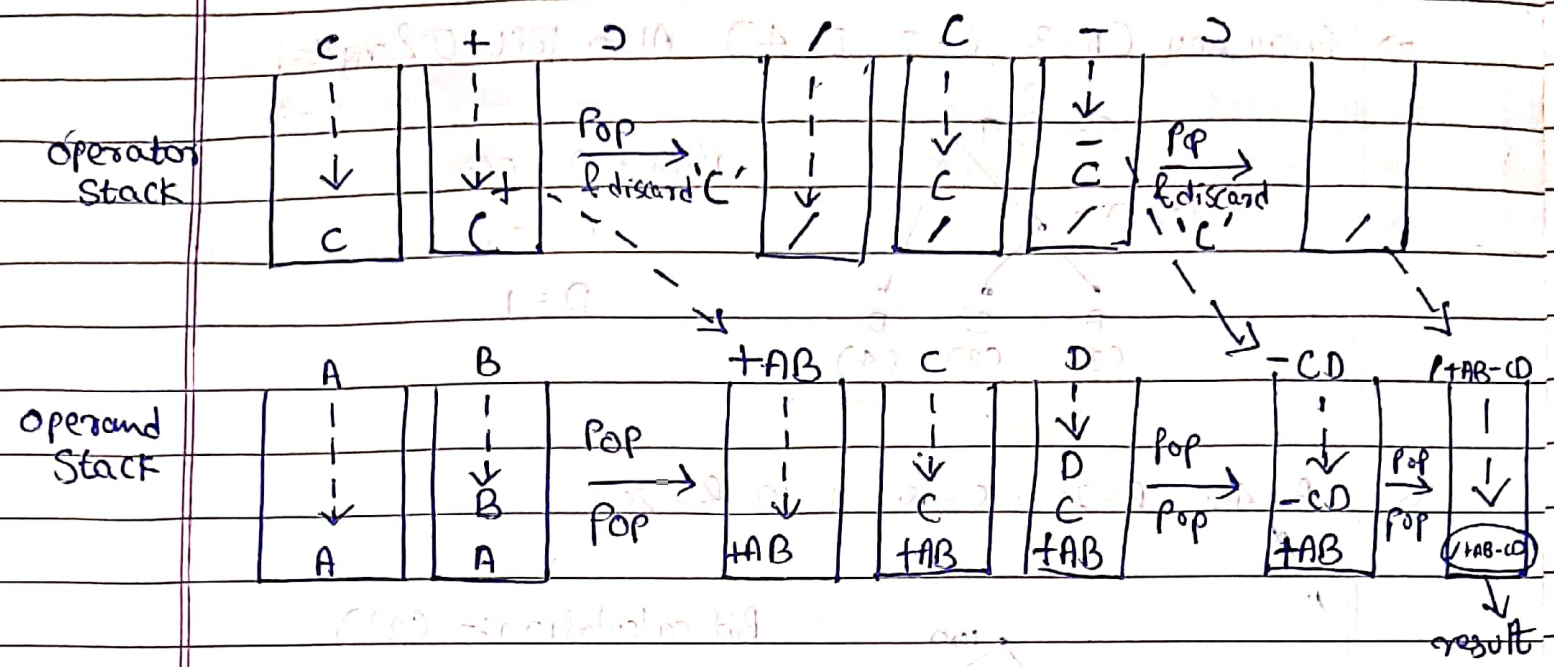
Data Structures

Infix → Prefix (A+B) / (C-D)



Trees

→ Recursive Functions

Preorder	PostOrder	Inorder

Input: x is root
if x ≠ null
then o/p key(x);
preorder(left(x));
preorder(right(x));
	I/p: x is root
	if x ≠ null
	then postorder(left(x));
	postorder(right(x));
	o/p key(x);
		I/p: x is root
		if x ≠ null
		then Inorder(left(x));
		o/p key(x);
		Inorder(right(x));

→ Non-Recursive Functions

Inorder

```
do{
 while(P!=null)
 { push(stack,P);
   P=P->left;
 }
 if(!empty(stack))
 { p=pop(stack);
   print(P->info);
   P=P->right;
 }
}while(!empty(stack)
   || P!=null)
```

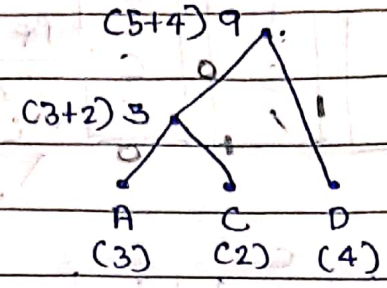
Preorder

```
do{
 while(P!=null)
 { print(P->info);
   push(stack,P);
   P=P->left;
 }
 if(!empty(stack))
 { P=pop(stack);
   P=P->right;
 }
}while(!empty(stack)
   || P!=null)
```

(*) Hoffman's Compression Algo

Q AACCDDADD

→ frequency (A=3, C=2, D=4) Also left=0 & right=1

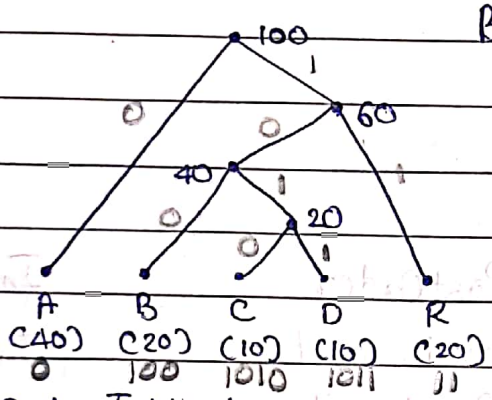


Path : A = 00

C = 01

D = 1

Q A=40 B=20 C=10 D=10 R=20



Bit calculation :- (2^n)

number of data = 5
(A, B, C, D, R)

