Теорема Akra-Bazzi.

№1

1) $T(x) = T\left(\frac{x}{2}\right) + \Theta(\log_2 x)$

$a = 1,\ b = \frac{1}{2},\ f(x) = \log_2 x$

$1 \cdot \left(\frac{1}{2}\right)^p = 1 \quad p = 0$

$$T(x) = \Theta\left(x^0 + x^0 \int_1^x \frac{\log_2 x}{x^{1+0}}\,dx\right) = \Theta\left(1 + \int_1^x \frac{\log_2 x}{x}\,dx\right) =$$

$$= \Theta(\log_2^2 x)$$

2) $T(x) = \frac{1}{2} \cdot T\left(\frac{x}{2}\right) + \Theta\left(\frac{1}{x}\right)$

$a = \frac{1}{2},\ b = \frac{1}{2},\ f(x) = \frac{1}{x}$

$\frac{1}{2} \cdot \left(\frac{1}{2}\right)^p = 1 \quad p = -1$

$$T(x) = \Theta\left(x^{-1} + x^{-1} \int_1^x \frac{1}{x} \cdot \frac{1}{x^{1-1}}\,dx\right) =$$

$$= \Theta\left(\frac{1}{x} + \frac{1}{x} \cdot \int_1^x \frac{1}{x}\,dx\right) = \Theta\left(\frac{1}{x} + \frac{\log_2 x}{x}\right) = \Theta\left(\frac{\log_2 x}{x}\right)$$

3) $T(x) = 4 \cdot T\left(\frac{x}{2}\right) + \Theta(x)$

$a = 4,\ b = \frac{1}{2},\ f(x) = x$

$4 \cdot \left(\frac{1}{2}\right)^p = 1 \quad p = 2$

$$T(x) = \Theta\left(x^2 + x^2 \int_1^x \frac{x}{x^{1+2}}\, dx\right) =$$

$$= \Theta\left(x^2 + x^2 \int_1^x \frac{1}{x^2}\, dx\right) = \Theta(x^2)$$

4) $T(x) = 2 \cdot T\left(\frac{2}{3} \cdot x\right)$

$a = 2,\ b = \frac{2}{3},\ f(x) = 0$

$2 \cdot \left(\frac{2}{3}\right)^p = 1 \qquad p = \frac{\ln 2}{\ln \frac{3}{2}}$

$$T(x) = \Theta\left(x^{\ln 2/\ln\frac{3}{2}} + x^{\ln 2/\ln \frac{3}{2}} \int_1^x \frac{0}{x^{\ln 2/\ln\frac{3}{2}}} = \Theta\left(x^{\ln 2/\ln\frac{3}{2}}\right)\right) =$$

$$\approx \Theta\left(x^{1,7095}\right)$$

5) $T(x) = T(x-1) + x$

т.к Akra-Bazzi не применима $\to$

$$\begin{cases} a_1 = 1 \\ a_n = a_{n-1} + n \end{cases}$$

$$a_n = 1^{n-1} \cdot 1 + 1^n \sum_{i=2}^{n} \left(\frac{1}{1}\right)^{i} \cdot i = 1 + \frac{1}{2}\left(n^2 + n - 2\right) =$$

$$= \Theta\left(\frac{n^2}{2} + \frac{n}{2}\right) = \Theta(n^2)$$

6) $T(x) = 3 \cdot T\left(\frac{x}{3}\right) + \frac{x}{\ln x}$

$a = 3,\ b = \frac{1}{3},\ f(x) = \frac{x}{\ln x}$

$3 \cdot \left(\frac{1}{3}\right)^p = 1 \qquad p = 1$

$$T(x) = \Theta\left(x^1 + x^1 \cdot \int_1^x \frac{x}{\ln x} \cdot \frac{1}{x^{1+1}}\, dx\right) =$$

$$= \Theta\left(x + x \cdot \int_1^x \frac{1}{x \ln x}\, dx\right) = \Theta(x \cdot \ln\ln(x))$$

№2

1) $T(n) = 2^n \cdot T\left(\frac{n}{2}\right) + n^n$

$\cancel{a = 2,\ b = \frac{1}{2},\ f(x) = n^n}$ Теорема Akra-Bazzi и основная теорема не применимы.

