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ANTI-ENTROPY AEROSPACE SOCIETY

Introduction to Composite Materials

About the lecturer

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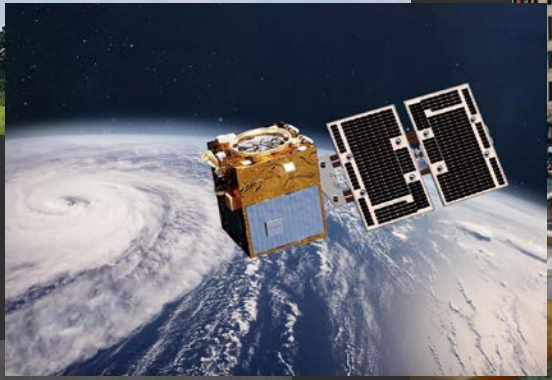
- 2018 – 2022 BSc AAE, NCKU, TW
  - Structure Engineer, Taiwan Innovative Space Inc.
  - Founder, Institute of Space Propulsion
  - Mechanical Engineer, NCKU SpaceLab
- 2023 – 2025 MSc ME, TU/e, NL
  - Mechanical Engineer, Tulip Tech
  - Member, AEAS
- Research area: Composite materials, Lightweight structures

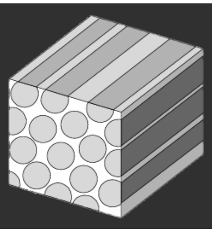
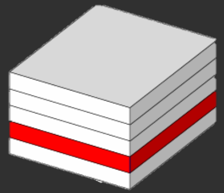
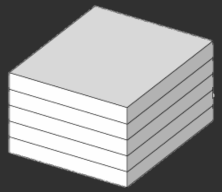
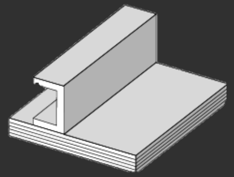
  Lin, T.-C., & Hsueh, T.-C. (2021, January 11). A Modular Structural Design for Payload Replaceable CubeSat. AIAA Scitech 2021 Forum. https://doi.org/10.2514/6.2021-1257

Composites in real-life



Nomenclature

As we will look at the composites on different length scales, the following icons will be used for indication:

	Micro-scale	The fibers and the matrix material are treated separatly. This is basically the length scale to discuss Homogenisation.
	Layer scale	An individual layer is considered as a homogeneous, orthotropic material.
	Laminate scale	A stack of layers (a laminate) is considered as a homogeneous material loaded by line loads and line moments.
	Structural scale	The laminate, including geometric effects (stringers, holes) external loads and boundary conditions.

Recap:
Linear Elasticity

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Constitutive relation: Hooke's law

- Linear elastic Hookes' law

$$\boldsymbol{\sigma} = {}^4\boldsymbol{C} : \boldsymbol{\varepsilon} \quad \text{or} \quad \boldsymbol{\varepsilon} = {}^4\boldsymbol{S} : \boldsymbol{\sigma}$$

${}^4\boldsymbol{C}$ - 4th order elastic stiffness tensor

${}^4\boldsymbol{S}$ - 4th order elastic compliance tensor

$${}^4\boldsymbol{S} = \left({}^4\boldsymbol{C}\right)^{-1}$$

- isotropic material: 2 elastic material constants

e.g. Young's modulus E and Poisson's ratio ν, or shear modulus G and bulk modulus K

$$\boldsymbol{\sigma} = \frac{E\nu}{(1+\nu)(1-2\nu)} tr(\boldsymbol{\varepsilon})\boldsymbol{I} + \frac{E}{1+\nu}\boldsymbol{\varepsilon}$$

where $${}^4\boldsymbol{C} = \frac{E\nu}{(1+\nu)(1-2\nu)}\boldsymbol{II} + \frac{E}{1+\nu}{}^4\boldsymbol{I}$$

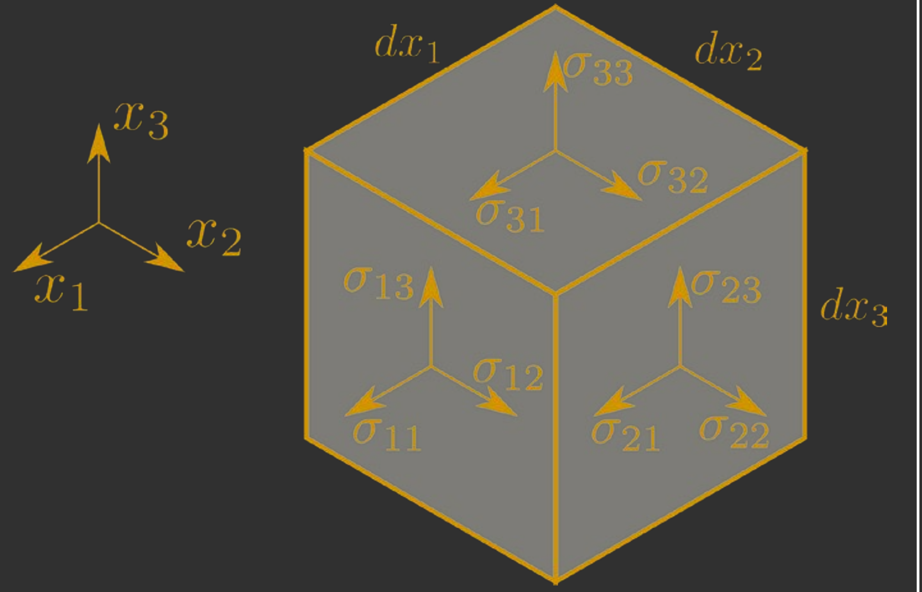
Voigt notation for 2nd and 4th-order tensors

- <u>second-order tensor</u>: in 3D has 9 components

$$\boldsymbol{\sigma} = \sigma_{ij}\overrightarrow{e_i}\,\overrightarrow{e_j} = \sigma_{11}\overrightarrow{e_1}\,\overrightarrow{e_1} + \sigma_{12}\overrightarrow{e_1}\,\overrightarrow{e_2} + \sigma_{13}\overrightarrow{e_1}\,\overrightarrow{e_3} +$$
$$\sigma_{21}\overrightarrow{e_2}\,\overrightarrow{e_1} + \sigma_{22}\overrightarrow{e_2}\,\overrightarrow{e_2} + \sigma_{23}\overrightarrow{e_2}\,\overrightarrow{e_3} +$$
$$\sigma_{31}\overrightarrow{e_3}\,\overrightarrow{e_1} + \sigma_{32}\overrightarrow{e_3}\,\overrightarrow{e_2} + \sigma_{33}\overrightarrow{e_3}\,\overrightarrow{e_3}$$

- Voigt notation:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{21} \\ \sigma_{13} \\ \sigma_{31} \\ \sigma_{23} \\ \sigma_{32} \end{bmatrix}$$



Voigt notation for 2nd and 4th-order tensors

- fourth-order tensor: in 3D has 81 components

$$^{4}\boldsymbol{C} = C_{ijkl}\vec{e_i}\,\vec{e_j}\,\vec{e_k}\,\vec{e_l}$$

- Voigt notation:

$$^{4}\boldsymbol{C} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1121} & C_{1112} & C_{1131} & C_{1113} & C_{1132} & C_{1123} \\ C_{2211} & C_{2222} & C_{2233} & C_{2221} & C_{2212} & C_{2231} & C_{2213} & C_{2232} & C_{2223} \\ C_{3311} & C_{3322} & C_{3333} & C_{3321} & C_{3312} & C_{3331} & C_{3313} & C_{3332} & C_{3323} \\ C_{1211} & C_{1222} & C_{1233} & C_{1221} & C_{1212} & C_{1231} & C_{1213} & C_{1232} & C_{1223} \\ C_{2111} & C_{2122} & C_{2133} & C_{2121} & C_{2112} & C_{2131} & C_{2113} & C_{2132} & C_{2123} \\ C_{1311} & C_{1322} & C_{1333} & C_{1321} & C_{1312} & C_{1331} & C_{1313} & C_{1332} & C_{1323} \\ C_{3111} & C_{3122} & C_{3133} & C_{3121} & C_{3112} & C_{3131} & C_{3113} & C_{3132} & C_{3123} \\ C_{2311} & C_{2322} & C_{2333} & C_{2321} & C_{2312} & C_{2331} & C_{2313} & C_{2332} & C_{2323} \\ C_{3211} & C_{3222} & C_{3233} & C_{3221} & C_{3212} & C_{3231} & C_{3213} & C_{3232} & C_{3223} \end{bmatrix}$$

Voigt notation for 2nd and 4th-order tensors

- double-dot product between second and fourth-order tensors:

$$\boldsymbol{\sigma} = {}^{4}\boldsymbol{C} : \boldsymbol{\varepsilon} \qquad \Longleftrightarrow \qquad \sigma_{ij} = C_{ijkl}\,\varepsilon_{lk}$$

- Voigt notation:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{21} \\ \sigma_{13} \\ \sigma_{31} \\ \sigma_{23} \\ \sigma_{32} \end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1121} & C_{1112} & C_{1131} & C_{1113} & C_{1132} & C_{1123} \\ C_{2211} & C_{2222} & C_{2233} & C_{2221} & C_{2212} & C_{2231} & C_{2213} & C_{2232} & C_{2223} \\ C_{3311} & C_{3322} & C_{3333} & C_{3321} & C_{3312} & C_{3331} & C_{3313} & C_{3332} & C_{3323} \\ C_{1211} & C_{1222} & C_{1233} & C_{1221} & C_{1212} & C_{1231} & C_{1213} & C_{1232} & C_{1223} \\ C_{2111} & C_{2122} & C_{2133} & C_{2121} & C_{2112} & C_{2131} & C_{2113} & C_{2132} & C_{2123} \\ C_{1311} & C_{1322} & C_{1333} & C_{1321} & C_{1312} & C_{1331} & C_{1313} & C_{1332} & C_{1323} \\ C_{3111} & C_{3122} & C_{3133} & C_{3121} & C_{3112} & C_{3131} & C_{3113} & C_{3132} & C_{3123} \\ C_{2311} & C_{2322} & C_{2333} & C_{2321} & C_{2312} & C_{2331} & C_{2313} & C_{2332} & C_{2323} \\ C_{3211} & C_{3222} & C_{3233} & C_{3221} & C_{3212} & C_{3231} & C_{3213} & C_{3232} & C_{3223} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{21} \\ \varepsilon_{13} \\ \varepsilon_{31} \\ \varepsilon_{23} \\ \varepsilon_{32} \end{bmatrix}$$

Voigt notation for 2nd and 4th-order tensors

- double-dot product between second and fourth-order tensors:

$$\boldsymbol{\sigma} = {}^{4}\boldsymbol{C} : \boldsymbol{\varepsilon} \qquad \Longleftrightarrow \qquad \sigma_{ij} = C_{ijkl}\,\varepsilon_{lk}$$

- Voigt notation:

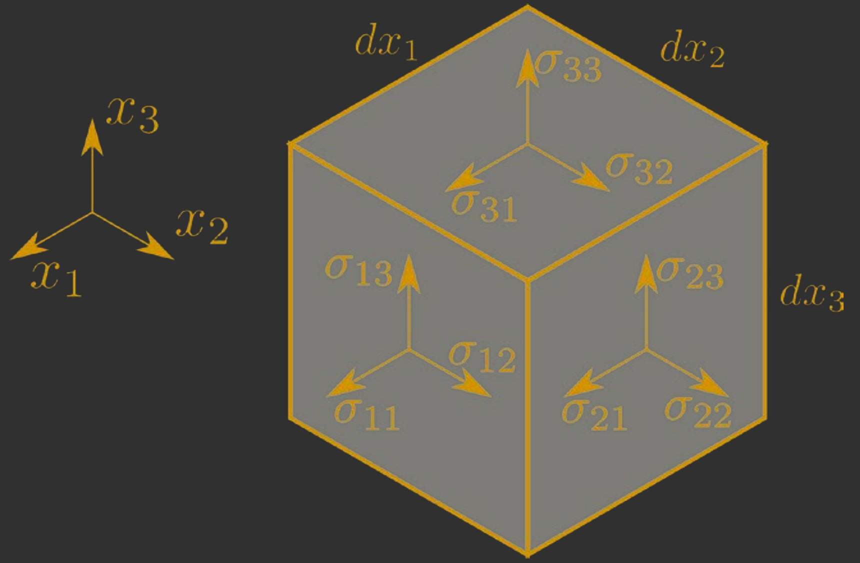
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \cancel{\sigma_{21}} \\ \sigma_{13} \\ \cancel{\sigma_{31}} \\ \sigma_{23} \\ \cancel{\sigma_{32}} \end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1121} & \cancel{C_{1112}} & C_{1131} & \cancel{C_{1113}} & C_{1132} & \cancel{C_{1123}} \\ C_{2211} & C_{2222} & C_{2233} & C_{2221} & \cancel{C_{2212}} & C_{2231} & \cancel{C_{2213}} & C_{2232} & \cancel{C_{2223}} \\ C_{3311} & C_{3322} & C_{3333} & C_{3321} & \cancel{C_{3312}} & C_{3331} & \cancel{C_{3313}} & C_{3332} & \cancel{C_{3323}} \\ C_{1211} & C_{1222} & C_{1233} & C_{1221} & \cancel{C_{1212}} & C_{1231} & \cancel{C_{1213}} & C_{1232} & \cancel{C_{1223}} \\ C_{2111} & C_{2122} & C_{2133} & C_{2121} & \cancel{C_{2112}} & C_{2131} & \cancel{C_{2113}} & C_{2132} & \cancel{C_{2123}} \\ C_{1311} & C_{1322} & C_{1333} & C_{1321} & \cancel{C_{1312}} & C_{1331} & \cancel{C_{1313}} & C_{1332} & \cancel{C_{1323}} \\ C_{3111} & C_{3122} & C_{3133} & C_{3121} & \cancel{C_{3112}} & C_{3131} & \cancel{C_{3113}} & C_{3132} & \cancel{C_{3123}} \\ C_{2311} & C_{2322} & C_{2333} & C_{2321} & \cancel{C_{2312}} & C_{2331} & \cancel{C_{2313}} & C_{2332} & \cancel{C_{2323}} \\ C_{3211} & C_{3222} & C_{3233} & C_{3221} & \cancel{C_{3212}} & C_{3231} & \cancel{C_{3213}} & C_{3232} & \cancel{C_{3223}} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \cancel{\varepsilon_{21}} \\ \varepsilon_{13} \\ \cancel{\varepsilon_{31}} \\ \varepsilon_{23} \\ \cancel{\varepsilon_{32}} \end{bmatrix}$$

Voigt notation for 2nd and 4th-order tensors

- strain and stress tensors: 6 components (symmetric)

$$\boldsymbol{\varepsilon}^T = [\varepsilon_{11} \quad \varepsilon_{22} \quad \varepsilon_{33} \quad \varepsilon_{12} \quad \varepsilon_{13} \quad \varepsilon_{23}]$$

$$\boldsymbol{\sigma}^T = [\sigma_{11} \quad \sigma_{22} \quad \sigma_{33} \quad \sigma_{12} \quad \sigma_{13} \quad \sigma_{23}]$$



- linear elastic Hooke's law :

$$\boldsymbol{\sigma} = {}^4\boldsymbol{C} : \boldsymbol{\varepsilon} \quad \Longleftrightarrow \quad \sigma_{ij} = C_{ijkl}\,\varepsilon_{lk}$$

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{13}\\ \sigma_{23}\end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1121} & C_{1131} & C_{1132}\\ C_{2211} & C_{2222} & C_{2233} & C_{2221} & C_{2231} & C_{2232}\\ C_{3311} & C_{3322} & C_{3333} & C_{3321} & C_{3331} & C_{3332}\\ C_{1211} & C_{1222} & C_{1233} & C_{1221} & C_{1231} & C_{1232}\\ C_{1311} & C_{1322} & C_{1333} & C_{1321} & C_{1331} & C_{1332}\\ C_{2311} & C_{2322} & C_{2333} & C_{2321} & C_{2331} & C_{2332}\end{bmatrix} \begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \varepsilon_{12}\\ \varepsilon_{13}\\ \varepsilon_{23}\end{bmatrix}$$

**Symmetric:*
No motion and rotation allowed

$$\sigma_{ij} = \sigma_{ji} \qquad \varepsilon_{kl} = \varepsilon_{lk}$$

Voigt notation for 2nd and 4th-order tensors

- Isotropic linear elastic Hooke's law

$$\boldsymbol{\sigma} = \frac{E\nu}{(1+\nu)(1-2\nu)} tr(\boldsymbol{\varepsilon})\boldsymbol{I} + \frac{E}{1+\nu}\boldsymbol{\varepsilon} \quad \text{with} \quad {}^4\boldsymbol{C} = \frac{E\nu}{(1+\nu)(1-2\nu)}\boldsymbol{II} + \frac{E}{1+\nu}{}^4\boldsymbol{I}$$

- Voigt notation:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{bmatrix}$$

Voigt notation for 2nd and 4th-order tensors

- Isotropic linear elastic Hooke's law

$$\begin{bmatrix}\sigma_{11}\\\sigma_{22}\\\sigma_{33}\\\sigma_{12}\\\sigma_{13}\\\sigma_{23}\end{bmatrix}=\begin{bmatrix}C_{1111} & C_{1122} & C_{1133} & C_{1121} & C_{1131} & C_{1132}\\C_{2211} & C_{2222} & C_{2233} & C_{2221} & C_{2231} & C_{2232}\\C_{3311} & C_{3322} & C_{3333} & C_{3321} & C_{3331} & C_{3332}\\C_{1211} & C_{1222} & C_{1233} & C_{1221} & C_{1231} & C_{1232}\\C_{1311} & C_{1322} & C_{1333} & C_{1321} & C_{1331} & C_{1332}\\C_{2311} & C_{2322} & C_{2333} & C_{2321} & C_{2331} & C_{2332}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\\varepsilon_{22}\\\varepsilon_{33}\\\varepsilon_{12}\\\varepsilon_{13}\\\varepsilon_{23}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_{11}\\\sigma_{22}\\\sigma_{33}\\\sigma_{12}\\\sigma_{13}\\\sigma_{23}\end{bmatrix}=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\\nu & 1-\nu & \nu & 0 & 0 & 0\\\nu & \nu & 1-\nu & 0 & 0 & 0\\0 & 0 & 0 & 1-2\nu & 0 & 0\\0 & 0 & 0 & 0 & 1-2\nu & 0\\0 & 0 & 0 & 0 & 0 & 1-2\nu\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\\varepsilon_{22}\\\varepsilon_{33}\\\varepsilon_{12}\\\varepsilon_{13}\\\varepsilon_{23}\end{bmatrix}$$

$$\Rightarrow \quad E=\frac{(A-B)(A+2B)}{(A+B)} \qquad \nu=\frac{B}{(A+B)}$$

where $A=C_{1111}$ or C_{2222} or C_{3333}

$B=C_{1122}$ or C_{1133} or C_{2233}

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Homogenization Concept

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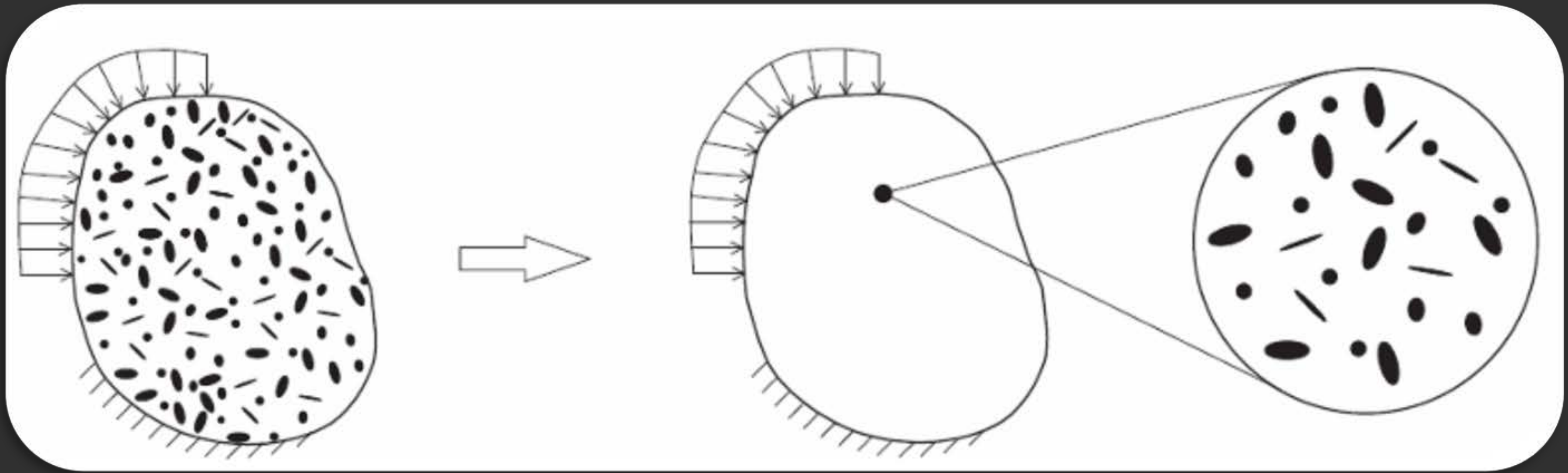
Homogenization Concept



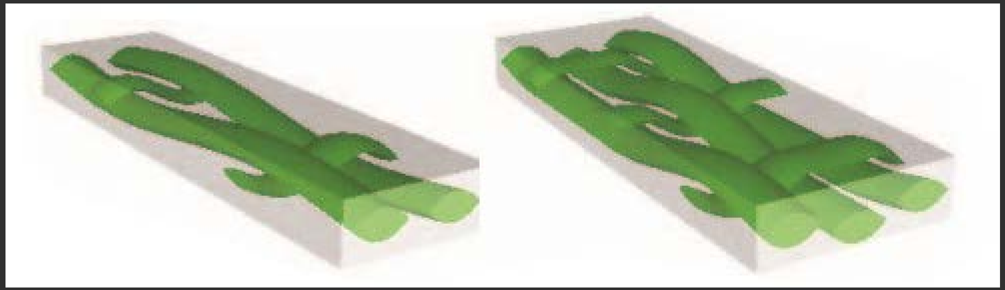
Heterogeneous material

Homogeneous material
with the properties obtained by
“homogenization” of microstructure

