MODULE ONE: PRESENTING AND DESCRIBING INFORMATION

TOPIC 3: NUMERICAL DESCRIPTIVE MEASURES

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+ Learning Objectives

At the completion of this topic, you should be able to:

- calculate and interpret numerical descriptive measures of central tendency, variation and shape for numerical data
- calculate and interpret descriptive summary measures for a population
- construct and interpret a box-and-whisker plot
- calculate and interpret the covariance and the coefficient of correlation for bivariate data

+Measures of Central Tendency, Variation and Shape

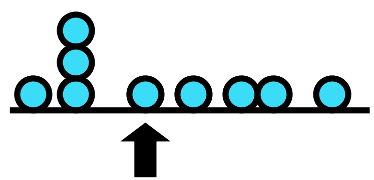
- Describing data by its central tendency, variation and shape
  - Central Tendency
    - Arithmetic Mean
    - Median
    - Mode
  - Quartiles
  - Measures of Variation
    - Range
    - Interquartile Range
    - Variance
    - Standard Deviation
    - Coefficient of Variation
  - Shape
    - Skewness

+Measures of Central Tendency

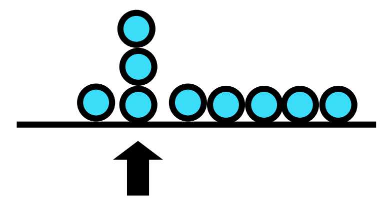
Central Tendency

- **Arithmetic Mean**
- **Median**
- **Mode**

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$



Midpoint of ranked values



Most frequently observed value

Arithmetic Mean

For a sample of size *n*, the sample mean, denoted $\overline{X}$, is calculated:

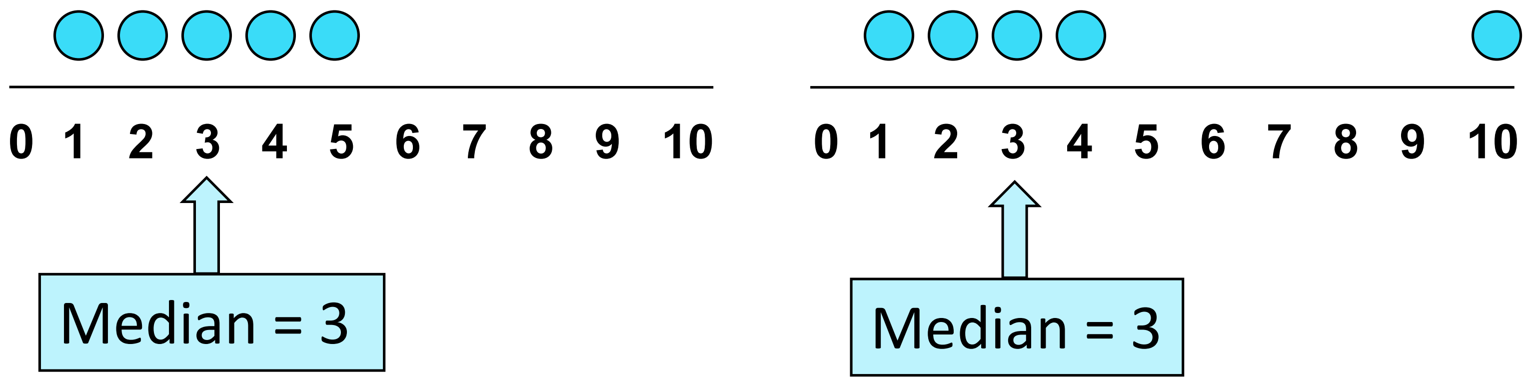
$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

X_i's are observed values

Where Σ means to sum or add up

+Median

In an ordered array, the median is the ‘middle’ number (50% above, 50% below)



Its main advantage over the arithmetic mean is that it is not affected by extreme values

+Median

The <u>location</u> of the median:

Median = $\frac{n+1}{2}$ ranked value

- Note that $\frac{n+1}{2}$ is not the **value** of the median, only the **position** of the median in the ranked data

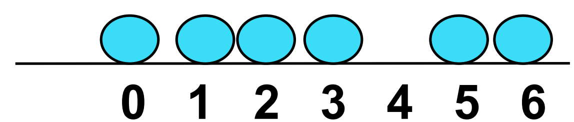
<u>*Rule 1:*</u> If the number of values in the data set is **odd**, the median is the **middle ranked value**

<u>*Rule 2:*</u> If the number of values in the data set is **even**, the median is the **mean** (average) of the **two middle ranked values**

