Marwadi
University

IMAGE PROCESSING
01CE0507

Unit - 4
Spatial Filters

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Outline

- Image Enhancement
  - Spatial Domain
  - Frequency Domain

Frequency Domain

- In Frequency Domain, we are processing images or Signals
- In frequency domain we don’t analyze signal with respect to time, but with respect of frequency.

Frequency Domain VS Spatial Domain

- In spatial domain, we deal with images as it is. The value of the pixels of the image change with respect to scene.
- Whereas in frequency domain, we deal with the rate at which the pixel values are changing in spatial domain.

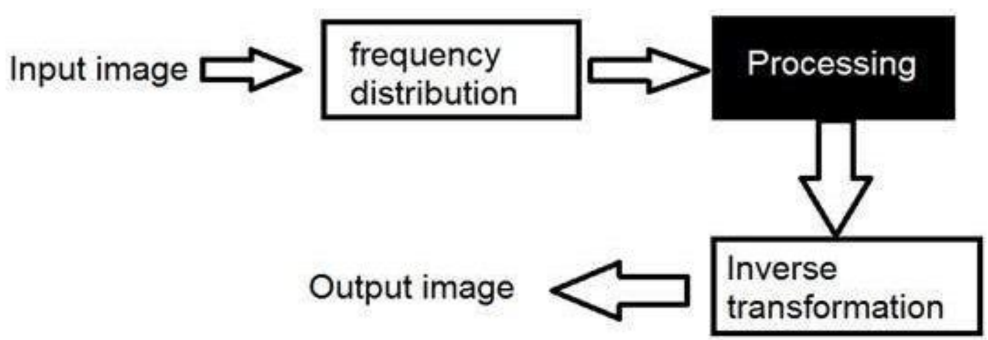
Frequency Domain VS Spatial Domain (Cont.)

Standards	Spatial Domain	Frequency domain
Computation Cost	Low	High
Robustness	Fragile	More Robust
Perceptual quality	High control	Low control
Computational complexity	Low	High
Computational Time	Less	More
Amplitude	High	Low
Example of Application	Mainly Authentication	Copy rights

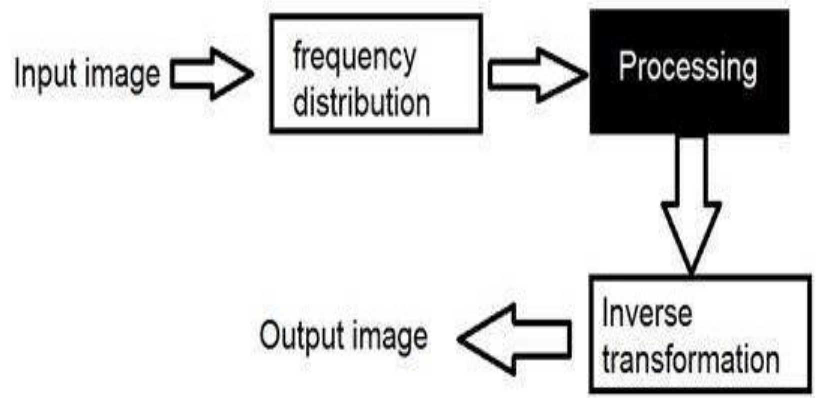
Frequency Domain VS Spatial Domain (Cont.)

- In simple spatial domain, we directly deal with the image matrix.

- Frequency domain analysis is used to indicate how signal energy can be distributed in a range of frequency.

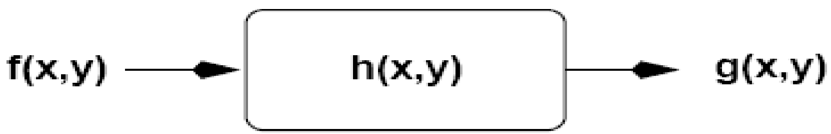


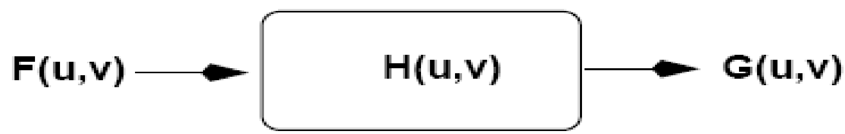
Frequency Domain VS Spatial Domain (Cont.)



