MIS772 Predictive Analytics

Linear Regression Models as estimation method

Refer to your textbook by Vijay Kotu and Bala Deshpande, *Data Science: Concepts and Practice*, 2nd ed, Elsevier, 2018.

Multiple regression

- Understanding a linear model coefficients, p-values, R²
- The fundamental assumptions of regression modelling
- Model diagnostics
- Attribute selection







Ames Real Estate Data Set (source: kaggle.com):

79 regular attributes describing (almost) every aspect of residential homes in Ames, Iowa, US

This competition challenges you to predict the label attribute, i.e. SALE PRICE of each home.

ExampleSet (2930 examples, 2 special attributes, 79 regular attributes)

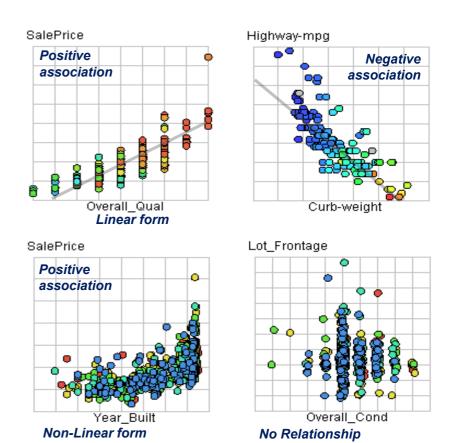
| Row No. | PID | SalePrice | MS_SubClass | MS_Zoning | Lot_Frontage | Lot_Area | Street | Alley | Lot_Shape | Land_Contour | Utilities | Lot_Config |
|---------|-----------|-----------|-------------|-----------|--------------|----------|--------|-------|-----------|--------------|-----------|------------|
| 1 | 526301100 | 215000 | 20 | RL | 141 | 31770 | Pave | NA | IR1 | Lvi | AllPub | Corner |
| 2 | 526350040 | 105000 | 20 | RH | 80 | 11622 | Pave | NA | Reg | LvI | AllPub | Inside |
| 3 | 526351010 | 172000 | 20 | RL | 81 | 14267 | Pave | NA | IR1 | LvI | AllPub | Corner |
| 4 | 526353030 | 244000 | 20 | RL | 93 | 11160 | Pave | NA | Reg | LvI | AllPub | Corner |
| 5 | 527105010 | 189900 | 60 | RL | 74 | 13830 | Pave | NA | IR1 | Lvi | AllPub | Inside |
| 6 | 527105030 | 195500 | 60 | RL | 78 | 9978 | Pave | NA | IR1 | Lvi | AllPub | Inside |
| 7 | 527127150 | 213500 | 120 | RL | 41 | 4920 | Pave | NA | Reg | LvI | AllPub | Inside |
| 8 | 527145080 | 191500 | 120 | RL | 43 | 5005 | Pave | NA | IR1 | HLS | AllPub | Inside |
| 9 | 527146030 | 236500 | 120 | RL | 39 | 5389 | Pave | NA | IR1 | LvI | AllPub | Inside |
| 10 | 527162130 | 189000 | 60 | RL | 60 | 7500 | Pave | NA | Reg | LvI | AllPub | Inside |
| 11 | 527163010 | 175900 | 60 | RL | 75 | 10000 | Pave | NA | IR1 | LvI | AllPub | Corner |
| 12 | 527165230 | 185000 | 20 | RL | ? | 7980 | Pave | NA | IR1 | Lvl | AllPub | Inside |

We can build a predictive model for the **Sale Price** based on <u>other attributes</u> (predictors) available in the data set, thanks to the **relationships** existing between them.

https://www.kaggle.com/c/house-prices-advanced-regression-techniques



- There exist many kinds of relationships between attributes.
- One such relationship is correlation.
- Attributes are correlated when an increase of value in one attribute is accompanied by the simultaneous increase (top left) or decrease (top right) in the value of another.
- The rate of such increase could indicate linear (top row) or non-linear dependency.
- Correlation does not imply causation, which indicates that changes to values of one attribute are in direct consequence of changes in another.
- Correlation can be described in terms of association, form and strength.
- Scatter plots are useful in visual identification of correlation in small data sets.
- In large data sets, scatter plots are very confusing and have little value.