+Mode

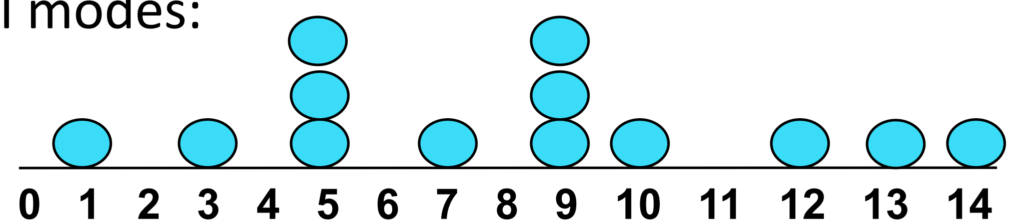
- A measure of central tendency
- Value that occurs most often (the most frequent)
- Not affected by extreme values
- Used for either numerical or categorical (nominal) data
- Unlike mean and median, there may be no unique (single) mode for a given data set

An example of no mode:



An example of several modes:

Modes = 5 and 9



+Quartiles

Similar to the median, we find a quartile by determining the value in the appropriate **position** in the **ranked** data, where:

First quartile position: $\mathbf{Q_1 = (n+1)/4}$

Second quartile position: $\mathbf{Q_2 = (n+1)/2}$ (the median)

Third quartile position: $\mathbf{Q_3 = 3(n+1)/4}$

where *n* is the number of observed values (sample size)

+Quartiles

Use the following rules to calculate the quartiles:

Rule 1 If the result is an integer, then the quartile is equal to the ranked value. For example, if the sample size is $n = 7$, the first quartile, Q1, is equal to the $(7 + 1)/4 = 2$, second-ranked value

Rule 2 If the result is a fractional half (2.5, 4.5, etc.), then the quartile is equal to the mean of the corresponding ranked values. For example, if the sample size is $n = 9$, the first quartile, Q1, is equal to the $(9 + 1)/4 = 2.5$ ranked value, halfway between the second- and the third-ranked values

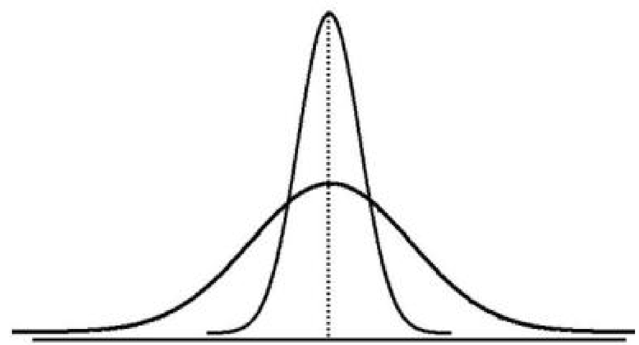
Rule 3 If the result is neither an integer nor a fractional half, round the result to the nearest integer and select that ranked value. For example, if the sample size is $n = 10$, the first quartile, Q1, is equal to the $(10 + 1)/4 = 2.75$ ranked value. Round 2.75 to 3 and use the third-ranked value

+Measures of Variation

Variation

- Range
- Interquartile Range
- Variance
- Standard Deviation
- Coefficient of Variation

Measures of variation gives information on the **spread** or **variability** of the data values



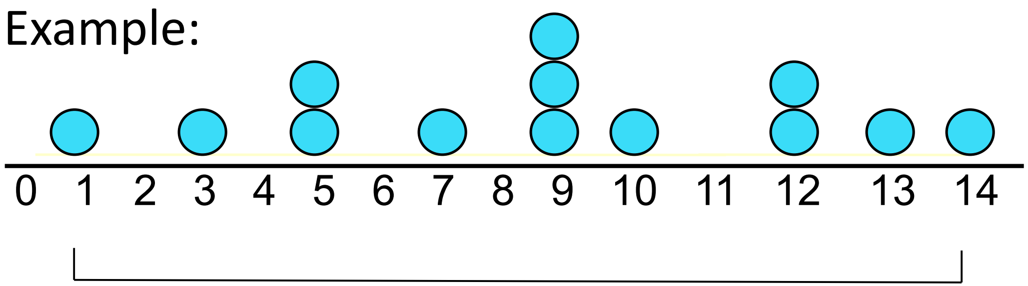
e.g. same centre, different variation

+Range

- Simplest measure of variation
- Difference between the largest and smallest values in data set
- Ignores the distribution of the data
- Like the Mean, the Range is sensitive to outliers

$$\boxed{\text{Range} = X_{largest} - X_{smallest}}$$

Example:



Range = 14 - 1 = 13

+Interquartile Range

Like the Median, Q_1 and Q_3, the IQR is a **resistant summary measure** (resistant to the presence of extreme values)