Spatial Domain

Frequency Domain





$g(x,y) = f(x,y) * h(x,y)$

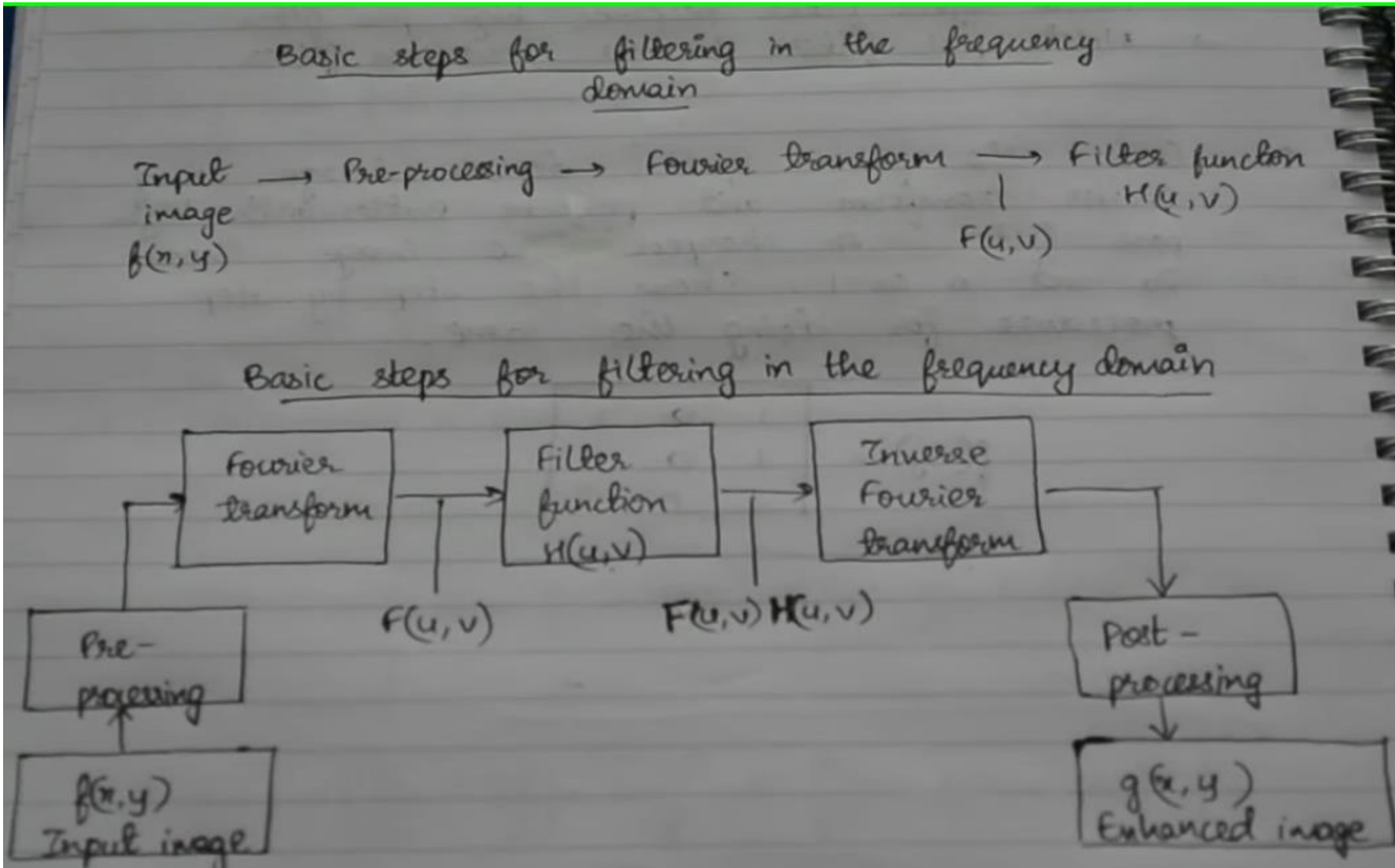
$h(x,y)$ = impulse response

$G(u,v) = F(u,v)\, H(u,v)$

$(g(x,y) = F^{-1} (F(u,v)\, H(u,v)))$

$H(x,y)$ = Transfer Function

Frequency Domain



Frequency Domain (Cont.)

