

# **Learning Representations for Reasoning: Generalizing Across Diverse Structures**

Zhaocheng Zhu

09/18/2024





**Why** is generalization essential?

**What** generalization do we need?

**Why** do we study structures?



perception



reasoning

# Are We Doing Well?

What is the answer to  $1 + 1 + 1 - 1 - 1$ ?

---



The answer to  $1 + 1 + 1 - 1 - 1$  is 1.

---



# Are We Doing Well?

What is my son's son's son's father's father?

---



Your son's son is your grandson. Your grandson's father is your son. Your son's father is you. So, the answer is you.

---



# Are We Doing Well?

## How do you get 4L water using 5L and 3L jugs?



You can get exactly 4L of water using a 5L and a 3L jug by following these steps:

1. Fill the 5L jug to the top.
2. Pour water from the 5L jug into the 3L jug until the 3L jug is full. This leaves you with 2L in the 5L jug.
3. Empty the 3L jug and pour the remaining 2L from the 5L jug into the 3L jug.
4. Fill the 5L jug to the top again.
5. Pour water from the 5L jug into the 3L jug until the 3L jug is full. Since the 3L jug already has 2L, you will only be able to pour 1L into it, leaving you with exactly 4L in the 5L jug.



Now you have exactly 4L of water in the 5L jug.

# Are We Doing Well?

How do you get **5L** water using **9L** and **5L** jugs?



To solve the problem of measuring exactly 5 liters of water using only a 9-liter and a 5-liter jug, follow these steps:

1. Fill the 9-liter jug to the top.
2. Use the 9-liter jug to fill the 5-liter jug. This leaves you with 4 liters in the 9-liter jug.
3. Empty the 5-liter jug.
4. Transfer the 4 liters from the 9-liter jug to the 5-liter jug.
5. Fill the 9-liter jug to the top again.
6. Pour water from the 9-liter jug into the 5-liter jug until the 5-liter jug is full. Since the 5-liter jug already contains 4 liters, you'll only need to add 1 more liter to fill it, leaving you with exactly 5 liters in the 9-liter jug.



$$9 - 1 = 5 ???$$



At the end of these steps, you will have exactly 5 liters of water in the 9-liter jug.

# Which Student is Smarter?

$$12 \times 15 = 180$$

---



Okay, I memorized  $12 \times 15 = 180$ .



Ones:  $2 \times 5 = 10$ . Carry 1 to the tens.

Tens:  $2 \times 1 = 2$ .  $1 \times 5 = 5$ .  $2 + 5 + 1 = 8$ .



Hundreds:  $1 \times 1 = 1$ . So  $12 \times 15 = 180$ .



# Which Student is Smarter?

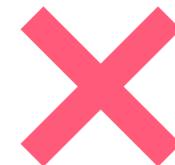
$$22 \times 15 = ?$$

---



It looks like  $12 \times 15$ .  $22 \times 15 = 180$ .

---



Ones:  $2 \times 5 = 10$ . Carry 1 to the tens.

Tens:  $2 \times 1 = 2$ .  $2 \times 5 = 10$ .  $2 + 10 = 12$ .

Carry 1 to the hundreds.

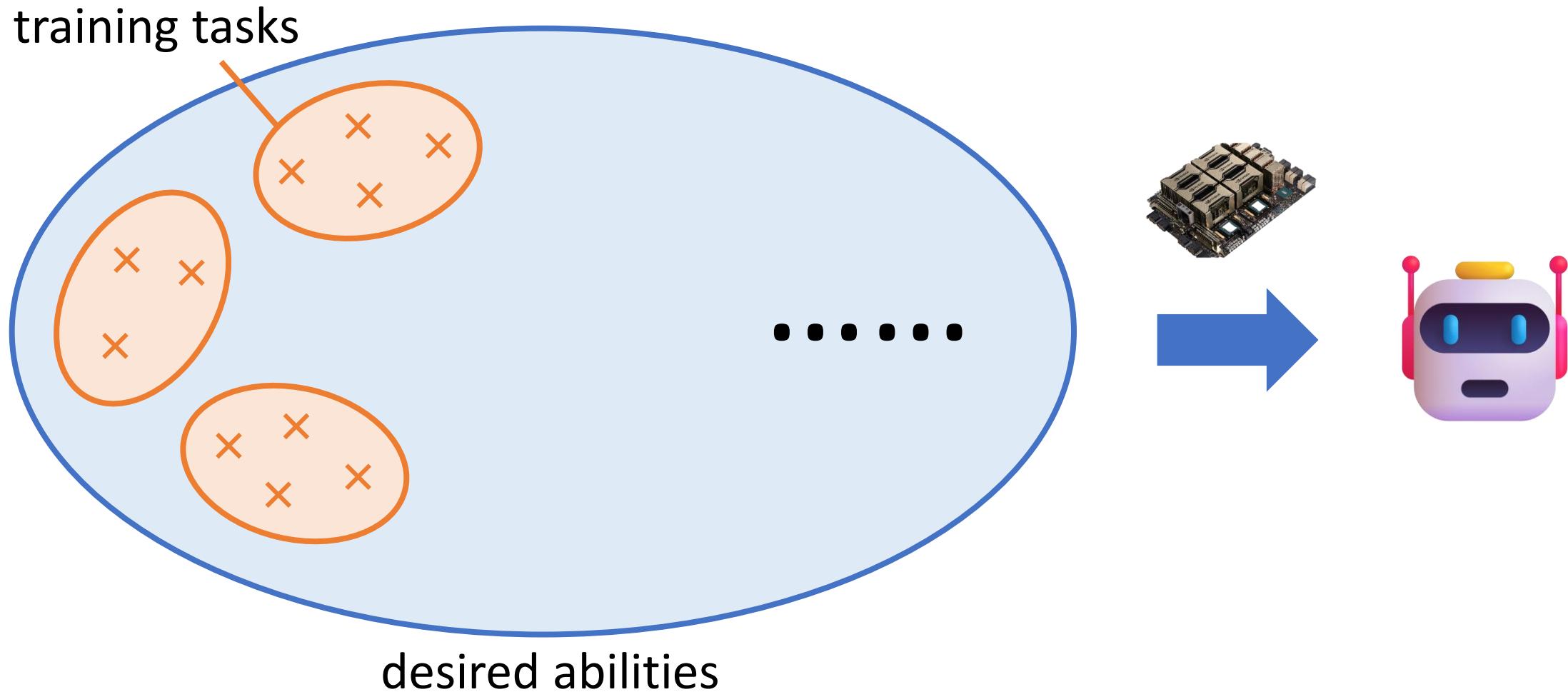
Hundreds:  $2 \times 1 = 2$ .  $2 + 1 = 3$ . So  $22 \times 15 = 330$ .

---

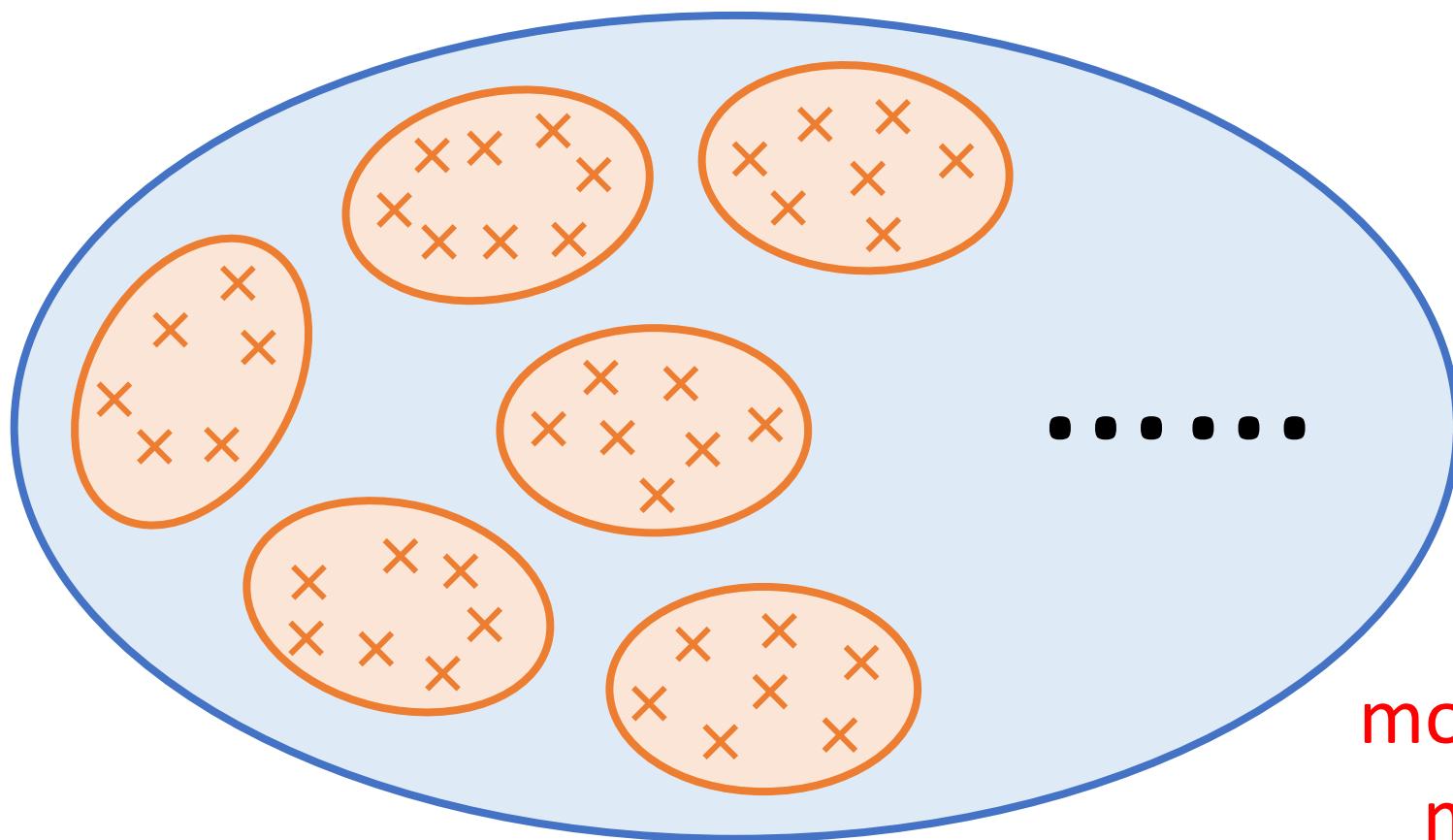


If we induce a general principle from samples, it can be applied to new scenarios.

# The Way We Build A(G)I

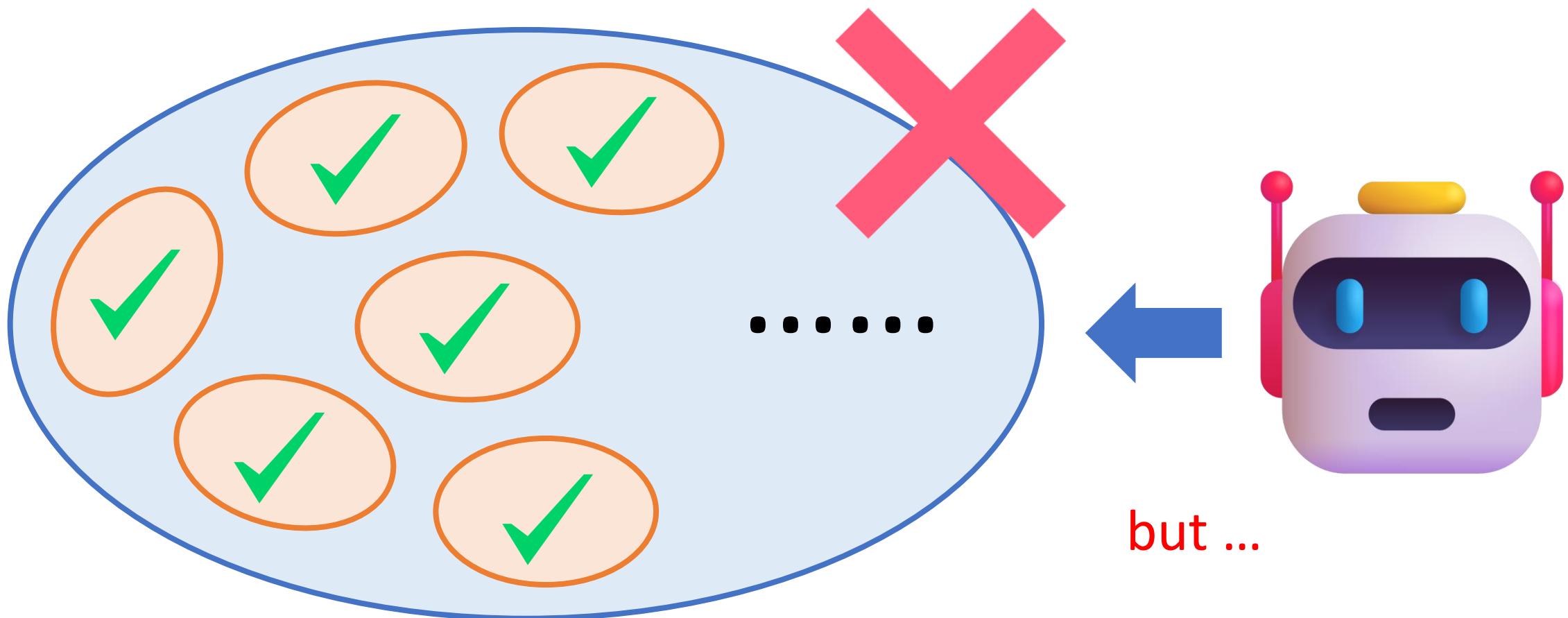


# The Way We Build A(G)I

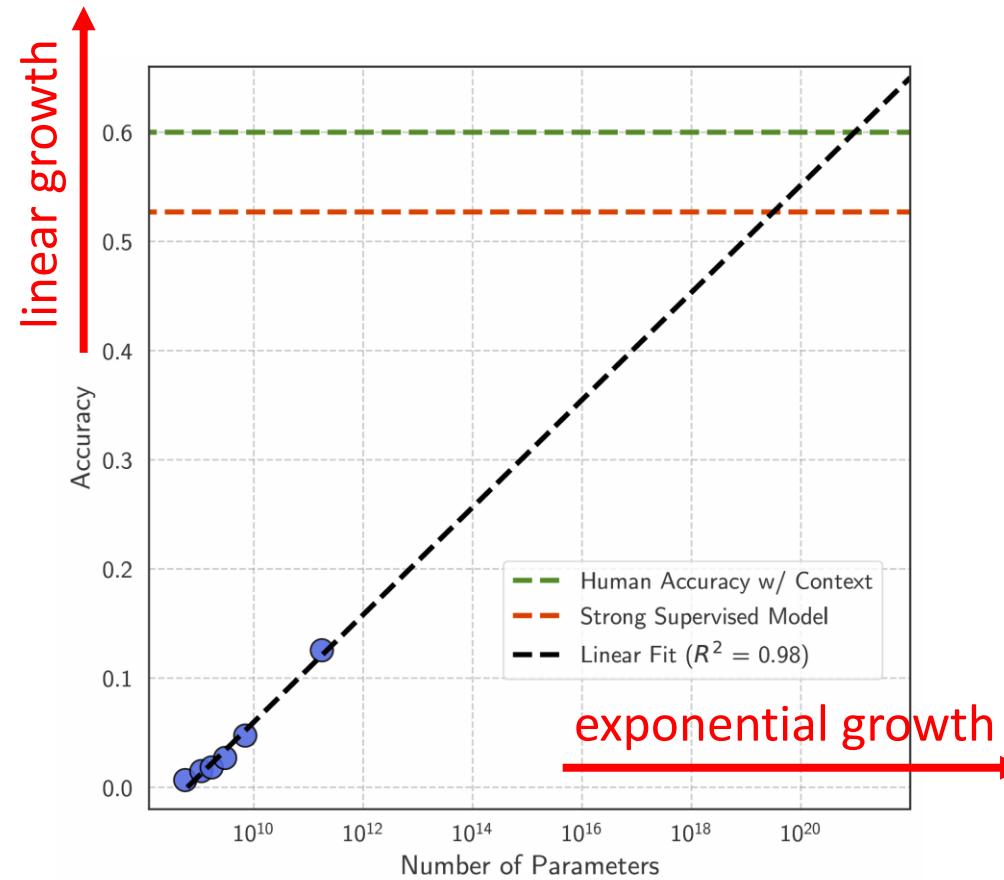


more training data  
more compute

# The Way We Build A(G)I

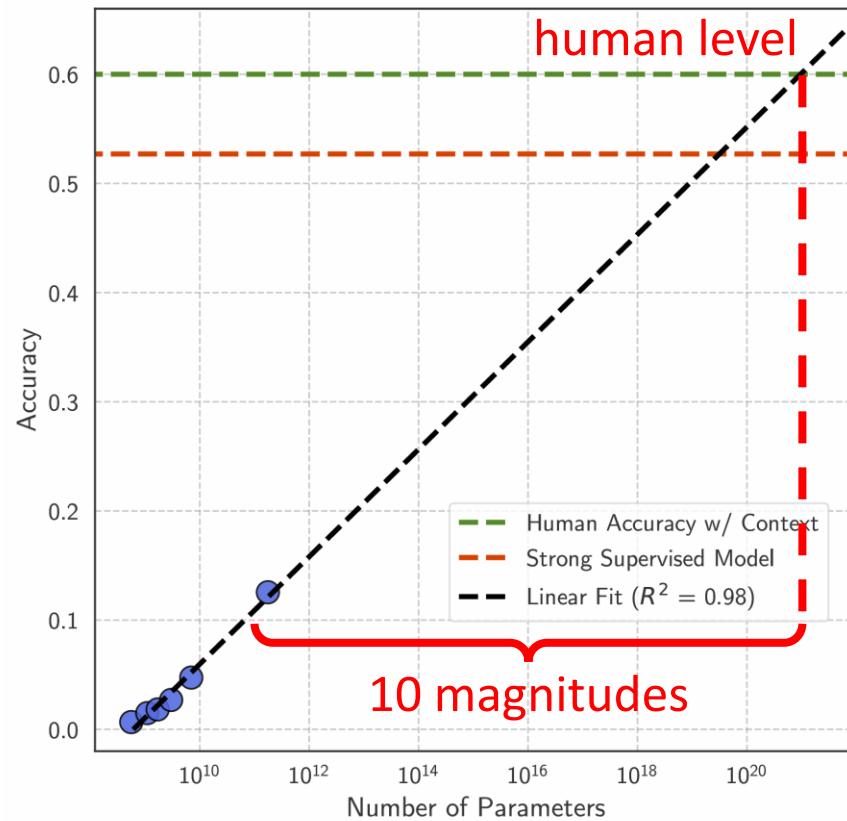


# Scaling Laws



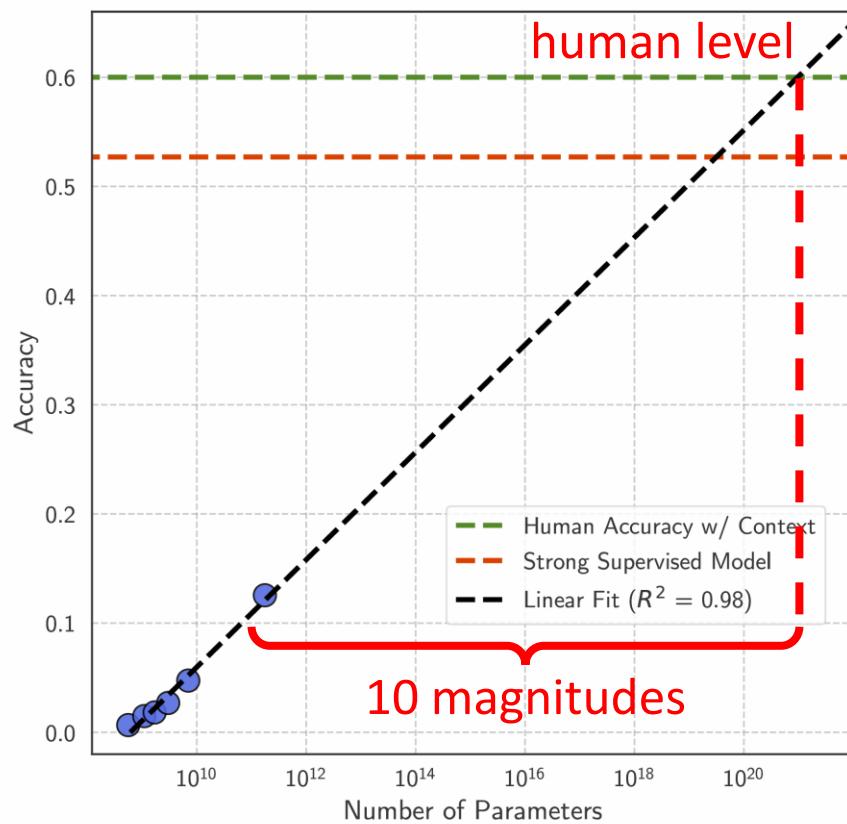
[1] Nikhil Kandpal, et al. Large Language Models Struggle to Learn Long-Tail Knowledge. ICML 2023.

# A Long Way to Go...

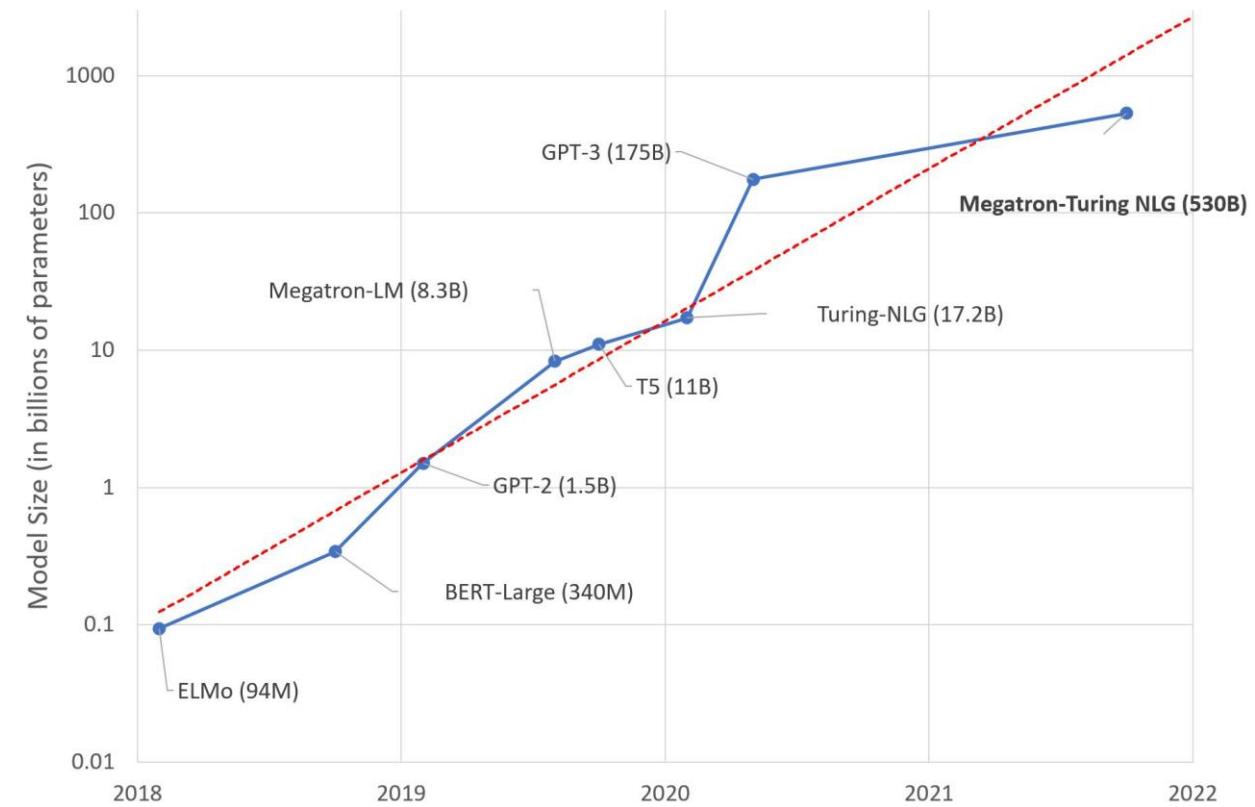


[1] Nikhil Kandpal, et al. Large Language Models Struggle to Learn Long-Tail Knowledge. ICML 2023.

# A Long Way to Go...

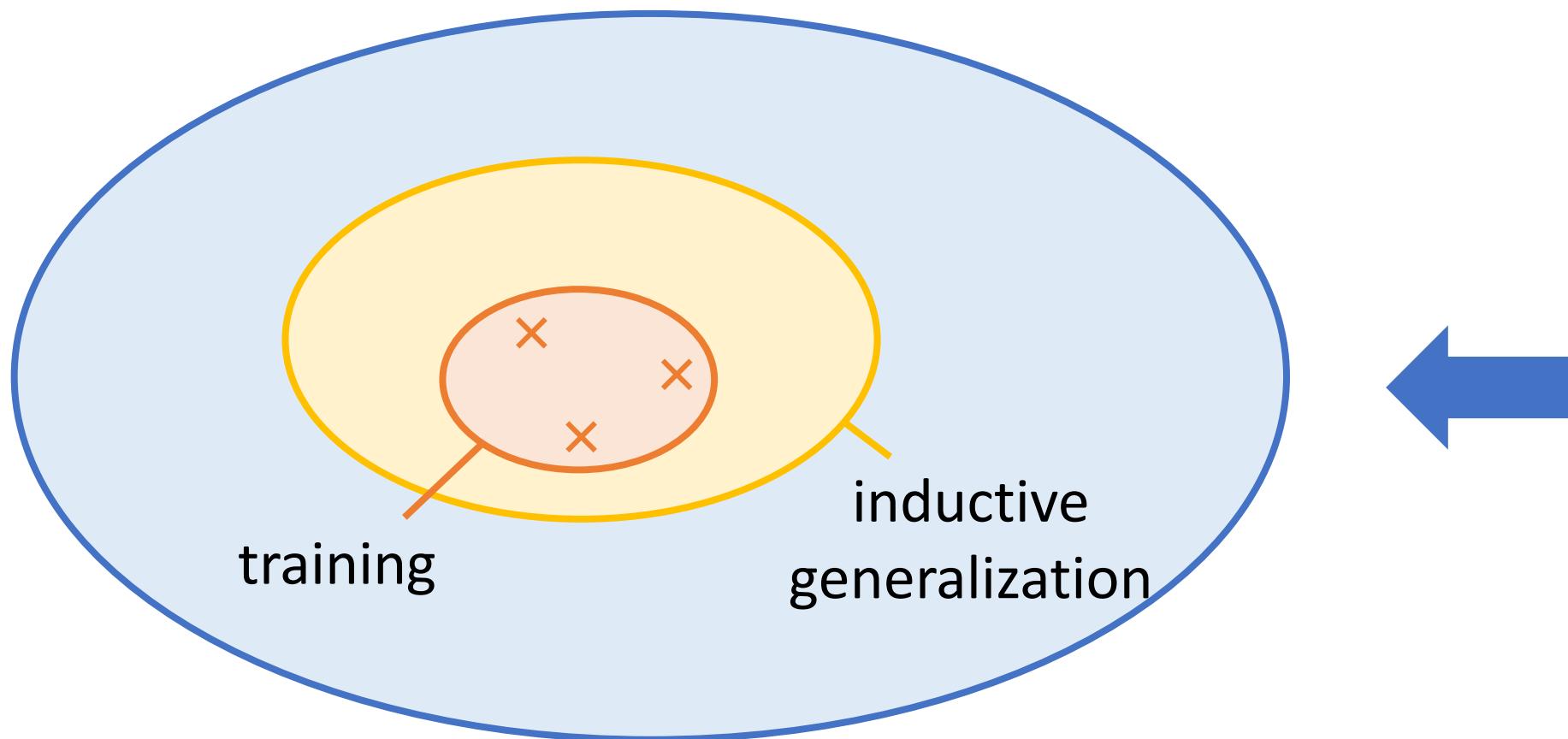


progress: 1 magnitude / year

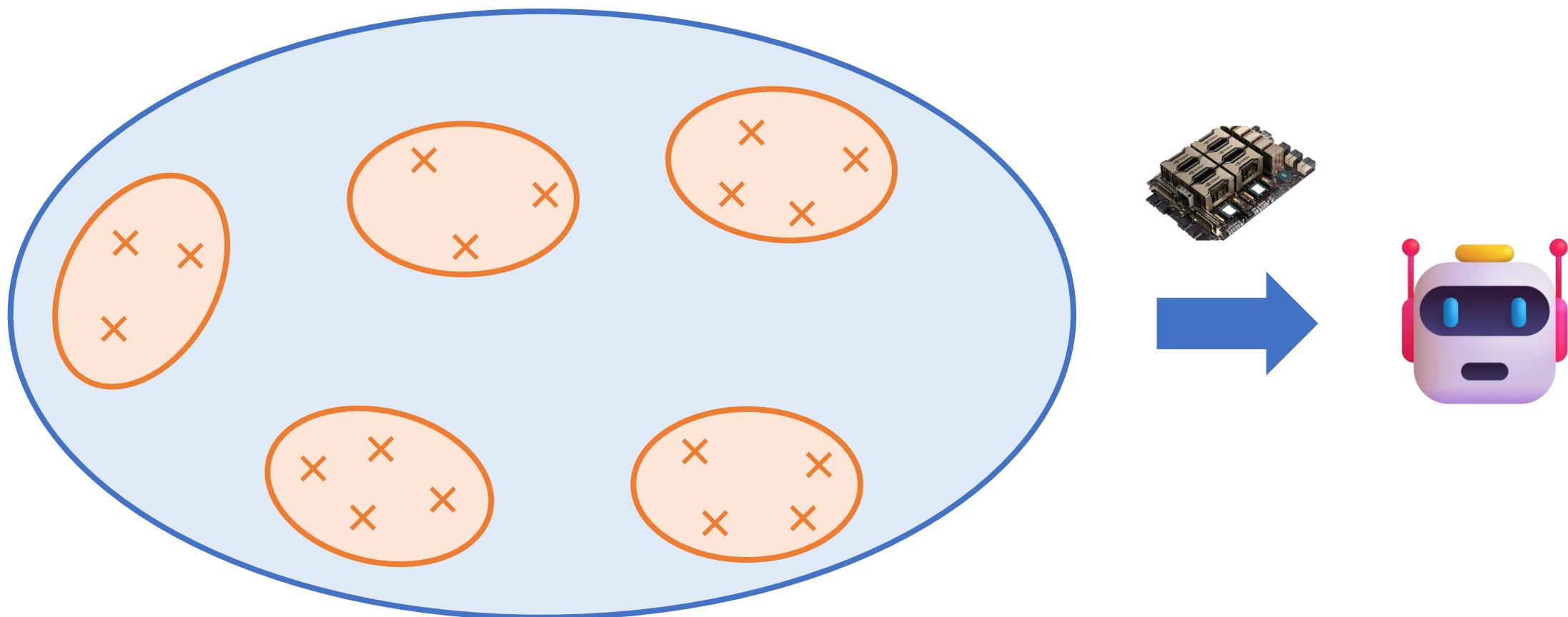


- [1] Nikhil Kandpal, et al. Large Language Models Struggle to Learn Long-Tail Knowledge. ICML 2023.
- [2] Julien Simon. Large Language Models: A New Moore's Law? HuggingFace blog. 2021.