$\therefore 2^1 = 2 >= 5$ ✗

$\therefore 2^3 = 8 >= 5$ ✓

Bits = n = 3

Data Table 1

letter	freq	bits	fre*bits
A	40	3	120
B	20	3	60
C	10	3	30
D	10	3	30
R	20	3	60
			300

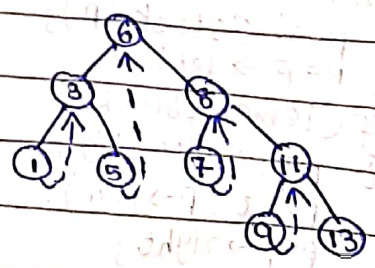
Data Table 2

letter	freq	path bits	freq*path
A	40	1	40
B	20	3	60
C	10	4	40
D	10	4	40
R	20	2	40
			220

Compression % = $\frac{\text{change}}{\text{original}} \times 100 = \frac{300-220}{300} \times 100 = 26.67\%$

(*) Threaded Tree

```
//Initialize ptr to the root
while(ptr!=null)
{ while(ptr->left!=null)
  { ptr = ptr->left; }
  print(ptr->info);
  while(ptr->right == thread)
  { ptr=ptr->right;
    print(ptr->info);
  }
  ptr = ptr->right;
}
```



O/P :- 1 3 5 6 7 8 9 11 13

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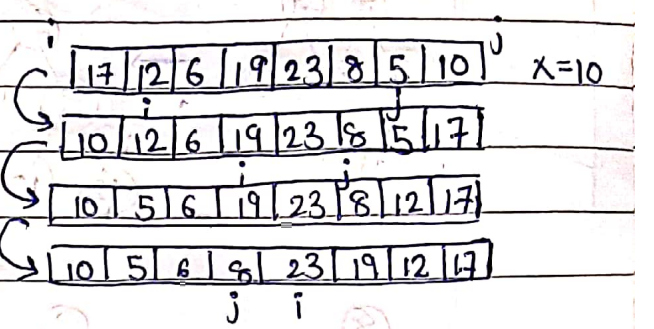
Linear Time Partitioning

→ Partition (A, p, r) // where r = last value in A[]

```
X = A[r]
i = p-1
j = r+1
while TRUE
    repeat j=j-1
    until A[j] ≤ X
    repeat i = i+1
    until A[i] > X
    if i<j   // i is left of j
     then exchange A[i] ↔ A[j]
    else return j
```

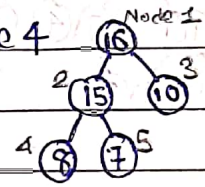


Heap Sort & Heapify Algo (Parent Node ≥ Child Node)

→ Left of node i = 2i Eg: Left of Node 2 = Node 4

→ Right of node i = 2i+1

→ Parent of node i = i/2



→ Heap Sort Algo :-

```
Parent(i)
    return (i/2)
left(i)
    return 2i
right(i)
    return 2i+1
```

Heap Property

A[parent(i)] ≥ A(i)

→ Heapify Algo :- (n = no of elements)

1) Heapify (A, i)
2) Left & Right sub-tree of i are heaps
3) Make sub-tree rooted at i a heap
4) l ← left l = 2i
5) r ← right r = 2i+1
6) if l ≤ n && A[l] > A[i]
   then largest ← l
7) else largest ← i
8) if r ≤ n && A[r] > A[largest]
   then largest ← r
9) if largest ≠ i then exchange A[i] & A[largest]
10) Heapify (A, largest)

Heap Sort (A)

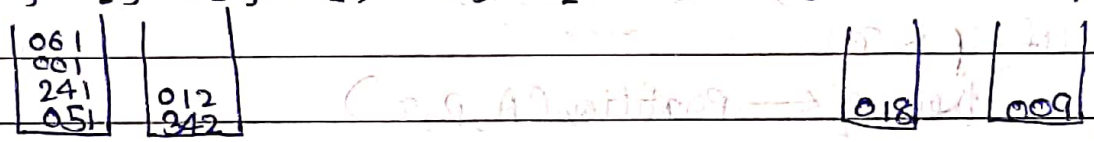
1) Build-heap (A)
2) for i ← n down to 2
3) exchange A[1] ↔ A[i]
4) n ← n-1
5) Heapify (A, 1)

⊛ Radix Sort & Radix Sort Reverse

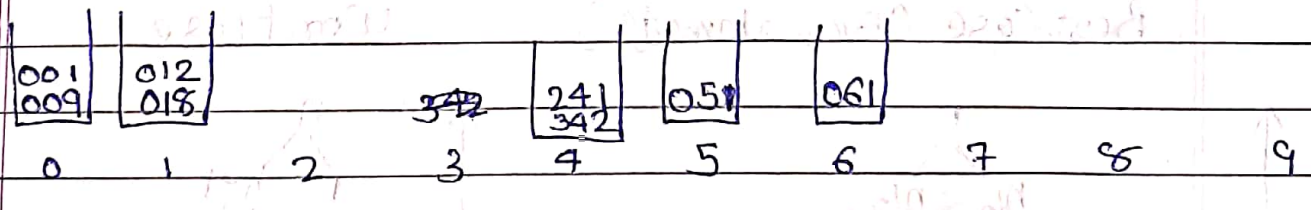
♡ 61, 12, 1, 342, 241, 51, 18, 9

Radix sort from left to right i.e 61 to 9 &
Radix sort reverse from right to left // gives ascending order

