(9) Morphological Image Processing

- Morphology
  → A branch of biology that deals with the form and structure of animals and plants

- Mathematical morphology
  → A tool to extract image components for representing and describing region shapes
  - E.g.: boundary, skeleton, convex hull…

(a) Basic set operations

- Definitions
  - If w is an element of set A: $w \in A$
  - If w is not an element of A: $w \notin A$
  - If set B of pixel coordinates satisfies a condition: $B = \{w \mid \text{condition}\}$
  - Complement of A: $A^c = \{w \mid w \notin A\}$

- Union of A and B: $A \cup B$
- Intersection of A and B: $A \cap B$
- Difference of A and B: $A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$
- Reflection of B: $\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$
- Translation of A by point $z = (z_1, z_2)$: $(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$

\- MATLAB set operations on binary images

Set Operation	MATLAB Expression	Name
$A \cap B$	A & B	AND
$A \cup B$	A | B	OR
A^c	~A	NOT
$A - B$	A & ~B	DIFFERENCE

(b) Dilation

\- Dilation: “grow” or “thicken” an object in a binary image
- Extent of thickening controlled by a *structuring element*

- Dilation of image A and structuring element B: $A \oplus B$

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

→ The set of all points z such that the intersection of $(\hat{B})_z$ with A is nonempty

- E.g., a five-pixel-long diagonal line with the origin at the center

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

				1
			1	
		$\boxed{1}$		
	1			
1				

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0	0	0
0	1	1	1	1	1	1	0	0	0	0
0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

→ When the structuring element overlaps 1-valued pixels, the pixel at the origin is marked 1

- Commutative: $A \oplus B = B \oplus A$

- Associative: $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

  * If $B = (B_1 \oplus B_2)$, then $A \oplus B = A \oplus (B_1 \oplus B_2) = (A \oplus B_1) \oplus B_2$

    → Dilate A by B_1, and then dilate the result by B_2 (decomposition)

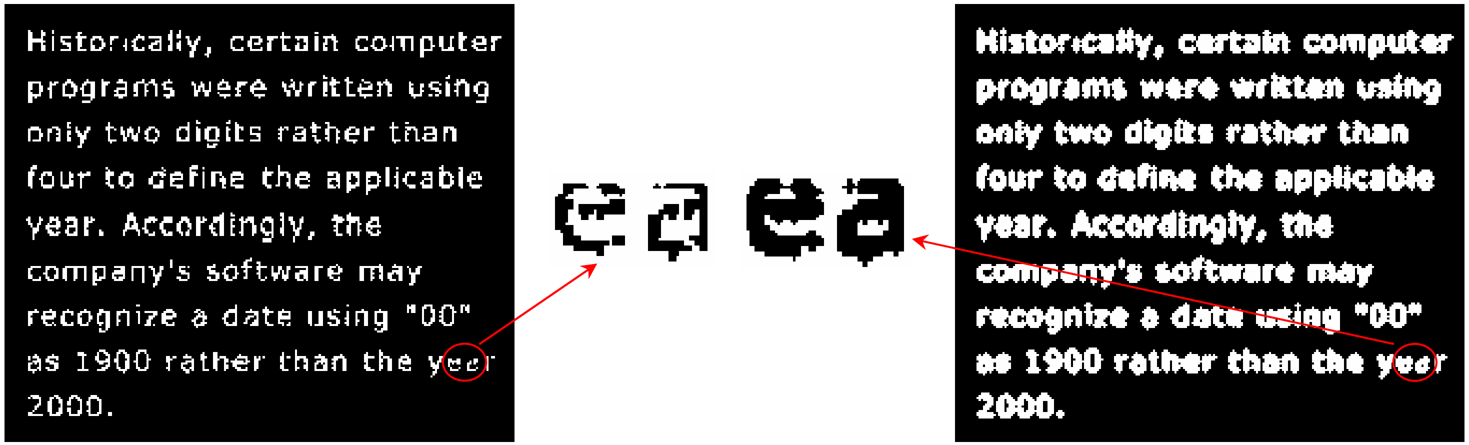
  * E.g., decomposing a structuring element saves computational cost

    → MATLAB decomposes structuring element automatically

$$\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix} \quad = \quad \begin{matrix} 1 & 1 & 1 & 1 & 1 \end{matrix} \quad \oplus \quad \begin{matrix} 1 \\ 1 \\ \boxed{1} \\ 1 \\ 1 \end{matrix}$$

- MATLAB: use dilation to bridge gaps

```
A = imread('text.tif'); B = [0 1 0; 1 1 1; 0 1 0]; A2 = imdilate(A, B); imshow(A2);
```



- strel function: create morphological structuring elements
  SE = strel(*shape*, parameters)
  * *shape*: 'arbitrary', 'diamond', 'disk', 'line', 'square', 'rectangle'…

(c) Erosion

- Erosion: "shrink" or "thin" an object in a binary image
  - Extent of shrinking controlled by a *structuring element*
  - Erosion of image A and structuring element B: $A \ominus B$

$$A \ominus B = \{z \mid (B)_z \cap A^c \neq \varnothing\}$$

→ The set of all points z such that the intersection of $(B)_z$ with A^c is nonempty