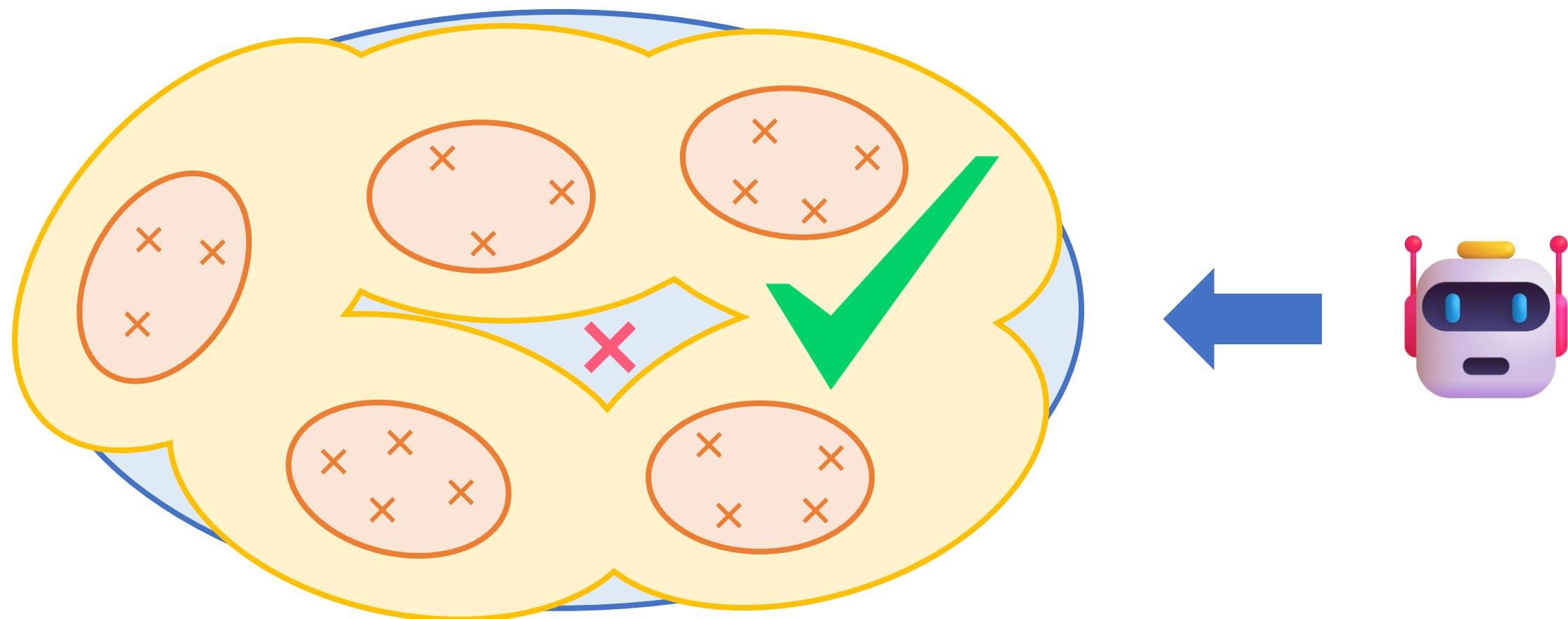
# The Way We Learn



# A Better Way to Build A(G)I

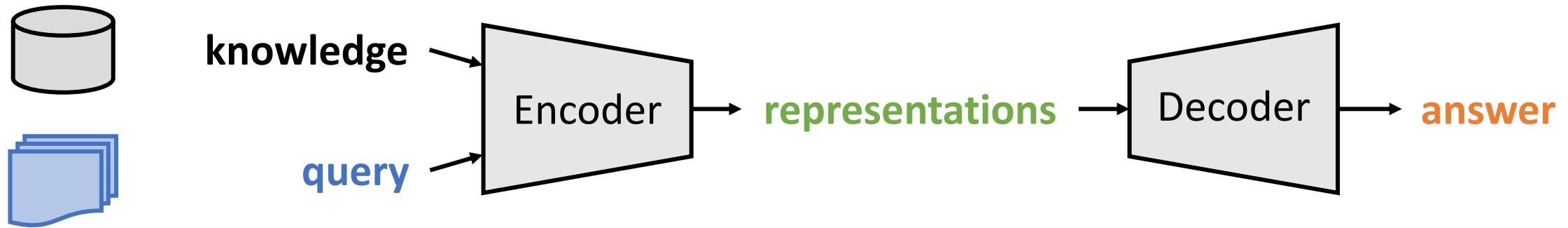


# A Better Way to Build $A(G)I$

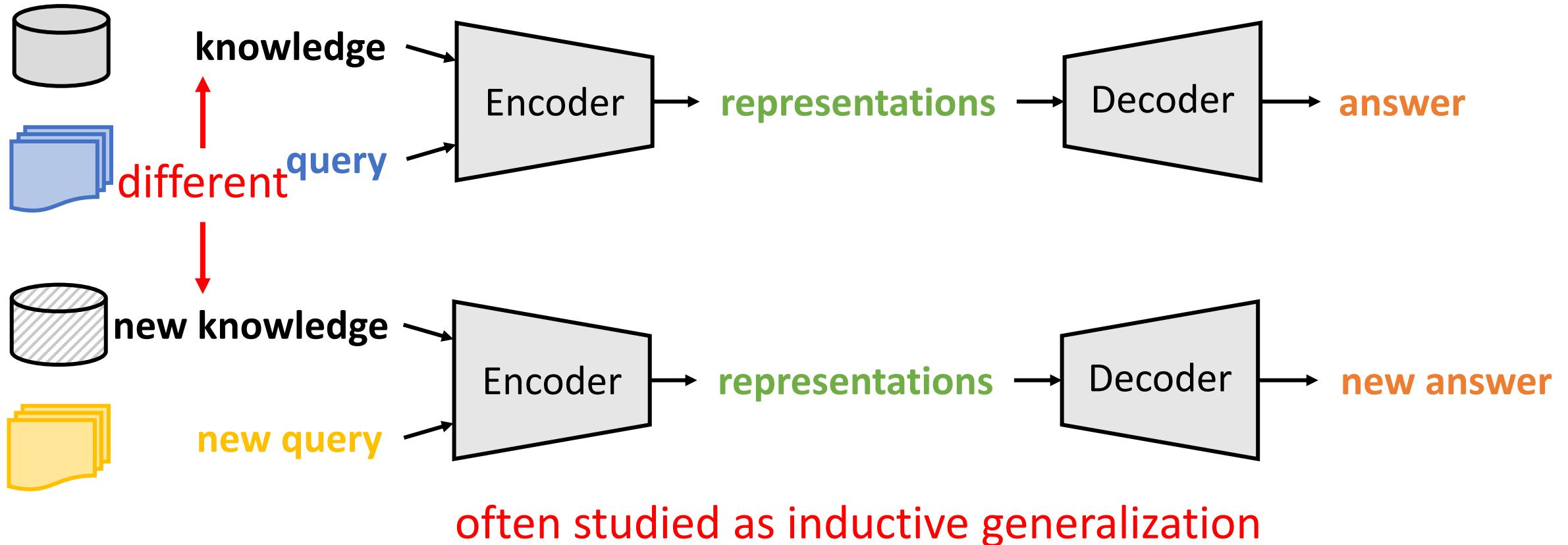


What **generalization** do we need for  
**representation learning models?**

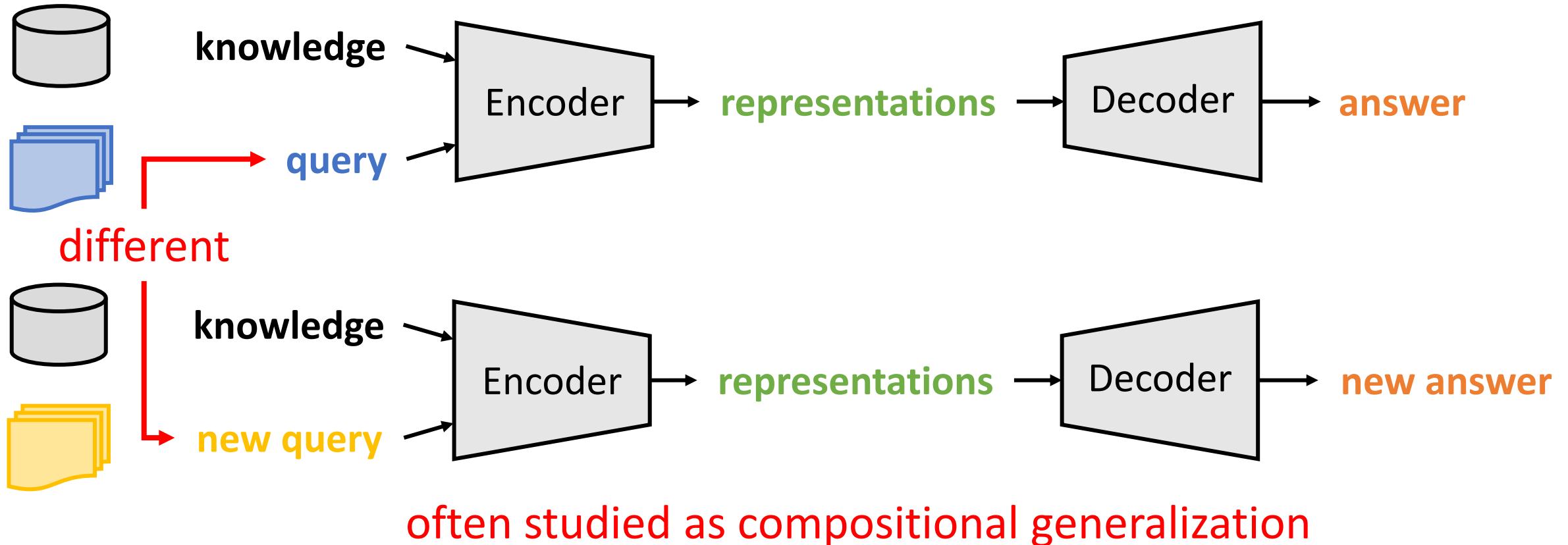
# Representation Learning



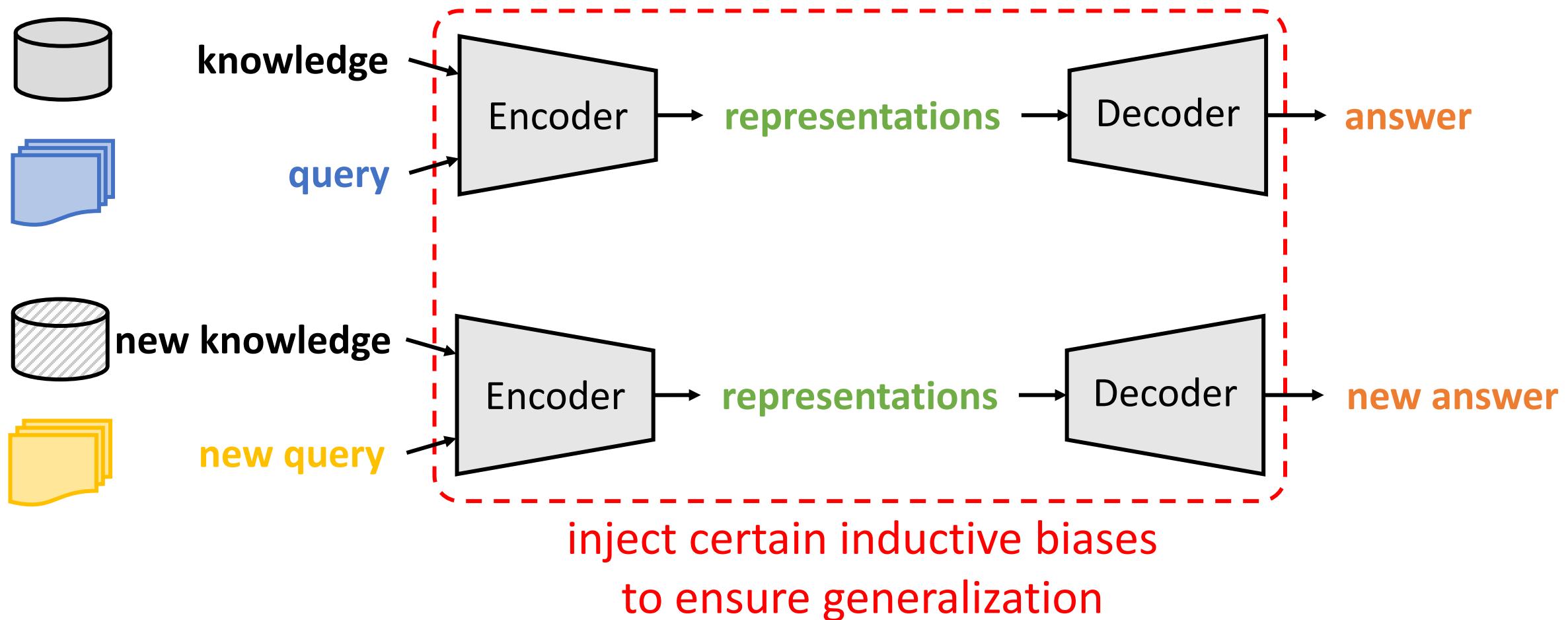
# Generalization to New Knowledge



# Generalization to New Queries



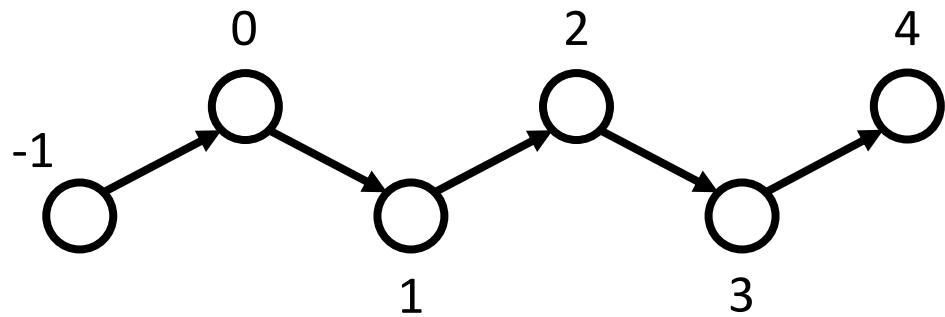
# Our Methodology



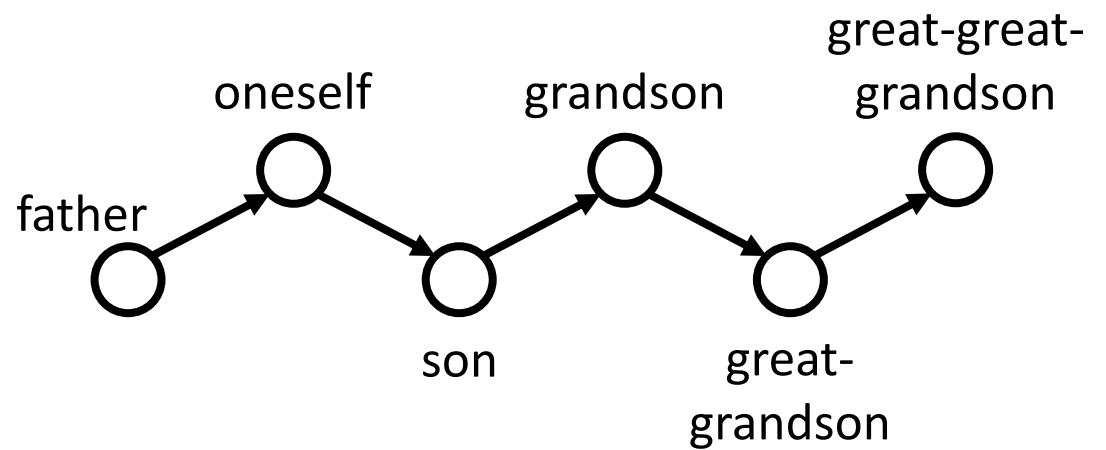
What kind of knowledge to generalize across?  
**Structure**

# Structure of Reasoning Problems

What is the answer to  
 $1 + 1 + 1 - 1 - 1$ ?

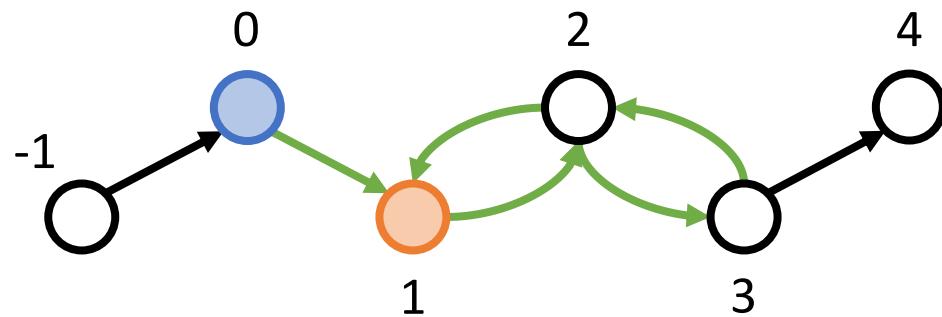


What is my son's son's  
son's father's father?

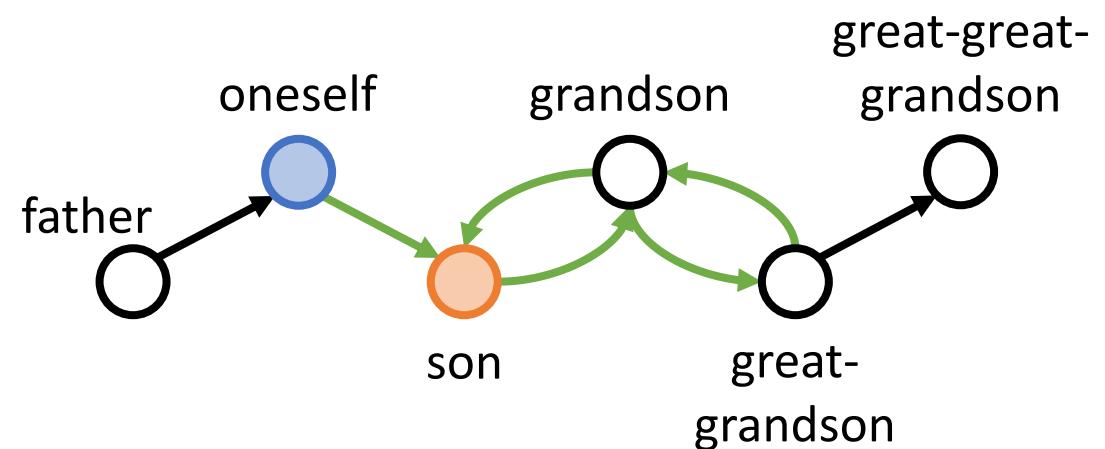


# Structure of Reasoning Problems

What is the answer to  
 $1 + 1 + 1 - 1 - 1$ ?



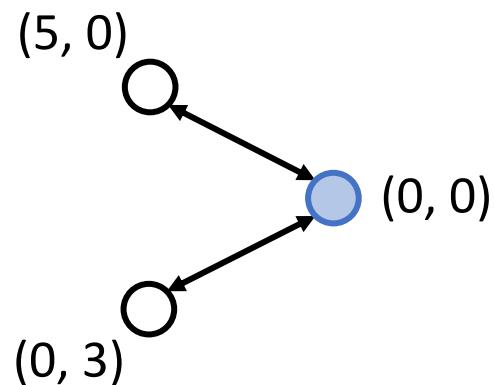
What is my son's son's  
son's father's father?



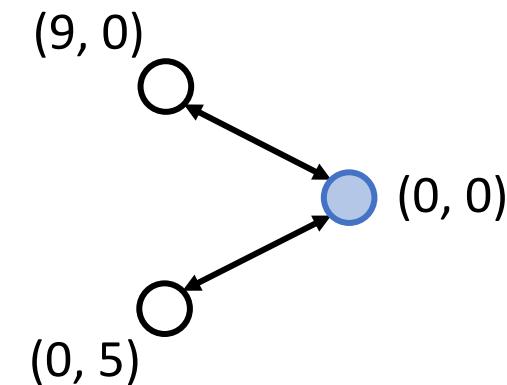
Both predict the ending node of a path!

# Structure of Reasoning Problems

How do you get 4L water  
using 5L and 3L jugs?

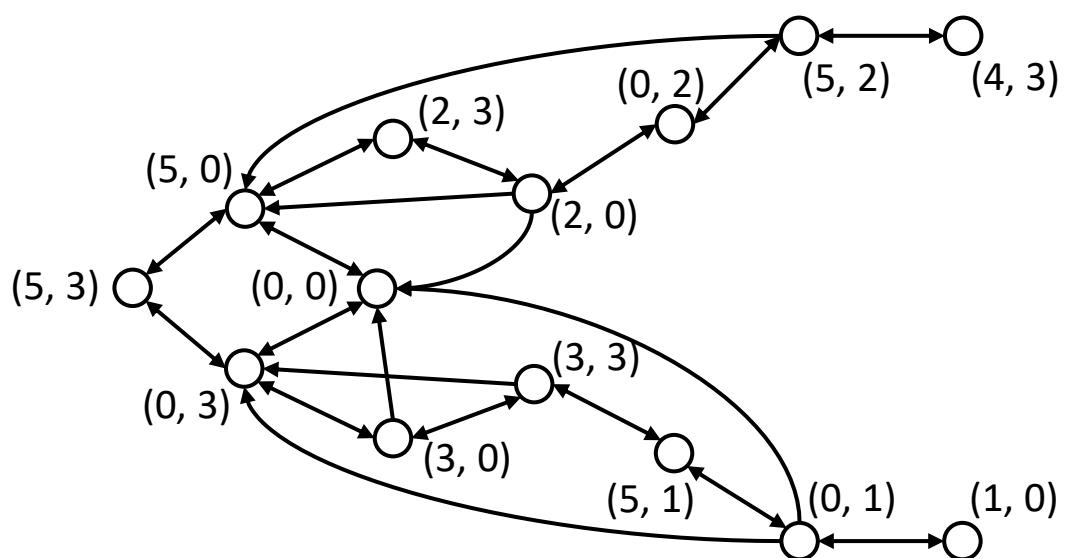


How do you get 5L water  
using 9L and 5L jugs?

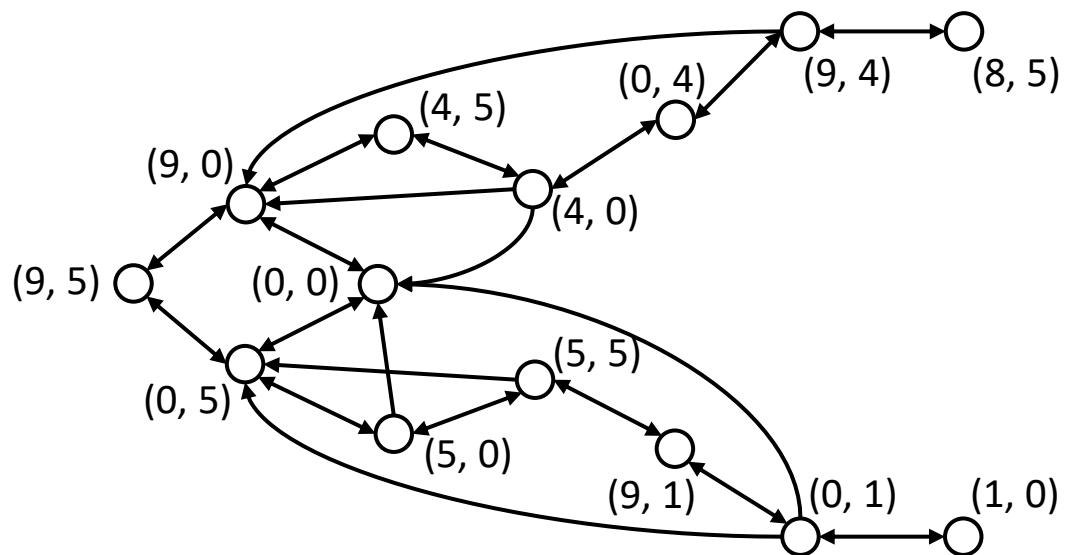


# Structure of Reasoning Problems

How do you get 4L water  
using 5L and 3L jugs?

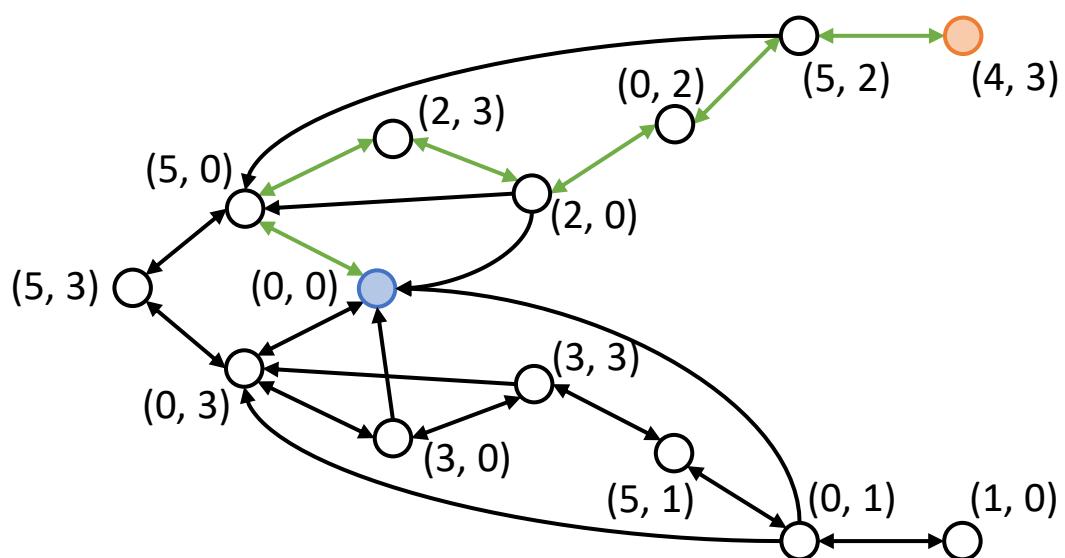


How do you get 5L water  
using 9L and 5L jugs?

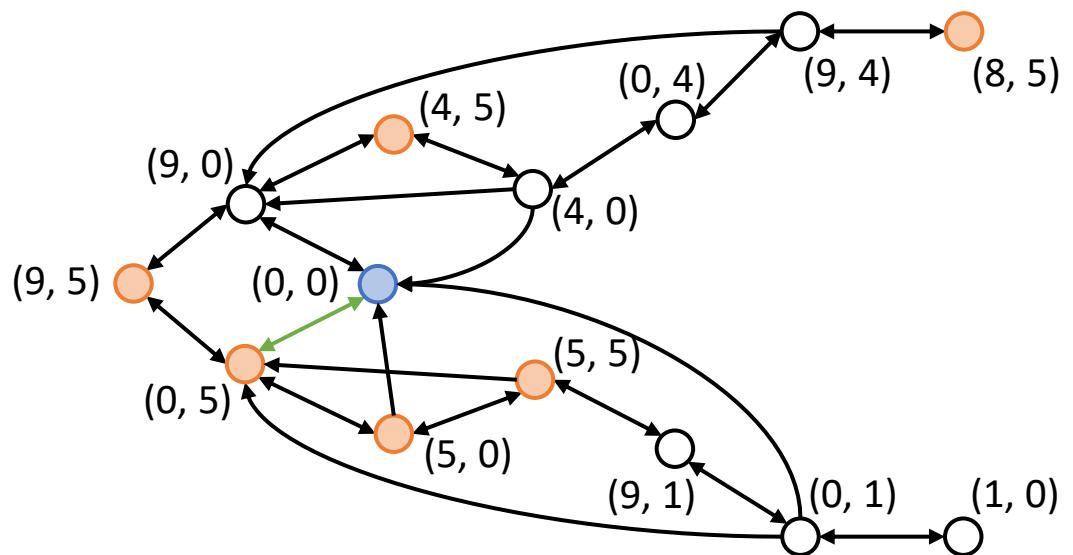


# Structure of Reasoning Problems

How do you get 4L water  
using 5L and 3L jugs?



How do you get 5L water  
using 9L and 5L jugs?



Both find a path to reach the target node(s)!



**How** to generalize across knowledge structures?

**How** to generalize across query structures?

**How** to make ML on structured data more accessible?

# Representation Learning Works

Method	Knowledge Structure	Query Structure	Entities	Generalization to New Relations	Multi-hop Queries
Embeddings NBFNet A*Net Ultra	Knowledge graph	Single-hop query			
	Knowledge graph	Single-hop query	✓		
	Knowledge graph	Single-hop query	✓		
	Knowledge graph	Single-hop query	✓	✓	
Embeddings GNN-QE UltraQuery	Knowledge graph	Multi-hop query			✓
	Knowledge graph	Multi-hop query	✓		✓✓
	Knowledge graph	Multi-hop query	✓	✓	✓✓
CoT	Natural language (latent)	Multi-step query			✓
HtT	Natural language (latent)	Multi-step query			✓✓

covered in this talk

# System Works



covered in this talk

simplifies development on structured data  
reduce the lines of code by 20×

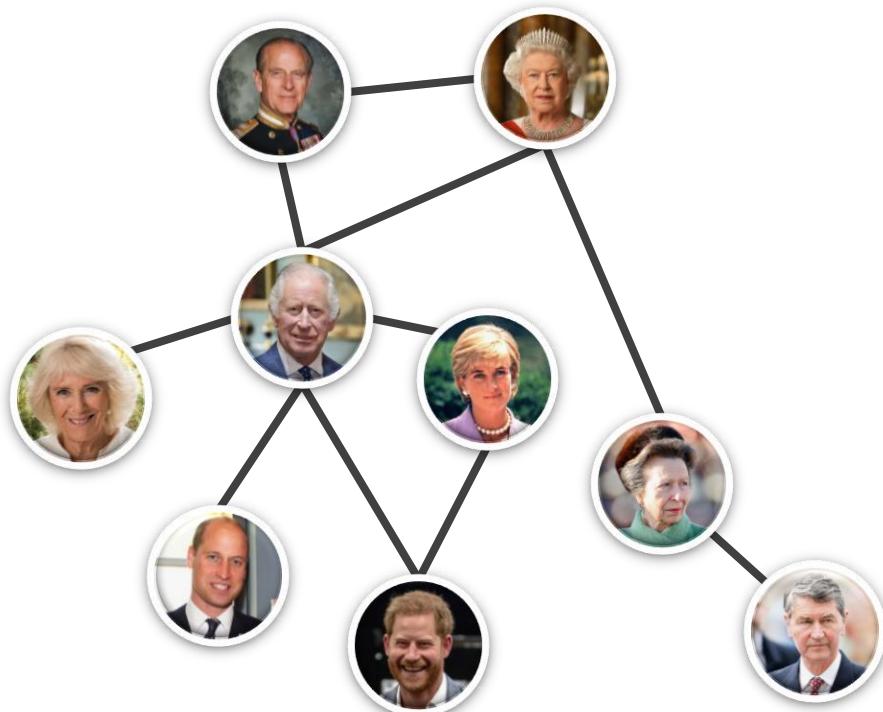
*GraphVite*

scales up training embedding methods  
speeds up by 51× on million-scale graphs

# NBFNet<sup>[1]</sup>: Learning **inductive representations** **of structures** by **encoding paths**

[1] **Zhaocheng Zhu**, Zuobai Zhang, Louis-Pascal Xhonneux, Jian Tang. Neural Bellman-Ford Networks: A General Graph Neural Network Framework for Link Prediction. NeurIPS 2021.

# A Simplified Setup: Knowledge Graphs



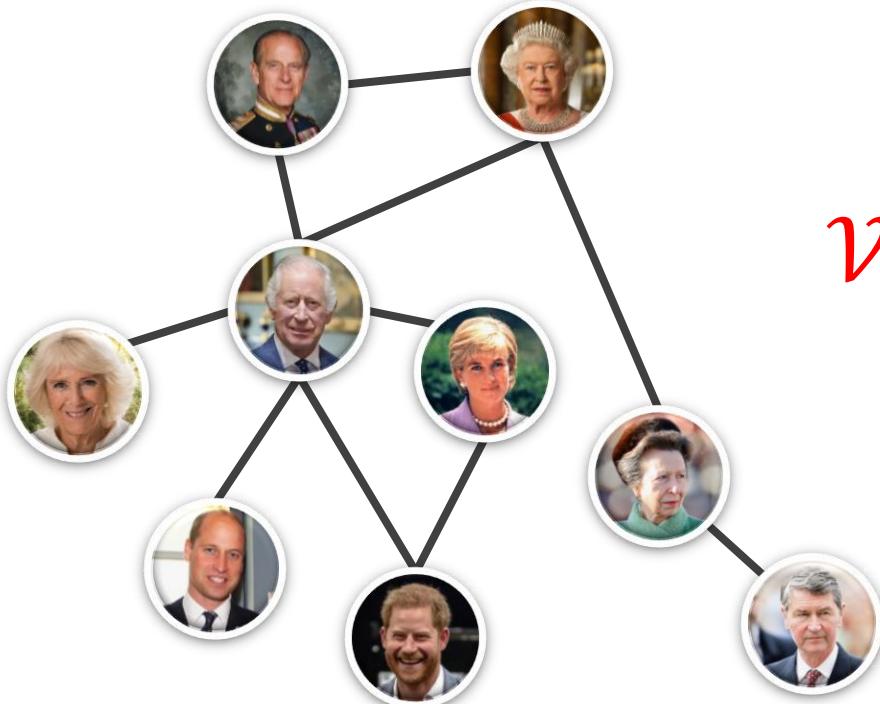
$$\text{Graph } \mathcal{G} = (\mathcal{V}, \mathcal{R}, \mathcal{E})$$

Entities  $\mathcal{V}$ : British royal family

Relations  $\mathcal{R}$ : {parent, spouse}

Edges  $\mathcal{E}$ : known family relationships

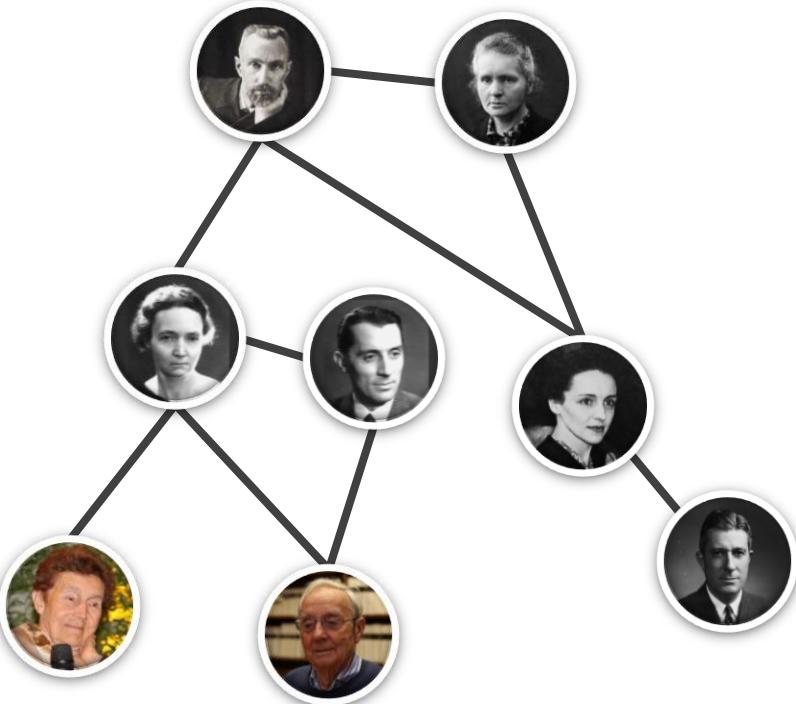
# Inductive Generalization on Structure



$$\mathcal{V}_{train} \neq \mathcal{V}_{test}$$

$\mathcal{V}$ : British royal family

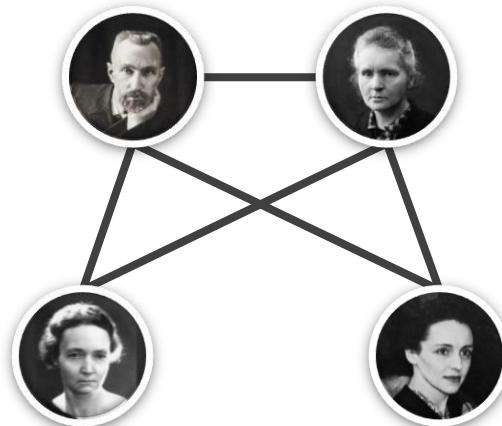
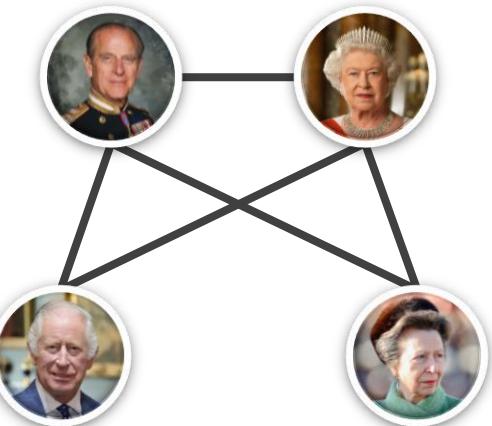
$\mathcal{R}$ : {parent, spouse}



