Part-B: Chi Square Test

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LEARNING OBJECTIVES

At the end of this section, you should be able to do the following:

- Perform a Chi Square test to see the difference between two proportions and difference between multiple proportions
- Perform a post hoc multiple comparison procedure (Marascuilo procedure)

CONTINGENCY TABLES

- Contingency tables are useful in situations involving multiple population proportions
- Used to classify sample observations according to two or more characteristics
- Also called a cross-classification table

	Column 1	Column 2	
Row 1			Subtotal
Row 2			Subtotal
	Subtotal	Subtotal	Grand total

CONTINGENCY TABLES

Example

- Sample results organised in a contingency table

- Sample size = n = 300
- 120 females, 12 were left-handed
- 180 males, 24 were left-handed

Gender	Hand Preference		
	Left	Right	
Female	12	108	120
Male	24	156	180
	36	264	300

χ2 TEST FOR THE DIFFERENCE BETWEEN TWO PROPORTIONS

H_0: $\pi_1 = \pi_2$

- (Proportion of females who are left-handed is equal to the proportion of males who are left-handed)

H_1: $\pi_1 \neq \pi_2$

- (The two proportions are not the same)

- If H_0 is true, then the proportion of left-handed females should be the same as the proportion of left-handed males. That is, the two proportions above should be the same as the proportion of left-handed people overall.

ESTIMATED OVERALL PROPORTION

- Represents the estimated overall proportion of successes for the two groups combined
- The estimate for the overall proportion is:

$$\overline{p} = \frac{X_1 + X_2}{n_1 + n_2}$$

- where X1 and X2 are the numbers from groups 1 and 2 with the characteristic of interest; i.e., number of 'successes' in groups 1 and 2

THE CHI-SQUARE TEST STATISTIC

- The Chi-square test statistic is:

$$\chi^2 = \sum_{all\ cells} \frac{(f_o - f_e)^2}{f_e}$$

Where:

f_o = observed frequency in a particular cell

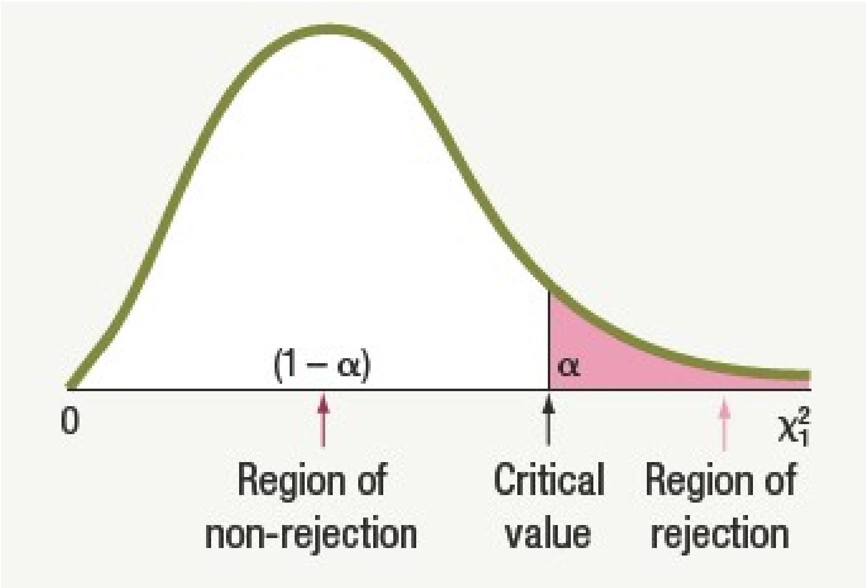
f_e = expected frequency in a particular cell if H_0 is true

$\chi 2$ for the 2 x 2 case has 1 degree of freedom

(Assumed: each cell in the contingency table has expected frequency of at least 5)

DECISION RULE

- The $\chi 2$ test statistic approximately follows a chi-squared distribution with one degree of freedom



- Decision Rule:
- If $\chi 2 > \chi 2U$, reject H_0, otherwise do not reject H_0

COMPUTING THE ESTIMATED OVERALL PROPORTION

- The overall proportion is: $\bar{p} = \dfrac{X_1 + X_2}{n_1 + n_2} = \dfrac{X}{n}$

- So, if 12 out of 120 females are left-handed and 24 out of 180 males were left handed, the computation would be:

$$\bar{p} = \frac{12 + 24}{120 + 180} = \frac{36}{300} = 0.12$$

- That is, the proportion of left-handers overall is 0.12 – i.e., 12%

FINDING EXPECTED FREQUENCIES

- To obtain the expected frequency for left-handed females, multiply the proportion left-handed ($\overline{p}$) by the total number of females
- To obtain the expected frequency for left-handed males, multiply the proportion left-handed ($\overline{p}$) by the total number of males

- If the two proportions are equal, then:

  P (Left-handed | Female) = P (Left-handed | Male) = 0.12
- That is, we would expect

  (0.12)(120) = 14.4 females to be left-handed

  (0.12)(180) = 21.6 males to be left-handed

OBSERVED VERSUS EXPECTED FREQUENCIES

Gender	Hand Preference		
	Left	Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

THE CHI-SQUARE TEST STATISTIC

Gender	Hand Preference		
	Left	Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

- The test statistic is:

