SIT718 Real world Analytics

Trimester 1, 2018

PRACTICE EXAM PAPER 1

Special Instructions

This examination is CLOSED BOOK.
Calculators are ALLOWED.
Writing time is 2 HOURS.

ONLY

Full marks can be obtained by correctly solving five (5) out of six (6) questions.
Formula sheet is attached at the end of the paper.

IMPORTANT NOTE:

- This is a PRACTICE EXAM PAPER - The aim is to give you an idea about the structure of the exam paper.
- The exam can include any part of the material. Programming with R and CPLEX is not required for the exam.

This examination question booklet must be handed in with any used answer booklets.

Question 1 *[20 marks] Aggregation functions - calculations*

(i) Answer the following questions about power means (PM). p is the parameter of power mean.

? $p=1$

TRANSL INVAR

- If $PM(9, 10, 17, 16) = 13$ for some value of p, can we work out the value of $PM(12, 13, 20, 19)$ and $PM(18, 20, 34, 32)$ without knowing p?

HOMOGENOUS

$= 13 \times 2 = 26$

(ii) Calculate the output for each of the following aggregation functions with respect to the input vectors, weighting vectors and additional input parameters indicated.

(a) $\text{WAM}(0.1, 0.9, 0.3, 0.2)$ with $\mathbf{w} = \langle 0.3, 0.6, 0.05, 0.05\rangle$.

(b) $\text{PM}_{\mathbf{w}}(0.7, 0.2, 0.5)$ with $\mathbf{w} = \langle 0.2, 0.25, 0.55\rangle$ and $p = -2$.

$PM_w(0.7, 0, 0.5)$

(c) $\text{GM}_{\mathbf{w}}(0.1, 0.2, 0.5, 0.2)$, with $\mathbf{w} = \langle 0.5, 0.3, 0.1, 0.1\rangle$.

(d) $\text{OWA}(0.2, 0.3, 0.1, 0.9)$, with $\mathbf{w} = \langle 7/16, 5/16, 3/16, 1/16\rangle$.

$x_\nearrow$ 0.1, 0.2, 0.3, 0.9 $\Rightarrow 0.1 \times 7/16 + 0.2 \times \frac{5}{16} + 0.3 \times \frac{3}{16} + 0.9 \times \frac{1}{16}$

(iii) Consider the power mean $\text{PM}_{\mathbf{w}}$ with weighting vector $\mathbf{w} = \langle 0.1, 0.2, 0.1, 0.6\rangle$ and $p = 3$. For an input vector $\mathbf{x} = \langle 0.67, 0.5, 0.39, 0.2\rangle$, which out of x_2, x_4 should we increase if we want to improve the output as much as possible. Explain.

PM_w - for given values → compute; chang x_2, x_4

Question 2 *[20 marks] Interpreting parameters and weights*

[Note: Transformations are within the scope though this practice exam paper does not have a question on this.]

An online website uses customer preferences in order to evaluate the suitability of potential houses. Each house is rated according to the following criteria: Price (x_1), proximity to high achieving Schools (x_2) and proximity to public transport (x_3). All of the ratings are scored on a scale between 0 and 1, with 0 indicating not suitable at all and 1 indicating very suitable.

(a) Interpret the behaviour of the following aggregation functions with respect to their weighting vectors (which would correspond with preferences entered by the user) and parameters. Be sure to comment on the perceived importance of each criterion, whether the function favours high or low inputs, any redundancy between the variables.

i. A weighted arithmetic mean with $\mathbf{w} = \langle 0.1, 0.7, 0.2\rangle$.

ii. A weighted power mean with $p = -2$ and $\mathbf{w} = \langle 0.4, 0.4, 0.2\rangle$.

iii. An OWA function with $\mathbf{w} = \langle 0.6, 0.2, 0.2\rangle$.

iv. A Choquet integral with $v(\{1\}) = 0.8$, $v(\{2\}) = 0.6$, $v(\{3\}) = 0.6$, $v(\{1,2\}) = 0.8$, $v(\{1,3\}) = 0.9$, $v(\{2,3\}) = 0.6$, $v(\{1,2,3\}) = 1$.

