Research on Combined Dynamic Traffic Assignment and Signal Control[1)]

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Abstract This paper presents a generalized bi-level programming model of combined dynamic traffic assignment and traffic signal control, and especially analyzes a procedure for determining the equilibrium queuing delays on saturated links for dynamic network signal control satisfying the FIFO (first-in-first-out) rule. The chaotic optimal algorithm proposed in this paper can not only present the optimal signal settings, but also calculate, at each interval, the link inflow rates and outflow rates for the dynamic user optimal problem, and provide real-time information for the travelers. Finally, a numerical example is given to illustrate the application of the proposed model and solution algorithm, and comparison shows that this model has better system performance.

Key words Dynamic user optimal, traffic optimal signal settings, chaotic optimal algorithm

1 Introduction

Dynamic traffic assignment (DTA) theory is one of the most crucial technologies in intelligent transportation systems (ITS), its theory and method have been obtained record-breaking attention in the world. Advanced traffic control system (ATCS), as one of the key subsystems of ITS, utilizes the real-time traffic information to optimize signal system and curve control and uses alterable information to guide traffic flow. Its success depends on the advanced traffic control model to determine the optimal traffic tactics with real-time data. Therefore, it is necessary to develop advanced model and algorithm to combine the DTA and traffic signal control.

Based on the close relation of traffic signal settings and network traffic flow distribution, Allsop first put forward the definition of equilibrium network traffic signal settings, which took account of traveler route choices when optimizing the signal settings[1]. Yang and Yager further formulated the traffic assignment and signal control problem in saturated road networks as a bi-level model[2]. Gao and Song formulated a traffic assignment/signal settings model of urban traffic network design problem which had better achievement at theory and application[3].

However, all the researches above are based on static traffic assignment. Since the traffic signal control system requires real-time route information, some researchers have considered the real-time traffic responsive signal control problem. Dion and Hellinga described the development and evaluation of a fully distributed, real-time, traffic-responsive model named signal priority procedure for optimization in real-time that explicitly considers the impact of transit vehicles[4]. Yang and Miller-Hooks addressed the problem of determining optimal routing decisions in signalized traffic networks, where arc travel times vary over time and are known only probabilistically[5]. Chang and Sun proposed a dynamic method to control an oversaturated traffic signal network by utilizing a bang-bang-like model for the oversaturated intersections and TRANSYT-7F for the undersaturated intersections[6].

In this study, considering the urban network traffic control during the peak period, we develop a generalized bi-level programming model on combined dynamic traffic assignment and traffic signal control to obtain the real-time traffic responsive signal control tactics. Taking account of the drivers′ route choice behavior in response to signal settings changes, the upper level problem determines signal settings when minimizing the total system cost (parking times, oil cost, *etc.*) and maximizing the total network capacity simultaneously, while the lower level problem represents an urban dynamic user optimal variational inequality problem, which predicts how drivers react to any given signal control

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pattern in real-time and satisfies the FIFO (first-in-first-out) rule. Furthermore, our study especially analyzes a procedure for determining the equilibrium queuing delay on saturated links during the peak period for dynamic network signal control. For solution, a chaotic optimal algorithm is developed for our model, which need not expand the time-space network and could be carried out on the original network. Finally, we set up a numerical example to evaluate the applications of the proposed model and solution algorithm. The results show that our model can not only give the real-time optimal signal settings for the traffic network system control, but also provide real-time travel information for the network users such as, at each interval, the link inflow rates, the link outflow rates, link flow and queuing delays following the dynamic user optimal principle precisely.

2 Link travel time function and exit function analysis

2.1 Notations and constraints

In a multi-destination and strong connected urban network G, N is defined as the set of nodes; r is one of the origin nodes, s is one of the destination nodes; $a=(l,m)$ is the link formed by nodes l and m; A is the set of arcs (links); A_l is the set of links leading to node l; B_l is the set of links leaving node l. Consider the fixed time period $[0,T]$, which is long enough to allow all travelers departing during the peak period to complete their trips. For $\forall t\in[0,T]$, the variables are defined as follows.

$u_a(t)$ the inflow rate on link a at time t; $u_a^s(t)$ is the inflow rate on link a to destination s at time t.