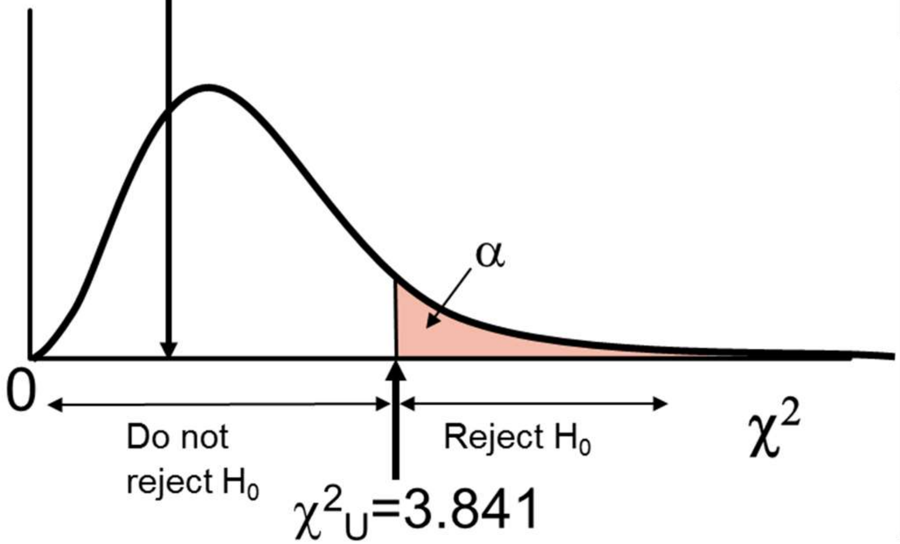
$$\chi^2 = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

$$= \frac{(12-14.4)^2}{14.4} + \frac{(108-105.6)^2}{105.6} + \frac{(24-21.6)^2}{21.6} + \frac{(156-158.4)^2}{158.4} = 0.7576$$

INTERPRETATION

The test statistic is $\chi^2 = \boxed{0.7576}$, χ^2_U with 1 d.f. $= 3.841$

Decision Rule:
If $\chi^2 > 3.841$, reject H_0, otherwise, do not reject H_0



Here,
$\chi^2 = 0.7576 < \chi^2_U = 3.841$, so we do not reject H_0 and conclude that there is not sufficient evidence that the two proportions are different at $\alpha = 0.05$

$\chi 2$ TEST FOR DIFFERENCES AMONG MORE THAN TWO PROPORTIONS

- Extend the $\chi 2$ test to the case with more than two independent populations
- H_0: $\pi_1 = \pi_2 = \ldots = \pi_c$
- H_1: Not all of the π_j are equal ($j = 1, 2, \ldots, c$)

THE CHI-SQUARE TEST STATISTIC

The Chi-square test statistic is:

$$\chi^2 = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

Where:

fo = observed frequency in a particular cell of the 2 x c table

fe = expected frequency in a particular cell if H_0 is true

c = number of independent populations under consideration

$\chi 2$ for the 2 x c case has c – 1 degrees of freedom

(Assumed: each cell in the contingency table has expected frequency of at least 1)

CALCULATING THE OVERALL PROPORTION

The overall proportion is:

$$\bar{p} = \frac{X_1 + X_2 + L + X_c}{n_1 + n_2 + \dots + n_c} = \frac{X}{n}$$

Expected cell frequencies for the c categories are calculated as in the 2 x 2 case, and the decision rule is the same

Decision Rule:

If $\chi 2 > \chi 2U$, reject H_0, otherwise, do not reject H_0

Where $\chi 2U$ is from the chi-squared distribution with c – 1 degrees of freedom

THE MARASCUILO PROCEDURE (1 OF 4)

Used when the null hypothesis of equal proportions is rejected and a post hoc multiple comparison procedure is needed (similar to Tukey–Kramer)

Enables you to make comparisons between all pairs of groups

Start with the observed differences, $p_j - p_{j'}$ (for $j \neq j'$) between all $c(c-1)/2$ pairs

Calculate corresponding critical ranges for the Marascuilo procedure then compare the absolute difference in sample proportions $| p_j - p_{j'} |$ to the calculated critical range

THE MARASCUILO PROCEDURE (2 OF 4)

Critical range for the Marascuilo procedure

$$\text{Critical range} = \sqrt{\chi_U^2} \sqrt{\frac{p_j(1 - p_j)}{n_j} + \frac{p_{j'}(1 - p_{j'})}{n_{j'}}}$$

(Note: the critical range is different for each pairwise comparison)

A particular pair of proportions is significantly different if:

$| p_j - p_{j'} | >$ critical range for j and j'

THE MARASCUILO PROCEDURE (3 OF 4)

Example:

A university is thinking of switching to a completely online mode. A random sample of 100 administrators, 50 students and 50 faculty members was surveyed.

Opinion	Admin	Students	Faculty
Favour	63	20	37
Oppose	37	30	13
	100	50	50

Using a 1% level of significance, which groups have a different attitude?

THE MARASCUILO PROCEDURE (4 OF 4)

Solution:

Marascuilo Procedure for difference in attitude			
Level of Significance	0.01		
Square Root of Critical Value	3.0349		
Group Sample Proportions			
1: Admin	0.6300		
2: Students	0.4000		
3: Faculty	0.7400		
MARASCUILO TABLE			
Proportions	Absolute Differences	Critical Range	
| Group 1 - Group 2 |	0.2300	0.2563	Not significant
| Group 1 - Group 3 |	0.1100	0.2386	Not significant
| Group 2 - Group 3 |	0.3400	0.2822	Significant

At 1% level of significance, there is evidence of a difference in attitude between students and faculty

QUESTIONS?

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