(b) Use *either* the power mean (given in ii) or the OWA (given in iii) to compare two houses (a and b) with ratings $\mathbf{x}_a = \langle 0.8, 0.2, 0.5\rangle$ and $\mathbf{x}_b = \langle 0.6, 0.5, 0.4\rangle$ and discuss the results.

$OWA_a = 0.38$ $OWA_b = 0.46$

Question 3 *[20 marks] Data Analysis*

A personal fitness company is looking into factors that affect the success of its Gyms in different suburbs across Melbourne. They collect data on three features:

Feature 1: ***Age*** - The age of residents (adults) within 5 km of the gym.

Feature 2: ***Personal income*** - The average income of residents within 5km of the gym.

Feature 3: ***Fitness centres*** - The number of fitness and recreation centres (including pools, stadiums etc) within 5km of the gym location.

Based on their 34 Gyms, the fitness company uses linear programming to learn aggregation weights associated with a weighted mean, an OWA function, a power mean ($p = 2$) and the Choquet integral, where the output corresponds with the number of gym memberships. They also calculate the accuracy of an unweighted arithmetic mean. The data was first transformed, with a high score in the 'age' feature indicating a higher number of residents near the gym were in the 19 - 35 age bracket (the most popular for gym memberships), higher incomes were associated with a high personal income score and a high value in 'fitness centres' indicated that there were very few fitness centres in the nearby area.

Function	Learned weights	L1 error	RMSE
AM	n/a	0.2153	0.2711
WAM	$\langle 0.318, 0.511, 0.171 \rangle$	0.1633	0.1839
PM ($p = 2$)	$\langle 0.200, 0.501, 0.299 \rangle$	0.1943	0.2388
OWA	$\langle 0.390, 0.396, 0.214 \rangle$	0.1201	0.1371
Choquet	$v(\{0\}) = 0$ $v(\{1\}) = 0.219$ $v(\{2\}) = 0.121$ $v(\{3\}) = 0$ $v(\{1,2\}) = 0.388$ $v(\{1,3\}) = 0.891$ $v(\{2,3\}) = 0.722$ $v(\{1,2,3\}) = 1$	0.1091	0.1135

$v\{1\} + v\{2\} =$; $v\{1,3\} > v\{1\} + v\{3\}$; $v\{2,3\} > v\{2\} + v\{3\}$

Interpret the results with regard to:

(a) Differences in the goodness of fit measures.

(b) The fitted weighting vectors and what they imply about (i) the importance of each variable in predicting the success of a gym (ii) whether the models tend towards high or low values, (iii) redundancies or complementary effects between any of the variables.

Question 4 *[20 marks] Linear Programming with 2 variables - graphical methods*

[Note: Linear Programming with more variables is within the scope though this practice exam paper does not have a question on this.]

Mary has been diagnosed to be having a large malignant tumour in the bladder area. She will receive radiation therapy. The treatment team has decided that radiation will be delivered from 2 beam angles for the treatment. The areas of the body that must be considered are listed in the first column of the table below. Column 2 provides the fraction of the radiation dose at the entry point for Beam 1 that is absorbed by the respective areas on average. E.g., if the dose level at the entry point for Beam 1 is 10 (Gray) Gy, then an average of 4 Gy; 3Gy; 5Gy; and 7Gy will be absorbed by the healthy anatomy; nearby critical tissues; entire tumour region; and the centre of the tumour respectively. Similar data are presented for Beam 2 in Column 3.

The last column of the table presents the restrictions on the total dosage from both beams that is absorbed on average by the respective areas of the body. The average dosage absorption for the healthy anatomy should be as small as possible, that for the critical tissues must not exceed 27 Gy, the average over the entire tumour region must be no less than 60 Gy, and the centre of the tumour must be at least 70 Gy. The decision problem is to decide how much radiation (in Gy) is to be delivered for the two beam angles.

