MODULE THREE: DETERMINING CAUSE AND MAKING RELIABLE FORECASTS

TOPIC 10: TIME SERIES FORECASTING AND INDEX NUMBERS

Learning Objectives

At the completion of this topic, you should be able to:

- explain the importance of business forecasting
- disaggregate the components of a time series
- use moving averages and exponential smoothing
- apply linear trend, quadratic trend and exponential trend time-series models
- estimate the Holt–Winters forecasting model
- use autoregressive models
- choose an appropriate forecasting model
- estimate a forecasting model for seasonal data
- calculate various price indices

+The Importance of Business Forecasting

Governments forecast unemployment, interest rates and expected revenues from income taxes for policy purposes

Marketing executives forecast demand, sales and consumer preferences for strategic planning

College administrators forecast enrolments to plan for facilities and for faculty recruitment

Retail stores forecast demand to control inventory levels, hire employees and provide training

+Common Approaches to Forecasting

Common Approaches to Forecasting

- Qualitative forecasting methods
  - Used when historical data are unavailable
  - Considered highly subjective and judgmental
- Quantitative forecasting methods
  - Time Series
    - Use past data to predict future values
  - Causal

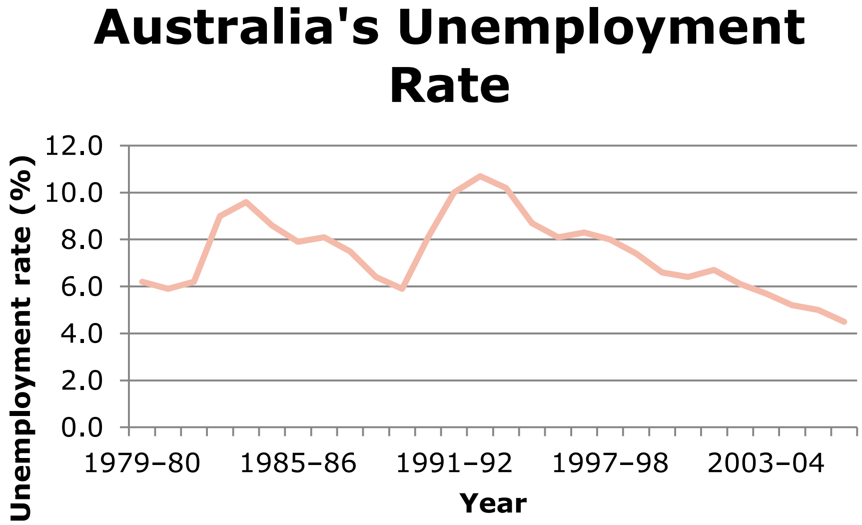
+Time-Series Data and Plot

- Numerical data obtained at regular time intervals
- The time intervals can be annually, quarterly, daily, hourly, etc.

A time-series plot is a two-dimensional plot of time series data

The vertical axis measures the variable of interest

The horizontal axis corresponds to the time periods



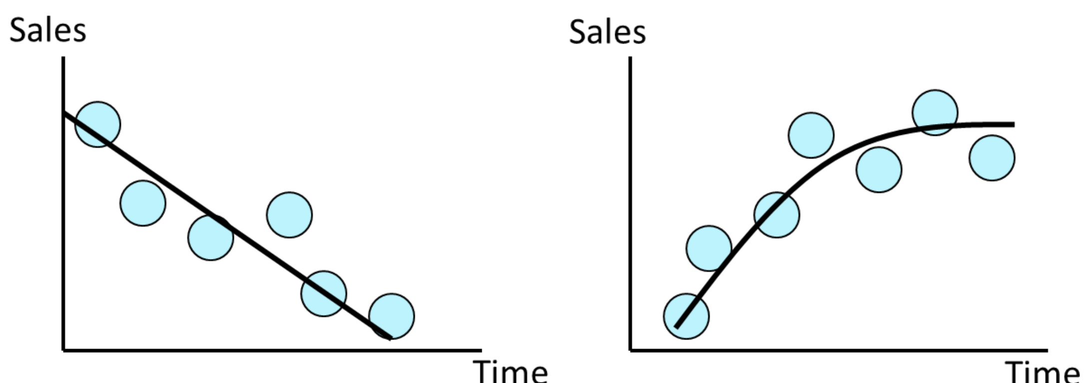
Classical Multiplicative Time-Series Model Components