$v_a(t)$ the outflow rate on link a at time t; $v_a^s(t)$ is the outflow rate on link a to destination s at time t.

$x_a(t)$ the flow on link a at time t (state variable); $x_a^s(t)$ is the flow on link a to destination s at time t (state variable).

$q_l^s(t)$ the flow rate generated at node l to destination s at time t (given).

$c_a(t)$ the instantaneous travel time on link a at time t.

$\tau_a(t)$ the actual travel time on link a at time t.

$d_a(t)$ the queuing delay on link a at time t.

$\lambda_a(t)$ the green time ratio on link a at time t; $\boldsymbol{\lambda}(t)$ denotes the vector of $\lambda_a(t)$.

2.2 Link travel time and queuing delay function

In our study, the link travel time function $c_a(t)$ is decomposed into three components, *i.e.*,

$$c_a(t)=c_a^0+c_a'(t)+d_a(t) \tag{1}$$

where c_a^0 denotes the free flow travel time on link a, and $c_a'(x_a(t),u_a(t),v_a(t),\lambda(t+c_a^0))$ denotes the signal delay (or the general cost including parking times, oil cost, *etc.*) function which is determined by a formula developed from local traffic conditions[2]. We combine these two components with $c_a^1(t)$ (*e.g.*, the well-known FHWA travel time function[7]). $d_a(t)$ denotes the queuing delay due to link capacity on the saturated link which, in our study, is determined by the dynamic network user optimal conditions.

Considering an urban traffic network with signal control, the link capacity is $\lambda_a(t)s_a$ (the flow rate capacity), s_a is the saturated flow rate of link a, then we have the following relations[2]

$$v_a(t)\leqslant\lambda_a(t)s_a,\quad\forall t \tag{2}$$

As is known, in a saturated road network, the queuing delay is apt to occur. However, this queuing delay is different from the signal delay because the former is due to the limited capacity and should be determined from network equilibrium conditions. Here we discuss the calculation of the queuing delay $d_a(t)$ due to the limited link capacity (2) on a saturated link particularly.

Although the dynamical system provides a continuous adjustment process, a discrete time process is required for actual computational purpose. We discretize the time period $[0,T]$ into K small time intervals, *i.e.*, $k=1,2,3,\cdots,K$, each interval is a unit of time.

For the instantaneous link travel time function, the delay formulation is given as[8]

$$d_a(k)=\frac{l_a(k)}{\lambda_a(k)s_a},\quad\forall a,\forall k \tag{3}$$

where $l_a(k)$ is the number of vehicles queuing on link a during interval k.

For the continuous time variables i and t, the vehicles entering link a at time i will arrive at the queue at time $t = i + c_a^1(i)$. Thus, the state equation for the queue on link a is[8]

$$\frac{\mathrm{d}l_a(t)}{\mathrm{d}t} = u_a(i) - v_a(t), \quad \forall a, \forall t \tag{4}$$

After discretization, (4) can be transformed into

$$l_a(k) - l_a(k-1) = \sum_{i^0 \leqslant i \leqslant i^1} u_a(i) - v_a(k), \quad \forall a, \forall k \tag{5}$$

where, for some k, let i^0 and i^1 be the smallest index and the largest index respectively, which satisfy the equation $i + \bar{c}_a^1(i) = k$. For every interval i, $\bar{c}_a^1(i)$ are modified in the following round-off method

$$\bar{c}_a^1(i) = n, \text{ if } n - 0.5 \leqslant c_a^1(i) < n + 0.5, \quad \forall a \tag{6}$$

where n is an integer and $0 \leqslant n \leqslant K$. Here this formulation satisfies the FIFO condition[9].