Design Mary's radiation therapy plan by formulating the decision problem as a Linear Programming model in order to decide how much radiation (in Gy) should each beam-angle deliver, and solve the LP using the graphical method.

	Fraction of entry dose absorbed by area (average)		
Area	Beam 1	Beam 2	Restriction on total avg dosage (Gy)
Healthy anatomy	0.4	0.5	Minimize
Critical tissues	0.3	0.1	≤ 27
Entire tumour region	0.5	0.5	≥ 60
Centre of tumour	0.7	0.35	≥ 70

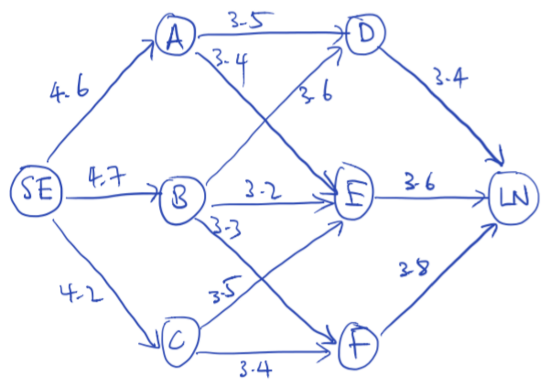
Question 5 *[20 marks] Game theory, Shortest Path, critical path Problems*

[Note: LP models of games with no saddle point are within the scope. Critical path problems are within the scope of the exam as well, even though this practice exam paper does not have a question on this.]

(i) Show that the following game has an optimal pure strategy pair. identify the optimal pure strategies for each of the players.

	A_1	A_2	A_3	A_4
a_1	5	2	1	3
a_2	3	3	-2	4
a_3	-1	3	-1	-3
a_4	2	-4	1	-2

(ii) One of Disunited Airlines' flights is about to take off from Seattle for a nonstop flight to London. There is some flexibility in choosing the precise route to be taken,



depending upon weather conditions. The following network depicts the possible routes under consideration, where SE and LN are Seattle and London, respectively, and other nodes represent various intermediate locations.

The winds along each arc greatly affect the flying time (and so the fuel consumption). Based on current meteorological reports, the flying times (in hours) for this particular flight are shown next to the arcs. because the fuel consumed is so expensive, the management of Disunited Airlines has established a policy of choosing the route that minimizes the total flight time.

Use the Dijkstra's algorithm to find the path from Seattle to London with the shortest flight time.

Question 6 *[20 marks]Transportation Problems*

One of the main products of iHatePeas Company is canned peas. The peas are prepared at 3 canneries (near Goulburn Valley, Victoria; Mitchellstown, Whiteheads Creek, and Locksley), and then shipped to four major warehouses in Australia; Melbourne, Sydney, Brisbane, and Adelaide. As shipping costs are a major expense, management is initiating a study to reduce them as much as possible. The table below details the shipping costs, the supply from each cannery, and the demand of each city.

Total Supply =

Total Demand =

	Shipping costs ($) per Truckload				
Canneries	Melbourne	Sydney	Brisbane	Adelaide	Supply
Mitchellstown	464 x_{11}	513 x_{12}	654 x_{13}	867 x_{14}	75
Whiteheads Creek	352	416	690	791	125
Locksley	995	682	388	685	100
Demand	80	65	70	85	

Formulate an LP model to assign shipments to the various canneries–warehouses combinations that would minimize the total shipment cost.

