Pearson

Chapter 10: Hypothesis testing: Two-sample tests

After studying this chapter you should be able to:
1. conduct hypothesis tests for the means of two independent populations
2. conduct hypothesis tests for the means of two related populations
3. conduct hypothesis tests for the variances of two independent populations
4. conduct hypothesis tests for two population proportions

10.1 $$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(72-66)-0}{\sqrt{\frac{10^2}{30} + \frac{15^2}{30}}} = 1.8229$$

10.2 Z = 1.8229

Decision rule: If Z_{calc} > 1.96 or Z < –1.96, reject H_0.

Decision: Since Z_{calc} = 1.8229 is greater than the critical bound of 1.96, reject H_0. There is enough evidence to conclude that the first population mean is different to the second population mean.

10.3 p-value = 2(1.0–0.9656) = 0.0688, where 0.9656 is a cumulative probability for Z = 1.82.

10.4 (a) $$S_p^2 = \frac{(n_1-1)\,S_1^2 + (n_2-1)\,S_2^2}{(n_1-1)+(n_2-1)} = \frac{(20)\,4^2 + (17)\,5^2}{19+16} = 21.29$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{(53-48)-0}{\sqrt{21.29\left(\frac{1}{20}+\frac{1}{17}\right)}} = 3.29$$

(both formula updated)

(b) $df = (n_1 - 1) + (n_2 - 1)$ = 19 + 16 = 35

(c) Decision rule: df = 35. If t_{calc} > 2.4377, reject H_0.

(d) Decision: Since t = 3.29 is greater than the critical bound of XXXX, reject H_0. There is enough evidence to conclude that the first population mean is larger than the second population mean.

10.5 Assume that you are sampling from two independent normal distributions with equal variances.

10.6 $$(\bar{X}_1 - \bar{X}_2) \pm t\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = (53-48) \pm 2.031\sqrt{21.29\left(\frac{1}{20}+\frac{1}{17}\right)}$$ formula changed

$1.81 \le \mu_1 - \mu_2 \le 8.09$ changed

10.7 question needs to say assume unequal variances

(a) $H_0: \mu_1 - \mu_2 \geq 0$ The mean estimated amount of calories in the cheeseburger is not lower for the people who thought about the cheesecake first than for the people who thought about the organic fruit salad first

$H_0: \mu_1 - \mu_2 < 0$ The mean estimated amount of calories in the cheeseburger is lower for the people who thought about the cheesecake first than for the people who thought about the organic fruit salad first

(b)

PHStat output:

Separate-Variances *t* Test for the Difference Between Two Means	
(assumes unequal population variances)	
Data	
Hypothesized Difference	**0**
Level of Significance	**0.01**
Population 1 Sample	
Sample Size	**20**
Sample Mean	**780**
Sample Standard Deviation	**128**
Population 2 Sample	
Sample Size	**20**
Sample Mean	**1041**
Sample Standard Deviation	**140**
Intermediate Calculations	
Numerator of Degrees of Freedom	3237120.6400
Denominator of Degrees of Freedom	85867.8232
Total Degrees of Freedom	37.6989
Degrees of Freedom	37
Separate Variance Denominator	42.4170
Difference in Sample Means	-261
Separate-Variance *t* Test Statistic	**-6.1532**
Lower-Tail Test	
Lower Critical Value	**-2.4314**
***p*-Value**	**0.0000**
Reject the null hypothesis	

Decision rule: if $t_{calc} < -2.4314$, reject H_0

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = -6.1532$$

Decision: Since -6.1532 < 2.4314, reject H_0. There is evidence that the mean estimated amount of calories in the cheeseburger is lower for the people who thought about the cheesecake first than for the people who thought about the organic fruit salad first

10.8 (a) H_0: μ1 – μ2 ≤ 0
H_1: μ1 – μ2 > 0

where population 1 = private school students population 2 = public school students.

$df = (n_1 - 1) + (n_2 - 1) = 53 + 31 = 84$

Decision rule: If $t_{calc} > 2.3733$, reject H_0.

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(53)12^2 + (31)10^2}{53 + 31} = 127.76$$

Test statistic: $$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{76.5 - 74.7}{\sqrt{127.76\left(\frac{1}{54} + \frac{1}{32}\right)}} = 0.7138$$

Decision: Since 0.7138 < 2.3733, do not reject H_0. There is not enough evidence that private school students outperform public school students.

10.9 Assuming that the variance e of the weight loss of the high-protein diet and high-carbohydrate diet are the same, the appropriate test to perform is the pooled-variance test.

(a) $H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 \neq 0$

(b) A Type I error is committed when one concludes that there is a difference in mean weight loss between the two diets when there is no significant difference.

(c) A Type II error is committed when one concludes that there is no significant difference in mean weight loss between the two diets when there is indeed significant difference.

(d) $$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(99)3.2^2 + (99)3.9^2}{(199)} = 12.725$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(7.6 - 6.7) - 0}{\sqrt{12.725\left(\frac{1}{100} + \frac{1}{100}\right)}} = 1.7840$$

Critical values = ±1.9720

Decision: Since $t_{calc} = 1.7840$ is between the critical bounds of ±1.9720, do not reject H_0. There is no evidence of a difference between the mean weight loss of obese patients in the high-protein and high-carbohydrate diets.

10.10 $H_0: \mu_1 - \mu_2 \leq 0$

$H_1: \mu_1 - \mu_2 > 0$

where Campbelltown = population 1 and rest of Sydney = population 2

$df = (n_1 - 1) + (n_2 - 1) = 222 + 222 = 444$

Decision rule: If $t_{calc} > 1.645$, reject H_0.

$$S_p^2 = \frac{(n_1-1)\,S_1^2 + (n_2-1)\,S_2^2}{(n_1-1)+(n_2-1)} = \frac{(222)12.1^2 + (222)12.7^2}{222+222} = 153.85$$

$$t_{calc} = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{140.1 - 131.4}{\sqrt{153.85\left(\frac{1}{222} + \frac{1}{222}\right)}} = 7.3898$$

Decision: Since t_{calc} = 7.3898 > 1.645, reject H_0. There is evidence to suggest that the average petrol price in Campbelltown is greater than the rest of Sydney.

10.11 (a) Populations 1 = female, 2 = male

$$H_0 : \mu_1 - \mu_2 = 0$$
$$H_1 : \mu_1 - \mu_2 \neq 0$$

Decision rule: *df* = 58. If t > 2.0017 or < -2.0017, reject H_0.

$$S_p^2 = \frac{(n_1-1)\,S_1^2 + (n_2-1)\,S_2^2}{(n_1-1)+(n_2-1)} = \frac{(29)57.78^2 + (29)205.72^2}{29+29} = 22830.24 \text{ new}$$

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(79 - 157.17) - 0}{\sqrt{22830.24\left(\frac{1}{30} + \frac{1}{30}\right)}} = -2.0036 \text{ new}$$

Decision: Since t_{calc} = -2.0036 is less than the lower critical bound of -2.0017, reject H_0. There is sufficient evidence to conclude thatthere is enough evidence of a difference in the mean time spent on Facebook per day between males and females

(assumes equal population variances)

Data	
Hypothesized Difference	0
Level of Significance	0.05
Population1Sample	
Sample Size	30
Sample Mean	79
Sample StandardDeviati	57.78079862
Population2Sample	
Sample Size	30
Sample Mean	157.1666667
Sample StandardDeviati	205.7227936
Intermediate Calculations	
Population 1Sample Deg	29
Population 2Sample Deg	29
Total Degrees of Freedo	58
Pooled Variance	22830.24425
Standard Error	39.0130
Difference in Sample Me	-78.16666667
t Test Statistic	-2.0036
Two-Tail Test	
Lower Critical Value	-2.0017
Upper Critical Value	2.0017
p-Value	0.0498
Reject the null hypothesis	

Confidence Interval Estimate forthe Difference BetweenTwoMeans	
Data	
Confidence Level	95%
Intermediate Calculations	
Degrees of Freedom	58
t Value	2.0017
Interval Half Width	78.0931
Confidence Interval	
Interval Lower Limit	-156.2597
Interval Upper Limit	-0.0736

(b) You must assume that each of the two independent populations is normally distributed.

10.12 (a) $H_0: \mu_1 - \mu_2 = 0$ where populations 1 = line A, 2 = line B

$H_1: \mu_1 - \mu_2 \neq 0$

Decision rule: *df* = 25. If $|t_{calc}| > 2.0595,$ reject *H*0.

$$S_p^2 = \frac{(n_1 - 1)\,S_1^2 + (n_2 - 1)\,S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$S_p^2 = \frac{(10)\,0.615 + (15)\,0.706}{10 + 15} = 0.0751$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t = \frac{(410.25 - 409.85) - 0}{\sqrt{0.0751\left(\frac{1}{11} + \frac{1}{16}\right)}} = 3.7276$$

Since *t* = 3.7276 > 2.0595 or *p*-value = 0.0010 < 0.05, reject *Ho*. There is sufficient evidence of a difference in the mean weight of cans filled on the two lines.

(b) $H_0: \mu_1 - \mu_2 = 0$ where populations 1= line A, 2 = line B

$H_1: \mu_1 - \mu_2 \neq 0$

Decision rule: *df* = 12. If $|t_{calc}| > 2.1788,$ reject *H*0.

Test statistic:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$t = \frac{(410.25 - 409.89) - 0}{\sqrt{\frac{0.615^2}{11} + \frac{0.706^2}{16}}} = 1.5625$$

Since *t* = 1.5625 < 2.1788 or *p*-value = 0.1441 > 0.05, do not reject *H*0. There is not sufficient evidence of a difference in the mean weight of cans filled on the two lines.

(c) The results from (a) and (b) are different. The results obtained from (b) may be more reliable because the sample variances from both samples suggest that the two population variances are not likely to be equal.

10.13 (a) $H_0: \mu_1 - \mu_2 \geq 0$

$H_1: \mu_1 - \mu_2 < 0$

$$S_p^2 = \frac{(n_1-1)\,S_1^2 + (n_2-1)\,S_2^2}{(n_1-1)+(n_2-1)} = \frac{(33)(6.25)+(44)(25)}{77} = 16.96$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_P^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{(8-9.5)-0}{\sqrt{16.96\left(\frac{1}{34}+\frac{1}{45}\right)}} = 1.603$$

df = 45 + 34 – 2 = 77
Decision rule: Reject H_0 if $t_{calc} < -1.6649$.

Since $t_{calc} = -1.603 > -1.6649$, do not reject H_0.
There is not enough evidence that the mean waiting time at the Bank of Singapore is lower than that at the competitor's bank.

(b) $$\left(\bar{X}_1 - \bar{X}_2\right) \pm t\sqrt{S_P^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = (8-9.5) \pm 1.99\sqrt{16.96\left(\frac{1}{34}+\frac{1}{45}\right)}$$

$-3.362 \le \mu_1 - \mu_2 \le 0.362$

We are 95% confident that the difference in mean time waiting time between Bank of Singapore and the competitor's bank is between –3.362 and 0.362.

10.14

$H_0 : \mu_1 - \mu_2 \ge 0$

$H_0 : \mu_1 - \mu_2 < 0$

Degrees of freedom = 68

$$t_{calc} = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\left(\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right)}}$$

$$t = \frac{(9.5-8)-0}{\sqrt{\frac{6.25}{34}+\frac{25}{25}}} = -1.744$$

Decision rule: Reject H_0 if $t_{calc} < -1.667$.

Since $t_{calc} = -1.744 < -1.667$, do not reject H_0.
There is sufficient evidence that the mean waiting time at the Bank of Singapore is lower than that at the competitor's bank.
The outcomes of the tests in 10.13(a) and 10.14 are different. The value of pooled t_{calc} in 10.13(a) is slightly lower and should be more reliable as variances at the two banks appear to be different.

10.15 (a) $H_0 : \mu_1 - \mu_2 = 0$ M Mean times to clear problems at Office I and Office II are the same.