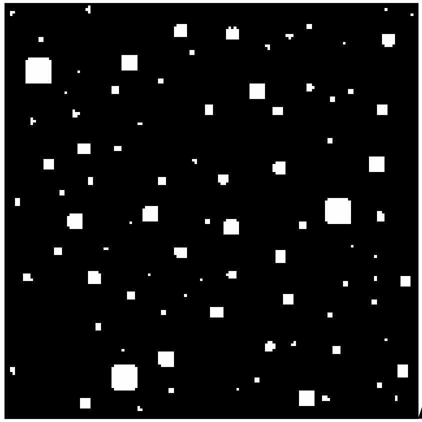
  - E.g., a three-pixel-long vertical line with the origin at the center

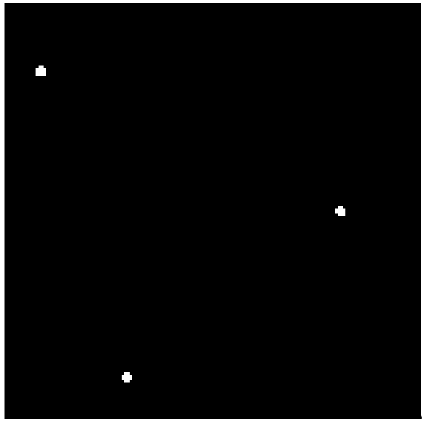
$$\begin{matrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&1&1&1&1&1&0&0 \\ 0&0&1&1&1&1&1&0&0 \\ 0&0&1&1&1&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{matrix} \qquad \begin{matrix} 1 \\ \boxed{1} \\ 1 \end{matrix} \qquad \begin{matrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&1&1&1&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{matrix}$$

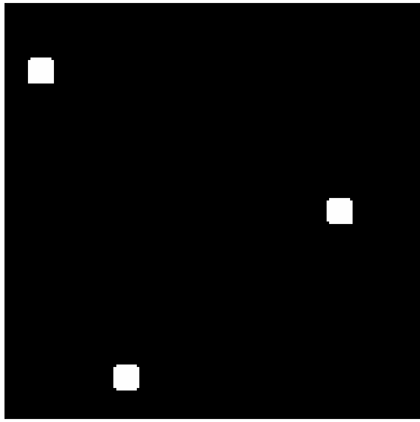
→ When the structuring element overlaps *only* 1-valued pixels, the pixel at the origin is marked 1 (i.e., does not overlap background)

- MATLAB: use erosion to eliminate irrelevant details

```
A = imread('dots.tif'); B = ones(7);
A1 = imerode(A, B); A2 = imdilate(A1, B);
```


A


A1


A2

(d) Opening and closing

- Opening: smooths the contour, breaks narrow isthmuses, and eliminates thin protrusions

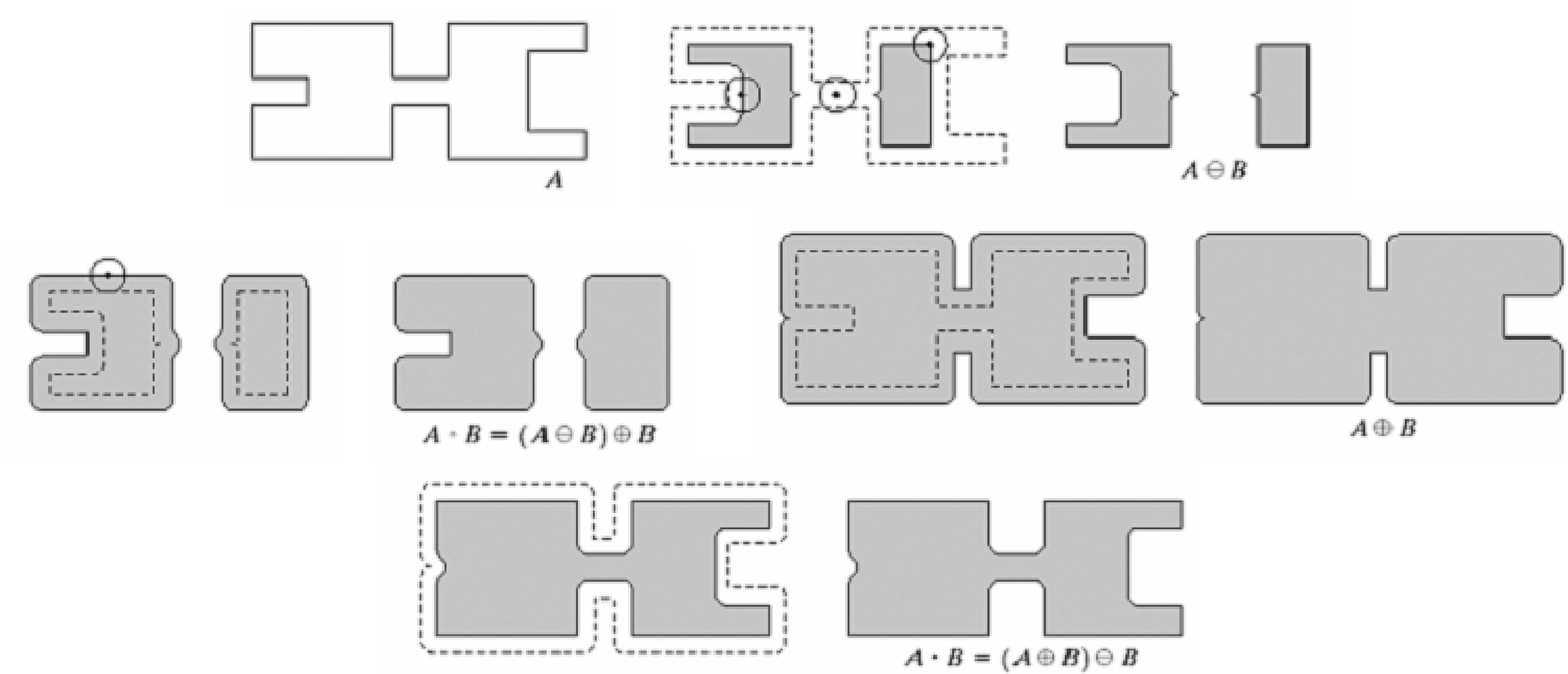
$$(A \circ B) = (A \ominus B) \oplus B = \cup\{(B)_z \mid (B)_z \subseteq A\}$$

