MATH268:Assignment 2

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Exercise 1 *John is gambling on the results of football games. He can bet any amount up to the total amount of money he currently has. For any amount that he bets, he can either gain the same amount he is betting or lose the amount he bet with probability $p \in (0,1)$ and $1-p$, respectively.*

John is allowed to bet on n football games. He wants to maximize the expectation of the logarithm of his final fortune.

What is the strategy needed to maximize the expectation of the logarithm of his final fortune when $p \leq 1/2$? What is the optimal value if the gambler has initial fortune $x > 0$?

[30]

Exercise 2 *Let A_n be the price of a banking share on the nth day ($n \geq 1$).*

The relation between the price in different days is

$$A_{n+1} = A_n + Y_{n+1} = A_0 + \sum_{i=1}^{n+1} Y_i,$$

where $Y_1, Y_2, \ldots$ are i.i.d. with common distribution function B. Assume $Y_1, Y_2, \ldots$ are independent on the initial price A_0 and the mean $E[Y_i] := \mu_B < 0, i = 1, 2, \cdots$ is finite. (We allow the "stock" price to fall below 0.)

Suppose that you own an option to buy one share of the stock at a fixed price, say a, and you have K days in which to exercise the option. You are not obliged to exercise it, but if you do at a time when the stock's price is s, then your profit is $s - c$ (i.e., buy a share of stock at c and immediately sell it at the price s). You are interested in the policy maximizes your expected profit.

(a) *Write down the optimality equation. (You may formulate a suitable dynamic programming problem with the following meaning of states: s_n is the stock price, and the option has not been exercise, at the day n.)*

[40]

(b) *Prove that $V_n(s) - s$ is decreasing in s.*

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