2) $T(n) = 2 \cdot T\left(\frac{n}{2}\right) + \frac{n}{\ln(n)}$

$a = 2, \quad b = \frac{1}{2}, \quad f(x) = \frac{n}{\ln(n)}$

$$T(x) = \Theta\left(x + x \int_2^x \frac{x}{x^2 \ln(x)}\right) = \Theta(x \cdot \ln\ln(x))$$

3) $T(x) = \frac{1}{2} T\left(\frac{x}{2}\right) + \frac{1}{x}$

$a = \frac{1}{2}, \quad b = \frac{1}{2}, \quad f(x) = \frac{1}{x}$

$\frac{1}{2} \cdot \left(\frac{1}{2}\right)^p = 1 \qquad p = -1$

$$T(x) = \Theta\left(x^{-1} + x^{-1} \int_1^x \frac{1}{x} \cdot \frac{1}{x^{1-1}}\, dx\right) =$$

$$= \Theta\left(\frac{1}{x} + \frac{1}{x} \int_1^x \frac{1}{x}\, dx\right) = \Theta\left(x + \frac{\ln(x)}{x}\right. = \Theta\left(\frac{\ln(x)}{x}\right)$$

4) $T(x) = 64 \cdot T\left(\frac{x}{8}\right) - x^2 \cdot \ln(x)$

Теорема Akra-Bazzi не применима.

5) $T(x) = 2 \cdot T\left(\frac{x}{2}\right) + \log_2 x$

$a = 2, \quad b = \frac{1}{2}, \quad f(x) = \log_2 x$

$2 \cdot \left(\frac{1}{2}\right)^p = 1 \qquad p = 1$

$T(x) = \Theta\left(x^1 + x^1 \int_1^x \frac{\log_2 x}{x^2}\, dx\right) =$

6) $T(x) = T\left(\frac{x}{2}\right) + x \cdot (2 - \cos(x))$

Теорема Akra-Bazzi не применима

$\lim\limits_{x \to \infty} \frac{x \cdot (2 - \cos(x))}{x^{\alpha}} = \lim\limits_{x \to \infty} \frac{2 - \cos(x)}{x^{\alpha - 1}} = 0 \Rightarrow$ $g(x) = o(x^{\alpha})$, если $\alpha > 1$.

$a = 1, \quad b = \frac{1}{2}, \quad f(x) = x \cdot (2 - \cos(x))$

$1 \cdot \left(\frac{1}{2}\right)^p = 1 \qquad p = 0$

$x^{1+p} = x^{1+0} = x$

№5

1) $T(x) = 2 \cdot T\left(\frac{x}{3}\right) + \frac{3}{2} T\left(\frac{x}{4}\right) + 5 \cdot T\left(\frac{x}{2}\right) + \Theta(x^2)$

$a_1 = 2 \qquad a_2 = \frac{3}{2} \qquad a_3 = 5$

$b_1 = \frac{1}{3} \qquad b_2 = \frac{1}{4} \qquad b_3 = \frac{1}{2} \qquad f(x) = x^2$

$p \approx 2{,}5754359$.

$$T(x) = \Theta\left(x^{2,574} + x^{2,574} \int_1^x \frac{x^2}{x^{1+2,574}}\,dx\right) =$$

$$= \Theta\left(x^{2,574} + (-x^2 \cdot 1,74216)\right) = \Theta\left(x^{2,574}\right)$$

2) $T(x) = 2 \cdot T\left(\frac{x}{5}\right) + T\left(\frac{x}{6}\right) + \Theta(x^2)$

$a_1 = 2 \quad a_2 = 1$

$b_1 = \frac{1}{5} \quad b_2 = \frac{1}{6} \quad f(x) = x^2$

$p \approx 0,658$

$$T(x) = \Theta\left(x^{0,658} + x^{0,658} \int_1^x \frac{x^2}{x^{1+0,658}}\,dx\right) =$$

$$= \Theta\left(x^{0,658} + x^2 \cdot 0,745156\right) = \Theta(x^2)$$

3) $T(x) = \frac{3}{4} T\left(\frac{x}{2}\right) + T\left(\frac{x}{3}\right) + T\left(\frac{x}{6}\right) + T\left(\frac{x}{8}\right) + \Theta(x)$

$a_1 = \frac{3}{4} \quad a_2 = 1 \quad a_3 = 1 \quad a_4 = 1$

$b_1 = \frac{1}{2} \quad b_2 = \frac{1}{3} \quad b_3 = \frac{1}{6} \quad b_4 = \frac{1}{8} \quad . \quad f(x) = x$

$\cancel{p \approx}$ $p = 1$

$$T(x) = \Theta\left(x + x \int_1^x \frac{x}{x^{1+1}}\,dx\right) = \Theta(x + x\ln(x)) =$$

$$= \Theta(x \cdot \ln(x))$$

4) $T(x) = \frac{4}{3} \cdot T\left(\frac{x}{2}\right) + 3 \cdot T\left(\frac{x}{2}\right) + \frac{16}{3} \cdot T\left(\frac{x}{4}\right) + \Theta\left(x^2 \cdot \ln\ln(x)\right)$

