

ENGINEERING DRAWING

[PLANE AND SOLID GEOMETRY]

[IN FIRST-ANGLE PROJECTION METHOD]



by

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FOREWORD

It gives me pleasure to introduce this text-book on Engineering Drawing by Prof. N. D. Bhatt of the Birla Vishvakarma Mahavidyalaya to students of Engineering. Prof. Bhatt has been teaching this subject for over twenty years and has deservedly earned the reputation of being one of the best teachers in the subject.

This book covers the syllabus usually prescribed for the Pre-engineering and First Year of the Degree and Diploma courses in Engineering and deals with the fundamental principles of this basic subject which have been treated by Prof. Bhatt with his characteristic lucidity.

I am sure the book will prove a boon to students and help them to acquire a sound knowledge of the subject without which a really satisfactory progress cannot be achieved in any branch of Engineering.

June 6, 1958

S. B. JUNNARKAR

M.B.E., B.A., B.Sc. Hons. (Eng.), London
POONA

ELEMENTARY ENGINEERING DRAWING

PREFACE

From the very early days, man realized that if he had to construct any structure or machine correctly and methodically, he must first record his ideas before starting construction work. These recorded ideas become more vivid and forceful if they are shown on paper in form of a drawing of the structure or machine. Such a drawing will be of very great help to the man who looks after the construction of this structure or machine.

Indeed, "technical drawing is the language of engineering". Without the good knowledge of drawing, an engineer is nowhere and he could not have constructed the various magnificent structures or intricate machines. Evidently, any one connected in any way, with engineering construction must understand this language of engineers. Technical drawing is, therefore, indispensable today and shall continue to be so as long as engineering and technology continue to be of use in the activities of man.

By means of drawing, the shape, size, finish, colour and construction of any object (no matter how complex) can be described accurately and clearly. The engineer should develop his skill, in two phases of technical drawing; first, he must be able to draw clearly and rapidly, the freehand technical sketches; secondly, he must be proficient in drawing to scale the instrumental drawing. The purpose of the present volume is to give the basic principles of the instrumental drawing only.

The book covers the syllabi in Engineering Drawing of many University Colleges and Polytechnics in India and has been written keeping in view the difficulties of a beginner in the subject of Engineering Drawing. I am quite hopeful that this book will serve its purpose very well for young and budding engineers.

I am highly indebted to Principal S. B. Junnarkar for his valuable guidance and for his kindness to write a suitable foreword for the book. I am also thankful to Prof. V. B. Priyani of Birla Vishvakarma Mahavidyalaya for going through the initial manuscript and for offering constructive suggestions. Finally, I feel grateful to the following:

(i) The authorities of the Universities of Bombay, Poona and Gujarat, and the Department of Technical Education, Bombay, for their kind permission to include a few questions set at their examinations. (ii) Mr. N. M. Panchal and Mr. M. D. Bhatt for their help in preparing pencil sketches. (iii) Mr. L. D. Bhatt for preparing the excellent typed manuscript. (iv) Mr. Ramanbhai C. Patel of Charotar Book Stall for careful proof-reading and for his efforts to see the book out in proper time. (v) The Anand Press authorities for the care and interest shown in the printing and get-up of this book. (vi) The Prabhat Process Studio for the promptness and good work of block-making.

Any suggestion to improve the value of this book will be gratefully received and will be incorporated in subsequent editions after due scrutiny.

June 30, 1958

N. D. BHATT

EIGHTH EDITION

Upto six editions, the book hardly underwent any fundamental change. Under the inspiration of the Indian Standards Institution, the industries in a large measure have switched on to the third-angle projection system. So have a vast majority of Technical Institutions. "Elementary Engineering Drawing", too, must keep step with the changes. There was a phased change-over beginning with the seventh edition. This edition has completed the change. Nevertheless, the first-angle projection system has not been altogether discarded. The fundamentals of both the systems are juxtaposed and elucidated.

The chapters on (i) Development of surfaces, (ii) Isometric projection and (iii) Conversion of pictorial views into orthographic views have been treated in some greater depth. This and the increase of the illustrative problems and practice exercises have evidently enlarged the size of the book and inevitably led to a little rise in price.

I should not miss this opportunity to record my gratitude to the numerous teachers for their very useful suggestions and the students for their excellent response to the book, without which this edition should not have come into being.

January 26, 1970

N. D. BHATT

FORTY-SECOND EDITION

The first edition was originally published in 1958 and now has entered in its 42nd Edition during 42 years, proves its popularity and utility among the teachers as well as the students of Engineering Institutions of our country.

This edition has been entirely revised and enlarged by adding the following *four* chapters:

- (1) Screw Threads
- (2) Screwed Fastenings
- (3) Riveted Joints and Welded Joints
- (4) Computer Aided Drafting.

It was thought desirable to include this fourth chapter on 'Computer Aided Drafting' which has now acquired an important place in this subject. We are thankful to Dr. S. S. Khandare of Y. C. College of Engineering, Nagpur for contributing this chapter. We are also highly obliged to Shri R. C. Patel of Charotar Publishing House, Anand for correcting the Proofs.

This book provides a wealth of information and describes an exciting new paradigm and is also a valuable tool for all the Engineering students, who wish to deepen and acquire a sound knowledge of this important subject (Language of Engineers) without which a really satisfactory progress cannot be achieved in any branch of Engineering.

We shall feel obliged to receive comments, suggestions and opinions from the readers to enhance the utility of the book.

August 15, 2000

N. D. BHATT
V. M. PANCHAL

FIFTIETH EDITION

We are inundated with joy to present before you the FIFTIETH EDITION of this most standard text-book. At the outset, we would like to mention that splendid response to earlier editions is prodigious. Despite such favourable responses, it was felt that the utility of the book could be further enhanced. This is one of the most comprehensive revisions since the book was first published. As a result, all the drawings have been redrawn with utmost intelligibility. Many new examples, drawings are incorporated along with some new text matter. This text-book is thoroughly revised, extensively enlarged, completely updated. Chapter on Computer Aided Drafting (CADr) is entirely rewritten with inclusion of 50 *self-interactive and self-learning practice modules*. This book accompanied by a computer CD as a novel pedagogical concept, containing 51 selected *audiovisual animation modules* presented for better visualization and understanding of the subject of Engineering Drawing.

We take this opportunity to thank Prof. V. M. Panchal, Former Professor in Mechanical Engineering, Faculty of Technology and Engineering, M. S. University of Baroda for revising the entire book and adding many new typical examples. We express our hearty gratitude to Prof. Pramod R. Ingle, Lecturer in Mechanical Engineering, B and B Institute of Technology, Vallabh Vidyanagar for redraw all the drawings with precision, rewriting the chapter on Computer Aided Drafting (CADr) and revising the entire book.

We are also indebted to Prof. R. S. Bhatt, Associate Professor in Mechanical Engineering, Birla Vishvakarma Mahavidyalaya (Engineering College), Vallabh Vidyanagar, Anand; Prof. Mukesh A. Bulsara, Assistant Professor in Mechanical Engineering, G. H. Patel College of Engineering and Technology (GCET), Vallabh Vidyanagar, Prof. N. V. Patel, Ex-Professor, L. D. Engineering College, Ahmedabad who were kind enough to send their suggestions to us for the improvement of the book. We are also grateful to Prof. S. H. Makwana, Lecturer in Mechanical Engineering, B & B Institute of Technology, Vallabh Vidyanagar, who spared no efforts in the tedious task of diligently correcting the final proofs. We are sincerely thankful to Cognifront, Nasik for preparing a unique CD and Repro India Ltd., Mumbai for their hearty co-operation and excellent printing of the book.

In spite of all the pains taken, it is possible that some errors may have escaped our attention. We shall be grateful if they are brought to our notice by sending e-mail at charotar@cphbooks.com, so that they can be corrected in subsequent edition. We strongly urge the readers to send their comments, suggestions and opinions to enhance the utility of the book.

August 24, 2010

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NOTE ABOUT THE CD ACCOMPANYING THE BOOK

This book is accompanied by a CD, which contains audiovisual modules for better visualization and understanding of the subject.

It has been found in our research done over hundreds of students and dozens of colleges and universities that visualization IQ is lacking in different quantities among male and female students. It is therefore necessary to aid learning process by use of high quality computer animations as a novel pedagogical concept.

This CD contains 51 modules and CD is provided FREE with 50th Edition of the book.

Chapter No.	Chapter Name	Module No.	Particulars
1	Drawing Instruments and their uses	Module 01	Introduction of the subject and Various Drawing Instruments.
2	Sheet layout and freehand sketching	Module 02	Introduction of sheets and sheet layout
3	Lines, Lettering and Dimensioning	Module 03	Type of lines, Lettering and Dimensioning
5	Geometrical Construction	Module 04	To draw parallel line : To draw perpendicular to a given line: Problem 5-5
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Chapter

1



DRAWING INSTRUMENTS AND THEIR USES

1-1. INTRODUCTION



Drawing instruments are used to prepare drawings easily and accurately. The accuracy of the drawings depends largely on the quality of instruments. With instruments of good quality, desirable accuracy can be attained with ease. It is, therefore, essential to procure instruments of as superior quality as possible.

Below is the list of minimum drawing instruments and other drawing materials which every student must possess:

1. Drawing board
2. T-square
3. Set-squares — 45° and 30° - 60°
4. Drawing instrument box, containing:
 - (i) Large-size compass with inter-changeable pencil and pen legs
 - (ii) Lengthening bar
 - (iii) Small bow compass
 - (iv) Large-size divider
 - (v) Small bow divider
 - (vi) Small bow ink-pen
 - (vii) Inking pen
5. Scales
6. Protractor
7. French curves
8. Drawing papers
9. Drawing pencils
10. Sand-paper block
11. Eraser (Rubber)
12. Drawing pins, clips or adhesive tapes
13. Duster
14. Drafting machine
15. Roll-n-draw.

We shall now describe each of the above in details with their uses:



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 1 for Introduction of the subject and various drawing instruments.

1-2. DRAWING BOARD (FIG. 1-1)



Drawing board is rectangular in shape and is made of strips of well-seasoned soft wood about 25 mm thick. It is cleated at the back by two battens to prevent warping. One of the edges of the board is used as the *working edge*, on which the T-square is made to slide. It should, therefore, be perfectly straight. In some boards, this edge is grooved throughout its length and a perfectly straight ebony edge is fitted inside this groove. This provides a true and more durable guide for the T-square to slide on.

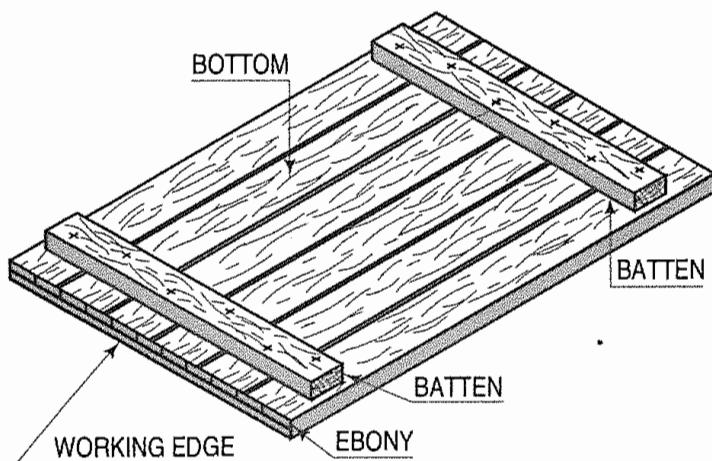


FIG. 1-1

**TABLE 1-1
SIZES OF DRAWING BOARDS**

Designation	Size (mm)
B0	1000 × 1500
B1	700 × 1000
B2	500 × 700
B3	350 × 500

Drawing board is made in various sizes. Its selection depends upon the size of the drawing paper to be used. The sizes of drawing boards recommended by the Bureau of Indian Standards (IS:1444-1989) are tabulated in table 1-1.

For use in schools and colleges, the last two sizes of the drawing boards are more convenient. Large-size boards are used in drawing offices of engineers and engineering firms. The drawing board is placed on the table in front of the student, with its working edge on his left side. It is more convenient if the table-top is sloping downwards towards the student. If such a table is not available, the necessary slope can be obtained by placing a suitable block of wood under the distant longer edge of the board.

1-3. T-SQUARE (FIG. 1-2)



A T-square is made up of hard-quality wood. It consists of two parts — the stock and the blade — joined together at right angles to each other by means of screws and pins. The stock is placed adjoining the working edge of the board and is made to slide on it as and when required. The blade lies on the surface of the board. Its distant edge which is generally bevelled, is used as the working edge and hence, it should be perfectly straight. The nearer edge of the blade is never used. The length of the blade is selected so as to suit the size of the drawing board. Now-a-days T-square is also available of celluloid or plastic with engraved scale.

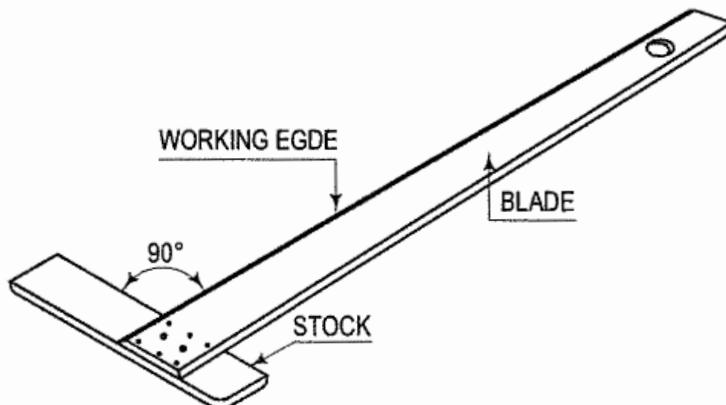


FIG. 1-2

Uses:

- (i) The T-square is used for drawing horizontal lines. The stock of the T-square is held firmly with the left hand against the working edge of the board, and the line is drawn from left to right as shown in fig. 1-3. The pencil should be held slightly inclined in the direction of the line (i.e. to the right) while the pencil point should be as close as possible to the working edge of the blade. Horizontal parallel lines are drawn by sliding the stock to the desired positions.

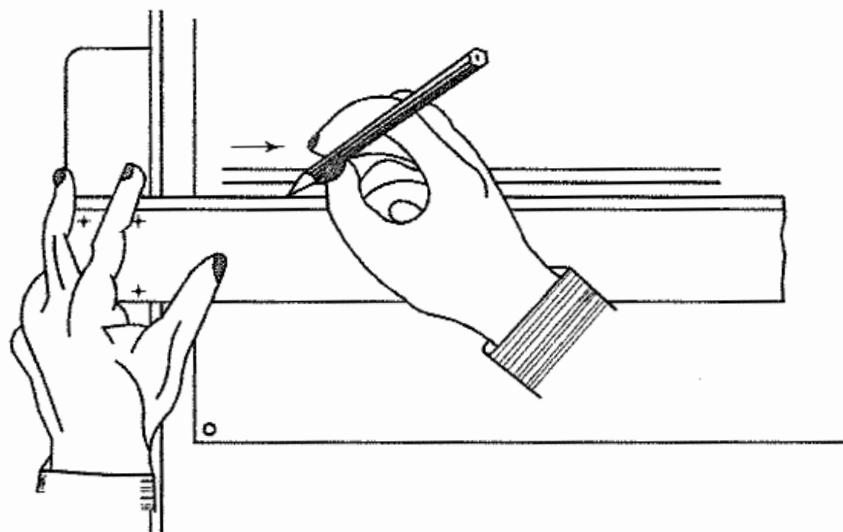


FIG. 1-3

- (ii) The working edge of the T-square is also used as a base for set-squares to draw vertical, inclined or mutually parallel lines. A pencil must be rotated while drawing lines for uniform wear of lead. The T-square should never be used on edge other than the working edge of the board. It should always be kept on the board even when not in use.
- (iii) Testing the straightness of the working edge of the T-square: Mark any two points *A* and *B* (fig. 1-4) spaced wide apart and through them, carefully draw a line with the working edge. Turn the T-square upside down as shown by dashed lines and with the same edge, draw another line passing through the same two points. If the edge is defective the lines will not coincide. The error should be rectified by planing or sand-papering the defective edge.

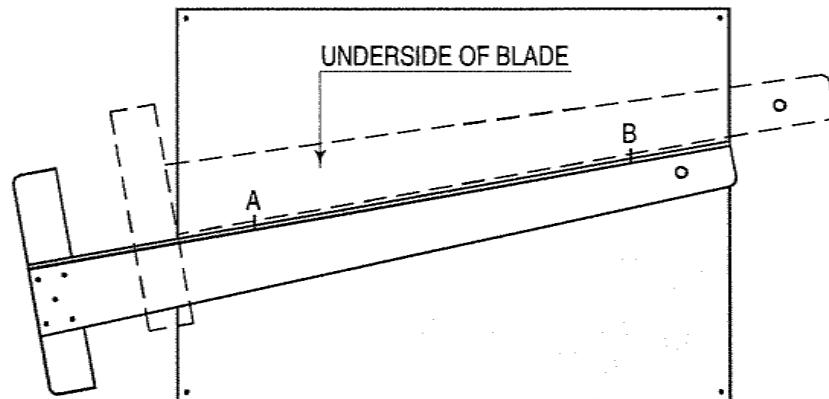


FIG. 1-4

1-4. SET-SQUARES (FIG. 1-5)

The set-squares are made of wood, tin, celluloid or plastic. Those made of transparent celluloid or plastic are commonly used as they retain their shape and accuracy for a longer time. Two forms of set-squares are in general use. A set-square is triangular in shape with one of the angles as right angle. The 30° - 60° set-square of 250 mm length and 45° set-square of 200 mm length are convenient sizes for use in schools and colleges.

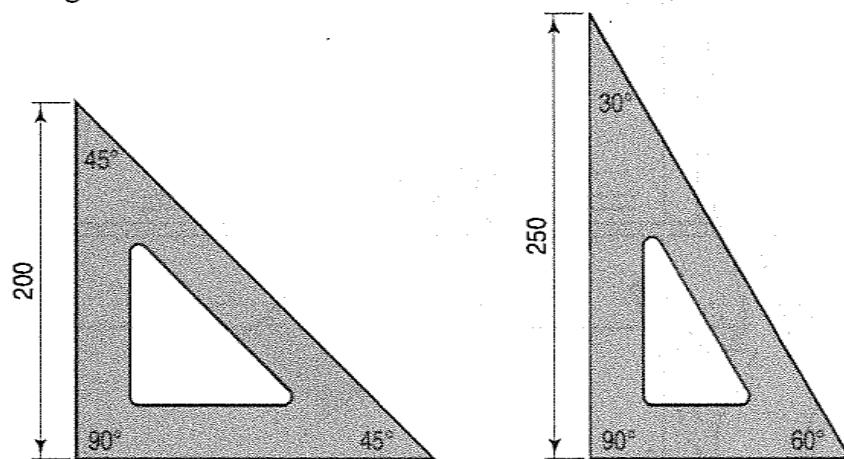


FIG. 1-5

Uses:

- (i) Set-squares are used for drawing all straight lines except the horizontal lines which are usually drawn with the T-square. Vertical lines can be drawn with the T-square and the set-square.
- (ii) In combination with the T-square, lines at 30° or 60° angle with vertical or horizontal lines can be drawn with 30° - 60° set-square and 45° angle with 45° set-square. The two set-squares used simultaneously along with the T-square will produce lines making angles of 15° , 75° , 105° etc.
- (iii) Parallel straight lines in any position, not very far apart, as well as lines perpendicular to any line from any given point within or outside it, can also be drawn with the two set-squares.
- (iv) A circle can be divided in six, eight, twelve and twenty four equal parts by using set-squares and T-square.

Problem 1-1. To draw a line perpendicular to a given horizontal line from a given point within it.

- (i) Place the T-square a little below the given line (fig. 1-6).
- (ii) Arrange any one set-square with one of the edges containing the right angle touching the working edge of the T-square, and the other edge passing through the given point.
- (iii) Hold the T-square and the set-square in this position firmly with the left hand.
- (iv) With the right hand, draw the required line through the given point in the upward direction as shown by the arrow. The pencil point should always be in contact with the edge of the set-square. A perpendicular from any given point outside the line can also be drawn in the same manner. Vertical parallel lines may be drawn by sliding the set-square along the edge of the T-square to the required positions.

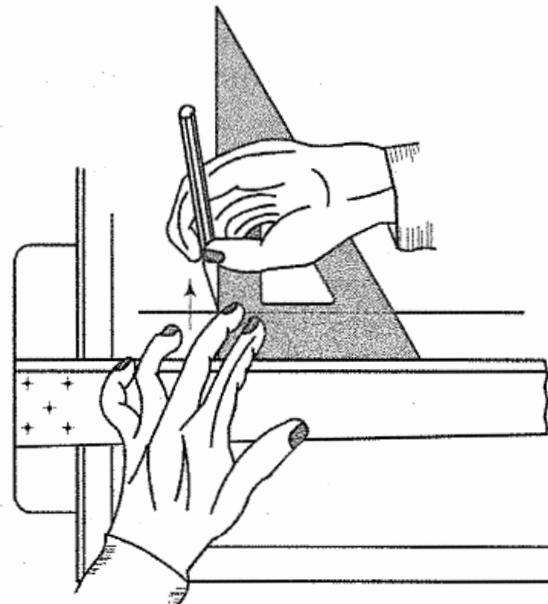


FIG. 1-6

Problem 1-2. To draw a line inclined at 45° , 30° or 60° to a given horizontal line from a given point.

- (i) Place the edge containing the right angle of the 45° set-square on the edge of the T-square (fig. 1-7).
- (ii) Slide it so that its longest edge (hypotenuse) passes through the given point and then draw the required line. The same line will make 45° angle with the vertical line passing through that point.
- (iii) By turning the set-square upside down, the line making 45° angle in the other direction will be drawn. The lines can also be drawn by placing the set-square so that its longest edge coincides with the edge of the T-square and the other edge passes through the given point. A circle can similarly be divided into eight equal parts by lines passing through its centre (fig. 1-8).

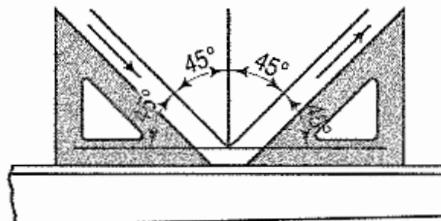


FIG. 1-7

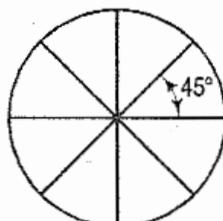


FIG. 1-8

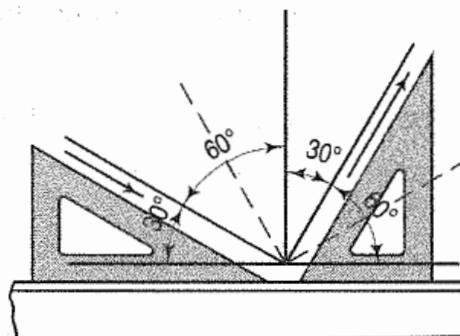


FIG. 1-9

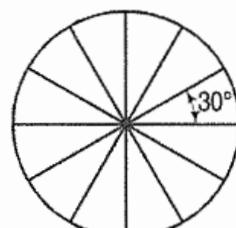


FIG. 1-10

Lines inclined at 30° or 60° to a given horizontal line can similarly be drawn with the aid of a 30° - 60° set-square (fig. 1-9). A circle may be divided into twelve equal divisions in the same manner (fig. 1-10).

Problem 1-3. To draw a line inclined at 15° to a given horizontal line from a given point.

- Place the 30° - 60° set-square with its longer edge containing the right angle, coinciding with the edge of the T-square (fig. 1-11).
- Arrange the 45° set-square with its longest edge on the longest edge of the 30° - 60° set-square.
- Slide the 45° set-square so that one of its edges containing the right angle passes through the given point, and draw the required line. The line drawn with the other edge will make 15° angle with the vertical line and 105° or 75° angles with the horizontal line. A circle may thus be divided into 24 equal parts with the aid of the set-squares (fig. 1-12).

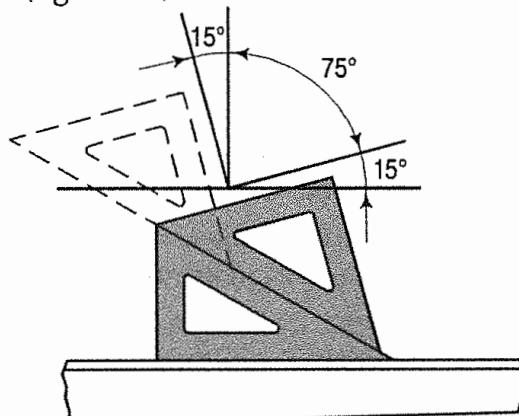


FIG. 1-11

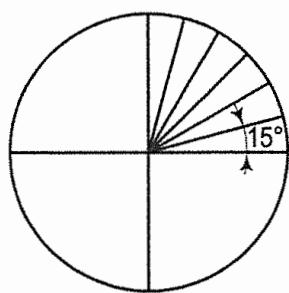


FIG. 1-12

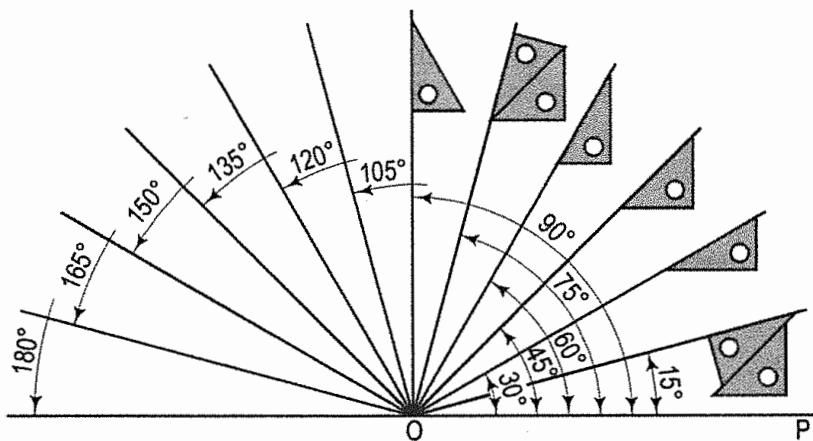


FIG. 1-13

Fig. 1-13 shows methods of drawing lines (with the aid of the T-square and set-squares) making angles with the horizontal line in multiples of 15° upto 180° .

Problem 1-4. To draw a line parallel to a given straight line through a given point.

The line AB and the point P are given (fig. 1-14).

- Arrange an edge of a set-square coinciding with AB .
- Place the other set-square as a base for the first.
- Hold the second set-square firmly and slide the first, till its arranged edge is along the point P .
- Draw the line CD through P . CD is the required parallel line.

By keeping the edge of the T-square as base for the set-square, parallel lines, long distances apart, can be drawn.

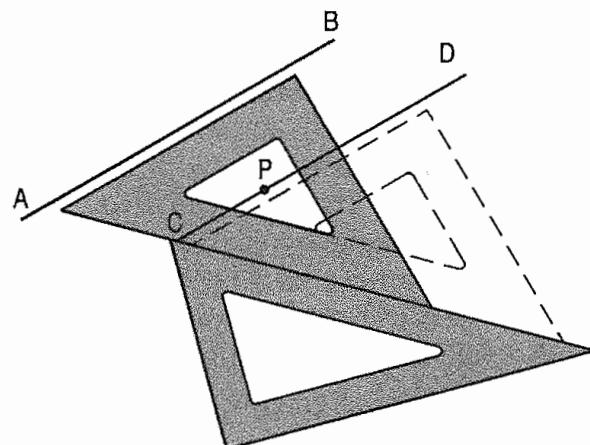


FIG. 1-14

Problem 1-5. To draw a line perpendicular to a given line through a point within or outside it.

The line PQ and the point O are given (fig. 1-15).

Method I:

- Arrange the longest edge of one set-square along PQ .
- Place the second set-square or T-square as base along one of the edges containing the right angle.
- Holding the base set-square firmly, rotate the first set-square so that its other edge containing the right angle coincides with the edge of the base set-square.
- Slide the first set-square till its longest edge is on the point O and draw the required line AB .

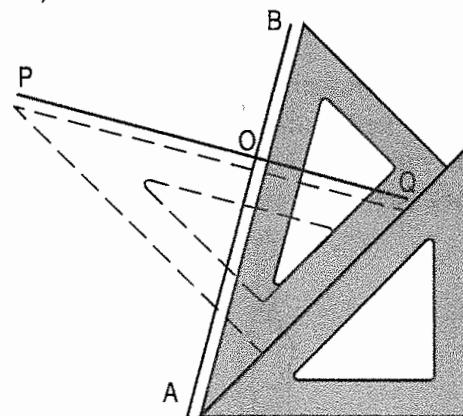


FIG. 1-15

Method II:

- Arrange one set-square with an edge containing the right angle along the line PQ (fig. 1-16).
- Place the second set-square or T-square as a base under the longest edge.
- Slide the first set-square on the second till the other edge containing the right angle is on the point O and draw the required line AB .

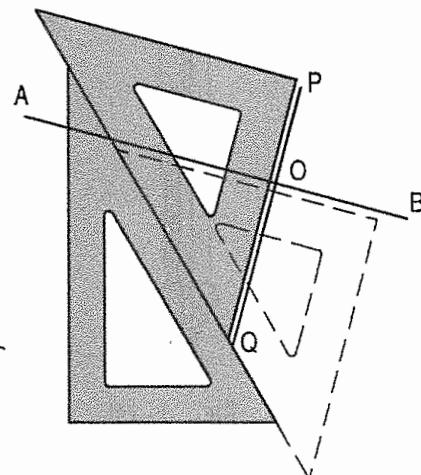


FIG. 1-16

Problem 1-6. To draw a line parallel to a given straight line at a given distance, say 20 mm from it (fig. 1-17).

Let AB be the given line.

- From any point P in AB , draw a line PQ perpendicular to AB (Problem 1-5).
- Mark a point R such that $PR = 20$ mm.
- Through R , draw the required line CD parallel to AB (Problem 1-4).

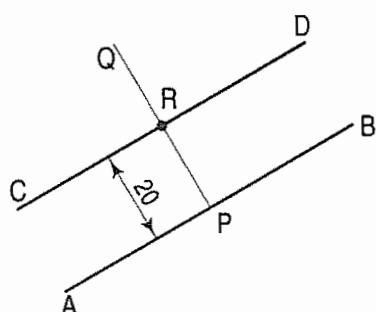


FIG. 1-17

1-5. DRAWING INSTRUMENT BOX

The drawing instrument box contains the following as mentioned earlier:

- | | |
|---|-----------------------|
| (1) Large-size compass with interchangeable pencil and pen legs | (5) Small bow divider |
| (2) Lengthening bar | (6) Small bow ink-pen |
| (3) Small bow compass | (7) Inking pen. |
| (4) Large-size divider | |

(1) Large-size compass with interchangeable pencil and pen legs (fig. 1-18): The compass is used for drawing circles and arcs of circles. It consists of two legs hinged together at its upper end. A pointed needle is fitted at the lower end of one leg, while a pencil lead is inserted at the end of the other leg. The lower part of the pencil leg is detachable and it can be interchanged with a similar piece containing an inking pen. Both the legs are provided with knee joints. Circles upto about 120 mm diameter can be drawn with the legs of the compass kept straight. For drawing larger circles, both the legs should be bent at the knee joints so that they are perpendicular to the surface of the paper (fig. 1-19).



FIG. 1-18

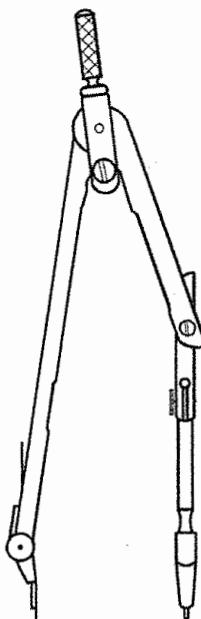


FIG. 1-19

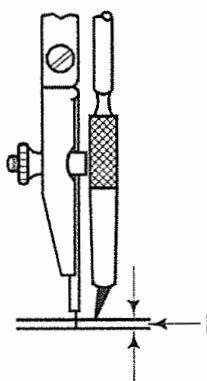


FIG. 1-20

As the needle is required to be inserted slightly inside the paper, it is kept longer than the lead point. *The setting of the pencil-lead relative to the needle, and the shape to which the lead should be ground are shown in fig. 1-20.*

To draw a circle, adjust the opening of the legs of the compass to the required radius. Hold the compass with the thumb and the first two fingers of the right hand and place the needle point lightly on the centre, with the help of the left hand. Bring the pencil point down on the paper and swing the compass about the needle-leg with a twist of the thumb and the two fingers, in clockwise \curvearrowright direction, until the circle is completed. The compass should be kept slightly inclined in the direction of its rotation. *While drawing concentric circles, beginning should be made with the smallest circle.*

(2) Lengthening bar: Circles of more than 150 mm radius are drawn with the aid of the lengthening bar. The lower part of the pencil leg is detached and the lengthening bar is inserted in its place. The detached part is then fitted at the end of the lengthening bar, thus increasing the length of the pencil leg (fig. 1-21).

It is often necessary to guide the pencil leg with the other hand, while drawing large circles.

(3) Small bow compass: For drawing small circles and arcs of less than 25 mm radius and particularly, when a large number of small circles of the same diameter are to be drawn, small bow compass is used (fig. 1-22).

Curves drawn with the compass should be of the same darkness as that of the straight lines. It is difficult to exert the same amount of pressure on the lead in the compass as on a pencil.

It is, therefore, desirable to use slightly softer variety of lead (about one grade lower, HB or H) in the compass than the pencil used for drawing straight lines, to maintain uniform darkness in all the lines.

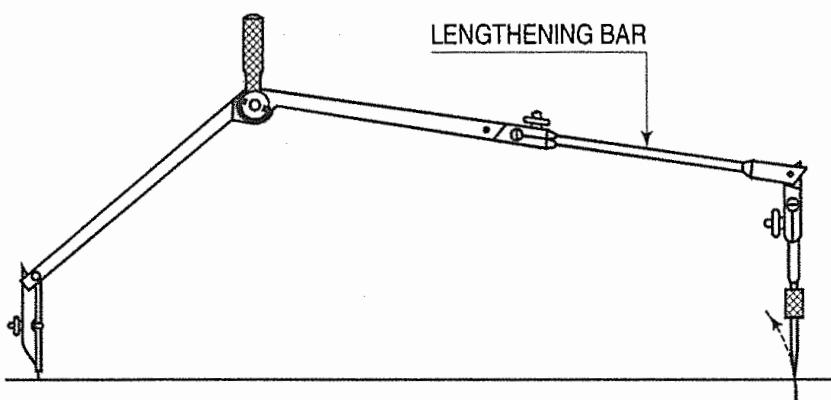


FIG. 1-21

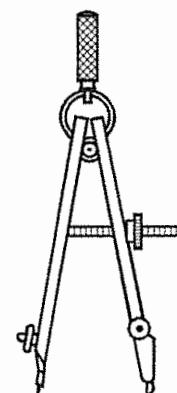


FIG. 1-22



FIG. 1-23

(4) **Large-size divider:** The divider has two legs hinged at the upper end and is provided with steel pins at both the lower ends, but it does not have the knee joints (fig. 1-23).

In most of the instrument boxes, a needle attachment is also provided which can be interchanged with the pencil part of the compass, thus converting it into a divider.

The dividers are used:

- to divide curved or straight lines into desired number of equal parts,
- to transfer dimensions from one part of the drawing to another part, and
- to set-off given distances from the scale to the drawing.

They are very convenient for setting-off points at equal distances around a given point or along a given line.

(5) **Small bow divider:** The small bow divider is adjusted by a nut and is very convenient for marking minute divisions and large number of short equal distances.

Problem 1-7. To divide a straight line into a number of equal parts — say 3.

The straight line AB is given (fig. 1-24).

- Set the legs of the divider so that the steel pins are approximately $\frac{1}{3}$ of the length of the line apart.

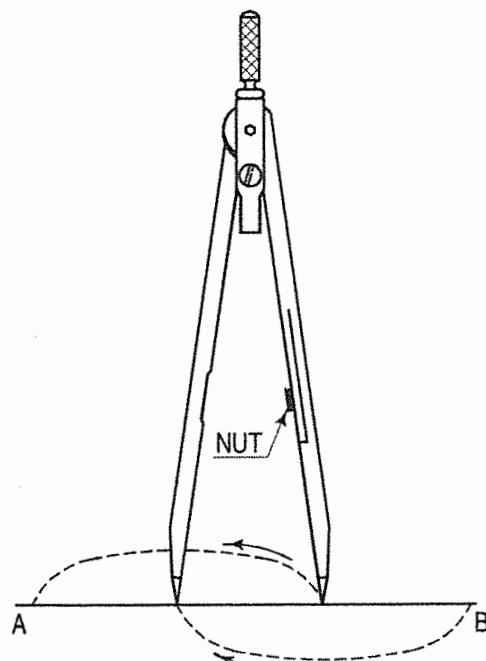


FIG. 1-24

- (ii) Step this distance lightly from one end of the line, say *B*, turning the divider first in one direction and then in the other. If the last division falls short, increase the set distance by approximately $\frac{1}{3}$ of the difference by means of the nut, keeping the other point of the divider on the paper. If the last division goes beyond the end of the line, decrease the set distance by $\frac{1}{3}$ of the difference.
- (iii) Re-space the line, beginning from the starting point, and adjusting the divider until the required setting is obtained.

With some practice, it will be possible to obtain the desired result with less trials and in short time. The trial divisions should be set-off as lightly as possible so that the paper is not pricked with large and unnecessary holes.

Any arc or a circle can similarly be divided into any number of equal divisions.

(6) **Small bow ink-pen:** It is used for drawing small circles and arcs in ink.

(7) **Inking pen (fig. 1-25):** This is used for drawing straight lines and non-circular arcs in ink. It consists of a pair of steel nibs fitted to a holder made of metal or ivory. Ink is filled between the two nibs to about 6 mm length by means of a quill which is usually fitted to the cork of the ink bottle. The gap between the nibs through which the ink flows and upon which the thickness of the line depends is adjusted by means of the screw *S*.

The pen should be kept sloping at about 60° with the paper in the direction of drawing the line and the ends of the nibs should be slightly away from the edge of the T-square or set-square. The screw should be on the side, farther from the T-square.

As the ink dries rapidly, the pen should be used immediately after it is filled. The inside faces of the nibs should be frequently cleaned for the ink to flow freely and to maintain uniformity in thickness of lines. Ink should never be allowed to dry within the pen. There should be no ink on the outside of the nibs and hence, the pen should never be dipped in ink.

For drawing large circles and circular arcs, inking attachment should be fitted in place of the pencil leg in the compass.

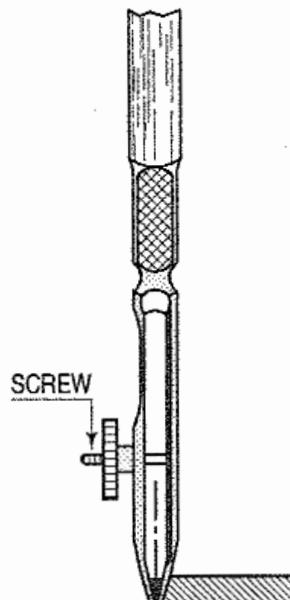


FIG. 1-25

1-6. SCALES (FIG. 1-26)



Scales are made of wood, steel, celluloid or plastic or card board. Stainless-steel scales are more durable. Scales may be flat or of triangular cross-section. 15 cm long and 2 cm wide or 30 cm long and 3 cm wide flat scales are in common use. They are usually about 1 mm thick. Scales of greater thickness have their longer edges bevelled. This helps in marking measurements from the scale to the drawing paper accurately. Both the longer edges of the scales are marked with divisions of centimetres, which are sub-divided into millimetres.

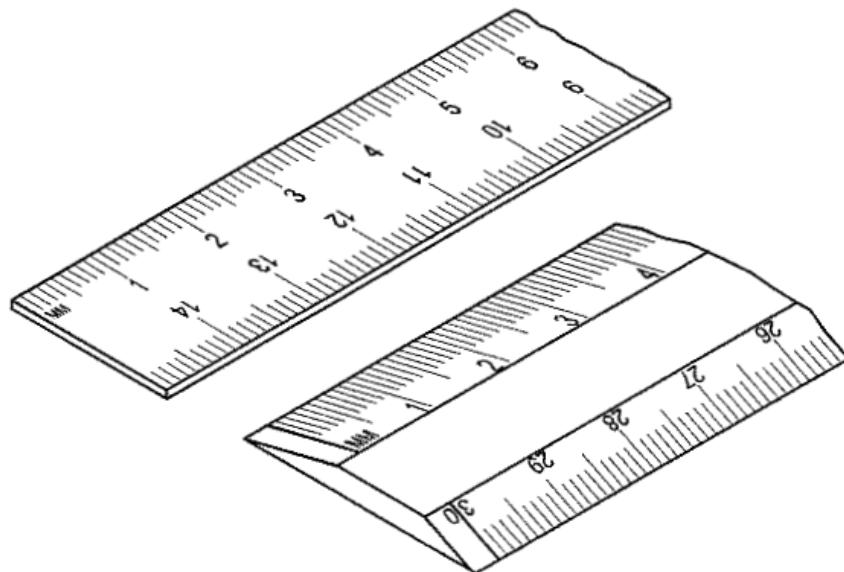


FIG. 1-26

Various other types of scales are described in chapter 4.

The scale is used to transfer the true or relative dimensions of an object to the drawing. It is placed with its edge on the line on which measurements are to be marked and, looking from exactly above the required division, the marking is done with a fine pencil point. *The scale should never be used as a straight-edge for drawing lines.* The card-board scales are available in a set of eight scales. They are designated from M1 to M8 as shown in table 1-2.

TABLE 1-2
STANDARD SCALES

Designation	Description	Scale
M1	Full size 50 cm to a metre	1:1 1:2
M2	40 cm to a metre 20 cm to a metre	1:2.5 1:5
M3	10 cm to a metre 5 cm to a metre	1:10 1:20
M4	2 cm to a metre 1 cm to a metre	1:50 1:100
M5	5 mm to a metre 2 mm to a metre	1:200 1:500
M6	3.3 mm to a metre 1.66 mm to a metre	1:300 1:600
M7	2.5 mm to a metre 1.25 mm to a metre	1:400 1:800
M8	1 mm to a metre 0.5 mm to a metre	1:1000 1:2000

1-7. PROTRACTOR (FIG. 1-27)

Protractor is made of wood, tin or celluloid. Protractors of transparent celluloid are in common use. They are flat and circular or semi-circular in shape. The commonest type of protractor is semi-circular and of about 100 mm diameter. Its circumferential edge is graduated to 1° divisions, is numbered at every 10° interval and is readable from both the ends. The diameter of the semi-circle (viz. straight line 0- 180°) is called the base of the protractor and its centre O is marked by a line perpendicular to it.

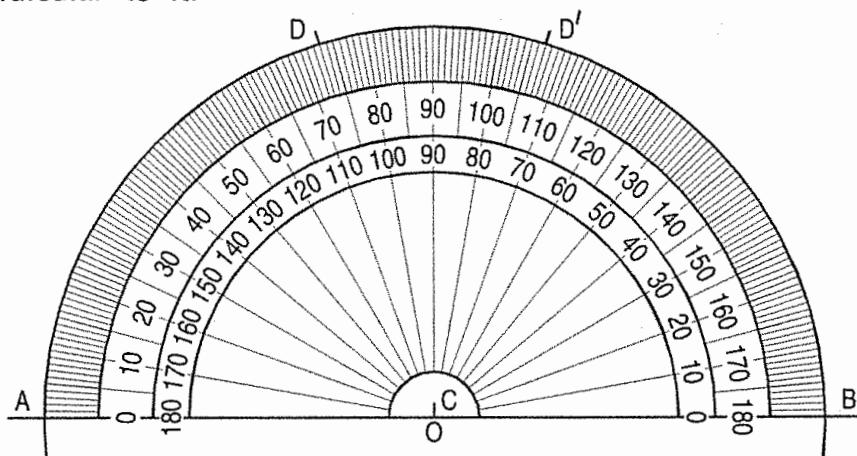


FIG. 1-27

The protractor is used to draw or measure such angles as cannot be drawn with the set-squares. A circle can be divided into any number of equal parts by means of the protractor.

Problem 1-8. To draw a line making an angle of 73° with a given line through a given point in it.

Let AB be the line and C the point in it.

- (i) Set the protractor with its base coinciding with AB (fig. 1-28) and its centre exactly on the point C.
- (ii) Mark a point D opposite to the 73° division and join C with D. Then $\angle ACD = 73^\circ$ (fig. 1-28). Another point D' can be marked against the reading from the other side. In this case $\angle BCD' = 73^\circ$ while $\angle ACD' = 107^\circ$.

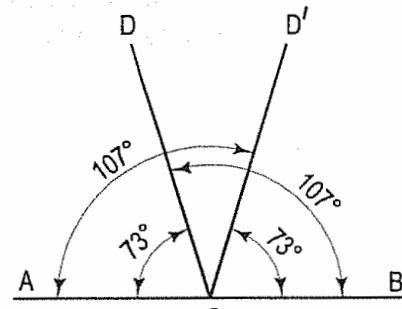


FIG. 1-28

1-8. FRENCH CURVES

French curves are made of wood, plastic or celluloid. They are made in various shapes, one of which is shown in fig. 1-29. Some set-squares also have these curves cut in their middle.

French curves are used for drawing curves which cannot be drawn with a compass. Faint freehand curve is first drawn through the

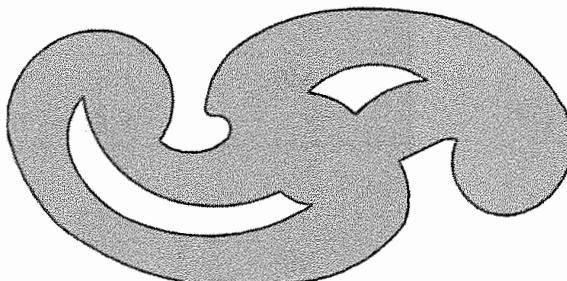


FIG. 1-29

known points. Longest possible curves exactly coinciding with the freehand curve are then found out from the french curve. Finally, neat continuous curve is drawn with the aid of the french curve. Care should be taken to see that no corner is formed anywhere within the drawn curve.

1-9. DRAWING PAPERS

Drawing papers are available in many varieties. For ordinary pencil-drawings, the paper selected should be tough and strong. It should be uniform in thickness and as white as possible. When the rubber eraser is used on it, its fibres should not disintegrate. Good quality of paper with smooth surface should be selected for drawings which are to be inked and preserved for a long time. It should be such that the ink does not spread. Thin and cheap quality paper may be used for drawings from which tracings are to be prepared. The standard sizes of drawing papers recommended by the Bureau of Indian Standards (B.I.S.) are given in table 2-1.

Surface area of A0 size is one square metre. Successive format sizes (from A0 to A5) are obtained by halving along the length or doubling along the width. The areas of the two subsequent sizes are in the ratio 1:2. See fig. 1-30.

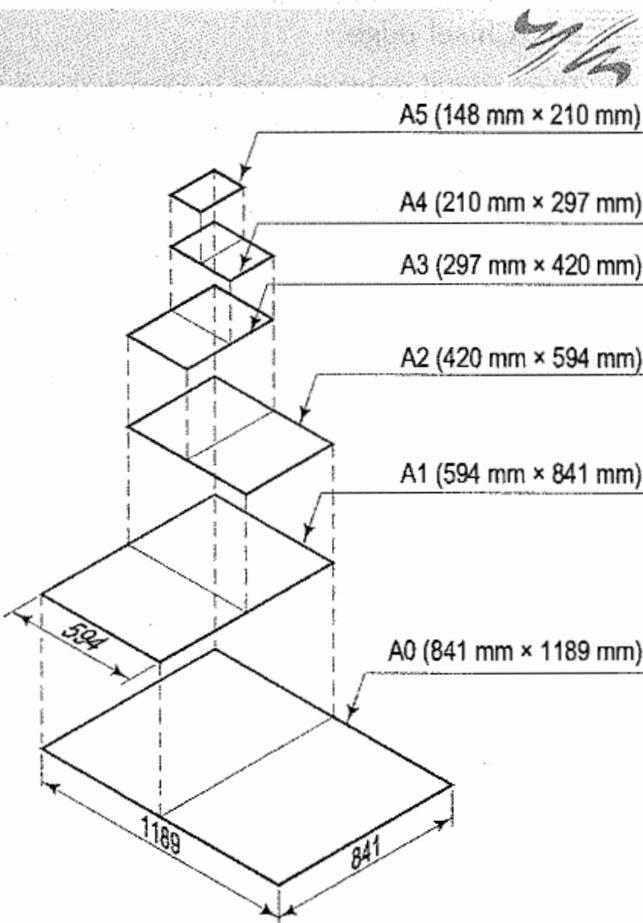


FIG. 1-30

1-10. DRAWING PENCILS

The accuracy and appearance of a drawing depend very largely on the quality of the pencils used. With cheap and low-quality pencils, it is very difficult to draw *lines of uniform shade and thickness*. The grade of a pencil lead is usually shown by figures and letters marked at one of its ends. Letters HB denote the medium grade. The increase in hardness is shown by the value of the figure put in front of the letter H, viz. 2H, 3H, 4H etc. Similarly, the grade becomes softer according to the figure placed in front of the letter B, viz. 2B, 3B, 4B etc.

Beginning of a drawing should be made with H or 2H pencil using it very lightly, so that the lines are faint, and unnecessary or extra lines can be easily erased. The final fair work may be done with *harder pencils*, e.g. 3H and upwards. Lines of uniform thickness and darkness can be more easily drawn with hard-grade pencils.

H and HB pencils are more suitable for lettering and dimensioning. For freehand sketching, where considerable erasing is required to be done, soft-grade pencils such as HB should be used.

Great care should be taken in mending the pencil and sharpening the lead, as the uniformity in thickness of lines depends largely on this. The lead may be sharpened to two different forms:

- (i) Conical point and
- (ii) Chisel edge.

The conical point is used in sketch work and for lettering etc. With the chisel edge, long thin lines of uniform thickness can be easily drawn and hence, it is suitable for drawing work.

To prepare the pencil lead for drawing work, the wood around the lead from the end, other than that on which the grade is marked, is removed with a pen-knife, leaving about 10 mm of lead projecting out, as shown in fig. 1-31(a).

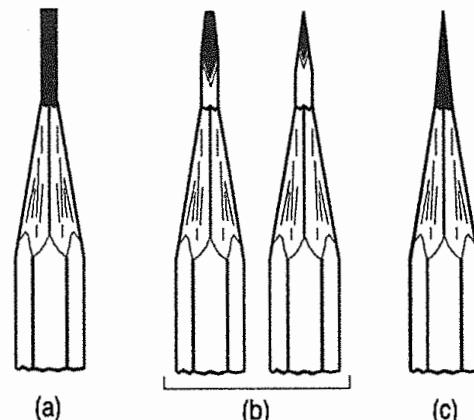


FIG. 1-31

The chisel edge [fig. 1-31(b)] is prepared by rubbing the lead on a sand-paper block, making it flat, first on one side and then on the other by turning the pencil through a half circle. For making the conical end [fig. 1-31(c)] the pencil should be rotated between the thumb and fingers, while rubbing the lead.

The pencil lead should occasionally be rubbed on the sand-paper block (while doing the drawing work) to maintain the sharpness of the chisel edge or the pointed end.

Instead of wooden pencils, Mechanical clutch pencils with a different lead size and grade like 5 mm, 4 mm and H, 2H, HB etc., are also available. Sharpening is not required in such pencils.

1-11. ERASER (RUBBER)



Soft India-rubber is the most suitable kind of eraser for pencil drawings. It should be such as not to spoil the surface of the paper. Frequent use of rubber should be avoided by careful planning.

1-12. DRAWING PINS, CLIPS OR ADHESIVE TAPES



These are used to fix the drawing paper on the drawing board. The needle part of the pin is generally made of steel, while the head may be of plated mild steel or brass. Pins of about 15 mm to 20 mm diameter and about 1 mm thick flat heads made of brass are quite convenient, as they do not rust. Pins should be so inserted that the heads sit on the surface of the paper. Clips or adhesive tapes are often used instead of the pins. (Refer fig. 1-32).

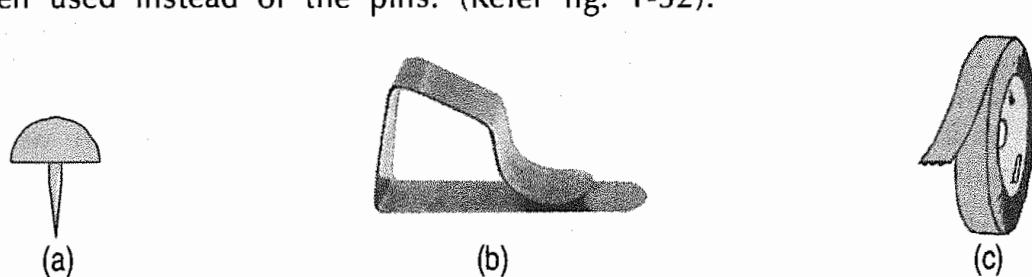


FIG. 1-32

1-13. SAND-PAPER BLOCK

It consists of a wooden block about 150 mm \times 50 mm \times 12 mm thick with a piece of sand-paper pasted or nailed on about half of its length, as shown in fig. 1-33.

The sand-paper, should be replaced by another, when it becomes dirty or worn out. This block should always be kept within easy reach for sharpening the pencil lead every few minutes.

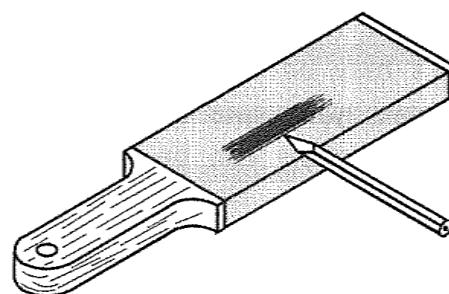


FIG. 1-33

1-14. DUSTER

Duster should preferably be of towel cloth of convenient size. Before starting work, all the instruments and materials should be thoroughly cleaned with the duster. The rubber crumbs formed after the use of the rubber should be swept away by the duster and not by hand. The underside of the T-square and the set-squares or the drafting machine which continuously rub against the paper should be frequently cleaned.

1-15. DRAFTING MACHINE (FIG. 1-34)

The uses and advantages of the T-square, set-squares, scales and the protractor are combined in the drafting machine. Its one end is clamped by means of a screw, to the distant longer edge of the drawing board. At its other end, an adjustable head having protractor markings is fitted. Two blades of transparent celluloid accurately set at right angles to each other are attached to the head.

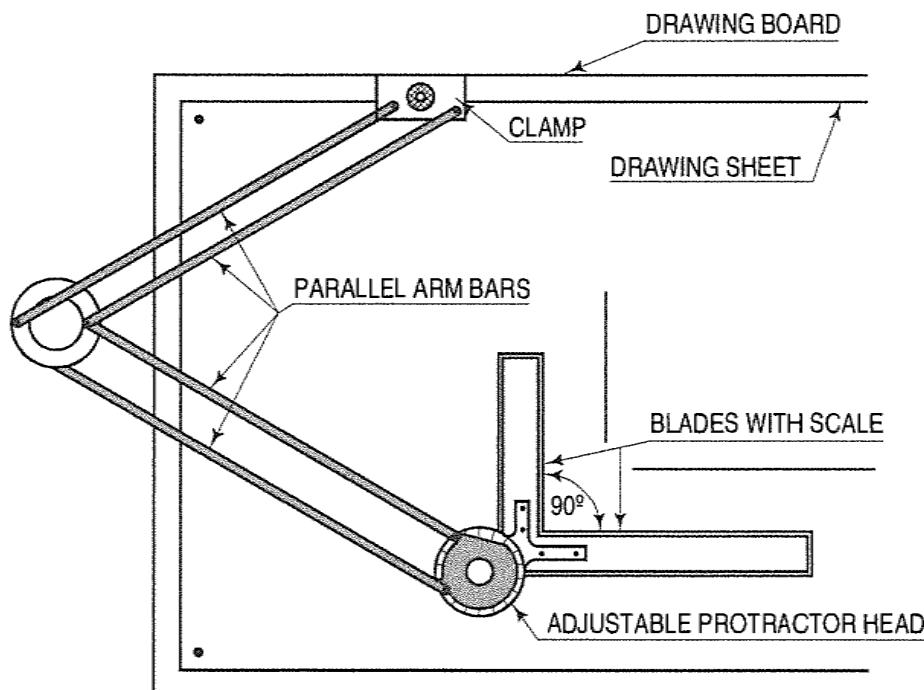


FIG. 1-34

The machine has a mechanism which keeps the two blades always parallel to their respective original position, wherever they may be moved on the board. The blades have scales marked on them and are used as straight edges. In some machines, the blades are removable and hence a variety of scales can be used. The blades may be set at any desired angle with the help of the protractor markings.

Thus, by means of this machine, horizontal, vertical or inclined parallel lines of desired lengths can be drawn anywhere on the sheet with considerable ease and saving of time. Drafting machines are common among the college students and draughtsmen.

1-16. ROLL-N-DRAW (FIG. 1-35)



It consists of graduated roller, scale of 16 centimeter and protactor. It is ideal for drawing vertical lines, horizontal lines, parallel lines, angles and circles.

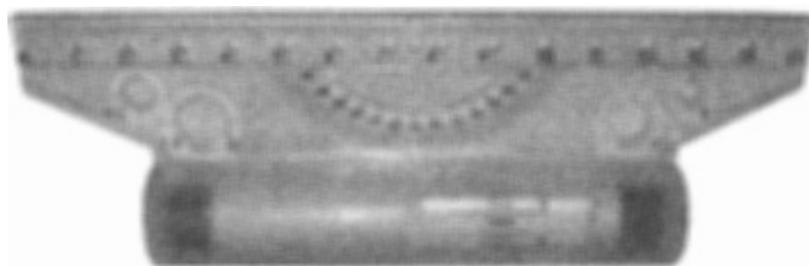


FIG. 1-35

1-17. GENERAL SUGGESTIONS FOR DRAWING A SHEET



(1) Cleaning the instruments: Clean the drawing board and the T-square and place them on the table, with the working edge of the board on your left-hand side and the stock of the T-square attached to that working edge. Clean all other instruments and materials and place them on a neat piece of paper by the side of the board. When a drafting machine is used, clean the drafting machine before fixing on drawing board.

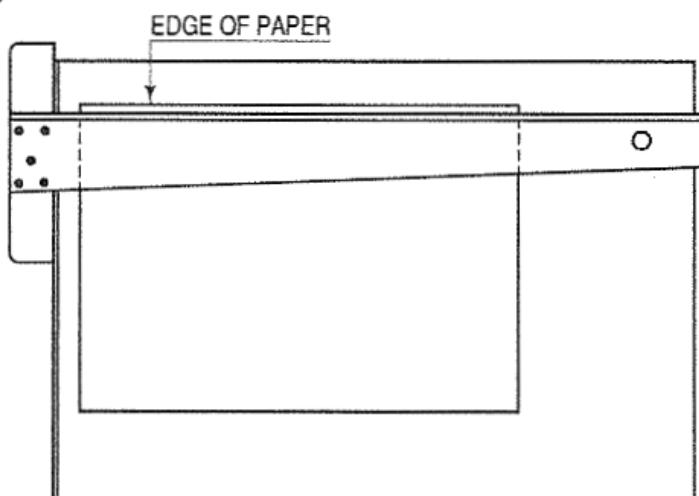


FIG. 1-36

(2) Pinning the paper to the drawing board: Place the paper at about equal distances from the top and bottom edges of the board and one of its shorter edges at about 25 mm from the working edge of the board. When the paper is

of a much smaller size than that of the board, it may be placed with its lower edge at about 50 mm from the bottom edge of the board. Insert a pin in the left-hand top corner of the paper and at about 10 mm from its edges. Adjust the paper with the right hand, bringing its upper edge in line with the working edge of the T-square (fig. 1-36).

Stretch the paper gently to make it perfectly flat and insert the second pin at the right-hand bottom corner. In the same manner, fix two more pins at the remaining corners. Push the pins down firmly till their heads touch the surface of the paper.

(3) Border lines: Perfectly rectangular working space is determined by drawing the border lines. These may be drawn at equal distances of about 20 mm to 25 mm from the top, bottom and right-hand edges of the paper and at about 25 mm to 40 mm from the left-hand edge. More space on the left-hand side is provided to facilitate binding of the drawing sheets in a book-form, if so desired.

To draw the border lines (fig. 1-37): [Dimensions shown are in mm.]

- Mark points along the left-hand edge of the paper at required distances from the top and bottom edges and through them, draw horizontal lines with the T-square or by mini-drafter.
- Along the upper horizontal line, mark two points at required distances from the left-hand and right-hand edges, and draw vertical lines through them by mini-drafter.
- Erase the extra lengths of lines beyond the points of intersection.
- One more horizontal line at about 10 mm to 20 mm from the bottom border line may also be drawn and the space divided into three blocks. A title block as shown in fig. 2-2 must be drawn in left-hand bottom corner above block-3; in which
 - name of the institution,
 - title of the drawing, and
 - name, class etc. of the student may be written.

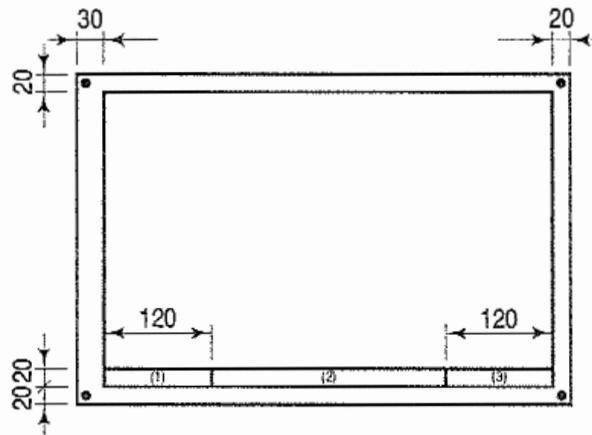


FIG. 1-37

(4) Spacing of drawings: When only one drawing or figure is to be drawn on a sheet, it should be drawn in the centre of the working space. For more than one figure, the space should be divided into suitable blocks and each figure should be drawn in the centre of its respective block.

Important: The subject of Engineering Drawing cannot be learnt only by reading the book. The student must have practice in drawing. With more practice he can attain not only the knowledge of the subject but also the speed. He may possess drawing instruments of the best quality. But to gain proficiency in the subject he should pay a lot of attention to accuracy, draftsmanship i.e. uniformity in thickness and shade of lines according to its type, nice lettering and above all general neatness.

EXERCISES 1

1. In an A2 size sheet, copy fig. 1-38(a) to fig. 1-38(g) as per layout shown in fig. 1-39. [All dimensions are in mm.]

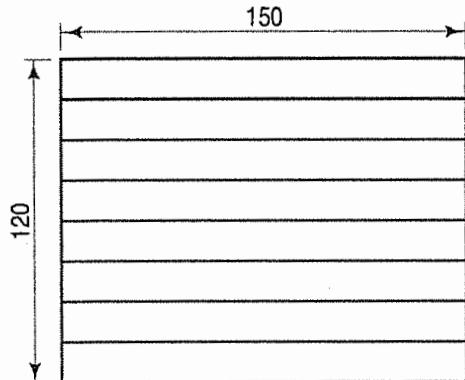


FIG. 1-38(a)

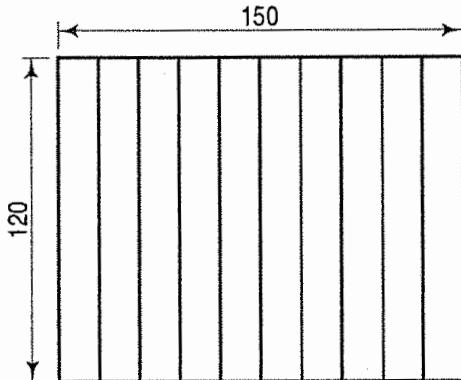


FIG. 1-38(b)

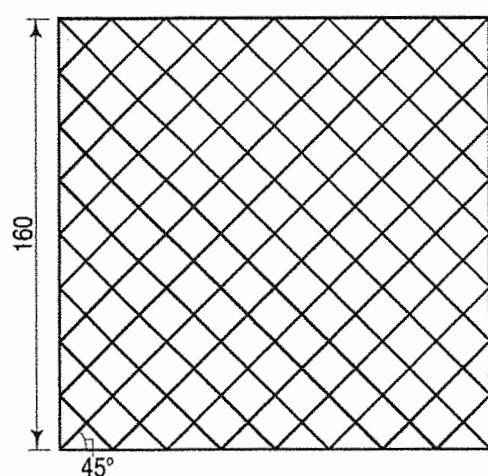


FIG. 1-38(c)

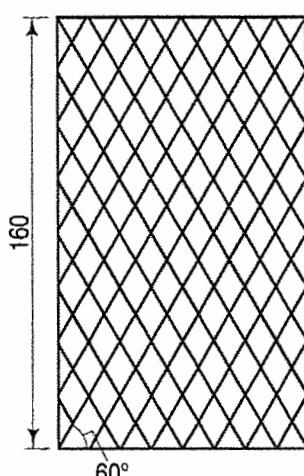


FIG. 1-38(d)

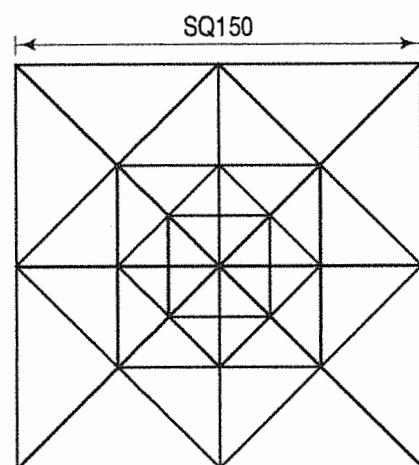


FIG. 1-38(e)

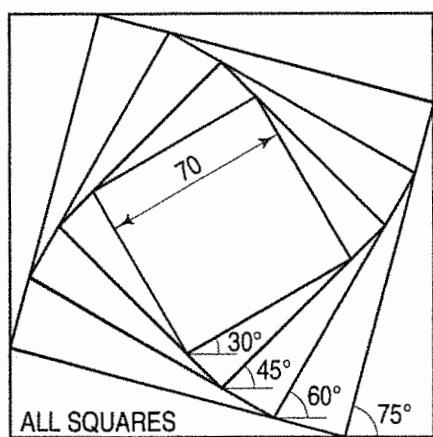


FIG. 1-38(f)

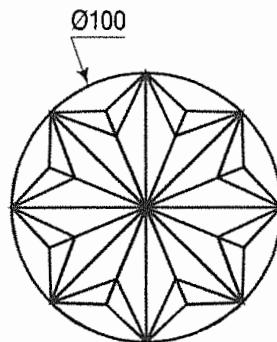


FIG. 1-38(g)

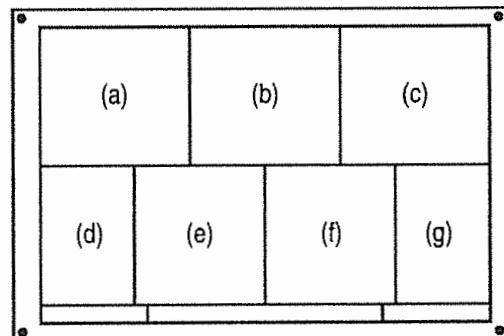


FIG. 1-39

2. Draw a circle of 75 mm diameter. With the aid of T-square and set-squares only, draw lines passing through its centre, dividing it into
 (i) eight, (ii) twelve and (iii) twenty four equal parts.

3. Without using a protractor, draw triangles having following base angles on a 75 mm long line as base:
 - (i) 75° and 15° , (ii) 60° and 75° , (iii) 135° and 15° , (iv) 105° and 45° .
4. Draw a line 125 mm long and divide it into seven equal parts by means of a divider.
5. Copy fig. 1-40(a) to fig. 1-40(d) as per layout shown in fig. 1-41.
[All dimensions are in mm.]

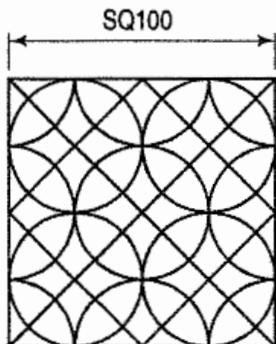


FIG. 1-40(a)

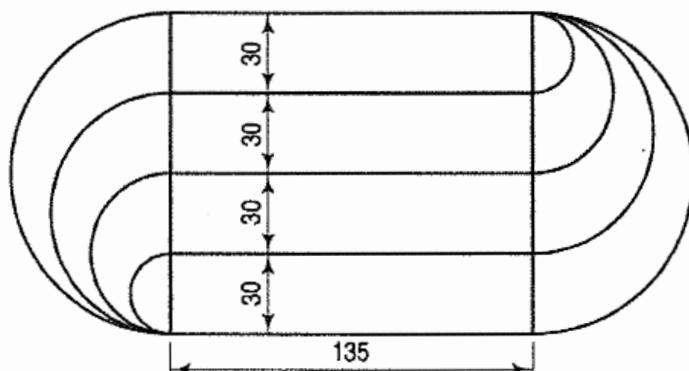


FIG. 1-40(b)

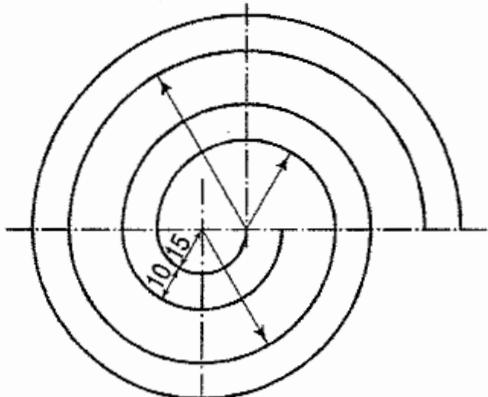


FIG. 1-40(c)

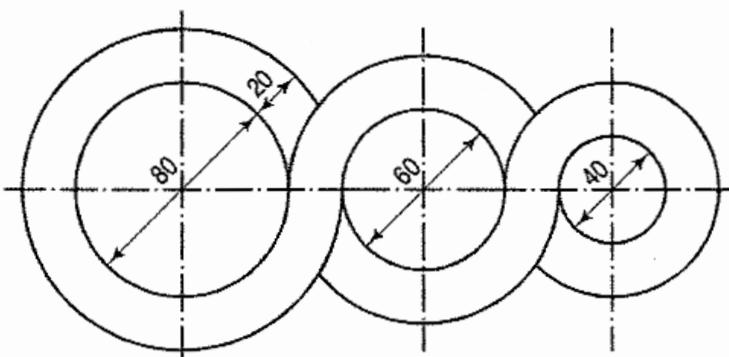


FIG. 1-40(d)

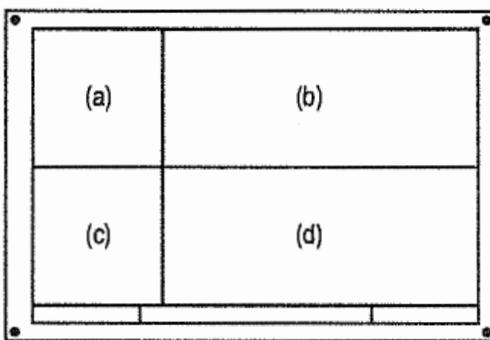


FIG. 1-41

6. Draw lines (using the protractor) meeting at the end A of a line AB and making with it the angles: (1) 27° , (2) 49° , (3) 115° , (4) 151° .
7. Draw a straight line AB of any length. Mark any point O on it. Through O, draw all possible angles in multiples of 15° , with the aid of the T-square and set-squares.
8. Fill up the blanks with appropriate words selected from the list given:
 - (a) The edge of the board on which T-square is made to slide is called its _____.