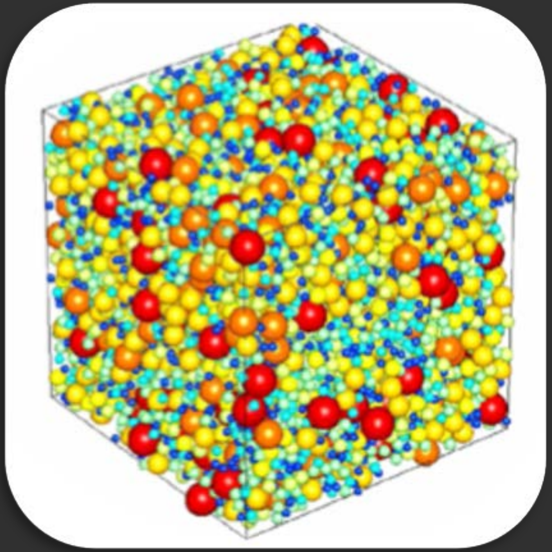
Representative Volume Element (RVE)

- RVE is a model of the microstructure of the material that sufficiently accurately represents geometrical and material statistics relevant for the problem of interest
  - large enough → sufficient statistics
  - small enough → feasible computational time

textile carbon fiber reinforced composite

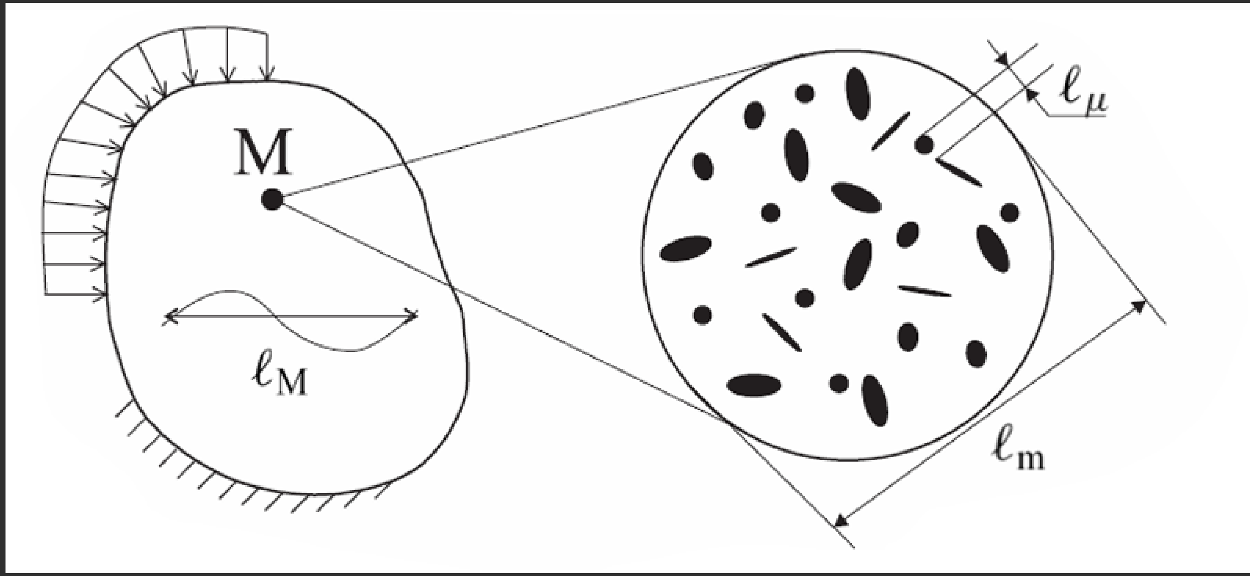


plane weave twill weave



polydisperse particle composite
(e.g. solid rocket propellant)
[from K. Matous et al (2007)]

Key principle: separation of scales



$$\ell_\mu < \ell_m \ll \ell_M$$

“The microscopic length scale is much smaller than the characteristic length over which the macroscopic loading varies in space”

Key principle: separation of scales

- Infinitesimal linear strain tensor

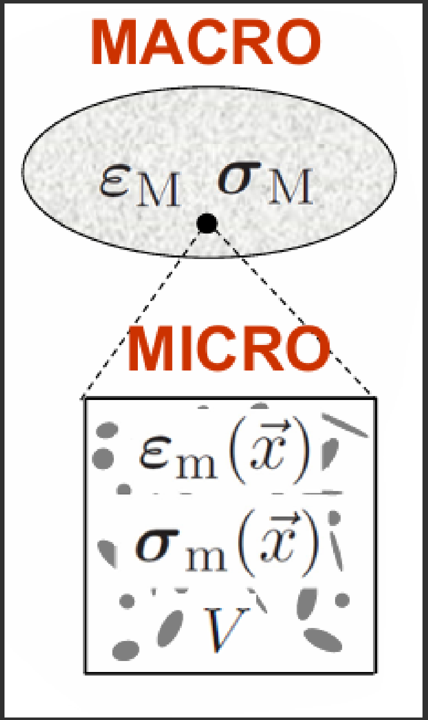
$$\boldsymbol{\varepsilon}_M = \frac{1}{V}\int_V \boldsymbol{\varepsilon}_m(\vec{x})\, dV$$

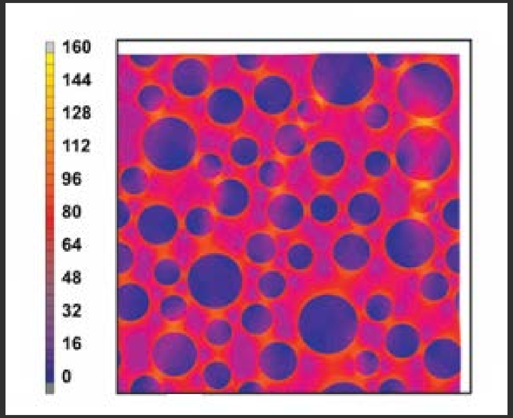
- Stress tensor

$$\boldsymbol{\sigma}_M = \frac{1}{V}\int_V \boldsymbol{\sigma}_m(\vec{x})\, dV$$

- Mechanical power

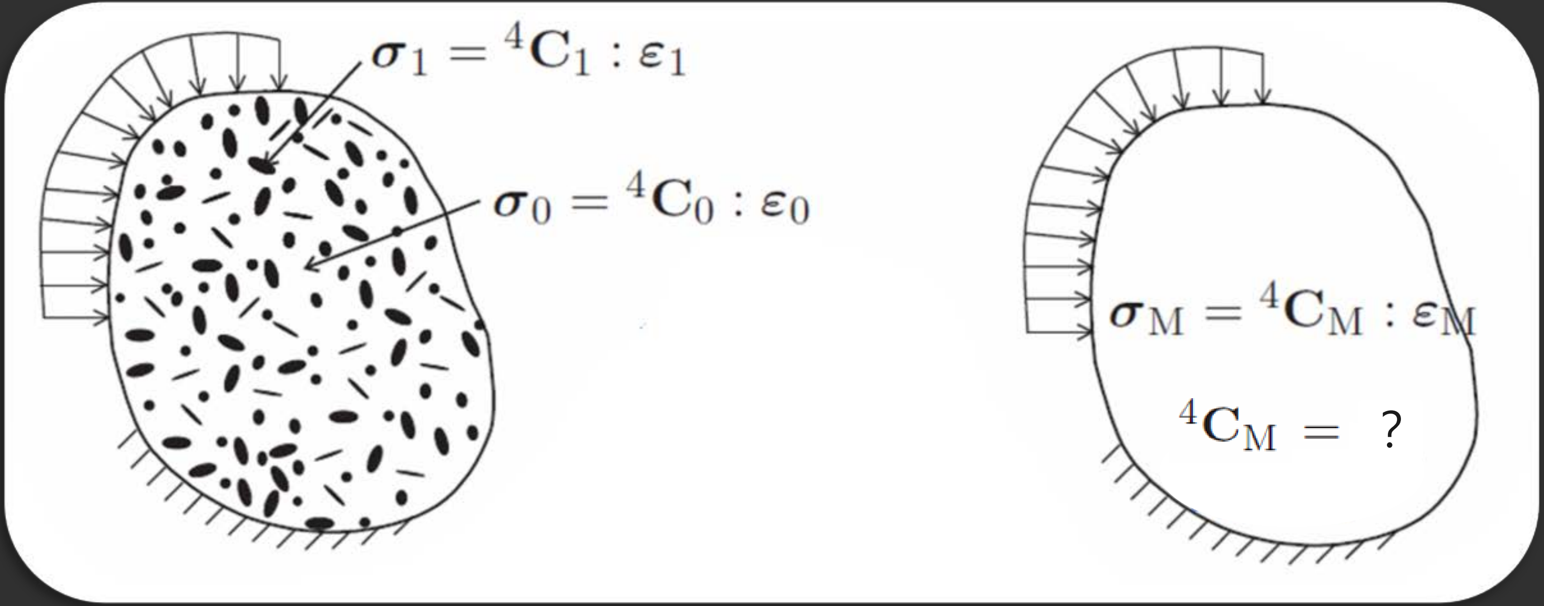
$$\boldsymbol{\sigma}_M : \delta\boldsymbol{\varepsilon}_M = \frac{1}{V}\int_V \boldsymbol{\sigma}_m(\vec{x}) : \delta\, \boldsymbol{\varepsilon}_m(\vec{x})\, dV$$





Equivalent von Mises stress distribution [MPa]
(R. Hill, 1963, 1984; S. Nemat-Nasser 1999)

Homogenization for linear elastic materials



${}^4C_M \Rightarrow$ Elastic constants

$$E_M = \frac{(A-B)(A+2B)}{(A+B)}$$

$$\nu_M = \frac{B}{(A+B)}$$

where $A = C_{M1111}$ $B = C_{M1122}$

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Rules of mixture

- Voigt average
- Reuss average

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Voigt average (iso-strain)

- Assumption: strains in all components are constant, equal to each other and to the macroscopic strain

$$\boldsymbol{\varepsilon}_0 = \boldsymbol{\varepsilon}_1 = \boldsymbol{\varepsilon}_M$$

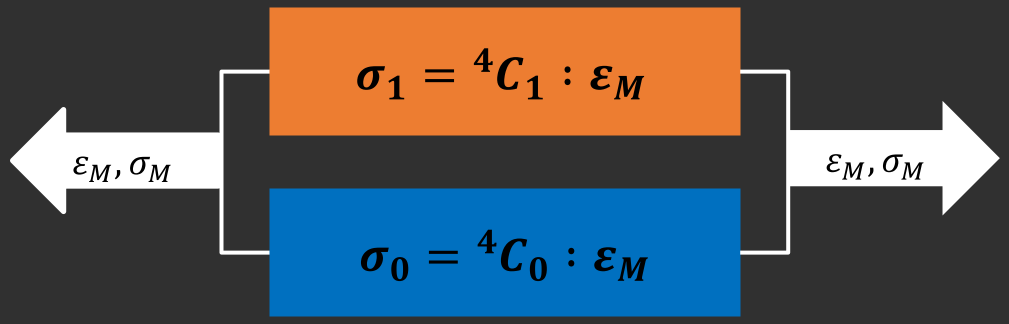
- Stresses:

$$\boldsymbol{\sigma}_M = f_1 \boldsymbol{\sigma}_1 + (1 - f_1)\boldsymbol{\sigma}_0$$

$$\begin{aligned}
\boldsymbol{\sigma}_M &= f_1\, {}^4\boldsymbol{C}_1 : \boldsymbol{\varepsilon}_1 + (1 - f_1)\, {}^4\boldsymbol{C}_0 : \boldsymbol{\varepsilon}_0 \\
&= f_1\, {}^4\boldsymbol{C}_1 : \boldsymbol{\varepsilon}_M + (1 - f_1)\, {}^4\boldsymbol{C}_0 : \boldsymbol{\varepsilon}_M \\
&= \underbrace{\left(f_1\, {}^4\boldsymbol{C}_1 + (1 - f_1)\, {}^4\boldsymbol{C}_0\right)} : \boldsymbol{\varepsilon}_M
\end{aligned}$$

$${}^4\boldsymbol{C}_M^V = f_1\, {}^4\boldsymbol{C}_1 + (1 - f_1)\, {}^4\boldsymbol{C}_0$$

Homogenized stiffness tensor



Phase 1 volume fraction:

$$f_1 = \frac{V_1}{V}$$

Phase 0 volume fraction:

$$f_0 = (1 - f_1)$$

Reuss average (iso-stress)

- Assumption: <u>stresses</u> in all components are constant, equal to each other and to the macroscopic stress

$$\boldsymbol{\sigma_0} = \boldsymbol{\sigma_1} = \boldsymbol{\sigma_M}$$

Phase 1 volume fraction:

$$f_1 = \frac{V_1}{V}$$

Phase 0 volume fraction:

$$f_0 = (1 - f_1)$$

- Strains:

$$\boldsymbol{\varepsilon_M} = f_1 \boldsymbol{\varepsilon_1} + (1 - f_1)\boldsymbol{\varepsilon_0}$$

$$\begin{aligned} \boldsymbol{\varepsilon_M} &= f_1 {}^4\boldsymbol{S_1} : \boldsymbol{\sigma_1} + (1 - f_1){}^4\boldsymbol{S_0} : \boldsymbol{\sigma_0} \\ &= f_1 {}^4\boldsymbol{S_1} : \boldsymbol{\sigma_M} + (1 - f_1){}^4\boldsymbol{S_0} : \boldsymbol{\sigma_M} \\ &= \underbrace{\left(f_1 {}^4\boldsymbol{S_1} + (1 - f_1){}^4\boldsymbol{S_0}\right)}_{{}^4\boldsymbol{S_M^R}} : \boldsymbol{\sigma_M} \end{aligned}$$

Homogenized compliance tensor

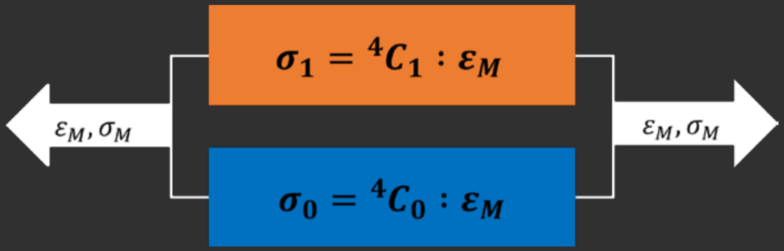
- Homogenized stiffness tensor:

$$\begin{aligned} {}^4\boldsymbol{C_M^R} = \left({}^4\boldsymbol{S_M^R}\right)^{-1} &= \left(f_1 {}^4\boldsymbol{S_1} + (1 - f_1){}^4\boldsymbol{S_0}\right)^{-1} \\ &= \left(f_1\left({}^4\boldsymbol{C_1}\right)^{-1} + (1 - f_1){}^4\boldsymbol{C_0}\right)^{-1} \end{aligned}$$

Quiz

- Voigt model: $^4C_M^V \rightarrow E_M^V$

$\boldsymbol{\varepsilon_0 = \varepsilon_1 = \varepsilon_M}$



- Reuss model: $^4C_M^R \rightarrow E_M^R$

$\boldsymbol{\sigma_0 = \sigma_1 = \sigma_M}$

- Question:

For an arbitrary composite material, what will be the relation between E_M^V and E_M^R?

a) $E_M^V = E_M^R$

b) $E_M^V > E_M^R$

c) $E_M^V < E_M^R$

d) no general relation exists, it will depend on the exact properties of the composite constituents

Exercise

- Compute the effective elastic stiffness tensor ${}^{4}\boldsymbol{C}_{M}$ of the Al-SiC composite for varying volume fraction of the SiC particles using the homogenization methods

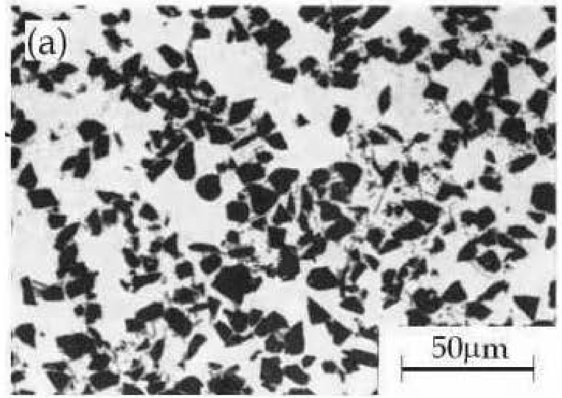


Figure 1: Typical microstructure of Al-SiC composite: aluminium matrix is light and SiC particles are black. *From: Watanabe and Sato, in Nanocomposites with Unique Properties and Applications in Medicine and Industry, J. Cuppoletti (Ed.),(2011)*

Table 1: Material properties of the constituents of Al-SiC composite

	E[GPa]	ν	ρ[g/cm^3]
Al	70	0.3	2.7
SiC	450	0.17	3.2

MATLAB example

```matlab
clear; close all; clc;

% Parameters
E_Al = 70; % GPa
E_SiC = 450; % GPa
nu_Al = 0.3;
nu_SiC = 0.17;
rho_Al = 2.7;
rho_SiC = 3.2;

ParticleVolFrac = linspace(0, 1, 101);


% Pre-allocate arrays
CV_M3333 = zeros(1, 101);
EV_M = zeros(1, 101);
CR_M3333 = zeros(1, 101);
ER_M = zeros(1, 101);
```

$$ {}^4\boldsymbol{C} = \frac{E\nu}{(1+\nu)(1-2\nu)}\boldsymbol{II} + \frac{E}{1+\nu}{}^4\boldsymbol{I} $$

```matlab
%% Function to calculate stiffness matrix C
function C = CalculateMatrixC(E, nu)
    I2 = eye(3); % Second-order identity tensor (3x3)
    I4 = eye(6); % Fourth-order identity tensor (6x6) in Voigt notation

    % Dyadic product I ⊗ I in Voigt notation
    % II_Voigt = I2 ⊗ I2
      II_Voigt = [1 1 1 0 0 0;
                  1 1 1 0 0 0;
                  1 1 1 0 0 0;
                  0 0 0 0 0 0;
                  0 0 0 0 0 0;
                  0 0 0 0 0 0];

    % Compute stiffness tensor using the given formula
    C = (E * nu / ((1 + nu) * (1 - 2 * nu))) * II_Voigt ...
      + (E / (1 + nu)) * I4;
end
```

```matlab
%% Function to calculate EM
function E_M = CalculateEM(C_M)
    A = C_M(1,1);
    B = C_M(1,2);
    E_M = ((A - B) * (A + 2 * B)) / (A + B);
end
```

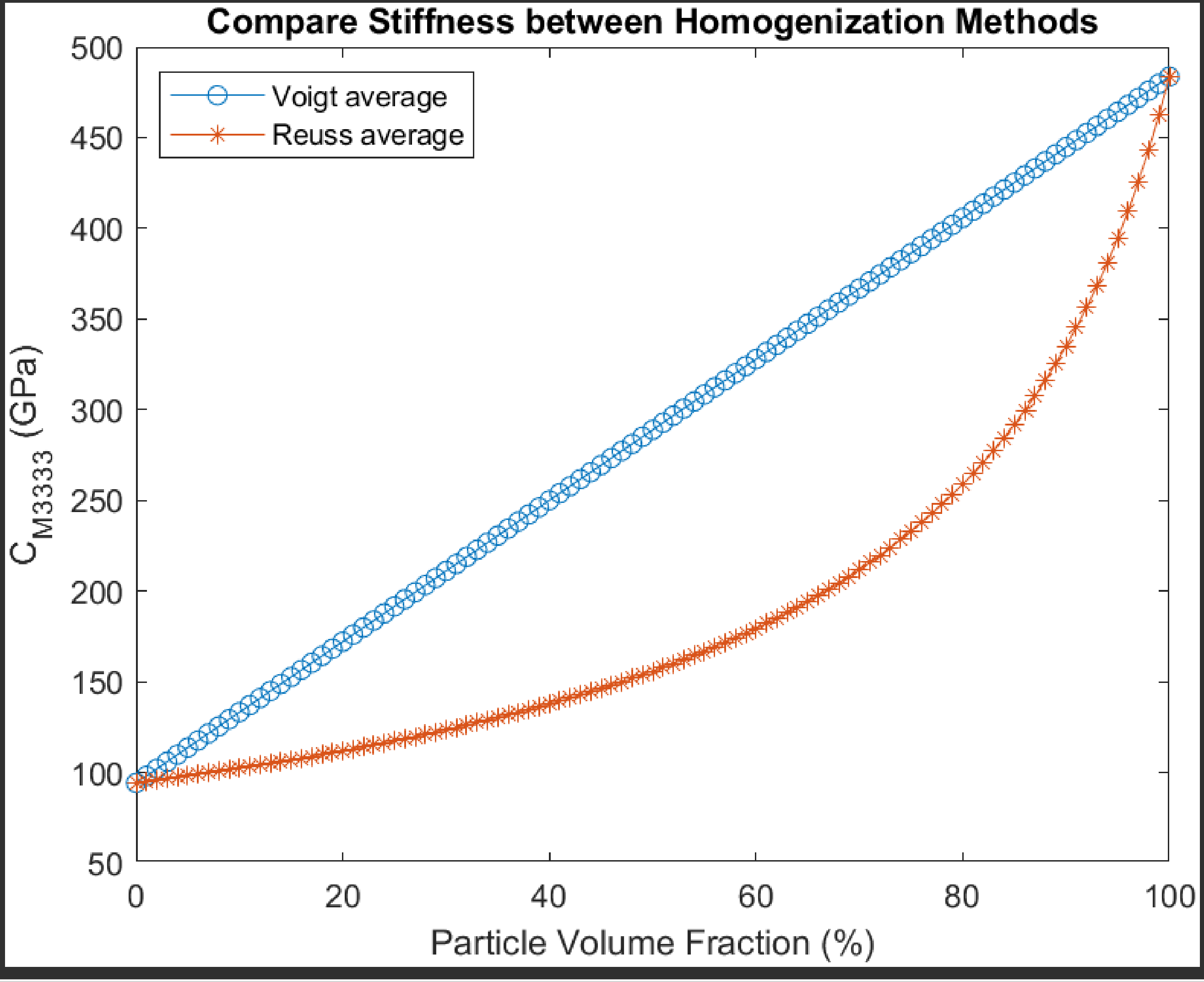
MATLAB example

```
% 1. Voigt Average
% Stiffness tensor C
C_Al = CalculateMatrixC(E_Al, nu_Al);
C_SiC = CalculateMatrixC(E_SiC, nu_SiC);

for i = 1:101
    Vol_SiC = ParticleVolFrac(i);
    Vol_Al = 1 - Vol_SiC;
    CV_M = Vol_Al * C_Al + Vol_SiC * C_SiC;
    CV_M3333(i) = CV_M(3,3);
    EV_M(i) = CalculateEM(CV_M);
end

% 2. Reuss Average
% Compliance tensor S
I = eye(6);
S_Al = C_Al\I;
S_SiC = C_SiC\I;

for j = 1:101
    Vol_SiC = ParticleVolFrac(j);
    Vol_Al = 1 - Vol_SiC;
    SR_M = Vol_Al * S_Al + Vol_SiC * S_SiC;
    CR_M = SR_M\I;
    CR_M3333(j) = CR_M(3,3);
    ER_M(j) = CalculateEM(CR_M);
end
```