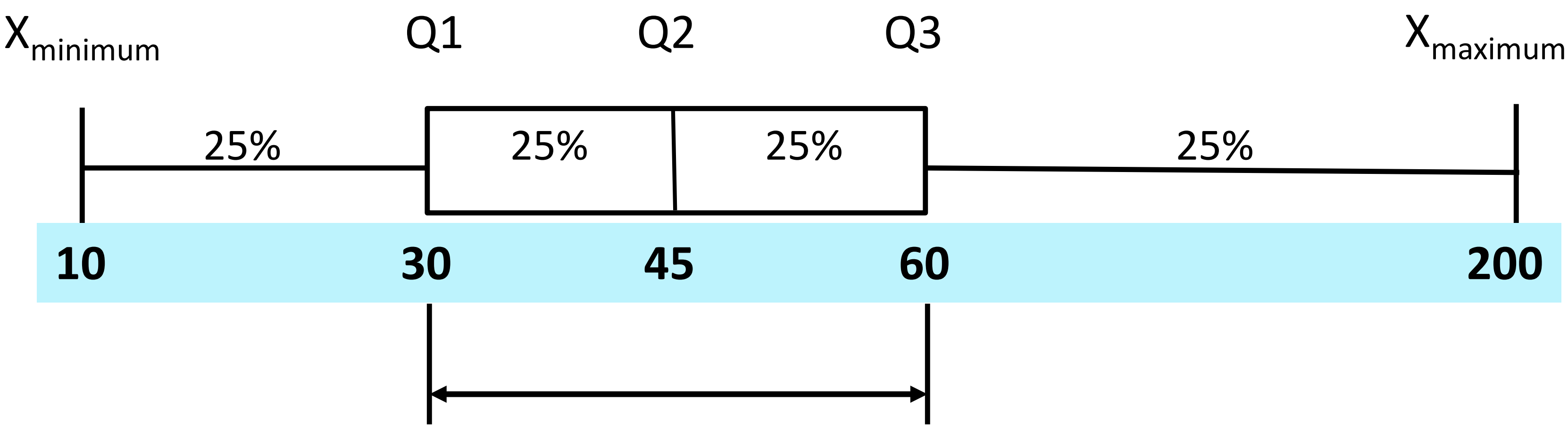
Eliminates outlier problems by using the **interquartile range**, as high- and low-valued observations are removed from calculations

IQR = 3rd quartile – 1st quartile

$$\mathbf{IQR = Q_3 - Q_1}$$

+Interquartile Range

Example: Range = 200–10 = 190 (misleading)



IQR = 60 – 30 = 30

+Variance and Standard Deviation

The **Sample Variance** – S^2

- Measures average scatter around the mean
- Units are also squared

$$S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

Where:
$\bar{X}$ = sample mean
n = sample size
X_i = i^{th} value of the variable X

+Variance and Standard Deviation

The **Sample Standard Deviation** – S

- Most commonly used measure of variation
- Shows variation about the mean
- Has the same units as the original data

$$S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n - 1}}$$

Where:

