

SIT718 Real world Analytics
Assessment Task 3
Total Marks = 100, Weighting - 30%

1. A food factory is making a beverage for a customer from mixing two different existing products A and B. The compositions of A and B and prices (\$/L) are given as follows,

	Amount (L) in /100 L of A and B			
	Lime	Orange	Mango	Cost (\$/L)
A	2	6	4	5
B	7	4	8	15

The customer requires that there must be at least 5 Litres (L) Orange and at least 5 Litres of Mango concentrate per 100 Litres of the beverage respectively, but no more than 6 Litres of Lime concentrate per 100 Litres of beverage. The customer needs at least 150 Litres of the beverage per week.

- a) Explain why a linear programming model would be suitable for this case study.

[5 marks]

- b) Formulate a Linear Programming (LP) model for the factory that minimises the total cost of producing the beverage while satisfying all constraints.

[5 marks]

- c) Use the graphical method to find the optimal solution. Show the feasible region and the optimal solution on the graph. Annotate all lines on your graph.

[5 marks]

Note: you can use graphical solvers available online but make sure that your graph is clear, all variables involved are clearly represented and annotated, and each line is clearly marked and related to the corresponding equation.

- d) What is the range for the cost (\$) of A that can be changed without affecting the optimum solution obtained above?

[5 marks]

2. A factory makes three products called Spring, Autumn, and Winter, from three materials containing Cotton, Wool and Silk. The following table provides details on the sales price, production cost and purchase cost per ton of products and materials respectively.

	Sales price	Production cost	Purchase price	
Spring	\$60	\$5	Cotton	\$30
Autumn	\$55	\$3	Wool	\$45
Winter	\$60	\$5	Silk	\$50

The maximal demand (in tons) for each product, the minimum cotton and wool proportion in each product is as follows:

	Demand	min Cotton proportion	min Wool proportion
Spring	3600	55%	30%
Autumn	3300	45%	40%
Winter	4000	30%	50%

- a) Formulate an LP model for the factory that maximises the profit, while satisfying the demand and the cotton and wool proportion constraints. There is no penalty for the shortage.

[20 Marks]

- b) Solve the model using R/R Studio. Find the optimal profit and optimal values of the decision variables.

[20 Marks]

Hints:

You may refer to Week 8.7 Example - Blending Crude Oils into Gasolines. For example, let $x_{ij} \geq 0$ be a decision variable that denotes the number of tons of products j for $j \in \{1 = \text{Spring}, 2 = \text{Autumn}, 3 = \text{Winter}\}$ to be produced from Materials $i \in \{C=\text{Cotton}, W=\text{Wool}, S=\text{Silk}\}$.

3. Consider the following parlor game to be played between two players. Each player begins with three chips: one red, one white, and one blue. Each chip can be used only once. To begin, each player selects one of her chips and places it on the table, concealed. Both players then uncover the chips and determine the payoff to the winning player. In particular, if both players play the same kind of chip, it is a draw; otherwise, the following table indicates the winner and how much she receives from the other player. Next, each player selects one of her two remaining chips and repeats the procedure, resulting in another payoff according to the following table. Finally, each player plays her one remaining chip, resulting in the third and final payoff.

Winning Chip	Payoff (\$)
Red beats white	250
White beats blue	100
Blue beats red	50
Matching colors	0

- (a) Formulate the payoff matrix for the game and identify possible saddle points. [10 Marks]
- (b) Construct a linear programming model for each player in this game. [10 Marks]
- (c) Produce an appropriate code to solve the linear programming model for this game. [10 Marks]
- (d) Solve the game for both players using the linear programming model. [10 Marks]

[Hint: Each player has the same strategy set. A strategy must specify the first chip chosen, the second and third chips chosen. Denote the white, red and blue chips by W, R and B respectively. For example, a strategy “WRB” indicates first choosing the white and then choosing the red, before choosing blue at the end.]