 (1) working edge (2) straight edge (3) chisel edge.

- (b) To prevent warping of the board _____ are cleated at its back.
 (1) packings (2) wooden blocks (3) battens.
- (c) The two parts of the T-square are _____ and _____.
 (1) vertical and horizontal edge (2) straight edge (3) stock (4) blade.
- (d) The T-square is used for drawing _____ lines.
 (1) vertical (2) curve (3) horizontal.
- (e) Angles in multiples of 15° are constructed by the combined use of _____ and _____.
 (1) T-square (2) set-squares (3) protractor.
- (f) To draw or measure angles, _____ is used.
 (1) set-squares (2) T-square (3) protractor.
- (g) For drawing large-size circles, _____ is attached to the compass.
 (1) straight bar (2) bow compass (3) lengthening bar.
- (h) Circles of small radii are drawn by means of a _____.
 (1) lengthening bar (2) bow divider (3) bow compass.
- (i) Measurements from the scale to the drawing are transferred with the aid of a _____.
 (1) scale (2) compass (3) divider.
- (j) The scale should never be used as a _____ for drawing straight lines.
 (1) set-squares (2) working edge (3) straight edge.
- (k) _____ is used for setting-off short equal distances.
 (1) compass (2) bow divider (3) scale.
- (l) For drawing thin lines of uniform thickness the pencil should be sharpened in the form of _____.
 (1) chisel edge (2) conical (3) pointed.
- (m) Pencil of _____ grade sharpened in the form of _____ is used for sketching and lettering.
 (1) soft (2) low (3) conical point (4) chisel.
- (n) _____ are used for drawing curves which cannot be drawn by a compass.
 (1) bow compass (2) protractor (3) French curves.
- (o) To remove unnecessary lines _____ is used.
 (1) duster (2) chalk (3) sand box (4) eraser.
- (p) Uses of the T-square, set-squares, scale and protractor are combined in the _____.
 (1) set-squares (2) drafting machine (3) compass.
- (q) Circles and arcs of circles are drawn by means of a _____.
 (1) lengthening bar (2) divider (3) compass.
- (r) Inking pen is used for drawing _____ in ink.
 (1) writing (2) curves (3) straight lines.
- (s) Set-squares are used for drawing _____, _____ and _____ lines.
 (1) horizontal (2) vertical (3) inclined (4) parallel.

Answer to Ex. 8:

a-1, b-3, c-3 and 4, d-3, e-1 and 2, f-3, g-3, h-3, i-3, j-3, k-2, l-1, m-1 and 3, n-3, o-4, p-2, q-3, r-3, s-2, 3 and 4.

Chapter 2



SHEET LAYOUT AND FREE-HAND SKETCHING

2-1. SHEET LAYOUT

(1) **Sheet sizes:** The preferred sizes of the drawing sheets recommended by the Bureau of Indian Standards (B.I.S.) are given below as per SP : 46 (2003). Refer fig. 1-30:

TABLE 2-1

Sheet designation	Recommended sizes	Unrecommended sizes
A0	841 × 1189	880 × 1230
A1	594 × 841	625 × 880
A2	420 × 594	450 × 625
A3	297 × 420	330 × 450
A4	210 × 297	240 × 330
A5	148 × 210	165 × 240

The layout of the drawing on a drawing sheet should be done in such a manner as to make its reading easy and speedy. Fig. 2-1(a) and fig. 2-1(b) shows an A1 size sheet layout. All dimensions are in millimetres.

(2) **Margin:** Margin is provided in the drawing sheet by drawing margin lines [fig. 2-1(a)]. Prints are trimmed along these lines. After trimming, the prints would be of the recommended trimmed sizes of the trimmed sheets.

(3) **Border lines:** Clear working space is obtained by drawing border lines as shown in [fig. 2-1(a)]. More space is kept on the left-hand side for the purpose of filing or binding if necessary. When prints are to be preserved or stored in a cabinet without filing, equal space may be provided on all sides (fig. 2-3).

(4) **Borders and frames:** SP : 46 (2003) recommends the borders of 20 mm width for the sheet sizes A0 and A1, and 10 mm for the sizes A2, A3, A4 and A5. Frame shows the clear space available for the drawing purpose.

(5) **Orientation mark:** Four centring marks are drawn as shown in fig. 2-1(b) to facilitate positioning of the drawing for the reproduction purpose. The *orientation mark* will coincide with one of centring marks which can be used for the orientation of drawing sheet on the drawing board.

(6) **Grid reference system (zones system):** The *grid reference system* is drawn on the sheet to permit easy location on the drawing such as details, alterations or additions. The rectangle of grid along the length should be referred by numerals 1, 2, 3... etc. and along the width by the capital letters A, B, C, D etc. as shown in fig. 2-1(b).

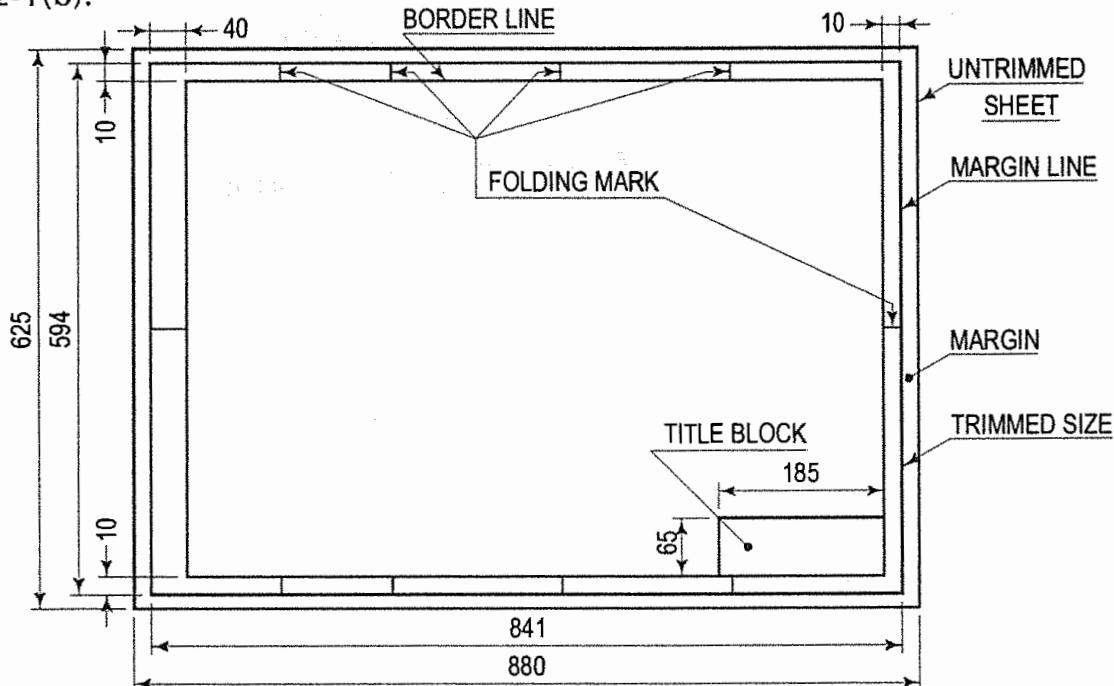


FIG. 2-1(a)

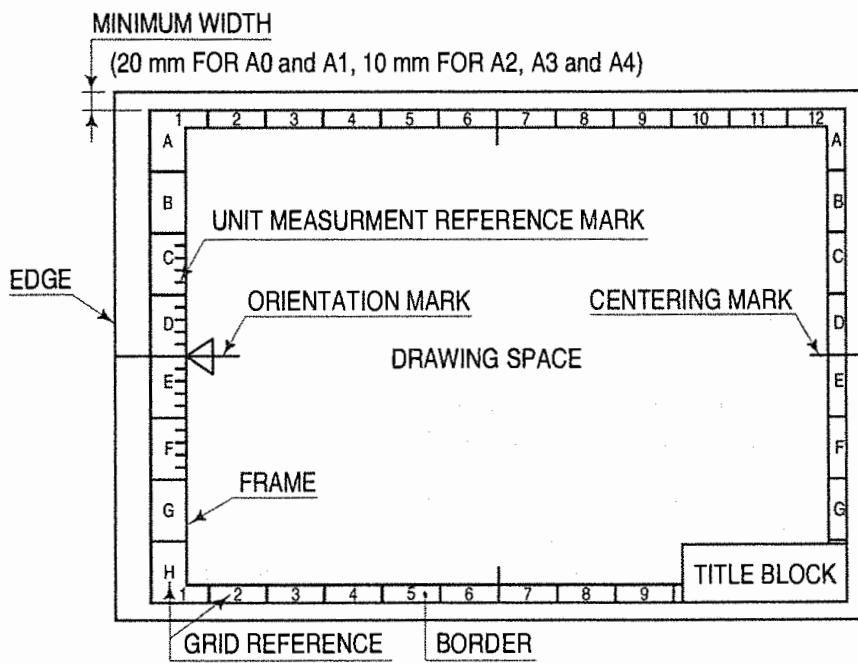


FIG. 2-1(b)

(7) **Title block:** Space for the title block must be provided in the bottom right-hand corner of the drawing sheet as shown in fig. 2-1(a) and fig. 2-1(b). The size of the title block as recommended by the B.I.S. is 185 mm × 65 mm for all designations of the drawing sheets. Fig. 2-2 shows the simplest type of a title block. All title blocks should contain at least the particulars as shown in table 2-2.

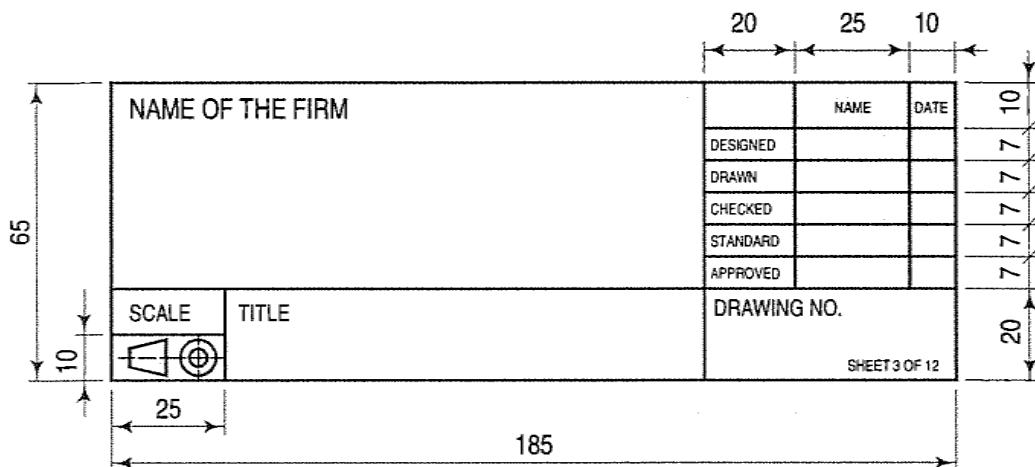


FIG. 2-2

TABLE 2-2
PARTICULARS OF TITLE BLOCK

1.	Name of the firm.
2.	Title of the drawing.
3.	Scale.
4.	Symbol for the method of projection.
5.	Drawing number.
6.	Initials with dates of persons who have designed, drawn, checked, standards and approved.
7.	No. of sheet and total number of sheets of the drawing of the object.

(8) **List of parts or the bill of materials:** When drawings of a number of constituent parts of an object are drawn in a single drawing sheet, a list of these parts should be placed above or beside the title block in a tabular form. It should provide the following minimum particulars for each part:

Part no., name or description, no. off i.e. quantity required, material and sometime stock size of raw material, remarks.

Additional information such as job and order number, instructions regarding finish, heat-treatment, tolerances (general) and references pertaining to jigs, fixtures, tools, gauges etc. may be, if necessary included in the title block or given separately in tabular form.

(9) **Revisions of drawing:** For locating a portion of the drawing for the purpose of revision etc., the sides of the three larger sizes of the drawing sheets viz. A0, A1 and A2 are divided into a number of equal zones.

Fig. 2-3 shows an A1 size sheet with zones marked on it. The zones along the length l are designated by numerals, while those along the width w are designated by letters. The location D8 is a rectangle formed by the intersection of the zones D and 8. The number of zones suggested by the B.I.S. for A0, A1 and A2 sizes of drawing sheets along the lengths l are 16, 12 and 8 respectively, while those along the widths w are 12, 8 and 6 respectively.

A revision panel is drawn either attached to the title block above it or in the top right-hand corner of the sheet. The revisions are recorded in it giving the revision number, date, zone etc. and also the initials of the approving authority.

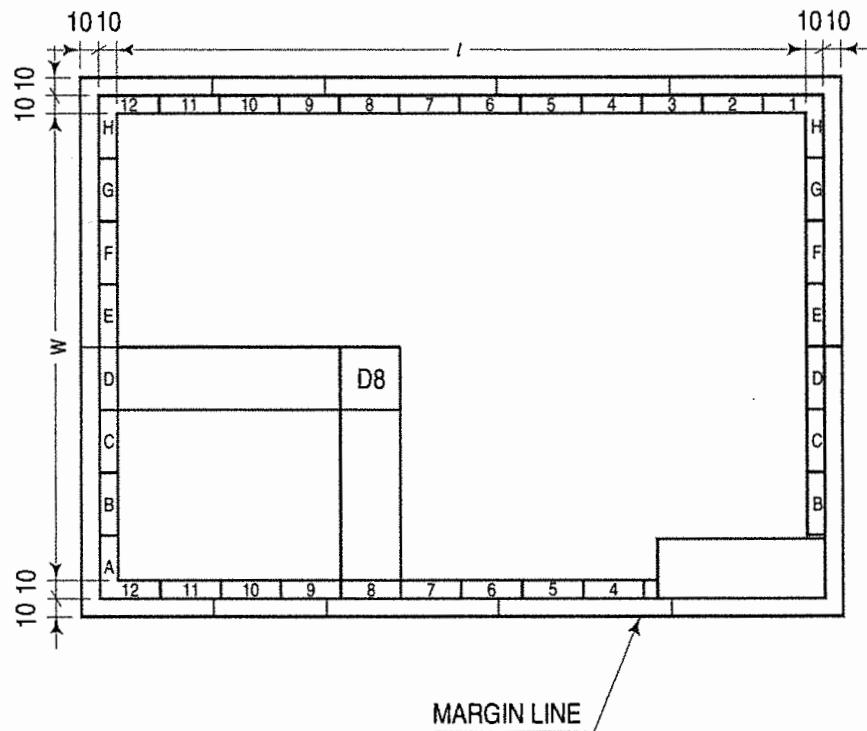


FIG. 2-3

(10) Folding marks: Folding marks are made in the drawing sheet as shown in fig. 2-1(a). They are helpful in folding of prints in proper and easy manner. Two methods of folding of prints are in general use. Method I is suitable for prints which are to be filed or bound. It allows prints to be unfolded or refolded without removing them from the files.

Fig. 2-4(i) shows the folding diagram for folding an A1 size sheet by method I. It is folded in two stages, viz. lengthwise [fig. 2-4(ii)] and crosswise [fig. 2-4(iii)].

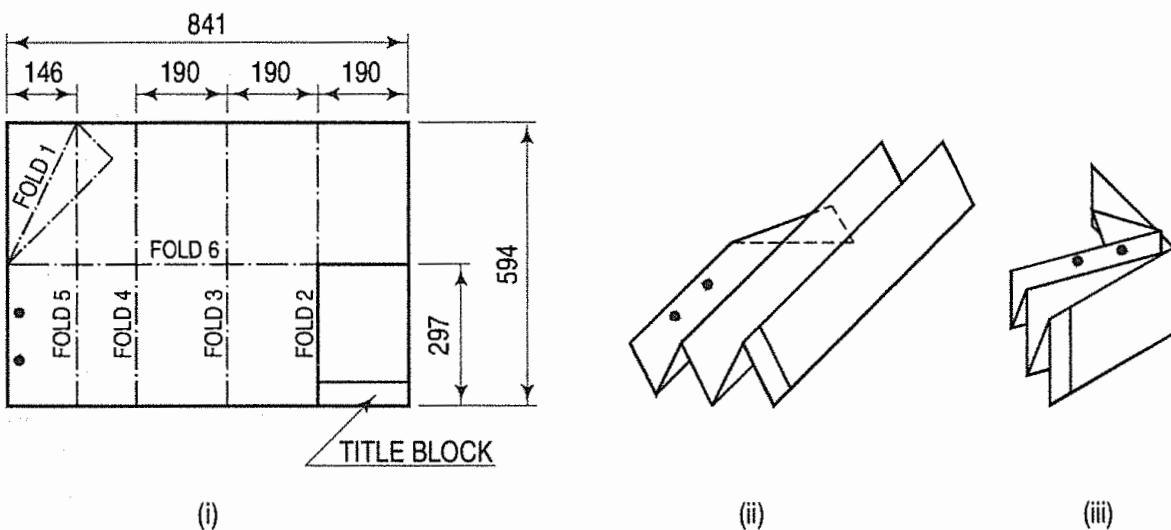


FIG. 2-4

When prints are to be stored and preserved in cabinets they are folded by method II. The folding diagram for folding an A1 size sheet by method II is shown in fig. 2-5. The two stages of folding are similar to those for method I.

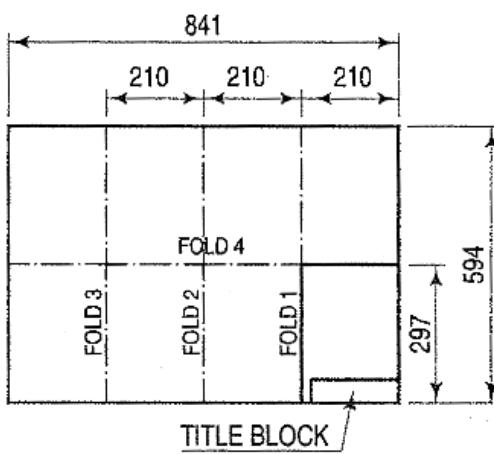


FIG. 2-5

Dimensions for folding of various sizes of drawing sheets by the two methods are given below:

Method I**Method II**

Sheet size	Horizontal dimensions from left (mm)	Vertical dimensions from bottom (mm)	No. of folds	Sheet size	Horizontal dimensions from left (mm)	Vertical dimensions from bottom (mm)	No. of folds
A0	$130 + 109 + 190 \times 5$	$297 \times 2 + 247$	9	A0	$139 + 210 \times 5$	$297 + 297 + 247$	7
A1	$146 + 125 + 190 \times 3$	$297 + 297$	6	A1	$211 + 210 \times 3$	$297 + 297$	4
A2	$116 + 96 \times 3 + 190$	297	1	A2	$174 + 210 \times 2$	$297 + 123$	3
A3	$125 + 105 + 190$	297	1	A3	$210 + 210$	297	1

The final size of the folded print in method I will be 297 mm \times 190 mm, while that in method II will be 297 mm \times 210 mm. In either case the title block is visible in the top part of the folded print.

(11) **Scales and scale drawing:** Sometime machine part is required to draw larger or smaller than their actual size. For example a crankshaft of an engine would be drawn to a reduced scale, while connecting rod's bolt is to be drawn to enlarged scale. The scale of drawing must be indicated in the title block as shown in fig. 2-2. When details are drawn to the different scale in the same drawing sheet corresponding scale should be mentioned under each such detail.

Table 2-3 shows the scales recommended in SP : 46 (2003).

TABLE 2-3

	Reducing scales	1 : 2	1 : 5	1 : 10
		1 : 20	1 : 50	1 : 100
		1 : 200	1 : 500	1 : 1000
		1 : 2000	1 : 5000	1 : 10000
	Enlarging scales	50 : 1	20 : 1	10 : 1
		5 : 1	2 : 1	
	Full size scales			1 : 1

2-2. TYPES OF MACHINE DRAWINGS



Machine drawings are prepared for various purposes and they are further classified as under:

(1) **Production drawing:** A production drawing is legal document of company. It is used by the technicians on the shop-floor for manufacturing the parts. It must provide informations about part number, dimensions, tolerance, surface finish, material and stock size, manufacturing process, special finishing process if required, and no. off required for each assembly. It is further sub-classified as:

- (i) Part drawing or detailed drawing
- (ii) Assembly drawing.

(2) **Exploded assembly drawing:** It represents the details of machine in a pictorial form as it is assembled. It helps the mechanics for dismantling machine for repairing purpose.

(3) **Schematic assembly drawing:** This type of assembly drawing is used for explaining working principle of any machine.

(4) **Drawing for instruction manual:** This is assembly drawing without dimensions. Each part of machine is numbered so that it can be easily dismantled or assembled if required. This is also used for explaining working principle of each part.

(5) **Drawing for installation:** This is assembly drawing with overall dimensions. It is used for the preparation of foundation for installing machine.

(6) **Drawing for catalogue:** Special assembly drawings are prepared for catalogues, with overall and principal dimensions.

(7) **Tabular drawing:** This is part drawing. It is used when components of same shape but different dimensions are to be manufactured.

(8) **Patent drawing:** It is generally assembly drawing either in pictorial form or principal view of orthographic projection of machine. It is used for obtaining patent of the machine.

2-3. FREE-HAND SKETCHING



(1) **Sketching or freehand:** Sketching or freehand is a first step to the preparation of a scale-drawing, i.e., a drawing drawn with the aid of instruments. A designer records his ideas initially in the form of sketches which are later converted into drawings. Similarly, views of actual objects are in the first instance, sketched freehand. Scale-drawings are then prepared from these sketches. Ideas and objects can be described in words, but the description is made more expressive with the aid of sketches. Thus sketching is of great importance in engineering practice.

Sketching is always done freehand. It is in fact a freehand drawing made in correct proportions, but not to scale. A sketch should be so prepared as to give to others a clear idea, complete information and true impression of the object to be constructed. It should never be drawn too small. The size of a sketch should be such that all the features of the object, together with their dimensions, explanatory notes etc. are clearly incorporated in it. Proficiency in sketching can be achieved with constant practice only.

(2) Sketching materials:

- (i) A soft-grade pencil, preferably HB or H sharpened to a conical point.
- (ii) A soft rubber-eraser.
- (iii) A paper in form of a sketch-book or a pad.

Above the *three* things are absolutely essential for sketching.

Sometimes, cross-sectioned graph paper ruled with light lines is also used instead of a plain paper. These lines and squares help in drawing straight lines and also maintaining proportions.

As such papers may not always be readily available, it is advisable to learn sketching without their aid.

(3) To sketch straight lines: Horizontal lines are sketched with the motion of the wrist and the fore-arm. They are sketched from left to right [fig. 2-6(i)]. To sketch a horizontal line, mark the end points. Hold the pencil at about 30 mm distance from the lead point. Swing it from left to right and backwards, between the two points and without touching the surface of the paper, till the correct direction is achieved. Then begin to draw the line (with the wrist-motion) with short and light strokes. Shift the hand after each stroke. Keep your eyes on the point at which the line is to end. Finish finally with a dark and firm line. Take proper care to maintain straightness and correct direction of the line.

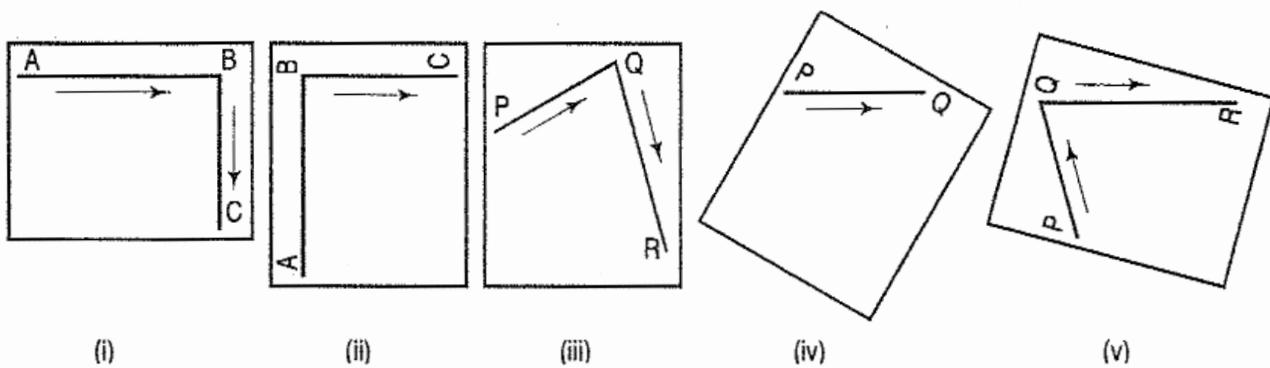


FIG. 2-6

Vertical lines are sketched downwards [fig. 2-6(ii)] with the movement of fingers. They may also be sketched by converting them into horizontal lines by revolving the paper as shown in fig. 2-6(iv). Vertical or horizontal lines near the edges of a sketch-book may be drawn by sliding fingers along those edges, thus using them as guides. Inclined lines, when they are nearly horizontal, are sketched from left to right. When they are nearly vertical, they are sketched downwards [fig. 2-6(iii)]. These lines also may be sketched as horizontal lines by revolving the paper as shown in fig. 2-6(iv) and fig. 2-6(v).

(4) To sketch circles and arcs: Mark the centre and through it, draw horizontal and vertical centre lines [fig. 2-7(i)]. Add four radial lines between them. Mark points on these lines at radius-distance from the centre, judging by the eye or using a slip of paper as a trammel, on which the radius-distance has been approximately marked [fig. 2-7(ii)]. Complete the circle with light strokes. The paper may be revolved after about each quarter-circle for easy wrist motion. Erase the additional radial lines completely. Dim all the lines before fairing the circle with a thin and black outline [fig. 2-7(iii)]. Keep the centre lines thin and light.

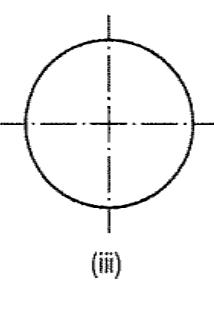
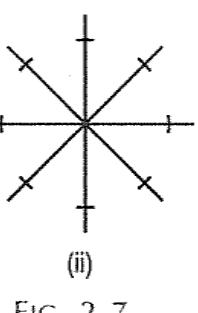
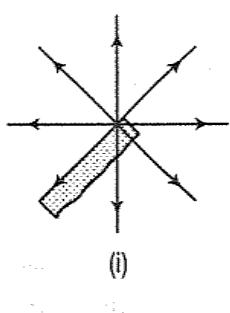


FIG. 2-7

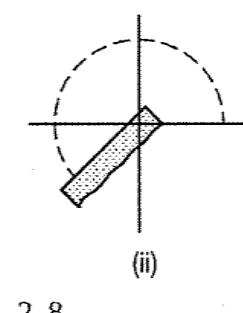
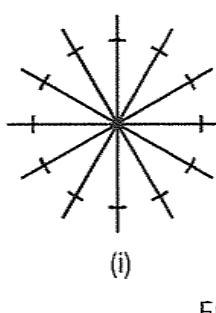


FIG. 2-8

Large circles may be sketched as described above by adding a few extra radial lines [fig. 2-8(i)]. An easier method is to mark a number of points by means of a trammel (at radius-distance from the centre) and to sketch the circle through these points [fig. 2-8(ii)].

Large circles can also be drawn by making a compass of fingers and a pencil. Keep the little finger as a pivot at the centre. Hold the pencil stationary so that its point is at radius-distance from the centre and touches the paper. Rotate the paper with the other hand. The pencil-point will mark the circle on the paper. Two pencils may also be used as a compass. One pencil is held as a pivot, while the other describes the circle as the paper is rotated.

A circle of small radius can be sketched within a square. Sketch the circumscribing square (length of the side equal to the diameter of the circle) and mark the diagonals [fig. 2-9(i)]. Mark the mid-points of the sides of the square and four points on the diagonals at radius-distance from the centre [fig. 2-9(ii)]. Sketch a neat circle through the eight points [fig. 2-9(iii)].

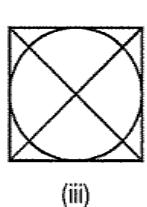
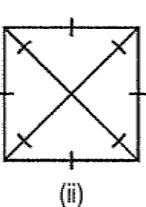
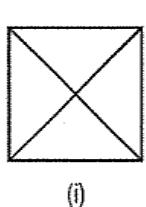


FIG. 2-9

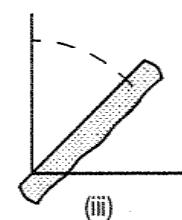
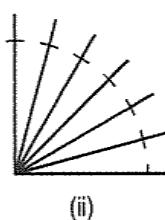
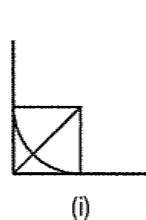


FIG. 2-10

Arches of small radii are conveniently drawn by constructing squares [fig. 2-10(i)]. Large-radius arches may be drawn by one of the methods described above for large circles. Radial-line and trammel methods are shown in fig. 2-10(ii) and fig. 2-10(iii) respectively.

(5) **Sketching procedure:** Sketches should never be prepared with the aid of a scale or a straight-edge. All lines must be absolutely free-hand and their measurements must be in proportion only. A sketch is considered to be good when its features are shown in correct proportions. Its outlines must be black and thin but rigidly firm. Dimension lines and centre lines should be comparatively light. Dimension figures must be inserted with good care, as if they are printed. Lettering also should be done in a similar manner.

(6) **Steps in sketching:** Following are the steps in sketching three orthographic views of a shaft-support, shown (in pictorial view) in fig. 2-11.

- Determine the over-all dimensions of the views and sketch rectangles for the same in good proportions and correct projection, keeping sufficient space between them and from the border lines [fig. 2-12(i)].

- (ii) Insert centre lines for arcs and circles, and block-in the main shapes [fig. 2-12(ii)].
- (iii) Sketch arcs and circles, and build-up the details [fig. 2-12(iii)].
- (iv) Erase all construction lines. Dim all the lines and then make them bold and firm [fig. 2-12(iv)] starting with the curves. Keep the centre lines fainter. Insert all dimensions. Print the title.

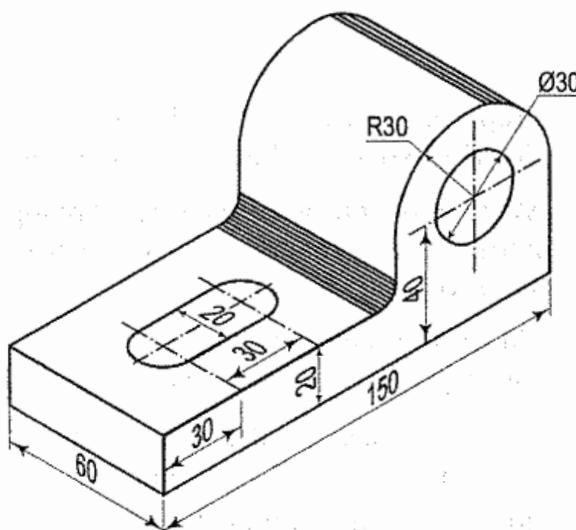


FIG. 2-11

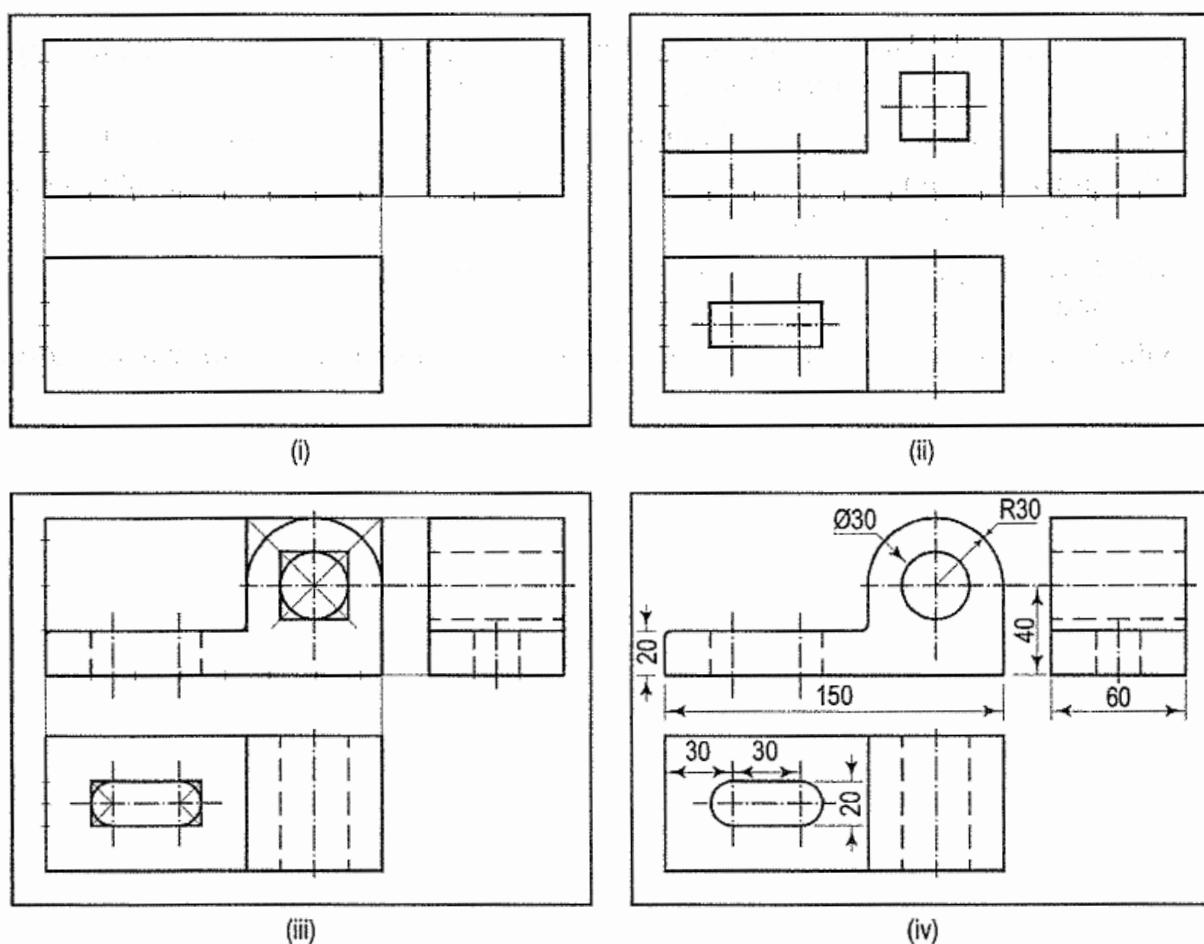


FIG. 2-12

Please also refer topic 20-7 "Procedure for preparing a scale-drawing".

EXERCISES 2



1. Prepare, giving all details, a neat sketch of a simple title block provided in a drawing sheet. What is its size, as recommended by the B.I.S.?
2. Make a sketch of an A1 size drawing sheet, showing in it, the border lines, margin lines, folding marks, zones and the title block. Give the untrimmed and trimmed sizes of this sheet.
3. Fill-up the blanks in the following by appropriate words or numerals selected from those given in the list as shown below.
 - (i) The size of the title block for all sizes of drawing sheets is _____ mm × _____ mm.
 - (ii) The zones along the length of the sheet are designated by _____, while those along its width by _____.
 - (iii) For locating a portion of a drawing, the sheet is divided into a number of _____.
 - (iv) For A1 size sheet the number of zones suggested by B.I.S. along the length are _____ while those along the width are _____.
 - (v) The drawing sheet is so folded that the _____ is always on the top.
 - (vi) Horizontal lines are sketched from _____ to _____ while vertical lines are sketched from _____ to _____.
 - (vii) Three things absolutely essential for sketch-work are _____, _____ and _____.
 - (viii) A sketch is considered to be good when its features are shown in correct _____.

List of words and numerals for Ex. 3:

1. Title block	7. Proportions	13. Eraser	19. Top
2. Pencil	8. Right	14. Left	20. Compass
3. 210	9. 85	15. 16	21. 65
4. Numerals	10. Letters	16. Position	22. Blocks
5. Paper	11. Bottom	17. Zones	23. 8
6. 12	12. 6	18. 185	

Answer to Ex. 3:

- (i) 18 and 21, (ii) 4 and 10, (iii) 17, (iv) 6 and 23,
- (v) 1, (vi) 14, 8, 19 and 11, (vii) 2, 13 and 5, (viii) 7.
4. Study chapter 7 and 20 carefully and then solve the exercises given at the end of each, by means of sketches.
5. What are the different types of machine drawings?
6. Differentiate clearly production drawing and installation drawing.
7. Prepare freehand sketches of the views given in figures 2-13 to 2-26. Assume each unit to be of 10 mm length. Insert all dimensions.

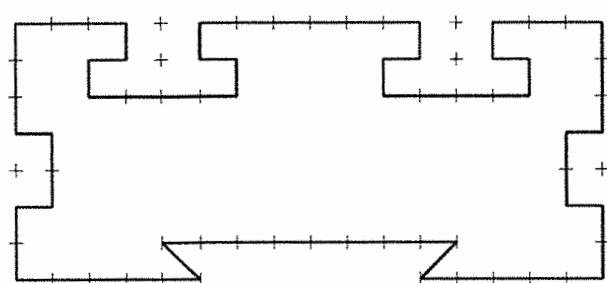


FIG. 2-13

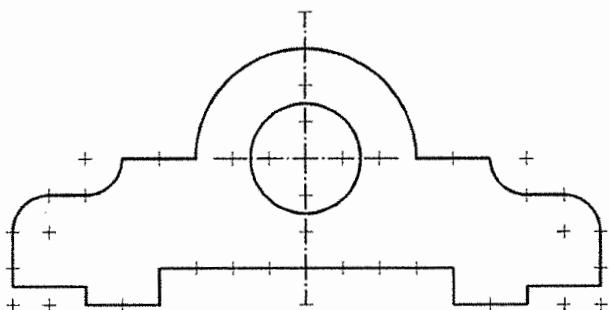


FIG. 2-14

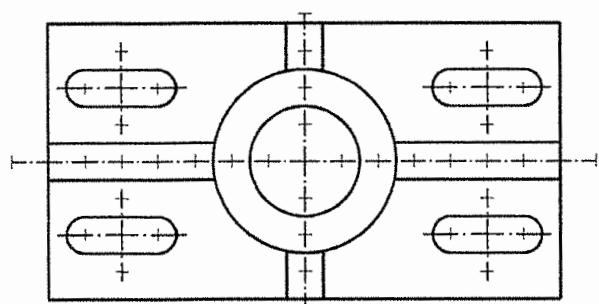


FIG. 2-15

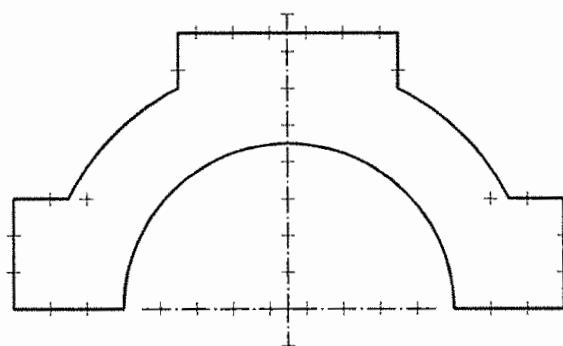


FIG. 2-16

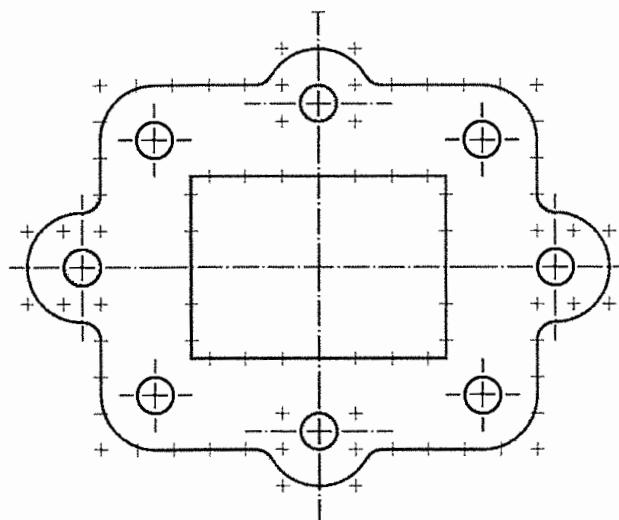


FIG. 2-17

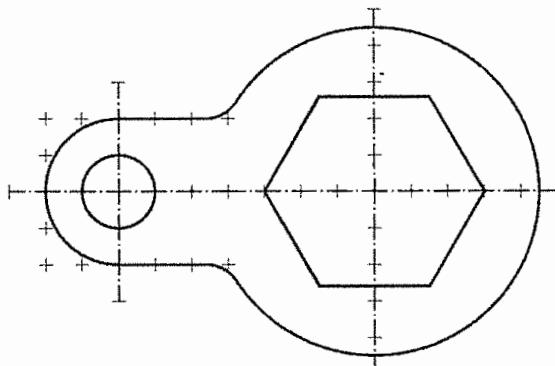


FIG. 2-18

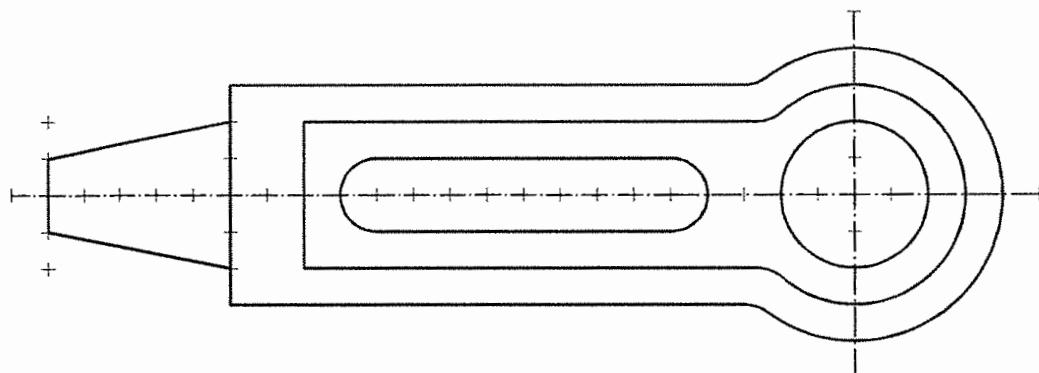


FIG. 2-19

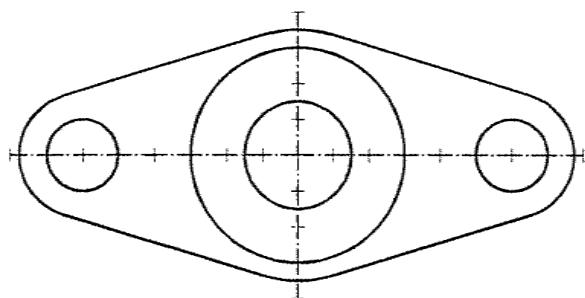


FIG. 2-20

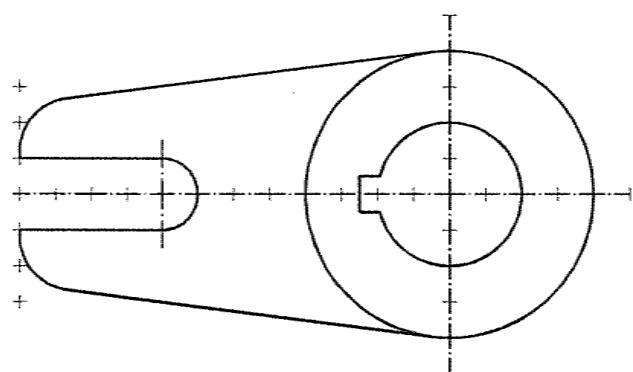


FIG. 2-21

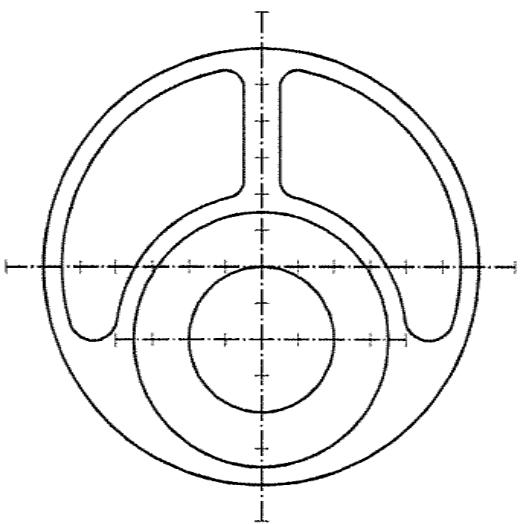


FIG. 2-22

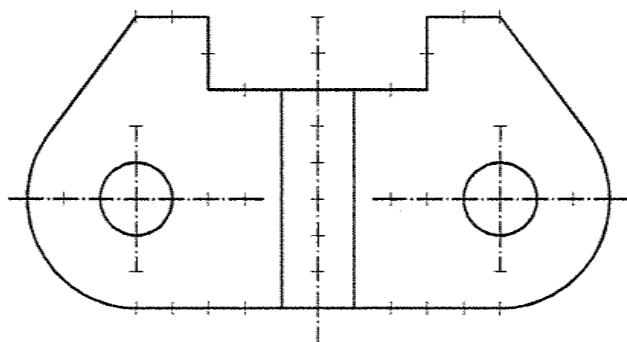


FIG. 2-23

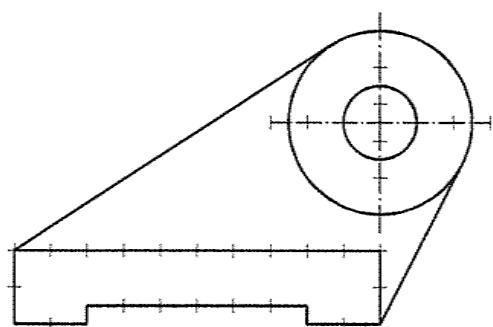


FIG. 2-24

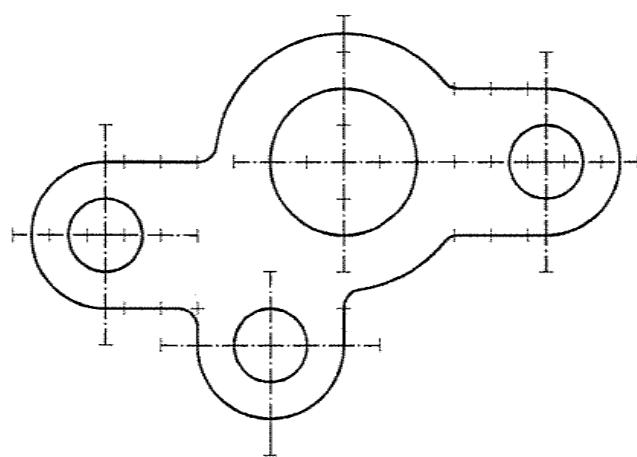


FIG. 2-25

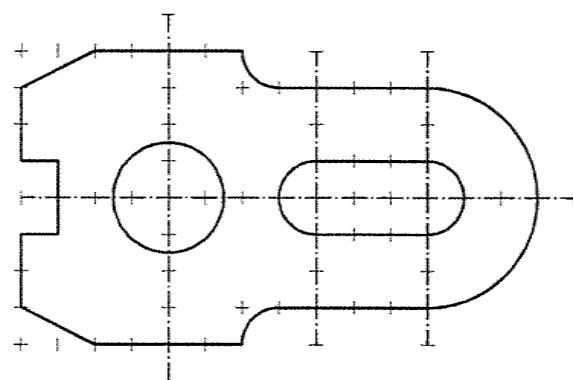


FIG. 2-26

Chapter 3



LINES, LETTERING AND DIMENSIONING

3-0. INTRODUCTION

This chapter deals with various types of lines, lettering and dimensioning which are used in engineering drawing.

3-1. LINES

Various types of lines used in general engineering drawing are shown in fig. 3-1 as described by SP 46:2003.

(1) **Line thickness:** The thicknesses of lines are varied according to the drawing and are finalized either *by ink* or *by pencil*.

(2) **Inked drawings:** The thicknesses of lines in various groups are shown in table 3-1. The line-group is designated according to the *thickness of the thickest line*. For any particular drawing a line-group is selected according to its size and type. All lines should be sharp and dense so that good prints can be reproduced.

TABLE 3-1

Line-group mm	Type of line						
	A	B	C	D	E	F	G
1.2	1.2	0.6	0.4	0.4	1.2	0.4	0.4
0.8	0.8	0.4	0.3	0.3	0.8	0.3	0.3
0.5	0.5	0.3	0.2	0.2	0.5	0.2	0.2
0.3	0.3	0.2	0.1	0.1	0.3	0.1	0.1

(3) **Pencil drawings:** For drawings finalized with pencil, the lines can be divided into two line-groups as shown in table 3-2. It is important to note that in the *finished drawing*, all lines except construction lines should be *dense, clean and uniform*. Construction lines should be drawn very *thin* and *faint* and should be hardly visible in the finished drawing.

TABLE 3-2

Line-group mm	Thickness	Lines
0.2	Medium	Out lines, dotted lines, cutting plane-lines
0.1	Thin line	Centre lines, section lines, dimension lines, extension lines, construction lines, leader lines, short-break lines and long-break lines.

TABLE 3-3

Lines	Pencil
Initial work and construction lines	H
Outlines, dotted lines, section-plane lines, dimension lines, arrowheads	2H
Centre lines, section lines	3H or 4H

Line	Description	General applications
A ——————	Continuous thick or Continuous wide	Visible outlines, visible edges; crests of screw threads; limits of length of full depth thread, lines of cuts and section arrows; parting lines of moulds in views; main representations in diagrams, maps, flow charts; system lines (structural metal engg.)
B ——————	Continuous thin (narrow) (straight or curved)	Imaginary lines of intersection; grid, dimension, extension, projection, short centre, leader, reference lines; hatching; outlines of revolved sections; root of screw threads; interpretation lines of tapered features; framing of details; indication of repetitive details;
C ~~~~~	Continuous thin (narrow) freehand	Limits of partial or interrupted views and sections, if the limit is not a chain thin line
D ——— —— ——	Continuous thin (narrow) with zigzags (straight)	Long-break line
E - - - - -	Dashed thick (wide)	Line showing permissible of surface treatment
F - - - - -	Dashed thin (narrow)	Hidden outlines; hidden edges
G - - - - -	Chain thin Long-dashed dotted (narrow)	Centre line; lines of symmetry; trajectories; pitch circle of gears, pitch circle of holes,
H THICK — THIN — THICK	Chain thin (narrow) with thick (wide) at the ends and at changing of position	Cutting planes
J - - - - -	Chain thick or Long-dashed dotted (wide)	Indication of lines or surfaces to which a special requirement applies
K - - - - -	Chain thin double-dashed or long-dashed double-dotted (narrow)	Outlines of adjacent parts Alternative and extreme positions of movable parts Centroidal lines Initial outlines prior to forming Parts situated in front of the cutting plane

FIG. 3-1

Drawing pencils are graded according to increase in relative hardness. They are marketed with the labels as H, 2H, 3H, 4H, 5H, 6H etc. Students are advised to use pencils as recommended in table 3-3, for getting accurate, clean and neat drawings.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 3 for Type of lines, lettering and dimensioning.

3-1-1. TYPES OF LINES (FIG. 3-2 AND FIG. 3-3)



(1) Outlines (A): Lines drawn to represent visible edges and surface boundaries of objects are called outlines or principal lines. They are continuous thick or wide lines (fig. 3-2).

(2) Margin lines (A): They are continuous thick or wide lines along which the prints are trimmed [fig. 2-1(a)].

(3) Dimension lines (B): These lines are continuous thin lines. They are terminated at the outer ends by pointed arrowheads touching the outlines, extension lines or centre lines (fig. 3-2).

(4) Extension or projection lines (B): These lines also are continuous thin lines. They extend by about 3 mm beyond the dimension lines (fig. 3-2).

(5) Construction lines (B): These lines are drawn for constructing figures. They are shown in geometrical drawings only. They are continuous thin light lines.

(6) Hatching or section lines (B): These lines are drawn to make the section evident. They are continuous thin lines and are drawn generally at an angle of 45° to the main outline of the section. They are uniformly spaced about 1 mm to 2 mm apart (fig. 3-2).

(7) Leader or pointer lines (B): Leader line is drawn to connect a note with the feature to which it applies. It is a continuous thin line (fig. 3-2).

(8) Border lines (B): Perfectly rectangular working space is determined by drawing the border lines [fig. 2-1(a)]. They are continuous thin lines.

(9) Short-break lines (C): These lines are continuous, thin and wavy. They are drawn freehand and are used to show a short break, or irregular boundaries (fig. 3-3).

(10) Long-break lines (D): These lines are thin ruled lines with short zigzags within them. They are drawn to show long breaks (fig. 3-3).

(11) Hidden or dotted lines (E or F): Interior or hidden edges and surfaces are shown by hidden lines. They are also called dotted lines. They are of medium thickness and made up of short dashes of approximately equal lengths of about 2 mm spaced at equal distances of about 1 mm. When a hidden line meets or intersects another hidden line or an outline, their point of intersection or meeting should be clearly shown (fig. 3-2).

(12) Centre lines (G): Centre lines are drawn to indicate the axes of cylindrical, conical or spherical objects or details, and also to show the centres of circles and arcs. They are thin, long, chain lines composed of alternately long and dot spaced approximately 1 mm apart. The long dashes are about 9 to 12 mm. Centre lines should extend for a short distance beyond the outlines to which they refer. For the purpose of dimensioning or to correlate the views they may be extended as required. The point of intersection between two centre lines must always be indicated. Locus lines, extreme positions of movable parts and pitch circles are also shown by this type of line (fig. 3-2 and fig. 3-3).

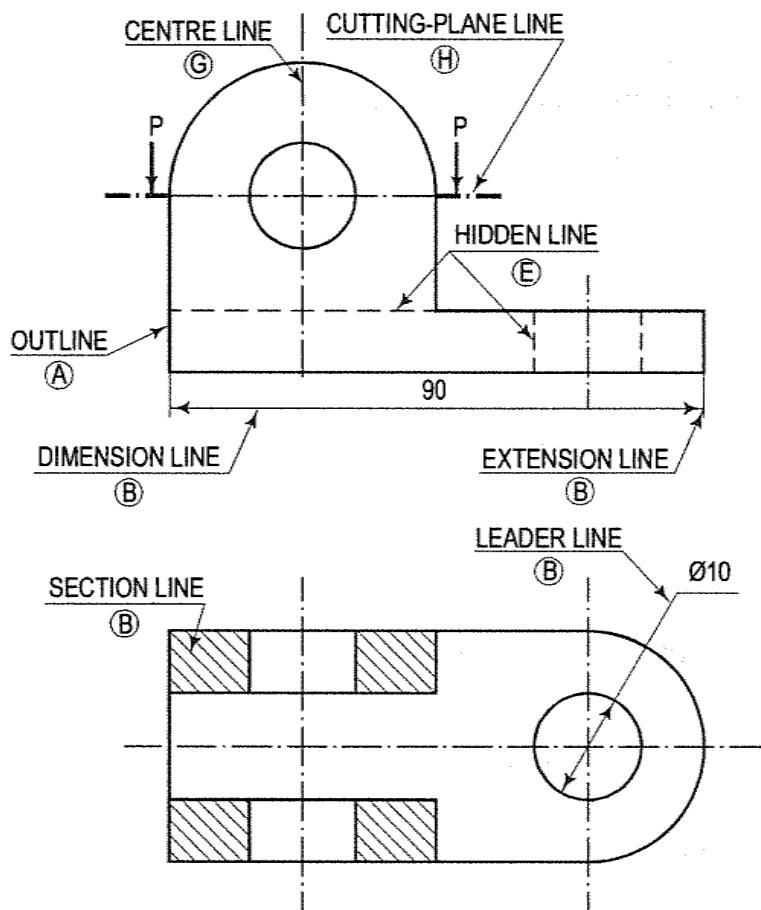


FIG. 3-2

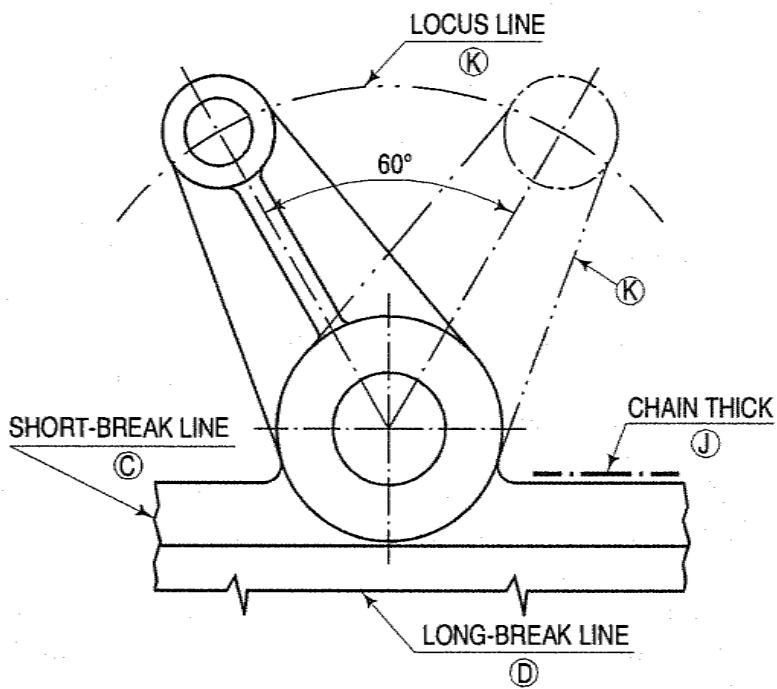


FIG. 3-3

(13) Cutting-plane lines (H): The location of a cutting plane is shown by this line. It is a long, thin, chain line, thick at ends only (fig. 3-2).

(14) Chain thick (J): These lines are used to indicate special treatment on the surface.

(15) Chain thick double-dots (K): This is chain thin double-dot line.

3-2. LETTERING (IS : 9609-2001)



Writing of titles, dimensions, notes and other important particulars on a drawing is called *lettering*. It is an important part of a drawing. However accurate and neat a drawing may be drawn. Its appearance is spoiled and sometimes, its usefulness is impaired by poor lettering. Lettering should, therefore, be done properly in clear, legible and uniform style. It should be in plain and simple style so that it could be done *freehand* and *speedily*.

Note: Use of drawing instruments in lettering takes considerable time and hence, it should be avoided. Efficiency in the art of lettering can be achieved by careful and continuous practice.

(1) **Single-stroke letters:** The Bureau of Indian Standards (IS : 9609-2001) recommends single-stroke lettering for use in engineering drawing. These are the simplest forms of letters and are usually employed in most of the engineering drawings.

The word *single-stroke* should not be taken to mean that the letter should be made in one stroke without lifting the pencil.

It actually means that the thickness of the line of the letter should be such as is obtained in one stroke of the pencil. The horizontal lines of letters should be drawn from left to right and vertical or inclined lines, from top to bottom.

Single-stroke letters are of two types:

- (i) vertical and
- (ii) inclined.

Inclined letters lean to the right, the slope being 75° with the horizontal. The size of a letter is described by its height. According to the height of letters, they are classified as:

- (i) Lettering 'A' (refer to table 3-4 and fig. 3-8)
- (ii) Lettering 'B' (refer to table 3-5 and fig. 3-8).

TABLE 3-4

$$\text{LETTERING A } \left(d = \frac{h}{10} \right)$$

Characteristic	Ratio	Dimensions (mm)							
Lettering height Height of capitals	$\left(\frac{14}{14} \right) h$	2.5	3.5	5	7	10	14	20	
Height of lower-case letters	$\left(\frac{10}{14} \right) h$	-	2.5	3.5	5	7	10	14	20
Spacing between characters	$\left(\frac{2}{14} \right) h$	0.35	0.5	0.7	1	1.4	2	2.8	
Minimum spacing of base lines	$\left(\frac{20}{14} \right) h$	3.5	5	7	10	14	20	28	
Minimum spacing between words	$\left(\frac{6}{14} \right) h$	1.05	1.5	2.1	3	4.2	6	8.4	
Thickness of lines	$\left(\frac{1}{14} \right) h$	0.18	0.25	0.35	0.5	0.7	1	1.4	

TABLE 3-5
LETTERING B ($d = \frac{h}{10}$)

Characteristic		Ratio	Dimensions (mm)							
Lettering height		$\left(\frac{10}{10}\right) h$	2.5	3.5	5	7	10	14	20	
Height of capitals	h	$\left(\frac{10}{10}\right) h$	—	2.5	3.5	5	7	10	14	20
Height of lower-case letters	c	$\left(\frac{7}{10}\right) h$	—	2.5	3.5	5	7	10	14	20
Spacing between characters	a	$\left(\frac{2}{10}\right) h$	0.5	0.7	1	1.4	2	2.8	4	
Minimum spacing of base lines	b	$\left(\frac{14}{10}\right) h$	3.5	5	7	10	14	20	28	
Minimum spacing between words e	e	$\left(\frac{6}{10}\right) h$	1.5	2.1	3	4.2	6	8.4	12	
Thickness of lines	d	$\left(\frac{1}{10}\right) h$	0.25	0.35	0.5	0.7	1	1.4	2	

In lettering 'A' type, the height of the capital letter is divided into 14 parts, while in lettering 'B' type it is divided into 10 parts. The height of the letters and numerals for engineering drawing can be selected from 2.5, 3.5, 5, 7, 10, 14 and 20 mm according to the size of drawing. The ratio of height to width varies but in case of most of the letters it is 6 : 5.

Lettering is generally done in capital letters. Different sizes of letters are used for different purposes.

The main titles are generally written in 6 mm to 8 mm size, sub-titles in 3 mm to 6 mm size, while notes, dimension figures etc. in 3 mm to 5 mm size. The drawing number in the title block is written in numerals of 10 mm to 12 mm size.

Fig. 3-4 shows single-stroke vertical capital letters and figures with approximate proportions.



FIG. 3-4

Single-stroke inclined capital letters and figures are shown in fig. 3-5. The lower-case letters are usually used in architectural drawings.

Vertical and inclined lower-case alphabets are shown in fig. 3-6 and fig. 3-7 respectively. The width of the majority of letters is equal to the height.

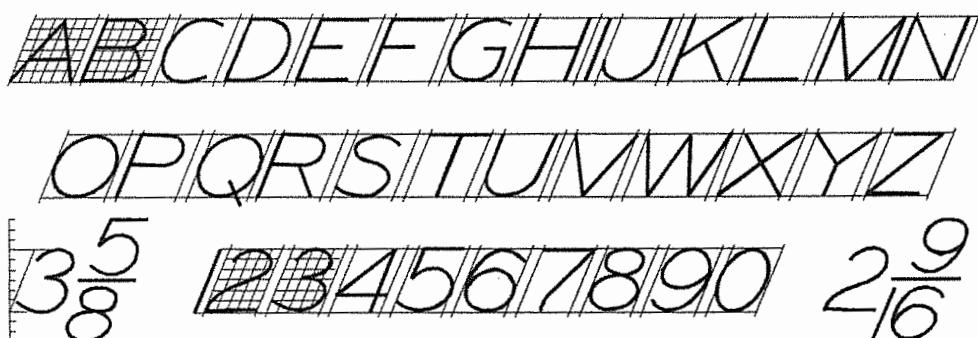


FIG. 3-5

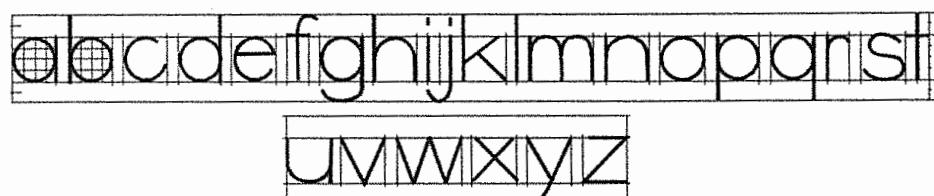


FIG. 3-6

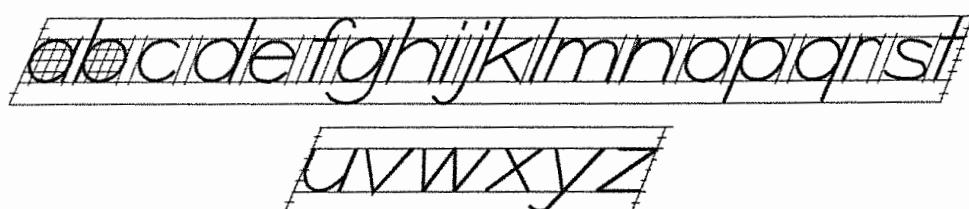


FIG. 3-7

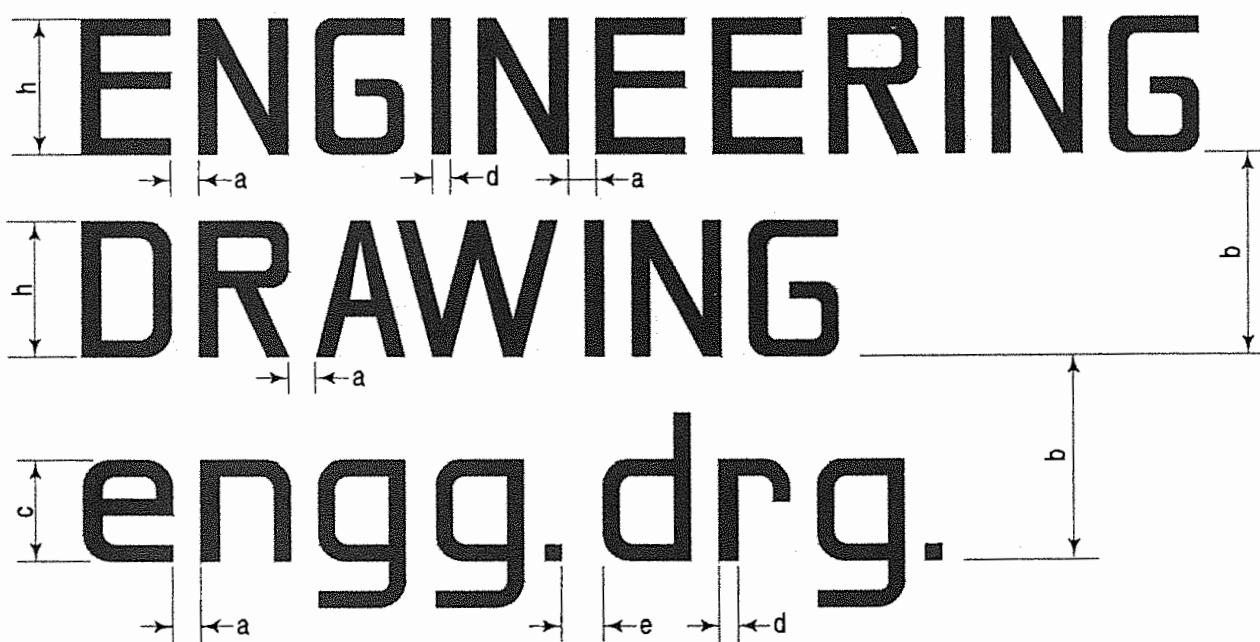


FIG. 3-8

All letters should be uniform in shape, slope, size, shade and spacing. The shape and slope of every letter should be uniform throughout a drawing. For maintaining uniformity in size, thin and light guide-lines may first be drawn, and lettering may then be done between them. The shade of every letter must be the same as that of the outlines of drawings, i.e. intensely black.

Therefore, H or HB grade of pencil is recommended for this purpose. The spacing between two letters should not necessarily be equal. The letters should be so spaced that they do not appear too close together or too much apart.

Judging by the eye, the back ground areas between the letters should be kept approximately equal. The distance between the words must be uniform and at least equal to the height of the letters. Refer to fig. 3-8.

Lettering should be so done as can be read from the front with the main title horizontal, i.e. when the drawing is viewed from the bottom edge.

All sub-titles should be placed below but not too close to the respective views. Lettering, except the dimension figures, should be underlined to make them more prominent.

(2) Gothic letters: Stems of single-stroke letters, if given more thickness, form what are known as *gothic* letters. These are mostly used for main titles of ink-drawings. The outlines of the letters are first drawn with the aid of instruments and then filled-in with ink.

The thickness of the stem may vary from $\frac{1}{5}$ to $\frac{1}{10}$ of the height of the letters.

Fig. 3-9 shows the alphabet and figures in gothic with thickness equal to $\frac{1}{7}$ of the height.

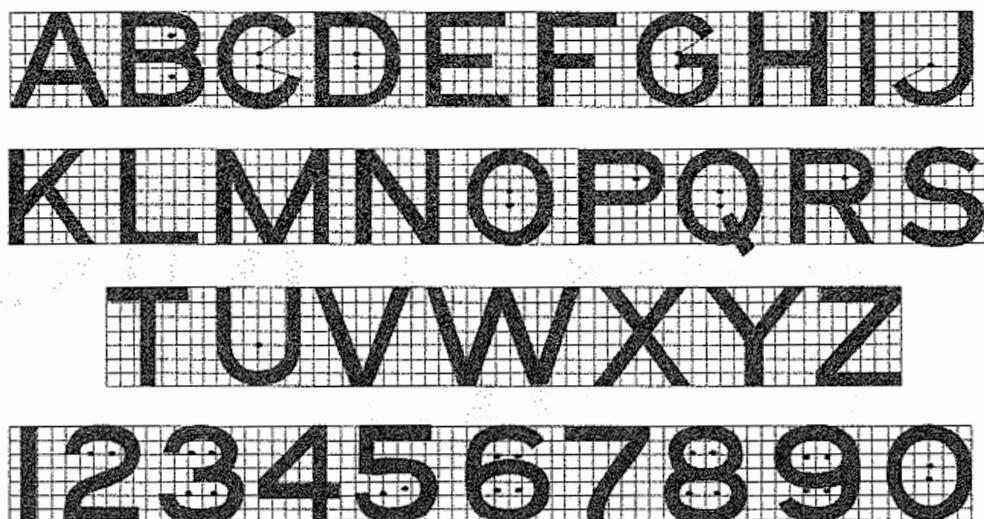


FIG. 3-9

3-3. DIMENSIONING

Every drawing, whether a scale drawing or a freehand drawing, besides showing the true shape of an object, must supply its exact length, breadth, height, sizes and positions of holes, grooves etc. and such other details relating to the manufacture of that object.

Providing this information on a drawing is called dimensioning. Lines, figures, numerals, symbols, notes etc. are used for this purpose.

Types of dimensions (fig. 3-10): Two types of dimensions needed on a drawing are:

- (i) size or functional dimensions and
- (ii) location or datum dimensions (shown by letters F and L respectively).