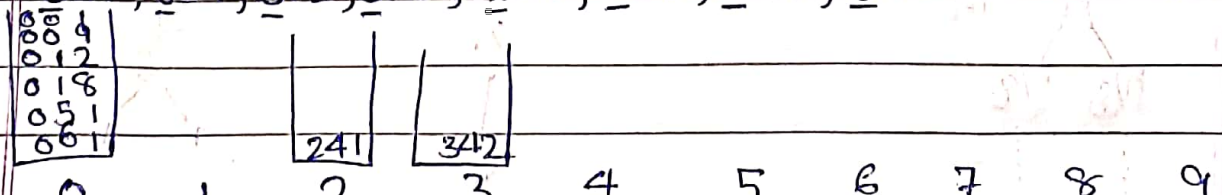
061, 012, 001, 342, 241, 051, 018, 009 // Unit's place



pop → 061, 001, 241, 051, 012, 342, 018, 009 // ten's place



pop → 001, 009, 012, 018, 241, 342, 051, 061 // Hundred's place



pop → 001, 009, 012, 018, 051, 061, 241, 342

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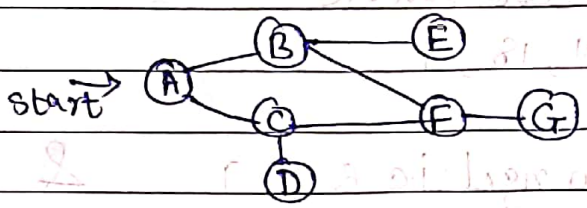
BFS Algo

→ while queue Q not empty
  dequeue first vertex u from Q
  for each vertex directly reachable from u
  if v is unvisited
    enqueue v to Q
    mark v as visited



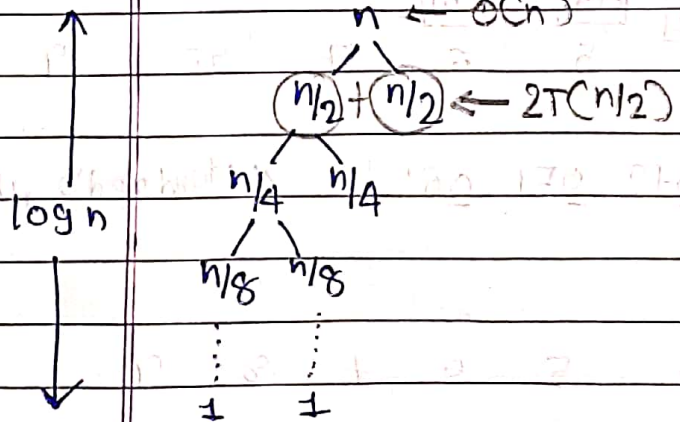
O/P :-

A B C E F D G

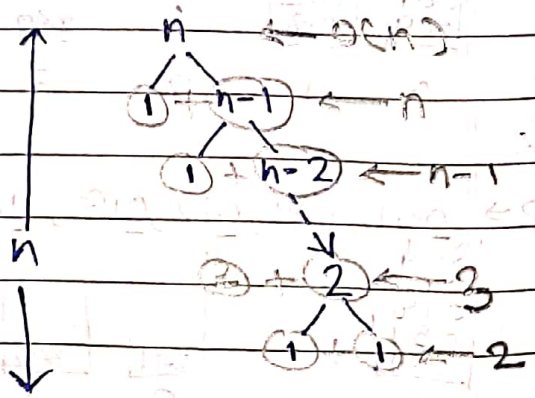
Quick Sort

→ Initial call QuickSort (A, p, r)
if p < r
  then q ← partition(A, p, r)
    QuickSort(A, p, q)
    QuickSort(A, q+1, r)

Best Case (Even elements)



Worst Case



Complexity = $O(n^2)$

$T(n) = 2T(n/2) + O(n)$

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DFS Algo

// ~~Initially all vertices are unvisited~~

```
DFS (vertex v)
{   mark v as visited
    for each vertex v directly reachable from v
    if v is unvisited
        DFS (v)
}
```

Bubble Sort Algo

// Two pointers

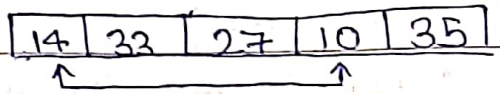
→	60	42	42	42	42	
→	42	→ 60	60	60	60	First pass complete
	75	→ 75	→ 75	75	75	
	83	83	→ 83	→ 83	27	
	27	27	27	→ 27	83	

Result after Second pass

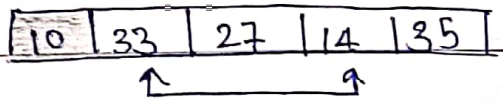
// Third pass	// Fourth Pass	
42	27	
27	42	
60	60	
75	75	Done
83	83	

Selection Sort

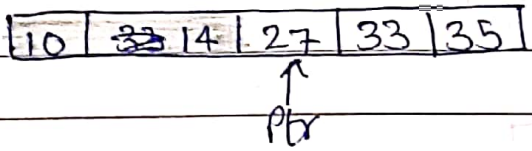
Q



// firstly scan all elements and exchange min val with first ele.