$H_1 : \mu_1 - \mu_2 \ne 0$ Mean times to clear problems at Office I and Office II are different.

$$S_P^2 = \frac{(n_1-1)\,S_1^2 + (n_2-1)\,S_2^2}{\sqrt{(n_1-1)+(n_2-1)}} = 3.265 \text{ new}$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_1)}{\sqrt{S_P^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 0.354 \text{ new}$$

Reject H_0 if $t_{calc} > 2.024$ or $t_{calc} < -2.024$.
Since $t_{calc} = 0.354$, that is < 2.024, do not reject.

(b)

t Test for Differences in Two Means	
Data	
Hypothesized Difference	**0**
Level of Significance	**0.05**
Population 1 Sample	
Sample Size	**20**
Sample Mean	**2.214**
Sample Standard Deviation	**1.718039**
Population 2 Sample	
Sample Size	**20**
Sample Mean	**2.0115**
Sample Standard Deviation	**1.891706**
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	19
Population 2 Sample Degrees of Freedom	19
Total Degrees of Freedom	38
Pooled Variance	3.265105
Difference in Sample Means	0.2025
t-Test Statistic	0.354386
Two-Tailed Test	
Lower Critical Value	**-2.02439**
Upper Critical Value	**2.024394**
***p*-Value**	**0.725009**
Do not reject the null hypothesis	

p-value = 0.725. The probability of obtaining a sample that will yield a *t* test statistic more extreme than 0.354 is 0.725 if, in fact, the mean times for Office I and II are the same.

(c)We need to assume that the two populations are normally distributed.

(d) $(\bar{X}_1 - \bar{X}_2) \pm t\sqrt{S_P^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = (2.214 - 2.0115) \pm 2.024\sqrt{3.265\left(\frac{1}{20} + \frac{1}{20}\right)}$ new

$-0.9543 \le \mu_1 - \mu_2 \le 1.3593$

Since the Confidence Interval contains 0, we cannot claim that there's a difference between the two means

10.16 $H_0 : \mu_{I} - \mu_{II} = 0$ Mean times to answer queries by Team I and Team II are the same.
$H_1 : \mu_{I} - \mu_{II} \neq 0$ Mean times to answer queries by Team I and Team II are different.

Degrees of freedom =38

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$t = \frac{(2.719 - 2.6615) - 0}{\sqrt{\frac{2.482}{20} + \frac{2.813}{20}}} = 0.112$$

Since t_{calc} < 2.024 (same as in 10.15) do not reject the null.
There is not enough evidence to conclude that the time to answer queries by the two groups is different. The conclusions from both the pooled-variance *t* test and the separate variance *t* test are exactly the same.

10.17 (a) $H_0 : \mu_1 - \mu_2 \geq 0$
$H_1 : \mu_1 - \mu_2 < 0$

Where females = population 1 and males = population 2

Excel output:

t-Test: Two-Sample Assuming Equal Variances

	Female	*male*
Mean	49925.94444	77478.18182
Variance	489142253.2	673473852.5
Observations	18	22
Pooled Variance	591009716	
Hypothesized Mean Difference	0	
df	38	
t Stat	-3.565965675	
P(*T*<=*t*) one-tail	0.00049959	
t Critical one-tail	1.68595446	
P(*T*<=*t*) two-tail	0.000999181	
t Critical two-tail	2.024394164	

Decision rule: Reject H_0 if $t_{calc} < -1.686$
Decision: Since $t_{calc} = -3.566$ is less than -1.686, reject H_0. There is evidence that male graduate salaries exceed those of females.

(b) *p*-value = 0.0005. The probability of obtaining two samples with a mean difference of -3.566 or less is 0.0005 if the mean female salaries are equal to those of males.

(c) Since both sample sizes are smaller than 30, you need to assume that the population of male and female graduate salaries is normally distributed.

(d) $$\left(\overline{X}_1 - \overline{X}_2\right) \pm t\sqrt{S_P^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = (49925.94 - 77478.18) \pm 1.686\sqrt{591009716\left(\frac{1}{18} + \frac{1}{22}\right)}$$

$-40578.7 \leq \mu_1 - \mu_2 \leq -14525.8$

10.18 $H_0 : \mu_1 - \mu_2 \geq 0$

$H_1 : \mu_1 - \mu_2 < 0$

Excel output:

t-Test: Two-Sample Assuming Unequal Variances

	female	*male*
Mean	49925.94444	77478.18182
Variance	489142253.2	673473852.5
Observations	18	22
Hypothesized Mean Difference	0	
df	38	
t Stat	−3.624446816	
P(T<=t) one-tail	0.000422668	
t Critical one-tail	1.68595446	
P(T<=t) two-tail	0.000845335	
t Critical two-tail	2.024394164	

Decision rule: reject H_0 if $t_{calc} < -1.686$

Decision: Since t_{calc} = -3.624 is less than the-1.686, reject H_0. The value of pooled-variance *t* test statistic and the separate-variance *t* test statistic are almost identical.

10.19 (a) Population 1 = computer-assisted individual-based, 2 = team-based.

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$

$$S_P^2 = \frac{(n_1 - 1)\, S_1^2 + (n_2 - 1)\, S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$S_P^2 = \frac{(20)\, 1.9333^2 + (20)\, 4.5767^2}{20 + 20} = 12.3419$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_P^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t = \frac{(17.5571 - 198905) - 0}{\sqrt{12.3419\left(\frac{1}{21} + \frac{1}{21}\right)}} = -2.1522$$

Decision rule: *df* = 40. If $t_{calc} < -2.0211$ or > 2.0211, reject H_0.

Decision: Since $t_{calc} = -2.1522$ is below the lower critical bound of –2.0211, reject H_0. There is enough evidence to conclude that the mean assembly times in seconds are different between employees trained in a computer-assisted, individual-based program and those in a team-based program.

(b) You must assume that each of the two independent populations is normally distributed.

(c) $H_0 : \mu_1 - \mu_2 = 0$
$H_1 : \mu_1 - \mu_2 \neq 0$

***t* Test: Two-Sample Assuming Unequal Variances**	**Computer-assisted program**	**Team-based program**
Mean	17.55714286	19.89047619
Variance	3.737571429	20.94590476
Observations	21	21
Hypothesized Mean Difference	0	
Df	27	
t Stat	−2.152203195	
P(T<=t) one-tail	0.020240852	
t Critical one-tail	1.703288035	
P(T<=t) two-tail	0.040481703	
t Critical two-tail	2.051829142	

$$t = \frac{(17.5571 - 19.8905) - 0}{\sqrt{\frac{1.9333^2}{21} + \frac{4.5767^2}{21}}} = -2.1522$$

Decision rule: df = 27. If $t_{calc} < -2.052$ or > 2.052, reject H_0.

Decision: Since $t_{calc} = -2.1522$ is below the lower critical bound of −2.052, reject H_0. There is enough evidence to conclude that the mean assembly times in seconds are different between employees trained in a computer-assisted, individual-based program and those in a team-based program.

(d) The results in (a) and (c) are the same.

(e) $$\left(\bar{X}_1 - \bar{X}_2\right) \pm t\sqrt{S_P^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \left(17.557 - 19.89\right) \pm 2.021\sqrt{12.3419\left(\frac{1}{21} + \frac{1}{21}\right)}$$

$-4.524 \leq \mu_1 - \mu_2 \leq -0.142$

10.20 df = n − 1 = 12 − 1 = 11

10.21 df = n − 1 = 11 − 1 = 10

10.22 (a) Population 1 = June 2011 daily room rates, 2 = March 2015 daily rates.

$H_0 : \mu_D = 0$
$H_1 : \mu_D \neq 0$

$$\bar{D} = \frac{\sum_{i=1}^{n} D_1}{n} = 27.39042 \qquad S_D^2 = \sum_{i=1}^{n}\left(D_i - \bar{D}\right)^2 \Big/ n = 750.2353$$

$$t = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} = \frac{7.333 - 0}{27.39/\sqrt{18}} = 1.1359$$

$$df = (n-1) = 17$$

$$t_{0.05} = 2.1098$$

2015	2011		Diffs			
120	173		−53			
139	133		6			
105	90		15			
156	167		−11			
170	139		31			
139	141		−2		Mean	7.333333
201	180		21		SD	27.39042
247	223		24		n	18
122	116		6			
125	167		−42		t	1.135897
156	142		14			
316	273		43			
148	143		5			
177	124		53			
165	135		30			
191	176		15			
163	159		4			
235	262		−27			

Decision: Since t_{calc} = 1.1359 is less than the upper critical value of 2.1098, do not reject H_0. There is insufficient evidence to conclude that there is a difference in the mean daily hotel rates in 2011 and 2015.

(b) You must assume that the distribution of the differences between the hotel daily room rate in 2011 and 2015 is approximately normal.

(c) *p*-value is 0.27. The probability of obtaining a mean difference in daily hotel rates that gives rise to a test statistic that deviates from 0 by 1.1359 or more in either direction is 0.27 if there is no difference in the mean daily hotel rate in 2011 and 2015.

(d) $\bar{D} \pm t\frac{S_D}{\sqrt{n}} = 7.333 \pm 2.1098\frac{27.39}{\sqrt{18}} \quad -6.2876 \le \mu_D \le 20.9536$

You are 95% confident that the mean difference in hotel rate between 2011 and 2015 is somewhere between −$6.29 and $20.95.

10.23 (a) $H_0: \mu_D = 0$ vs. $H_1: \mu_D \ne 0$

Excel Output:

Paired *t* Test	

Data	
HypothesizedMeanDifference	0
Level of significance	0.05

Intermediate Calculations	
Sample Size	13
DBar	-8.9231
Degrees of Freedom	12
S_D	3.0403
Standard Error	0.8432
t Test Statistic	-10.5820

Two-Tail Test	
Lower Critical Value	-2.1788
Upper Critical Value	2.1788
p-Value	0.0000
Reject the null hypothesis	

Test statistic: $t = \dfrac{\bar{D} - \mu_D}{\dfrac{S_D}{\sqrt{n}}}$ = -10.582

Decision: Since t_{calc} = −10.582 falls below the lower critical values -2.1788, reject H_0. There is enough evidence of a difference in the mean service rating between TV and phone.

(b) You must assume that the distribution of the differences between the mean measurements is approximately normal.

(c)

Both the boxplot and normal probability plot suggest that the distribution does not deviate too far from normal.

(d) $\bar{D} \pm t\,\dfrac{S_D}{\sqrt{n}} = -8.9231 \pm 2.1788\,\dfrac{3.0403}{\sqrt{13}}$ new

$-10.76 \le \mu_D \le -7.09$

You are 95% confident that the difference in the mean service rating between TV and phone is between -10.76 and -7.09

10.24 Excel output:

	Cola A Adindex	*Cola B(Test Cola) Adindex*
Mean	18.55263158	21.31578947
StandardError	0.978937044	0.822086011
Median	18	21

Mode	24	21
StandardDeviation	6.034573222	5.067678519
Sample Variance	36.41607397	25.68136558
Kurtosis	-0.640865482	-0.294923931
Skewness	-0.077015645	-0.173917096
Range	24	21
Minimum	6	9
Maximum	30	30
Sum	705	810
Count	38	38
First Quartile	15	18
Third Quartile	24	24
Interquartile Range	9	6
1.33*StdDev	8.025982385	6.740012431
5*StdDev	30.17286611	25.3383926

From the descriptive statistics provided in the Microsoft Excel output there does not seem to be any violation of the assumption of normality. The mean and median are similar and the skewness value is near 0. Without observing other graphical devices such as a stem-and-leaf display, boxplot, or normal probability plot, the fact that the sample size ($n = 38$) is not very small enables us to assume that the paired t test is appropriate here.

PHStat output:

Paired *t* Test	
Data	
HypothesizedMeanDifference	**0**
Level of significance	**0.05**
Intermediate Calculations	
Sample Size	38
DBar	-2.7632
Degrees of Freedom	37
S_D	6.6309
Standard Error	1.0757
***t* Test Statistic**	**-2.5688**
Lower-Tail Test	
Lower Critical Value	**-1.6871**
***p*-Value**	**0.0072**
Reject the null hypothesis	

The PHStat output for the paired t test indicates the p-value is 0.0072 < 0.05, and, hence, reject H_0 that the mean Cola A Adindex is no less than Cola B (Test Cola) Adindex. There is enough evidence that the cola video ad is significant in raising the Adindex of the test Cola.