The former indicate sizes, viz. length, breadth, height, depth, diameter etc. The latter show locations or exact positions of various constructional details within the object. The letter F represents functional dimensions, while NF represents non-functional dimensions.

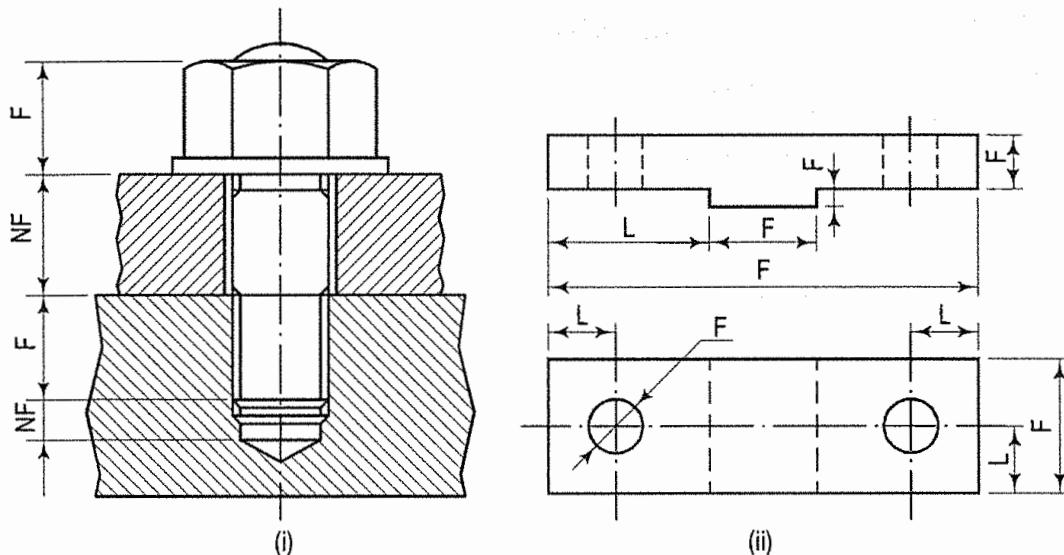


FIG. 3-10

3-4. DIMENSIONING TERMS AND NOTATIONS



(1) **Dimension line** (fig. 3-11): Dimension line is a thin continuous line. It is terminated by arrowheads touching the outlines, extension lines or centre lines.

(2) **Extension line** (fig. 3-11): An extension line is also a thin continuous line drawn in extension of an outline. (Formerly, the B.I.S. had recommended that a gap of about 1 mm should be kept between the extension line and an outline or object boundary.) It extends by about 3 mm beyond the dimension line.

(3) **Arrowhead** (fig. 3-11): An arrowhead is placed at each end of a dimension line. Its pointed end touches an outline, an extension line or a centre line. The size of an arrowhead should be proportional to the thickness of the outlines. *The length of the arrowhead should be about three times its maximum width.* It is drawn freehand with two strokes made in the direction of its pointed end. The space between them is neatly filled up. Different types of arrow heads are shown in fig. 3-11(a). Generally closed and filled arrowhead is widely used in engineering drawing.

Note (fig. 3-11): A note gives information regarding specific operation relating to a feature. It is placed outside a view but adjacent to the feature concerned. It is so written that it may be read when the drawing is viewed from the bottom edge.

(4) **Leader** (fig. 3-11): A leader or a pointer is a thin continuous line connecting a note or a dimension figure with the feature to which it applies. One end of the leader terminates either in an arrowhead or a dot. The arrowhead touches the outline, *while the dot is placed within the outline of the object* [fig. 2-1(a) and fig. 3-11(b)].

The other end of the leader is terminated in a horizontal line at the bottom level of the first or the last letter of the note. *The leader is never drawn vertical or horizontal or curved. It is drawn at a convenient angle of not less than 30° to the line to which it touches.* When pointing to a circle or an arc it is drawn radially. Use of common leaders for more than one feature should never be made.

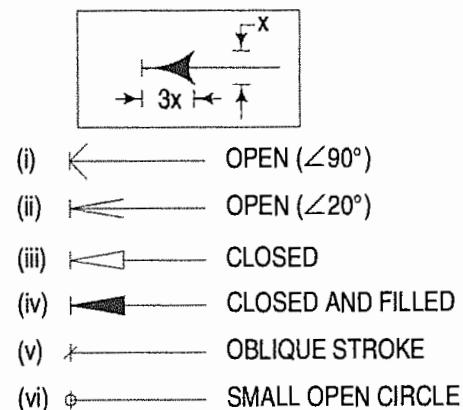
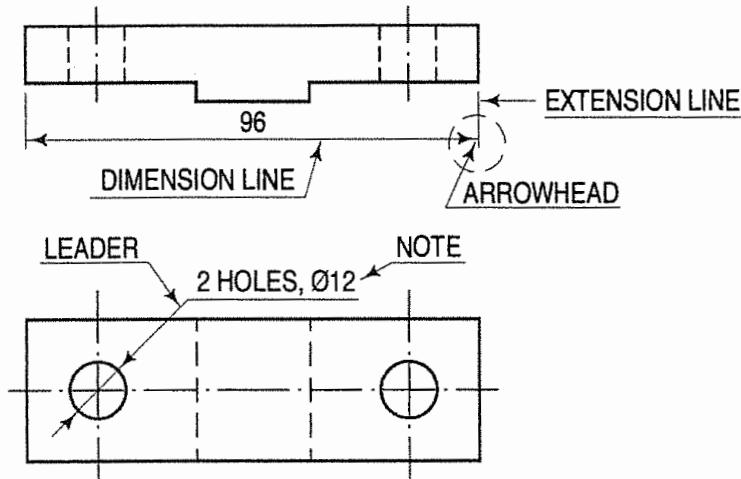


FIG. 3-11(a)

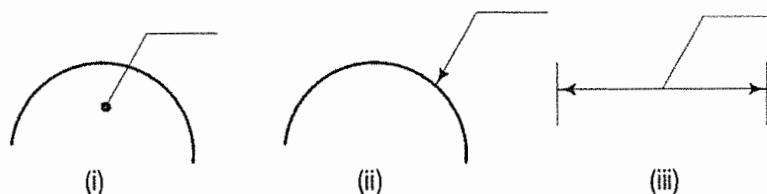


FIG. 3-11(b)

3-5. PLACING OF DIMENSIONS

The two systems of placing dimensions are:

- (1) Aligned system and
- (2) Unidirectional system.

(1) **Aligned system** (fig. 3-12): In the aligned system the dimension is placed perpendicular to the dimension line in such a way that it may be read from the bottom edge or the right-hand edge of the drawing sheet. The dimensions should be placed near the middle and above, but clear of the dimension lines.

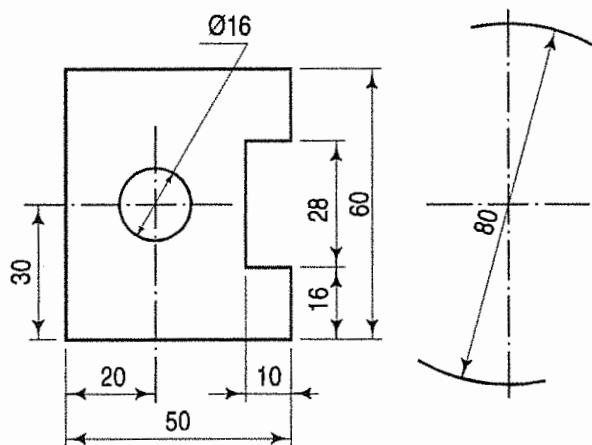


FIG. 3-12

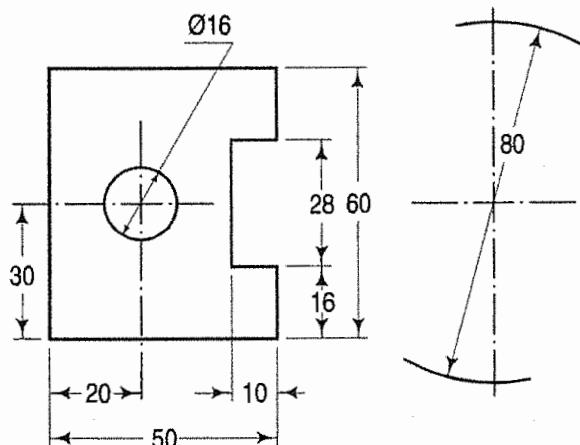


FIG. 3-13

(2) **Unidirectional system** (fig. 3-13): In unidirectional system all dimensions are so placed that they can be read from the bottom edge of the drawing sheet. The dimension lines are broken near the middle for inserting the dimensions. This system is mainly used on large drawings — as of aircrafts, automobiles etc. where it is inconvenient to read dimensions from the right-hand side.

3-6. UNIT OF DIMENSIONING

As far as possible all dimensions should be given in *millimetres*, omitting the abbreviation mm. Even when it is not convenient to give dimensions in millimetres and another unit is used, only the dimension figures are written. But a foot note such as '*all dimensions are in centimetres*' is inserted in a prominent place near the title block. The *height* of the *dimension figures* (as stated earlier) should be from 3 mm to 5 mm. The decimal point in a dimension should be quite distinct and written in line with the bottom line of the figure (fig. 3-27). A zero must always precede the decimal point when the dimension is less than unity.

3-7. GENERAL RULES FOR DIMENSIONING

- (1) Dimensioning should be done so completely that further calculation or assumption of any dimension, or direct measurement from the drawing is not necessary.
- (2) Every dimension must be given, but none should be given more than once.
- (3) A dimension should be placed on the view where its use is shown more clearly.
- (4) Dimensions should be placed outside the views, unless they are clearer and more easily read inside.
- (5) Mutual crossing of dimension lines and dimensioning between hidden lines should be avoided. Dimension lines should not cross any other line of the drawing.
- (6) An outline or a centre line should never be used as a dimension line. A centre line may be extended to serve as an extension line (fig. 3-13).
- (7) Aligned system of dimensioning is recommended.

3-8. PRACTICAL HINTS ON DIMENSIONING

- (1) Dimension lines should be drawn at least 8 mm away from the outlines and from each other.
- (2) Dimensions in a series may be placed in any one of the following two ways:
 - (i) *Continuous or chain dimensioning* (fig. 3-14): Dimensions are arranged in a straight line. An overall dimension is placed outside the smaller dimensions. One of the smaller dimensions (the least important) is generally omitted.
 - (ii) *Progressive or parallel dimensioning* (fig. 3-15): All dimensions are shown from a common base line. Cumulative error is avoided by this method. This method is preferable.
- (3) Smaller dimensions should be placed nearer the view and the larger further away so that extension lines do not cross dimension lines. Extension lines may cross each other (fig. 3-16) or the outlines (fig. 3-13).

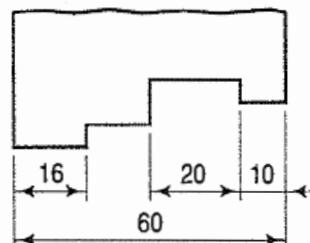


FIG. 3-14

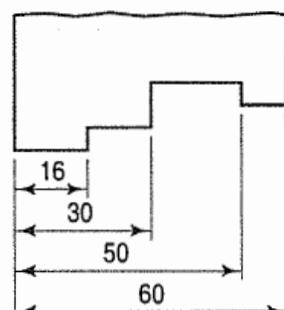


FIG. 3-15

- (4) When a number of parallel dimension lines are to be shown near each other, the dimensions should be staggered (fig. 3-16).
- (5) Dimensions should be shown where the shape is easily identified.
- (6) Arrowheads should ordinarily be drawn within the limits of the dimensioned feature. But when the space is too narrow, they may be placed outside (fig. 3-17). A dot may also be used to replace an arrowhead. Due to lack of space, the dimension figure may be written above the extended portion of the dimension line, but *preferably on the right-hand side* (fig. 3-17).

- (7) Dimensions of cylindrical parts should as far as possible be placed in the views in which they are seen as rectangles (fig. 3-18). The dimension indicating a diameter should always be preceded by the symbol \varnothing . Dimension of a cylinder should not be given as a radius.

Fig. 3-19 shows various methods of dimensioning different sizes of circles. Dimensions should be shown in one view only, the same dimension must not be repeated in other view.

Holes should be dimensioned in the view in which they appear as circles (fig. 3-20). They should be located by their centre lines.

As far as possible, all dimensions for one particular operation, such as diameter and depth of a drilled hole (fig. 3-18), or size and depth of a threaded hole (fig. 3-29) should be given in one view only.

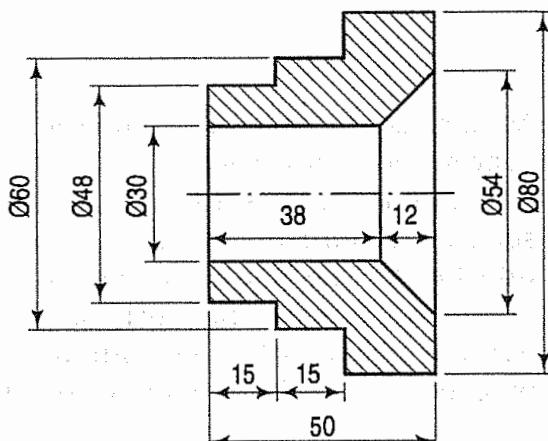


FIG. 3-16

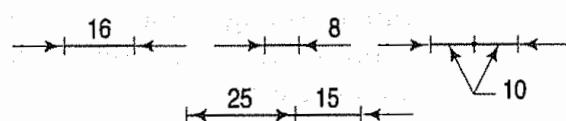


FIG. 3-17

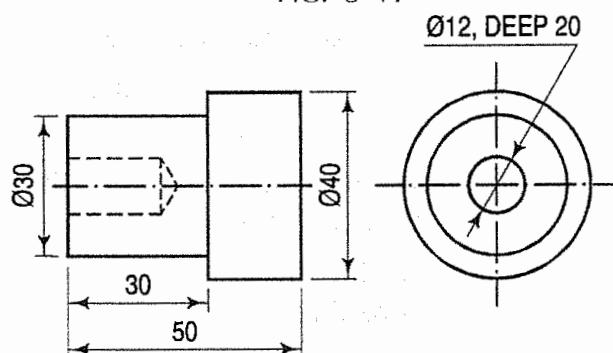


FIG. 3-18

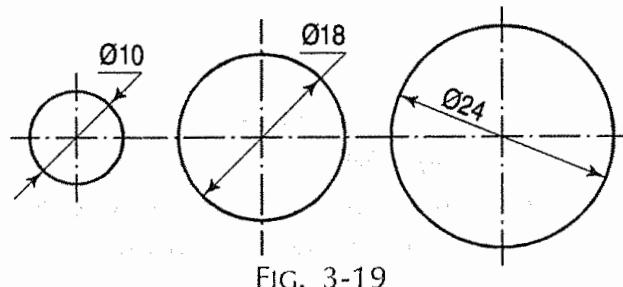


FIG. 3-19

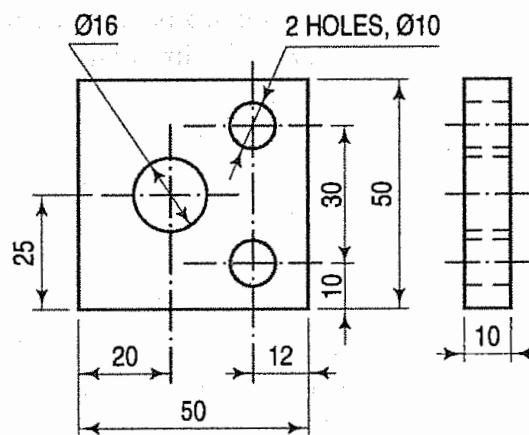


FIG. 3-20

In case of a large-size bore or a pitch circle, the dimension may be shown by a diagonal diameter (fig. 3-21).

But (in aligned system) a dimension should not be placed within 30° zone of the vertical centre line as shown by the shaded space in fig. 3-21.

Holes on pitch circles when equally spaced should be dimensioned as shown in fig. 3-21. When holes are not equally or uniformly spaced on the pitch circle, they should be located by angles with one of the two main centre lines (fig. 3-22).

Arches of circles should be dimensioned by their respective radii. Dimension line for the radius should pass through the centre of the arc. The dimension figure must be preceded by the letter R. Fig. 3-23 shows different methods of showing the radii of arcs.

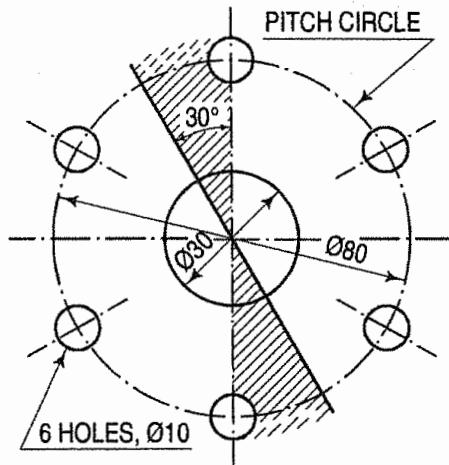


FIG. 3-21

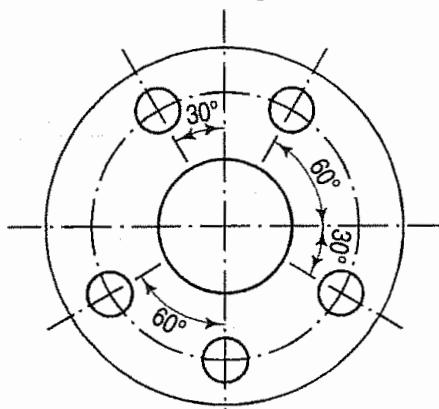


FIG. 3-22

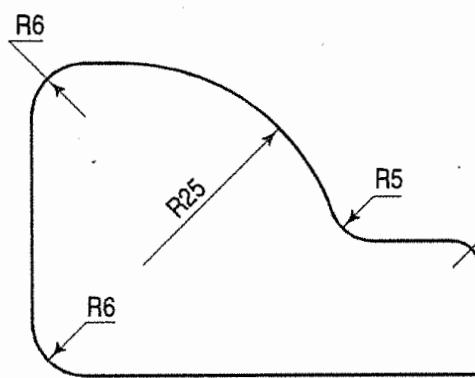


FIG. 3-23

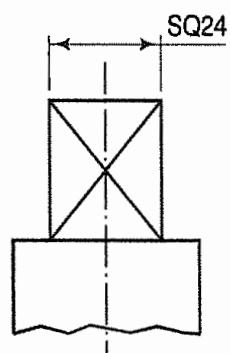


FIG. 3-24

- (8) Letters **SQ** should precede the dimension for a rod of square cross-section (fig. 3-24). The word **SPHERE** should be placed before the dimension (radius **R** or diameter **Ø**) of a spherical part (fig. 3-25).
- (9) Angular dimensions may be given by any one of the methods shown in fig. 3-26.
- (10) Fig. 3-27 shows a method of dimensioning a countersunk hole. The maximum diameter is also sometimes given.

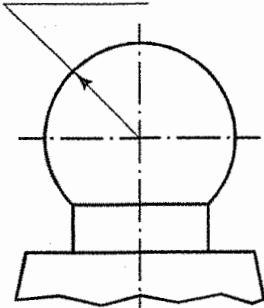
SPHERE R20

FIG. 3-25

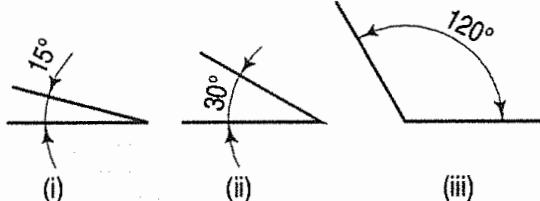


FIG. 3-26

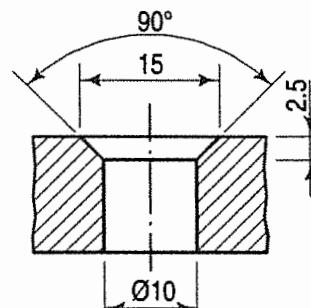


FIG. 3-27

(11) Methods of dimensioning a chamfer is shown in fig. 3-28.

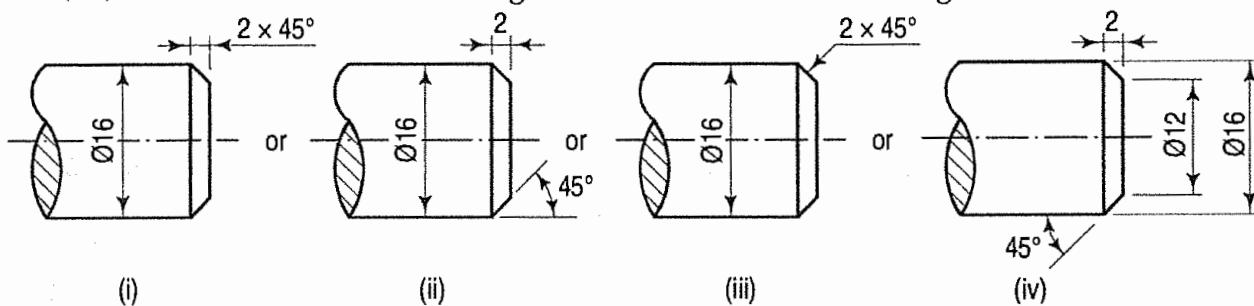


FIG. 3-28

(12) Designation and size, along with the useful length must be given while dimensioning an external screw thread (fig. 3-29). In case of internal screw thread, in addition to the size and type, the depth of the drilled hole before tapping must also be given (fig. 3-30).

(13) Left-hand thread and multiple-start thread should be dimensioned as shown in fig. 3-31 and fig. 3-32 respectively.

(14) A slope or taper is defined as unit alteration in a specified length. The specified length is measured along the base line in case of flat pieces and along the axis in case of shafts.

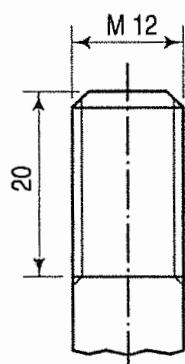


FIG. 3-29

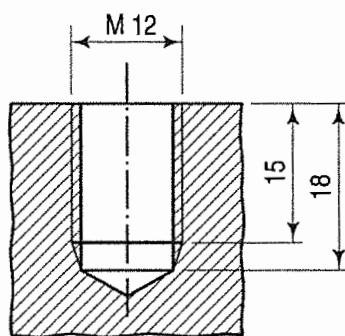


FIG. 3-30

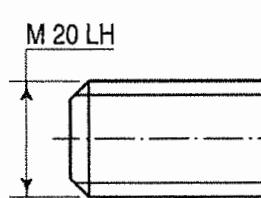


FIG. 3-31

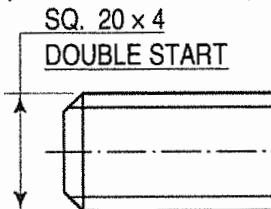


FIG. 3-32

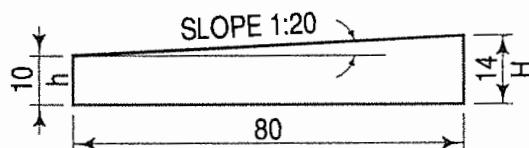


FIG. 3-33

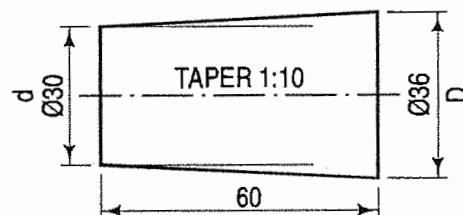


FIG. 3-34

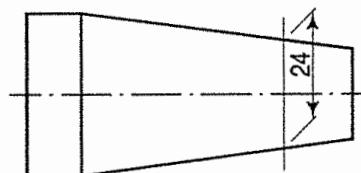


FIG. 3-35

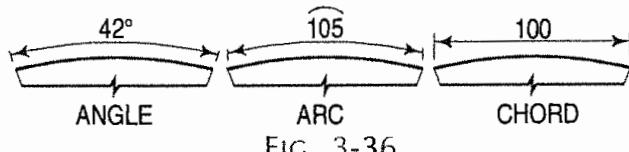


FIG. 3-36

(15) Fig. 3-33 shows the method of indicating slope on a flat piece. It is written parallel to the sloping line.

$$\text{Flat taper} = \frac{H - h}{L} = \frac{1}{20}.$$

(16) The taper on a shaft is indicated along the centre line and is accompanied by one or both the diameters (fig. 3-34).

$$\text{Taper on diameter} = \frac{D - d}{L} = \frac{1}{10}.$$

(17) Fig. 3-35 shows method of dimensioning for frustum. It is drawn obliquely but parallel to each other.

(18) Fig. 3-36 shows methods of dimensioning chords, arcs and angles.

(19) Study carefully fig. 3-37 for correct dimensioning.

Incorrect dimensioning is shown by cross (X) in the figure.

	CORRECT	INCORRECT	REASONS FOR INCORRECT
(i)	<p>Dimensions should be placed outside view</p>		<ul style="list-style-type: none"> 1. Arrow head not proportionate. 2. Hole dimension shown in figure. Leader line not ends horizontally. 3. Dimension '40' is too close. 4. Placing dimensions methods mix. Dimension '40' is according to aligned method.
(ii)	<p>Dim. should be marked from visible outlines</p>		<ul style="list-style-type: none"> 1. A key-way is shown with dotted line where the dimensions are placed. 2. Leader line for the shaft diameter is drawn horizontal touching the boundary line.
(iii)	<p>Dimensions should be given from the outlines (finished surface) or a centre line of a hole</p>		<ul style="list-style-type: none"> 1. Dimensions are given form the mid-line of the object. 2. Dimensions of holes are shown inside the figure. 3. Dimensions are shown in vertical line. 4. Smaller dimensions (25 mm) precedes the larger dimensions (30 mm). 5. Fillet radius is not shown.
(iv)	<p>Dimensions should be given from the outlines (finished surface) or a centre line of a hole</p>		<ul style="list-style-type: none"> 1. Dimension lines are used as extension. 2. Dimensions are placed inside the view. 3. Dimension 27 and 50 not written according to aligned system.
(v)			Section overlap the dimension 21.
(vi)			The outlines of the object are used as the extension lines.
(vii)			<ul style="list-style-type: none"> 1. Smaller circle is designated with radius. 2. Convention 'Ø' for diameter is placed after dimension. 3. Leader has arrow and it is drawn horizontal.

FIG. 3-37

EXERCISES 3



1. Write freehand, in single-stroke (i) vertical capital letters and (ii) inclined capital letters of 3 mm height, the following paragraph from page 39.
'All letters should _____ i.e. intensely black'.
2. Write freehand, in single-stroke vertical lower-case letters of 3 mm height, the following two paragraphs from page 37.
'Writing of titles, _____ freehand and speedily'.
3. Write freehand, the paragraphs stated in Ex. 2 above, in single-stroke inclined lower-case letters of 3 mm height.
4. Print in gothic letters of 12 mm height, the titles of Chapters 1, 2 and 3.
5. Fill-in the blanks with appropriate words selected from the list given below:
 - (a) Writing of titles, notes etc. on a drawing is called _____.
 - (b) Efficiency in the art of lettering is achieved by continuous _____.
 - (c) Lettering should be in plain and simple style so that it could be done _____ and _____.
 - (d) For maintaining uniformity in size, thin and light _____ may first be drawn.
 - (e) All letters should be uniform in _____, _____, _____, _____ and _____.
 - (f) The inclined letters slope to the _____ at an angle of _____ degrees.
 - (g) The size of the letter is described by its _____.
 - (h) Main title of inked drawing is generally written in _____ letters.
 - (i) The two types of single-stroke letters are _____ and _____.
 - (j) Lettering is usually done in _____ letters.
 - (k) Lower-case letters are usually used in _____ drawings.
 - (l) Lettering should be so done as can be read from the _____ with the main title _____.

List of words for Ex. 5:

- | | | | |
|---------------|------------------|-----------------|-------------------|
| 1. Inclined | 6. Vertical | 11. Right | 16. Practice |
| 2. Shade | 7. Freehand | 12. Front | 17. Lettering |
| 3. Horizontal | 8. Shape | 13. Guide lines | 18. Spacing |
| 4. Speedily | 9. Gothic | 14. Slope | 19. Architectural |
| 5. Height | 10. Seventy five | 15. Capital | 20. Size. |
6. Complete each statement given in A, by selecting an appropriate one from those given in B:
 - A (a) Outlines or principal lines are drawn as _____
 - (b) Lines for hidden edges are drawn as _____
 - (c) Thin and long chain line is made up of _____
 - (d) Centre lines, locus lines and pitch circles are drawn as _____
 - (e) Dashed line of medium thickness is made up of _____
 - (f) Dimension lines, hatching and extension lines are drawn as _____
 - (g) The position of cutting plane is shown by _____
 - (h) Irregular boundaries and short breaks are shown by _____
 - (i) Long breaks are shown by _____

- B (1) long and thin chain lines
 (2) thick and long chain line, thick at ends only
 (3) thick continuous lines
 (4) short dashes of approximately equal lengths
 (5) thin continuous lines
 (6) continuous, thin and wavy lines, drawn freehand
 (7) thin ruled lines with short zigzags within them
 (8) alternately long and short dashes
 (9) dashed lines of medium thickness.
7. Fill-up the blanks in the following sentences by appropriate words selected from the list of words given below:
- Two types of dimensions needed on a drawing are _____ and _____ dimensions.
 - The two systems of placing dimensions are _____ and _____.
 - In _____ system, the dimension is placed _____ to and near the _____ but clear of dimension line.
 - In _____ system, all dimensions are so placed that they are readable from the _____ edge of the drawing sheet. Dimension lines are _____ near the middle for inserting dimensions.
 - As far as possible dimensions should be given in one unit, preferably in _____.
 - Dimension lines should be drawn at least 8 mm away from the _____ and from _____.
 - An _____ or _____ should never be used as a dimension line.
 - Mutual _____ of dimension lines and dimensioning between _____ should be avoided.
 - All dimensions are shown from a common base line in _____ dimensioning.
 - In _____ dimensioning, dimensions are arranged in a straight line, and an _____ dimension is placed outside the small dimensions.
 - When a number of parallel dimensions are to be shown near each other, the dimensions should be _____.
 - The extension line should extend about 3 mm beyond the _____.
 - The line connecting a view to a note is called a _____.
 - The dimension indicating a diameter should be _____ by the symbol _____.
 - The symbol _____ should be _____ by the dimension indicating a radius.
 - Dimensions of cylindrical parts should as far as possible be shown in the views in which they are seen as _____.
 - Dimension of a cylinder should never be given as a _____.
 - The taper on a shaft is indicated along the _____.
 - The specified length for taper is measured along the _____ in case of a flat piece and along the _____ in case of a shaft.
 - The section lines are continuous _____ lines and are drawn at an _____ of _____ to the main outline of the section.

List of words for Ex. 7:

- | | | | |
|----------------|--------------------|-------------------|---------------------|
| 1. Thin | 9. Crossing | 17. Millimetres | 25. Leader |
| 2. Axis | 10. Dimension line | 18. Overall | 26. Radius |
| 3. Aligned | 11. Dashed line | 19. Outlines | 27. Angle |
| 4. Broken | 12. Each other | 20. Outline | 28. R |
| 5. Base line | 13. Followed | 21. Perpendicular | 29. 45° |
| 6. Bottom | 14. Location | 22. Progressive | 30. Size |
| 7. Centre line | 15. Middle | 23. Rectangles | 31. Staggered |
| 8. Continuous | 16. Ø | 24. Preceded | 32. Unidirectional. |

Answers to Ex. 5, 6 and 7:

- Ex. 5. (a) 17 (d) 13 (g) 5 (j) 15
 (b) 16 (e) 20, 8, 14, 2 and 18 (h) 9 (k) 19
 (c) 7 and 4 (f) 11 and 10 (i) 6 and 1 (l) 12 and 3.

Ex. 6. (a) 3 (d) 1 (g) 2
 (b) 9 (e) 4 (h) 6
 (c) 8 (f) 5 (i) 7.

Ex. 7. (a) 30 and 14 (f) 19 and 12 (k) 31 (p) 23
 (b) 3 and 32 (g) 20 and 7 (l) 10 (q) 26
 (c) 3, 21 and 15 (h) 9 and 11 (m) 25 (r) 7
 (d) 32, 6 and 4 (i) 22 (n) 24 and 16 (s) 5 and 2
 (e) 17 (j) 8 and 18 (o) 28 and 13 (t) 1, 27 and 29.

8. What are the two systems of placing dimensions on a drawing? Illustrate your answer with sketches.

9. Show by sketches the difference between (i) continuous or chain dimensioning and (ii) progressive or parallel dimensioning. What are the advantages of one above the other?

10. The orthographic views of the objects are drawn according to the First-angle projection method to 1 : 2 scale size and shown below in fig. 3-38(i), fig. 3-38(ii) and fig. 3-38(iii). Insert dimensions.

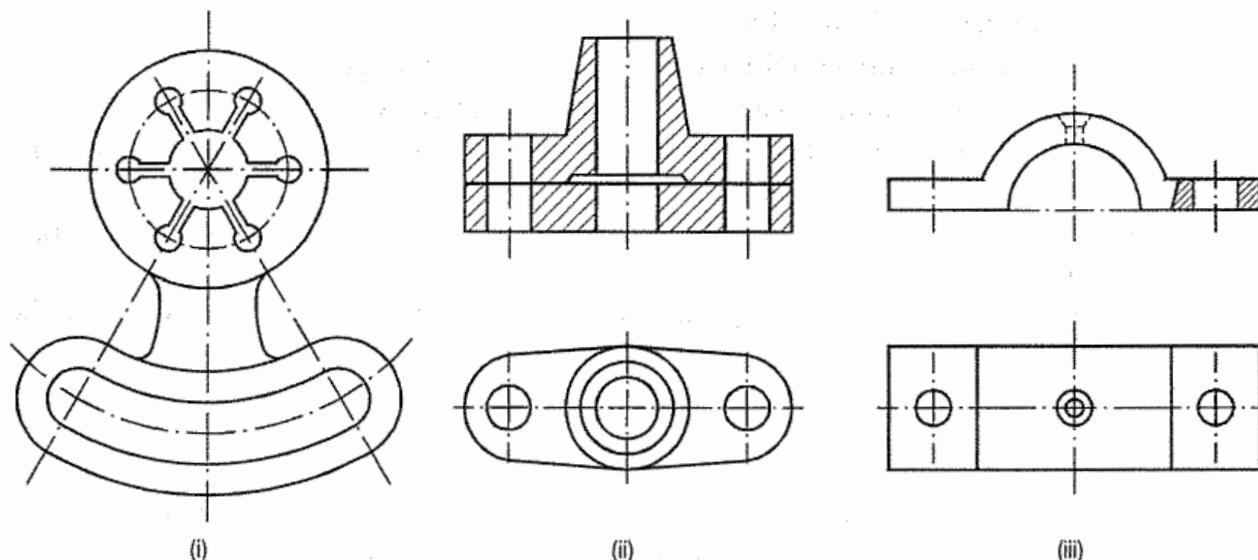


FIG. 3-38



4-1. INTRODUCTION

Drawings of small objects can be prepared of the same size as the objects they represent. A 150 mm long pencil may be shown by a drawing of 150 mm length. Drawings drawn of the same size as the objects, are called full-size drawings. The ordinary full-size scales are used for such drawings.

A scale is defined as the ratio of the linear dimensions of element of the object as represented in a drawing to the actual dimensions of the same element of the object itself.

4-2. SCALES

The scales generally used for general engineering drawings are shown in table 4-1 [SP : 46].

TABLE 4-1

(i)	Reducing scales	1 : 2 1 : 20 1 : 200 1 : 2000	1 : 5 1 : 50 1 : 500 1 : 5000	1 : 10 1 : 100 1 : 1000 1 : 10000
(ii)	Enlarging scales	50 : 1 5 : 1	20 : 1 2 : 1	10 : 1 1 : 1
(iii)	Full size scales			

All these scales are usually 300 mm long and sub-divided throughout their lengths. The scale is indicated on the drawing at a suitable place near the title. The complete designation of a scale consists of word scale followed by the ratio, i.e. scale 1 : 1 or scale, full size.

It may not be always possible to prepare full-size drawings. They are, therefore, drawn proportionately smaller or larger. When drawings are drawn smaller than the actual size of the objects (as in case of buildings, bridges, large machines etc.) the scale used is said to be a *reducing scale* (1 : 5). Drawings of small machine parts, mathematical instruments, watches etc. are made larger than their real size. These are said to be drawn on an *enlarging scale* (5 : 1).

The scales can be expressed in the following three ways:

(1) **Engineer's scale:** In this case, the relation between the dimension on the drawing and the actual dimension of the object is mentioned numerically in the style as 10 mm = 5 m etc.

(2) Graphical scale: The scale is drawn on the drawing itself. As the drawing becomes old, the engineer's scale may shrink and may not give accurate results.

However, such is not the case with graphical scale because if the drawing shrinks, the scale will also shrink. Hence, the graphical scale is commonly used in survey maps.

(3) Representative fraction: The ratio of the length of the object represented on drawing to the actual length of the object represented is called the Representative Fraction (i.e. R.F.).

$$\text{R.F.} = \frac{\text{Length of the drawing}}{\text{Actual length of object}}$$

When a 1 cm long line in a drawing represents 1 metre length of the object, the R.F. is equal to $\frac{1 \text{ cm}}{1 \text{ m}} = \frac{1 \text{ cm}}{1 \times 100 \text{ cm}} = \frac{1}{100}$ and the scale of the drawing will be

$1 : 100$ or $\frac{1}{100}$ full size. *The R.F. of a drawing is greater than unity when it is drawn on an enlarging scale.* For example, when a 2 mm long edge of an object is shown in a drawing by a line 1 cm long, the R.F. is $\frac{1 \text{ cm}}{2 \text{ mm}} = \frac{10 \text{ mm}}{2 \text{ mm}} = 5$. Such a drawing is said to be drawn on scale $5 : 1$ or *five times full-size*.

4-3. SCALES ON DRAWINGS

When an unusual scale is used, it is constructed on the drawing sheet. To construct a scale the following information is essential:

- (1) The R.F. of the scale.
- (2) The units which it must represent, for example, millimetres and centimetres, or feet and inches etc.
- (3) The maximum length which it must show.

The length of the scale is determined by the formula:

Length of the scale = R.F. \times maximum length required to be measured.

It may not be always possible to draw as long a scale as to measure the longest length in the drawing. The scale is therefore drawn 15 cm to 30 cm long, longer lengths being measured by marking them off in parts.

4-4. TYPES OF SCALES

The scales used in practice are classified as under:

- | | |
|------------------------|----------------------|
| (1) Plain scales | (4) Vernier scales |
| (2) Diagonal scales | (5) Scale of chords. |
| (3) Comparative scales | |

(1) Plain scales: A plain scale consists of a line divided into suitable number of equal parts or units, the first of which is sub-divided into smaller parts. Plain scales represent either two units or a unit and its sub-division.

In every scale,

- (i) The zero should be placed at the end of the first main division, i.e. between the unit and its sub-divisions.

- (ii) From the zero mark, the units should be numbered to the right and its sub-divisions to the left.
- (iii) The names of the units and the sub-divisions should be stated clearly below or at the respective ends.
- (iv) The name of the scale (e.g. scale, 1 : 10) or its R.F. should be mentioned below the scale.

Problem 4-1. (fig. 4-1): Construct a scale of 1 : 4 to show centimetres and long enough to measure upto 5 decimetres.

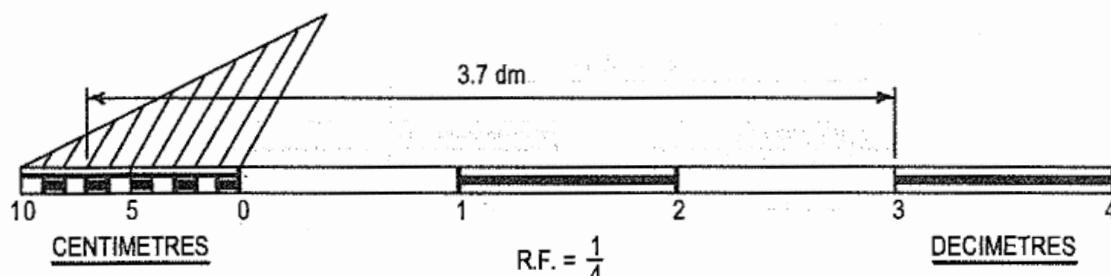


FIG. 4-1

- (i) Determine R.F. of the scale. Here it is $\frac{1}{4}$.

- (ii) Determine length of the scale.

$$\text{Length of the scale} = \text{R.F.} \times \text{maximum length} = \frac{1}{4} \times 5 \text{ dm} = 12.5 \text{ cm.}$$

- (iii) Draw a line 12.5 cm long and divide it into 5 equal divisions, each representing 1 dm.
- (iv) Mark 0 at the end of the first division and 1, 2, 3 and 4 at the end of each subsequent division to its right.
- (v) Divide the first division into 10 equal sub-divisions, each representing 1 cm.
- (vi) Mark cms to the left of 0 as shown in the figure.

To distinguish the divisions clearly, show the scale as a rectangle of small width (about 3 mm) instead of only a line. Draw the division-lines showing decimetres throughout the width of the scale. Draw the lines for the sub-divisions slightly shorter as shown. Draw thick and dark horizontal lines in the middle of all alternate divisions and sub-divisions. This helps in taking measurements. Below the scale, print DECIMETRES on the right-hand side, CENTIMETRES on the left-hand side, and the R.F. in the middle.

To set-off any distance, say 3.7 dm, place one leg of the divider on 3 dm mark and the other on 7 cm mark. The distance between the ends of the two legs will represent 3.7 dm.

Problem 4-2. (fig. 4-2): Draw a scale of 1 : 60 to show metres and decimetres and long enough to measure upto 6 metres.

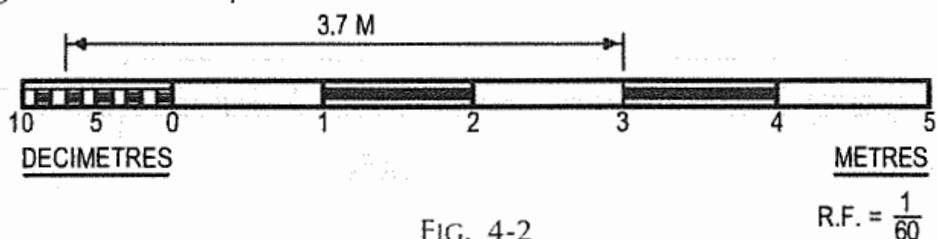


FIG. 4-2

$$\text{R.F.} = \frac{1}{60}$$

(i) Determine R.F. of the scale, here $R.F. = \frac{1}{60}$.

(ii) Determine length of the scale.

$$\text{Length of the scale} = \frac{1}{60} \times 6 \text{ m} = \frac{1}{10} \text{ metre} = 10 \text{ cm.}$$

(iii) Draw a line 10 cm long and divide it into 6 equal parts.

(iv) Divide the first part into 10 equal divisions and complete the scale as shown. The length 3.7 metres is shown on the scale.

Problem 4-3. (fig. 4-3): Construct a scale of 1.5 inches = 1 foot to show inches and long enough to measure upto 4 feet.

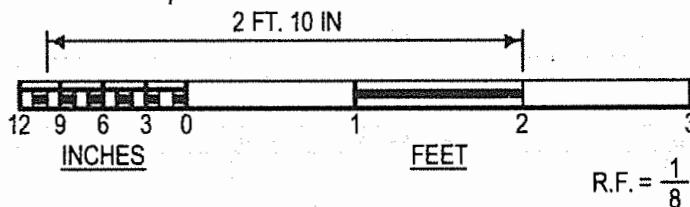


FIG. 4-3

(i) Determine R.F. of the scale. $R.F. = \frac{1.5 \text{ inches}}{1 \times 12 \text{ inches}} = \frac{1}{8}$.

(ii) Draw a line, $1.5 \times 4 = 6$ inches long.

(iii) Divide it into four equal parts, each part representing one foot.

(iv) Divide the first division into 12 equal parts, each representing 1". Complete the scale as explained in problem 4-1. The distance 2'-10" is shown measured in the figure.

Problem 4-4. (fig. 4-4): Construct a scale of $R.F. = \frac{1}{60}$ to read yards and feet, and long enough to measure upto 5 yards.

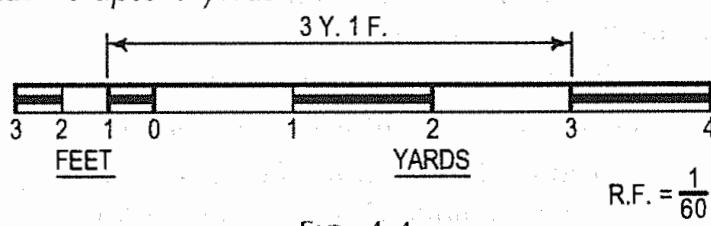


FIG. 4-4

(i) Length of the scale = $R.F. \times \text{max. length} = \frac{1}{60} \times 5 \text{ yd}$

$$= \frac{1}{12} \text{ yd} = 3 \text{ inches.}$$

(ii) Draw a line 3 inches long and divide it into 5 equal parts.

(iii) Divide the first part into 3 equal divisions.

(iv) Mark the scale as shown in the figure.

Problem 4-5. (fig. 4-5): Construct a scale of $R.F. = \frac{1}{84480}$ to show miles and furlongs and long enough to measure upto 6 miles.

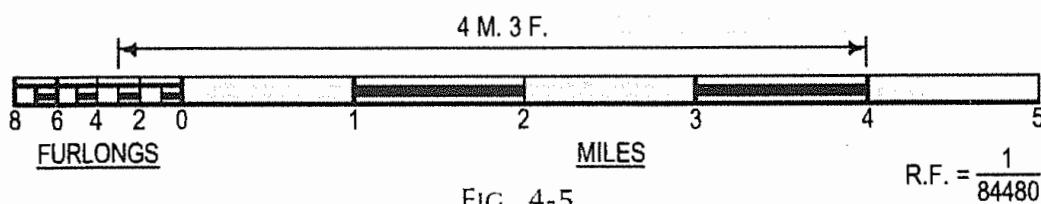


FIG. 4-5

$$(i) \text{ Length of the scale} = \frac{1}{84480} \times 6 = \frac{1}{14080} \text{ miles} = 4\frac{1}{2}''.$$

- (ii) Draw a line $4\frac{1}{2}''$ long and divide it into 6 equal parts. Divide the first part into 8 equal divisions and complete the scale as shown.

The distance 4 miles and 3 furlongs is shown measured in the figure.

(2) **Diagonal scales:** A diagonal scale is used when very minute distances such as 0.1 mm etc. are to be accurately measured or when measurements are required in three units; for example, dm, cm and mm, or yard, foot and inch.

Small divisions of short lines are obtained by the principle of diagonal division, as explained below.

Principle of diagonal scale: To obtain divisions of a given short line AB in multiples of $\frac{1}{10}$ its length, e.g. $0.1 AB$, $0.2 AB$, $0.3 AB$ etc. (fig. 4-6).

- (i) At one end, say B , draw a line perpendicular to AB and along it, step-off ten equal divisions of any length, starting from B and ending at C .
- (ii) Number the division-points, 9, 8, 7, ..., 1 as shown.
- (iii) Join A with C .
- (iv) Through the points 1, 2 etc. draw lines parallel to AB and cutting AC at $1'$, $2'$ etc. It is evident that triangles $1'1C$, $2'2C$, ..., ABC are similar.

Since $C5 = 0.5BC$, the line $5'5 = 0.5AB$.

Similarly, $1'1 = 0.1AB$, $2'2 = 0.2AB$ etc.

Thus, each horizontal line below AB becomes progressively shorter in length by $\frac{1}{10} AB$ giving lengths in multiples of $0.1AB$.

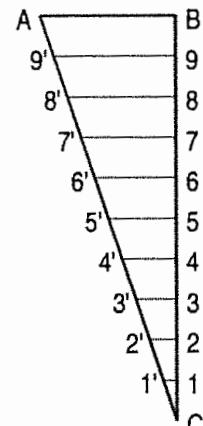


FIG. 4-6

Problem 4-6. (fig. 4-7): Construct a diagonal scale of $3 : 200$ i.e. $1 : 66\frac{2}{3}$ showing metres, decimetres and centimetres and to measure upto 6 metres.

$$\text{Length of the scale} = \frac{3}{200} \times 6 \text{ m} = 9 \text{ cm.}$$

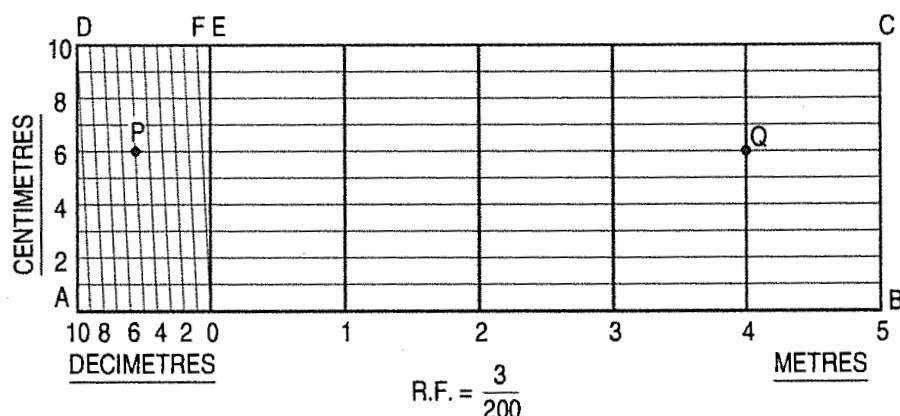


FIG. 4-7

- (i) Draw a line AB 9 cm long and divide it into 6 equal parts. Each part will show a metre.
- (ii) Divide the first part $A0$ into 10 equal divisions, each showing a decimetre or 0.1 m.
- (iii) At A erect a perpendicular and step-off along it, 10 equal divisions of any length, ending at D . Complete the rectangle $ABCD$.
- (iv) Erect perpendiculars at metre-divisions 0, 1, 2, 3 and 4.
- (v) Draw horizontal lines through the division-points on AD .
- (vi) Join D with the end of the first division along $A0$, viz. the point 9.
- (vii) Through the remaining points i.e. 8, 7, 6 etc. draw lines parallel to $D9$.

In $\triangle OFE$, FE represents 1 dm or 0.1 m. Each horizontal line below FE progressively diminishes in length by $0.1FE$. Thus, the next line below FE is equal to $0.9FE$ and represents $0.9 \text{ A } 1 \text{ dm} = 0.9 \text{ dm or } 0.09 \text{ m or } 9 \text{ cm}$.

Any length between 1 cm or 0.01 m and 6 m can be measured from this scale. To show a distance of 4.56 metres, i.e. 4 m, 5 dm and 6 cm, place one leg of the divider at Q where the vertical through 4 m meets the horizontal through 6 cm and the other leg at P where the diagonal through 5 dm meets the same horizontal.

Problem 4-7. (fig. 4-8): Construct a diagonal scale of R.F. = $\frac{1}{4000}$ to show metres and long enough to measure upto 500 metres.

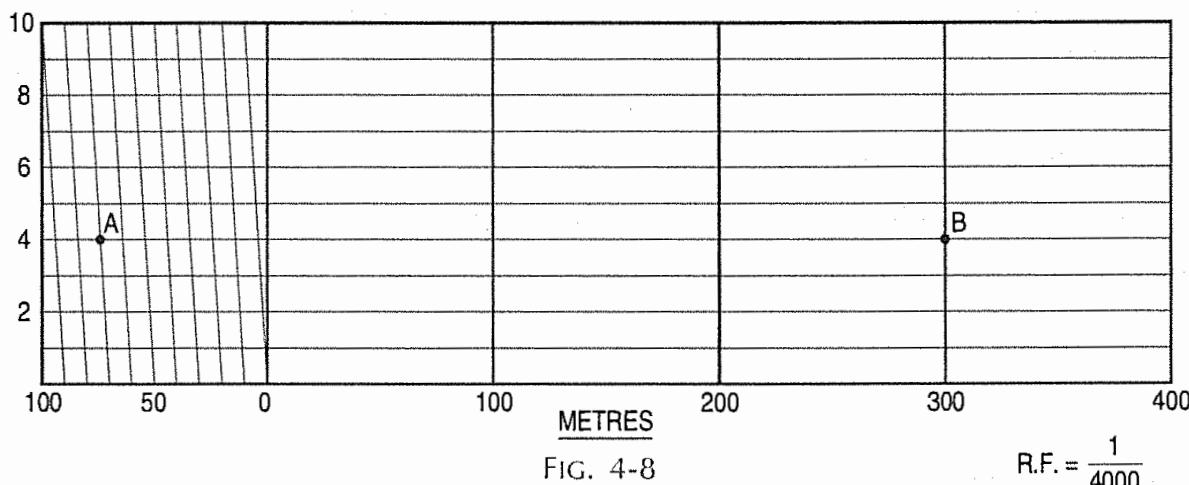


FIG. 4-8

$$\text{R.F.} = \frac{1}{4000}$$

$$\text{Length of the scale} = \frac{1}{4000} \times 500 \text{ m} = \frac{1}{8} \text{ metre} = 12.5 \text{ cm.}$$

- (i) Draw a line 12.5 cm long and divide it into 5 equal parts. Each part will show 100 metres.
- (ii) Divide the first part into ten equal divisions. Each division will show 10 metres.
- (iii) At the left-hand end, erect a perpendicular and on it, step-off 10 equal divisions of any length.
- (iv) Draw the rectangle and complete the scale as explained in problem 4-6.

The distance between points A and B shows 374 metres.

Problem 4-8. (fig. 4-9): Draw a diagonal scale of 1 : 2.5, showing centimetres and millimetres and long enough to measure upto 20 centimetres.

$$\text{Length of the scale} = \frac{1}{2.5} \times 20 \text{ cm} = 8 \text{ cm.}$$

- (i) Draw a line 8 cm long and divide it into 4 equal parts. Each part will represent a length of 5 cm.
- (ii) Divide the first part into 5 equal divisions. Each division will show 1 cm.
- (iii) At the left-hand end of the line, draw a vertical line and on it step-off 10 equal divisions of any length.

Complete the scale as explained in problem 4-6. The distance between points C and D shows 13.4 cm.

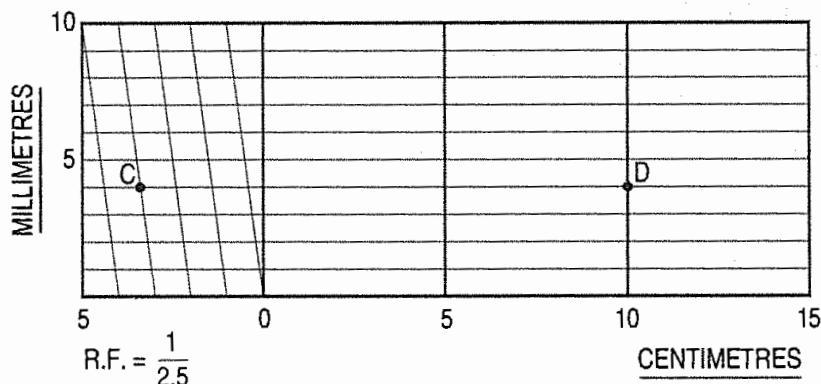


FIG. 4-9

Problem 4-9. (fig. 4-10): Construct a diagonal scale of $R.F. = \frac{1}{32}$ showing yards, feet and inches and to measure upto 4 yards.

$$\text{Length of the scale} = \frac{1}{32} \times 4 \text{ yd} = 18 \text{ yd} = 4\frac{1}{2} \text{ "}.$$

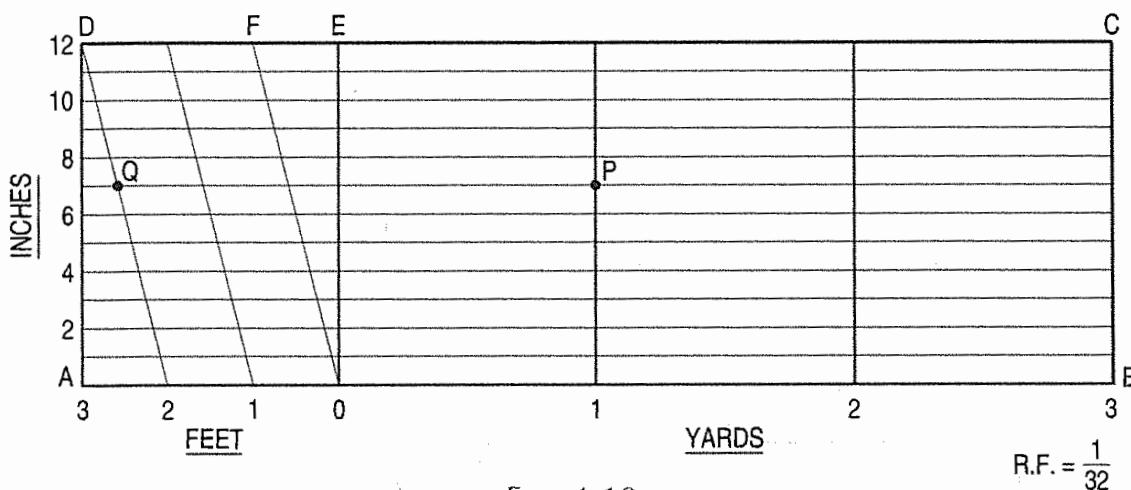


FIG. 4-10

- (i) Draw a line AB $4\frac{1}{2}$ " long.
- (ii) Divide it into 4 equal parts to show yards. Divide the first part A0 into 3 equal divisions showing feet.
- (iii) At A, erect a perpendicular and step-off along it, 12 equal divisions of any length, ending at D. Complete the scale as explained in problem 4-6.

To show a distance of 1 yard, 2 feet and 7 inches, place one leg of the divider at P, where the horizontal through 7" meets the vertical from 1 yard and the other leg at Q where the diagonal through 2' meets the same horizontal.

Problem 4-10. (fig. 4-11): Draw a scale of full-size, showing $\frac{1}{100}$ inch and to measure upto 5 inches.

- Draw a line AB 5" long and divide it into five equal parts. Each part will show one inch.
- Sub-divide the first part into 10 equal divisions. Each division will measure $\frac{1}{10}$ inch.
- At A , draw a perpendicular to AB and on it, step-off ten equal divisions of any length, ending at D .
- Draw the rectangle $ABCD$ and complete the scale as explained in problem 4-6. The line QP shows 2.68 inches.

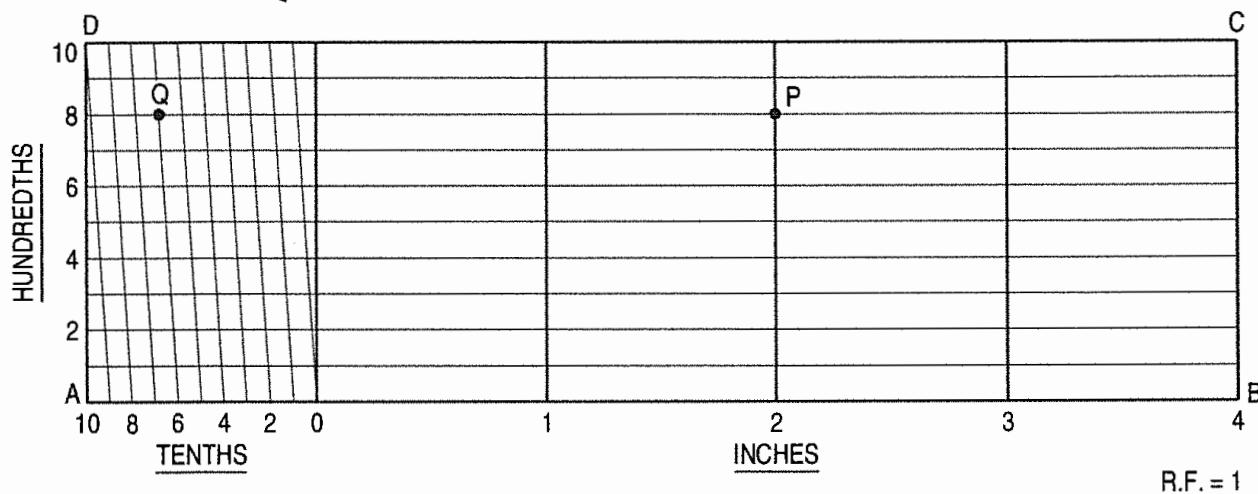


FIG. 4-11

Problem 4-11. (fig. 4-12): The area of a field is 50,000 sq m. The length and the breadth of the field, on the map is 10 cm and 8 cm respectively. Construct a diagonal scale which can read upto one metre. Mark the length of 235 metre on the scale. What is the R.F. of the scale?

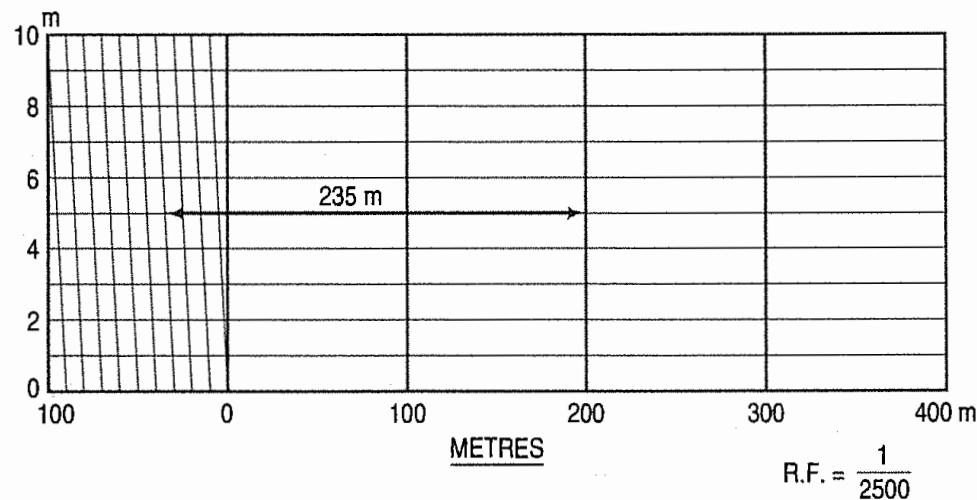


FIG. 4-12

The area of the field = 50,000 sq m.

The area of the field on the map = $10 \text{ cm} \times 8 \text{ cm} = 80 \text{ cm}^2$.

$$\therefore 1 \text{ sq cm} = \frac{50000}{80} = 625 \text{ sq m.}$$

$$\therefore 1 \text{ cm} = 25 \text{ m.}$$

$$\text{Now representative fraction} = \frac{1 \text{ cm}}{25 \text{ m}} = \frac{1}{2500}.$$

$$\text{Length of the scale} = \frac{1}{2500} \times \frac{500 \times 100}{1} = \frac{50000}{2500} = 20 \text{ cm.}$$

Take 20 cm length and divide it into 5 equal parts. Complete the scale as shown in fig. 4-12.

(3) Comparative scales: Scales having same representative fraction but graduated to read different units are called *comparative scales*. A drawing drawn with a scale reading inch units can be read in metric units by means of a metric comparative scale, constructed with the same representative fraction. Comparative scales may be plain scales or diagonal scales and may be constructed separately or one above the other.

Problem 4-12. [fig. 4-13(i) and fig. 4-13(ii)]: A drawing is drawn in inch units to a scale $\frac{3}{8}$ full size. Draw the scale showing $\frac{1}{8}$ inch divisions and to measure upto 15 inches. Construct a comparative scale showing centimetres and millimetres, and to read upto 40 centimetres.

(i) Inch scale:

Length of the scale

$$= \frac{3}{8} \times 15 = \frac{45}{8}$$

$$= 5 \frac{5}{8} \text{ inches.}$$

Construct the diagonal scale as shown in fig. 4-13(i).

(ii) Comparative scale:

Length of the scale

$$= \frac{3}{8} \times 40$$

$$= 15 \text{ cm.}$$

Construct the diagonal scale as shown in fig. 4-13(ii).

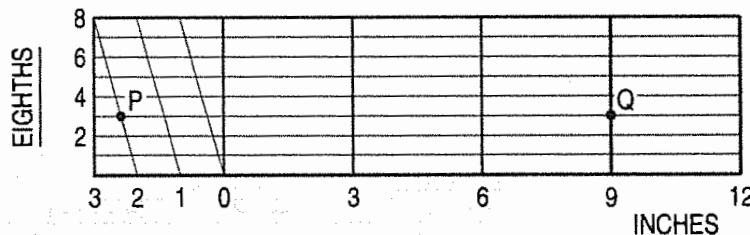


FIG. 4-13(i)

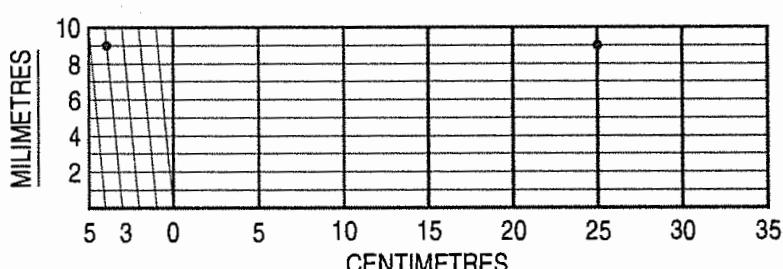


FIG. 4-13(ii)

The line PQ on the inch scale shows a length equal to $11 \frac{3}{8}$. Its equivalent, when measured on the comparative scale is 28.9 cm.

Problem 4-13. (fig. 4-14): Draw comparative scales of R.F. = $\frac{1}{485000}$ to read upto 80 kilometres and 80 versts. 1 verst = 1.067 km.

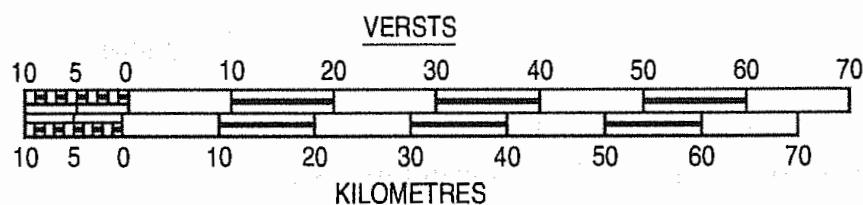


FIG. 4-14

$$\text{Length of kilometre scale} = \frac{1}{485000} \times 80 \times 1000 \times 100 = 16.5 \text{ cm.}$$

$$\text{Length of verst scale} = \frac{1}{485000} \times 80 \times 1.067 \times 1000 \times 100 = 17.6 \text{ cm.}$$

Draw the two scales one above the other as shown in the figure.

Problem 4-14. (fig. 4-15): On a road map, a scale of miles is shown. On measuring from this scale, a distance of 25 miles is shown by a line 10 cm long. Construct this scale to read miles and to measure upto 40 miles. Construct a comparative scale, attached to this scale, to read kilometres upto 60 kilometres. 1 mile = 1.609 km.

(i) **Scale of miles:**

$$\text{Length of the scale} = \frac{10 \times 40}{25} = 16 \text{ cm.}$$

Draw a line 16 cm long and construct a plain scale to show miles.

(ii) **Scale of kilometres:**

$$\text{R.F.} = \frac{10}{25 \times 1.609 \times 1000 \times 100} = \frac{1}{402250}$$

$$\text{Length of the scale} = \frac{1}{402250} \times 60 \times 1000 \times 100 = 14.9 \text{ cm.}$$

Construct the plain scale 14.9 cm long, above the scale of miles and attached to it, to read kilometres.

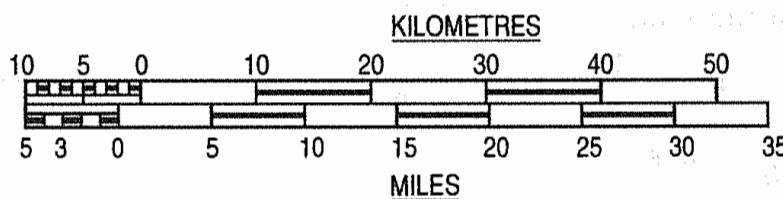


FIG. 4-15

Problem 4-15. (fig. 4-16): The distance between Bombay and Poona is 180 km. A passenger train covers this distance in 6 hours. Construct a plain scale to measure time upto a single minute. The R.F. of the scale is $\frac{1}{200000}$. Find the distance covered by the train in 36 minutes.

$$\text{Speed of the train} = \frac{180}{6} = 30 \text{ km/hour.}$$

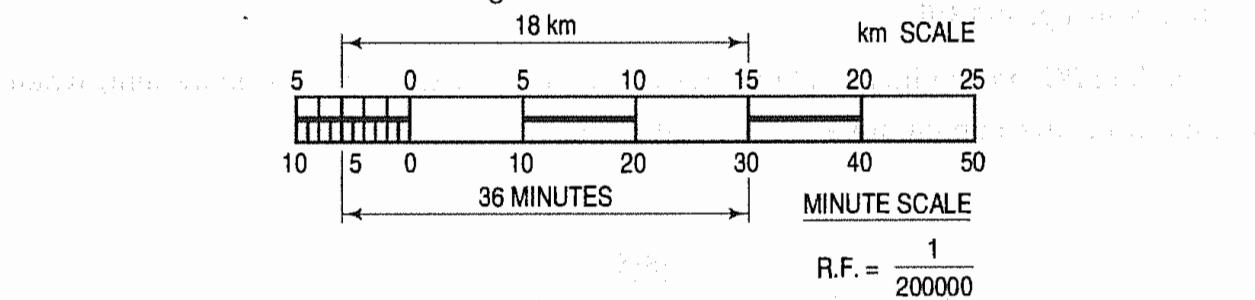


FIG. 4-16

(i) **Distance scale (kilometres scale):**

$$\text{Length of the scale} = \text{R.F.} \times \text{maximum distance}$$

$$= \frac{1}{200000} \times 30 \times 1000 \times 100 = 15 \text{ cm.}$$

(ii) Time scale (minute scale):

Speed of the train = 30 km/hour.

i.e. 30 km is covered in 60 minutes.

As length of the scale of 15 cm represents 30 km, 60 minutes which is the time required to cover 30 km, can be represented on the same length of the scale.

- (iii) Draw a line 15 cm long and divide it into 6 equal parts. Each part represents 5 km for the distance scale and 10 minutes for the time scale.
- (iv) Divide the first part of the distance scale and the time scale into 5 and 10 equal parts respectively. Complete the scales as shown. The distance covered in 36 minutes is shown on the scale.

Problem 4-16. (fig. 4-17): On a Russian map, a scale of versts is shown. On measuring it with a metric scale, 150 versts are found to measure 15 cm. Construct comparative scales for the two units to measure upto 200 versts and 200 km respectively. 1 verst = 1.067 km.

(i) Scale of verst:

$$\text{Length of the scale} = \frac{15 \times 200}{150} = 20 \text{ cm.}$$

Draw a line 20 cm long and construct a plain scale to show versts.

(ii) Scale of kilometres:

$$\text{R.F.} = \frac{15}{150 \times 1.609 \times 1000 \times 10} = \frac{1}{160900}$$

$$\text{Length of the scale} = \frac{1}{160900} \times 200 \times 1000 \times 10 = 12.4 \text{ cm.}$$

Construct the plain scale 12.4 cm long, above the scale of versts and attached to it, to read kilometres (fig. 4-17).

