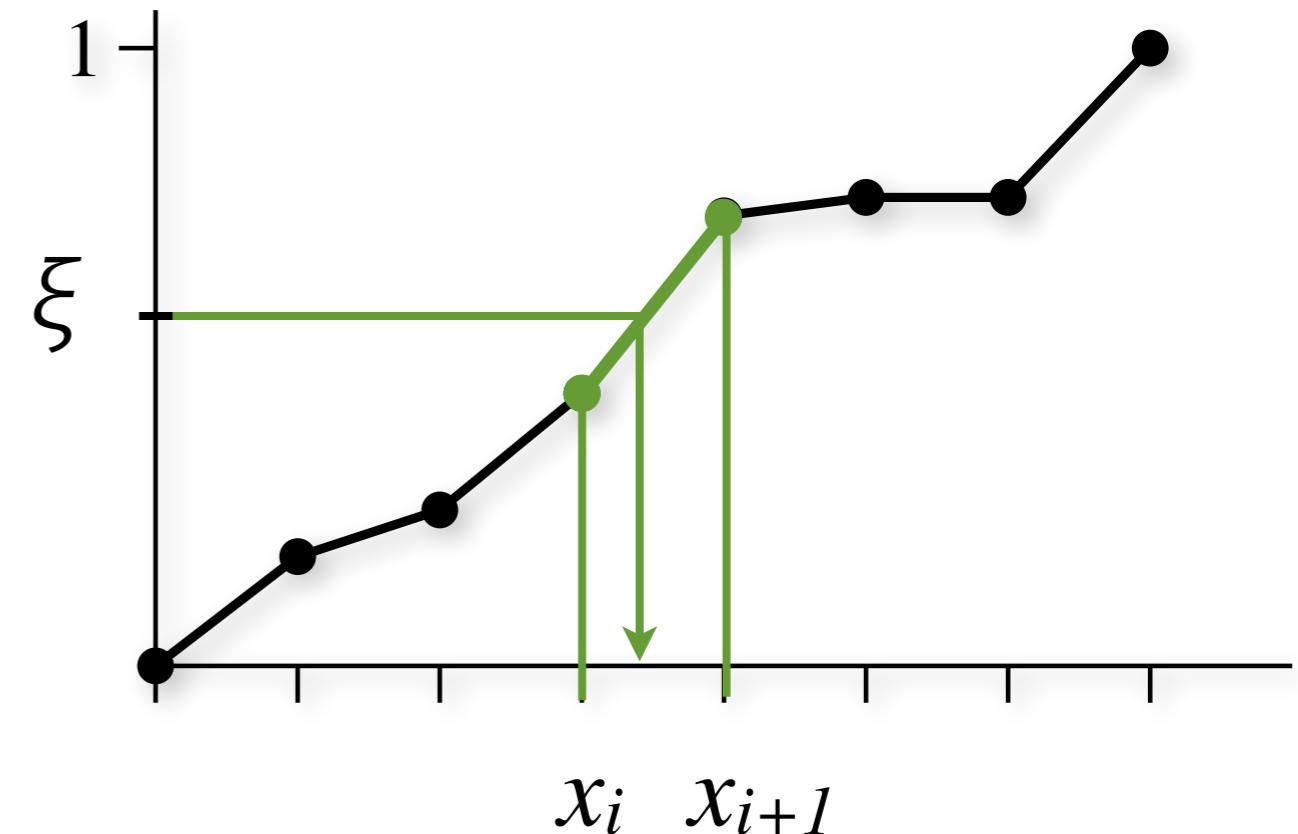
PDF



CDF

# Sampling Discrete 1D PDFs

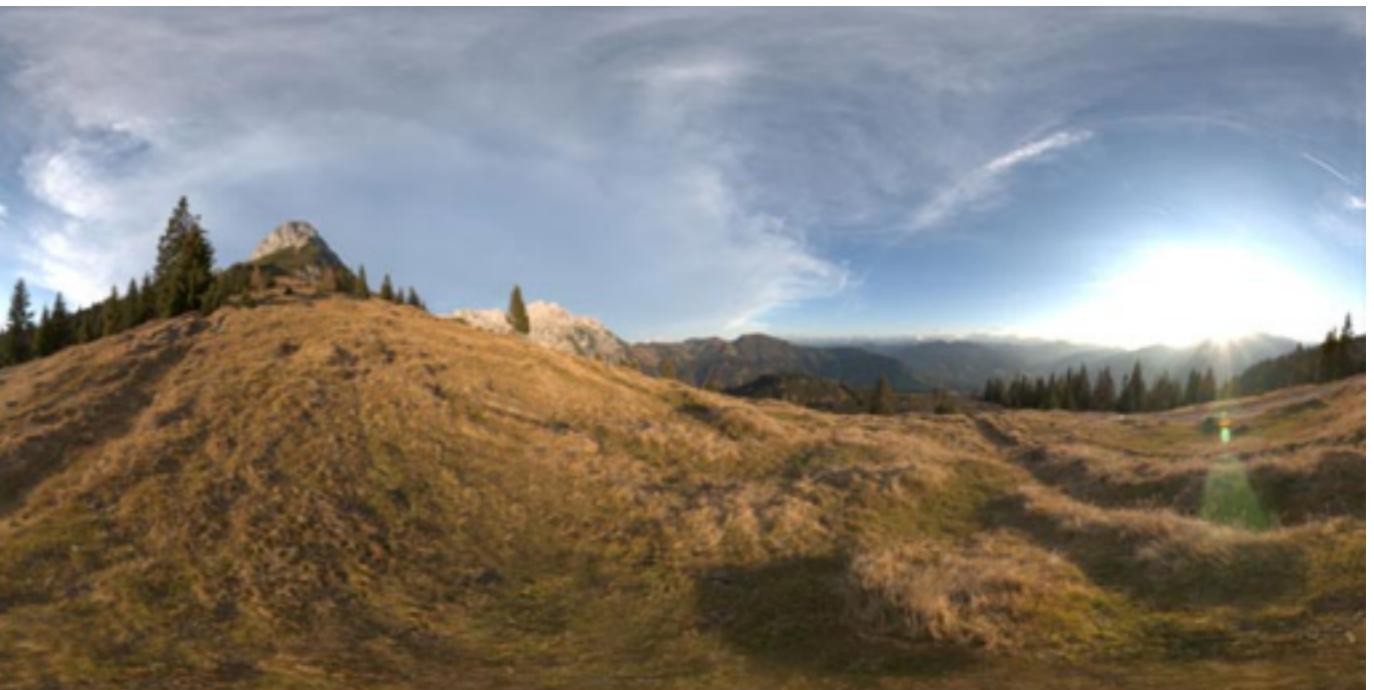
- Given a uniform random value  $\xi$
- Using binary search find  $x_i$  and  $x_{i+1}$
- Linearly interpolate to find  $x$



# Resulting Sample Distribution

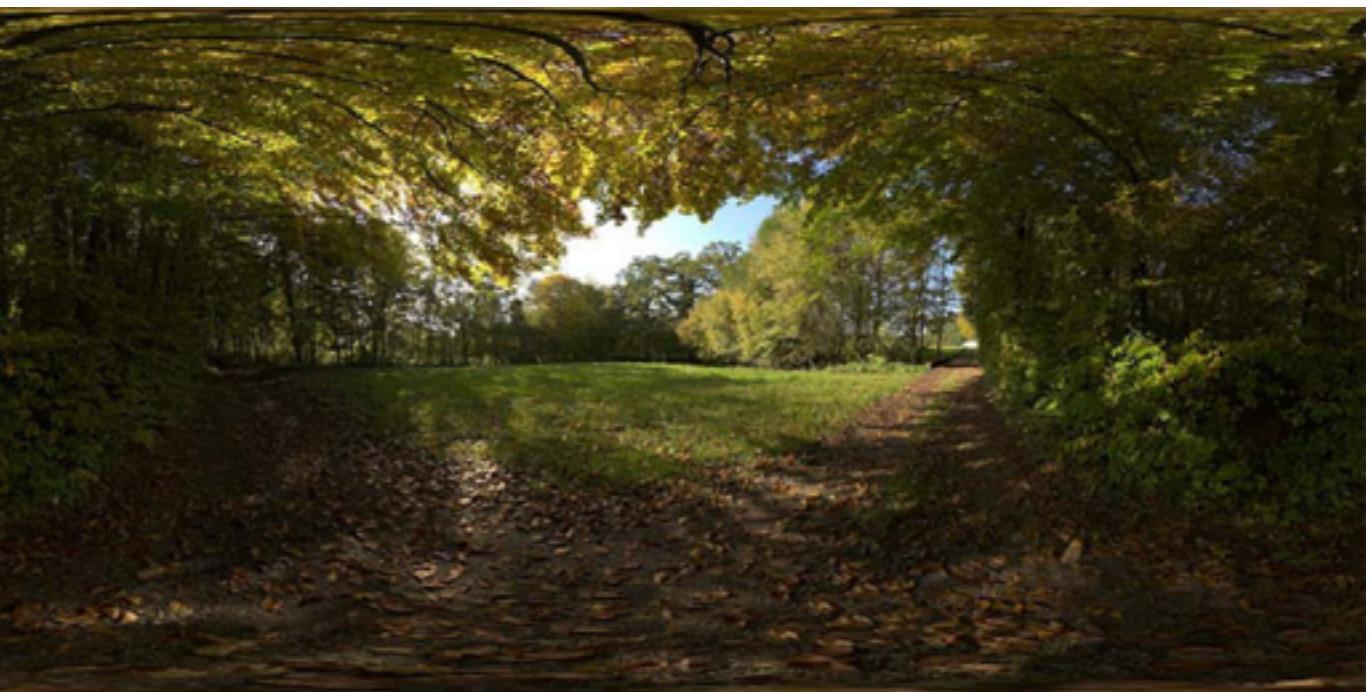


# Visual Break



[openfootage.net](http://openfootage.net)

# Visual Break



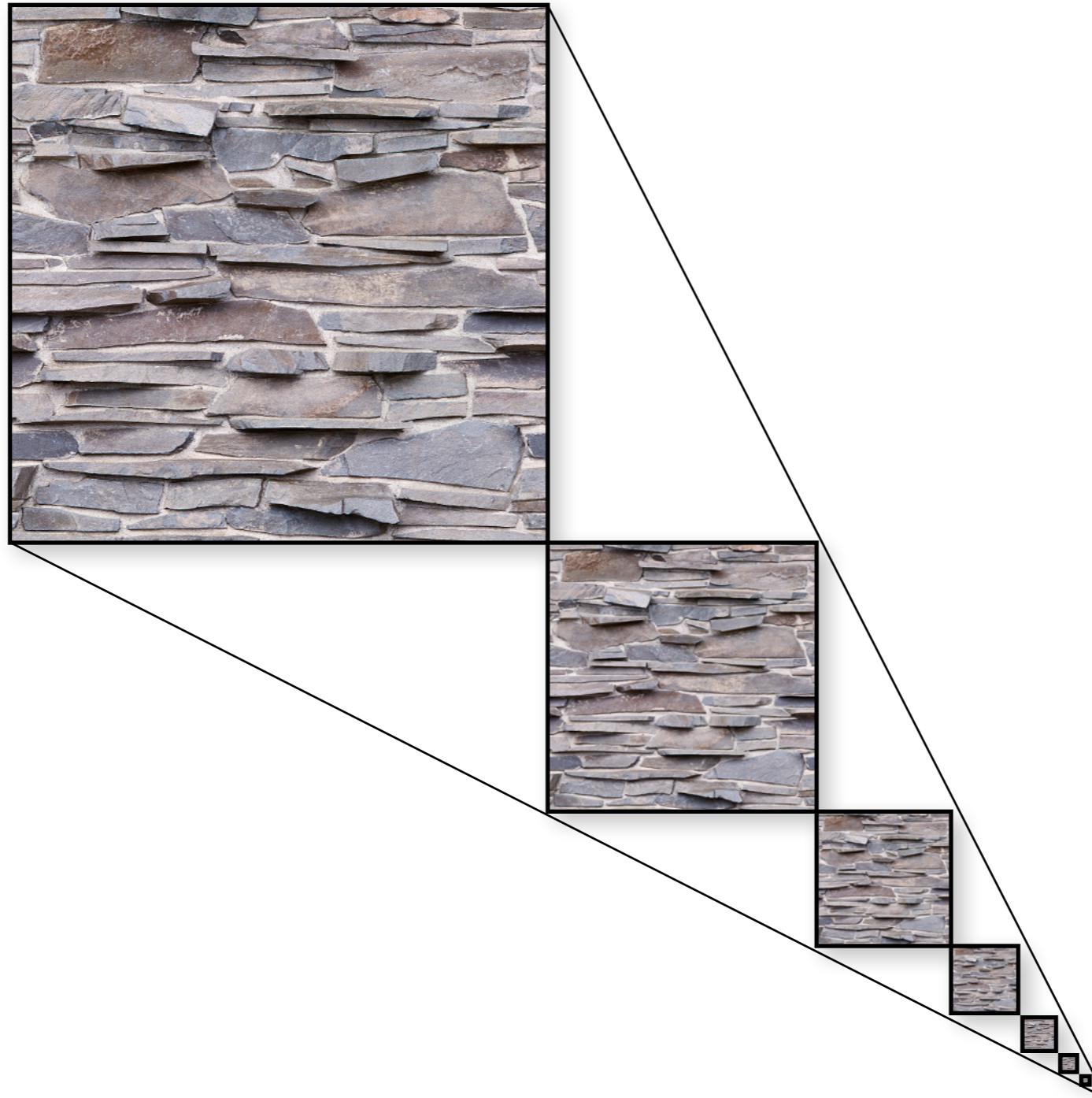
[openfootage.net](http://openfootage.net)

# Hierarchical Sample Warping

Clarberg, Jarosz, Akenine-Möller, Jensen. "Wavelet Importance Sampling," 2005.

