Demo [Analytics vs Numeric]

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The goal of this demo is to apply eNDM to mouse data. Uses numerical solutions for models using ode solvers in objective functions. Several consistency checks are provided.

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Created: Feb/6/2020

Last Modified: Feb/19/2020

0. Setup

```
% Initialization
    clear all; close all; clc;

% Add folder with raw data to path
    addpath([pwd '/raw_data_mouse'])

% Add library with eNDM functions to path
    addpath([pwd '/lib_eNDM_numeric'])
    addpath([pwd '/lib_eNDM_analytic'])

% Load dataset of interest
   load MouseDataForPedro.mat

% Select connectome C (426x426)
    C = Networks.nd;        % symmmetric
    %C = Networks.ret;      % non-symmetric
    %C = Networks.ant;     % non-symmetric

% Normalize C
    cmax = max(max(C));
    cmin = min(min(C));
    C = (C - cmin)./(cmax-cmin);
```

```
% Define number of regions of interest (nroi)
    nroi = size(C,1);

% Load time stamps, pathology measurements, and the seed_location

        % Hippocampus Injection
           time_stamps = [1,3,6];
           pathology = data426.IbaHippInj;
           seed_location = seed426.IbaHippInj;

        % Striatal Injection
%            time_stamps = [1,3,6];
%            pathology = data426.IbaStrInj;
%            seed_location = seed426.IbaStrInj;
```

1. [Analytics vs Numeric] NDM, fmincon minimizing MSE

```
% Set ndm_comp = 1 to run
ndm_comp = 1;
if ndm_comp==1

% Parameters to fit
    % param(1) = beta
    % param(2) = x0_value

% Extra input to required by objective function
    % seed_location
    % pathology
    % time_stamps
    % C

% Set initial guess for beta
    init_guess_params(1) = 1;

% Set initial guess for x0_value
    init_guess_params(2) = nansum(pathology(:,1));

% Set lower bounds (lb) for parameters
    lb = [0,0];

% Set upper bounds (ub) for parameters
%    ub = [10,nansum(pathology(:,1))];
ub = [3,3];
% Apply fmincon w/ numeric
    [param_num, fval_num] =
 fmincon(@(param)objfun_NDM_numeric(param,seed_location,pathology,time_stamps,C),.
                            init_guess_params,[],[],[],[],lb,ub,[]);

% Solve NDM with the optimal parameters
    beta_num = param_num(1);
    x0_num = seed_location*param_num(2);
```

```
    ynum = NDM_numeric(x0_num,time_stamps,C,beta_num);

    % Apply fmincon w/ analytic
    [param_ana, fval_ana] =
 fmincon(@(param)objfun_NDM_analytic(param,seed_location,pathology,time_stamps,C),
                           init_guess_params,[],[],[],[],lb,ub,[]);

% Solve NDM with the optimal parameters
    beta_ana = param_ana(1);
    x0_ana = seed_location*param_ana(2);
    yana = NDM_analytic(x0_ana,time_stamps,C,beta_ana);

    %DEBUG
    %beta_ana = param_num(1);
    %x0_ana = seed_location*param_num(2);
    %yana = NDM_analytic(x0_ana,time_stamps,C,beta_ana);
```

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than the default value of the step size tolerance and constraints are satisfied to within the default value of the constraint tolerance.

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.

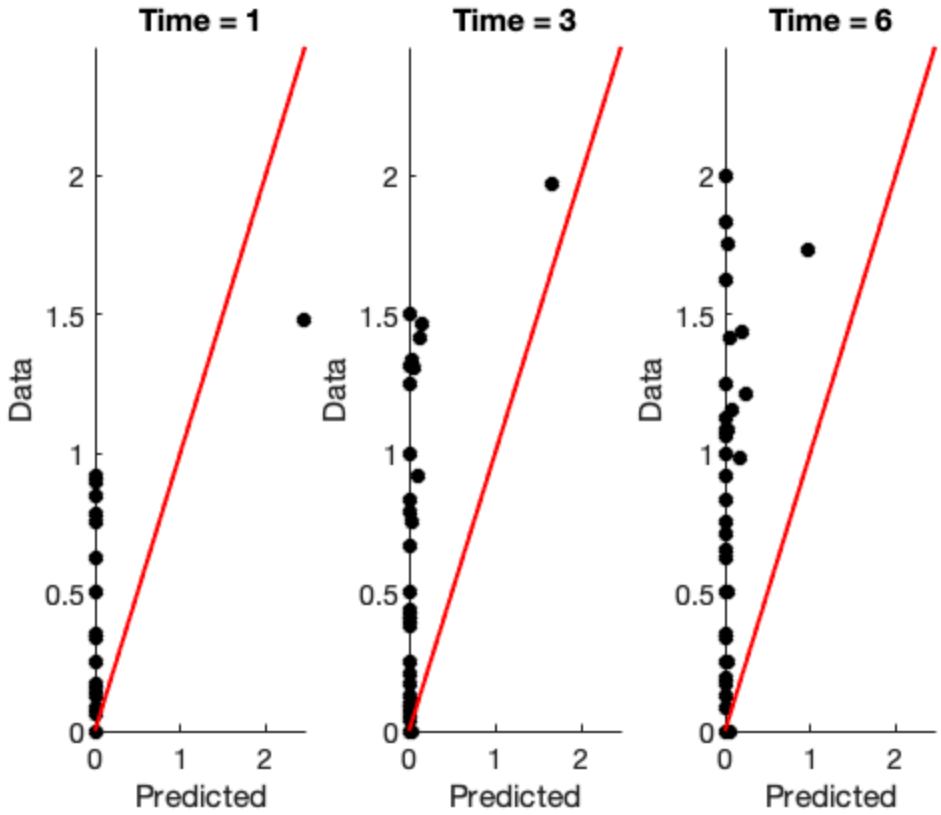
Display Numeric Results

Save Rvalues in a matrix

```
            for jj = 1:length(time_stamps)
                 Rvalues(jj,:) =
 corr(ynum(:,jj),pathology(:,jj), 'rows','complete');
            end

 % Display results
    disp('-------------------------------------------------')
    disp('NDM minimizing quadratic error at all time stamps with
 fmincon');
    disp(' ')
```

```
    disp(['NUMERIC Beta = ' num2str(param_num(1)) ', x0 value = '
 num2str(param_num(2))])
    disp(' ')
    disp(['R values at each time stamp'])
    disp(Rvalues)
    disp(' ')
    disp('Square error')
    disp(fval_num)

% Plot prediction vs data using optimal parameters
plot_pred_vs_data(ynum,pathology,time_stamps)
```

```
-----------------------------------------------------
NDM minimizing quadratic error at all time stamps with fmincon

NUMERIC Beta = 0.32422, x0 value = 2.4629

R values at each time stamp
    0.5415
    0.4776
    0.4166


Square error
   59.1826
```