Voigt average: ${}^4\boldsymbol{C}_M^V = f_1\,{}^4\boldsymbol{C}_1 + (1 - f_1)\,{}^4\boldsymbol{C}_0$

Reuss average: ${}^4\boldsymbol{C}_M^R = \left({}^4\boldsymbol{S}_M^R\right)^{-1} = \left(f_1\,{}^4\boldsymbol{S}_1 + (1 - f_1)\,{}^4\boldsymbol{S}_0\right)^{-1}$

Comparison between C_M



- In general, Voigt overestimates effective material stiffness, while Reuss underestimates
- Distance between two estimates depends on the contrast (ratio) in properties between microstructural constituents
  i.e. the larger the contrast, the larger the difference

Clone the file from:
https://github.com/Raistlinwolf/IntroComposite.git

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Classical Laminate Theory

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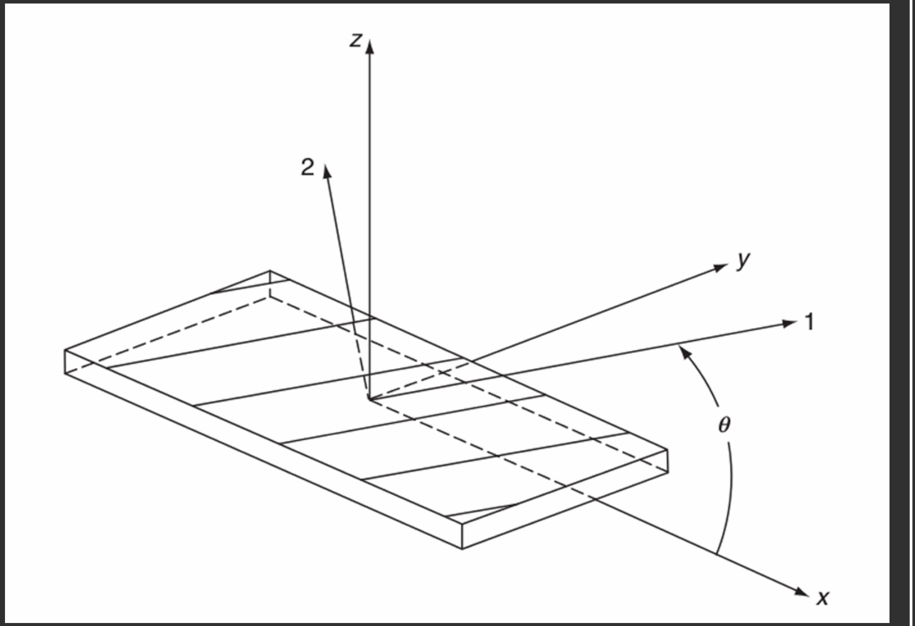
Voigt notation for 2nd and 4th-order tensors

- linear elastic Hooke's law :

$$\boldsymbol{\sigma} = {}^4\boldsymbol{C} : \boldsymbol{\varepsilon} \quad \Longleftrightarrow \quad \sigma_{ij} = C_{ijkl}\,\varepsilon_{lk}$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1121} & C_{1131} & C_{1132} \\ C_{2211} & C_{2222} & C_{2233} & C_{2221} & C_{2231} & C_{2232} \\ C_{3311} & C_{3322} & C_{3333} & C_{3321} & C_{3331} & C_{3332} \\ C_{1211} & C_{1222} & C_{1233} & C_{1221} & C_{1231} & C_{1232} \\ C_{1311} & C_{1322} & C_{1333} & C_{1321} & C_{1331} & C_{1332} \\ C_{2311} & C_{2322} & C_{2333} & C_{2321} & C_{2331} & C_{2332} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{bmatrix}$$



**Symmetric:*
No motion and rotation allowed

$$\sigma_{ij} = \sigma_{ji} \qquad \varepsilon_{kl} = \varepsilon_{lk}$$

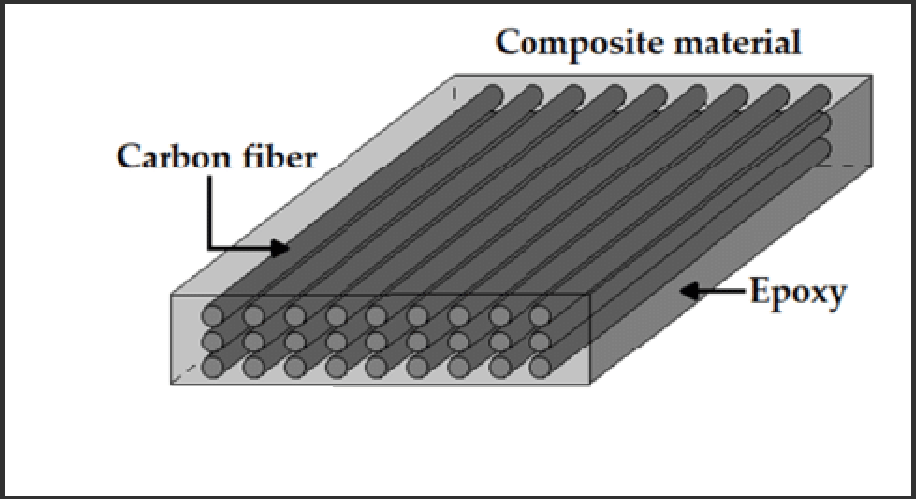
Orthotropic material

- Stiffness matrix:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{13}\\ \sigma_{23}\end{bmatrix}=\begin{bmatrix}C_{1111} & C_{1122} & C_{1133} & 0 & 0 & 0\\ & C_{2222} & C_{2233} & 0 & 0 & 0\\ & & C_{3333} & 0 & 0 & 0\\ & & & C_{1221} & 0 & 0\\ & Sym. & & & C_{1331} & 0\\ & & & & & C_{2332}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \varepsilon_{12}\\ \varepsilon_{13}\\ \varepsilon_{23}\end{bmatrix}$$

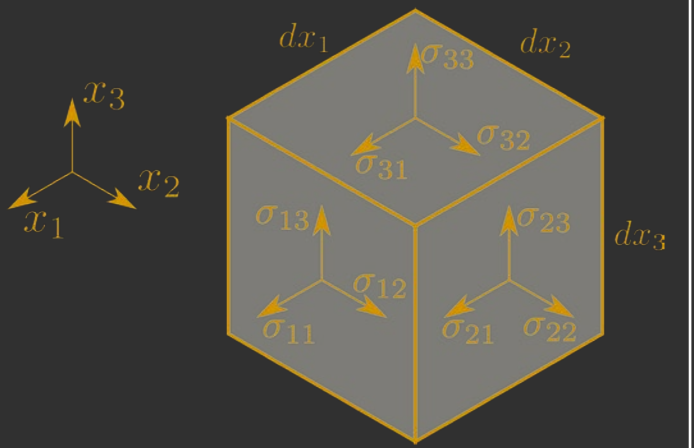
- Compliance matrix:

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \varepsilon_{12}\\ \varepsilon_{13}\\ \varepsilon_{23}\end{bmatrix}=\begin{bmatrix}S_{1111} & S_{1122} & S_{1133} & 0 & 0 & 0\\ & S_{2222} & S_{2233} & 0 & 0 & 0\\ & & S_{3333} & 0 & 0 & 0\\ & & & S_{1221} & 0 & 0\\ & Sym. & & & S_{1331} & 0\\ & & & & & S_{2332}\end{bmatrix}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{13}\\ \sigma_{23}\end{bmatrix}$$

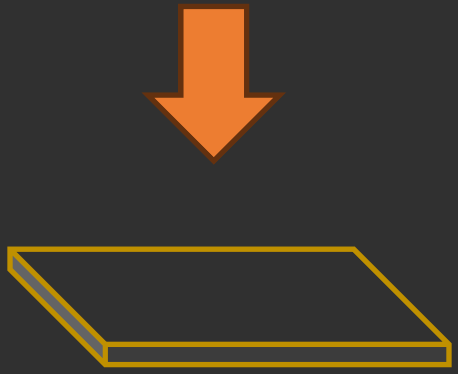


Classical laminate theory

- Assumptions:
  - Layers are thin compared to the other dimensions
  - Generalised plane stress state is assumed
  - All through-the-thickness stress terms are neglected, i.e. $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$



$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \cancel{\varepsilon_{33}} \\ \varepsilon_{12} \\ \cancel{\varepsilon_{13}} \\ \cancel{\varepsilon_{23}} \end{bmatrix} = \begin{bmatrix} S_{1111} & S_{1122} & \cancel{S_{1133}} & 0 & \cancel{0} & \cancel{0} \\ & S_{2222} & \cancel{S_{2233}} & 0 & \cancel{0} & \cancel{0} \\ & & \cancel{S_{3333}} & \cancel{0} & \cancel{0} & \cancel{0} \\ & & & S_{1221} & \cancel{0} & \cancel{0} \\ & Sym. & & & \cancel{S_{1331}} & \cancel{0} \\ & & & & & \cancel{S_{2332}} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \cancel{\sigma_{33}} \\ \sigma_{12} \\ \cancel{\sigma_{13}} \\ \cancel{\sigma_{23}} \end{bmatrix}$$



Classical laminate theory

- Assumptions:
  - Layers are thin compared to the other dimensions
  - Generalised plane stress state is assumed
  - All through-the-thickness stress terms are neglected, i.e. $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix} = \boldsymbol{S\sigma}$$

Where:

$$S_{11} = S_{1111} = \frac{1}{E_1} \qquad S_{22} = S_{2222} = \frac{1}{E_2} \qquad S_{66} = S_{1221} = \frac{1}{G_{12}}$$

$$S_{12} = S_{1122} \qquad S_{21} = S_{2211} \qquad S_{12} = S_{21} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}$$

Classical laminate theory

- Stiffness matrix $\boldsymbol{Q} = \boldsymbol{S}^{-1}$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{bmatrix} = \boldsymbol{Q\varepsilon}$$

Where:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = Q_{21} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}}$$

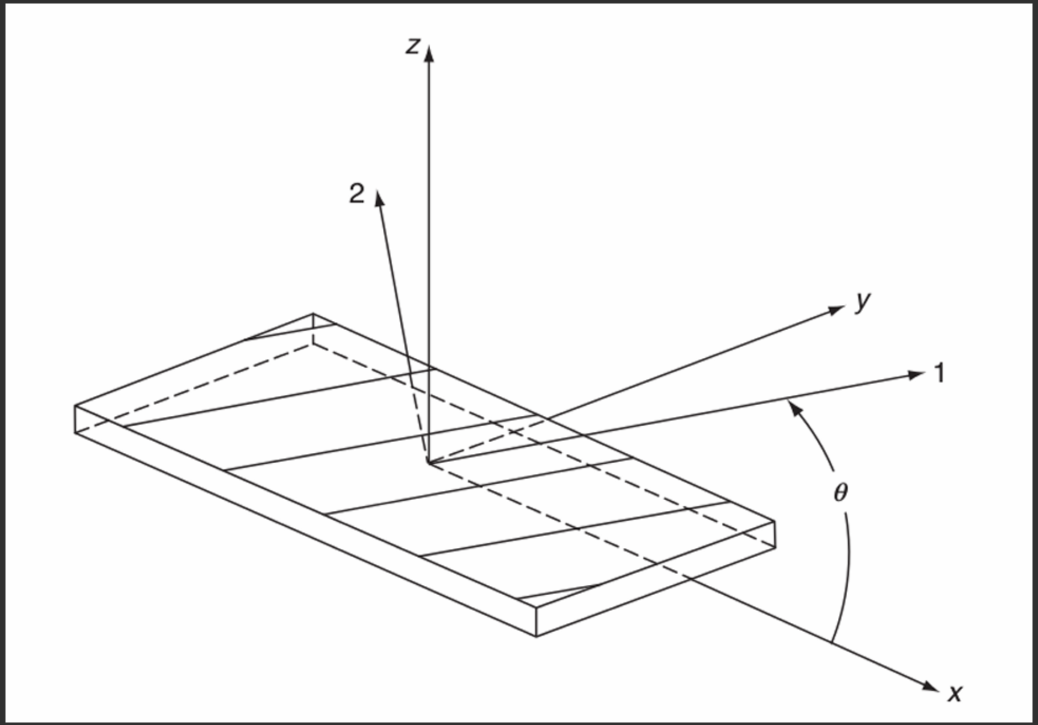
$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

Stress transform to principle axis

$$\sigma_{11} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$
$$\sigma_{22} = \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$
$$\tau_{12} = \left(\sigma_{yy} - \sigma_{xx}\right)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$



Or write into tensor form:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{bmatrix} = \mathcal{T}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}$$

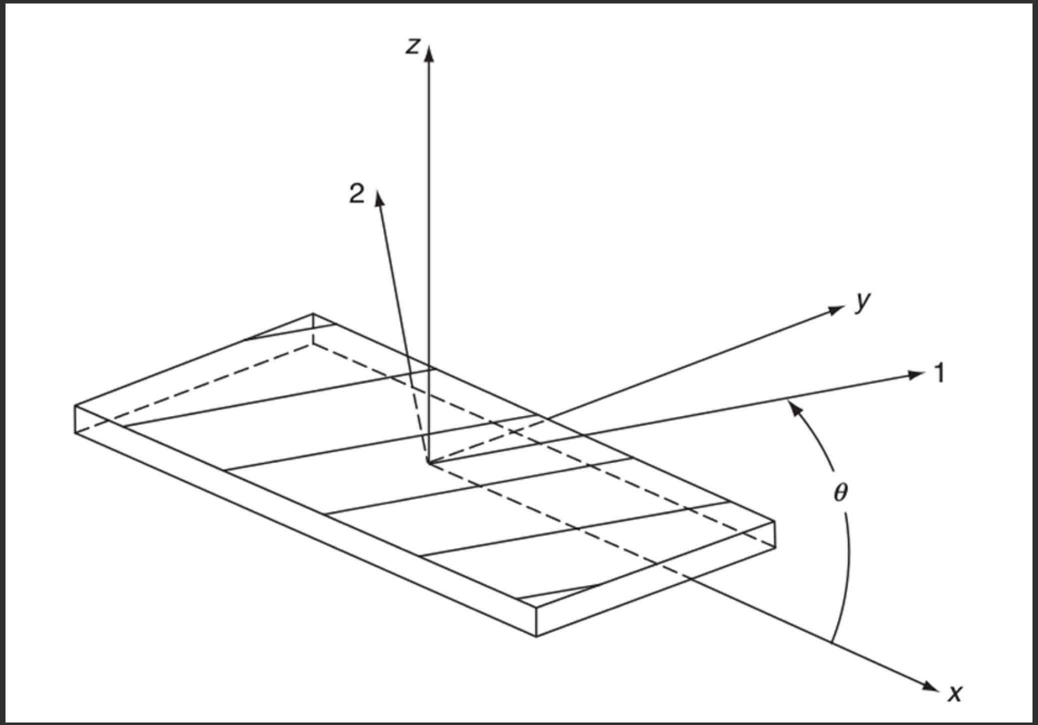
Where:

$$\mathcal{T} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix}$$

Where $c = \cos\theta$ and $s = \sin\theta$

Strain transform to principle axis

$$\varepsilon_{11} = \varepsilon_{xx}\cos^2\theta + \varepsilon_{yy}\sin^2\theta + \gamma_{xy}\cos\theta\sin\theta$$
$$\varepsilon_{22} = \varepsilon_{xx}\sin^2\theta + \varepsilon_{yy}\cos^2\theta - \gamma_{xy}\cos\theta\sin\theta$$
$$\gamma_{12} = 2\left(\varepsilon_{yy} - \varepsilon_{xx}\right)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$



Or write into tensor form:

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{bmatrix} = \mathcal{R}\mathcal{T}\mathcal{R}^{-1}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}$$

Where $\mathcal{R}$ is Reuter's matrix:

$$\mathcal{R} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 2\end{bmatrix}$$

Compliance matrix (Global)

$\bar{\boldsymbol{S}}$ is the compliance matrix with respect to the **global** coordinate system.

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}\bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16}\\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26}\\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66}\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} = \bar{\boldsymbol{S}}\boldsymbol{\sigma}$$

Where

$$\bar{S}_{11} = \frac{1}{E_{xx}} = S_{11}\cos^4\theta + (2S_{12} + S_{66})\sin^2\theta\cos^2\theta + S_{22}\sin^4\theta$$

$$\bar{S}_{12} = \frac{-\nu_{xy}}{E_{xx}} = S_{12}(\sin^4\theta + \cos^4\theta) + (S_{11} + S_{22} - S_{66})\sin^2\theta\cos^2\theta$$

$$\bar{S}_{22} = \frac{1}{E_{yy}} = S_{11}\sin^4\theta + (2S_{12} + S_{66})\sin^2\theta\cos^2\theta + S_{22}\cos^4\theta$$

$$\bar{S}_{16} = (2S_{11} - 2S_{12} - S_{66})\sin\theta\cos^3\theta - (2S_{22} - 2S_{12} - S_{66})\sin^3\theta\cos\theta$$

$$\bar{S}_{26} = (2S_{11} - 2S_{12} - S_{66})\sin^3\theta\cos\theta - (2S_{22} - 2S_{12} - S_{66})\sin\theta\cos^3\theta$$

$$\bar{S}_{66} = \frac{1}{G_{xy}} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})\sin^2\theta\cos^2\theta + S_{66}(\sin^4\theta + \cos^4\theta)$$

Stiffness matrix (Global)

$\bar{\boldsymbol{Q}}$ is the stiffness matrix with respect to the **global** coordinate system.

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} = \bar{\boldsymbol{Q}}\boldsymbol{\varepsilon}$$

Where

$$\bar{Q}_{11} = Q_{11}\cos^4\theta + (2Q_{12} + Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta$$

$$\bar{Q}_{12} = Q_{12}(\sin^4\theta + \cos^4\theta) + (Q_{11} + Q_{22} - Q_{66})\sin^2\theta\cos^2\theta$$

$$\bar{Q}_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta$$

$$\bar{Q}_{16} = (Q_{11} - Q_{22} - 2Q_{66})\sin\theta\cos^3\theta - (Q_{11} - Q_{22} + 2Q_{66})\sin^3\theta\cos\theta$$

$$\bar{Q}_{26} = (Q_{11} - Q_{22} - 2Q_{66})\sin^3\theta\cos\theta - (Q_{11} - Q_{22} + 2Q_{66})\sin\theta\cos^3\theta$$

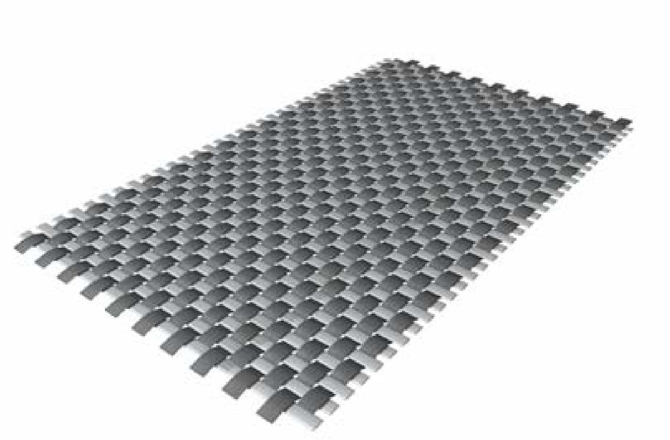
$$\bar{Q}_{66} = (Q_{11} - Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta)$$

Material properties for a single orthotropic layer

- In classical laminate theory, the elastic properties of a layer (both unidirectional and isotropic) is characterised by the following 4 parameters
  - E_1 : the elastic constant in the stiffest direction (fiber direction)
  - E_2 : the elastic constant in the orthogonal direction
  - ν_{12} : the poisson's ratio
  - G_{12} : the shear modulus

➢ Note that $\nu_{21} = \nu_{12}\frac{E_2}{E_1}$

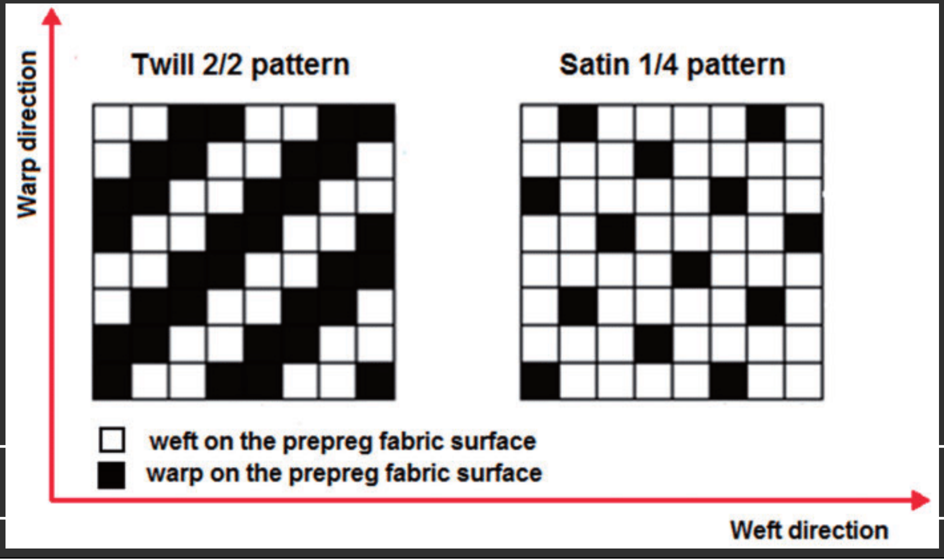
➢ The Poisson's ratio can be larger than 0.5

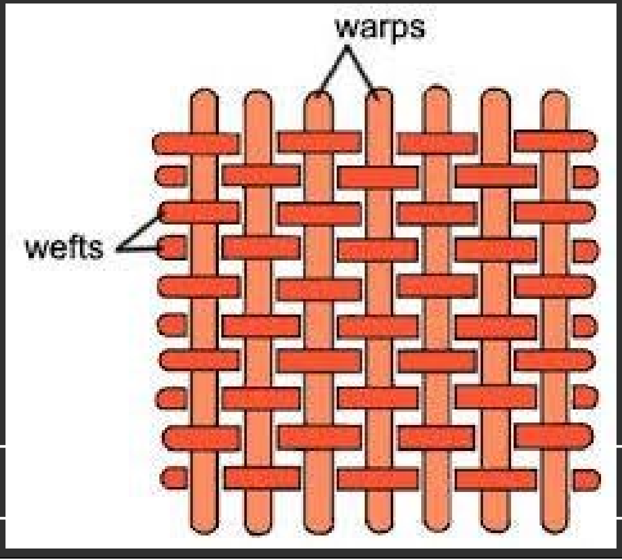


Material properties for a single woven layer

- Woven layers can be characterised by the same material parameters:
  - E_1 : the elastic constant in the stiffest direction (warp direction)
  - E_2 : the elastic constant in the orthogonal direction (weft direction)
  - ν_{12} : the poisson's ratio
  - G_{12} : the shear modulus

➢ Note that for a balanced woven composite, $E_1 = E_2$. However, even in this case, the relation $G = \frac{E}{2(1+\nu)}$ still does not hold (we call this a quasi-isotropic material).