- Given:
  - input point set, and
  - hierarchical representation of density function (mip-map)

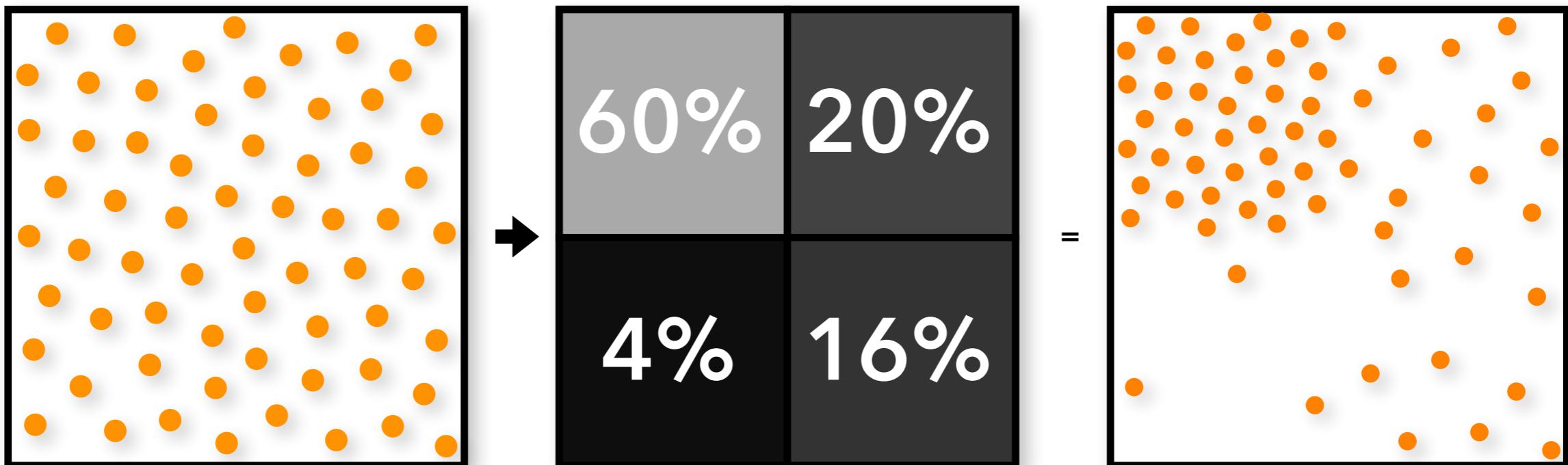
# Mip-Map

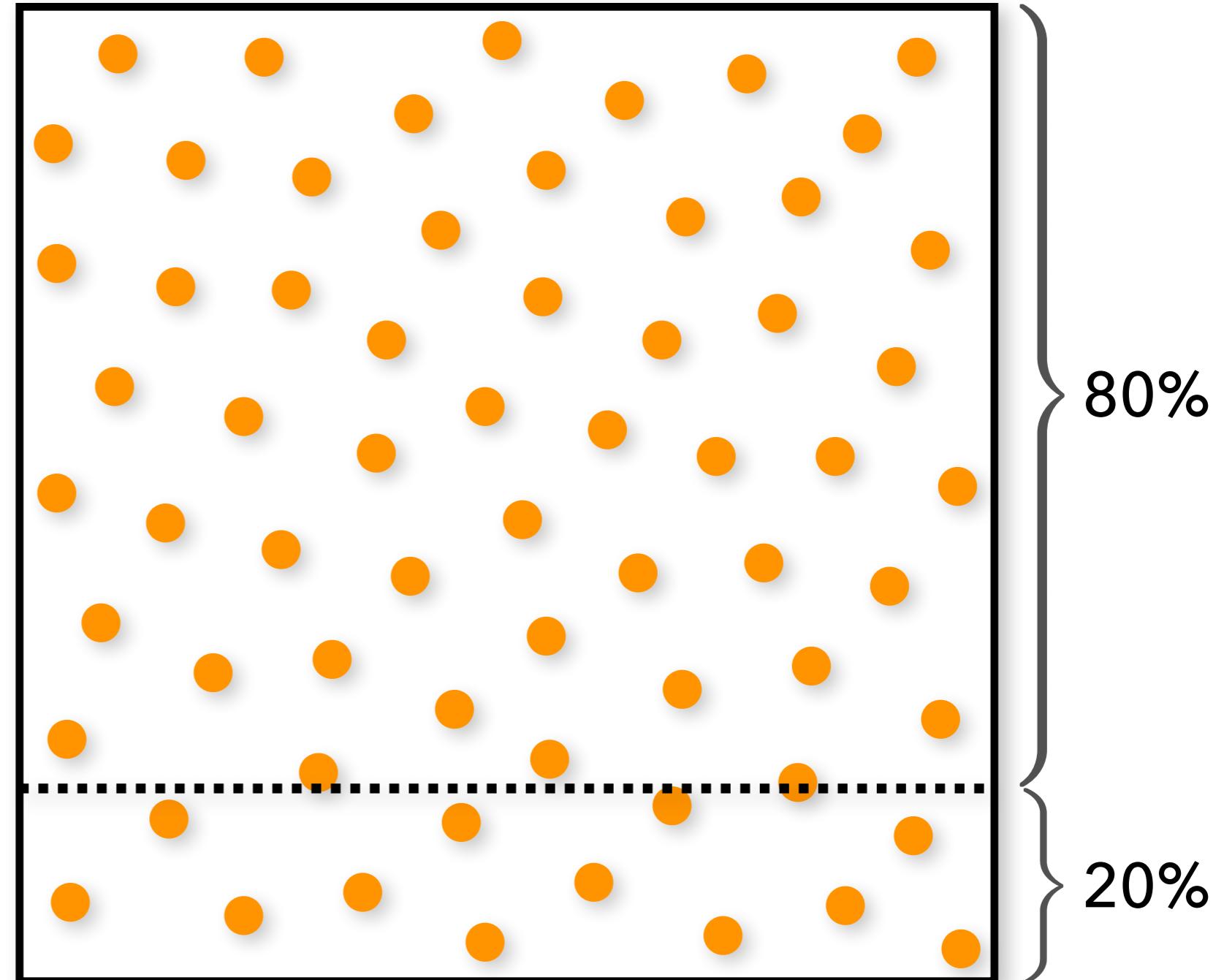
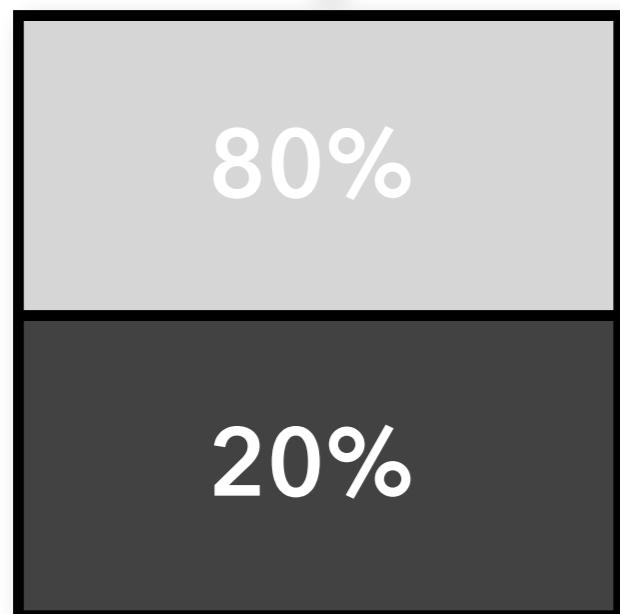
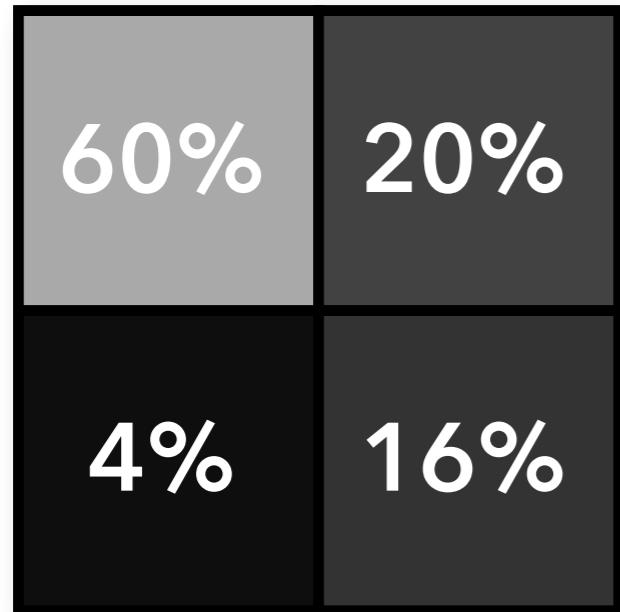


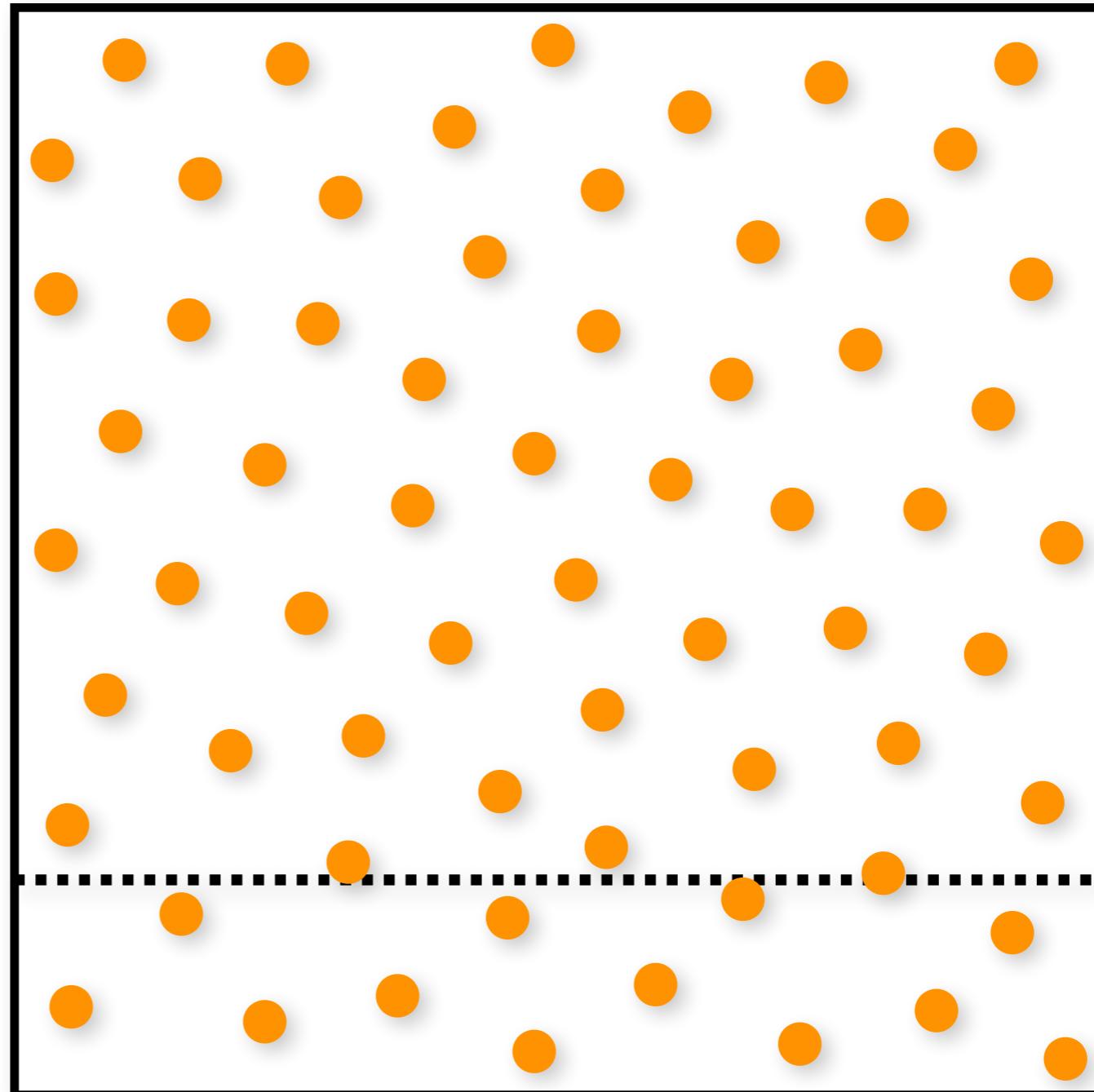
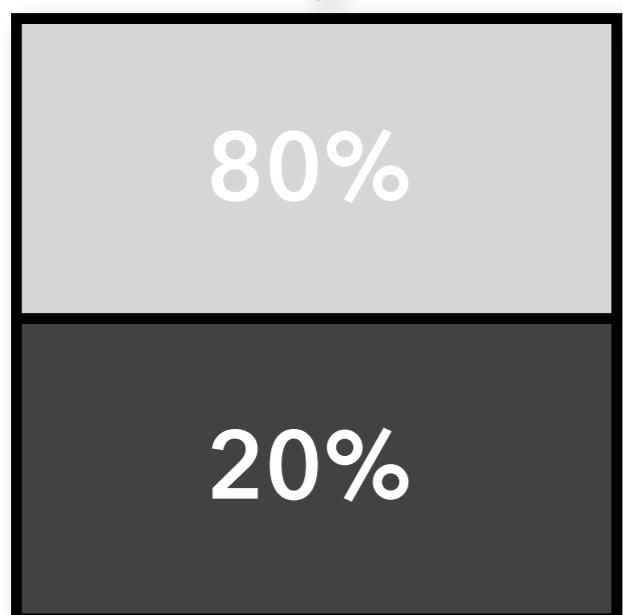
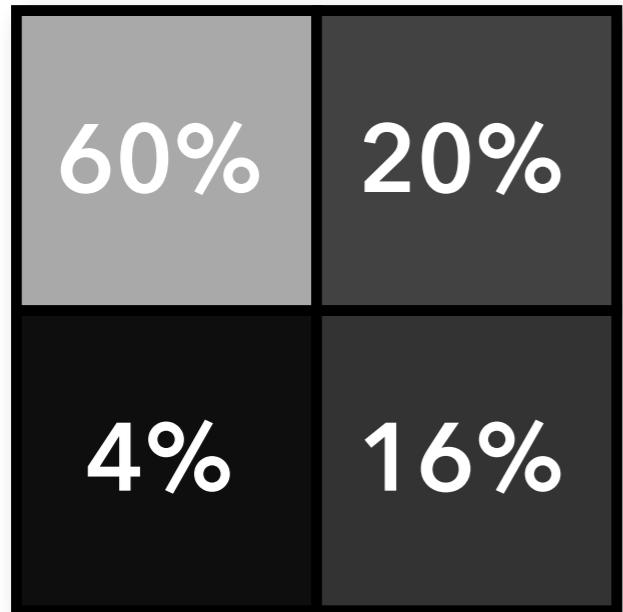
# Hierarchical Sample Warping