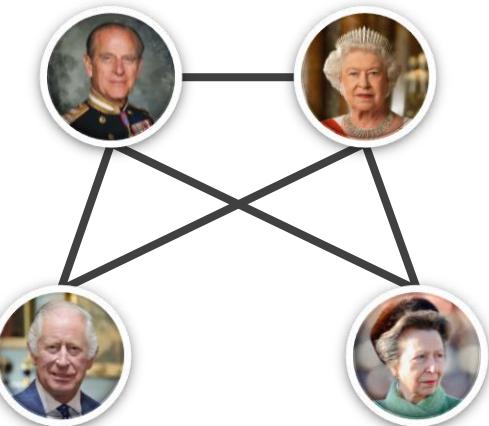
$\mathcal{V}$ : Curie family

$\mathcal{R}$ : {parent, spouse}

# What Is an Inductive Function on Structure?

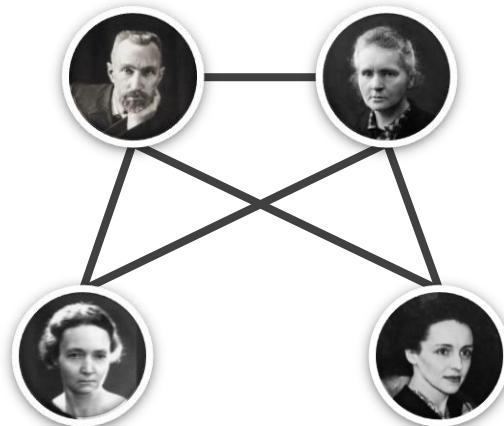


# What Is an Inductive Function on Structure?



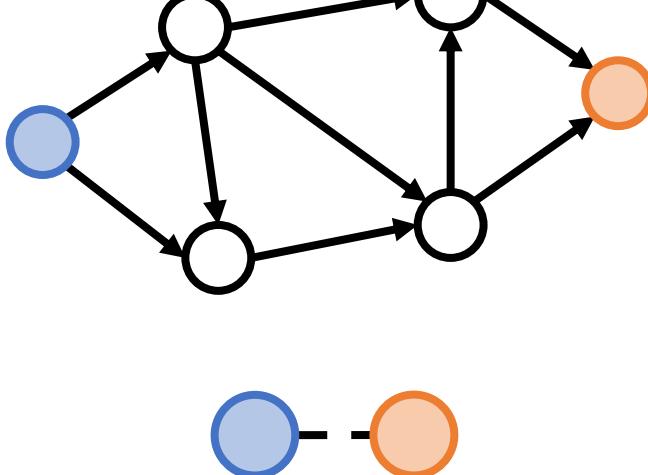
distance: 2  
#shortest path: 2  
PageRank: 0.154

same structure  
same value



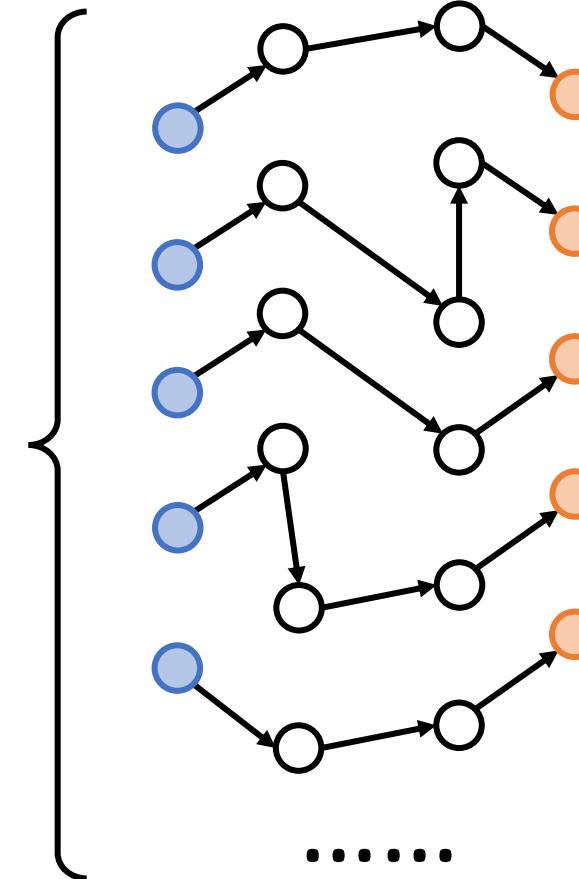
distance: 2  
#shortest path: 2  
PageRank: 0.154

# Path-based Methods



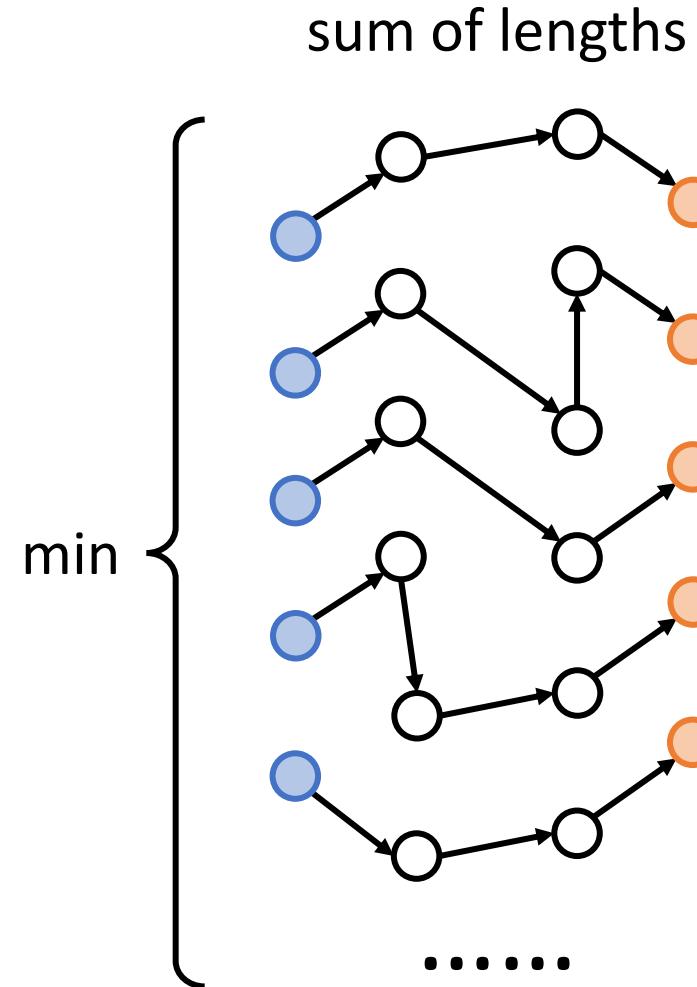
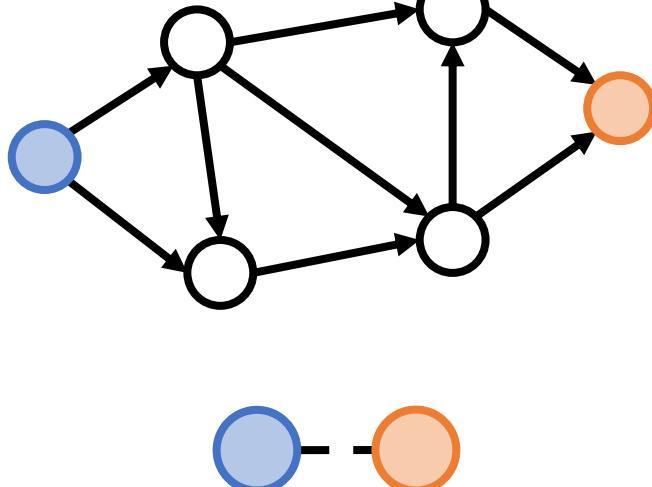
aggregation

path representations



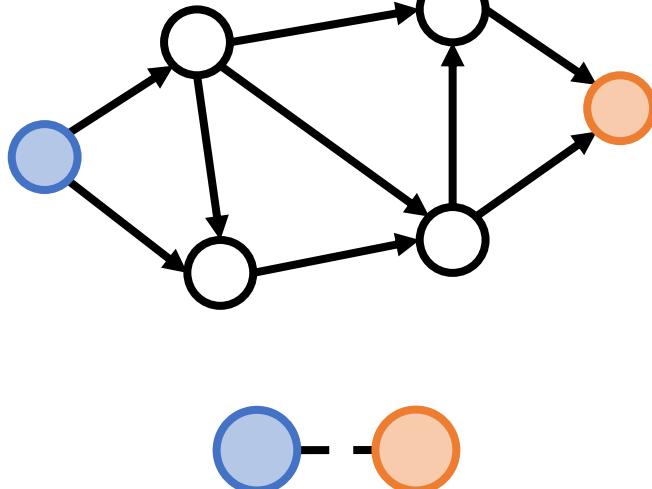
# Path-based Methods

Graph distance

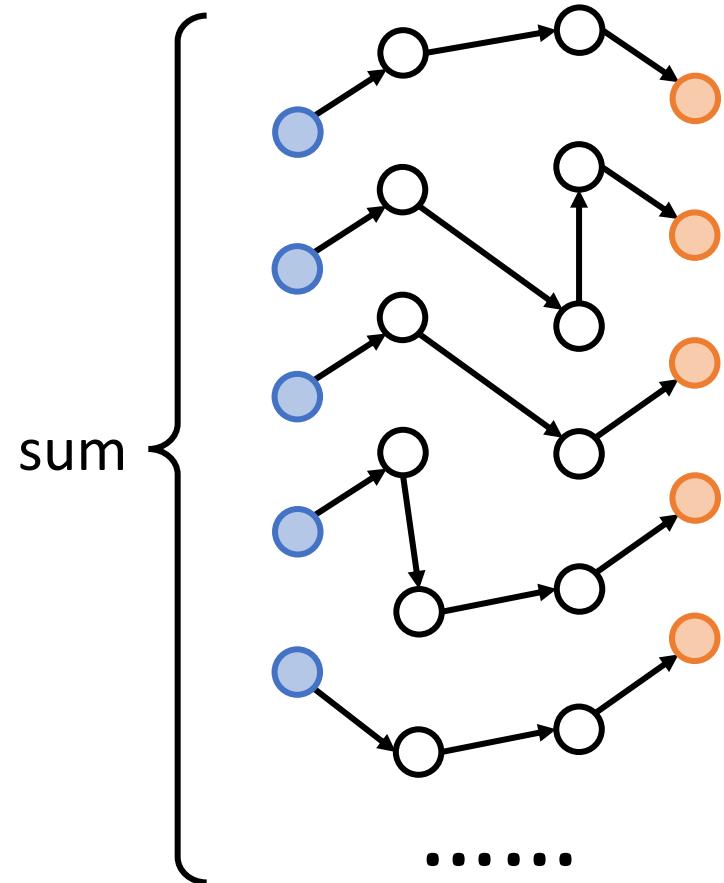


# Path-based Methods

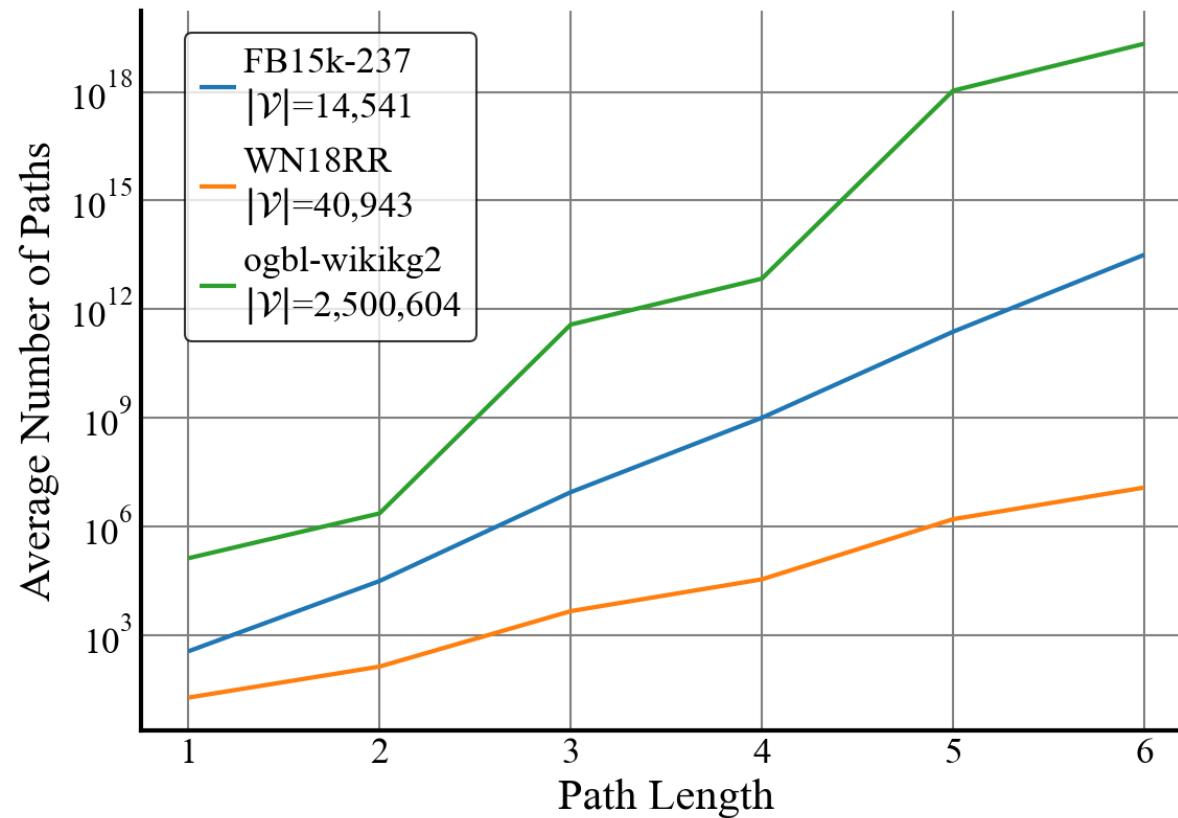
## Personalized PageRank



product of transition probabilities



# Scalability Issue



# Dynamic Programming

To compute paths of length  $T$

graph distance

DFS



Bellman-Ford

Personalized PageRank

random walk



power iteration

exponential in  $T$

$O(T|\mathcal{E}|)$

# Dynamic Programming

To compute paths of length  $T$

graph distance

Personalized PageRank

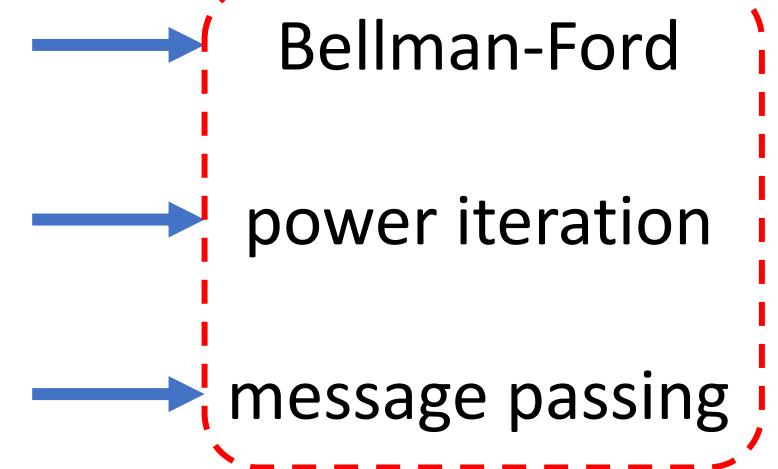
representation learning

DFS

random walk

encode each path

instances of the generalized  
Bellman-Ford algorithm

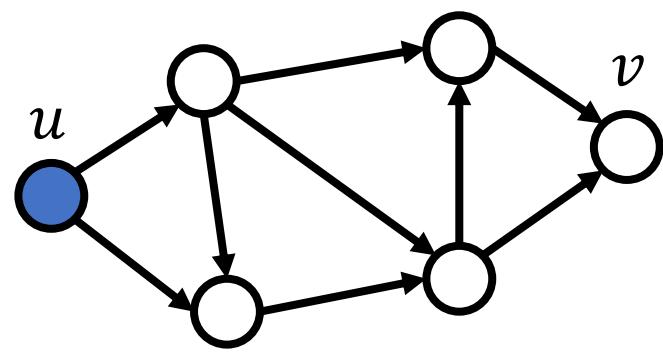


exponential in  $T$

$O(T|\mathcal{E}|)$

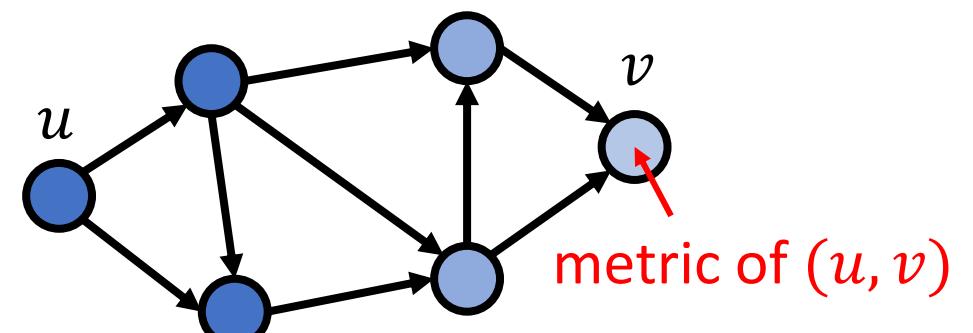
# Generalized Bellman-Ford Algorithm

Message passing with a **single-source input**



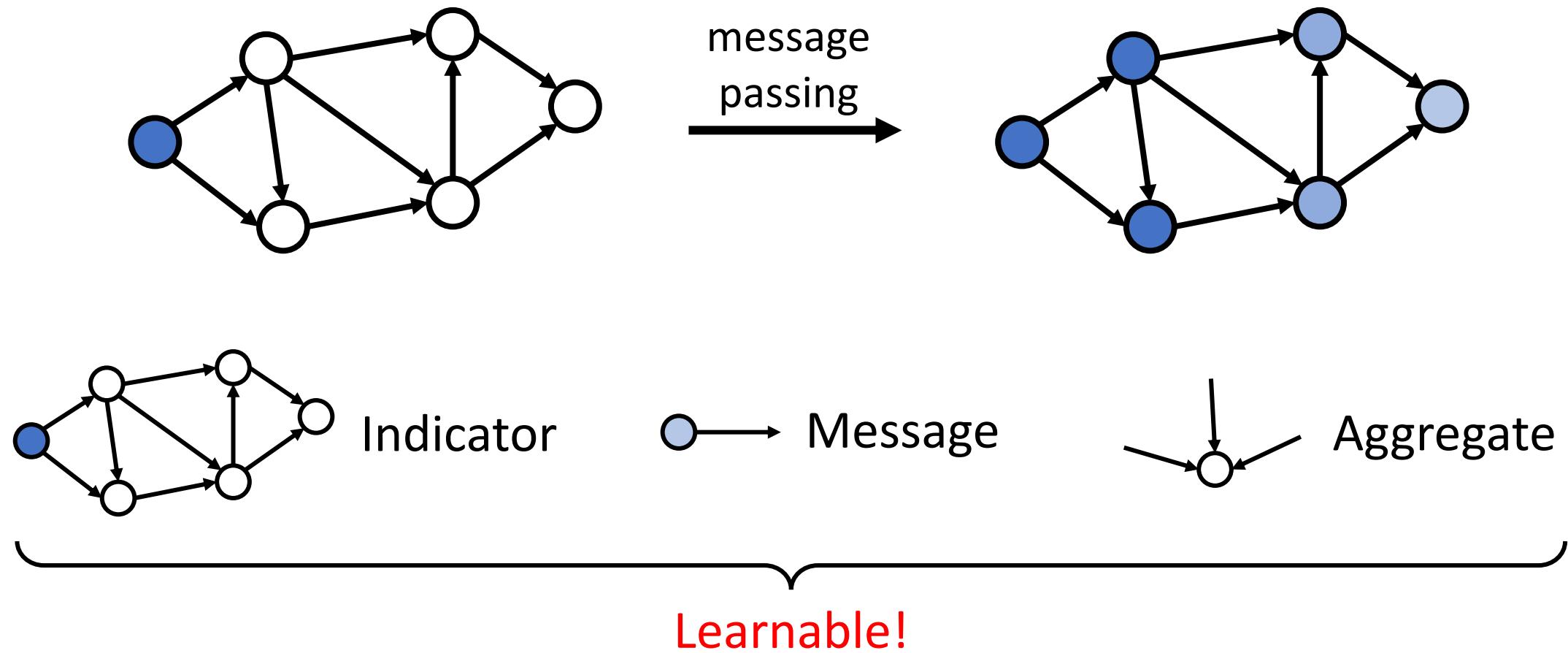
Input

message  
passing

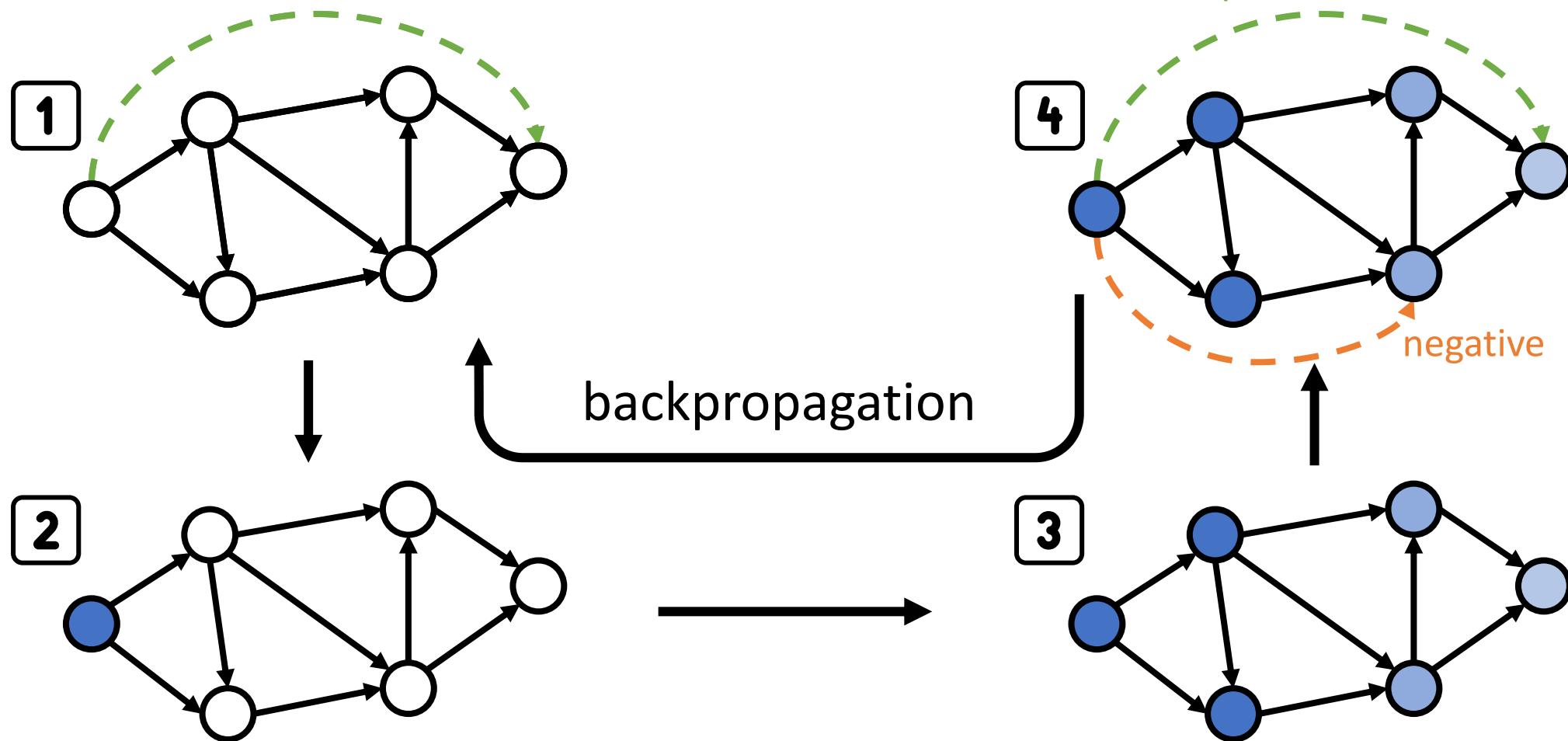


Output

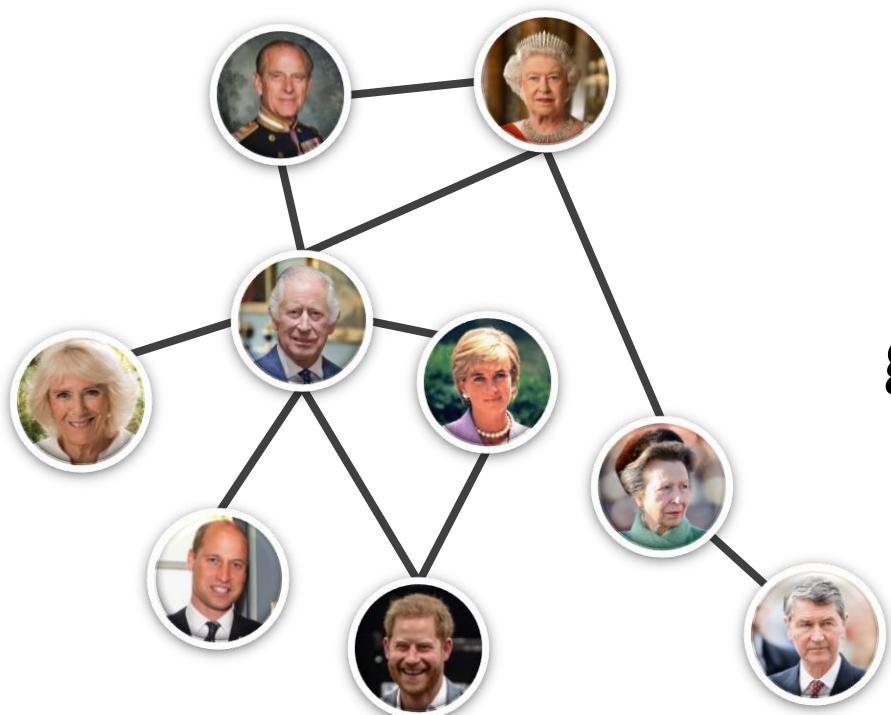
# Neural Bellman-Ford Networks



# Learning Neural Bellman-Ford Networks

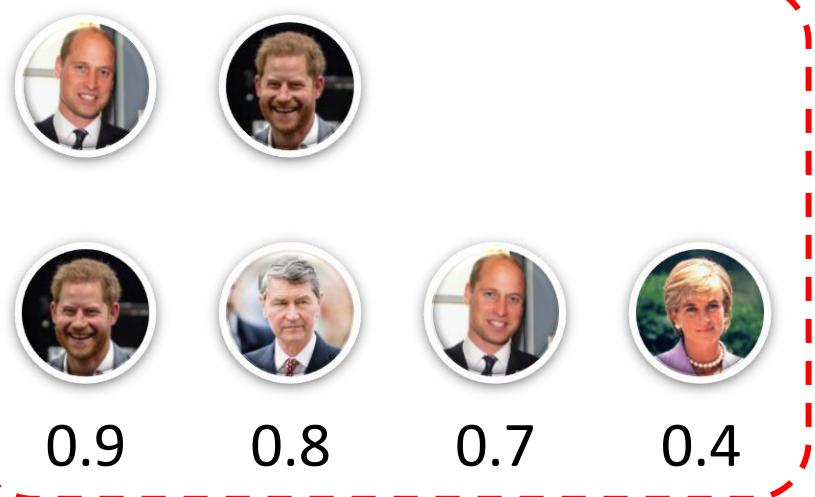


# Evaluation: Knowledge Graph Completion



(  , grandson, ? )

ground truth:



prediction:

ranking metrics

# Knowledge Graphs ( $\mathcal{V}_{train} = \mathcal{V}_{test}$ )

Class	Method	FB15k-237					WN18RR				
		MR↓	MRR↑	H@1↑	H@3↑	H@10↑	MR↓	MRR↑	H@1↑	H@3↑	H@10↑
Path-based	Path Ranking	3521	0.174	0.119	0.186	0.285	22438	0.324	0.276	0.360	0.406
	NeuralLP	-	0.240	-	-	0.362	-	0.435	0.371	0.434	0.566
	DRUM	-	0.343	0.255	0.378	0.516	-	0.486	0.425	0.513	0.586
Embeddings	TransE	357	0.294	-	-	0.465	3384	0.226	-	-	0.501
	DistMult	254	0.241	0.155	0.263	0.419	5110	0.43	0.39	0.44	0.49
	ComplEx	339	0.247	0.158	0.275	0.428	5261	0.44	0.41	0.46	0.51
	RotatE	177	0.338	0.241	0.375	0.533	3340	0.476	0.428	0.492	0.571
	HAKE	-	0.346	0.250	0.381	0.542	-	0.497	0.452	0.516	0.582
	LowFER	-	0.359	0.266	0.396	0.544	-	0.465	0.434	0.479	0.526
GNNs	RGCN	221	0.273	0.182	0.303	0.456	2719	0.402	0.345	0.437	0.494
	GraIL	2053	-	-	-	-	2539	-	-	-	-
	NBFNet	<b>114</b>	<b>0.415</b>	<b>0.321</b>	<b>0.454</b>	<b>0.599</b>	<b>636</b>	<b>0.551</b>	<b>0.497</b>	<b>0.573</b>	<b>0.666</b>

# Knowledge Graphs ( $\mathcal{V}_{train} \neq \mathcal{V}_{test}$ )

metric: H@10↑

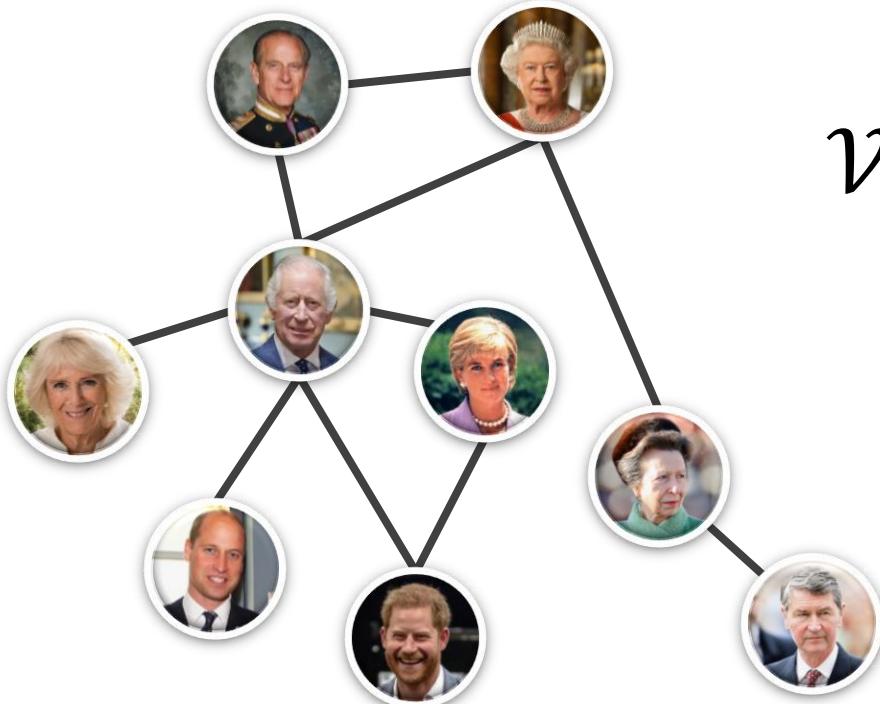
Class	Method	FB15k-237				WN18RR			
		v1	v2	v3	v4	v1	v2	v3	v4
Path-based	NeuralLP	0.529	0.589	0.529	0.559	0.744	0.689	0.462	0.671
	DRUM	0.529	0.587	0.529	0.559	0.744	0.689	0.462	0.671
	RuleN	0.498	0.778	0.877	0.856	0.809	0.782	0.534	0.716
GNNs	GraIL	0.642	0.818	0.828	0.893	0.825	0.787	0.584	0.734
	NBFNet	<b>0.834</b>	<b>0.949</b>	<b>0.951</b>	<b>0.960</b>	<b>0.948</b>	<b>0.905</b>	<b>0.893</b>	<b>0.890</b>

(Drinking Water)

# Ultra<sup>[1]</sup>: Generalizing to **any knowledge graph** with **inductive relation representations**

[1] Mikhail Galkin, Xinyu Yuan, Hesham Mostafa, Jian Tang, Zhaocheng Zhu. Towards Foundation Models for Knowledge Graph Reasoning. ICLR 2024.