→ Erosion followed by dilation ($\cup$: union of all the sets inside the braces), MATLAB: imopen( )

- Closing: smooths the contour, fuses narrow breaks and long thin gulfs, and eliminates small holes

$$(A \bullet B) = (A \oplus B) \ominus B = \{z \mid (B)_z \cap A \neq \emptyset\}$$

→ Dilation followed by erosion, MATLAB: imclose( )



(e) Hit-or-miss transformation

- Hit-or-miss transformation: identify special configuration of pixels

$$A \otimes B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

- E.g., identify $\begin{array}{ccc} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{array} \rightarrow$ B_1 $\begin{array}{ccc} & 1 & \\ 1 & \boxed{1} & 1 \\ & 1 & \end{array}$ B_2 $\begin{array}{ccc} 1 & & 1 \\ & \boxed{} & \\ 1 & & 1 \end{array}$

A:

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	1	1	1	1	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	1	0	0	0	1	1	0	0
0	0	1	0	1	1	1	0	0	1	1	1	0
0	0	0	0	0	1	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

$A \ominus B_1$:

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

A^c:

1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	0	0	0	0	1	1
1	1	0	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1	1	0	0	1	1
1	1	0	1	0	0	0	1	1	0	0	0	1
1	1	1	1	1	0	1	1	1	1	0	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1

$A^c \ominus B_2$:

1	0	1	0	1	1	0	0	0	0	0	0	1
1	0	1	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	1	0	0	0	0	0	0	1
1	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	1	0	0	0	0	1
1	0	1	0	0	0	0	0	0	0	0	0	0
1	1	1	1	0	1	0	1	1	0	1	0	1

$A \otimes B_1$:

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

- MATLAB: `C = bwhitmiss(A, B1, B2);`

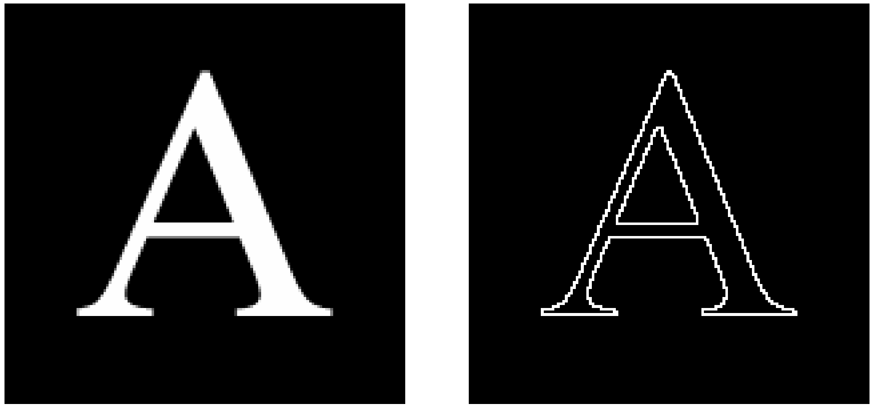
(f) Basic morphological operations

- Boundary extraction: extract the boundary of an object

$$\beta(A) = A - (A \ominus B)$$

- MATLAB

```
A = imread('A.tif'); B = ones(3); A1 = A - imerode(A, B);
```



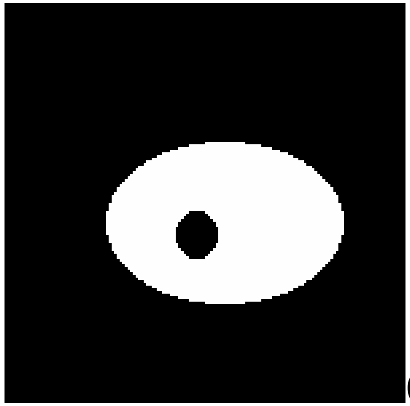
- Region filling

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1, 2, 3\ldots$$

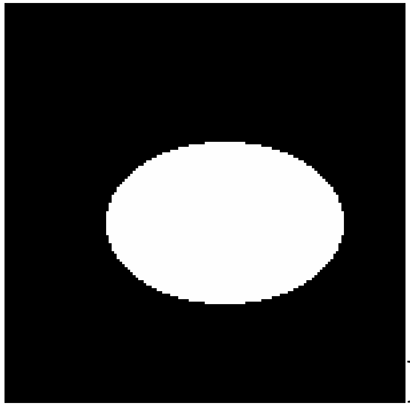
- X_0: a background point inside the object; converged when $X_k = X_{k-1}$

- MATLAB

```
A = im2bw(imread('eye.tif')); B = [0 1 0; 1 1 1; 0 1 0];
Xk = zeros(size(A)); Xk1 = Xk; Xk(85, 70) = 1;
while any(Xk(:) ~= Xk1(:))
    Xk1 = Xk;
    Xk = imdilate(Xk1, B) & ~A;
end
A1 = Xk | A;
```



eye.tif



Region filled

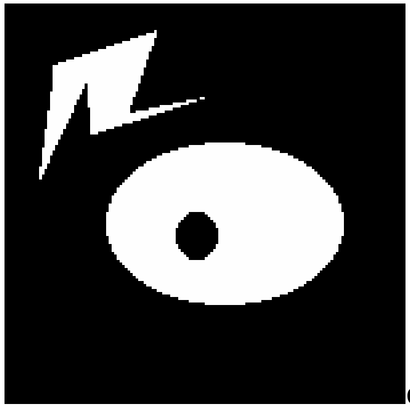
* Problem: need to find the initial point
→ Solution?