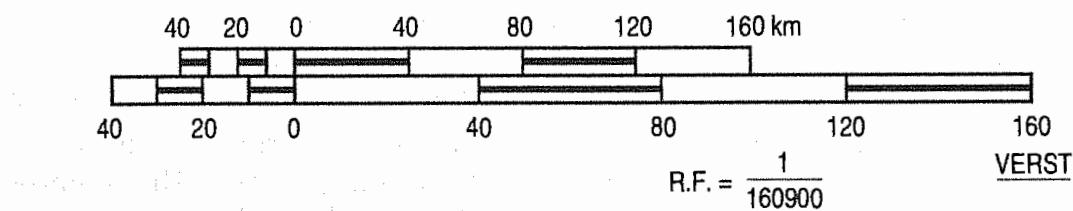


FIG. 4-17

(4) Vernier scales: Vernier scales, like diagonal scales, are used to read to a very small unit with great accuracy. A vernier scale consists of two parts — a primary scale and a vernier. The primary scale is a plain scale fully divided into minor divisions.

As it would be difficult to sub-divide the minor divisions in the ordinary way, it is done with the help of the vernier. The graduations on the vernier are derived from those on the primary scale.

(a) *Principle of vernier:* Fig. 4-18 shows a part of a plain scale in which the length A0 represents 10 cm. If we divide A0 into ten equal parts, each part will represent 1 cm. It would not be easy to divide each of these parts into ten equal divisions to get measurements in millimetres.

Now, if we take a length B_0 equal to $10 + 1 = 11$ such equal parts, thus representing 11 cm, and divide it into ten equal divisions, each of these divisions will represent

$$\frac{11}{10} = 1.1 \text{ cm or } 11 \text{ mm.}$$

The difference between one part of A_0 and one division of B_0 will be equal $1.1 - 1.0 = 0.1 \text{ cm or } 1 \text{ mm.}$

Similarly, the difference between two parts of each will be $0.2 \text{ cm or } 2 \text{ mm.}$

The *upper scale* B_0 is the vernier. The combination of the plain scale and the vernier is the vernier scale.

In general, if a line representing n units is divided into n equal parts, each part will show $\frac{n}{n} = 1$ unit. But, if a line equal to $n + 1$ of these units is taken and then divided into n equal parts, each of these parts will be equal to $\frac{n+1}{n} = 1 + \frac{1}{n}$ units.

The difference between one such part and one former part will be equal to $\frac{n+1}{n} - \frac{n}{n} = \frac{1}{n}$ unit.

Similarly, the difference between two parts from each will be $\frac{2}{n}$ unit.

(b) **Least count of a vernier:** It is the difference of 1 primary scale division and 1 vernier scale division. It is denoted by $\angle C$.

$\angle C = 1$ primary scale division – 1 vernier scale division.

The vernier scales are classified as under:

- (i) **Forward vernier:** In this case, the length of one division of the vernier scale is smaller than the length of one division of the primary scale. The vernier divisions are marked in the same direction as that of the main scale.
- (ii) **Backward vernier:** The length of each division of vernier scale is greater than the length of each division of the primary scale. The numbering is done in the opposite direction as that of the primary scale.

Problem 4-17. (fig. 4-19): Draw a vernier scale of R.F. = $\frac{1}{25}$ to read centimetres upto 4 metres and on it, show lengths representing 2.39 m and 0.91 m.

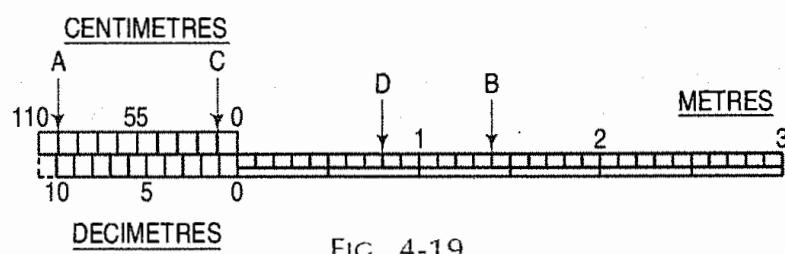


FIG. 4-19

$$\text{Length of the scale} = \frac{1}{25} \times 4 \times 100 = 16 \text{ cm.}$$

- (i) Draw a line 16 cm long and divide it into 4 equal parts to show metres. Divide each of these parts into 10 equal parts to show decimetres.
- (ii) To construct a vernier, take 11 parts of dm length and divide it into 10 equal parts. Each of these parts will show a length of 1.1 dm or 11 cm.

To measure a length representing 2.39 m, place one leg of the divider at A on 99 cm mark and the other leg at B on 1.4 m mark. The length AB will show 2.39 metres ($0.99 + 1.4 = 2.39$).

Similarly, the length, CD shows 0.91 metre ($0.8 + 0.11 = 0.91$).

The necessity of dividing the plain scale into minor divisions throughout its length is quite evident from the above measurements.

Problem 4-18. (fig. 4-20): Construct a full-size vernier scale of inches and show on it lengths 3.67", 1.54" and 0.48".

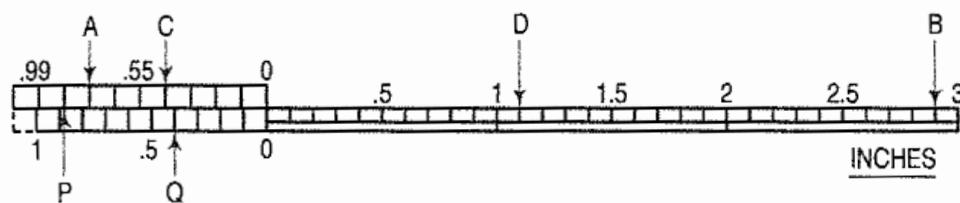


FIG. 4-20

- (i) Draw a plain full-size scale 4" long and divide it fully to show 0.1" lengths.
- (ii) Construct a vernier of length equal to $10 + 1 = 11$ parts and divide it into 10 equal parts. Each of these parts will $\frac{11 \times 0.1}{10} = 0.11"$.

The line AB shows a length of 3.67" ($0.77" + 2.9" = 3.67"$). Similarly, lines CD and PQ show lengths of 1.54" ($0.44" + 1.1" = 1.54"$) and 0.48" ($0.88" - 0.4" = 0.48"$) respectively.

Problem 4-19. (fig. 4-21): Construct a vernier scale of R.F. = $\frac{1}{80}$ to read inches and to measure upto 15 yards.

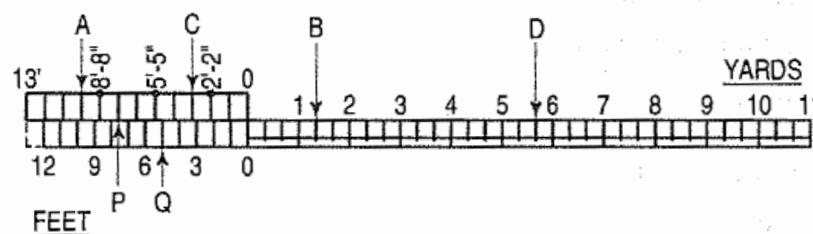


FIG. 4-21

$$\text{Length of the scale} = \frac{1}{80} \times 15 \text{ yd} = \frac{3}{16} \text{ yd} = 6\frac{3}{4} \text{".}$$

- (i) Draw the plain scale $6\frac{3}{4}$ " long and divide it fully to show yards and feet.
- (ii) To construct the vernier, take a length of $12 + 1 = 13$ feet-divisions and divide it into 12 equal parts. Each part will represent $\frac{13}{12}$ ft or 1'-1".

Lines AB, CD and PQ show respectively lengths representing 4 yd 1 ft 9 in ($9' - 9" + 4'$), 6 yd 2 ft 3 in ($3' - 3" + 17'$) and 0 yd 2 ft 7 in ($7' - 7" - 5"$).

(c) **Circular vernier:** The circular vernier are used in surveying instruments to measure angle to the required accuracy. In the case of mechanical engineering, it is used in measuring instruments such as micrometer.

Problem 4-20. A theodolite has main scale plate of 240 mm diameter graduated to 0° to 360° with the vernier scale to read degree and minute. The main scale can read to accuracy of 0.5° and the vernier is to read one minute. Draw the scale.

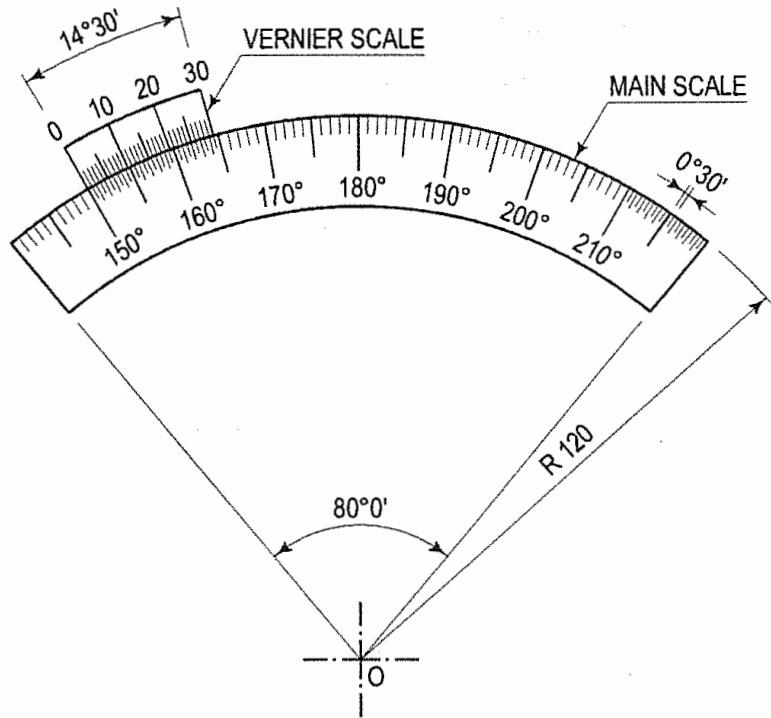


FIG. 4-22

- Draw an arc of radius 120 mm. (Diameter 240 mm)
- Mark with the protractor graduation as shown on the main scale, each division is $\frac{1}{2}^\circ$.
- Now one division of the main scale will read to accuracy of $\frac{1}{2}^\circ$.
- The vernier can read upto 1 minute
i.e. 1 minute = $\frac{1}{60}^\circ = \left(\frac{1}{30}\right) \left(\frac{1}{2}\right)$
One vernier scale can read = $\frac{1}{30}$ of main scale division.
- But least count of the vernier

$$\angle C = (1 \text{ m.s.d} - 1 \text{ v.s.d}) \quad \text{where m.s.d.} = \text{main scale division}$$

$$\therefore \angle C = 1 \text{ m.s.d.} - \frac{1}{30} \text{ m.s.d.} \quad \text{v.s.d.} = \text{vernier scale division}$$

$$1 \text{ v.s.d.} = \frac{29}{30} \text{ m.s.d.}$$

$$\therefore 30 \text{ division of v.s.d.} = 29 \text{ division of m.s.d.}$$
- Draw clockwise arc from 150° above the main scale of the convenient radius and arc length equal 29 divisions of m.s.d (i.e. $14^\circ - 30'$).
- Divide this arc (vernier) into 30 division by protractor or divider. The last division will coincide in this case at $164^\circ - 30'$.

(5) Scale of chords: The scale of chords is used to set out or measure angles when a protractor is not available. It is based on the lengths of chords of different angles measured on the same arc and is constructed as shown below.

- (i) Draw a line AB of any length (fig. 4-23).
- (ii) At B , erect a perpendicular.
- (iii) With B as centre, describe an arc AC cutting the perpendicular at a point C . Then, the arc AC (or the chord AC) subtends an angle of 90° at the centre B .
- (iv) Divide AC into nine equal parts. This may be done
 - (a) by dividing the arc AC into three equal parts by drawing arcs with centres A and C and radius AB , and then
 - (b) by dividing each of these parts into three equal parts by trial and error method. Each of the nine equal parts subtends an angle of 10° at the centre B .
- (v) Transfer each division-point from the arc to the straight line AB -produced, by taking A as centre and radii equal to chords $A-10$, $A-20$ etc.
- (vi) Complete the scale by drawing a rectangle below AD . The divisions obtained are unequal, decreasing gradually from A to D . It is quite evident that the distance from A to a division-point on the scale is equal to the length of the chord of the angle subtended by it at the centre B . It may be noted that the chord $A-60$ is equal to the radius AB .

The scale may be fully divided, i.e. each division divided into ten equal parts to show degrees. In the figure, degrees are shown in multiples of 5.

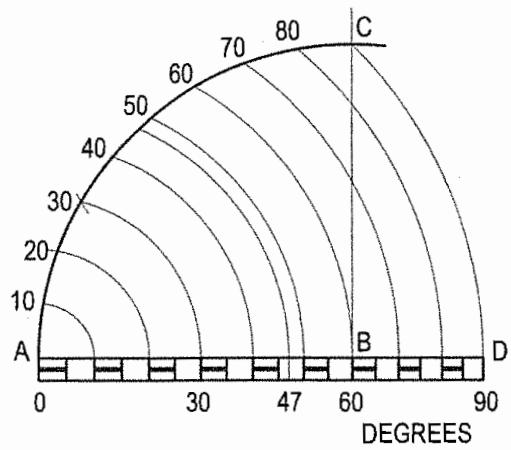


FIG. 4-23

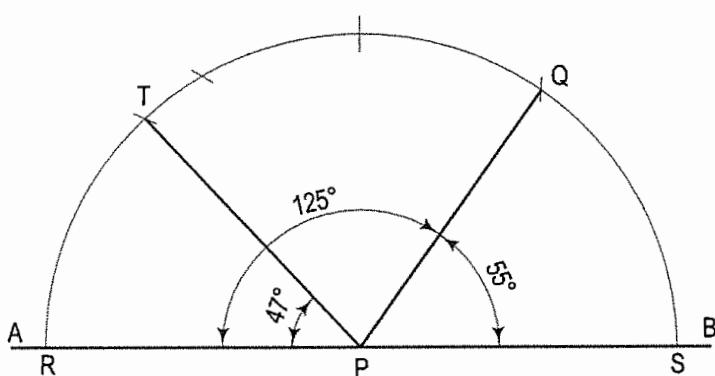


FIG. 4-24

Problem 4-21. (fig. 4-24): Construct angles of 47° and 125° by means of the scale of chords.

- (i) Draw any line AB .
- (ii) With any point P on it as centre and radius equal to $0-60$ (from the scale of chords), draw an arc cutting AP at a point R .
- (iii) With R as centre and radius equal to $0-47$ (chord of 47°) cut the arc at a point T .
- (iv) Draw a line joining P with T . Then $\angle RPT = 47^\circ$.

As the scale of chords gives angles upto only 90° , angle of 125° may be set-off in two parts viz. $60^\circ + 65^\circ$ or $90^\circ + 35^\circ$ as shown in the figure.

It may also be obtained by setting-off a chord SQ of 55° ($180^\circ - 125^\circ$) from the right side.

Problem 4-22. (fig. 4-25): Measure the given angle PQR by means of the scale of chords.

- With Q as centre and radius equal to 0-60, draw an arc cutting PQ at A and RQ at B .
- Take the chord length AB and apply it to the scale of chords which shows the angle to be of 37° .

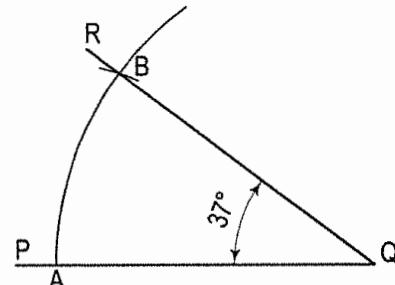


FIG. 4-25

EXERCISES 4

- Fill-up the blanks in the following sentences, using appropriate words, selected from those given in the brackets:
 - The ratio of the length of the drawing of the object to the actual length of the object is called _____ (resulting fraction, representative figure, representative fraction).
 - When the drawing is drawn of the same size as that of the object, the scale used is _____ (diagonal scale, full-size scale, vernier scale).
 - For drawings of small instruments, watches etc. _____ scale is always used (reducing, full-size, enlarging).
 - Drawings of buildings are drawn using _____ (full-size scale, reducing scale, scale of chords).
 - The R.F. in case of (b), (c) and (d) above would be, _____, _____, and _____ respectively (equal to 1, less than 1, greater than 1).
 - When measurements are required in three units _____ scale is used (diagonal, plain, comparative).
 - The scale of chords is used to set out or measure _____ (chords, lines, angles).

Answers to Ex. (1):

- a-3, b-2, c-3, d-2, e-1, 3 and 2, f-1, g-3.
- Construct a scale of 1:5 to show decimetres and centimetres and to read upto 1 metre. Show the length of 7.6 dm on it.
 - Construct a scale of $1.5 \text{ cm} = 1 \text{ dm}$ to read upto 1 metre and show on it a length of 0.6 metre.
 - Draw a diagonal scale of $\text{R.F.} = \frac{3}{100}$, showing metres, decimetres and centimetres, and to measure upto 5 metres. Show the length of 3.69 metres on it.
 - Draw a scale of 1:50 showing metres and decimetres, and to measure upto 8 metres.

6. A 3.2 cm long line represents a length of 4 metres. Extend this line to measure lengths upto 25 metres and show on it units of metre and 5 metres. Show the length of 17 metres on this line.
7. Construct a diagonal scale of R.F. = $\frac{1}{6250}$ to read upto 1 kilometre and to read metres on it. Show a length of 653 metres on it.
8. On a map, the distance between two points is 14 cm. The real distance between them is 20 km. Draw a diagonal scale of this map to read kilometres and hectametres, and to measure upto 25 km. Show a distance of 17.6 km on this scale.
9. An area of 144 sq cm on a map represents an area of 36 sq km on the field. Find the R.F. of the scale for this map and draw a diagonal scale to show kilometres, hectametres and decametres and to measure upto 10 kilometres. Indicate on the scale a distance of 7 kilometres, 5 hectametres and 6 decametres.
10. Construct the following scales and show below each, its R.F. and the units which its divisions represent:
 - (a) Scale of $1\frac{1}{4}''$ = 1 foot, to measure upto 5 feet and showing feet and inches.
 - (b) Scale of $\frac{3}{4}''$ = 1 yard, to measure upto 10 yards and showing yards and feet.
 - (c) Scale of $1\frac{3}{8}''$ = 1 mile, to measure upto 4 miles and showing miles and furlongs.
11. Construct a scale of $1''$ = 1 foot to read upto 6 feet and show on it, 4' - 7" length.
12. The R.F. of a scale showing miles, furlongs and chains is $\frac{1}{50688}$. Draw a scale to read upto 5 miles and show on it, the length representing 3 m 5 f 3 ch.
13. Draw a 4" long diagonal scale of $1''$ = 1" and show on it, the length of 2.14" and 3.79".
14. The distance between two points on a map is $5\frac{3}{4}''$. The points are actually 20 miles apart. Construct a diagonal scale of the map, showing miles and furlongs and to read upto 25 miles.
15. Construct comparative diagonal scales of metres and yards having R.F. = $\frac{1}{2700}$ to show upto 400 metres. 1 metre = 1.0936 yards.
16. Define simple and comparative scales. What is the R.F. of a scale which measures 2.5 inches to a mile? Draw a comparative scale of kilometres to read upto 10 km. 1 mile = 1.609 km.
17. On a map showing a scale of kilometres, 60 km are found to equal 4.5 inches. What is the R.F.? Construct a comparative scale of English miles. 1 km = 1093.6 yards.
18. On a Russian map, a scale of versts is shown. On measuring it with a metric scale, 120 versts are found to measure 10 cm. Construct comparative scales for the two units to measure upto 150 versts and 150 km respectively. 1 verst = 1.067 km.

19. Prepare a scale of knots comparative to a scale of $1 \text{ cm} = 5 \text{ km}$. Assume suitable lengths. 1 knot = 1.85 km.
20. Draw a full-size vernier scale to read $\frac{1}{8}$ " and $\frac{1}{64}$ " lengths and mark on it lengths of $5\frac{7}{32}"$, $2\frac{51}{64}"$ and $\frac{29}{64}"$.
21. Construct a scale of R.F. = $\frac{1}{2.5}$ to show decimetres and centimetres and by a vernier to read millimetres, to measure upto 4 decimetres.
22. Construct a vernier scale to show yards, the R.F. being $\frac{1}{3300}$. Show the distance representing 2 furlongs 99 yards.
23. Construct a scale of chords showing 5° divisions and with its aid set-off angles of 25° , 40° , 55° and 130° .
24. Draw a triangle having sides 8 cm, 9 cm and 10 cm long respectively and measure its angles with the aid of a scale of chords.
25. The distance between Vadodara and Surat is 130 km. A train covers this distance in 2.5 hours. Construct a plain scale to measure time upto a single minute. The R.F. of the scale is $\frac{1}{260000}$. Find the distance covered by the train in 45 minutes.
26. On a building plan, a line 20 cm long represents a distance of 10 m. Devise a diagonal scale for the plan to read upto 12 m, showing metres, decimetres and centimetres. Show on your scale the lengths 6.48 m and 11.14 m.
27. A room of 1728 m^3 volume is shown by a cube of 216 cm^3 volume. Find R.F. and construct a plain scale to measure upto 42 m. Mark a distance of 22 m on the scale.
28. An old plan was drawn to a scale of $1 \text{ cm} = 24 \text{ m}$. It has shrunk so that actual length of 100 m at site now works out to 96 m as per scale on plan. Find out the shrinkage factor and the corrected R.F. of the plan. (Hint: Shrinkage factor = present length on scale/original length on scale.)
29. The actual length of 500 m is represented by a line of 15 cm on a drawing. Construct a vernier scale to read upto 600 m. Mark on the scale a length of 549 m.
30. Draw a vernier scale of R.F. = 5 to read $\frac{1}{5} \text{ cm}$ and $\frac{1}{25} \text{ cm}$ and to measure upto 5 cm. Mark on the scale distances of 2.12 cm.
31. A car is running at a speed of 50 km/hour. Construct a diagonal scale to show 1 km by 3 cm and to measure upto 6 km. Mark also on the scale the distance covered by the car in 5 minutes 28 seconds.

Chapter

5



GEOMETRICAL CONSTRUCTION

5-0. INTRODUCTION



In this chapter, we shall deal with problems on geometrical construction which are mostly based on plane geometry and which are very essential in the preparation of engineering drawings. They are described as under:

- (1) Bisecting a line
- (2) To draw perpendiculars
- (3) To draw parallel lines
- (4) To divide a line
- (5) To divide a circle
- (6) To bisect an angle
- (7) To trisect an angle
- (8) To find the centre of an arc
- (9) To construct an ogee or reverse curve
- (10) To construct equilateral triangles
- (11) To construct squares
- (12) To construct regular polygons
- (13) Special methods of drawing regular polygons
- (14) Regular polygons inscribed in circles
- (15) To draw regular figures using T-square and set-squares
- (16) To draw tangents
- (17) Lengths of arcs
- (18) Circles and lines in contact
- (19) Inscribed circles.

5-1. BISECTING A LINE



Problem 5-1. To bisect a given straight line (fig. 5-1).

- (i) Let AB be the given line. With centre A and radius greater than half AB , draw arcs on both sides of AB .

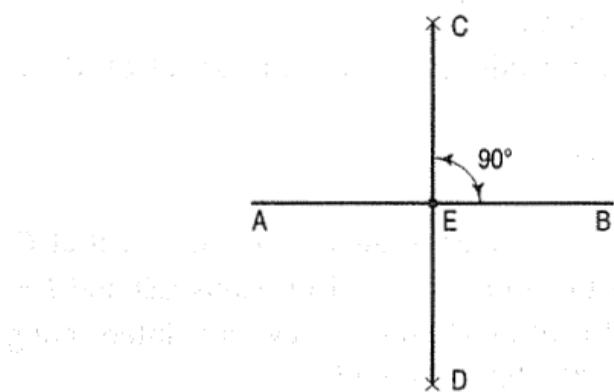


FIG. 5-1

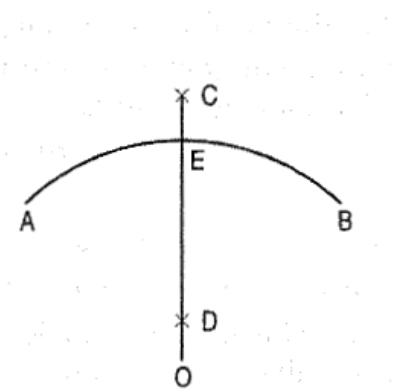


FIG. 5-2

- (ii) With centre B and the same radius, draw arcs intersecting the previous arcs at C and D .
 (iii) Draw a line joining C and D and cutting AB at E .

Then $AE = EB = \frac{1}{2} AB$.

Further, CD bisects AB at right angles.

Problem 5-2. To bisect a given arc (fig. 5-2).

Let AB be the arc drawn with centre O . Adopt the same method as shown in problem 5-1. The bisector CD , if produced, will pass through the centre O .

5-2. TO DRAW PERPENDICULARS



Problem 5-3. To draw a perpendicular to a given line from a point within it.

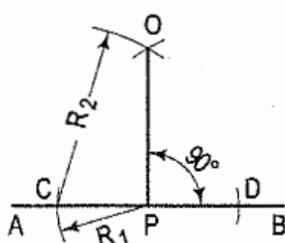


FIG. 5-3

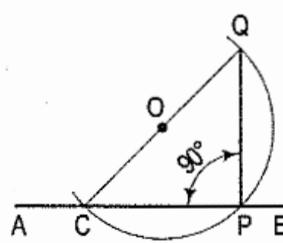


FIG. 5-4

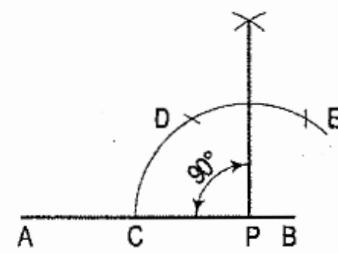


FIG. 5-5

Method I (fig. 5-3):

- (a) When the point is near the middle of the line.

Let AB be the given line and P the point in it.

- (i) With P as centre and any convenient radius R_1 , draw an arc cutting AB at C and D .
 (ii) With any radius R_2 greater than R_1 and centres C and D , draw arcs intersecting each other at O .
 (iii) Draw a line joining P and O .

Then PO is the required perpendicular.

- (b) When the point is near an end of the line.

Let AB be the given line and P the point in it.

Method II (fig. 5-4):

- (i) With any point O as centre and radius equal to OP , draw an arc greater than the semi-circle, cutting AB at C .
 (ii) Draw a line joining C and O , and produce it to cut the arc at Q . Draw a line joining P and Q .

Then PQ is the required perpendicular.

Method III (fig. 5-5):

- (i) With P as centre and any convenient radius, draw an arc cutting AB at C .
 (ii) With the same radius cut (from the arc) two equal divisions CD and DE .
 (iii) Again with the same radius and centres D and E , draw arcs intersecting each other at Q . Draw a line joining P and Q .

Then PQ is the required perpendicular.

Problem 5-4. To draw a perpendicular to a given line from a point outside it.

- (a) When the point is nearer the centre than the end of the line (fig. 5-6).

Let AB be the given line and P the point.

- With centre P and any convenient radius, draw an arc cutting AB at C and D .
- With any radius greater than half CD and centres C and D , draw the arcs intersecting each other at E .
- Draw a line joining P and E and cutting AB at Q .

Then PQ is the required perpendicular.

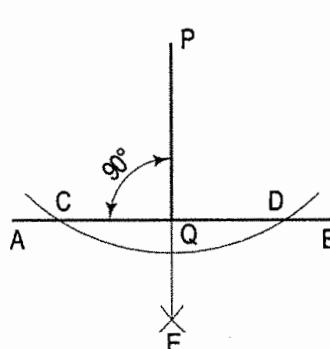


FIG. 5-6

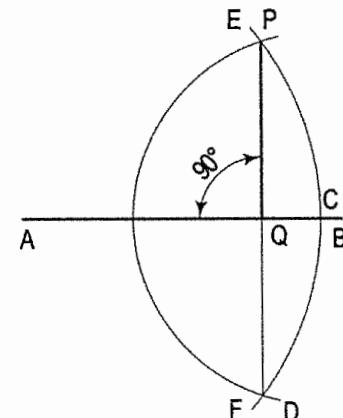


FIG. 5-7

- (b) When the point is nearer the end than the centre of the line (fig. 5-7).

Let AB be the given line and P the point.

- With centre A and radius equal to AP , draw an arc EF cutting AB or AB -produced, at C .
- With centre C and radius equal to CP , draw an arc cutting EF at D .
- Draw a line joining P and D and intersecting AB at Q .

Then PQ is the required perpendicular.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 4 for the following problem.

Problem 5-5. To draw perpendicular to a given line from a point outside it (fig. 5-8).

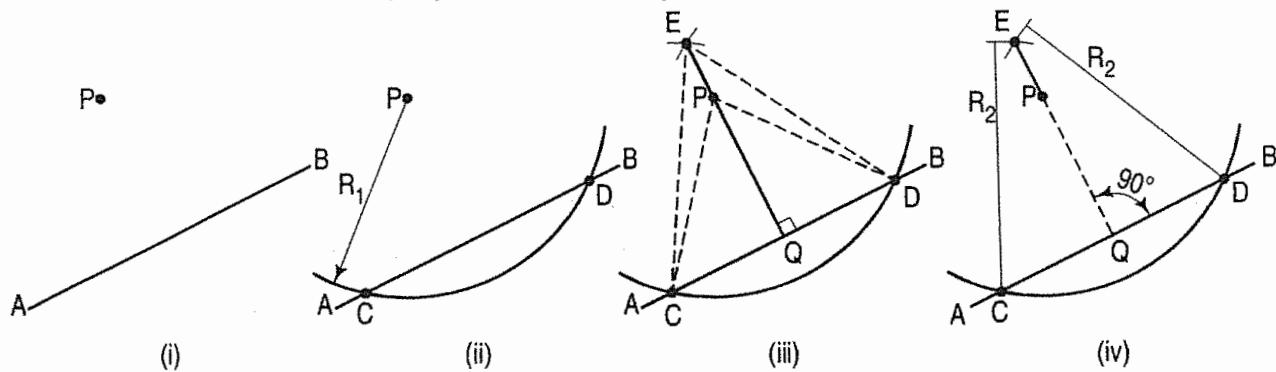


FIG. 5-8

When the point is nearer the centre than the end of the line.

Let AB be the given line and P the point.

- (i) Using compass and with P as centre draw an arc of any radius R_1 cutting the line AB at points C and D .
- (ii) With points C and D as centres, and with a larger radius $R_2 (> R_1)$, draw arcs to cut on the side of the line AB in which perpendicular is to draw. The arcs intersect in point E .
- (iii) Now join points E and P . (If required) Line EP may be extended to meet the line AB at point Q . Line EPQ will be perpendicular to line AB .
- (iv) Verify the angle AQP or BQP using a protractor. The angle AQP or BQP is the required perpendicular.

5-3. TO DRAW PARALLEL LINES



Problem 5-6. To draw a line through a given point, parallel to a given straight line (fig. 5-9).

Let AB be the given line and P the point.

- (i) With centre P and any convenient radius, draw an arc CD cutting AB at E .
- (ii) With centre E and the same radius, draw an arc cutting AB at F .
- (iii) With centre E and radius equal to FP , draw an arc to cut CD at Q .
- (iv) Draw a straight line through P and Q . Then this is the required line.

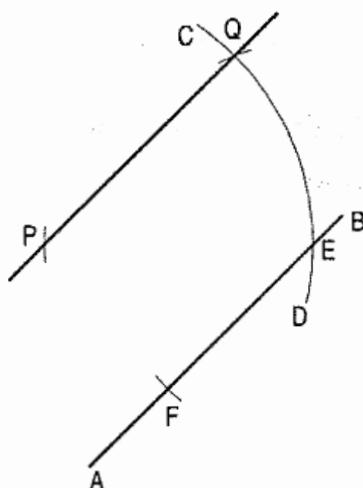


FIG. 5-9

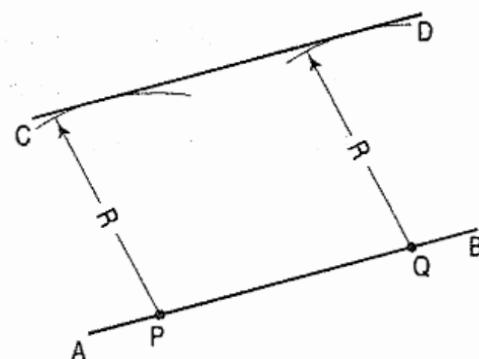


FIG. 5-10

Problem 5-7. To draw a line parallel to and at a given distance from a given straight line (fig. 5-10).

Let AB be the given line and R the given distance.

- (i) Mark points P and Q on AB , as far apart as convenient.
- (ii) With P and Q as centres and radius equal to R , draw arcs on the same side of AB .
- (iii) Draw the line CD , just touching the two arcs. CD is the required line.

5-4. TO DIVIDE A LINE



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 5 for the following problem.

Problem 5-8. To divide a given straight line into any number of equal parts (fig. 5-11).

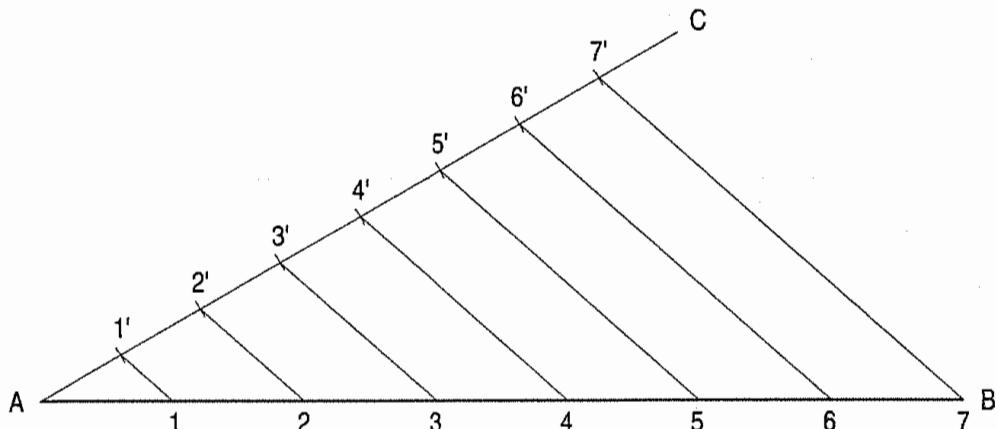


FIG. 5-11

Let AB be the given line to be divided into say, seven equal parts.

- Draw the line AB of given length.
- Draw another line AC making an angle of less than 30° with AB .
- With the help of dividers mark 7 equal parts of any suitable length on line AC and mark them by points $1'$, $2'$, $3'$, $4'$, $5'$, $6'$ and $7'$ as shown.
- Join the last point $7'$ with point B of the line AB .
- Now, from each of the other marked points $6'$, $5'$, $4'$, $3'$, $2'$ and $1'$, draw lines parallel to $7'B$ cutting the line AB at 6 , 5 , 4 , 3 , 2 and 1 respectively.
- Now the line AB has been divided into 7 equal parts. You can verify this by measuring the lengths.

Problem 5-9. To divide a given straight line into unequal parts (fig. 5-12).

Let AB be the given line to be divided into unequal parts say $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{2}$.

- Draw a line AB of given length, say, 120 mm.
- Erect perpendicular AD and BC at the ends A and B . Complete rectangular $ABCD$.
- Join diagonals AC and BD intersecting at E .
- Draw perpendicular from E on AB as shown.
- Then $AF = \frac{1}{2} AB$.
- Join D and F . The line FD intersects the diagonal AC at G . Drop perpendicular from G to AB . Then $AH = \frac{1}{3} AB$.
- Similarly make construction in figure, for obtaining $\frac{1}{4} AB$, $\frac{1}{5} AB$ and $\frac{1}{6} AB$ as shown.

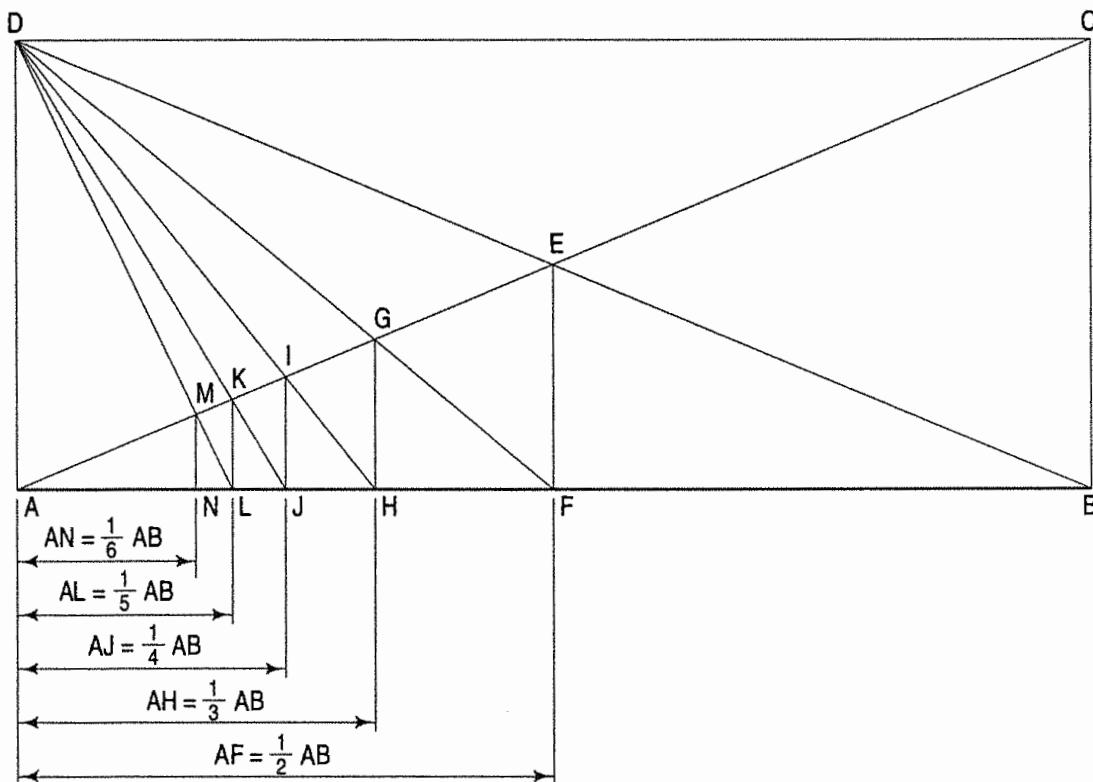


FIG. 5-12

5-5. TO DIVIDE A CIRCLE



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 6 for the following problem.

Problem 5-10. To divide a circle of a given radius into N equal parts (fig. 5-13).

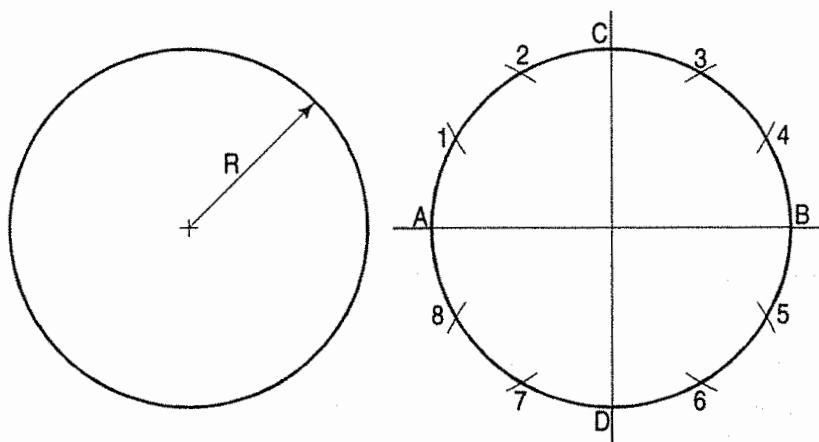


FIG. 5-13

Given a circle of radius R , to divide it into 12 equal parts along its circumference, the geometric construction procedure is as follows.

- Draw two diagonals AB and CD at right angles to each other cutting the circle at A , B and C , D .

- (ii) With A as centre, and radius equal to the given radius of the circle, draw arcs cutting the circle at 2 and 7.
- (iii) Similarly, with B as centre, and with the given circle radius, draw arcs to cut the circle at 3 and 6.
- (iv) Similarly, from points C and D as centre, and with the given radius of circle, draw arcs to cut the circle at 1, 4 and 5, 8 respectively.
- (v) Thus dividing it at the cuts are the required twelve equal parts of the circle.

5-6. TO BISECT AN ANGLE

Problem 5-11. To bisect a given angle (fig. 5-14).

Let ABC be the given angle.

- (i) With B as centre and any radius, draw an arc cutting AB at D and BC at E .
- (ii) With centres D and E and the same or any convenient radius, draw arcs intersecting each other at F .
- (iii) Draw a line joining B and F . BF bisects the angle ABC , i.e. $\angle ABF = \angle FBC$.

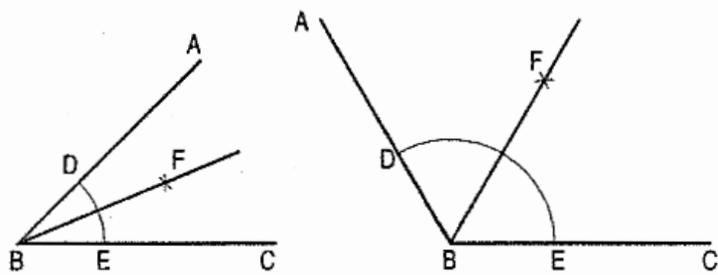


FIG. 5-14

Problem 5-12. To draw a line inclined to a given line at an angle equal to a given angle (fig. 5-15).

Let PQ be the given line and AOB the given angle.

- (i) With O as centre and any radius, draw an arc cutting OA at C and OB at D .
- (ii) With the same radius and centre P , draw an arc EF cutting PQ at F .
- (iii) With F as centre and radius equal to CD , draw an arc cutting the arc EF at G .
- (iv) From P , draw a line passing through G . This is the required line.

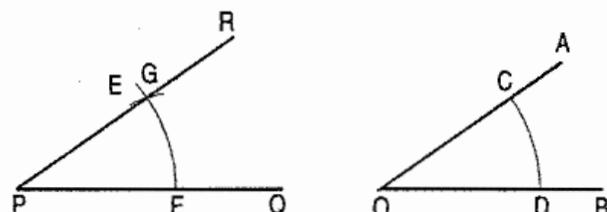


FIG. 5-15

5-7. TO TRISECT AN ANGLE



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 7 for the following problem.

Problem 5-13. To trisect a given right angle (fig. 5-16).

Let ABC be the given right angle.

- (i) With centre B and any radius, draw an arc cutting AB at D and BC at E .
- (ii) With the same radius and centres D and E , draw arcs cutting the arc DE at points Q and P .

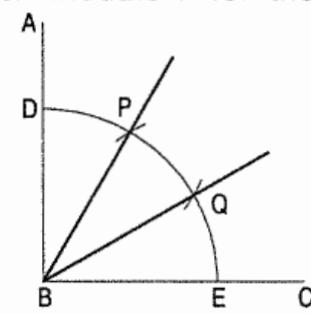


FIG. 5-16

- (iii) Draw lines joining B with P and Q . BP and BQ trisect the right angle ABC .
 Thus, $\angle ABP = \angle PBQ = \angle QBC = \frac{1}{3} \angle ABC$.

5-8. TO FIND THE CENTRE OF AN ARC

Problem 5-14. To find the centre of a given arc (fig. 5-17).

Let AB be the given arc.

- In AB , draw two chords CD and EF of any lengths.
- Draw perpendicular bisectors of CD and EF intersecting each other at O . Then O is the required centre.

Problem 5-15. To draw an arc of a given radius, touching a given straight line and passing through a given point (fig. 5-18).

Let AB be the given line, P the point and R the radius.

- Draw a line CD parallel to and at a distance equal to R from AB (Problem 5-7).
- With P as centre and radius equal to R , draw an arc cutting CD at O .
- With O as centre, draw the required arc.

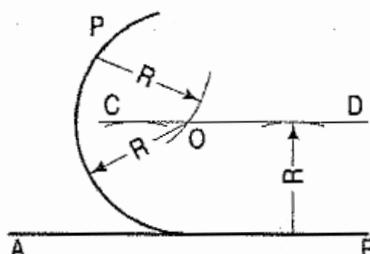


FIG. 5-18

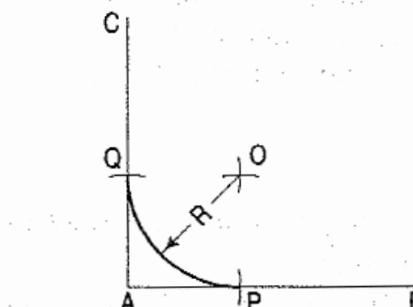


FIG. 5-19

Problem 5-16. To draw an arc of a given radius touching two given straight lines at right angles to each other (fig. 5-19).

Let AB and AC be the given lines and R the given radius.

- With centre A and radius equal to R , draw arcs cutting AB at P and AC at Q .
- With P and Q as centres and the same radius, draw arcs intersecting each other at O .
- With O as centre and radius equal to R , draw the required arc.

Problem 5-17. To draw an arc of a given radius touching two given straight lines which make any angle between them (fig. 5-20 and fig. 5-21).

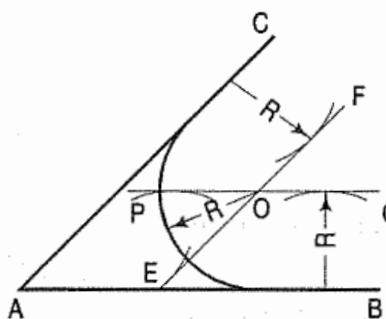


FIG. 5-20

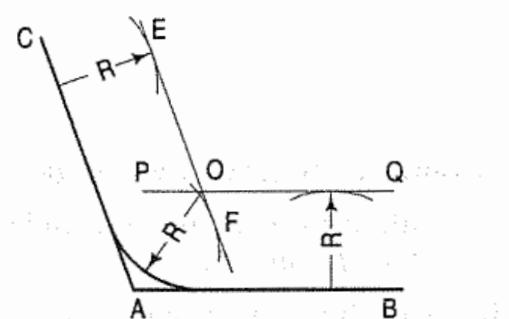


FIG. 5-21

Let AB and AC be the given lines and R the given radius.

- Draw a line PQ parallel to and at a distance equal to R from AB .
- Similarly, draw a line EF parallel to and at a distance equal to R from AC , intersecting PQ at O .
- With O as centre and radius equal to R , draw the required arc.

Problem 5-18. To draw an arc of a given radius touching a given arc and a given straight line.

Case I: (fig. 5-22): Let AB be the given line, CD the given arc drawn with centre O and radius equal to R_1 , and R_2 the given radius.

- With O as centre and radius equal to $(R_1 - R_2)$, draw an arc EF .
- Draw a line parallel to and at a distance equal to R_2 from AB and intersecting EF at a point P .
- With P as centre and radius equal to R_2 , draw the required arc.

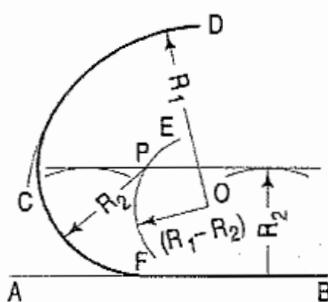


FIG. 5-22

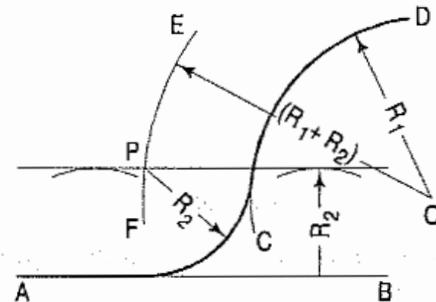


FIG. 5-23

Case II: (fig. 5-23): Let AB be the given line, CD the given arc drawn with centre O and radius equal to R_1 , and R_2 the given radius.

- With O as centre and radius equal to $(R_1 + R_2)$, draw an arc EF .
- Draw a line parallel to and at a distance equal to R_2 from AB and intersecting EF at a point P .
- With P as centre and radius equal to R_2 , draw the required arc.

Problem 5-19. To draw an arc of a given radius touching two given arcs.

Let AB be the given arc drawn with centre O and radius equal to R_1 ; CD the arc drawn with centre P and radius equal to R_2 , and R_3 the given radius.

Case I (fig. 5-24):

- With O as centre and radius equal to $(R_1 + R_3)$, draw an arc EF .
- With O as centre and radius equal to $(R_2 + R_3)$, draw an arc intersecting EF at a point Q .
- With Q as centre and radius equal to R_3 , draw the required arc.

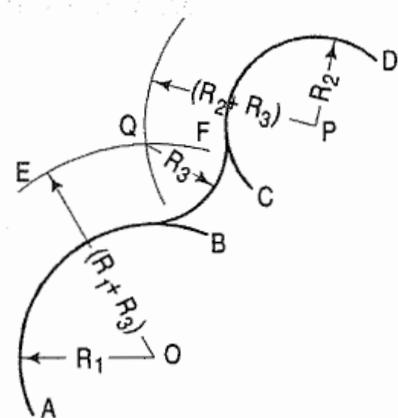


FIG. 5-24

Case II (fig. 5-25):

- With O as centre and radius equal to $(R_1 - R_3)$, draw an arc EF .
- With P as centre and radius equal to $(R_2 + R_3)$, draw an arc intersecting EF at a point Q .
- With Q as centre and radius equal to R_3 , draw the required arc.

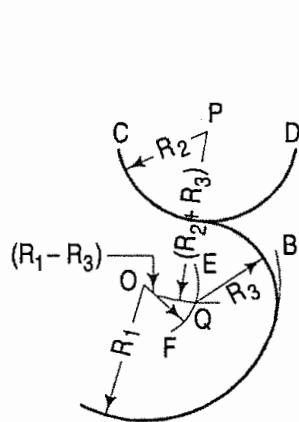


FIG. 5-25

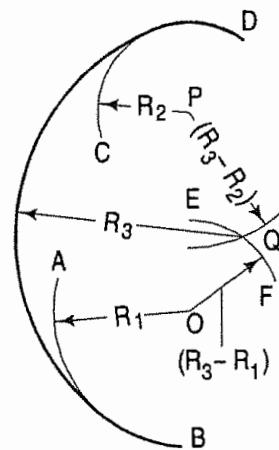


FIG. 5-26

Case III (fig. 5-26):

- With O as centre and radius equal to $(R_3 - R_1)$, draw an arc EF .
- With P as centre and radius equal to $(R_3 - R_2)$, draw an arc intersecting EF at a point Q .
- With Q as centre and radius equal to R_3 , draw the required arc.

Problem 5-20. To draw an arc passing through three given points not in a straight line (fig. 5-27).

Let A , B and C be the given points.

- Draw lines joining B with A and C .
- Draw perpendicular bisectors of AB and BC intersecting each other at a point O .
- With O as centre and radius equal to OA or OB or OC , draw the required arc.

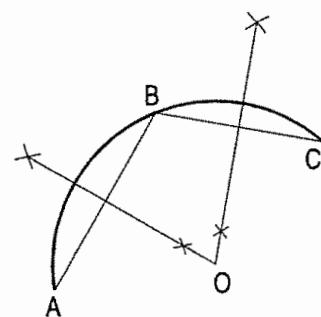


FIG. 5-27

Problem 5-21. To draw a continuous curve of circular arcs passing through any number of given points not in a straight line (fig. 5-28).

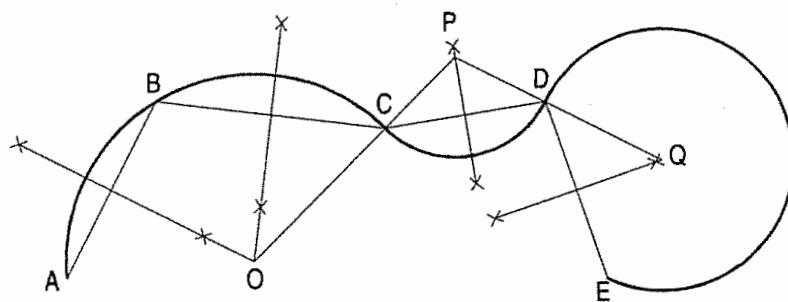


FIG. 5-28

Let A, B, C, D and E be the given points.

- Draw lines joining A with B , B with C , C with D etc.
- Draw perpendicular bisectors of AB and BC intersecting at O .
- With O as centre and radius equal to OA , draw an arc ABC .
- Draw a line joining O and C .
- Draw the perpendicular bisector of CD intersecting OC or OC produced, at P .
- With P as centre and radius equal to PC , draw an arc CD .
- Repeat the same construction. Note that the centre of the arc is at the intersection of the perpendicular bisector and the line, or the line-produced, joining the previous centre with the last point of the previous arc.

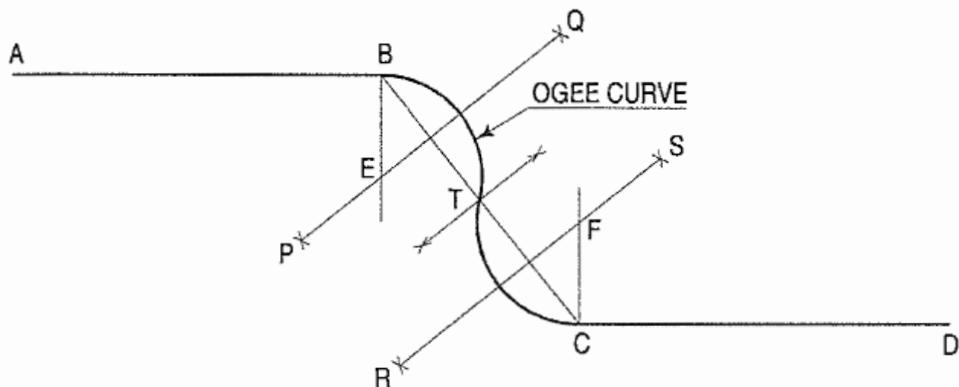
5-9. TO CONSTRUCT AN OGEET OR REVERSE CURVE

An ogee curve or a reverse curve is a combination of two same curves in which the second curve has a reverse shape to that of the first curve. In other words, any curve or line or mould consist of a continuous double curve with the upper part convex and lower part concave, to some extent having shape of "S".



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 8 for the following problem.

Problem 5-22. To draw an ogee shaped arc tangent between two parallel lines (fig. 5-29).



Ogee shaped arc tangent to two parallel lines

FIG. 5-29

- Let AB and CD be the two given parallel lines which are to be connected by an ogee curve.
- Join points B and C . Bisect the line BC and obtain its centre point T .
- Bisect line segment BT and draw the bisector line PQ . Similarly, bisect segment CT and draw the bisector line RS .
- From point B , draw a perpendicular line to cut the bisector line PQ at E .

- (v) Similarly, from point C , draw a perpendicular line to cut the bisector line RS at F .
- (vi) Points E and F are the centre points of the ogee curve. With E as centre draw an arc BT . With F as centre draw another arc CT . Arc BTC is the required ogee curve.

5-10. TO CONSTRUCT EQUILATERAL TRIANGLES



Problem 5-23. To construct an equilateral triangle, given the length of the side (fig. 5-30 and fig. 5-31).

- (a) *With T-square and set-square only* (fig. 5-30).
 - (i) With the T-square, draw a line AB of given length.
 - (ii) With 30° - 60° set-square and T-square, draw a line through A making 60° angle with AB .
 - (iii) Similarly, through B , draw a line making the same angle with AB and intersecting the first line at C .

Then ABC is the required triangle.

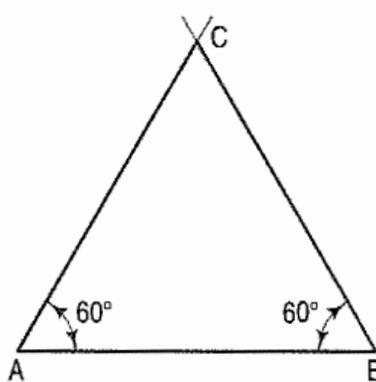


FIG. 5-30

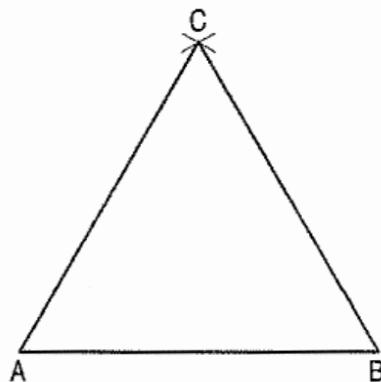


FIG. 5-31

- (b) *With the aid of a compass* (fig. 5-31).
 - (i) With centres A and B and radius equal to AB , draw arcs intersecting each other at C .
 - (ii) Draw lines joining C with A and B .

Then ABC is the required triangle.

Problem 5-24. To construct an equilateral triangle of a given altitude (fig. 5-32 and fig. 5-33).

- (a) *With T-square and set-square only* (fig. 5-32).
 - (i) With the T-square, draw a line AB of any length.
 - (ii) From a point P in AB , draw with a set-square, the vertical PQ equal to the given altitude.

- (iii) With T-square and 30° - 60° set-square, draw lines through Q on both sides of and making 30° angles with PQ and cutting AB in R and T.

Then QRT is the required triangle.

- (b) *With the aid of a compass* (fig. 5-33).

- Draw a line AB of any length.
- At any point P in AB, draw the perpendicular PQ equal to the given altitude (Problem 5-3).
- With centre Q and any radius, draw an arc intersecting PQ at C.
- With centre C and the same radius, draw arcs cutting the first arc at E and F.
- Draw bisectors of CE and CF to intersect AB at R and T respectively.

Then QRT is the required triangle.

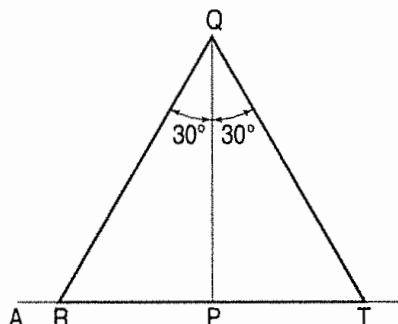


FIG. 5-32

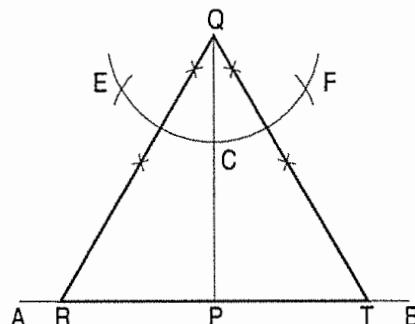


FIG. 5-33

5-11. TO CONSTRUCT SQUARES

Problem 5-25. *To construct a square, length of a side given* (fig. 5-34 and fig. 5-35).

- (a) *With T-square and set-square only* (fig. 5-34).

- With the T-square, draw a line AB equal to the given length.
- At A and B, draw verticals AE and BF.
- From point A draw a line inclined at 45° to AB, cutting BF at C.
- From point B draw a line inclined at 45° to AB, cutting AE at D.
- Draw a line joining C with D.

Then ABCD is the required square.

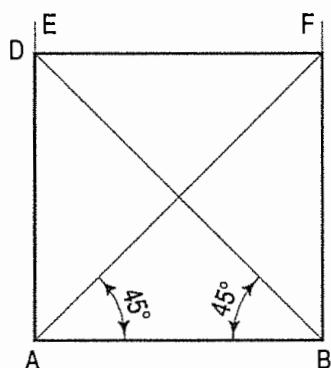


FIG. 5-34

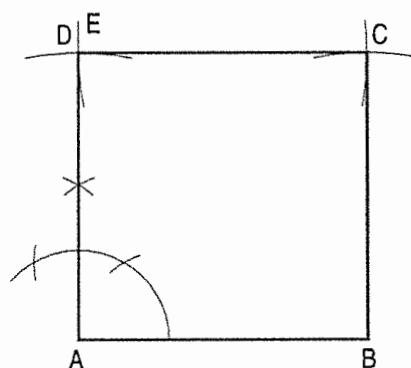


FIG. 5-35

- (b) *With the aid of a compass* (fig. 5-35).
- Draw a line AB equal to the given length.
 - At A , draw a line AE perpendicular to AB . (Refer problem 5-3, Method III, fig. 5-5).
 - With centre A and radius AB , draw an arc cutting AE at D .
 - With centres B and D and the same radius, draw arcs intersecting at C .
 - Draw lines joining C with B and D .
- Then $ABCD$ is the required square.

5-12. TO CONSTRUCT REGULAR POLYGONS



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 9 for the following problem.



Problem 5-26. *To construct a regular polygon, given the length of its side.*

Let the number of sides of the polygon be seven (i.e. heptagon).

Method I: (fig. 5-36 and fig. 5-37):

- Draw a line AB equal to the given length.
- With centre A and radius AB , draw a semi-circle BP .
- With a divider, divide the semi-circle into seven equal parts (same as the number of sides). Number the division-points as 1, 2, etc. starting from P .
- Draw a line joining A with the second division-point 2.

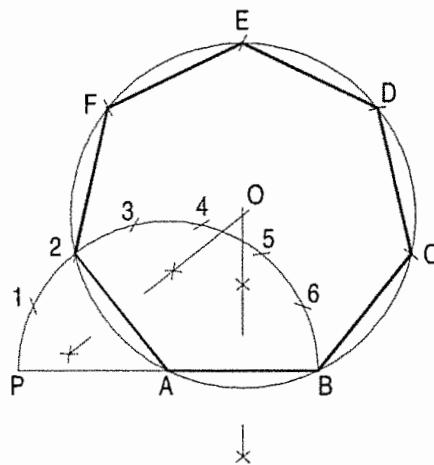


FIG. 5-36

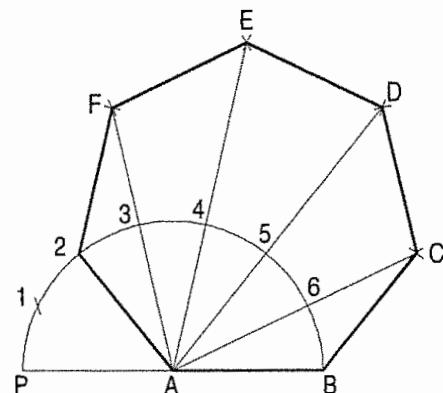


FIG. 5-37

(a) *Inscribe circle method* (fig. 5-36).

- Draw perpendicular bisectors of $A2$ and AB intersecting each other at O .
- With centre O and radius OA , describe a circle.
- With radius AB and starting from B , cut the circle at points, $C, D, \dots, 2$.
- Draw lines BC, CD etc. thus completing the required heptagon.

(b) *Arc method* (fig. 5-37).

- (i) With centre B and radius AB , draw an arc cutting the line $A6$ -produced at C .
- (ii) With centre C and the same radius, draw an arc cutting the line $A5$ -produced at D .
- (iii) Find points E and F in the same manner.
- (iv) Draw lines BC , CD etc. and complete the heptagon.

Method II: General method for drawing any polygon (fig. 5-38):

- (i) Draw a line AB equal to the given length.
 - (ii) At B , draw a line BP perpendicular and equal to AB .
 - (iii) Draw a line joining A with P .
 - (iv) With centre B and radius AB , draw the quadrant AP .
 - (v) Draw the perpendicular bisector of AB to intersect the straight line AP in 4 and the arc AP in 6.
- (a) A square of a side equal to AB can be inscribed in the circle drawn with centre 4 and radius $A4$.
 - (b) A regular hexagon of a side equal to AB can be inscribed in the circle drawn with centre 6 and radius $A6$.
 - (c) The mid-point 5 of the line 4-6 is the centre of the circle of the radius $A5$ in which a regular pentagon of a side equal to AB can be inscribed.
 - (d) To locate centre 7 for the regular heptagon of side AB , step-off a division 6-7 equal to the division 5-6.
 - (i) With centre 7 and radius equal to $A7$, draw a circle.
 - (ii) Starting from B , cut it in seven equal divisions with radius equal to AB .
 - (iii) Draw lines BC , CD etc. and complete the heptagon.

Regular polygons of any number of sides can be drawn by this method.

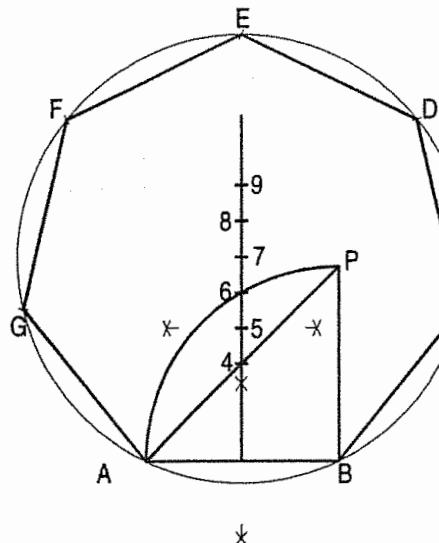


FIG. 5-38

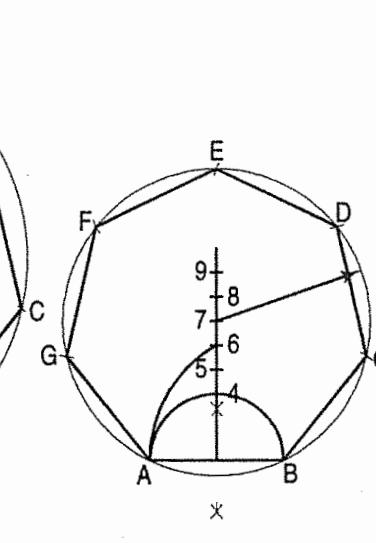


FIG. 5-39

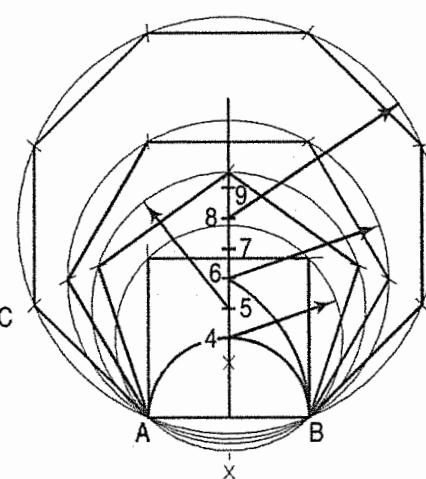


FIG. 5-40

Alternative method (fig. 5-39 and fig. 5-40):

- On AB as diameter, describe a semi-circle.
- With either A or B as centre and AB as radius, describe an arc on the same side as the semi-circle.
- Draw a perpendicular bisector of AB cutting the semi-circle at point 4 and the arc at point 6.
- Obtain points 5, 7, 8 etc. as explained in method II.

Fig. 5-40 shows a *square*, a *regular pentagon*, a *regular hexagon* and a *regular octagon*, all constructed on AB as a common side.

5-13. SPECIAL METHODS OF DRAWING REGULAR POLYGONS



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 10 for the following problem.

Problem 5-27. To construct a pentagon, length of a side given.

Method I: (fig. 5-41):

- Draw a line AB equal to the given length.
- With centre A and radius AB , describe a circle-1.
- With centre B and the same radius, describe a circle-2 cutting circle-1 at C and D .
- With centre C and the same radius, draw an arc to cut circle-1 and circle-2 at E and F respectively.
- Draw a perpendicular bisector of the line AB to cut the arc EF at G .
- Draw a line EG and produce it to cut circle-2 at P .
- Draw a line FG and produce it to cut circle-1 at R .
- With P and R as centres and AB as radius, draw arcs intersecting each other at Q .
- Draw lines BP , PQ , QR and RA , thus completing the pentagon.

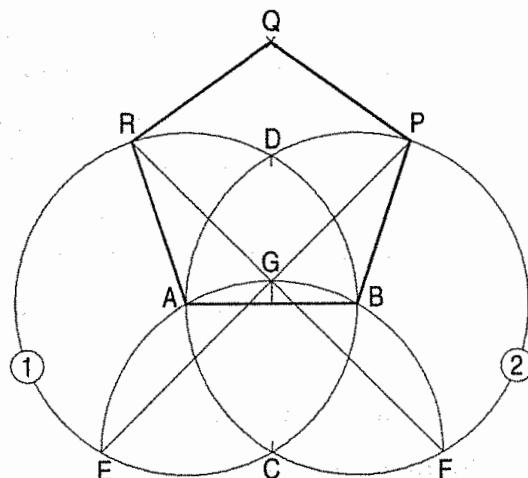


FIG. 5-41

Method II: (fig. 5-42):

- Draw a line AB equal to the given length.
- Bisect AB in a point C .
- Draw a line BD perpendicular and equal to AB .

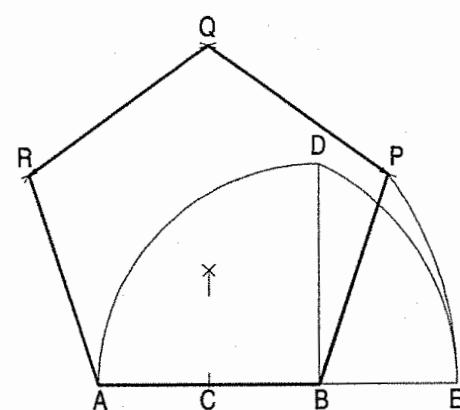


FIG. 5-42

- (iv) With centre C and radius CD , draw an arc to intersect the line AB -produced at E .
- (v) Then AE is the length of the diagonal of the pentagon.
- (vi) Therefore, with centre A and radius AB , draw an arc intersecting the arc drawn with centre B and radius AE at R .
- (vii) Again with centre A and radius AE , draw an arc intersecting the arc drawn with centre B and radius AB at P .
- (viii) With centres A and B and radius AE , draw arcs intersecting each other at Q .
- (ix) Draw lines BP , PQ , QR and RA , thus completing the pentagon.

Problem 5-28. To construct a hexagon, length of a side given (fig. 5-43 and fig. 5-44).

- (a) With T-square and 30° - 60° set-square only (fig. 5-43).
 - (i) Draw a line AB equal to the given length.
 - (ii) From A , draw lines A_1 and A_2 making 60° and 120° angles respectively with AB .
 - (iii) From B , draw lines B_3 and B_4 making 60° and 120° angles respectively with AB .
 - (iv) From O the point of intersection of A_1 and B_3 , draw a line parallel to AB and intersecting A_2 at F and B_4 at C .
 - (v) From F , draw a line parallel to BC and intersecting B_3 at E .
 - (vi) From C , draw a line parallel to AF and intersecting A_1 at D .
 - (vii) Draw a line joining E and D .

Then $ABCDEF$ is the required hexagon.