10.25 (a) Population 1 = Fruit Shop, 2 = Supermarket

H_0: $\mu_D \geq 0$

H_1: $\mu_D < 0$

Excel output

t-Test: Paired Two Sample for Means

	Thursday	*Friday*
Mean	130.9	148
Variance	94.54444	177.3333
Observations	10	10
Pearson Correlation	0.496847	
Hypothesized Mean Difference	0	
df	9	
t Stat	-4.51864	
P(T<=t) one-tail	0.000725	
t Critical one-tail	2.821438	
P(T<=t) two-tail	0.00145	
t Critical two-tail	3.249836	

df = 10-1 = 9

Decision rule: Reject H_0 is $t_{calc} < -2.821$

Decision: Since $t = -4.519$ is less than the lower critical value of –2.821, reject H_0. There is sufficient evidence at α = 0.01 to conclude that petrol prices increase on public holidays.

(b) The *p*-value of 0.0007 indicates that there is a 0.0007 probability of observing a calculated value of −4.519 or less if petrol prices on public and non-public holidays are equal.

10.26 (a) Population 1 = Before Lumosity, 2 = After Lumosity

H_0: $\mu_D \geq 0$

H_1: $\mu_D < 0$

Excel output

t-Test: Paired Two Sample for Means

	Before	*After*
Mean	108.2857	111.4286
Variance	299.5714	300.2857
Observations	7	7
Pearson Correlation	0.955863	
Hypothesized Mean Difference	0	
df	6	
t Stat	-1.61602	
P(T<=t) one-tail	0.078609	
t Critical one-tail	3.142668	
P(T<=t) two-tail	0.157218	
t Critical two-tail	3.707428	

Decision rule: Reject H_0 if $t_{calc} < -3.143$

Decision: Since $t = -1.616$ is not less than the lower critical value of -3.143, do not reject H_0. There is insufficient evidence at $\alpha = 0.01$ to conclude that the mean IQ has increased after using Lumosity.

(b) The differences between the IQ before and after using Lumosity is approximately normally distributed.

(c) The *p*-value for this test is 0.079; it needs to be less than 0.01 in order to reject H_0.

10.27 (a) change to 0.05 $\alpha = 0.05$, $n_1 = 8$, $n_2 = 7$, $F_U = 4.21$, $F_L = 0.2375$ i.e. $df_1 = 7$, $df_2 = 6$

(b) $\alpha = 0.05$, $n_1 = 9$, $n_2 = 6$, $F_U = 4.82$, $F_L = 0.2075$

(c) $\alpha = 0.025$, $n_1 = 7$, $n_2 = 5$, $F_U = 9.20$, $F_L = 0.1087$

(d) $\alpha = 0.01$, $n_1 = 9$, $n_2 = 9$, $F_U = 6.03$, $F_L = 0.1658$

10.28

(a) $\alpha = 0.05$, $n_1 = 8$, $n_2 = 7$, $F_{0.05} = 4.21$, i.e. $df_1 = 7$, $df_2 = 6$

(b) $\alpha = 0.025$, $n_1 = 9$, $n_2 = 6$, $F_{0.025} = 6.76$

(c) $\alpha = 0.01$, $n_1 = 7$, $n_2 = 5$, $F_{0.01} = 15.21$

(d) $\alpha = 0.005$, $n_1 = 9$, $n_2 = 9$, $F_{0.005} = 7.50$

10.29

(a) $\alpha = 0.05$, $n_1 = 16$, $n_2 = 21$, $F_{0.95} = 0.4296$, i.e. $df_1 = 15$, $df_2 = 20$

(b) $\alpha = 0.025$, $n_1 = 16$, $n_2 = 21$, $F_{0.975} = 0.3629$

(c) $\alpha = 0.01$, $n_1 = 16$, $n_2 = 21$, $F_{0.99} = 0.2966$

(d) $\alpha = 0.005$, $n_1 = 16$, $n_2 = 21$, $F_{0.995} = 0.2576$

10.30

$F_{calc} = \frac{S_2^2}{S_1^2} = \frac{161.9}{133.7} = 1.2109$ (or alternatively, $F_{calc} = \frac{S_2^2}{S_1^2} = \frac{133.7}{161.9} = 0.8258$)

10.31 There are $\upsilon_2 = 15$ and $\upsilon_1 = 10$ degrees of freedom respectively in numerator and denominator of $F = \frac{S_2^2}{S_1^2}$ (or alternatively, 10 and 15 in $F = \frac{S_1^2}{S_2^2}$)

10.32 $F_U = 3.52$ and $F_L = 0.327$ (or alternatively $F_U = 3.06 = 1/0.327$ and $F_L = 0.826 = 1/3.52$)

10.33 Since $F_{calc} = 1.2109 < F_U = 3.52$ and $F_{calc} = 1.2109 > F_L = 0.327$, then F_{calc} is not in the rejection region. Therefore, do not reject H_0.

10.34 No, since the *F* test is very sensitive to the normality assumption, it cannot be validly used when that assumption is clearly violated by the statement that the data, unlike normally distributed data, is very skewed.

10.35 (a) $F_{calc} = S_1^2 / S_2^2 = 47.3/36.4 = 1.299$ which is considerably greater than 1, suggesting that $\sigma_1^2/\sigma_2^2 > 1$, that is, $\sigma_1^2 > \sigma_2^2$.

(b) $F_U = F(\alpha, \upsilon_1, \upsilon_2) = F(0.05, 15, 12) = 2.62$ where $\upsilon_1 = df_1 - 1 = 16 - 1$ and $\upsilon_2 = df_2 - 1 = 13 - 1$. Thus, since $F_{calc} = 1.299 < F_U = 2.62$, then do not reject H_0: $\sigma_1^2 = \sigma_2^2$ in favour of H_1: $\sigma_1^2 > \sigma_2^2$. There is little evidence to support the claim that $\sigma_1^2 > \sigma_2^2$.

(c) $F_L = F(1-\alpha, \upsilon_1, \upsilon_2) = F(0.95, 15, 12) = 0.4040$ and since $F_{calc} = 1.299$ is not even less than 1, it is certainly not in the lower rejection region, i.e. do not reject H_0: $\sigma_1^2 = \sigma_2^2$ in favour of H_1: $\sigma_1^2 < \sigma_2^2$. What little evidence there is that $\sigma_1^2 \neq \sigma_2^2$ is that $\sigma_1^2 > \sigma_2^2$.

10.36

(a) $H_0: \sigma_1^2 - \sigma_2^2 = 0$

$H_1: \sigma_1^2 - \sigma_2^2 \neq 0$

PHStat2 output

F Test for Differences in Two Variances

Data	
Level of Significance	**0.05**
Larger-Variance Sample	
Sample Size	**54**
Sample Variance	**144**
Smaller-Variance Sample	
Sample Size	**32**
Sample Variance	**100**

Intermediate Calculations	
***F* Test Statistic**	**1.4400**
Population 1 Sample Degrees of Freedom	53
Population 2 Sample Degrees of Freedom	31

Two-Tail Test	
Upper Critical Value	**1.9409**
***p*-Value**	**0.2780**
Do not reject the null hypothesis	

Since the p-value = 0.278 > 0.05, then do reject H_0, i.e. underlying variances are equal.

(b) The test assumes that the two populations are both normally distributed.

(c) The pooled variance t test is appropriate.

10.37 (a) H_0: $\sigma_1^2 = \sigma_2^2$ The population variances are the same.

H_1: $\sigma_1^2 \neq \sigma_2^2$ The population variances are different.

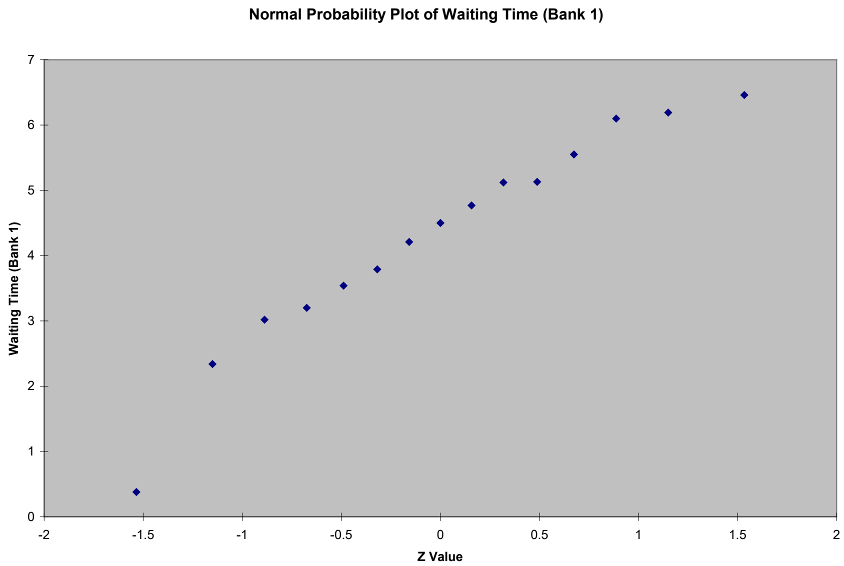
Decision rule: If $F_{calc} > 2.9786$, reject H_0.

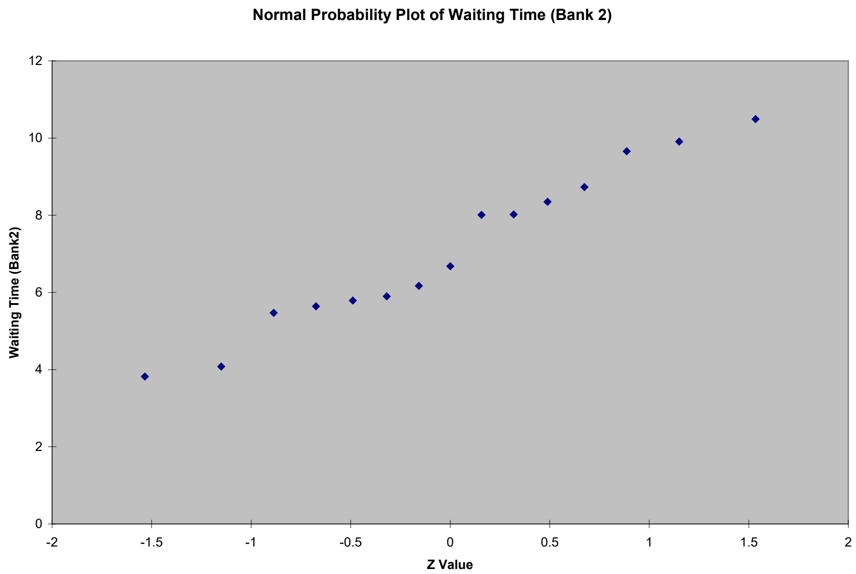
Test statistic: $F = \dfrac{S_1^2}{S_2^2} = \dfrac{2.0822^2}{1.6380^2} = 1.6159$

Decision: Since $F_{calc} = 1.6159$ is below the upper critical bound of $F_{\alpha/2} = 2.9786$, do not reject H_0. There is not enough evidence to conclude that the two population variances are different.

(b) p-value = 0.715. The probability of obtaining a sample that yields a test statistic more extreme than 1.6159 is 0.715 if the null hypothesis that there is no difference in the two population variances is true.

(c) The test assumes that the two populations are both normally distributed.





(c) cont.

(d) Based on the results of (b), it is not appropriate to use the pooled-variance *t*-test to compare the means of the two branches. That is, the *F*-ratio test for testing equality of variances is not justified and since we are not able to assume that the two population variances are equal, we cannot pool the sample variances.