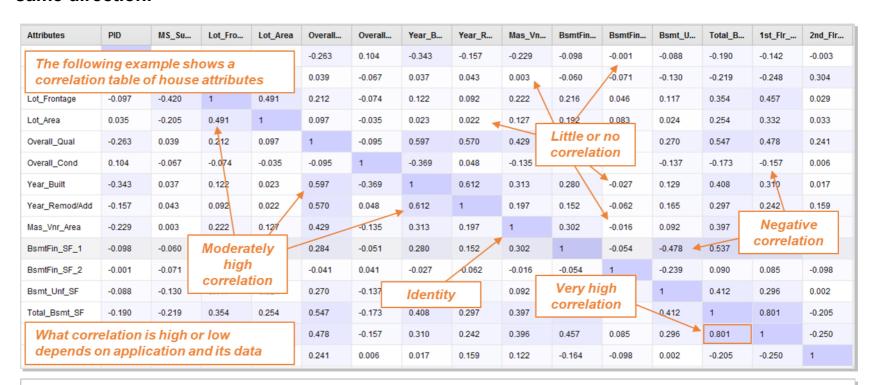
Selected charts from Houses and Cars data sets

Also see KD 3.3 on Correlation



- The most common measure of correlation is Pearson's correlation, which measures linear dependency between numerical variables.
- Pearson's correlation coefficient for two normally distributed variables indicates that if one variable's value increases the other also changes values consistently.
- Correlation is positive when values of two variables fluctuate together and grow in the same direction.

- Correlation of +1 indicates identity (x=x).
- Correlation is negative when two variables fluctuate together, but values of one variable grows while the other decreases.
- Correlation of -1 indicates -identity (x=-x).
- Correlation is close to zero when there is little relationship between two variables.
- Correlation of 0 indicates completely random pairing of variables' values.



Some commonly used types of correlation coefficients:

- Pearson r (linear relationship, assumes normal distribution, sensitive to outliers)
- Spearman rho (monotonic relationship, non-parametric, based on deviations, not linear, no normality)
- Kendall tau (between any ordered vars, non-parametric, based on concordance same order, good for small samples)



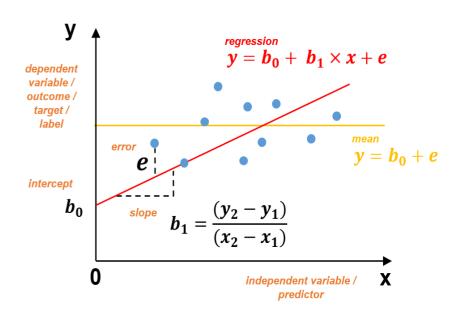
- Relationships between two variables can often be approximated by an equation describing their linear combination, which defines a linear model.
- In case of two variables, the equation describes a line, which is referred to as a regression line or simple regression.

$$y = b_0 + b_1 \times x + e$$

- The regression line can be defined by a mathematical formula for a line, defined by its *intercept* with the axis of the outcome variable (where x=0), its *slope* (proportion between x and y) and *error term*.
- When we have more variables we describe a multiple regression.

$$y = b_0 + b_1 \times x_1 + b_2 \times x_2 + \cdots + b_n \times x_n + e$$

- Regression analysis is the most commonly used predictive model.
- When predicted values are calculated using the regression formula, the total error, or differences between the expected values and the actual values gives an idea as to the model quality.