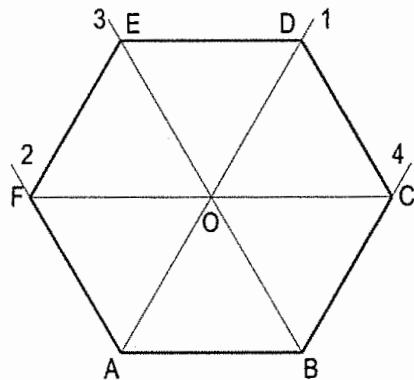


FIG. 5-43

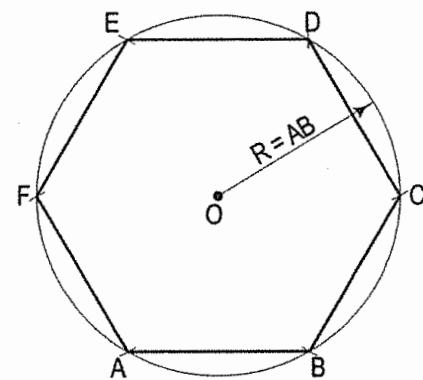


FIG. 5-44

- (b) With the aid of a compass (fig. 5-44).
 - (i) With Point O as centre, draw a circle of radius equal to the given side length of the required polygon.
 - (ii) Draw a horizontal line passing through the centre of the circle and cutting the circle at opposite ends, say at points F and C . Mark the centre of circle as O .

- (iii) Starting with either F or C as centre and side as length, go on marking the points on the circumference, A , B , D and E .
- (iv) Join points $A-B-C-D-E-F$. You will get the required Hexagon (6 sided polygon).



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 11 for the following problem.

Problem 5-29. To inscribe a regular octagon in a given square (fig. 5-45).

- (i) Draw the given square $ABCD$.
- (ii) Draw diagonals AC and BD intersecting each other at O .
- (iii) With centre A and radius AO , draw an arc cutting AB at 2 and AD at 7.
- (iv) Similarly, with centres B , C and D and the same radius, draw arcs and obtain points 1, 3, 4 etc. as shown.

Draw lines 2-3, 4-5, 6-7 and 8-1, thus completing the octagon.

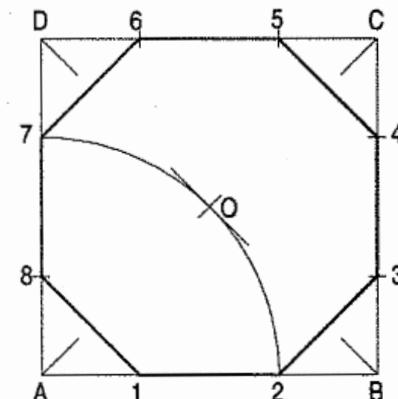


FIG. 5-45

5-14. REGULAR POLYGONS INSCRIBED IN CIRCLES

Problem 5-30. To inscribe a regular polygon of any number of sides, say 5, in a given circle (fig. 5-46).

- (i) With centre O , draw the given circle.
- (ii) Draw a diameter AB and divide it into five equal parts (same number of parts as the number of sides) and number them as shown.
- (iii) With centres A and B and radius AB , draw arcs intersecting each other at P .
- (iv) Draw a line P_2 and produce it to meet the circle at C . Then AC is the length of the side of the pentagon.
- (v) Starting from C , step-off on the circle, divisions CD , DE etc., equal to AC .
- (vi) Draw lines CD , DE etc., thus completing the pentagon.

Problem 5-31. To inscribe a square in a given circle (fig. 5-47).

- (i) With centre O , draw the given circle.
- (ii) Draw diameters AB and CD perpendicular to each other.
- (iii) Draw lines AC , CB , BD and DA , thus completing the square.

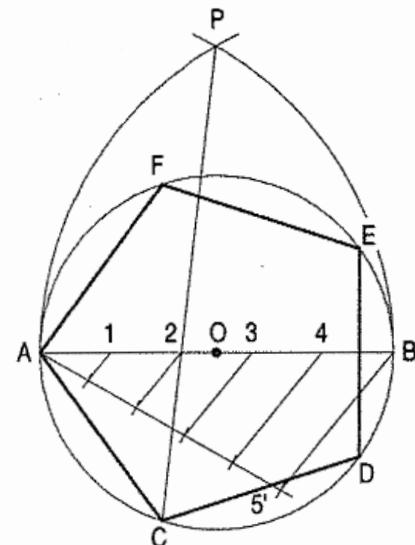


FIG. 5-46

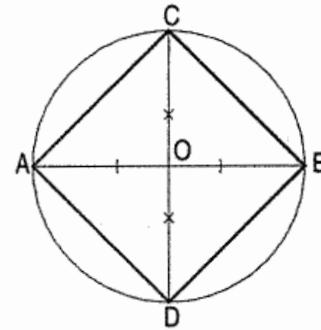


FIG. 5-47

Problem 5-32. To inscribe a regular pentagon in a given circle (fig. 5-48).

- (i) With centre O , draw the given circle.
- (ii) Draw diameters AB and CD perpendicular to each other.
- (iii) Bisect AO in a point P . With centre P and radius PC , draw an arc cutting OB in Q .
- (iv) With centre C and radius CQ , draw an arc cutting the circle in E and F .
- (v) With centres E and F and the same radius, draw arcs cutting the circle in G and H respectively.
- (vi) Draw lines CE , EG , GH , HF and FC , thus completing the required pentagon.

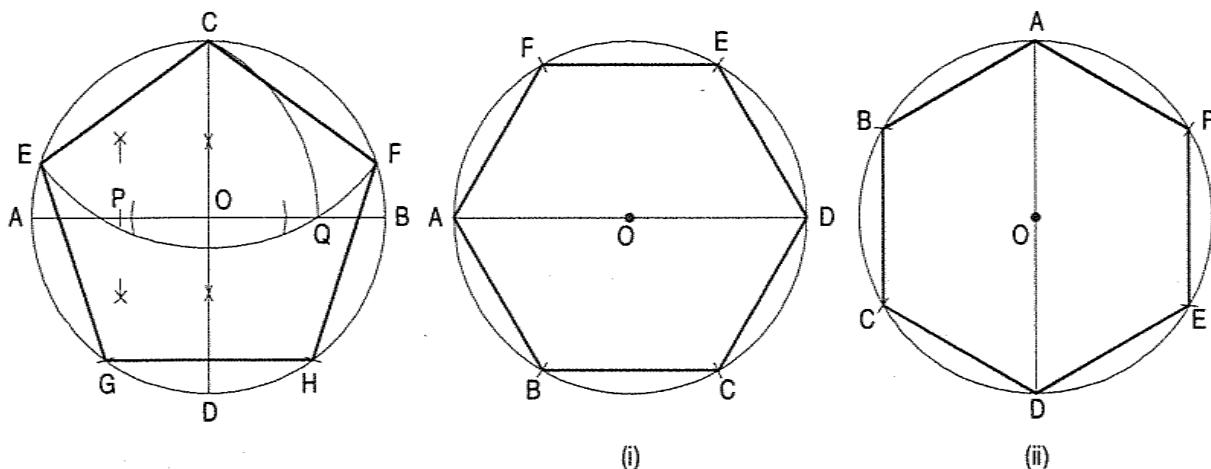


FIG. 5-48

FIG. 5-49

Problem 5-33. To inscribe a regular hexagon in a given circle (fig. 5-49).

Apply the same method as shown in Problem 5-28(b).

- Note:* (a) When two sides of the hexagon are required to be horizontal the starting point for stepping-off equal divisions should be on an end of the horizontal diameter.
 (b) If they are to be vertical, the starting point should be on an end of the vertical diameter.

In either case, to avoid inaccuracy, the points should be joined with the aid of T-square and 30° - 60° set-square.

Problem 5-34. To inscribe a regular heptagon in a given circle (fig. 5-50).

- (i) With centre O , draw the given circle.
- (ii) Draw a diameter AB . With centre A and radius AO , draw an arc cutting the circle at E and F .
- (iii) Draw a line EF , cutting AO in G .

Then EG or FG is the length of the side of the heptagon.

Therefore, from any point on the circle, say A , step-off divisions equal to EG , around the circle. Join the division-points and obtain the heptagon.

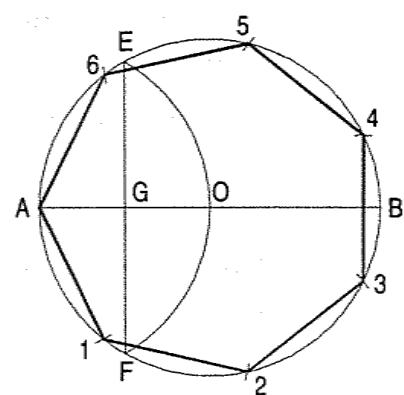


FIG. 5-50

Problem 5-35. To inscribe a regular octagon in a given circle (fig. 5-51).

- With centre O , draw the given circle.
- Draw diameters AB and CD at right angles to each other.
- Draw diameters EF and GH bisecting angles AOC and COB .
- Draw lines AE , EC etc. and complete the octagon.

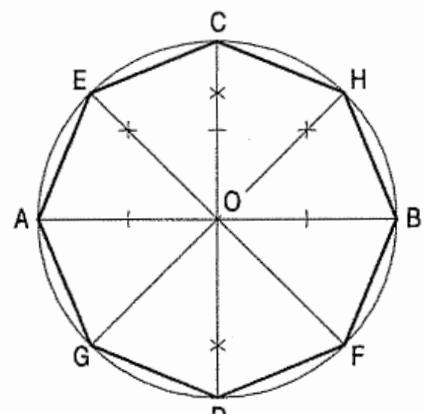


FIG. 5-51

5-15. TO DRAW REGULAR FIGURES USING T-SQUARE AND SET-SQUARES

Problem 5-36. To describe an equilateral triangle about a given circle (fig. 5-52).

- With centre O , draw the given circle.
- Draw a vertical radius OA .
- Draw radii OB and OC with a 30° - 60° set-square, such that $\angle AOB = \angle AOC = 120^\circ$.
- At A , B and C , draw tangents to the circle, i.e. a horizontal line EF through A , and lines FC and GE through B and C respectively with a 30° - 60° set-square.

Then EFG is the required triangle.

Problem 5-37. To draw a square about a given circle (fig. 5-53).

- With centre O , describe the given circle.
- Draw diameters AB and CD at right angles to each other as shown.
- At A and B , draw vertical lines, and at C and D , draw horizontal lines intersecting at E , F , G and H .

$EFGH$ is the required square.

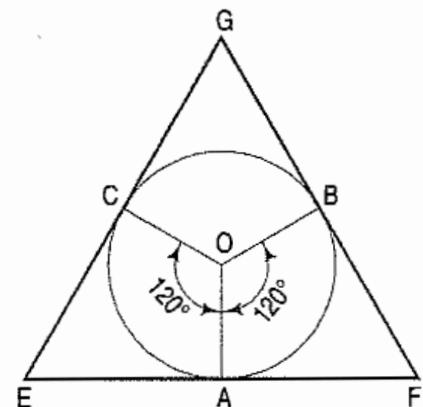


FIG. 5-52

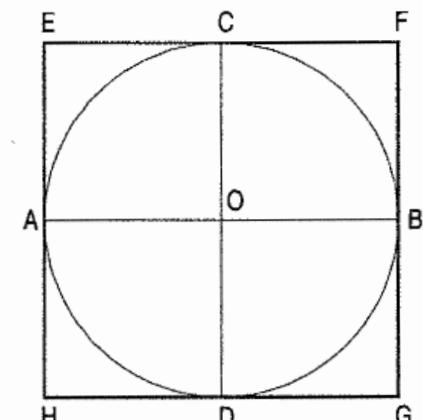


FIG. 5-53

Problem 5-38. To describe a regular hexagon about a given circle (fig. 5-54).

- With centre O , draw the given circle.
- Draw horizontal diameter AB , and diameters CD and EF making 60° angle with AB .
- Draw tangents at all the six ends, i.e. verticals at A and B , and lines with a 30° - 60° set-square at the remaining points intersecting at 1, 2, ..., 6.

A hexagon with two sides horizontal can be drawn by drawing a vertical diameter AB and the other lines as shown in fig. 5-55.

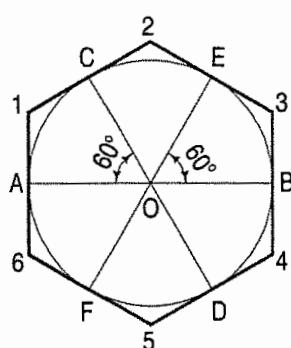


FIG. 5-54

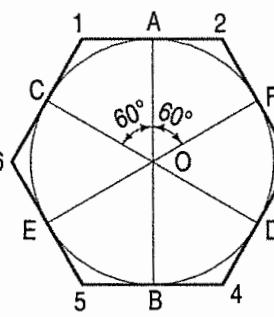


FIG. 5-55

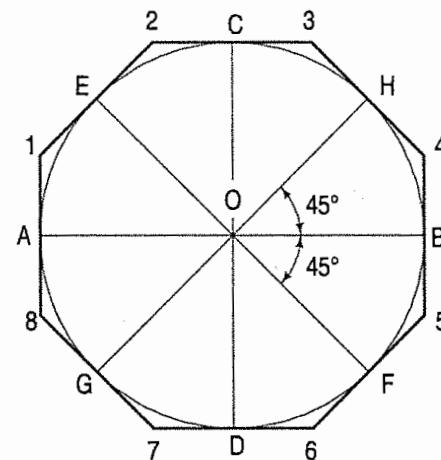


FIG. 5-56

Problem 5-39. To describe a regular octagon about a given circle (fig. 5-56).

- With centre O , describe the given circle.
- Draw a horizontal diameter AB , a vertical diameter CD and diameters EF and GH at 45° to the first two.
- Draw tangents at the eight points A, B, \dots, H intersecting one another at $1, 2, \dots, 8$. Then $1, 2, \dots, 8$ is the required octagon.

5-16. TO DRAW TANGENTS

Problem 5-40. (fig. 5-57): To draw a tangent to a given circle at any point on it.

- With centre O , draw the given circle and mark a point P on it.
- Draw a line joining O and P .
- Produce OP to Q so that $PQ = OP$.
- With centres O and Q and with any convenient radius, draw arcs intersecting each other at R .
- Draw a line through P and R . Then this line is the required tangent.

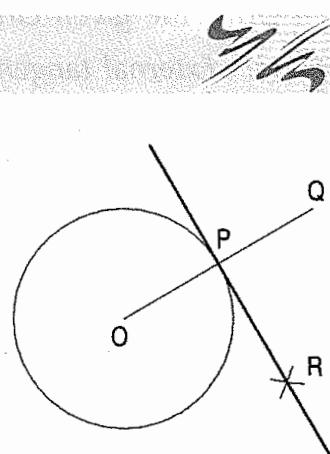


FIG. 5-57

Problem 5-41. (fig. 5-58): To draw a tangent to a given circle from any point outside it.

- With centre O , draw the given circle.
- Mark a point P outside it.
- Draw a line joining O and P .
- With OP as diameter, draw a semi-circle cutting the given circle at R and R_1 .
- Draw a line through P and R . Then this line is the required tangent. The line through P and R_1 is the other tangent which can be drawn from the same point.

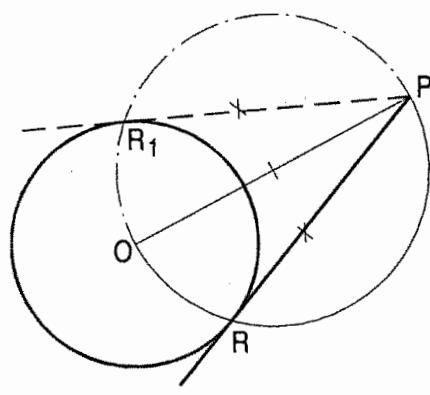


FIG. 5-58

Problem 5-42. (fig. 5-59): To draw a tangent to a given arc of inaccessible centre at any point on it.

Let AB be the given arc and P the point on it.

- With centre P and any radius, draw arcs cutting the arc AB at C and D . Draw EF , the bisector of the arc CD . It will pass through P .
- Through P , draw a line RS perpendicular to EF . RS is the required tangent.

Problem 5-43. (fig. 5-60): To draw a tangent to a given circle and parallel to a given line.

The circle with centre O and the line AB are given.

- From O , draw a line perpendicular to AB and cutting the circle at a point P or Q .
- Through P or Q , draw the required tangent CD or $C_1 D_1$ (problem 5-40).

Problem 5-44. To draw a common tangent to two given circles of equal radii (fig. 5-61).

Draw the given circles with centres O and P .

- (a) *External tangents* (fig. 5-61):

- Draw a line joining O and P .
- At O and P , erect perpendiculars to OP on the same side of it and intersecting the circles at A and B .
- Draw a line through A and B . This line is the required tangent. $A_1 B_1$ is the other tangent.

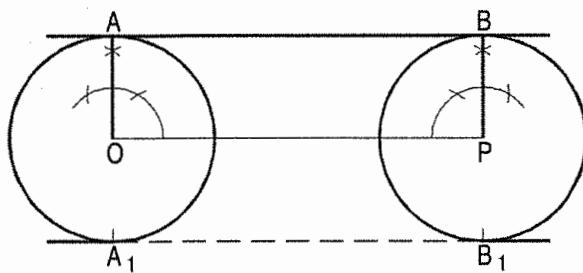


FIG. 5-61

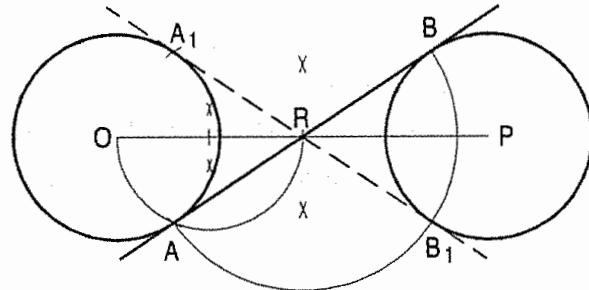


FIG. 5-62

- (b) *Internal tangents* (fig. 5-62):

- Draw a line joining O and P .
- Bisect OP in R . Draw a semi-circle with OR as diameter to cut the circle at A .
- With centre R and radius RA , draw an arc to intersect the other circle on the other side of OP at B .
- Draw a line through A and B . This line is the required tangent.

The other tangent through A_1 and B_1 can also be similarly drawn.

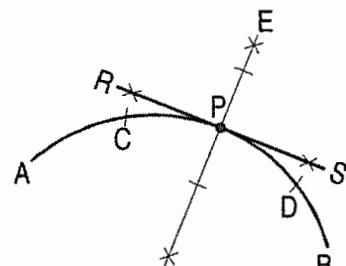


FIG. 5-59

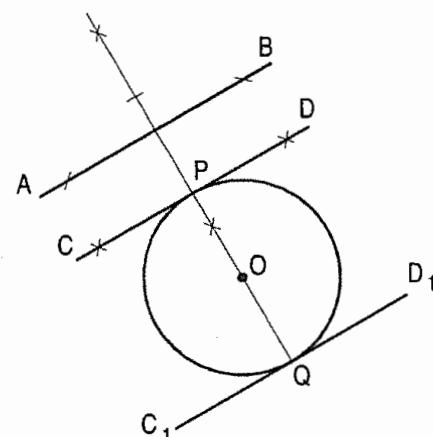


FIG. 5-60

Problem 5-45. To draw a common tangent to two given circles of unequal radii (fig. 5-63 and fig. 5-64).

Draw the given circles with centres O and P , and radii R_1 and R_2 respectively, of which R_1 is greater than R_2 .

(a) *External tangents* (fig. 5-63):

- Draw a line joining centres O and P .
- With centre O and radius equal to $(R_1 - R_2)$, draw a circle.
- From P , draw a tangent PT to this circle (Problem 5-41).
- Draw a line OT and produce it to cut the outer circle at A .
- Through P , draw a line PB parallel to OA , on the same side of OP and cutting the circle at B .
- Draw a line through A and B . Then this line is the required tangent.

The other similar tangent will pass through A_1 and B_1 .

(b) *Internal tangents* (fig. 5-64):

- Draw a line joining the centres O and P .
- With centre O and radius equal to $(R_1 + R_2)$, draw a circle.
- From P , draw a line PT tangent to this circle.
- Draw a line OT cutting the circle at A .
- Through P , draw a line PB parallel to OA , on the other side of OP and cutting the circle at B .
- Draw a line through A and B . Then this line is the required tangent. The second tangent will pass through A_1 and B_1 .

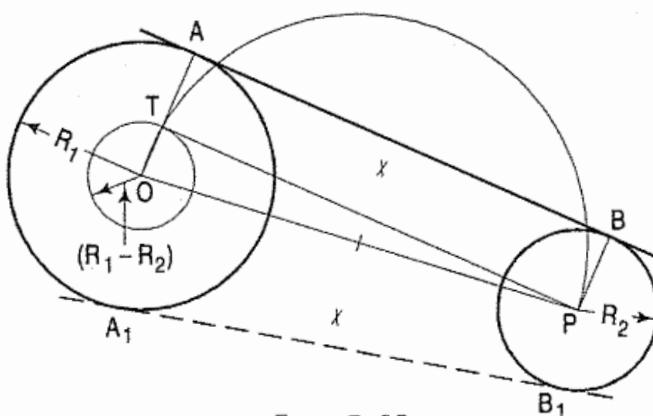


FIG. 5-63

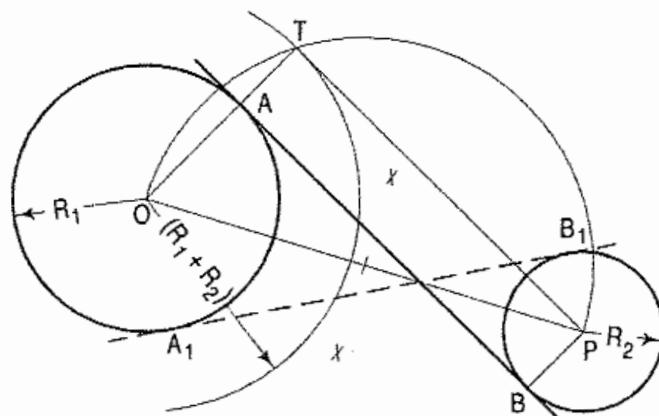


FIG. 5-64

5-17. LENGTHS OF ARCS



Problem 5-46. To determine the length of a given arc (fig. 5-65).

Let AB be the given arc drawn with centre O .

- At A , draw a tangent to the arc.
- Draw the chord AB and produce it beyond A to a point C such that $AC = \frac{1}{2} AB$.
- With centre C and radius equal to CB , describe an arc cutting the tangent at D .
- Then the length AD is approximately equal to the length of the arc AB .

This method is satisfactory for arcs which subtend at the centre, angles smaller than 60° .

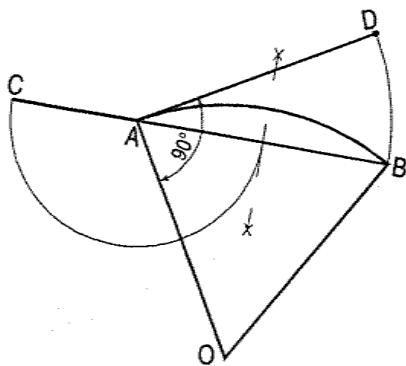


FIG. 5-65

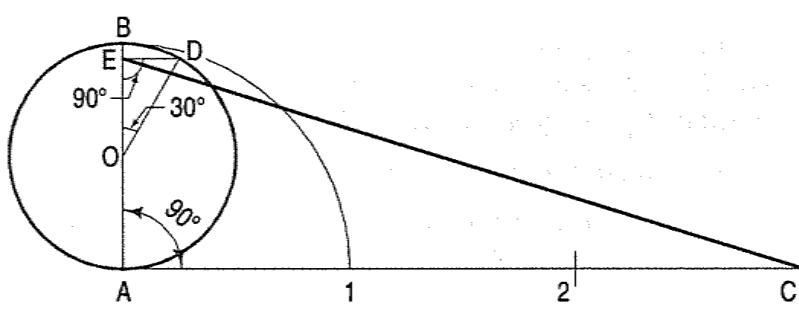


FIG. 5-66

Problem 5-47. To determine the length of the circumference of a given circle (fig. 5-66).

Let the circle with centre O be given.

- Draw a diameter AB.
- At A, draw a tangent AC equal to 3 times AB.
- Draw a radius OD making an angle of 30° with OB.
- From D, draw a line DE perpendicular to OB.
- Draw a line joining E and C. Then EC is approximately equal in length to the circumference of the circle.

5-18. CIRCLES AND LINES IN CONTACT

Problem 5-48. To draw a circle passing through a given point and tangent to a given line at a given point on it (fig. 5-67).

A point P and a line AB with a point Q in it are given. At Q, draw a line perpendicular to AB.

- Draw a line joining P and Q.
- Draw a perpendicular bisector of PQ to intersect the perpendicular from Q at O.
- With centre O and radius OP or OQ, draw the required circle.

Problem 5-49. To draw a circle passing through a given point and touching a given circle at a given point on it (fig. 5-68).

A point P, a circle with centre A and a point Q on the circle are given.

- Draw a line joining P and Q.
- Draw a perpendicular bisector of PQ, to intersect the line through A and Q at O.
- With centre O and radius OP, draw the required circle.

The required circle includes the given circle when the point is in a position such as P'.

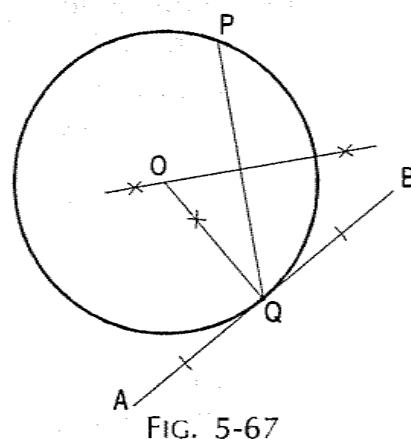


FIG. 5-67

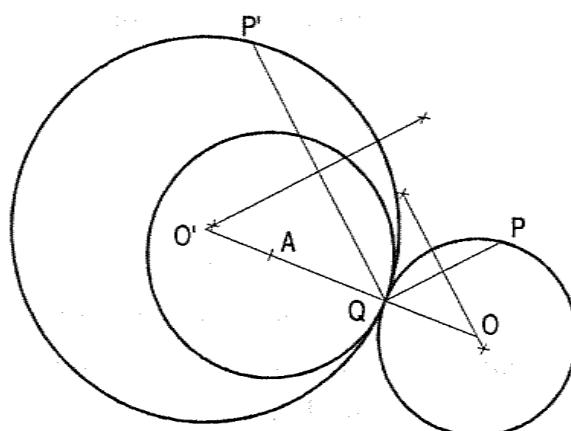


FIG. 5-68

Problem 5-50. To draw a circle to touch a given line and a given circle at a given point on it (fig. 5-69).

A line AB , a circle with centre C and a point P on the circle are given.

From P , draw a tangent to the circle intersecting AB in D .

- Draw a bisector of $\angle PDB$, to intersect the line through C and P at O . With centre O and radius OP , draw the required circle.
- Draw a bisector of $\angle PDA$ to meet the line through C and P at O' . Then O' is the centre of another circle which will include the given circle within it.

Problem 5-51. To draw a circle to touch a given circle and a given line at a given point on it (fig. 5-70).

A circle with centre C and a line AB with a point P in it are given.

Through C , draw a line perpendicular to AB and cutting the circle in E or F .

- Draw a line joining P and F and intersecting the circle at G . At P , draw a perpendicular to AB intersecting the line through C and G at O . With centre O and radius OP , draw the required circle.
- Draw a line through P and E and obtain centre O' for another circle in the same manner. It will include the given circle within it.

Problem 5-52. To draw a circle touching two given circles, one of them at a given point on it (fig. 5-71).

Circles with centres A and B , and a point P on the circle A are given (fig. 5-71).

- Draw a line joining A and P .
- Through B , draw a line parallel to AP and intersecting the circle in C .
- Draw a line PC and produce it (if necessary) to cut the circle (with centre B) in D .
- Draw a line through D and B to intersect AP or AP -produced, at O .
- With centre O and radius OP , draw the required circle.

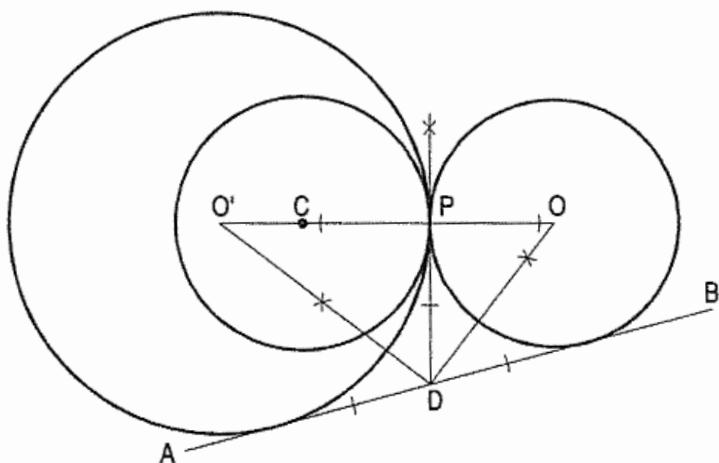


FIG. 5-69

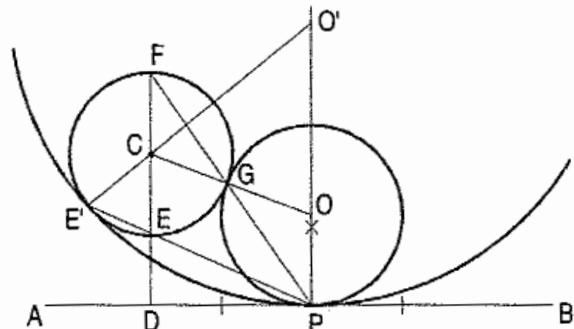


FIG. 5-70

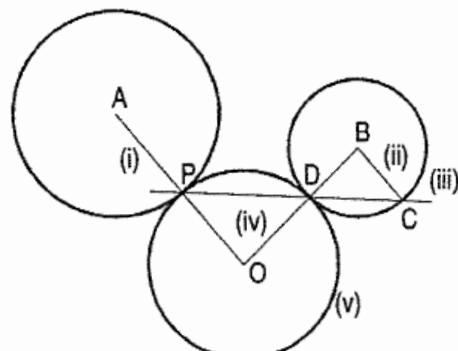


FIG. 5-71

Problem 5-53. To draw a circle touching two given circles, one of them at a given point on it [fig. 5-72].

- (i) Circles with centres A and B , and a point P on the circle A are given.
- (ii) Draw a line joining centre A and the point P .
- (iii) Through B , draw a line parallel to AP (if extended) and intersecting circle B in C'' and D' .
- (iv) Join PC'' and extend to intersect circle B at D'' .
- (v) Draw a line through D'' and B to intersect the line AP at O'' .
- (vi) Join PD' which intersect circle B at C' . Join $C'B$ and extend to intersect AP at O' . Draw a circle with O' as centre and $O'P$ as radius. It is the circle (1).
- (vii) Draw another circle with centre as O'' and the radius $O''P$. It is the circle (2).

Circle-(1) which includes one of the given circles, and circle-(2) which includes both of them (fig. 5-72).

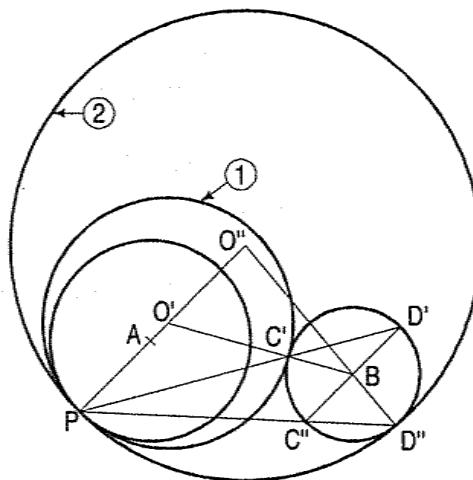


FIG. 5-72

5-19. INSCRIBED CIRCLES

Problem 5-54. To inscribe a circle in given triangle (fig. 5-73).

Let ABC be the triangle.

- (i) Bisect any two angles by lines intersecting each other at O .
- (ii) Draw a perpendicular from O to any one side of the triangle, meeting it at P .
- (iii) With centre O and radius OP , describe the required circle.

Problem 5-55. To draw a circle touching three lines inclined to each other but not forming a triangle (fig. 5-74).

Let AB , BC and AD be the given lines.

- (i) Draw bisectors of the two angles intersecting each other at O .
- (ii) From O , draw a perpendicular to any one line intersecting it at P .
- (iii) With centre O and radius OP , draw the required circle.

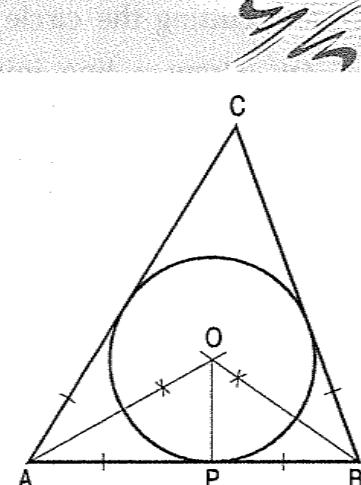


FIG. 5-73

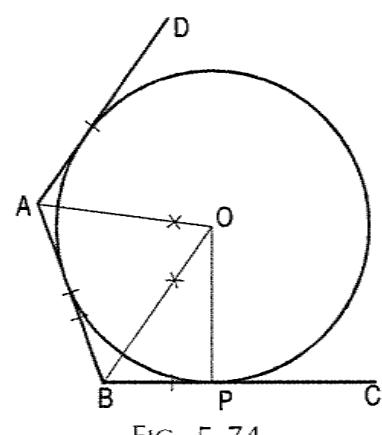


FIG. 5-74

Problem 5-56. To inscribe a circle in a regular polygon of any number of sides, say a pentagon (fig. 5-75).

Let $ABCDE$ be the pentagon.

- Bisect any two angles by lines intersecting each other at O .
- From O , draw a perpendicular to any one side of the pentagon cutting it at P .
- With centre O and radius OP , draw the required circle.

Problem 5-57. To draw in a regular polygon, the same number of equal circles as the sides of the polygon, each circle touching one side of the polygon and two of the other circles (fig. 5-76).

- Let $ABCD$ be the given square.
- Draw bisectors of all the angles of the square. They will meet at O , thus dividing the square into four equal triangles.

In each triangle inscribe a circle (Problem 5-54). Each circle will touch a side of the square and two other circles as required.

Fig. 5-77 shows five equal circles inscribed in a regular pentagon in the same manner.

Problem 5-58. To draw in a regular polygon, the same number of equal circles as the sides of the polygon, each circle touching two adjacent sides of the polygon and two of the other circles (fig. 5-78).

Let $ABCDEF$ be the given hexagon.

- Draw the perpendicular bisectors of all sides of the hexagon. They will meet at O and will divide the hexagon into six equal quadrilaterals.
- Inscribe a circle in each quadrilateral as shown in case of $A_1 O_2$ and as explained below.
- Bisect any two adjacent angles with bisectors intersecting each other at P .
- From P , draw a perpendicular to any one side of the quadrilateral, meeting it at Q . With centre P and radius PQ , draw one of the required circles.
- Draw other circles in the same manner.

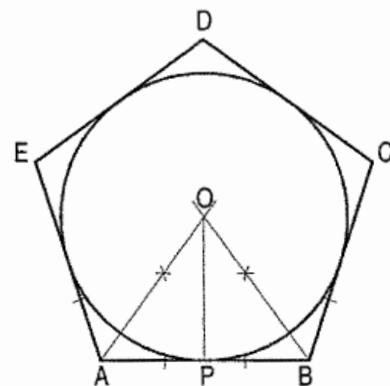


FIG. 5-75

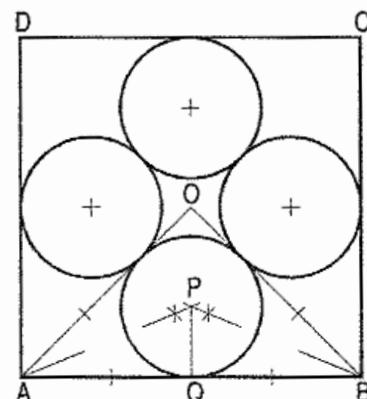


FIG. 5-76

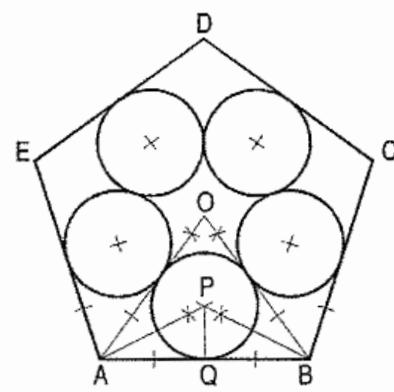


FIG. 5-77

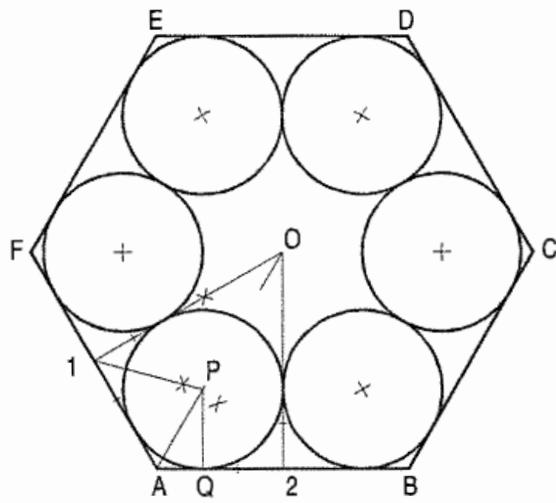


FIG. 5-78

Problem 5-59. To draw in a given regular hexagon, three equal circles, each touching one side and two other circles (fig. 5-79).

- Draw the given hexagon.
- Draw perpendicular bisectors of its two alternate sides, to intersect each other at O and to meet the middle side produced on both sides at 1 and 2.
- Inscribe a circle in triangle $O 1 2$. Similarly, draw the other two required circles.

Problem 5-60. To draw in a given circle, any number of equal circles, say four, each touching the given circle and two of the other circles (fig. 5-80).

- Divide the given circle into four equal parts by diameters AB and CD .
- Draw a tangent to the circle at D . Draw lines bisecting $\angle AOD$ and $\angle BOD$ and meeting the tangent at 1 and 2.
- Inscribe a circle in the triangle $O 1 2$.

Draw the other circles in the same manner. The centres for the remaining circles may also be determined by drawing a circle with centre O and radius OP to cut the diameters at the required points.

Problem 5-61. To draw outside a given regular polygon, the same number of equal circles as the sides of the polygon, each circle touching one side and two of the other circles (fig. 5-81).

- Let $ABCDE$ be the given pentagon.
- Draw bisectors of two adjacent angles, say $\angle A$ and $\angle B$, and produce them outside the pentagon.
- Draw a circle touching the extended bisectors and the side AB (Problem 5-55). Obtain the other four required circles in the same manner.

Problem 5-62. To draw outside a given circle any number of equal circles, say six, each touching the given circle and two other circles (fig. 5-82).

- Draw the given circle and describe a regular hexagon about it.
- Draw the required six equal circles outside the hexagon as shown in the previous problem.

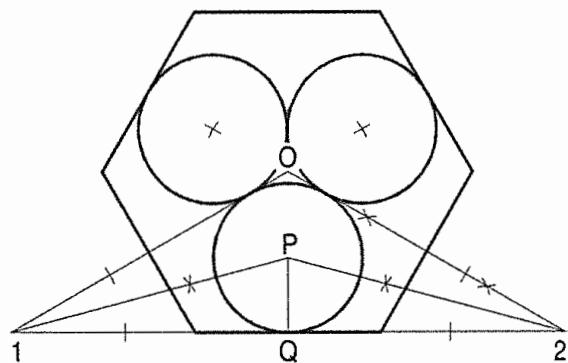


FIG. 5-79

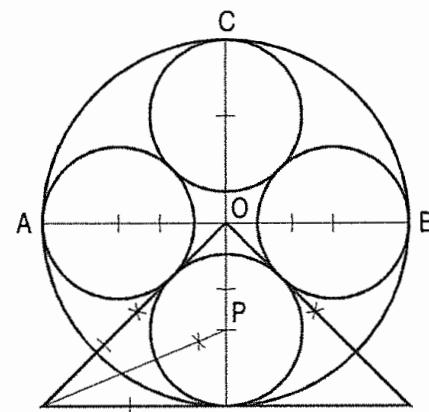


FIG. 5-80

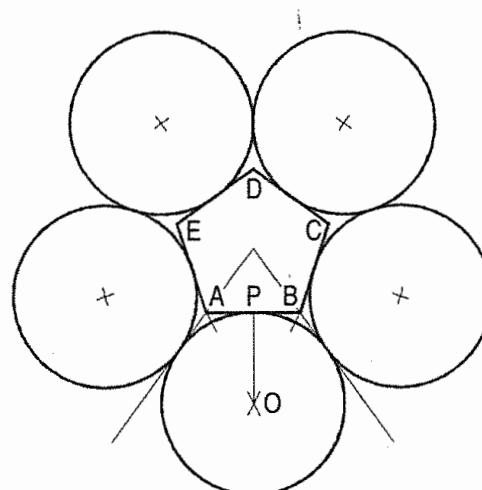


FIG. 5-81

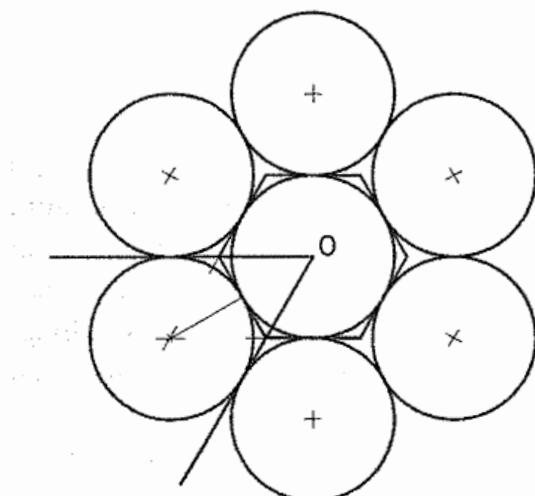


FIG. 5-82

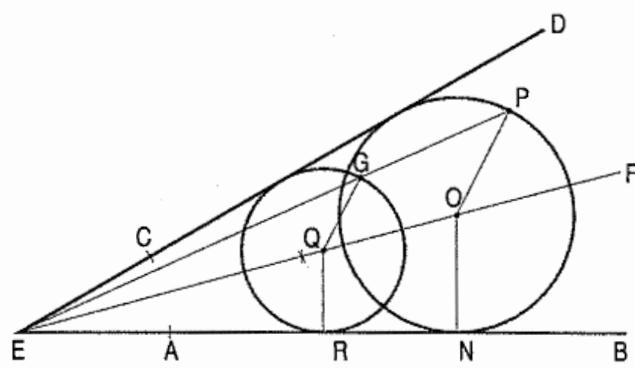


FIG. 5-83

Problem 5-63. To draw a circle touching two converging lines and passing through a given point between them (fig. 5-83).

- (i) Lines AB and CD, and the point P are given.
- (ii) Produce lines AB and CD to intersect at a point E. Draw the bisector EF of $\angle AEC$.
- (iii) Mark any point Q on EF and from it draw a perpendicular QR on AB.
- (iv) With Q as centre and QR as radius draw a circle which will touch the line CD also.
- (v) Draw a line joining P with E, cutting the circle at a point G.
- (vi) Draw the line QG.
- (vii) From P, draw a line parallel to QG intersecting EF at a point O.
- (viii) From O, draw a perpendicular ON to either AB or CD.
- (ix) With O as centre and ON as radius, draw the required circle.

Problem 5-64. To draw two circles touching each other and two converging lines, the smaller circle being of given radius (fig. 5-84).

- (i) Lines AB and CD and radius R of the smaller circle are given.
- (ii) Produce lines AB and CD to intersect at a point E. Draw the bisector EF of $\angle AEC$.
- (iii) Draw a line parallel to and at distance R from AB to intersect EF in a point Q.
- (iv) Draw the perpendicular QP on the line AB.
- (v) With Q as centre and QP as radius, draw the smaller circle.
- (vi) Mark points T and N at which the circle cuts EF.
- (vii) Draw the line joining T with P. Draw a line NS parallel to TP intersecting AB in the point S.

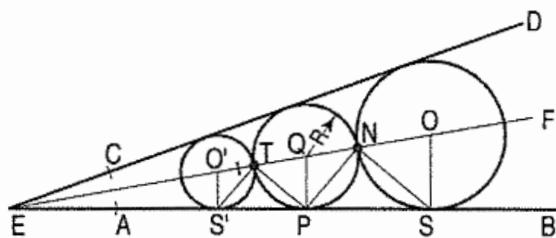


FIG. 5-84

From S, draw the perpendicular to AB cutting EF in the point O. With O as centre and OS as radius, draw the required circle.

O' is the centre of smaller circle, obtained in the same manner, touching the two given lines and the given circle.

EXERCISES 5

1. Draw a line 125 mm long and quadrisection it.
2. Draw a line AB 80 mm long and divide it into five parts, one of them 20 mm long and the remaining each 15 mm long, by the method of bisection.
3. With centre O and radius equal to 50 mm, draw two arcs of any lengths on opposite sides of O . Bisect the two arcs and produce the bisectors till they meet.
4. Draw a line AB 75 mm long. At B , erect a perpendicular BC 100 mm long. Draw a line joining A and C , and measure its length. Construct a square on each line as a side.
5. Draw a line PQ 100 mm long. At any point O in it near its centre, erect a perpendicular OA 65 mm long. Through A , draw a line parallel to PQ .
6. Mark any point O . Draw a line AB , such that its shortest distance from O is 50 mm.
7. Draw a line AB 75 mm long. Mark a point C , 65 mm from A and 90 mm from B . Join C with A and B . Through the points A , B and C , draw lines (i) perpendicular and (ii) parallel to their opposite lines.
7. Construct a rectangle of sides 65 mm and 40 mm long.
9. Construct a square of 75 mm side. Draw the diagonals intersecting at O . From O , draw lines perpendicular to the sides of the square.
10. Draw a circle of 50 mm radius. Divide it (i) into 8 equal parts by continued bisection and (ii) into 12 equal parts by bisection of a line and trisection of a right angle methods.
11. Draw two lines AB and AC making an angle of 75° . Draw a circle of 25 mm radius touching them.
12. Construct a right angle PQR . Describe a circle of 20 mm radius touching the sides PQ and QR .
13. Draw a line AB of any length. Mark a point O at a distance of 25 mm from AB . With O as centre, draw a circle of 40 mm diameter. Describe another circle (i) of 20 mm radius, touching the circle and AB ; (ii) of 35 mm radius, touching AB and the circle, and including the circle within it.
14. Draw two circles of 20 mm and 30 mm radii respectively with centres 65 mm apart. (i) Describe a third circle of 50 mm radius touching the two circles and (a) outside them; (b) including 20 mm circle; (c) including 30 mm circle. (ii) Describe a circle of 75 mm radius, touching both circles and including both of them within it.
15. Mark points A and B , 50 mm apart. Mark a third point 75 mm from both A and B . Describe a circle passing through the three points.
16. Draw the machine handle shown in fig. 5-85. All dimensions are in millimetres.

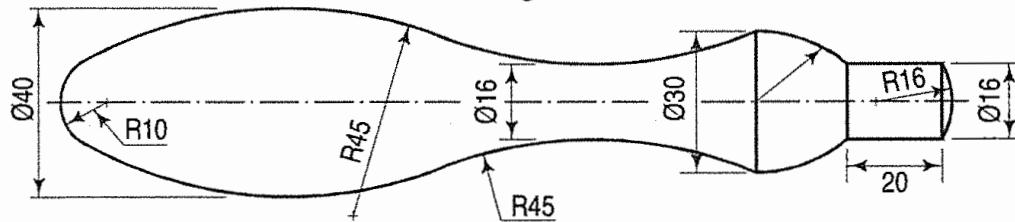


FIG. 5-85

17. The distance between the centres of two circles of 65 mm and 90 mm diameters is 120 mm. Draw an internal and an external common tangent to the two circles.
18. Draw a circle with centre O and radius equal to 30 mm. From a point P , 75 mm from O , draw a line joining P and O , and produce it to cut the circle at Q . From P and Q draw tangents to the circle.
19. Two shafts carry pulleys of 900 mm and 1350 mm diameters respectively. The distance between their centres is 2700 mm. Draw the arrangement showing the two pulleys connected by (i) direct belt (ii) crossed belt. Take 1 mm = 20 mm.
20. An arc AB drawn with 50 mm radius subtends an angle of 45° at the centre. Determine approximately the length of AB .
21. Determine the length of the circumference of a 75 mm diameter circle.
22. A point P is 25 mm from a line AB . Q is a point in AB and is 50 mm from P . Draw a circle passing through P and touching AB at Q .
23. Construct an equilateral triangle ABC of 40 mm side. Construct a square, a regular pentagon and a regular hexagon on its sides AB , BC and CA respectively.
24. The centre O of a circle of 30 mm diameter is 25 mm from a line AB . Draw a circle (i) to touch the given circle and the line AB at a point P , 50 mm from O ; (ii) to touch AB and the given circle at a point Q , 20 mm from AB .
25. Two circles of 40 mm and 50 mm diameters have their centres 60 mm apart. Draw a circle to touch both circles and (i) to include the bigger circle, the point of contact on it being 75 mm from the centre of the other circle; (ii) to include both the circles, the point of contact being the same as in (i).
26. Construct a regular pentagon of 30 mm side by three different methods.
27. On a line AB 40 mm long, construct a regular heptagon by two different methods.
28. Construct a regular octagon of 40 mm side. Inscribe another octagon with its corners on the mid-points of the sides of the first octagon.
29. Construct the following regular polygons in circles of 100 mm diameter, using a different method in each case: (i) Pentagon (ii) Heptagon.
30. Draw the following regular figures, the distance between their opposite sides being 75 mm: (i) Square; (ii) Hexagon; (iii) Octagon.
31. Construct a regular octagon in a square of 75 mm side.
32. Describe a regular pentagon about a circle of 100 mm diameter.
33. Construct a triangle having sides 25 mm, 30 mm and 40 mm long. Draw three circles, each touching one of the sides and the other two sides produced.
34. Inscribe a circle in a triangle having sides 50 mm, 65 mm and 75 mm long.
35. Construct a regular heptagon of 25 mm side and inscribe a circle in it.
36. Construct a regular hexagon of 40 mm side and draw in it, six equal circles, each touching one side of the hexagon and two other circles.
37. Construct a square of 50 mm side and draw in it, four equal circles, each touching two adjacent sides and two other circles.
38. In a regular octagon of 40 mm side, draw four equal circles, each touching one side of the octagon and two other circles.
39. Draw a circle of 125 mm diameter and draw in it, five equal circles, each touching the given circle and two other circles.

40. Construct a square of 25 mm side. Draw outside it four equal circles, each touching a side of the square and two other circles.
41. Outside a circle of 25 mm diameter, draw five equal circles, each touching the given circle and two other circles.
42. Two lines converge to a point making an angle of 30° between them. Draw three circles to touch both these lines, the middle circle being of 25 mm radius and touching the other two circles.
43. Two lines converge to a point making an angle of 30° between them. A point P is between these lines 15 mm from one line and 25 mm from the other. Draw a circle to touch both the lines and pass through P .
44. Draw a series of four circles, each touching the preceding circle and two converging lines which make an angle of 25° between them. Take the radius of the smallest circle as 10 mm.
45. A vertical straight line AB is at a distance of 90 mm from the centre of a circle of 75 mm diameter. A straight line PQ passes through the centre of the circle and makes an angle of 60° with the vertical. Draw circles having their centres on PQ and to touch the straight line AB and the circle. Measure the radius of each circle.
46. Draw a semi-circle of 125 mm diameter and inscribe in it the largest equilateral triangle having a corner at the centre. The semi-circle is the development of a cone and the triangle that of a line on its surface. Draw the projections of the cone resting on its base on the ground showing the line in both views.
47. Construct a lever as shown in fig. 5-86.

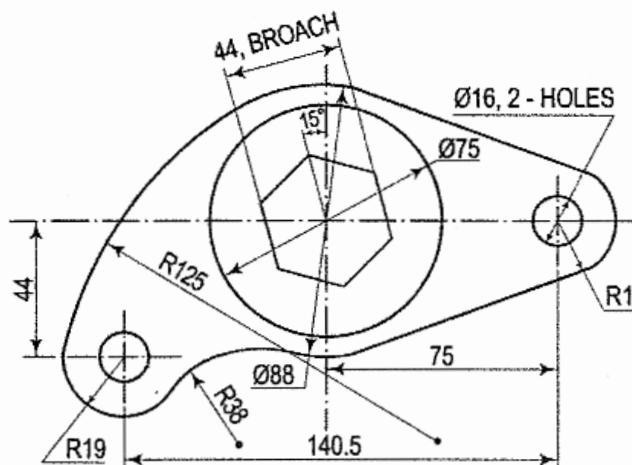


FIG. 5-86

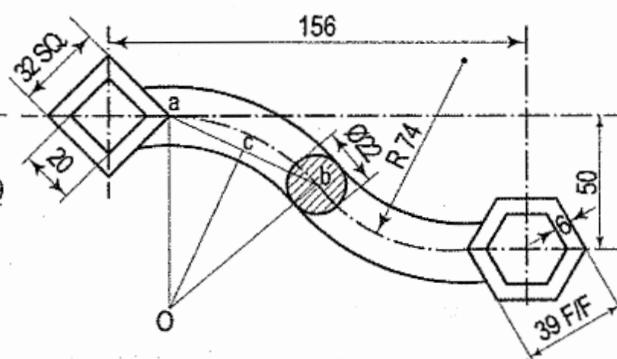


FIG. 5-87

48. Construct a special spanner as shown in fig. 5-87.
 [Hint: The method of drawing curve is shown on left-hand side. The same is to be followed on the right-hand side. This curve is known as ogee (reverse) curve.]
49. Two shafts, 1200 mm apart are connected by flat belt. The flat belt pulleys of 300 mm diameter and 600 mm diameter are fixed on the shafts. Draw the arrangement and determine approximately length of the belt.
50. Draw plan-view of a hexagonal nut of 20 mm using standard dimensions.
51. (i) Draw a number '8' of height 105 mm and 15 mm thick.
 (ii) Draw an alphabet 'S' of height 105 mm and 15 mm thick.

Chapter

6



CURVES USED IN ENGINEERING PRACTICE

6-0. INTRODUCTION

The profile of number of objects consists of various types of curves. This chapter deals with various types of curves which are commonly used in engineering practice as shown below:

- 1. Conic sections
- 2. Cycloidal curves
- 3. Involute
- 4. Evolutes
- 5. Spirals
- 6. Helix.

We shall now discuss the above in details with reference to their construction and applications.

6-1. CONIC SECTIONS

The sections obtained by the intersection of a right circular cone by a plane in different positions relative to the axis of the cone are called *conics*. Refer to fig. 6-1.

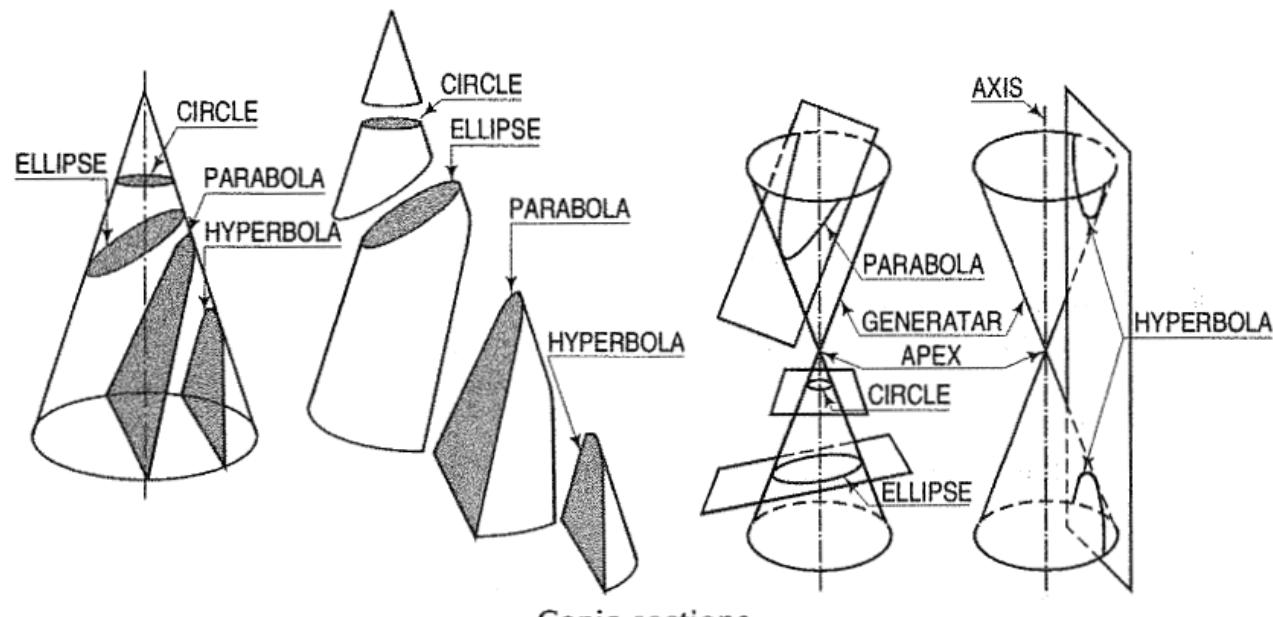


FIG. 6-1

- (i) When the section plane is inclined to the axis and cuts all the generators on one side of the apex, the section is an *ellipse* [fig. 6-1].

- (ii) When the section plane is inclined to the axis and is parallel to one of the generators, the section is a *parabola* [fig. 6-1].
- (iii) A *hyperbola* is a plane curve having two separate parts or branches, formed when two cones that point towards one another are intersected by a plane that is parallel to the axes of the cones.

The conic may be defined as the locus of a point moving in a plane in such a way that the ratio of its distances from a fixed point and a fixed straight line is always constant. The fixed point is called the *focus* and the fixed line, the *directrix*.

The ratio $\frac{\text{distance of the point from the focus}}{\text{distance of the point from the directrix}}$ is called *eccentricity* and is denoted by e . It is always less than 1 for ellipse, equal to 1 for parabola and greater than 1 for hyperbola i.e.

- (i) ellipse : $e < 1$
- (ii) parabola : $e = 1$
- (iii) hyperbola : $e > 1$.

The line passing through the focus and perpendicular to the directrix is called the *axis*. The point at which the conic cuts its axis is called the *vertex*.

6-1-1. ELLIPSE



Use of elliptical curves is made in arches, bridges, dams, monuments, manholes, glands and stuffing-boxes etc. Mathematically an ellipse can be described by equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Here 'a' and 'b' are half the length of major and minor axes of the ellipse and x and y co-ordinates.

(1) General method of construction of an ellipse:



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 12 for the following problem.

Problem 6-1. (fig. 6-2): To construct an ellipse when the distance of the focus from the directrix is equal to 50 mm and eccentricity is $\frac{2}{3}$.

- (i) Draw any vertical line AB as directrix.
- (ii) At any point C on it, draw the axis perpendicular to the AB (directrix).
- (iii) Mark a focus F on the axis such that $CF = 50$ mm.
- (iv) Divide CF into 5 equal divisions (sum of numerator and denominator of the eccentricity).
- (v) Mark the vertex V on the third division-point from C .

$$\text{Thus, eccentricity, } e = \frac{VF}{VC} = \frac{2}{3}.$$

- (vi) A scale may now be constructed on the axis (as explained below), which will directly give the distances in the required ratio.
- (vii) At V , draw a perpendicular VE equal to VF . Draw a line joining C and E .

$$\text{Thus, in triangle } CVE, \frac{VE}{VC} = \frac{VF}{VC} = \frac{2}{3}.$$

- (viii) Mark any point 1 on the axis and through it, draw a perpendicular to meet CE -produced at $1'$.
(ix) With centre F and radius equal to $1-1'$, draw arcs to intersect the perpendicular through 1 at points P_1 and P'_1 .

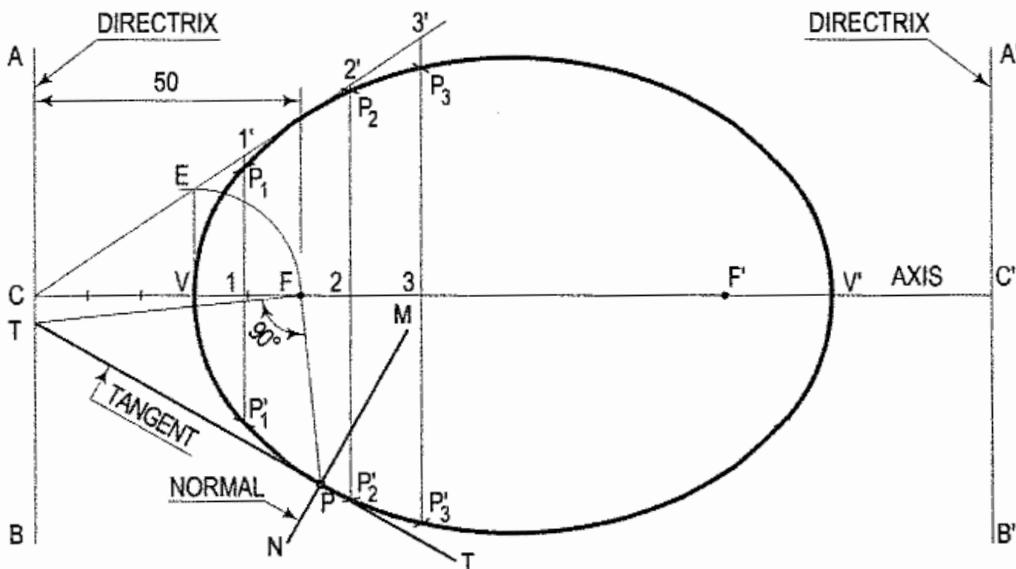
These are the points on the ellipse, because the distance of P_1 from AB is equal to C_1 ,

$$P_1 \ F = 1-1'$$

$$\text{and } \frac{1-1'}{C1} = \frac{VF}{VC} = \frac{2}{3}.$$

Similarly, mark points 2, 3 etc. on the axis and obtain points P_2 and P'_2 , P_3 and P'_3 etc.

- (x) Draw the ellipse through these points. It is a closed curve having two foci and two directrices.



Directrix and focus

FIG. 6-2

(2) Construction of ellipse by other methods:

Ellipse is also defined as a curve traced out by a point, moving in the same plane as and in such a way that the sum of its distances from two fixed points is always the same.

- (i) Each of the two fixed points is called the *focus*.
 - (ii) The line passing through the two foci and terminated by the curve, is called the *major axis*.
 - (iii) The line bisecting the major axis at right angles and terminated by the curve, is called the *minor axis*.

Conjugate axes: Those axes are called conjugate axes when they are parallel to the tangents drawn at their extremities.

In fig. 6-3, AB is the major axis, CD the minor axis and F_1 and F_2 are the foci. The foci are equidistant from the centre O .

The points A, P, C etc. are on the curve and hence, according to the definition,

$$(AF_1 + AF_2) = (PF_1 + PF_2) = (CF_1 + CF_2) \text{ etc.}$$

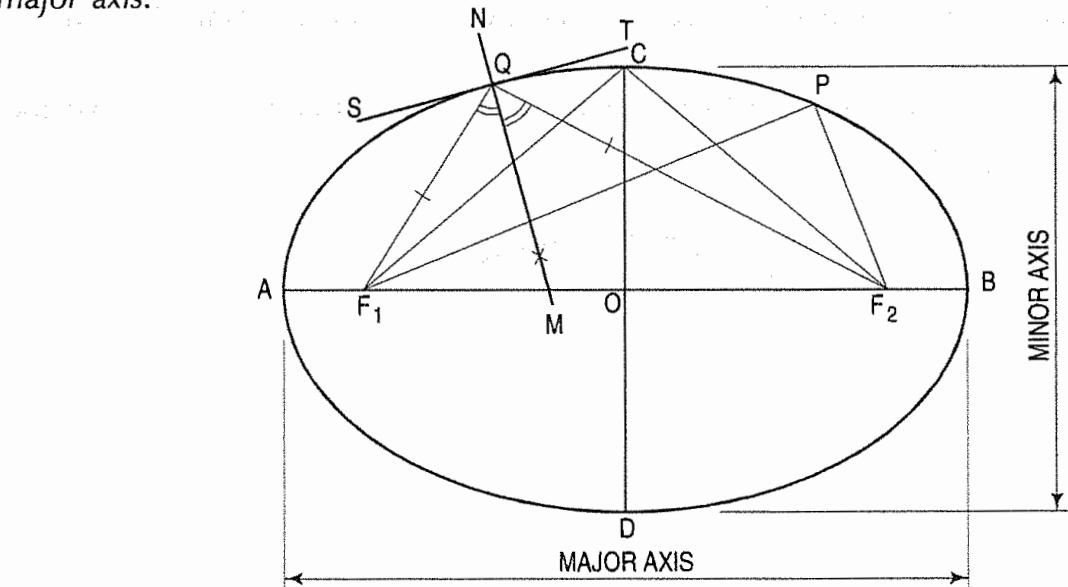
But $(AF_1 + AF_2) = AB$. $\therefore (PF_1 + PF_2) = AB$, the major axis.

Therefore, the sum of the distances of any point on the curve from the two foci is equal to the major axis.

Again, $(CF_1 + CF_2) = AB$.

$$\text{But } CF_1 = CF_2 \quad \therefore CF_1 = CF_2 = \frac{1}{2} AB.$$

Hence, the distance of the ends of the minor axis from the foci is equal to half the major axis.



Conjugate axes

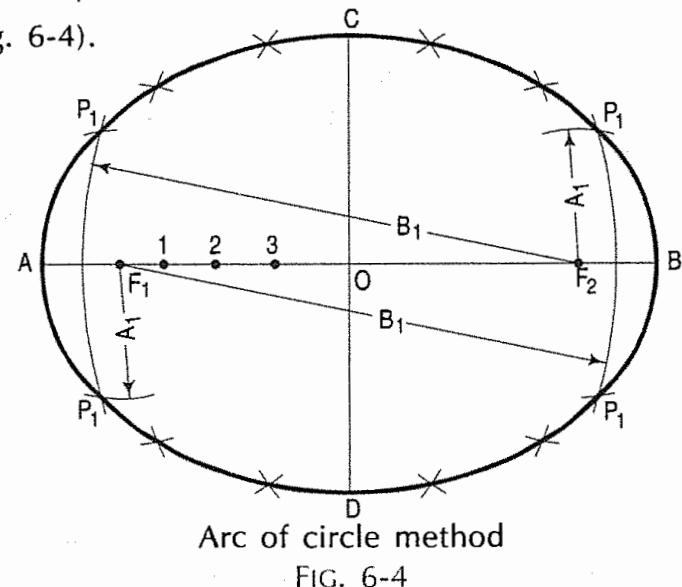
FIG. 6-3

Problem 6-2. To construct an ellipse, given the major and minor axes.

The ellipse is drawn by, first determining a number of points through which it is known to pass and then, drawing a smooth curve through them, either freehand or with a french curve. Larger the number of points, more accurate the curve will be.

Method I: Arcs of circles method (fig. 6-4).

- Draw a line AB equal to the major axis and a line CD equal to the minor axis, bisecting each other at right angles at O .
- With centre C and radius equal to half AB (i.e. AO) draw arcs cutting AB at F_1 and F_2 , the foci of the ellipse.
- Mark a number of points 1, 2, 3 etc. on AB .
- With centres F_1 and F_2 and radius equal to A_1 , draw arcs on both sides of AB .



Arc of circle method

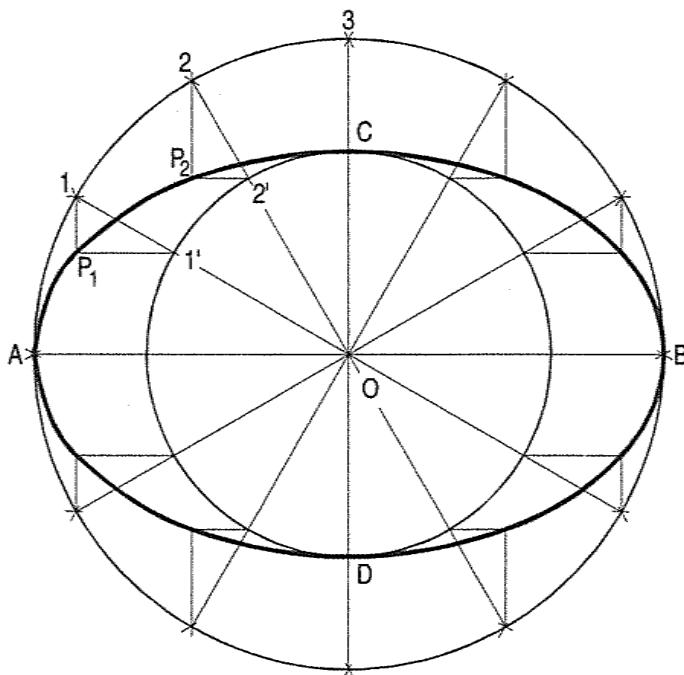
FIG. 6-4

- (v) With same centres and radius equal to B_1 , draw arcs intersecting the previous arcs at four points marked P_1 .
- (vi) Similarly, with radii A_2 and B_2 , A_3 and B_3 etc. obtain more points.
- (vii) Draw a smooth curve through these points. This curve is the required ellipse.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 13 for the following method II.

Method II: Concentric circles method (fig. 6-5).



Concentric circle method

FIG. 6-5

- (i) Draw the major axis AB and the minor axis CD intersecting each other at O .
- (ii) With centre O and diameters AB and CD respectively, draw two circles.
- (iii) Divide the major-axis-circle into a number of equal divisions, say 12 and mark points 1, 2 etc. as shown.
- (iv) Draw lines joining these points with the centre O and cutting the minor-axis-circle at points 1', 2' etc.
- (v) Through point 1 on the major-axis-circle, draw a line parallel to CD , the minor axis.
- (vi) Through point 1' on the minor-axis-circle, draw a line parallel to AB , the major axis. The point P_1 , where these two lines intersect is on the required ellipse.
- (vii) Repeat the construction through all the points. Draw the ellipse through A , P_1 , P_2 ... etc.

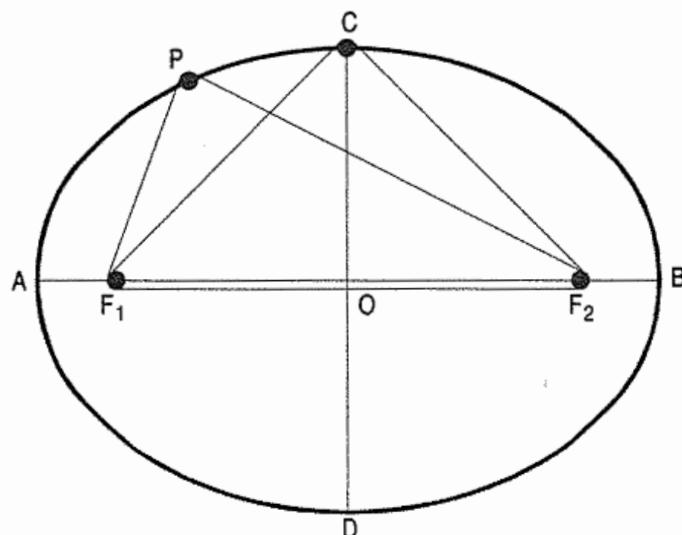
Method III: Loop of the thread method (fig. 6-6).