10.38 (a) $H_0: \sigma_1^2 - \sigma_2^2 = 0$

$H_1: \sigma_1^2 - \sigma_2^2 \neq 0$

PHStat2 output:

F Test for Differences in Two Variances

Data	
Level of Significance	**0.05**
Larger-Variance Sample	
Sample Size	**11**
Sample Variance	**51.09090909**
Smaller-Variance Sample	
Sample Size	**11**
Sample Variance	**20.47272727**

Intermediate Calculations	
***F* Test Statistic**	**2.4956**
Population 1 Sample Degrees of Freedom	10
Population 2 Sample Degrees of Freedom	10

Two-Tail Test	
Upper Critical Value	**3.7168**
***p*-Value**	**0.1653**
Do not reject the null	

hypothesis	

Decision: Since the *p*-value is 0.1653 > 0.05, do not reject H_0. There is not enough evidence to conclude that the two population variances are different.

(b) The *p*-value = 0.1653. If the population variances of both groups were equal, the probability of a sample *F* ratio falling in the lower or upper rejection regions is 0.1653.

(c) The test assumes that the two populations are both normally distributed.

(d) The pooled-variance *t* test can be validly carried out.

10.39 (a) $H_0: \sigma_1^2 - \sigma_2^2 = 0$ The population variances in petrol prices are the same for Campbelltown and the rest of Sydney.

$H_1: \sigma_1^2 - \sigma_2^2 \neq 0$ The population variances in petrol prices are not the same for Campbelltown and the rest of Sydney..

Decision rule: if *F* < 0.74 or *F* > 1.35, reject the null hypothesis (using n=120 as a proxy).

Test statistics $F = \dfrac{S_1^2}{S_2^2} = \dfrac{146.41}{161.29} = 0.91$.

Decision: Since *F* = 0.74 < 0.91 < 1.35 we do not reject the null hypothesis. There is not enough evidence to conclude the population variances in petrol prices are different.

(b) Assuming the underlying normality in the two populations is met, based on the results obtained in part (a), it is more appropriate to use the pooled-variance *t* test to compare petrol prices for Campbelltown and the rest of Sydney .

10.40 (a)

$$H_0: \pi_1 - \pi_2 = 0$$
$$H_1: \pi_1 - \pi_2 \neq 0$$
new

$$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\overline{p} = \dfrac{X_1 + X_2}{n_1 + n_2} = \dfrac{55+30}{120+65} = \dfrac{85}{185} = 0.459$ new

$$= \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{\left(\frac{55}{120} - \frac{30}{65}\right) - 0}{\sqrt{0.459(1-0.459)\left(\frac{1}{120} + \frac{1}{65}\right)}}$$ new

$$= -0.04$$

Decision rule: if Z > 1.96 or , -1.96reject H_0

Since -0.04 > -1.96, do not reject H_0.. Thus there is no evidence that the two group population proportions are not equal.

(b)

$$(p_1 - p_2) \pm Z\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$= \left(\frac{55}{120} - \frac{30}{65}\right) \pm 1.96\sqrt{\frac{\frac{55}{120}\left(1-\frac{55}{120}\right)}{120} + \frac{\frac{30}{65}\left(1-\frac{30}{65}\right)}{65}} \text{ new}$$

$$= -0.003 \pm 0.1505$$

10.41 (a)

$$H_0: \pi_1 - \pi_2 = 0$$
$$H_1: \pi_1 - \pi_2 \neq 0$$

new

$$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where } \overline{p} = \frac{x_1 - x_2}{n_1 + n_2} = \frac{45+25}{100+50} = \frac{70}{150} = 0.467$$

$$= \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{\left(\frac{45}{100} - \frac{25}{50}\right) - 0}{\sqrt{0.467(1-0.467)\left(\frac{1}{100} + \frac{1}{50}\right)}}$$

$$= -0.5787$$

$$Z_{0.005} = \pm 2.576$$

Thus, fail to reject H_0 as the calculated z is in the non-rejection region. Thus there is not enough evidence that the two group population proportions are unequal.

(b)

$$(p_1 - p_2) \pm Z\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$= \left(\frac{45}{100} - \frac{25}{50}\right) \pm 2.576\sqrt{\frac{\frac{45}{100}\left(1-\frac{45}{100}\right)}{100} + \frac{\frac{25}{50}\left(1-\frac{25}{50}\right)}{50}}$$

$$= -0.05 \pm 0.0132$$

$$= [-0.27272,\ 0.172716]$$

10.42 (a) $H_0 : \pi_1 - \pi_2 \geq 0$
$H_1 : \pi_1 - \pi_2 < 0$

Where population 1 = Christchurch and population 2 = Brisbane

$$p_1 = \frac{12}{365} = 0.0329, p_2 = \frac{32}{890} = 0.0360, \overline{p} = \frac{12+32}{365+890} = 0.0348$$

Using the 0.01 level of significance
Decision rule: Reject null hypothesis if Z_{calc} < −2.33.

$$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = -0.2703$$

Since −0.2703 > −2.33, we do not reject the null hypothesis at 1% level and conclude that there is not enough evidence that there is a significant improvement in the rate of resignations of Christchurch vs Brisbane factories.

(b)

$$-0.003078 \pm -2.33\sqrt{\frac{0.0329(1-0.0329)}{365} + \frac{0.0360(1-0.0360)}{890}} = -0.003078 \pm 0.011228$$

= [−0.014306, 0.00815]

10.43 (a) $H_0 : \pi_1 - \pi_2 = 0$
new
$H_1 : \pi_1 - \pi_2 \neq 0$

$p_1 = 0.38, p_2 = 0.33, \overline{p} = 0.34$

Using the 0.05 level of significance
Decision, reject null hypothesis if Z_{calc} < −1.96 or > 1.96.

$$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 2.6765 \text{ new}$$

Since 2.6765 > 1.96, we reject the null hypothesis. There is sufficient evidence of a significant difference in the proportion of technology crowd-funding projects and games crowd-funding projects that were successful.

ZTest for Differences inTwoProportions	
Data	
HypothesizedDifference	**0**
Level of Significance	**0.05**
Group1	
Number of Items of Interest	**316**
Sample Size	**831**
Group2	
Number of Items of Interest	**923**
Sample Size	**2796**
Intermediate Calculations	
Group 1Proportion	0.380264741
Group 2Proportion	0.330114449
Difference in Two Proportions	0.050150292
Average Proportion	0.3416
***Z* Test Statistic**	**2.6765**
Two-Tail Test	
Lower Critical Value	**-1.9600**
Upper Critical Value	**1.9600**
***p*-Value**	**0.0074**
Reject the null hypothesis	

(b) *p*-value = 0.0074. The probability of obtaining a difference in proportions that gives rise to a test statistic that deviates from 0 in either direction by 2.6765 or more in either direction is virtually 0 if there is no difference in the proportion of technology crowd- funding projects and games crowd-funding projects that were successful

10.44 (a) $H_0: \pi_1 - \pi_2 = 0$ new
$H_1: \pi_1 - \pi_2 \neq 0$

where Populations: 1 = males, 2 = females
Decision rule: If $Z_{calc} < -2.58$ or > 2.58, reject H_0.
Zcalc = -3.5080
Decision: Since $Z_{calc} = -3.5080$ is less than the lower critical bound, reject H_0. There is sufficient evidence to conclude that a significant difference exists in the proportion of males and females who enjoy shopping clothing for themselves

Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	**0**
Level of Significance	**0.01**
Group 1	
Number of Items of Interest	**218**

Sample Size	**542**
Group 2	
Number of Items of Interest	**276**
Sample Size	**543**
Intermediate Calculations	
Group 1 Proportion	0.402214022
Group 2 Proportion	0.508287293
Difference in Two Proportions	-0.106073271
Average Proportion	0.455299539
Z Test Statistic	**-3.50802898**
Two-Tail Test	
Lower Critical Value	**-2.575829304**
Upper Critical Value	**2.575829304**
***p*-Value**	**0.00045144**
Reject the null hypothesis	

(b)
p-value = 0.0005. The probability of obtaining a difference in two sample proportions of 0.1061 or more in either direction when the null hypothesis is true is 0.0005.

(c) Change question to 99%-0.1835 ≤ $\pi_1 - \pi_2$ ≤ -0.0286

You are 99% confident that the difference in the proportions of males and females who enjoy shopping clothing for themselves is between -0.1835 and - 0.0286.

Confidence Interval Estimate	
of the Difference Between Two Proportions	
Data	
Confidence Level	**99%**
Intermediate Calculations	
Z Value	-2.575829304
Std. Error of the Diff. between two Proportions	0.030064781
Interval Half Width	0.077441743
Confidence Interval	
Interval Lower Limit	**-0.183515014**
Interval Upper Limit	**-0.028631527**

√

10.45 $H_0 : \pi_1 - \pi_2 \le 0$ The proportion of car drivers in Malaysia that have converted to LPG fuel is no more than the proportion of car drivers in Singapore.

$H_1 : \pi_1 - \pi_2 > 0$ The proportion of car drivers in Malaysia that have converted to LPG fuel is greater than the proportion of car drivers in Singapore.

$p_1 = 0.4, p_2 = 0.4, \overline{p} = 0.4$

Using the 0.01 level of significance, reject H_0, if Z_{calc} > 2.326.

$$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 0.00$$

Since 0.00 < 2.326, we do not reject the null hypothesis, there is not enough evidence to show that the proportion of car drivers in Malaysia that have converted to LPG fuel is different as the proportion of car drivers in Singapore.

10.46 $H_0 : \pi_M - \pi_F \geq 0$ The proportion of males who prefer margarine is at least the proportion of females who prefer margerine

$H_1 : \pi_M - \pi_F < 0$ The proportion of males that prefer margarine is less than the proportion of females that prefer margarine

$\boldsymbol{p_M = 0.4024, p_F = 0.4887, \bar{p} = 0.4558}$

Using the 0.05 level of significance

Decision, reject null hypothesis if Z_{calc} < –1.645.

$$Z = \frac{(p_M - p_F) - (\pi_M - \pi_F)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_M} + \frac{1}{n_F}\right)}} = -1.235$$

Since –1.235 > –1.645, we do not reject the null hypothesis at 5% level, there is insufficient evident to conclude that the proportion of males who prefer margarine is less than the proportion of females who prefer margarine.

10.47 Among the criteria to be used in selecting a particular hypothesis test are the type of data, whether the samples are independent or paired, whether the test involves central tendency or variation, whether the assumption of normality is valid and whether the variances in the two populations are equal.

10.48 The separate variance *t* test is used when the variances of independent populations are unequal.

10.49 The *F* test can be used to examine the differences in two variances when each of the two populations is assumed to be normally distributed.

10.50 With independent populations, the outcomes in one population do not depend on the outcomes in the second population. With two related populations, either repeated measurements are obtained on the same set of items or individuals, or items or individuals are paired or matched according to some characteristic.

10.51 Repeated measurements represent two measurements on the same items or individuals, while paired measurements involve matching items according to a characteristic of interest.

10.52 When you have obtained data from either repeated measurements or paired data.

10.53 They are two different ways of investigating the concern of whether there is a significant difference between the means of two independent populations. If the hypothesised value of 0 for the difference in two population means is not in the confidence interval, then, assuming a two-tailed test is used, the null hypothesis of no difference in the two population means can be rejected.

10.54 When parametric assumptions can be met that the data is normally distributed and measured at least at the interval scale.

