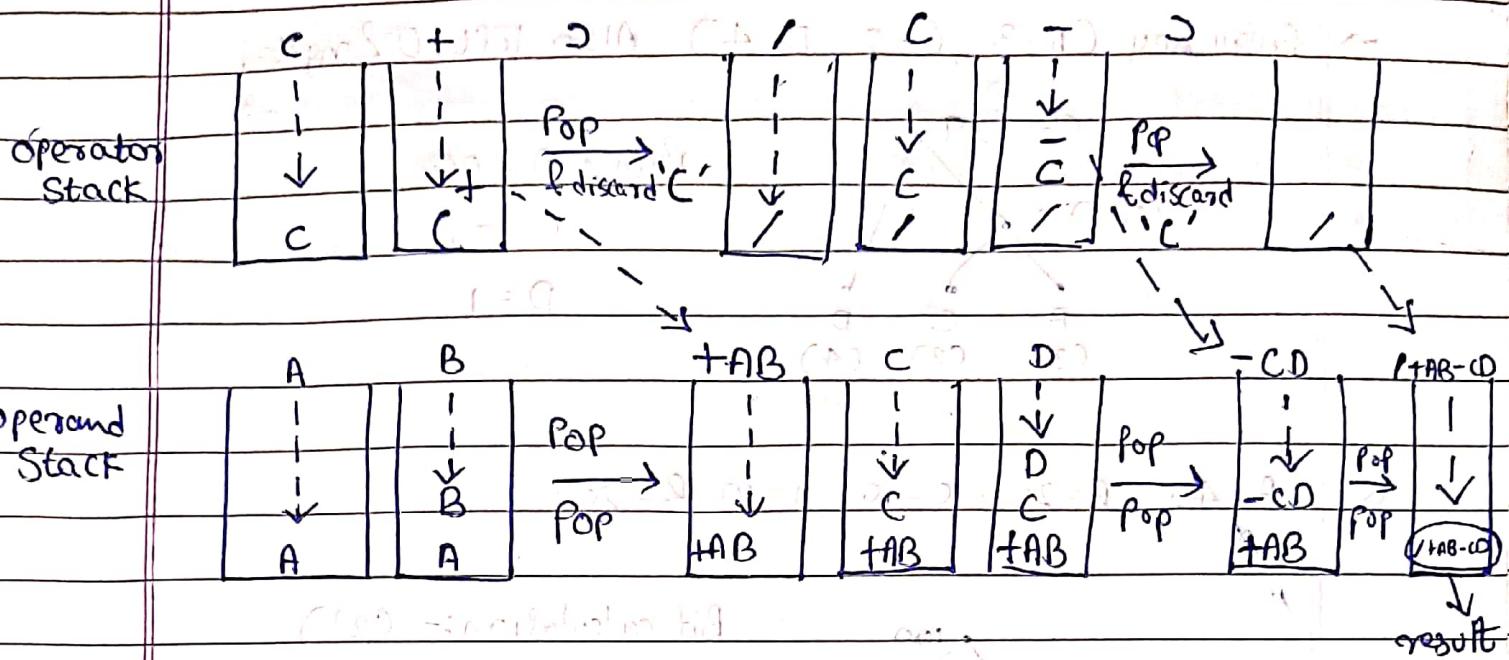


# Data Structures

(\*) Infix  $\rightarrow$  Prefix

$(A+B)/CC-D)$



(\*) Trees

Full Binary Tree  
(Left, Right, Root)

→ Recursive Functions

Preorder

Input:  $x$  is root

if  $x \neq \text{null}$

then O/P key( $x$ );  
preorder(left( $x$ ));  
preorder(right( $x$ ));

PostOrder

I/P:  $x$  is root

if  $x \neq \text{null}$

then postorder(left( $x$ ));  
postorder(right( $x$ ));  
O/P key( $x$ );

Inorder

I/P:  $x$  is root

if  $x \neq \text{null}$

then inorder(left( $x$ ));  
O/P key( $x$ );  
inorder(right( $x$ ));

→ Non-Recursive Functions

Inorder

```
do {
    while( $P \neq \text{null}$ )
        if push(stack,  $P$ );
         $P = P \rightarrow \text{left}$ ;
    if (!empty(stack))
         $p = pop(stack)$ ;
        print( $P \rightarrow \text{info}$ );
         $P = P \rightarrow \text{right}$ ;
} while(!empty(stack))
||  $p \neq \text{null}$ 
```