\- Extraction of connected components

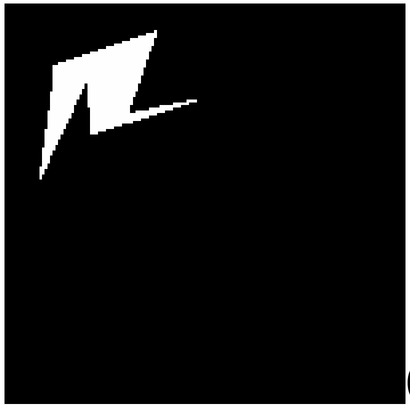
$$X_k = (X_{k-1} \oplus B) \cap A \qquad k = 1, 2, 3\ldots$$

- X_0: a point of the object; converged when $X_k = X_{k-1}$
- MATLAB

```
A = im2bw(imread('a.tif')); B = ones(3);
Xk = zeros(size(A)); Xk1 = Xk; Xk(30, 40) = 1;
while any(Xk(:) ~= Xk1(:))
    Xk1 = Xk;
    Xk = imdilate(Xk1, B) & A;
end
A1 = Xk;
```



a.tif

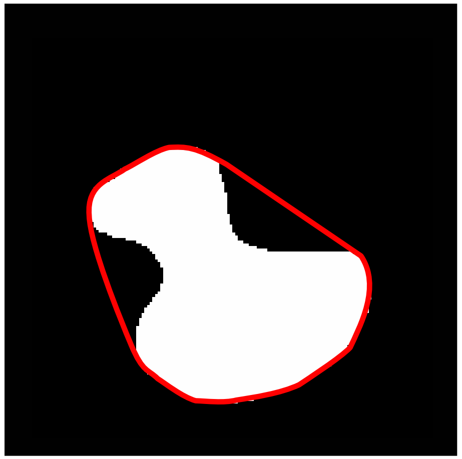


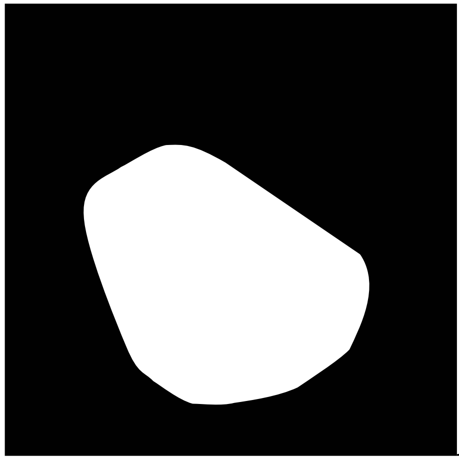
One component found

- MATLAB bwlabel: find all connected components [label number] = bwlabel(im, 4); or [label number] = bwlabel(im, 8);

* label: output image with labeled objects (4- or 8-connectivity)
* number: the number of labeled objects

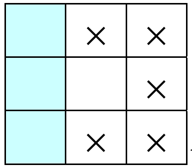
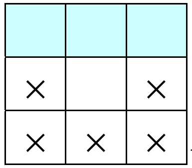
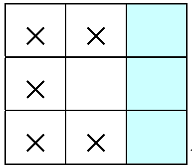
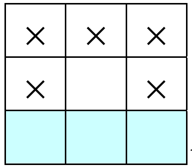
- Convex hull
  - A set A is *convex* if the straight line segment joining any two points in A lies entirely within A
  - *Convex hull H* of set S is the smallest convex set containing S
  - $H - S$: convex deficiency of S





H

  - Four structuring elements: B^i, i = 1, 2, 3, 4, (×: don't care)


B^1

B^2

B^3

B^4

- Convex hull of A: $C(A)$

$$X_k^i = (X_{k-1}^i \otimes B^i) \cup A, \quad i = 1, 2, 3, 4$$

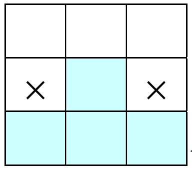
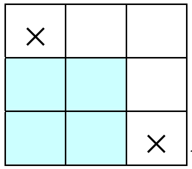
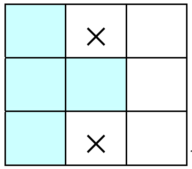
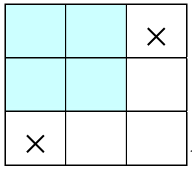
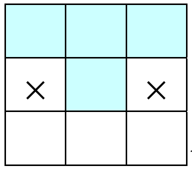
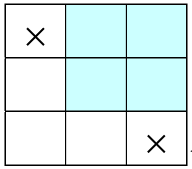
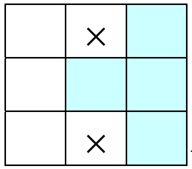
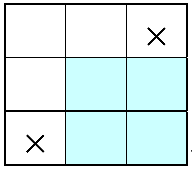
$$D^i = X_{\text{conv}}^{\text{i}}, \quad C(A) = \bigcup_{i=1}^{4} D^i$$