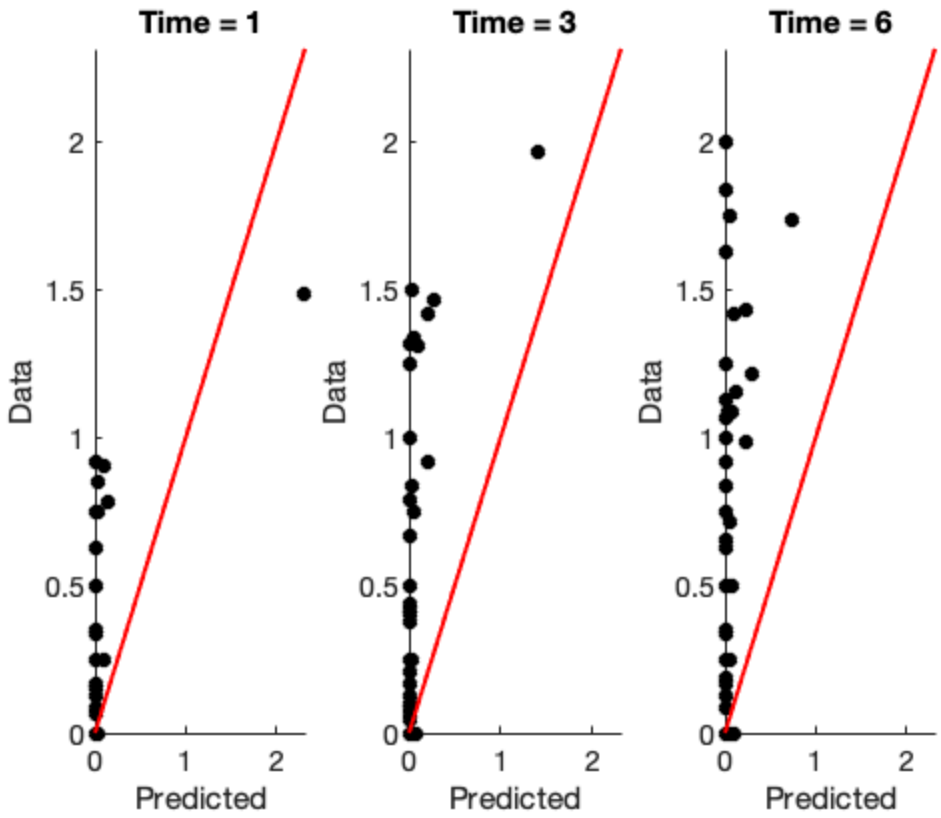


Display Analytic Results

Save Rvalues in a matrix

```
            for jj = 1:length(time_stamps)
                 Rvalues(jj,:) =
 corr(yana(:,jj),pathology(:,jj), 'rows','complete');
            end

 % Display results
    disp('--------------------------------------------------')
    disp('NDM minimizing quadratic error at all time stamps with
 fmincon');
    disp(' ')
    disp(['ANALYTIC Beta = ' num2str(param_ana(1)) ', x0 value = '
 num2str(param_ana(2))])
    disp(' ')
    disp(['R values at each time stamp'])
    disp(Rvalues)
    disp(' ')
    disp('Square error')
    disp(fval_ana)

% Plot prediction vs data using optimal parameters
plot_pred_vs_data(yana,pathology,time_stamps)
clearvars y
```

```
--------------------------------------------------
NDM minimizing quadratic error at all time stamps with fmincon

ANALYTIC Beta = 0.42779, x0 value = 3

R values at each time stamp
    0.5851
    0.5516
    0.4866


Square error
   57.2228
```