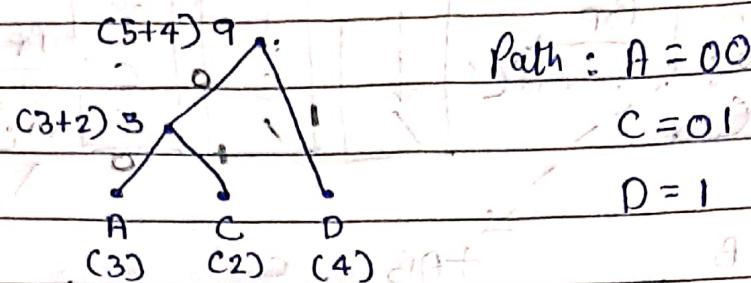
Preorder

```
do {
    while( $P \neq \text{null}$ )
        print( $P \rightarrow \text{info}$ );
        push(stack,  $P$ );
         $P = P \rightarrow \text{left}$ ;
    if (!empty(stack))
         $p = pop(stack)$ ;
         $P = P \rightarrow \text{right}$ ;
} while(!empty(stack))
||  $p \neq \text{null}$ 
```

## \* Huffman's Compression Algo

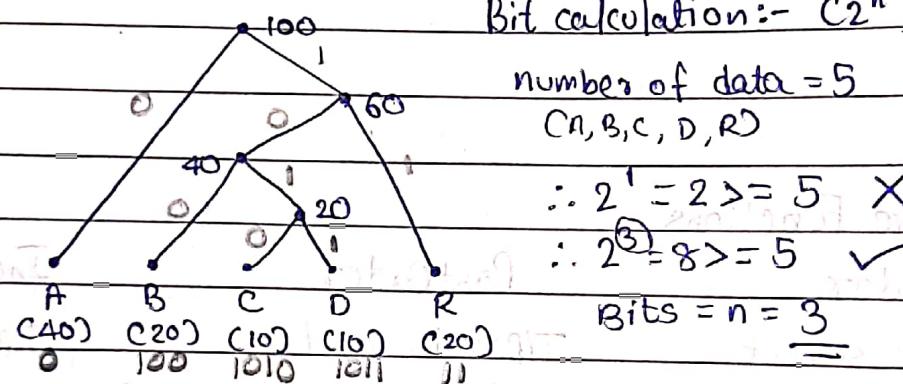
Q AACCDADD

→ frequency (A=3, C=2, D=4) Also left=0 & right=1



Q A=40 B=20 C=10 D=10 R=20

Bit calculation:-  $C2^n$ )



number of data = 5

(A, B, C, D, R)

$$\therefore 2^1 = 2 >= 5 \times \text{X}$$

$$\therefore 2^3 = 8 >= 5 \checkmark$$

$$\text{Bits} = n = 3$$

Data Table 1

Data Table 2

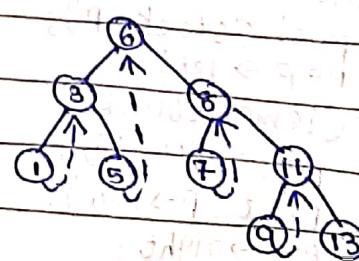
letter	freq	bits	freq * bits
A	40	3	120
B	20	3	60
C	10	3	30
D	10	3	30
R	20	3	60
			300

letter	freq	path	bits	freq * path
A	40	1	1	40
B	20	3	3	60
C	10	4	4	40
D	10	4	4	40
R	20	2	2	40
				220

$$\text{Compression \%} = \frac{\text{change}}{\text{original}} \times 100 = \frac{300 - 220}{300} \times 100 = 26.67\%$$

## \* Threaded Tree

```
//Initialize ptr to the root
while (ptr != null)
{
    while (ptr->left != null)
    {
        ptr = ptr->left;
        print (ptr->info);
        while (ptr->right == thread)
        {
            ptr = ptr->right;
            print (ptr->info);
            ptr = ptr->right;
        }
    }
}
```



O/P:- 1356789||13

### (\*) Linear Time Partitioning

→ Partition ( $A, p, r$ )