$\overline{X}$ = sample mean

n = sample size

X_i = i^{th} value of the variable X

+Variance and Standard Deviation

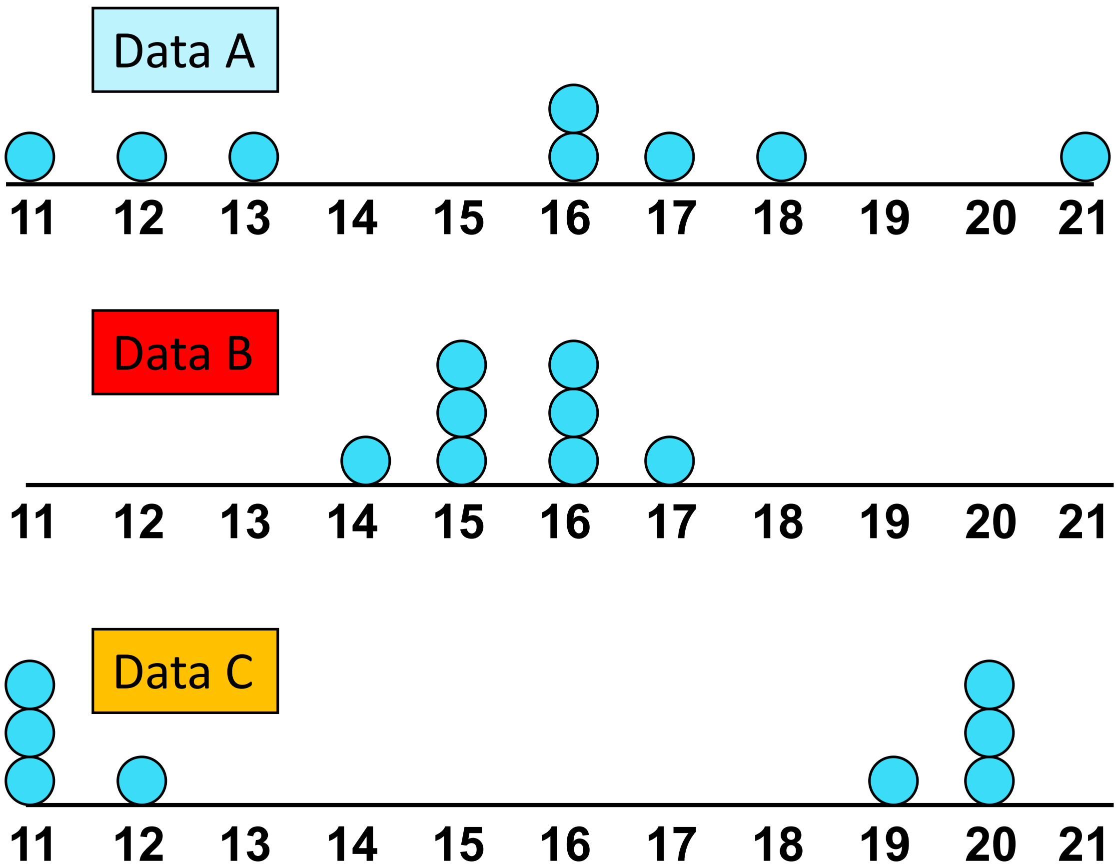
Advantages

- Each value in the data set is used in the calculation
- Values far from the mean are given extra weight as deviations from the mean are squared

Disadvantages

- Sensitive to extreme values (outliers)
- Measures of absolute variation not relative variation

+Comparing Standard Deviations



Mean = 15.5

S = 3.338

Mean = 15.5

S = 0.926

Mean = 15.5

S = 4.567

+Coefficient of Variation

Measures relative variation

- i.e. shows variation relative to mean

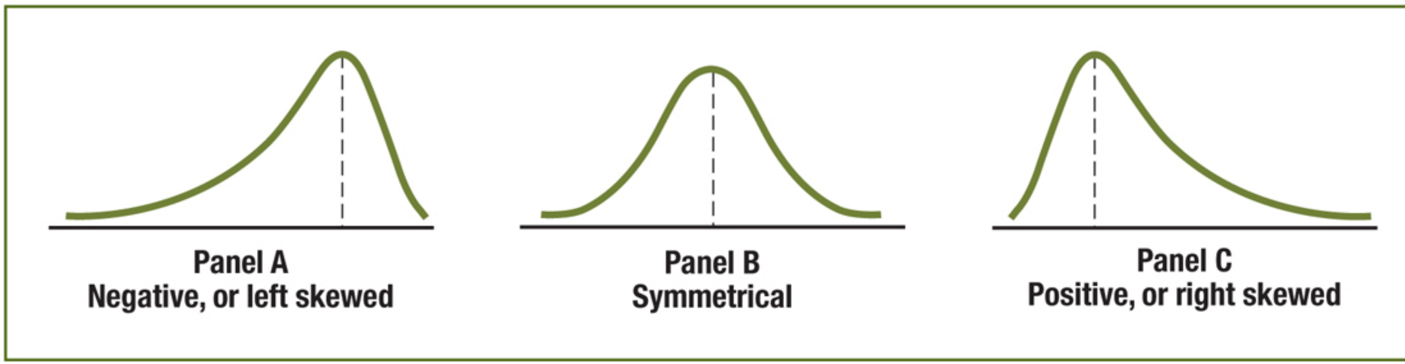
Can be used to compare two or more sets of data measured in different units

Always expressed as percentage (%)