Component	Classification of component	Definition	Reason for influence	Duration
Trend	Systematic	Overall or persistent, long-term upward or downward pattern of movement	Changes in technology, population, wealth, value	Several years
Seasonal	Systematic	Fairly regular periodic fluctuations that occur within each 12-month period year after year	Weather conditions, social customs, religious customs, school schedules	Within 12 months (or monthly or quarterly data)
Cyclical	Systematic	Repeating up-and-down swings or movements through four phases: from peak (prosperity) to contraction (recession) to trough (depression) to expansion (recovery or growth)	Interactions of numerous combinations of factors that influence the economy	Usually 2–10 years, with differing intensity for a complete cycle
Irregular	Unsystematic	The erratic or 'residual' fluctuations in a series that exist after taking into account the systematic effects	Random variations in data due to unforeseen events such as strikes, natural disasters and wars	Short duration and non-repeating

Table 14.1
Factors influencing time-series data

+Trend Component

- Long-run increase or decrease over time (overall upward or downward movement)
- Data taken over a long period of time
- Trend can be upward or downward
- Trend can be linear or non-linear

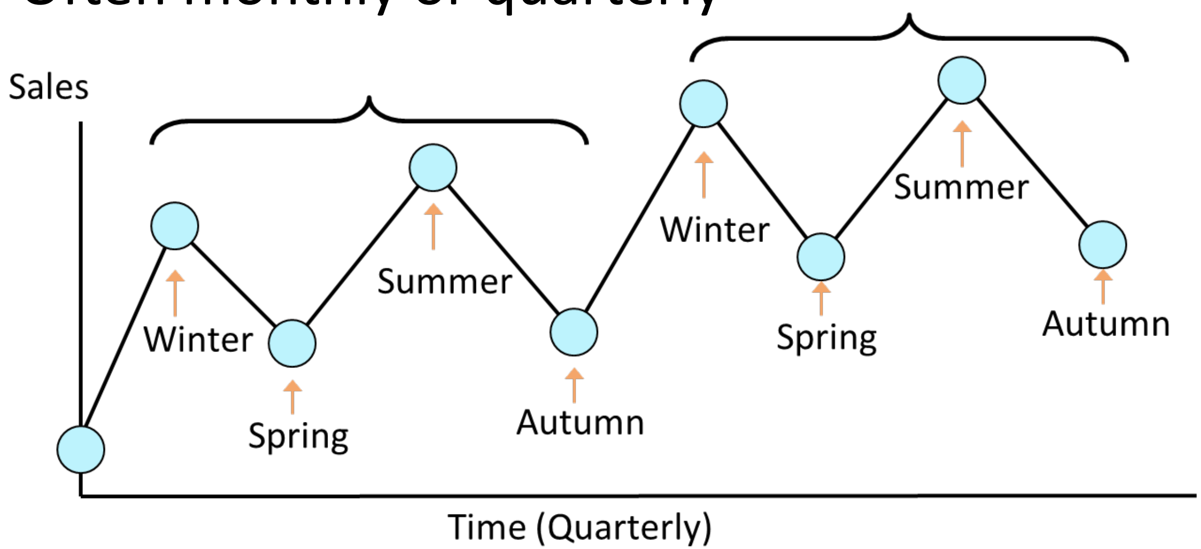


Downward linear trend

Upward non-linear trend

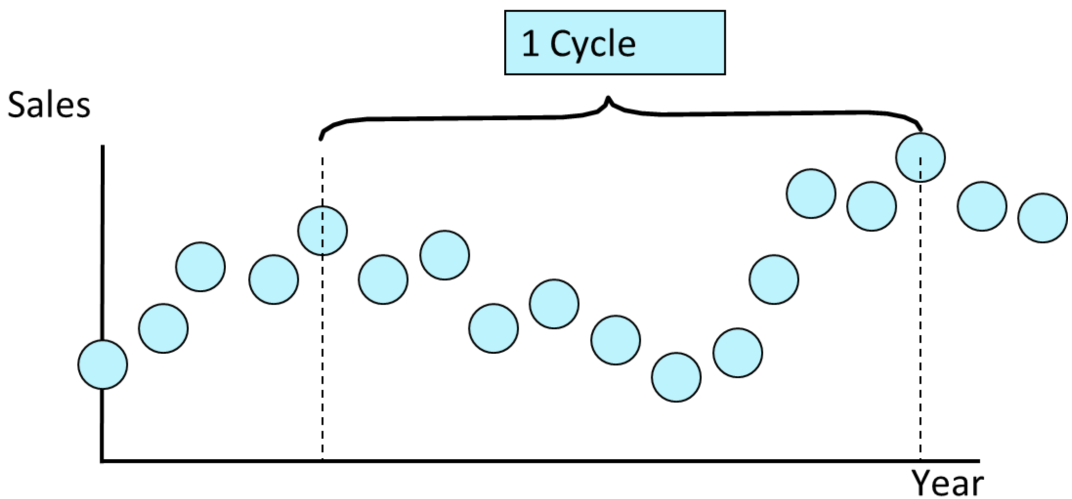
+Seasonal Component

- Short-term regular wave-like patterns
- Observed within 1 year
- Often monthly or quarterly