\- Thinning

$$A \odot B = A - (A \otimes B) = A \cap (A \otimes B)^c$$

where $\{B\} = \{B^1, B^2, \ldots, B^n\}$

- Eight structuring elements: B^i, $i = 1, 2, \ldots, 8$


B^1

B^2

B^3

B^4

B^5

B^6

B^7

B^8

• Problem: connectivity not guaranteed

- Thickening

$$A \odot B = A \cup (A \otimes B)$$

where $\{B\} = \{B^1, B^2, \ldots, B^n\}$

• Eight structuring elements are the same as those of thinning

- Skeletonization
  • Repeatedly delete the *contour points* provided the following conditions are satisfied
    * End points are not deleted
    * Connectivity are not broken
    * Do not cause excessive erosion of the region
  • Algorithm: repeat following steps until no contour points
    (1) Delete all contour points according to Definition 1
    (2) Delete all contour points according to Definition 2
  • Definition 1: right, bottom, and upper left corner contour points

$(a)\ 2 \le N(p_1) \le 6$
$(b)\ T(p_1) = 1$
$(c)\ p_2 \cdot p_4 \cdot p_6 = 0$
$(d)\ p_4 \cdot p_6 \cdot p_8 = 0$

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

* $N(p_1)$: number of 1's in the neighborhood of p_1

$$N(p_1) = \sum_{i=2}^{9} p_i$$

* $T(p_1)$: number of 0-1 transitions in the ordered sequence $p_2, p_3, \ldots, p_8, p_9, p_2$ (clockwise)

E.g.:

$N(p_1) = 4,\ T(p_1) = 3$
$p_2 \cdot p_4 \cdot p_6 = 0$
$p_4 \cdot p_6 \cdot p_8 = 0$

0	0	1
1	p_1	0
1	0	1

- Definition 2: left, top, and lower right corner contour points

(a) and (b) are the same as those in definition 1
$(c')\ p_2 \cdot p_4 \cdot p_8 = 0;$

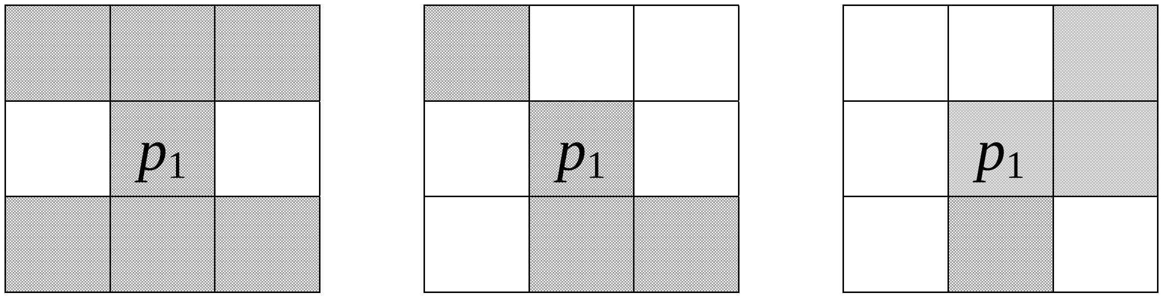
(d') $p_2 \cdot p_6 \cdot p_8 = 0$;

- Description:

  * (a): if there is only one 1 in the neighborhood, p_1 is an end point and should not be deleted; if there are seven 1's, deleting p_1 would cause erosion; if there are eight 1's, p_1 is not a contour point

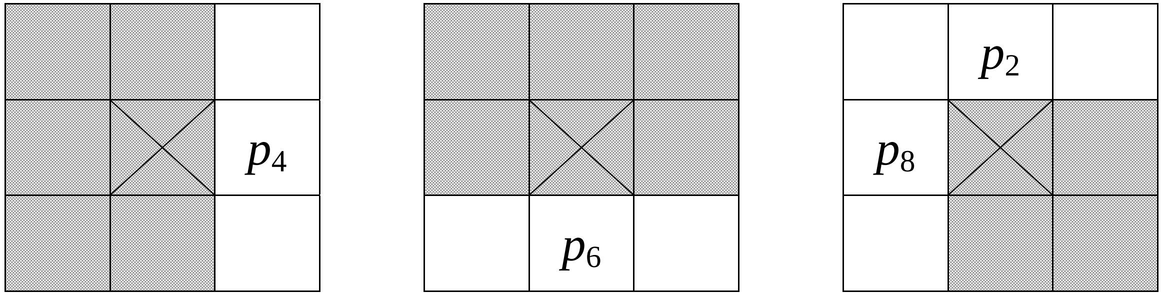
  * (b): $T(p_1) \neq 1$: p_1 is an arc point and deleting p_1 would break the connectivity

    → If the mask consists of only <u>two</u> connected regions, $T(p_1) = 1$



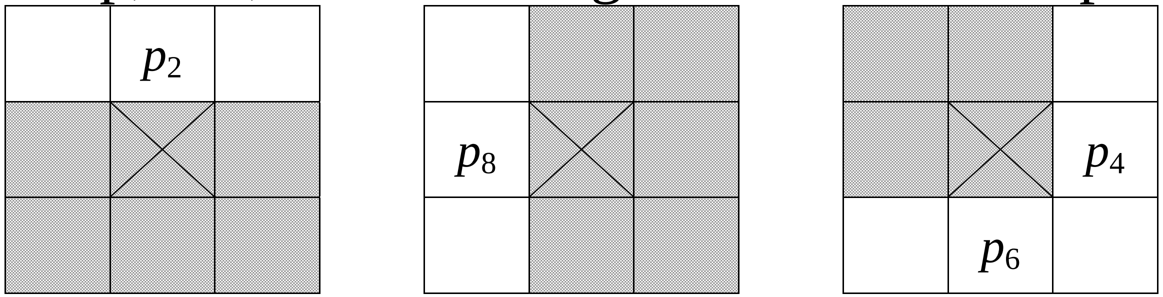
  * (c) and (d): $p_4 = 0$ or $p_6 = 0$ or $p_2 = p_8 = 0$

