COMS20011 – Data-Driven Computer Science

Problem Sheet Solutions MM04

1 – How would low pass filtering be achieved using the Fourier domain? In your answer describe what is meant by Cut-off Frequency.

Answer:

Low pass filtering can be achieved by removing higher frequency information in the Fourier space, i.e. by retaining and letting lower frequencies pass through a filtering operation. Example filters are the ideal low pass filter and the Butterworth low pass filter. Some filter types have an abrupt cut-off point above which no higher frequencies are passed through, while others, like a Butterworth or Gaussian based filters, are more smoothly varying and do not have an abrupt cut-off point.

2 – Consider you are given the Fourier Transform space of an image. Using simple descriptions or sketches to illustrate your answer, how would you select relevant regions to extract spectral features from

(a) only low frequency regions,
(b) only the very high frequency regions corresponding to prominent variations in intensity in the image that are at around 45 degrees to the horizontal,
(c) all approximately mid-range frequencies.

Answer:

Using conjugate symmetry, we can ignore the bottom half of the Fourier space and extract features from only the top half. We can then extract features from regions defined as

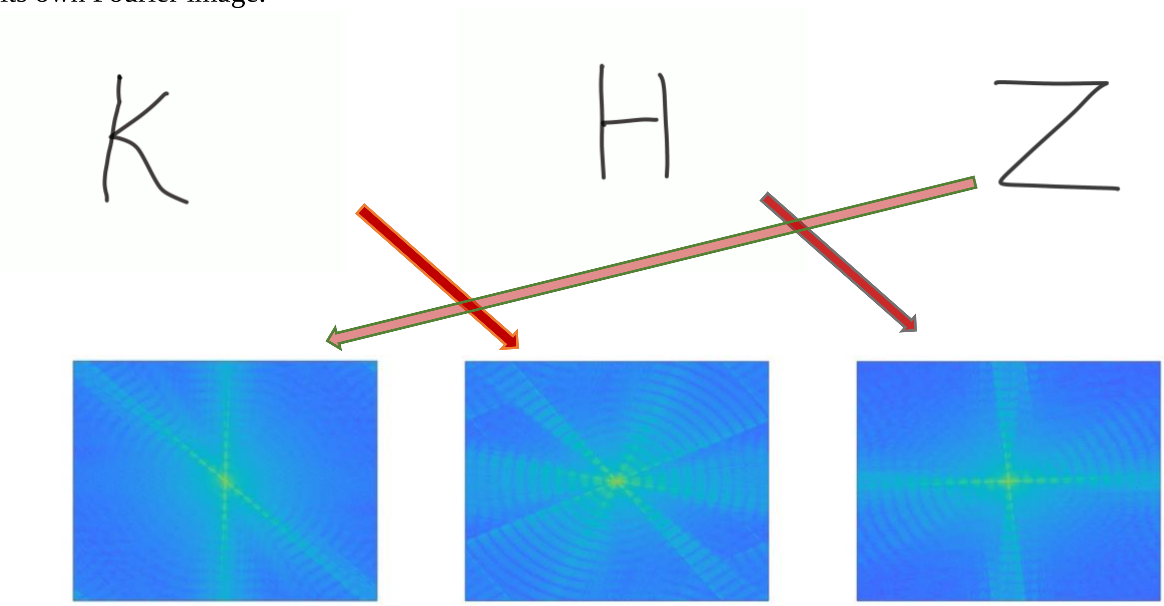
(a) for example a half
disc-shaped region with its centre at the centre of the Fourier space, i.e. at (u=0,v=0)

(b) a bar-shaped region with a small width, say 10, starting at around
(u= - max(ufreq.)/2, v= max(vfreq.)/2) angled at 135 degrees in the Fourier space

(c) a half-ring of a reasonable width, up to a maximum of 2*max(ufreq)/3, and starting from around (u=max(ufreq)/3, v=0).

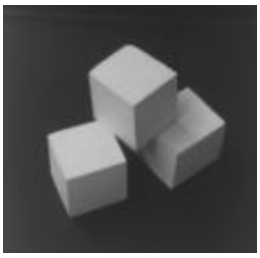
NOTE: Exact u,v coordinates are not necessary, but approximations plus sketches should give the right indication, e.g. the half-disc must clearly be said to be at the centre of the space.

3 – Here are images of three handwritten letters. Their Fourier spaces are randomly shown. Match each image with its own Fourier image.



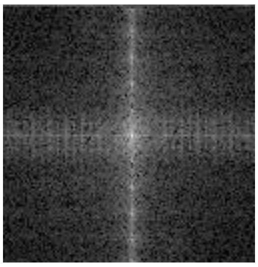
4 – Similar to the previous question, consider the two images (Sugar and Bricks) on the left. Identify which of the Fourier spaces (FS1 and FS2) on the right belongs to which image and explain briefly why.