+Cyclical Component

- Long-term wave-like patterns
- Usually occur every 2-10 years
- Often measured peak-to-peak or trough-to-trough



+Irregular Component

Unpredictable, random, ‘residual’ fluctuations

Due to random variations of:

- nature
- accidents or unusual events

‘Noise’ in the time series

Usually short duration and non-repeating

+Multiplicative Time-Series Model for Annual Data

Used primarily for forecasting

Observed value in time-series is the product of components

$$\boxed{Y_i = T_i \times C_i \times I_i}$$

Where: T_i = value of trend component

C_i = value of cyclical component

I_i = value of irregular (random) component

+Multiplicative Time-Series Model with Seasonal Component

Used primarily for forecasting

Allows consideration of seasonal variation

$$\boxed{Y_i = T_i \times S_i \times C_i \times I_i}$$

Where:

T_i = value of trend component

S_i = value of seasonal component

C_i = value of cyclical component

I_i = value of irregular (random) component

+Smoothing the Annual Time-Series – Moving Averages

A series of arithmetic means over time

Calculate moving averages to get an overall impression of the pattern of movement over time

Moving averages can be used for smoothing: averages of consecutive time series values for a chosen period of length L

Result dependent upon choice of L (length of period for computing means)

Examples:

- For a 5-year moving average, L = 5
- For a 7-year moving average, L = 7 etc.

Moving Averages

Example: Five-year moving average (L=5)

First average

$$MA(5) = \frac{Y_1 + Y_2 + Y_3 + Y_4 + Y_5}{5}$$

Second average

$$MA(5) = \frac{Y_2 + Y_3 + Y_4 + Y_5 + Y_6}{5}$$

+Exponential Smoothing

A **weighted** moving average:

- weights decline exponentially
- most recent observation weighted most

Used for smoothing and short-term forecasting (often one period into the future)

The weight (smoothing coefficient) is **W**:

- subjectively chosen
- range from 0 to 1
- smaller W gives more smoothing, larger W gives less smoothing

The weight is:

- close to 0 for smoothing out unwanted cyclical and irregular components
- close to 1 for forecasting

+Exponential Smoothing Model

$$\boxed{E_1 = Y_1}$$

$$\boxed{E_i = WY_i + (1 - W)E_{i-1}}$$

For i = 2, 3, 4, …

Where:

- E_i = exponentially smoothed value for period i
- E_i-1 = exponentially smoothed value already calculated for period i - 1
- Y_i = observed value in period i
- W = weight (smoothing coefficient), 0 < W < 1

+Exponential Smoothing Example using W = 0.2:

Time Period (i)	Sales (Y_i)	Forecast from prior period (E_{i-1})	Exponentially Smoothed Value for this period (E_i) using $E_i = WY_i + (1-W)E_{i-1}$
1	23	--	23*
2	40	23	(.2)(40)+(.8)(23)=26.4
3	25	26.4	(.2)(25)+(.8)(26.4)=26.12
4	27	26.12	(.2)(27)+(.8)(26.12)=26.296
5	32	26.296	(.2)(32)+(.8)(26.296)=27.437
6	48	27.437	(.2)(48)+(.8)(27.437)=31.549
7	33	31.549	(.2)(48)+(.8)(31.549)=31.840
8	37	31.840	(.2)(33)+(.8)(31.840)=32.872
9	37	32.872	(.2)(37)+(.8)(32.872)=33.697
10	50	33.697	(.2)(50)+(.8)(33.697)=36.958
etc.	etc.	etc.	etc.