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Mechanical properties of a laminate

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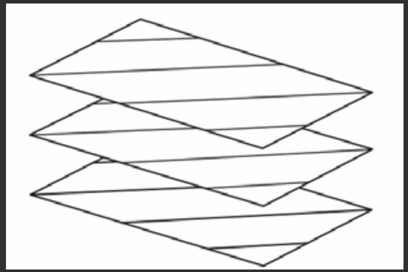
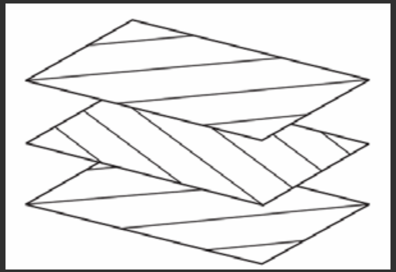
A laminate

- Definition
  - A laminate is a stack of thin layers with different fiber orientations and sometimes different composite materials or weave architectures

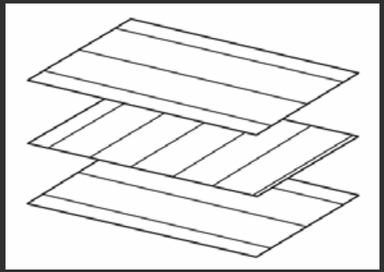
- ***Unidirectional laminate:*** the orientation angles θ are identical in each layer
- ***Angle-ply laminate:*** the fiber orientations in alternate layers are $\theta / -\theta / \theta / -\theta$, (where $\theta \neq 0°$ or $90°$)
- ***Cross-ply laminate***: fiber orientations in alternate layers are $0/ 90 / 0 / 90$
- ***Symmetric laminate***: the fiber orientations of the layer are symmetrical about the center line of the laminate.



Unidrectional laminate



Angle-ply laminate



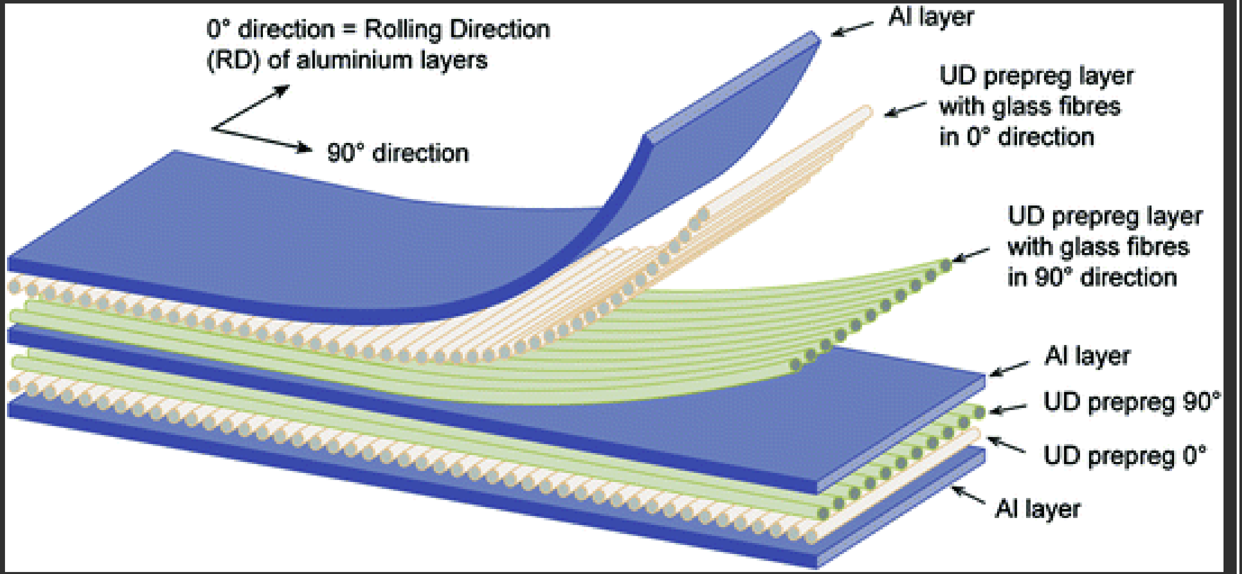
Cross-ply laminate

Stacking sequences, notation

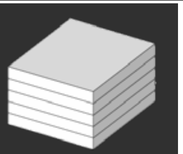
- 6 layers, symmetric, UD material at angles $\theta = 0°,\ 45°,\ 90°$
  Notation: $[\ 0/\ 45/\ 90/\ 90/\ 45/\ 0\ ]$
  Short form: $[\ 0/\ 45/\ 90\ ]_S$
- 5 layers, symmetric, UD material at angles $\theta = 0°,\ 45°,\ 90°$
  Notation: $[\ 0/\ 45/\ 90/\ 45/\ 0\ ]$
  Short form: $[\ 0/\ 45/\ \overline{90}\ ]_S$
- 7 layers, symmetric, UD material at angles $\theta = 0°,\ 45°,\ -45°,\ 90°$
  Notation: $[0/\ +45/\ -45/\ 90/\ -45/\ +45/\ 0]$
  Short form: $[\ 0/\ \pm 45/\ \overline{90}\ ]_S$
- 8 layers, UD material in groups of 2 layers at angles $\theta = 0°,\ 90°,\ 90°,\ 0°$
  Notation: $[0/\ 0/\ 90/\ 90/\ 90/\ 0/\ 0]$
  Short form: $[0_2/90_4/0_2]$ or $[0_2/90_2]_S$
- 8 layers, UD material symmetric at angles $\theta = 0°,\ 90°,\ 90°,\ 0°,\ 0°,\ 90°,\ 90°,\ 0°$
  Notation: $[0/\ 90/\ 90/\ 0/\ 90/\ 90/\ 0]$
  Short form: $[0/90]_{2S}$

Stacking sequences, notation (continued)

- 8 layers, symmetric, Woven at 0° on the outside, e.g.
  Notation: $[0w/\ 90/\ 45/\ 0/0/45/\ 90/\ 0w]$
  Short form: $[0w/\ 90/\ 45/\ 0]_S$
- ARALL (1981), symmetric, aluminum and AFRP:
  Notation: $[Al/\ 0/\ 90/0/Al\underline{/0}/90/0/\ Al]$
  Short form: $[Al/\ 0/\ 90/0/\overline{Al}]_S$
- GLARE (1987), symmetric, aluminum and GFRP:
  Notation: $[Al/\ 0/\ 90/\ Al\underline{/}90/0/\ Al]$
  Short form: $[Al/\ 0/\ 90/\ \overline{Al}]_S$



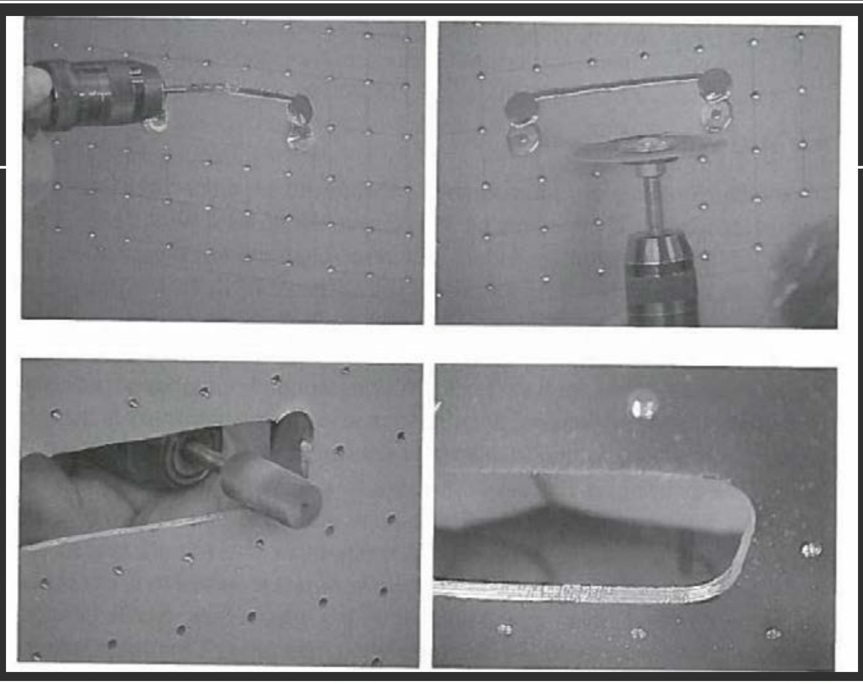
Fiber-Metal laminates (FMLs)

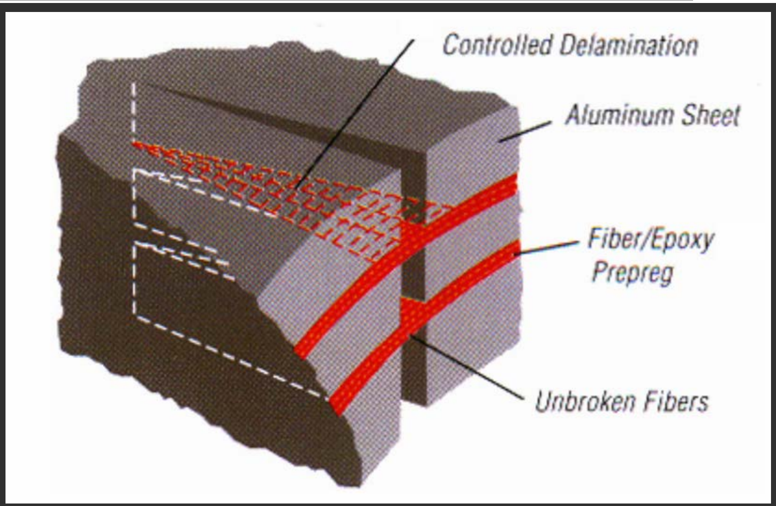


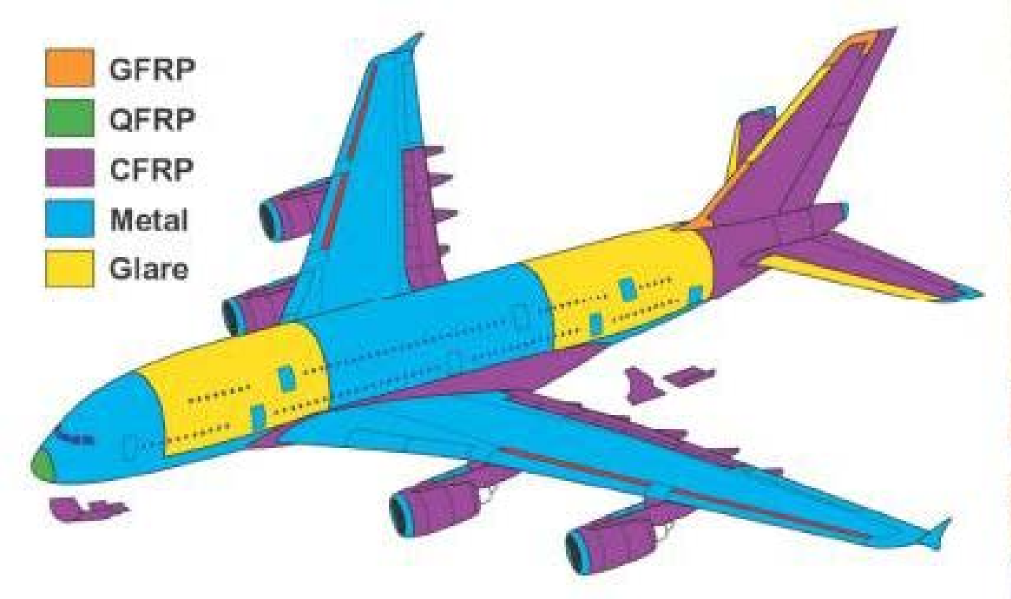
- GLARE Materials
  - Aluminum panels (0.3mm)
    - Al. 2024 or 7075-T6 Alloys
    - Surface treatment for better adhesion
    - Always on the outside
  - Glass fiber epoxy (0.2mm)
    - Prepreg material (uni-directional)
    - Laser cutting, manual stacking
- GLARE Advantages
  - Excellent fatigue performances
  - Fire resistant
  - Good impact resistance (aluminum layer shields)
  - Allows for old-fashioned tooling
  - Plastic deformation
- GLARE Disadvantages
  - Fits well into classical design (stringer stiffened panels), but does not allow for alternative design strategies
  - Compressive strength has been an issue

Grade (ply orientations, in degrees)	Advantages	Application on A380
1 (0°/0°)	Fatigue, strength, yield stress	
2A (0°/0°)	Fatigue, strength	stringers
2B (90°/90°)	Fatigue, strength	butt straps
3A (0°/90°)	Fatigue, impact	
3B (90°/0°)	Fatigue, impact	fuselage skins
4A (0°/90°/0°)	Fatigue, strength in 0° direction	
4B (90°/0°/90°)	Fatigue, strength in 90° direction	~~fuselage skins~~
5A (0°/90°/90°/0°)	impact	
5B (90°/0°/0°/90°)	impact	fuselage skins
6A (+45°/-45°)	Shear, off-axis properties	
6B (-45°/+45°)	Shear, off-axis properties	

GLARE




Controlled Delamination
Aluminum Sheet
Fiber/Epoxy Prepreg
Unbroken Fibers

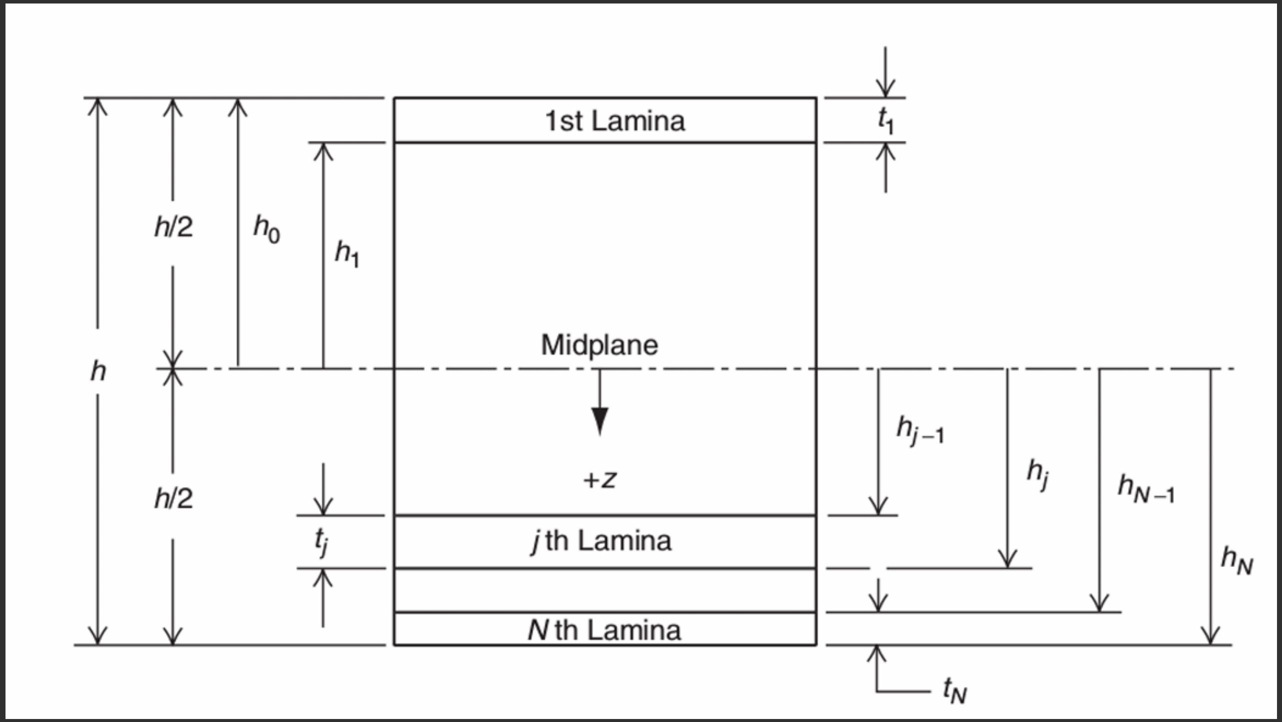

GFRP
QFRP
CFRP
Metal
Glare

Lamination theory

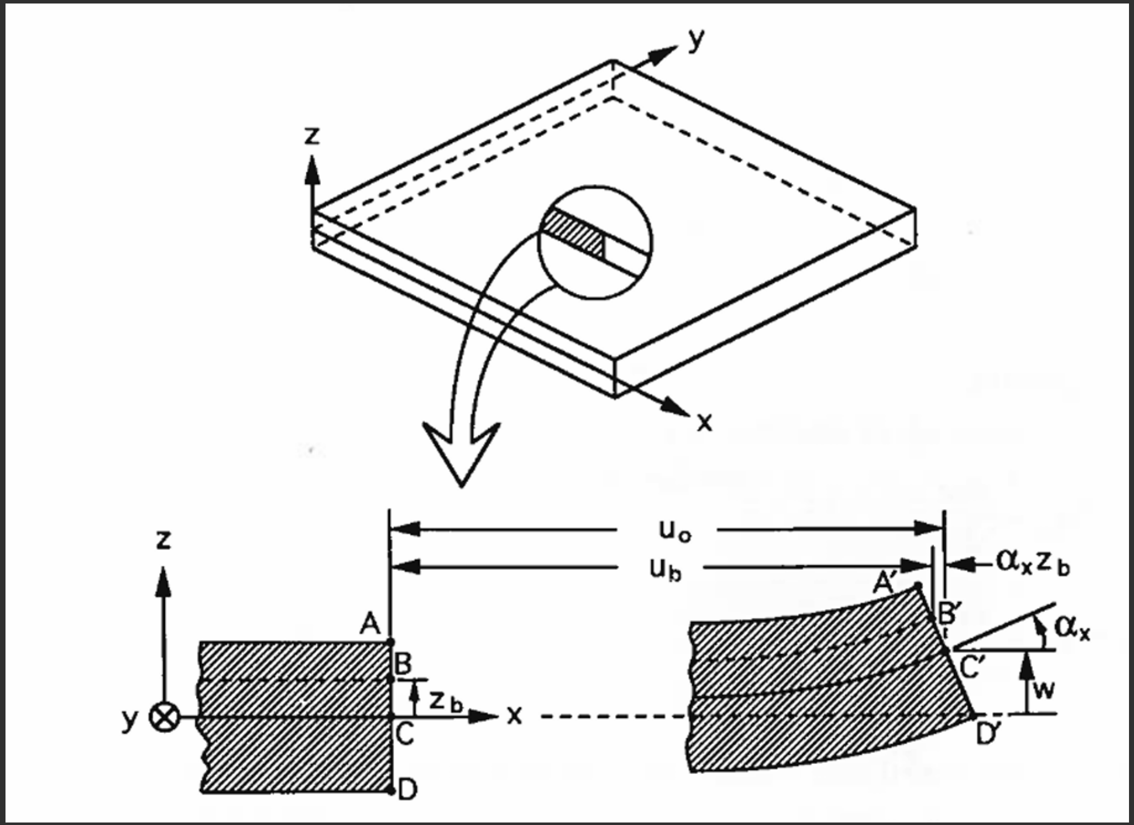
- Assumptions:
  - Laminate is thin and wide (width $\gg$ thickness)
  - A perfect interlaminar bond exists between various laminae
  - Strain distribution in the thickness direction is linear
  - All laminas are macroscopically homogeneous and behave in a linearly elastic manner
  - All displacements are small compared to the thickness
  - Transverse strains ε_{zz}, ε_{xz} and ε_{yz} are negligible

Lamination theory

- To calculate the laminate properties, you need to know:
  - The elastic properties of each layer j (E_1, E_2, ν_{12}, G_{12})
  - The thickness t_j of each layer j
  - The orientation angle θ_j of each layer (in case of an orthotropic layer material)



Strain-displacement relation



$$\Delta x_b = z_b \tan\alpha_x \quad \Longrightarrow \quad \tan\alpha_x = \frac{\Delta x_b}{z_b}$$

Assume $\alpha_x \to 0$, $\tan\alpha_x = \alpha_x$

$\Longrightarrow \Delta x_b = z_b \alpha_x$

$\Longrightarrow u_0 = u_b + z_b \alpha_x$

$$\boxed{\begin{aligned} u_b &= u_0 - \alpha_x z_b \\ v_b &= v_0 - \alpha_y z_b \end{aligned}}$$

Where:

$\alpha_x = \frac{\partial w}{\partial x}$, $\alpha_y = \frac{\partial w}{\partial y}$

$$\boxed{\begin{aligned} u &= u_0 - z\frac{\partial w}{\partial x} \\ v &= v_0 - z\frac{\partial w}{\partial y} \end{aligned}}$$

Strains

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} = \varepsilon_{xx}^0 + z\kappa_{xx}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2} = \varepsilon_{yy}^0 + z\kappa_{yy}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y} = \gamma_{xy}^0 + z\kappa_{xy}$$

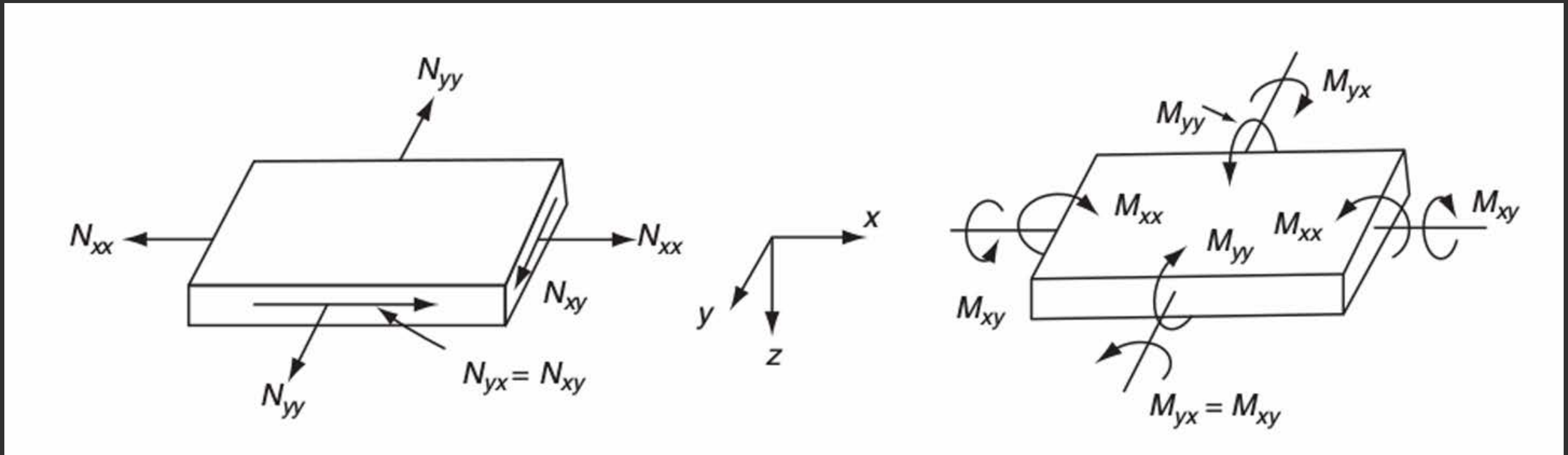
Where by definition:

$$\kappa_{xx} = -\frac{\partial^2 w}{\partial x^2} \qquad \kappa_{yy} = -\frac{\partial^2 w}{\partial y^2} \qquad \kappa_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$$