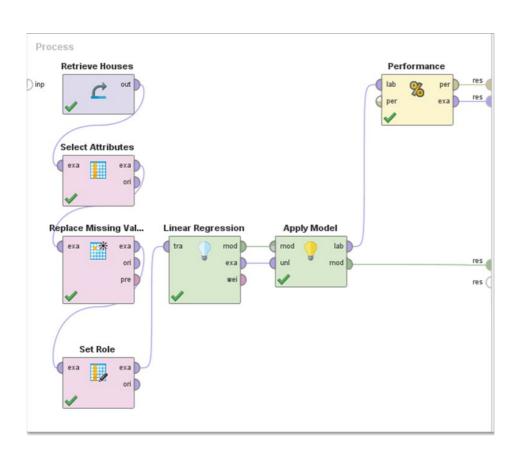
- The following measures are commonly used:
 - MAE (mean absolute error),
 - RMSE (root mean square error)
- Regression also assesses the quality of its predictions. It calculates a metric which indicates how much variance in the label attribute the model can explain from the input predictors, this is called the coefficient of determination:
 - R² (coefficient of determination).



Your task is: Create an estimator (estimation model) to predict the house price in Ames

Plan of action:

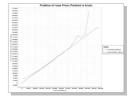
- Acquire data
- Study variables involved
- Create a simple regression model
- Create a multiple regression model
- Create and interpret various regression diagnostic charts



Regression model's assumptions:

- All variables are numeric
- No missing/bad values
- No extreme cases
- All predictors are independent (no multi-collinearities)
- Prediction errors (residuals) are normally distributed





There are several diagnostic plots of ensuring regression model quality (RapidMiner)



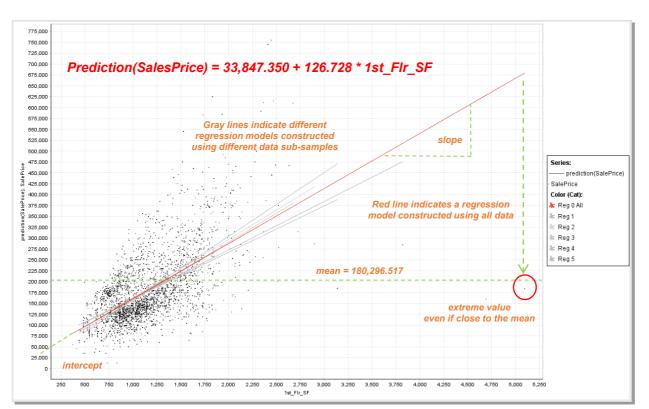
Pros

- Most common approach for modelling numeric data
- Can be (and is being) adapted to model almost any data
- Provides estimates of the strength and size of the relationships among independent variables and a dependent variable
- Can be visualised (not easy in case of multiple regression)

Cons

- Makes strong assumptions about data
- The model's form must be specified by the user in advance
- Cannot handle missing data
- Only works with numeric features, so categorical attributes may need dummy encoding

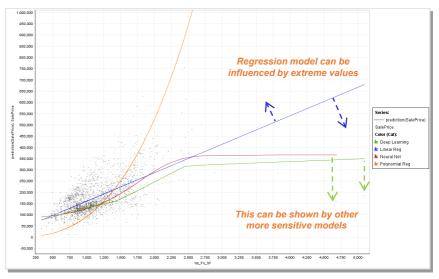




Example:

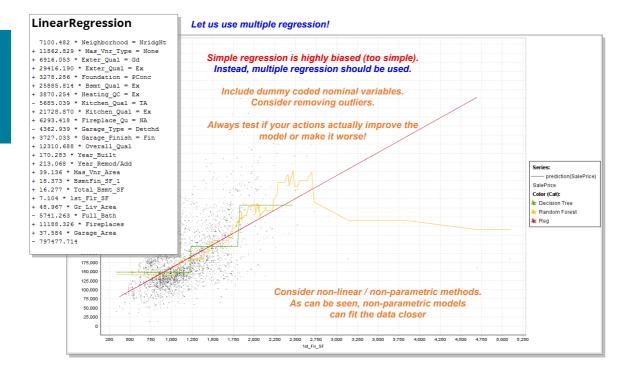
A scatter plot showing the relationship btw. the floor area vs the house price A training sample defines a regression line, which could significantly vary! (gray lines - left) Cross-validation (or bootstrap validation) is thus very important!