// where  $r = \text{last value in } A[r]$

$$x = A[r]$$

$$i = p - 1$$

$$j = r + 1$$

while TRUE

$$\text{repeat } j = j - 1$$

$$\text{until } A[i] \leq x$$

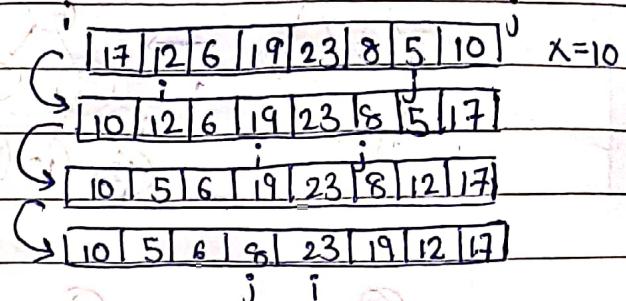
$$\text{repeat } i = i + 1$$

$$\text{until } A[i] > x$$

$$\text{if } i < j \quad // i \text{ is left of } j$$

then exchange  $A[i] \leftrightarrow A[j]$

else return  $j$

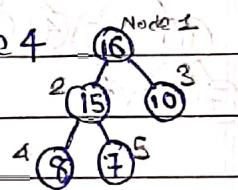


### (\*) Heap Sort & Heapify Algo (Parent Node $\geq$ Child Node)

→ Left of node  $i = 2i$  Eg: Left of Node 2 = Node 4

→ Right of node  $i = 2i + 1$

→ Parent of node  $i = i/2$



→ Heap Sort Algo :-

Parent( $i$ )

return  $i/2$

left( $i$ )

return  $2i$

right( $i$ )

return  $2i + 1$

Heap Property

$A[\text{parent}(i)] \geq A[i]$

→ Heapify Algo :- ( $n = \text{no of elements}$ )

1) Heapify ( $A, i$ )

2) Left & right sub-tree of  $i$  are heaps

3) Make sub-tree rooted at  $i$  a heap

4)  $l \leftarrow \text{left} \quad l = 2i$

5)  $r \leftarrow \text{right} \quad r = 2i + 1$

6) if  $l \leq n \text{ and } A[l] > A[i]$

then largest  $\leftarrow l$

7) else largest  $\leftarrow i$

8) if  $r \leq n \text{ and } A[r] > A[\text{largest}]$

then largest  $\leftarrow r$

9) if largest  $\neq i$  then exchange  $A[i] \leftrightarrow A[\text{largest}]$

10) Heapify ( $A, \text{largest}$ )

## Heap Sort (4)

- 1) Build-heap (A)
- 2) for  $i \leftarrow n$  down to 2
  - a) exchange  $A[i] \leftrightarrow A[1]$
  - b)  $n \leftarrow n - 1$
  - c) Heapify ( $A, 1$ )

## \* Radix Sort & Radix Sort Reverse

61, 12, 1, 342, 241, 51, 18, 9

Radix sort from left to right i.e 61 to 9 &

Radix sort reverse from right to left // gives ascending order

061, 012, 001, 342, 241, 051, 018, 009 // unit's place

061	001	241	342	012	051	018	009
0	1	2	3	4	5	6	7

Pop  $\rightarrow$  061, 001, 241, 051, 012, 342, 018, 009 // ten's place

001	012	342	241	051	061
0	1	2	3	4	5

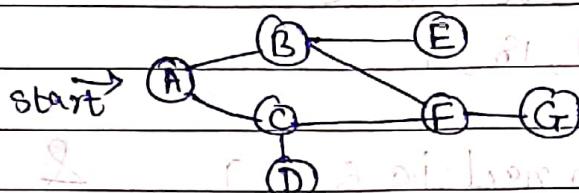
Pop  $\rightarrow$  001, 009, 012, 018, 241, 342, 051, 061 // Hundred's place

004							
012							
018							
051							
061							
0	1	2	3	4	5	6	7

Pop  $\rightarrow$  001, 009, 012, 018, 051, 061, 241, 342

### (\*) BFS Algo

→ while queue Q not empty  
 dequeue first vertex v from Q  
 for each vertex directly reachable from v  
 if v is unvisited  
 enqueue v to Q  
 mark v as visited