In this way, the queuing vehicle number can be given by

$$l_a(k) = l_a(k-1) + \sum_{i^0 \leqslant i \leqslant i^1} u_a(i) - v_a(k), \quad \forall a, \forall k \tag{7}$$

Both sides of (7) are divided by $\lambda_a(k)s_a$, then with (3) the resulting queuing delay function becomes

$$d_a(k) = \frac{l_a(k-1)}{\lambda_a(k)s_a} + \frac{\sum\limits_{i^0 \leqslant i \leqslant i^1} u_a(i) - v_a(k)}{\lambda_a(k)s_a}, \quad \forall a, \forall k \tag{8}$$

2.3 Exit function

When a vehicle arrives at the exit queue of link a at interval k, it has to wait if there is a queue $l_a(k) > 0$ and the link outflow rate at interval k is then at its capacity $\lambda_a(k)s_a$. On the other hand, if the arrival rate to the exit queue at interval k, *i.e.*, $\sum\limits_{i^0 \leqslant i \leqslant i^1} u_a(i)(i + \bar{c}_a^1(i) = k)$ is higher than the capacity, then the link outflow rate is $\lambda_a(k)s_a$ since the departure rate can not exceed the exit capacity. The vehicles will pass the exit without delay if there is no queue and the flow rate is less than the exit capacity[8].

Thereupon, if $l_a(k) > 0$, we have $v_a(k) = \lambda_a(k)s_a$. Together with (7), it follows that

$$l_a(k-1) + \sum_{i^0 \leqslant i \leqslant i^1} u_a(i) - \lambda_a(k)s_a > 0 \tag{9}$$

which is equivalent to

$$l_a(k-1) + \sum_{i^0 \leqslant i \leqslant i^1} u_a(i) > \lambda_a(k)s_a \tag{10}$$

And if $\sum_{i^0 \leqslant i \leqslant i^1} u_a(i) \geqslant \lambda_a(k)s_a$, the inequality above can also be satisfied.

If $l_a(k) = 0$, with (7) it follows that

$$v_a(k) = l_a(k-1) + \sum_{i^0 \leqslant i \leqslant i^1} u_a(i) \tag{11}$$

To sum up, the outflow rate function is stated as

$$v_a(k) = \begin{cases} \lambda_a(k)s_a, & \text{if } \sum\limits_{i^0 \leqslant i \leqslant i^1} u_a(i) + l_a(k-1) > \lambda_a(k)s_a \\ \sum\limits_{i^0 \leqslant i \leqslant i^1} u_a(i) + l_a(k-1), & \text{otherwise} \end{cases}, \ \forall a, \forall k \tag{12}$$

where i satisfies the equation $k = i + \bar{c}_a^1(i)$.

From the above, we know that $v_a(k)$ is a function of $u_a(i)$, $l_a(k-1)$ and $\lambda_a(k)$. $l_a(k-1)$ can be obtained from the data of interval $k-1$. Therefore, if $\lambda_a(k)$ is given, $v_a(k)$ also can be determined by the data of previous intervals. Consequently, the queuing delay $d_a(k)$ according to (8) can be calculated based on the data of previous intervals.

2.4 Multi-destination exit function

For the inflow rate and outflow rate to different destinations s on link a at time t, the following equation must hold in order to implement the FIFO rule[10~12]

$$v_a^s(t+\tau_a(t)) = \frac{u_a^s(t)}{\mathrm{d}\tau_a(t)/\mathrm{d}t+1}, \quad \forall t \tag{13}$$

It is easy to see that summing (13) over s gives

$$v_a(t+\tau_a(t)) = \frac{u_a(t)}{\mathrm{d}\tau_a(t)/\mathrm{d}t+1}, \quad \forall t \tag{14}$$

Noting that $v_a(t+c_a(t))$ must be positive, it is true to have

$$\mathrm{d}\tau_a(t)/\mathrm{d}t > -1, \quad \forall t \tag{15}$$

However, this inequality is equivalent to

$$t+\tau_a(t) > t-\varepsilon+\tau_a(t-\varepsilon) \text{ for small } \varepsilon>0, \ \forall t \tag{16}$$

which implies that the vehicle first entering link a must first depart the link, *i.e.*, the FIFO rule.

Comparing (13) and (14), we have

$$\frac{u_a^s(t)}{u_a(t)} = \frac{v_a^s(t+\tau_a(t))}{v_a(t+\tau_a(t))}, \quad \forall t \tag{17}$$

According to the discrete FIFO conditions[9], the out flow rate for the destination s from link a during interval k can be calculated by the following equation

$$v_a^s(k) = v_a(k)\frac{\sum\limits_{i^0\leqslant i\leqslant i^1} u_a^s(i)}{\sum\limits_{i^0\leqslant i\leqslant i^1} u_a(i)}, \quad \forall k \tag{18}$$

where i satisfies the equation $k=i+\bar{\tau}_a(i)$.