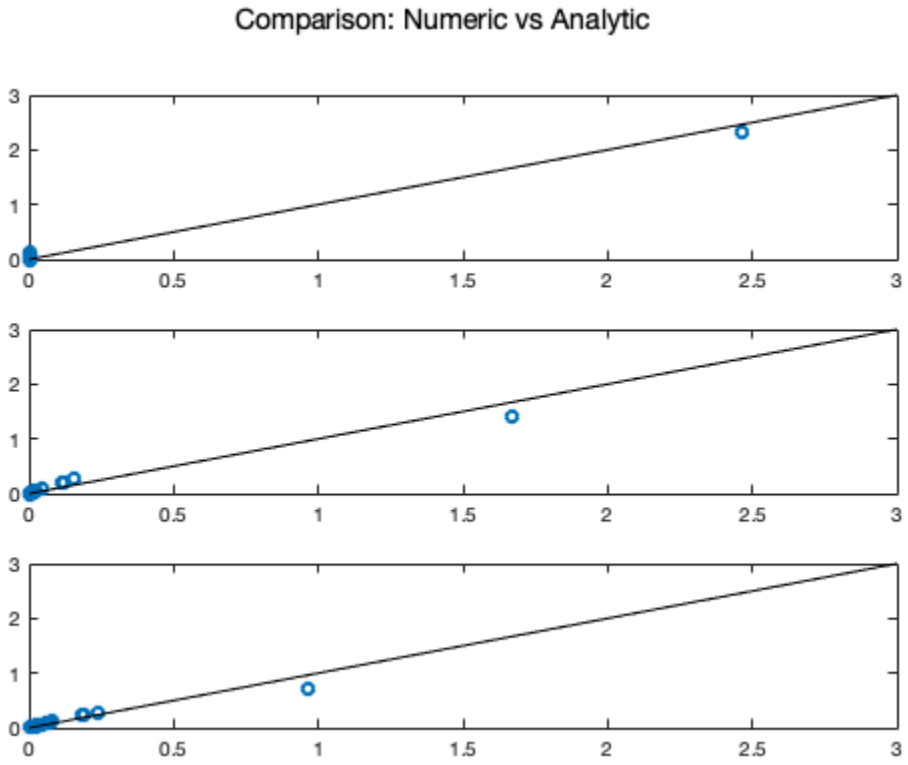


Compare values

```
%table(ynum,yana)
figure
subplot(3,1,1)
plot(ynum(:,1),yana(:,1),'o')
hold on
line([0 3], [0 3],'Color','k')
%
subplot(3,1,2)
plot(ynum(:,2),yana(:,2),'o')
hold on
line([0 3], [0 3],'Color','k')
%
subplot(3,1,3)
plot(ynum(:,3),yana(:,3),'o')
hold on
line([0 3], [0 3],'Color','k')
%
suptitle('Comparison: Numeric vs Analytic')

disp('rel error at t1')
disp(    norm(ynum(:,1)-yana(:,1),2) /norm(ynum(:,1),2)      )

disp('rel error at t2')
```

```
disp(    norm(ynum(:,2)-yana(:,2),2) /norm(ynum(:,2),2)      )


disp('rel error at t3')
disp(    norm(ynum(:,3)-yana(:,3),2) /norm(ynum(:,3),2)      )
```

```
rel error at t1
    0.1072

rel error at t2
    0.1900

rel error at t3
    0.2581
```




```
end
```

2. [Analytics vs Numeric] NDMwS, fmincon minimizing MSE

```
% Set ndmws_comp = 1 to run
ndmws_comp = 1;

if ndmws_comp ==1
```

```
% Parameters to fit
    % param(1) = beta
    % param(2) = x0_value
    % param(3) = alpha1 = linear growth/clearance term in x

% Extra input to required by objective function
    % seed_location
    % pathology
    % time_stamps
    % C

% Set initial guess for beta
    init_guess_params(1) = 1;

% Set initial guess for x0_value
    init_guess_params(2) = nansum(pathology(:,1));

% Set initial guess for alpha1 and alpha2
    init_guess_params(3) = .5;

% Set lower bounds (lb) for parameters
    lb = [0,0,-5];

% Set upper bounds (ub) for parameters
%    ub = [10,nansum(pathology(:,end)),5];
    ub = [3,3,3];

% Apply fmincon w/ numeric
    [param_num, fval_num] =
 fmincon(@(param)objfun_NDMwS_numeric(param,seed_location,pathology,time_stamps,C)
                           init_guess_params,[],[],[],[],lb,ub,[]);

% Solve NDMwS with the optimal parameters
    beta_num = param_num(1);
    x0_num = seed_location*param_num(2);
    alpha1_num = param_num(3);
    ynum = NDMwS_numeric(x0_num,time_stamps,C,beta_num,alpha1_num);

    % Apply fmincon w/ analytic
    [param_ana, fval_ana] =
 fmincon(@(param)objfun_NDMwS_analytic(param,seed_location,pathology,time_stamps,C
                           init_guess_params,[],[],[],[],lb,ub,[]);

% Solve NDMwS with the optimal parameters
    beta_ana = param_ana(1);
    x0_ana = seed_location*param_ana(2);
    alpha1_ana = param_ana(3);
    yana = NDMwS_analytic(x0_ana,time_stamps,C,beta_ana,alpha1_ana);

    %DEBUG
    %beta_ana = param_num(1);
    %x0_ana = seed_location*param_num(2);
    %yana = NDM_analytic(x0_ana,time_stamps,C,beta_ana);
```

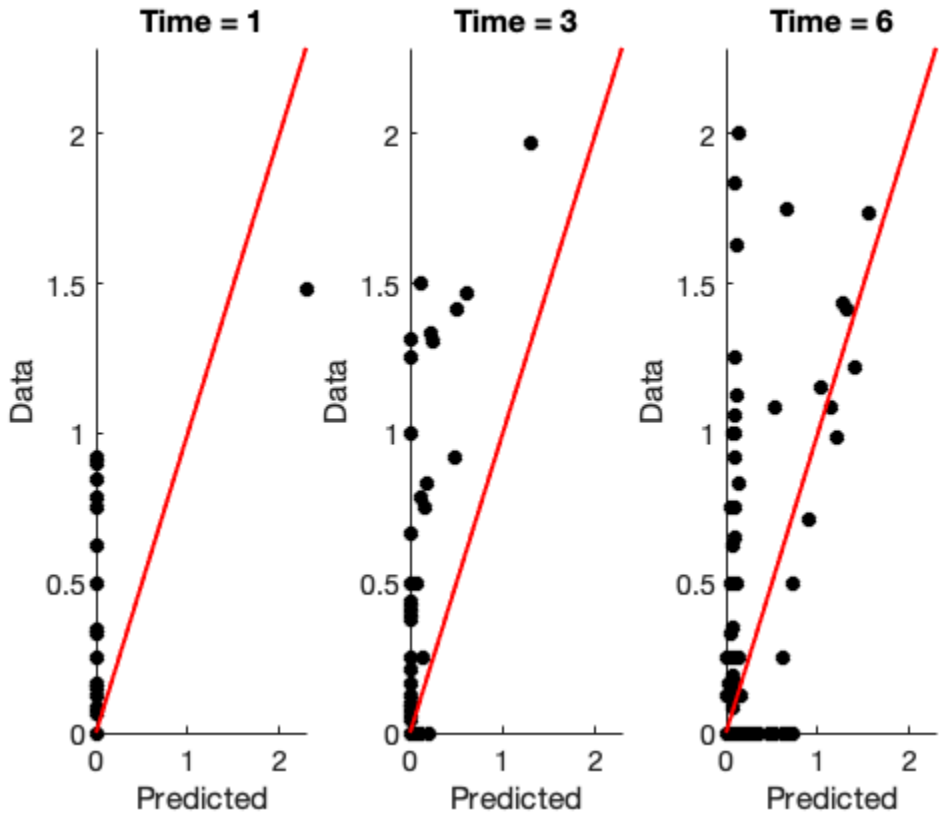
```
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the default value of the step size tolerance and constraints are
satisfied to within the default value of the constraint tolerance.



Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-
decreasing in
feasible directions, to within the default value of the optimality
 tolerance,
and constraints are satisfied to within the default value of the
 constraint tolerance.
```