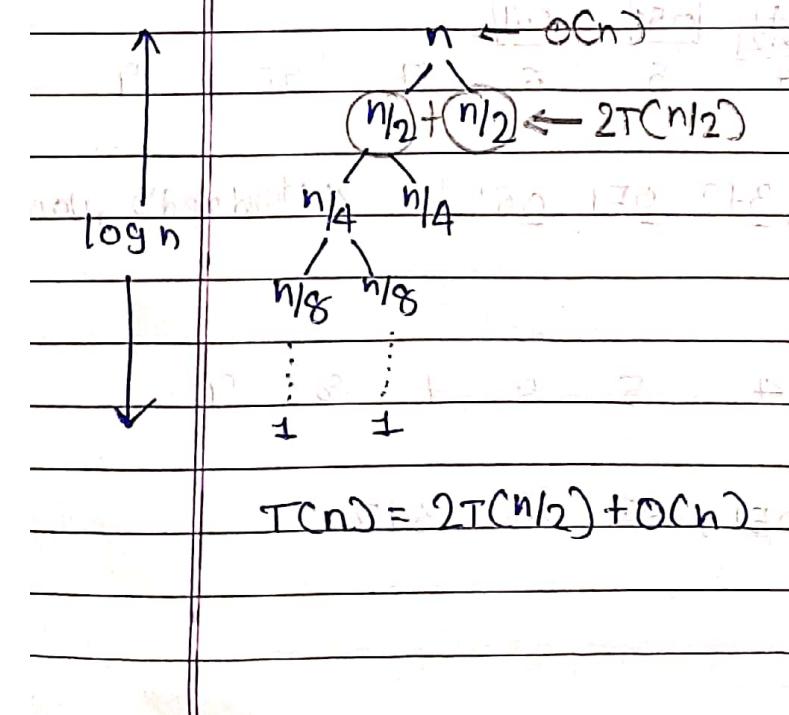
O/P:-

A B C E F D G

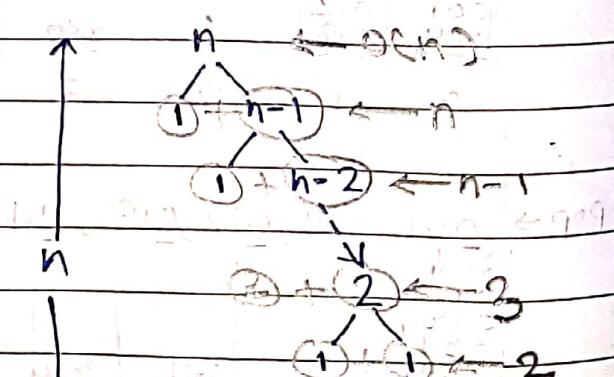
### (\*) Quick Sort

→ Initial call Quicksort(A, p, r)  
 if  $p < r$   
 then  $q \leftarrow \text{partition}(A, p, r)$   
 $T(n) = T(p, q) + T(q+1, r)$

Best Case (Even elements)



Worst Case



Complexity =  $O(n^2)$

### (\*) DFS Algo

DFS (vertex u)

mark u as visited

for each vertex v directly reachable from u

if v is unvisited

DFS Cv

### (\*\*) Bubble Sort Algo

// Two pointers

→ 60 42 42 42 42

→ 42 → 60 60 60 60 First Pass Complete

75 → 75 → 75 75 75

83 83 → 83 → 83 27

27 27 27 → 27 83

Result after

Second pass

" Third pass.

" Fourth Pass

42

42

27

60

27

42

27

60

60

75

75

75

83

83

83

done

### (\*) Selection Sort

(\*)

14 33 27 10 35

// firstly scan all elements and exchange min val with first ele.