10.55 One year return $H_0: \sigma_1^2 - \sigma_2^2 = 0$
$H_1: \sigma_1^2 - \sigma_2^2 \neq 0$

F Test for Differences in Two Variances	
Data	
Level of Significance	**0.05**
Larger-Variance Sample	
Sample Size	**10**
Sample Variance	**8.925161111**
Smaller-Variance Sample	
Sample Size	**10**
Sample Variance	**3.29496**
Intermediate Calculations	
***F* Test Statistic**	**2.7087**
Population 1 Sample Degrees of Freedom	9
Population 2 Sample Degrees of Freedom	9
Two-Tail Test	
Upper Critical Value	**4.0260**
***p* -Value**	**0.1538**
Do not reject the null hypothesis	

Since p-value > 0.05, do not reject H_0. There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance t test

$H_0: \mu_1 - \mu_2 = 0$
$H_1: \mu_1 - \mu_2 \neq 0$
Populations: 1 = short-term, 2 = long-term

Pooled-Variance *t* Test for the Difference Between Two	
(assumes equal population variances)	
Data	
Hypothesized Difference	**0**
Level of Significance	**0.05**
Population 1 Sample	
Sample Size	**10**
Sample Mean	**5.046**
Sample Standard Deviation	**1.815202468**
Population 2 Sample	
Sample Size	**10**
Sample Mean	**11.365**
Sample Standard Deviation	**2.987500814**

Intermediate Calculations	
Population1Sample Degrees of Freedom	9
Population2Sample Degrees of Freedom	9
Total Degrees of Freedom	18
Pooled Variance	6.1101
Standard Error	1.1054
Difference in Sample Means	-6.3190
***t* Test Statistic**	**-5.7162**

Two-Tail Test	
Lower Critical Value	**-2.1009**
Upper Critical Value	**2.1009**
***p*-Value**	**0.0000**
Reject the null hypothesis	

Since the p-value = 0.0000 is less than 0.05, reject H_0. There is sufficient evidence to conclude that the mean 1-year return is different between the long-term and short-term bond funds

Three year return

$H_0: \sigma_1^2 - \sigma_2^2 = 0$

$H_1: \sigma_1^2 - \sigma_2^2 \neq 0$

F Test for Differences inTwoVariances	
Data	
Level of Significance	**0.05**
Larger-Variance Sample	
Sample Size	**10**
Sample Variance	4.965444444
Smaller-Variance Sample	
Sample Size	**10**
Sample Variance	2.279555556
Intermediate Calculations	
***F* Test Statistic**	**2.1783**
Population1Sample Degrees of Freedom	9
Population2Sample Degrees of Freedom	9
Two-Tail Test	
UpperCritical Value	**4.0260**
***p*-Value**	**0.2617**
Donot reject the null hypothesis	

Since p-value > 0.05, do not reject H_0. There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance t test

$H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 \neq 0$

Populations: 1 = short-term, 2 = long-term

Pooled-Variance *t* Test for the Difference Between Two	
(assumes equal population variances)	
Data	
Hypothesized Difference	**0**
Level of Significance	**0.05**
Population 1 Sample	
Sample Size	**10**
Sample Mean	**4.82**
Sample Standard Deviation	**1.50981971**
Population 2 Sample	
Sample Size	**10**
Sample Mean	**11.59**
Sample Standard Deviation	**2.228327724**
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	9
Population 2 Sample Degrees of Freedom	9
Total Degrees of Freedom	18
Pooled Variance	3.6225
Standard Error	0.8512
Difference in Sample Means	-6.7700
***t* Test Statistic**	**-7.9537**
Two-Tail Test	
Lower Critical Value	**-2.1009**
Upper Critical Value	**2.1009**
***p*-Value**	**0.0000**
Reject the null hypothesis	

Since the p-value < 0.05, reject H_0. There is sufficient evidence to conclude that the mean 3-year return is different between the long-term and short-term funds

10.56 (a) $H_0 : \sigma_1^2 - \sigma_2^2 = 0$

$H_1 : \sigma_1^2 - \sigma_2^2 \neq 0$

F Test for Differences in Two Variances	
Data	
Level of Significance	**0.05**
Larger-Variance Sample	
Sample Size	**20**
Sample Standard Deviation	**5.714421**
Smaller-Variance Sample	
Sample Size	**38**
Sample Standard Deviation	**5.406387**
Intermediate Calculations	

***F* Test Statistic**	**1.117198**
Population 1 Sample Degrees of Freedom	19
Population 2 Sample Degrees of Freedom	37
Two-Tail Test	
Upper Critical Value	**2.11685**
***p*-Value**	**0.749246**
Do not reject the null hypothesis	

Since the p value > 0.05 there is not enough evidence of any difference in the variance of the study time for male students and female students

(b) Since there is not enough evidence of any difference in the variance of the study time for male students and female students, a pooled-variance *t* test should be used

(c)

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$

Pooled-Variance *t* Test for the Difference Between Two Means	
(assumes equal population variances)	
Data	
Hypothesized Difference	**0**
Level of Significance	**0.05**
Population 1 Sample	
Sample Size	**20**
Sample Mean	**16.625**
Sample Standard Deviation	**5.714421**
Population 2 Sample	
Sample Size	**38**
Sample Mean	**11.02632**
Sample Standard Deviation	**5.406387**
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	19
Population 2 Sample Degrees of Freedom	37
Total Degrees of Freedom	56
Pooled Variance	30.39127
Difference in Sample Means	5.598684
***t* Test Statistic**	**3.676244**
Two-Tail Test	
Lower Critical Value	**-2.00324**
Upper Critical Value	**2.003241**
***p*-Value**	**0.000532**
Reject the null hypothesis	

(d) Since the p value < 0.05 there is enough evidence of a difference in the mean study time for male and female students

10.57 (a) At the 5% significance level, there is not enough evidence that the means of responses to 'I have been sunburnt at least once during the summer' are not equal.

(b) At the 5% significance level, there is enough evidence to conclude that the mean responses to 'I would willing to pay more for a sunscreen that I know will be more effective while I am swimming' are not equal.

(c) At the 5% significance level, there is enough evidence that the means of responses for males and females to 'skin cancer due to sun exposure is something I want to prevent' are not equal.

(d) At the 5% significance level, there is not enough evidence to conclude that the means of responses for males and females to 'I was not aware that sunscreen needs to be applied at least twenty minutes before exposure to the sun' are not equal.

(e) The means of the responses for males and females to two out of the four questions being asked in the survey are significantly different, at the 5% significance level.

10.58 (a) $H_0 : \mu_A - \mu_S = 0$ Mean petrol is the same in Adelaide and rural South Australia

$H_1 : \mu_A - \mu_S \neq 0$ Mean petrol is different in Adelaide and rural South Australia

Assuming that the samples are from underlying normal populations with equal variances, we can use pooled-variance *t* test. The *t* test statistics follow a *t* distribution with 44 degrees of freedom. Using a level of significance of 0.01, the critical values are –2.692 and 2.692.

Reject H_0 if $t_{calc} < -2.692$ or > 2.692
Test statistics:

$$t = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where:

$$S_p^2 = \frac{\left(n_1 - 1\right)S_1^2 + \left(n_2 - 1\right)S_2^2}{\left(n_1 - 1\right) + \left(n_2 - 1\right)} = \frac{24\left(0.23^2\right) + 20\left(0.74^2\right)}{24 + 20} = 0.278$$

$$t = \frac{\left(1.23 - 1.43\right) - 0}{\sqrt{0.278\left(\frac{1}{25} + \frac{1}{21}\right)}} = -1.281$$

Since, −1.281 > −2.692, we do not reject the null hypothesis and at 1% significance level, we conclude that there is not enough evidence that mean price petrol in Adelaide is higher than that of rural South Australia.

(b)

$H_0 : \sigma_A^2 - \sigma_S^2 = 0$ The population variances for petrol in Adelaide and rural South Australia are the same

$H_1: \sigma_A^2 - \sigma_S^2 \neq 0$ The population variances for petrol in Adelaide and rural South Australia are different
Decision rule: Reject null if *F* > 3.22, or *F* < 0.33

Test statistics: $F = \frac{S_A^2}{S_S^2} = \frac{0.23^2}{0.74^2} = 0.097$

Decision: Since *F* = 0.097 is less than 0.33, we reject H_0. There is enough evidence to conclude that Adelaide and rural South Australia have different population variances for mean petrol prices.

(c)

$$\left(\bar{X}_1 + \bar{X}_2\right) \pm t_{n1+n2-2}\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$\left(1.23 - 1.43\right) \pm 2.692\sqrt{\left(0.278\right)\left(\frac{1}{25} + \frac{1}{21}\right)}$$

$$-0.62 \leq \mu_1 - \mu_2 \leq 0.22$$

We are 95% confident that the difference in mean petrol prices between Adelaide and rural South Australia is between −0.62 to 0.22. From a hypothesis-testing perspective, since the interval includes zero, we do not reject the null hypothesis of no difference between the means of the two populations.

10.59 (a)

$H_0: \mu_R \leq 6$: The mean processing time in the research department is not greater than 6 seconds
$H_0: \mu_R > 6$: The mean processing time in the research department is greater than 6 seconds
Decision rule: *df* = 5. If t_{calc} > 2.015, reject null hypothesis.

Test statistics: $t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{85 - 6}{\frac{3.1464}{\sqrt{6}}} = 1.9463$

Decision: Since *t* = 1.9643 < 2.015, do not reject the null hypothesis. There is not enough evidence to conclude that the mean processing time in the research department is greater than 6 seconds.

(b) $H_0: \sigma_A^2 - \sigma_R^2 = 0$: The population variances for processing times are the same for the accounting department and research department
$H_1: \sigma_A^2 - \sigma_R^2 \neq 0$: The population variances for processing times are different for the accounting department and the research department
Decision rule: if *F* < 0.107 or *F* > 7.39, reject the null hypothesis.

Test statistics $F = \frac{S_A^2}{S_R^2} = \frac{3.2711^2}{3.1464^2} = 1.08$.

Decision: Since *F* = 1.08 is between critical bounds of 0.107 and 7.39, do not reject the null hypothesis. There is not enough evidence to conclude that the population variances for processing times are different for the accounting department and the research department.

(c) $H_0 : \mu_A - \mu_R = 0$: The two departments have the same mean processing time
$H_1 : \mu_A - \mu_R \neq 0$: The two departments have different mean processing time
Assuming that the samples are from underlying normal populations with equal variances, we can use pooled-variance *t* test. The *t* test statistics follow a *t* distribution with 9 degrees of freedom. Using a level of significance of 0.05, the critical values are –2.2622 and 2.2622.

Reject H_0 if $t_{calc} < -2.2622$ or > 2.2622
Test statistics:

$$t = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where:

$$S_p^2 = \frac{\left(n_1 - 1\right)S_1^2 + \left(n_2 - 1\right)S_2^2}{\left(n_1 - 1\right) + \left(n_2 - 1\right)} = \frac{4\left(3.2711^2\right) + 5\left(3.1464^2\right)}{4+5} = 10.2556$$

$$t = \frac{\left(7.8 - 8.5\right) - 0}{\sqrt{10.2556\left(\frac{1}{5} + \frac{1}{6}\right)}} = -0.3610$$

Since, –0.3610 > –2.2622, we do not reject the null hypothesis and at 5% significance level, we conclude that there is not enough evidence that the two departments have different mean processing times.

(d) (a) *p*–value = 0.0545. Since the *p*-value > 0.05, the probability of obtaining a *t* test statistic value that is 1.9643 or greater is 5.46% if the mean processing time in the research department is no more than 6 seconds.
(b) Given *F* = 1.08, numerator *df* = 4 and denominator *df* = 5 for a two-tailed hypothesis test:

$$P\left(\frac{1}{F_{5,4}} < \frac{1}{1.08}\right) = 0.4551 \quad \boldsymbol{P\left(F_{5,4} > 1.08\right) = 0.4551}$$

$$p-value = P\left(\frac{1}{F_{5,4}} < \frac{1}{1.08}\right) + P\left(F_{5,4} > 1.08\right) = 0.9102$$

The probability of obtaining the *F* statistic value that is smaller than 1.108 or larger than 1.08 is 91.02% if the population variances for processing times are the same for the accounting department and the research department.

(c) Given *t* = -0.3610, *df* = 9 for a two-tailed hypothesis test, the *p*-value = 0.7264 using excel.

(e) $$\left(\bar{X}_A - \bar{X}_R\right) \pm t_{n1+n2-2}\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$-0.7 \pm 2.2622\sqrt{10.2556\left(\frac{1}{5}+\frac{1}{6}\right)} = -5.0867 \le \mu_A - \mu_R \le 3.6867$$

You are 95% confident that the mean difference in the mean processing times between the accounting and research departments is between –5.0867 and 3.6867 seconds.