*$E_1 = Y_1$ since no prior information exists

+Forecasting Time Period i + 1

The smoothed value in the current period (i) is used as the forecast value for the next period (i + 1)

$$\boxed{\hat{Y}_{i+1} = E_i}$$

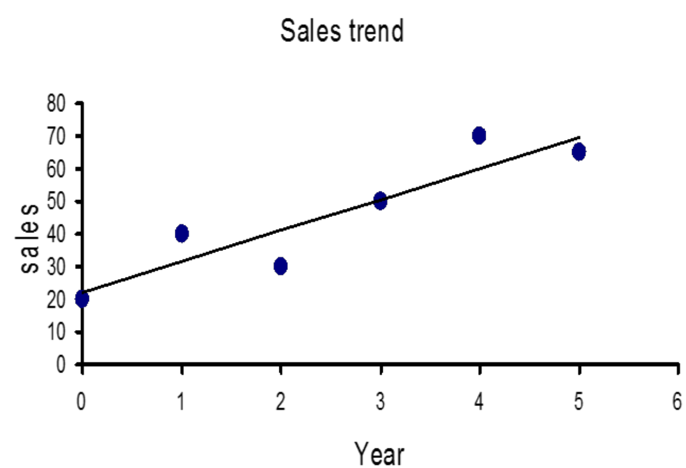
+Least-squares Trend-fitting

Estimate a trend line using regression analysis

Year	Time Period (X)	Sales (Y)
2010	0	20
2011	1	40
2012	2	30
2013	3	50
2014	4	70
2015	5	65

$$\hat{Y} = b_0 + b_1 X$$

$$\hat{Y}_i = 21.905 + 9.5714\, X_i$$



Non-linear Trend Forecasting

A non-linear regression model can be used when the time series exhibits a non-linear trend

Quadratic form is one type of a non-linear model

$$\boxed{Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \varepsilon_i}$$

Compare adjusted r^2 and standard error to that of linear model to see if this is an improvement

Can try other functional forms to get best fit

+Exponential Trend Model

Another non-linear trend model

$$\boxed{Y_i = \beta_0 \beta_1^{X_i} \varepsilon_i}$$

Transform to linear form using base-10 logs (alternatively use base e)

$$\boxed{\log(Y_i) = \log(\beta_0) + X_i \log(\beta_1) + \log(\varepsilon_i)}$$

+Exponential Trend Model (Cont)

Exponential trend forecasting equation

$$\boxed{\log(\hat{Y}_i) = b_0 + b_1 X_i}$$

Where:

b_0 = estimate of $\log(\beta_0)$
b_1 = estimate of $\log(\beta_1)$

Interpretation:

$(\hat{\beta}_1 - 1) \times 100\%$ is the estimated annual compound growth rate in %

+Model Selection

Use a **linear trend** model if the first differences are approximately constant

$$(Y_2 - Y_1) = (Y_3 - Y_2) = \cdots = (Y_n - Y_{n-1})$$

Use a **quadratic trend** model if the second differences are approximately constant

$$[(Y_3 - Y_2) - (Y_2 - Y_1)] = [(Y_4 - Y_3) - (Y_3 - Y_2)] \\ = \cdots = [(Y_n - Y_{n-1}) - (Y_{n-1} - Y_{n-2})]$$

Use an **exponential trend** model if the percentage differences are approximately constant

$$\frac{(Y_2 - Y_1)}{Y_1} \times 100\% = \frac{(Y_3 - Y_2)}{Y_2} \times 100\% = \cdots = \frac{(Y_n - Y_{n-1})}{Y_{n-1}} \times 100\%$$

+The Holt-Winters Method

The Holt-Winters method extends exponential smoothing by including the future trend

This method requires updated estimates of the time-series value (the smoothed value E_i) and the trend value (T_i) in each period

Level: $E_i = U(E_{i-1} + T_{i-1}) + (1 – U)\ Y_i$

Trend: $T_i = VT_{i-1} + (1 – V)(E_i – E_{i-1})$

Where:

E_i = level of the smoothed series calculated in time period i (i = 3,4,…,n)

E_{i-1} = level of the smoothed series already calculated in time period i – 1

T_i = value of the trend component being calculated in time period i

T_{i-1} = value of the trend component already calculated in time period i – 1

Y_i = observed value in time period i

U = subjectively assigned smoothing constant ($0 < U < 1$)

V = subjectively assigned smoothing constant ($0 < V < 1$)

+The Holt-Winters Method (Cont)

To begin, define $E_2 = Y_2$ and $T_2 = Y_2 - Y_1$

Choose smoothing constants **U** and **V**

Calculate E_i and T_i for all i years, i = 3, 4, ..., n

Note:

- Smaller U values give more weight to more recent levels of the time-series
- Smaller V values give more weight to the current trend and less weight to past trends in the series