Display Numeric Results

Save Rvalues in a matrix

```
            for jj = 1:length(time_stamps)
                 Rvalues(jj,:) =
 corr(ynum(:,jj),pathology(:,jj), 'rows','complete');
            end

 % Display results
    disp('-----------------------------------------------------')
    disp('NDMwS minimizing quadratic error at all time stamps with
 fmincon');
    disp(' ')
    disp(['NUMERIC Beta = ' num2str(param_num(1)) ', x0 value = '
 num2str(param_num(2)) ...
         ', alpha1 = ' num2str(param_num(3))])
    disp(' ')
    disp(['R values at each time stamp'])
    disp(Rvalues)
    disp(' ')
    disp('Square error')
    disp(fval_num)

% Plot prediction vs data using optimal parameters
plot_pred_vs_data(ynum,pathology,time_stamps)

-----------------------------------------------------
NDMwS minimizing quadratic error at all time stamps with fmincon

NUMERIC Beta = 1.5128, x0 value = 2.2857, alpha1 = 0.53475
```

```
R values at each time stamp
    0.5415
    0.7023
    0.5911


Square error
   43.9155
```



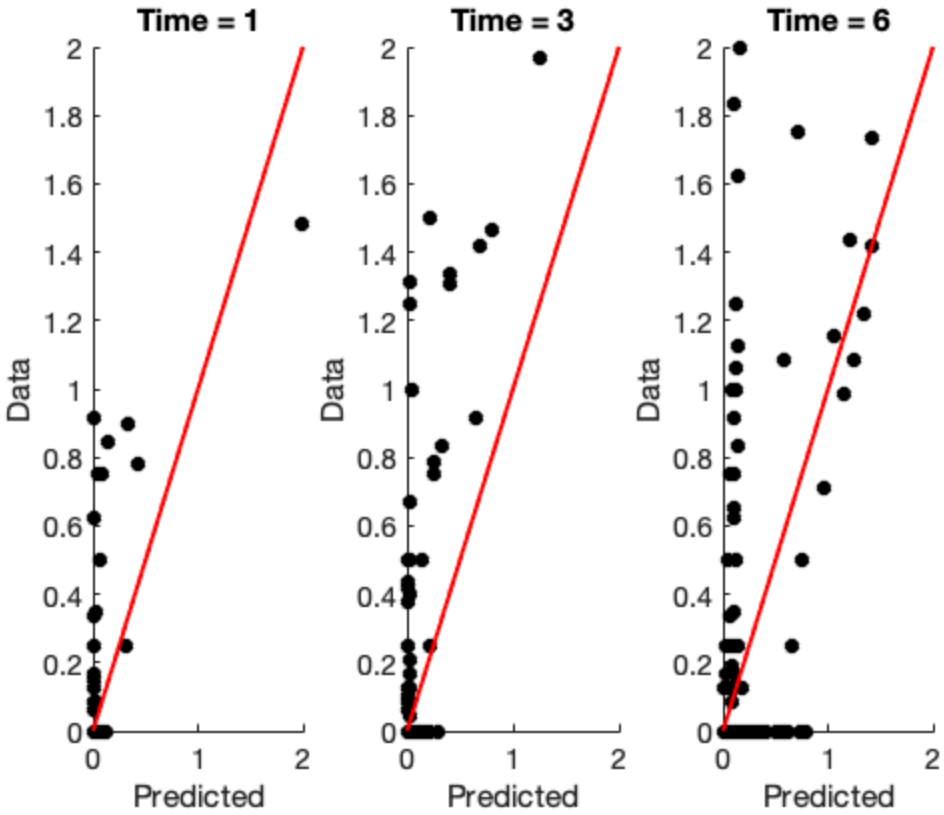
Display Analytic Results

Save Rvalues in a matrix

```
            for jj = 1:length(time_stamps)
                 Rvalues(jj,:) =
 corr(yana(:,jj),pathology(:,jj), 'rows','complete');
            end

 % Display results
    disp('-------------------------------------------------')
    disp('NDMwS minimizing quadratic error at all time stamps with
 fmincon');
    disp(' ')
    disp(['ANALYTIC Beta = ' num2str(param_ana(1)) ', x0 value = '
 num2str(param_ana(2)) ...
```

```
        ', alpha1 = ' num2str(param_ana(3))])
    disp(' ')
    disp(['R values at each time stamp'])
    disp(Rvalues)
    disp(' ')
    disp('Square error')
    disp(fval_ana)

% Plot prediction vs data using optimal parameters
plot_pred_vs_data(yana,pathology,time_stamps)
clearvars y
```

```
--------------------------------------------------
NDMwS minimizing quadratic error at all time stamps with fmincon

ANALYTIC Beta = 1.4161, x0 value = 3, alpha1 = 0.41649

R values at each time stamp
    0.6798
    0.7469
    0.5890


Square error
   40.0727
```



