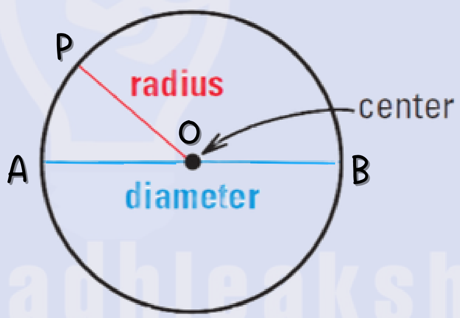
Circles

Introduction

- A circle is a collection of all points in a plane which are at a constant distance (radius) from a fixed point (centre).

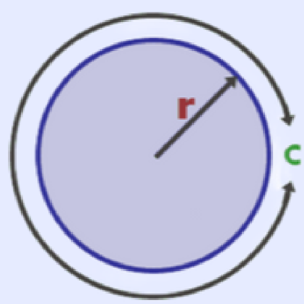


- In the above diagram, the point O is the centre of circle, OP is the radius of circle and AOB is the diameter of the circle.

Some Basic Terms :

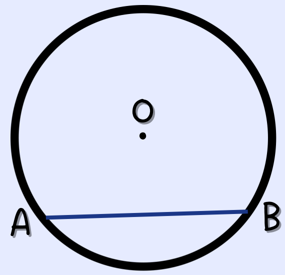
(1) Circumference of a Circle :

Circumference of the circle or perimeter of the circle is the measurement of the boundary of the circle. It is calculated by the formula $2\pi r$, where r is the radius of the circle.



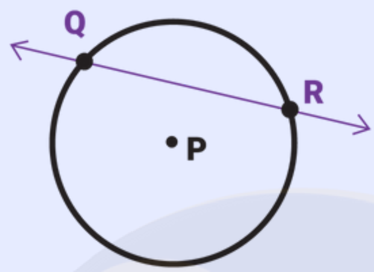
(2) Chord of a Circle :

The line segment within the circle joining any two points on the circumference of the circle is called "chord of the circle". Diameter is the longest chord of a circle.



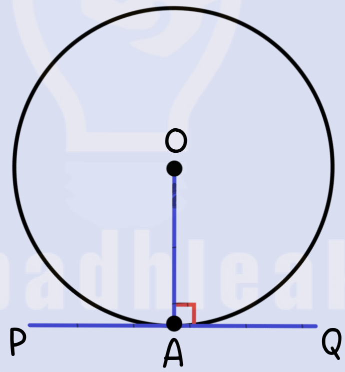
(3) Secant of a Circle:

A line intersecting a circle in two distinct points is called "Secant of circle".



(4) Tangent of a Circle:

A tangent to a circle is a line that intersects or touches the circle at only one point.



(5) Point of Contact :

The common point of the tangent and the circle is called the 'point of contact' and the tangent is said to 'touch' the circle at the common point.

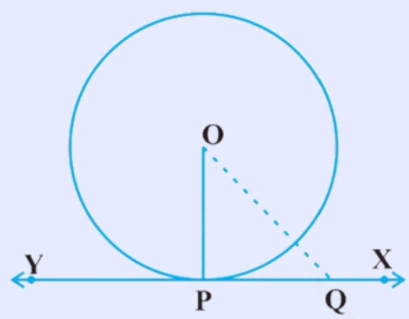
THEOREM 10.1 :

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given : A circle with Centre O and a tangent XY at a point P to the circle.

To Prove : OP is perpendicular to XY

Construction : Take a point Q on XY and join OQ.



Proof : We need to show that line OP is the shortest of all other lines joined from O to XY.