Substituting this in the layer stress-strain relations gives:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}_j = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_j \begin{bmatrix}\varepsilon_{xx}^0\\ \varepsilon_{yy}^0\\ \gamma_{xy}^0\end{bmatrix} + \boldsymbol{z}\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_j \begin{bmatrix}\kappa_{xx}\\ \kappa_{yy}\\ \kappa_{xy}\end{bmatrix}$$

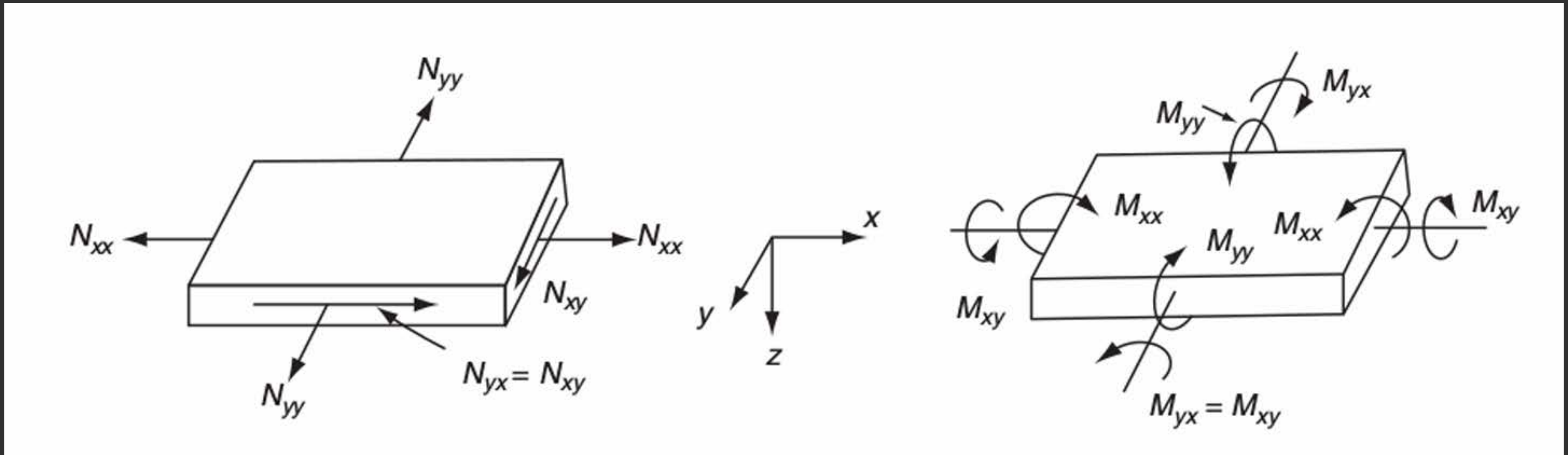
Constitutive relation on laminate level



- $\mathcal{N}$ and $\mathcal{M}$ are line forces and moments respectively
- They can vary over the width of the panel, but are constant in height
- Their units are: $\mathcal{N} = [N/m]$ and $\mathcal{M} = [Nm/m] = [N]$

$$\mathcal{N} = \int_{-h/2}^{h/2} \boldsymbol{\sigma}\, dz \qquad\qquad \mathcal{M} = \int_{-h/2}^{h/2} \boldsymbol{\sigma} z\, dz$$

Constitutive relation on laminate level



$$\boldsymbol{\mathcal{N}} = \int_{-h/2}^{h/2} \overline{\boldsymbol{Q}}\boldsymbol{\varepsilon}^{0} + z\overline{\boldsymbol{Q}}\boldsymbol{\kappa}\, dz \qquad \boldsymbol{\mathcal{M}} = \int_{-h/2}^{h/2} z\overline{\boldsymbol{Q}}\boldsymbol{\varepsilon}^{0} + z^{2}\overline{\boldsymbol{Q}}\boldsymbol{\kappa}\, dz$$

$$\boldsymbol{\mathcal{N}} = \sum_{j=1}^{N} \int_{h_{j-1}}^{h_j} \overline{\boldsymbol{Q}_j}\boldsymbol{\varepsilon}^{0} + z\overline{\boldsymbol{Q}_j}\boldsymbol{\kappa}\, dz \qquad \boldsymbol{\mathcal{M}} = \sum_{j=1}^{N} \int_{h_{j-1}}^{h_j} z\overline{\boldsymbol{Q}_j}\boldsymbol{\varepsilon}^{0} + z^{2}\overline{\boldsymbol{Q}_j}\boldsymbol{\kappa}\, dz$$

Constitutive relation on laminate level

- Since ε and κ are independent of z, we can write:

$$\mathcal{N} = \sum_{j=1}^{N} \int_{h_{j-1}}^{h_j} \overline{\boldsymbol{Q}}_j dz\, \boldsymbol{\varepsilon}^0 + \sum_{j=1}^{N} \int_{h_{j-1}}^{h_j} z\overline{\boldsymbol{Q}}_j dz\, \boldsymbol{\kappa} = \boldsymbol{\mathcal{A}}\boldsymbol{\varepsilon}^0 + \boldsymbol{\mathcal{B}}\boldsymbol{\kappa}$$

$$\mathcal{M} = \sum_{j=1}^{N} \int_{h_{j-1}}^{h_j} z\overline{\boldsymbol{Q}}_j dz\, \boldsymbol{\varepsilon}^0 + \sum_{j=1}^{N} \int_{h_{j-1}}^{h_j} z^2\overline{\boldsymbol{Q}}_j dz\, \boldsymbol{\kappa} = \boldsymbol{\mathcal{B}}\boldsymbol{\varepsilon}^0 + \boldsymbol{\mathcal{D}}\boldsymbol{\kappa}$$

- **Extensional stiffness matrix**
- **Coupling stiffness matrix**
- **Bending stiffness matrix**

$$\boldsymbol{\mathcal{A}} = \sum_{j=1}^{N} \overline{\boldsymbol{Q}}_j \left(h_j - h_{j-1}\right)$$

$$\boldsymbol{\mathcal{B}} = \frac{1}{2}\sum_{j=1}^{N} \overline{\boldsymbol{Q}}_j \left(h_j^2 - h_{j-1}^2\right)$$

$$\boldsymbol{\mathcal{D}} = \frac{1}{3}\sum_{j=1}^{N} \overline{\boldsymbol{Q}}_j \left(h_j^3 - h_{j-1}^3\right)$$

Relations

- Balanced lay-up (e.g. $[45/-45]$):
  - For each layer with angle θ there is a layer of the same material and thickness and with opposite angle $-\theta$
  - In that case $\mathcal{A}_{16} = \mathcal{A}_{26} = 0$
- Symmetric lay-up (e.g. $[45/-45]_s$):
  - For each layer there is a layer of the same material, thickness and angle θ on the opposite side of the lay-up
  - In that case $\boldsymbol{\mathcal{B}} = \mathcal{O}$
- Anti-symmetric lay-up (e.g. $[45/\,0/-45]$):
  - For each layer θ there is a layer with opposite angle $-\theta$ on exactly the other side of the mid-plane
  - In that case $\mathcal{D}_{16} = \mathcal{D}_{26} = 0$

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How to calculate?

課程回饋表單

Mid-plane strains and curvatures

- Remember:

$$\boldsymbol{\mathcal{N}} = \boldsymbol{\mathcal{A}}\boldsymbol{\varepsilon}^{\mathbf{0}} + \boldsymbol{\mathcal{B}}\boldsymbol{\kappa} \quad \text{and} \quad \boldsymbol{\mathcal{M}} = \boldsymbol{\mathcal{B}}\boldsymbol{\varepsilon}^{\mathbf{0}} + \boldsymbol{\mathcal{D}}\boldsymbol{\kappa}$$

- Which is:

$$\begin{bmatrix} \mathcal{N}_{xx} \\ \mathcal{N}_{yy} \\ \mathcal{N}_{xy} \end{bmatrix} = \boldsymbol{\mathcal{A}} \begin{bmatrix} \varepsilon_{xx}^{0} \\ \varepsilon_{yy}^{0} \\ \gamma_{xy}^{0} \end{bmatrix} + \boldsymbol{\mathcal{B}} \begin{bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathcal{M}_{xx} \\ \mathcal{M}_{yy} \\ \mathcal{M}_{xy} \end{bmatrix} = \boldsymbol{\mathcal{B}} \begin{bmatrix} \varepsilon_{xx}^{0} \\ \varepsilon_{yy}^{0} \\ \gamma_{xy}^{0} \end{bmatrix} + \boldsymbol{\mathcal{D}} \begin{bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{bmatrix}$$

- Can be written into:

$$\begin{bmatrix} \mathcal{N}_{xx} \\ \mathcal{N}_{yy} \\ \mathcal{N}_{xy} \\ \mathcal{M}_{xx} \\ \mathcal{M}_{yy} \\ \mathcal{M}_{xy} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{A}} & \boldsymbol{\mathcal{B}} \\ \boldsymbol{\mathcal{B}} & \boldsymbol{\mathcal{D}} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx}^{0} \\ \varepsilon_{yy}^{0} \\ \gamma_{xy}^{0} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{bmatrix}$$

Mid-plane strains and curvatures

- The inverse relation is:

$$\begin{bmatrix}\varepsilon_{xx}^0\\ \varepsilon_{yy}^0\\ \gamma_{xy}^0\end{bmatrix} = \boldsymbol{A_1}\begin{bmatrix}\mathcal{N}_{xx}\\ \mathcal{N}_{yy}\\ \mathcal{N}_{xy}\end{bmatrix} + \boldsymbol{B_1}\begin{bmatrix}\mathcal{M}_{xx}\\ \mathcal{M}_{yy}\\ \mathcal{M}_{xy}\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}\kappa_{xx}\\ \kappa_{yy}\\ \kappa_{xy}\end{bmatrix} = \boldsymbol{C_1}\begin{bmatrix}\mathcal{N}_{xx}\\ \mathcal{N}_{yy}\\ \mathcal{N}_{xy}\end{bmatrix} + \boldsymbol{D_1}\begin{bmatrix}\mathcal{M}_{xx}\\ \mathcal{M}_{yy}\\ \mathcal{M}_{xy}\end{bmatrix}$$

- Where:

$$\boldsymbol{A_1} = \boldsymbol{\mathcal{A}}^{-1} + \boldsymbol{\mathcal{A}}^{-1}\boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{D}}^*)^{-1}\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{A}}^{-1}$$

$$\boldsymbol{B_1} = -\boldsymbol{\mathcal{A}}^{-1}\boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{D}}^*)^{-1}$$

$$\boldsymbol{C_1} = -(\boldsymbol{\mathcal{D}}^*)^{-1}\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{A}}^{-1} = \boldsymbol{B_1}^{\mathrm{T}}$$

$$\boldsymbol{\mathcal{D}}^* = \boldsymbol{\mathcal{D}} - \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{A}}^{-1}\boldsymbol{\mathcal{B}}$$

$$\boldsymbol{D_1} = (\boldsymbol{\mathcal{D}}^*)^{-1}$$

**Only hold for the standard cases, without thermal strains*

Mid-plane strains and curvatures

- In case $\boldsymbol{\mathcal{B}} = \mathcal{O}$, then $\boldsymbol{B_1} = \boldsymbol{C_1} = \mathcal{O}$, thus:

$$\begin{bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{bmatrix} = \boldsymbol{A_1} \begin{bmatrix} \mathcal{N}_{xx} \\ \mathcal{N}_{yy} \\ \mathcal{N}_{xy} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{bmatrix} = \boldsymbol{D_1} \begin{bmatrix} \mathcal{M}_{xx} \\ \mathcal{M}_{yy} \\ \mathcal{M}_{xy} \end{bmatrix}$$

- Where:

$$\boldsymbol{A_1} = \boldsymbol{\mathcal{A}}^{-1}$$

$$\boldsymbol{D_1} = \boldsymbol{\mathcal{D}}^{-1}$$

Stress in a single layer

- Once the mid-plane strains and curvatures are known, the strain in layer j is equal to:

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}_j = \begin{bmatrix}\varepsilon_{xx}^0\\ \varepsilon_{yy}^0\\ \gamma_{xy}^0\end{bmatrix} + z_j \begin{bmatrix}\kappa_{xx}\\ \kappa_{yy}\\ \kappa_{xy}\end{bmatrix}$$

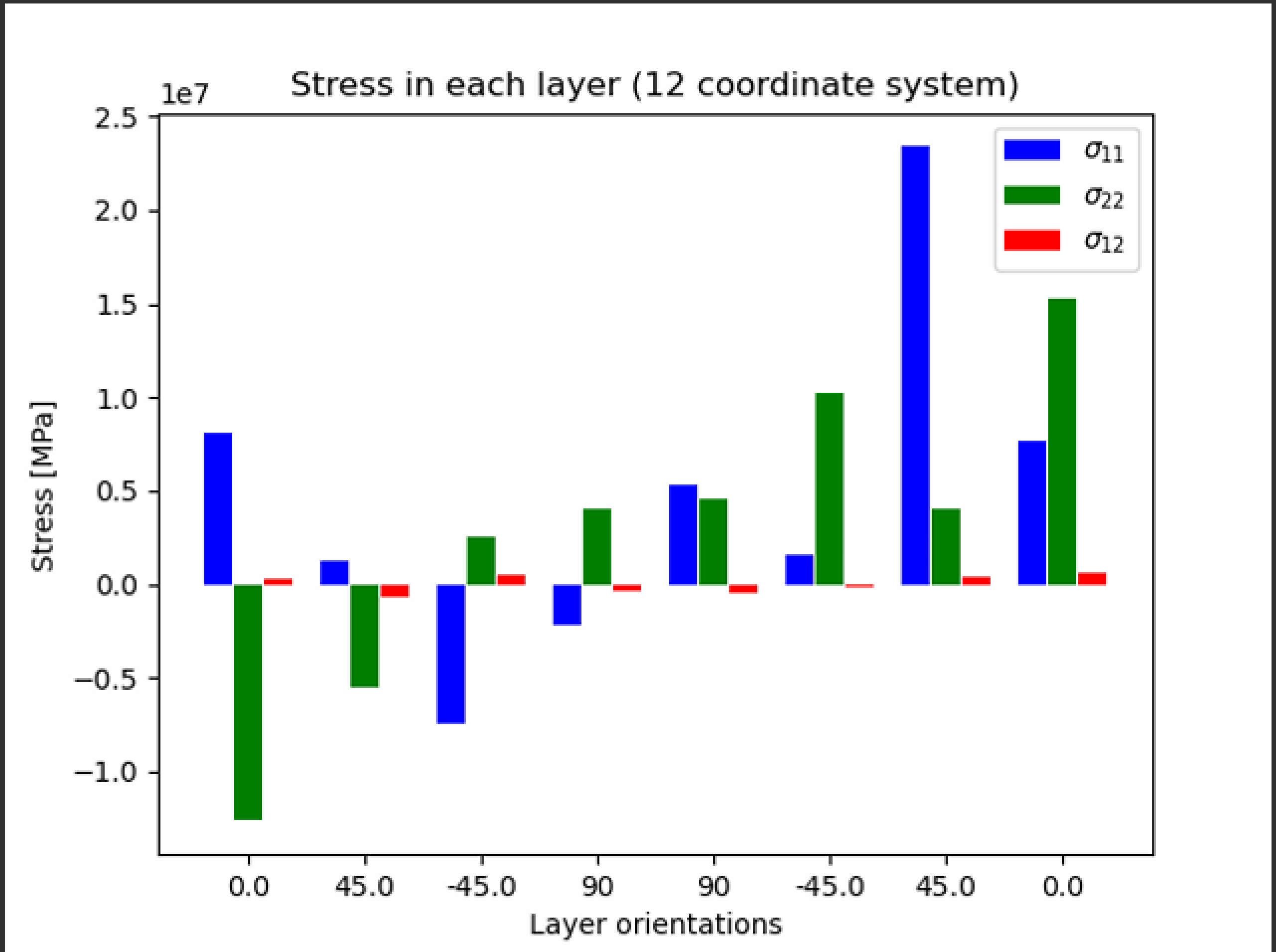
- The stress in that layer is equal to:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}_j = \bar{\boldsymbol{Q}}_j \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}_j = \bar{\boldsymbol{Q}}_j \begin{bmatrix}\varepsilon_{xx}^0\\ \varepsilon_{yy}^0\\ \gamma_{xy}^0\end{bmatrix} + z_j \bar{\boldsymbol{Q}}_j \begin{bmatrix}\kappa_{xx}\\ \kappa_{yy}\\ \kappa_{xy}\end{bmatrix}$$

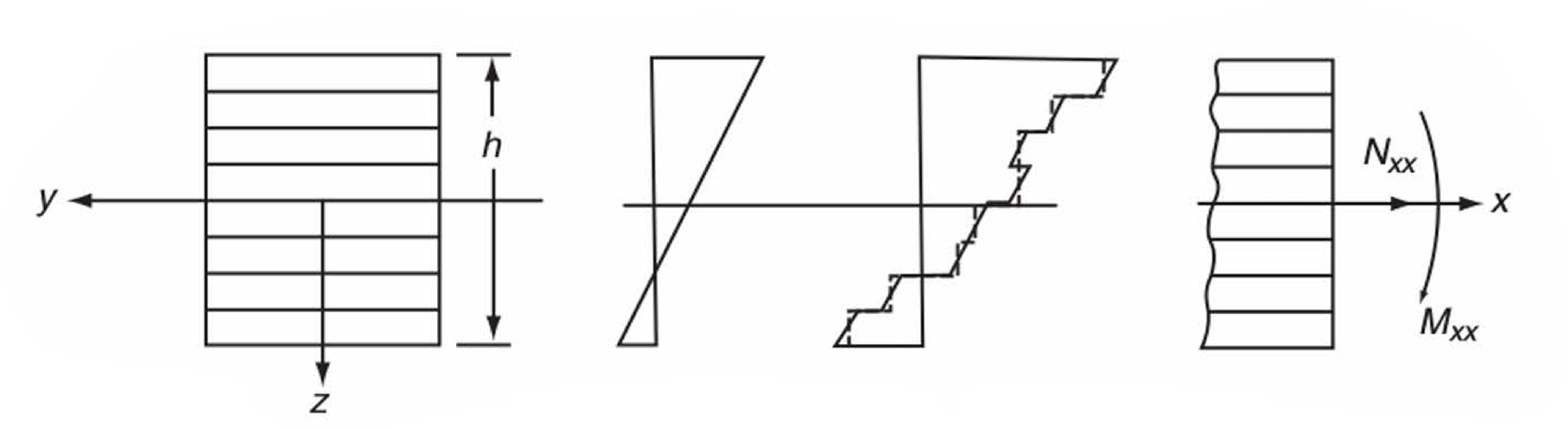
- Note that these stresses are defined in the global(x - y) coordinate system. To obtain the stresses in the material (1 - 2) coordinate system:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{bmatrix} = \mathcal{T} \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} \quad \text{where} \quad \mathcal{T} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix}$$

Python Example



Stress and strain distribution



a) Laminate.

b) Strain distribution

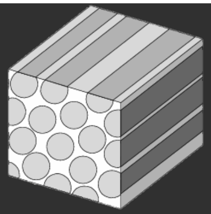
$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^0 + z\boldsymbol{\kappa}$$

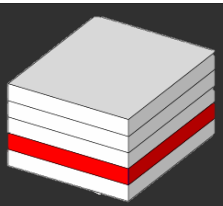
c) Stress distribution

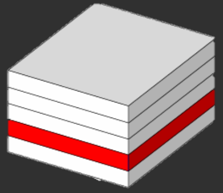
d) Normal force and bending moment resultants

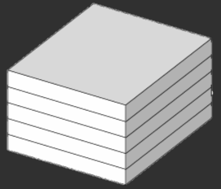
Stress in a single layer

Please use with care. You might encounter situation where additional calculations must be performed, or where steps can be skipped.

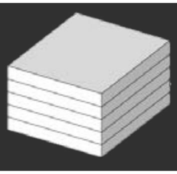
1.  Calculate engineering properties E_1, E_2, G_{12} and ν_{12} from each material in the laminate.

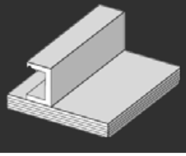
2.  Calculate layer stiffnesses in the local frame of reference: $\boldsymbol{Q}$.

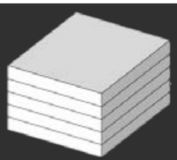
3.  Calculate transformed stiffnesses in the global frame of reference $\overline{\boldsymbol{Q}}$ using the fiber orientation angle θ.

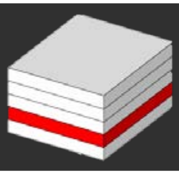
4.  Calculate stiffness matrices $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{D}$ using the layer thicknesses.