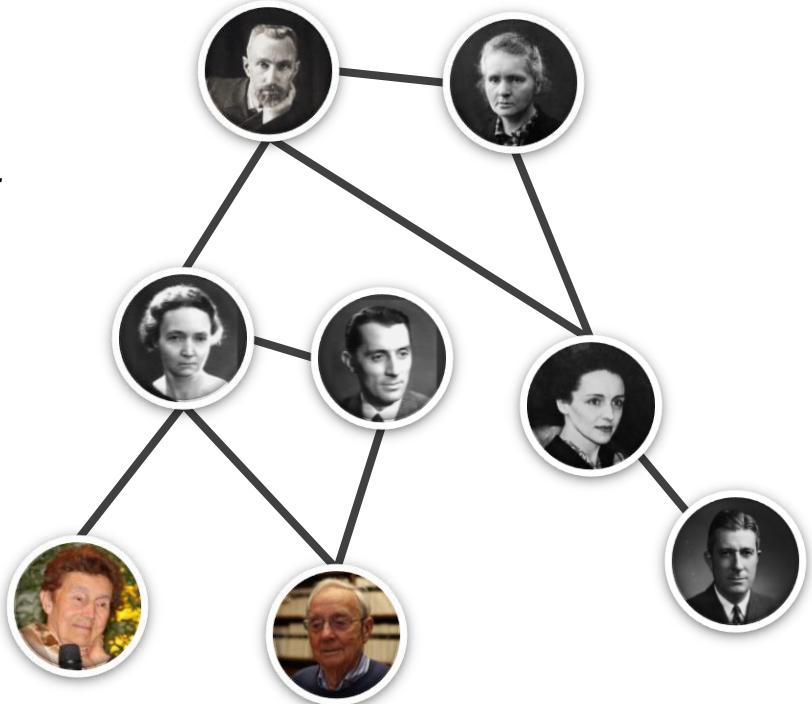
# Inductive Generalization on Structure



$\mathcal{V}$ : British royal family

$\mathcal{R}$ : {parent, spouse}

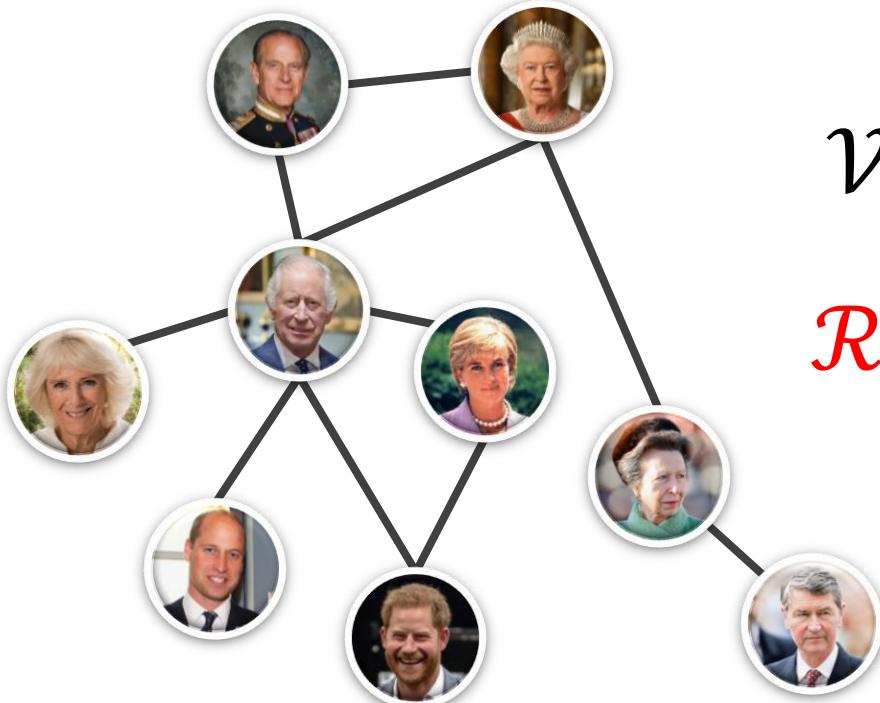
$$\mathcal{V}_{train} \neq \mathcal{V}_{test}$$



$\mathcal{V}$ : Curie family

$\mathcal{R}$ : {parent, spouse}

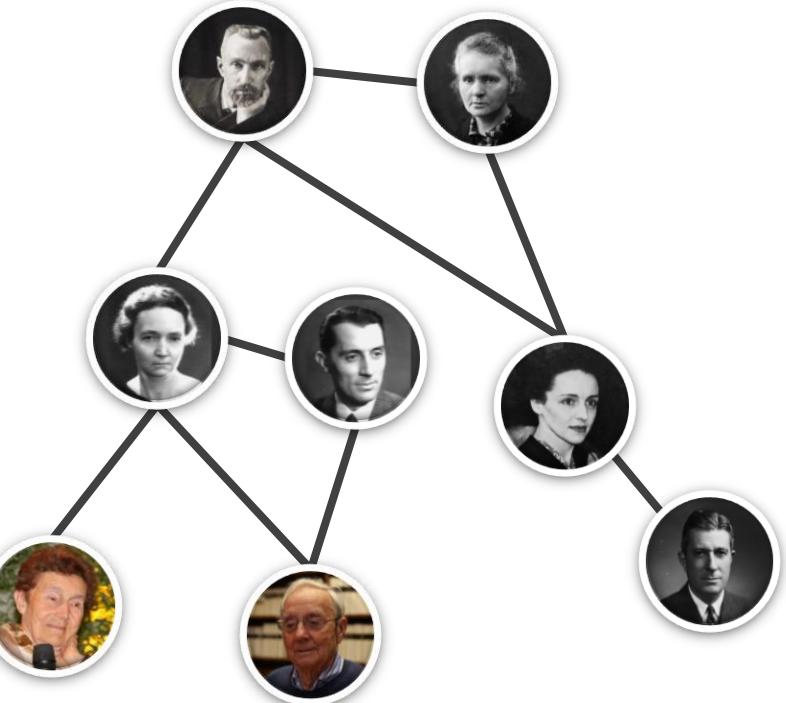
# Inductive Generalization on Structure



$\mathcal{V}$ : British royal family

$\mathcal{R}$ : {parent, spouse}

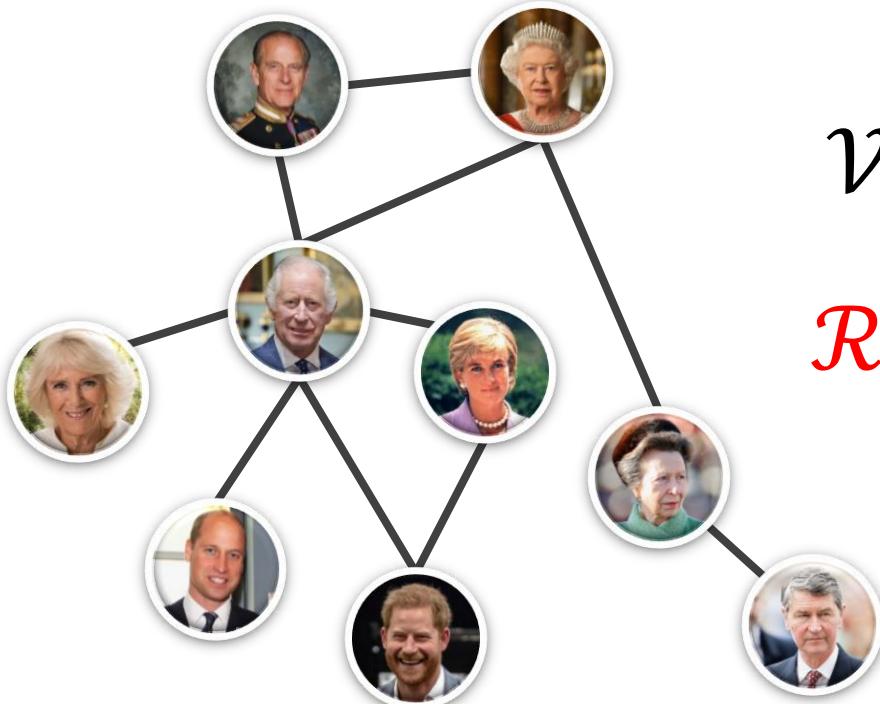
$$\mathcal{V}_{train} \neq \mathcal{V}_{test}$$
$$\mathcal{R}_{train} = \mathcal{R}_{test}$$



$\mathcal{V}$ : Curie family

$\mathcal{R}$ : {parent, spouse}

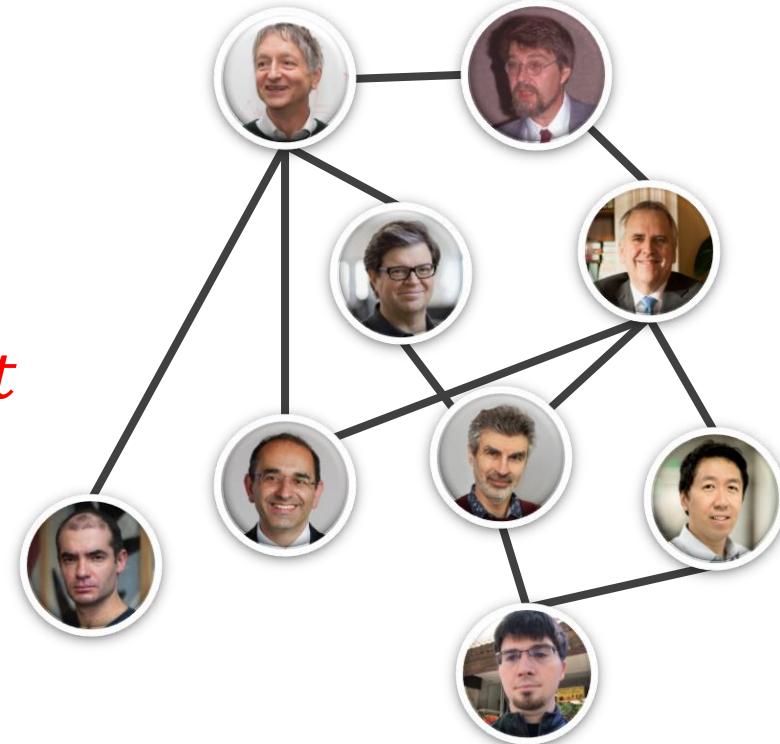
# Inductive Generalization on Structure



$\mathcal{V}$ : British royal family

$\mathcal{R}$ : {parent, spouse}

$$\mathcal{V}_{train} \neq \mathcal{V}_{test}$$
  
$$\mathcal{R}_{train} \neq \mathcal{R}_{test}$$



$\mathcal{V}$ : deep learning researchers

$\mathcal{R}$ : {supervisor, collaborator}

# What Generalizes for Entities?

$\mathcal{V}$ : British royal family

$\mathcal{R}$ : {parent, spouse}

$\mathcal{V}$ : Curie family

$\mathcal{R}$ : {parent, spouse}

Elizabeth II



Marie Curie

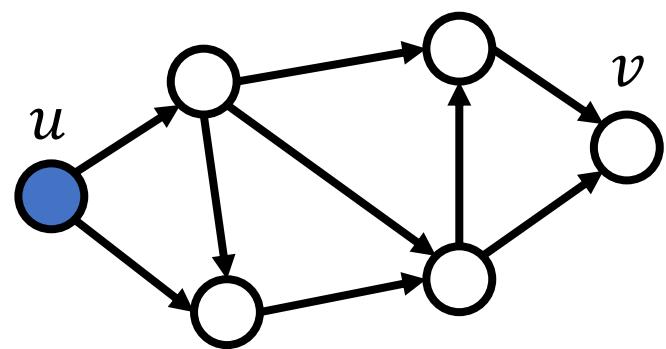
Elizabeth II - Princess Anne



Marie Curie - Irene Curie

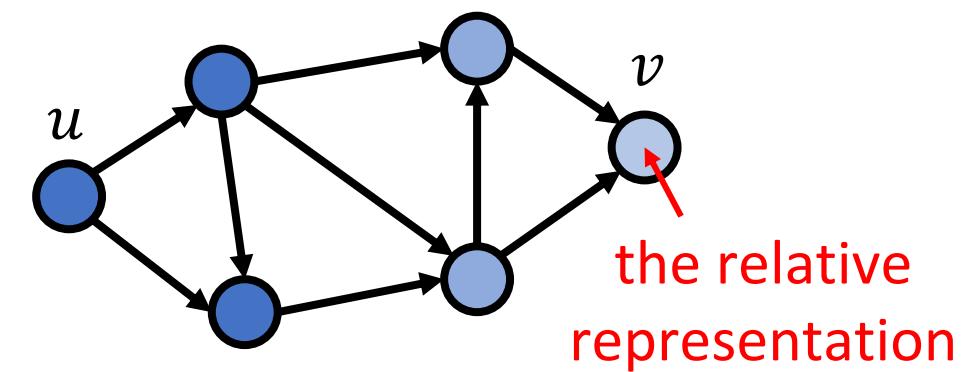
# Relative Entity Representations

encode  $v - u$  on graph  $\mathcal{G}$



Input

message  
passing



Output

the relative  
representation

# What Generalizes for Relations?

$\mathcal{V}$ : British royal family

$\mathcal{R}$ : {parent, spouse}

$\mathcal{V}$ : deep learning researchers

$\mathcal{R}$ : {supervisor, collaborator}

parent



supervisor

parent - spouse



supervisor - collaborator

# Relative Relation Representations

relative entity: encode  $v - u$  on graph  $\mathcal{G}$

relative relation: encode  $r - q$  on what?

# Relative Relation Representations

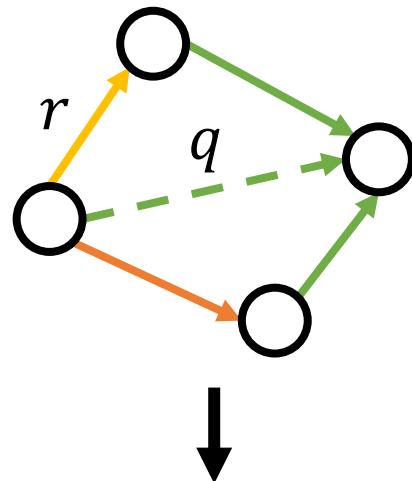
relative entity: encode  $v - u$  on graph  $\mathcal{G}$

relative relation: encode  $r - q$  on what?

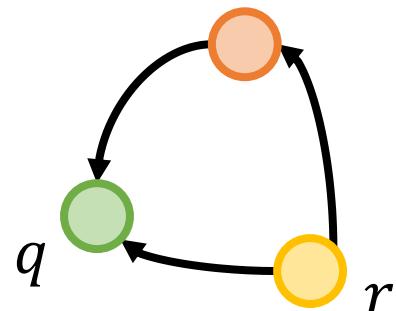
Construct a relation graph to  
capture relation interactions!

# Relative Relation Representations

knowledge graph  $\mathcal{G}$

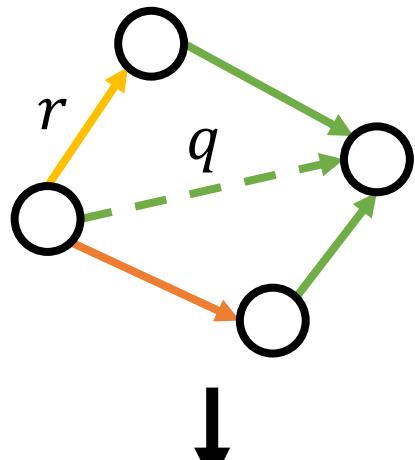


relation graph  $\mathcal{G}_r$

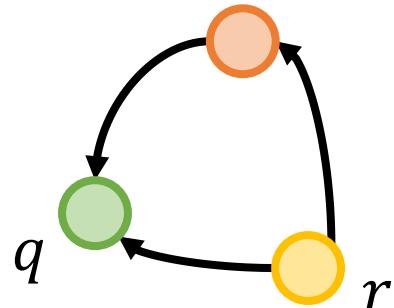


# Relative Relation Representations

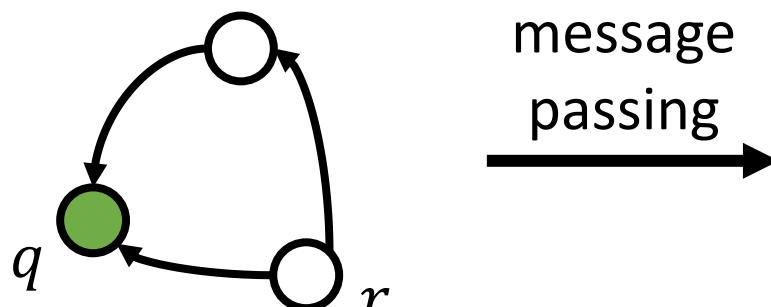
knowledge graph  $\mathcal{G}$



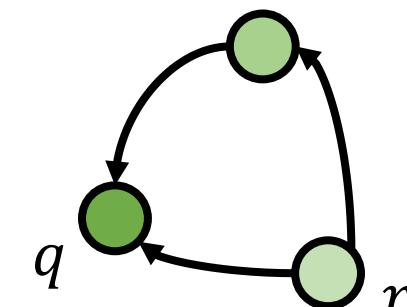
relation graph  $\mathcal{G}_r$



Encode  $r - q$  on  $\mathcal{G}_r$



Input



Output

# Relation Graph

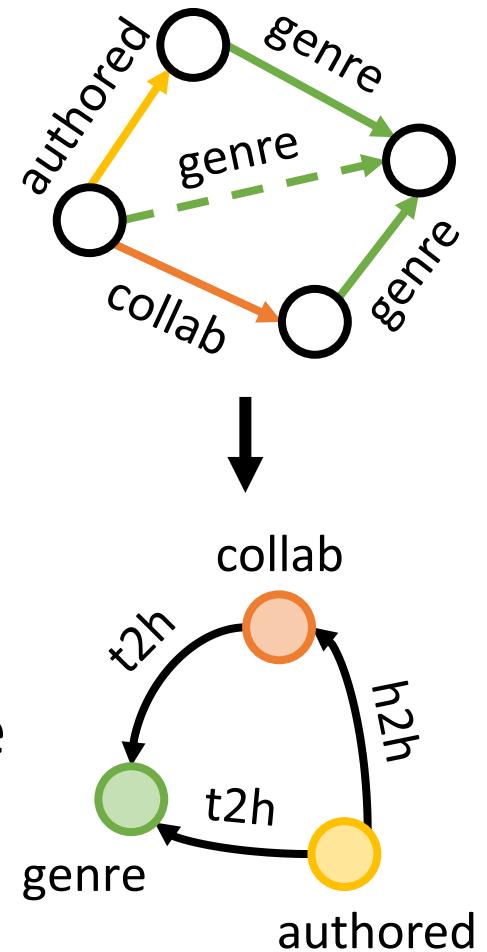
# Relation interactions:

*head2head, head2tail, tail2head, tail2tail*

## Example:

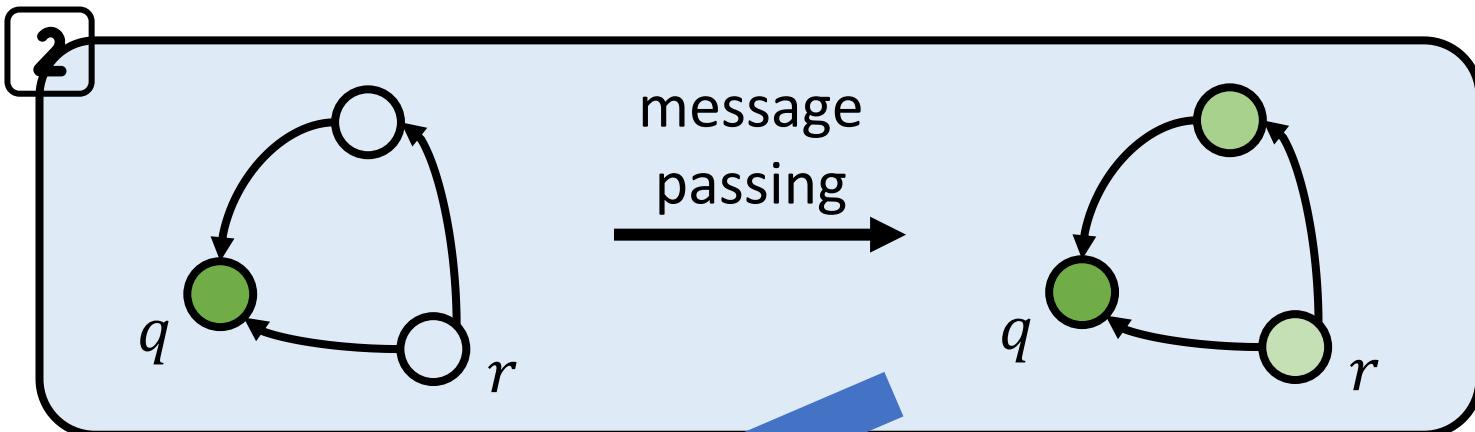
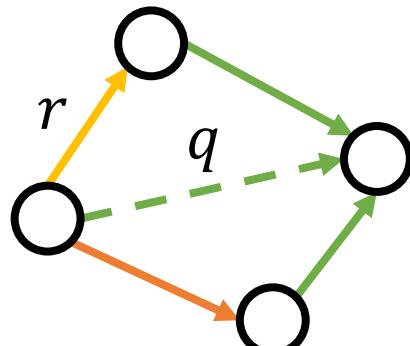
*(author, t2h, genre)*

Anything that has an author is likely to have a genre



# Ultra: Unified, Learnable, Transferable

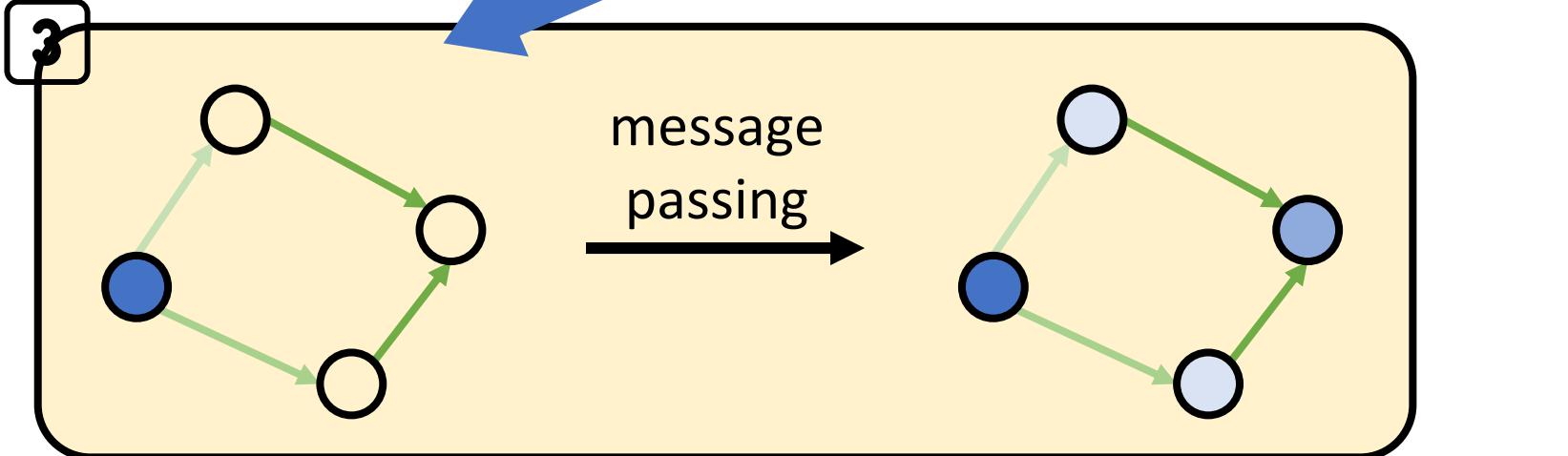
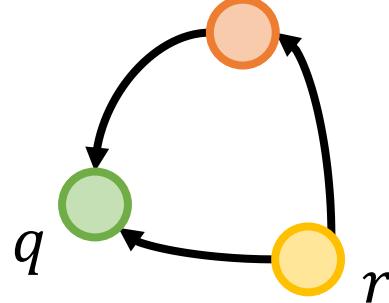
knowledge graph  $\mathcal{G}$



1

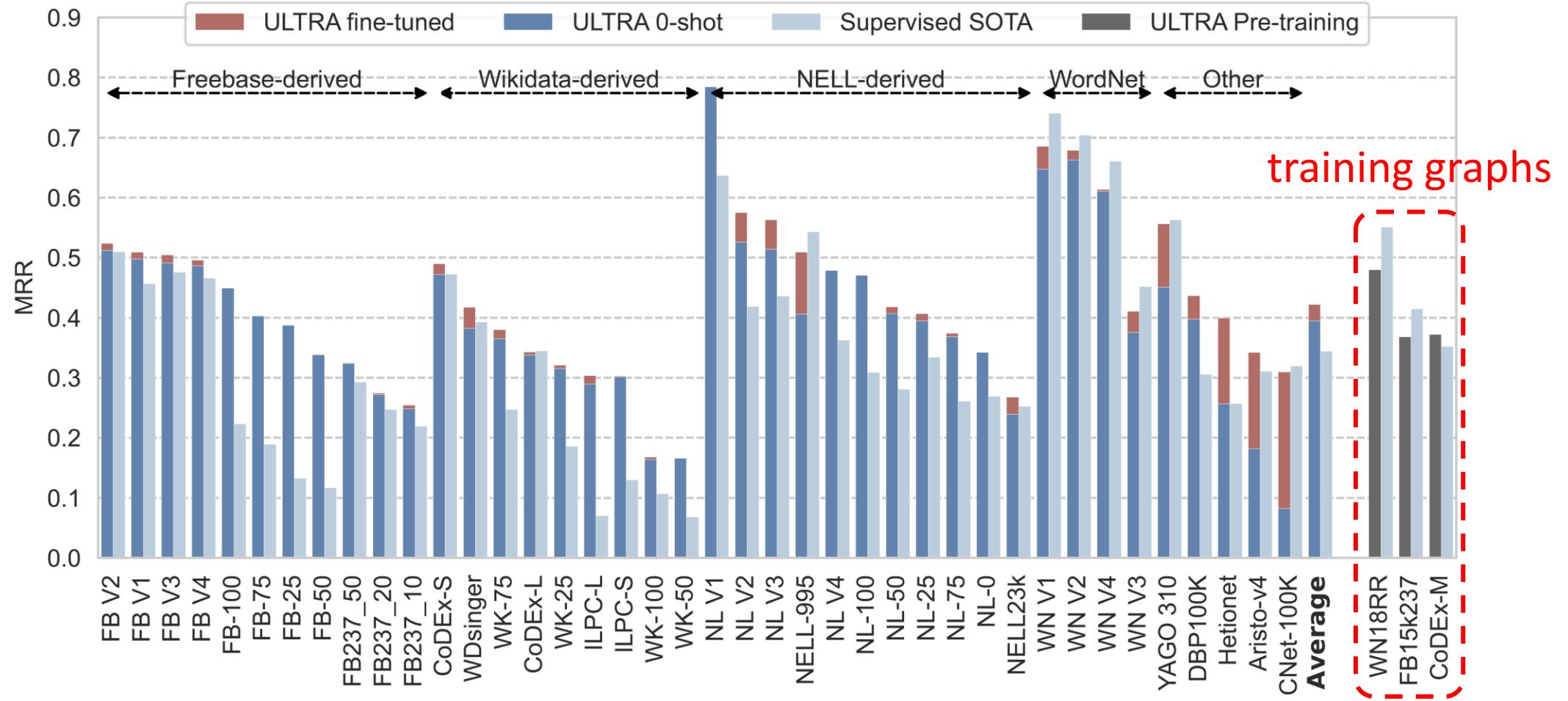


relation graph  $\mathcal{G}_r$

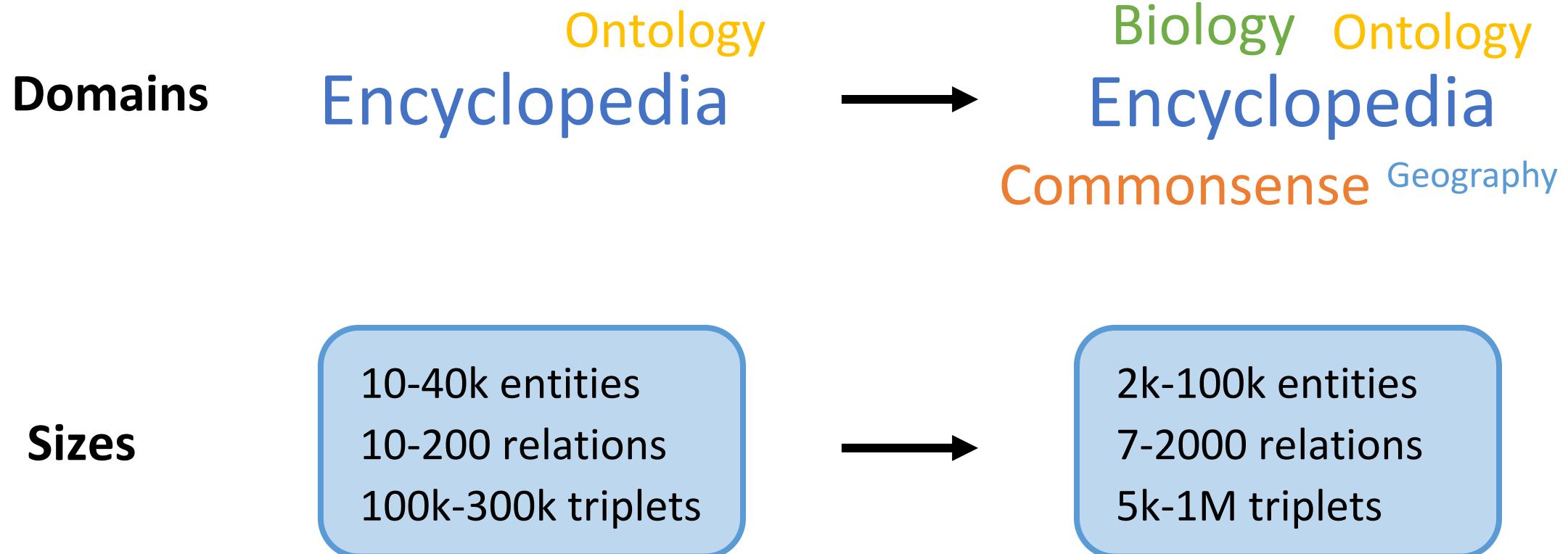


initialize relation representations

# 0-shot Inference on any Knowledge Graph



# Surprising Generalization Ability



# GNN-QE<sup>[1][2][3]</sup>: Solving **multi-hop queries** with **inductive models and logical operations**

[1] **Zhaocheng Zhu**, Mikhail Galkin, Zuobai Zhang, Jian Tang. Neural-Symbolic Models for Logical Queries on Knowledge Graphs. ICML 2022.

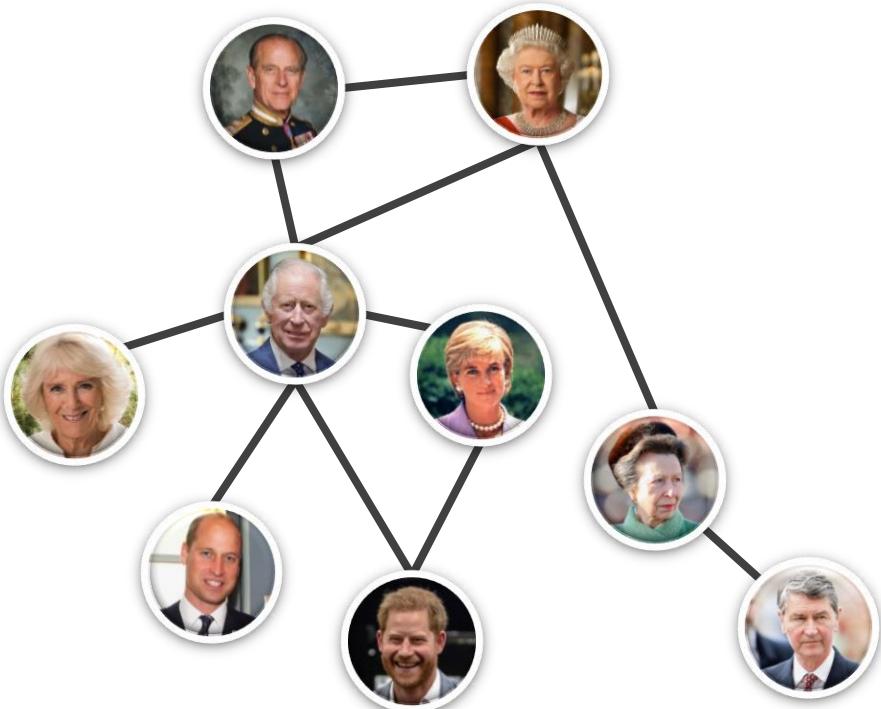
[2] Mikhail Galkin, **Zhaocheng Zhu**, Hongyu Ren, Jian Tang. Inductive Logical Query Answering in Knowledge Graphs. NeurIPS 2022.

[3] Mikhail Galkin, Jincheng Zhou, Bruno Ribeiro, Jian Tang, **Zhaocheng Zhu**. Zero-shot Logical Query Reasoning on any Knowledge Graph. arXiv 2024.

# Knowledge Graph Completion

**Input:** a head entity, a relation

**Output:** one or many tail entities

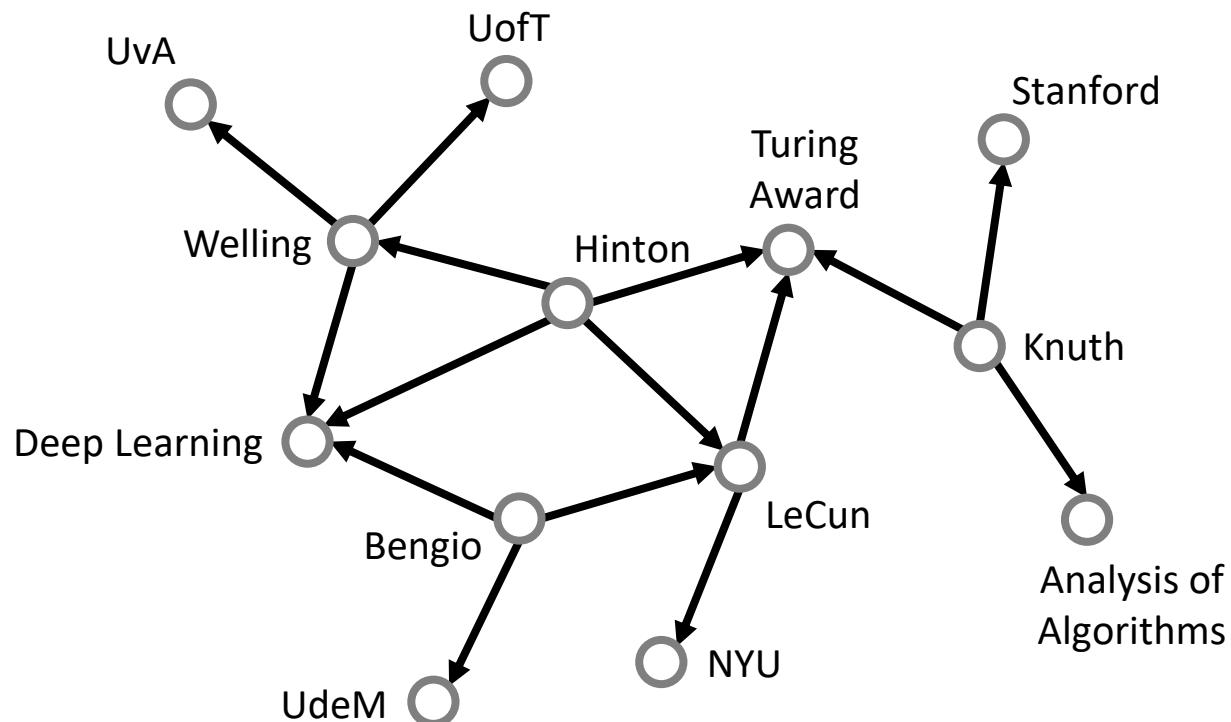


(  , grandson, ? )

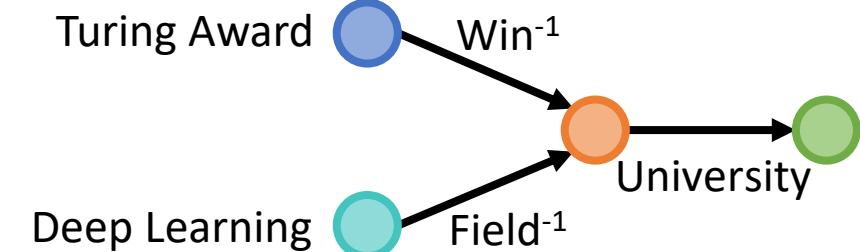
# Multi-hop Logical Queries

**Input:** one or several entities, several relations, logical operations

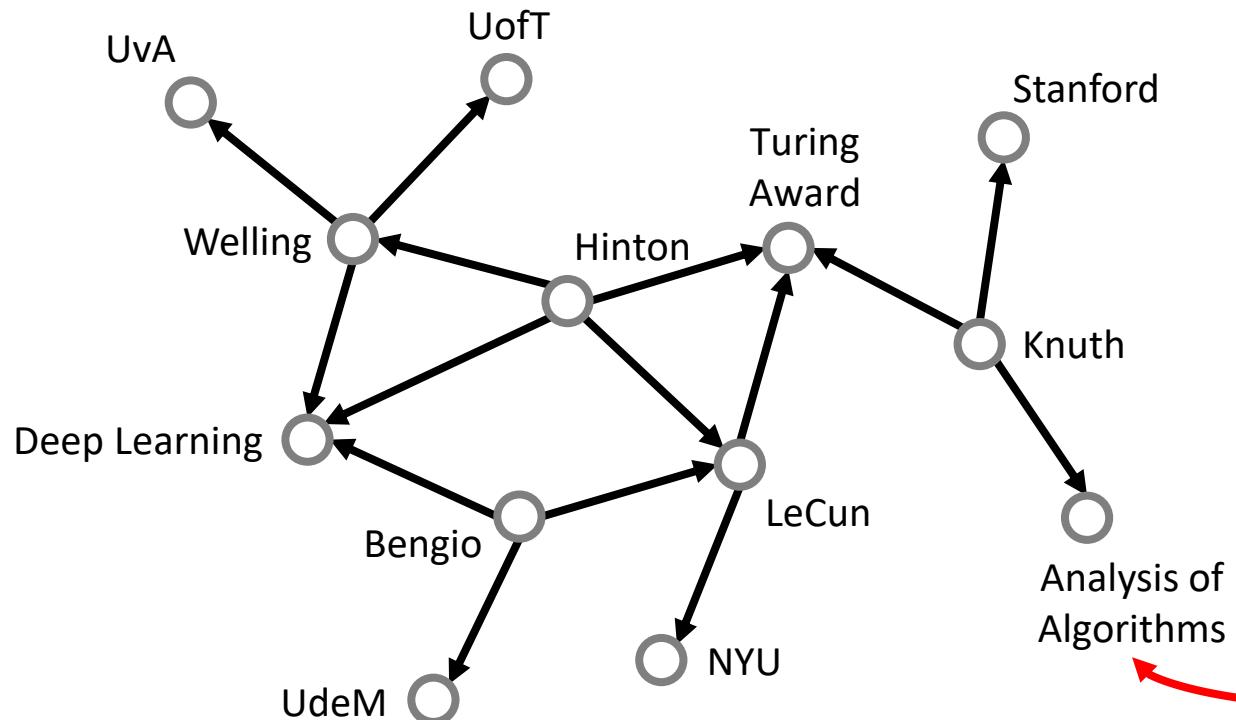
**Output:** one or many tail entities



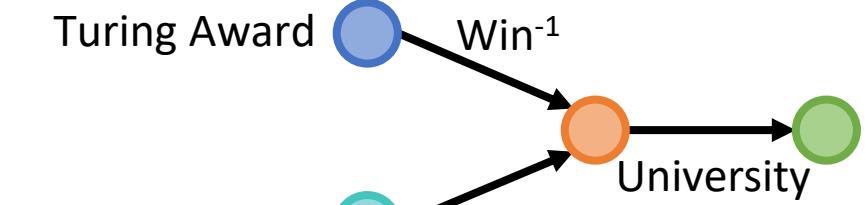
*At what universities do the Turing Award winners in the field of deep learning work?*