Compare values

```
%table(ynum,yana)
figure
subplot(3,1,1)
plot(ynum(:,1),yana(:,1),'o')
hold on
line([0 3], [0 3],'Color','k')
%
subplot(3,1,2)
plot(ynum(:,2),yana(:,2),'o')
hold on
line([0 3], [0 3],'Color','k')
%
subplot(3,1,3)
plot(ynum(:,3),yana(:,3),'o')
hold on
line([0 3], [0 3],'Color','k')
%
suptitle('Comparison: Numeric vs Analytic')

disp('rel error at t1')
disp(    norm(ynum(:,1)-yana(:,1),2) /norm(ynum(:,1),2)      )

disp('rel error at t2')
disp(    norm(ynum(:,2)-yana(:,2),2) /norm(ynum(:,2),2)      )

disp('rel error at t3')
disp(    norm(ynum(:,3)-yana(:,3),2) /norm(ynum(:,3),2)      )


table(ynum,yana)

rel error at t1
    0.3286

rel error at t2
    0.3223

rel error at t3
    0.0810


ans =

  426×2 table

                    ynum                                    yana

    ____________________________________
  ______________________________________
```

```
         0     0.0020308      0.039729    0.00033375     0.0056007
  0.050215
         0       0.01262        0.1013      0.004237      0.025203
   0.11411
         0     0.0069717       0.08913     0.0015194      0.016585
    0.1062
         0     0.0022583      0.037707    0.00043069     0.0058465
  0.046852
         0     0.0037108      0.066407    0.00061237      0.010056
  0.082051
         0      0.014534       0.12564     0.0052721       0.02909
   0.14462
         0   0.00075822      0.016169    0.00012617     0.0021257
  0.020869
         0      0.003525      0.049175    0.00075463     0.0085606
  0.059739
         0     0.0028585      0.050071    0.00053365     0.0075202
  0.062563
         0     0.0067207      0.089243     0.0014523      0.016154
    0.10702
         0      0.020894       0.14751     0.0086588      0.038867
   0.16554
         0   1.9977e-05     0.0011665    1.5076e-06    8.1076e-05
 0.0017084
         0   7.7508e-06    0.00039944    6.4396e-07    2.9887e-05
0.00057988
         0    8.801e-06    0.00044416    7.5882e-07    3.3475e-05
0.00064582
         0   2.1455e-06    0.00016015    1.4704e-07    9.3483e-06
0.00024524
         0   2.4719e-05    0.00094529    2.8879e-06    8.4117e-05
 0.0013334
         0   1.9615e-05    0.00069281    2.8288e-06    6.3062e-05
0.00098369
         0   0.00042583     0.0046369    0.00010401    0.00095005
 0.0054726
         0   4.2752e-07    2.8015e-05    3.1739e-08    1.7765e-06
 4.228e-05
         0   9.2907e-06    0.00057816    6.8566e-07    3.8354e-05
0.00085799
         0   0.00011143     0.0044947    1.2572e-05    0.00039101
 0.0062967
         0   5.8639e-06    0.00039258    4.0179e-07    2.4949e-05
0.00058713
         0   2.0415e-05     0.0011652    1.5638e-06    8.2171e-05
 0.0017029
         0    0.0058827       0.068144     0.0013255      0.013609
   0.079862
         0     0.011119       0.12449     0.0024925      0.025622
   0.14418
         0      0.01007        0.1233     0.0022219      0.023702
    0.1455
         0      0.25622        1.0307      0.14144         0.3966
    1.0623
```

```
 2.2857        1.3113        1.5542        1.9925        1.2522
 1.4274
      0       0.22113        1.3154      0.093432       0.39662
 1.4154
      0       0.51313        1.2691       0.33557       0.68982
 1.2145
      0      0.083455       0.53206      0.028959       0.15835
0.56661
      0       0.20413        0.6295       0.12145       0.29402
0.62161
      0      0.045031       0.33041      0.017767      0.085445
0.37068
      0      0.011323       0.15743     0.0023957      0.027625
0.18992
      0      0.008862       0.11399     0.0019221      0.021169
 0.1355
      0      0.075459       0.28682      0.041787       0.11453
0.29638
      0       0.12262       0.67403      0.052934       0.21554
0.71406
      0     0.0054865      0.076981     0.0011609      0.013404
0.093164
      0   0.00018638     0.0071213   1.8837e-05    0.00065299
0.009833
      0     0.0010603      0.018289    0.00020687     0.0027459
0.023024
      0     0.0085493      0.094258     0.0019728       0.01953
 0.10914
      0     0.0072294       0.10479      0.001478      0.017963
 0.12663
      0     0.0052175      0.043348     0.0020149      0.010164
0.050214
      0      0.045247       0.35278      0.017649      0.087201
  0.4001
      0   0.00023112     0.0045528    4.175e-05    0.00062649
0.0058379
      0     0.0032162      0.050949    0.00064642     0.0081479
0.062943
      0      0.011583       0.10582      0.004196      0.023463
 0.12309
      0   0.00022657     0.0037287    4.5691e-05    0.00057621
0.0046748
      0     0.0019754      0.030286     0.0004054     0.0049423
0.037321
      0     0.0012342      0.022256    0.00023213     0.0032583
0.028083
      0   5.4419e-06    0.00024491    4.8519e-07    2.0083e-05
0.00034825
      0     0.0012176      0.018572    0.00025745     0.0030132
0.023063
      0   0.00041295     0.0060052    8.7914e-05     0.0010114
0.0073786
      0     0.0002313      0.0080087    2.7118e-05    0.00077573
0.010947
```

```
         0      0.0045307       0.043665      0.0016564      0.0092231
0.051797
         0       0.002641       0.042551     0.00052557      0.0067309
0.052671
         0     0.00053616       0.009823      0.0001053      0.0014001
0.012593
         0      0.0036107       0.046089     0.00078742      0.0085635
0.055093
         0      0.0048275       0.045872      0.0019534      0.0095872
0.054806
         0      0.0016452       0.032188     0.00025865      0.0045857
0.040362
         0      0.0028011       0.045786     0.00049674      0.0073252
0.056111
         0      0.0043257       0.066727     0.00073504       0.011303
0.080039
         0      0.0038265        0.04126     0.00093981      0.0085033
0.048596
         0      0.0020017       0.029358     0.00041711      0.0049399
0.035962
         0      0.0035829       0.039924     0.00086572      0.0080454
0.047245
         0      0.0070643       0.076578      0.0016516       0.015991
0.088844
         0      0.0011263       0.025927     0.00014565      0.0033594
0.033056
         0      0.0020695       0.031932     0.00040563      0.0052327
0.039198
         0      0.0084104       0.077083      0.0029688        0.01716
  0.0894
         0       0.002483       0.031242     0.00060032      0.0057259
0.037833
         0      0.0037842       0.042686     0.00093098      0.0084789
0.050702
         0      0.0015297       0.030946      0.0002433      0.0042966
0.039033
         0      0.0018609       0.036703     0.00031169      0.0051515
 0.04627
         0      0.0016819       0.032441     0.00028379      0.0046149
0.040838
         0       0.011298       0.099814      0.0028812       0.023997
  0.1124
         0      0.0018426       0.033828     0.00033829      0.0049182
0.042552
         0      0.0060474       0.049562      0.0025596       0.011524
0.057802
         0       0.004591       0.045313      0.0016829      0.0093883
0.054023
         0      0.0024826        0.04227     0.00045275      0.0065505
0.052082
         0      0.0089007       0.086675      0.0022034        0.01945
0.099041
         0      0.0057574       0.041858       0.002406       0.010566
 0.04835
```

```
         0     0.0012541      0.024383     0.00020834      0.0034549
 0.030752
         0     0.0012652      0.023562     0.00022275       0.003409
 0.029675
         0    0.00053029      0.012683     8.5999e-05      0.0015345
 0.016679
         0     0.0030005      0.037668     0.00066852      0.0070482
  0.04512
         0     0.0018759      0.028343     0.00036566      0.0047035
 0.034804
         0      0.003855      0.031141      0.0019498       0.006995
 0.037158
         0      0.004175      0.049879     0.00093305      0.0097235
 0.058928
         0    0.00083531      0.018271     0.00013364      0.0023742
 0.023568
         0     0.0029922      0.044697     0.00059964      0.0075027
 0.054384
         0     0.0038422      0.051536     0.00082256      0.0092772
 0.061833
         0     0.0064445       0.06461      0.0020675       0.013636
 0.075758
         0      0.012556      0.086454      0.0053741       0.023023
 0.097192
         0      0.001914      0.031282     0.00038072      0.0048848