- Large houses cost more: given the floor surface area we can estimate the price
- Visually, the relationship appears to have a linear trend, which may suggest using a linear model to estimate sales price as a function of floor surface area
- And yet, on closer look by applying more sensitive models, we can find that extreme values (i.e. outliers) in data could distort this view significantly









- When we deal with many predictors, we need to create a multiple regression model
- It is important that all predictors are independent
- Multi-collinearity of attributes implies that predictors are actually closely related (i.e. one predictor can be estimated from the other predictors). This is problematic predictive modelling.
- The seminars this week will illustrate how multi-collinearity can be identified in RapidMiner.
- RapidMiner can eliminate attribute multi-collinearities from multiple regression
- There are also many methods of attribute selection for regression, e.g. greedy or M5 Prime

Improving Performance of linear models:

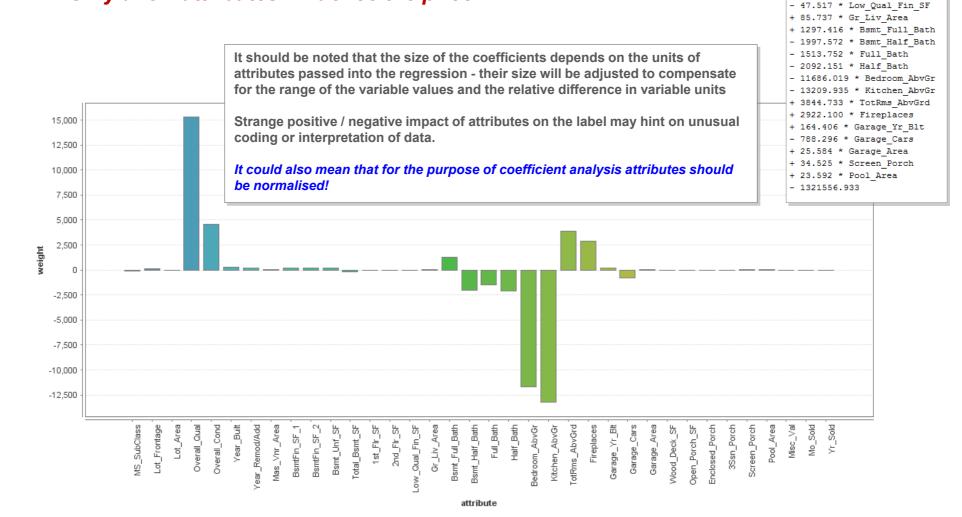
Consider:

- metrics (R², RMSE, etc.)
- The business context
- Logic of why a predictor(s) would influence the predicted attribute
- Parsimonious (less is more!) models are preferred in many business contexts



Analysis of Regression Coefficients

- The coefficients represent the amount of change to expect in the label if there was a one-unit change in the predictor attribute.
- The largest positive coefficients are "Overall_Qual" and "Overall_Cond" → better quality / condition, higher the price
- Only a few attributes influence the price