# Multi-hop Logical Queries

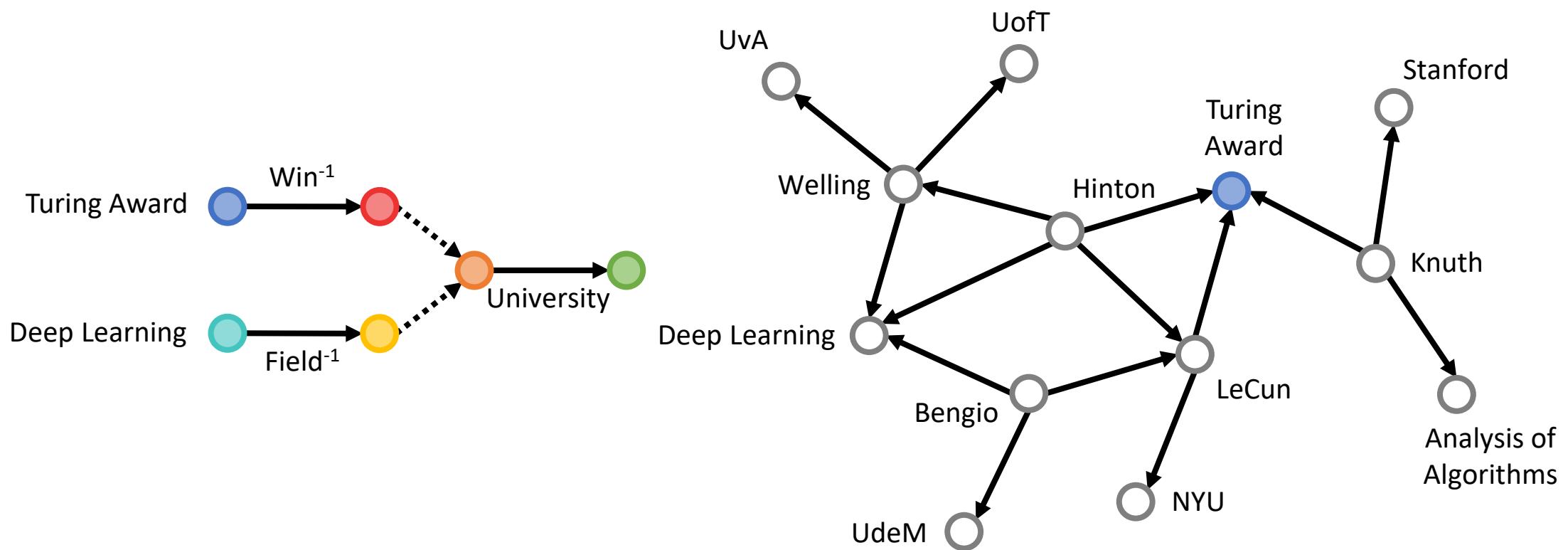


*At what universities do the Turing Award winners in the field of deep learning work?*

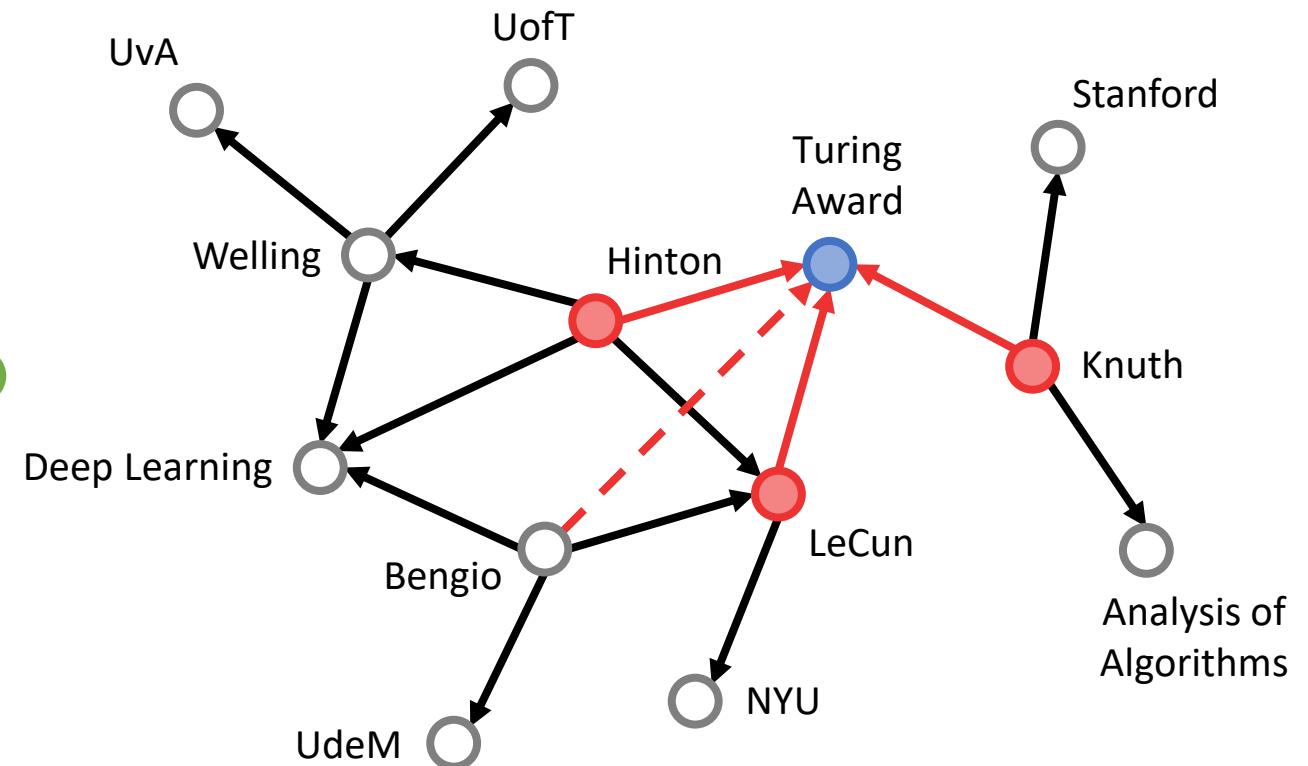
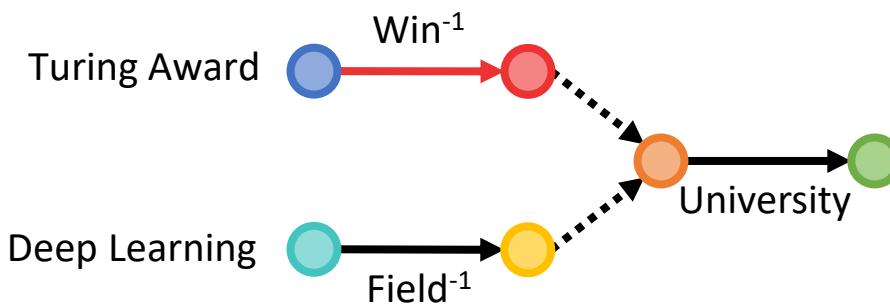


search this subgraph

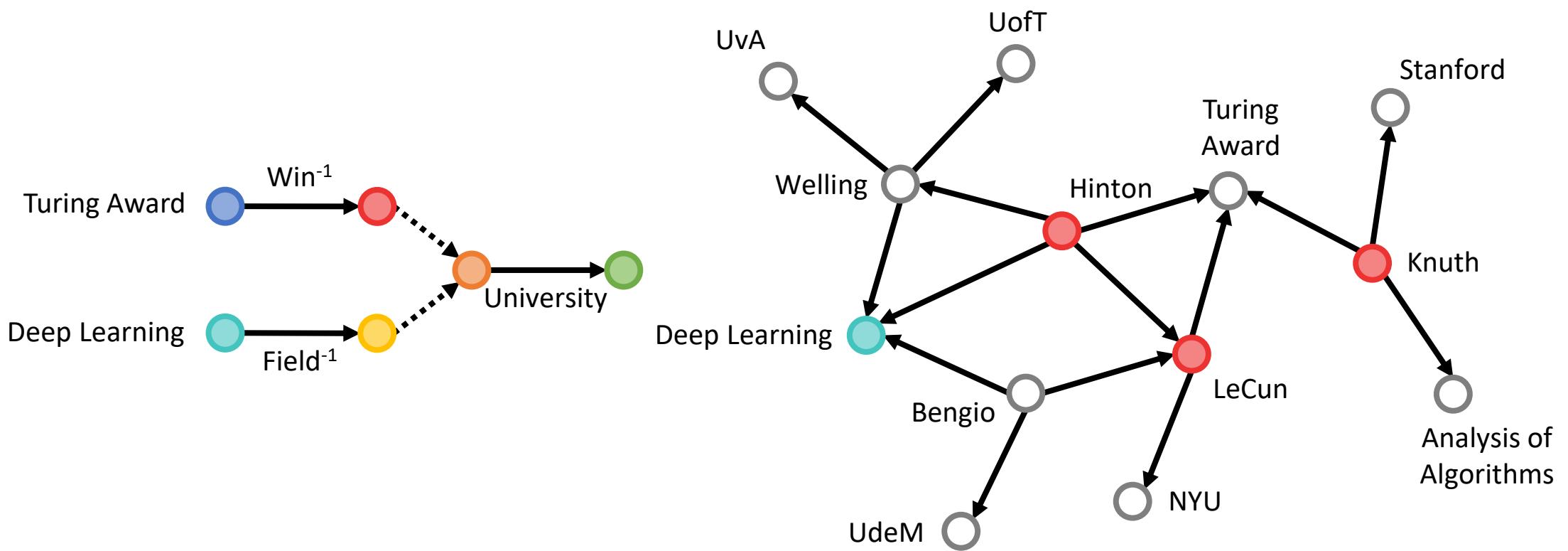
# Subgraph Matching



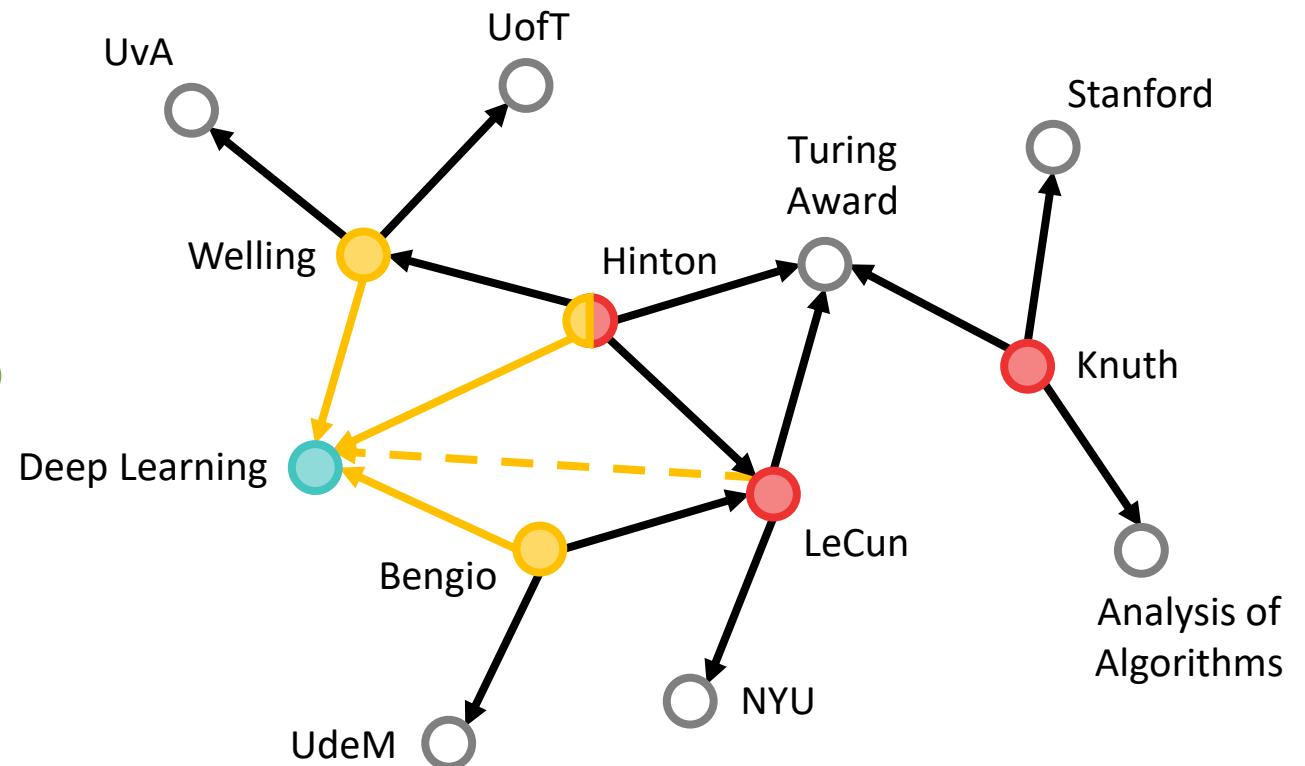
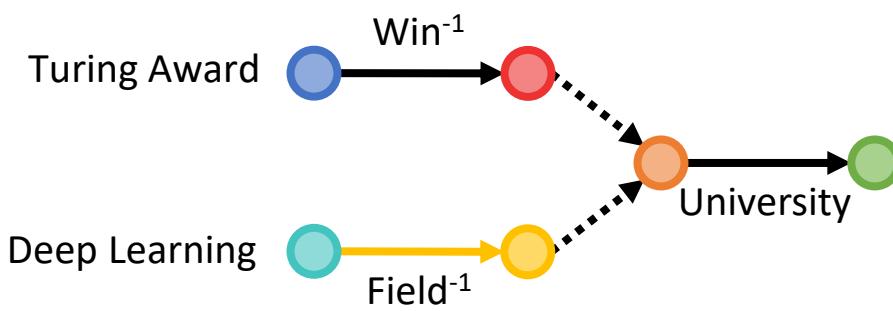
# Subgraph Matching



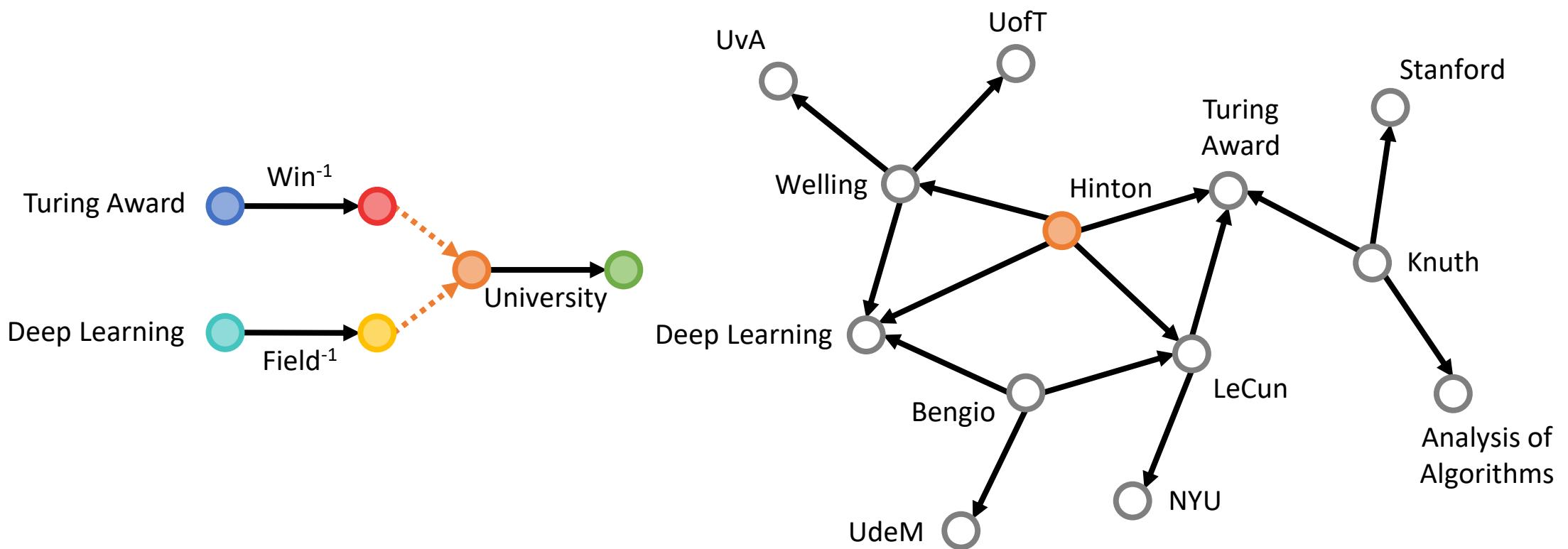
# Subgraph Matching



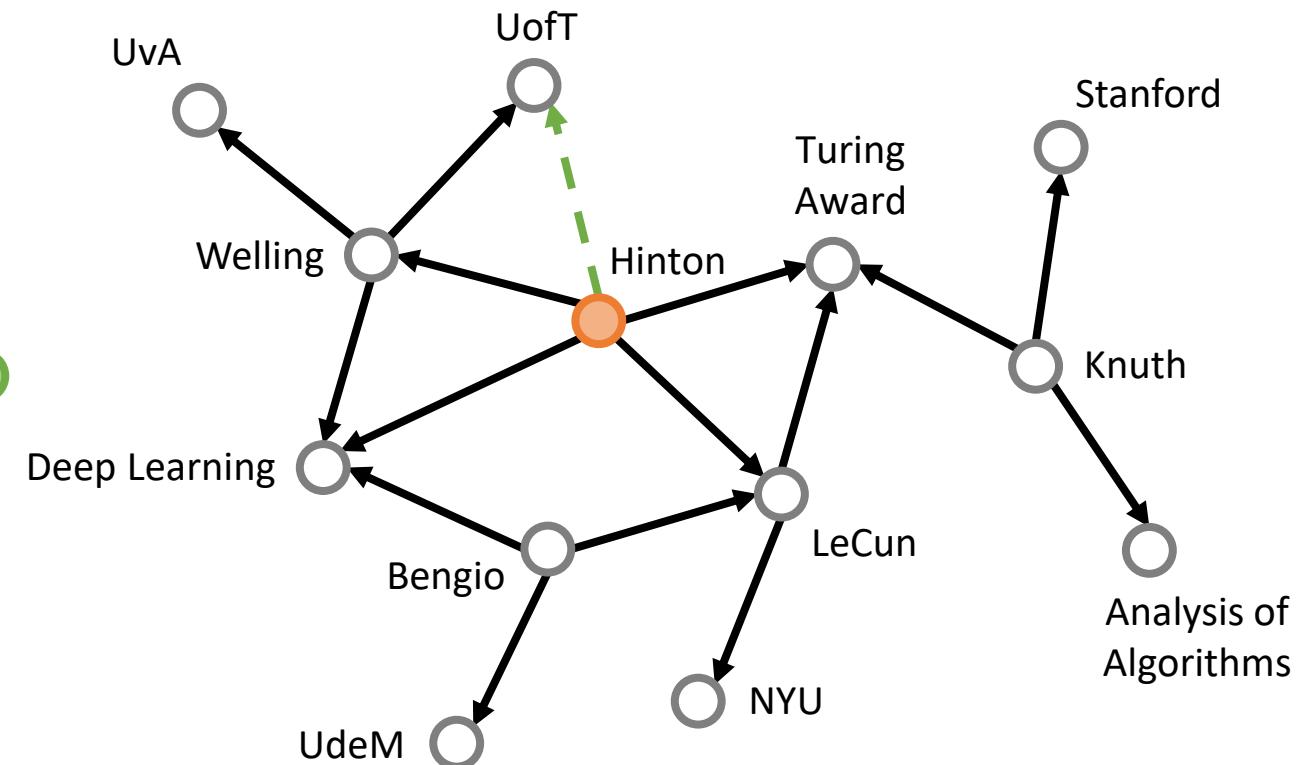
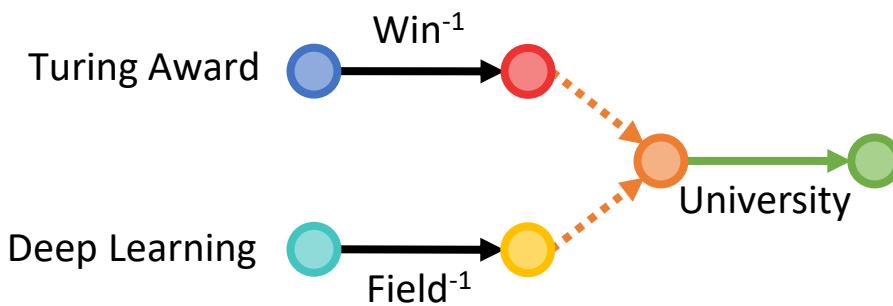
# Subgraph Matching



# Subgraph Matching

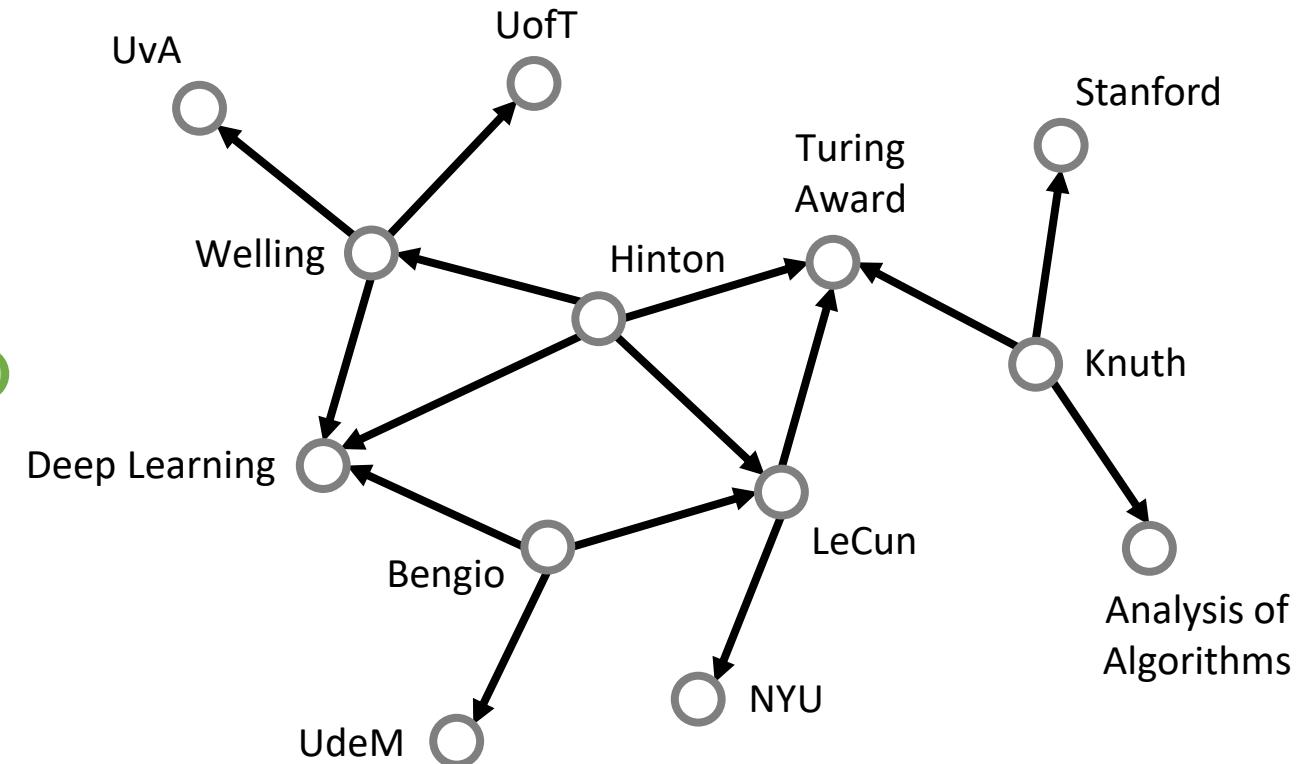
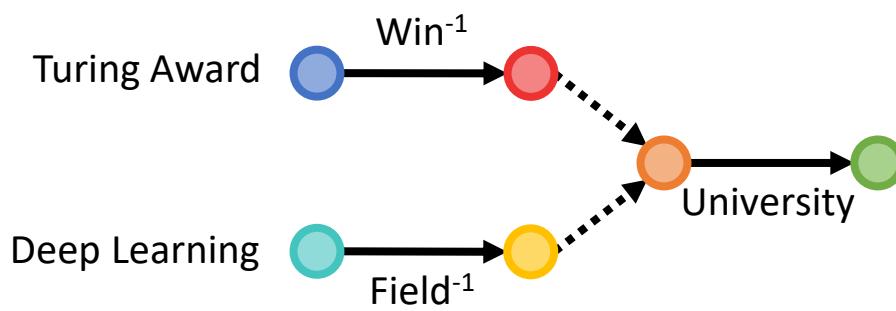


# Subgraph Matching



# Subgraph Matching

No answer!



Subgraph matching is **inductive**, but it **can't reason about missing links**.

# Subgraph Matching

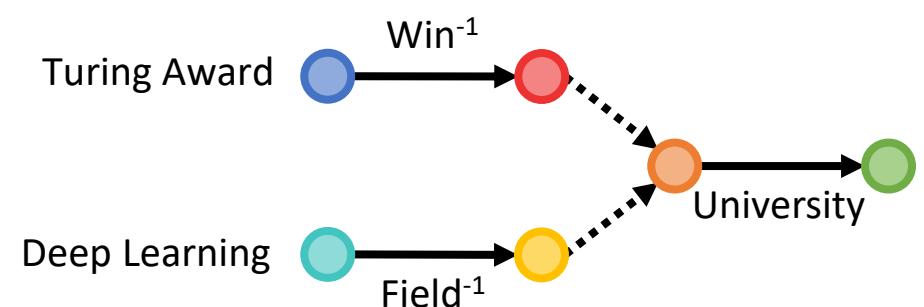
$\mathcal{X} = \{\text{Hinton, Lecun, Bengio}\} \in 2^{\mathcal{V}}$

**Relation Projection:**  $\mathcal{Y} = \text{University}(\mathcal{X})$

**Conjunction:**  $\mathcal{X} \cap \mathcal{Y}$

**Disjunction:**  $\mathcal{X} \cup \mathcal{Y}$

**Negation:**  $\mathcal{V} \setminus \mathcal{X}$



# Relax to Fuzzy Sets

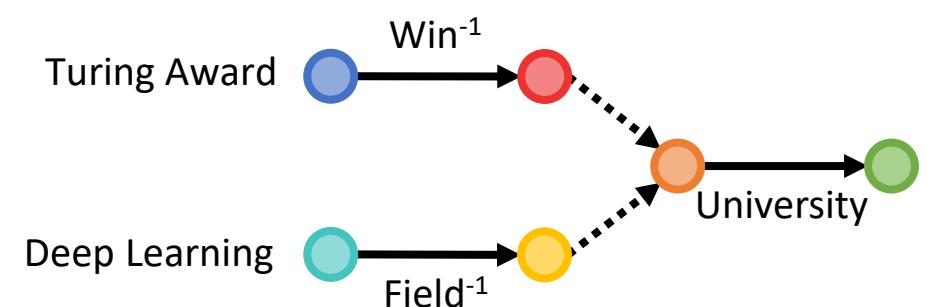
$$x = \{\text{Hinton: } 0.81, \text{Lecun: } 0.56, \text{Bengio: } 0.64\} \in [0,1]^\nu$$

**Relation Projection:**  $y = \text{University}(x)$

**Conjunction:**  $x \odot y$

**Disjunction:**  $x + y - x \odot y$

**Negation:**  $1 - x$



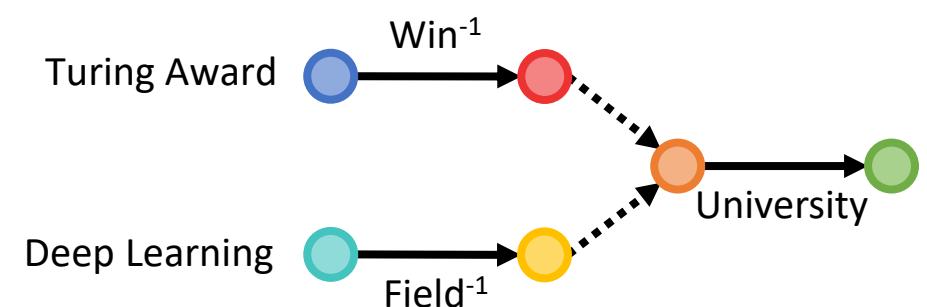
# Relax to Fuzzy Sets

$$x = \{\text{Hinton: } 0.81, \text{Lecun: } 0.56, \text{Bengio: } 0.64\} \in [0,1]^\mathcal{V}$$

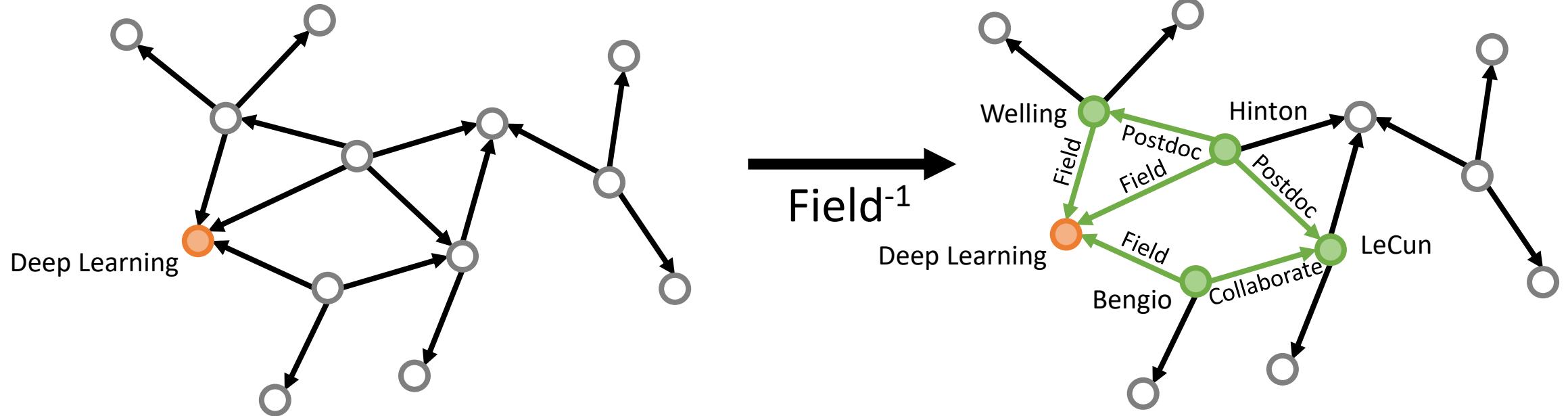
**Relation Projection:**  $y = University(x)$

**Conjunction:**  $x \odot y$   
**Disjunction:**  $x + y - x \odot y$   
**Negation:**  $1 - x$

} Inductive!