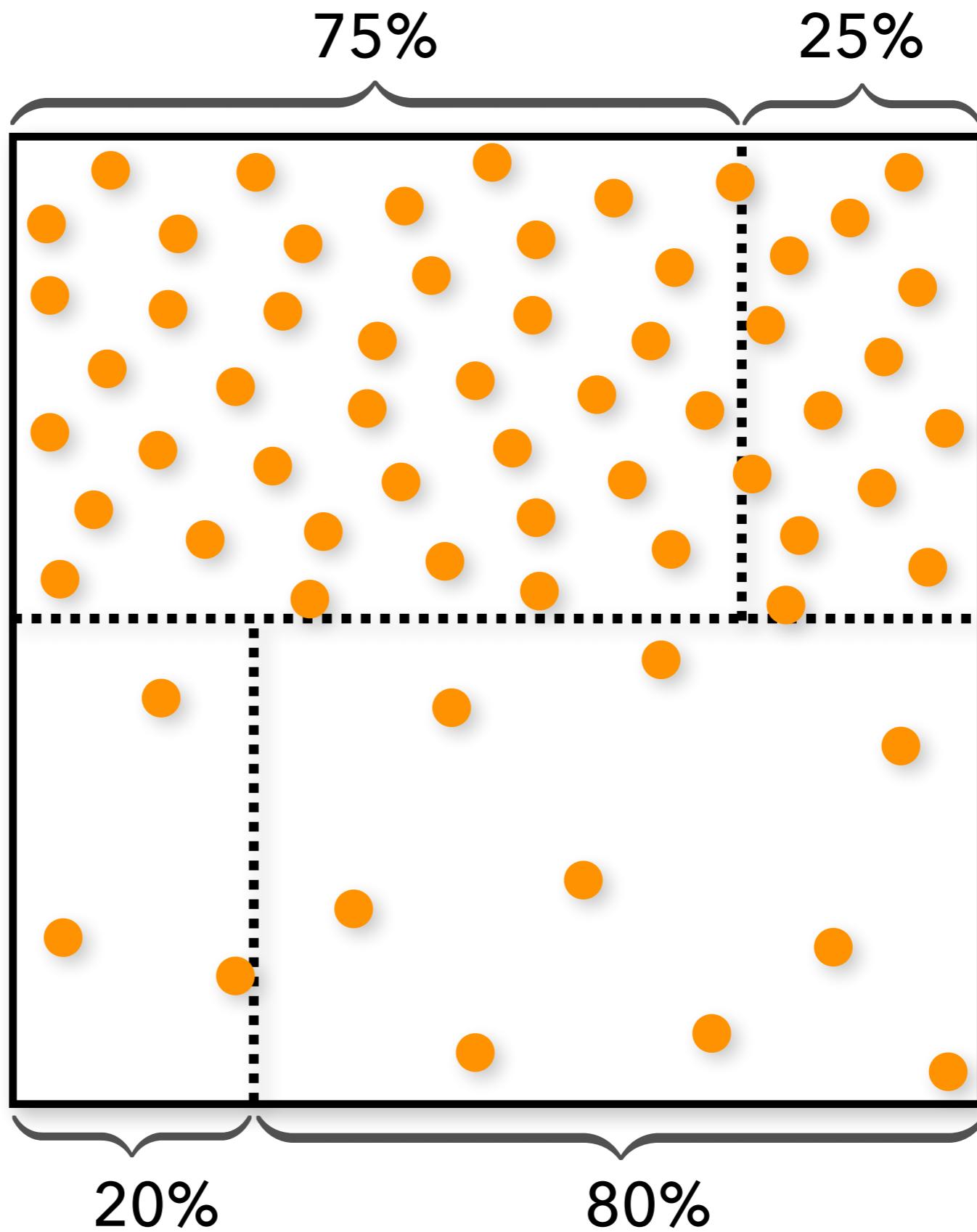
Clarberg, Jarosz, Akenine-Möller, Jensen. "Wavelet Importance Sampling," 2005.

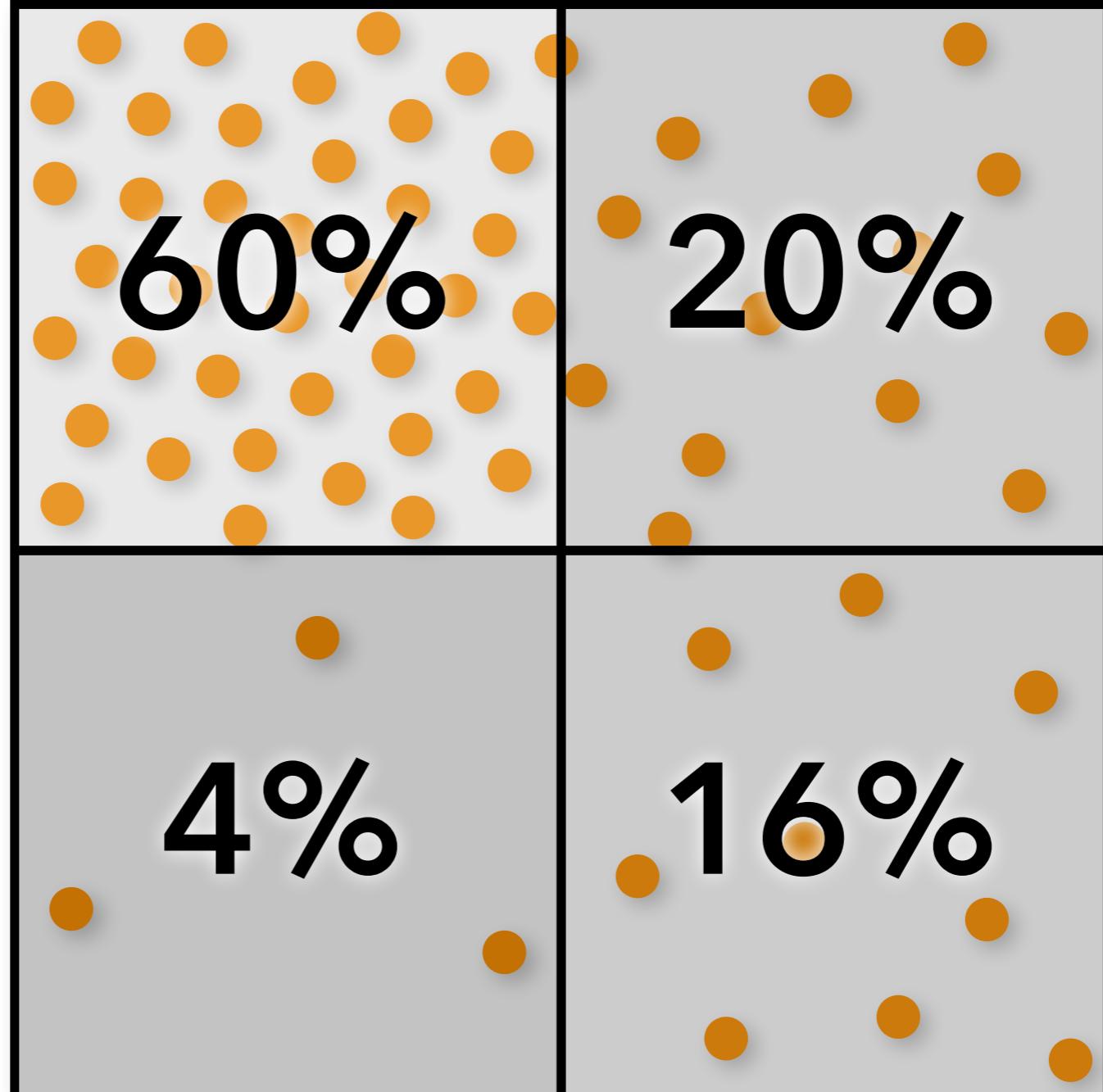
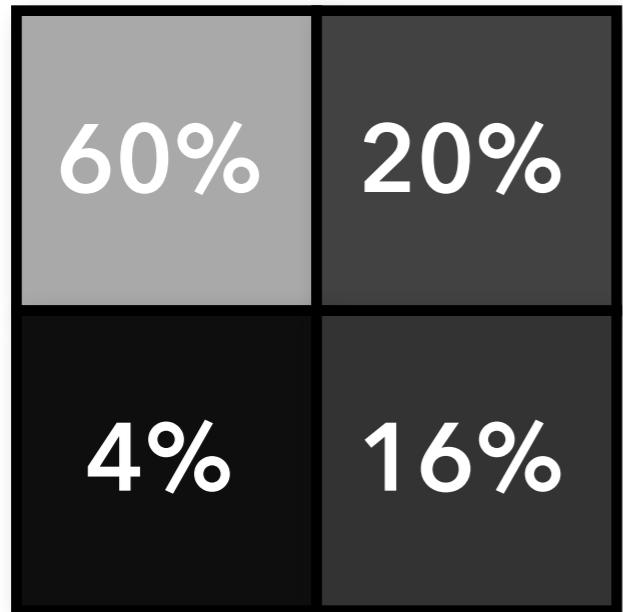
- Given:
  - input point set, and
  - hierarchical representation of density function (mip-map)
- Recursively warp point set to match the importance function