LinearRegression

- 101.898 * MS SubClass

+ 147.582 * Lot_Frontage + 0.576 * Lot_Area + 15301.046 * Overall Qual

+ 4606.237 * Overall_Cond + 273.158 * Year Built

+ 199.149 * Year_Remod/Add + 27.907 * Mas Vnr Area

+ 223.492 * BsmtFin SF 1

+ 203.531 * BsmtFin SF 2

+ 202.005 * Bsmt Unf SF

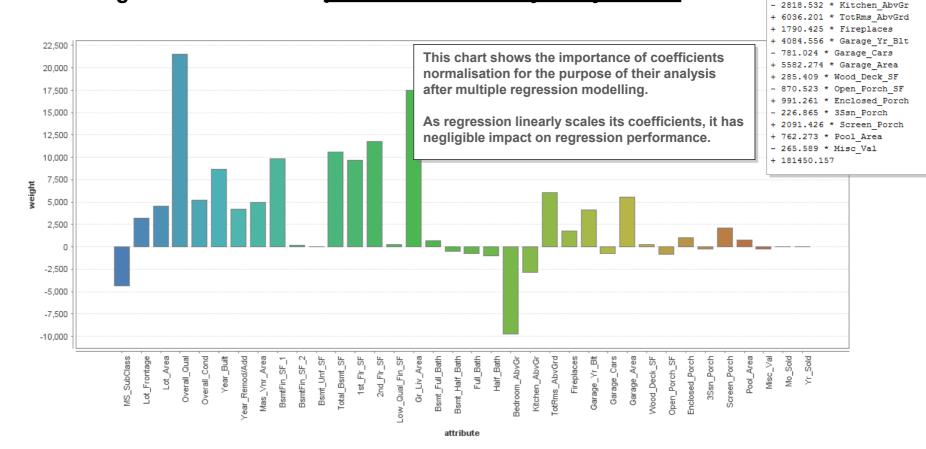
- 23.947 * 2nd Flr SF

- 177.893 * Total_Bsmt_SF - 26.764 * 1st Flr SF



Analysis of Regression Coefficients

- To analyse the impact of regression coefficients, attributes should be initially assumed to be of equal importance
- This can be done by normalising their values (e.g. Z-transform)
- This has negligible impact on the model performance!
- The largest positive coefficients "Overall_Qual" and "Gr_Liv_Area"
 → better overall quality and larger living area, higher the price
- The figure shows that the <u>price is influenced by many factors</u>