10.60 (a) $H_0: \mu_Z - \mu_A \le 0$ New Zealand watches no more television on average than Australia

$H_1: \mu_Z - \mu_A > 0$ New Zealand has the higher average than Australia

Assuming that the samples are from underlying normal populations with equal variances, we can use pooled-variance t test. The t test statistics follow a t distribution with 100 degrees of freedom. Use a level of significance of 0.05. The critical value is 2.692.

Reject H_0 if t_{calc} > 2.692.

Test statistics: $t = \dfrac{(\bar{X}_Z - \bar{X}_A) - (\mu_Z - \mu_A)}{\sqrt{S_p^2\left(\frac{1}{n_Z}+\frac{1}{n_A}\right)}}$

Where: $S_p^2 = \dfrac{(n_Z-1)\,S_Z^2 + (n_A-1)\,S_A^2}{(n_Z-1)+(n_A-1)} = \dfrac{40(34^2)+60(23^2)}{100} = 779.8$

$$t = \frac{(245-220)-(\mu_Z-\mu_A)}{\sqrt{779.8\left(\frac{1}{41}+\frac{1}{61}\right)}} = 4.43$$

Since 4.43 > 2.692, we reject the null hypothesis and we conclude that at 5% significance level, there is enough evidence that New Zealand has a higher mean of watching television compared to Australia.

(b) $H_0: \mu_Z - \mu_A \le 0$ New Zealand watches no more television on average than Australia

$H_1: \mu_Z - \mu_A > 0$ New Zealand has the higher average than Australia

Test statistics:

df = 64

$$t = \frac{(\bar{X}_Z - \bar{X}_A) - (\mu_Z - \mu_A)}{\sqrt{\frac{S_Z^2}{n_Z}+\frac{S_A^2}{n_A}}} = \frac{25-0}{6.072} = 4.117$$

Decision: reject H_0 if t_{calc} > 1.669.

Since 4.117 > 1.669, we reject the null hypothesis. We conclude that at 5% significance level, there is enough evidence that New Zealand has a higher mean of watching television compared to Australia.

(c) $H_0: \sigma_Z^2 - \sigma_A^2 = 0$: The population variances for mean time to watch TV are the same for New Zealand and Australia

$H_1: \sigma_Z^2 - \sigma_A^2 \ne 0$ The population variances for mean time to watch TV are different for New Zealand and Australia

Decision rule: if F_{calc} < 0.556 or > 1.74, reject the null hypothesis.

Test statistics: $F = \dfrac{S_Z^2}{S_A^2} = \dfrac{34^2}{23^2} = 2.185$.

Decision: Since F = 2.185>1.74 we reject the null hypothesis. There is enough evidence to conclude that the population variances for mean time to watch TV are different for New Zealand and Australia.

(d) Test in part (b) is appropriate since the variances of Australia and New Zealand are different.

(e) As illustrated in part (d) in which there is enough evidence that the population variances are different for the mean time of watching television for Australia and New Zealand, the *t* test with unequal variances is appropriate in this case. The *p*-value is virtually zero. The probability of observing a sample t_{calc} that is greater than 4.117 is 0% which is an unlikely event.
The test in (a) is not appropriate since based on the result of hypothesis testing in (d), we know that the variances are unequal. The *p*-value in this case is also virtually zero. In this case, two tests give us the same result.

10.61 (a) $H_0: \sigma_1^2 - \sigma_2^2 = 0$

$H_1: \sigma_1^2 - \sigma_2^2 \neq 0$

Population 1 = Pinterest, 2 = Facebook

F Test for Differences inTwoVariances	
Data	
Level of Significance	**0.05**
Larger-Variance Sample	
Sample Size	**500**
Sample Variance	**22500**
Smaller-Variance Sample	
Sample Size	**500**
Sample Variance	**6400**
Intermediate Calculations	
***F* Test Statistic**	**3.5156**
Population1Sample Degrees of Freedom	499
Population2Sample Degrees of Freedom	499
Two-Tail Test	
Upper Critical Value	**1.1921**
***p*-Value**	**0.0000**
Reject the null hypothesis	

Since the *p*-value < 0.05, reject H_0. There is enough evidence of a difference in the variances of the order values between Pinterest shoppers and Facebook shoppers. Hence, a separate-variance *t* test is appropriate

(b) $H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 \neq 0$

Separate-Variances *t* Test for the Difference Between T	
(assumes unequal population variances)	
Data	
Hypothesized Difference	**0**
Level of Significance	**0.05**
Population 1 Sample	
Sample Size	**500**
Sample Mean	**153**
Sample Standard Deviation	**150.0000**
Population 2 Sample	
Sample Size	**500**
Sample Mean	**85**
Sample Standard Deviation	**80.0000**
Intermediate Calculations	
Numerator of Degrees of Freedom	3340.8400
Denominator of Degrees of Freedom	4.3865
Total Degrees of Freedom	761.6268
Degrees of Freedom	761
Standard Error	7.6026
Difference in Sample Means	68.0000
Separate-Variance *t* Test Statistic	**8.9443**
Two-Tail Test	
Lower Critical Value	**-1.9631**
Upper Critical Value	**1.9631**
***p*-Value**	**0.0000**
Reject the null hypothesis	

Since the p-value is virtually zero, reject H_0. There is enough evidence of a difference in the mean order value between Pinterest shoppers and Facebook shoppers

(c) $53.0754 \le \mu_1 - \mu_2 \le 82.92462003$

10.62 (a) $H_0 : \pi_1 - \pi_2 = 0$

$H_1 : \pi_1 - \pi_2 \neq 0$

where Populations: 1 = Males, 2 = Females

PHStat2 output:

ZTest for Differences inTwoProportions

Data	
HypothesizedDifference	**0**
Level of Significance	**0.05**
Group1	
Number of Items of Interest	**50**
Sample Size	**300**
Group2	
Number of Items of Interest	**96**
Sample Size	**330**
Intermediate Calculations	
Group 1Proportion	0.166666667
Group 2Proportion	0.290909091
Difference in Two Proportions	-0.12424242
Average Proportion	0.2317
***Z* Test Statistic**	**-3.6911**
Two-Tail Test	
Lower Critical Value	**-1.9600**
Upper Critical Value	**1.9600**
***p*-Value**	**0.0002**
Reject the null hypothesis	

Since the *p*-value is smaller than 0.05, reject H_0. There is enough evidence of a difference between males and females in the proportion who order dessert

Decision rule: Reject H_0 if $Z_{calc} > 1.645$
Decision: Since $Z_{calc} = 1.07 < 1.645$, we do not reject H_0. There is insufficient evidence to suggest a greater proportion of younger voters vote for the Greens compared to older voters.

(b) From the PHStat2 output, the *p*-value is 0.1422.

(c)

F Test Two-Sample for Variances

	Male	*Female*
Mean	88683.23684	74575.17544
Variance	3834187011	2243135437
Observations	114	114
df	113	113
F	1.709298042	
P(*F*<=*f*) one-tail	0.002354401	
F Critical one-tail	1.553351444	

$H_0 : \sigma_M^2 - \sigma_F^2 = 0$: The population variances are the same

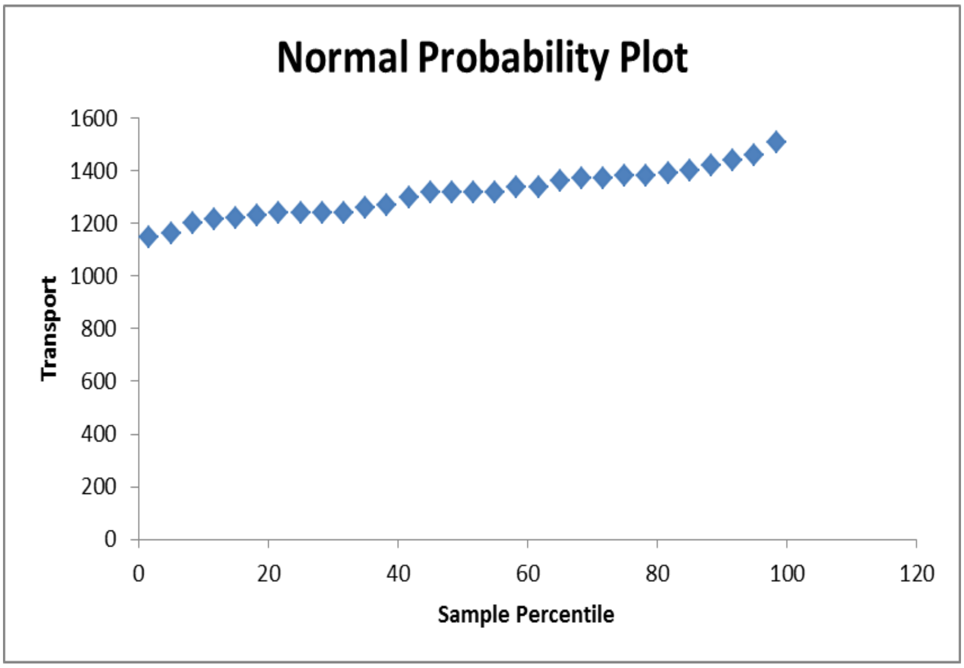
$H_1 : \sigma_M^2 - \sigma_F^2 \neq 0$: The population variances are different

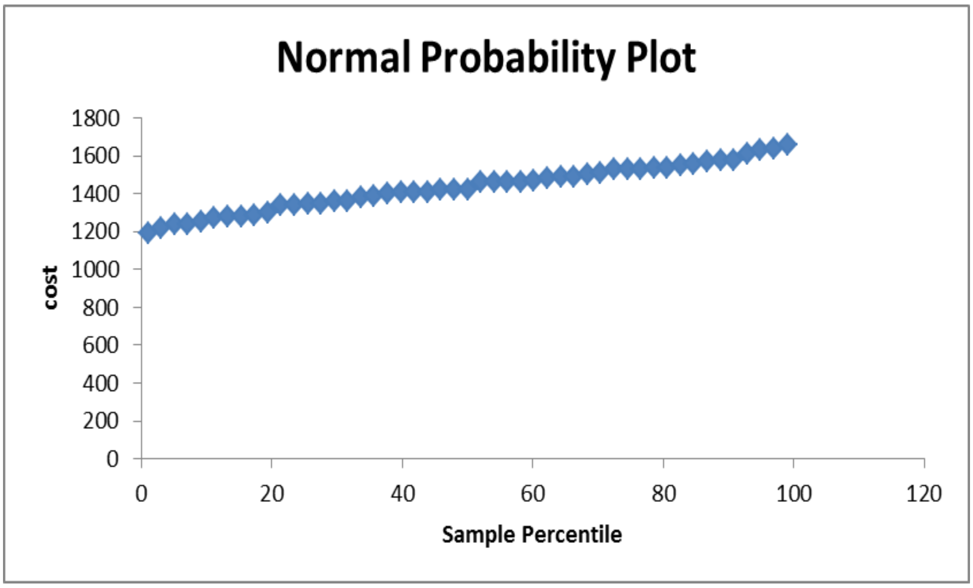
Decision rule: if $F_{calc} < 0.614$ or > 1.629, reject null hypothesis.

Test statistics: $F = \dfrac{S_M^2}{S_F^2} = 1.709$

Decision: Since $F = 1.709 > 1.629$, reject null hypothesis. There is enough evidence to conclude that the two population variances are different.

10.63





The two normal probability plots do not suggest any departure from the normality assumption. You can perform *F* test on the different variances.