$$CV = \left(\frac{S}{\overline{X}} \right) \cdot 100\%$$

+Shape

Figure 3.1
A comparison of three data sets differing in shape



+Shape

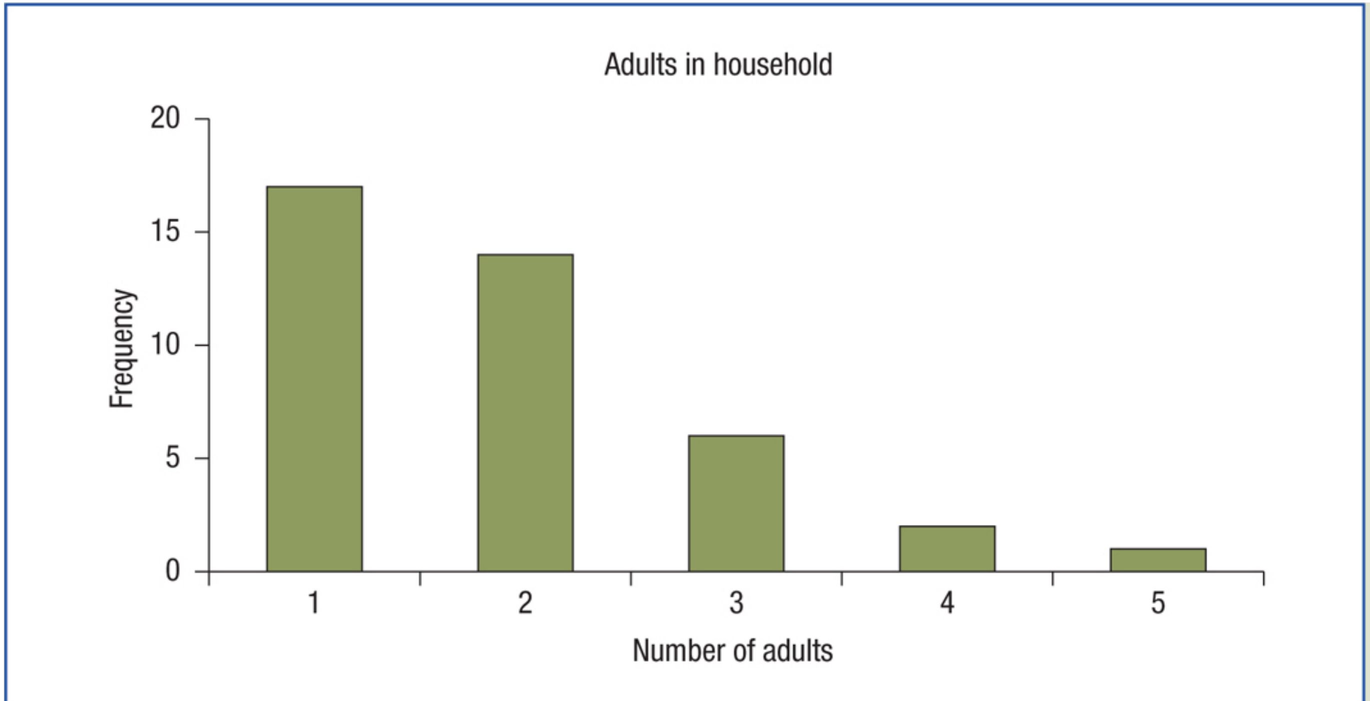


Figure 3.2
Column chart for number of adults in household

+Microsoft Excel Descriptive Statistics Output

	A	B
1	**Festival spending – international visitors**	
2		
3	Mean	743.75
4	Standard error	74.9867
5	Median	744
6	Mode	#N/A
7	Standard deviation	259.761
8	Sample variance	67476
9	Kurtosis	–1.41411
10	Skewness	–0.13236
11	Range	776
12	Minimum	343
13	Maximum	1119
14	Sum	8925
15	Count	12

Figure 3.3 Microsoft Excel summary statistics for festival expenditure

+Numerical Descriptive Measures for a Population

- Population summary measures are called parameters
- The population mean is the sum of the values in the population divided by the population size, N

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N} = \frac{X_1 + X_2 + \cdots + X_N}{N}$$

+Population Variance and Standard Deviation

Population Variance:

- the average of the squared deviations of values from the mean

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

μ = population mean; N = population size; X_i = i^{th} value of the variable X

Population Standard Deviation:

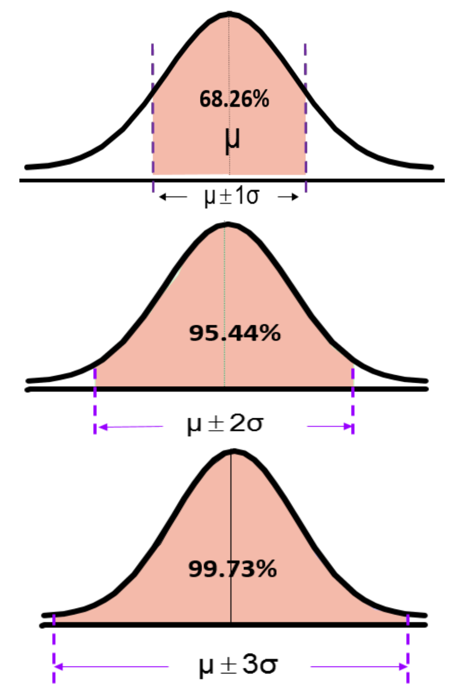
- shows variation about the mean
- is the square root of the population variance
- has the same units as the original data

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}}$$

+The Empirical Rule

If the data distribution is approximately bell-shaped, then the interval:

- $\mu \pm 1\sigma$ contains about 68.26% of values of the population
- $\mu \pm 2\sigma$ contains about 95.44% of values of the population
- $\mu \pm 3\sigma$ contains about 99.73% of values of the population



+The Chebyshev Rule

	% of values found in intervals around the mean	
Interval	Chebyshev (any distribution)	Empirical rule (bell-shaped distribution)
$(\mu - \sigma, \mu + \sigma)$	At least 0%	Approximately 68%
$(\mu - 2\sigma, \mu + 2\sigma)$	At least 75%	Approximately 95%
$(\mu - 3\sigma, \mu + 3\sigma)$	At least 88.89%	Approximately 99.7%