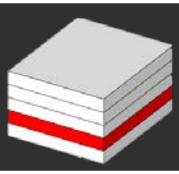
Stress in a single layer

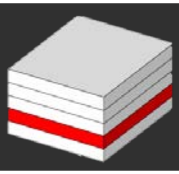
5.  Calculate the compliance matrices $\boldsymbol{A_1}$, $\boldsymbol{B_1}$, $\boldsymbol{C_1}$, and $\boldsymbol{D_1}$

6.  Enter loads $[\mathcal{N}_{xx} \quad \mathcal{N}_{yy} \quad \mathcal{N}_{xy}]$ and $[\mathcal{M}_{xx} \quad \mathcal{M}_{yy} \quad \mathcal{M}_{xy}]$

7.  Calculate strains and curvatures $\boldsymbol{\varepsilon^0}$ and $\boldsymbol{\kappa}$

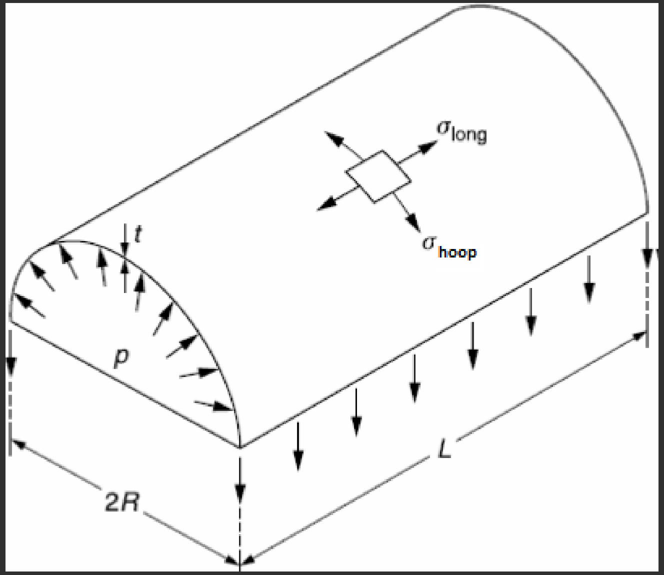
8.  Calculate layer strains: $[\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy}]_{\boldsymbol{j}}$.

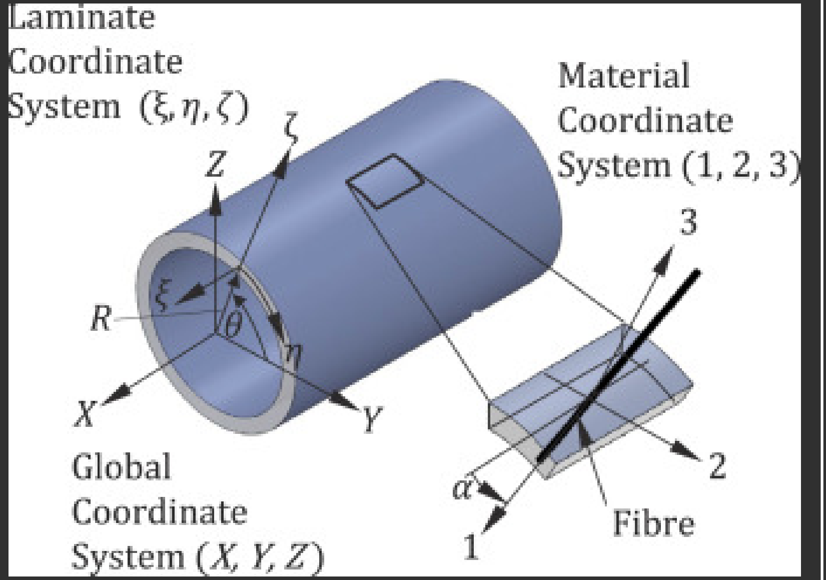
9.  Transform layer strains in local direction $[\varepsilon_{11} \quad \varepsilon_{22} \quad \gamma_{12}]_{\boldsymbol{j}}$

10.  Calculate layer stresses $[\sigma_{11} \quad \sigma_{22} \quad \tau_{12}]_{\boldsymbol{j}} = \boldsymbol{Q_j}[\varepsilon_{11} \quad \varepsilon_{22} \quad \gamma_{12}]_{\boldsymbol{j}}$

Exercise

- Calculate the stress in each layer in the wall. The vessel is subjected to an internal pressure p = 5 bar= 500000Pa.
- The wall of the gas tank as shown in the figure consists of 48 uni-directional layers UCHSC200 SE84 CFRP with the following lay-up: $[35/-35]_{12S}$.
- The vessel's longitudinal direction is the 0° direction. Assume that the vessel is initially stress-free. The diameter of the mid plane of the wall of the tank is 280mm





(S. Li, 2023)

Stress in thin-walled pressurized COPV

- Longitudinal Tensile Force ($\mathcal{N}_{xx}$)

$$2\pi R\mathcal{N}_{xx} = \pi R^2 P$$

$$\mathcal{N}_{xx} = \frac{\mathrm{PR}}{2}$$

- Hoop Force ($\mathcal{N}_{yy}$)

$$2\mathcal{N}_{yy}L = 2RPL$$

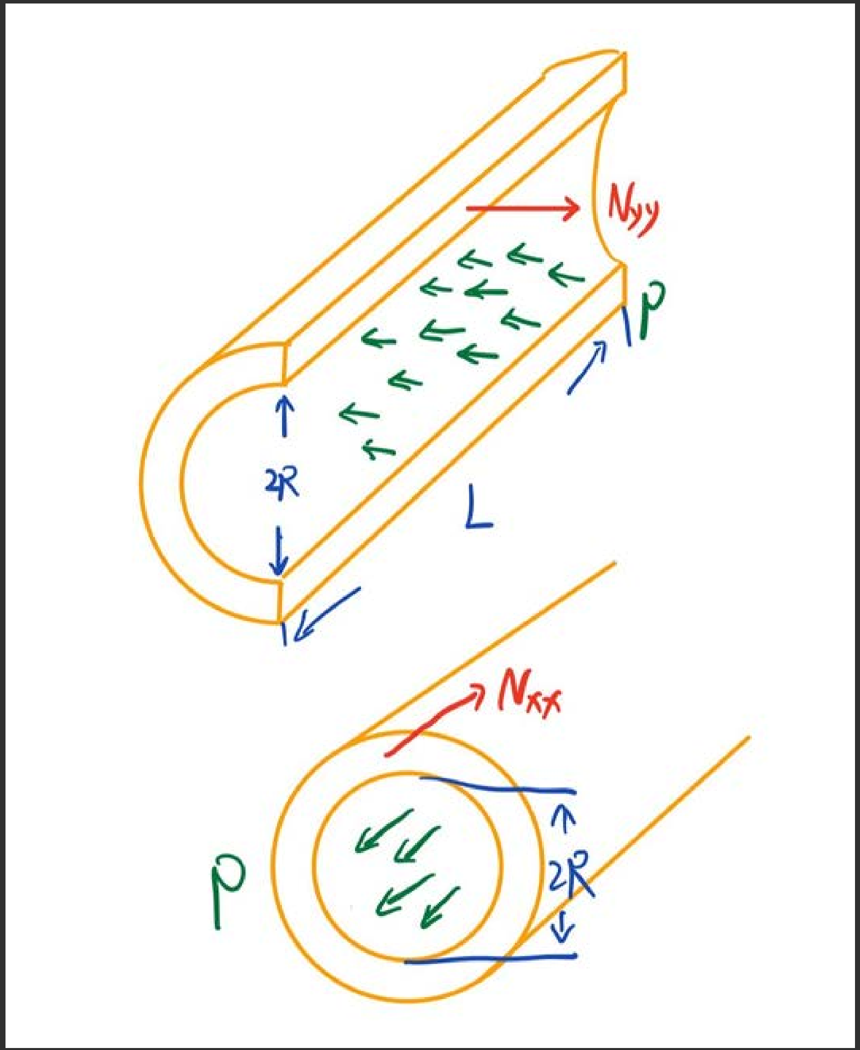
$$\mathcal{N}_{yy} = PR$$

- Then we get:

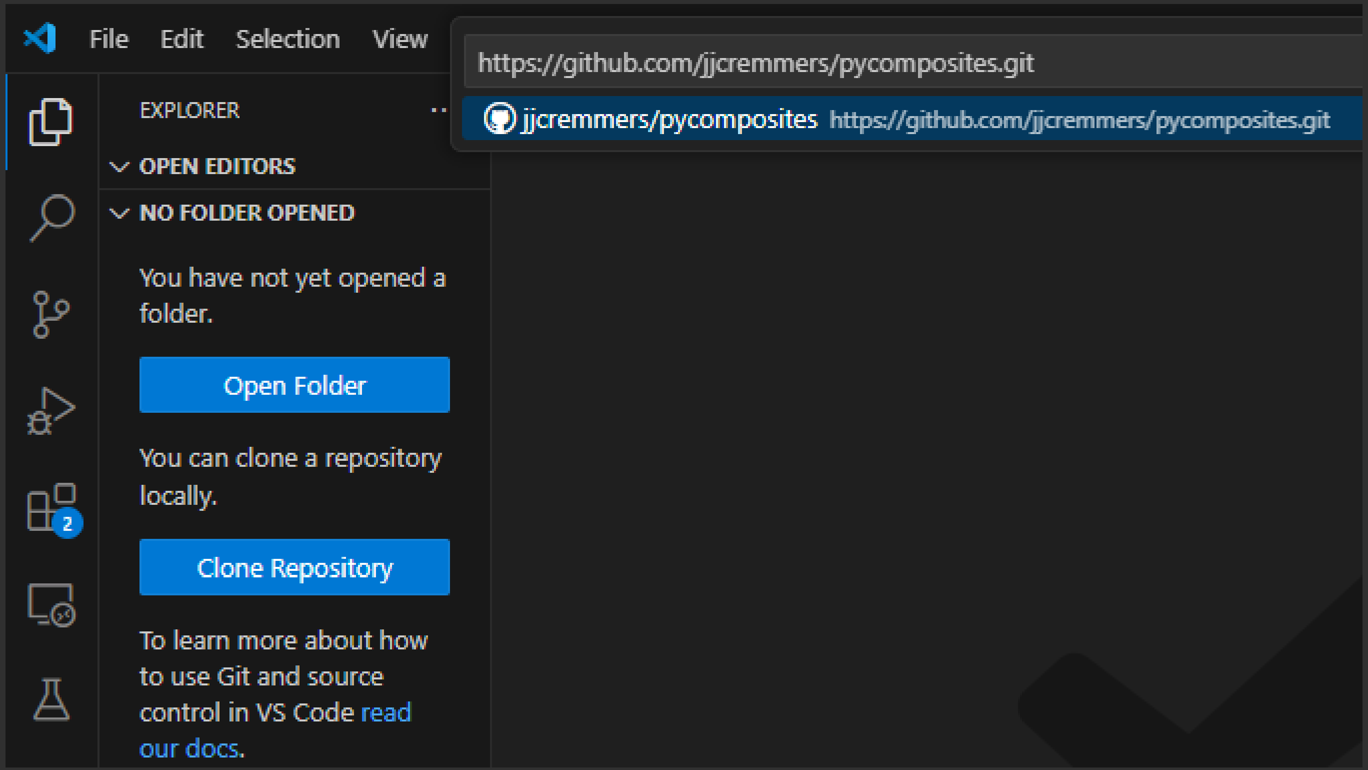
$$\boldsymbol{\mathcal{N}} = \begin{bmatrix} \mathcal{N}_{xx} \\ \mathcal{N}_{yy} \\ \mathcal{N}_{xy} \end{bmatrix} = \begin{bmatrix} PR/2 \\ PR \\ 0 \end{bmatrix}$$

$$\boldsymbol{\mathcal{M}} = \begin{bmatrix} \mathcal{M}_{xx} \\ \mathcal{M}_{yy} \\ \mathcal{M}_{xy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Unit: $\mathcal{N} = [N/m]$, $\mathcal{M} = [Nm/m]$



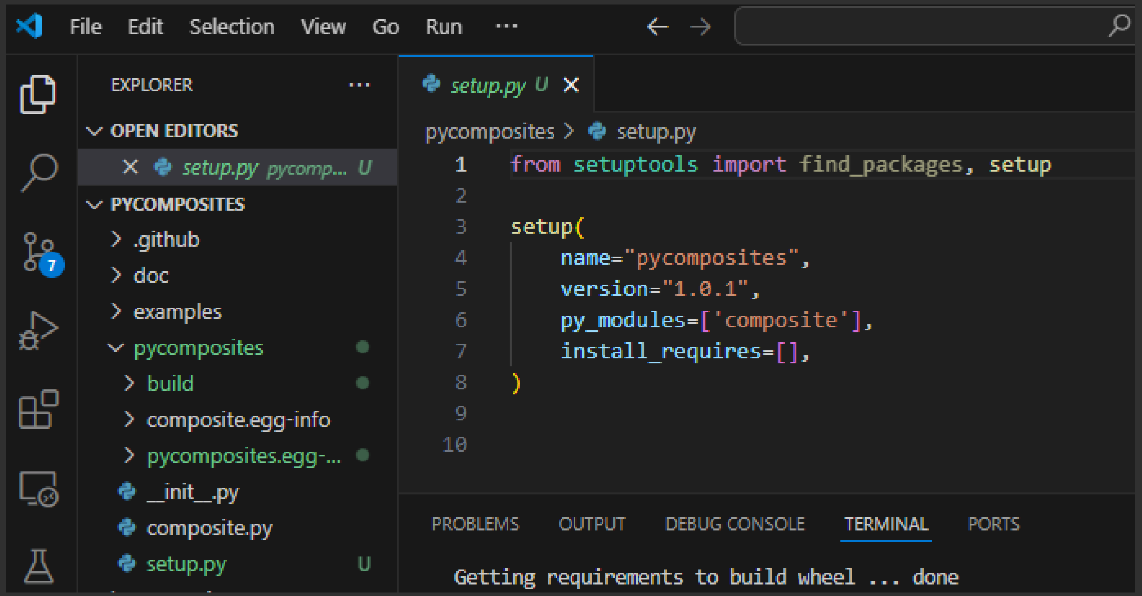
Clone the pycomposites

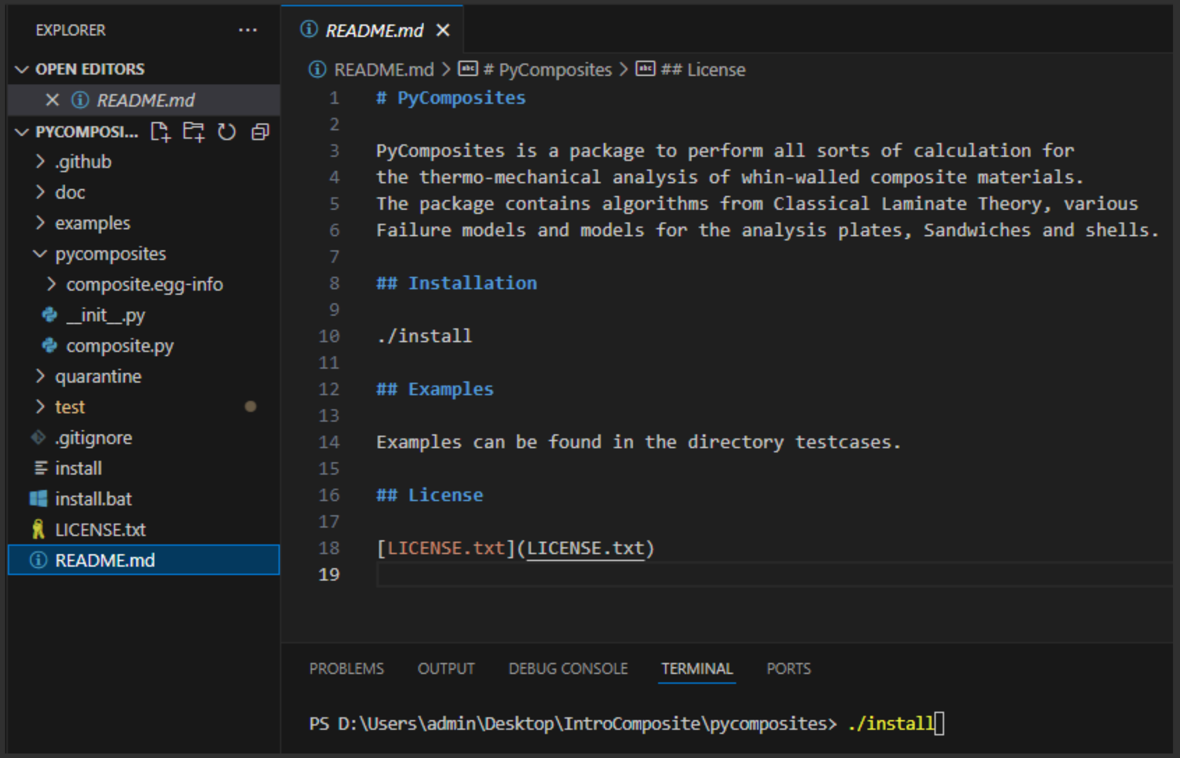


Clone the pycomposites from: https://github.com/jjcremmers/pycomposites.git
Clone the test file from: https://github.com/Raistlinwolf/IntroComposite.git

Install the pycomposites

- Add a ‘setup.py’ under the folder “pycomposites”, and then install from terminal





Example

We first define the Uni-Directional CFRP material properties

```python
from composite import TransverseIsotropic
XT = 1433.6e6
XC = 1003.3e6
YT = 32.5e6
YC = 108.3e6
S = 76.1e6

E1 = 130.33e9
E2 = 7.220e09
nu12 = 0.337
G12 = 4.230e09

UD = TransverseIsotropic([E1,E2],nu12,G12)

print(UD)
```

[26] ✓ 0.0s

- The response:

```
Elastic Properties:
------------------------------------------------------------
E1      :     1.303e+11 , E2      :     7.220e+09
nu12    :          0.34 , G12     :     4.230e+09

Thermal expansion coefficients:
------------------------------------------------------------
alpha1 :     0.000e+00 , alpha2 :     0.000e+00
```

Example

From the assigned fiber orientations, we define the stackup

```
from composite import Laminate

lam = Laminate()

lam.addMaterial( 'UD' , UD )

orientations = [ 35 , -35 , 35 , -35 , 35 , -35 , 35 , -35 , 35 , -35 , 35 , -35 , 35 , -35 , 35 , -35 , 35 , -35 , 35 , -35 , 35 , -35 , 35 , -35 , -35
```

[28] ✓ 0.0s

Example

Then we calculate the forces (N) and moments (M) from the internal pressure

```
P = 50e5
R = 0.28/2

Nxx = P * R/2
Nyy = P * R
Nxy = 0
Mxx = 0
Myy = 0
Mxy = 0
```

[27] ✓ 0.0s

```
t = 0.0003 #layer thickness

for angle in orientations:
    lam.addLayer( 'UD' , angle , t )

print ("\nA matrix:\n",lam.getA())
print ("\nB matrix:\n",lam.getB())
print ("\nD matrix:\n",lam.getD())
```

[29] ✓ 0.0s

- The response:

```
A matrix:
 [[ 9.e+08  4.e+08 -3.e-08]
 [ 4.e+08  3.e+08 -2.e-08]
 [-3.e-08 -2.e-08  4.e+08]]

B matrix:
 [[-1.e-10 -7.e-11  2.e-10]
 [-7.e-11 -7.e-11  9.e-11]
 [ 2.e-10  9.e-11 -2.e-10]]

D matrix:
 [[16088.  7014.  598.]
 [ 7014.  5545.  307.]
 [  598.   307.  7458.]]
```

$= 0$ *Balance*

$= 0$

Symmetric

$\neq 0$

Example

```python
# Calculate the stresses in each layer (expressed in the material coordinate system)

# First, calculate the inverse matrices
A1,B1,C1,D1 = lam.getInverseMatrices()

import numpy as np

N = np.array([Nxx, Nyy, Nxy])
M = np.array([Mxx, Myy, Mxy])

eps0  = np.dot( A1 , N ) + np.dot( B1 , M )
kappa = np.dot( C1 , N ) + np.dot( D1 , M )

print("The midplane strains are : ",eps0)
print("The curvatures are       : ",kappa)
```

✓ 0.0s

- The response:

```
The midplane strains are :  [-1.e-03  4.e-03  5.e-20]
The curvatures are       :  [-1.e-17  5.e-17 -2.e-17]
```

Example

```python
from composite import stressTransformation

sigmaplt = []

for iLay,angle in enumerate(orientations):
    epsilon = eps0 + lam.getZcoord( iLay ) * kappa
    sigma   = np.dot( lam.getQbar(iLay) , epsilon )
    sigmaplt.append(stressTransformation( sigma , angle ))

print("The stresses : ",sigmaplt)
```

✓ 0.0s

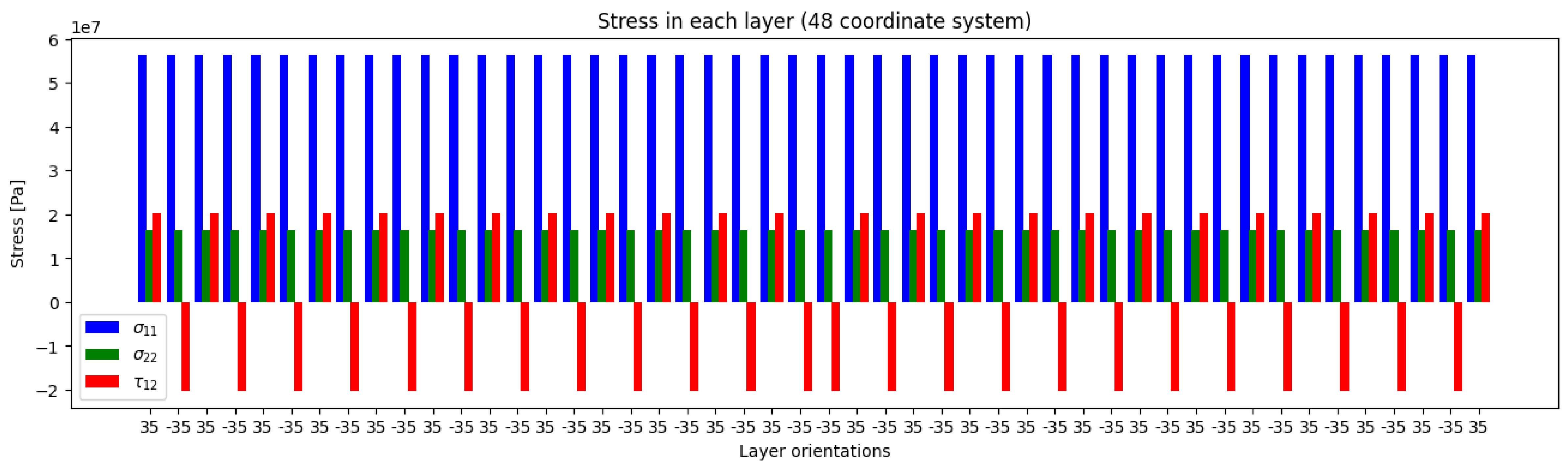
Example

```python
import matplotlib.pyplot as plt

X = np.arange(48)
# Create a figure with a custom size
fig = plt.figure(figsize=(16, 4))

plt.bar(X - 0.26, [s[0] for s in sigmaplt] , color = 'b', width = 0.3, label = "$\\sigma_{11}$")
plt.bar(X       , [s[1] for s in sigmaplt] , color = 'g', width = 0.3, label = "$\\sigma_{22}$")
plt.bar(X + 0.26, [s[2] for s in sigmaplt] , color = 'r', width = 0.3, label = "$\\tau_{12}$")

plt.xticks(X, orientations)

plt.xlabel("Layer orientations")
plt.ylabel("Stress [Pa]")
plt.title("Stress in each layer (48 coordinate system)")

plt.legend()
plt.show()
```