// ptr moves to second ele scan min value & exchange



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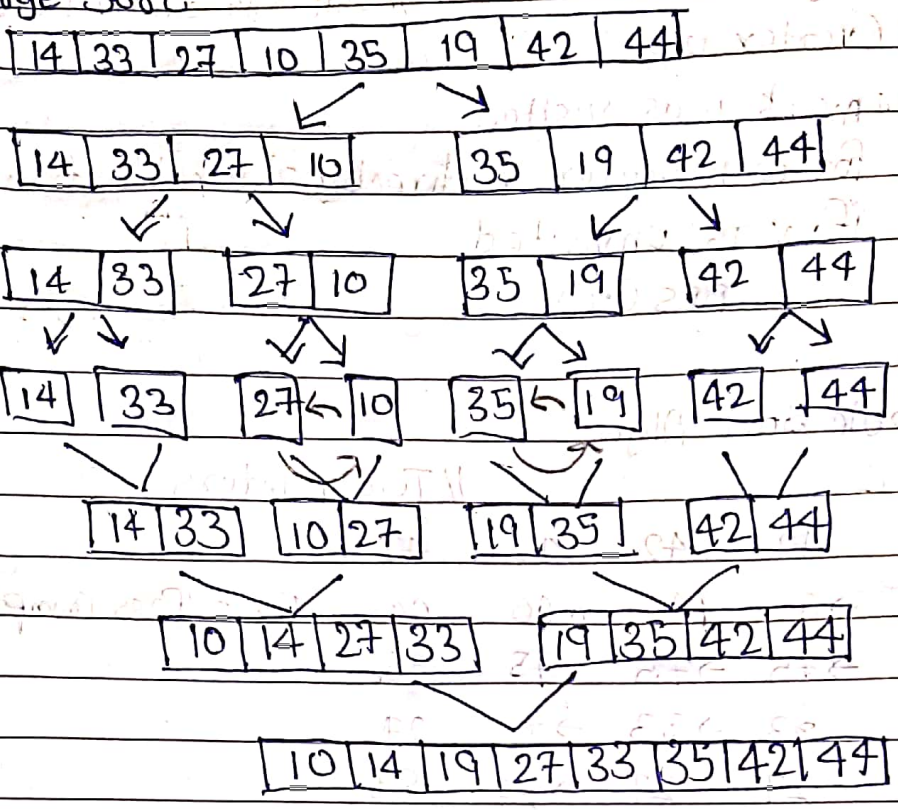
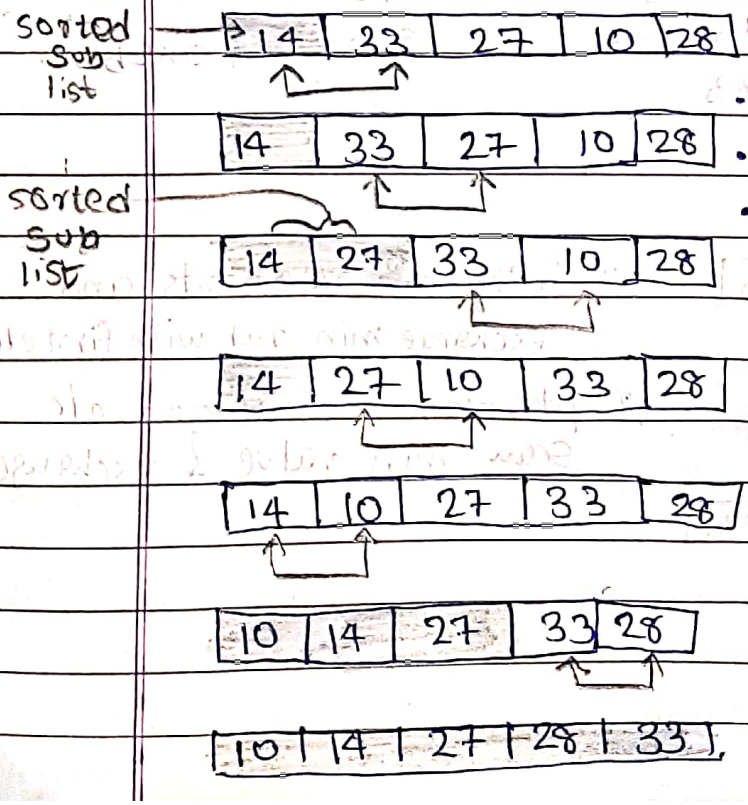
⊛ Merge Sort



Algo :- Divide list into 2 parts till it can no more be divided
Merge smaller lists into sorted order.

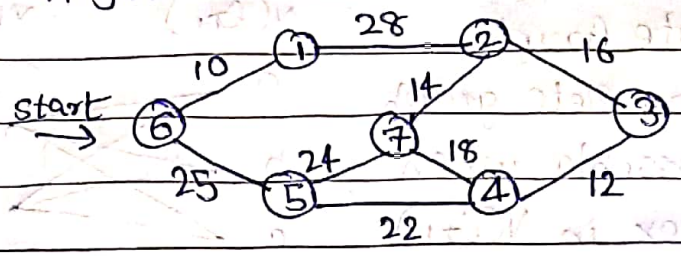
⊛ Insertion Sort :-



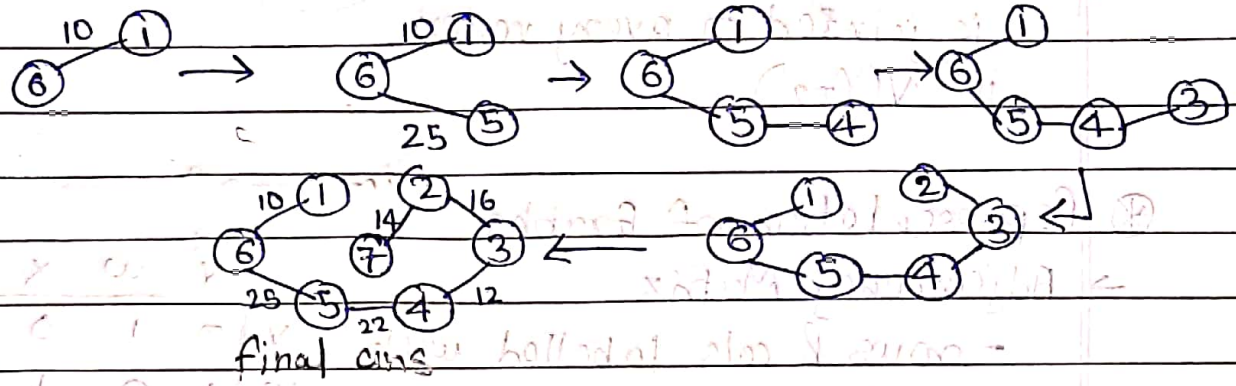
Algo :-
- Sort first 2 elements
- Go to next 2 elements
- Compare with all elements in sorted-sub list
- Repeat until sorted.