LinearRegression

- 4375.632 * MS_SubClass + 3174.601 * Lot Frontage

+ 4525.996 * Lot_Area + 21517.558 * Overall_Qual + 5193.555 * Overall Cond

+ 8689.946 * Year_Built + 4230.949 * Year Remod/Add

+ 4997.577 * Mas_Vnr_Area + 9818.219 * BsmtFin SF 1

+ 184.897 * BsmtFin_SF_2 + 10619.638 * Total_Bsmt_SF + 9692.426 * 1st Flr SF

+ 11769.446 * 2nd_Flr_SF + 218.693 * Low Qual Fin SF

+ 17521.622 * Gr_Liv_Area + 676.707 * Bsmt Full Bath

- 463.973 * Bsmt_Half_Bath - 740.567 * Full_Bath - 990.634 * Half Bath

- 9736.961 * Bedroom AbvGr



Diagnostic Charts vs Goodness of Fit

| Attribute | Coefficient | Std. Error | Std. Coefficient | Tolerance | t-Stat | p-Value | Code | |
|------------------------|-------------|------------|------------------|-----------|--------|---------|-------|-----------------------|
| Neighborhood = NridgHt | 5030.569 | 2705.756 | 0.015 | 0.790 | 1.859 | 0.063 | * | П |
| Mas_Vnr_Type = None | 10949.336 | 1562.229 | 0.067 | 0.793 | 7.009 | 0.000 | **** | |
| Exter_Qual = Gd | 7920.205 | 5417.798 | 0.047 | 0.777 | 1.462 | 0.144 | | |
| Exter_Qual = TA | 3397.221 | 5079.672 | 0.021 | 0.601 | 0.669 | 0.504 | | 75 |
| Exter_Qual = Ex | 34154.604 | 6636.864 | 0.080 | 0.752 | 5.146 | 0.000 | **** | 70 |
| Foundation = PConc | 2205.366 | 1663.621 | 0.014 | 0.701 | 1.326 | 0.185 | | 65 |
| Bsmt_Qual = Ex | 26953.961 | 2507.857 | 0.097 | 0.651 | 10.748 | 0 | *** | 55 |
| Bsmt_Qual = TA | -867.878 | 1539.929 | -0.005 | 0.776 | -0.564 | 0.573 | | ePrice) |
| Heating_QC = Ex | 4705.045 | 1327.494 | 0.029 | 0.779 | 3.544 | 0.000 | *** | average(SalePrice) |
| Kitchen_Qual = Ex | 24560.037 | 2772.996 | 0.080 | 0.710 | 8.857 | 0 | **** | |
| Kitchen_Qual = TA | -5570.479 | 1524.418 | -0.035 | 0.700 | -3.654 | 0.000 | *** | prediction(SalePrice) |
| Fireplace_Qu = Gd | 3896.583 | 1545.896 | 0.021 | 0.845 | 2.521 | 0.012 | ** | ion(Sal |
| Fireplace_Qu = NA | 5644.848 | 2558.791 | 0.035 | 0.732 | 2.206 | 0.027 | ** | pard 25 |
| Garage_Finish = Fin | 3528.981 | 1471.162 | 0.019 | 0.808 | 2.399 | 0.017 | ** | 15 |
| Garage_Finish = Unf | -1189.953 | 1431.747 | -0.007 | 0.800 | -0.831 | 0.406 | | 10 |
| Overall_Qual | 10564.290 | 688.659 | 0.187 | 0.351 | 15.340 | 0 | **** | : |
| Mas_Vnr_Area | 38.437 | 4.410 | 0.085 | 0.740 | 8.716 | 0 | **** | |
| BsmtFin_SF_1 | 21.585 | 1.459 | 0.119 | 0.846 | 14.793 | 0 | **** | |
| Total_Bsmt_SF | 15.672 | 2.346 | 0.083 | 0.547 | 6.681 | 0.000 | *** | |
| 1st_Flr_SF | 7.683 | 2.585 | 0.036 | 0.548 | 2.972 | 0.003 | *** | П |
| Gr_Liv_Area | 59.985 | 2.430 | 0.370 | 0.488 | 24.681 | 0 | **** | |
| Full_Bath | -4120.002 | 1432.609 | -0.028 | 0.634 | -2.876 | 0.004 | *** | |
| TotRms_AbvGrd | -2177.602 | 596.085 | -0.042 | 0.696 | -3.653 | 0.000 | **** | |
| Fireplaces | 8960.712 | 1944.569 | 0.072 | 0.761 | 4.608 | 0.000 | *** | |
| Garage_Cars | 906.981 | 1673.757 | 0.009 | 0.529 | 0.542 | 0.588 | | |
| Garage_Area | 27.067 | 5.670 | 0.073 | 0.550 | 4.774 | 0.000 | *** | |
| Year_Built | 225.608 | 34.598 | 0.086 | 0.673 | 6.521 | 0.000 | *** | |
| Year_Remod/Add | 209.689 | 37.723 | 0.055 | 0.703 | 5.559 | 0.000 | **** | |
| Garage_Yr_Blt | -24.679 | 38.042 | -0.008 | 0.700 | -0.649 | 0.517 | | |
| (Intercept) | -848706.078 | 92466.655 | ? | ? | -9.179 | 0 | *oksk | |

Coefficients should
have small
p-values < 0.05
Not always – it depends on
var selection methods!