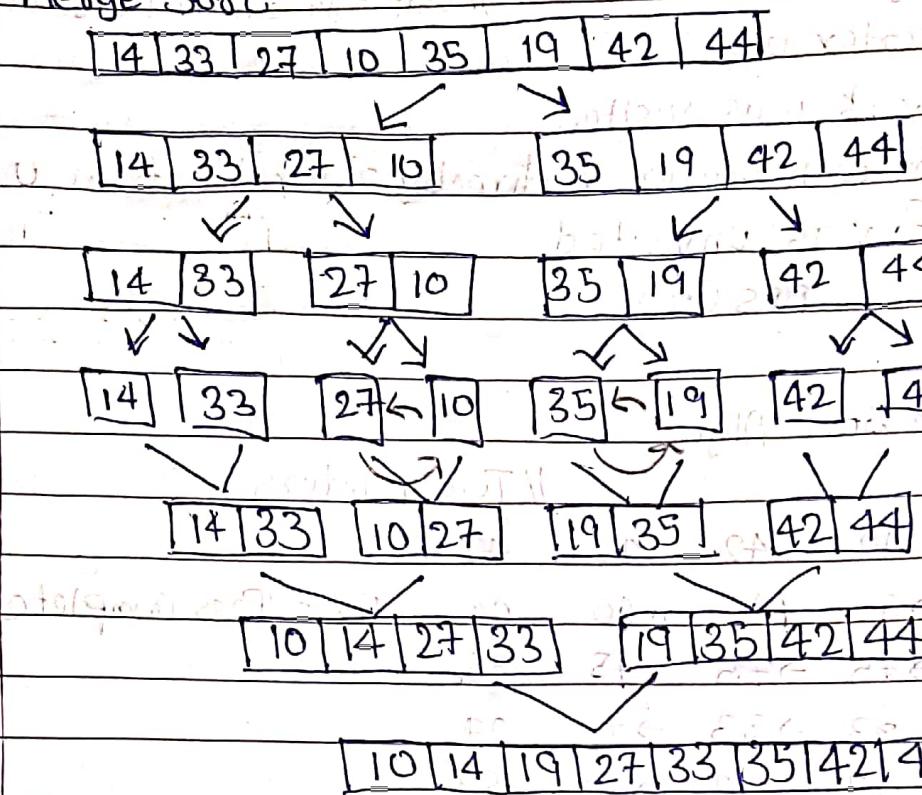
10 33 27 14 35

// ptr moves to second ele  
scan min value & exchange

10 27 14 27 33 35

ptr

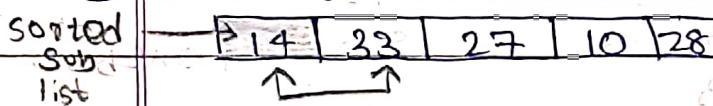
### (\*) Merge Sort



Algo :- Divide list into 2 parts till it can no more be divided

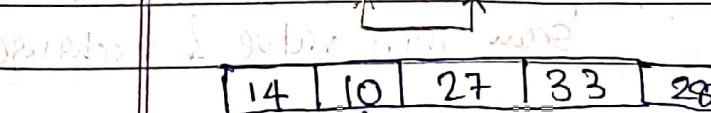
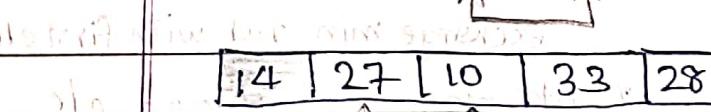
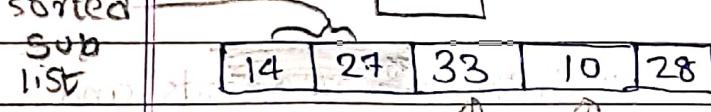
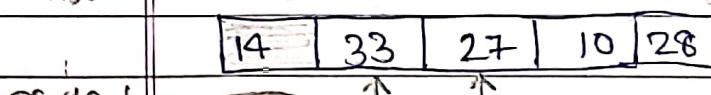
Merge smaller lists into sorted order.

### (\*) Insertion Sort:-



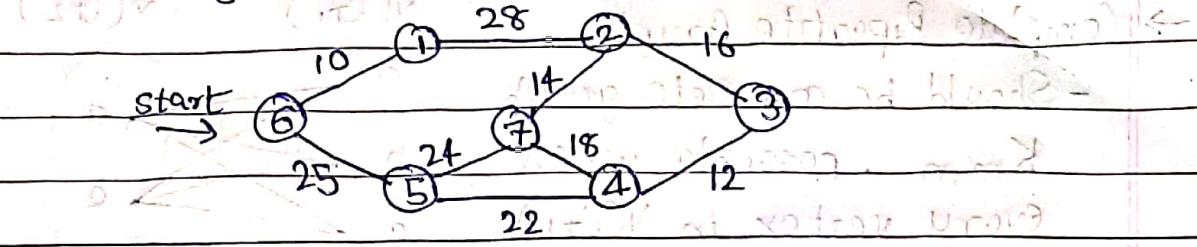
Algo:-

- Sort first 2 elements
- Go to next 2 elements
- Compare with all elements in sorted-sub list
- Repeat until sorted.