✱ Prim's Algorithm

Q



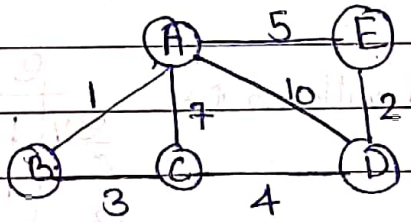
Ans.



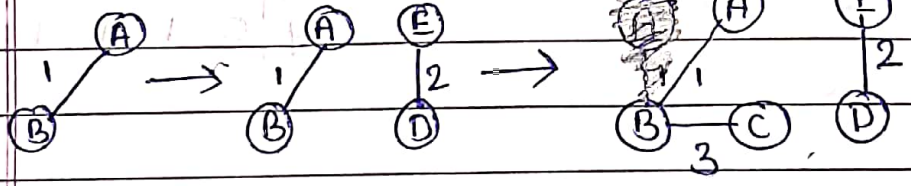
Min cost = 10+25+22+12+16+14 = 99

✱ Krushkal's Algo

Q



Ans.



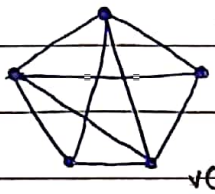
// Since all nodes are covered we stop

Min cost = 1+2+3 = 6

✱ Graphs

→ Complete graph (K_n)
- every pair of vertices is joined by an edge

Eg :-



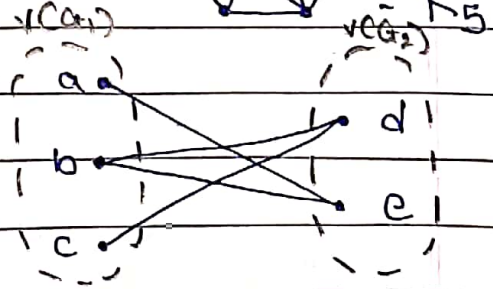
→ Bipartite Graphs

$V(G) = V(G_1) \cup V(G_2)$

$|V(G_1)| = m$, $|V(G_2)| = n$

$V(G_1) \cap V(G_2) = \phi$

- No edges exists b/w any 2 vertices in same subset.
  Eg :- Edge b/w a & b should not exist.



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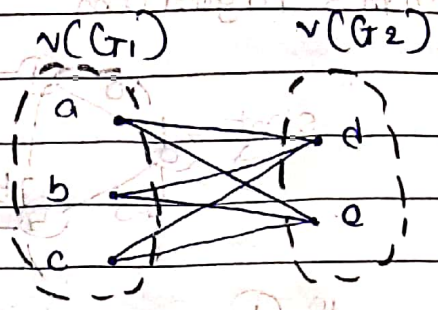
→ Complete Bipartite Graph

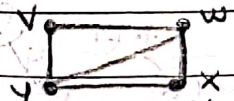
- Should be complete graph $K_{m,n}$, possible when every vertex in $V(G_1)$ is jointed to every vertex in $V(G_2)$.



⊛ Representation of Graphs

→ Adjancency Matrix

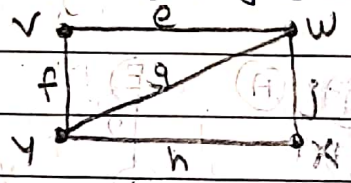
- rows & cols labelled with ordered vertices.



	v	w	x	y
v	0	1	0	1
w	1	0	1	1
x	0	1	0	1
y	1	1	1	0

→ Incidence Matrix

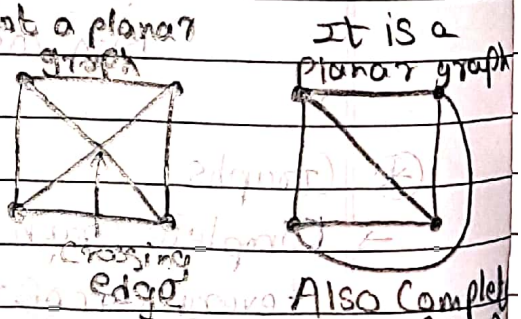
- rows are labelled with vertices
- cols by edges



	e	f	g	h	j
v	1	1	0	0	0
w	1	0	1	0	1
x	0	0	0	1	1
y	0	1	1	1	0

⊛ Planar Graphs :-

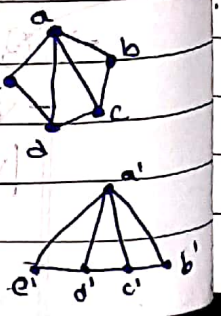
- A graph is planar if it can be drawn in the plane without crossing-edges.