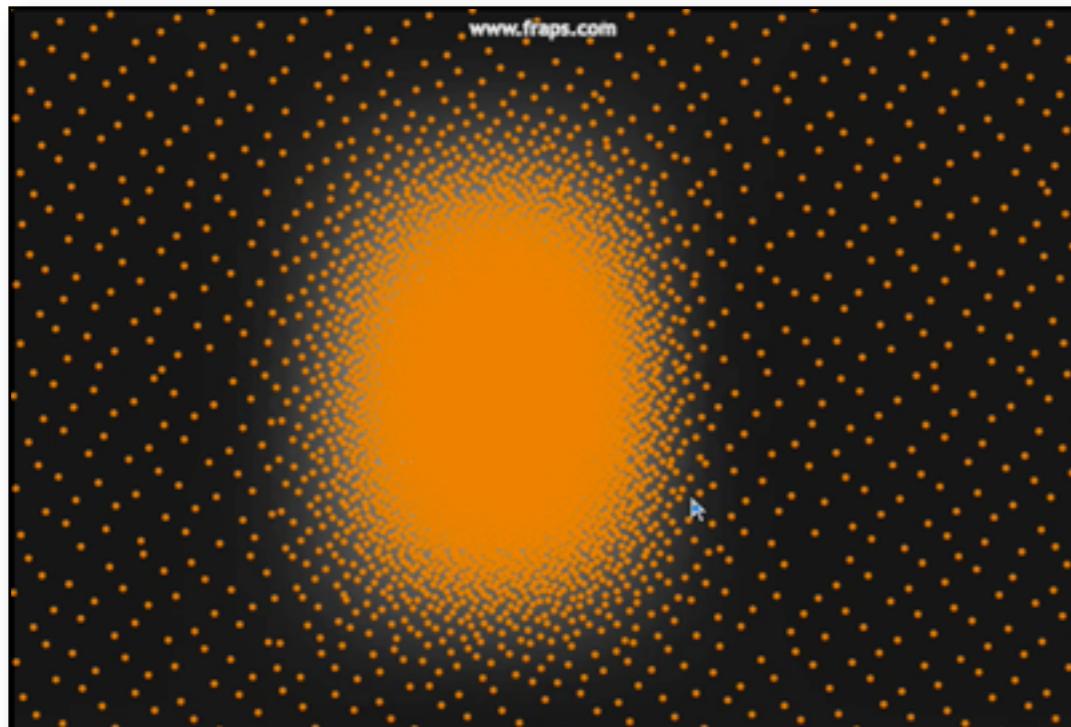
repeat on each quadrant recursively

# Environment Lighting

$$\begin{aligned} L_r(\mathbf{x}, \vec{\omega}_r) &= \int_{\Omega} f_r(\vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i \\ &= \int_{\Omega} f_r(\vec{\omega}_i, \vec{\omega}_r) L_{\text{env}}(\vec{\omega}_i) V(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i \\ &\approx \frac{1}{N} \sum_{i=1}^N \frac{f_r(\vec{\omega}_i, \vec{\omega}_r) L_{\text{env}}(\vec{\omega}_i) V(\mathbf{x}, \vec{\omega}_i) \cos \theta_i}{\text{pdf}(\vec{\omega}_i)} \end{aligned}$$

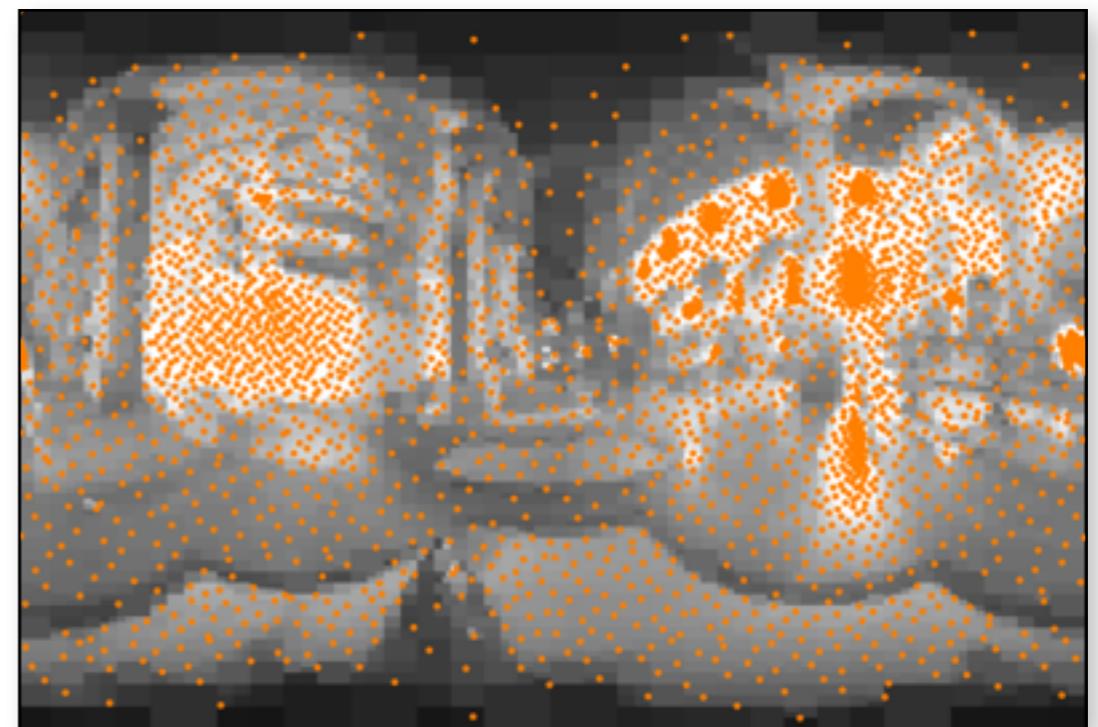
- What pdf should/can we use?
  - prop. to  $f_r$
  - prop. to  $L_{\text{env}}$