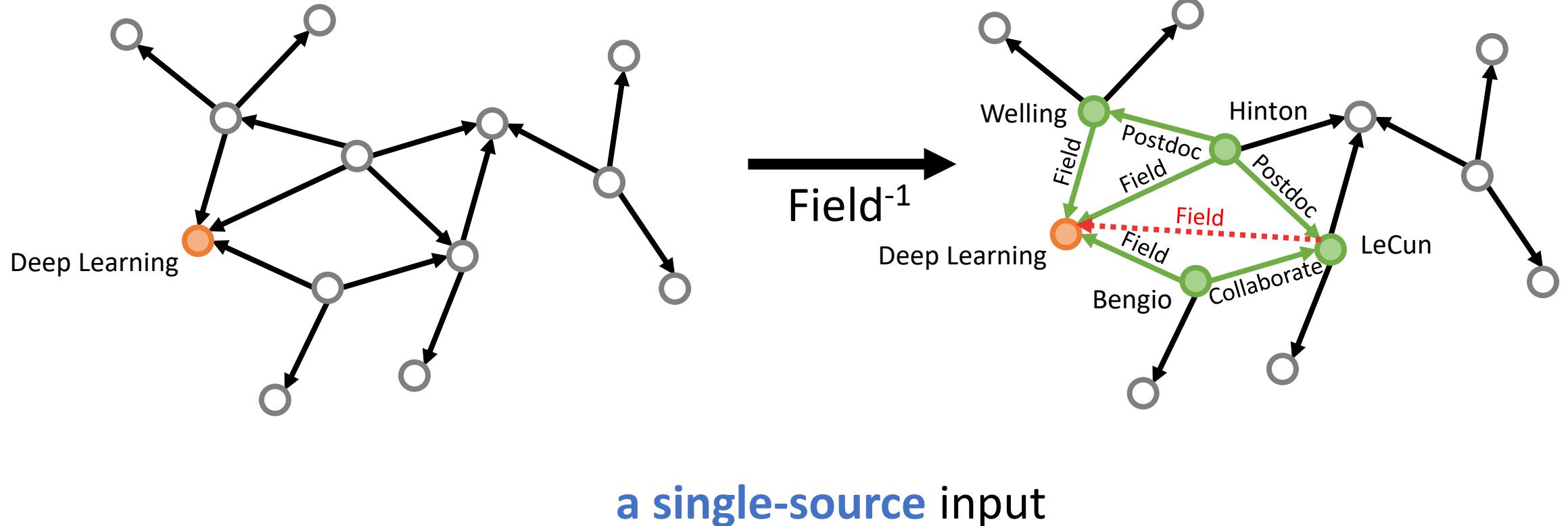


# Refresher: NBFNet

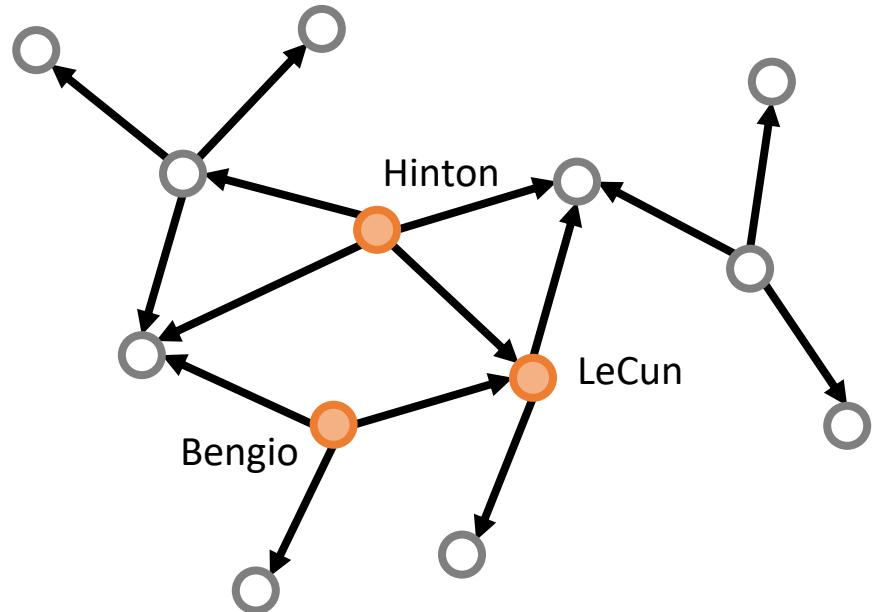


a single-source input

# Refresher: NBFNet

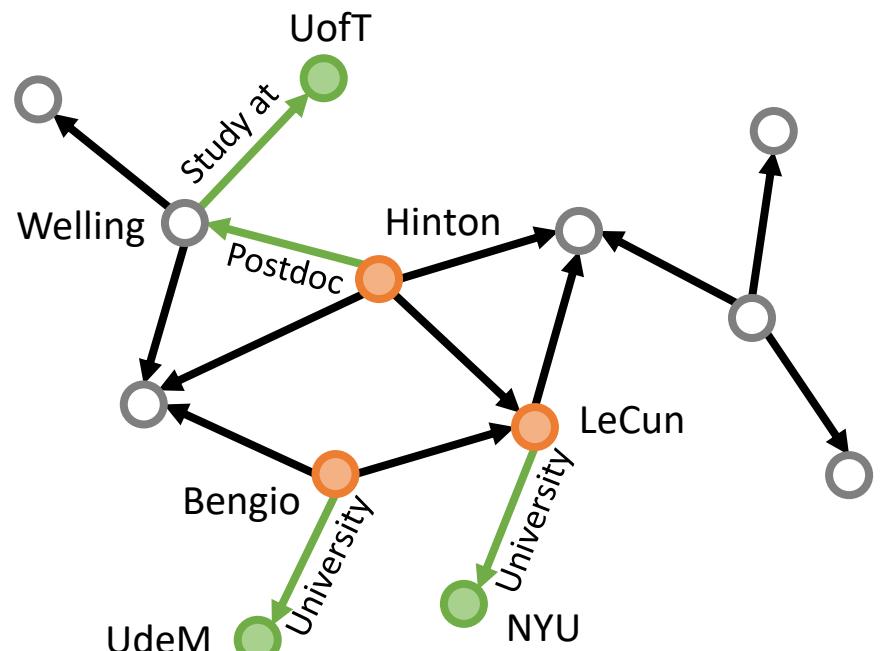


# Relation Projection

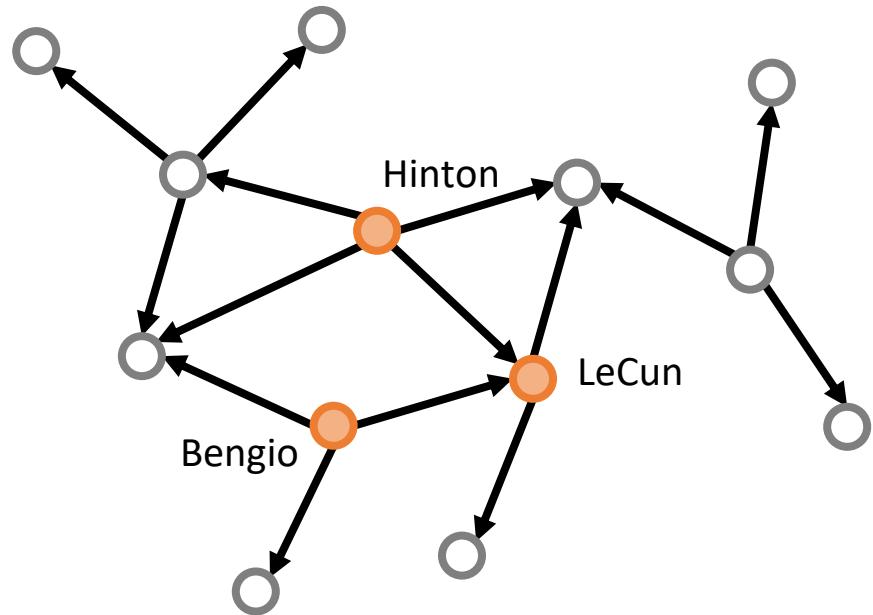


University

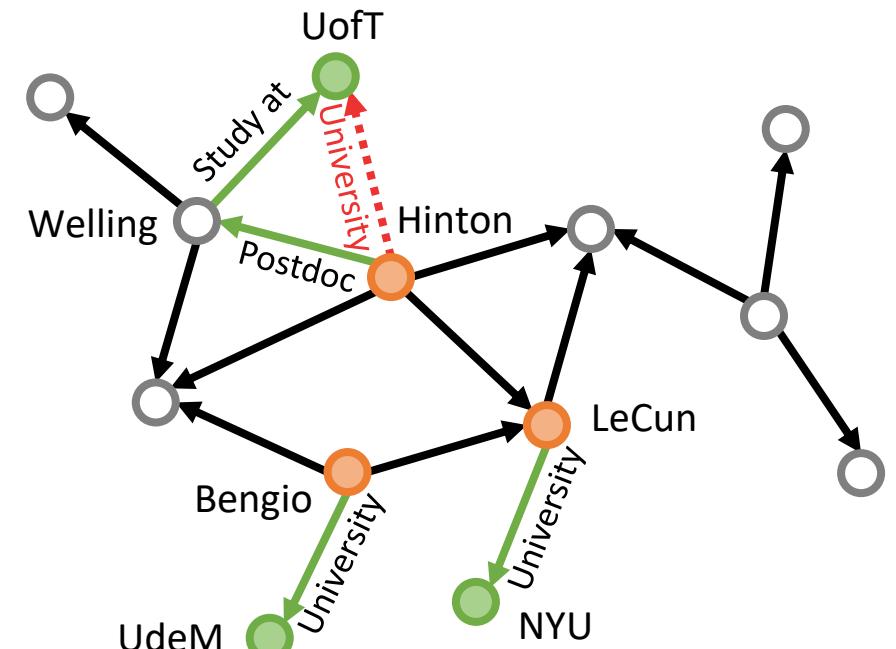
a fuzzy set input



# Relation Projection



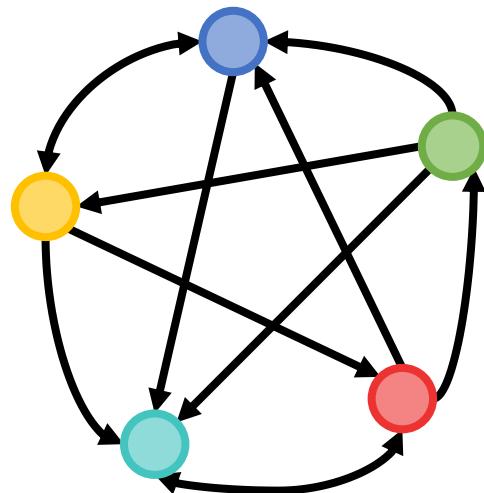
University



a fuzzy set input

# 0-shot Relation Projection

relation graph



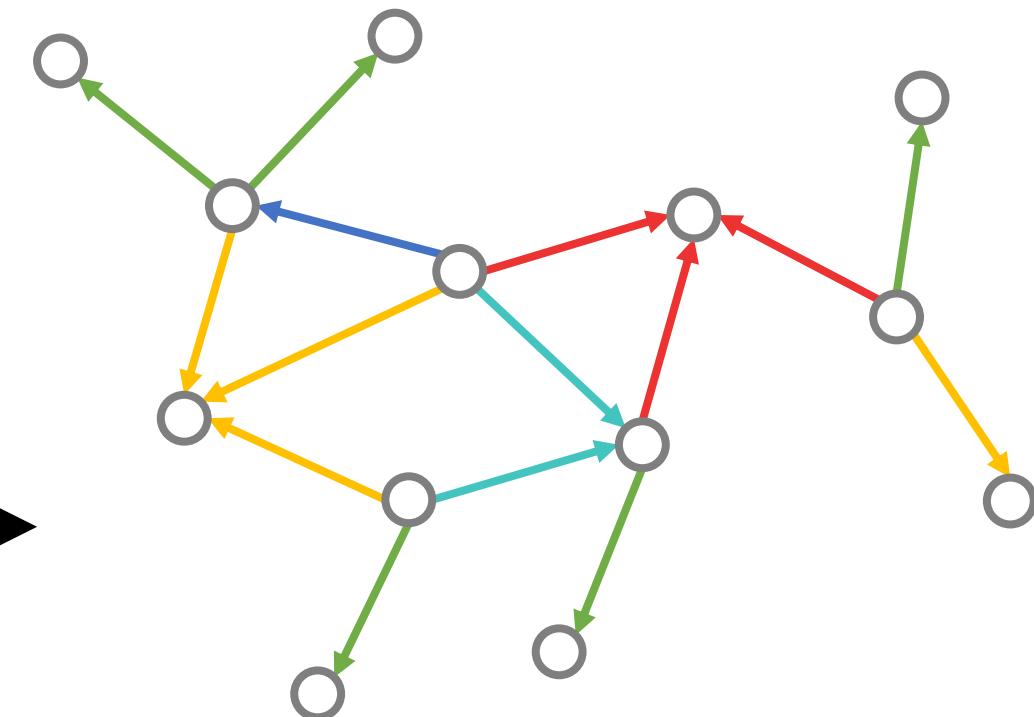
1

construct

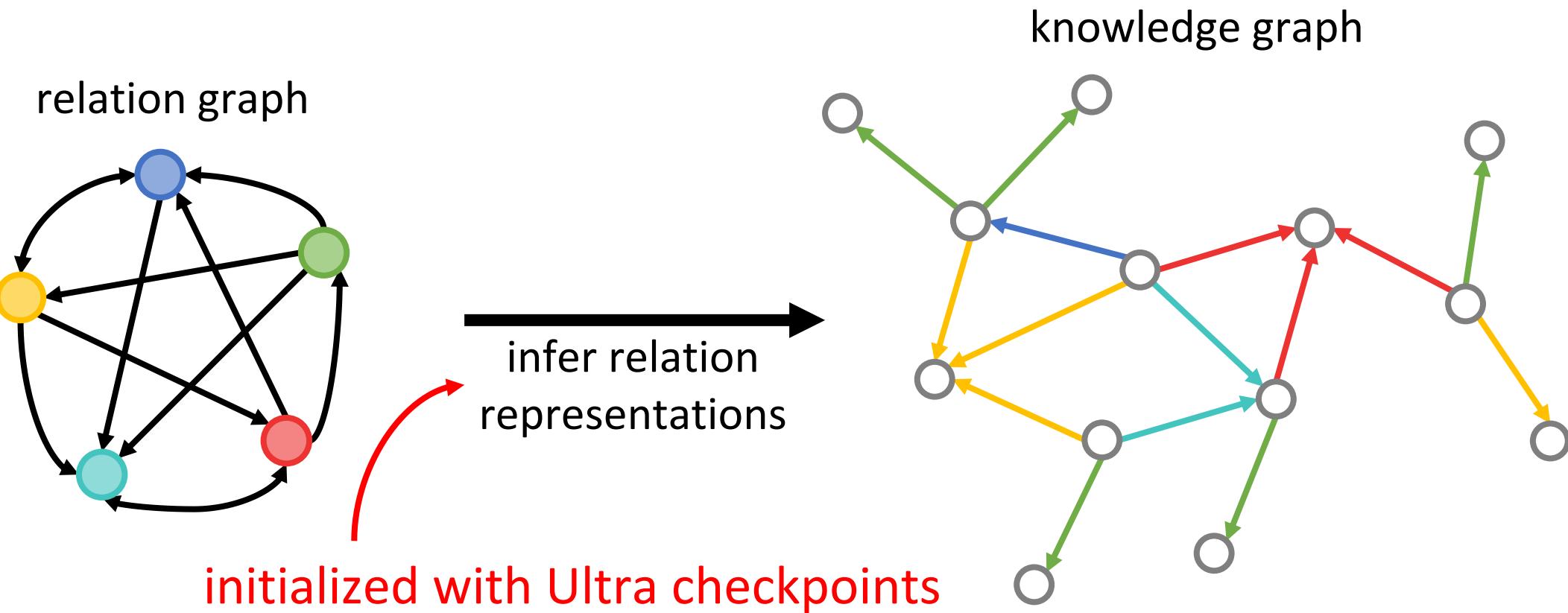
2

infer relation  
representations

knowledge graph



# 0-shot Relation Projection



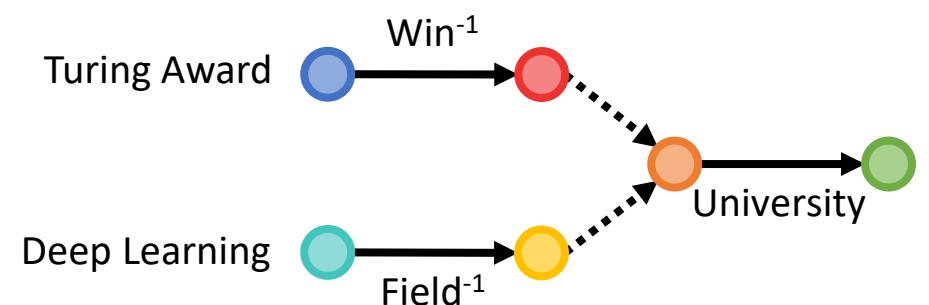
# Graph Neural Network Query Executor

$$x = \{\text{Hinton: 0.81, Lecun: 0.56, Bengio: 0.64}\} \in [0,1]^{\mathcal{V}}$$

**Relation Projection:**  $y = \text{University}(x)$  Inductive!

**Conjunction:**  $x \odot y$   
**Disjunction:**  $x + y - x \odot y$   
**Negation:**  $1 - x$

} Inductive!



# Multi-hop Logical Queries ( $\mathcal{V}_{train} = \mathcal{V}_{test}$ )

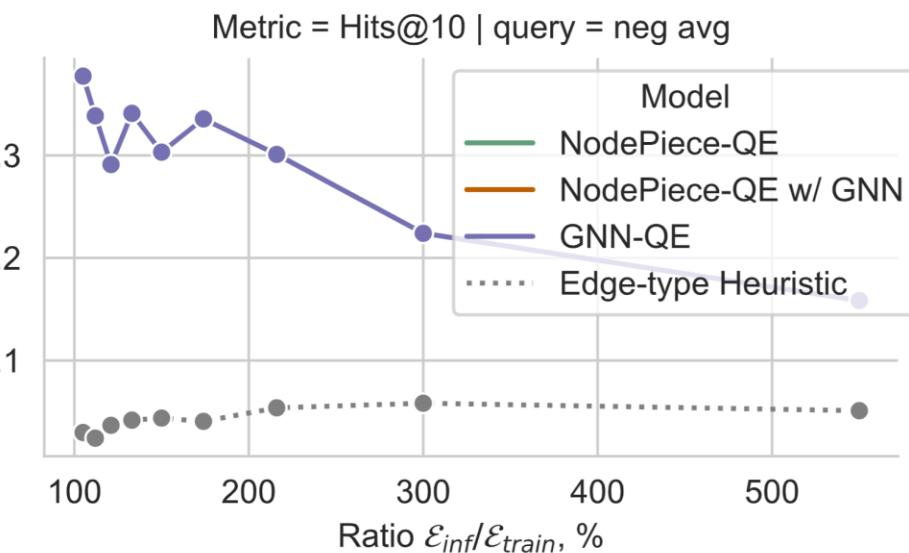
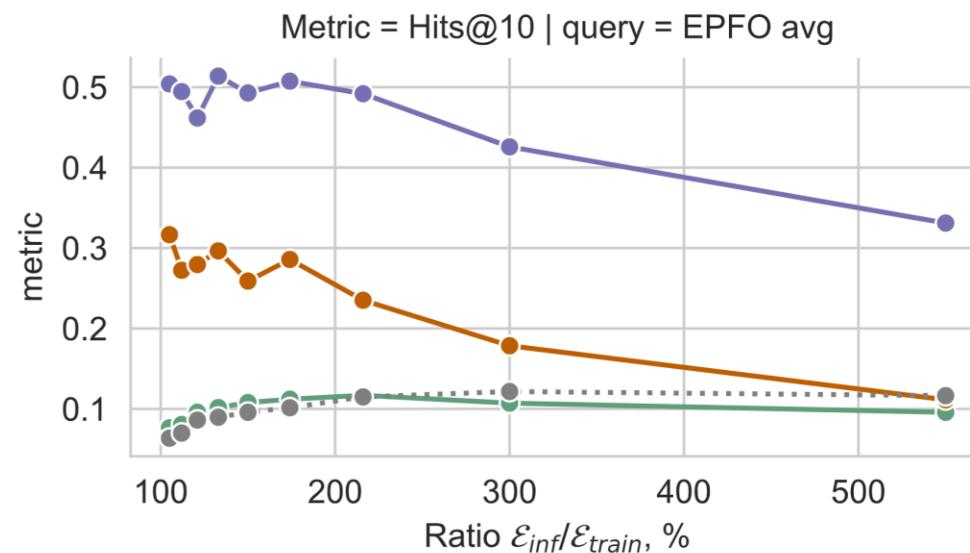
metric: MRR↑

Model	avg <sub>p</sub>	avg <sub>n</sub>	1p	2p	3p	2i	3i	pi	ip	2u	up	2in	3in	inp	pin	pni
FB15k																
GQE	28.0	-	54.6	15.3	10.8	39.7	51.4	27.6	19.1	22.1	11.6	-	-	-	-	-
Q2B	38.0	-	68.0	21.0	14.2	55.1	66.5	39.4	26.1	35.1	16.7	-	-	-	-	-
BetaE	41.6	11.8	65.1	25.7	24.7	55.8	66.5	43.9	28.1	40.1	25.2	14.3	14.7	11.5	6.5	12.4
CQD-CO	46.9	-	<b>89.2</b>	25.3	13.4	74.4	78.3	44.1	33.2	41.8	21.9	-	-	-	-	-
CQD-Beam	58.2	-	<b>89.2</b>	54.3	28.6	74.4	78.3	58.2	67.7	42.4	30.9	-	-	-	-	-
ConE	49.8	14.8	73.3	33.8	29.2	64.4	73.7	50.9	35.7	55.7	31.4	17.9	18.7	12.5	9.8	15.1
GNN-QE	<b>72.8</b>	<b>38.6</b>	88.5	<b>69.3</b>	<b>58.7</b>	<b>79.7</b>	<b>83.5</b>	<b>69.9</b>	<b>70.4</b>	<b>74.1</b>	<b>61.0</b>	<b>44.7</b>	<b>41.7</b>	<b>42.0</b>	<b>30.1</b>	<b>34.3</b>
FB15k-237																
GQE	16.3	-	35.0	7.2	5.3	23.3	34.6	16.5	10.7	8.2	5.7	-	-	-	-	-
Q2B	20.1	-	40.6	9.4	6.8	29.5	42.3	21.2	12.6	11.3	7.6	-	-	-	-	-
BetaE	20.9	5.5	39.0	10.9	10.0	28.8	42.5	22.4	12.6	12.4	9.7	5.1	7.9	7.4	3.5	3.4
CQD-CO	21.8	-	<b>46.7</b>	9.5	6.3	31.2	40.6	23.6	16.0	14.5	8.2	-	-	-	-	-
CQD-Beam	22.3	-	<b>46.7</b>	11.6	8.0	31.2	40.6	21.2	18.7	14.6	8.4	-	-	-	-	-
FuzzQE	24.0	7.8	42.8	12.9	10.3	33.3	46.9	26.9	17.8	14.6	10.3	8.5	11.6	7.8	5.2	5.8
ConE	23.4	5.9	41.8	12.8	11.0	32.6	47.3	25.5	14.0	14.5	10.8	5.4	8.6	7.8	4.0	3.6
GNN-QE	<b>26.8</b>	<b>10.2</b>	42.8	<b>14.7</b>	<b>11.8</b>	<b>38.3</b>	<b>54.1</b>	<b>31.1</b>	<b>18.9</b>	<b>16.2</b>	<b>13.4</b>	<b>10.0</b>	<b>16.8</b>	<b>9.3</b>	<b>7.2</b>	<b>7.8</b>

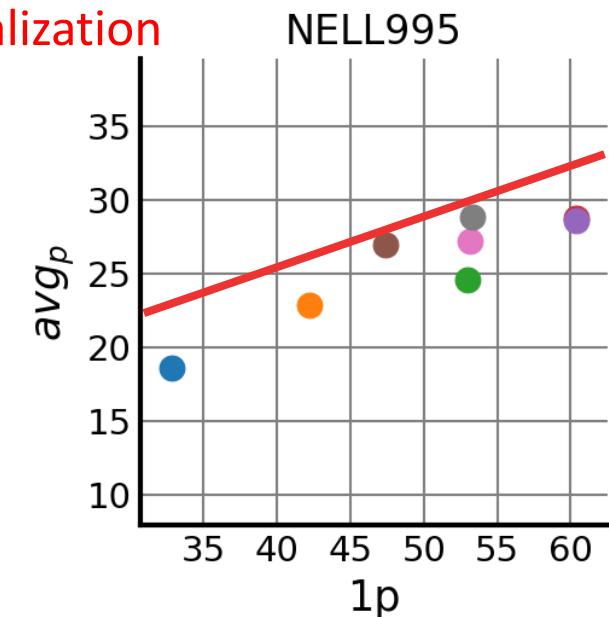
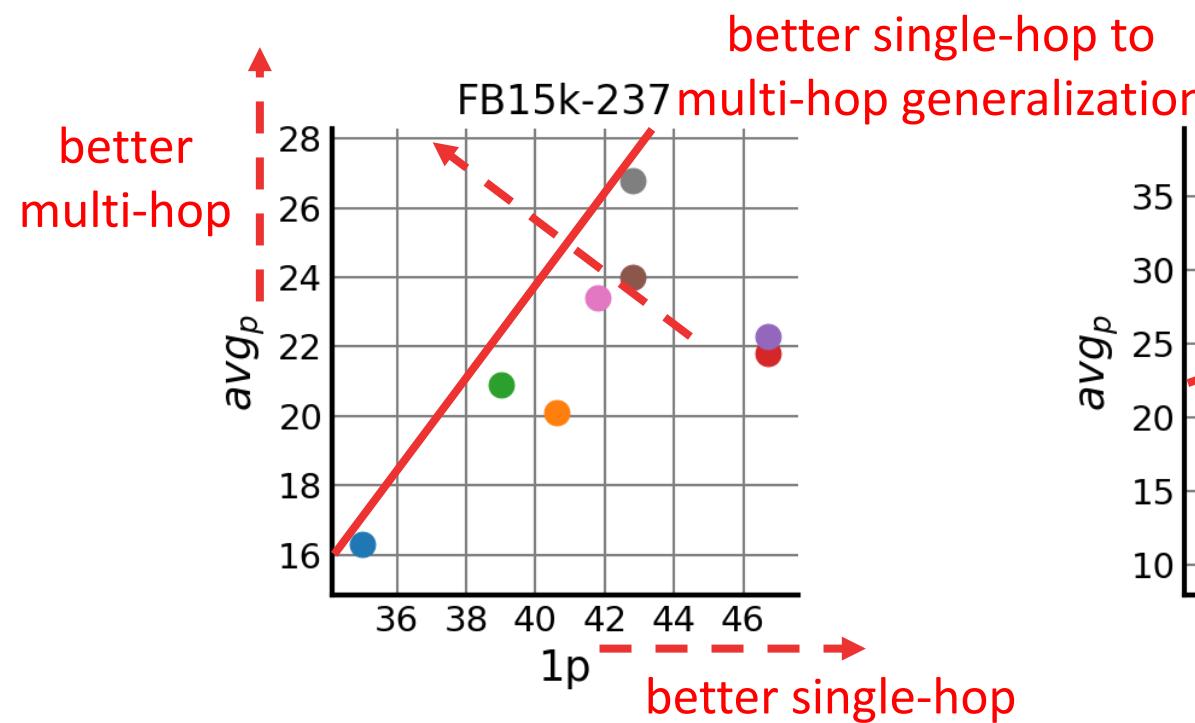
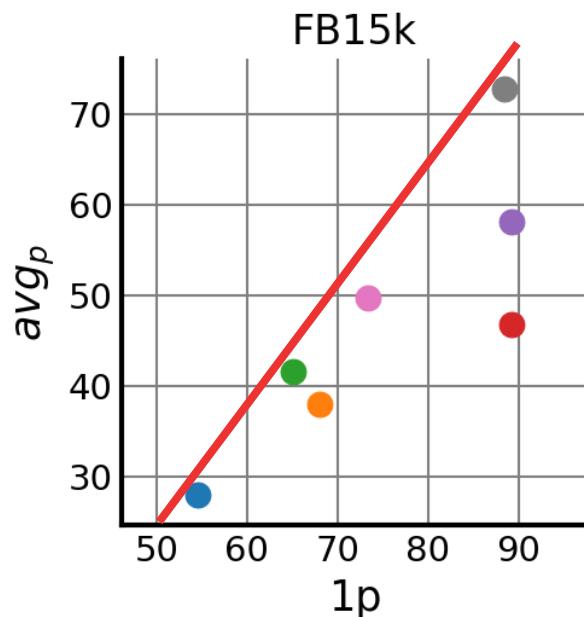
# Multi-hop Logical Queries ( $\mathcal{V}_{train} \neq \mathcal{V}_{test}$ )

metric: H@10↑

Class	Model	avg <sub>p</sub>	avg <sub>n</sub>	1p	2p	3p	2i	3i	pi	ip	2u	up	2in	3in	inp	pin	pni
FB15k-237																	
Inference-only	Edge-type Heuristic	10.1	4.1	17.7	8.2	9.9	10.7	13.0	9.8	8.2	5.3	8.5	2.6	2.9	8.4	3.8	2.7
	NodePiece-QE	11.2	-	25.5	8.2	8.4	12.4	13.9	9.9	8.7	7.0	6.8	-	-	-	-	-
	NodePiece-QE w/ GNN	28.6	-	45.9	19.2	11.5	39.9	48.8	29.4	22.6	25.3	14.6	-	-	-	-	-
Trainable	GNN-QE	<b>50.7</b>	<b>33.6</b>	<b>65.4</b>	<b>36.3</b>	<b>31.6</b>	<b>73.8</b>	<b>84.3</b>	<b>56.5</b>	<b>41.5</b>	<b>39.3</b>	<b>28.0</b>	<b>33.3</b>	<b>46.4</b>	<b>29.2</b>	<b>24.9</b>	<b>34.0</b>

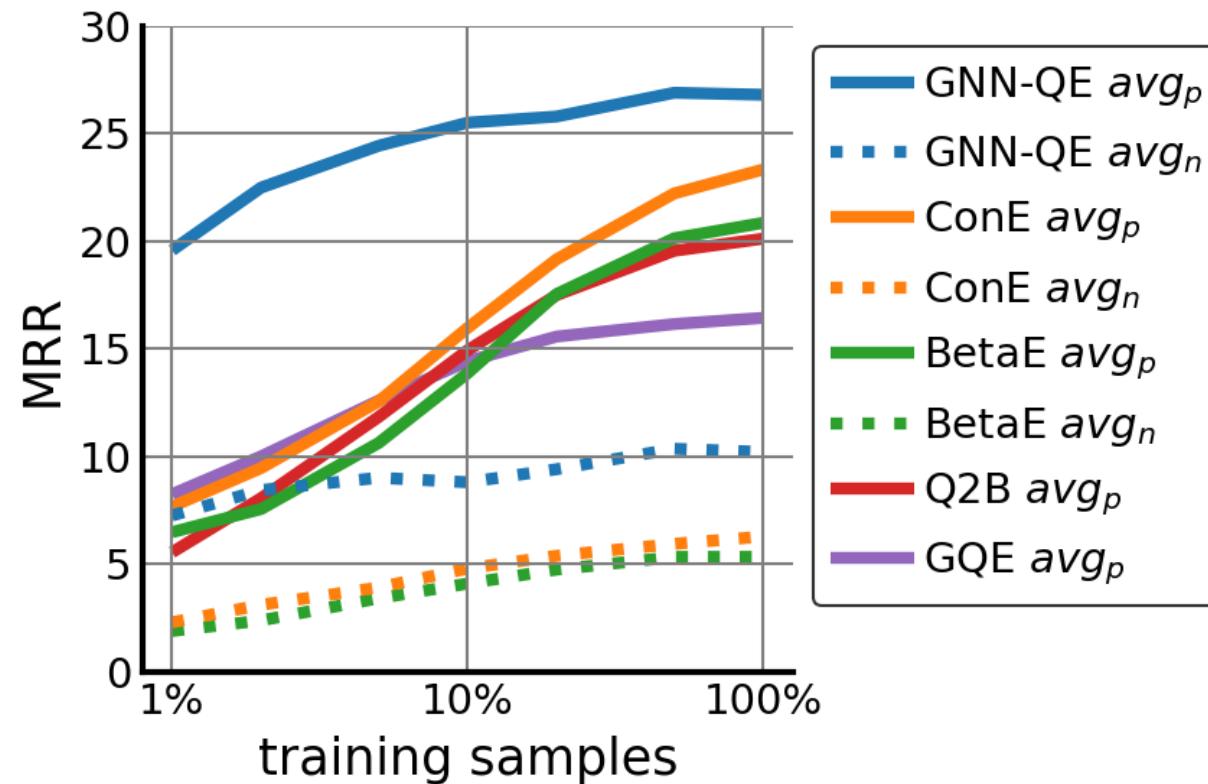


# Better Compositional Generalization

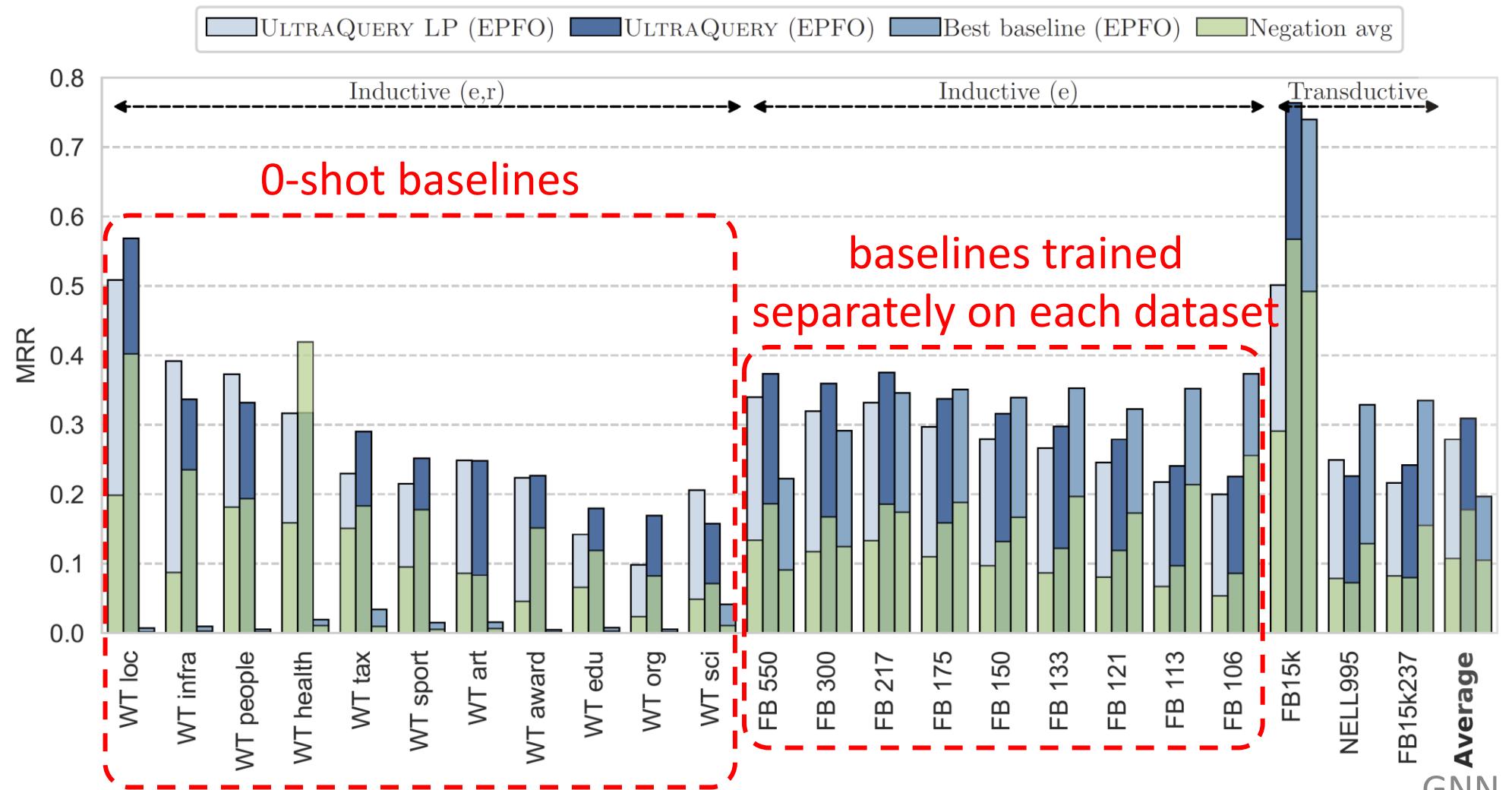


● GQE   ● Q2B   ● BetaE   ● CQD-CO   ● CQD-Beam   ● FuzzQE   ● ConE   ● GNN-QE

# Effective for Small Training Data



# 0-shot Inference of Multi-hop Queries



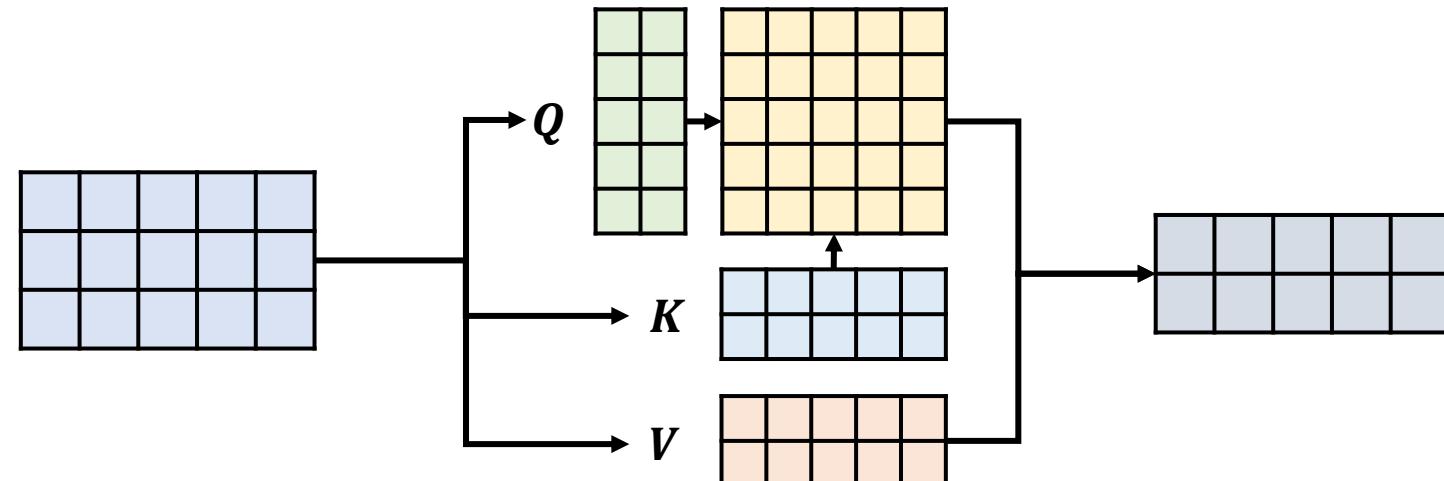
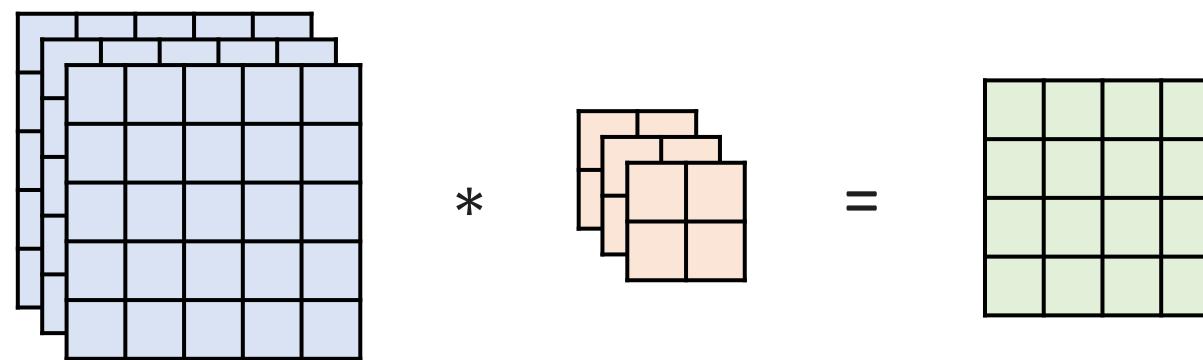
# TorchDrug<sup>[1]</sup>: Simplifying **development** on **structured data** and related applications

[1] Zhaocheng Zhu, Chence Shi, Zuobai Zhang, Shengchao Liu, Minghao Xu, Xinyu Yuan, Yangtian Zhang, Junkun Chen, Huiyu Cai, Jiarui Lu, Chang Ma, Runcheng Liu, Louis-Pascal Xhonneux, Meng Qu, Jian Tang. TorchDrug: A Powerful and Flexible Machine Learning Platform for Drug Discovery. arXiv 2022.

