

BM-101

MATHEMATICS

Time Allotted: 3 Hours

Full Marks: 70

The questions are of equal value.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP A

(Multiple Choice Type Questions)

1. Answer any *ten* questions.

10×1 = 10

(i) An element x in a ring R is a zero divisor if

(A) $x \cdot b = 0$

(B) $x \cdot b = 0$, for some non zero element in R

(C) $x \cdot b \neq 0$, for all elements b in R

(D) none of these

(ii) If $A = \{2, 4, 6\}$ and $B = \{1, 3, 5, 7\}$ then $A \cup B$ is

(A) $\{0\}$

(B) $\{1, 2, 3, 4, 5, 6, 7\}$

(C) $\{1, 2, 4, 5, 6, 7\}$

(D) $\{0, 2\}$

(iii) The polar form of the equation $x^2 + y^2 - 8y = 0$ is

(A) $r = 8\cos\theta$

(B) $r = 8\sin\theta$

(C) $r^2 = 8\cos\theta$

(D) none of these

(iv) The diagonal elements of a real skew symmetric matrix are

(A) 1

(B) -1

(C) 2

(D) 0

(v) If α, β, γ are the roots of the equation $x^3 - 3x^2 + 6x - 2 = 0$ then, $\alpha + \beta + \gamma$ is

(A) 0

(B) 1

(C) 3

(D) -2

(vi) The value of t for which the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{bmatrix}$ is singular is

- (A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) 2 (D) -2

(vii) The function $f(x) = |x|$ then

- (A) continuous and differentiable at $x = 0$
 (B) continuous everywhere but differentiable at $x = 0$
 (C) discontinuous and not differentiable at $x = 0$
 (D) none of these

(viii) Which of the following function obeys Rolle's theorem in $[0, \pi]$

- (A) x (B) $\sin x$ (C) $\cos x$ (D) $\tan x$

(ix) By 3rd order Maclaurin's theorem we have $\sin x = f(x) - \frac{x^3}{6} \cos \theta x$, then $f(x)$ equal to

- (A) x^2 (B) $-x^2$ (C) x (D) $-x$

(x) If $f(x, y) = x^2y$ then df equal to

- (A) $2x^2 dx + dy$ (B) $x - 2 dy$ (C) $x + dy$ (D) $2xy dx + x^2 dy$

(xi) $f(x, y) = |x| + |y|$ then $f_x(0, 0)$ equal to

- (A) 0 (B) 1 (C) does not exist (D) none of these

(xii) The value of $\int_1^2 \frac{e^{\log x}}{x} dx$

- (A) 1 (B) -1 (C) 2 (D) 0

GROUP B
(Short Answer Type Questions)

Answer any *three* questions.

3×5 = 15

2. If α, β are the roots of the equation $x^2 - px + q = 0$ then find the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
3. Solve the following equations by matrix inversion method
 $x + y + z = 2, x - y + 2z = 6, 3x + 5y + 7z = 14$
4. Give the definition of commutative group and show that $\{1, \omega, \omega^2\}$ where $\omega^3 = 1$ forms a commutative group w.r.t. multiplication.
5. If $y = \cos(m \sin^{-1} x)$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.
6. If $u = \tan^{-1} \frac{x^2 - y^2}{x - y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$.

GROUP C
(Long Answer Type Questions)

Answer any *three* questions.

3×5 = 15

7. (a) In a class of 50 students, 15 read Physics, 20 read Chemistry and 20 read Mathematics, 3 read Physics and Chemistry, 6 read Chemistry and Mathematics and 5 read Physics and Mathematics, 7 read none of the subjects. How many students read all the subjects? 7
- (b) Discuss the nature of the conic represented by $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$ by reducing to its canonical form. 8
8. (a) Apply Descartes's rule of sign to show that the equation $x^4 + 2x^2 - 7x - 5 = 0$ has two real roots and two non real roots. 5
- (b) Verify Rolle's theorem for the function $f(x) = |x|, -1 \leq x \leq 1$ 5

- (c) Discuss the continuity of the following function $f(x) = x - [x]$, where $[x]$ denotes the greatest integer not greater than x . 5
9. (a) Using the mean value theorem prove the following inequalities $x < \sin^{-1}x < \frac{x}{\sqrt{1-x^2}}$ if $0 < x < 1$ 6
- (b) Show that $z = \log\{(x-a)^2 + (y-b)^2\}$ satisfies the relation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ except at (a, b) 6
- (c) Evaluate $\int \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$ 3
- 10.(a) Solve $x^3 - 9x + 28 = 0$ using Cardan's method. 6
- (b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ 4
- (c) Find the nature of the conic $\frac{8}{r} = 4 - 5\cos\theta$ 5
- 11.(a) Find $\frac{dy}{dx}$ when $x = y \log(xy)$ 5
- (b) Give the definition of a ring with two binary composition. Let H be the set of all matrices $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 1 \right\}$ 5
- Prove that H forms a non-commutative group with respect to matrix multiplication.
- (c) If by a transformation of motion of co-ordinate axis, the expression $ax^2 + 2hxy + by^2$ changes into $a'x'^2 + 2h'x'y' + b'y'^2$ then show that $a + b = a' + b'$ 5