 0.038948
         0     0.0013096      0.023356     0.00024982      0.0034388
 0.029438
         0    1.6204e-06    0.00012858    1.0293e-07    7.2568e-06
 0.00019789
         0    1.5481e-05    0.00085107    1.1999e-06    6.1587e-05
 0.0012374
         0      0.000188     0.0056128    2.4855e-05    0.00059519
 0.0075524
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         0    5.1986e-05     0.0018121    7.4841e-06    0.00016706
0.0025533
         0    2.0388e-05    0.00094211    2.1698e-06    7.4344e-05
0.0013552
         0    1.2641e-05    0.00069663    1.2519e-06    4.8716e-05
0.0010295
         0    3.0908e-05     0.0012094    3.7115e-06    0.00010532
0.0017158
         0    3.5768e-06    0.00022068    2.8748e-07    1.4407e-05
0.00033222
         0    0.00040791      0.011071    5.9872e-05     0.0012452
 0.014707
         0     0.0015262       0.03089    0.00026213     0.0042226
 0.039337
         0    9.9269e-05     0.0041314    9.7696e-06     0.0003575
0.0057759
         0    2.7687e-05     0.0011182    2.7986e-06    9.8302e-05
0.0015629
         0    8.5847e-05     0.0030248    1.1384e-05    0.00028197
0.0042189
         0    0.00010069     0.0037863    1.1937e-05    0.00034327
0.0052801
         0    0.00046123      0.012743    5.8089e-05      0.001454
 0.016708
         0      0.010528       0.10227     0.0036112       0.02177
  0.12048
         0    0.00013999     0.0040234    2.0722e-05    0.00043012
0.0054427
         0      0.001107      0.021065    0.00019185     0.0030212
 0.026493
         0    0.00017911     0.0048031    2.8031e-05    0.00053514
0.0064579
         0     0.0001398     0.0044823    1.8844e-05    0.00044882
0.0061324
         0     0.0017319      0.029464    0.00031238     0.0045737
 0.036354
         0    0.00065835      0.014964    0.00010924     0.0018794
 0.019477
         0    0.00018109     0.0064447    1.8483e-05    0.00062456
0.0087629
         0    0.00024858      0.008851    2.5947e-05    0.00085648
 0.012029
         0     0.0012828      0.022368    0.00023705     0.0033875
 0.027871
         0     0.0010766      0.018832    0.00020809     0.0028077
 0.023681
```
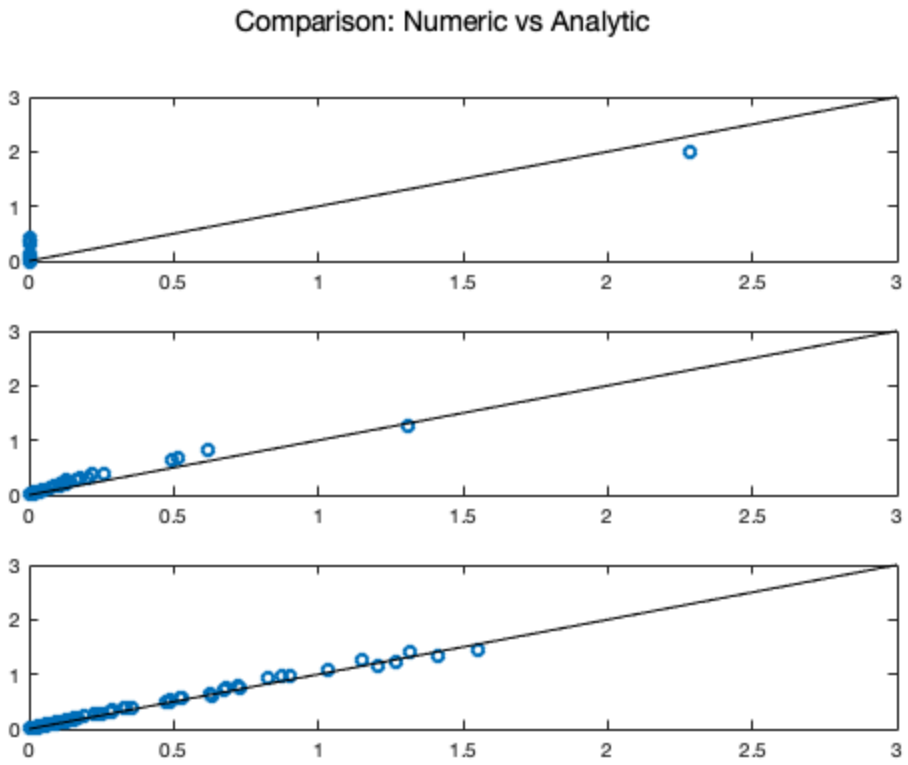

```
         0     0.00033184       0.010876     3.4894e-05      0.0011124
  0.014625
         0     0.00042416       0.012644     5.0013e-05      0.0013639
  0.016866
         0      0.0012057       0.026455      0.00019789     0.0034332
  0.034009
         0     0.00028138      0.0094678     2.9289e-05      0.0009494
  0.012826
         0      0.0064354       0.084325      0.0013308      0.015564
   0.10036
         0       0.001118       0.023566      0.00017728     0.0031594
  0.030122
         0      0.0032479       0.064151      0.00048014     0.009228
  0.079642
         0     0.00073617       0.018902     9.2002e-05      0.0022605
  0.024674
         0       0.012559        0.10958      0.0030767       0.02641
   0.12434
         0      0.0056193       0.079106      0.0011144      0.01392
  0.09512
         0      0.0076804        0.10682      0.0014534      0.019251
   0.12654
         0      0.0024312       0.044337      0.00042385     0.0065554
  0.05529
         0     0.00052736       0.014445     7.1519e-05      0.0016326
  0.019105
         0      0.0016906       0.024462      0.00034862     0.004154
  0.029979
         0     0.00077031       0.023178     8.7055e-05      0.0025089
  0.030762
         0        0.11953        0.72505       0.051785       0.21366
   0.78843
         0       0.042988        0.33742       0.013415       0.086791
   0.37287
         0      0.0030344       0.054733      0.00048368     0.0083045
   0.06735
         0       0.031644        0.24754       0.012651       0.060426
   0.28391
         0       0.014945        0.11492         0.00573      0.028754
   0.13027
         0     0.00036008      0.0084046     5.8362e-05      0.0010374
  0.010999
         0     0.00019466      0.0068255      2.335e-05     0.00065409
 0.0093477
         0     9.3081e-05      0.0039714      8.942e-06     0.00033867
 0.0055646
         0     0.00062167        0.01347     0.00010674      0.0017419
  0.017458
         0     0.00022966      0.0055085     3.6286e-05     0.00067086
  0.007208
         0     0.00036931       0.010201     4.9927e-05      0.0011477
  0.013489
         0     0.00042808       0.011267      6.264e-05      0.0012969
  0.014876
```

```
         0    0.00019782     0.0049235    3.2262e-05     0.0005759
0.0065542
         0    0.00019253     0.0051006    2.9639e-05    0.00057448
0.0068348
         0    7.6667e-06    0.00034119    7.2272e-07    2.7812e-05
0.00048915
         0    0.00017346     0.0050243    2.4129e-05    0.00054087
0.0067532
         0    6.1841e-05     0.0016598    9.5897e-06    0.00018444
0.0022416
         0      0.012452      0.12883     0.0026825      0.028501
  0.14576
         0     0.0034034      0.038989     0.0008571     0.0076227
 0.046555
         0    0.00066162      0.015087    9.7914e-05      0.001921
  0.01954
         0       0.12567      0.82371      0.054546       0.22724
  0.91733
         0        0.1296      0.62618      0.061297       0.21711
  0.65334
         0    1.0827e-05    0.00045578    9.2969e-07    3.9785e-05
0.00063402
         0     0.0058265      0.086423    0.00099978      0.015043
   0.1031
         0      0.030841      0.22541      0.012895      0.057426
  0.25667
         0     0.0044861      0.042862     0.0017188     0.0089908
 0.051281
         0     0.0016777      0.031345    0.00030058     0.0045201
 0.039483
         0     0.0002121     0.0050799    3.2518e-05    0.00062114
0.0066484
         0     0.0016555      0.030348    0.00030278     0.0044176
 0.038232
         0    0.00024614     0.0078845    2.9492e-05    0.00080702
 0.010653
         0    0.00041208      0.010344    6.4477e-05     0.0012129
  0.01371
         0    0.00041921      0.014846    3.9214e-05     0.0014666
 0.019958
         0      0.019354      0.11848     0.0089089      0.033935
  0.13168
         0      0.019283       0.1446     0.0068008      0.037623
  0.16085
         0     0.0051427      0.038011     0.0021258     0.0095998
 0.043525
         0    0.00017288     0.0054855    2.2185e-05    0.00055888
0.0074441
         0    0.00085786      0.021738    0.00011461     0.0026189
 0.028199
         0    0.00066119      0.020082    7.5626e-05     0.0021633
 0.026597
         0    0.00024223     0.0081428    2.8599e-05    0.00080676
 0.011071
```

```
         0    6.3804e-05     0.0017108    1.0638e-05    0.00018796
0.0023219
         0    0.00040039      0.010633    5.3735e-05     0.0012288
 0.01402
         0    0.00054449      0.013935    8.2394e-05     0.0016248
0.018417
         0    0.00028812      0.009045    3.5005e-05     0.0009339
0.012226
         0    0.00088947      0.018128    0.00016407     0.0024277
0.023339
         0     0.0001178     0.0037161    1.5968e-05    0.00037631
0.005074
         0     0.0004294      0.010847    6.6216e-05     0.0012709
0.014351
         0     0.0026607      0.044036    0.00051532     0.0068617
0.054633
         0    6.2855e-05      0.002611     6.302e-06    0.00022489
0.0036662
         0       0.00901       0.11268     0.0019527      0.021411
 0.13302
         0    0.00042956      0.008287    7.7921e-05     0.0011583
0.010583
         0    0.00081227       0.01904    0.00012498     0.0023747
0.024681
         0    0.00020513     0.0070977     2.419e-05    0.00068729
0.0097196
         0    5.6775e-05     0.0020936    6.1107e-06    0.00019427
0.0029017
         0    3.4974e-05     0.0014095     3.535e-06    0.00012432
0.0019676
         0    0.00025993     0.0081706    3.4936e-05    0.00083357
 0.011092
         0    0.00068515      0.017985     8.794e-05     0.0021162
0.023484
         0    0.00048556      0.014363    5.9672e-05     0.0015555
0.019111
         0    0.00087067      0.016287    0.00017482     0.0022829
0.020899
         0    0.00065323      0.012342    0.00013474     0.0016989
0.015999
         0    8.9794e-05      0.002974    1.3297e-05    0.00028489
0.0041414
```




```
end
```