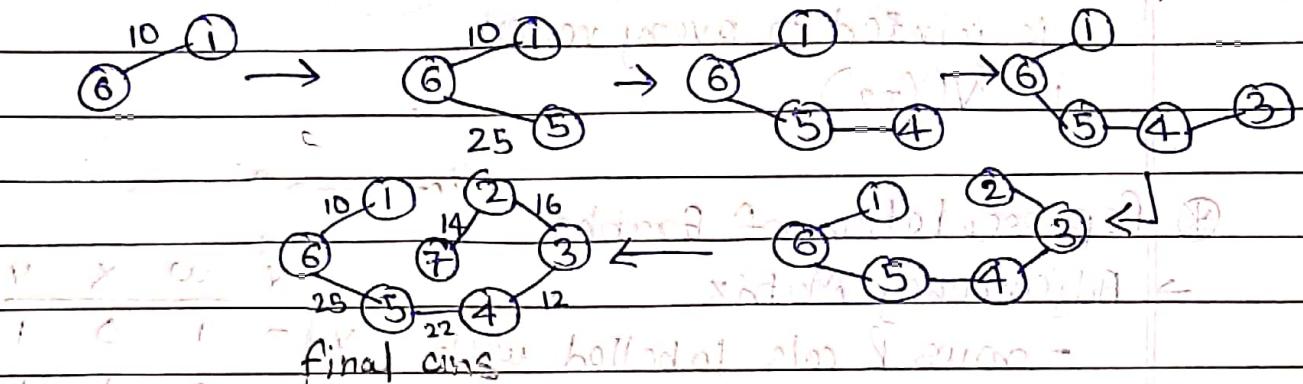




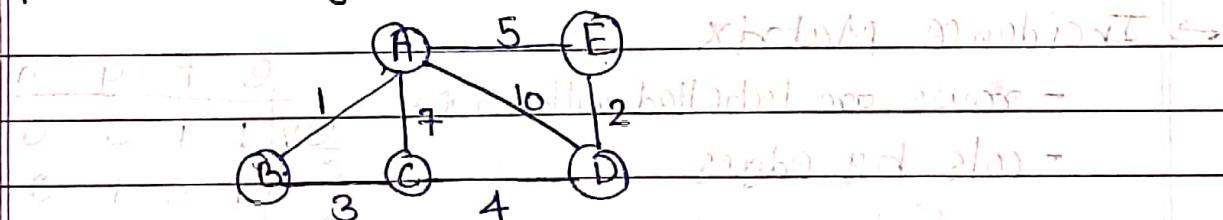
## Prim's Algorithm



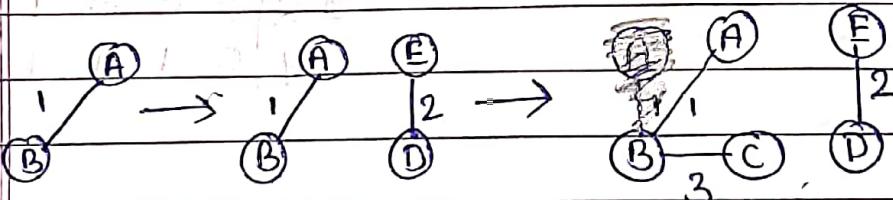
Ans.



## Kruskal's Algo



Ans.



// Since all nodes are covered we stop

$$\text{Min cost} = 1 + 2 + 3 = \underline{6}$$



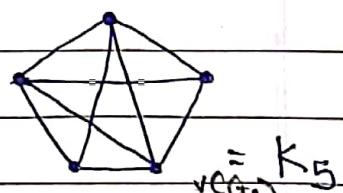
## Graphs

→ Complete graph ( $K_n$ )

- every pair of vertices

is joined by an edge

EG :-



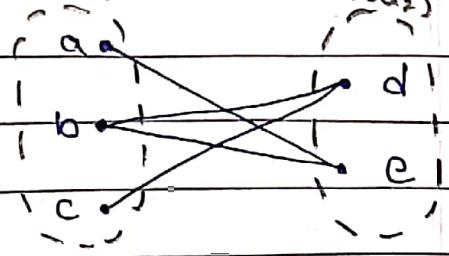
→ Bipartite Graphs

-  $V(G) = V(G_1) \cup V(G_2)$

$|V(G_1)| = m$ ,  $|V(G_2)| = n$

$V(G_1) \cap V(G_2) = \emptyset$

- No edges exists b/w any 2 vertices in same subset.  
Ex:- Edge b/w a & b should not exist.

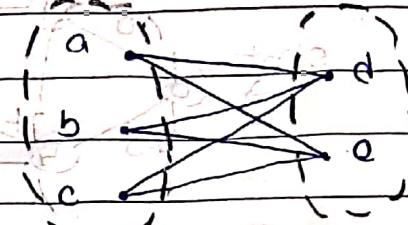


→ Complete Bipartite Graph

- Should be complete graph

$K_{m,n}$ , possible when every vertex in  $V(G_1)$

$$V(G_1) \quad V(G_2)$$



is jointed to every vertex in  $V(G_2)$ .