This is practical application of the first method.

- (i) Draw the two axes AB and CD intersecting at O . Locate the foci F_1 and F_2 .
- (ii) Insert a pin at each focus-point and tie a piece of thread in the form of a loop around the pins, in such a way that the pencil point when placed in the loop (keeping the thread tight), is just on the end C of the minor axis.

- (iii) Move the pencil around the foci, maintaining an even tension in the thread throughout and obtain the ellipse.

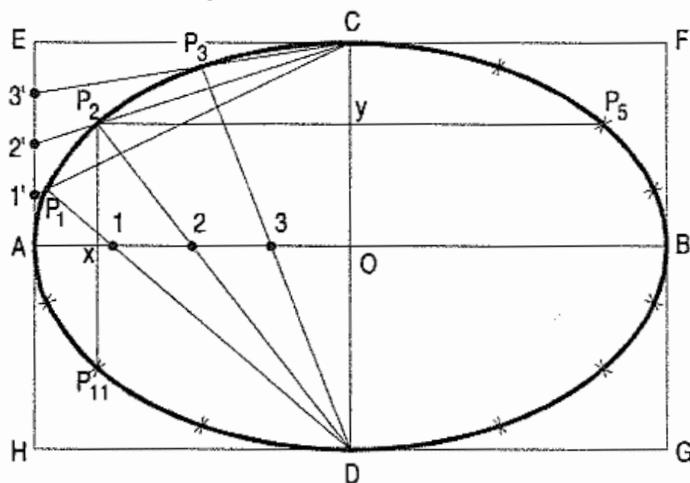
It is evident that $PF_1 + PF_2 = CF_1 + CF_2$ etc.



Loop of the thread method

FIG. 6-6

Method IV: Oblong method (fig. 6-7).



Oblong method

FIG. 6-7

- (i) Draw the two axes AB and CD intersecting each other at O .
- (ii) Construct the oblong $EFGH$ having its sides equal to the two axes.
- (iii) Divide the semi-major-axis AO into a number of equal parts, say 4, and AE into the same number of equal parts, numbering them from A as shown.
- (iv) Draw lines joining $1'$, $2'$ and $3'$ with C .
- (v) From D , draw lines through 1 , 2 and 3 intersecting C'_1 , C'_2 and C'_3 at points P_1 , P_2 and P_3 respectively.
- (vi) Draw the curve through A , P_1 C . It will be one quarter of the ellipse.
- (vii) Complete the curve by the same construction in each of the three remaining quadrants.

As the curve is symmetrical about the two axes, points in the remaining quadrants may be located by drawing perpendiculars and horizontals from P_1 , P_2 etc. and making each of them of equal length on both the sides of the two axes.

For example, $P_2x = xP_{11}$ and $P_2y = yP_5$.

An ellipse can be inscribed within a parallelogram by using the above method as shown in fig. 6-8.

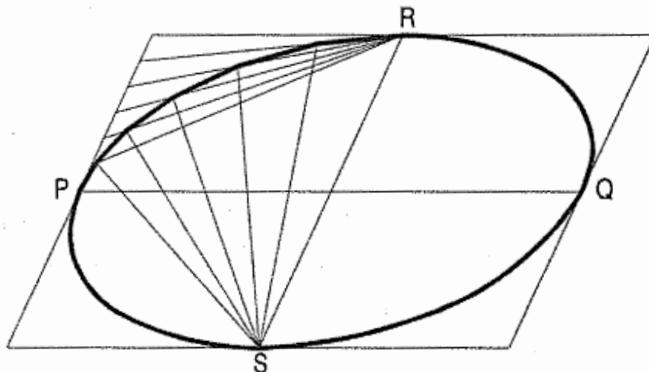
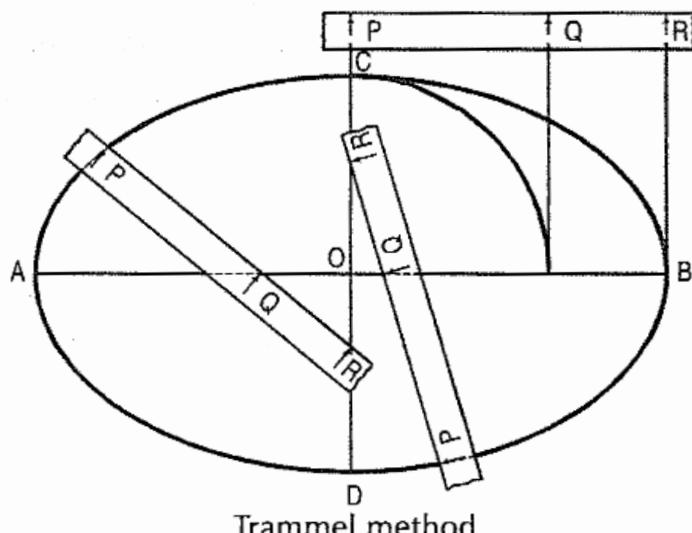


FIG. 6-8

Lines PQ and RS , joining the mid-points of the opposite sides of the parallelogram are called conjugate axes.

Method V: Trammel method (fig. 6-9).



Trammel method

FIG. 6-9

- Draw the two axes AB and CD intersecting each other at O . Along the edge of a strip of paper which may be used as a trammel, mark PQ equal to half the minor axis and PR equal to half the major axis.
- Place the trammel so that R is on the minor axis CD and Q on the major axis AB . Then P will be on the required ellipse. By moving the trammel to new positions, always keeping R on CD and Q on AB , obtain other points. Draw the ellipse through these points.

Problem 6-3. (fig. 6-10): ABC is a triangle such that $AB = 100 \text{ mm}$, $AC = 80 \text{ mm}$ and $BC = 60 \text{ mm}$. Draw an ellipse passing through points A , B and C .

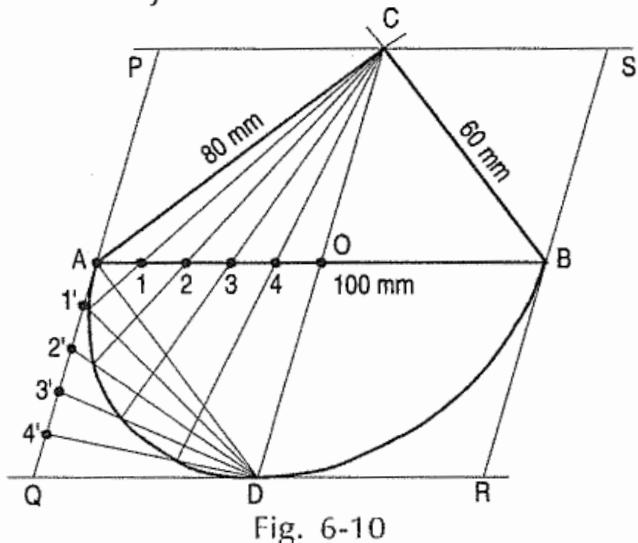


Fig. 6-10

- (i) Draw horizontal line $AB = 100$ mm. Take A as centre draw an arc of 80 mm. Similarly B as centre and the radius equal to 60 mm, draw the arc such that it intersects previously drawn arc at the point C . Join ABC to complete triangle.
- (ii) Mark the mid point of AB such that $OA = OB = 50$ mm. Join OC and extend CO such that $CO = OD$.
- (iii) Draw parallel lines from C and D to the line AB . Similarly draw parallel lines from A and B to the line CD and complete the rhombus ($PQRS$).
- (iv) Divide AO into convenient number of equal parts $A_1 = 12 = 23 = 34 = 4O$ and AQ to same number of equal parts $A_1' = 1' 2' = 2' 3' = 3' 4' = 4' Q$. Join $A, 1', 2', 3', 4'$ with D . Join C_1 and extend it to intersect line D_1' . Similarly join C_2, C_3, C_4 and extend it to intersect D_2', D_3', D_4' respective. Draw smooth curve passing through all intersection.
- (v) Complete the ellipse by above method for the remaining part.

(3) **Normal and tangent to an ellipse:** The normal to an ellipse at any point on it bisects the angle made by lines joining that point with the foci.

The tangent to the ellipse at any point is perpendicular to the normal at that point.

Problem 6-4. (fig. 6-3): *To draw a normal and a tangent to the ellipse at a point Q on it.*

Join Q with the foci F_1 and F_2 .

- (i) Draw a line NM bisecting $\angle F_1 QF_2$. NM is the normal to the ellipse.
- (ii) Draw a line ST through Q and perpendicular to NM . ST is the tangent to the ellipse at the point Q .

Problem 6-5. (fig. 6-11): *To draw a curve parallel to an ellipse and at distance R from it.*

This may be drawn by two methods:

- (a) A large number of arcs of radius equal to the required distance R , with centres on the ellipse, may be described. The curve drawn touching these arcs will be parallel to the ellipse.
- (b) It may also be obtained by drawing a number of normals to the ellipse, making them equal to the required distance R and then drawing a smooth curve through their ends.

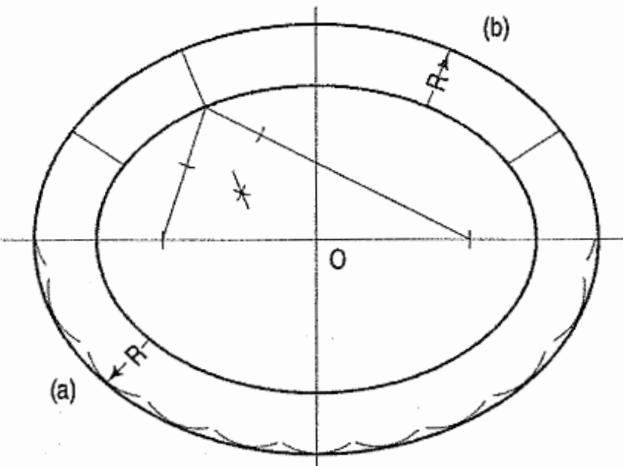


FIG. 6-11

Problem 6-6. (fig. 6-12): *To find the major axis and minor axis of an ellipse whose conjugate axes and angle between them are given.*

Conjugate axes PQ and RS , and the angle α between them are given.

- (i) Draw the two axes intersecting each other at O .
- (ii) Complete the parallelogram and inscribe the ellipse in it as described in problem 6-2, method (iv).

- (iii) With O as centre and OR as radius, draw the semi-circle cutting the ellipse at a point E .
- (iv) Draw the line RE .
- (v) Through O draw a line parallel to RE and cutting the ellipse at points C and D . CD is the minor axis.
- (vi) Through O , draw a line perpendicular to CD and cutting the ellipse at points A and B . AB is the major axis.

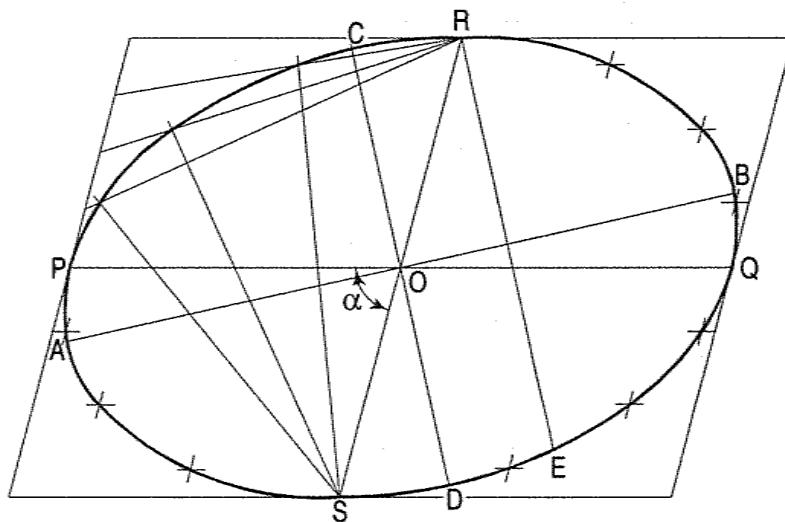


FIG. 6-12

Problem 6-7. (fig. 6-13): To find the centre, major axis and minor axis of a given ellipse.

- (i) Draw any two chords 1-2 and 3-4 parallel to each other.
- (ii) Find their mid-points P and Q , and draw a line passing through them, cutting the ellipse at points R and S . Bisect the line RS in the point O which is the centre of the ellipse.

With O as centre and any convenient radius, draw a circle cutting the ellipse in points E , F , G and H . Complete the rectangle $EFGH$. Through O , draw a line parallel to EF cutting the ellipse in points A and B . Again through O , draw a line parallel to FG cutting the ellipse at points C and D . AB and CD are respectively the major axis and the minor axis.

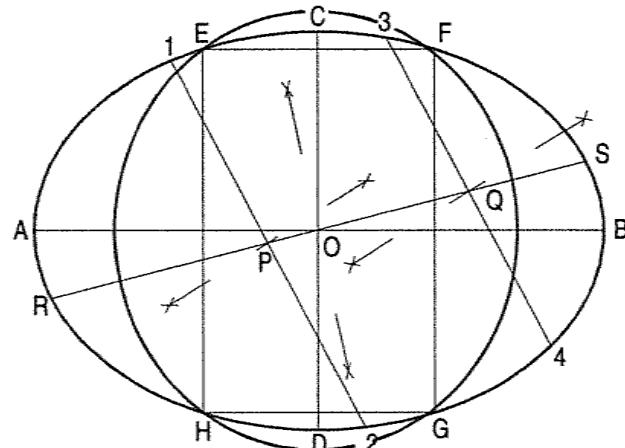


FIG. 6-13

6-1-2. PARABOLA

Use of parabolic curves is made in arches, bridges, sound reflectors, light reflectors etc. Mathematically a parabola can be described by an equation $y^2 = 4ax$ or $x^2 = 4ay$.

(1) General method of construction of a parabola:

Problem 6-8. (fig. 6-14): To construct a parabola, when the distance of the focus from the directrix is 50 mm.

- (i) Draw the directrix AB and the axis CD .
- (ii) Mark focus F on CD , 50 mm from C .
- (iii) Bisect CF in V the vertex (because eccentricity = 1).
- (iv) Mark a number of points 1, 2, 3 etc. on the axis and through them, draw perpendiculars to it.
- (v) With centre F and radius equal to $C1$, draw arcs cutting the perpendicular through 1 at P_1 and P'_1 .
- (vi) Similarly, locate points P_2 and P'_2 , P_3 and P'_3 etc. on both the sides of the axis.
- (vii) Draw a smooth curve through these points. This curve is the required parabola. It is an open curve.

Problem 6-9. (fig. 6-15): To find the axis of a given parabola.

- (i) Draw any two chords AB and CD across the parabola, parallel to each other and any distance apart.
- (ii) Bisect AB and CD in points E and F respectively and draw a line GH passing through them. The line GH will be parallel to the axis.
- (iii) Draw a chord PQ , perpendicular to GH .
- (iv) Bisect PQ in the point O and through it draw a line XY parallel to GH .

Then XY is the required axis of the parabola.

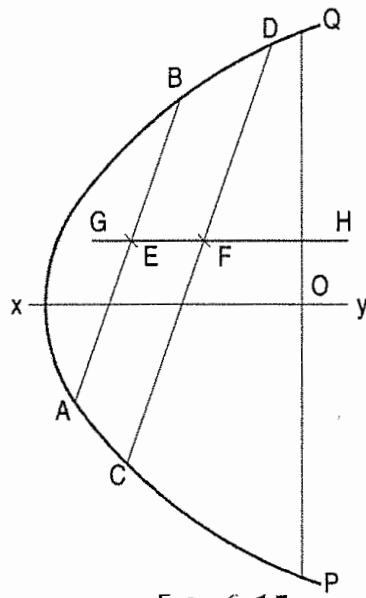


FIG. 6-15

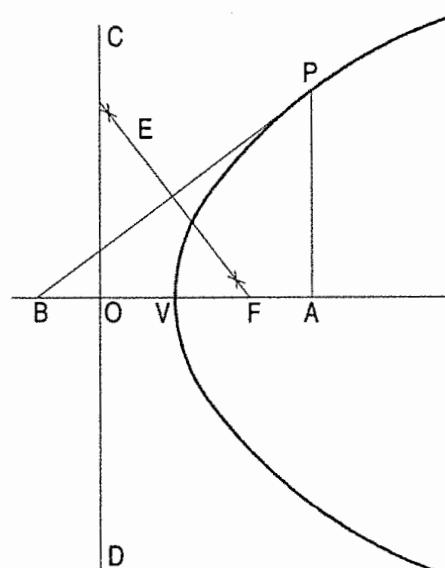
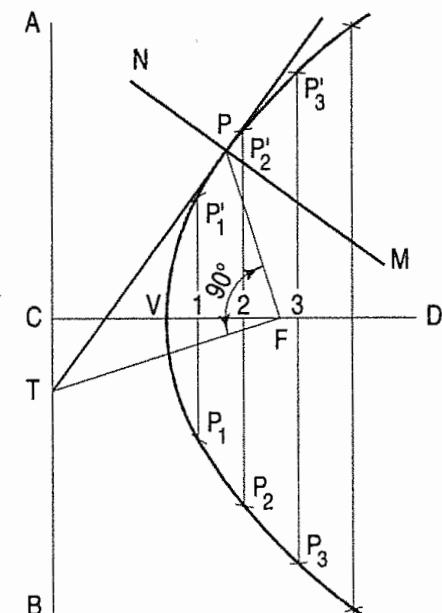


FIG. 6-16



Directrix and focus
FIG. 6-14

Problem 6-10. (fig. 6-16): To find the focus and the directrix of a parabola whose axis is given.

- (i) Mark any point P on the parabola and draw a perpendicular PA to the axis.
Mark a point B on the axis such that $BV = VA$.
- (ii) Draw a line joining B with P .
- (iii) Draw a perpendicular bisector EF of BP , intersecting the axis at a point F .
Then F is the focus of the parabola.
- (iv) Mark a point O on the axis such that $OV = VF$. Through O , draw a line CD perpendicular to the axis. Then CD is the directrix of the parabola.

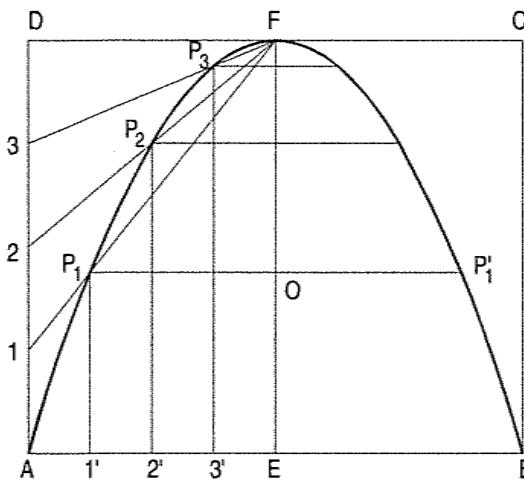
(2) Construction of parabola by other methods:

Method I: Rectangle method (fig. 6-17).

Problem 6-11. To construct a parabola given the base and the axis.

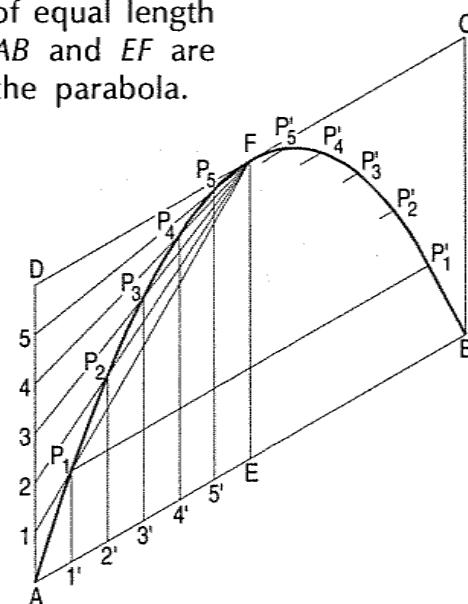
- (i) Draw the base AB .
- (ii) At its mid-point E , draw the axis EF at right angles to AB .
- (iii) Construct a rectangle $ABCD$, making side BC equal to EF .
- (iv) Divide AE and AD into the same number of equal parts and name them as shown (starting from A).
- (v) Draw lines joining F with points $1'$, $2'$ and $3'$. Through $1'$, $2'$ and $3'$, draw perpendiculars to AB intersecting $F1$, $F2$ and $F3$ at points P_1 , P_2 and P_3 respectively.
- (vi) Draw a curve through A , P_1 , P_2 etc. It will be a half parabola.

Repeat the same construction in the other half of the rectangle to complete the parabola. Or, locate the points by drawing lines through the points P_1 , P_2 etc. parallel to the base and making each of them of equal length on both the sides of EF , e.g. $P_1O = OP'_1$. AB and EF are called the base and the axis respectively of the parabola.



Rectangle method

FIG. 6-17



Parallelogram method

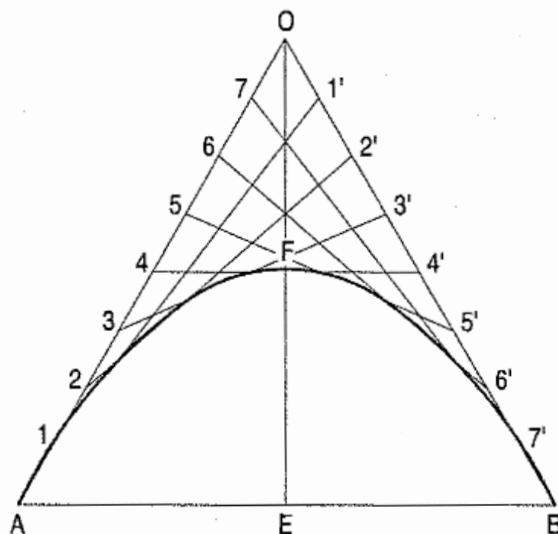
FIG. 6-18

Fig. 6-18 shows a parabola drawn in a parallelogram by this method.

Method II: Tangent method (fig. 6-19).

- (i) Draw the base AB and the axis EF . (These are taken different from those in method I.)
- (ii) Produce EF to O so that $EF = FO$.

- (iii) Join O with A and B. Divide lines OA and OB into the same number of equal parts, say 8.
- (iv) Mark the division-points as shown in the figure.
- (v) Draw lines joining 1 with 1', 2 with 2' etc. Draw a curve starting from A and tangent to lines 1-1', 2-2' etc. This curve is the required parabola.



Tangent method

FIG. 6-19

6.1.3. HYPERBOLA

Use of hyperbolical curves is made in cooling towers, water channels etc.

Rectangular hyperbola: It is a curve traced out by a point moving in such a way that the product of its distances from two fixed lines at right angles to each other is a constant. The fixed lines are called *asymptotes*.

This curve graphically represents the Boyle's Law, viz. $P \times V = a$, P = pressure, V = volume and a is constant. It is also useful in design of water channels.

General method of construction of a hyperbola:

Mathematically, we can describe a hyperbola by

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \text{ (Fig. 6-20 and fig. 6-21.)}$$

Problem 6-12. (fig. 6-20): Construct a hyperbola, when the distance of the focus from the directrix is 65 mm and eccentricity is $\frac{3}{2}$.

- (i) Draw the directrix AB and the axis CD .
- (ii) Mark the focus F on CD and 65 mm from C .
- (iii) Divide CF into 5 equal divisions and mark V the vertex, on the second division from C .

$$\text{Thus, eccentricity} = \frac{VF}{VC} = \frac{3}{2}.$$

To construct the scale for the ratio $\frac{3}{2}$ draw a line VE perpendicular to CD such that $VE = VF$. Join C with E .

$$\text{Thus, in triangle } CVE, \frac{VE}{VC} = \frac{VF}{VC} = \frac{3}{2}.$$

- (iv) Mark any point 1 on the axis and through it, draw a perpendicular to meet CE -produced at $1'$.
- (v) With centre F and radius equal to $1-1'$, draw arcs intersecting the perpendicular through 1 at P_1 and P'_1 .
- (vi) Similarly, mark a number of points 2, 3 etc. and obtain points P_2 and P'_2 , P_3 and P'_3 etc.
- (vii) Draw the hyperbola through these points.

Problem 6-13. (fig. 6-21): To draw a hyperbola when its foci and vertices are given, and to locate its asymptotes.

- (i) Draw a horizontal line as axis and on it, mark the given foci F and F_1 , and vertices V and V_1 .
- (ii) Mark any number of points 1, 2, 3 etc. to the right of F_1 .
- (iii) With F and F_1 as centres and radius, say V_2 , draw four arcs.
- (iv) With the same centres and radius V_{12} , draw four more arcs intersecting the first four arcs at points P_2 . Then these points lie on the hyperbola.
- (v) Repeat the process with the same centres and radii V_1 and V_{11} , V_3 and V_{13} etc. Draw the required hyperbola through the points thus obtained.
- (vi) With FF_1 as diameter, draw a circle.
- (vii) Through the vertices V and V_1 , draw lines perpendicular to the axis, cutting the circle at four points A . From O , the centre of the circle, draw lines passing through points A . These lines are the required asymptotes.

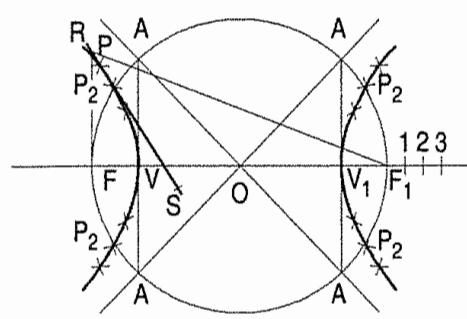


FIG. 6-21

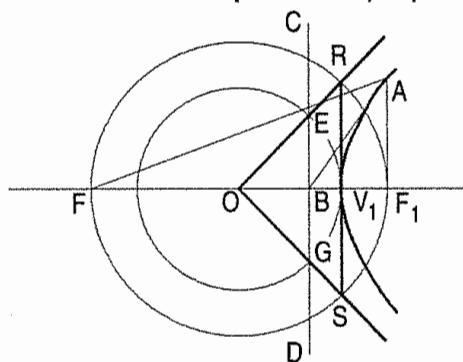


FIG. 6-22

Problem 6-14. (fig. 6-22): To locate the directrix and asymptotes of a hyperbola when its axis and foci are given.

From the focus F_1 , draw a perpendicular to the axis intersecting the hyperbola at a point A .

- (i) Draw a line joining A with the other focus F . Draw the bisector of $\angle FAF_1$, cutting the axis at a point B .
- (ii) Through B , draw a line CD perpendicular to the axis. CD is the required directrix.
- (iii) With O the mid-point of FF_1 as centre and OV_1 as radius, draw a circle cutting CD at points E and G .
- (iv) Lines drawn from O and passing through E and G are the required asymptotes.
- (v) The asymptotes will also pass through the points of intersection (R and S) between the circle of radius OF_1 and the vertical through V_1 .

Problem 6-15. (fig. 6-23): To draw a rectangular hyperbola, given the position of a point P on it.

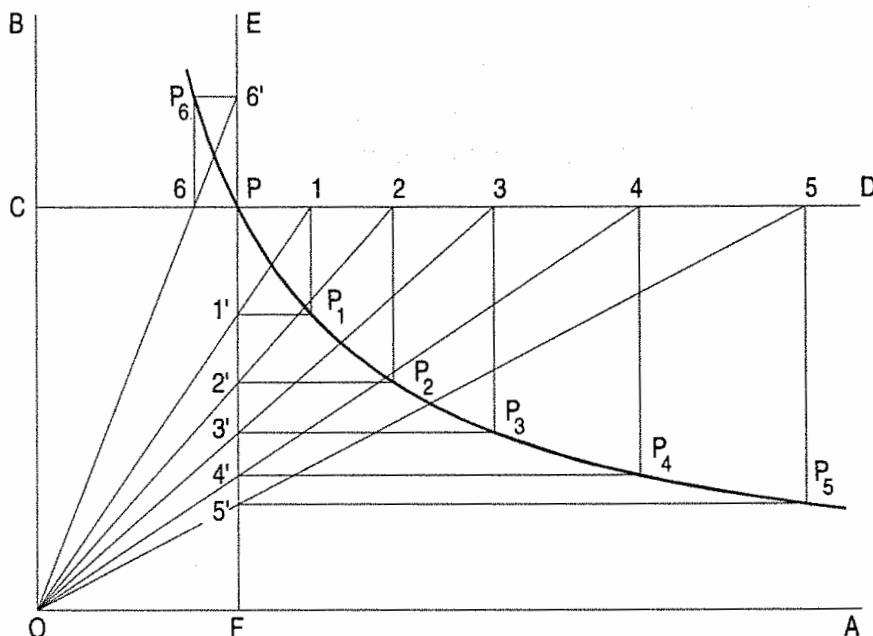


FIG. 6-23

- (i) Draw lines OA and OB at right angles to each other.
- (ii) Mark the position of the point P .
- (iii) Through P , draw lines CD and EF parallel to OA and OB respectively.
- (iv) Along PD , mark a number of points $1, 2, 3$ etc. not necessarily equidistant.
- (v) Draw lines $O1, O2$ etc. cutting PF at points $1', 2'$ etc.
- (vi) Through point 1 , draw a line parallel to OB and through $1'$, draw a line parallel to OA , intersecting each other at a point P_1 .
- (vii) Obtain other points in the same manner.

For locating the point, say P_6 , to the left of P , the line $O6$ should be extended to meet PE at $6'$. Draw the hyperbola through the points P_6, P, P_1 etc.

A hyperbola, through a given point situated between two lines making any angle between them, can similarly be drawn, as shown in fig. 6-24.

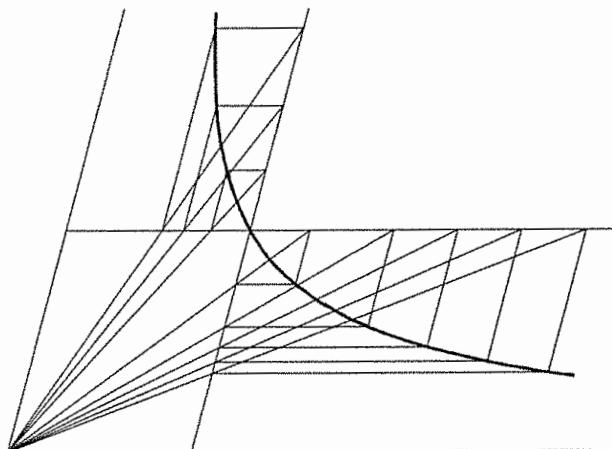


FIG. 6-24

6-1-4. TANGENTS AND NORMALS TO CONICS



(1) General method:

The common rule for drawing tangents and normals:

When a tangent at any point on the curve is produced to meet the directrix, the line joining the focus with this meeting point will be at right angles to the line joining the focus with the point of contact.

The normal to the curve at any point is perpendicular to the tangent at that point.

Problem 6-16. To draw a tangent at any point P on the conics (fig. 6-2, fig. 6-14 and fig. 6-20).

- (i) Join P with F .
 - (ii) From F , draw a line perpendicular to PF to meet AB at T .
 - (iii) Draw a line through T and P . This line is the tangent to the curve.
 - (iv) Through P , draw a line NM perpendicular to TP . NM is the normal to the curve.

(2) Other methods of drawing tangents to conics:

Method I:

Problem 6-17. (fig. 6-25): To draw a tangent to an ellipse at any point P on it.

- (i) With O , the centre of the ellipse as centre, and one half the major axis as radius, draw a circle.
(ii) From P , draw a line parallel to the minor axis, cutting the circle at a point Q .

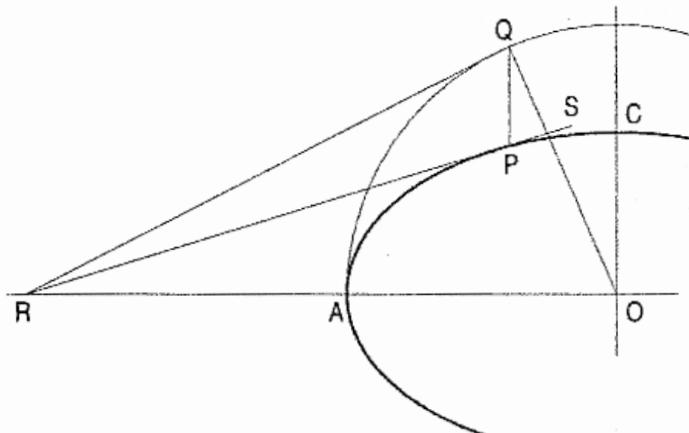


FIG. 6-25

- (iii) Draw a tangent to the circle at the point Q cutting the extended major axis at a point R .

From R , draw a line RS passing through P . RS is the required tangent to the ellipse.

Method II: When an axis of parabola is given.

Problem 6-18. To draw a tangent to a parabola at any point P on it.

- (i) From P , draw a perpendicular PA to the axis, intersecting it at a point A (fig. 6-26).

Mark a point B on the axis such that $BV = VA$. Draw a line from B passing through P .

Then this line is the required tangent.

- (ii) Through P , draw a line PQ parallel to the axis (fig. 6-27).

Draw two lines AB and CD parallel to, equidistant from and on opposite sides of PQ , and cutting the parabola at points A and C . Draw a line joining A with C .

Through P , draw a line RS parallel to AC . RS is the required tangent.

Method III: When the focus and the directrix are given (fig. 6-28).

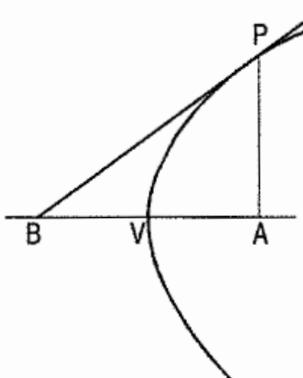


FIG. 6-26

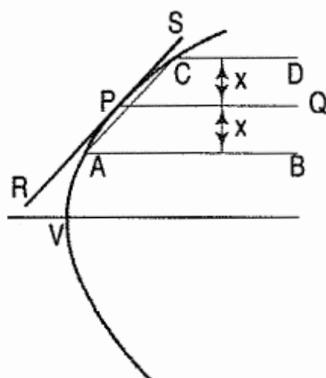


FIG. 6-27

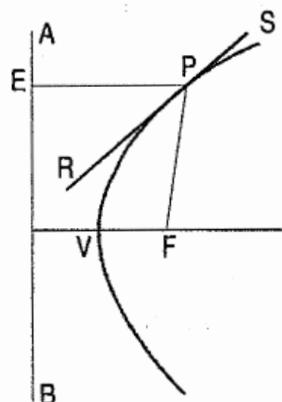


FIG. 6-28

- From P draw a line PE perpendicular to the directrix AB , meeting it at a point E .
 - Draw a line joining P with the focus F .
 - Draw the bisector RS of angle EPF .
- Then RS is the required tangent.

Problem 6-19. (fig. 6-21): To draw a tangent to a hyperbola at a point P on it when the axis and the foci are given.

Draw lines joining P with foci F and F_1 . Draw the bisector RS of $\angle FPF_1$. RS is the required tangent to the hyperbola.

Note: In fig. 6-22, the line AB is the tangent to the hyperbola at the point A .

6-2. CYCLOIDAL CURVES

These curves are generated by a fixed point on the circumference of a circle, which rolls without slipping along a fixed straight line or a circle. The rolling circle is called *generating circle* and the fixed straight line or circle is termed *directing line* or *directing circle*. Cycloidal curves are used in tooth profile of gears of a dial gauge.

6-2-1. CYCLOID

Cycloid is a curve generated by a point on the circumference of a circle which rolls along a straight line. It can be described by an equation,

$$y = a(1 - \cos\theta) \text{ or } x = a(\theta - \sin\theta).$$

Problem 6-20. (fig. 6-29): To construct a cycloid, given the diameter of the generating circle.

- With centre C and given radius R , draw a circle. Let P be the generating point.

- (ii) Draw a line PA tangential to and equal to the circumference of the circle.
- (iii) Divide the circle and the line PA into the same number of equal parts, say 12, and mark the division-points as shown.
- (iv) Through C , draw a line CB parallel and equal to PA .
- (v) Draw perpendiculars at points 1, 2 etc. cutting CB at points C_1, C_2 etc.

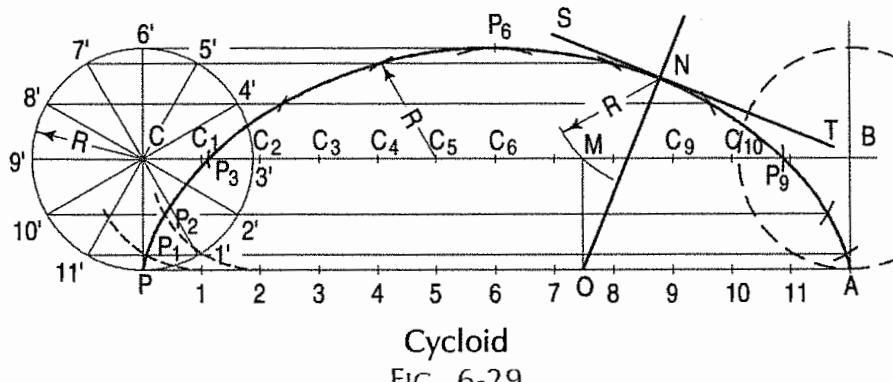


FIG. 6-29

Assume that the circle starts rolling to the right. When point $1'$ coincides with 1, centre C will move to C_1 . In this position of the circle, the generating point P will have moved to position P_1 on the circle, at a distance equal to P'_1 from point $1'$. It is evident that P_1 lies on the horizontal line through $1'$ and at a distance R from C_1 .

Similarly, P_2 will lie on the horizontal line through $2'$ and at the distance R from C_2 .

Construction:

- (vi) Through the points $1', 2'$ etc. draw lines parallel to PA .
- (vii) With centres C_1, C_2 etc. and radius equal to R , radius of generating circle, draw arcs cutting the lines through $1', 2'$ etc. at points P_1, P_2 etc. respectively. Draw a smooth curve through points $P, P_1, P_2 \dots A$. This curve is the required cycloid.

Normal and tangent to a cycloid curve: The rule for drawing a normal to all cycloidal curves:

The normal at any point on a cycloidal curve will pass through the corresponding point of contact between the generating circle and the directing line or circle.

The tangent at any point is perpendicular to the normal at that point.

Problem 6-21. (fig. 6-29): *To draw a normal and a tangent to a cycloid at a given point N on it.*

- (i) With centre N and radius equal to R , draw an arc cutting CB at M .
 - (ii) Through M , draw a line MO perpendicular to the directing line PA and cutting it at O .
- O is the point of contact and M is the position of the centre of the generating circle, when the generating point P is at N .
- (iii) Draw a line through N and O . This line is the required normal.
 - (iv) Through N , draw a line ST at right angles to NO . ST is the tangent to the cycloid.

Problem 6-22. (fig. 6-30): A thin circular disc of 50 mm diameter is allowed to roll without slipping from upper edge of sloping plank which is inclined at 15° with the horizontal plane. Draw the curve traced by the point on the circumference of the disc.

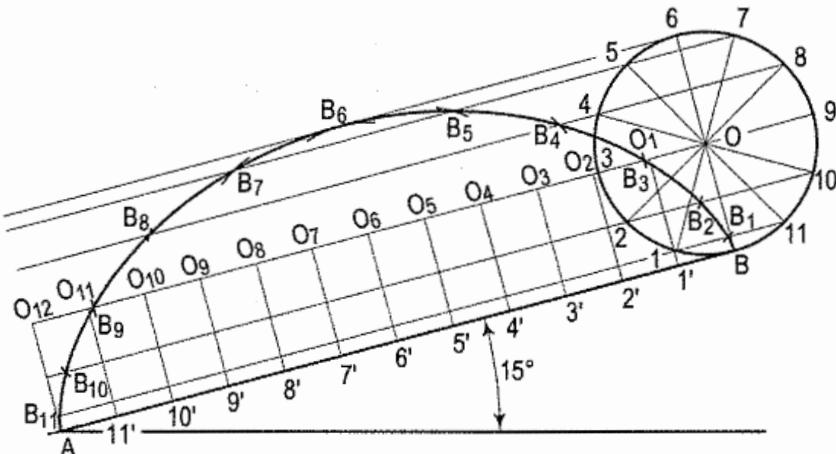


FIG. 6-30

- Draw a line AB of length 157 mm (i.e. πD) at angle of 15° with the horizontal.
- Draw a circle of 50 mm diameter at upper edge B as shown and divide the circumference of the circle in twelve equal parts. Name 1, 2, 3, 4 to 11. Draw parallel lines from 1, 2, 3, 4 to 11 to the AB . Take B as generating point on the circumference.
- Draw a parallel line from the centre of circle O to line AB and equal to length of AB . Divide this line in 12 parts. Name O_1 to O_{12} as shown.
- As the disc rolls from position B , the centre of disc simultaneously moves from O to O_1 . Draw an arc with radius as 25 mm and centre as O_1 to intersect the line drawn from the point 1 parallel to AB . Mark intersection point as B_1 . This point is on the cycloid.
- Similarly draw arc from the points O_2 to O_{12} such that it intersects each line drawn from 2 to 11 respectively. Join the points by smooth curve. This curve is the cycloid.

6-2-2. TROCHOID



Trochoid is a curve generated by a point fixed to a circle, within or outside its circumference, as the circle rolls along a straight line.

When the point is within the circle, the curve is called an *inferior trochoid* and when outside the circle, it is termed a *superior trochoid*.

Problem 6-23. (fig. 6-31, fig. 6-32 and fig. 6-33): To draw a trochoid, given the rolling circle and the generating point.

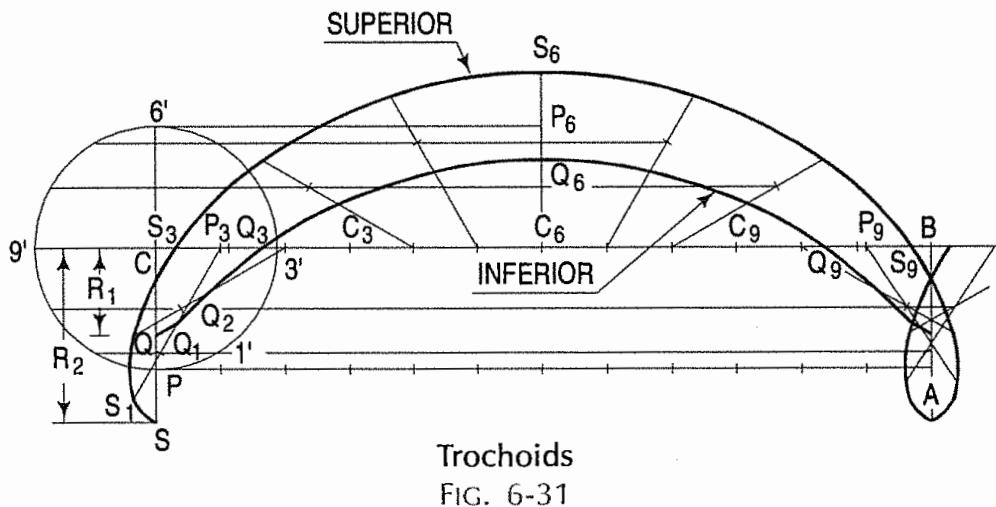
(a) *Inferior trochoid:*

Let Q be the point within the circle and at a distance R_1 from the centre C .

- Draw the circle and mark a point Q on the line CP and at a distance R_1 from C .
- Draw a tangent PA equal to the circumference of the circle and a line CB equal and parallel to PA .
- Divide the circle and the line CB into 12 equal parts.

Method I: (fig. 6-31): Determine the positions P_1, P_2 etc. for the cycloid as shown in problem 6-20. Draw lines C_1P_1, C_2P_2 etc. With centres C_1, C_2 etc. and radius equal to R_1 , draw arcs cutting C_1P_1, C_2P_2 etc. at points Q_1, Q_2 etc. respectively.

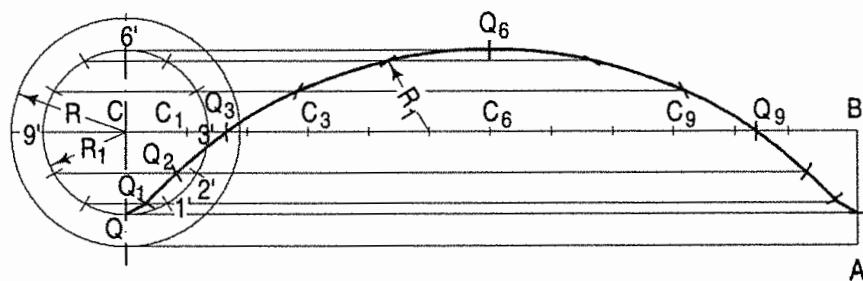
Draw a curve through these points. This curve is the inferior trochoid.



Trochoids
FIG. 6-31

Method II: (fig. 6-32): With centre C and radius equal to R_1 , draw a circle and divide it into 12 equal parts.

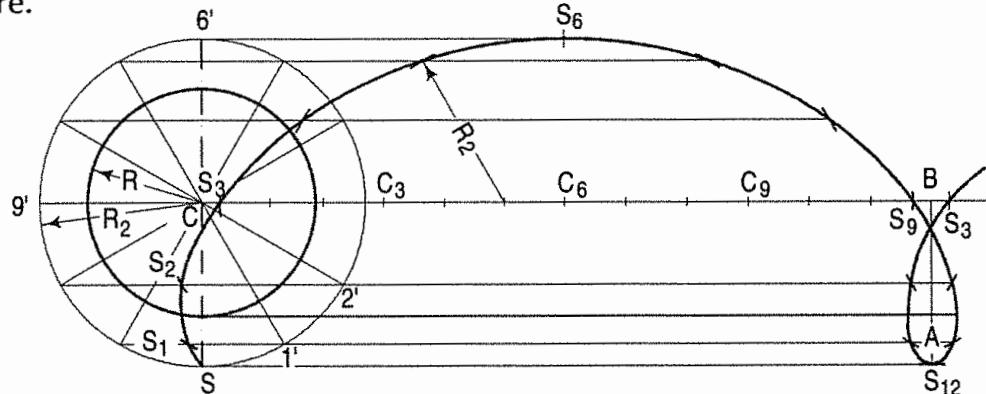
Through the division-points, draw lines parallel to PA . With centres C_1, C_2 etc. and radius equal to R_1 , draw arcs to cut the lines through $1', 2'$ etc. at points Q_1, Q_2 etc. Draw the trochoid through these points.



Inferior trochoid
FIG. 6-32

(b) Superior trochoid:

Let S be the point outside circumference of the circle and at a distance R_2 from the centre.



Superior trochoid
FIG. 6-33

Method I: (fig. 6-31): Adopt the same method as method I used for inferior trochoid. Point S will lie on the line CP -produced, at distance R_2 from C . Points S_1, S_2 etc. are obtained by cutting the lines C_1P_1 -produced, C_2P_2 -produced etc. with arcs drawn with centres C_1, C_2 etc. and radius equal to R_2 . S, S_1, S_2 etc. are the points on the superior trochoid.

Method II: (fig. 6-33): Same as method II for inferior trochoid. Note that the radius of the circle is equal to R_2 . A loop is formed when the circle rolls for more than one revolution.

6-2-3. EPICYCLOID AND HYPOCYCLOID



The curve generated by a point on the circumference of a circle, which rolls without slipping along another circle *outside* it, is called an *epicycloid*. The epicycloid can be represented mathematically by

$$x = (a + b) \cos\theta - a \cos\left(\frac{a+b}{a}\theta\right),$$

$$y = (a + b) \sin\theta - a \sin\left(\frac{a+b}{a}\theta\right)$$

where a is the radius of rolling circle.

When the circle rolls along another circle *inside* it, the curve is called a *hypocycloid*. It can be represented by mathematically $x = a \cos^3\theta, y = a \sin^3\theta$.

Problem 6-24. To draw an epicycloid and a hypocycloid, given the generating and directing circles of radii r and R respectively.

Epicycloid (fig. 6-34): With centre O and radius R , draw the directing circle (only a part of it may be drawn). Draw a radius OP and produce it to C , so that $CP = r$.

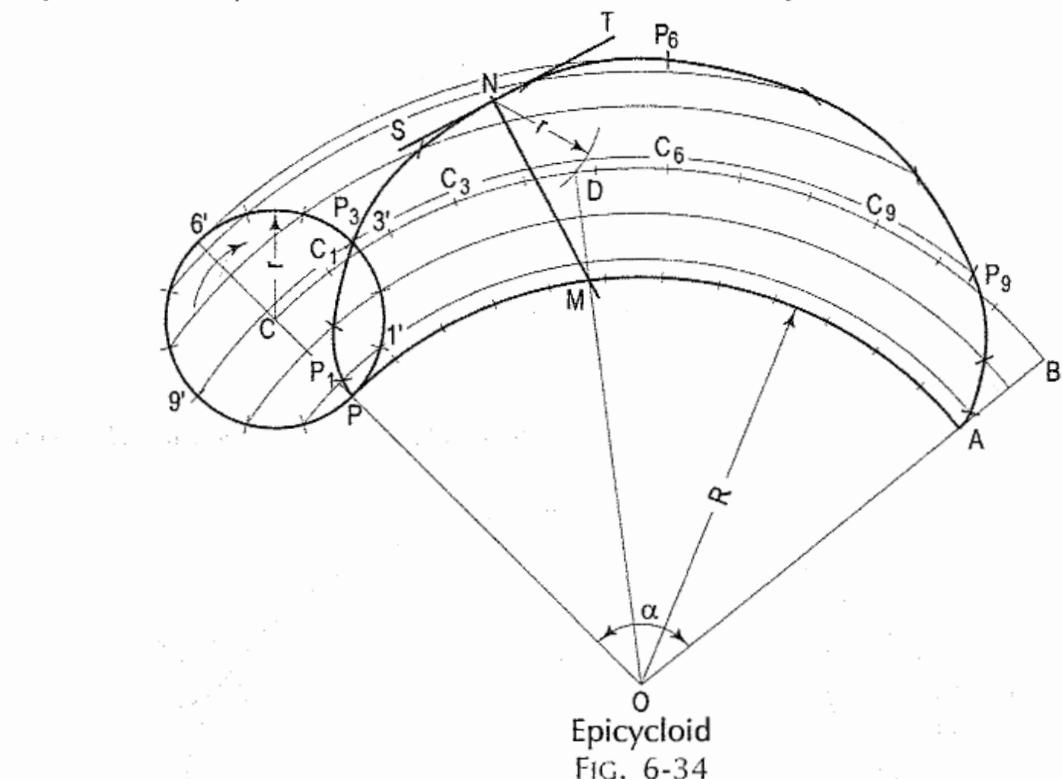


FIG. 6-34

With C as centre, draw the generating circle. Let P be the generating point.

In one revolution of the generating circle, the point P will move to a point A , so that the arc PA is equal to the circumference of the generating circle.

The position of A may be located by calculating the angle subtended by the arc PA at centre O , by the formula,

$$\frac{\angle POA}{360^\circ} = \frac{\text{arc } PC}{\text{circumference of directing circle}} = \frac{2\pi r}{2\pi R} = \frac{r}{R}$$

$$\therefore \angle POA = 360^\circ \times \frac{r}{R}.$$

(i) Set-off this angle and obtain the position of A .

(ii) With centre O and radius equal to OC , draw an arc intersecting OA -produced at B . This arc CB is the locus of the centre C .

(iii) Divide CB and the generating circle into twelve equal parts.

(iv) With centre O , describe arcs through points $1'$, $2'$, $3'$ etc.

(v) With centres C_1 , C_2 etc. and radius equal to r , draw arcs cutting the arcs through $1'$, $2'$ etc. at points P_1 , P_2 etc.

Draw the required hypocycloid through the points P , P_1 , P_2 A .

Hypocycloid (fig. 6-35): The method for drawing the hypocycloid is same as for epicycloid. Note that the centre C of the generating circle is inside the directing circle.

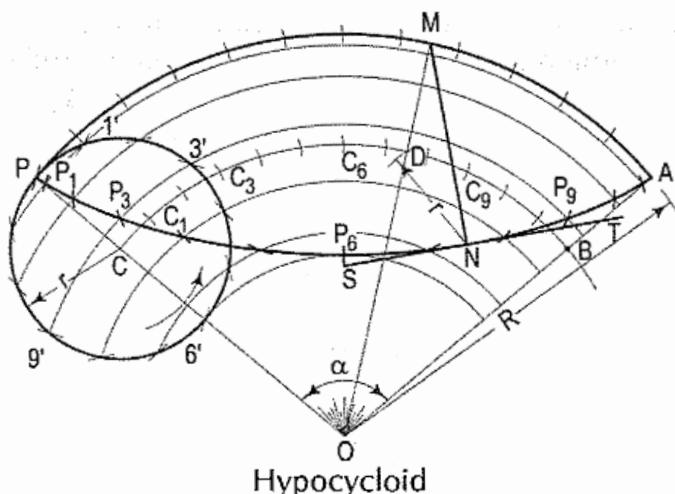
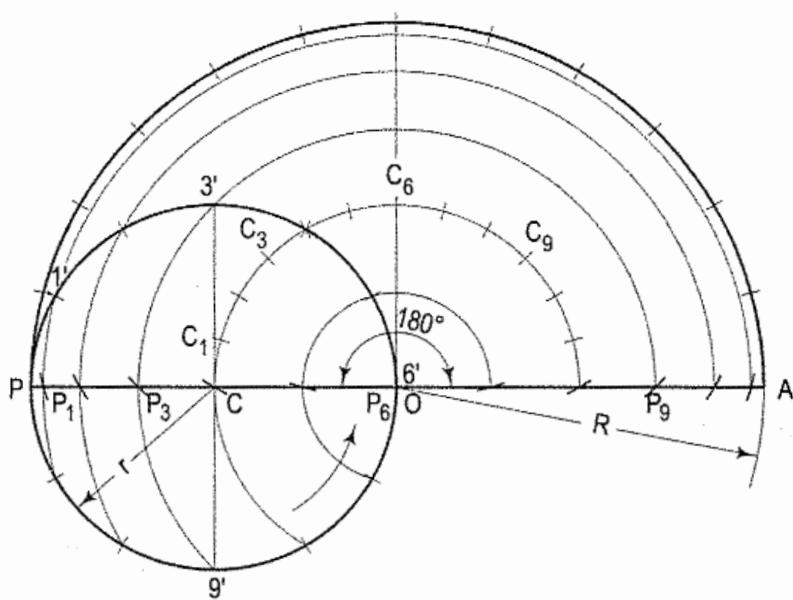


FIG. 6-35



Hypocycloid

FIG. 6-36

When the diameter of the rolling circle is half the diameter of the directing circle, the hypocycloid is a straight line and is a diameter of the directing circle (fig. 6-36).

Normal and tangent to an epicycloid and a hypocycloid:

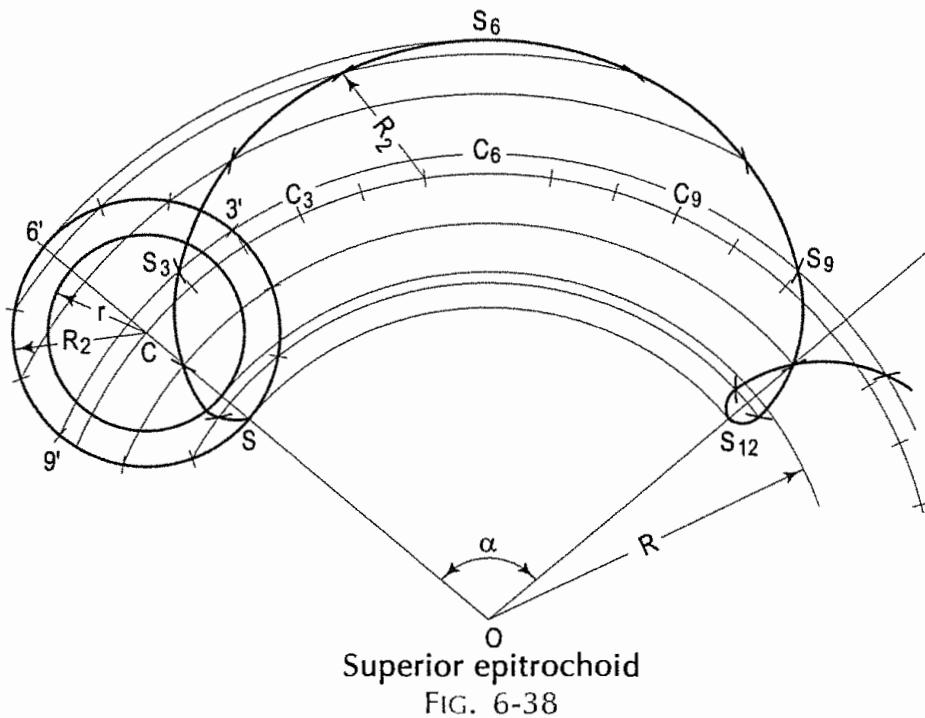
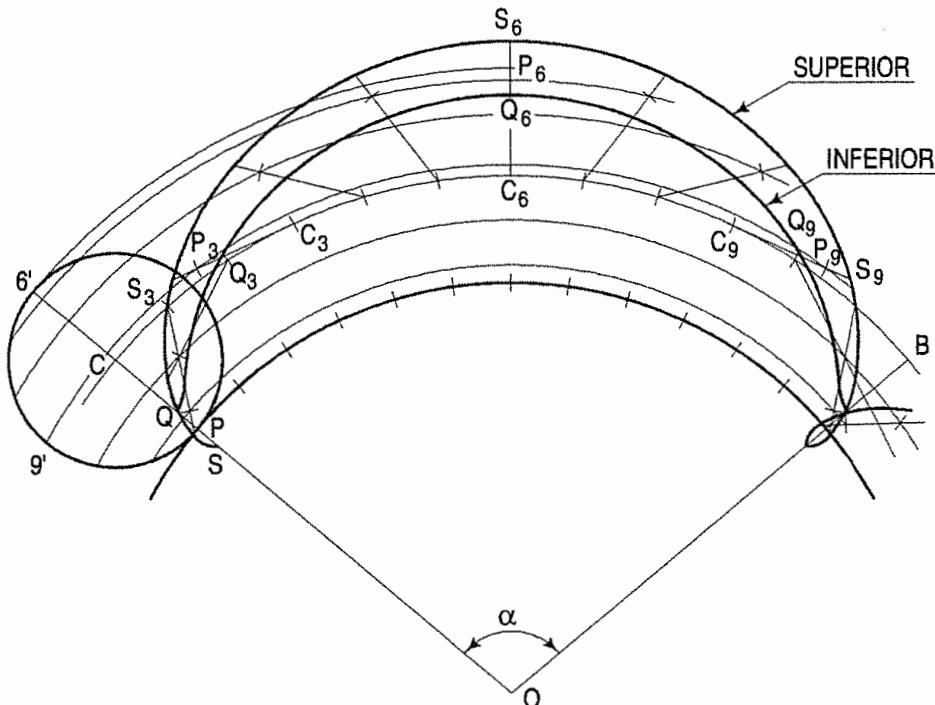
Problem 6-25. (fig. 6-34 and fig. 6-35): To draw a normal and a tangent to an epicycloid and a hypocycloid at a point N in each of them.

- With centre N and radius equal to r , draw an arc cutting the locus of the centre C at a point D .
- Draw a line through O and D , cutting the directing circle at M .
- Draw a line through N and M . This line is the normal. Draw a line ST through N and at right angles to NM . ST is the tangent.

6-2-4. EPITROCHOID (FIG. 6-37 AND FIG. 6-38)



Epitrochoid is a curve generated by a point fixed to a circle (within or outside its circumference, but in the same plane) rolling on the outside of another circle.

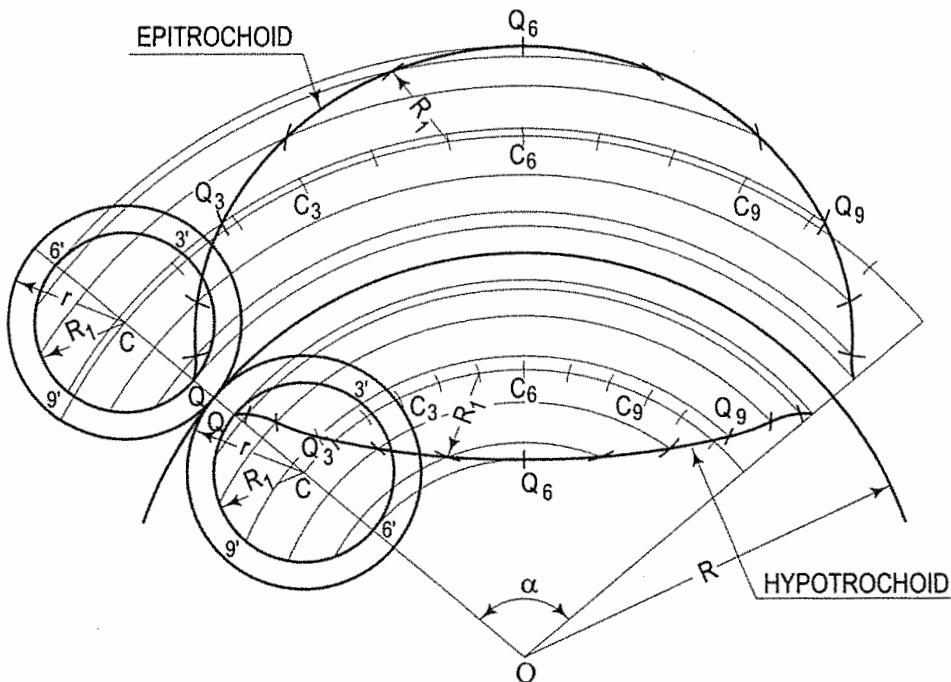


6-2-5. HYPOTROCHOID (FIG. 6-39)



When the circle rolls inside another circle, the curve is called a hypotrochoid.

The curve is termed inferior or superior, according to the position of the point being inside or outside the rolling circle.



Inferior epitrochoid and hypotrochoid

FIG. 6-39

Problem 6-26. To draw an epitrochoid and a hypotrochoid, given the rolling and directing circles and the generating points.

These curves are drawn by applying the methods used for trochoids. Note that arcs are drawn instead of horizontal lines.

Epitrochoids:

Method I: Superior and inferior

– see fig. 6-37

Method II: Superior – see fig. 6-38

Inferior – see fig. 6-39

Hypotrochoids:

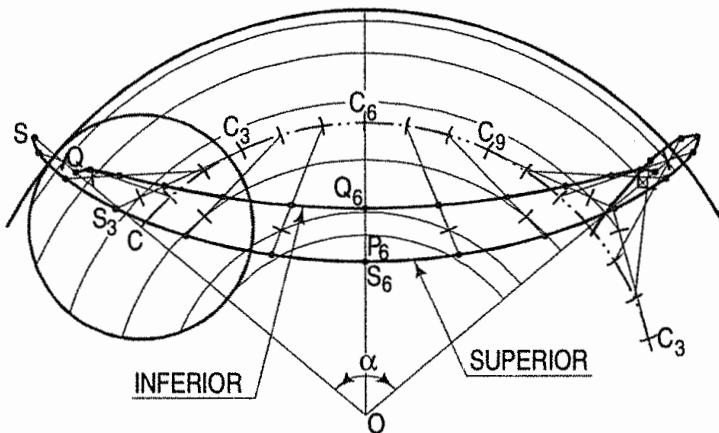
Method I: Superior and inferior

– see fig. 6-40

Method II: Superior – see fig. 6-41

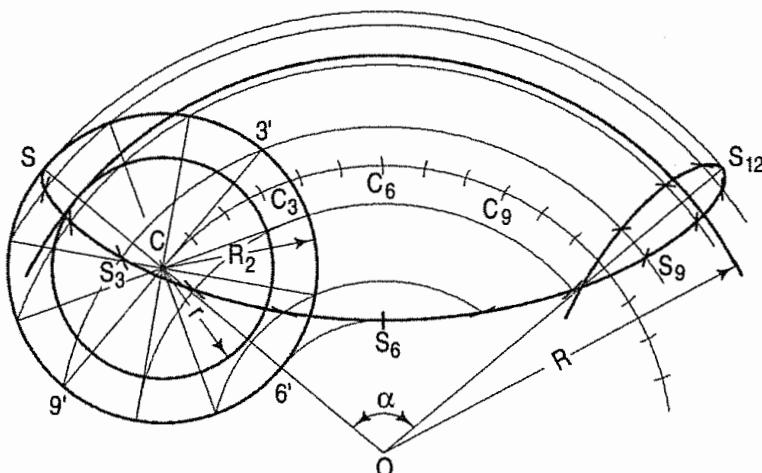
Inferior – see fig. 6-39

Note: When the diameter of the rolling circle is half the diameter of the directing circle, the hypotrochoid will be an ellipse.



Hypotrochoids

FIG. 6-40



Superior hypotrochoids

FIG. 6-41

6-3. INVOLUTE

The involute is a curve traced out by an end of a piece of thread unwound from a circle or a polygon, the thread being kept tight. It may also be defined as a curve traced out by a point in a straight line which rolls without slipping along a circle or a polygon. Involute of a circle is used as teeth profile of gear wheel. Mathematically it can be described by $x = r\cos\theta + r\theta\sin\theta$, $y = r\sin\theta - r\theta\cos\theta$, where "r" is the radius of a circle.

Problem 6-27. (fig. 6-42): *To draw an involute of a given circle.*

With centre C, draw the given circle. Let P be the starting point, i.e. the end of the thread.

Suppose the thread to be partly unwound, say upto a point 1. P will move to a position P_1 such that $1P_1$ is tangent to the circle and is equal to the arc $1P$. P_1 will be a point on the involute.

Construction:

- Draw a line PQ , tangent to the circle and equal to the circumference of the circle.
- Divide PQ and the circle into 12 equal parts.

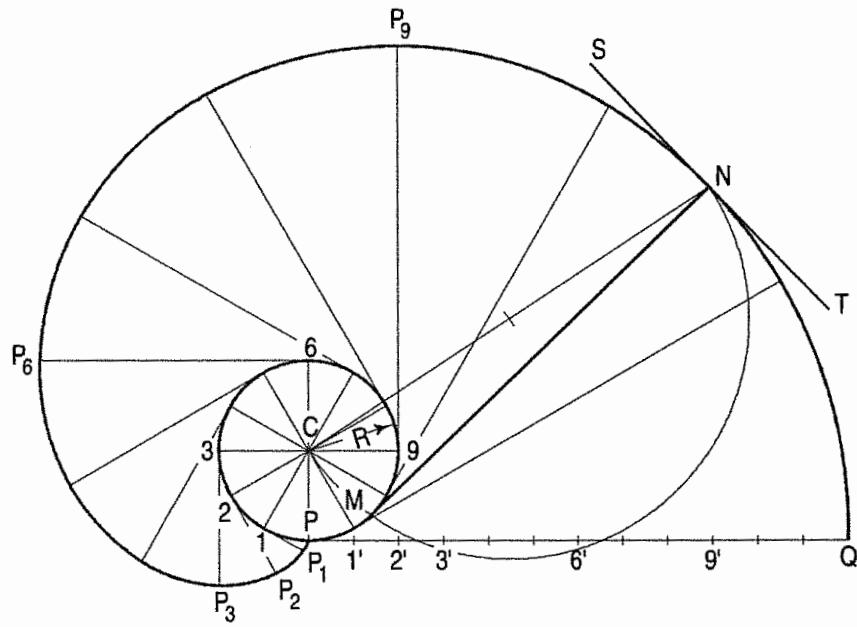
- (iii) Draw tangents at points 1, 2, 3 etc. and mark on them points P_1, P_2, P_3 etc. such that $1P_1 = P_1' = 2P_2 = P_2' = 3P_3 = P_3'$ etc.

Draw the involute through the points P, P_1, P_2, \dots etc.

Normal and tangent to an involute:

The normal to an involute of a circle is tangent to that circle.

Problem 6-28. (fig. 6-42): *To draw a normal and a tangent to the involute of a circle at a point N on it.*



Involute

FIG. 6-42

- Draw a line joining C with N.
- With CN as diameter describe a semi-circle cutting the circle at M.
- Draw a line through N and M. This line is the normal. Draw a line ST, perpendicular to NM and passing through N. ST is the tangent to the involute.

Problem 6-29. (fig. 6-43): *To draw an involute of a given square.*

Let ABCD be the given square.

- With centre A and radius AD, draw an arc to cut the line BA-produced at a point P_1 .
- With centre B and radius BP_1 (i.e. $BA + AD$) draw an arc to cut the line CB-produced at a point P_2 . Similarly, with centres C and D and radii CP_2 (i.e. $CB + BA + AD$) and DP_3 (i.e. $DC + CB + BA + AD = \text{perimeter}$) respectively, draw arcs to cut DC-produced at a point P_3 and AD-produced at a point P_4 .

The curve thus obtained is the involute of the square.

Problem 6-30. (fig. 6-44): *Trace the paths of the ends of a straight line AP, 100 mm long, when it rolls, without slipping, on a semi-circle having its diameter AB, 75 mm long. (Assume the line AP to be tangent to the semi-circle in the starting position.)*

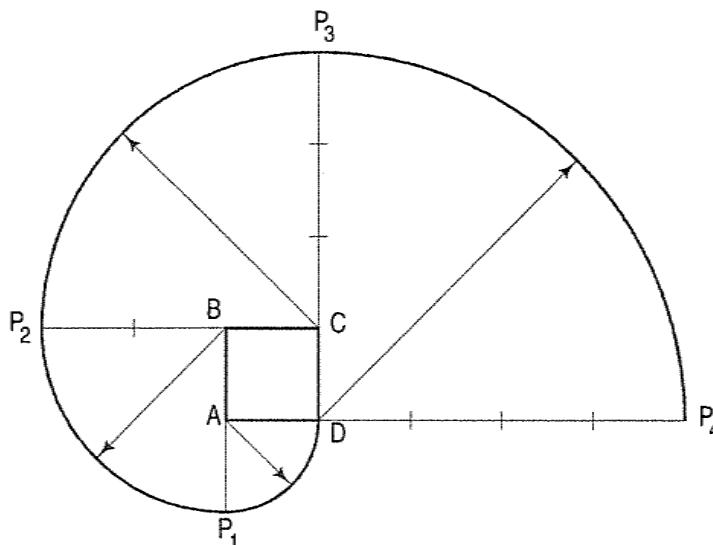


FIG. 6-43

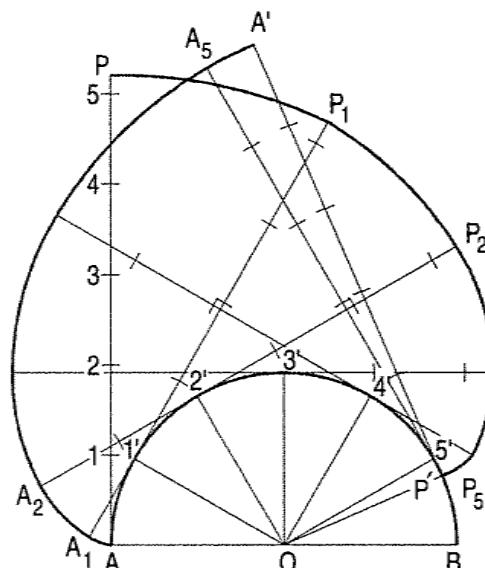


FIG. 6-44

- Draw the semi-circle and divide it into six equal parts.
- Draw the line AP and mark points 1, 2 etc. such that $A_1 = \text{arc } A_1'$, $A_2 = \text{arc } A_2'$ etc. The last division $5P$ will be of a shorter length. On the semi-circle, mark a point P' such that $5'P' = 5P$.
- At points 1', 2' etc. draw tangents and on them, mark points P_1, P_2 etc. such that $1'P_1 = 1P, 2'P_2 = 2P, \dots$ and $5'P_5 = 5'P_6 = 5P$. Similarly, mark points A_1, A_2 etc. such that $A_11' = A_1, A_22' = A_2, \dots$ and $A'P' = AP$. Draw the required curves through points P, P_1, \dots, P' , and through points A, A_1, \dots, A' .