⊛ Isomorphic Graphs :-

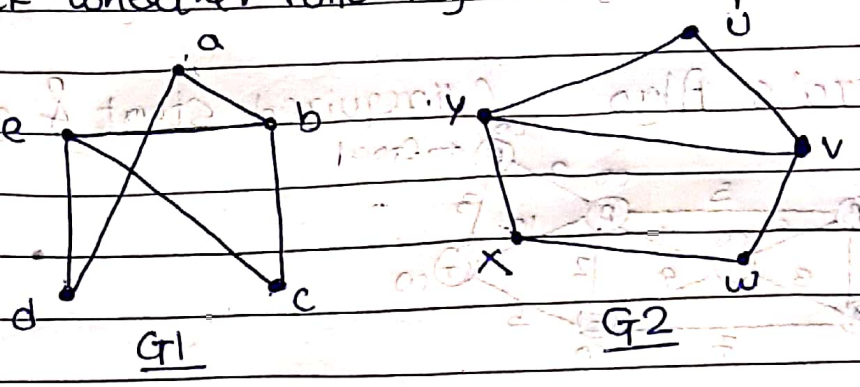
- if there exists one-to-one onto functions $f: V(G_1) \to V(G_2)$ & $g: E(G_1) \to E(G_2)$ such that

- edge e is adjacent to vertices v, w in G_1 if & only if $g(e)$ is adjacent to $f(v)$ & $f(w)$ in G_2



⊛ Isomorphic Graphs with adjacency matrix

Q Check wheather following are isomorphic graphs :-

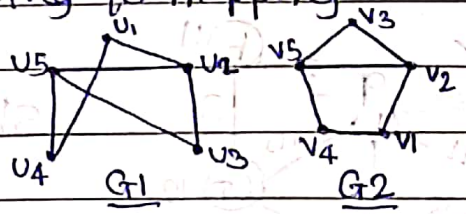


Ans. No of vertices in G1 = 5 in G2 = 5 ✓ satisfied
No of edges in G1 = 6 in G2 = 6 ✓ satisfied
degree sequence G1 : 2,3,2,2,3 G2 : 2,3,2,2,3
Mapping :- • we have two degree 3 vertices i.e e,b & y,v
lets map e ↔ y & b ↔ v

- Now check neighbours degree w has 2 neighbour with degree 2 & 3 same for x. Similarly in graph G1 we have d & a.
- But u & c has two neighbours with degree 3 Hence map u ↔ c. So remaining w ↔ a & x ↔ d
- Sort according to mapping :-

U_1	U_2	U_3	U_4	U_5
a	b	c	d	e
w	v	u	x	y
V_1	V_2	V_3	V_4	V_5

- New graphs :-



- Get adjacency Matrix :-

G1 :

	U_1	U_2	U_3	U_4	U_5
U_1	0	1	0	1	0
U_2	1	0	1	0	1
U_3	0	1	0	0	1
U_4	1	0	0	0	1
U_5	0	1	1	1	0

G2 :

	V_1	V_2	V_3	V_4	V_5
V_1	0	1	0	1	0
V_2	1	0	1	0	1
V_3	0	1	0	0	1
V_4	1	0	0	0	1
V_5	0	1	1	1	0

Since adj matrix for G1 & G2 are same
∴ G1 & G2 are isomorphic Graphs.

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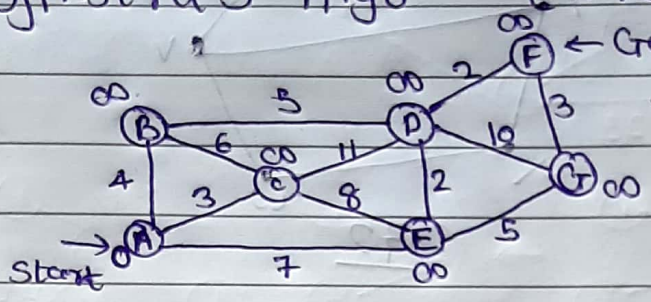
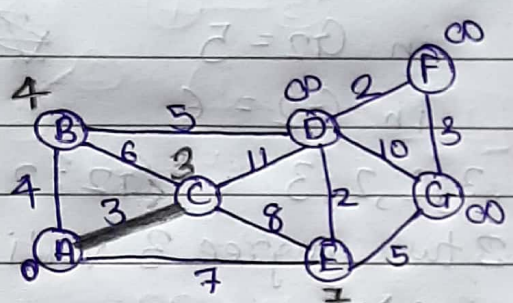
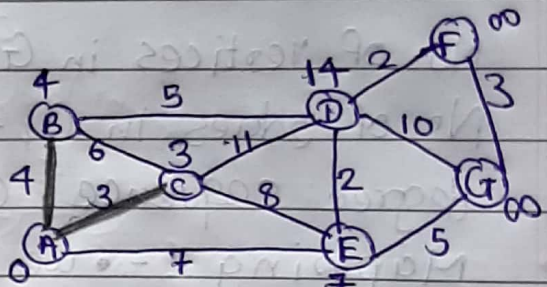
(*) Shortest path Algos :-

→ Djikstra's Algo (If required start & goal node)