$a_1 2 =$ $\quad a_2 \cancel{=}$ $\quad p=2$

$$T(x) = \Theta\left(x^2 + x^2 \int_1^x \frac{x^2 \cdot \ln\ln(x)}{x^{1+2}} dx\right) =$$

$$= \Theta(x^2 + x^2 \cdot \ln(x) \cdot \ln\ln(x)) = \Theta(x^2 \cdot \ln(x) \cdot \ln\ln(x))$$

5) $T(x) = 2 \cdot T\left(\frac{x}{2}\right) + \Theta(x \cdot \ln^2 \ln(x))$; $p=1$

$$T(x) = \Theta\left(x + x \int_1^x \frac{x \cdot \ln^2 \ln(x)}{x^2} dx\right) =$$

$$= \Theta(x + x \cdot \ln(x) \cdot \ln^2 \ln(x) - 2 \cdot x \cdot \ln(x) \cdot \ln\ln(x) + 2 \cdot$$
$$\cdot x \cdot \ln(x) = \Theta(x \cdot \ln(x) \cdot \ln^2 \ln(x))$$

№6

1) $T(x) = 2 \cdot T\left(\frac{x}{4}\right) + 3 \cdot T\left(\frac{x}{6}\right) + \Theta(x \cdot \log_2 x)$,

$p=1$

$$T(x) = \Theta\left(x + x \int_1^x \frac{x \log_2 x}{x^2} dx\right) =$$

$$= \Theta\left(x + x \cdot \frac{\log_2^2 x}{2\log(2)}\right) = \Theta\left(\cancel{\frac{x^2}{\log_2 x}}\, x \cdot \log^2(x)\right)$$

2) $T(x) = 2 \cdot T\left(\frac{x}{2}\right) + \frac{8}{9} \cdot T\left(\frac{2x}{4}\right) + \Theta\left(\frac{x^2}{\log_2 x}\right)$,

$p=2$

$$T(x) = \Theta\left(x^2 + x^2 \int_1^x \frac{x^2}{\log_2 x} \cdot \frac{1}{x^3} dx\right) =$$

$= \Theta(x^2 + x^2 \cdot \log_2 \log_2 x) = \Theta(x^2 \cdot \log_2 \log_2 x)$

3) $T(x) = 3 \cdot T(\frac{x}{2}) + 2 \cdot T(\frac{x}{4}) + \Theta(x)$, $p = 1,832$

$T(x) = x^{1,832} + x^{1,822} \int_1^x \frac{x}{x^{1+1,832}} dx) =$

$= \Theta(x^{1,832} + (-1,20192 \cdot x)) = \Theta(x^{1,832})$

4) $T(x) = 3 \cdot T(\frac{x}{2}) + 5 \cdot T(\frac{x}{4}) + T(\frac{x}{8}) + \Theta(x)$, $p = 2,082$

$T(x) = \Theta(x^{2,082} + x^{2,082} \int_1^x \frac{x}{x^{1+2,082}} dx) =$

$= \Theta(x^{2,082} + (-0,924214 \cdot x)) = \Theta(x^{a2,082})$

Сравнение ассимптотической скорости решений рекуррентных соотношений декомпозиции.

Время работы алгоритма в

$T(n) = \alpha\, T(\frac{n}{4})_{n/4} + n^2$, $\alpha > 0$

Время работы алгоритма а

$T(n) = 7 \cdot T_{n/2} + n^2$

Алгоритм в

$T(x) = \Theta(x^{\log_4 \alpha} + x^{\log_4 \alpha} \cdot \int_1^x \frac{x^2}{x^{1+\log_4 \alpha}} dx) =$

$= \Theta(x^{\log_4 \alpha} + x^2)$

Алгоритм В а $\quad p = \log_2 7$

$$T(x) = \Theta\left(x^{\log_2 7} + x^{\log_2 7} \int_1^x \frac{x^2}{x^{1+\log_2 7}} dx\right) = \Theta(x^{\log_2 7})$$

$$0 = \lim_{x \to \infty} \cdot \frac{x^{\log_4 \alpha} + x^2}{x^{\log_2 7}} \Rightarrow \lim_{x \to \infty} (x^{\log_4 \alpha - \log_2 7} + x^{2 - \log_2 7}) = 0$$

$$\begin{cases} \log_4 \alpha - \log_2 7 < 0 \\ 2 - \log_2 7 < 0 \end{cases} \qquad \begin{cases} \log_4 \alpha < \log_2 7 \\ 2,807 > 2 \end{cases}$$

$\alpha < 4^{\log_2 7} \Rightarrow \alpha = 49$