# Sample Distributions

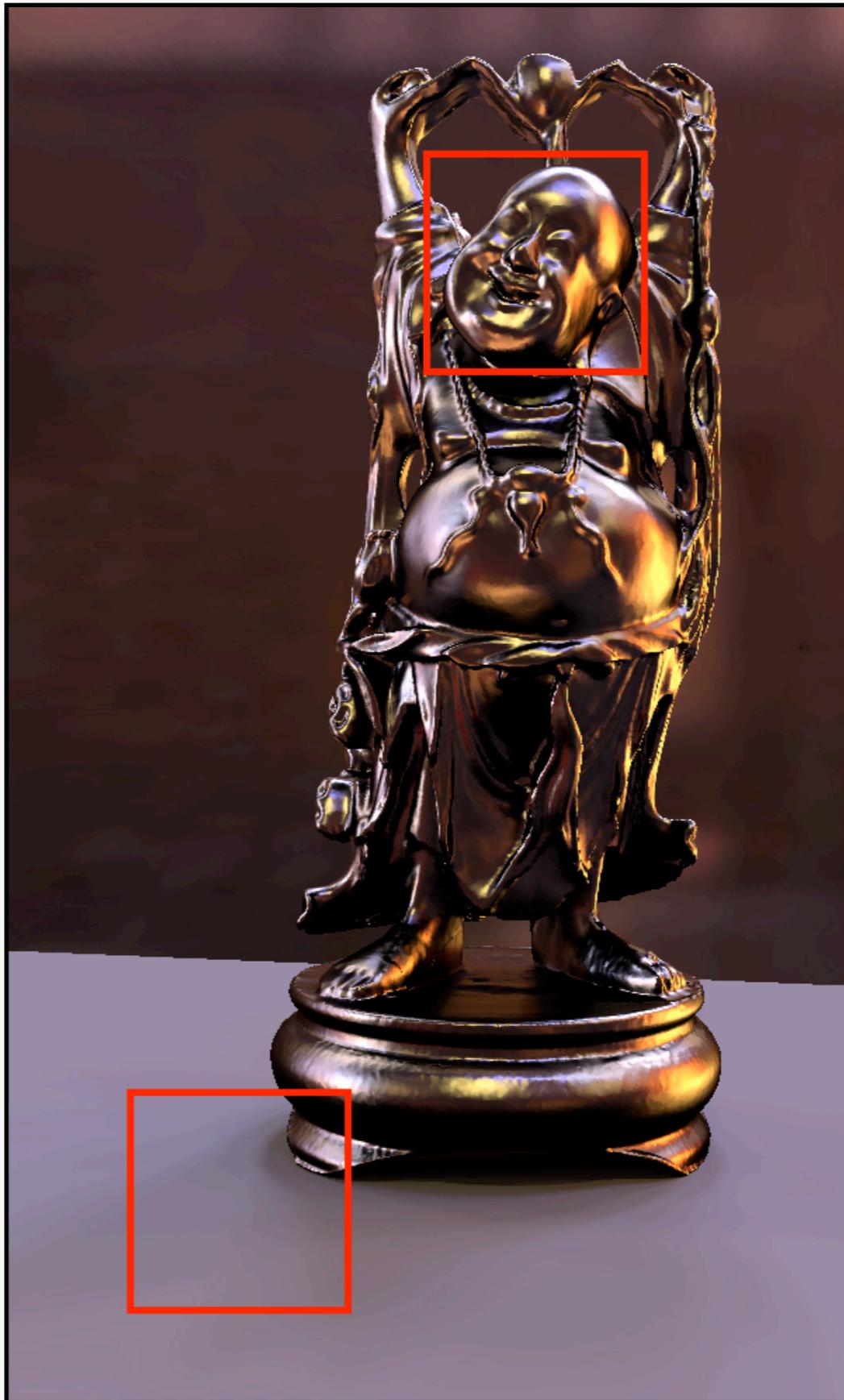


BRDF

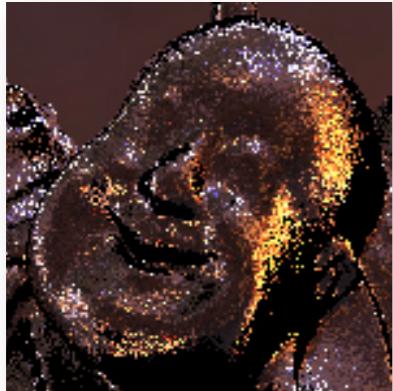
or



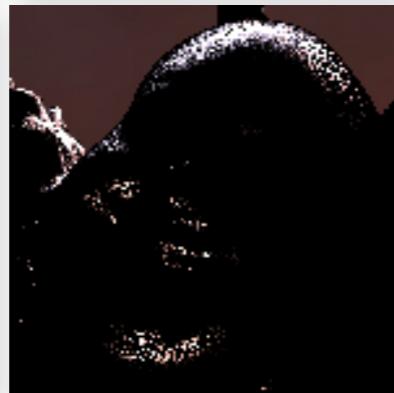
Lighting



BRDF



Lighting



1

3

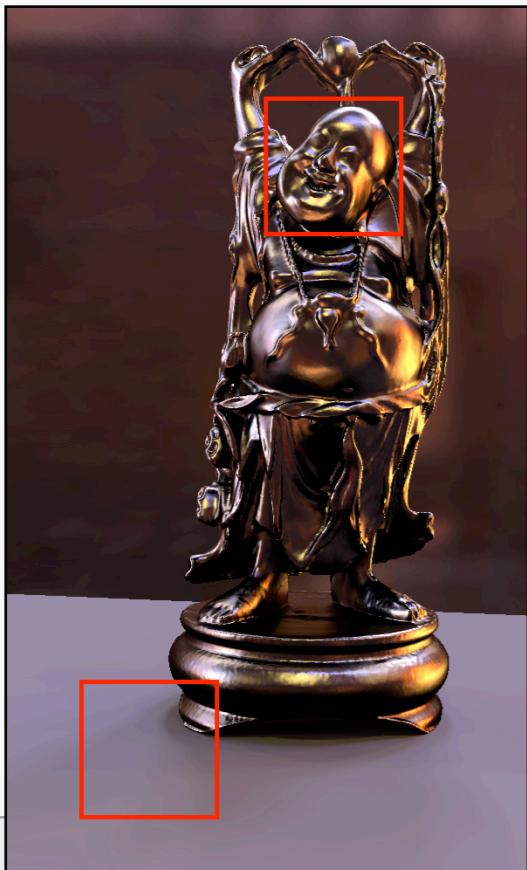
10

30

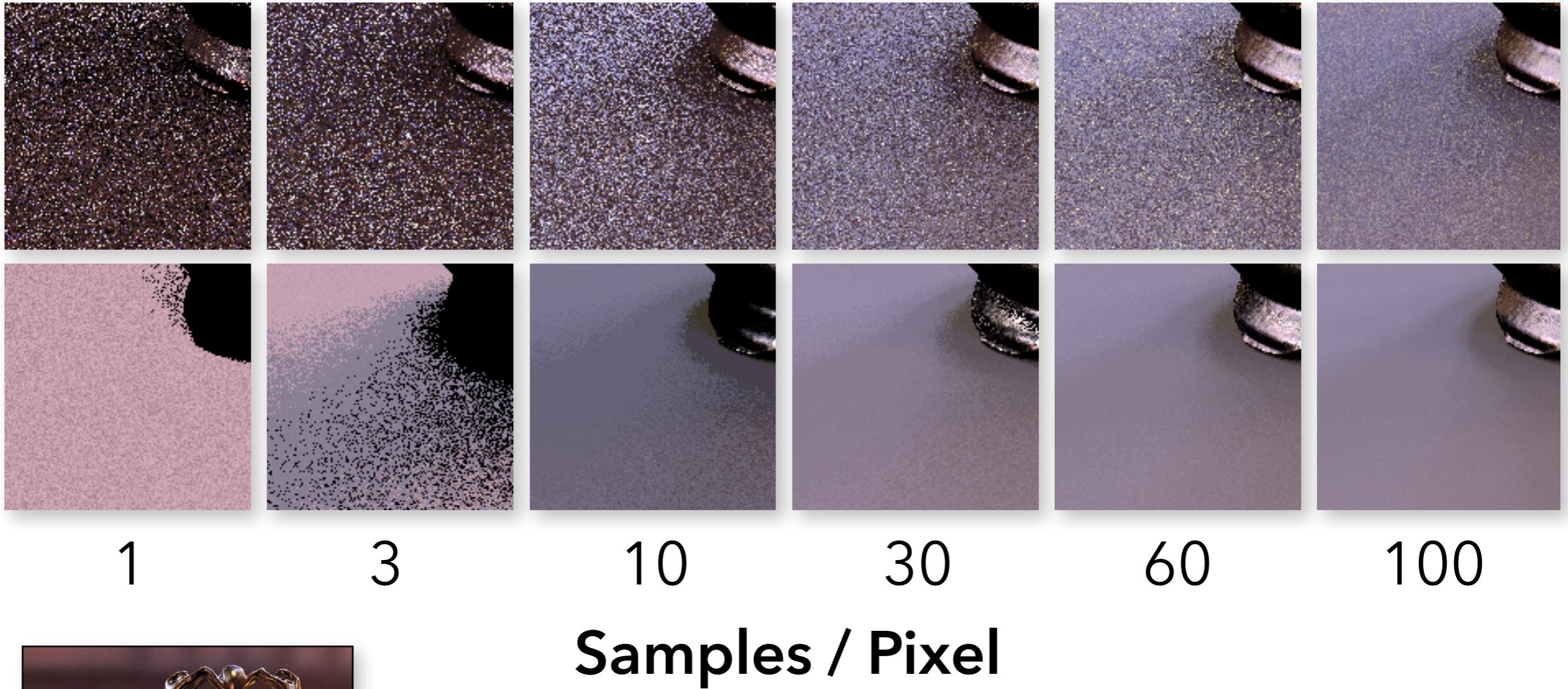
60

100

Samples / Pixel



# Lighting BRDF



# Product Sampling

# Environment Lighting

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- Could use MIS
  - pdf is a linear combination of  $f_r$  and  $L_{\text{env}}$
- Can we sample the *product* ( $f_r \cdot L_{\text{env}} \cdot \cos \theta_i$ ) directly?

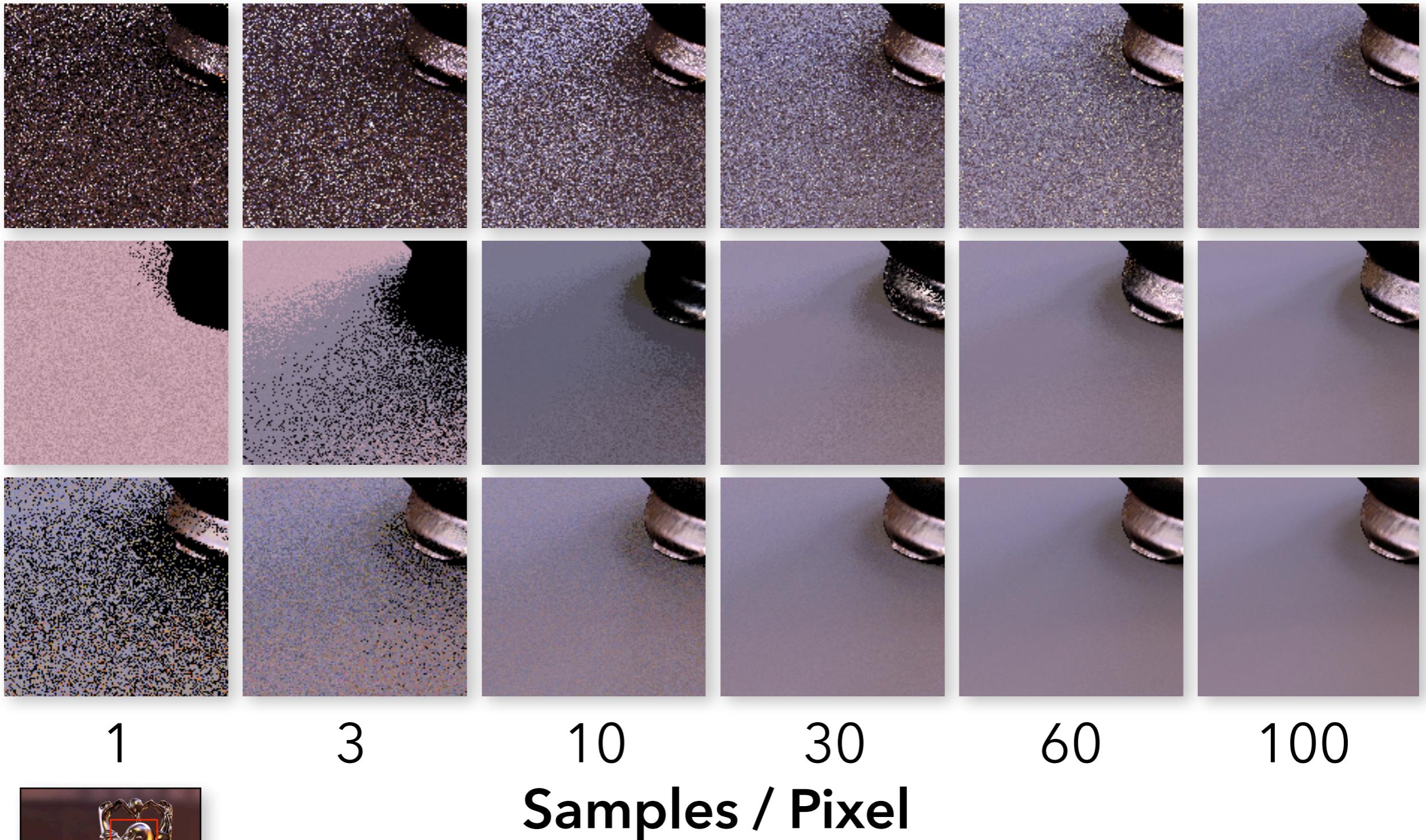
Clarberg, Jarosz, Akenine-Möller, Jensen. "Wavelet Importance Sampling," 2005.

Subr and Arvo. "Steerable Importance Sampling," 2007.

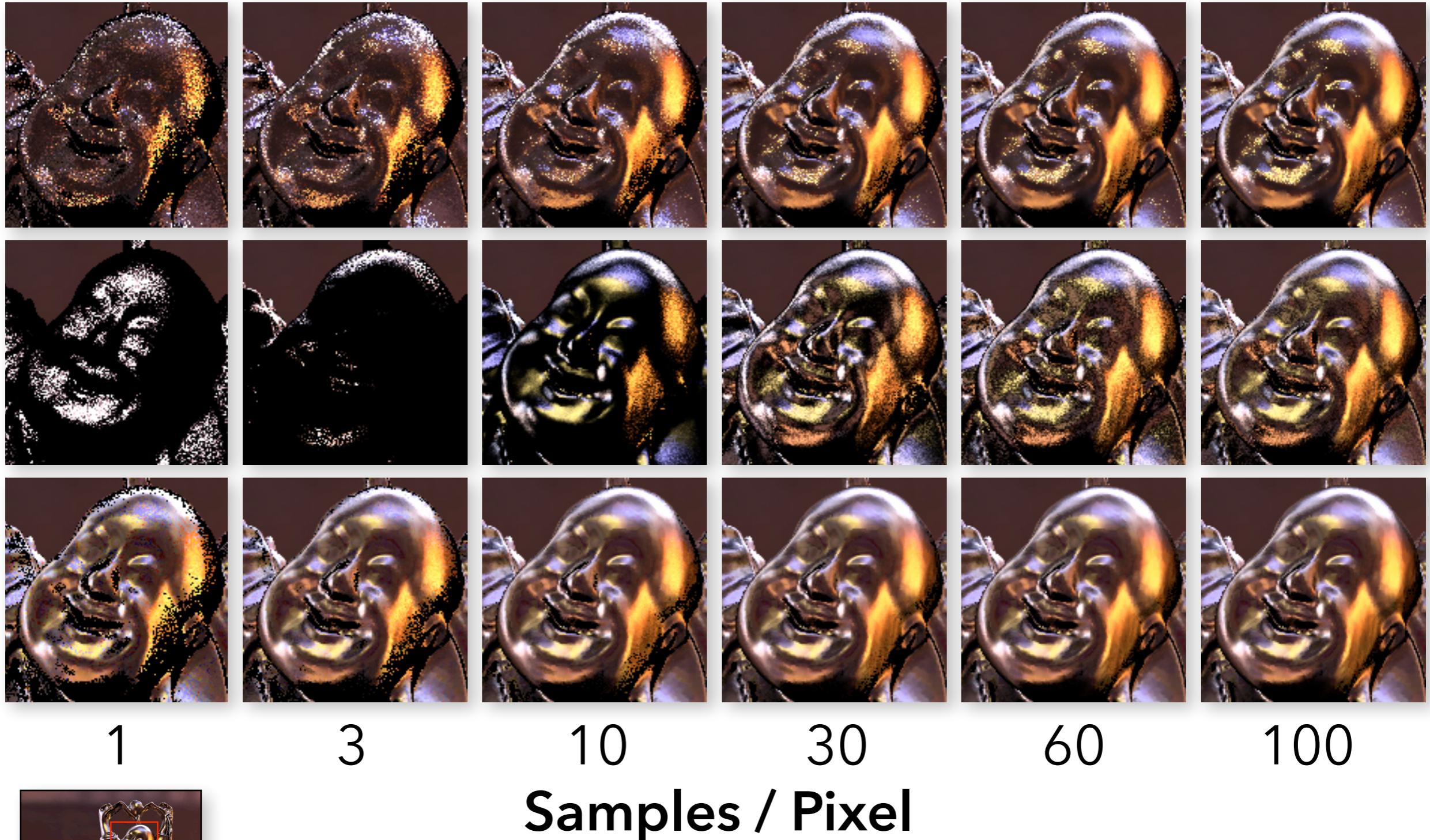
Jarosz, Carr, Jensen. "Importance Sampling Spherical Harmonics," 2009.

many more...