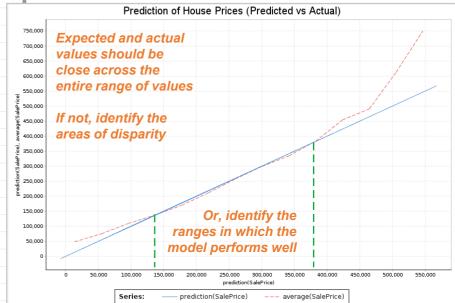
PerformanceVector

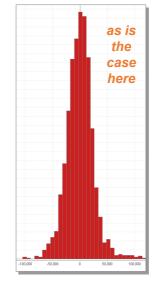
Low RMSE and High R² (from Cross-Validation)

PerformanceVector:

root_mean_squared_error: 26850.937 +/- 7817.039 (mikro: 27960.905 +/- 0.000)
correlation: 0.939 +/- 0.028 (mikro: 0.935)

squared correlation: 0.883 +/- 0.051 (mikro: 0.873)

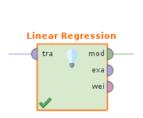


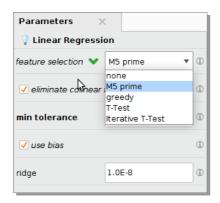


Residuals should be near-normally distributed



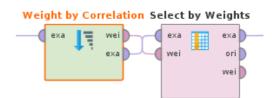
 Regression method require some way of selecting attributes for the model creation.





- Regression models commonly use a stepwise ("greedy") method of attributes selection, i.e. adding or removing attributes to improve the model performance.
- Greedy (or locally optimised) selection is not guaranteed to find the optimum solution.
- Stepwise selection or elimination of attributes usually also leads to model over-fitting the training data.
- An alternative to greedy attribute selection is "M5 prime", which is very effective and a default in RM. It uses regression trees to select the best attributes.

- Selection of the best attributes is called *feature engineering*, including:
 - Feature weighing
 - Feature selection/generation (optimised)



- The simplest is the feature weighing.
- In this approach, attributes are weighed against the label using correlation (high weight = high label and predictor correlation), then you can select top k attributes.
- One way to guarantee selection of the best attributes is to use *brute force*, i.e. trying all possible attrs combinations. This is computationally prohibitive.
- A better approach is to use evolutionary feature engineering, which aims to optimise the model by selecting or generating best features (e.g. using a genetic algorithm).
- However, never discount the simple way
 of selecting attributes –the simplest
 approach can sometimes give the best
 result (or close to optimal).

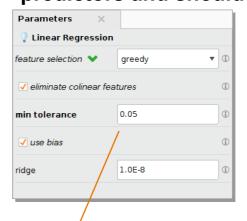


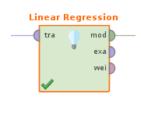
- One of the regression requirements is independence of predictor attributes.
- Analysis of pairwise correlation between predictors in not always sufficient for determining their independence.
- It is possible that a predictor is a linear combination of other predictors; in other words, it is possible to create a regression model to predict one predictor using the others, so that its coefficient of determination R² is high, i.e. R² > 0.8.
- The majority of systems calculate the following statistics for each predictor:

Tolerance =
$$(1 - R^2)$$

VIF = $1 / (1 - R^2)$
VIF = $1 / Tolerance$

- R², tolerance and VIF (variance inflation factor) are clearly related.
- When for some attribute R² > 0.8 then tolerance < 0.2 and VIF > 5, if lower threshold of tolerance is required, e.g. for R2 > 0.95, then tolerance < 0.05 and VIF > 20. In all such cases the attribute would be considered multicollinearily dependent on other predictors and should be removed.