From the results (12) together with (18), we conclude that the outflow rate to different destinations s on link a during interval k can be obtained from the data of previous intervals i, given the estimated value of $\tau_a(t)(\forall t)$[11].

3 Model of combined dynamic traffic assignment and traffic signal control

3.1 Dynamic user optimal assignment model

We give the following definition of instantaneous dynamic user optimal state[11].

Definition. If, for each decisive node and each destination, at any instant of time, the instantaneous travel times for all routes being used equal to the minimal instantaneous route travel cost, the dynamic traffic flow over the network is in a travel-time-based instantaneous DUO state.

From the above definition, we have the following relationships

$$u_a(k) = \sum_s u_a^s(k), \quad \forall a, \forall k \tag{19}$$

$$v_a(k) = \sum_s v_a^s(k), \quad \forall a, \forall k \tag{20}$$

Then we give the related discrete constraints on link $a=(l,m)$, such as the flow conservation constraint

$$\sum_{a\in A_l} v_a^s(k) + q_l^s(k) = \sum_{a\in B_l} u_a^s(k), \quad \forall l\neq s, \forall s, \forall k \tag{21}$$

Note. If l is an origin node, $\sum_{a\in A_l} v_a^s(k)$, $\forall s$ can be viewed as zero.

The outflow rate satisfies

$$v_a(k)=\begin{cases}\lambda_a(k)s_a, & \text{if } \sum\limits_{i^0\leqslant i\leqslant i^1} u_a(i)+l_a(k-1)>\lambda_a(k)s_a \\ \sum\limits_{i^0\leqslant i\leqslant i^i} u_a(i)+l_a(k-1), & \text{otherwise}\end{cases}, \ \forall a, \forall k \tag{22}$$

where l satisfies the equation $k=i+\bar{c}_a^1(i)$.

The discrete state equation is

$$x_a(k+1)=x_a(k)+u_a(k)-v_a(k), \quad \forall a, \forall k \tag{23}$$

All variables must be nonnegative all the time

$$x_a(k)\geqslant 0, \ u_a(k)\geqslant 0, \ v_a(k)\geqslant 0, \ \forall a, \forall k \tag{24}$$

The initial values at time $t=0$ are assumed to be zero

$$x_a(0)=0, \quad v_a(0)=0, \quad \forall a \tag{25}$$

From (12) and these constraints, we can see that if $\lambda_a(k)$ is given, $x_a(k)$ and $v_a(k)$ can be obtained by using the data of the previous intervals. Thus, during interval k, $x_a^s(k)$ and $v_a^s(k)$ are given, and $u_a(k)$ are the only nonnegative variables.

Here, the symbol Ω denotes the set of flows that satisfy constraints (21)~(25). The equivalent variational inequality of the IDUO state may be stated as follows.

Theorem. The dynamic traffic flow pattern in the feasible flow set Ω is in an instantaneous DUO route choice state if and only if it satisfies the following inequality problem (VIP)

$$\sum_s\sum_a[\eta_m^{s^*}(k)+c_a^*(k)-\eta_l^{s^*}(k)][u_a^s(k)-u_a^{s^*}(k)]\geqslant 0, \quad \forall u_a^s(t)\in\Omega, \ \forall k \tag{26}$$

where $c_a^*(k)$ is the link instantaneous travel cost corresponding to link $a=(l,m)$; $\eta_l^{s^*}(k)$ and $\eta_m^{s^*}(k)$ are defined as the minimal instantaneous route travel cost, at time k, from nodes l and m to destination s, respectively.

Proof. Refer to [9].