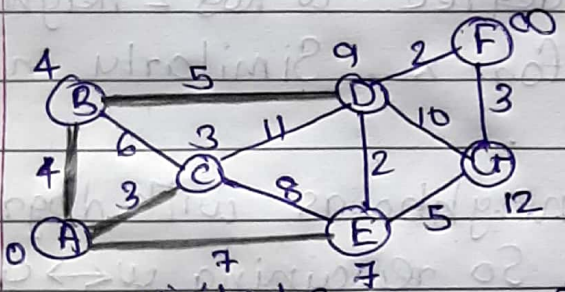




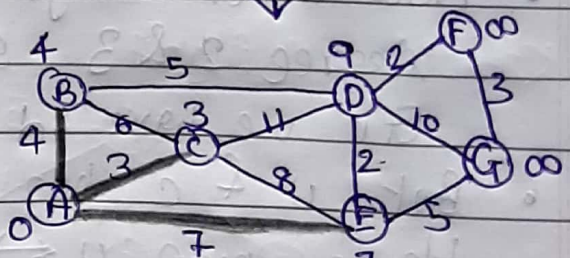
visited {A, C}



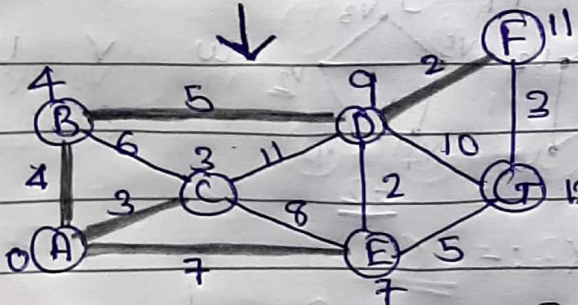
visited {A, B, C}



visited {A, B, C, D, E}



visited {A, B, C, E}



visited {A, B, C, D, E, F}

Shortest path = A - B - D - F
= 11 units

⊛ Circular Queue

→ Queue Size :- (maxsize - front + rear + 1) % maxSize

→ class byQueue
{ private int maxsize, front, rear;
private int[] ~~Queue~~ queueArray;

```
public myQueue (int s)
{   maxsize = s;
    queueArray = new int [maxsize];
    front = -1;
}

public boolean isEmpty()
{ return (front == -1);
}

public boolean isFull()
{ return((rear+1)% maxsize == front);
}

public int getQueueSize()
{ return((maxsize-front+rear+1)%maxsize);
}

public void enqueue (int data)
{   if (isFull() == true)
        SOP("Queue Full");
    else
    { rear = (rear+1) % maxsize;
      queueArray [rear] = data;
      if (front == -1)
          front = rear;
    }
}

public int deQueue()
{   if (isEmpty() == true)
        SOP("Queue Empty");
    int data = queueArray [front];
    queueArray [front] = -1;
    if (front == rear)
    { front = -1;
      rear = -1;
    }
    else if (front == maxsize-1)
        front = 0;
    else
        front++;
    return data;
}
```

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⊛ Complexity

→ Master Theorem

$T(n) = aT\left(\frac{n}{2}\right) + f(n)$

find $n^{\log_b a}$

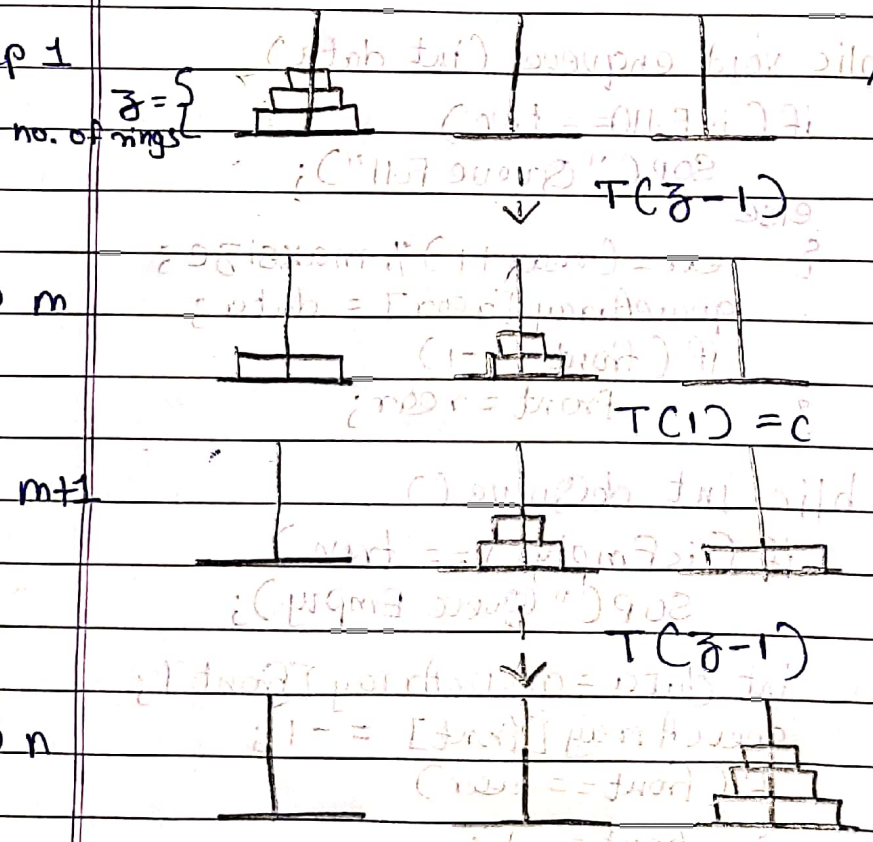
- case 1 : $n^{\log_b a} > f(n)$
  $T(n) = O(n^{\log_b a})$
- case 2 : $n^{\log_b a} < f(n)$
  $T(n) = O(f(n))$
- case 3 : $n^{\log_b a} = f(n)$
  $T(n) = (\log n \cdot f(n))$

→ Subtract & Conquer

$T(n) = aT(n-b) + f(n)$ where $n \geq 1$

- case 1 : $a < 1$
  $T(n) = O(n^k)$
- case 2 : $a = 1$
  $T(n) = O(n^{k+1})$
- case 3 : $a > 1$
  $T(n) = O(n^k a^{n/b})$

→ Tower of Hanoi



$\therefore T(z) = 2T(z-1) + c$

$T(n) = 2T(n-1) + c$

// By Subtract & Conquer

$a = 2 \quad b = 1 \quad k = 0$

Since $a > 1$

$\therefore T(n) = O(n^k a^{n/b})$

$\therefore T(n) = O(n^0 \cdot 2^{n/1})$

$\therefore T(n) = O(2^n)$

Q1 $T(n) = 3T\left(\frac{n}{2}\right) + n^2$ $\quad a=3 \quad f(n) = n^2$

$b=2$

$n^{\log_b a} = n^{\log_2 3}$

Here $n^{\log_2 3} \quad n^2$

Since $n^{\log_2 3} < n^2$

$T(n) = O(f(n)) = O(n^2)$

Q2 $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$n^{2\log_2 2} = n^2$

$n^2 = n^2$

$\therefore T(n) = O(n^2 \cdot \log n)$

Q3 $T(n) = 16T\left(\frac{n}{4}\right) + n$

~~log~~ $n^{\log_4 4^2} \quad n$

$n^2 > n$

$T(n) = O(n^2)$

$f(n) = n^2$

$\therefore$ ~~log~~ $n^{\log_2 4} = n^{2\log_2 2} = n^2$

Q4 $T(n) = 2T(n/4) + n^{0.51}$

$n^{\log_4 2}$

$= n^{\frac{\log 2}{\log 4}} = n^{\frac{\log 2}{2\log 2}} = n^{1/2} = n^{0.5}$

Since $n^{0.50} < n^{0.51}$

$T(n) = O(n^{0.51})$

Q5 $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$n^{\log_3 3} = n$

$n \geq n/2$ (Big-O)