- We first transform the image to its frequency distribution.
- Then our black box system perform what ever processing it has to performed, and the output of the black box in this case is not an image, but a transformation.
- After performing inverse transformation, it is converted into an image which is then viewed in spatial domain.

Transformation

- A signal can be converted from time domain into frequency domain using mathematical operators called transforms.
- There are many kind of transformation that does this.
- Some of them are given below.
  - Fourier Series
  - Fourier transformation
  - Laplace transform
  - Z transform

Fourier Series and Transform

- Fourier series and Fourier transform are used to convert a signal to frequency domain.

Fourier

- Fourier was a mathematician in 1822.
- He give Fourier series and Fourier transform to convert a signal into frequency domain.

Fourier Series VS Fourier Transform

- Difference between Although both Fourier series and Fourier transform are given by Fourier , but the difference between them is **Fourier series is applied on periodic signals and Fourier transform is applied for non periodic signals**

Fourier Series VS Fourier Transform

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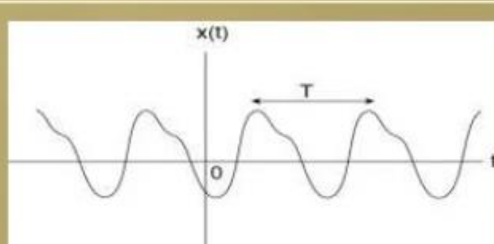
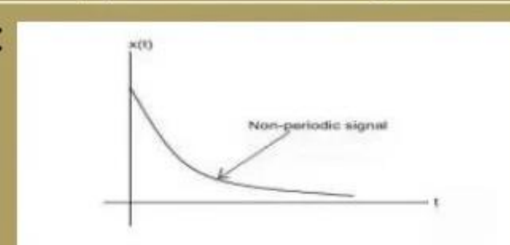
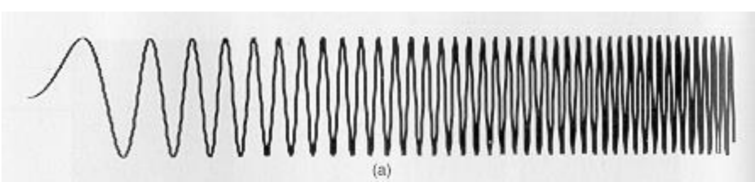
Periodic Signal	Aperiodic Signal
❑ A signal which repeats itself after a specific interval of time is called periodic signal.	❑ A signal which does not repeat itself after a specific interval of time is called aperiodic signal.
❑ A signal that repeats its pattern over a period is called periodic signal	❑ A signal that does not repeats its pattern over a period is called aperiodic signal or non periodic.
❑ They can be represented by a mathematical equation	❑ They cannot be represented by any mathematical equation
❑ Their value can be determined at any point of time	❑ Their value cannot be determined with certainty at any given point of time
❑ They are deterministic signals	❑ They are random signals
❑ Example: sine cosine square sawtooth etc	❑ Example: sound signals from radio , all types of noise signals
❑ Figure: 	❑ Figure: 

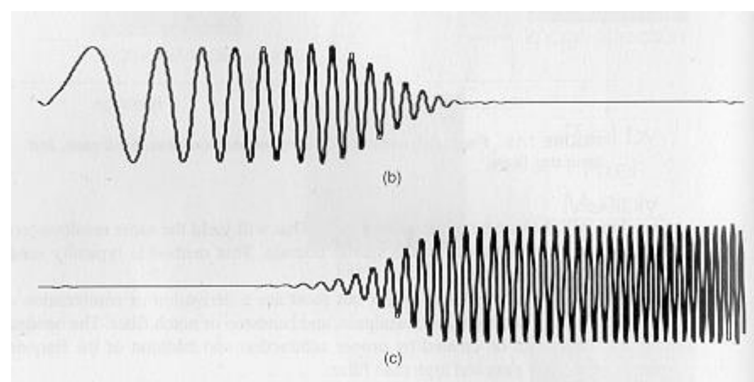
Which one is applied on images

- Which one is applied on images
  - Images are non – periodic.
  - And since the images are non periodic, so **Fourier transform** is used to convert them into frequency domain.