✓ 0.7s

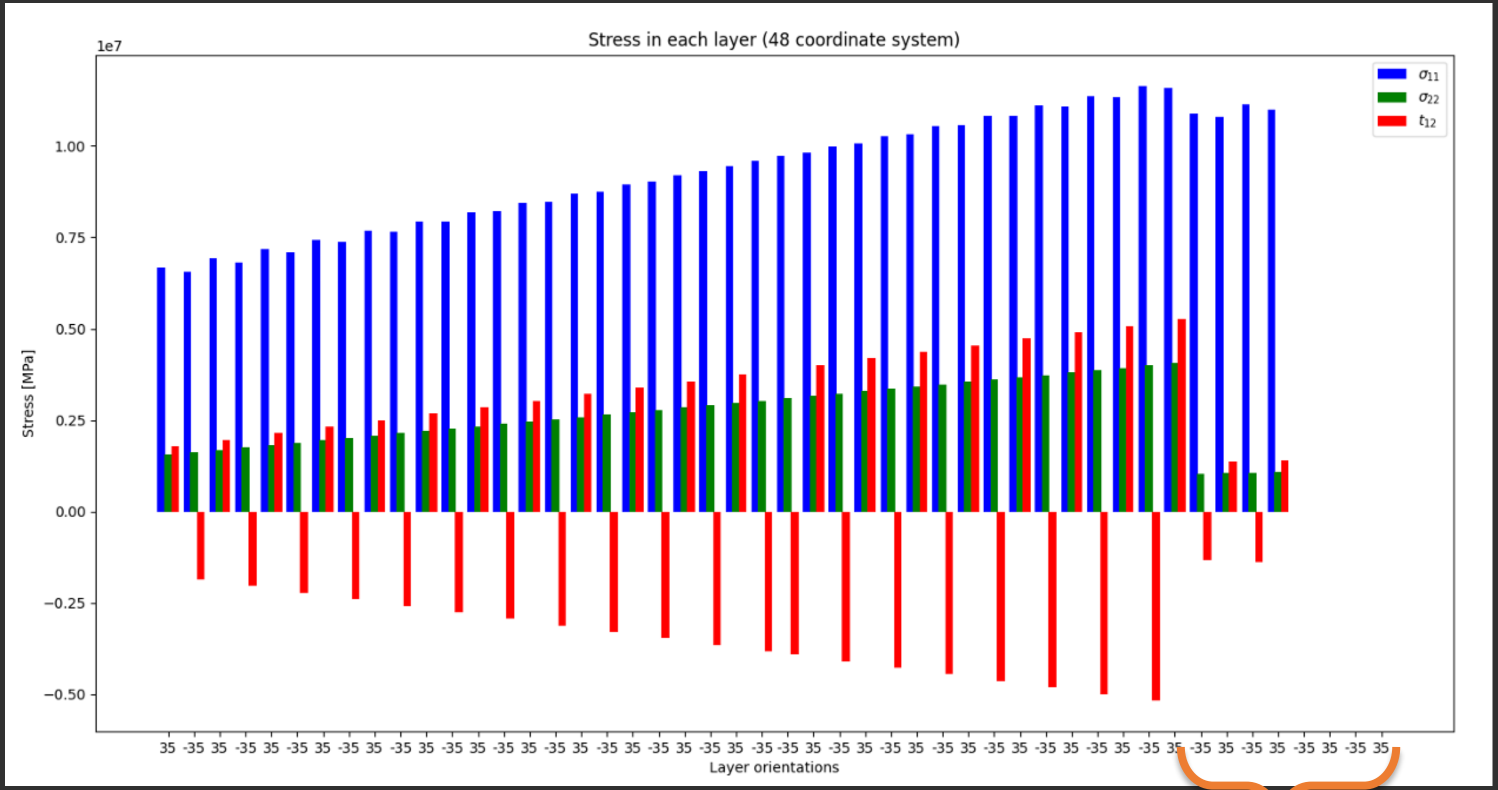
Stresses from internal pressure

- Determine the stresses in each layer of the vessel in the cylindrical part.



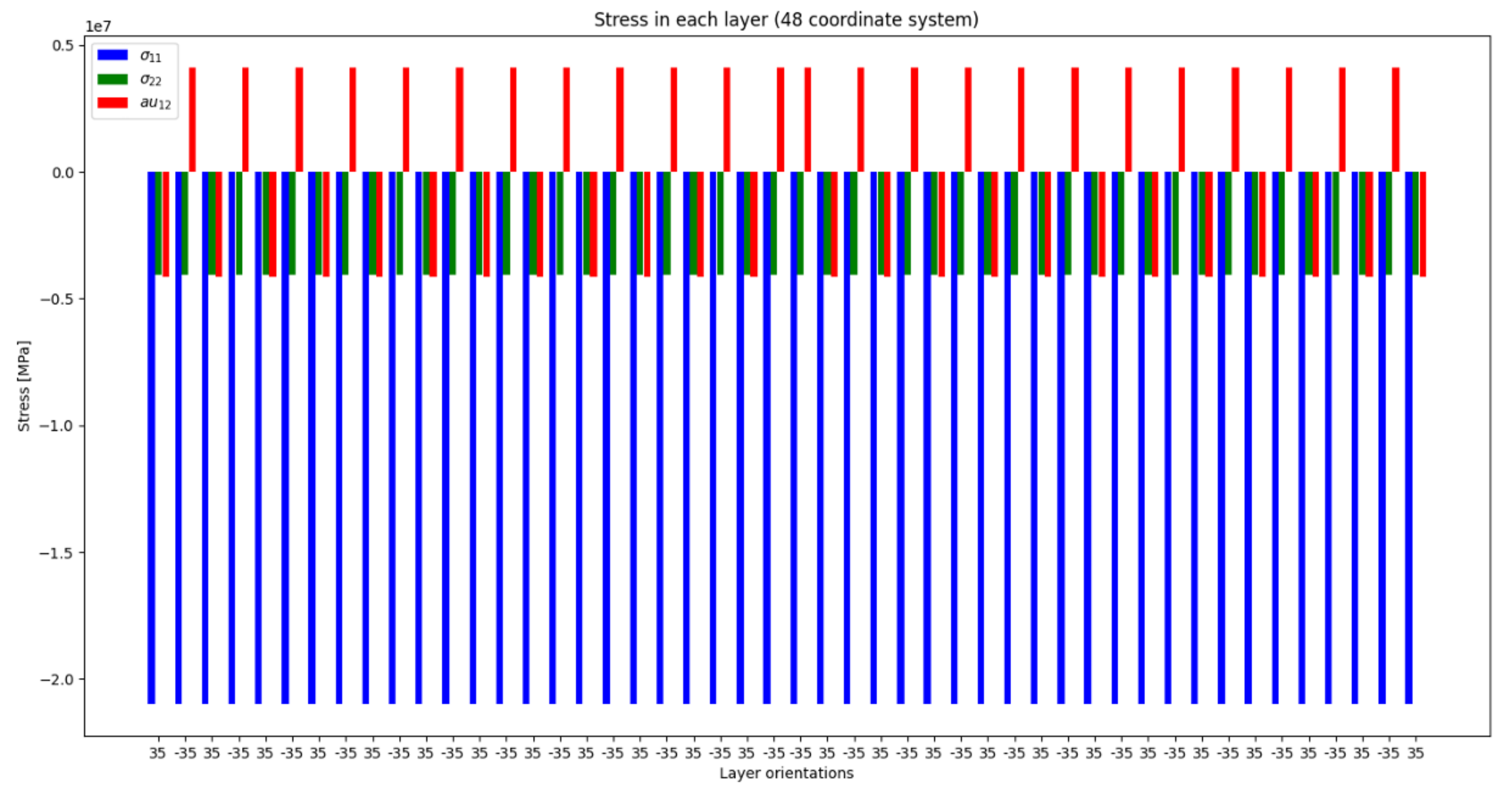
Stresses change due to damage

- 8 outer layers are damaged in the same position on the vessel. (p=5 bar)
  - In the first 4 layers, both the fibres and the matrix are damaged (fibre breakage, intralaminar cracks and debonding)
  - In the following 4 layers, only the matrix material is damaged (intralaminar cracks and debonding)



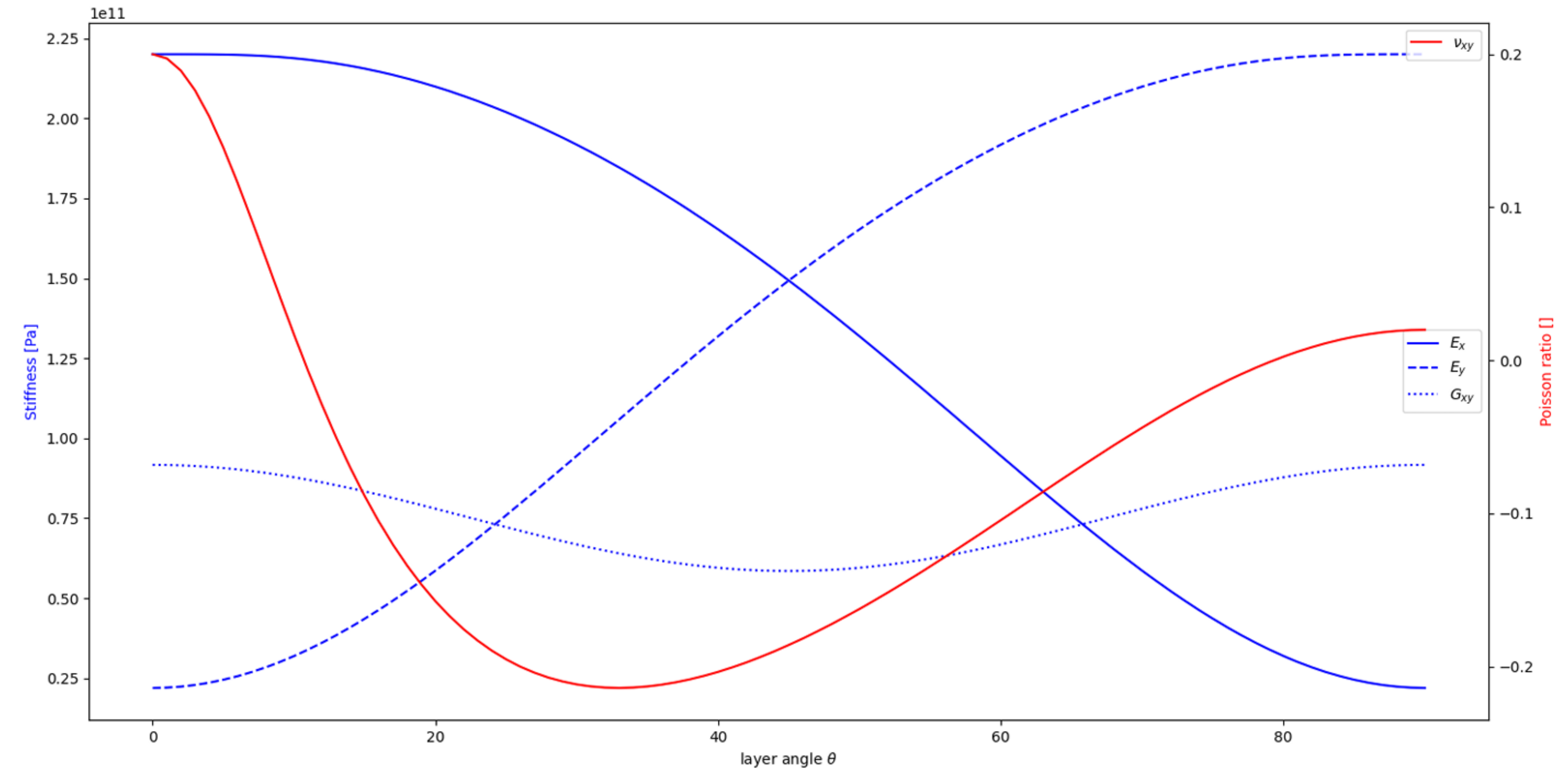
Stresses from temperature difference

- Determine the stresses in each layer of the material when the vessel is cooled down by $\Delta T = -60°$ and the vessel is not pressurised.



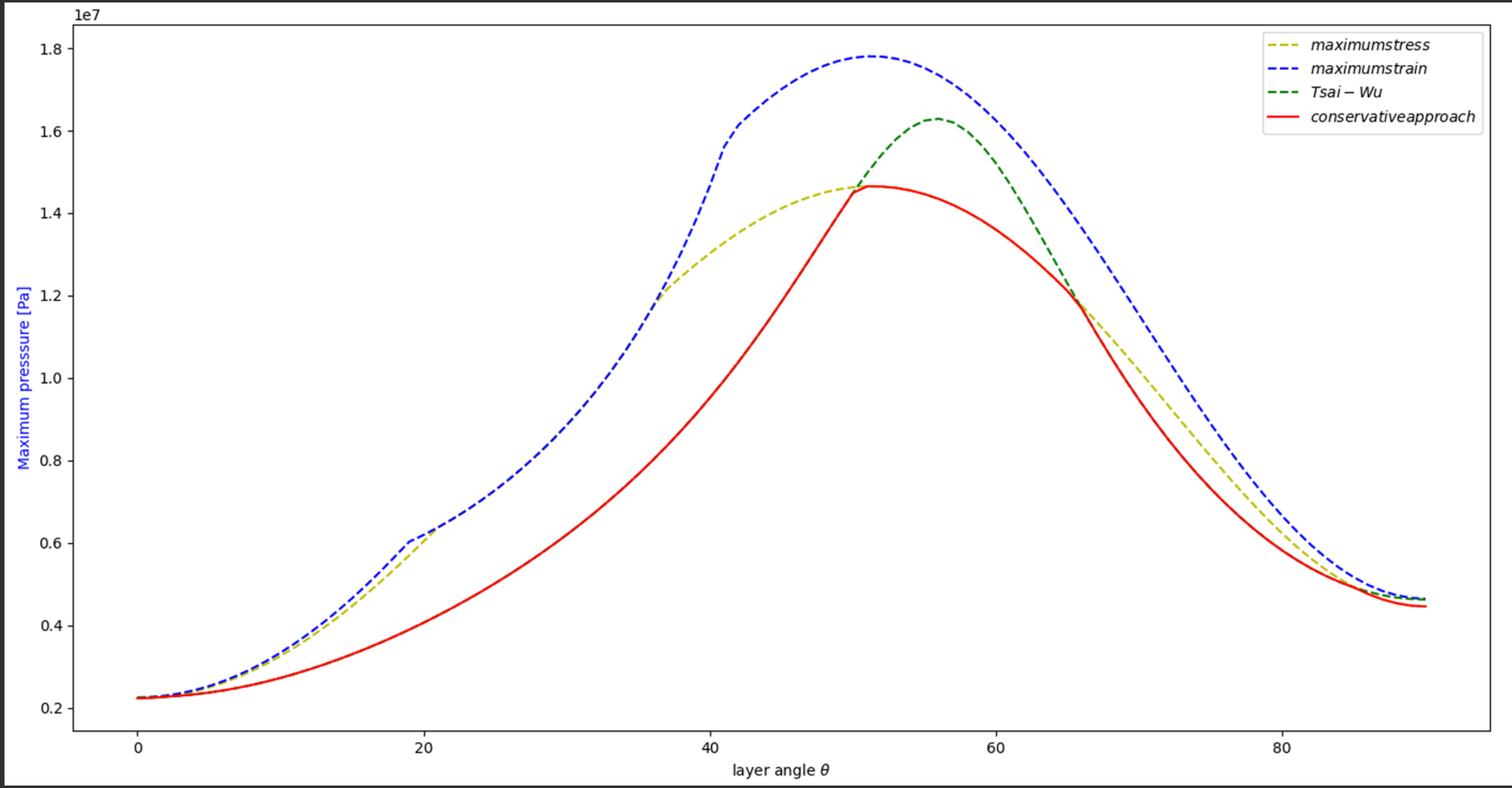
Properties according to angle

- For the same $[(\theta/-\theta)]_{12S}$ stacking sequence, show the mechanical properties with different θ



Maximum allowable pressure

- Determine the angle θ that has the highest maximum allowable pressure from failure criterias



$P_{max} = 146.5$ bar,
at angle $\theta = 51°$

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Composite Overwrapped Pressure Vessels (COPVs)

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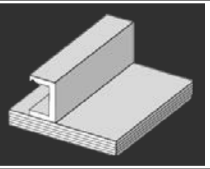
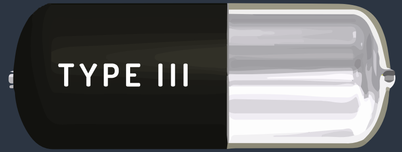
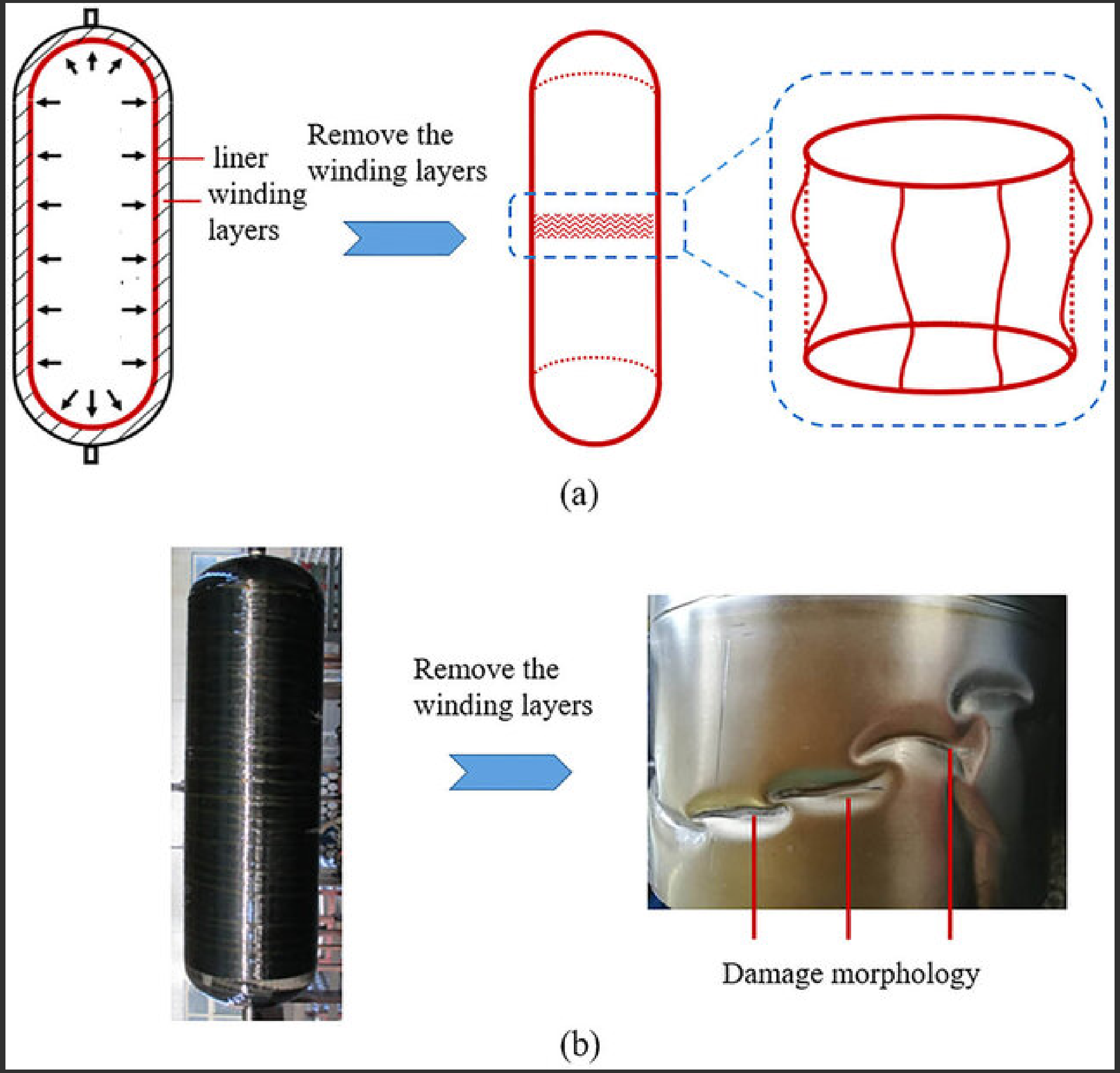
Types of COPV



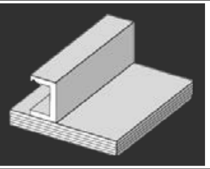
CLASS	COMPOSITION	DRAWBACKS
 TYPE V	Fiber Reinforced Shell, with Metallic or Composite End fittings	**Permeation** (similar to Type IV) **Low Heat Transfer**
 TYPE IV	Plastic Liner with Fiber Reinforced Shell	**Liner Permeation** **Liner Collapse** **Liner Embrittlement**
 TYPE III	4130 Steel or 6061 Aluminum Full Fiber Reinforced Shell	**Liner Fatigue** **Liner Corrosion** **Liner Weight**
 TYPE II	4130 Steel or 6061 Aluminum Fiber Reinforced Center	**Corrosion** **Weight** **Pressure Limitations**
 TYPE I	4130 Steel or 6061 Aluminum	**Corrosion** **Weight** **Pressure Limitations**

Liner buckling in type III COPV



(G. Zhang, 2021)

Rules of thumb for tube/vessel design



- $0°$ (axial)
  - Resists to longitudinal bending and axial tension/compression
- $90°$ (hoop)
  - Resists to internal and external pressure
  - Helps a tube to stay round and provides consolidation in conventional filament winding
- $\pm 45°$
  - Resists to pure torsion
- $\pm 55°$
  - For internal pressure, the hoop stress is twice the longitudinal stress
- $\pm 65°$
  - For External pressure as above but to resist buckling
- $\pm 22.5°$ & $\pm 67.5°$ alternate
  - For Quasi-Isotropic laminate
- $\pm 5°$ to $\pm 25°$
  - For bending with torsion angles

General design rules

1. The lay-up should be balanced ($\mathcal{A}_{16} = 0$ and $\mathcal{A}_{26} = 0$) to avoid normal/shear coupling
2. The lay-up should be symmetric ($\boldsymbol{\mathcal{B}} = \mathcal{O}$) to avoid warping after curing.
3. The lay-up shouldn't be anti-symmetric ($\mathcal{D}_{16} = 0$ and $\mathcal{D}_{26} = 0$) to avoid bending-twisting coupling
4. At least 10% of the fibers should be lined up with each of the 4 principal directions ($0, -45, +45, 90$). This is the so-called 10% rule to account for unexpected loading states.
5. Many consecutive layers with the same orientation of uni-directional plies should be avoided (to reduce the risk of delamination)
6. To improve bending stiffness, place $0°$ layers as far away from the neutral axis as possible
7. Put woven plies on the outside to prevent splitting and damage from impact