+Using the Holt-Winters Method for Forecasting

$$\hat{Y}_{n+j} = E_n + j(T_n)$$

Where: $\hat{Y}_{n+j}$ = forecast value j years into the future

E_n = level of the smoothed series calculated in the most recent time period n

T_n = value of the trend component calculated in the most recent time period n

j = number of years into the future

+Autoregressive Modelling

Used for forecasting

Takes advantage of autocorrelation

- 1st order - correlation between consecutive values
- 2nd order - correlation between values two periods apart

p^{th} order autoregressive model

$$\boxed{Y_i = A_0 + A_1 Y_{i-1} + A_2 Y_{i-2} + \cdots + A_p Y_{i-p} + \delta_i}$$

Non-auto-correlated random error

+Autoregressive Model Example:

The Office Concept Limited has acquired a number of office units (in hundreds of square metres) over a period of eight years. Develop the second order autoregressive model

Year	Units
2008	4
2009	3
2010	2
2011	3
2012	2
2013	2
2014	4
2015	6

+Autoregressive Model Solution:

- Develop the 2nd order table
- Use Excel to estimate a regression model

Year	Y_i	Y_{i-1}	Y_{i-2}
08	4	--	--
09	3	4	--
00	2	3	4
11	3	2	3
12	2	3	2
13	2	2	3
14	4	2	2
15	6	4	2

Excel Output

	Coefficients
Intercept	3.5
X Variable 1	0.8125
X Variable 2	-0.9375

$$\hat{Y}_i = 3.5 + 0.8125 Y_{i-1} - 0.9375 Y_{i-2}$$

Autoregressive Model Example:

Use the second-order equation to forecast number of units for 2016:

$$\hat{Y}_i = 3.5 + 0.8125Y_{i-1} - 0.9375Y_{i-2}$$

$$\begin{aligned}\hat{Y}_{2016} &= 3.5 + 0.8125(Y_{2015}) - 0.9375(Y_{2014}) \\ &= 3.5 + 0.8125(6) - 0.9375(4) \\ &= 4.625\end{aligned}$$

+Autoregressive Modelling Steps

1. Choose a value for p (noting that t test for significance has df = n – 2p – 1)
2. Form a series of p ‘lagged predictor’ variables

   $$Y_{i-1}, Y_{i-2}, \ldots, Y_{i-p}$$

3. Use Excel to run a multiple regression model using all p lagged predictor variables
4. Test significance of A_p
   - if null hypothesis rejected, this model is selected
   - if null hypothesis not rejected, discard the p^{th} variable and repeat steps 3 and 4 with evaluation of new highest order parameter whose predictor variable lags by p-1 years