$H_0: \sigma_C^2 - \sigma_S^2 = 0$, $H_1: \sigma_C^2 - \sigma_S^2 \neq 0$

F Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	1313.566667	1435
Variance	8088.529885	14939.79592
Observations	30	50
df	29	49
F	0.541408325	
P(F<=f) one-tail	0.039847629	

F Critical one-tail	0.562623603	

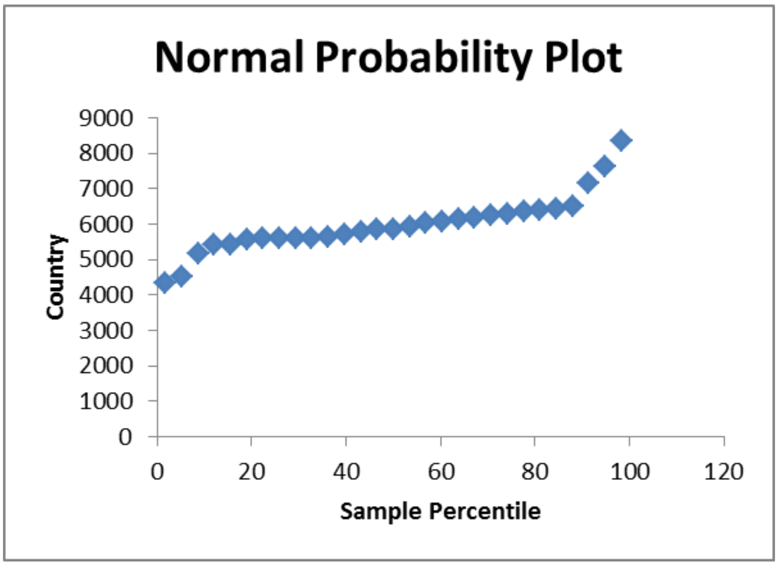
Since the *p*-value = 0.07965 > 0.05, we do not reject the null hypothesis. There is not sufficient evidence to conclude that two variances are different. We can perform pooled variance *t* test for differences in means.

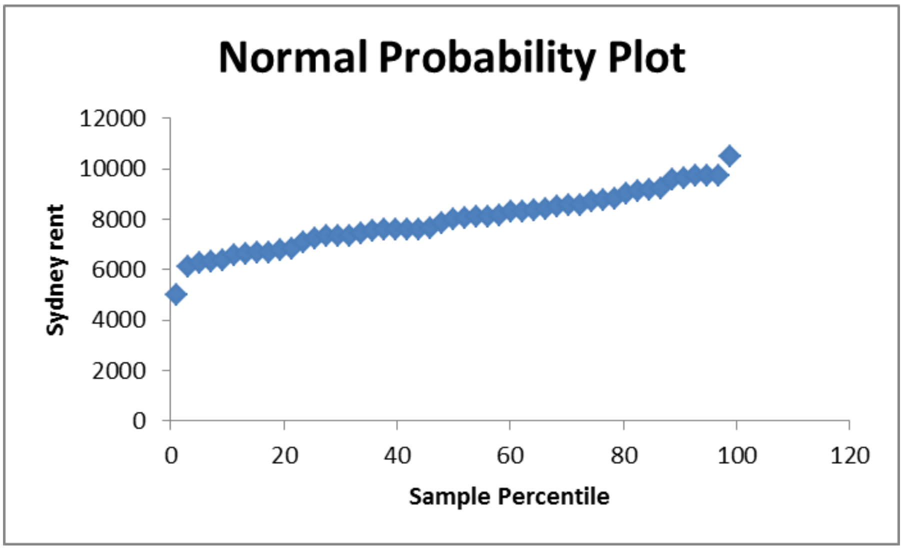
$H_0: \mu_C - \mu_S = 0$

$H_1: \mu_C - \mu_S \neq 0$

t Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	1313.566667	1435
Variance	8088.529885	14939.79592
Observations	30	50
Pooled Variance	12392.53034	
Hypothesized Mean Difference	0	
df	78	
t Stat	-4.72344168	
P(*T*<=*t*) one-tail	5.03164E-06	
t Critical one-tail	1.664624645	
P(*T*<=*t*) two-tail	1.00633E-05	
t Critical two-tail	1.990847069	

Since the *p*-values are essentially zero, reject the null hypothesis. There is sufficient evidence to conclude that means of transport costs are different for NSW and Sydney trainees.





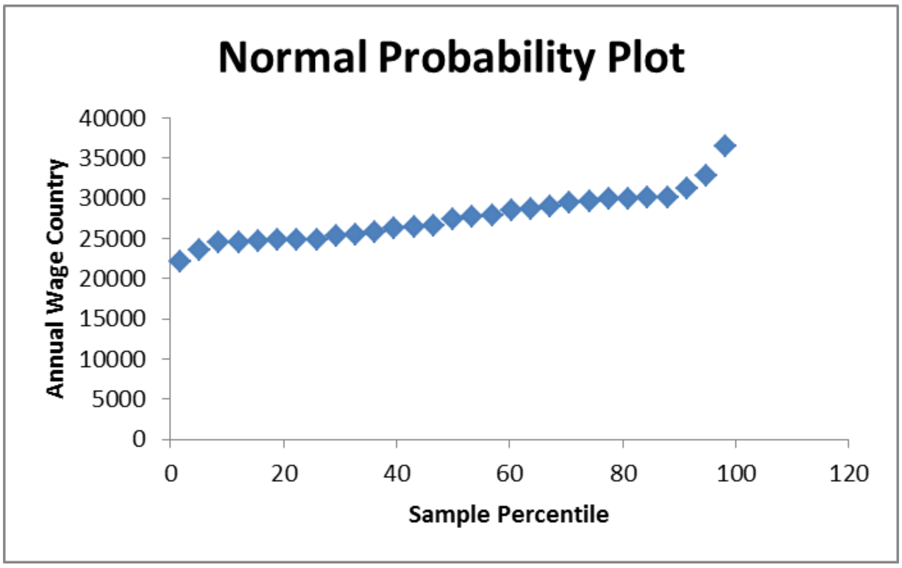
Both normal probability plots suggest that both distributions are not normal. It is inappropriate to perform an *F* test on the difference in variances. Values on sample variances, $S_C^2 = 617974.01$ and $S_S^2 = 1306292.396$, suggest that a separate variance *t* test is more appropriate.

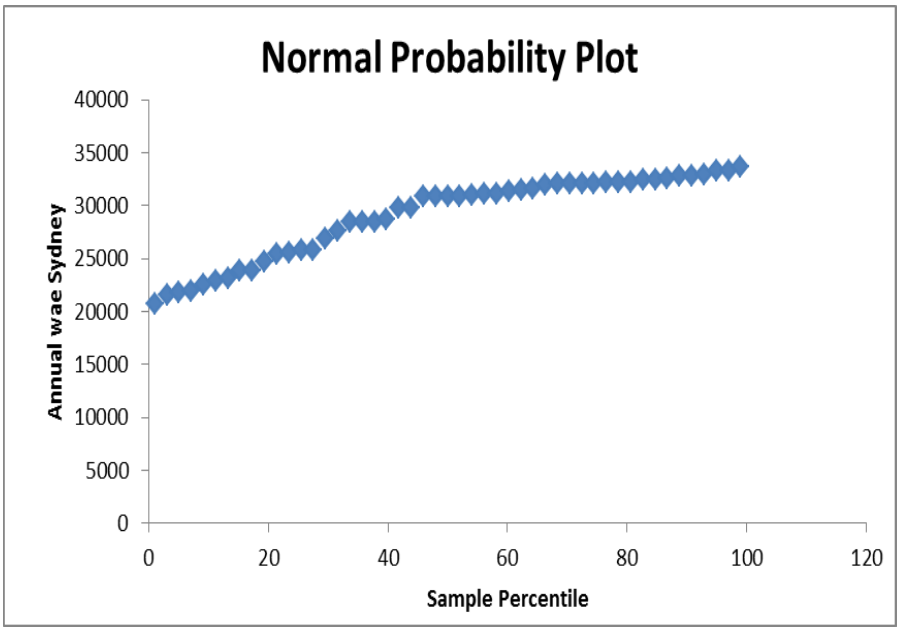
$H_0 : \mu_C - \mu_S = 0$

$H_1 : \mu_C - \mu_S \neq 0$

t Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	5986.7	7917.18
Variance	617974.0103	1306292
Observations	30	50
Hypothesized Mean Difference	0	
Df	76	
t Stat	−8.93080768	
P(*T*<=*t*) one-tail	9.05559E-14	
t Critical one-tail	1.665151353	
P(*T*<=*t*) two-tail	1.81112E-13	
t Critical two-tail	1.99167261	

Since the *p*-value is essentially zero, reject the null hypothesis. There is sufficient evidence to conclude that the means of the rent cost are different for NSW country and Sydney.





Both normal probability plots suggest that both distributions are not normal. It is inappropriate to perform an *F* test on the difference in variances. Values on sample variances, $S_C^2 = 9410128.179$ and $S_S^2 = 15180521.27$, suggest that a separate variance *t* test is more appropriate.

$$H_0: \mu_C - \mu_S = 0$$
$$H_1: \mu_C - \mu_S \neq 0$$

t Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	27478.4	29124.44
Variance	9410128	15180521
Observations	30	50
Hypothesized	0	

Mean Difference		
df	72	
t Stat	-2.09507	
P(*T*<=*t*) one-tail	0.019841	
t Critical one-tail	1.666294	
P(*T*<=*t*) two-tail	0.039683	
t Critical two-tail	1.993464	

Since the *p*-values for a two-tail test is 0.0397 < 0.05, reject the null hypothesis. There is sufficient evidence to conclude that the means of the annual wages are different for NSW country and Sydney trainees.

10.64

A		*B*	
Mean	7377.325	Mean	8260.9
Standard Error	135.3817224	Standard Error	143.8566051
Median	7316.5	Median	8140.5
Mode	8416	Mode	#N/A
Standard Deviation	856.2291925	Standard Deviation	909.8290569
Sample Variance	733128.4301	Sample Variance	827788.9128
Kurtosis	−1.010573077	Kurtosis	−1.311672475
Skewness	-0.17096025	Skewness	0.047944167
Range	3187	Range	3043
Minimum	5544	Minimum	6701
Maximum	8731	Maximum	9744
Sum	295093	Sum	330436
Count	40	Count	40

From the descriptive statistics above, we know that both data seem to have come from rather symmetrical distributions that are quite normally distributed since the value of skewness is close to zero.

F Test Two-Sample for Variances		
	A	*B*
Mean	7377.325	8260.9
Variance	733128.4301	827788.9
Observations	40	40
df	39	39
F	0.885646593	
P(*F*<=*f*) one-tail	0.35321677	
F Critical one-tail	0.586694336	

The following *F* test *p*-value = 0.7064. Do not reject the null hypothesis. There is insufficient evidence that the two population variances are significantly different at 5% significance level.

Since both data are drawn from independent populations, the most appropriate test for any difference in the life of the bulbs between two manufacturers is the pooled variance *t* test.

t Test: Two-Sample Assuming Equal Variances		
	A	*B*
Mean	7377.325	8260.9
Variance	733128.4301	827788.9
Observations	40	40
Pooled Variance	780458.6715	
Hypothesized Mean Difference	0	
df	78	
t Stat	−4.472841019	
P(T<=t) one-tail	1.29478E-05	
t Critical one-tail	1.664624645	
P(T<=t) two-tail	2.58957E-05	
t Critical two-tail	1.990847069	

Since the *p*-value is virtually zero, at the 5% significance level, there is sufficient evidence to reject the null hypothesis of no difference in the mean life of the bulbs between two manufacturers. Based on the above analysis we can conclude that there is significant difference in the life of the bulbs between two manufacturers.