In this study, the users are assumed to choose their paths by reactive or instantaneous travel time. Supposing route p consists of links $a_1, a_2, \cdots, a_n$, the instantaneous travel cost on route p is given by

$$\xi_P(t)=c_{a_1}(t)+c_{a_2}(t)+\cdots+c_{a_n}(t), \quad \forall t \tag{27}$$

Then, the time interval during which a vehicle arrives at the entry of one link can be determined by summing up all the traveled links′ time. In model (26), $\eta_l^{s^*}(k)=\min\limits_a\{c_a^*(k)+\eta_m^{s^*}(k)\}$; similarly, $\eta_m^{s^*}(k)$ can be obtained by calculating the shortest route successively based on the link instantaneous travel cost from node m.

3.2 Dynamic network signal control model

To avoid the queue on oversaturated link prolonging to the upstream intersection, the queuing vehicle number ought to be restricted[2] as follows

$$l_a(k)\leqslant Q_a^{\max}, \quad \forall a\in A, \ \forall k \tag{28}$$

where $Q_a^{\max}$ is the maximum storage capacity (vehicles) on link a.

From the signal settings knowledge, we also have

$$\lambda_a^{\min}\leqslant\lambda_a(k)\leqslant\lambda_a^{\max}, \quad \forall a\in A, \ \forall k \tag{29}$$

where $\lambda_a^{\min}$ and $\lambda_a^{\max}$ denote the minimum and maximum green time ratios on link a, respectively.

Here, we suppose the intersection without signal control has only one green phrase. Then, each link at this intersection is provided with only one phrase and always has green time. In this way, all the intersections in a traffic network may be regard as those with signal control.

Let $\Lambda_{m,i}(k)$ be the green time ratio at intersection m during phrase i and interval k. The total green time ratio on link a equals to the sum of green time ratio during all the phrases on link a[13]. Then we have

$$\lambda_a(k) = \sum_m \sum_i \delta^a_{m,i} \cdot \Lambda_{m,i}(k), \quad \forall a \in A, \forall k \tag{30}$$

If link a is at intersection m during phrase i, $\delta^a_{m,i} = 1$; otherwise, $\delta^a_{m,i} = 0$. Moreover, for every intersection m, there is

$$\sum_i \Lambda_{m,i}(k) = 1 - a_m, \quad \forall m, \forall k \tag{31}$$

where α_m denotes the total loss time of one cycle length at intersection m.

The symbol Θ denotes the feasible region that satisfies constraints (28)~(31).

Considering the constraints above, we formulate an urban dynamic network signal control model to obtain the optimal signal settings tactics. In this model, in response to the current approaching flow pattern, the system optimal principle is adopted to minimize the total system cost (parking times, oil cost, *etc*) and maximize the network traffic capacity, which is the following optimization problem

$$\min_{\boldsymbol{\lambda}} \sum_k \sum_{a\in A} u_a(k)(c^1_a(k) + d_a(k)) - \theta \sum_k \sum_{a\in A} \lambda_a(k) s_a, \quad \forall \lambda_a(k) \in \Theta \tag{32}$$

where the second item is negative to maximize the total network traffic capacity and θ is the conversion coefficient according to the traffic authority′s different programming preference.

3.3 A generalized bi-level programming model on combined dynamic traffic assignment and traffic signal control

With the above discussion, we formulate a generalized bi-level programming model. Formulation (32) is the upper level problem which is a dynamic signal control model to minimize the total system cost (parking times, oil cost, *etc*) and maximize the total network traffic capacity considering the traveler route choice; while the urban dynamic user optimal variational inequality problem (26) is the lower level problem, which can describe the network users′ route choices reacting to the given signal control tactics in real-time.

4 Solution method

Due to the complexity of the dynamic network bi-level problem, we develop a chaotic optimal algorithm to solve the generalized bi-level model. The chaotic search method is used to solve the upper level problem. Since chaotic states have ergodicity in a certain area and the chaotic search behavior does not depend on the properties of objective functions and constraints such as the differentiability and continuity, it seems that the chaos search method performs better than the traditional ones[14]. For the lower level VI problem, we adopt the Frank-Wolfe method to complete DTA. Here, a discrete time process is still needed and the detailed solution algorithm is presented as follows.

Step 1. Initialization. Let $n = 1$, and the optimal value.