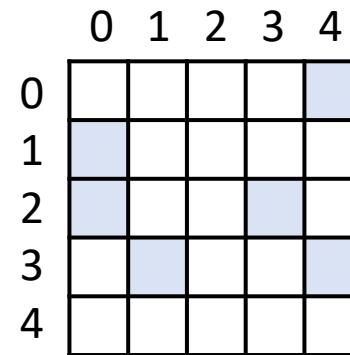
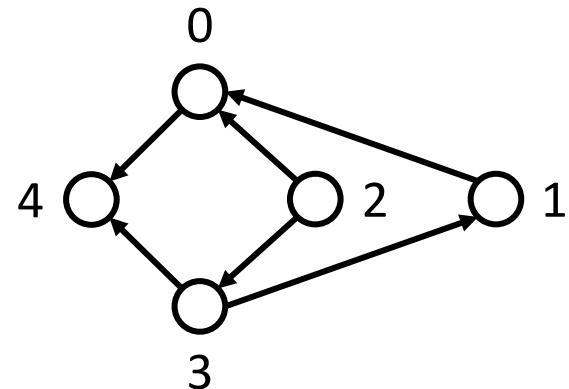
# ML Implementation = Tensor Operations



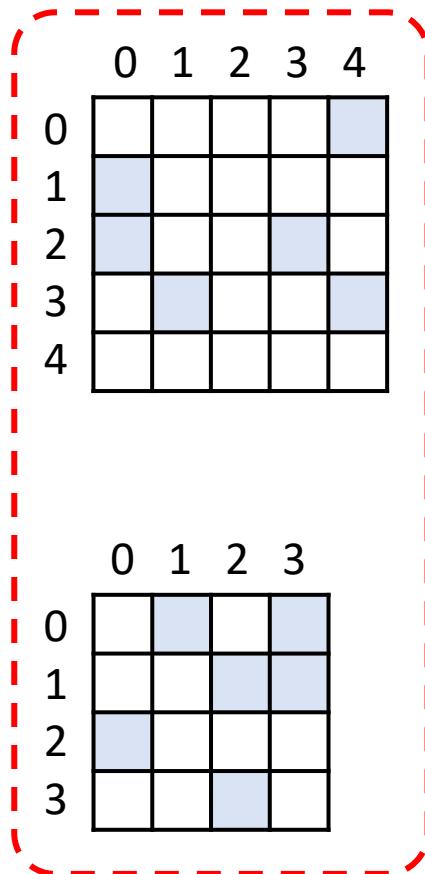
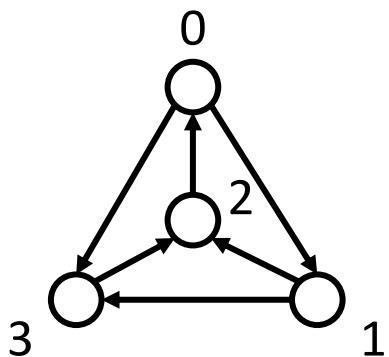
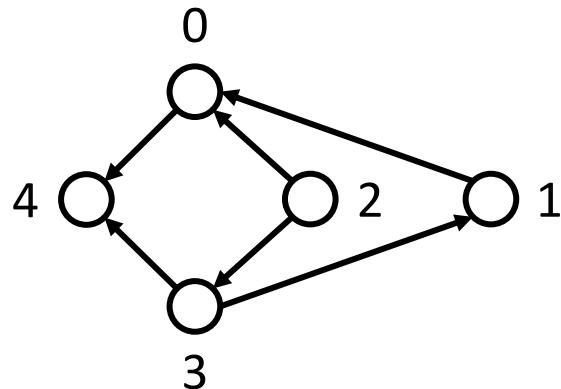
She sells sea shells.



# Structured Data Meets Tensors

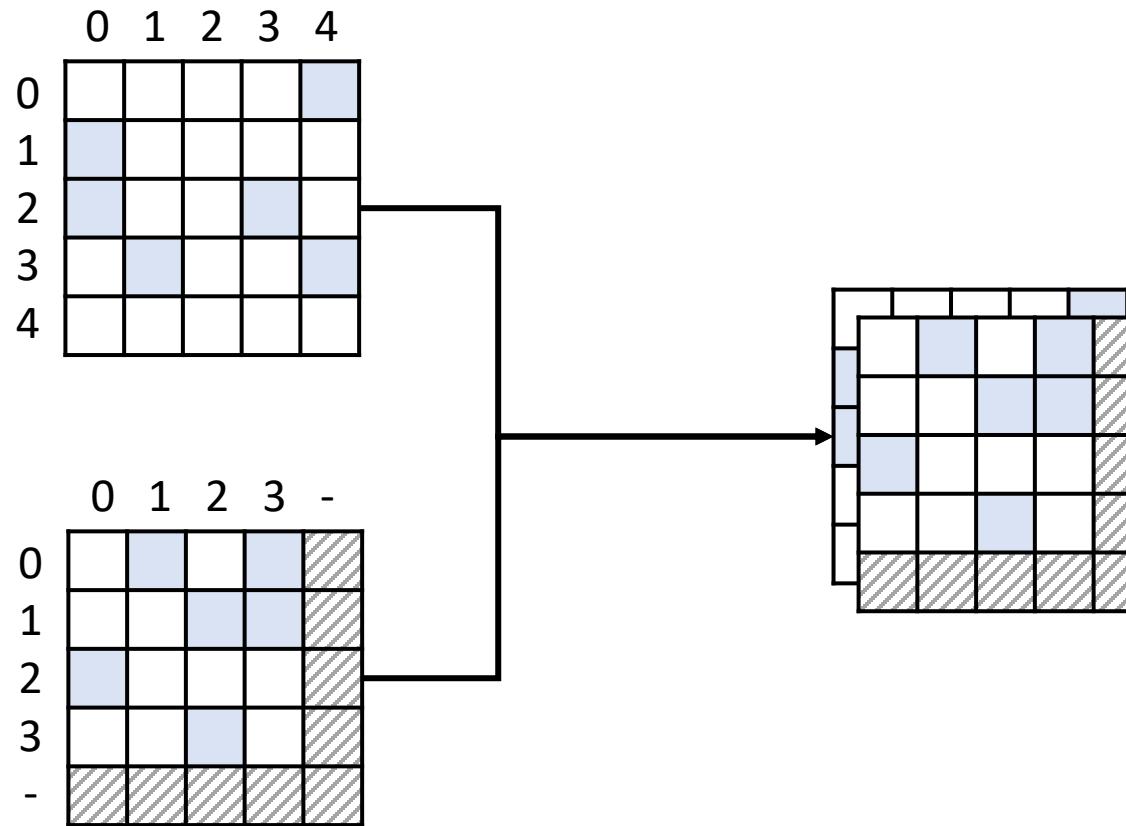
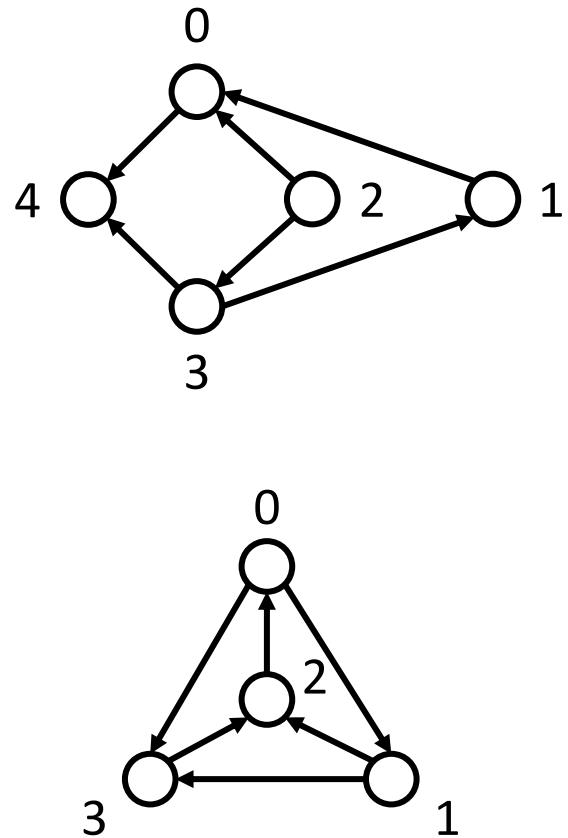


# Structured Data Meets Tensors

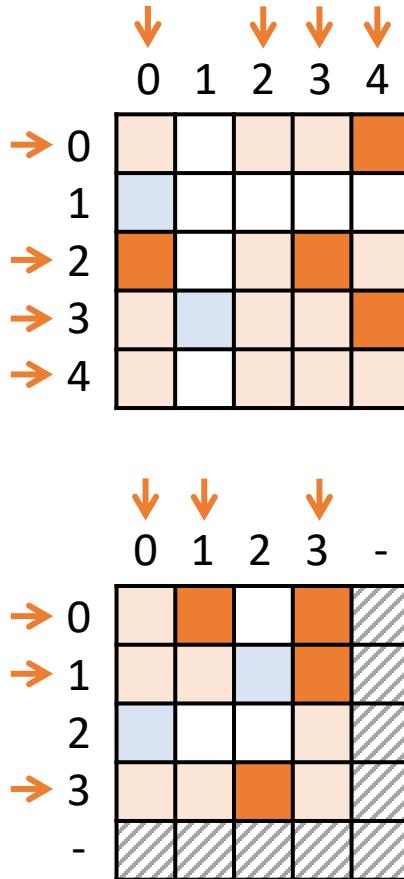
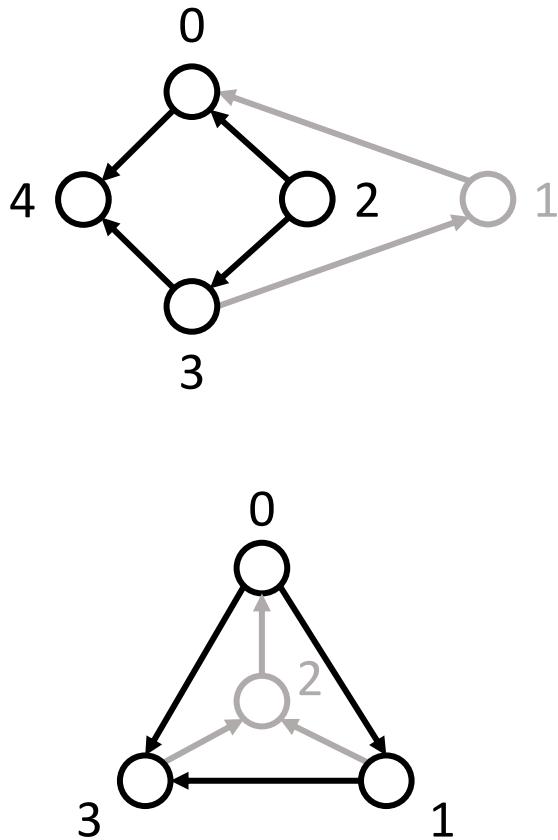


How to batch tensors  
of different shapes?

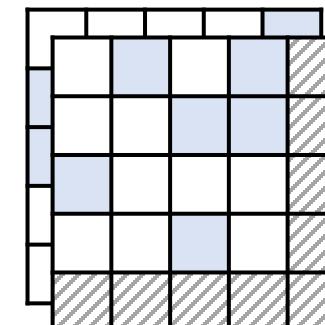
# Naïve Solution: Padding



# Naïve Solution: Padding



How to perform operations  
on batched tensors?



# Solutions



arrays



easy to implement  
preprocessing  
**very slow**



dense tensors



on-the-fly  
**not scalable**

# Solutions



arrays



easy to implement  
preprocessing  
very slow



dense tensors



on-the-fly  
not scalable



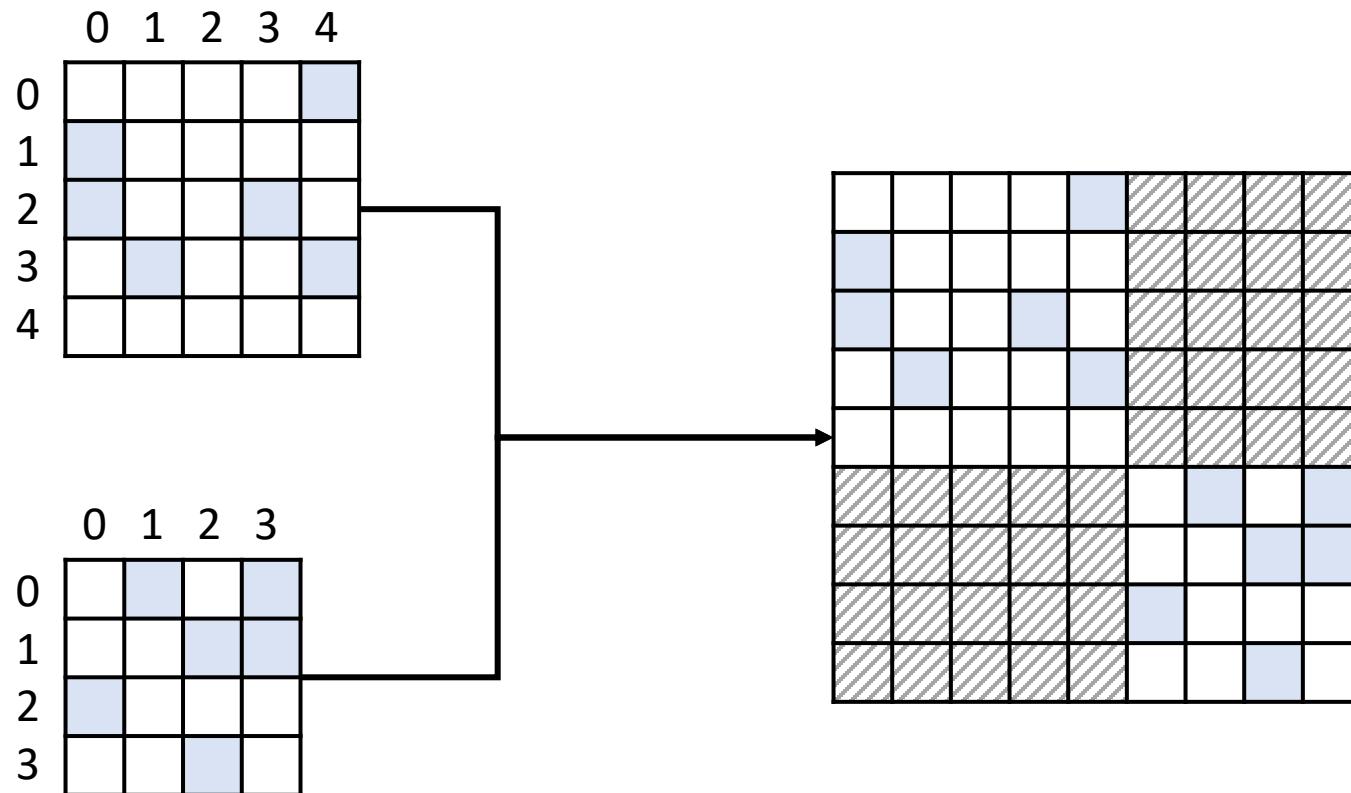
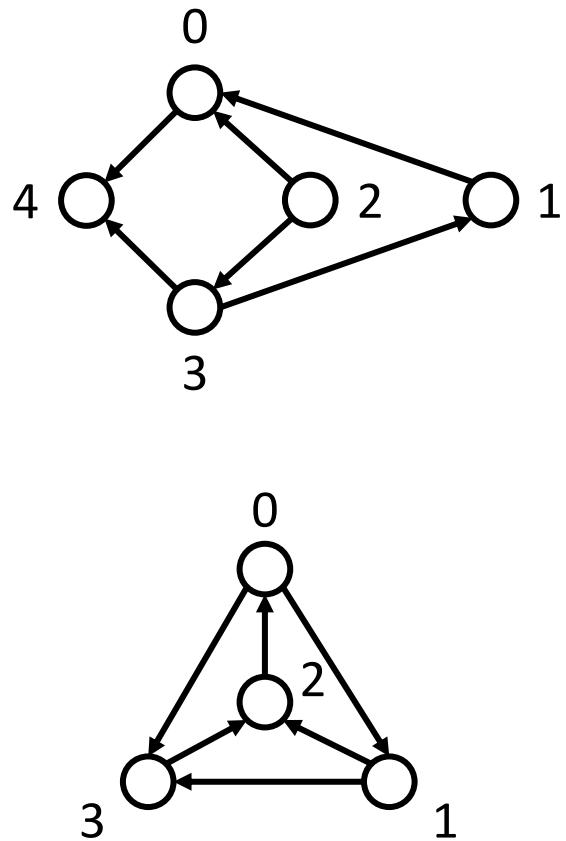
sparse tensors



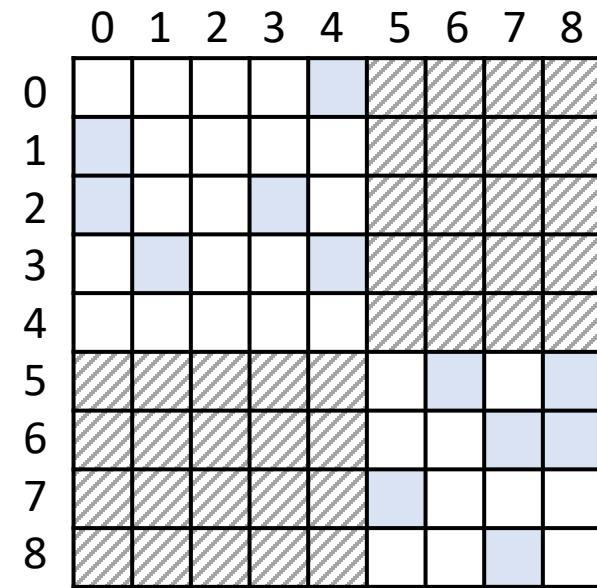
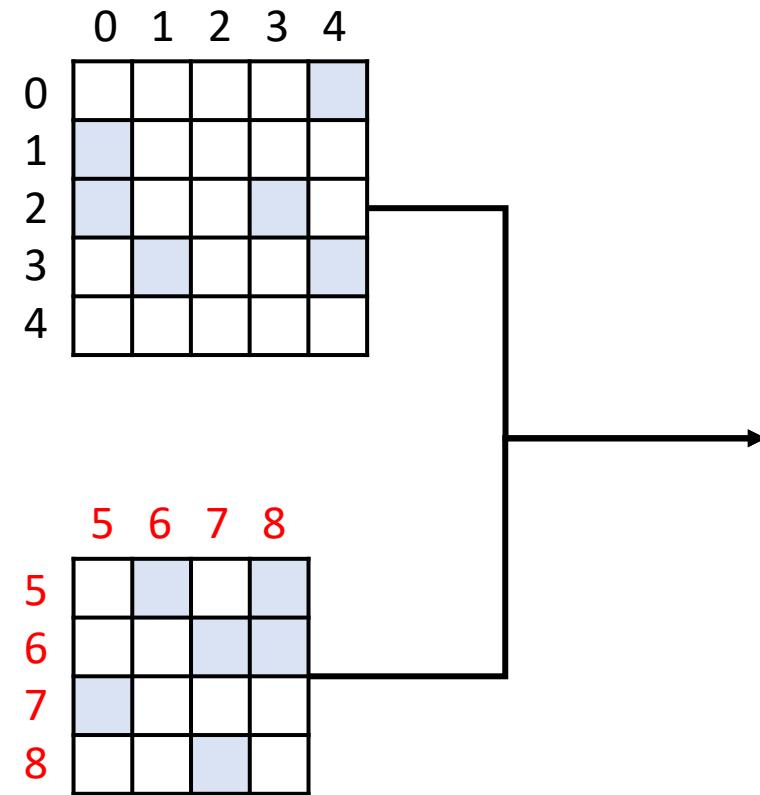
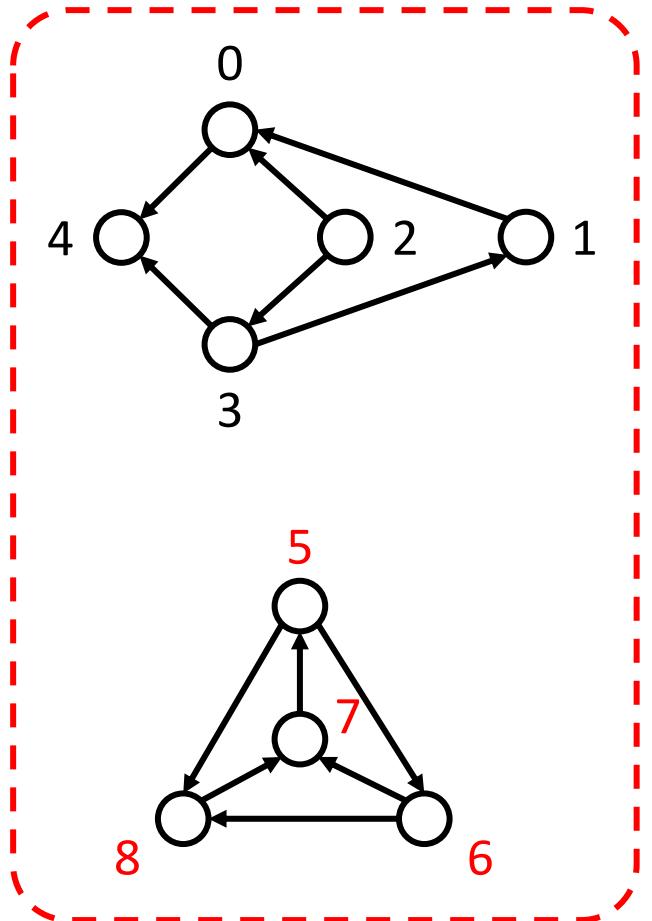
on-the-fly  
scalable

How to implement?

# The High-Level Idea

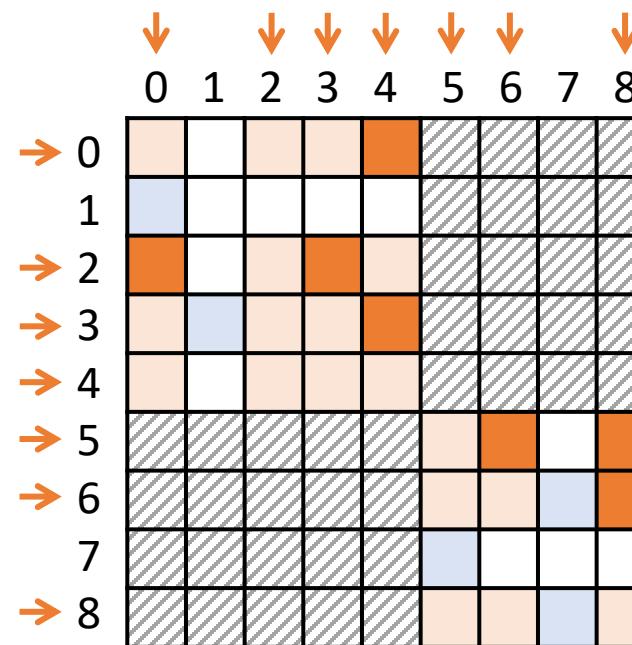
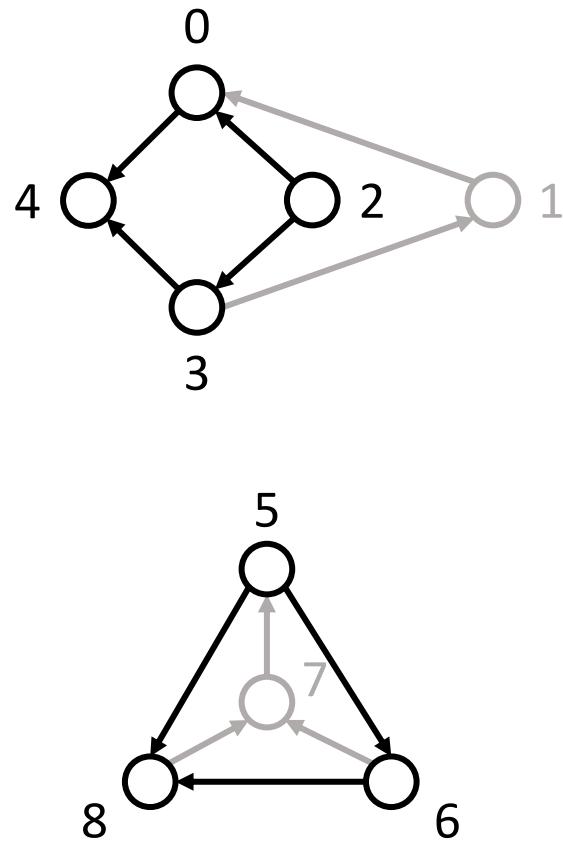


# The High-Level Idea



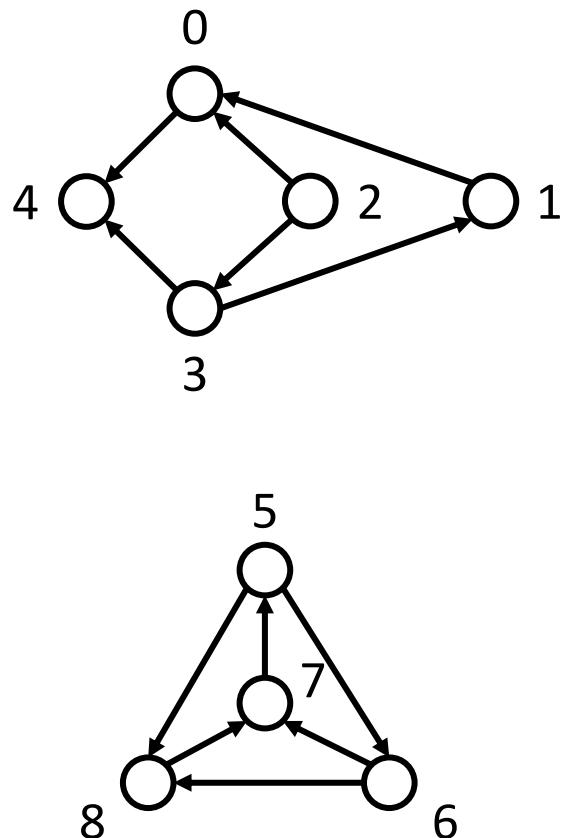
a graph of two connected components

# The High-Level Idea



easy to implement!

# Data Structure



`torchdrug.data.PackedGraph`

edge list

0	1	2	2	3	3	5	5	6	6	7	8
4	0	0	3	1	4	6	8	7	8	5	7

#nodes

5	4
---	---

#edges

6	6
---	---

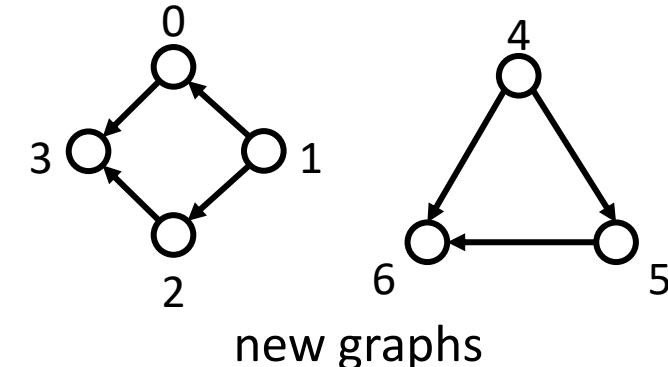
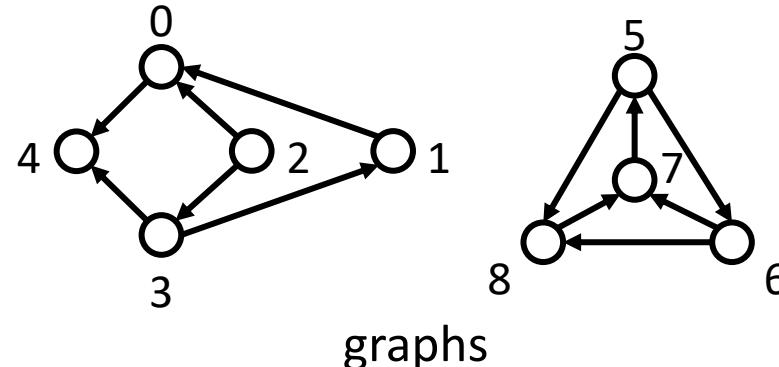
node/edge/graph  
attributes

.....

predefined or user-registered

# Graph Operations

```
new_graphs = graphs.subgraph(node_index)
```



edge list	<table border="1"><tr><td>0</td><td>1</td><td>2</td><td>2</td><td>3</td><td>3</td><td>5</td><td>5</td><td>6</td><td>6</td><td>7</td><td>8</td></tr><tr><td>4</td><td>0</td><td>0</td><td>3</td><td>1</td><td>4</td><td>6</td><td>8</td><td>7</td><td>8</td><td>5</td><td>7</td></tr></table>	0	1	2	2	3	3	5	5	6	6	7	8	4	0	0	3	1	4	6	8	7	8	5	7
0	1	2	2	3	3	5	5	6	6	7	8														
4	0	0	3	1	4	6	8	7	8	5	7														
#nodes	<table border="1"><tr><td>5</td><td>4</td></tr></table>	5	4																						
5	4																								
#edges	<table border="1"><tr><td>6</td><td>6</td></tr></table>	6	6																						
6	6																								

node index 

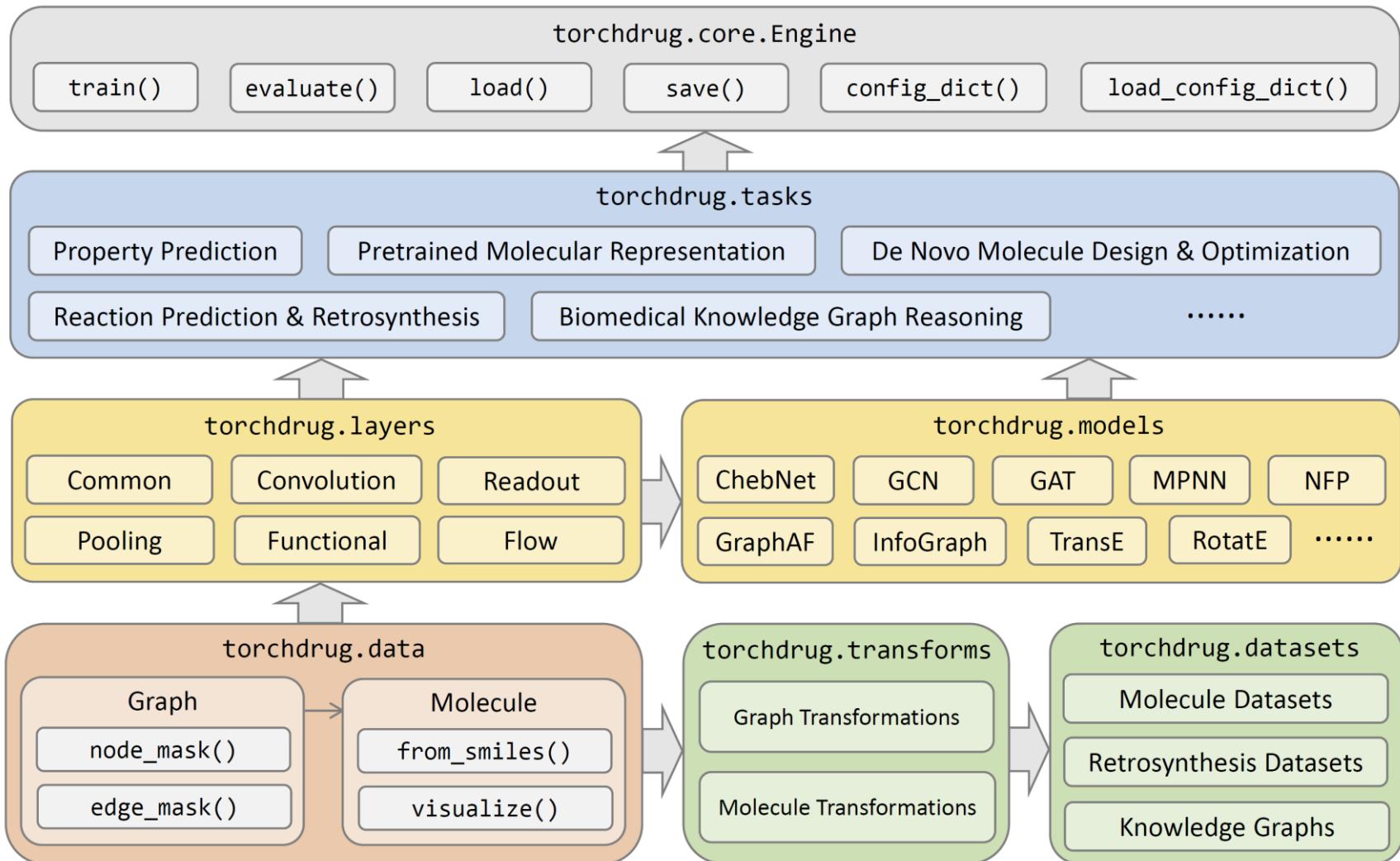
0	2	3	4	5	6	8
---	---	---	---	---	---	---

edge list	<table border="1"><tr><td>0</td><td>1</td><td>1</td><td>2</td><td>4</td><td>4</td><td>5</td></tr><tr><td>3</td><td>0</td><td>2</td><td>3</td><td>5</td><td>6</td><td>6</td></tr></table>	0	1	1	2	4	4	5	3	0	2	3	5	6	6
0	1	1	2	4	4	5									
3	0	2	3	5	6	6									
#nodes	<table border="1"><tr><td>4</td><td>3</td></tr></table>	4	3												
4	3														
#edges	<table border="1"><tr><td>4</td><td>3</td></tr></table>	4	3												
4	3														