Table 3.4
How data vary around the mean

+Z Scores

The difference between a given observation and the mean, divided by the standard deviation

$$Z = \frac{X - \bar{X}}{S}$$

For example:

- A Z score of 2.0 means that a value is 2.0 standard deviations from the mean
- A Z score above 3.0 or below -3.0 is considered an outlier (symmetrical distribution)

+Calculating Numerical Descriptive Measures from a Frequency Distribution

Sometimes only a frequency distribution is available, not the raw data

Use the midpoint of a class interval to approximate the values in that class

$$\overline{X} = \frac{\sum_{j=1}^{c} m_j f_j}{n}$$

where: n = number of values or sample size

c = number of classes in the frequency distribution

m_j = midpoint of the j^{th} class

f_j = number of values in the j^{th} class

Calculating Numerical Descriptive Measures from a Frequency Distribution

Approximating the Standard Deviation

$$S = \sqrt{\frac{\sum_{j=1}^{c} (m_j - \bar{X})^2 f_j}{n - 1}}$$

$$S = \sqrt{\frac{\sum_{j=1}^{c} f_j m_j^2 - n\bar{X}^2}{n - 1}}$$

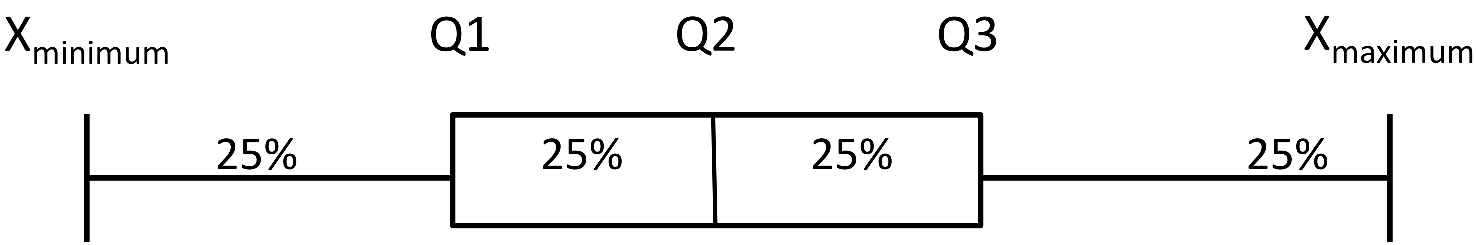
Note: Assume that all values within each class interval are located at the midpoint of the class

+Calculating Numerical Descriptive Measures from a Frequency Distribution (cont)

Table 3.6
Calculations needed to calculate approximations of the mean and standard deviation of the real estate prices

Asking price ($)	Frequency	Mid-point in $000s	$f_j m_j$	$f_j m_j^2$
300,000 to < 350,000	8	325	2,600	845,000
350,000 to < 400,000	17	375	6,375	2,390,625
400,000 to < 450,000	21	425	8,925	3,793,125
450,000 to < 500,000	20	475	9,500	4,512,500
500,000 to < 550,000	16	525	8,400	4,410,000
550,000 to < 600,000	6	575	3,450	1,983,750
600,000 to < 650,000	7	625	4,375	2,734,375
650,000 to < 700,000	3	675	2,025	1,366,875
700,000 to < 750,000	0	725	0	0
750,000 to < 800,000	0	775	0	0
800,000 to < 850,000	2	825	1,650	1,361,250
Totals	100		47,300	23,397,500

+ Five-Number Summary and Box-and-Whisker Plot



Minimum($X_{smallest}$) -- Q1 -- Median -- Q3 -- Maximum ($X_{largest}$)

+Five Number Summary

Comparison	Type of distribution: Left skewed	Symmetrical	Right skewed
Distance from $X_{smallest}$ to the median versus the distance from the median to $X_{largest}$.	The distance from $X_{smallest}$ to the median is greater than the distance from the median to $X_{largest}$.	Both distances are the same.	The distance $X_{smallest}$ to the median is less than the distance from the median to $X_{largest}$.
Distance from $X_{smallest}$ to Q_1 versus the distance from Q_3 to $X_{largest}$.	The distance from $X_{smallest}$ to Q_1 is greater than the distance from Q_3 to $X_{largest}$.	Both distances are the same.	The distance from $X_{smallest}$ to Q_1 is less the distance from Q_3 to $X_{largest}$.
Distance from Q_1 to the median versus the distance from the median to Q_3.	The distance from Q_1 to the median is greater than the distance from the the median to Q_3.	Both distances are the same.	The distance from Q_1 to the median is less than the distance from the the median to Q_3.