$\therefore T(n) = O(n \log n)$

11

Q5
```
for ( i=1; i<=n)
{
    i = i*2;
}
```

iteration	i	i ≤ n (Assume n=1000)
1	$2=2^1$	$2 \le 1000$ ✓
2	$4=2^2$	$4 \le 1000$ ✓
3	$8=2^3$	$8 \le 1000$ ✓
	$16=2^4$	⋮
K	$i=2^K$	$i > 1000$ ✗
		$\therefore 2^K > 1000$

→ loop runs till $2^K > 1000$ i.e $\rightarrow n$

$\therefore 2^K \geq n$

$\log 2^K \geq \log n$

$K \log 2 \geq \log n$

$K > \frac{\log n}{\log 2}$

$T(n) = O(K) = O\left(\frac{\log n}{\log 2}\right) = O(\log n)$ ← constant (Big O)

★

Q6
```
void f (int n) {
    int i, j, k; ~~cons~~ count = 0;
    for (i = n/2 ; i <= n; i++ )        ⟶ n/2 times
    {
        for (j = 1; j + n/2 <= n; j++ )   ⟶ n/2 times
        {
            for (k = 1; k <= n; k *= 2 )   ⟶ log n times
            {
                count++;                   ⟶ constant
            }
        }
    }
}
```

→ $T(n) = O\left(\frac{n}{2} * \frac{n}{2} * (\log n)\, c\right)$

$= O\left(\frac{n^2}{4} (\log n)\right)$ ← constant

$= O(n^2 \log n)$

Since loops are dependent on each other we multiply complexities.

Else we add complexities for non-dependent loops.

Q7
```
void f(int n)
{
    int i=1; s=1;        // const1, const2
    while (s<=n)
    {
        i++;             // c3
        s=s+i;
    }
    sop("*");
}
```

iteration	i	s
0	1	1
1	2	1+2
2	3	1+2+3
⋮	⋮	⋮
k	k	1+2+3+...+k

loop will stop when $S_n > n$

$T(n) = \frac{k(k+1)}{2} > n$

$= \frac{k^2+k}{2} > n$

$= k^2+k > 2n$

$= k^2 > 2n-k$

$\therefore T(n) = O(k^2) = O(\sqrt{2n-k})$

$T(n) = O(\sqrt{n})$

Q8
```
f()
{
    int n=5;        // c1
    int p=n^2;      // c2
    sop(p);         // c3
}
```

$T(n) = c_1+c_2+c_3$

$= O(c) = O(1)$

Q9
```
A(n)
{
    if(n<=1) return;
    else
    return 15A(n/2)+3A(n/2)+n^2;
}
```
f(n)

Note:-

In if-else construct, the complexity is of which takes more time i.e if or else

In this case else takes more time.

Q1

a = A(n/2) } T(n/2)
b = 15 * a } → c1 i.e O(1)
a = A(n/2) }
d = 3 * a → c2 i.e O(1)
e = n² → c3 i.e O(1)

T(n) = T(n/2) + c1 + T(n/2) + c2 + c3
(c1, c2, c3 — constants)

= 2T(n/2) + c ← in form aT(n/b) + f(n)

a = 2 b = 2

$n^{\log_2 2} = n$ n > f(n)

T(n) = O(n)

Q10 A(n)

if (n ≤ i) return 1;

else return A(n/2) + A(n/2) + A(n/2)

T(n) = T(n/2) + a + T(n/2) + b + T(n/2) + c
(a, b, c — constants)

= 3T(n/2) + c

a = 3 b = 2 f(n) = n⁰ // form aT(n/b) + f(n)

$n^{\log_2 3} > f(n)$

$T(n) = O(n^{\log_2 3})$

✱ Master Thm for Subtract & Conquer

* If recursion eqn is of format $T(n) = c$ where $n \leq 1$

$= aT(n-b) + f(n)$ where $n > 1$

$T(n) = O(n^k)$ if $a < 1$

$= O(n^{k+1})$ if $a = 1$

$= O(n^k a^{n/b})$ if $a > 1$

Q1) $T(n) = 3T(n-1) + n$

$a = 3 \quad b = 1 \quad f(n) = n^1 \; k=1$, Here $a > 1$, hence.

$T(n) = O(n^k a^{n/b})$

$= O(n^1 a^{n/1}) = O(na^n)$ Since $a = 3$

$= O(n3^n)$

Q2)
```
A(n)
{ if (n==0 || n==1) return 1;
  else return n * A(n-1);
}
```

n → a, A(n-1) → b

$a = n \quad b = A(n-1) \quad d = a * b = O(c)$

$O(1) \quad O(n-1)$

$c_1 + T(n-1) + c_2$

$T(n-1) + c \qquad a = 1 \quad b = 1 \quad f(n) = n^0 \; k = 0$

Here $a = 1 \; \therefore \; O(n^{k+1})$

$= O(n^{0+1}) = O(n)$

Similarly $T(n-1)$ = Go down the chain