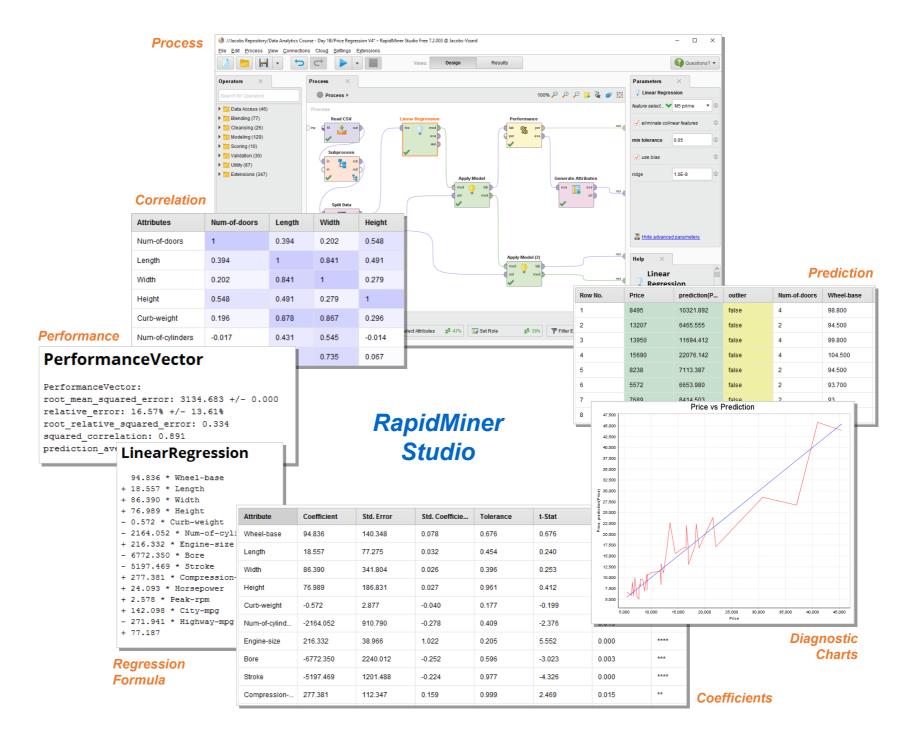


| Attribute | Coefficient | Std. Error | Std. Coeffic | Tolerance | t-Stat | p-Value | Code |
|------------------------|-------------|------------|--------------|-----------|--------|---------|------|
| MS_Zoning = RM | -7932.628 | 1627.039 | -0.036 | 0.922 | -4.875 | 0.000 | **** |
| Lot_Shape = Reg | -2174.858 | 1102.222 | -0.013 | 0.912 | -1.973 | 0.049 | ** |
| Neighborhood = NridgHt | 7350.332 | 2578.598 | 0.022 | 0.797 | 2.851 | 0.004 | *** |
| Neighborhood = NoRidge | 32651.604 | 3642.117 | 0.062 | 0.932 | 8.965 | 0 | **** |
| Exterior_1st = VinylSd | -2134.778 | 1354.327 | -0.013 | 0.866 | -1.576 | 0.115 | |
| Mas_Vnr_Type = None | 9669.770 | 1527.897 | 0.059 | 0.802 | 6.329 | 0.000 | **** |
| Mas_Vnr_Type = Stone | 5738.905 | 2033.239 | 0.020 | 0.901 | 2.823 | 0.005 | *** |
| Exter_Qual = Gd | 3524.525 | 1742.496 | 0.021 | 0.785 | 2.023 | 0.043 | ** |
| Exter_Qual = Ex | 27810.095 | 3885.745 | 0.065 | 0.771 | 7.157 | 0.000 | **** |
| Foundation = CBlock | -2021.385 | 1291.248 | -0.012 | 0.871 | -1.565 | 0.118 | |

In RapidMiner default minimum tolerance is 0.05

It means that RapidMiner is VERY tolerant!







- What are regression model assumptions / requirements?
- What is correlation?
 How different is correlation from causation?
- Why is Pearson correlation useful in regression analysis?
- Can Pearson correlation be applied to nominal attributes?
- Explain regression terms: intercept, slope and residuals.
- What is the difference between Pearson correlation (r) and coefficient of determination (R²)?
- Explain regression pros and cons.
- How can you use scatter plot in regression analysis.

- What are extreme values?
 What is their impact on regression modelling?
- What is multi-collinearity?
 Is it the same as correlation?
 What needs to be done about it?
- What is tolerance?How is it used?
- What is dummy encoding?
 What should be done when we get too many dummy variables?
- Explain the role and method of coefficient analysis.
- Explain the main issues of regression model preparation in business contexts.