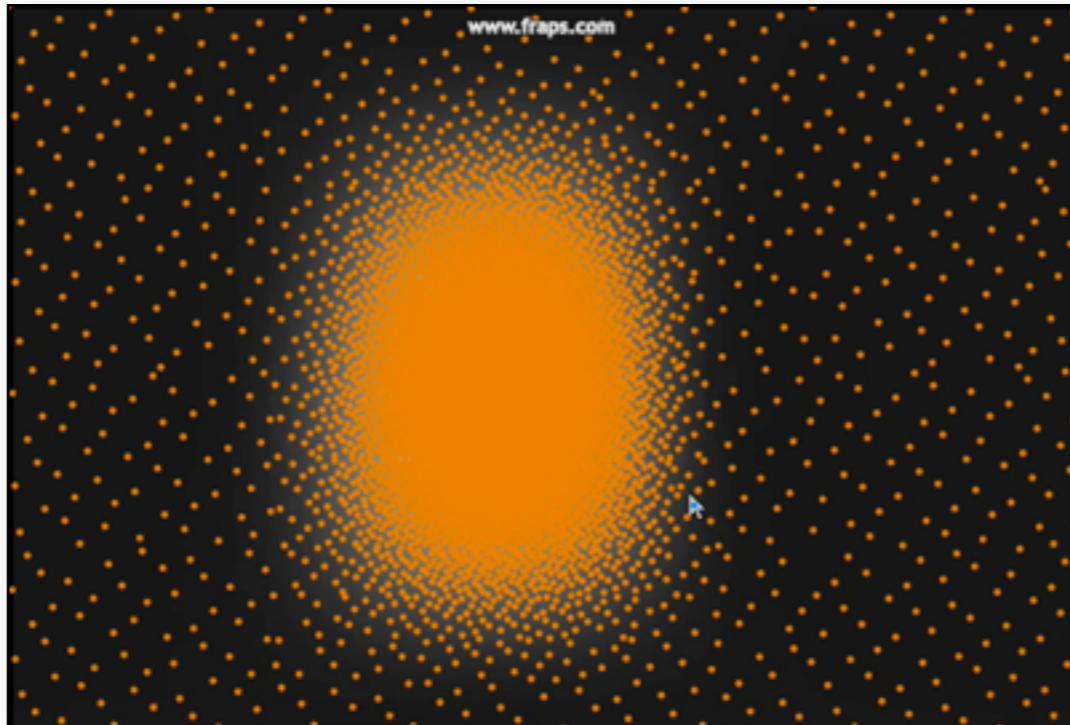
# Product Lighting BRDF



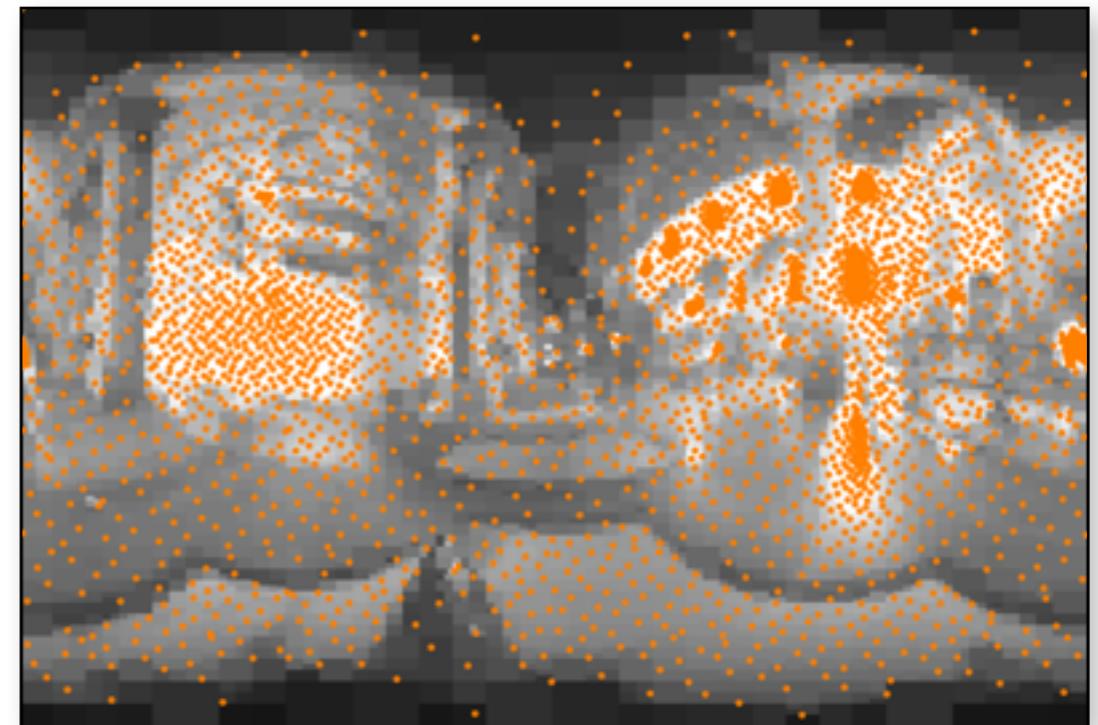
BRDF  
Lighting  
Product



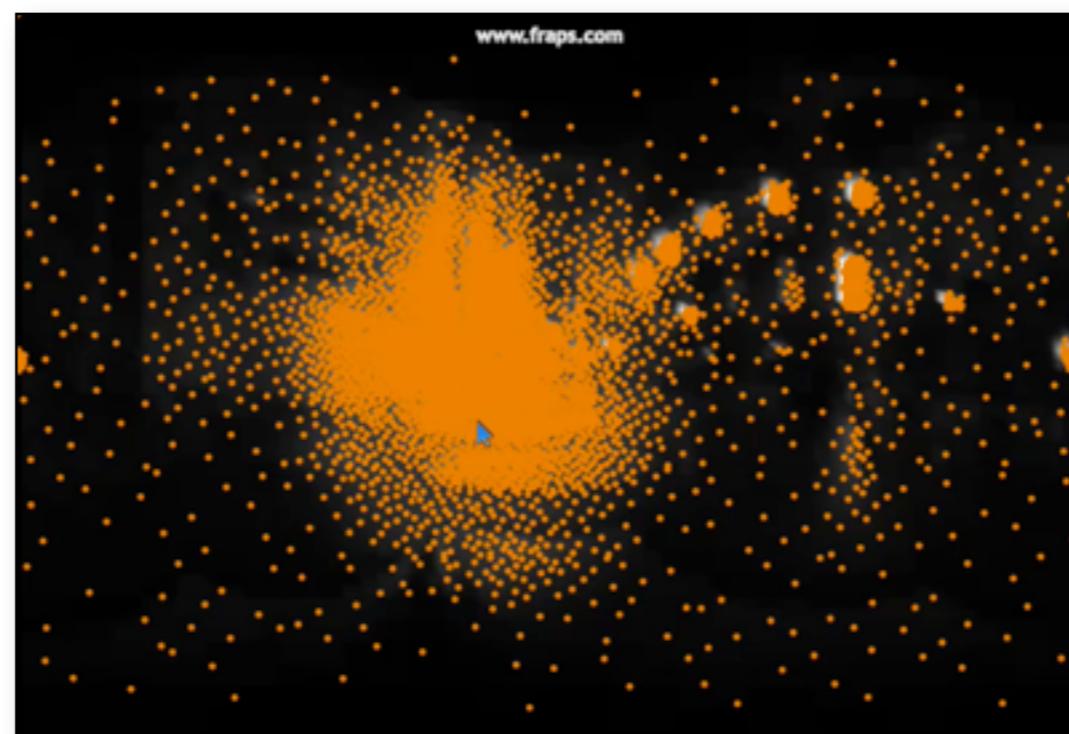
# Sample Distributions



BRDF



Lighting



BRDF x Lighting



Blue Fabric

Grace Cathedral

# Questions?

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# The Avengers (2012)