If AP is an inelastic string with the end A attached to the semi-circle, the end P will trace out the same curve PP' when the string is wound round the semi-circle.

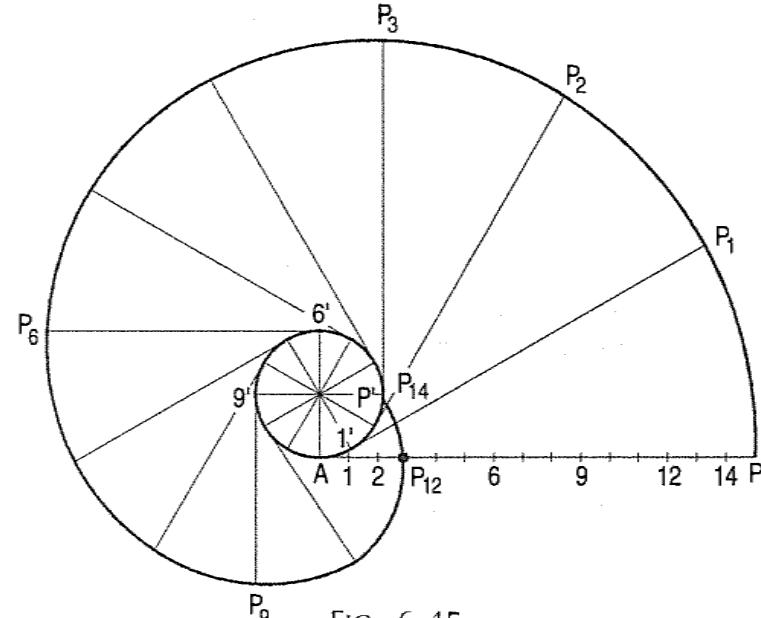


FIG. 6-45

Fig. 6-45 shows the curve traced out by the end of a thread which is longer than the circumference of the circle on which it is wound.

Note that the tangent $1'P_1 = 1P, 2'P_2 = 2P$ etc. and lastly $2'P' = 2'P_{14} = 14P$.

Problem 6-31. AP is a rope 1.50 metre long, tied to peg at A as shown in fig. 6-46. Keeping it always tight, the rope is wound round the pole. Draw the curve traced out by the end P . Use scale 1:20.

- Draw given figure to the scale.

- (ii) From A, draw a line passing through 1. A as centre and AP as radius, draw the arc intersecting extended line A1' at P_0 . Extend the side 1-2, 1 as centre and $1'P_0$ as radius, draw the arc to intersect extended line 1-2 at P_1 .
- (iii) Divide the circumference of the semicircle into six equal parts and label it as 2, 3, 4, 5, 6, 7 and 8.
- (iv) Draw a tangent to semicircle from 2 such that $2'-P_1 = 2'-P_2$. Mark 8' on this tangent such that $2'-8' = \pi R$. Divide $2'-8'$ into six equal parts.
- (v) Similarly draw tangents at 3, 4, 5, 6, 7 and 8 in anti-clockwise direction such that $3-P_3 = 3'-9'$, $4-P_4 = 4'-9'$, $5-P_5 = 5'-9'$, $6-P_6 = 6'-9'$, $7-P_7 = 7'-9'$, $8-P_8 = 8'-9'$ and $8-P_9 = 8'-9'$ respectively.
- (vi) Join P_1, P_2, \dots, P_9 with smooth curve.

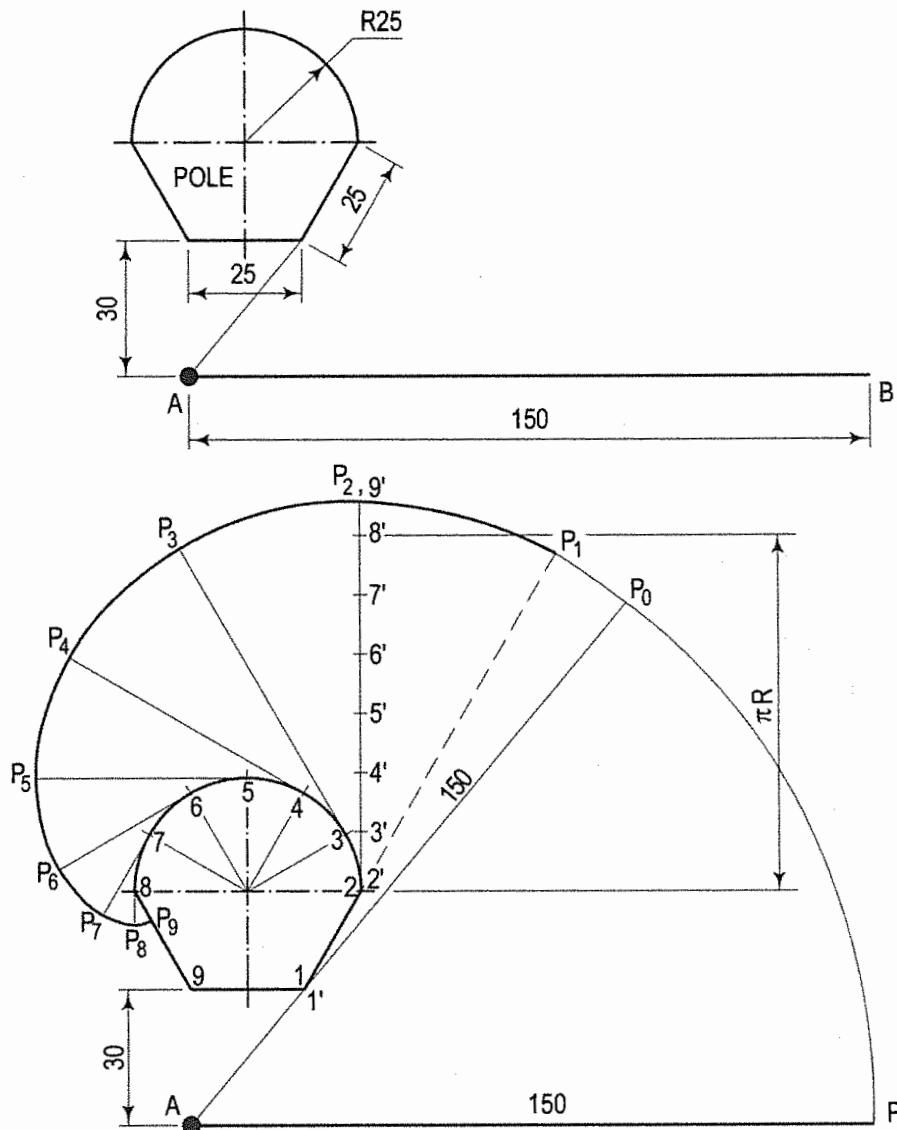


Fig. 6-46

Problem 6-32. A thin semi-circular plate with C as centre and radius equal to 32 mm is fixed. OP is the inelastic rope of 140 mm horizontal length. End O of the rope is fixed. The end P is 20 mm above and 20 mm on the left of C. The rope is wound in anti-clockwise direction around the circumference of the plate. Draw the locus of free end P of the rope.

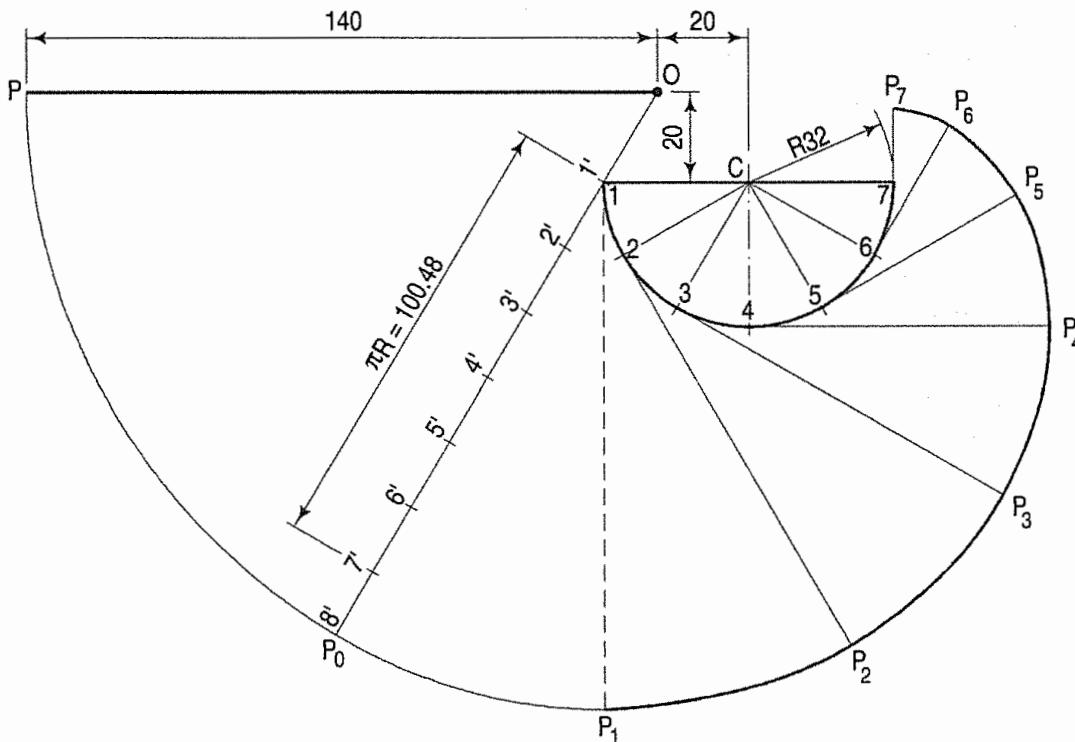


Fig. 6-47

- Taking C as centre, draw the semicircle of radius 32 mm as shown in fig. 6-47. Draw the horizontal line 20 mm above C. Mark O, 20 mm on left of C. Draw OP parallel to the diameter of the plate equal to 140 mm.
- Divide the semi-circle into six equal parts and label it 1, 2, 3, 4, 5, 6 and 7. Join all points with C. Now rotate the line OP about O, till it touches the semicircular plate at point 1. Mark the point 1 as point 1' on the rope.
- Mark 7' on line $1'P_0$ from 1' such that $1'-7' = \pi R = 100.48$ mm. Divide $1'-7'$ into six equal parts and name it as 1', 2', 3', 4', 5', 6' and 7'.
- Draw tangents on semicircular plate at 1, 2, 3, 4, 5, 6 and 7 in anticlockwise direction such that $1-P_1 = 18'$, $2-P_2 = 2'8'$, $3-P_3 = 3'8'$, $4-P_4 = 4'8'$, $5-P_5 = 5'8'$, $6-P_6 = 6'8'$, and $7-P_7 = 7'8'$ respectively.
- Join points P_1, P_2, \dots, P_7 by smooth curve. It is involute.

Problem 6-33. (fig. 6-48): A thin triangular equilateral plate of 20 mm side is pinned at its centroid O. An inelastic string circumscribes complete perimeter of the plate. One end of the string is attached to one of the apexes of the plate. Draw the curve traced out by other end of the string keeping it tight, when the string is unwounded.

- Draw an equilateral triangle of 20 mm sides. Determine its centroid by drawing perpendicular at mid point of each sides.
- Consider starting point P. Taking $PR = 20$ mm as radius and R as centre, draw the arc to intersect the extended line QR at point 1, of 40 mm as radius.
- At the point Q, extend the line PQ equal to $2QR = 40$ mm. Q as centre draw the arc to cut at the point 2.
- Extend line RP equal to $3PQ = 60$ mm P as centre, 60 mm as radius draw the arc to intersect the extended line RP at 3.
- Thus the curve obtained is involute.

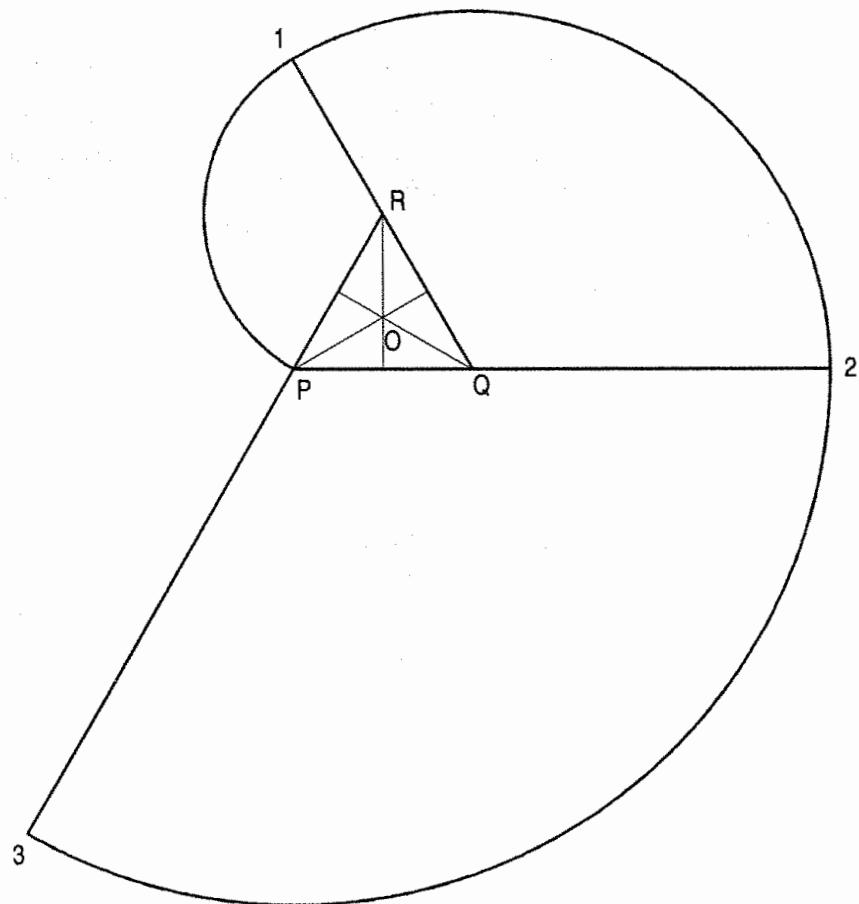


FIG. 6-48

Problem 6-34. A regular pentagonal plate of 20 mm side is fixed at its centre. An inelastic rope is circumscribed along the perimeter of the pentagonal. Draw the path of free end of the rope when it is unwound keeping, tight for one complete turn.

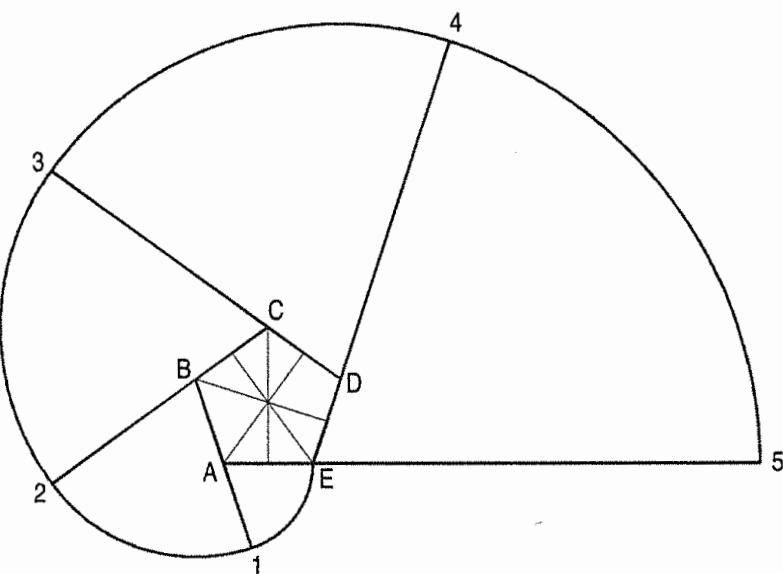


FIG. 6-49

- Construct pentagon of 20 mm side as shown fig. 6-49
- Name the corners (apex) as A, B, C, D , and E .

- (iii) Consider starting point E . Extend line BA through A . A as centre, AE as radius draw an arc starting from E and intersecting the extended line BA at 1.
- (iv) Similarly at B , C , and D extend lines. At B as centre and $B1 = (AE + AB)$ as radius, draw the arc cutting extended line CB at 2. At C and $C3 = 3A1$ as radius draw arc cutting extended line CB at 3. Similarly draw arcs for extended line ED and AE cutting at 4, 5 respectively.
- (v) Thus the curve obtained is involute.

6-4. EVOLUTES



APB is a given curve (fig. 6-50). O is the centre of a circle drawn through three points C , P and D on this curve. If the points C and D are moved to converge towards P , until they are indefinitely close together, then in the limit, the circle becomes the *circle of curvature* of the curve APB at P . The centre O of the circle of curvature lies on the normal to the curve at P . This centre is called the *centre of curvature* at P . The locus of the centre of curvature of a curve is called the *evolute* of the curve. A curve has only one evolute.

Problem 6-35 (fig. 6-51): *To determine the centre of curvature at a given point on a conic.*

Let P be the given point on the conic and F , the focus.

- (i) Join P with F .
- (ii) At P , draw a normal PN , cutting the axis at N .
- (iii) Draw a line NR perpendicular to PN and cutting PF or PF -produced at R .
- (iv) Draw a line RO perpendicular to PR and cutting PN -produced at O . Then O is the centre of curvature of the conic at the point P .

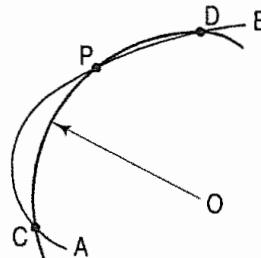


FIG. 6-50

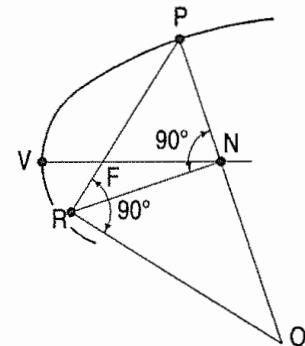


FIG. 6-51

The above construction does not hold good when the given point coincides with the vertex. As the point P approaches the vertex, the points R , N and O move nearer to one another, so that when P is at the vertex, the three points coincide on the axis.

In a parabola, PF will be equal to FR . Hence, when P is at V the vertex, the centre of curvature O is on the axis so that $OF = VF$.

In an ellipse or a hyperbola, the ratio of the distances of P from the foci is equal to the ratio of focal distances of N , i.e. $\frac{PF}{PF_1} = \frac{NF}{NF_1}$.

Hence, when P coincides with V the vertex, the ratio becomes $\frac{VF}{VF_1} = \frac{OF}{OF_1}$.

Similarly, when P coincides with the other vertex V_1 , the ratio becomes $\frac{V_1F_1}{V_1F} = \frac{O_1F_1}{O_1F}$.

Problem 6-36. To determine the centre of curvature O , when the point P is at the vertex V of a conic.

- (a) **Parabola** (fig. 6-52): Mark the centre of curvature O on the axis such that $OF = VF$.
- (b) **Ellipse** (fig. 6-53):

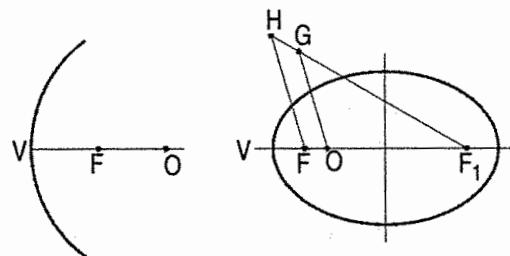


FIG. 6-52

FIG. 6-53

Method I:

- (i) Draw a line F_1G inclined to the axis and equal to VF_1 .
- (ii) Produce F_1G to H so that $GH = VF$. Join H with F .
- (iii) Draw a line GO parallel to HF and intersecting the axis at O . Then O is the required centre of curvature.

Method II: (fig. 6-54):

- (i) Draw a rectangle $AOCE$ in which
 $AO = \frac{1}{2}$ major axis and
 $CO = \frac{1}{2}$ minor axis.

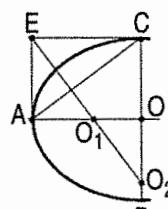


FIG. 6-54

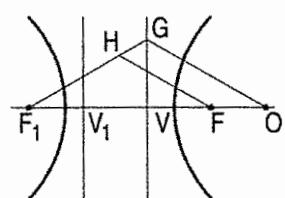


FIG. 6-55

- (ii) Join A with C .
- (iii) Through E , draw a line perpendicular to AC and cutting the major axis at O_1 and the minor axis O_2 . Then O_1 and O_2 are the centres of curvature when the point P is at A and C respectively.

(c) **Hyperbola** (fig. 6-55):

- (i) Draw a line F_1G inclined to the axis and equal to FV_1 .
- (ii) On F_1G , mark a point H such that $HG = VF$. Join H with F .
- (iii) Draw a line GO parallel to HF and cutting the axis at O . Then O is the centre of curvature at the vertex V .

Problem 6-37. (fig. 6-56 and fig. 6-57): To draw the evolute of an ellipse.

The ellipse with major axis AB and minor axis CD is given.

- (i) Mark a number of points on the ellipse.
- (ii) Determine the centres of curvatures at these points (as shown at the point P) and draw a smooth curve through them. This is the evolute of the ellipse. The evolute may sometimes go outside the ellipse as shown in fig. 6-57. The centres of curvature at points A and C are shown by method II of problem 6-36.

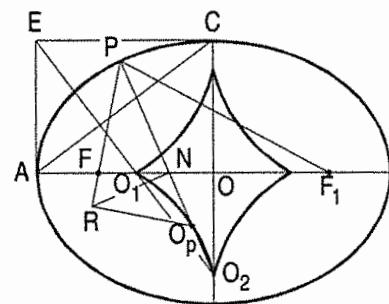


FIG. 6-56

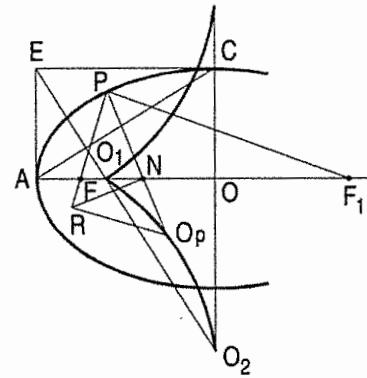


FIG. 6-57

Problem 6-38. (fig. 6-58): *To draw the evolute of a parabola.*

- Mark a number of points on the parabola and determine the centres of curvature at these points (as shown at the point P).
- Draw the evolute through these centres. Note that $PF = FR$.

Problem 6-39. (fig. 6-59): *To draw the evolute of a hyperbola.*

- Mark a number of points on the hyperbola and determine the centres of curvature at these points (as shown at the point P).
- Draw the evolute through these centres. To obtain the centre of curvature at the vertex, the position of the other focus F_1 must be found.

It is determined by making use of the following rule:

The tangent at any point on the curve bisects the angle made by lines joining that point with the two foci, i.e. $\angle F_1QC = \angle FQC$.

Problem 6-40. (fig. 6-60): *To draw the evolute of a cycloid.*

- Mark a point P on the cycloid and draw the normal PN to it (Problem 6-21).
- Produce PN to O_p so that $NO_p = PN$. O_p is the centre of curvature at the point P .
- Similarly, mark a number of points on the cycloid and determine centres of curvature at these points. The curve drawn through these centres is the evolute of the cycloid. It is an equal cycloid.

Problem 6-41. (fig. 6-61): *To draw the evolute of an epicycloid.*

- Mark a point P on the epicycloid and draw the normal PN to it (Problem 6-24 and problem 6-25).
- Draw the diameter PQ of the rolling circle. Join Q with O , the centre of the directing circle.
- Produce PN to cut QO at O_p , which is the centre of curvature at the point P .
- Mark a number of points on the epicycloid and similarly, obtain centres of curvature at these points. The curve drawn through these centres is the evolute of the epicycloid.

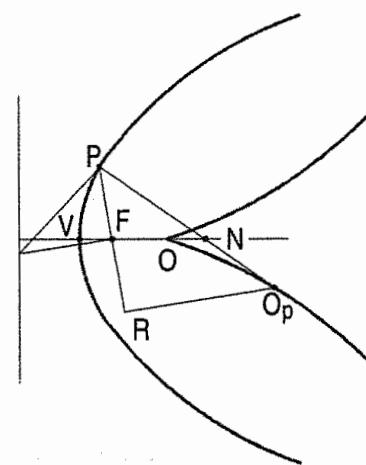


FIG. 6-58

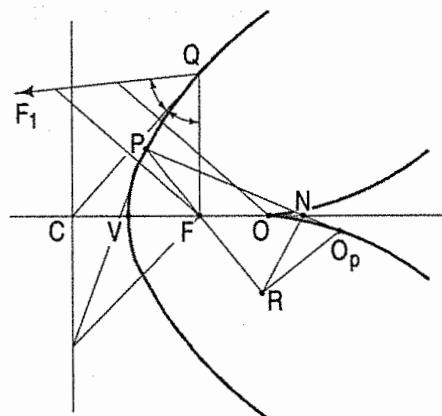


FIG. 6-59

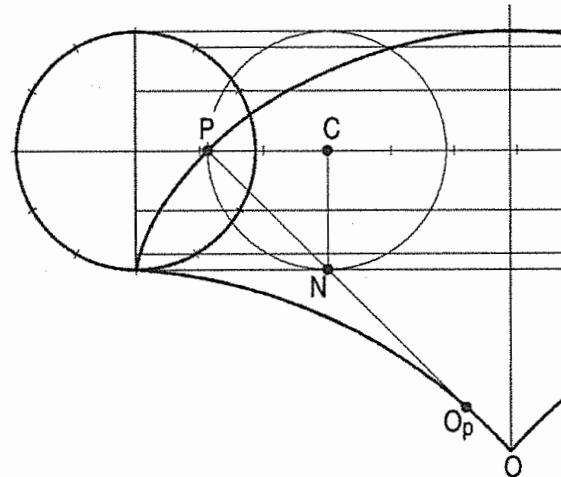


FIG. 6-60

Through O_p , draw a line perpendicular to PO_p and intersecting the line joining C (the centre of the rolling circle) with O at a point R. The evolute is the epicycloid of the circle of diameter NR, rolling along the circle of radius OR.

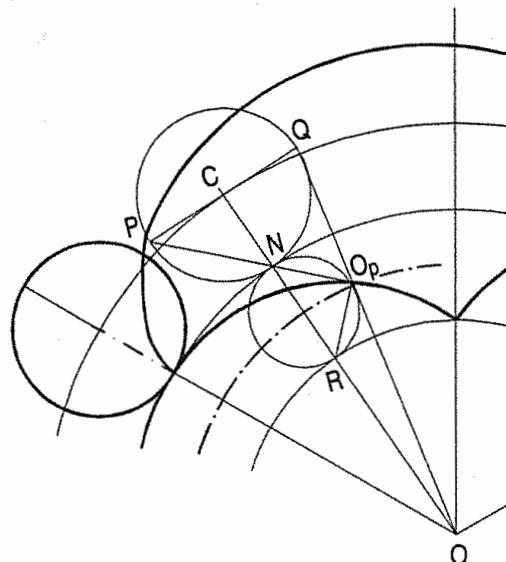


FIG. 6-61

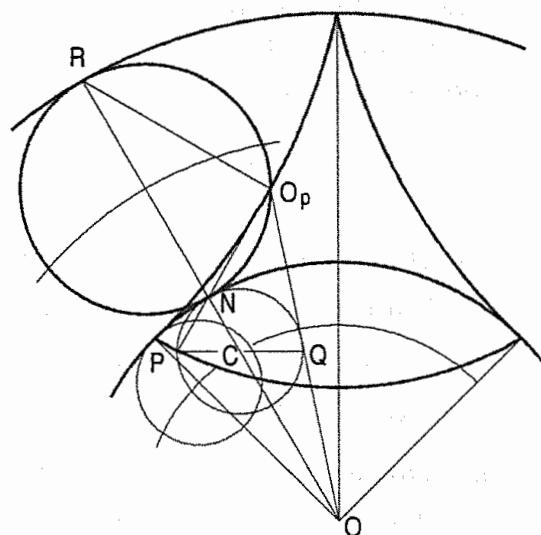


FIG. 6-62

Problem 6-42. (fig. 6-62): *To draw the evolute of a hypocycloid.*

- Mark a point P on the hypocycloid and draw the normal PN to it (Problem 6-24 and problem 6-25).
- Draw the diameter PQ of the rolling circle. Join Q with O, the centre of the directing circle.
- Produce PN to cut OQ -produced at O_p , which is the centre of curvature at the point P .
- Mark a number of points on the hypocycloid and similarly, obtain centres of curvature at these points. The curve drawn through these centres is the evolute of the hypocycloid.

Through O_p , draw a line perpendicular to PO_p and intersecting OC -produced at a point R. The evolute is the hypocycloid of the circle of diameter NR rolling along the circle of radius OR.

Problem 6-43. (fig. 6-42): *To draw the evolute of an involute of a circle.*

In the involute of a circle, the normal NM at any point N is tangent to the circle at the point of contact M. M is the centre of curvature at the point N. Hence, the evolute of the involute is the circle itself.

6-5. SPIRALS



If a line rotates in a plane about one of its ends and if at the same time, a point moves along the line continuously in one direction, the curve traced out by the moving point is called a *spiral*. The point about which the line rotates is called a *pole*.

The line joining any point on the curve with the pole is called the *radius vector* and the angle between this line and the line in its initial position is called the *vectorial angle*.

Each complete revolution of the curve is termed the *convolution*. A spiral may make any number of convolutions before reaching the pole.

6-5-1. ARCHEMEDIAN SPIRAL



It is a curve traced out by a point moving in such a way that its movement towards or away from the pole is uniform with the increase of the vectorial angle from the starting line.

The use of this curve is made in teeth profiles of helical gears, profiles of cams etc.

Problem 6-44. (fig. 6-63): *To construct an Archimedean spiral of $1\frac{1}{2}$ convolutions, given the greatest and the shortest radii.*

Let O be the pole, OP the greatest radius and OQ the shortest radius.

(i) With centre O and radius equal to OP, draw a circle. OP revolves around O for $1\frac{1}{2}$ revolutions. During this period, P moves towards O, the distance equal to $(OP - OQ)$ i.e. QP.

(ii) Divide the angular movements of OP, viz $1\frac{1}{2}$ revolutions i.e. 540° , and the line QP into the same number of equal parts, say 18 (one revolution divided into 12 equal parts).

When the line OP moves through one division, i.e. 30° , the point P will move towards O by a distance equal to one division of QP to a point P_1 .

(iii) To obtain points systematically draw arcs with centre O and radii O₁, O₂, O₃ etc. intersecting lines O_{1'}, O_{2'}, O_{3'} etc. at points P_1 , P_2 , P_3 etc. respectively.

In one revolution, P will reach the 12th division along QP and in the next half revolution it will be at the point P_Q on the line 18'-O.

(iv) Draw a curve through points P, P_1 , P_2 , P_Q . This curve is the required Archimedean spiral.

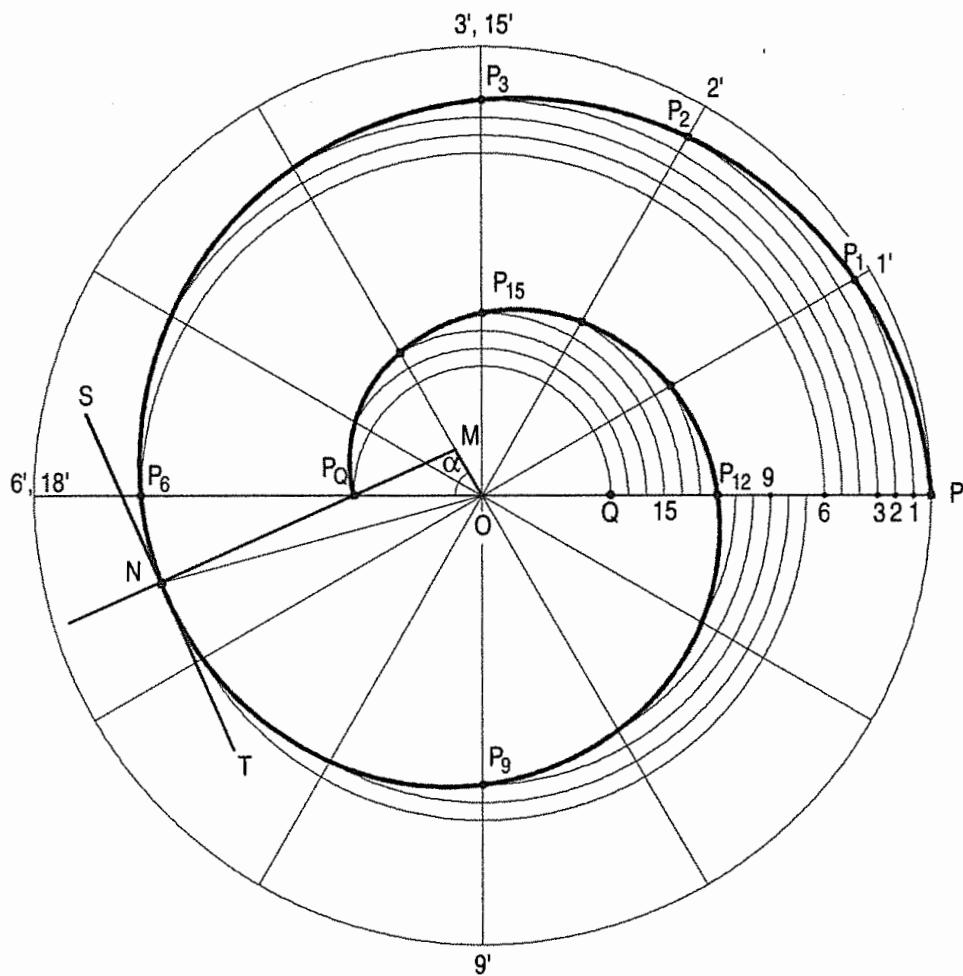
Normal and tangent to an Archimedean spiral: The normal to an Archimedean spiral at any point is the hypotenuse of the right-angled triangle having the other two sides equal in length to the *radius vector* at that point and the *constant of the curve* respectively.

The constant of the curve is equal to the difference between the lengths of any two radii divided by the circular measure of the angle between them.

OP and OP₃ (fig. 6-63) are two radii making 90° angle between them. In circular measure in radian, $90^\circ = \frac{\pi}{2} = 1.57$. Therefore, the constant of the curve, C = $\frac{OP - OP_3}{1.57}$.

Problem 6-45. (fig. 6-63): *To draw a normal to the Archimedean spiral at a point N on it.*

- Draw the radius vector NO.
- Draw a line OM equal in length to the constant of the curve C and perpendicular to NO.
- Draw the line NM which is the normal to the spiral.
- Through N, draw a line ST perpendicular to NM. ST is the tangent to the spiral.



Archimedean spiral

FIG. 6-63

Problem 6-46. (fig. 6-64): A link 225 mm long, swings on a pivot O from its vertical position of rest to the right through an angle of 75° and returns to its initial position at uniform velocity. During that period, a point P moving at uniform speed along the centre line of the link from a point at a distance of 25 mm from O , reaches the end of the link. Draw the locus of the point P .

- Draw a vertical line OA , 225 mm long.
- With centre O and radius equal to OA , draw an arc.
- Draw a line OB making $\angle AOB$ equal to 75° and cutting the arc at B .
- Mark a point P along OA and at a distance of 25 mm from O .
- Divide the angular movement of the link and the line PA into the same number of equal parts, say 8.

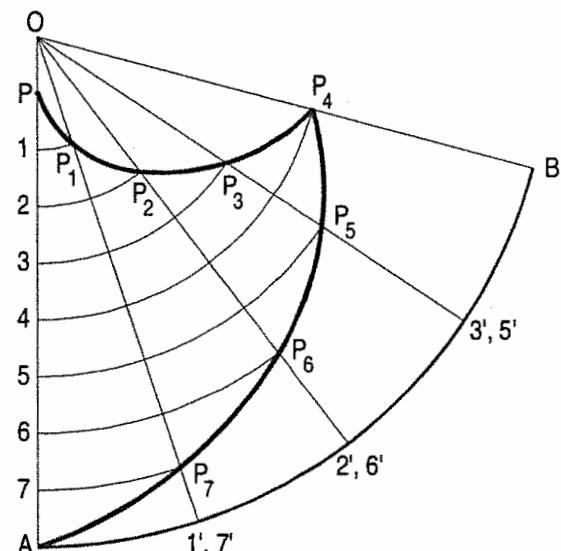


FIG. 6-64

The end A of the link moves to B and returns to its original position. Hence, the arc AB must be divided into four equal parts.

- (vi) With centre O and radii O_1, O_2, O_3 etc., draw arcs intersecting lines O_1', O_2', O_3' etc. at points P_1, P_2, P_3 etc. respectively. Draw a curve through $P, P_1, \dots, P_4, \dots, A$. This curve is the locus of the point P .

Problem 6-47. (fig. 6-65): A straight link PQ of 60 mm length revolves one complete revolution with uniform motion in anti-clockwise direction about hinged P . During this period an insect moves along the link from P to Q and Q to P with uniform linear motion. Draw the path of the insect and name the curve.

- Draw circle of 60 mm radius at P as centre and divide into twelve equal parts. Name in anti-clockwise direction 1, 2, 3, 4, 5, ..., 12.
- Divide line PQ into six equal part and mark as 1', 2', 3', 4', 5' and 6' and from Q as 7' to 12'.
- Mark 1", 2", 3", 4", 5" and 6" along radius P_1, P_2, P_3, P_4, P_5 and P_6 respectively increasing each division of the line PQ as shown in fig 6-65.
- Bottom portion is the mirror image of top portion. So drop the respective point on bottom radii.
- Join them with the smooth curve. The curve is arched median spiral.

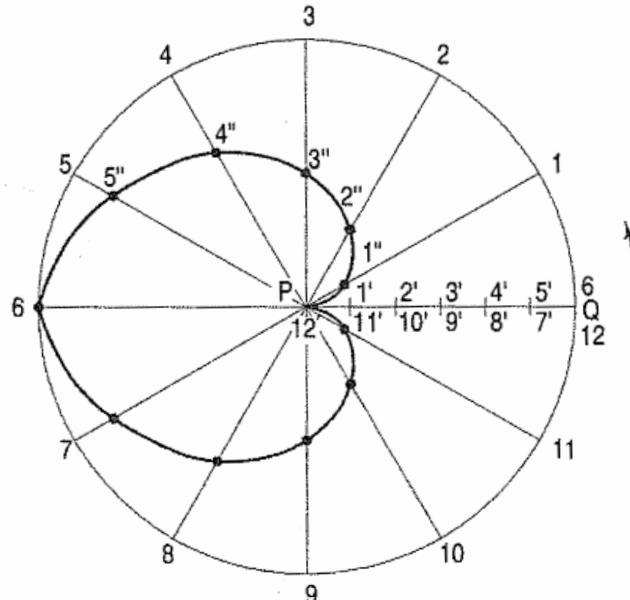


FIG. 6-65

6-5-2. LOGARITHMIC OR EQUIANGULAR SPIRAL

In a *logarithmic spiral*, the ratio of the lengths of consecutive radius vectors enclosing equal angles is always constant. In other words the values of vectorial angles are in *arithmetical progression* and the corresponding values of *radius* vectors are in *geometrical progression*.

The logarithmic spiral is also known as *equiangular spiral* because of its property that the angle which the tangent at any point on the curve makes with the radius vector at that point is constant.

Problem 6-48. (fig. 6-66): To construct a logarithmic spiral of one convolution, given the length of the shortest radius and the ratio of the lengths of radius vectors enclosing an angle θ .

Assume the shortest radius be 1 cm long, θ equal to 30° and the ratio $\frac{10}{9}$.

The lengths of radius vectors are determined from the scale which is constructed as shown below:

- Draw lines AB and AC [fig. 6-66(ii)] making an angle of 30° between them.
- On AB , mark AD , 1 cm long. On AC , mark a point E such that $AE = \frac{10}{9} \times 1 \text{ cm} = \frac{10}{9} \text{ cm}$.

- (iii) Draw a line joining D with E .
- (iv) With centre A and radius AE , draw an arc cutting AB at a point 1. Through 1, draw a line 1-1' parallel to DE and cutting AC at 1'. Again, with centre A and radius $A1'$ draw an arc cutting AB at 2. Through 2, draw a line 2-2' parallel to DE and cutting AC at 2'. Repeat the construction and obtain points 3, 4.....12. $A1, A2$ etc. are the lengths of consecutive radius vectors.
- (v) Draw a horizontal line OQ and on it, mark OP , 1 cm long [fig. 6-66(i)]. Through O , draw radial lines making 30° angles between two consecutive lines. These are the radius vectors.
- (vi) Mark points P_1, P_2, \dots, P_{12} on consecutive radius vectors such that $OP_1 = A1, OP_2 = A2, \dots, OP_{12} = A12$.
- (vii) Draw a smooth curve through P_1, P_2, \dots, P_{12} . This curve is the required logarithmic spiral.

The equation to the logarithmic spiral is $r = a^\theta$ where r is the radius vector, θ is the vectorial angle and a is a constant.

Hence, $\log r = \theta \log a$.

If $\theta = 0, \log r = O \therefore r = 1$.

When $\theta = 30^\circ = \frac{\pi}{6}$ radians, $r = \frac{10}{9} \times 1 = \frac{10}{9} \therefore \log \frac{10}{9} = \frac{\pi}{6} \log a$

i.e. $\log a = \frac{6}{\pi} \log \frac{10}{9} \therefore a = 1.22$.

Problem 6-49. [fig. 6-66(i)]: To draw a normal and a tangent at a given point N on the logarithmic spiral in problem 6-48.

The value of the tangent to the constant angle is obtained by the formula given below:

$$\tan \alpha = \frac{\log e}{\log a}, \quad \text{where } e = 2.718$$

$$\text{i.e. } \tan \alpha = \frac{\log 2.718}{\frac{6}{\pi} \log \frac{10}{9}} \quad \therefore \alpha = 78^\circ - 38'$$

- (i) Draw a line joining N with O .
- (ii) Draw a line NT making an angle of $78^\circ - 38'$ with NO . NT is the required tangent. NR drawn perpendicular to NT is the normal to the spiral.

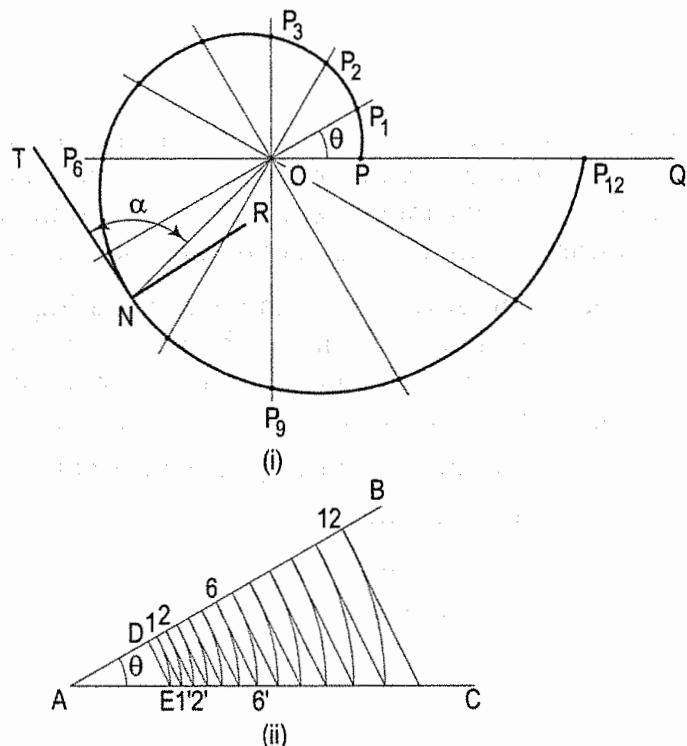


FIG. 6-66

6-6. HELIX



Helix is defined as a curve, generated by a point, moving around the surface of a right circular cylinder or a right circular cone in such a way that, its axial advance, i.e. its movement in the direction of the axis of the cylinder or the cone is uniform with its movement around the surface of the cylinder or the cone.

The axial advance of the point during one complete revolution is called the *pitch* of the helix. If the pitch is say 20 mm and the point starts upwards from the base of the cylinder, in one-fourth of a revolution, the point will move up a distance of 5 mm from the base. We shall now learn in the articles to follow:

1. A method of drawing a helical curve
2. Helical springs
3. Screw threads
4. Helix upon a cone.

6-6-1. A METHOD OF DRAWING A HELICAL CURVE



Problem 6-50. (fig. 6-67): To draw a helix of one convolution, given the pitch and the diameter of the cylinder. Also to develop the surface of the cylinder showing the helix in it.

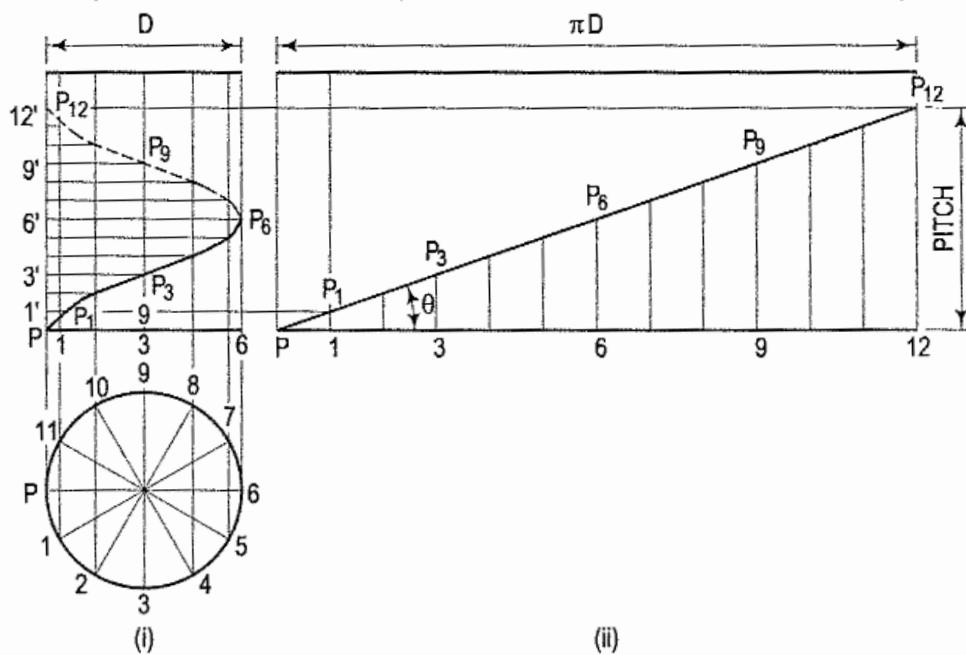


FIG. 6-67

- (i) Draw the projections of the cylinder [fig. 6-67(i)]. Divide the circle (in the top view) into any number of equal parts, say 12.
- (ii) Mark a length $P - 12'$ equal to the given pitch, along a vertical side of the rectangle in the front view and divide it into the same number of equal parts, viz. 12.
- (iii) Assume the point P to move upwards and in anti-clockwise direction. When it has moved through 30° around the circle, it will have moved up by one division. To locate this position, draw a vertical line through the point 1 and a horizontal line through the point 1', both intersecting each other at a point P_1 , which will be on the helix.

- (iv) Obtain other points in the same manner and draw the helix through them. The portion of the curve from P_6 to P_{12} will be on the back side of the cylinder and hence it will not be visible.

Fig. 6-67(ii) shows the development of the surface of the cylinder. The helix is seen as a straight line and is the hypotenuse of a right-angled triangle having base equal to the circumference of the circle and the vertical side equal to the pitch of the helix. The angle θ which it makes with the base, is called the *helix angle*. The helix angle can be expressed as $\tan\theta = \frac{\text{pitch}}{\text{circumference of the circle}}$.

6-6-2. HELICAL SPRINGS



In a spring having a wire of square cross-section, the outer two corners of the section may be assumed to be moving around the axis, on the surface of a cylinder having a diameter equal to the outside diameter of the spring. The inner two corners of the section will move on the surface of a cylinder having a diameter, equal to the inside diameter of the spring. The pitch in case of each corner will be the same.

- (1) Helical spring of a wire of square cross-section.
- (2) Helical spring of a wire of circular cross-section.

(1) Helical spring of a wire of square cross-section.

Problem 6-51. (fig. 6-68): Draw the projections of two complete turns of a spring of a wire of square section of 20 mm side. Outside diameter of the spring = 110 mm; pitch = 60 mm.

Instead of full circles, semi-circles of 110 mm and 70 mm diameters for the outside and the inside diameters of the spring may be drawn in the top view.

- (i) Divide each semi-circle into $\frac{12}{2}$, i.e. 6 equal parts.
- (ii) Project up the division-points as lines in the front view. Let PQSR be the square section of the wire.
- (iii) In one revolution, the point P will move to P' so that $PP' = 60$ mm. Similarly, RR' , QQ' and SS' will each be equal to 60 mm.
- (iv) Divide these distances into 12 equal parts and plot the helices traced out by P and R on the outer surface and by Q and S on the inner surface, as shown. The outer helices will be parallel to each other. Note carefully the visible and the hidden parts of the curves.

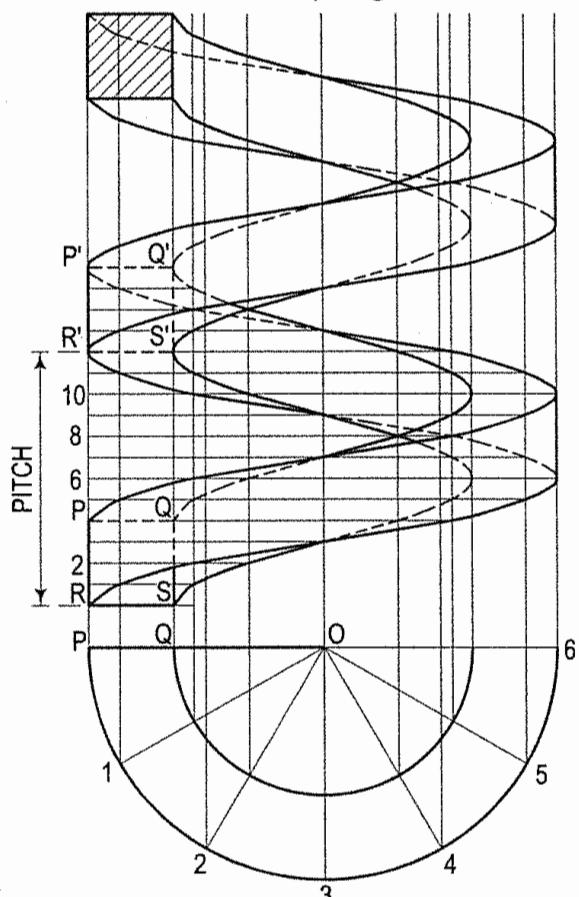


FIG. 6-68

(2) Helical spring of a wire of circular cross-section.

When the wire is of circular cross-section, a helical curve for the centre of the cross-section is first traced out. A number of circles of diameter equal to that of the cross-section are then drawn with a number of points on this curve as centres. Curves, tangent to these circles, will give the front view of the spring.

Problem 6-52. (fig. 6-69): Draw two complete turns of a helical spring of circular cross-section of 20 mm diameter. Spring index and pitch are 6 and 60 mm respectively.

$$\text{Spring index} = \frac{\text{Mean diameter of a coil}}{\text{Diameter of wire}}$$

$$\begin{aligned} \text{(a) Mean diameter of a coil} &= \text{Spring index} \times \text{Diameter of wire} \\ &= 6 \times 20 \\ &= 120 \text{ mm.} \end{aligned}$$

$$\begin{aligned} \text{(b) Outside diameter of a coil} &= \text{Mean diameter} + \text{Diameter of wire} \\ &= 120 \text{ mm} + 20 \text{ mm} \\ &= 140 \text{ mm.} \end{aligned}$$

$$\begin{aligned} \text{(c) Inside diameter of a coil} &= \text{Mean diameter} - \text{Diameter of wire} \\ &= 120 \text{ mm} - 20 \text{ mm} \\ &= 100 \text{ mm.} \end{aligned}$$

- (i) Draw the three concentric circles of diameters 140 mm, 120 mm and 100 mm representing the outside, mean and the inside circles of coil of the spring respectively in the top view.
- (ii) Divide the circles into 12 divisions. Mark the divisions on the circle of the mean diameter as 1, 2, 3 etc.
- (iii) Draw the projectors passing through these divisions in the front view. Mark the pitch length along one of the projectors which is parallel to the axis of the spring.
- (iv) Divide the pitch length into 12 divisions and name these division points as 1', 2', 3' etc. Draw the horizontal lines intersecting the previously drawn projectors from 1, 2, 3 etc.
- (v) With the intersection point as centre and radius equal to 10 mm, draw circles. Draw the curve touching the top and bottom of circle of the wire as shown in fig. 6-69. Repeat the process for one more pitch length.

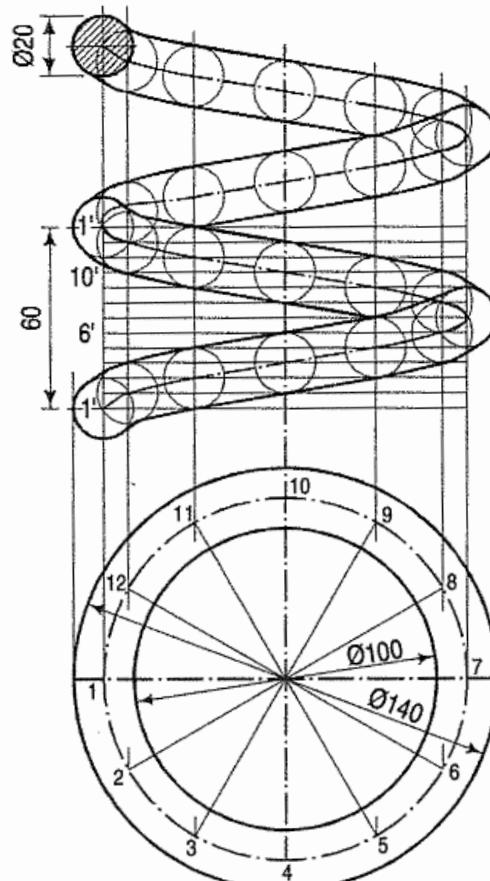


FIG. 6-69

6-6-3. SCREW THREADS



These are also constructed on the principle of the helix.

In a screw thread, the *pitch* is defined as the distance from a point on a thread to a corresponding point on the adjacent thread, measured parallel to the axis. The axial advance of a point on a thread, per revolution, is called the *lead* of the screw.

In the single-threaded screws, which are most commonly used in practice, the pitch is equal to lead. Therefore, the pitch of the screw is equal to the pitch of the helix. Unless stated otherwise, screws are always assumed to be single-threaded.

Problem 6-53. (fig. 6-70): Project two complete turns of a square thread having outside diameter 120 mm and pitch 45 mm.

In a square thread, the thickness of the thread = the depth of the thread = $0.5 \times \text{pitch}$. Hence, the section of the thread will be a square of 22.5 mm side and the diameter at the bottom of the thread will be 75 mm.

Project the threads in the same manner as the spring in problem 6-51. The screw differs from the spring in having a solid cylinder inside, which completely hides the back portions of the curves.

In double-threaded screws, two threads of the same size run parallel to each other. The axial advance per revolution, viz. the lead, is made twice the lead of the single-threaded screw, the pitch of the thread being kept the same in both cases.

Hence, in double-threaded screws,

Pitch of the helix = Lead = Twice the pitch of the screw.

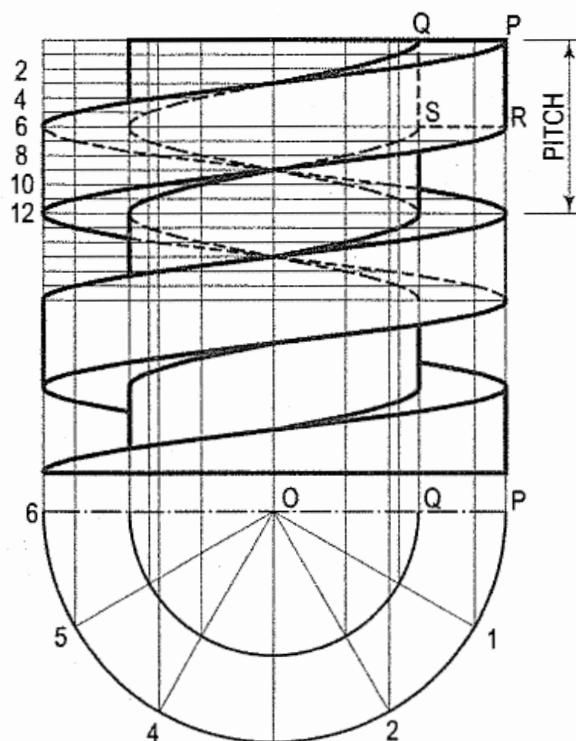


FIG. 6-70

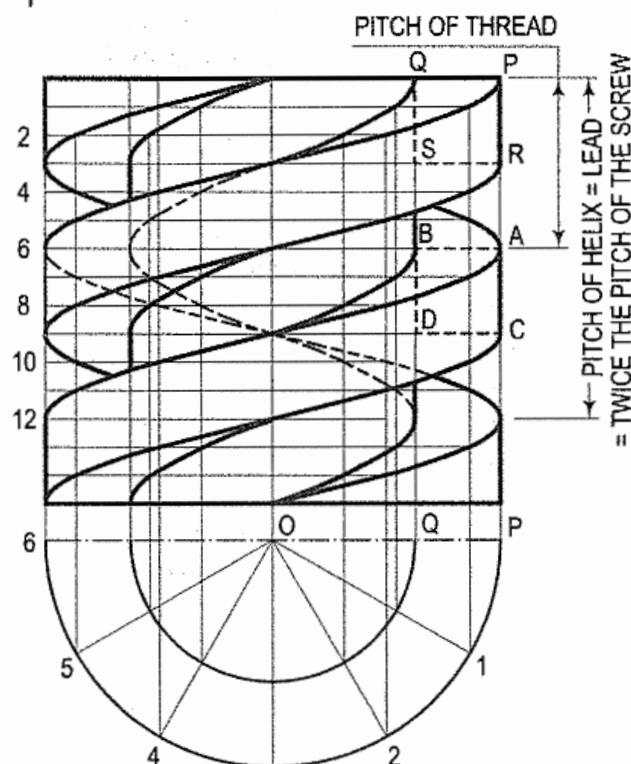


FIG. 6-71

Fig. 6-71 shows a double-threaded screw, having the same cross-section and the same pitch of the screw as in problem 6-53.

6-6-4. HELIX UPON A CONE



This curve is traced out by a point which, while moving around the axis and on the surface of the cone, approaches the apex. The movement around the axis is uniform with its movement towards the apex, measured parallel to the axis. The pitch of the helix is measured parallel to the axis of the cone.

As the whole surface of the cone is visible in its top view, the helix will be fully seen in it.

Problem 6-54. (fig. 6-72): Draw a helix of one convolution upon a cone, diameter of base 75 mm, height 100 mm and pitch 75 mm. Also develop the surface of the cone, showing the helix on it.

- Draw the projections of the cone as shown. Divide the circle into twelve equal parts and join points 1, 2 etc. with o . Project these points to the base-line in the front view and join them with o' .
- Mark a point A on the axis at a distance of 75 mm from the base. Draw a horizontal line through A to cut the generators $o'P$ at A' . Divide PA' into twelve equal parts.
- Let P be the starting point. When it has moved around through 30° , it will have moved up through one division to a point p'_1 on the generator $o'1'$, obtained by drawing a horizontal line through $1''$. p'_1 will be a point on the helix in the front view. Its projection to p_1 on o_1 , will be the position of the point in the top view.
- Obtain all the points in this manner and draw smooth curves through them in both the views.
- Draw the development of the surface of the cone and locate points p_1, p_2 etc.

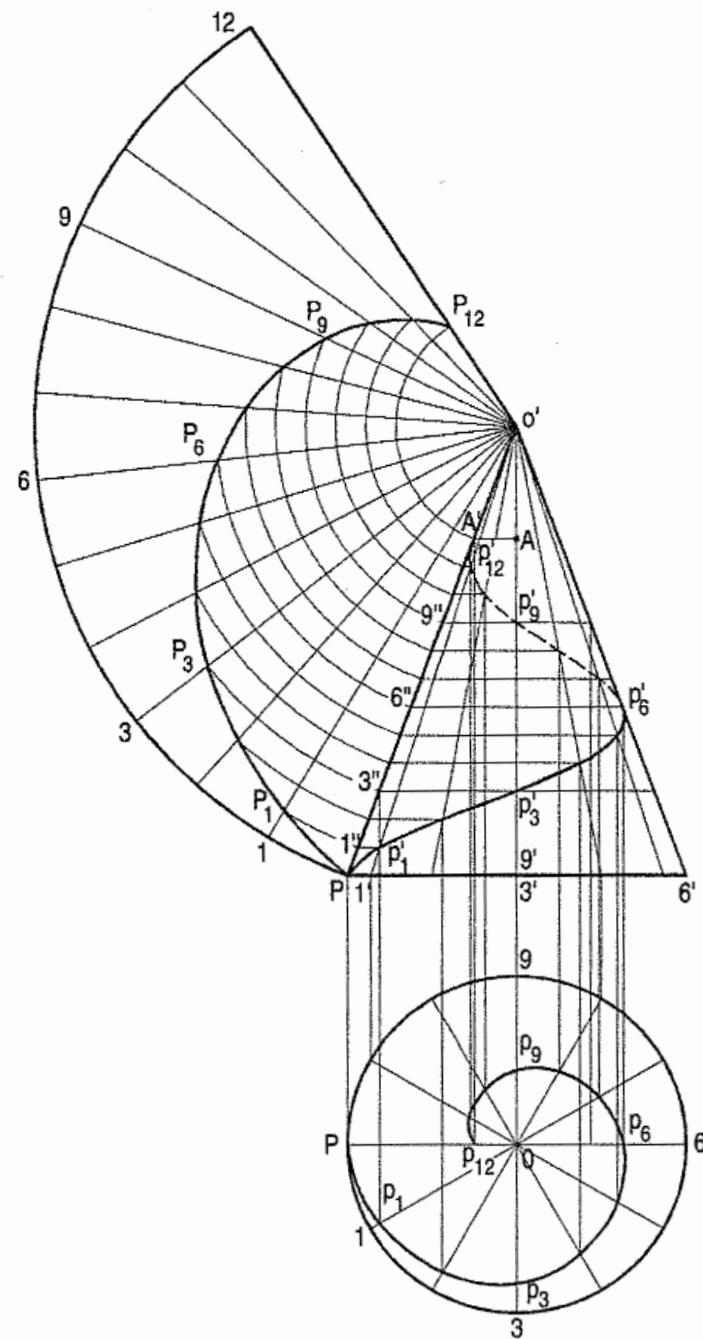


FIG. 6-72

Problem 6-55. (fig. 6-73): A conical spring of a bicycle's seat has following specifications. Draw the front view and the top view of the spring. Only one-turn is sufficient.

- (i) Outer diameter of the coil at the bottom 72 mm
- (ii) Outer diameter of the coil at the top 42 mm
- (iii) Wire diameter 10 mm
- (iv) Pitch 60 mm.
 - (a) Draw a circle of mean diameter of 72 mm in the top view.
 - (b) Divide the circle into 12 divisions. Mark the divisions 1, 2, 3 etc. as shown in fig. 6-73.
 - (c) Draw the front view of the cone with the base length 72 mm.
 - (d) Draw pitch length 60 mm along the line parallel to the axis of the cone. Divide this pitch length into 12 divisions and number them as 1', 2', 3' etc.
 - (e) Draw the vertical projectors from these points 1, 2, 3 etc. to intersect the horizontal lines from the points 1', 2', 3' etc. With the intersection point as centre and radius equal to 5 mm, draw the circles and the smooth curve touching to the top and bottom of the circles as shown in fig. 6-73.

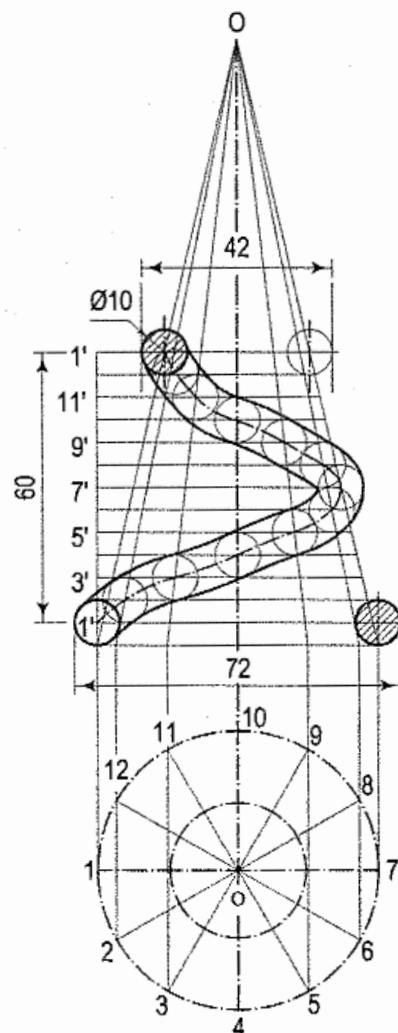


FIG. 6-73

6-7. CAM

A cam is a machine-part which, while rotating at uniform velocity, imparts reciprocating linear motion or oscillating motion to another machine-part called a follower. The motion imparted may be either uniform or variable, depending upon the shape of the cam profile.

The shape of the cam to transmit uniform linear motion is determined by the application of the principle of Archimedean spiral, as shown in problem 6-44.

The cams are widely used in automates, printing machines, an I.C. engines etc. The shape of cam depends upon the motion required for the followers. The terminology of the cam and follower is shown in fig. 6-74.

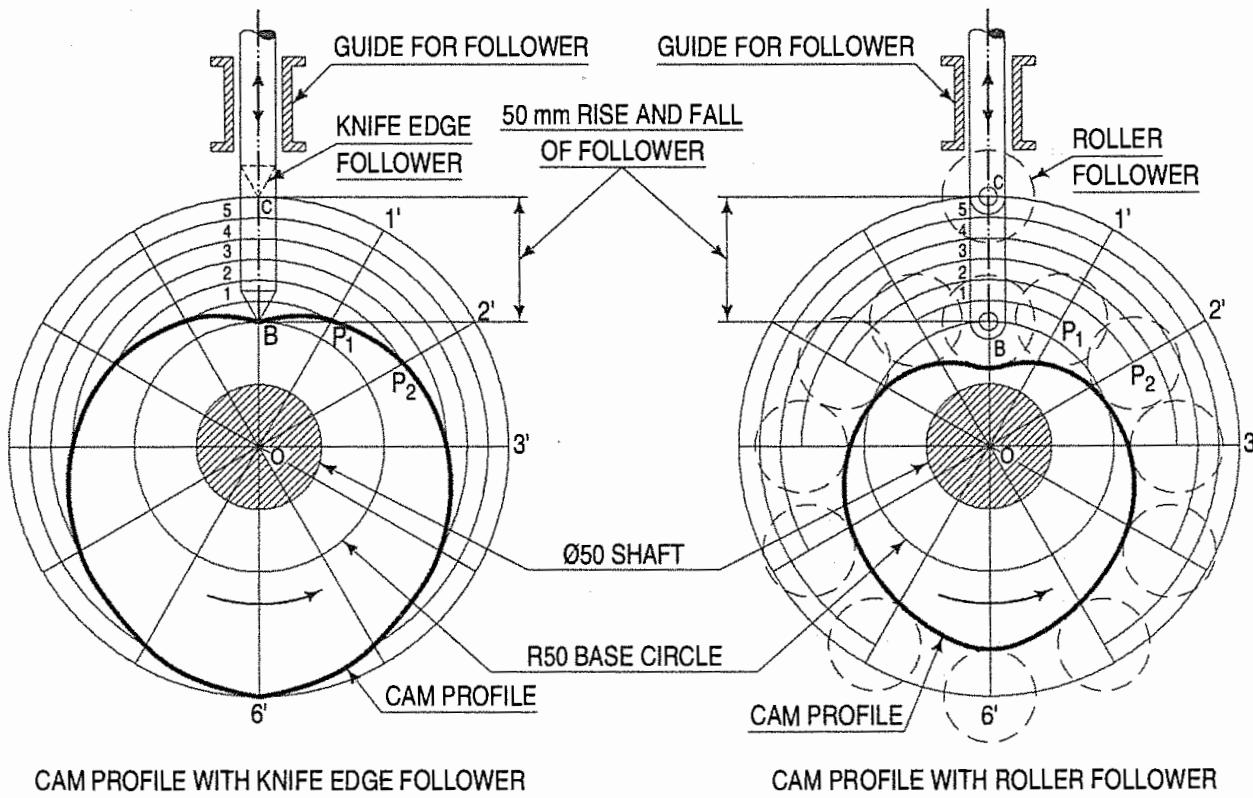
Problem 6-56. (fig. 6-74): Draw the shape of a cam to give a uniform rise and fall of 50 mm to a point, during each revolution of the cam. Diameter of the shaft = 50 mm; least radius of the cam = 50 mm.

- (i) Draw a circle (for the shaft) with centre O. With the same centre and radius OB equal to 50 mm (the least radius of the cam), draw another circle.
- (ii) Produce OB to C making BC equal to 50 mm (the rise of the point). The point is to be uniformly raised during $\frac{1}{2}$ revolution of the shaft.
- (iii) Therefore, divide BC and angle 180° into the same number of equal parts, say 6.

Obtain points P_1 , P_2 etc. as in the Archimedean spiral and draw the curve through them.

The point is to fall the same distance during the same period. Hence, the curve will be exactly of the same shape as for the rise.

The followers are generally provided with rollers to give smooth working. In such cases, the profile of the cam is designed initially, to transmit motion to a point (the centre of the roller) and then, a parallel curve is drawn inside it at a distance equal to the radius of the roller. This is done by first drawing a number of arcs with points on the curve as centres and radius equal to radius of the roller, and then drawing a smooth curve touching these arcs, as shown in fig. 6-74.



EXERCISES 6

1. Draw a straight line AB of any length. Mark a point F , 65 mm from AB . Trace the paths of a point P moving in such a way, that the ratio of its distance from the point F , to its distance from AB is (i) 1; (ii) 3:2; (iii) 2:3. Plot at least 8 points. Name each curve. Draw a normal and a tangent to each curve at a point on it, 50 mm from F .
2. A fixed point is 75 mm from a fixed straight line. Draw the locus of a point P moving such a way that its distance from the fixed straight line is (i) twice its distance from the fixed point; (ii) equal to its distance from the fixed point. Name the curves.
3. The vertex of a hyperbola is 65 mm from its focus. Draw the curve if the eccentricity is $\frac{3}{2}$. Draw a normal and a tangent at a point on the curve, 75 mm from the directrix.