Step 2. Carrier wave. Generate the chaotic variable $\boldsymbol{y}(k)^n$ of $\boldsymbol{\lambda}(k)$ by the following Logistic mapping:

$$\boldsymbol{y}^n = 4\boldsymbol{y}^{n-1}(1 - \boldsymbol{y}^{n-1}), \quad n = 1, 2, \ldots, \quad \boldsymbol{y}^0 \in [0, 1]$$

Then adjust the chaotic variables $\boldsymbol{y}(k)^n$ to the upper level feasible region and obtain the upper level variable $\boldsymbol{\lambda}(k)^n$.

Step 3. Solving the low level problem. To find the solution to the lower level VI problem, we use the Frank-Wolfe method to solve its equivalent NLP[9]

$$\min_{\boldsymbol{u}^s} Z(\boldsymbol{u}(t)) = \sum_a \int_0^{u_a(k)} c_a(w)\mathrm{d}w + \sum_a \sum_s u^s_a(k)[\bar{\eta}^s_m(k) - \bar{\eta}^s_l(k)]$$

Fix the upper level variable $\boldsymbol{\lambda}(k)^n$ and solve the lower level DTA problem, in particular:

Step 3.1. Let $k = 1$.

Step 3.1.1. Initialization. Select a initial feasible link flow pattern $\{u_a^0(k)\}$ based on the network constructed by the estimated actual link travel time $\tau_a(t)(\forall a)$. Let $i=0$.

Step 3.1.2. Finding descend direction. When using the shortest route algorithm to find the feasible solution $\boldsymbol{p}^s(k)$ for linear sub-problem, every link travel cost is corresponding to $\dfrac{\partial Z(\boldsymbol{u})}{\partial u_a^s(k)} = c_a[u_a(k)] + \bar{\eta}_m^s(k) - \bar{\eta}_l^s(k)$, where $\bar{\eta}_l^s(k)$ and $\bar{\eta}_m^s(k)$ are the minimal instantaneous travel cost fixed temporarily according to (6) from nodes l and m to destination s, respectively. After loading the inflows $\sum\limits_{a\in B_l} u_a^s(k)(\forall l \neq s, s)$ (If $k=0$, they are $q_l^s(0)(\forall l \neq s, s)$ to network with the "all-or-nothing" method, we get the feasible inflows $\{p_a^{s^i}(k)\}$ and the feasible directions $\{p_a^{s^i}(k) - u_a^{s^i}(k)\}$.

Step 3.1.3. Determining move size α^i. Solve the following one-dimensional problem

$$\min_{0\leqslant\alpha\leqslant 1} Z = \sum_a \int_0^{u_a^i(k)+\alpha[p_a^i(k)-u_a^i(k)]} c_a(w)\mathrm{d}w + \sum_a\sum_s \{u_a^{s^i}(k) + \alpha[p_a^{s^i}(k) - u_a^{s^i}(k)]\}[\bar{\eta}_m^s(k) - \bar{\eta}_l^s(k)]$$

we get step α^i.

Step 3.1.4. Updating inflow rates.

Calculate $u_a^{s^{i+1}}(k) = u_a^{s^i}(k) + \alpha^i[p_a^{s^i}(k) - u_a^{s^i}(k)], \forall a, s$; then we obtain $\{u_a^{i+1}(k)\}$ with (19).

Step 3.1.5. Convergence test. If $\|\boldsymbol{u}^i(k) - \boldsymbol{u}^{i-1}(k)\| \leqslant \varepsilon$ (ε is the given convergence criterion), obtain $\{u_a(k)\}$ and go to Step 3.2; otherwise, set $i=i+1$ and go to Step 3.1.2.

Step 3.2. If $k=K$, go to Step 4; otherwise, set $k=k+1$ and go to Step 3.1.1.

Step 4. Calculate the value of the upper objective function Z^n, and compare it with the current optimal value Z^*: if $Z^n < Z^*$, let $Z^* = Z^n$; otherwise, give up the solution, set $n=n+1$ and go to Step 2. Loop Steps 2~4 until Z^* does not improve after a number of searching steps.

