MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BM-101

MATHEMATICS

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The value of $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ is

   a) 1 b) 4

   c) 0 d) 2.

   ii) The value of $\int_0^1 x^3 \, dx$ is

   a) $\frac{3}{4}$ b) 3

   c) $\frac{1}{4}$ d) 1.

iii) $\frac{\partial}{\partial x}(x^y) =$

a) 1 b) yx^y

c) $x^y \log x$ d) yx^{y-1}.

iv) $A = \{2, 4, 6\}$, $B = \{1, 3, 5, 7\}$ then $A \cup B$ is

a) $\{0\}$ b) $\{1, 2, 3, 4, 5, 6, 7\}$

c) $\{1, 2, 4, 5, 6, 7\}$ d) $\{0, 2\}$.

v) The value of $\lim_{x \to 0} (1+x)^{1/x}$ is

a) 1 b) e

c) ∞ d) 0.

vi) The inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 \\ -\frac{3}{2} & 3 \end{bmatrix}$

c) $\begin{bmatrix} -2 & 4 \\ -3 & 6 \end{bmatrix}$

d) Does not exit.

vii) If α, β, γ be the roots of the equation

$x^3 - 3x^2 + 6x - 2 = 0$, then $\sum \alpha \beta$ is

a) 3 b) 6

c) 2 d) none of these.

viii) The conic $\frac{l}{r} = 1 - e\cos\theta$ represents a parabola if

a) $e = 1$ b) $e > 1$

c) $e < 1$ d) none of these.

ix) If $x = at^2$, $y = 2at$, then $\frac{dy}{dx}$ at $t = 1$ is

a) 1 b) $2a$

c) -1 d) $2a^2$.

x) The degree of the polynomial $f(x) = x^2 + x - 2$ is

a) 0 b) 1

c) 2 d) 3.

xi) If $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$ then the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

a) $\Delta > 0$ b) $\Delta < 0$

c) $\Delta = 0$ d) none of these.

xii) The polar form of the equation $x^2 + y^2 - 8y = 0$ is

a) $r = 8 \cos \theta$ b) $r = 8 \sin \theta$

c) $r^2 = 8 \cos \theta$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Express $\begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

3. Evaluate the integral $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$.

4. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0.$$

5. If α, β, γ be the roots of the equation $x^3 + px + q = 0$ then find the equation whose roots are $\frac{\beta+\gamma}{\alpha^2}, \frac{\gamma+\alpha}{\beta^2}, \frac{\alpha+\beta}{\gamma^2}$.

6. Prove that $G = \{1, -1, i, -i\}$ forms a commutative group under multiplication, where ω be the cube root of unity.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that the matrix $A = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal and hence find A^{-1}. 5

b) If $y = \sin(m \sin^{-1} x)$ then show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2 y = 0.$$ 5

c) Using mean value theorem prove that

$$\frac{x}{1+x} < \log(1+x) < x \text{ if } x > 0.$$ 5

8. a) Solve the following equations by matrix method : 5

$$x + y + z = 4$$

$$2x - y - 3z = 1$$

$$3x + 2y - z = 1$$

b) Solve $x^3 - 9x + 28 = 0$ using Carden's method. 5

c) Evaluate

$$\lim_{n \to \infty} \left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \ldots\ldots + \frac{n}{n^2+n^2} \right].$$ 5

9. a) State Descartes' rule of sign. Using this rule find the nature of the roots of the equation

$$x^4 - 7x^3 + 21x^2 - 9x + 21 = 0.$$ 7

b) Reduce the following equation is the canonical form and determine the nature of conic represented by it :

$$8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0.$$ 8

10. a) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0.$$ 5

b) A function $f(x)$ is defined as follows :

$f(x) = -x^2$ when $x \leq 0$

$= 5x^2$ when $0 < x < 1$

$= 4 + x^2$ when $x \geq 1$.

Show that $f(x)$ is continuous at $x = 0$ and $x = 1$.

5

c) If by a transformation of one rectangular axes to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$, prove that

$$a^2 + b^2 = a'^2 + b'^2.$$ 5

11. a) Find $\frac{dy}{dx}$ when $x = y \log(xy)$. 5

b) Find for what values of x, the following expression is maximum and minimum respectively :

$$2x^3 - 21x^2 + 36x - 20.$$ 5

c) Show that the set of rational numbers other than 1, Q' forms a group under the binary operation * defined by $a * b = a + b - ab : a, b \in Q$. 5