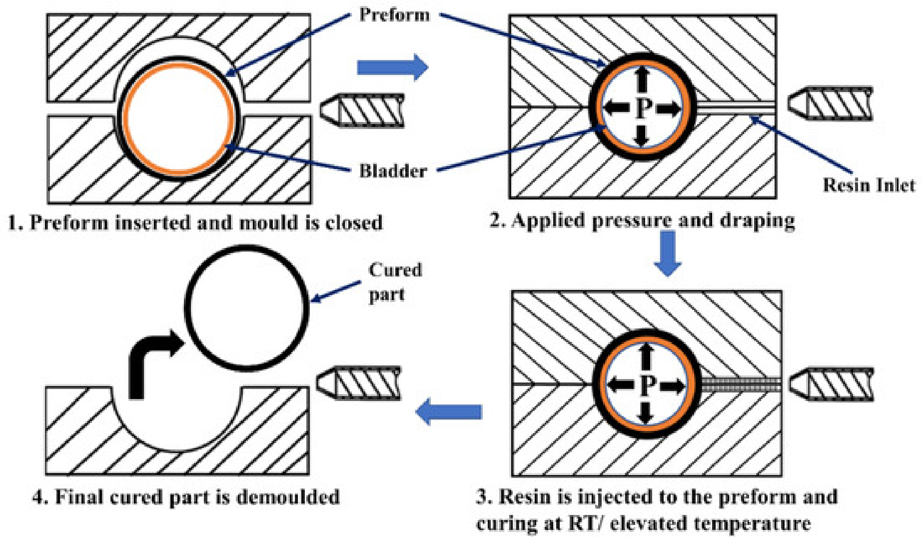
Production

- Bladder Assisted Composite Manufacturing (BACM)
- Vacuum-Assisted Resin Transfer Molding (VARTM)
- Resin Transfer Molding (RTM)
- Filament Winding (FW)
- Automated Fiber Placement (AFP)

課程回饋表單

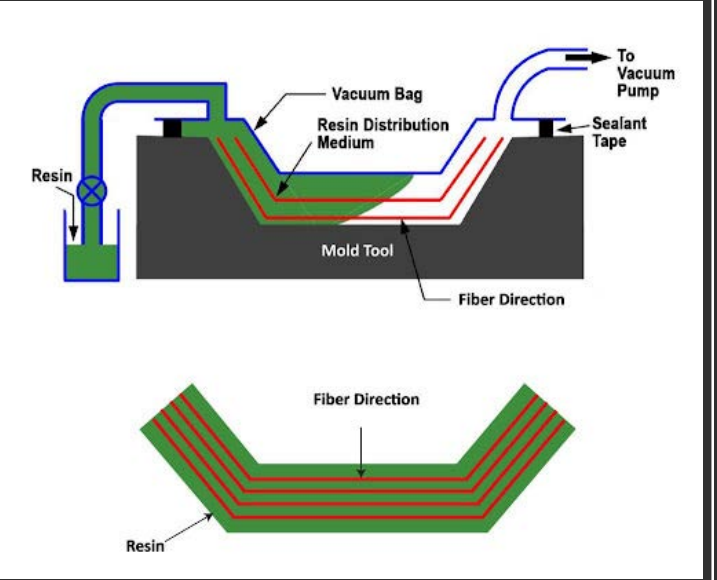
Bladder Assisted Composite Manufacturing

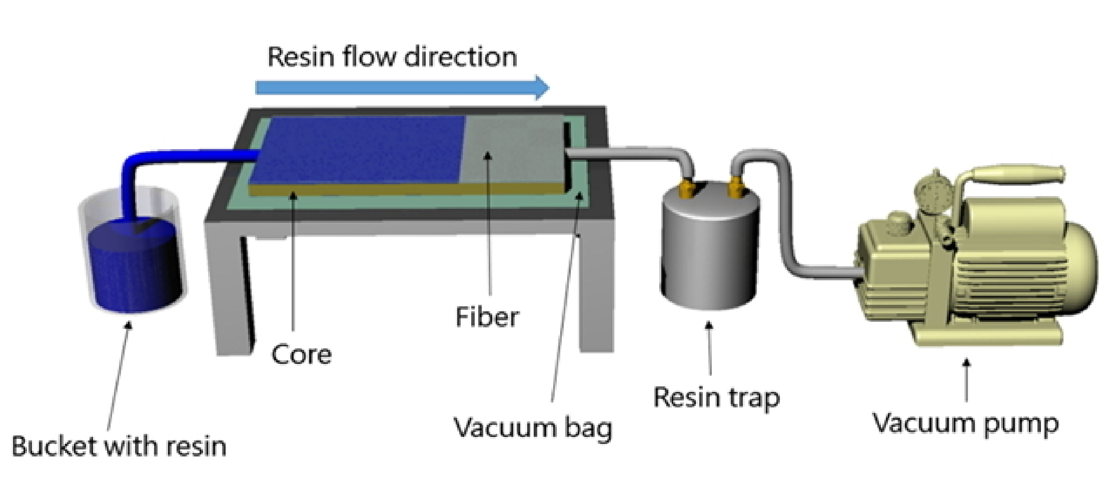
- **Forming Mechanism:**
  - Uses an **inflatable bladder** inside a **rigid mold**.
  - The bladder expands, **pressing fibers against the mold** for consolidation.
  - Resin is injected and cured under **internal pressure**.
- **Fiber Placement:**
  - Dry fibers or prepregs are **laid into a single-sided rigid mold**.
  - The **bladder is inserted** before sealing the mold.
- **Key Steps:**
  1. Lay dry fibers or prepregs inside the **rigid mold**.
  2. Place an **inflatable bladder** inside the mold.
  3. Seal the mold and **inflate the bladder**, compacting the fibers.
  4. Inject **resin** (if using dry fibers).
  5. **Cure and demold** the final part.



Vacuum Assisted Resin Transfer Molding

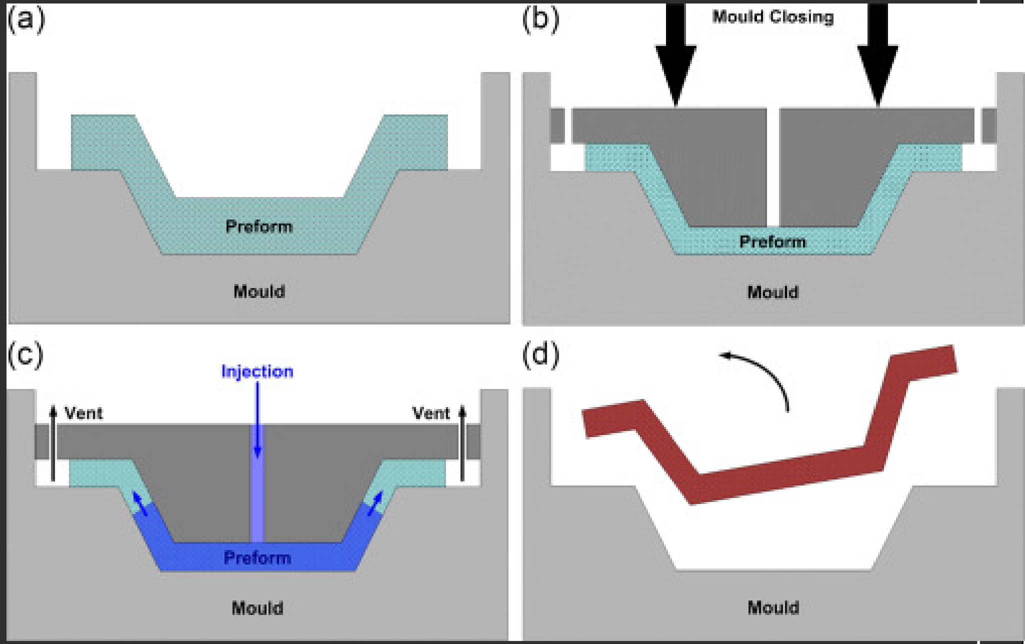
- **Forming Mechanism:**
  - Uses vacuum pressure (~1 bar) to pull resin into dry fiber reinforcements.
  - The vacuum bag encloses the fibers and mold to control resin flow.
  - Resin infusion occurs at atmospheric pressure, and curing follows.
- **Fiber Placement:**
  - Dry fibers are laid inside a rigid single-sided mold.
  - Vacuum bagging film is sealed over the mold.
- **Key Steps:**
  1. Place dry fibers into the rigid mold.
  2. Cover with a vacuum bag and seal the system.
  3. Apply vacuum pressure, removing air.
  4. Infuse resin, which spreads through capillary action.
  5. Cure under vacuum before demolding.



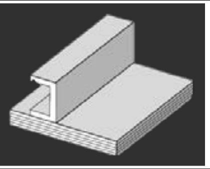


Resin Transfer Molding

- **Forming Mechanism:**
  - Uses a closed two-sided mold with external pressure (~2-7 bar).
  - Resin is injected into the mold and fully saturates the dry fibers.
  - High-pressure injection results in low void content and high-quality parts.
- **Fiber Placement:**
  - Dry fibers are **pre-placed inside the closed mold cavity**.
- **Key Steps:**
  1. Pre-place dry fiber layers into a two-sided rigid mold.
  2. Close the mold and seal it tightly.
  3. Inject resin under pressure, ensuring full saturation.
  4. Cure the part under controlled conditions.
  5. Open the mold and remove the final composite.



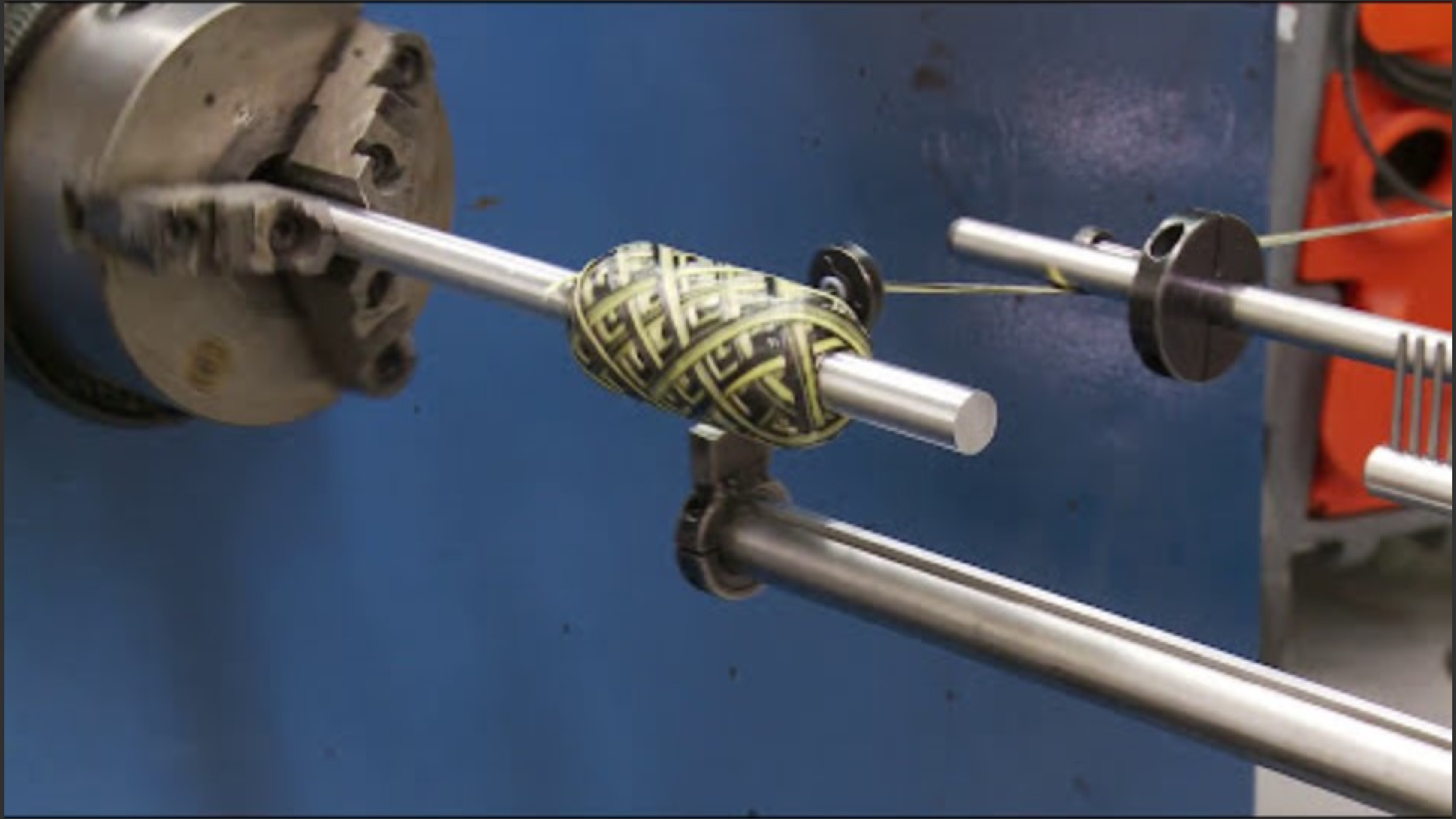
Filament Winding



- **Forming Mechanism:**
  - Uses continuous fiber strands wound around a rotating mandrel.
  - Fibers are impregnated with resin before winding.
  - The composite is cured, and the mandrel is removed (or remains as part of the structure).
- **Fiber Placement:**
  - Continuous fiber tows are wound in precise helical or hoop patterns.
  - Placement angles control mechanical properties (axial vs. hoop strength).
- **Key Steps:**
  1. Set up a rotating mandrel.
  2. Guide fiber tows through a resin bath (wet winding) or use prepreg tows.
  3. Wind fibers onto the mandrel with controlled tension.
  4. Cure in an oven or autoclave.
  5. Remove the mandrel (if necessary).

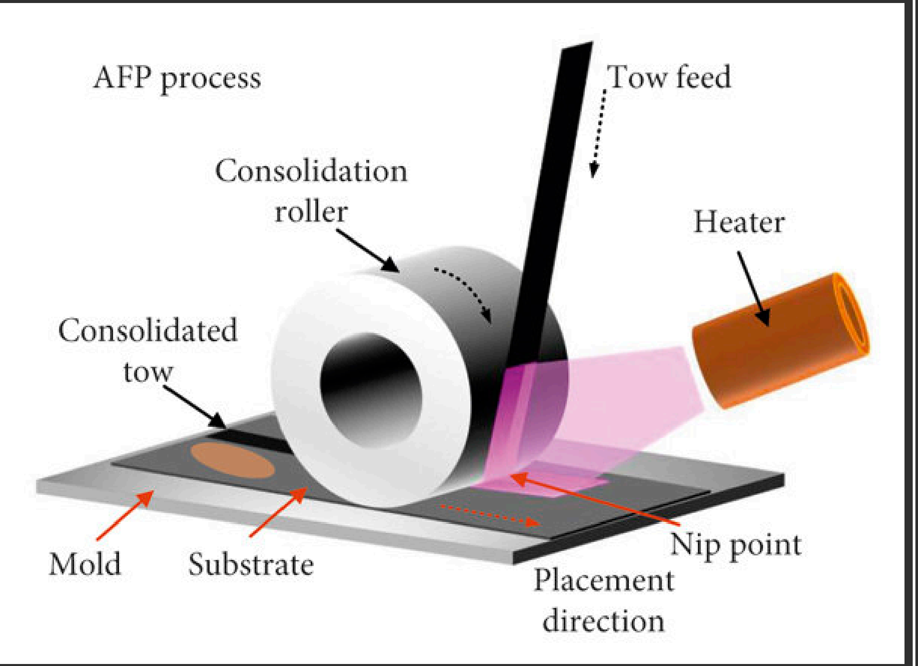
Filament Winding (FW)

Filament Winding (FW)

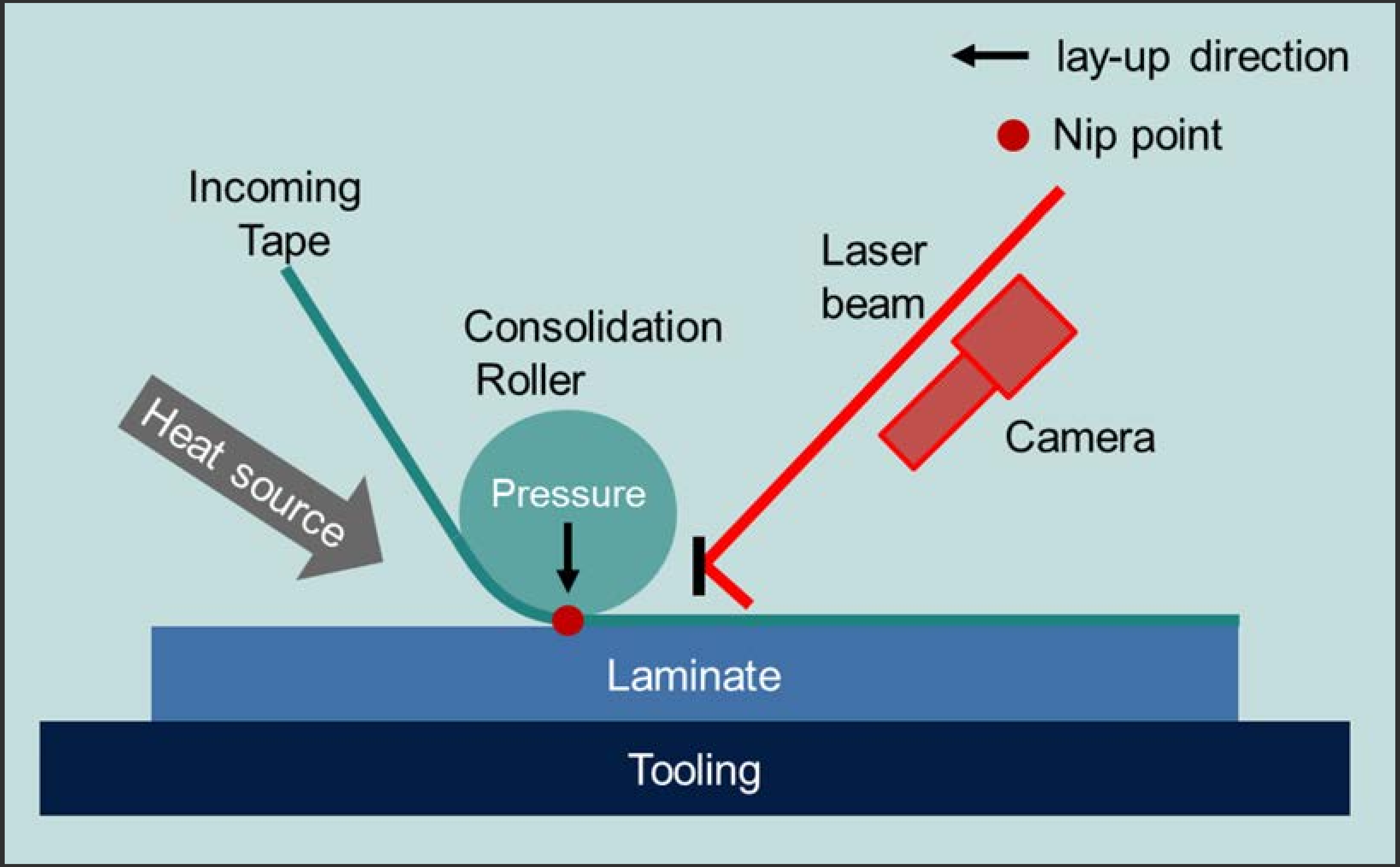


Automated Fiber Placement

- **Forming Mechanism:**
  - Uses robotic automation to precisely place fiber tows onto a mold.
  - Fibers are heated, compacted, and cured layer by layer.
  - Enables complex geometries and variable stiffness designs.
- **Fiber Placement:**
  - Prepreg fiber tows are individually placed by a robotic head.
  - Fibers are pressed down using rollers for consolidation.
- **Key Steps:**
  1. Load prepreg tows onto the AFP robotic system.
  2. Move the robotic head to precisely place fibers layer by layer.
  3. Use rollers and heat to consolidate each layer.
  4. Cure the part (in an autoclave or using in-situ curing).
  5. Inspect and trim the final composite structure.



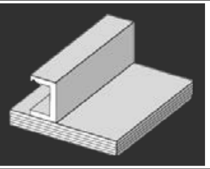
Thermoplastic AFP (T-AFP)



(A. Schuster, 2000)

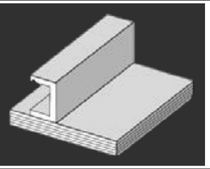
Thermoplastic AFP (T-AFP)

Comparison



	BACM	VARTM	RTM	FW	AFP
Equipment	1 rigid mold + 1 bladder	1 rigid mold + 1 vacuum bag	2 rigid molds	1 filament feeder + 1 rotating mandrel	1 ridid mold + 1 robotic arm
Compaction & Void Content	**Internal bladder pressure (2–5 bar)** enables high fiber volume fraction (FVF) and low voids.	**Vacuum pressure (~1 bar)** limits compaction, resulting in higher void content.	**External pressure (~2–7 bar)** ensures low void content.	**Tensioned fibers** provide good compaction, reducing voids.	**Mechanical rollers and heat-assisted compaction** ensure high consolidation quality.
Speed	Medium	Fast	Medium	High	Slow
Cost	Medium-High	Low	High	Medium	Very High
Main applications	**Hollow or enclosed shapes** (e.g., pressure vessels, bicycle frames).	**Large open structures** (e.g., wind turbine blades, marine hulls, automotive panels).	**Precision-molded parts** (e.g., aerospace and automotive components).	**Cylindrical and pressure-resistant parts** (e.g., COPVs, gas tanks).	**Complex geometries** (e.g., aircraft fuselage, wings, spars).

Surface treatment



- Transparent:
  - Matte
  - Sanding & Polishing
  - Clear Coating / paint
  - Plasma
  - Surface veil / barrier layer
  - Chemical Etching & Primers
- Opaque:
  - Gel coat
  - Paint

Or simply say you want the surface to be 1) weaving pattern or specific color, 2) polished or matte, 3) resist to certain environment

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Thank you for listening!
Question(s)?

The presentation is based on the course materials of *4MM00 Composite and Llightweight Materials Design and Analysis*, taught by Prof. Dr. Ir. Varvara Kouznetsova and Prof. Dr. Ir. Joris Remmers, from the Mechanics of Materials group in the Mechanical Engineering department at Eindhoven University of Technology.