Frequency Components

- We will divide frequency components into two major components.
  - High frequency components
    - High frequency components correspond to edges in an image.
  - Low frequency components
    - Low frequency components in an image correspond to smooth regions.





Frequency Domain Filters

- Low Pass Filters / Smoothing Filters / Blurring Masks
  - Ideal Lowpass Filters (ILPF)
  - Butterworth Lowpass Filters (BLPF)
  - Gaussian Lowpass Filters (GLPF)
- High Pass Filters / Sharpening Filters / Derivative Masks
  - Ideal Highpass Filters (IHPF)
  - Butterworth Highpass Filters (BHPF)
  - Gaussian Highpass Filters (GHPF)

Blurring Masks VS Derivative Masks

Blurring Masks	Derivative Masks
All the values in blurring masks are positive	A derivative mask have positive and as well as negative values
The sum of all the values is equal to 1	The sum of all the values in a derivative mask is equal to zero
The edge content is reduced by using a blurring mask	The edge content is increased by a derivative mask
As the size of the mask grow, more smoothing effect will take place	As the size of the mask grows , more edge content is increased

Low Pass Filters / Smoothing Filters / Blurring Masks

- A low pass filter is used to pass low-frequency signals.
- The strength of the signal is reduced and frequencies which are passed is higher than the cut-off frequency.
- The amount of strength reduced for each frequency depends on the design of the filter.
- Smoothing is low pass operation in the frequency domain.

High Pass Filters / Sharpening Filters / Derivative Masks

- A highpass filter is used for passing high frequencies but the strength of the frequency is lower as compared to cut off frequency.
- Sharpening is a highpass operation in the frequency domain.

High Pass Filters / Sharpening Filters / Derivative Masks (Cont.)

- Intended goal is to do the reverse operation of low-pass filters
  - When low-pass filer attenuates frequencies, high-pass filter passes them

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

  - When high-pass filter attenuates frequencies, low-pass filter passes them

Ideal Lowpass Filters (ILPF)

- The ideal lowpass filter is used to cut off all the high-frequency components of Fourier transformation.
- Below is the transfer function of an ideal lowpass filter.

$$H(u,v) = \begin{cases} 1 & if\ D(u,v) \le D0 \\ 0 & if\ D(u,v) > D0 \end{cases}$$

Where, D0 is Cutoff frequency
D(u,v) is the distance to the center freq.

$$D(u,v) = \left[\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2\right]^{\frac{1}{2}}$$

Ideal Highpass Filters (IHPF)

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Butterworth Lowpass Filter (BLPF)

- Butterworth Lowpass Filter is used to remove high-frequency noise with very minimal loss of signal components.

$$H(u,v) = \frac{1}{1+\left[\frac{D(u,v)}{D0}\right]^{2n}}$$

Where, D0 is Cutoff frequency
n is order of the Butterworth filter
D(u,v) is the Euclidean Distance from any point (u,v) to the frequency plain

$$D(u,v) = \sqrt{(u^2 + v^2)}$$

Butterworth Highpass Filter (BHPF)

- Butterworth Highpass Filter is used to remove low-frequency noise with very minimal loss of signal components.

$$\boxed{H(u,v) = \frac{1}{1+[D_0 / D(u,v)]^{2n}}}$$

Where, D0 is Cutoff frequency
n is order of the Butterworth filter
D(u,v) is the Euclidean Distance from any point (u,v) to the frequency plain

$$D(u,v) = \sqrt{(u^2 + v^2)}$$

Gaussian Lowpass Filters (GLPF)

- The transfer function of Gaussian Lowpass filters is ahown below:

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

Where, D0 is Cutoff frequency

D(u,v) is the Euclidean Distance from any point (u,v) to the frequency plain

$$D(u,v) = \sqrt{(u^2 + v^2)}$$

Gaussian Highpass Filters (GHPF)

- The transfer function of Gaussian Highpass filters is ahown below:

$$\boxed{H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}}$$

Where, D0 is Cutoff frequency
D(u,v) is the Euclidean Distance from any point (u,v) to the frequency plain

$$D(u,v) = \sqrt{(u^2 + v^2)}$$


Thank you