\* Representation of Graphs

→ Adjacency Matrix

- rows & cols labelled with

ordered vertices

	v	w	x	y
v	0	1	0	1
w	1	0	1	1
x	0	1	0	1
y	1	1	1	0

→ Incidence Matrix

- rows are labelled with vertices

- cols by edges

v -> e

v -> w

f -> e

f -> w

y -> e

y -> w

y -> x

g -> x

g -> w

h -> x

h -> w

h -> y

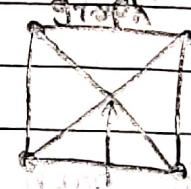
	e	f	g	h	j
e	1	1	0	0	0
w	1	0	1	0	1
g	0	0	0	1	1
y	0	1	1	1	0



Planar Graphs :-

- A graph is planar if it can be drawn in the plane without crossing-edges.

Not a planar graph



It is a planar graph



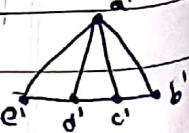
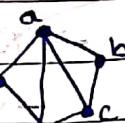
Isomorphic Graphs :-

- If there exists one-to-one onto functions  $f: V(G_1) \rightarrow V(G_2)$

&  $g: E(G_1) \rightarrow E(G_2)$

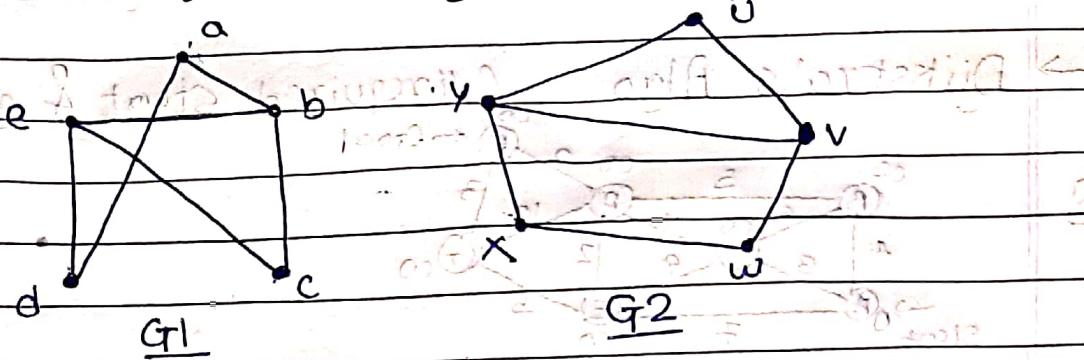
such that

- edge  $e$  is adjacent to vertices  $v, w$  in  $G_1$  if and only if  $g(e)$  is adjacent to  $f(v)$  &  $f(w)$  in  $G_2$



## \* Isomorphic Graphs with adjacency matrix

Q Check whether following are isomorphic graphs:-



Ans. No of vertices in  $G_1 = 5$  in  $G_2 = 5$   satisfied  
 No of edges in  $G_1 = 9$  in  $G_2 = 9$   satisfied

degree sequence  $G_1: 2, 3, 2, 2, 3$   $G_2: 2, 3, 2, 2, 3$

Mapping :- we have 2 two degree 3 vertices i.e. e, b & y, v  
 lets map  $e \leftrightarrow y$  &  $b \leftrightarrow v$

- Now check neighbour's degree  $w$  has 2 neighbours with degree 2 & 3 same for  $x$ . Similarly in graph  $G_1$  we have  $d$  &  $a$ .

- But  $u$  &  $c$  has two neighbours with degree 3  
 Hence map  $u \leftrightarrow c$ . So remaining  $w \leftrightarrow a$  &  $x \leftrightarrow d$

- Sort according to mapping  $\therefore a \ b \ c \ d \ e$

- New graphs :-

$G_1$	$G_2$

- Get adjacency Matrix :-

$G_1:$	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$v_1$	0	1	0	1	0
$v_2$	1	0	1	0	1
$v_3$	0	1	0	0	1
$v_4$	1	0	0	0	1
$v_5$	0	1	1	1	0