Table 3.7 Relationships between the five-number summary and the type of distribution

+Distribution Shape and Box-and-Whisker Plots

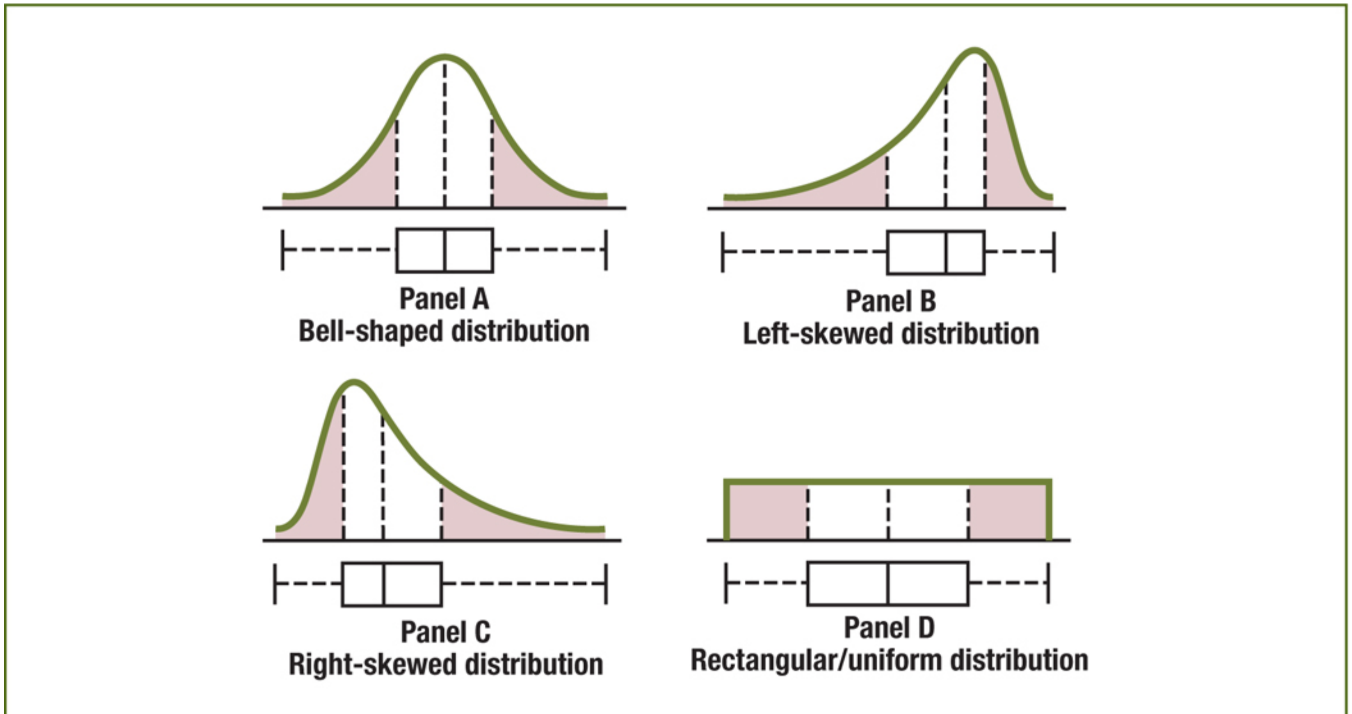


Figure 3.6
Box-and-whisker plots and corresponding polygons for four distributions

+Covariance

The covariance is a measure of the strength and direction of the linear relationship between two numerical variables (X and Y):

$$\text{cov}(X,Y) = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

As a covariance can have any value, it is difficult to use it as a measure of the relative strength of a linear relationship

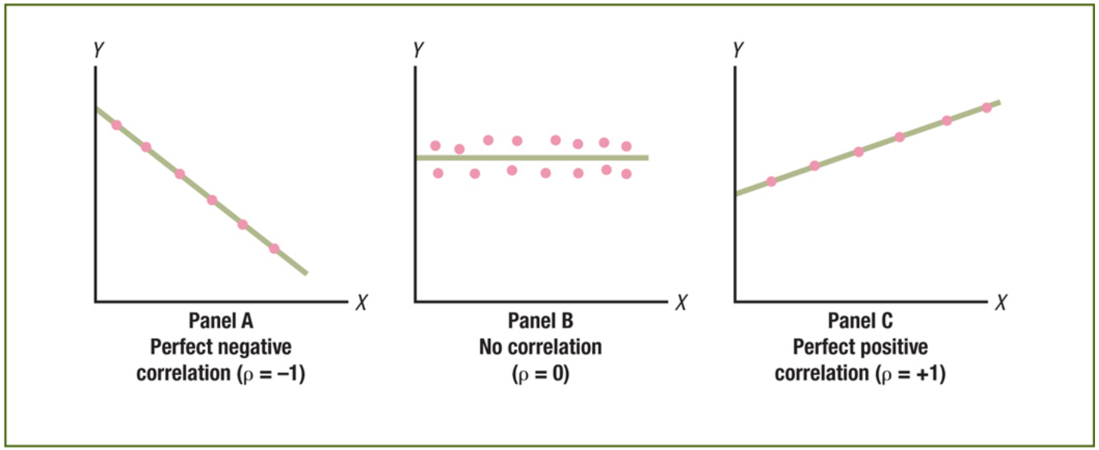
A better, and related, measure of the relative strength of a linear relationship is the Coefficient of Correlation, *r*

