SIT718 Real World Analytics

School of Information Technology

Deakin University

Prac. 8 Problems

Single Period Production Model

[Modified from Taha]
Haus' Clothing produces down jackets, fleece hoodies, thermo pants, and beanies. If there are unmet demands, a penalty cost will incur.

The time taken in the manufacturing processes (cutting, insulating, sewing, and packaging) for each of the products are listed in the table (see next page), so are the demands for each product, manufacturing capacities for each process, and the unit profit and shortage penalty.

Single Period Production Model

Process	Production time required per unit (hr) Down	Hoodies	Pants	Beanies	Process capacity (hr)
Cutting	0.3	0.3	0.25	0.15	1000
Insulating	0.25	0.35	0.3	0.1	1000
Sewing	0.45	0.5	0.4	0.22	1000
Packaging	0.15	0.15	0.1	0.05	1000
Demand	800	750	600	500	
Unit profit	\$30	\$40	\$20	\$10	
Unit penalty	\$15	\$20	\$10	\$8	

Multiple Period Production-Inventory Model

[Modified from Taha]
Haus & Sons manufactures dinning tables. The demand for the next six months are: 100, 250, 190, 140, 220, and 110.

Production cost per table varies over the months, and the estimated costs for the next 6 months are: \$50, \$45, \$55, \$48, \$52, and \$50.

If the number of tables made in a month exceeds the demand, they can be stored for sales in later months. The storage cost is \$8 per table.

Transportation Problem–Supply larger than Demand

If the total supply exceeds the total demand, we can always use a **dummy demand point** with a demand exactly the amount of excess supply. As "shipments" to the dummy demand point is not a real shipment, the shipment cost will be **zero**. Consider:

	To					
From	City 1	City 2	City 3	City 4	Dummy	Supply (mil kwh)
Plant 1	\$8	\$6	\$10	\$9	\$0	72
Plant 2	\$9	\$12	\$13	\$7	\$0	60
Plant 3	\$14	\$ 9	\$16	\$5	\$0	78
Demand (mil kwh)	45	70	30	55	10	

Task: Write a LP model for this problem and solve it by implementing it in R.

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Prac. 8 Solutions

Single Period Production Model

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Single Period Production Model - Solution

Let: x_1, x_2, x_3, and x_4 be the number of down jackets, hoodies, thermo pants, and beanies respectively.
Let: s_1, s_2, s_3, and s_4 be the number in shortage for down jackets, hoodies, thermo pants, and beanies respectively. (Notice that these shortages will incur a penalty cost).

The objective is to maximize:

Net profit = Total profit - Total penalty

$$\max z = 30x_1 + 40x_2 + 20x_3 + 10x_4 - (15s_1 + 20s_2 + 10s_3 + 8s_4)$$

Single Period Production Model - Solution

We also have the following capacity constraints:

$$\begin{aligned} 0.3x_1 + 0.3x_2 + 0.25x_3 + 0.15x_4 &\leq 1000 \\ 0.25x_1 + 0.35x_2 + 0.30x_3 + 0.1x_4 &\leq 1000 \\ 0.45x_1 + 0.5x_2 + 0.4x_3 + 0.22x_4 &\leq 1000 \\ 0.15x_1 + 0.15x_2 + 0.1x_3 + 0.05x_4 &\leq 1000 \end{aligned}$$

Shortage constraints:

$$s_1 = 800 - x_1; \quad s_2 = 750 - x_2; \quad s_3 = 600 - x_3; \quad s_4 = 500 - x_4,$$

and non-negativity constraints: $x_j, s_j \geq 0, \forall j = 1, 2, 3, 4.$

The optimal solution is: $z = 64,625;$
$x_1 = 800,\ x_2 = 750,\ x_3 = 387.5,\ x_4 = 500;$
$s_1 = s_2 = s_4 = 0,\ s_3 = 212.5.$

Multiple Period Production-Inventory Model

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Production cost per table varies over the months, and the estimated costs for the next 6 months are: $50, $45, $55, $48, $52, and $50.

If the number of tables made in a month exceeds the demand, they can be stored for sales in later months. The storage cost is $8 per table.

Multiple Period Production-Inventory Model - The LP model

In theory, this should be an Integer Programming (IP) model, as we can't make half, or three quarters of a table, for example. In any case, the objective function is obviously to minimize cost.

Let :
x_j be the number of tables to produce in Month j; and
I_j be the number of tables to be put in inventory in Month j, for $j = 1, \ldots, 6$.

The objective function is to minimize total production and storage costs:

$$\begin{aligned} \min\ z = \quad & 50x_1 + 45x_2 + 55x_3 + 48x_4 + 52x_5 + 50x_6 \\ & +8(I_1 + I_2 + I_3 + I_4 + I_5 + I_6) \end{aligned}$$

Multiple Period Production-Inventory Model - The Constraints

$$
\begin{aligned}
x_1 - I_1 &= 100 \quad \text{(Month 1)} \\
I_1 + x_2 - I_2 &= 250 \quad \text{(Month 2)} \\
I_2 + x_3 - I_3 &= 190 \quad \text{(Month 3)} \\
I_3 + x_4 - I_4 &= 140 \quad \text{(Month 4)} \\
I_4 + x_5 - I_5 &= 220 \quad \text{(Month 5)} \\
I_5 + x_6 &= 110 \quad \text{(Month 6)}
\end{aligned}
$$

The optimal solution is:
$x_1 = 100$, $x_2 = 440$, $x_3 = 0$, $x_4 = 140$, $x_5 = 220$, and $x_6 = 110$; and $I_1 = I_3 = \cdots = I_6 = 0$; $I_2 = 190$.

The optimal total cost is $z = 49980$.

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If the total supply exceeds the total demand, we can always use a **dummy demand point** with a demand exactly the amount of excess supply. As "shipments" to the dummy demand point is not a real shipment, the shipment cost will be **zero**. Consider:

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Optimal Solutions: $x_{12} = 47$; $x_{13} = 25$; $x_{21} = 45$; $x_{23} = 5$; $x_{32} = 23$; $x_{34} = 55$; $x_{25} = 10$.