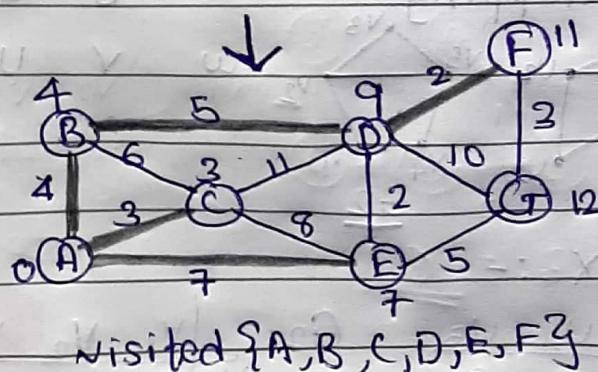
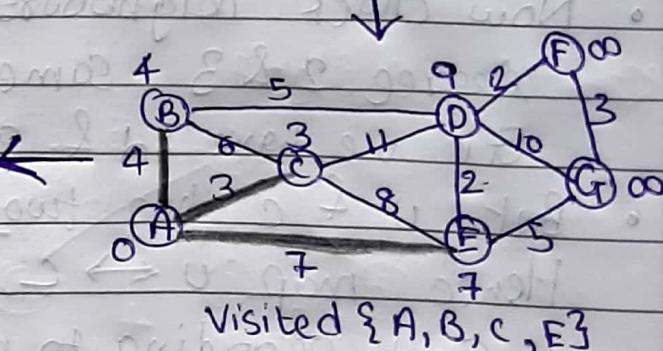
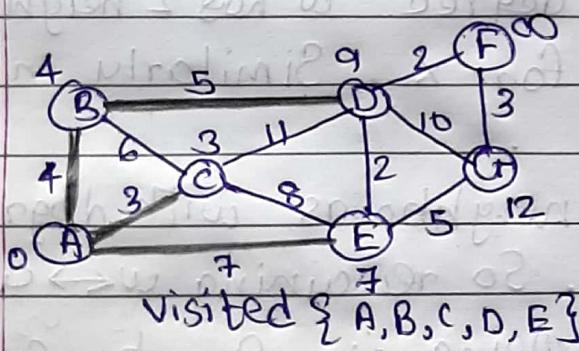
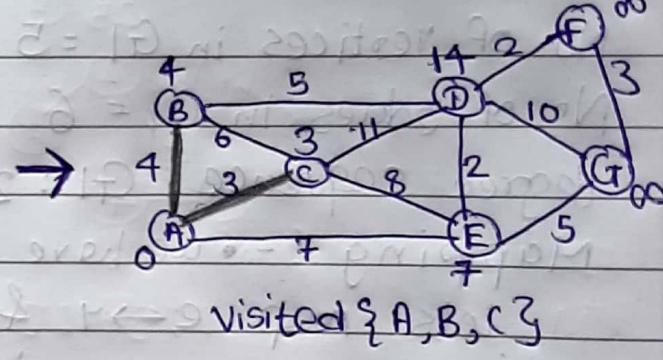
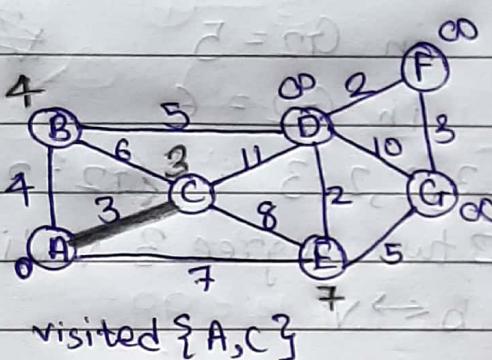
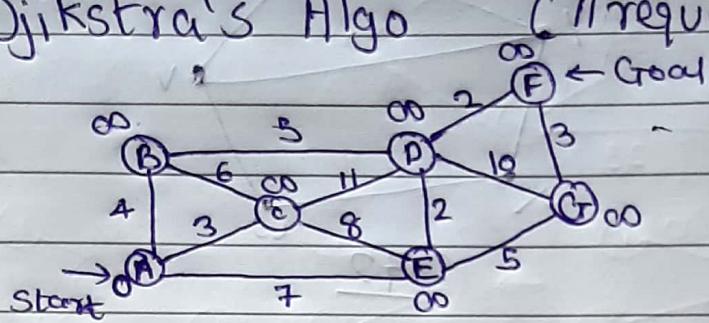
$G_2:$	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$v_1$	0	1	0	1	0
$v_2$	1	0	1	0	1
$v_3$	0	1	0	0	1
$v_4$	1	0	0	0	1
$v_5$	0	1	1	1	0

Since adj matrix for  $G_1$  &  $G_2$  are same

$\therefore G_1$  &  $G_2$  are isomorphic Graphs.

## \* Shortest path Algos :-

→ Dijkstra's Algo ( // required start & goal node )



Shortest path = A - B - D - F  
= 11 units