5 Numerical example

5.1 Input data

A simple network shown in Fig. 1 is used for testing. The test network consists of 9 nodes, 16 links and 12 routes, in which there are two O-D $2\rightarrow 8$ and $4\rightarrow 6$. We further assume node 5 is two-phase signalized intersection on which the east-west direction is designated as the first phase and the north-south direction as the second phase, and $0.10 \leqslant \lambda_i \leqslant 0.80$ $(\forall i)$. On the other nodes, no signal devices are assumed. The saturation flow rate is $s_a = 50$ $(\forall a)$ and free flow link travel time $c_a^0 = 1$ $(\forall a)$, the coefficients are $\beta = 0.8$ and $\theta = 0.05$. The link travel time function $c_a^1(t)$ is assumed as the dynamic transformation of BPR function, that is,

$$c_a^1(t) = c_a^0\left(1 + \beta\left(\frac{u_a(t)}{\lambda_a(t+c_a^0)s_a}\right)^2\right)$$

Traffic demand for origin 2 to destination 8 are 45 during interval $k=1$ and 30 during interval $k=2$, no demand for other intervals, and the same for origin 4 to destination 6. We load these demands on an empty traffic network shown in Fig. 1.

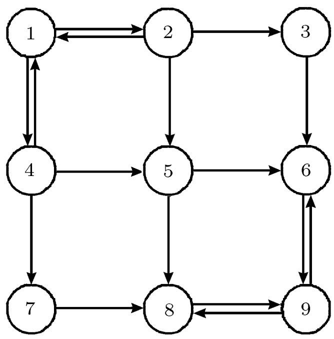

Fig. 1 Test network

5.2 Test results

The resulting flow pattern is summarized in Table 1. The instantaneous route travel time and the instantaneous queuing delay are also computed and shown in Table 2 and Table 4, respectively. The computed green time ratios associated with each phase are shown in Table 3.

From the tables above, we can verify that the used route/s always has/have the minimal route travel time whereas the unused ones have equal or higher route travel times, and the real-time traffic responsive signal control can also be obtained considering the queuing effects.

Table 1 Results for the model timing plan

Objective value=379.03						
Link	Entering interval	Inflow	Exit flow	Number of vehicle	Instantaneous link travel time	Exiting interval
$2 \to 5$	1	45	0	0	1.8816	3,4
	2	30	0	45	1.7319	4,5
	3	0	16.532	75	2.722	–
	4	0	35.921	58.468	1.6277	–
	5	0	22.547	22.547	1.0000	–
$2 \to 3$	3	20.447	0	0	1.1338	4
	4	4.0646	20.447	20.447	1.0053	5
	5	0	4.0646	4.0646	1.0000	–
$3 \to 6$	4	20.447	0	0	1.1338	5
	5	4.0646	20.447	20.447	1.0053	6
	6	0	4.0646	4.0646	1.0000	–
$4 \to 1$	1	20.447	0	0	1.1338	2
	2	4.0646	20.447	20.447	1.0053	3
	3	0	4.0646	4.0646	1.0000	–
$1 \to 2$	2	20.447	0	0	1.1338	3
	3	4.0646	20.447	20.447	1.0053	4
	4	0	4.0646	4.0646	1.0000	–
$4 \to 5$	1	3.5947	0	0	3.2743	4
	2	21.566	0	3.5947	3.0009	5,6,7
	3	0	0	25.1603	1.0000	–
	4	0	3.5947	25.1603	1.0000	–
	5	0	16.7692	1.5656	1.2861	–
	6	0	1.4226	4.7971	3.372	–
	7	0	3.3744	3.3744	1.0000	–
$4 \to 7$	1	20.959	0	0	1.1338	2
	2	4.3698	20.959	20.959	1.0053	3
	3	0	4.3698	4.3698	1.0000	–
$7 \to 8$	2	20.959	0	0	1.1338	3
	3	4.3698	20.959	20.959	1.0053	4
	4	0	4.3698	4.3698	1.0000	–
$5 \to 6$	4	3.5947	0	0	1.0041	5
	5	16.769	3.5947	3.5947	1.0900	6
	6	1.4226	16.769	16.7686	1.0006	7
	7	3.3744	1.4226	1.4226	1.0036	8
	8	0	3.3744	3.3744	1.0000	–
$5 \to 8$	3	16.532	0	0	1.0875	4
	4	35.921	16.532	16.532	1.4129	5
	5	22.547	35.921	35.921	1.1627	6
	6	0	22.547	22.547	1.0000	–
$8 \to 9$	3	20.959	0	0	1.1406	4
	4	4.3698	20.959	20.959	1.0061	5
	5	0	4.3698	4.3698	1.0000	–
$9 \to 6$	4	20.959	0	0	1.1406	5
	5	4.3698	20.959	20.959	1.0061	6
	6	0	4.3698	4.3698	1.0000	–