    → Right, bottom, and upper left corner contour points

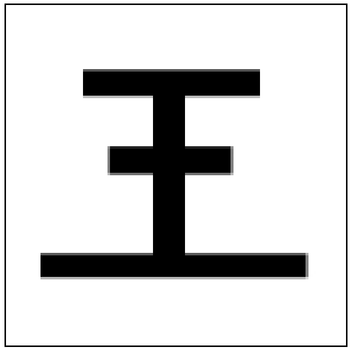
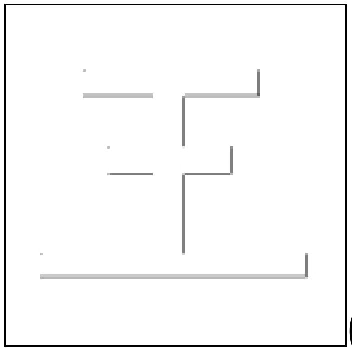
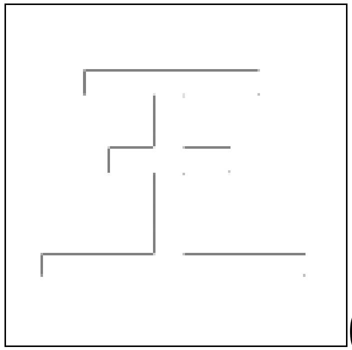


* (c') and (d'): $p_2 = 0$ or $p_8 = 0$ or $p_4 = p_6 = 0$

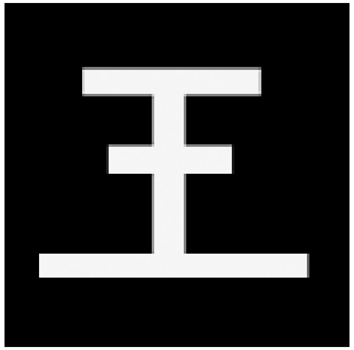
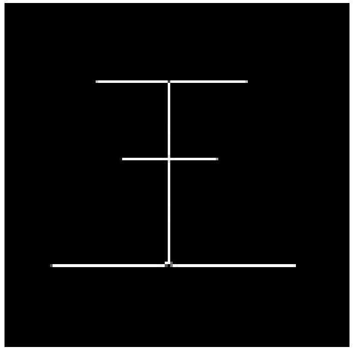
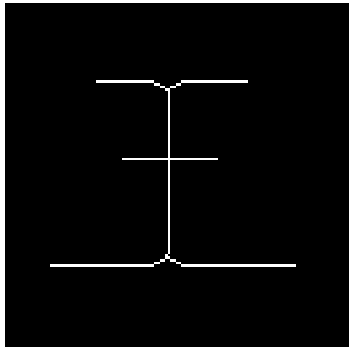
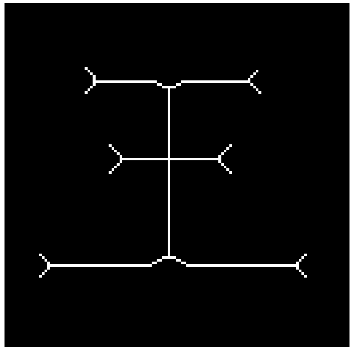
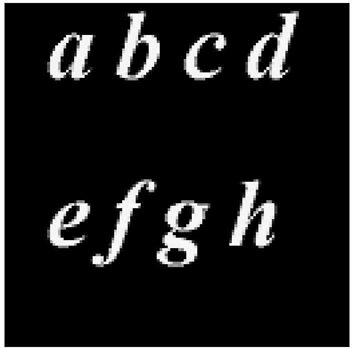
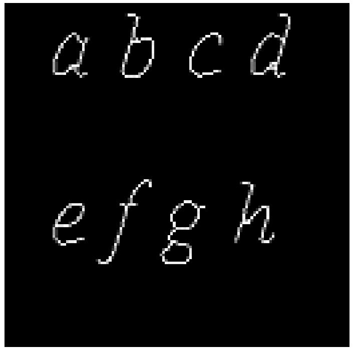
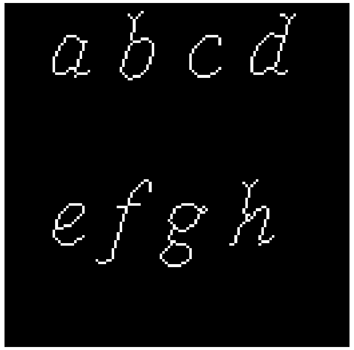
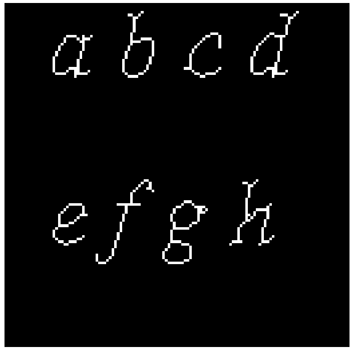
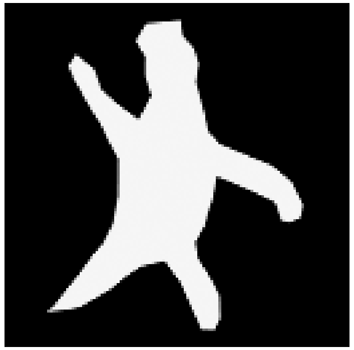
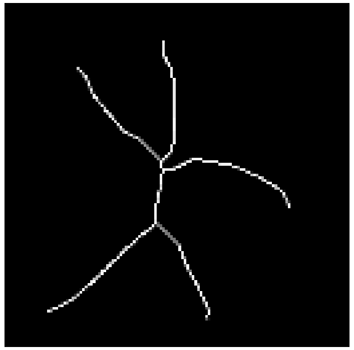
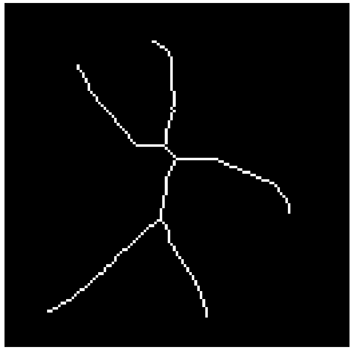
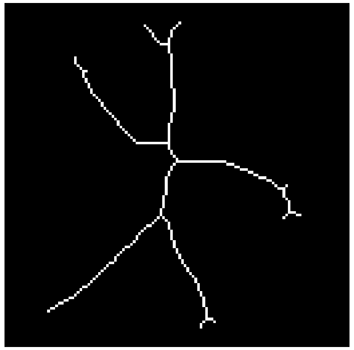
→ Top, left, and lower right corner contour points