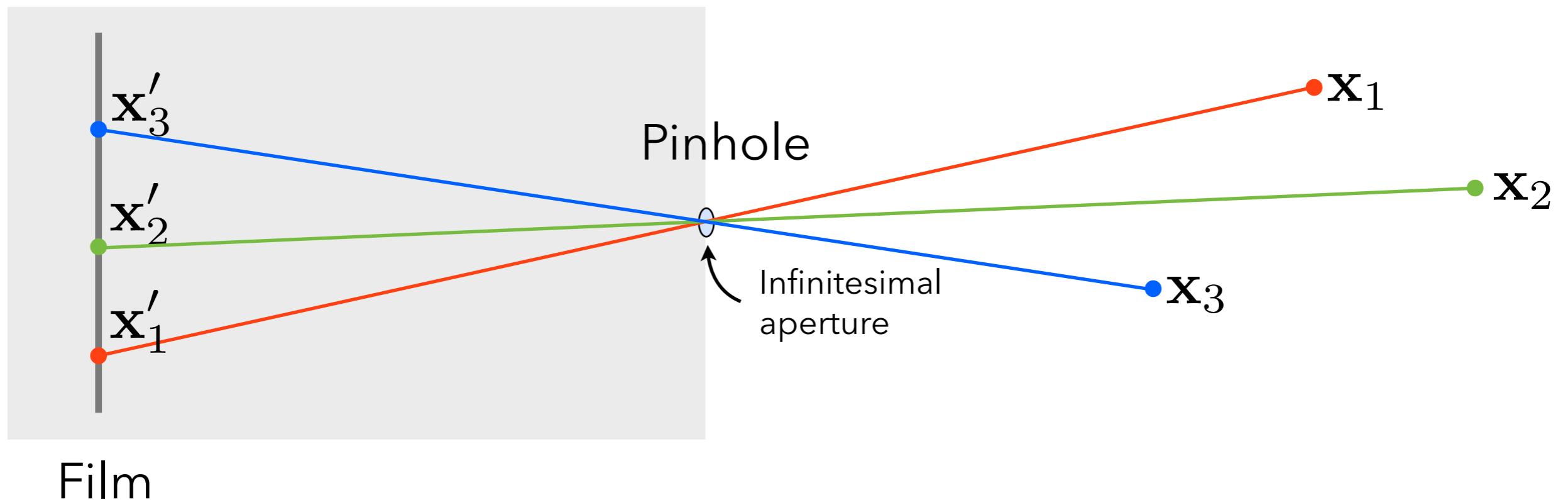
# Advanced Camera Models

# Pinhole Camera (Camera Obscura)



# Pinhole Camera

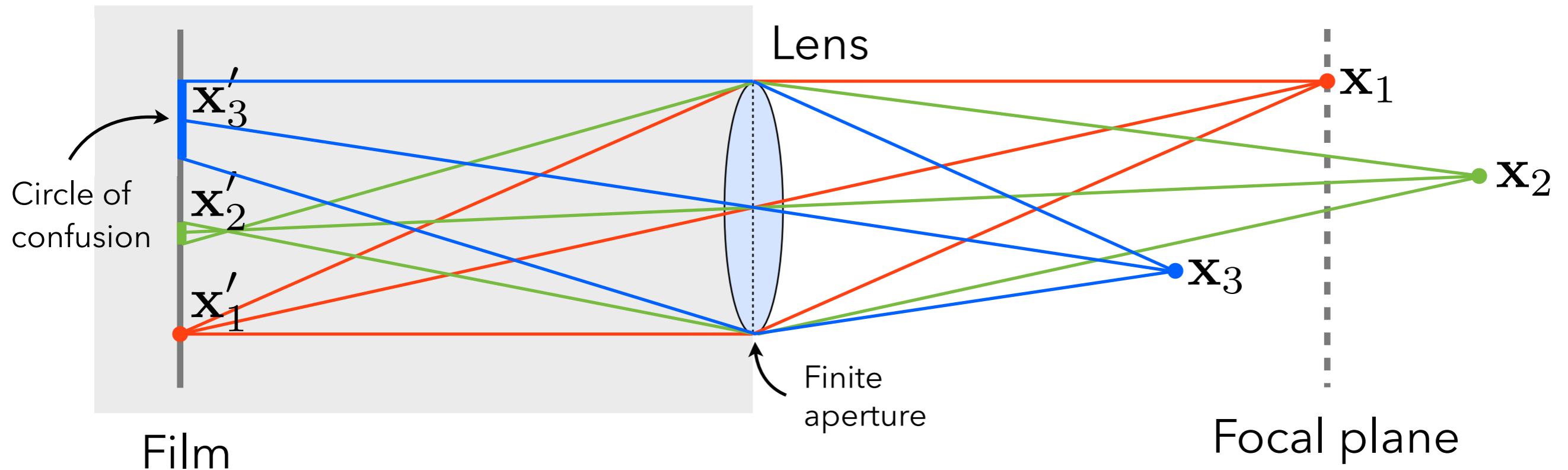
Camera



Film

# Thin Lens Camera

Camera

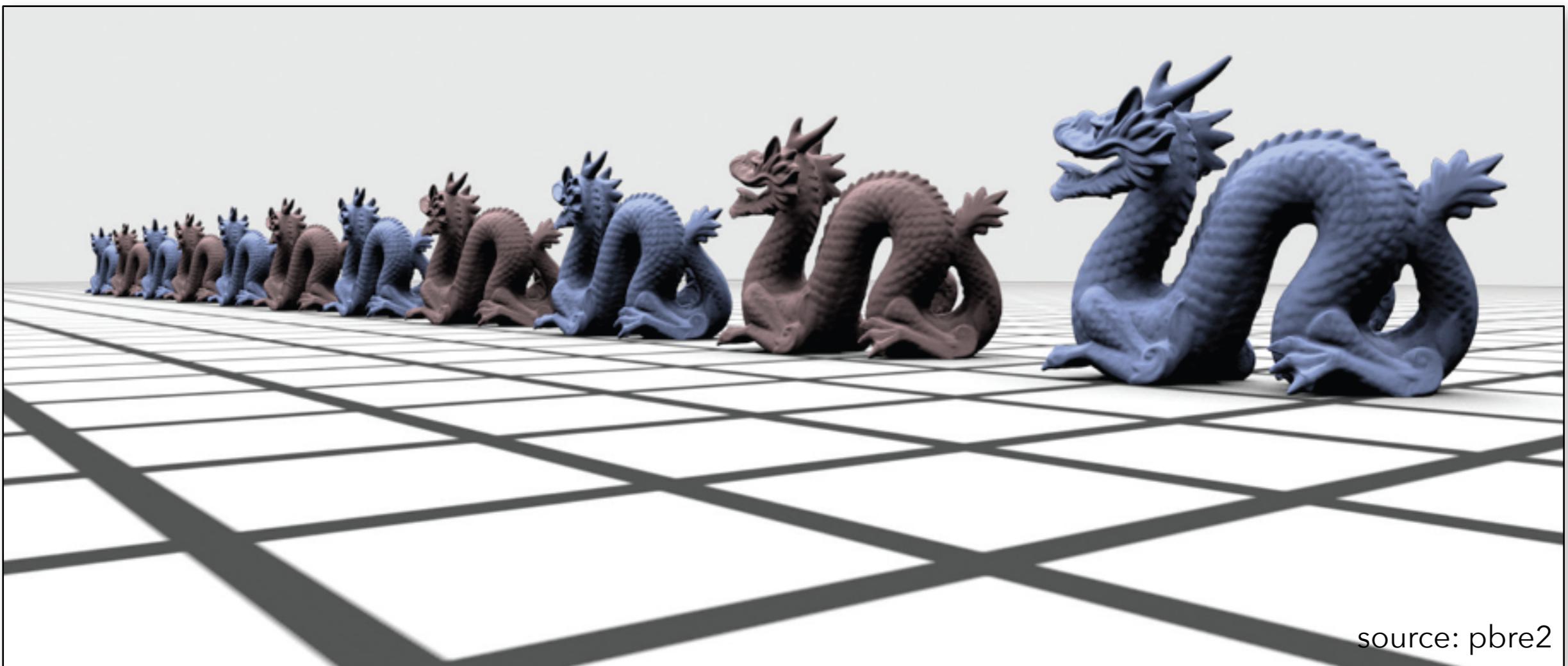


# Depth of Field



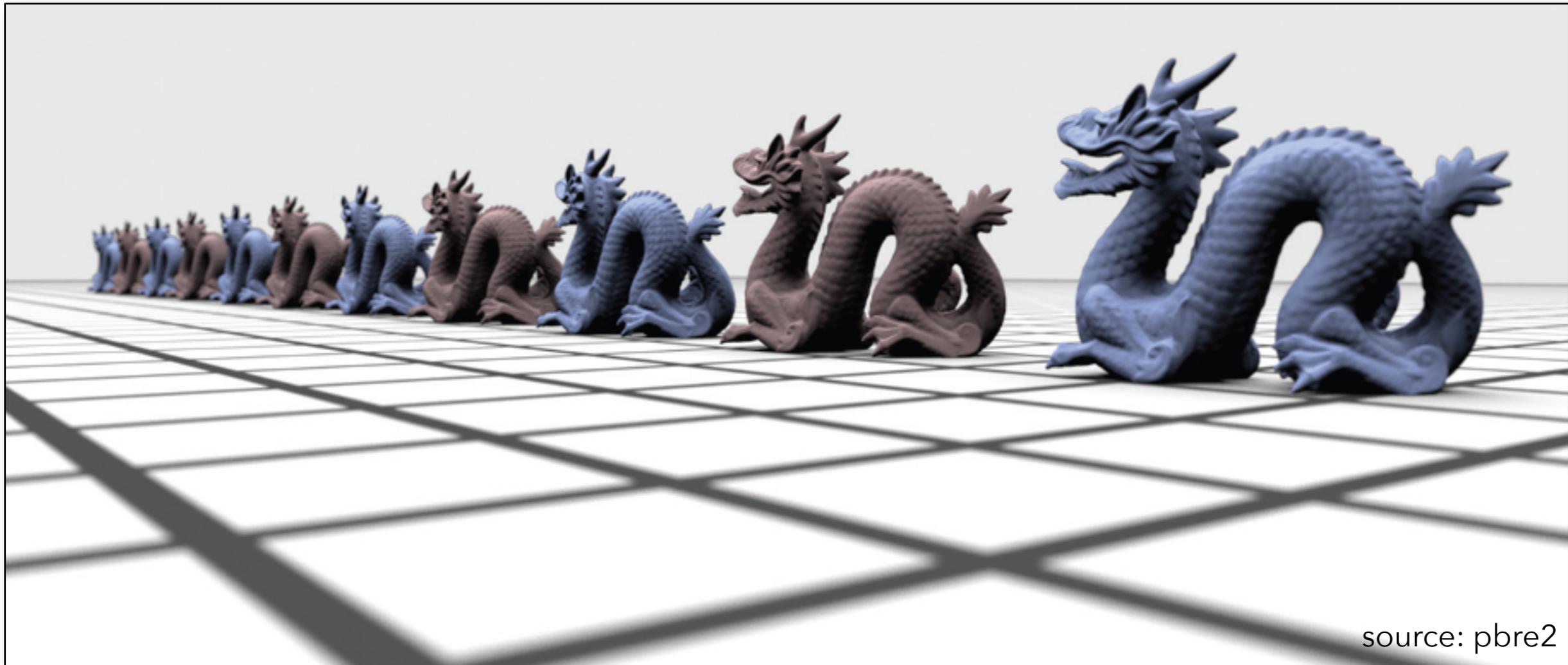
source: flickr

# Pinhole Camera

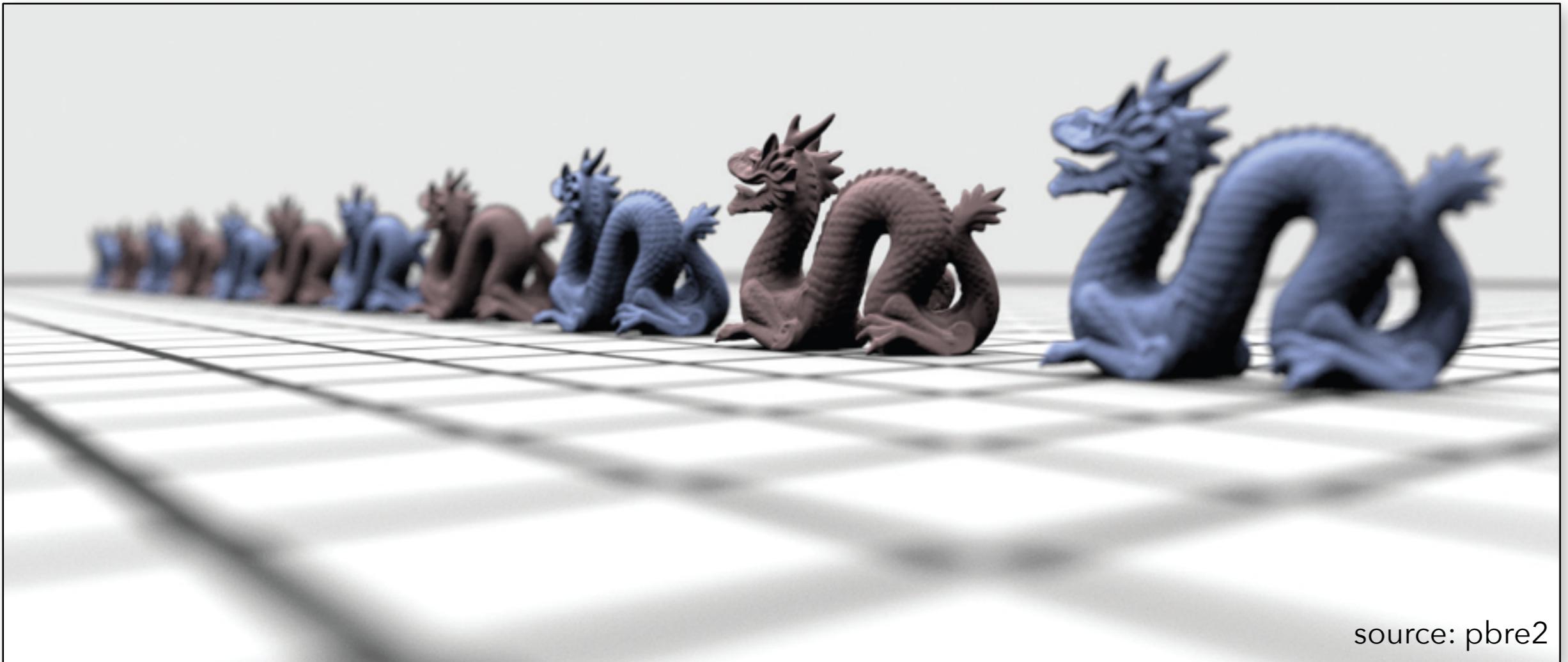


source: pbre2

# Thin-Lens Camera (small aperture)



# Thin-Lens Camera (larger aperture)



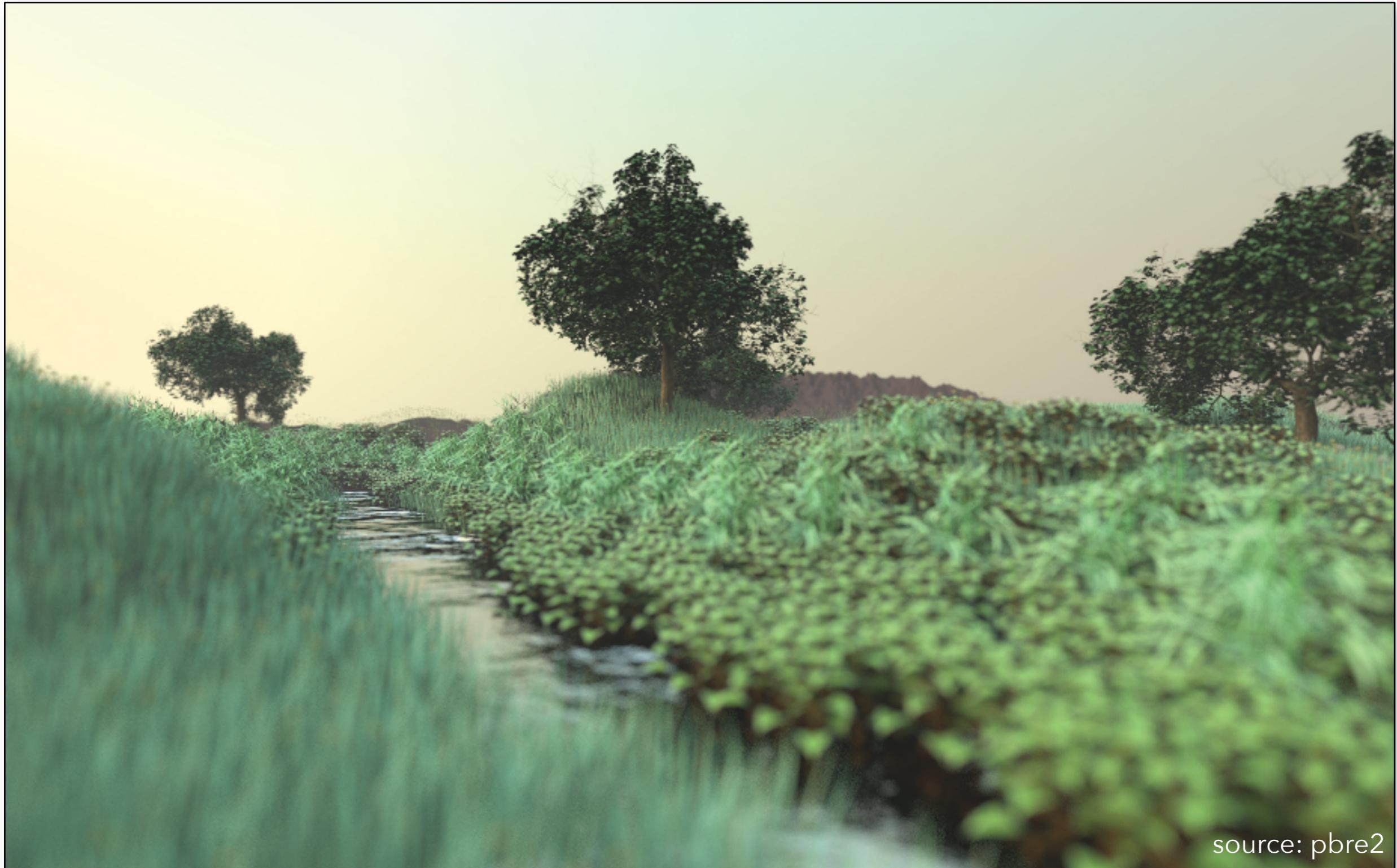
source: pbre2

# Thin-Lens Camera (huge aperture)



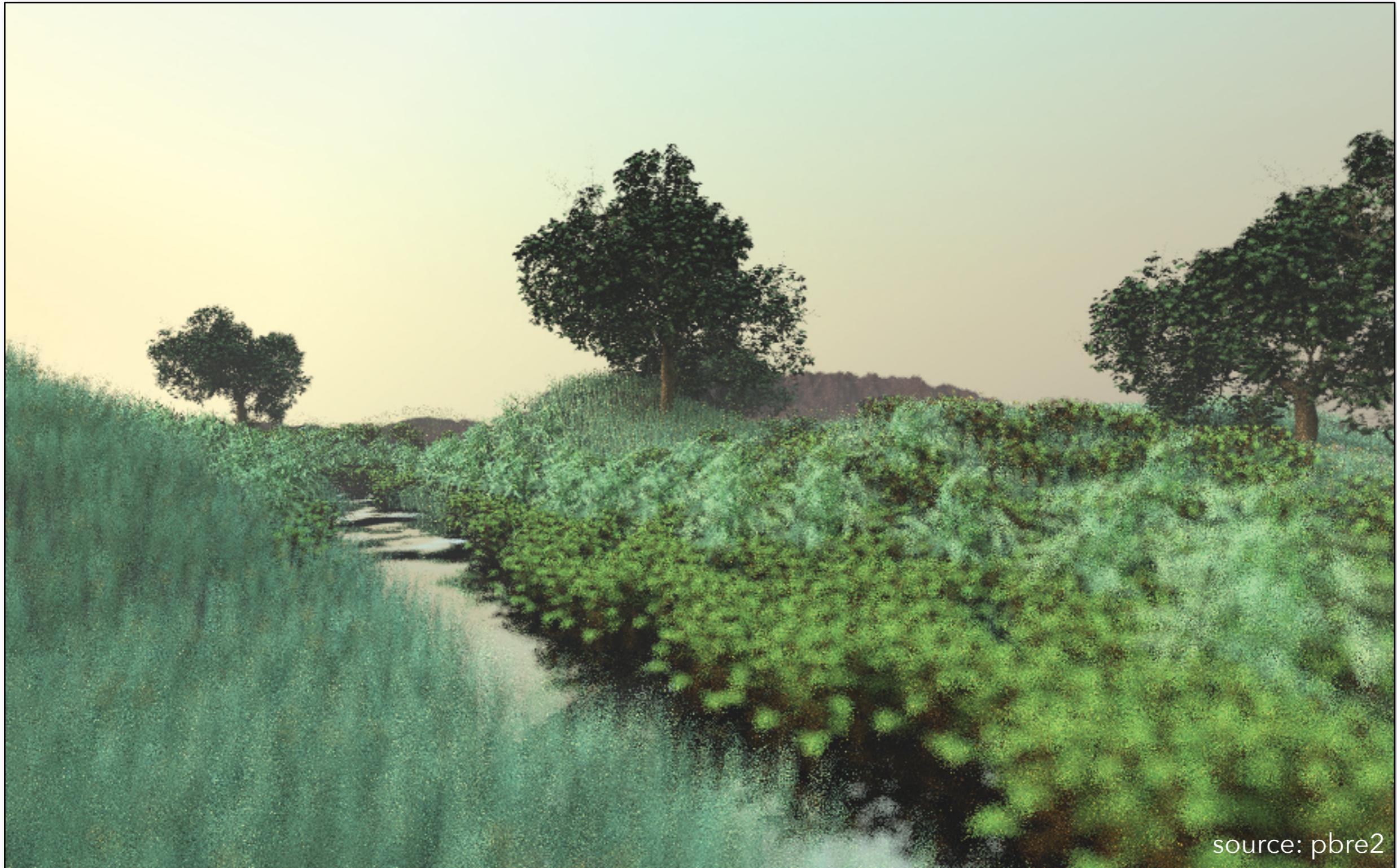
source: pbre2

# Depth of Field



source: pbre2

# Depth of Field



source: pbre2

# Depth of Field

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- *Depth of field* = the range of distances within which objects appear in focus
- Integration over pixel area and *aperture area*
- Typically, the lens geometry is defined by the *radius* and the *distance* to the focal plane
- Sample the aperture, figure out the outgoing direction so that the points on the focal plane are in focus