Table 2 The instantaneous route travel times for the model timing plan

Departure interval	Route			
Interval	$2 \to 5 \to 8$	$4 \to 1 \to 2 \to 3 \to 6$	$4 \to 5 \to 6$	$4 \to 7 \to 8 \to 9 \to 6$
$k = 1$	2.8816	4.2676	4.2743	4.2811
$k = 2$	2.7319	4.0106	4.0009	4.0122

The routes unused are not indicated

Table 3 Green time ratio allocation for the model timing plan

Green time ratio $\lambda_a(k)$								
Time interval	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$
Phase 1	0.8574	0.6273	0.3306	0.7184	0.5646	0.8716	0.2932	0.3438
Phase 2	0.0426	0.2727	0.5694	0.1816	0.3354	0.0285	0.6068	0.55619

Table 4 The instantaneous queuing delay for the model timing plan

The instantaneous queuing delay								
Link	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$
$2 \to 5$	0	0	1.7220	0.6277	0	0	0	0
$4 \to 5$	0	0	0	0	0.2861	2.3720	0	0

A more detailed examination about performance of our model shows that the objective value and the sum of two O-D′s minimal route travel time associated with our real-time traffic responsive signal settings plan are lower than those with the fixed time signal settings plan (see Fig. 2 and Fig. 3). Moreover, when the green time ratios are evenly distributed at 0.45 the whole queuing vehicle number is 129.328, whereas our real-time signal timing plan reduces it largely, giving 59.1865. This is because our real-time traffic responsive signal timing plan assigns more green times to those links with higher exit flows.

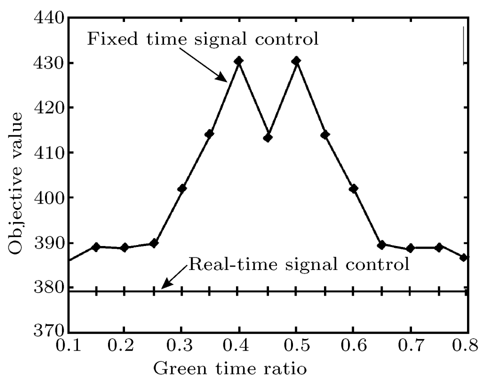

Fig. 2 Objective value comparison

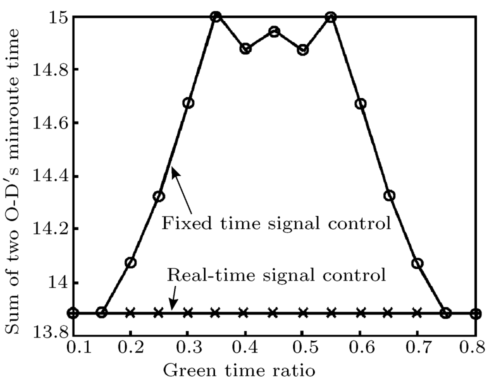

Fig. 3 Sum of two O-D′s minimal route travel time comparison

To summarize, our model is capable of providing the real-time traffic responsive signal control tactics while equilibrating traffic according to the instantaneous DUO conditions and thus better system performance than the fixed time signal control.

6 Conclusions

This paper develops a generalized bi-level programming model on combined dynamic traffic assignment and traffic signal control to obtain the real-time traffic responsive signal control tactics. Eespecially we analyze a procedure for determining the delays induced by the queuing on saturated links during the peak period. To solve the model, we employ a chaotic optimal algorithm for our model, avoiding the complexity of finding derivative information with traditional sensitive analysis method. The test example shows that our model has achieved better performance and will be valuable for the urban traffic control.

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