+Coefficient of Correlation

The coefficient of correlation measures the relative strength of a linear relationship between two numerical variables (X and Y)

Values range from -1 (perfect negative) to +1 (perfect positive)

Figure 3.7
Types of association between variables



+Coefficient of Correlation (cont)

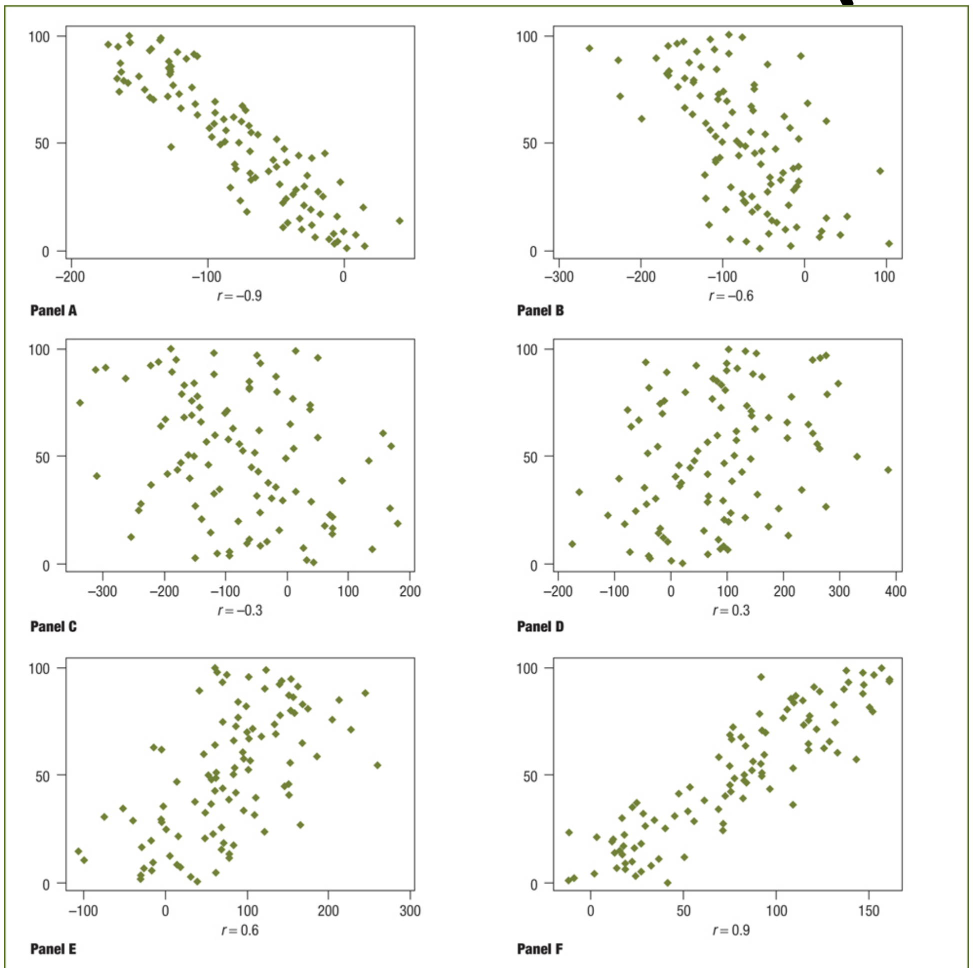


Figure 3.8 Six scatter diagrams and their sample coefficients of correlation, r

Coefficient of Correlation - Calculation

The sample coefficient of correlation is the sample covariance divided by the sample deviations of *X* and *Y*

$$r = \frac{\text{cov}(X,Y)}{S_X S_Y}$$

where:

$$\text{cov}(X,Y) = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{n-1}$$

$$S_X = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}} \qquad S_Y = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}{n-1}}$$

Pitfalls in Numerical Descriptive Measures and Ethical Issues

Data analysis is *objective*

- Should report the summary measures that best meet the assumptions about the data set

Data interpretation is *subjective*

- Should be done in fair, neutral and transparent manner
- Should document both good and bad results
- Results should be presented in a fair, objective and neutral manner
- Should not use inappropriate summary measures to distort facts
- Do not fail to report pertinent findings even if such findings do not support original argument