SIT718 Real World Analytics

School of Information Technology

Deakin University

Prac. 10 Problems

2-Person 0-Sum Game: Q1

Consider the following 2-person zero-sum game. Does the game have a pure strategy? If not, what is the range of possible values for the value of the game, v?

$$V = \begin{bmatrix} 3 & -1 & -3 \\ -2 & 4 & -1 \\ -5 & -6 & 2 \end{bmatrix}$$

Solve the game for both players using Linear Programming (you can use a solver to solve the LP).

2-Person 0-Sum Game: Q2

Does the following game have an optimal pure strategy pair? If yes, identify the optimal pure strategies for each of the players, and if not, present the optimal mixed strategies (you can use a solver to solve the LP).

	A_1	A_2	A_3	A_4
a_1	1	-2	1	2
a_2	3	2	5	4
a_3	0	4	-2	-3
a_4	2	3	-2	-4

2-Person non-zero Sum Game

Consider the following non-zero sum game and find an equilibrium point (if one exists in pure strategies).

	A_1	A_2
a_1	(3,3)	(11,-1)
a_2	(-1,11)	(7,7)

SIT718 Real World Analytics

Prac. 10 Problems

2-Person 0-Sum Game: Q1

Player I's game

$$
\begin{aligned}
\max \quad & z = v \\
\text{s.t.} \quad & v - (3x_1 - 2x_2 - 5x_3) \leq 0 \\
& v - (-x_1 + 4x_2 - 6x_3) \leq 0 \\
& v - (-3x_1 - x_2 + 2x_3) \leq 0 \\
& x_1 + x_2 + x_3 = 1 \\
& x_i \geq 0, \forall i = 1, 2, 3. \\
& v \text{ u.r.s.}
\end{aligned}
$$

Optimal strategies for Player I: $v = -0.90826;$
$x_1 = 0.3945; x_2 = 0.31193; x_3 = 0.29358$

Player II's game

$$
\begin{aligned}
\min \quad & z = v \\
\text{s.t.} \quad & v - (3y_1 - y_2 - 3y_3) \geq 0 \\
& v - (-2y_1 + 4y_2 - y_3) \geq 0 \\
& v - (-5y_1 - 6y_2 + 2y_3) \geq 0 \\
& y_1 + y_2 + y_3 = 1 \\
& y_i \geq 0, \forall i = 1, 2, 3. \\
& v \text{ u.r.s.}
\end{aligned}
$$

Optimal strategies for Player II: $v = -0.90826;$
$y_1 = 0.3211;\ y_2 = 0.082569;\ y_3 = 0.59633$

2-Person 0-Sum Game: Q2

Since min col max = 3 and max row min = 2, min col max $\neq$ max row min, the game does not have an optimal pure strategy pair. However, optimal mixed strategies exist for both players.

Player I's game:

$$
\begin{array}{rll}
\max & z = v & \\
\text{s.t.} & v - (x_1 + 3x_2 + 2x_4) & \leq 0 \\
& v - (-2x_1 + 2x_2 + 4x_3 + 3x_4) & \leq 0 \\
& v - (x_1 + 5x_2 - 2x_3 - 2x_4) & \leq 0 \\
& v - (2x_1 + 4x_2 - 3x_3 - 4x_4) & \leq 0 \\
& x_1 + x_2 + x_3 + x_4 & = 1 \\
& x_i \geq 0, \forall i = 1, \ldots, 4. & \\
& v \text{ u.r.s.} &
\end{array}
$$

The value of the game is: 2.4074. The optimal mixed strategy for Player I is: $x_1 = 0$, $x_2 = 0.77778$, $x_3 = 0.18519$, and $x_4 = 0.037037$.

Player II's game:

$$
\begin{array}{rll}
\min & w = v & \\
\text{s.t.} & v - (y_1 - 2y_2 + y_3 + 2y_4) & \geq 0 \\
& v - (3y_1 + 2y_2 + 5y_3 + 4y_4) & \geq 0 \\
& v - (4y_2 - 2y_3 - 3y_4) & \geq 0 \\
& v - (2y_1 + 3y_2 - 2y_3 - 4y_4) & \geq 0 \\
& y_1 + y_2 + y_3 + y_4 & = 1 \\
& y_i \geq 0, \forall i = 1, \ldots, 4. & \\
& v \text{ u.r.s.} &
\end{array}
$$

The value of the game is: 2.4074. The optimal mixed strategy for Player I is: $y_1 = 0.33333$, $y_2 = 0.62963$, $y_3 = 0$, and $y_4 = 0.037037$.

2-Person non-zero Sum Game

(3,3) is an equilibrium point.