3. [Numeric] NDMwC, fmincon minimizing MSE, no analytic counterpart

```
% Set ndmwc_comp = 1 to run
ndmwc_comp = 1;

if ndmwc_comp ==1

% Parameters to fit
    % param(1) = beta
    % param(2) = x0_value
    % param(3) = alpha0  cte growth/decay term independent of x

% Extra input to required by objective function
    % seed_location
    % pathology
    % time_stamps
    % C

% Set initial guess for beta
    init_guess_params(1) = 1;

% Set initial guess for x0_value
```

```
    init_guess_params(2) = nansum(pathology(:,1));

% Set initial guess for alpha1 and alpha2
    init_guess_params(3) = .5;

% Set lower bounds (lb) for parameters
    lb = [0,0,-3];

% Set upper bounds (ub) for parameters
%    ub = [10,nansum(pathology(:,end)),5];
    ub = [3,3,3];

% Apply fmincon w/ numeric
    [param_num, fval_num] =
 fmincon(@(param)objfun_NDMwC_numeric(param,seed_location,pathology,time_stamps,C)
                           init_guess_params,[],[],[],[],lb,ub,[]);

% Solve NDMwS with the optimal parameters
    beta_num = param_num(1);
    x0_num = seed_location*param_num(2);
    alpha0_num = param_num(3);
    ynum = NDMwC_numeric(x0_num,time_stamps,C,beta_num,alpha0_num);
```

```
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the default value of the step size tolerance and constraints are
satisfied to within the default value of the constraint tolerance.
```