- Examples of contour definitions 1 and 2:

 →  (Def. 1)  (Def. 2)

- E.g.:

Image	Thinning	bwmorph(..., 'thin', Inf)	bwmorph(..., 'skel', Inf)
			
			
			

- MATLAB morphing function

bwmorph(*bw*, *operation*) or bwmorph(*bw*, *operation*, *n*)

- *bw*: binary image, *n*: number of repeations of the operation
- *operation*:

'bothat'	"Bottom hat" operation using a 3×3 structuring element; use imbothat for other structuring elements
'erode'	Erosion using a 3×3 structuring element; use imerode for other structuring elements
'shrink'	Shrink objects with no holes to points; shrink objects with holes to rings
'bridge'	Connect pixels separated by single-pixel gaps
'fill'	Fill in single-pixel holes; use imfill for larger holes
'skel'	Skeletonize an image
'clean'	Remove isolated foreground pixels
'hbreak'	Remove H-connected foreground pixels
'spur'	Remove spur pixels
'close'	Closing using a 3×3 structuring element; use imclose for other structuring elements
'majority'	Makes pixel p a foreground pixel if $N_8(p) \geq 5$; otherwise make p a background pixel

'thicken'	Thicken objects without joining disconnected 1s
'diag'	Fill in around diagonally connected foreground pixels
'open'	Opening using a 3×3 structuring element; use imopen for other structuring elements
'thin'	Thin objects without holes to minimally connected strokes; thin objects with holes to rings
'dilate'	Dilation using a 3×3 structuring element; use imdilate for other structuring elements
'remove'	Remove "interior" pixels
'tophat'	"Top hat" operation using a 3×3 structuring element; use imtophat for other structuring elements

(g) Gray-scale morphology

- Morphology
  - Binary image: change the shape of the foreground objects
  - Gray-scale image: change the shape of the image surface (3D)

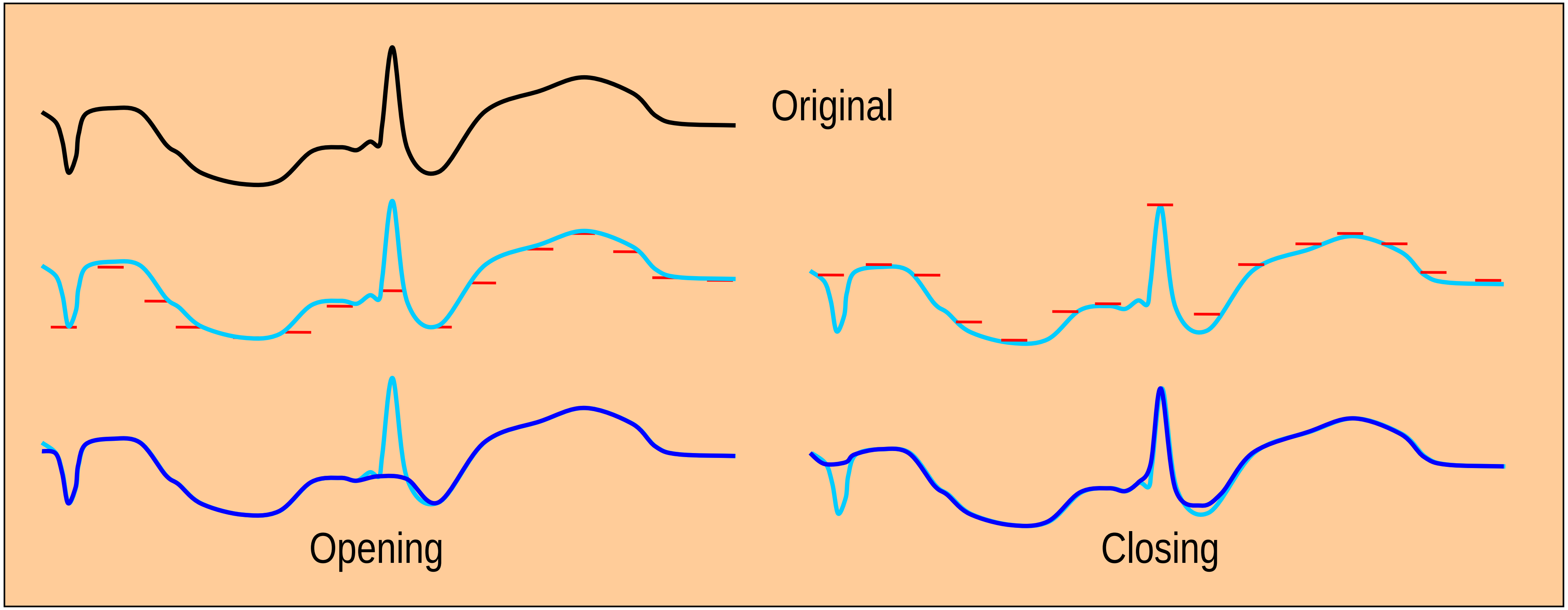
- Gray-scale dialtion and erosion
  - Dialtion: $f \oplus b(x, y) = \max\{f(x-x', y-y') + b(x', y') \mid (x', y') \in D_b\}$

