Homework3

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6.5 分别用带有前向检验、MRV和最少约束值启发式的回溯算法手工求解**图6.2**中的密码算数问题

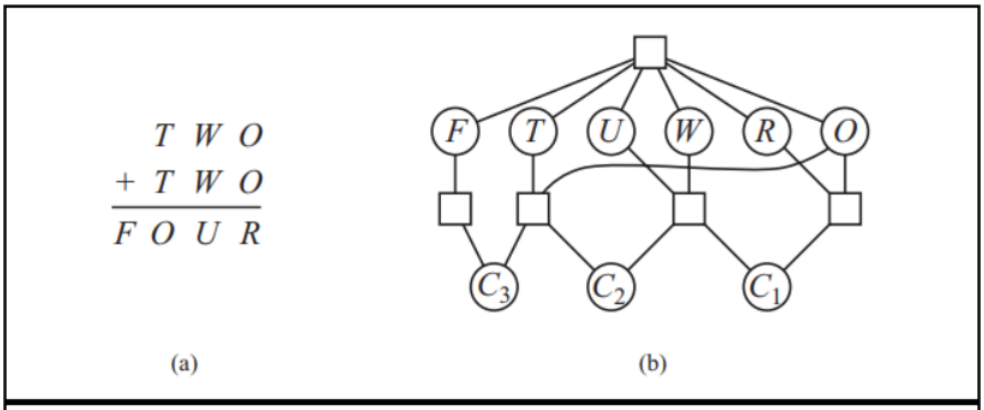

Figure 6.2 (a) A cryptarithmetic problem. Each letter stands for a distinct digit; the aim is to find a substitution of digits for letters such that the resulting sum is arithmetically correct, with the added restriction that no leading zeroes are allowed. (b) The constraint hypergraph for the cryptarithmetic problem, showing the *Alldiff* constraint (square box at the top) as well as the column addition constraints (four square boxes in the middle). The variables C_1, C_2, and C_3 represent the carry digits for the three columns.

A: 密码算数问题约束条件如下

1. $O + O = R + 10C_1$
2. $C_1 + W + W = U + 10C_2$
3. $C_2 + T + T = O + 10C_3$
4. $C_3 = F, T \neq 0, F \neq 0$
5. 不同的字母 F, T, U, W, R, O 数字各不相同

变量有：$F, T, U, W, R, O \in \{0, 1, 2\ldots, 9\}$, $C_1, C_2, C_3 \in \{0, 1\}$

采取MRV方法

	C_3	C_2	C_1	F	T	W	O	U	R
$Init$	**{1}**	{0,1}	{0,1}	{1}	{1,..,9}	{0,..,9}	{0,..,9}	{0,..,9}	{0,..,9}
$C_3 = 1$	1	{0,1}	{0,1}	**{1}**	{5,...,9}	{0,..,9}	{0,..,9}	{0,..,9}	{0,..,9}
$F = 1$	1	**{0,1}**	{0,1}	1	{5,...,9}	{0,..,9}	{0,..,9}	{0,..,9}	{0,..,9}
$C_2 = 0$	1	0	**{0,1}**	1	{5,...,9}	{0,..,4}	{0,2,4,6,8}	{0,..,9}	{0,..,9}
$C_1 = 0$	1	0	0	1	{5,6,7}	{0,2,3,4}	**{0,2,4}**	{0,4,6,8}	{0,4,8}
$O = 4$	1	0	0	1	{7}	{0,2,3,4}	4	{0,4,6,8}	**{8}**
$R = 8$	1	0	0	1	**{7}**	{0,3}	4	{0,6}	8
$T = 7$	1	0	0	1	7	**{0,3}**	4	{0,6}	8
$W = 0$	1	0	0	1	7	3	4	**{6}**	8

	C_3	C_2	C_1	F	T	W	O	U	R
U=6	1	0	0	1	7	3	4	6	8

6.11 用 AC-3 算法说明弧相容对图中问题能够检测出部分赋值{WA = red, V = blue}的不相容

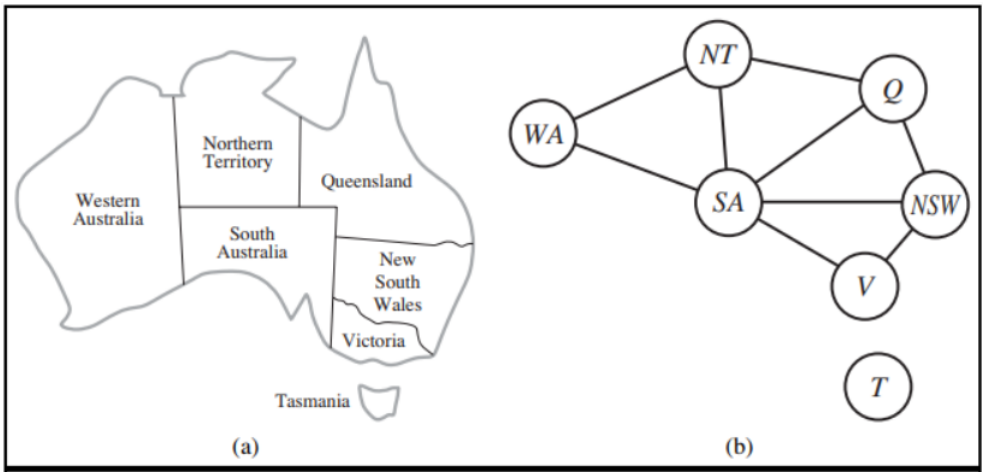

Figure 6.1 (a) The principal states and territories of Australia. Coloring this map can be viewed as a constraint satisfaction problem (CSP). The goal is to assign colors to each region so that no neighboring regions have the same color. (b) The map-coloring problem represented as a constraint graph.

A： 初始队列 {SA,WA},{SA,V},{NT,SA},{SA,NT}{Q,SA}{SA,Q}.....

	SA	**WA**	**NT**	**Q**	**NSW**	**V**
$Init$	{RGB}	{R}	{RGB}	{RGB}	{RGB}	{B}
{SA,WA}	{GB}	{R}	{RGB}	{RGB}	{RGB}	{B}
{SA,V}	{G}	{R}	{RGB}	{RGB}	{RGB}	{B}
{NT,SA},{Q,SA},{NSW,SA}	{G}	{R}	{RB}	{RB}	{RB}	{B}
{NT,WA}	{G}	{R}	{B}	{RB}	{RB}	{B}
{NSW,V}	{G}	{R}	{B}	{RB}	{R}	{B}
{Q,NT}	{G}	{R}	{B}	{R}	{R}	{B}
{Q,NSW}	{G}	{R}	{B}	{}	{R}	{B}

此时 Q 出现空集，则检查出 $WA = red, V = blue$ 不相容

6.12 用**AC-3**算法求解树结构**CSP**在最坏情况下的复杂度是多少?

A:

假设有n个顶点，每个顶点可以有D个取值，那么每次检查一条弧 (X_i, X_j)中 X_i 是否要修改其值域时，时间复杂度最坏为 $O(D^2)$；

在树状结构的情况下，弧的数目为 $O(n)$，而在树状结构中采用逆拓扑排序检验和集合存储的方式可以使得每条弧只会被检查 $O(1)$次

所以，最坏复杂度为 $O(nD^2)$