Sugar

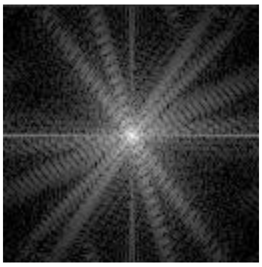


Bricks

FS1



FS2

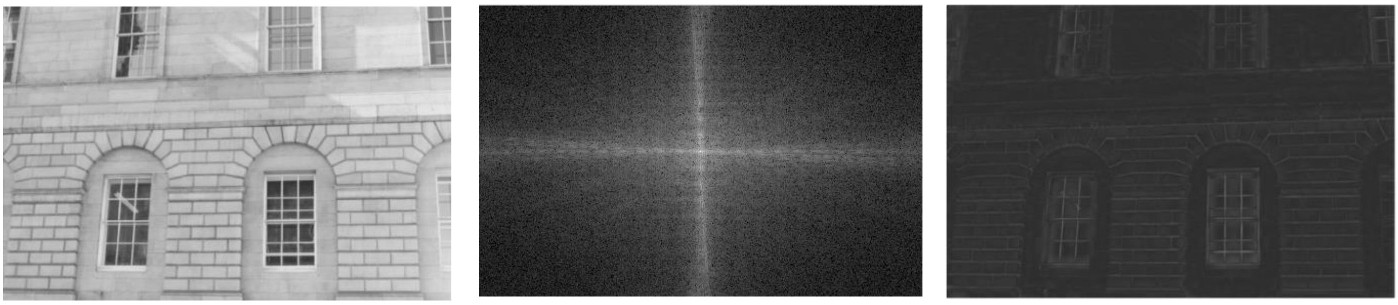


FS2 belongs to the sugar blocks image and FS1 belongs to the brick image. The high magnitude frequencies in FS1 are for the Brick image as they clearly signify the presence of very strong horizontal and vertical lines in that image. The angled lines in the sugar blocks image result in the strong non-horizontal and non-vertical directional lines in FS2.

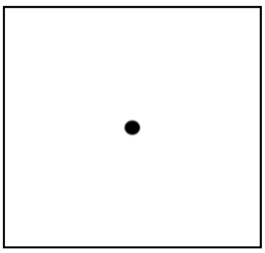
5 – Rotate an object, Fourier space rotates too. Translate an object, Fourier space translates too.

(a) Both statements are **True**.
(b) First statement is True and second one is False.
(c) First statement is **False** and the second one is **True**.
(d) Both statements are **False**.

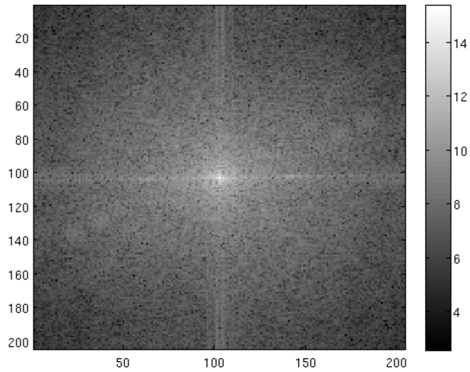
6 – The figure below on the left shows an image of a building wall, with its Fourier Space magnitudes shown in the middle. A reconstructed image (inverse FFT image), after some manipulation of the Fourier magnitudes, is shown on the right. How should the Fourier space be manipulated (e.g., what kind of a mask could have been applied to it) to achieve this reconstructed result? Include a sketch to illustrate your answer.



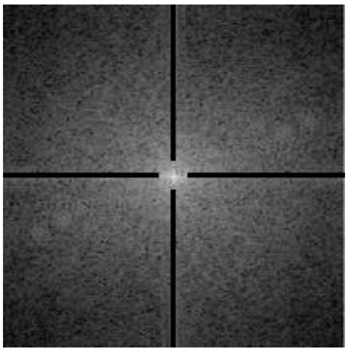
Much of the contrast has been removed and an almost edge-map of the image has resulted. Edges signify high frequency changes in the image pixels. Hence, all this evidence points to a loss of low frequency magnitudes. The mask applied to the Fourier space magnitudes is therefore something similar to this:



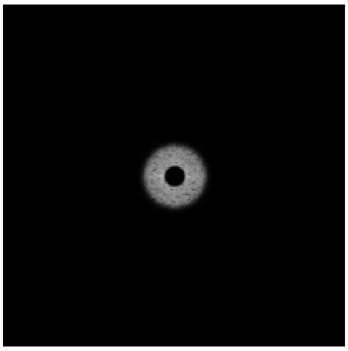
7 – Below is an original image of a clown and its Fourier space after an FFT operation.



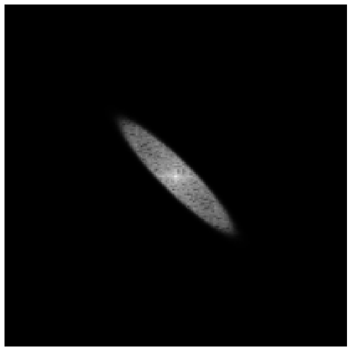
Next, there are three images, labelled (A;B;C), in each case after applying a specific mask to the Clown's Fourier space.



A



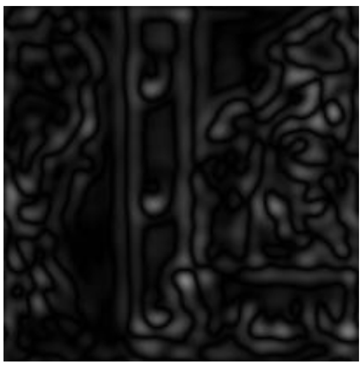
B



C

Below, there are three results, labelled (X;Y;Z) that represent in an arbitrary order, the inverse FFT of the Fourier spaces in (A;B;C) above. Explain which inverse FFT space corresponds to which filtered image.

X



Y

Z

A results in Z because it is clear that many horizontal and vertical frequencies are eroded except those around the centre. All other frequencies remain too. B results in Y because this band pass filter loses key low frequencies and much of the mid-range to high frequencies. The outcome shows little contrast and no sharp changes. Finally, C produces X. Contrast is somewhat preserved and sharpness is lost, but what exists is very directional as many frequencies are missing along other directions.

8 – What are the two 1D filters that can replace the 2D filter (in each example for W and X) if they were applied consecutively?

$$W = \frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} x \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} x \begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$$