+Choosing a Forecasting Model

Perform a residual analysis

- look for pattern or direction

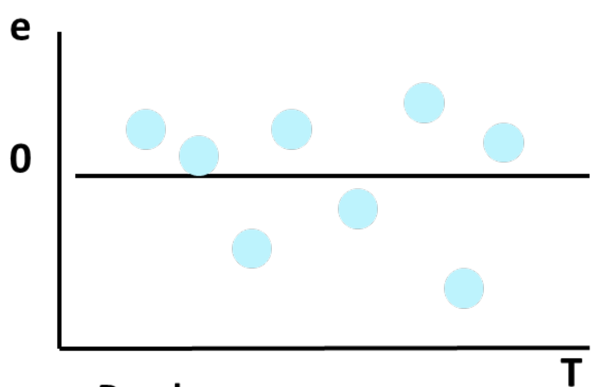
Measure magnitude of residual error

- using squared differences
- using absolute differences

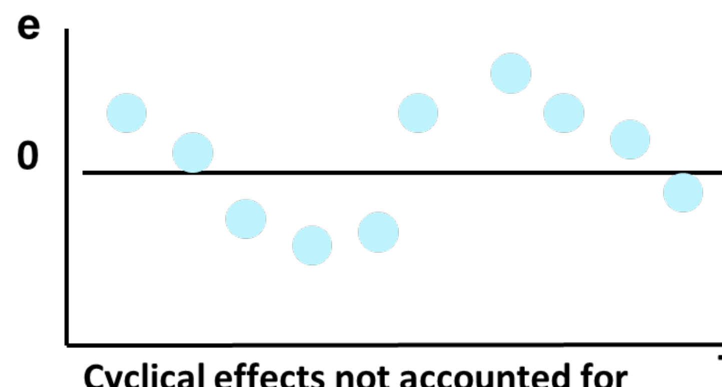
Use simplest model

- principle of parsimony

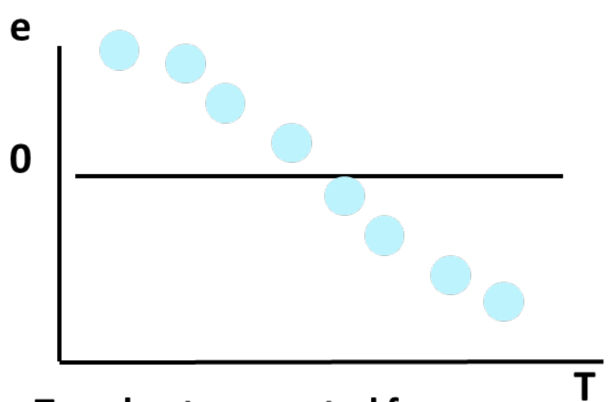
+Residual Analysis



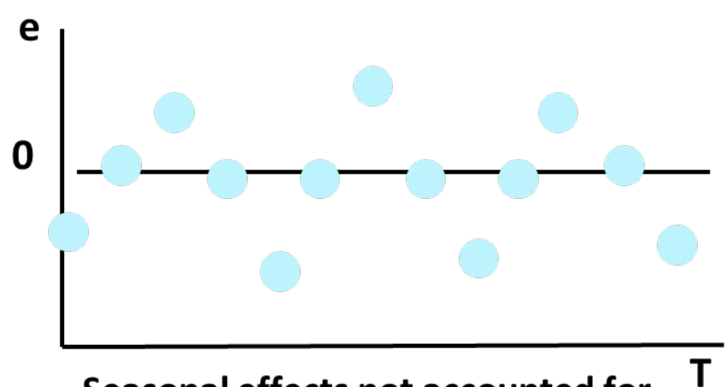
Random errors



Cyclical effects not accounted for



Trend not accounted for



Seasonal effects not accounted for

+Measuring Errors

Choose the model that gives the smallest measuring errors

Sum of squared errors (SSE)

$$SSE = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

Sensitive to outliers

Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{i=1}^{n}\left|Y_i - \hat{Y}_i\right|}{n}$$

Not sensitive to extreme observations

+The Principle of Parsimony

Suppose two or more models provide a good fit for the data

Select the simplest model:

- Simplest model types
  - least-squares linear
  - least-squares quadratic
  - 1st order autoregressive
- More complex types
  - 2nd and p^{th} order autoregressive
  - least-squares exponential
  - Holts-Winters

+Forecasting with Seasonal Data

Recall the classical time series model with seasonal variation

$$Y_i = T_i \times S_i \times C_i \times I_i$$

If the seasonality is quarterly, define three new dummy variables for quarters

Q1 = 1 if first quarter, 0 otherwise
Q2 = 1 if second quarter, 0 otherwise
Q3 = 1 if third quarter, 0 otherwise
Q4 is the default if Q1 = Q2 = Q3 = 0

+Exponential Model with Quarterly Data

$$Y_i = \beta_0 \beta_1^{X_i} \beta_2^{Q_1} \beta_3^{Q_2} \beta_4^{Q_3} \varepsilon_i$$

$(\beta_1 - 1) \times 100\%$ is the quarterly compound growth rate (in %)
β_i provides the multiplier for the i^{th} quarter relative to the 4th quarter (i = 2, 3, 4)

Transform to linear form:

$$\log(Y_i) = \log(\beta_0) + X_i \log(\beta_1) + Q_1 \log(\beta_2) + Q_2 \log(\beta_3) + Q_3 \log(\beta_4) + \log(\varepsilon_i)$$

+Estimating the Quarterly Model

Exponential forecasting equation:

$$\log(\hat{Y}_i) = b_0 + b_1 X_i + b_2 Q_1 + b_3 Q_2 + b_4 Q_3$$

Where: b_0 = estimate of $\log(\beta_0)$, so $10^{b_0} = \hat{\beta}_0$
b_1 = estimate of $\log(\beta_1)$, so $10^{b_1} = \hat{\beta}_1$
etc.