We see, $OP = OM$ (radii of same circle)

$OQ = OM + MQ$

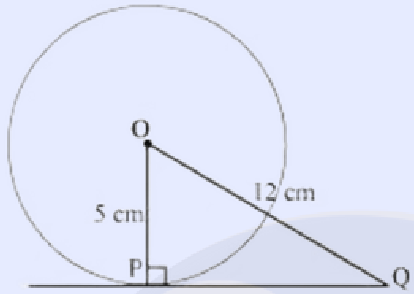
$OQ > OM$

$OQ > OP \quad (OP = OM)$

$\Rightarrow \boxed{OP \perp XY}$

Hence proved.

Example: A tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q so that OQ = 12cm. Find the length PQ?



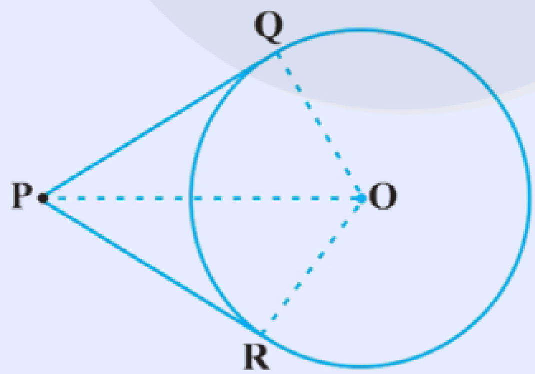
SOLUTION: By theorem 10.1, we know that $OP \perp PQ$

By applying Pythagoras theorem in ΔOPQ, we have

$$OP^2 + PQ^2 = OQ^2$$
$$(5)^2 + PQ^2 = (12)^2$$
$$PQ^2 = 144 - 25 = 119$$
$$PQ = \sqrt{119}\,cm$$

THEOREM 10.2 :

The lengths of tangents drawn from an external point to a circle are equal.



Given : A circle with centre O have two tangents PQ and PR drawn from an external point P.

To Prove : PQ = PR

Construction : Join OP, OQ and OR.

Proof : In ΔPOQ & ΔPOR

OQ = OR (radii of same circle)

$\angle OQP = \angle ORP$ (each 90° from theorem 10.1)

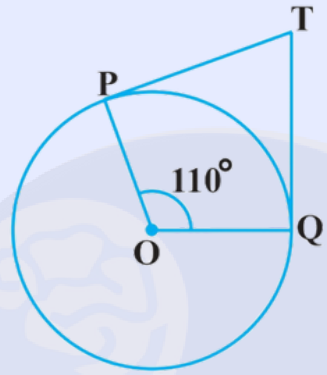
OP = OP (common side of both triangles)

$\therefore \Delta POQ \cong \Delta POR$ by RHS Criterion

=> PQ = PR [By CPCT]

Hence proved.

Example: In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PYQ$ is equal to



SOLUTION: From the figure, $OP \perp TP$ and $OQ \perp TQ$ (By theorem 10.1)

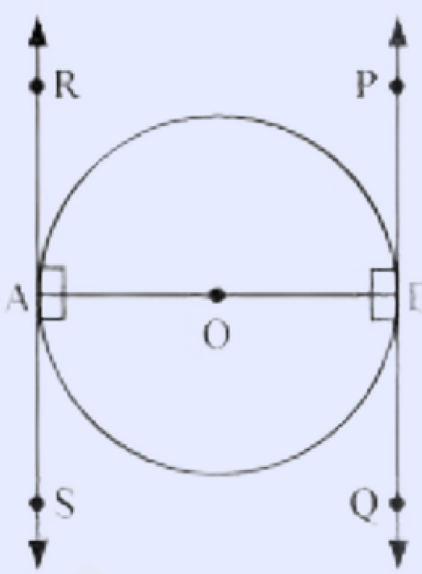
i.e $\angle OPT = 90°$ and $\angle OQT = 90°$

Now, OQTP is a Quadrilateral and Sum of all its interior angles is 360°

$$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360°$$
$$90° + 110° + 90° + \angle PTQ = 360°$$
$$290° + \angle PTQ = 360°$$
$$\angle PTQ = 360° - 290°$$
$$\boxed{\angle PTQ = 70°}$$

Example: Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Given : PQ and RS are two tangents and AB is the diameter.