[Note: Transportation problems with shortages or surplus are within the scope of the exam as well]

Formula Sheet:

Arithmetic Mean:

$$AM(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Geometric Mean:

$$GM(\mathbf{x}) = \left(\prod_{i=1}^{n} x_i\right)^{1/n} = (x_1 x_2 \cdots x_n)^{1/n}.$$

Harmonic Mean:

$$HM(\mathbf{x}) = n\left(\sum_{i=1}^{n} \frac{1}{x_i}\right)^{-1} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}.$$

Power Mean:

$$PM_p(\mathbf{x}) = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^p\right)^{\frac{1}{p}} = \left(\frac{x_1^p + x_2^p + \cdots + x_n^p}{n}\right)^{\frac{1}{p}}.$$

Weighted Arithmetic Mean:

$$WAM_{\mathbf{w}}(\mathbf{x}) = \sum_{i=1}^{n} w_i x_i = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n$$

Weighted Power Mean:

$$PM_{\mathbf{w}}(\mathbf{x}) = \left(\sum_{i=1}^{n} w_i x_i^p\right)^{\frac{1}{p}}$$

Weighted Geometric Mean:

$$GM_{\mathbf{w}}(\mathbf{x}) = \prod_{i=1}^{n} x_i^{w_i} = x_1^{w_1} \times x_2^{w_2} \times \cdots \times x_n^{w_n}$$

Weighted Harmonic Mean:

$$HM_{\mathbf{w}}(\mathbf{x}) = \left(\sum_{i=1}^{n} \frac{w_i}{x_i}\right)^{-1} = \frac{1}{\frac{w_1}{x_1} + \frac{w_2}{x_2} + \cdots + \frac{w_n}{x_n}}$$

Ordered Weighted Averaging (OWA):

$$OWA_{\mathbf{w}}(\mathbf{x}) = \sum_{i=1}^{n} w_i x_{(i)} = w_1 x_{(1)} + w_2 x_{(2)} + \cdots + w_n x_{(n)}$$

where $x_{(\cdot)}$ indicates that the arguments have been rearranged into non-decreasing order.

Orness of OWA$=\sum\limits_{i=1}^{n} w_i \frac{i-1}{n-1}$

Choquet integral (discrete):

$$C_v(\mathbf{x}) = \sum_{i=1}^{n} x_{[i]}(v(\{[i],[i+1],\ldots,[n]\}) - v(\{[i+1],[i+2],\ldots,[n]\}))$$

where $[i]$ means the inputs are arranged from lowest to highest and v are fuzzy measures.

Dijkstra's Algorithm:

Step 0

Set $=\emptyset$ (Initialise set of permanently labelled vertices)

Step 1

Set $P_1 = 0$ and $=\{1\}$ (Set permanent label of 1 to 0)
$\forall i \in V \setminus \{1\}$, s.t., $(1,i) \in A$, set $T_i = c_{1,i}$
$\forall i \in V \setminus \{1\}$, s.t., $(1,i) \notin A$, set $T_i = \infty$

Step 2

Repeat
Find $i^* = \arg\min_{i \in V\setminus}\{T_i\}$
Set $P_{i^*} = T_{i^*}$
Set $= \cup\{i^*\}$
$\forall i \in V\setminus$, s.t. $(i^*, i) \in A$, set $T_i = \min\{T_i, P_{i^*} + c_{i^*,i}\}$
Until $V\setminus = \emptyset$.

Transportation Problems–LP formulation:

$$\begin{aligned}
\min z = {} & (c_{11}x_{11} + c_{12}x_{12} + \cdots + c_{1n}x_{1n}) \\
& + (c_{21}x_{21} + c_{22}x_{22} + \cdots + c_{2n}x_{2n}) \\
& + \cdots + (c_{m1}x_{m1} + c_{m2}x_{m2} + \cdots + c_{mn}x_{mn}) \\
\text{s.t.} \quad & x_{i1} + x_{i2} + \cdots + x_{in} \leq \; s_i, \text{ one for each supply point } i \\
& x_{1j} + x_{2j} + \cdots + x_{mj} \geq \; d_j, \text{ one for each demand point } j \\
& x_{ij} \geq \; 0 \quad \text{ for each } i = 1\ldots, m; \; j = 1, \ldots, n
\end{aligned}$$

where x_{ij} is the number of units shipped from supply point i to demand point j.