Interpretation:

$(\hat{\beta}_1 - 1) \times 100\%$ = estimated quarterly compound growth rate (in %)
$\hat{\beta}_2$ = estimated multiplier for first quarter relative to fourth quarter
$\hat{\beta}_3$ = estimated multiplier for second quarter relative to fourth quarter
$\hat{\beta}_4$ = estimated multiplier for third quarter relative to fourth quarter

+Quarterly Model Example:

Suppose the forecasting equation is:

$$\boxed{\log(\hat{Y}_i) = 3.43 + .017X_i - .082Q_1 - .073Q_2 + .022Q_3}$$

$b_0 = 3.43$, so $10^{b_0} = \hat{\beta}_0 = 2691.53$

$b_1 = .017$, so $10^{b_1} = \hat{\beta}_1 = 1.040$

$b_2 = -.082$, so $10^{b_2} = \hat{\beta}_2 = 0.827$

$b_3 = -.073$, so $10^{b_3} = \hat{\beta}_3 = 0.845$

$b_4 = .022$, so $10^{b_4} = \hat{\beta}_4 = 1.052$

Quarterly Model (Cont) Example:

Value:	Interpretation:
$\hat{\beta}_0 = 2691.53$	Unadjusted trend value for first quarter of first year
$\hat{\beta}_1 = 1.040$	4.0% = estimated quarterly compound growth rate
$\hat{\beta}_2 = 0.827$	Average sales in Q_1 are 82.7% of average 4^{th} quarter sales, adjusting for the 4% quarterly growth rate
$\hat{\beta}_3 = 0.845$	Average sales in Q_2 are 84.5% of average 4^{th} quarter sales, adjusting for the 4% quarterly growth rate
$\hat{\beta}_4 = 1.052$	Average sales in Q_3 are 105.2% of average 4^{th} quarter sales, adjusting for the 4% quarterly growth rate

+Index Numbers

Index numbers allow relative comparisons over time

Index numbers are reported relative to a **Base Period Index**

Base period index = 100 by definition

Used for an individual item or measurement

+Simple Price Index

$$I_i = \frac{P_i}{P_{base}} \times 100$$

Where: I_i = index number for year i

P_i = price for year i

P_{base} = price for the base year

+Index Numbers Example:

Airplane ticket prices from 1995 to 2003

Year	Price	Index (base year = 2000)
1995	272	85.0
1996	288	90.0
1997	295	92.2
1998	311	97.2
1999	322	100.6
2000	320	100.0
2001	348	108.8
2002	366	114.4
2003	384	120.0

$$I_{1996} = \frac{P_{1996}}{P_{2000}} \times 100 = \frac{288}{320}(100) = 90$$

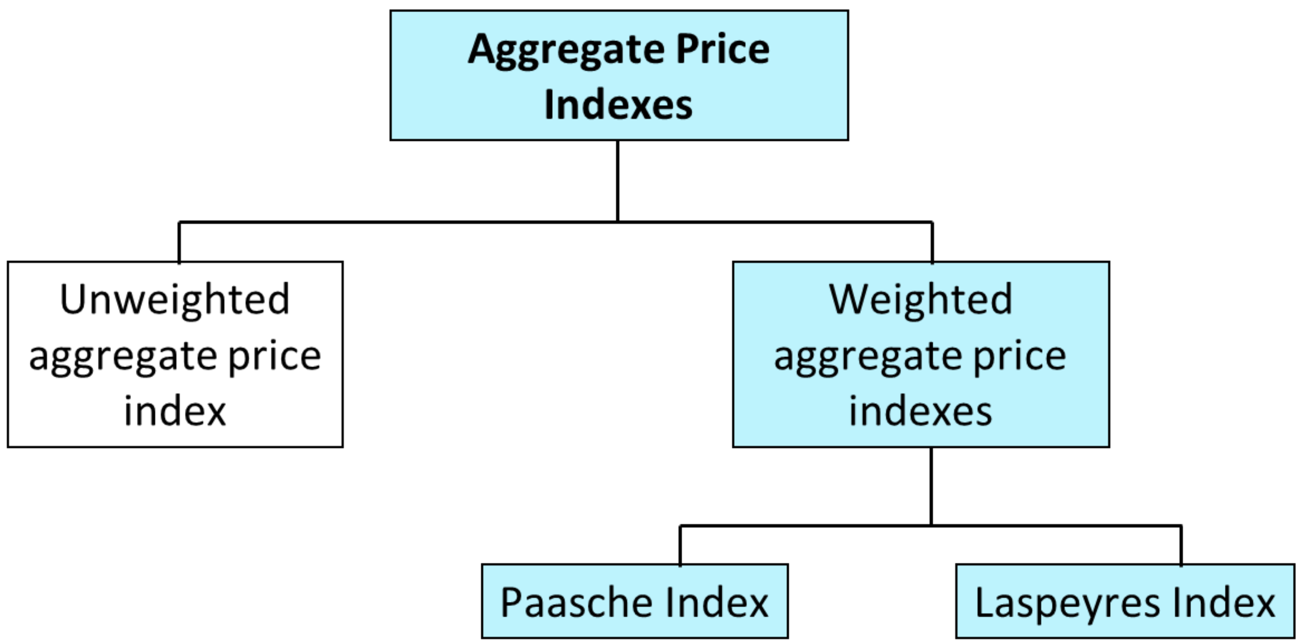
Base Year:

$$I_{2000} = \frac{P_{2000}}{P_{2000}} \times 100 = \frac{320}{320}(100) = 100$$

$$I_{2003} = \frac{P_{2003}}{P_{2000}} \times 100 = \frac{384}{320}(100) = 120$$

Aggregate Price Indexes

An aggregate index is used to measure the rate of change from a base period for a group of items