To Prove : $PQ \parallel RS$

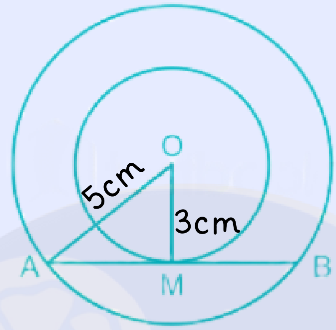
Proof : $\left.\begin{array}{l}\angle OAQ = 90° \text{ and} \\ \angle OBR = 90°\end{array}\right\}$ [By theorem 10.1]

Also, $\angle OAQ = \angle OBR$ [Alternate interior angles are equal]

$\Rightarrow$ $\boxed{PQ \parallel RS.}$

Hence proved.

Example: Two concentric circles are of radii 5cm and 3cm. Find the length of the chord of the larger circle which touches the smaller circle.



SOLUTION: We observe that PQ is a tangent to the smaller circle.

=> $\angle OMA = 90°$ [theorem 10.1]

Now, In ΔOMA applying Pythagoras theorem, we have

$OM^2 + AM^2 = OA^2$

$(3)^2 + AM^2 = (5)^2$

$9 + AM^2 = 25$

$AM^2 = 25 - 9 = 16$

$AM = (4)^2$

$AM = 4cm$

Now, In ΔOAB, $OM \perp AB$

MA = MB (perpendicular from centre of the circle bisects the chord)

$\therefore$ AB = 2MA = 2 × 4 = 8cm

Therefore, the length of the chord of the larger circle is 8cm.

Example: A quadrilateral ABCD is drawn to circumscribe a circle. Prove that

AB + CD = AD + BC

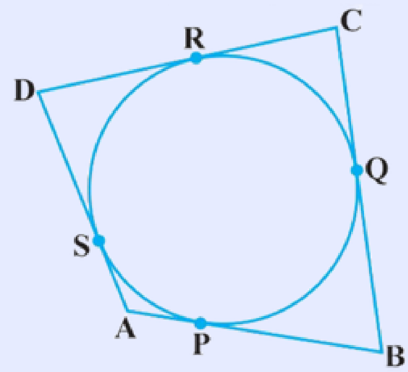
SOLUTION: It can be observed that

AP = AS (tangents from point A to circle)

BP = BQ (tangents from point B to circle)

CR = CQ (tangents from point C to circle)

DR = DS (tangents from point D to circle)



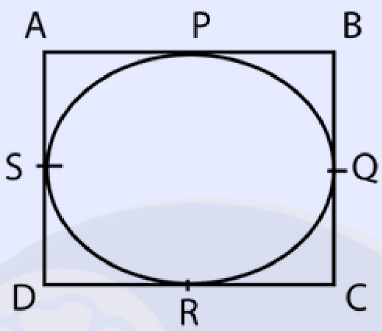
Adding all these 4 equations, we get

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

=> AB + CD = AD + BC

Hence proved.

Example: Prove that the parallelogram circumscribing a circle is a rhombus.



Given : ABCD is a parallelogram i.e. AB = CD & AD = BC

To Prove : ABCD is a rhombus i.e. AB = BC = CD = DA

Proof : From the figure, AP = AS [By theorem 10.2] ....(1)

BP = BQ [By theorem 10.2] ....(2)

CR = CQ [By theorem 10.2] ....(3)

DR = DS [By theorem 10.2] ....(4)

Adding all these above 4 equations, we get

AP + BP + CR + DR = AS + BQ + CQ + DS

=> AB + CD = AD + BC

=> AB + AB = BC + BC [AB = CD & AD = BC]

=> 2AB = 2BC

=> AB = BC

$\therefore$ We now have, AB = CD, AD = BC, AB = BC

$\therefore$ AB = BC = CD = AD

So, ABCD is a rhombus.

Hence proved.

*After studying from these notes