where D_b is the domain of b, and $f(x, y)$ is assumed to equal $-\infty$ outside the domain of f

- Erosion: $f \ominus b(x, y) = \min\{f(x+x', y+y') - b(x', y') \mid (x', y') \in D_b\}$

  where D_b is the domain of b, and $f(x, y)$ is assumed to equal $+\infty$ outside the domain of f

- Opening and closing



- Example: using opening to compensate for nonuniform background illumination
  - Fig. 1: rice grains on nonuniform background (darker towards bottom), `f = imread('rice.tif')`
  - Fig. 2: simple thresholding: `fbw = im2bw(f, graythresh(f))` causes grains improperly separated at the bottom portion
  - Fig. 3: opening with `se = strel('disk', 10); fo = imopen(f, se)`: since the size of `se` is set to be larger than the grains, only background remains
  - Fig. 4: subtracting from the original `f2 = imsubtract(f, fo)`: results in a more uniform background
  - Fig. 5: thresholding with `fbw = im2bw(f2, graythresh(f2))` obtains a better result
  - Subtracting an opened image from the original is called the *top-hat transformation*, a single step in Matlab: `f2 = imtophat(f, se)`

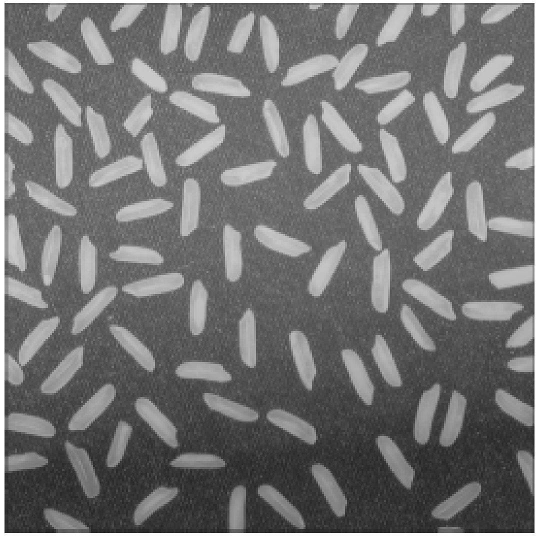


Fig. 1

Fig. 2

Fig. 3

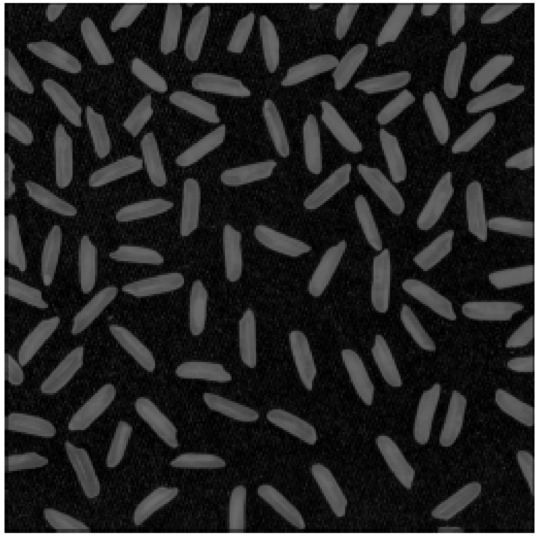


Fig. 4

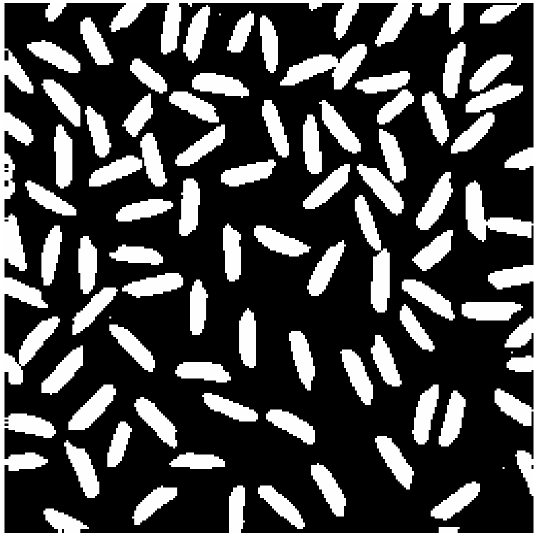


Fig. 5