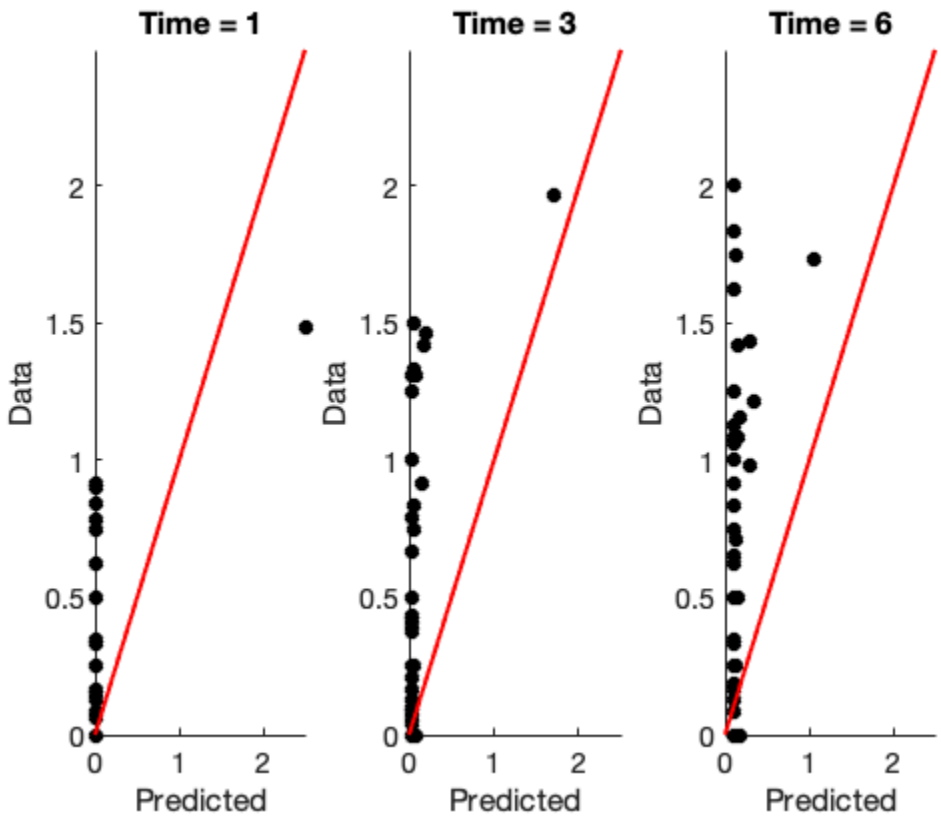
Display Numeric Results

Save Rvalues in a matrix

```
            for jj = 1:length(time_stamps)
                 Rvalues(jj,:) =
 corr(ynum(:,jj),pathology(:,jj), 'rows','complete');
            end

 % Display results
    disp('--------------------------------------------------')
    disp('NDMwC minimizing quadratic error at all time stamps with
 fmincon');
    disp(' ')
    disp(['NUMERIC Beta = ' num2str(param_num(1)) ', x0 value = '
 num2str(param_num(2)) ...
        ', alpha0 = ' num2str(param_num(3))])
    disp(' ')
    disp(['R values at each time stamp'])
    disp(Rvalues)
    disp(' ')
```

```
    disp('Square error')
    disp(fval_num)

% Plot prediction vs data using optimal parameters
plot_pred_vs_data(ynum,pathology,time_stamps)
clearvars y
```

```
--------------------------------------------------
NDMwC minimizing quadratic error at all time stamps with fmincon

NUMERIC Beta = 0.33666, x0 value = 2.4958, alpha0 = 0.020384

R values at each time stamp
    0.5415
    0.4806
    0.4218


Square error
   55.0033
```




```
end
```

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