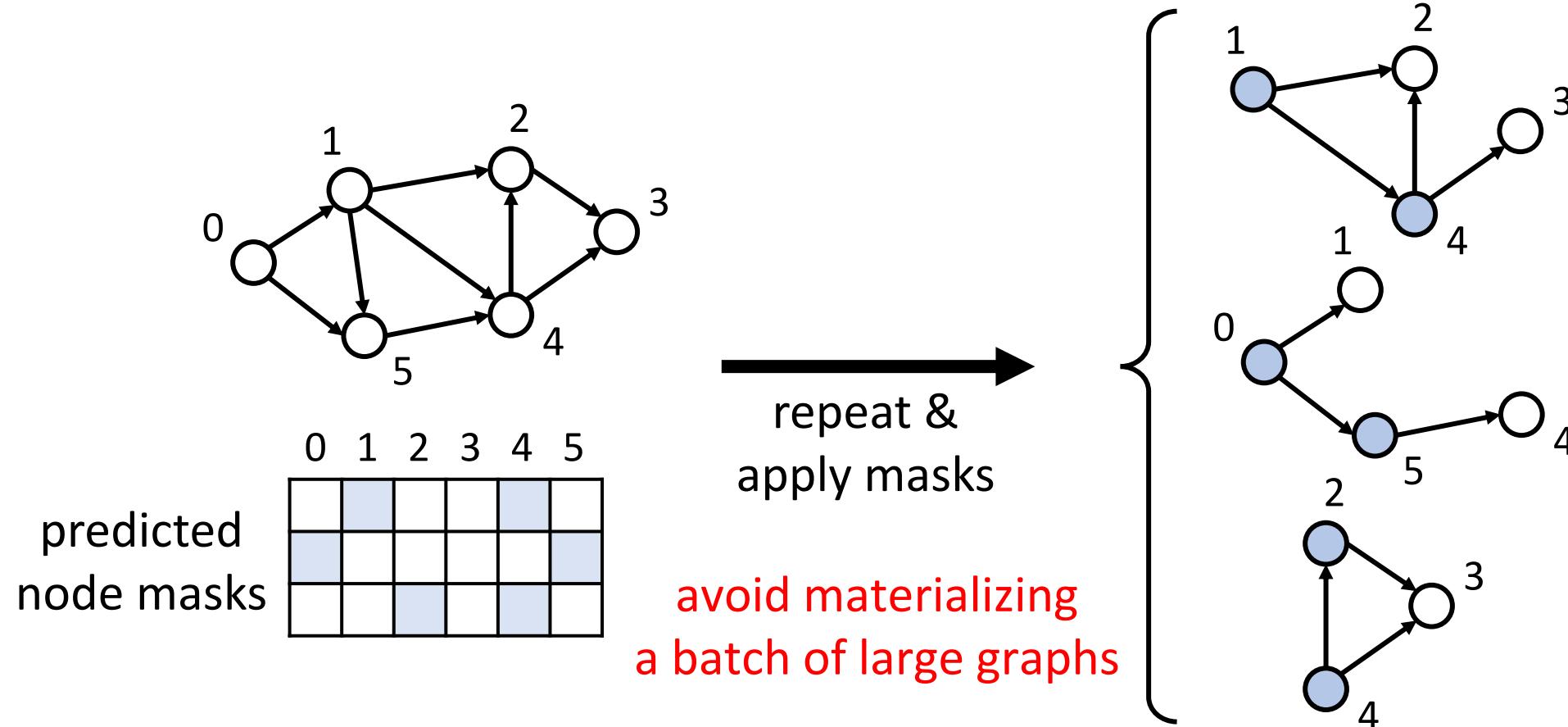
# Supported Operations

Class	API	Graph Operation
PyTorch-like	<code>data.Graph.clone</code>	Clone this graph
	<code>data.Graph.detach</code>	Detach this graph
	<code>data.Graph.cpu</code>	Move this graph to CPU
	<code>data.Graph.cuda</code>	Move this graph to GPU
	<code>data.Graph.copy_</code>	Copy data from another graph
	<code>data.Graph.full</code>	Return a fully connected graph over nodes
	<code>data.Graph.repeat</code>	Repeat this graph like <code>torch.repeat</code>
	<code>data.PackedGraph.repeat_interleave</code>	Repeat this graph like <code>torch.repeat_interleave</code>
Node-level	<code>data.Graph.node_mask</code>	Mask out some nodes from this graph
	<code>data.Graph.compact</code>	Remove isolated nodes
Edge-level	<code>data.Graph.edge_mask</code>	Mask out some edges from this graph
	<code>data.Graph.directed</code>	Return a directed version of this graph
	<code>data.Graph.undirected</code>	Return an undirected version of this graph
	<code>data.Graph.match</code>	Search specific edges in this graph
Graph-level	<code>data.Graph.connected_components</code>	Split a graph into connected components
	<code>data.Graph.split</code>	Split a graph into a batch of graphs
	<code>data.Graph.pack</code>	Pack multiple graphs into a batch
	<code>data.Graph.line_graph</code>	Return a line graph of this graph
	<code>data.PackedGraph.graph_mask</code>	Mask out some graphs from this batch
	<code>data.PackedGraph.merge</code>	Merge some graphs into a smaller batch
	<code>data.PackedGraph.unpack</code>	Unpack a batch into multiple graphs
Molecule	<code>data.Molecule.ion_to_molecules</code>	Convert ions to molecules
Protein	<code>data.Protein.residue_mask</code>	Mask out some residues from this protein

# Different Levels of Abstraction



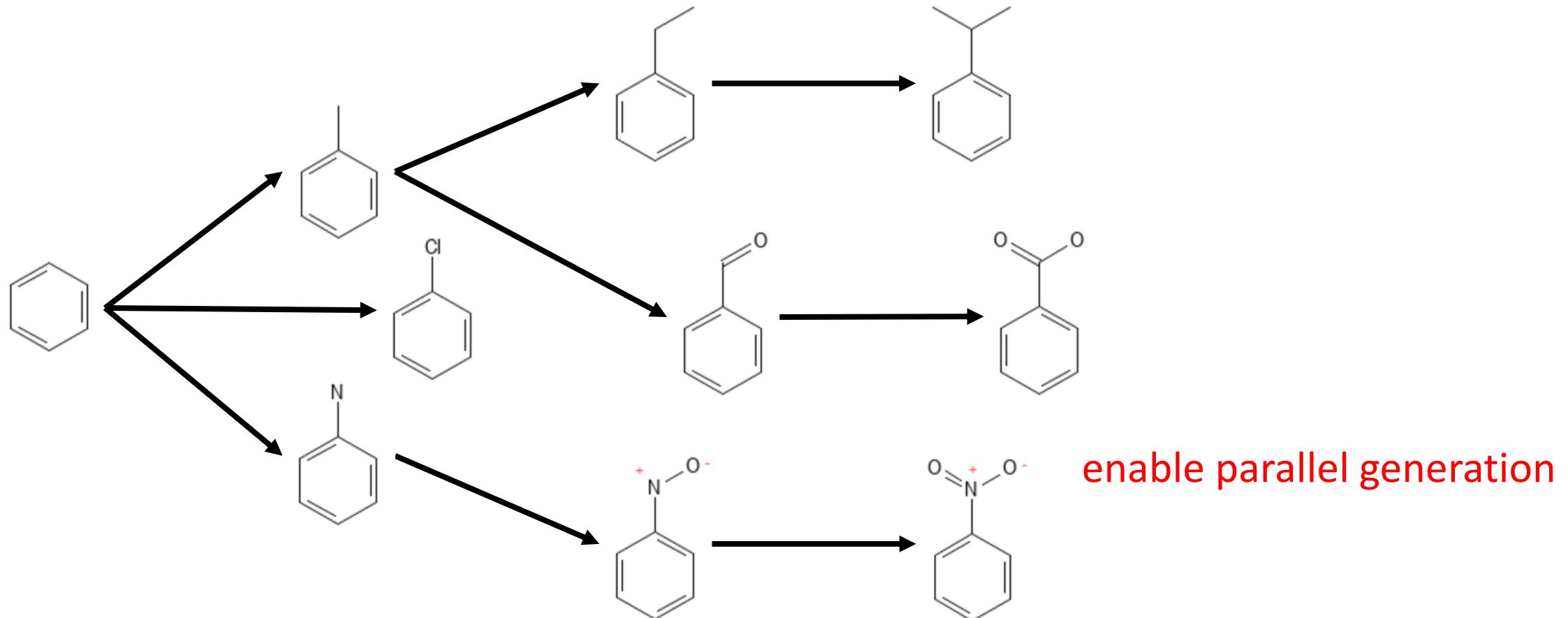
# Use Case: Adaptive Message Passing<sup>[1]</sup>



[1] Zhaocheng Zhu\*, Xinyu Yuan\*, Mikhail Galkin, Sophie Xhonneux, Ming Zhang, Maxime Gazeau, Jian Tang.

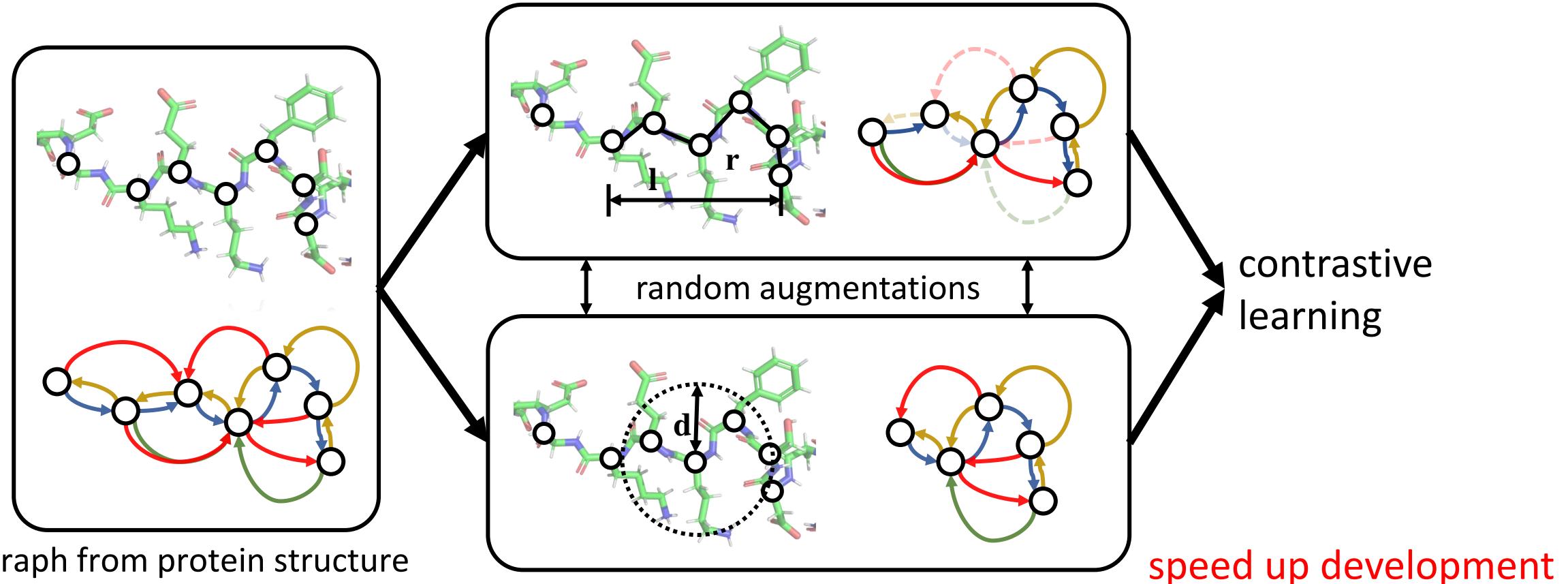
A\*Net: A Scalable Path-based Reasoning Approach for Knowledge Graphs. NeurIPS 2023.

# Use Case: Beam Search of Generation<sup>[1]</sup>



[1] Chence Shi, Minkai Xu, Hongyu Guo, Ming Zhang, Jian Tang. A Graph to Graphs Framework for Retrosynthesis Prediction. ICML 2020.

# Use Case: On-the-fly Graph Construction<sup>[1]</sup>



[1] Zuobai Zhang, Minghao Xu, Arian Jamasb, Vijil Chenthamarakshan, Aurelie Lozano, Payel Das, Jian Tang.  
Protein Representation Learning by Geometric Structure Pretraining. ICLR 2023.



**What** is the impact of our works?

**What** is the future for reasoning and generalization?

Direct impact: Accelerating **the transition  
from transductive models to inductive ones**

Lesson: Models with **inductive biases inspired by symbolic algorithms generalize better**

Belief: Many reasoning problems can be  
**unified**

# Inductive Generalization on Text

Train

What is the answer to  $1 + 1 + 1 - 1 - 1$ ?



Test

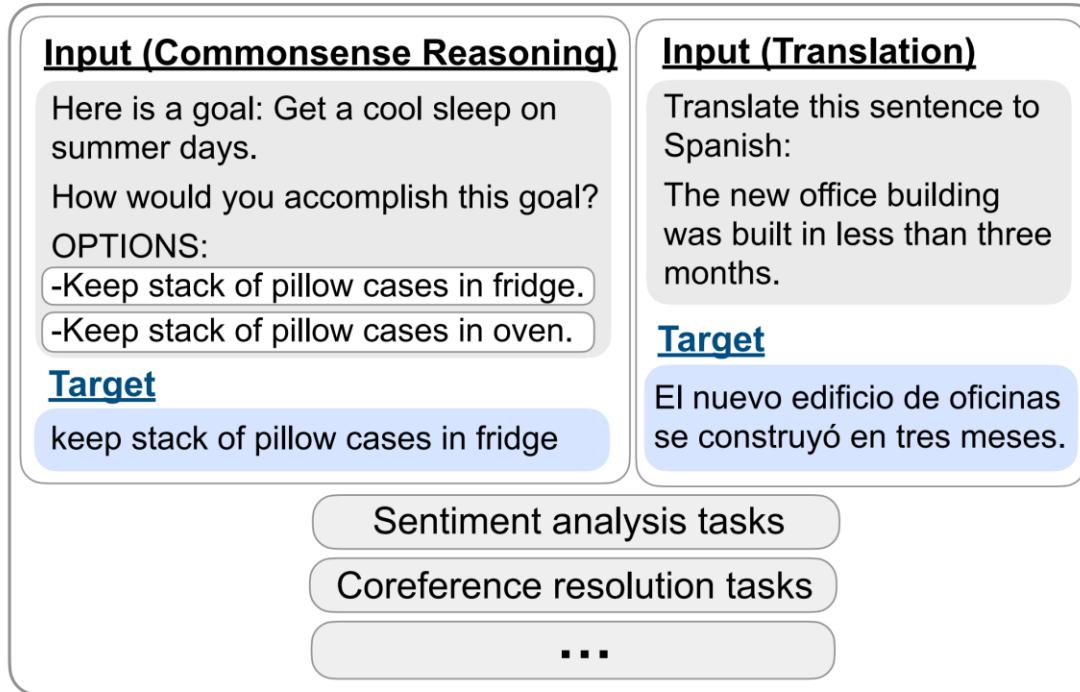


What is my son's son's son's father's father?

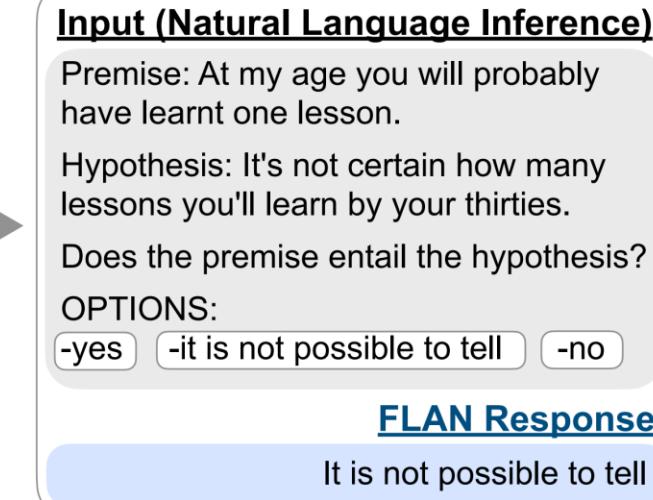


# The De Facto Approach: Instruction Tuning

Finetune on many tasks (“instruction-tuning”)



Inference on unseen task type

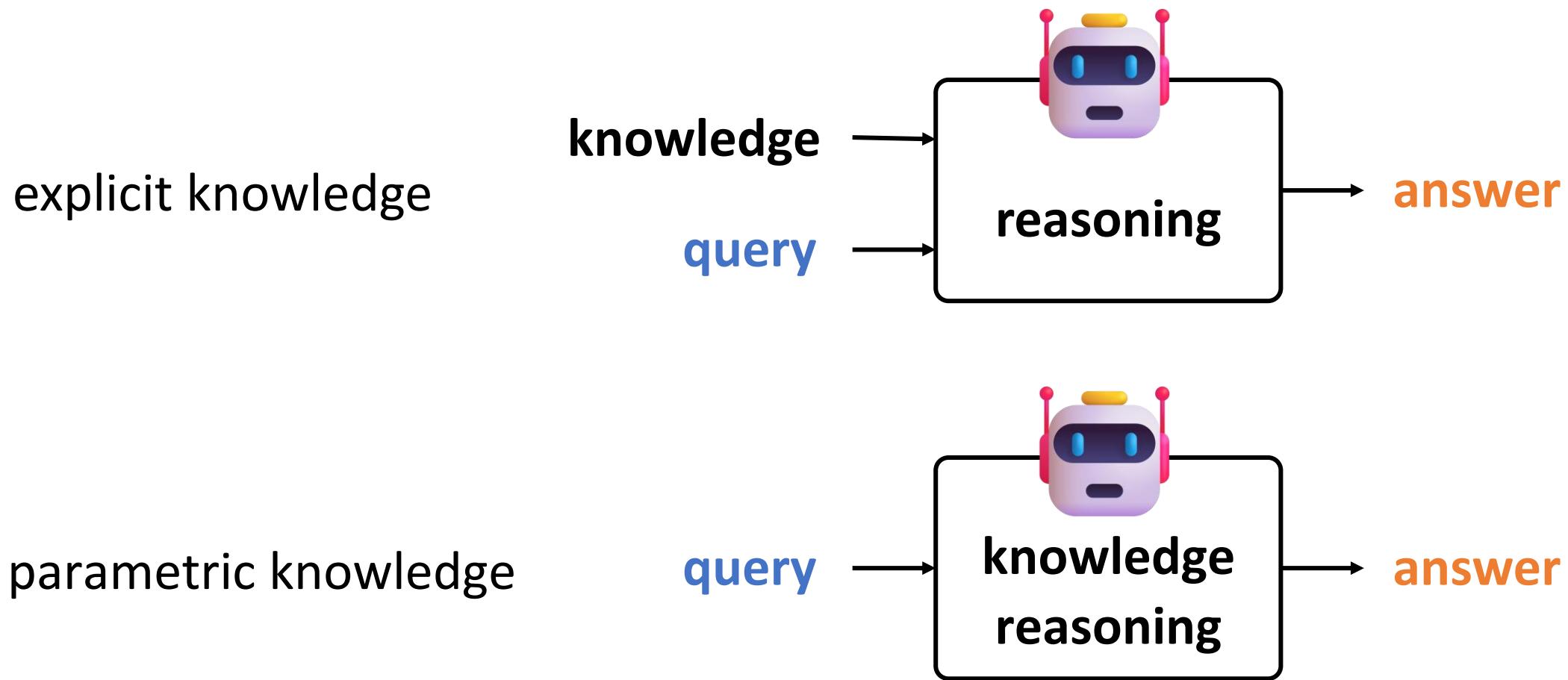


implicitly perform inductive generalization

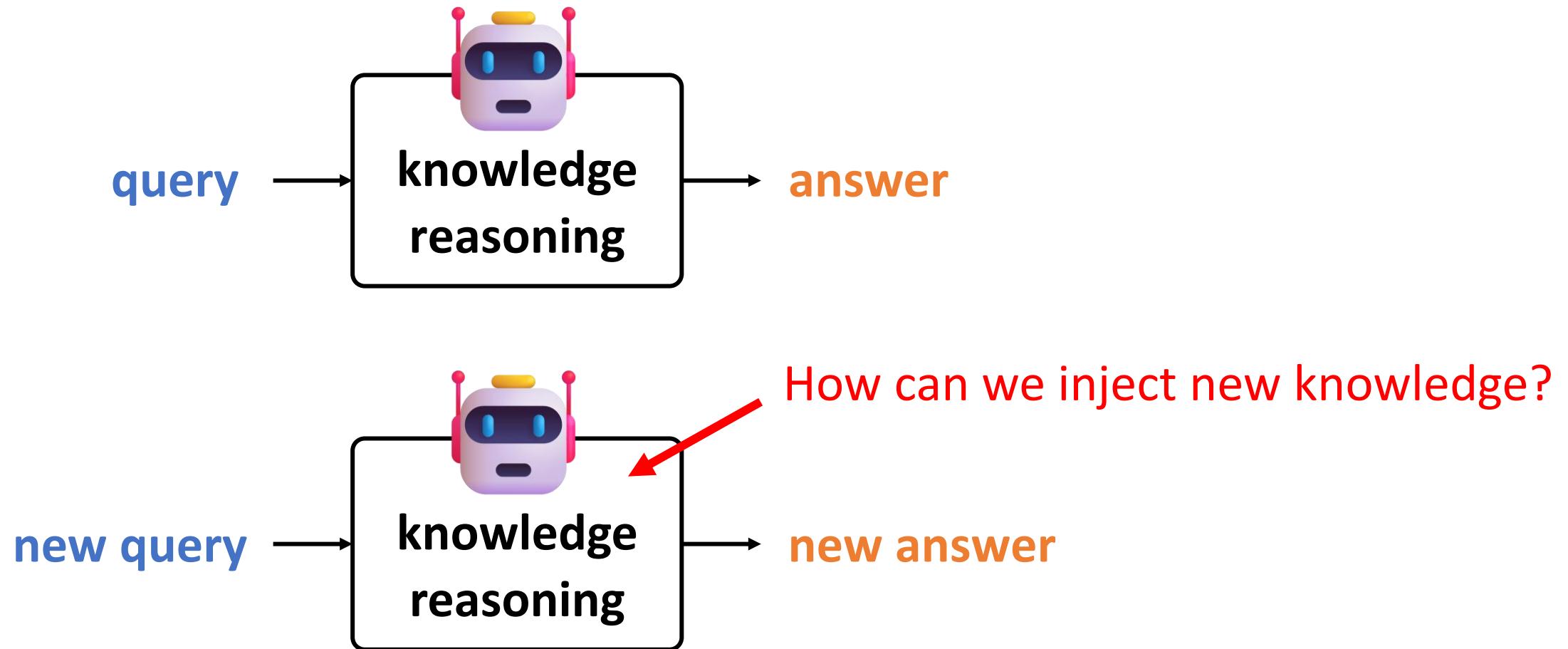
[1] Jason Wei, et al. Finetuned Language Models Are Zero-Shot Learners. ICLR 2022.

Conclusions: 5 / 10

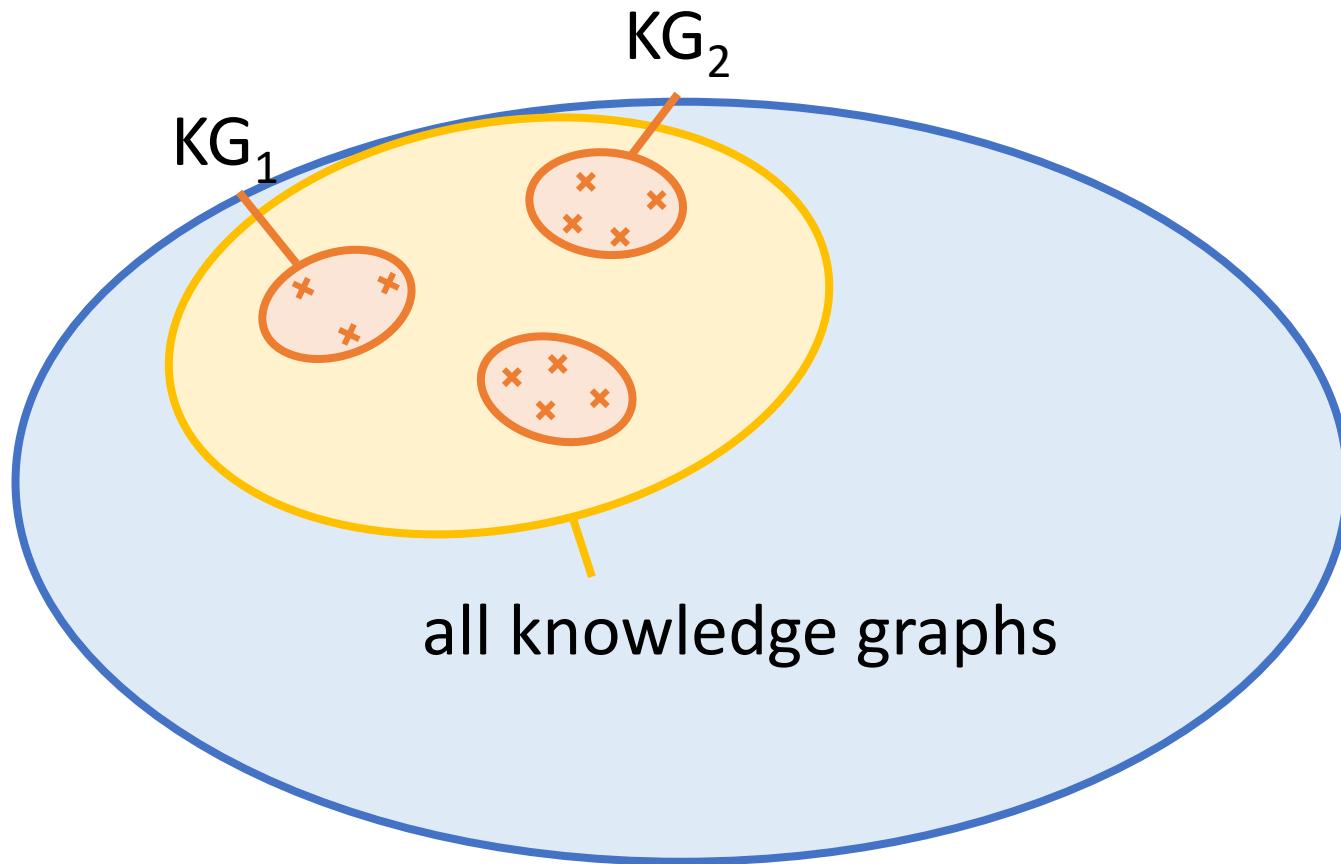
# Dealing with Parametric Knowledge



# Dealing with Parametric Knowledge

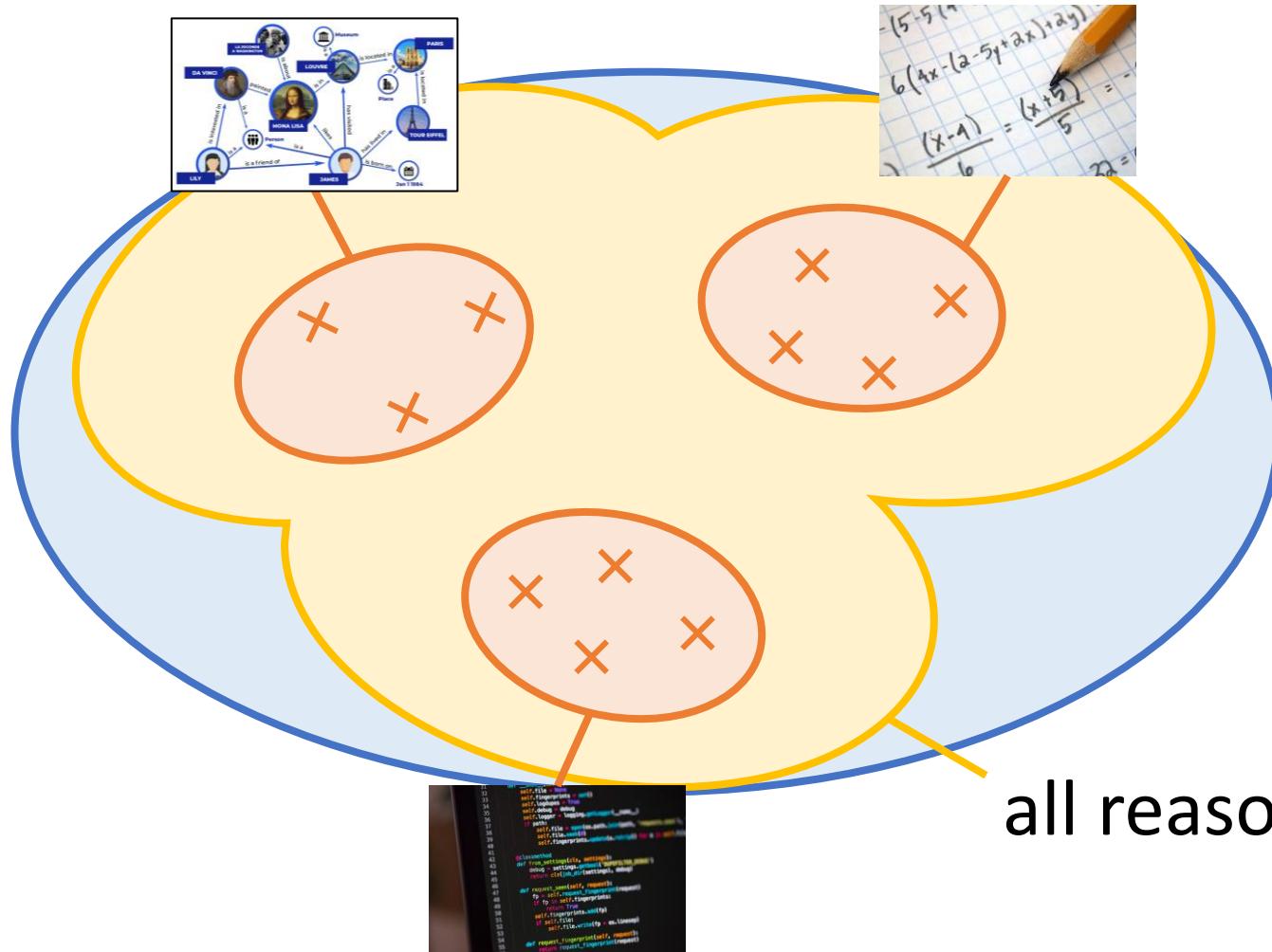


# Expand the Scope of Generalization



unify knowledge graphs

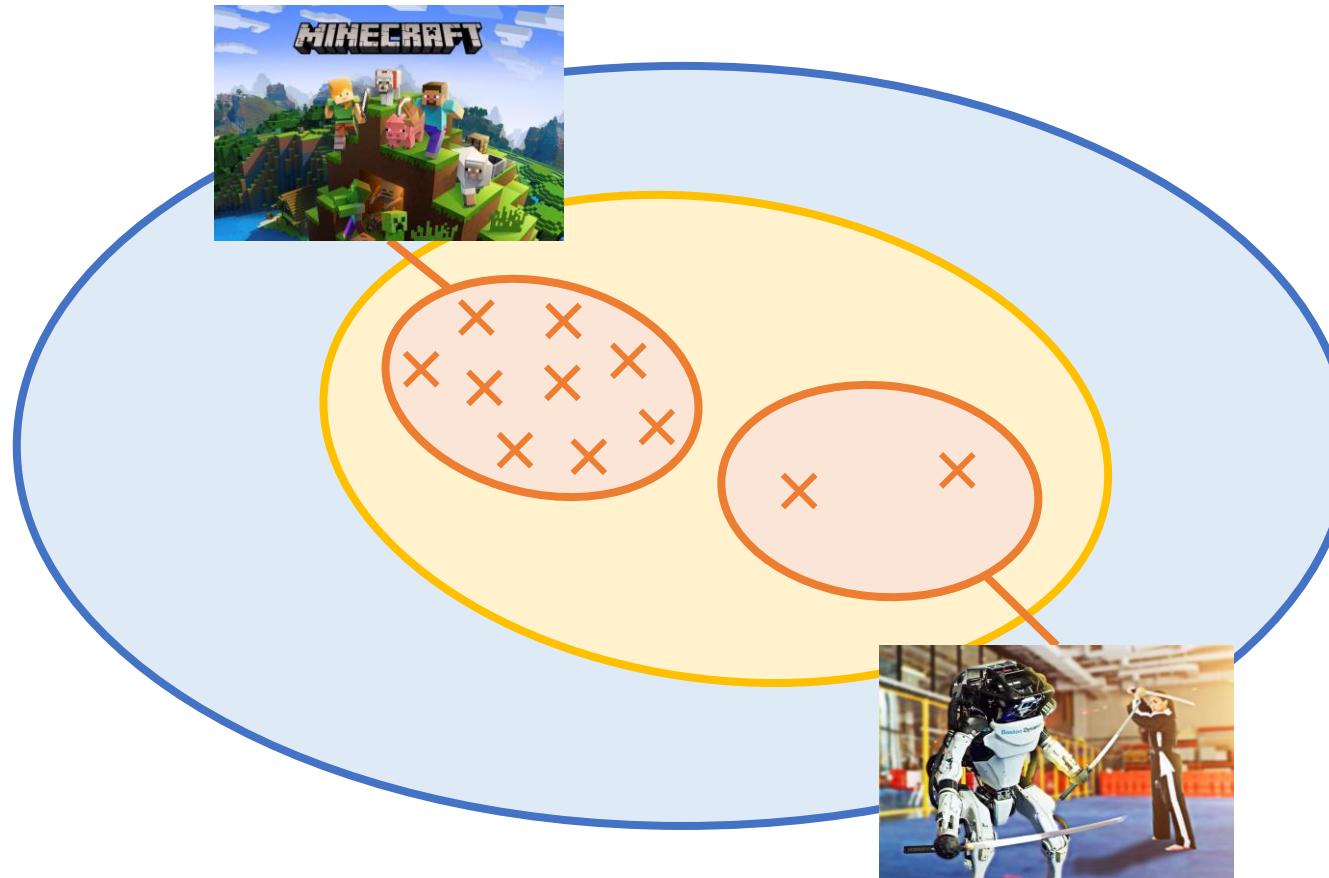
# Expand the Scope of Generalization



unify reasoning tasks

all reasoning tasks

# From Simulators to the Real World



save cost for data collection



Thank you! 