10.65
Female

Age		*Current points*		*Top speed*		*Season points*	
Mean	24.48387	Mean	184.9032	Mean	71.70968	Mean	3460.323
Standard Error	0.483871	Standard Error	2.599981	Standard Error	0.464187	Standard Error	105.6994
Median	24	Median	183	Median	72	Median	3505
Mode	23	Mode	184	Mode	72	Mode	3590
Standard Deviation	2.69408	Standard Deviation	14.47608	Standard Deviation	2.584486	Standard Deviation	588.5093
Sample Variance	7.258065	Sample Variance	209.557	Sample Variance	6.67957	Sample Variance	346343.2
Kurtosis	0.050729	Kurtosis	2.586411	Kurtosis	-0.51209	Kurtosis	-1.2441
Skewness	0.013505	Skewness	1.489346	Skewness	0.157465	Skewness	-0.17929
Range	12	Range	64	Range	10	Range	1830
Minimum	18	Minimum	163	Minimum	67	Minimum	2485
Maximum	30	Maximum	227	Maximum	77	Maximum	4315
Sum	759	Sum	5732	Sum	2223	Sum	107270
Count	31	Count	31	Count	31	Count	31

Male

Age		*Current points*		*Top speed*		*Season points*	
Mean	22.15556	Mean	187.9778	Mean	71	Mean	3391.722
Standard Error	0.456309	Standard Error	1.33931	Standard Error	0.331079	Standard Error	50.79488
Median	21	Median	189	Median	71	Median	3427.5
Mode	21	Mode	201	Mode	68	Mode	3990
Standard Deviation	4.328923	Standard Deviation	12.70581	Standard Deviation	3.140887	Standard Deviation	481.8825
Sample Variance	18.73958	Sample Variance	161.4377	Sample Variance	9.865169	Sample Variance	232210.8
Kurtosis	8.172932	Kurtosis	−0.29096	Kurtosis	−0.0965	Kurtosis	-0.50866
Skewness	2.47886	Skewness	−0.2567	Skewness	0.556294	Skewness	-0.29968
Range	24	Range	60	Range	14	Range	2165
Minimum	17	Minimum	155	Minimum	65	Minimum	2150
Maximum	41	Maximum	215	Maximum	79	Maximum	4315
Sum	1994	Sum	16918	Sum	6390	Sum	305255
Count	90	Count	90	Count	90	Count	90

From the descriptive statistics above, we can see that the value of skewness and kurtosis are quite different from normal distribution. The *F* test for the difference in variances, which is sensitive to departure from normal probability distribution assumption, will not be appropriate. The variances of all the variables between female and male cyclists are also quite different. Hence, you perform separate variance *t* tests on the difference on means.

$$H_0 : \mu_F - \mu_M = 0$$
$$H_1 : \mu_F - \mu_M \neq 0$$

t Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	24.48387	22.15556
Variance	7.258065	18.73958
Observations	31	90
Hypothesized Mean Difference	0	
Df	85	
t Stat	3.500737	
P(T<=t) one-tail	0.000371	
t Critical one-tail	1.662978	
P(T<=t) two-tail	0.000742	
t Critical two-tail	1.988268	

Since the *p*-value is 0.0007 < 0.05, we reject the null hypothesis. There is sufficient evidence to conclude that the mean ages are different between males and females.

Current points:

$H_0 : \mu_F - \mu_M = 0$

$H_1 : \mu_F - \mu_M \neq 0$

t Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	184.9032	187.9778
Variance	209.557	161.4377
Observations	31	90
Hypothesized Mean Difference	0	
df	47	
t Stat	−1.05125	
P(T<=t) one-tail	0.14926	
t Critical one-tail	1.677927	
P(T<=t) two-tail	0.298519	
t Critical two-tail	2.011741	

Since, the *p*-values = 0.298 > 0.05, we do not reject the null hypothesis. There is not enough evidence to conclude that the mean current points are different between males and females.

$H_0 : \mu_F - \mu_M = 0$

$H_1 : \mu_F - \mu_M \neq 0$

t Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	71.70968	71
Variance	6.67957	9.865169
Observations	31	90
Hypothesized Mean Difference	0	
df	63	
t Stat	1.244698	
P(T<=t) one-tail	0.108927	
t Critical one-tail	1.669402	
P(T<=t) two-tail	0.217853	
t Critical two-tail	1.998341	

Since *p*-value = 0.2179 > 0.05, we do not reject the null hypothesis. There is not enough evidence to conclude that the mean top speed is different between males and females.

Season points

$H_0 : \mu_F - \mu_M = 0$

$H_1 : \mu_F - \mu_M \neq 0$

t Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	3460.323	3391.722
Variance	346343.2	232210.8
Observations	31	90
Hypothesized Mean Difference	0	
Df	45	
t Stat	0.584973	
P(T<=t) one-tail	0.280744	
t Critical one-tail	1.679427	
P(T<=t) two-tail	0.561488	
t Critical two-tail	2.014103	

Since the *p*-values = 0.5615 > 0.05, we do not reject the null hypothesis. There is not sufficient evidence that the mean season points are different between males and females.

10.66

$H_0 : \mu_D = 0$

$H_1 : \mu_D \neq 0$

Choosing the level of significance = 0.1 and assuming the differences are normally distributed, use paired *t* test. Degrees of freedom = 7

Reject null hypothesis if t_{calc} >1.895 or <–1.895.

$$t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}} = \frac{-35.125}{44.95} = -0.781$$

Since –0.781 > –1.895, we do not reject the null hypothesis and there is not enough evidence that the husband and wife of a couple have different spending patterns at 10% significance level.

10.67
$H_0 : \sigma_1^2 - \sigma_2^2 = 0$

$H_1 : \sigma_1^2 - \sigma_2^2 \neq 0$

F Test for Differences in Two Variances	
Data	
Level of Significance	**0.01**
Larger-Variance Sample	
Sample Size	**100**
Sample Variance	**15625**
Smaller-Variance Sample	
Sample Size	**100**
Sample Variance	**10000**
Intermediate Calculations	
***F* Test Statistic**	**1.5625**

Population1SampleDegreesofFreedo	99
Population2SampleDegreesofFreedo	99

Two-Tail Test	
Upper Critical Value	**1.6854**
***p*-Value**	**0.0274**
Donot reject the null hypothesis	

Since $p > 0.01$ there is not enough evidence of a difference in the variances of the amount of time spent talking between women and men

(b)It is more appropriate to use a pooled-variance *t* test

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 - \mu_2 \neq 0$$

Pooled-Variance *t* Test for the Difference Betwee	
(assumes equal populationvariances)	
Data	
HypothesizedDifference	**0**
Level of Significance	**0.01**
Population1Sample	
Sample Size	**100**
Sample Mean	**818**
Sample StandardDeviation	**125**
Population2Sample	
Sample Size	**100**
Sample Mean	**716**
Sample StandardDeviation	**100**
Intermediate Calculations	
Population1SampleDegreesofFreedo	99
Population2SampleDegreesofFreedo	99
Total Degrees of Freedom	198
Pooled Variance	12812.5
Standard Error	16.0078
Difference in Sample Means	102
***t* Test Statistic**	**6.3719**
Two-Tail Test	
Lower Critical Value	**-2.6009**
Upper Critical Value	**2.6009**
***p*-Value**	**0.0000**
Reject the null hypothesis	

Since p value is virtually zero there is enough evidence of a difference in the mean amount of time spent talking between women and men.

(c) $H_0: \sigma_1^2 - \sigma_2^2 = 0$
$H_1: \sigma_1^2 - \sigma_2^2 \neq 0$

F Test for Differences in Two Variances	
Data	
Level of Significance	**0.01**
Larger-Variance Sample	
Sample Size	**100**
Sample Variance	**22500**
Smaller-Variance Sample	
Sample Size	**100**
Sample Variance	**15625**
Intermediate Calculations	
***F* Test Statistic**	**1.4400**
Population 1 Sample Degrees of Freedo	99
Population 2 Sample Degrees of Freedo	99
Two-Tail Test	
Upper Critical Value	**1.6854**
***p*-Value**	**0.0711**
Do not reject the null hypothesis	

Since p value > 0.01 there is not enough evidence of a difference in the variances of the number of text messages sent per month by women and men.

(d) It is more appropriate to use a pooled-variance *t* test.

$H_0: \mu_1 - \mu_2 = 0$
$H_1: \mu_1 - \mu_2 \neq 0$

Pooled-Variance *t* Test for the Difference Betwee

(assumes equal populationvariances)	
Data	
HypothesizedDifference	**0**
Level of Significance	**0.01**
Population1Sample	
Sample Size	**100**
Sample Mean	**716**
Sample StandardDeviation	**150**
Population2Sample	
Sample Size	**100**
Sample Mean	**555**
Sample StandardDeviation	**125**
Intermediate Calculations	
Population1SampleDegreesofFreedo	99
Population2SampleDegreesofFreedo	99
Total Degrees of Freedom	198
Pooled Variance	19062.5
Standard Error	19.5256
Difference in Sample Means	161
***t* Test Statistic**	**8.2456**
Two-Tail Test	
Lower Critical Value	**-2.6009**
Upper Critical Value	**2.6009**
***p*-Value**	**0.0000**
Reject the null hypothesis	

Since p value is virtually zero there is enough evidence of a difference in the mean number of text messages sent per month by women and men

10.68

$H_0:\sigma_L^2-\sigma_S^2\le 0$, $H_1:\sigma_L^2-\sigma_S^2>0$

Decision rule: If F_{calc} > 4.54, reject null hypothesis.

Test statistics: $F=\dfrac{S_l^2}{S_S^2}=1.345$

Decision: Since F = 1.345 < 4.54, we do not reject the null hypothesis. There is not enough evidence to conclude that two population variances are different at 1% significance level.

10.69 (a) As the data prices for the same items at two different stores, a paired *t* test is appropriate.

$H_0:\mu_C-\mu_W=0$ The mean price of stationery at Coles and Woolworths are the same in the week

$H_1:\mu_C-\mu_W\ne 0$ The mean price of stationery at Coles and Woolworths are different in the week

t Test: Paired Two-Sample for Means		
	Coles	*Woolworths*
Mean	0.719333	0.702667
Variance	0.37575	0.371807
Observations	15	15
Pearson Correlation	0.960864	
Hypothesized Mean Difference	0	
Df	14	
t Stat	0.377318	
P(T<=t) one-tail	0.355798	
t Critical one-tail	2.624494	
P(T<=t) two-tail	0.711595	
t Critical two-tail	2.976843	

Since t = 0.377 < 2.9768, we do not reject the null hypothesis and conclude that there is insufficient evidence that the mean price of stationery was different at Coles and Woolworths.

(b) The *p*-value for two-tail test is 0.7116 > 0.01, so we do not reject the null hypothesis. The *p*-value represents the probability of obtaining samples that will yield a test statistic more extreme than 0.3773 if the means are equal.

10.70

$H_0:\pi_p-\pi_r\le 0$

$H_1:\pi_p-\pi_r>0$

$\mathbf{p_p=0.74,\ p_r=0.4635,\ \bar{p}=0.5976}$

Using the 0.01 level of significance

Decision, reject null hypothesis if Z_{calc} > 2.326

$$Z=\frac{(p_p-p_r)-(\pi_p-\pi_r)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_p}+\frac{1}{n_r}\right)}}=11.521$$

Since 11.521 > 2.326, we reject the null hypothesis and conclude that there is enough evidence that the proportion of city dwellers who have access ADSL is higher than the proportion of regional dwellers.

10.71 $H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$

df = 20 + 20 -2 = 38

Decision rule: Reject H_0 if $t_{calc} < -2.0244$ or > 2.0244.

Since $t_{calc} = 5.1615 > 2.0244$, reject H_0.

There is enough evidence of a difference in the mean delivery time in the two wings of the hotel.

10.72 To construct the 95% confidence interval estimate in question 10.70

$$(0.74-0.4635)\pm 1.96\sqrt{\frac{0.1824}{800}+\frac{0.2487}{850}}=\left[0.2318{,}0.32125\right]$$

You have 95% confidence that the difference between the population proportion of city dwellers and regional dwellers who have access to ADSL is between 0.2315 and 0.3213.

10.73

$H_0 : \pi_p - \pi_F \leq 0$

$H_1 : \pi_p - \pi_F > 0$

$p_1 = 0.76, p_2 = 0.78, \bar{p} = 0.773$

Using the 0.1 level of significance

Decision, reject null hypothesis if $Z_{calc} > 2.326$

$$Z=\frac{(p_p-p_f)-(\pi_p-\pi_f)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_p}+\frac{1}{n_f}\right)}}=-0.323$$

Since −0.323 < 1.282, we reject the null hypothesis at 10% level and conclude that there is not enough evidence that the proportion of young males is higher than the proportion of males speeding on a regular basis.