# Depth of Field (Miniatrization)



source: [onebigphoto.com](http://onebigphoto.com)

# Depth of Field (Bokeh)



source: wikipedia

# DoF (Cheap lens/Bad Bokeh)



source: wikipedia

# Depth of Field (Bokeh)



source: pptbackgrounds.net

# Bokeh (Cat's Eyes)



source: flickr

# Bokeh (Cat's Eyes)



source: flickr

# Cat's Eyes



source: toothwalker.org

# Implementing a Thin-lens

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- Blackboard & Book

# Motion Blur

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- Real cameras have a finite exposure time (shutter speed)
- Integration over time

# Motion Blur



source: flickr

# Motion Blur



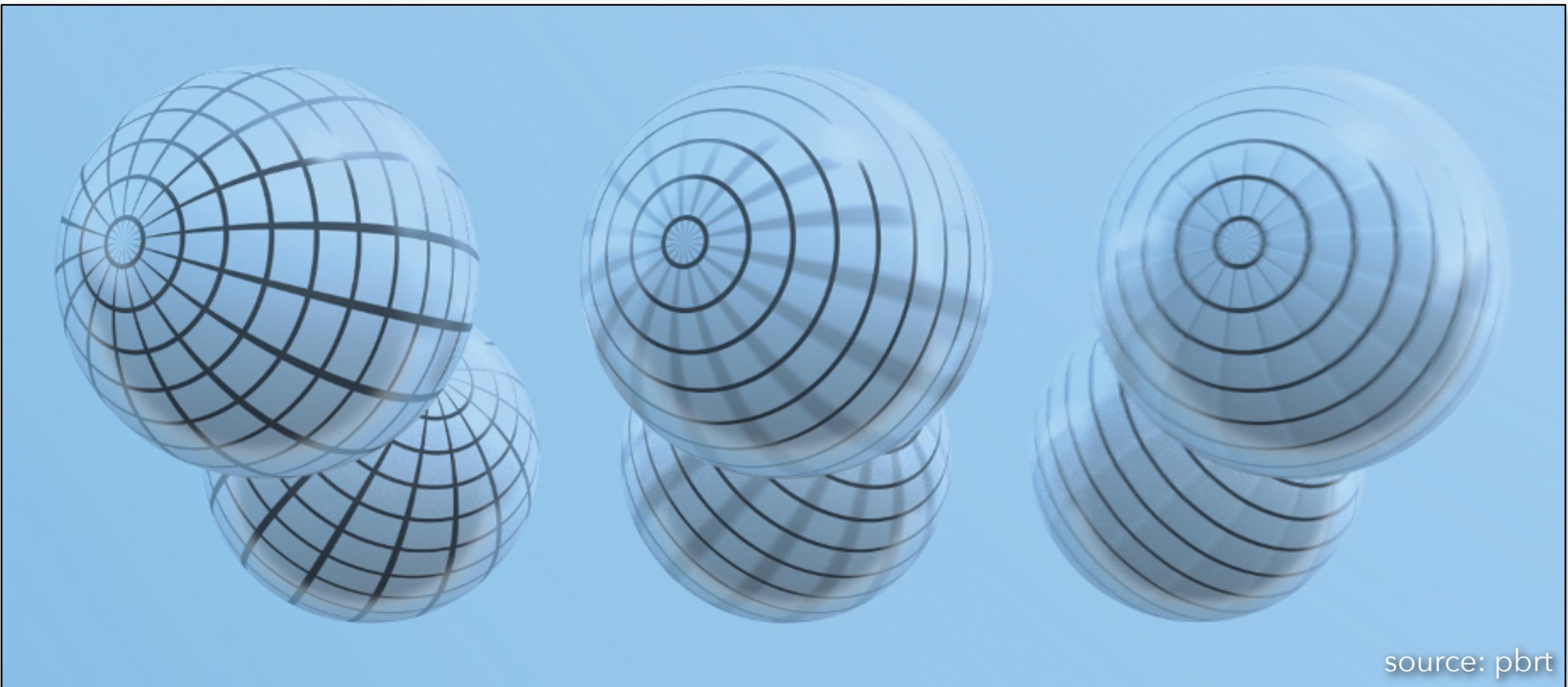
source: 1x.com

# Motion Blur



source: flicker

# Motion Blur



source: pbrt

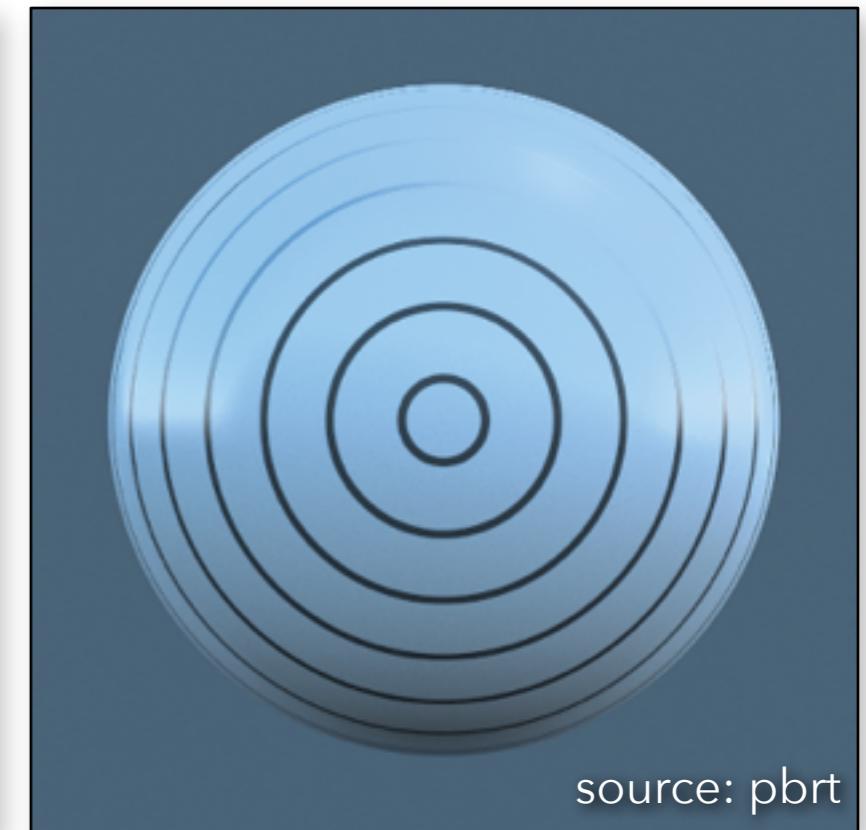
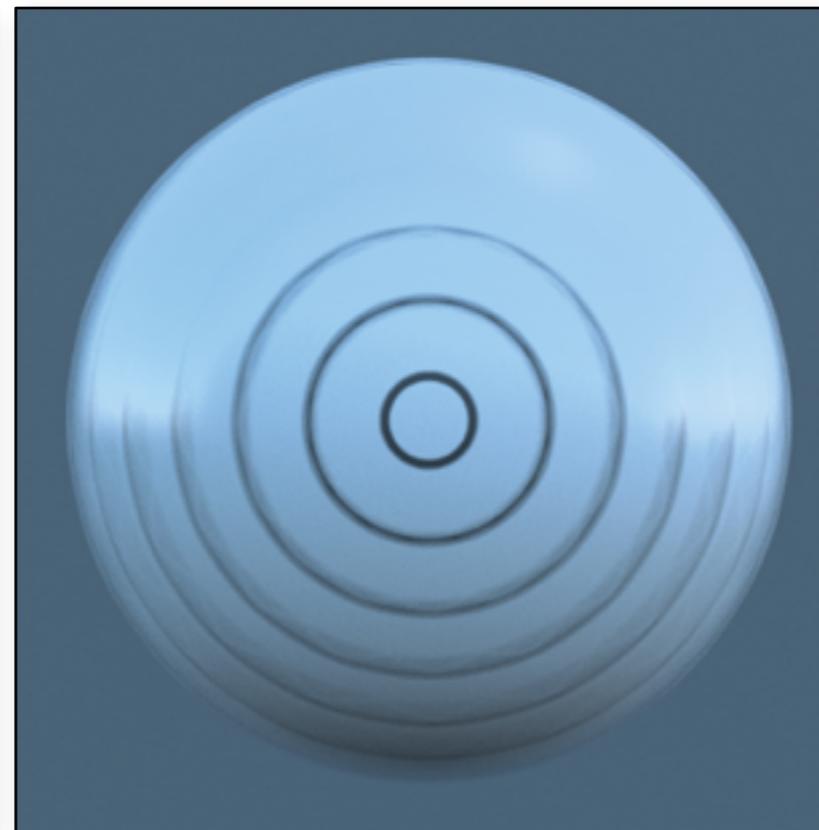
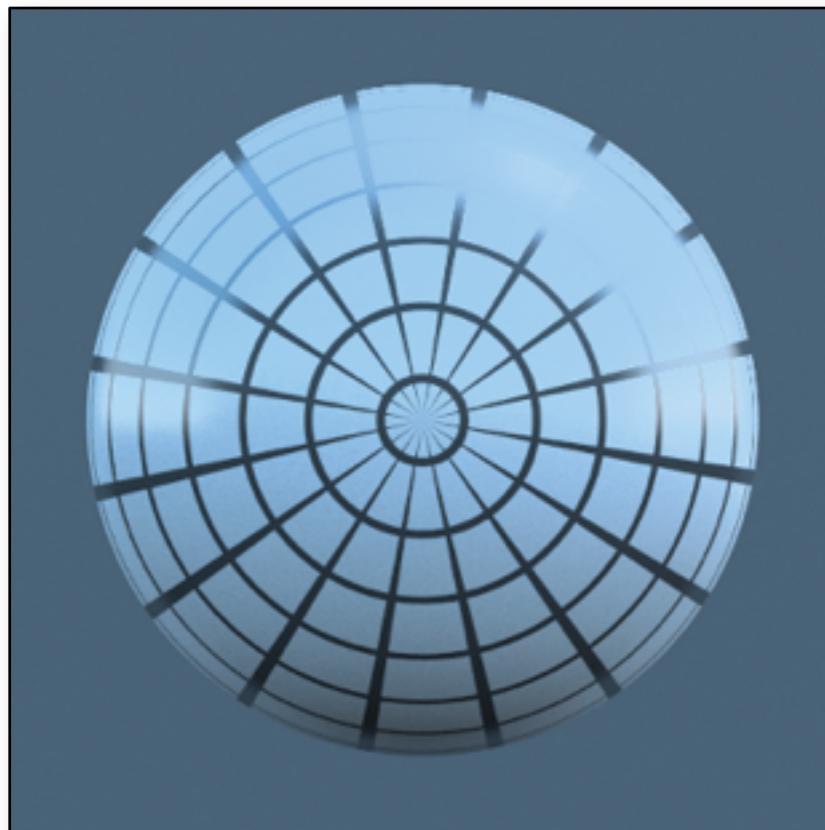
# Motion Blur

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- Moving camera (easy):
  - trace many rays/pixel, each ray gets a random time
  - origin/direction of camera ray based on time value
- Object motion blur?
  - each ray gets a random time
  - intersect with transformed objects based on time
    - acceleration structures?

# Motion Blur

- We typically specify object positions/ orientations using matrix transformations
- How can we interpolate two transformations?
- Naïve/bad: interpolate matrix entries



# Next time

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- More improvements to Monte Carlo