+Unweighted Aggregate Price Index

Unweighted aggregate price index formula

$$I_U^{(t)} = \frac{\sum_{i=1}^{n} P_i^{(t)}}{\sum_{i=1}^{n} P_i^{(0)}} \times 100$$

i = item (1,2,….,n)
t = time period (0, 1, 2,…)
n = total number of items

$I_U^{(t)}$ = unweighted price index at time t

$\sum_{i=1}^{n} P_i^{(t)}$ = sum of the prices for the group of items at time t

$\sum_{i=1}^{n} P_i^{(0)}$ = sum of the prices for the group of items in time period 0

+Unweighted Aggregate Price Index Example:

	Automobile Expenses: Monthly Amounts ($):				
Year	Lease payment	Fuel	Repair	Total	Index (2001=100)
2011	260	45	40	345	100.0
2012	280	60	40	380	110.1
2013	305	55	45	405	117.4
2014	310	50	50	410	118.8

$$I_{2014} = \frac{\sum P_{2014}}{\sum P_{2011}} \times 100 = \frac{410}{345}(100) = 118.8$$

Unweighted total expenses were 18.8% higher in 2014 than in 2011

+Unweighted Aggregate Price Indices

	Year				
	1994	1999	2004	2009	2014
Fruit	$P_i^{(0)}$	$P_i^{(1)}$	$P_i^{(2)}$	$P_i^{(3)}$	$P_i^{(4)}$
Kiwi fruit	2.60	3.10	3.38	3.52	3.91
Rockmelon	1.80	1.90	2.15	2.20	2.30
Watermelon	0.35	0.39	0.44	0.49	0.56

Table 14.6
Prices (in dollars per kilogram) for three fruit items

+Weighted Aggregate Price Indices

Fruit	1994 $P_i^{(0)}, Q_J^{(0)}$	1999 $P_i^{(1)}, Q_J^{(1)}$	2004 $P_i^{(2)}, Q_J^{(2)}$	2009 $P_i^{(3)}, Q_J^{(3)}$	2014 $P_i^{(4)}, Q_J^{(4)}$
Kiwi fruit	2.60, 5.2	3.10, 5.5	3.38, 4.9	3.52, 4.5	3.91, 4.7
Rockmelon	1.80, 21.6	1.90, 19.8	2.15, 20.5	2.20, 21.4	2.30, 22.4
Watermelon	0.35, 24.8	0.39, 25.9	0.44, 26.3	0.49, 27.7	0.56, 28.2

(Column headings grouped under "Year".)

Table 14.7
Prices (in dollars per kilogram) and quantities (annual per capita consumption in kilograms) for three fruit items

+Weighted Aggregate Price Indexes

Laspeyres index

$$I_L^{(t)} = \frac{\sum_{i=1}^{n} P_i^{(t)} Q_i^{(0)}}{\sum_{i=1}^{n} P_i^{(0)} Q_i^{(0)}} \times 100$$

t = time period (0, 1, 2 ,…)

i = item number (1, 2 ,…n)

n = total number of items under consideration

$Q_i^{(0)}$ = quantity of item i at time period 0

$I_L^{(t)}$ = value of the Laspeyres price index at time t

Paasche index

$$I_P^{(t)} = \frac{\sum_{i=1}^{n} P_i^{(t)} Q_i^{(t)}}{\sum_{i=1}^{n} P_i^{(0)} Q_i^{(t)}} \times 100$$

t = time period (0, 1, 2 ,…)

i = item number (1, 2,…n)

n = total number of items under consideration

$Q_i^{(t)}$ = quantity of item i at time period t

$I_P^{(t)}$ = value of the Paasche price index at time t

+Pitfalls in Time-series Analysis

Assuming the mechanism that governs the time-series behaviour in the past will still hold in the future

Using mechanical extrapolation of the trend to forecast the future without considering personal judgments, business experiences, changing technologies, changing habits and needs