4. The major axis of an ellipse is 150 mm long and the minor axis is 100 mm long. Find the foci and draw the ellipse by 'arcs of circles' method. Draw a tangent to the ellipse at a point on it 25 mm above the major axis.
5. The foci of an ellipse are 90 mm apart and the minor axis is 65 mm long. Determine the length of the major axis and draw half the ellipse by concentric-circles method and the other half by oblong method. Draw a curve parallel to the ellipse and 25 mm away from it.
6. The major axis of an ellipse is 100 mm long and the foci are at a distance of 15 mm from its ends. Find the minor axis. Prepare a trammel and draw the ellipse using the same.
7. Two fixed points A and B are 100 mm apart. Trace the complete path of a point P moving (in the same plane as that of A and B) in such a way that, the sum of its distances from A and B is always the same and equal to 125 mm. Name the curve. Draw another curve parallel to and 25 mm away from this curve.
8. Inscribe an ellipse in a parallelogram having sides 150 mm and 100 mm long and an included angle of 120° .
9. Two points A and B are 100 mm apart. A point C is 75 mm from A and 60 mm from B. Draw an ellipse passing through A, B and C.
10. Draw a rectangle having its sides 125 mm and 75 mm long. Inscribe two parabolas in it with their axis bisecting each other.
11. A ball thrown up in the air reaches a maximum height of 45 metres and travels a horizontal distance of 75 metres. Trace the path of the ball, assuming it to be parabolic.
12. A point P is 30 mm and 50 mm respectively from two straight lines which are at right angles to each other. Draw a rectangular hyperbola from P within 10 mm distance from each line.
13. Two straight lines OA and OB make an angle of 75° between them. P is a point 40 mm from OA and 50 mm from OB. Draw a hyperbola through P, with OA and OB as asymptotes, marking at least ten points.
14. Two points A and B are 50 mm apart. Draw the curve traced out by a point P moving in such a way that the difference between its distances from A and B is always constant and equal to 20 mm.
15. A circle of 50 mm diameter rolls along a straight line without slipping. Draw the curve traced out by a point P on the circumference, for one complete revolution of the circle. Name the curve. Draw a tangent to the curve at a point on it 40 mm from the line.
16. Two points Q and S lie on a straight line through the centre C of a circle of 50 mm diameter, rolling along a fixed straight line. Draw and name the curves traced out by the points Q and S during one revolution of the circle. $CQ = 20 \text{ mm}$; $CS = 35 \text{ mm}$.
17. A circle of 50 mm diameter rolls on the circumference of another circle of 175 mm diameter and outside it. Trace the locus of a point on the circumference of the rolling circle for one complete revolution. Name the curve. Draw a tangent and a normal to the curve at a point 125 mm from the centre of the directing circle.
18. Construct a hypocycloid, rolling circle 50 mm diameter and directing circle 175 mm diameter. Draw a tangent to it at a point 50 mm from the centre of the directing circle.

19. A circle of 115 mm diameter rolls on another circle of 75 mm diameter with internal contact. Draw the locus of a point on the circumference of the rolling circle for its one complete revolution.
20. A circle of 50 mm diameter rolls on another circle of 175 mm diameter. Draw and name the curves traced out by two points Q and S lying on a straight line through the centre C of the rolling circle and respectively 20 mm and 35 mm from it, when it rolls (i) outside and (ii) inside the other circle.
21. Show by means of a drawing that when the diameter of the directing circle is twice that of the generating circle, the hypocycloid is a straight line. Take the diameter of the generating circle equal to 50 mm.
22. A circle of 50 mm diameter rolls on a horizontal line for a half revolution and then on a vertical line for another half revolution. Draw the curve traced out by a point P on the circumference of the circle.
23. Draw an involute of a circle of 40 mm diameter. Also, draw a normal and a tangent to it at a point 100 mm from the centre of the circle.
24. An inelastic string 145 mm long, has its one end attached to the circumference of a circular disc of 40 mm diameter. Draw the curve traced out by the other end of the string, when it is completely wound around the disc, keeping the string always tight.
25. Draw a circle with diameter AB equal to 65 mm. Draw a line AC 150 mm long and tangent to the circle. Trace the path of A , when the line AC rolls on the circle, without slipping.
26. AB is a rope 1.6 metres long, tied to a peg at B (fig. 6-75). Keeping it always tight, the rope is wound round the pole O . Draw the curve traced out by the end A . Scale $\frac{1}{10}$ full size.
27. Draw an Archimedean spiral of two convolutions, the greatest and the least radii being 115 mm and 15 mm respectively. Draw a tangent and a normal to the spiral at a point, 65 mm from the pole.
28. A point P moves towards another point O , 75 mm from it and reaches it while moving around it once, its movement towards O being uniform with its movement around it. Draw the curve traced out by the point P .
29. A link OA , 100 mm long rotates about O in anti-clockwise direction. A point P on the link, 15 mm away from O , moves and reaches the end A , while the link has rotated through $\frac{2}{5}$ of a revolution. Assuming the movements of the link and the point to be uniform, trace the path of the point P .
30. Draw a cam to give the following uniform motions to a point: Rise 40 mm in 90° . In upper position for 75° . Fall the same distance in 120° . In lower position for the remaining period of the revolution. Diameter of shaft = 50 mm. Least metal = 20 mm.
31. A circle of 40 mm diameter rolls on the inside of a circle of 90 mm diameter. A point P lies within the rolling circle at a distance of 15 mm from its centre. Trace the path of the point P for one revolution of the circle.

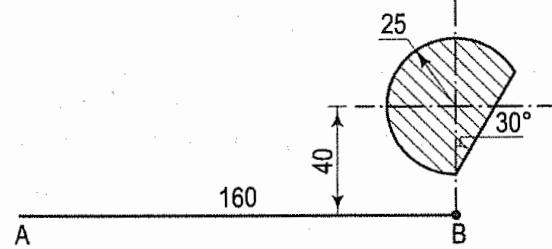


FIG. 6-75

32. The distance between the foci of an ellipse is 100 mm and the minor axis is 65 mm long. Draw the evolute of the ellipse.
33. Draw the evolutes of the two curves, the data of which is given in Ex. 2.
34. Construct the evolute of the hyperbola whose data is given in Ex. 3.
35. Draw the evolute of the cycloid obtained in Ex. 15.
36. Draw the epicycloid and hypocycloid, when the generating circle and the directing circle are of 50 mm and 175 mm diameters respectively. Construct the evolutes of the two curves.
37. In a logarithmic spiral, the shortest radius is 40 mm. The length of adjacent radius vectors enclosing 30° are in the ratio 9.8. Construct one revolution of the spiral. Draw a tangent to the spiral at any point on it.
38. Draw a triangle ABC with $AB = 30$ mm, $AC = 40$ mm and $\angle BAC = 45^\circ$. B and C are the points on an Archimedean spiral of one convolution of which A is the pole. Find the initial line and draw the spiral.
39. ABC is an equilateral triangle of side equal to 70 mm. Trace the loci of the vertices A , B and C when the circle circumscribing ABC rolls without slipping along a fixed straight line for one complete revolution.
40. Draw the shape of the cam to give the same motions as in Ex. 30 to a roller of 25 mm diameter.
41. A point is raised by a cam uniformly 25 mm in $\frac{3}{8}$ of a revolution; kept at the same height for $\frac{1}{8}$ of the revolution; then allowed to drop through 10 mm height and remain there for $\frac{1}{8}$ of the revolution; lowered uniformly to its original position in $\frac{1}{4}$ of the revolution and kept there for the rest of the revolution. Draw the shape of the cam. Diameter of shaft = 40 mm. Least metal = 25 mm.
42. Two pegs A and B are fixed on the ground 6 metres apart. A string, 8 metres long has its one end tied to the peg A , while the other end is passed through a small ring C and tied to the peg B . A link CD of 1 metre length is fixed to the ring C . The link with the ring slides along the string. Draw the boundary of the area of the ground beyond which D cannot reach, as C slides along the string AB .
43. Fill in the blanks in the following with appropriate words selected from the list given below:
- When a cone is cut by planes at different angles, the curves of intersection are called _____.
 - When the plane makes the same angle with the axis as do the generators, the curve is a _____.
 - When the plane is perpendicular to the axis, the curve is a _____.
 - When the plane is parallel to the axis, the curve is a _____.
 - When the plane makes an angle with the axis greater than what do the generators, the curve is a _____.
 - A conic is a locus of a point moving in such a way that the ratio of its distance from the _____ and its distance from the _____ is always constant. This ratio is called the _____. It is _____ in case of parabola, _____ in case of hyperbola and _____ in case of ellipse.

- (g) In a conic the line passing through the fixed point and perpendicular to the fixed line is called the _____.
- (h) The vertex is the point at which the _____ cuts the _____.
- (i) The sum of the distances of any point on the _____ from its two foci is always the same and equal to the _____.
- (j) The distance of the ends of the _____ of an ellipse from the _____ is equal to half the _____.
- (k) In a _____ the product of the distances of any point on it from two fixed lines at right angles to each other is always constant. The fixed lines are called _____.
- (l) Curves generated by a fixed point on the circumference of a circle rolling along a fixed line or circle are called _____.
- (m) The curve generated by a point on the circumference of a circle, rolling along another circle inside it, is called a _____.
- (n) The curve generated by a point on the circumference of a circle rolling along a straight line is called a _____.
- (o) The curve generated by a point on the circumference of a circle rolling along another circle outside it, is called _____.
- (p) The curve generated by a point fixed to a circle outside its circumference, as it rolls along a straight line is called _____.
- (q) The curve generated by a point fixed to a circle inside its circumference, as it rolls along a circle inside it, is called _____.
- (r) The curve generated by a point fixed to a circle outside its circumference as it rolls along a circle outside it, is called _____.
- (s) The curve traced out by a point on a straight line which rolls, without slipping, along a circle or a polygon is called _____.
- (t) The curve traced out by a point moving in a plane in one direction, towards a fixed point while moving around it is called a _____.
- (u) The line joining any point on the spiral with the pole is called _____.
- (v) In _____ the ratio of the lengths of consecutive radius vectors enclosing equal angles is always constant.

List of words for Ex. 43:

1. Asymptotes	9. Equal to 1	17. Hypotrochoid	25. Rectangular
2. Axis	10. Epitrochoid	18. Involute	26. Smaller than 1
3. Cycloidal	11. Ellipse	19. Inferior	27. Superior
4. Conic	12. Eccentricity	20. Logarithmic	28. Spiral
5. Circle	13. Focus	21. Minor axis	29. Trochoid
6. Cycloid	14. Greater than 1	22. Major axes	30. Conics
7. Directrix	15. Hyperbola	23. Parabola	31. Curves
8. Epicycloid	16. Hypocycloid	24. Radius vector	

Answers to Ex. 43:

- (a) – 30 (b) – 23 (c) – 5 (d) – 15 (e) – 11 (f) – 13, 7, 12, 9, 14 and 26 (g) – 2 (h) – 4 and 2 (i) – 11 and 22 (j) – 21, 13 and 22 (k) – 25, 15 and 1 (l) – 3 and 31 (m) – 16 (n) – 6 (o) – 8 (p) – 27 and 29 (q) – 19 and 17 (r) – 27 and 10 (s) – 18 (t) – 28 (u) – 24 (v) – 20 and 28.

Miscellaneous problems:

44. ABC is an equilateral triangle of side equal to 70 mm. Trace the loci of the vertices A , B and C , when the circle circumscribing ABC rolls without slipping along a fixed straight line for one half revolution.
45. The major-axis AB of an ellipse is 140 mm long with P as its mid-point. The foci F_1 and F_2 of the ellipse are 48 mm away from the mid-point P . Draw the ellipse and find the length of the minor axes.
46. Draw an ellipse by 'concentric circles method' and find the length of the minor axis with the help of the following data: (i) Major axes = 100 mm. (ii) Distance between foci 80 mm.
47. PQ is a diameter of a circle and is 75 mm long. A piece of string is tied tightly round the circumference of the semi-circle starting from P and finishing at Q . The end Q is then untied and the string, always kept taut, is gradually unwound from the circle, until it lies along the tangent at P . Draw the curve traced by the moving extremity of the string.
48. A half-cone is standing on its half base on the ground with the triangular face parallel to the V.P. An inextensible string is passed round the half-cone from a point on the periphery and brought back to the same point. Find the shortest length of the string. Take the base-circle diameter of the half-cone as 60 mm and height 75 mm.
49. One end of an inelastic thread of 120 mm length is attached to one corner of a regular hexagonal disc having a side of 25 mm. Draw the curve traced out by the other end of the thread when it is completely wound along the periphery of the disc, keeping the thread always tight.
50. A circus man rides a motor-bike inside a globe of 6 metres diameter. The motor-bike has the wheel of 1 metre diameter. Draw the locus of the point on the circumference of the motor-bike wheel for one complete revolution. Adopt suitable scale.
51. A fountain jet discharges water from ground level at an inclination of 45° to the ground. The jet travels a horizontal distance of 7.5 metre from the point of discharge and falls on the ground. Trace the path of the jet. Name the curve.
52. A coin of 35 mm diameter rolls over dinning table without slipping. A point on the circumference of the coin is in contact with the table surface in the beginning and after one complete revolution. Draw the curve traced by the point. Draw a tangent and a normal at any point on the curve.
53. Draw a rectangle of 130 mm \times 85 mm. Draw two parabolas in it with their axes bisecting each other.
54. Two concentric discs of 40 mm and 50 mm diameters roll on the horizontal line AB 150 mm long. Both discs start at the same point and roll in the same direction. Plot the curves for the movement of the points lying on their circumferences.
55. Draw a path of the end of string when it is wound on a circle of 40 cm diameter without slipping. The length of the string is 150 cm long. Name the curve and write practical applications.

56. A circle of 60 mm diameter rolls on a horizontal line for a half revolution and then on a vertical line for another half revolution. Draw the curve traced out by a point P on the circumference of the circle.
57. Draw the path traced out by a point on a circumference of circle when it rolls, without slip, on vertical surface, for the distance equal to the perimeter of the circle of diameter of 40 mm.
58. Draw a helix of pitch equal to 50 mm upon a cylinder of 75 mm diameter and develop the surface of the cylinder. Assume the starting point to be on the vertical centre line in the top view.
59. Draw the projections of a helix having a helix-angle of 30° , on a cylinder of 75 mm diameter.
60. A spiral is made of a wire of rectangular cross-section 25 mm \times 20 mm. Draw two complete turns of the spring. Outside diameter 100 mm, inside diameter 50 mm and pitch 50 mm.
61. Project one complete turn of a helical spring of outside diameter 75 mm and pitch 50 mm, the cross-section of wire being a circle of 20 mm diameter.
62. Draw the projections of three coils of a helical spring of steel wire 20 mm diameter. Outside diameter of the spring 100 mm and pitch 50 mm.
63. Project two complete turns of a triangular thread, outside diameter 125 mm, pitch 25 mm and angle 60° .
64. A screw having triple-start square thread has outside diameter 150 mm, lead 120 mm and pitch 40 mm. Draw its projections.
65. Draw a helix of one convolution upon a cone, diameter of the base 75 mm, axis 100 mm long and pitch 75 mm. Take apex as the starting point for the curve.
66. A point P , starting from the base-circle of a cone, reaches the apex, while moving around the axis through two complete turns. Assuming the movement of P towards the apex (measured parallel to the axis) to be uniform with its movement around the axis, draw the projections and the development of the surface of the cone showing the path of P in each. Diameter of the base of the cone 75 mm; axis 100 mm long.
67. A propeller screw of 20 mm diameter has a helical blade welded on its surface. The diameter of the helix is 80 mm and the pitch is 50 mm. Project two complete turns of the screw with the blade.
68. Two objects A and B , 10 m above and 7 m below the ground level respectively, are observed from the top of a tower 35 m high from the ground. Both the objects make an angle of depression of 45° with the horizon. The horizontal distance between A and B is 20 m. Draw to scale 1:250, the projections of the objects and the tower and find (a) the true distance between A and B , and (b) the angle of depression of another object C situated on the ground midway between A and B .
69. An ant is travelling on a cylindrical surface in a circumferential direction at a uniform angular speed in the clockwise direction as observed from the top and simultaneously advances at a uniform rate in the axial direction. If the diameter of the cylinder is 60 mm and the ant moves 80 mm in the axial direction during one turn, trace the path of the ant and name the curve.

Chapter

7



LOCI OF POINTS

7-0. INTRODUCTION

A machine is made of the various types of mechanisms for transmitting motions. These mechanisms consist of interrelated links moving on the required path. Dimensions (generally lengths) of the links of the mechanism can be determined by plotting the path of required points. This chapter deals with problems of plotting path of points on the links of different mechanisms.

7-1. LOCI OF POINTS

A locus (plural loci) is the path of a point which moves in space.

- (i) The locus of a point P moving in a plane about another point O in such a way that its distance from it is constant, is a circle of radius equal to OP (fig. 7-1).
- (ii) The locus of a point P moving in a plane in such a way that its distance from a fixed line AB is constant, is a line through P , parallel to the fixed line (fig. 7-2).
- (iii) When the fixed line is an arc of a circle, the locus will be another arc drawn through P with the same centre (fig. 7-3).

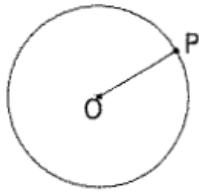


FIG. 7-1

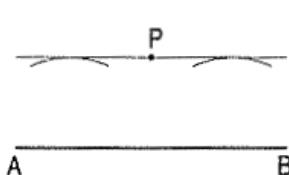


FIG. 7-2

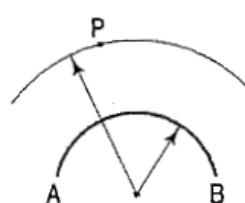


FIG. 7-3

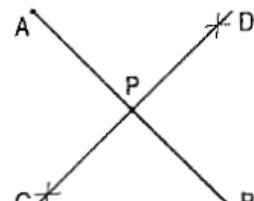


FIG. 7-4

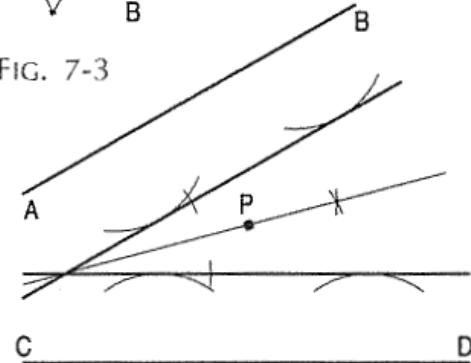


FIG. 7-5

- (iv) The locus of a point equidistant from two fixed points A and B in the same plane, is the perpendicular bisector of the line joining the two points (fig. 7-4).
- (v) The locus of a point equidistant from two fixed non-parallel straight lines AB and CD will be a straight line bisecting the angle between them (fig. 7-5).

Problem 7-1. (fig. 7-6): To draw the locus of a point equidistant from a fixed straight line and a fixed point.

Let AB be the given line and C the given point.

- From C , draw a line CD perpendicular to AB . The mid-point P of CD is equidistant from AB and C , and hence, it lies on the locus.
- To obtain more points, mark a number of points 1, 2 etc. on PC and through them, draw lines parallel to AB .
- With centre C and radius $D1$, draw an arc cutting the line through 1 at points P_1 and P'_1 .
- Similarly, obtain more points and through them, draw a smooth curve which will be the required locus.

Problem 7-2. (fig. 7-7): To draw the locus of a point equidistant from a fixed circle and a fixed point.

A circle with centre O and a point C are given.

- Draw a line joining O and C and cutting the circle at a point A . The mid-point P of the line AC will lie on the locus.
- Mark a number of points 1, 2 etc. on PC and through them, draw arcs with O as centre.
- With centre C and radius equal to $A1$, draw an arc cutting the arc through 1 at points P_1 and P'_1 .

Similarly, obtain more points and draw the required curve through them.

Fig. 7-6 shows the construction of the locus of a point equidistant from a fixed straight line and a fixed point. A horizontal line AB is given. A point C is also given. A line CD is drawn perpendicular to AB through C . The mid-point P of CD is marked. Points 1, 2, 3, ... are marked on PC . Lines parallel to AB are drawn through these points. With centre C and radius $D1$, an arc is drawn intersecting the line through 1 at points P_1 and P'_1 . The locus is a smooth curve passing through P_1 and P'_1 .

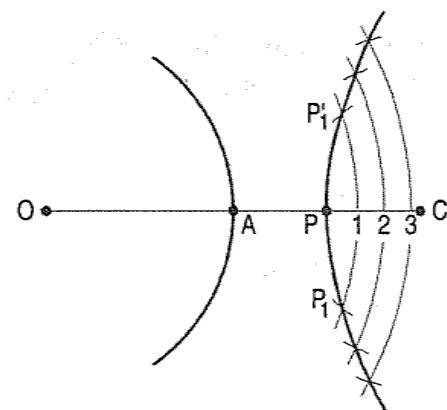


FIG. 7-7

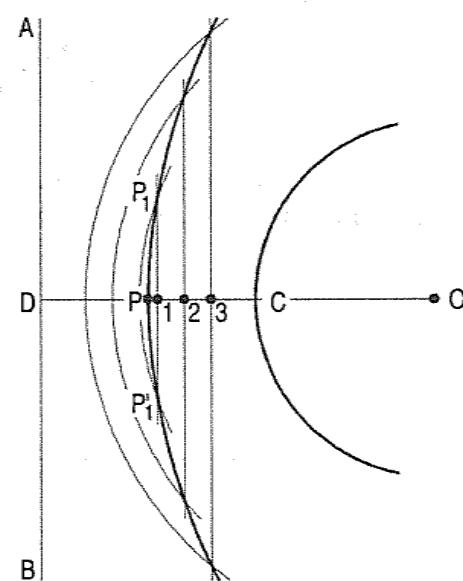


FIG. 7-8

Problem 7-3. (fig. 7-8): To draw the locus of a point equidistant from a fixed straight line and a fixed circle.

A line AB and a circle with centre O are given.

- From O , draw a line OD perpendicular to AB and cutting the circle at C .
- Find the mid-point P of the line DC . It will lie on the curve.

- (iii) Mark any point 1 on PC and through it, draw a line parallel to AB .
- (iv) With centre O and radius equal to $(OC + D1)$, draw an arc cutting the line through 1 at P_1 and P'_1 .
- (v) Similarly, obtain more points and draw the curve through them.

Problem 7-4. (fig. 7-9): *To draw the locus of a point equidistant from two given circles.*
Circles with centres A and B are given.

- (i) Draw a line joining A and B and cutting the circles at points C and D .
- (ii) Find the mid-point P of the line CD .
- (iii) Mark any point 1 on PD and through it, draw an arc with centre A .
- (iv) With centre B and radius equal to $(BD + C1)$, draw an arc, cutting the arc through 1 at points P_1 and P'_1 .
- (v) Similarly, locate more points and draw the curve through them.

The curves obtained in the above four problems are also the loci of centres of circles which will touch the given line, point or circles as the case may be.

Problem 7-5. (fig. 7-9): *To draw a circle touching two given circles and a given straight line.*

The circles with centres A and B and the line EF are given.

Draw the locus of a point equidistant from one of the circles, say the smaller circle, and the line EF .

The point of intersection of this curve, with the locus of the point equidistant from the two given circles, viz. O is the centre of the required circle.

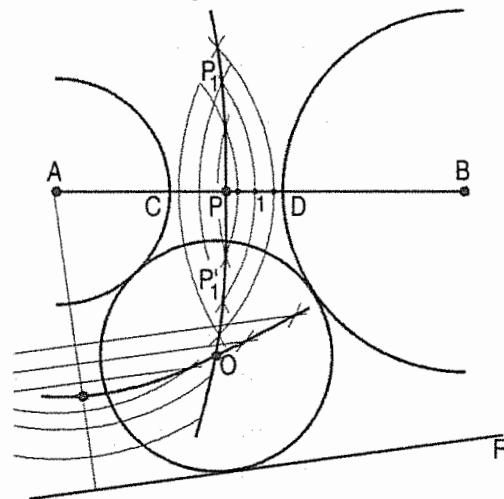


FIG. 7-9

7-2. SIMPLE MECHANISMS

In simple mechanisms, it is often necessary to know the paths of points on their moving parts. These are determined by assuming a number of different positions of the moving parts and then locating the corresponding positions of the points.

7-2-1. THE SLIDER CRANK MECHANISM

The slider crank mechanism shown diagrammatically in fig. 7-10 is one of the simplest mechanisms and it is used in internal combustion engine, sewing-machine and printing press etc. The end A of the connecting rod AB is connected to the crank OA which rotates about O . The other end B is attached to a slider which slides along a straight line. The locus of A will be a circle and that of the end B will be a straight line. The locus of any other point, say P , on the connecting rod will be neither a circle nor a straight line and may be determined:

- (i) by assuming various positions of the crank-end A ,
- (ii) by obtaining the corresponding positions of the end B and finally
- (iii) by locating the positions of P on the lines joining the first two positions.

A smooth curve drawn through the points thus located will be the locus of P . Such curves are known as *coupler curves*.

(1) Simple slider crank mechanism:



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 15 for the following problem of simple slider crank mechanism.

Problem 7-6. (fig. 7-10): *In a simple slider crank mechanism, the connecting rod AB is 750 mm long and the crank OA is 150 mm long. The end B moves along a straight line passing through O. Trace the locus of a point P, 300 mm from A along the rod, for one revolution of OA.*

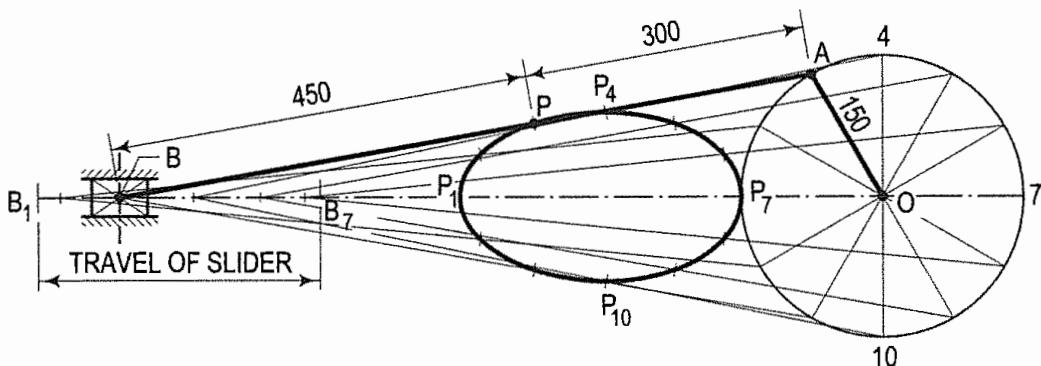


FIG. 7-10

- Divide the circle (path of A) into 12 equal parts.
- With centre 1 and radius AB, cut the path of B at a point B_1 .
- Draw a line joining 1 and B_1 . Again, with centre 1 and radius PA, cut the line $1B_1$ at a point P_1 .
- Obtain other points in similar manner and draw a smooth curve through these points. Then this curve is the locus of the point P.

Note that the distance B_1B_7 is the travel of the slider and is equal to twice the length of the crank. This distance is known as *stroke length*. But the movement of the slider is not uniform with that of the crank-end A.

(2) Offset slider crank mechanism:



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 16 for the following problem of offset slider crank mechanism.

Problem 7-7. (fig. 7-11): *In the offset slider crank mechanism shown in fig. 7-11, the slider-end B moves in guides along the line CD, 225 mm below the axis O of the crank-shaft. Plot the locus of a point P, 450 mm from A along AB and of a point Q along the extension of the rod, 300 mm beyond B. Determine also the travel of the slider.*

- Divide the circle into 12 equal parts and obtain the positions of the end B on its path, and of the point P as shown in problem 7-6.
- Produce lines $1B$, $2B$ etc. 300 mm further and mark positions Q_1 , Q_2 etc.

In addition to the above 12 points, it is also necessary to determine the limiting positions of the end B. They will occur when the connecting rod and the crank are in a straight line.

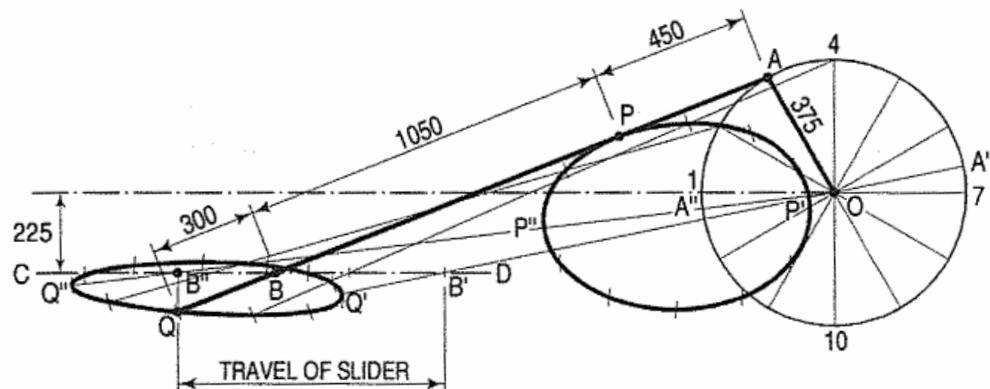


FIG. 7-11

- (iii) These are found by drawing arcs with centre O and radii (i) ($AB - AO$) and (ii) ($AB + AO$), and cutting the path of B at points B' and B'' . The travel of the slider is shown by the distance $B''B'$. The corresponding positions of the end A for the limiting positions of B , viz. A' and A'' will be on the lines $B'O$ -produced and $B''O$ respectively. The positions of P and Q , viz. P' , P'' and Q' , Q'' are obtained as already explained.

Problem 7-8. (fig. 7-12): In the mechanism shown in fig. 7-12, the connecting rod is constrained to pass through the guide at D. Trace the locus of the end B and of a point P on AB for one complete revolution of the crank. $AB = 1800 \text{ mm}$, $AO = 375 \text{ mm}$ and $AP = 750 \text{ mm}$.

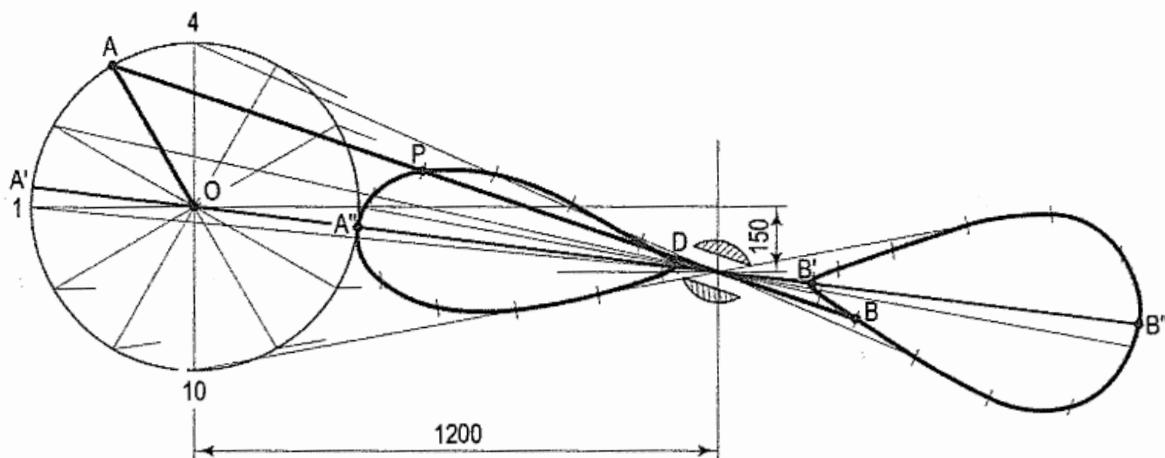


FIG. 7-12

- (i) Divide the circle into 12 equal parts.
 - (ii) Draw a line from the point 1, passing through D and obtain a point B_1 such that $1B_1 = AB$.
 - (iii) Similarly, locate other positions of the end B and draw a curve through them. The locus of P is drawn in the same manner as explained in the previous problem. The limiting positions B' and B'' (of the end B) are found by drawing a line through O and D and making OB' equal to $(AB - AO)$ and OB'' equal to $(AB + AO)$.

Note that $A'B' = A''B'' = AB$.

7-2-2. A FOUR-BAR MECHANISM



This mechanism consists of four links. It is widely used in locomotive, steering mechanism of the car, pantograph and straight-line mechanisms.

Problem 7-9. (fig. 7-13): Two equal cranks AB and CD connected by the link BD , rotate in opposite directions. Draw the locus of a point P on BD and of Q along extension of the rod beyond B for one revolution of AB .

$$AB = CD = 450 \text{ mm}; AC = BD = 1500 \text{ mm};$$

$$PD = 300 \text{ mm}; BQ = 300 \text{ mm}.$$

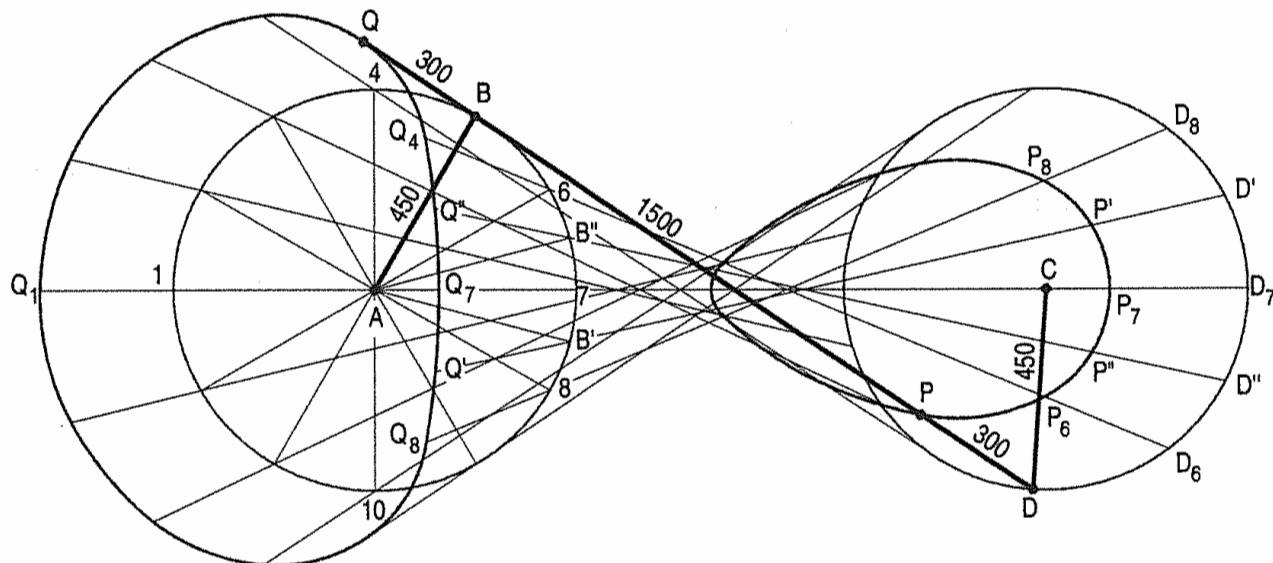


FIG. 7-13

- (i) Divide one of the circles (say path of B), into 12 equal parts.
- (ii) Determine the position of D on its path (the other circle), for every position of B and find the corresponding positions of P and Q for these positions as shown. It will be found that there is a wide gap between the points for positions 6, 7 and 8 of the end B . A few more positions such as AB' and AB'' etc. may be taken and points P' , P'' , Q' , Q'' etc. may be located.

Problem 7-10. (fig. 7-14): Two cranks AO and BQ oscillate about O and Q respectively. Trace the locus of the mid-point P of the connecting link AB . $AO = 450 \text{ mm}$; $BQ = 675 \text{ mm}$; $AB = 375 \text{ mm}$.

The limiting positions of the ends A and B will be when the link and each of the cranks are in straight lines. They are found as shown below:

- (i) With centre Q and radius $(BQ + AB)$, draw arcs cutting the path of A at points A' and A'' .
- (ii) With centre O and radius $(AO + AB)$, draw arcs cutting the path of B at points B' and B'' .

Assume A to be moving downwards. Then B will move towards B' . After B has reached B' , if A moves further towards A' , B will begin its return-motion. A will go upto A' and then move backwards. The movement will be repeated in a similar manner at B'' and A'' also.

- (i) To draw the locus of P , mark a number of points on the path of A .
(ii) With centre A_1 and radius AB , draw an arc cutting the path of B at B_1 .
(iii) Mark a point P_1 on A_1B_1 such that $A_1P_1 = AP$.
(iv) Similarly, locate other points during the complete oscillation of the crank OA , from A' to A'' . It is not necessary to draw the cranks in various positions.

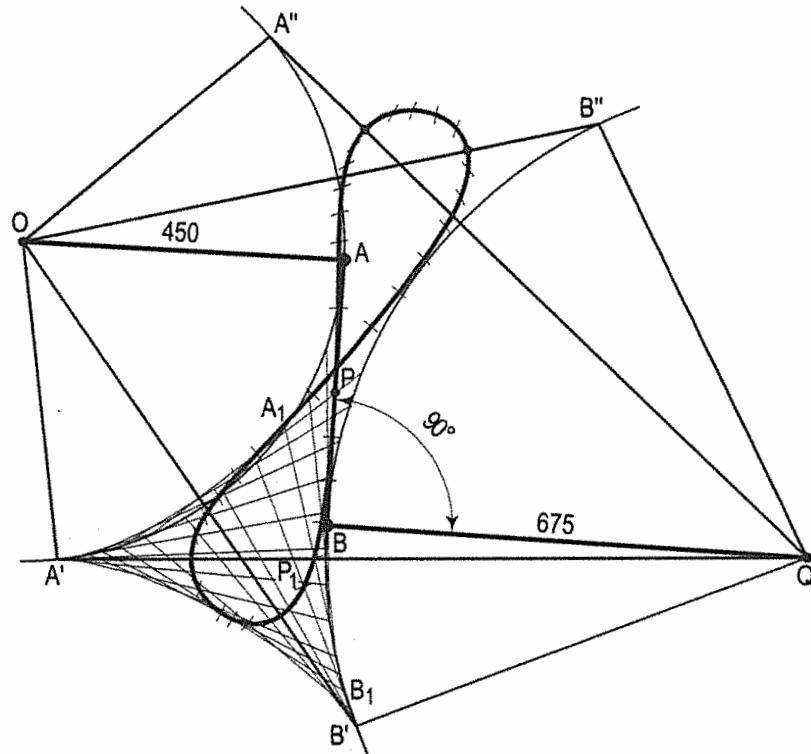


FIG. 7-14

Problem 7-11. (fig. 7-15): Two cranks AB and CD are connected by a link BD . AB rotates about A , while CD oscillates about C . Trace the locus of the mid-point of the link BD during one complete revolution of the crank AB . $AB = 450 \text{ mm}$, $CD = 750 \text{ mm}$, $BD = 1050 \text{ mm}$. Distance between A and C is equal to 900 mm .

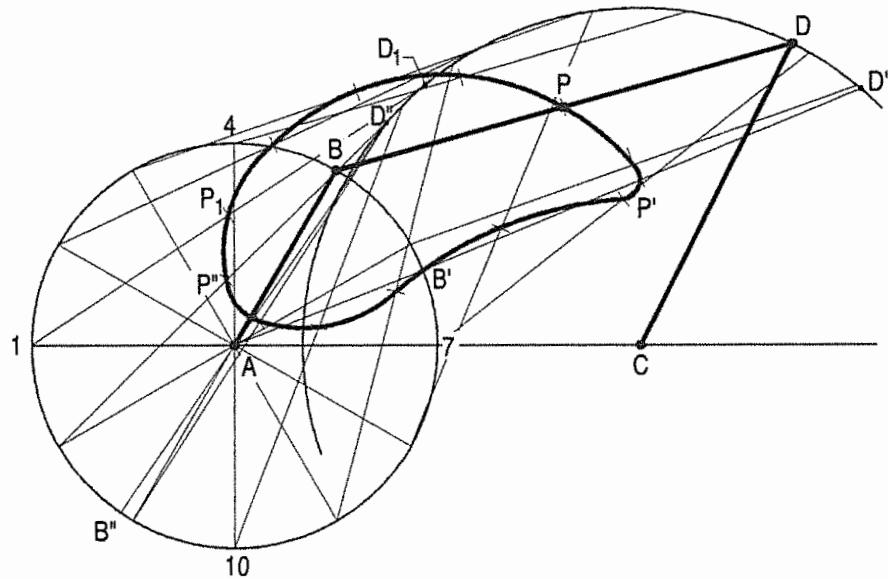


FIG. 7-15

- Divide the circle into 12 equal parts. With centre 1 and radius BD , cut the path of D at D_1 .
- Locate P_1 , the mid-point of $1D_1$. Similarly, find other points.
- In addition, find points P' and P'' for limiting positions, when $AD^1 = (BD + AB)$ and $AD'' = (BD - AB)$.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 17 for the following problem.

Problem 7-12. (fig. 7-16): The end A of a rod AB rotates about O , while the end B slides along a straight line. A crank CQ oscillates about Q . Draw the locus of the mid-point P of the connecting link CD for one revolution of the crank OA . $AB = 1500 \text{ mm}$, $CD = 750 \text{ mm}$, $OA = 450 \text{ mm}$ and $CQ = 1200 \text{ mm}$.

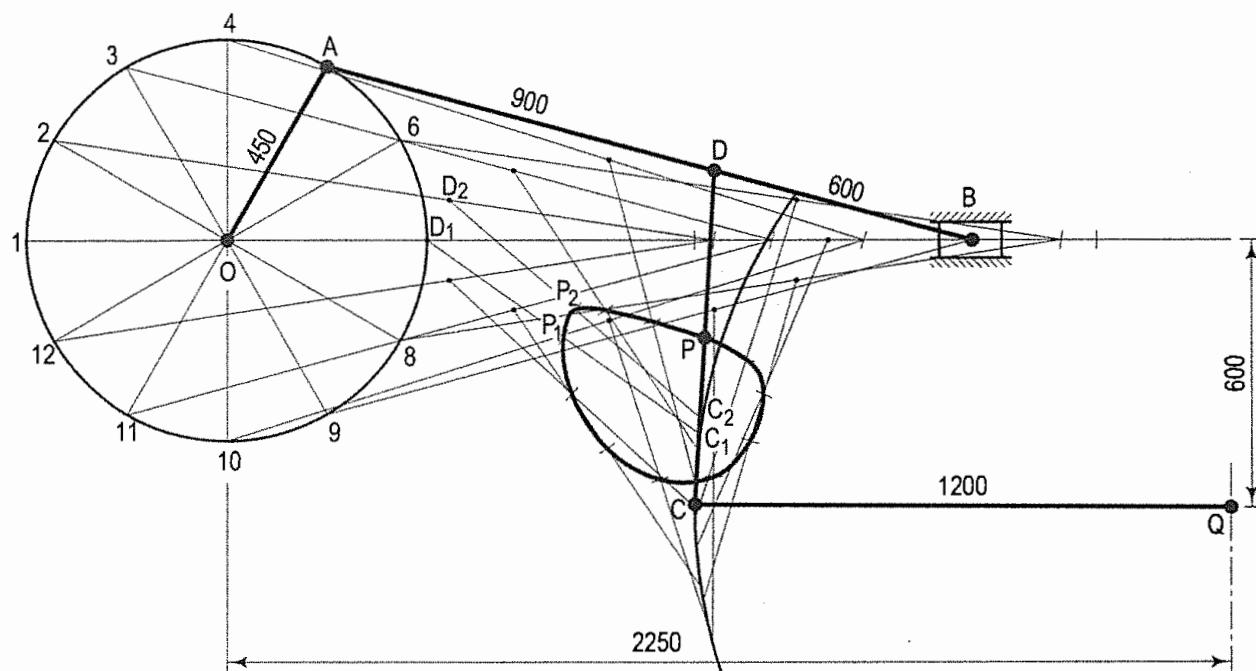


FIG. 7-16

- Determine points D_1 , D_2 etc. for various positions of the crank OA .
- With centres D_1 , D_2 etc. and radius CD , draw arcs cutting the path of C at C_1 , C_2 etc.
- Locate the mid-points P_1 , P_2 etc. of lines D_1C_1 , D_2C_2 etc. and draw the required curve through them.

Problem 7-13. (fig. 7-17): A circular disc of 100 mm diameter, revolves clockwise about its centre with uniform angular velocity. A point P is situated initially at the end A of the chord AB (60 mm long), travels along the chord towards the end B , with uniform velocity. As the disc completes one revolution, the point P reaches to the end B . Trace the path of the point P for one complete revolution of the disc.

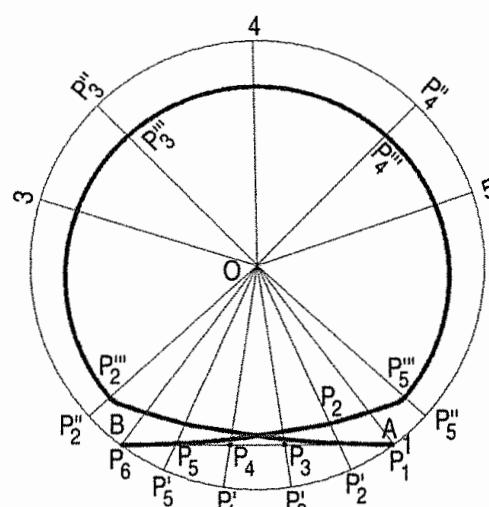


FIG. 7-17

- (i) Draw a circle of 100 mm diameter and divide it into 5 equal parts. Mark 1, 2, ..., 5.
- (ii) Draw a chord AB 60 mm long from the point 1. (The point 1 coincides with the point A .) Divide it into 5 equal parts and number P_1, P_2, \dots, P_6 . (The point P_6 coincides with the point B .)
- (iii) When the line OA moves through one division i.e. arc AB , the point P_1 will move towards B by a distance equal to one division of the chord AB (or $P_1 P_6$).
- (iv) To obtain points systematically, draw arcs with centre O and radii OP_2, OP_3, OP_4 etc. intersecting lines OP''_2, OP''_3, OP''_4 etc. at points P'''_2, P'''_3, P'''_4 etc. respectively. Draw a curve through points P_1, P'''_2, \dots, B .

Problem 7-14. (fig. 7-18): A mechanism as used on cup-board door is in fully open position. Draw the path of end A of the mechanism as the door moves to the fully closed position. Take $AB = 175$ mm.

- (i) Draw the mechanism to full size scale. Divide $\angle 120^\circ$ into equal divisions.
- (ii) Mark the successive position of the door. Mark the points B_1, B_2, \dots etc. at 10 mm away on the perpendicular line to the door position as shown. Draw lines $B_2O_2, B_3O_2, \dots, B_9O_2$ passing through O_2 .
- (iii) With a centre as B_1, B_2, \dots, B_9 and the radius equal to AB , draw arcs intersecting lines $B_1O_2, B_2O_2, \dots, B_9O_2$ at points $A_1, A_2, A_3, \dots, A_9$.
- (iv) Join the points A_1, A_2, \dots, A_9 by smooth curve.

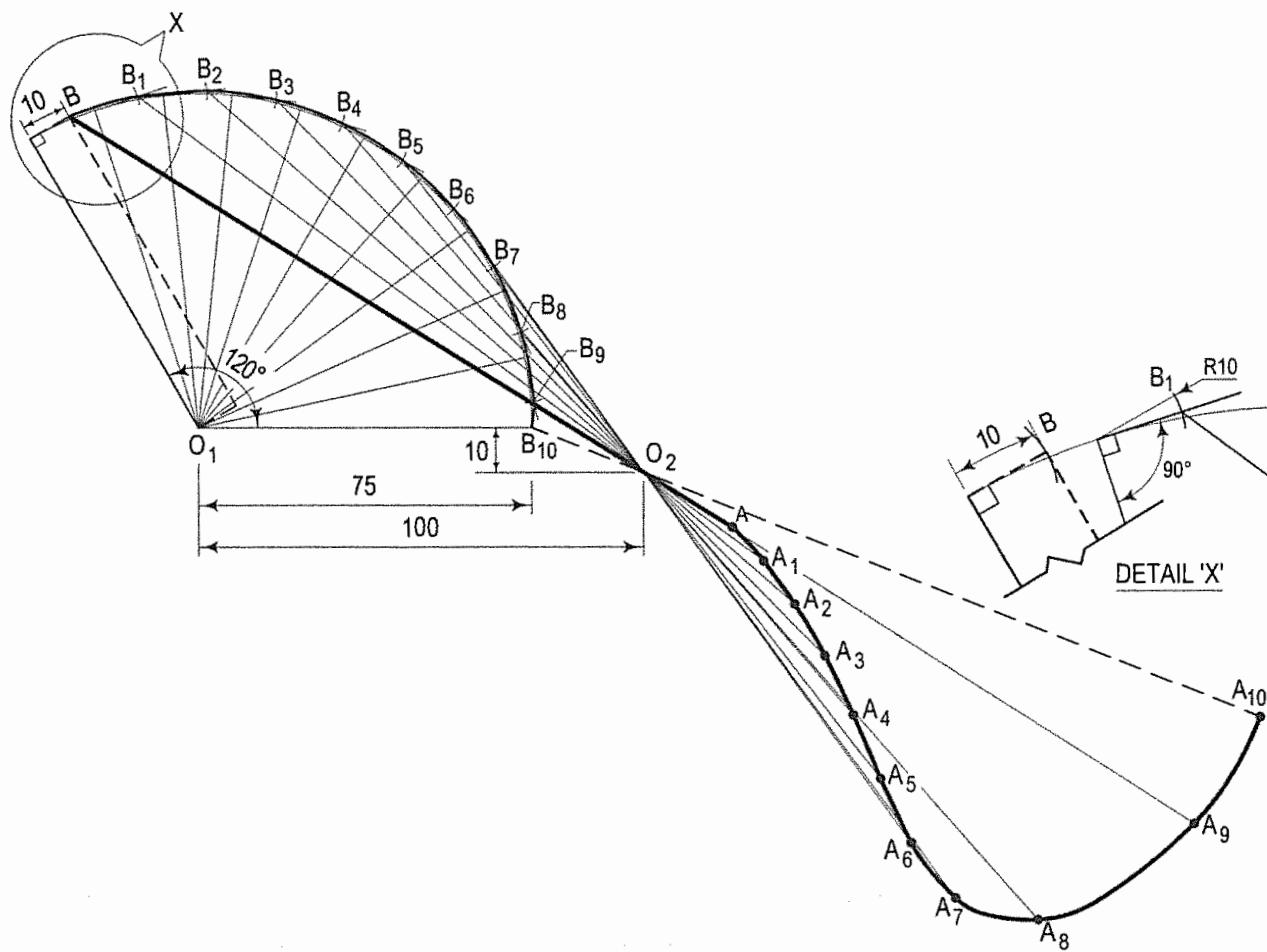


FIG. 7-18

Problem 7-15. (fig. 7-19): A link of shaping machine mechanism is shown in fig. 7-19. A sliding block moves towards O_4 along the oscillating link with the uniform motion.

Draw the path of the sliding block when the link O_4D rotates 40° in anti-clockwise direction. Use following data $O_4D = 160\text{ mm}$. $O_4B = 113\text{ mm}$. The movement of the sliding block along the link = 67 mm .

The construction of path of the sliding block is shown in fig. 7-19.

Problem 7-16. (fig. 7-20): A link PQ 100 mm long carries a circular disc of 30 mm diameter having centre Q . The link PQ oscillates about the hinged P from left to right and vice-versa (i.e. right to left) to maximum amplitude of 45° on either side from the vertical.

At the same time the disc rotates uniformly through one revolution in clockwise direction. Draw the path of point on the circumference of the disc.

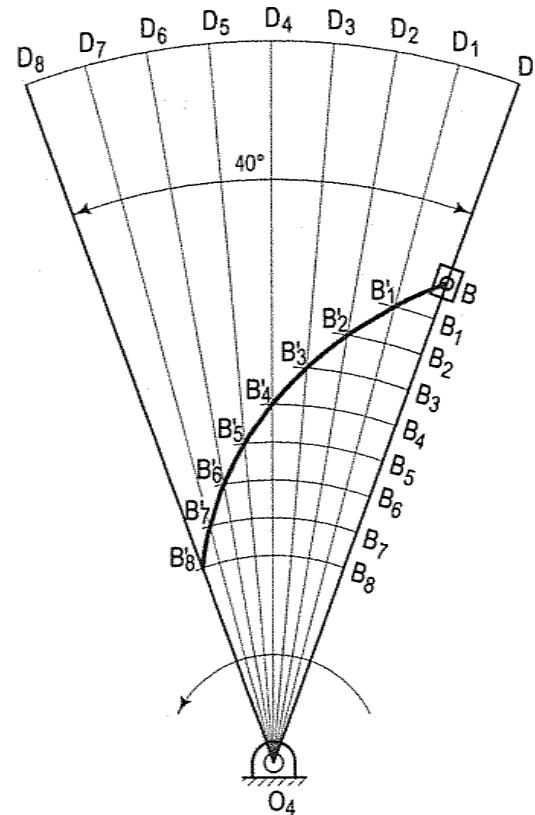


FIG. 7-19

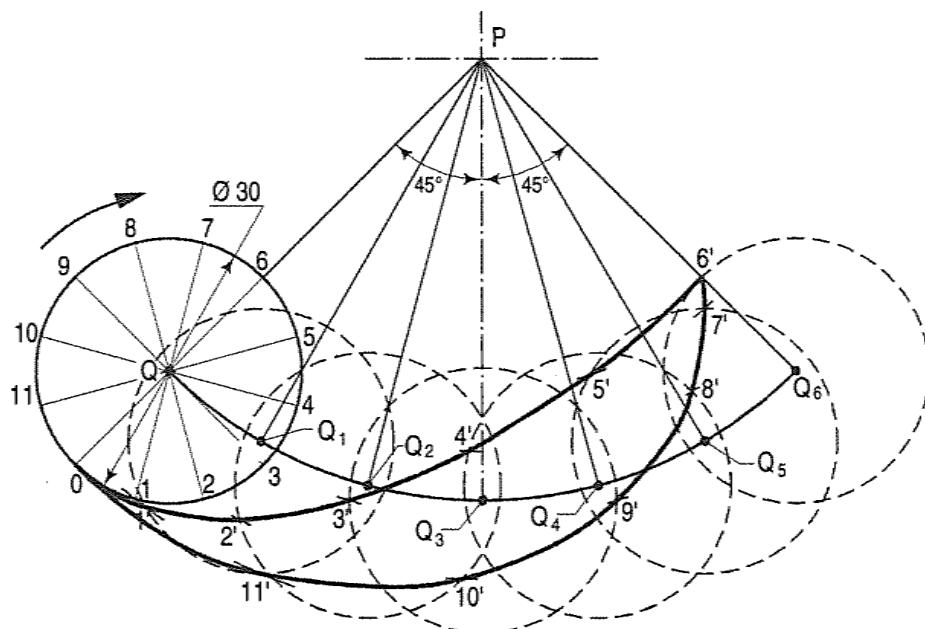


FIG. 7-20

- (i) Mark the point P . Draw vertical line from P . Draw the link PQ at angle of 45° with the vertical on the left side as well as on the right side.
 - (ii) Draw circle of radius 15 mm at Q and divide the circle into twelve equal parts. Number as 0, 1 to 11 from the division of extended PQ in anticlockwise direction as shown.

- (iii) The disc rotates as well as oscillates with the link PQ from left to right and then right to left for completing one cycle of oscillation. The angle moved by the link PQ is $(2 \times 90 = 180^\circ)$. Divide half cycle (i.e. 90°) into six equal parts described by the link. Each division will be of 15° .
- (iv) Now draw the arc joining initial position and left extreme position. Mark the intersection of radial lines drawn from P and this arc as $Q, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$ and for return cycle on the same arc $Q_7, Q_8, Q_9, Q_{10}, Q_{11}$.
- (v) When the disc rotates from 0 to 1, the rod moves PQ to PQ_1 . Therefore the centre of disc moves to $Q_1, Q_2, Q_3, \dots, Q_{11}$ successive position, the rod (radii) will occupy the position PQ_1, PQ_2, \dots, PQ_6 and PQ_7 to PQ_{11} .
- (vi) $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$ as centre, draw the circle for successive position of the disc. Take point O on its circumference of the circle for the initial position as shown.
- (vii) Now $1P$ as radius, draw arc intersecting the circle drawn from centre Q_1 . Name this point as $1'$. Similarly P as centre and radii $2P, 3P, 4P, 5P, 6P, \dots, 11P$ obtain other the intersection points. Join with all points by the smooth curve.
- (viii) Fig. 7-20 is drawn to half scale.

Problem 7-17. (fig. 7-21): A rod PQ 40 mm long rotates about R with a speed of 60 rpm, while Q moves along a straight line QR towards R with a speed of 108 mm per second (i.e. $QR = 108$ mm). Draw to full size scale the path traced out by the end P for one complete revolution of the rod. Assume that intial position of the rod PQ is along the extension of the line RQ .

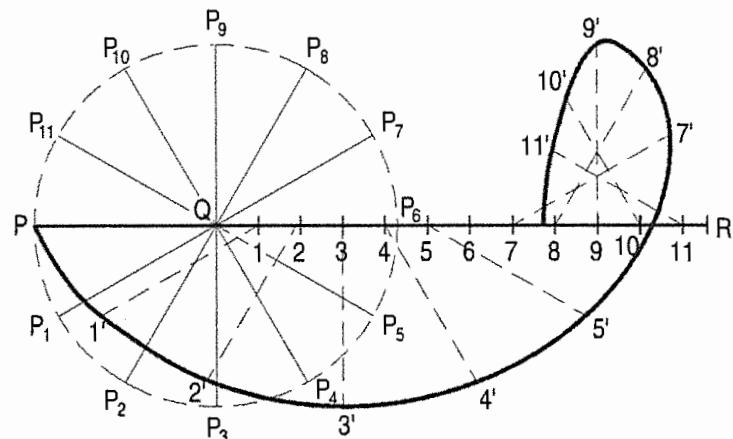


FIG. 7-21

- (i) Draw a line PQR of length $= 148$ mm, ($PQ + QR = 40 + 108$ mm)
- (ii) Now Q as centre $PQ = 40$ mm as radius, draw the circle and divide circle into twelve equal parts, label them P, P_1, \dots, P_{11} . Each part of the circle is covered by the rod in $\frac{1}{12}$ second. $\left(\frac{360}{30} = 12^\circ\right)$ Similarly the line QR is also divide in twelve parts. Mark division as $Q, 1, 2, \dots, 11, R$.
- (iii) From 1 draw a parallel line to QP_1 . Now 1 as centre and QP_1 as radius, cut the parallel line at $1'$. Similarly from 2, 3, 4, 5, ..., 11, R, draw parallel lines to the respective radii of the circle $QP_2, QP_3, QP_4, \dots, QP_{11}$.
- (iv) Draw parallel lines from $P_1, P_2, P_3, \dots, P_{11}$ to PR . Take radius equal to PQ and centre as 1, intersect the line drawn parallel to PQ from P_1 . Similarly with same radius and successive centres say 2, 3, ..., R, intersect the line drawn parallel to PQ from P_2, P_3, \dots, R .
- (v) Note that QP_6 will be along the line QR .
- (vi) Join points with the smooth curve.

Problem 7-18. (fig. 7-22): The crank O_1P_1 turns about O_1 and the connecting rod P_1Q_1 slides in the same plane on the curved surface of a shaft (with centre O_2) of 30 mm diameter. Trace the loci of the point Q_1 and R_1 which is extension of P_1Q_1 at 30 mm from the P_1 when O_1P_1 revolves one revolution. Take $O_1P_1 = 30 \text{ mm}$. $P_1Q_1 = 100 \text{ mm}$ and $O_1O_2 = 70 \text{ mm}$. Take scale 10 mm = 1 mm

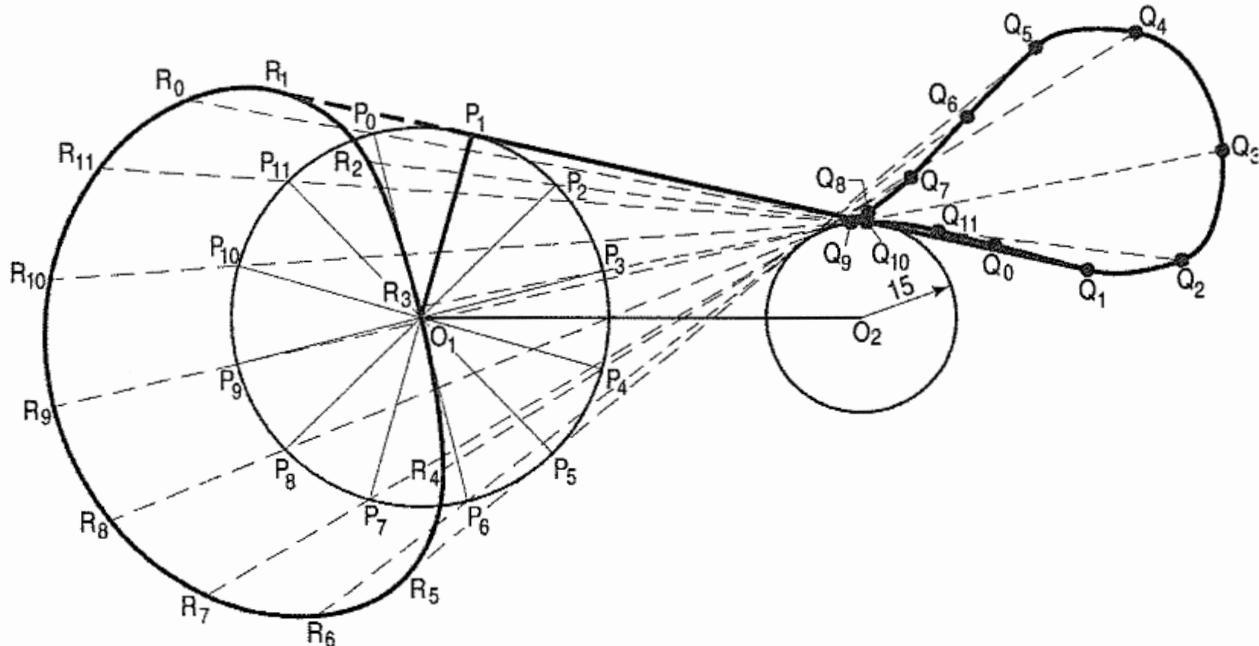


FIG. 7-22

- Draw a circle of 30 mm radius and centre O_1 .
- Mark $O_1O_2 = 70 \text{ mm}$ and draw the circle of radius 15 mm and the centre as O_2 the representing shaft.
- Mark any point P_1 on the circle of Q_1P_1 . Mark line $P_1Q_1 = 100 \text{ mm}$ making tangent to the circle of the shaft. Extend O_1P_1 to left side to R_1 such that $P_1R_1 = 30 \text{ mm}$.
- Divide the circle of radius O_1P into twelve divisions and number them $P_0, P_1, P_2, \dots, P_{11}$. From $P_0, P_1, P_2, \dots, P_{11}$ draw tangent on the same side to the circle of the shaft. On each extended tangent QP mark the points $P_0R_0, P_1R_1, \dots, P_{11}R_{11}$ as shown in fig. 7-22. Similarly from P_0, P_1, \dots, P_{11} mark the length 100 mm each tangent showing end $Q_1, Q_2, Q_3, \dots, Q_{11}$.
- Join all R points and Q points by the smooth curve.

Problem 7-19. (fig. 7-23): A mechanism O_1A rotates clockwise direction about fixed point O_1 . (O_1AB) is an offset slider crank mechanism. Link O_2E oscillates about fixed point O_2 . AB and CE are connecting links. Draw the loci of the points C and D for one complete revolution of the link (O_1A).

- First mark the points O_1 and O_2 2000 mm apart to some suitable scale.
- O_1 and O_2 as centres, draw the circle showing path of point A and point E . Mark the points A and E above and below O_1O_2 .
- Draw a line parallel to O_1O_2 at a distance of 1700 mm above as shown. A as centre and AB as radius, mark arc intersecting to parallel line drawn at B . Mark points C and D with given distances on AB and CE respectively.

- (iv) Divide the circle of $O_1 A$ into twelve parts. Number them as $A_1, A_2, A_3 \dots A_{11}$. Now A_1 as centre and AB distance as radius, draw intersection on the horizontal line MN at B_1 . From B_1 mark the distance of CB on A_1B_1 from B_1 at 1400 mm. This is new position of C call as C_1 . C as centre CE as radius, cut the arc at E_2 drawn from O_2 . O_2E as radius, mark the point D at given distance from E (680 mm). From $A_2, A_3 \dots A_{11}$ taking AB as radius draw arcs intersecting the path MN at $B_2, B_3 \dots B_{11}$. Join A_1B_1, A_2B_2 upto $A_{11}B_{11}$. On these lines mark $B_1C_1 = B_2C_2 = B_3C_3 = B_{11}C_{11}$. Join the points $C, C_1, C_2 \dots C_{11}$ by smooth curve. Similarly from $C_1, C_2, C_3 \dots C_{11}$, draw arc of radius CE intersecting points $E_1, E_2, E_3 \dots E_{11}$. Join $C_1E_1, C_2E_2, C_3E_3 \dots C_{11}E_{11}$. Along the lines $C_1E_1, C_2E_2, C_3E_3 \dots C_{11}E_{11}$, mark DE respectively. Let these points be $D_1, D_2, D_3 \dots D_{11}$. Join the points by smooth curve.

Data:

$O_1A = 400$	$EO_2 = 680$
$AB = 2800$	$DE = 680$
$BC = 1400$	$O_1O_2 = 2000$
$CE = 1600$	OFFSET = 1700

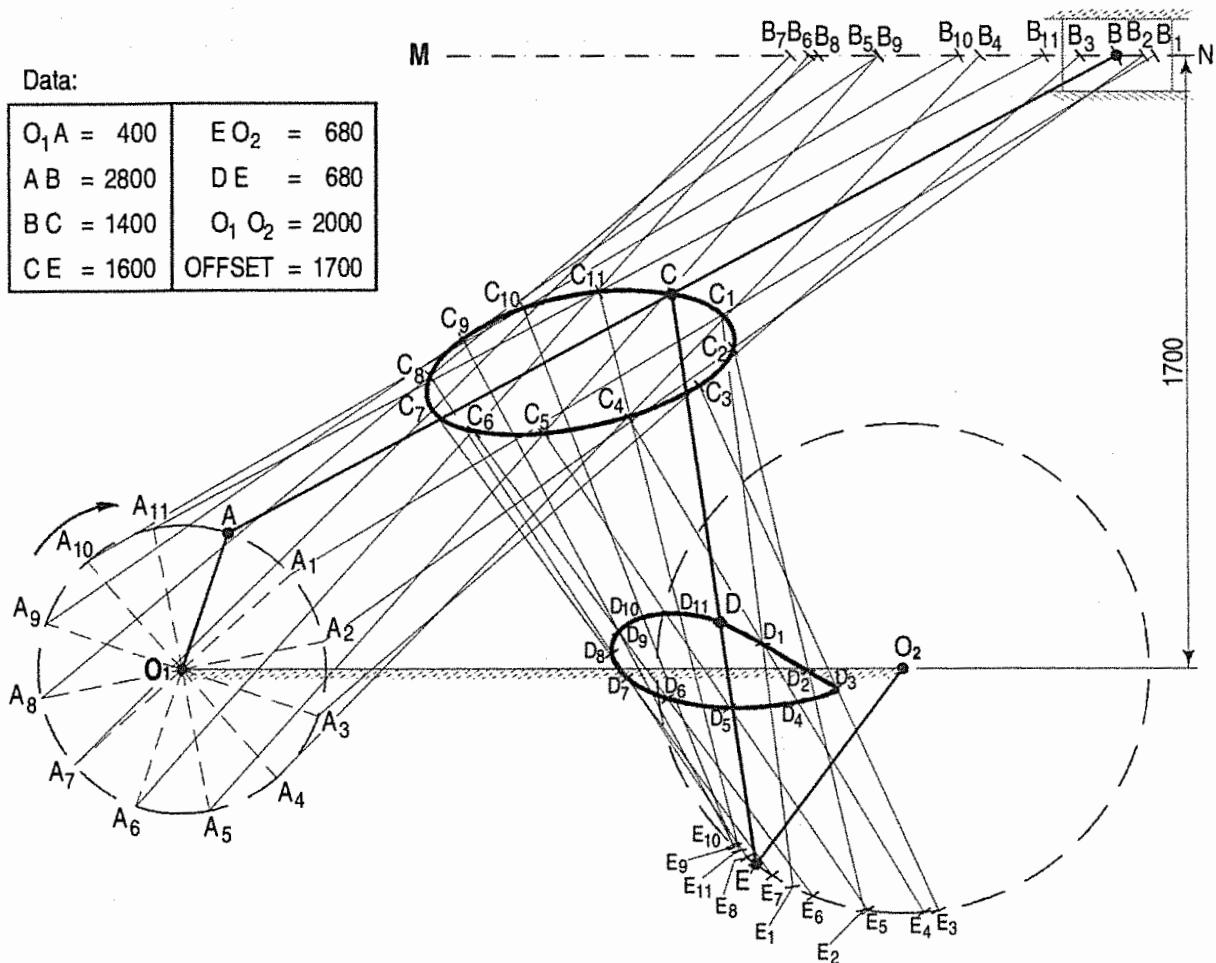


FIG. 7-23

Problem 7-20. (fig. 7-24): PQ and PR_1 are fastened rigidly at P at angle of 90° to each other. The ends P and R are sliding in the guides vertically and horizontally respectively. Draw the loci of the points Q and mid-point of the link PR when the link PR moves from vertically to the horizontal. Name the curve trace by the mid-point. Take $PQ = 40$ mm and $PR = 70$ mm.

- (i) Draw two links PQ and PR_1 of 40 mm and 70 mm respectively at right angle.

- (ii) Make slotted link of 10 mm of width and at P and R . Construct sliders of the convenient length at P and R_1 .
- (iii) Mark the mid point of PR_1 , say S_1 .
- (iv) Assume that the slider P moves vertically downward while R_1 will move horizontally.
- (v) Divide vertical line PR_1 into equal division and number them as 1, 2, 3, 4.....14, R_1 as shown (P , 1 are coinciding).
- (vi) Now 1 as centre and PR_1 as radius, draw arc to cut the horizontal line passing through R_1 . Let us call this point as R_1 . Join P_1R_1 , P_1 as centre mark the mid-point of P_1R_1 , say this point S_1 , similarly from the point 2, 3..... 14, draw arc cutting the horizontal line and number them respectively $R_2, R_3..... R_{14}$. Join all points Q and S respective respectively. Mark on these lines the mid-points $S_2, S_3, S_4..... S_{14}$. Join them with smooth curve. The curve is quarter portion of ellipse. The mechanism is known as *trammel mechanism*.
- (vii) The path of Q is simple to trace as shown in the fig. 7-24.

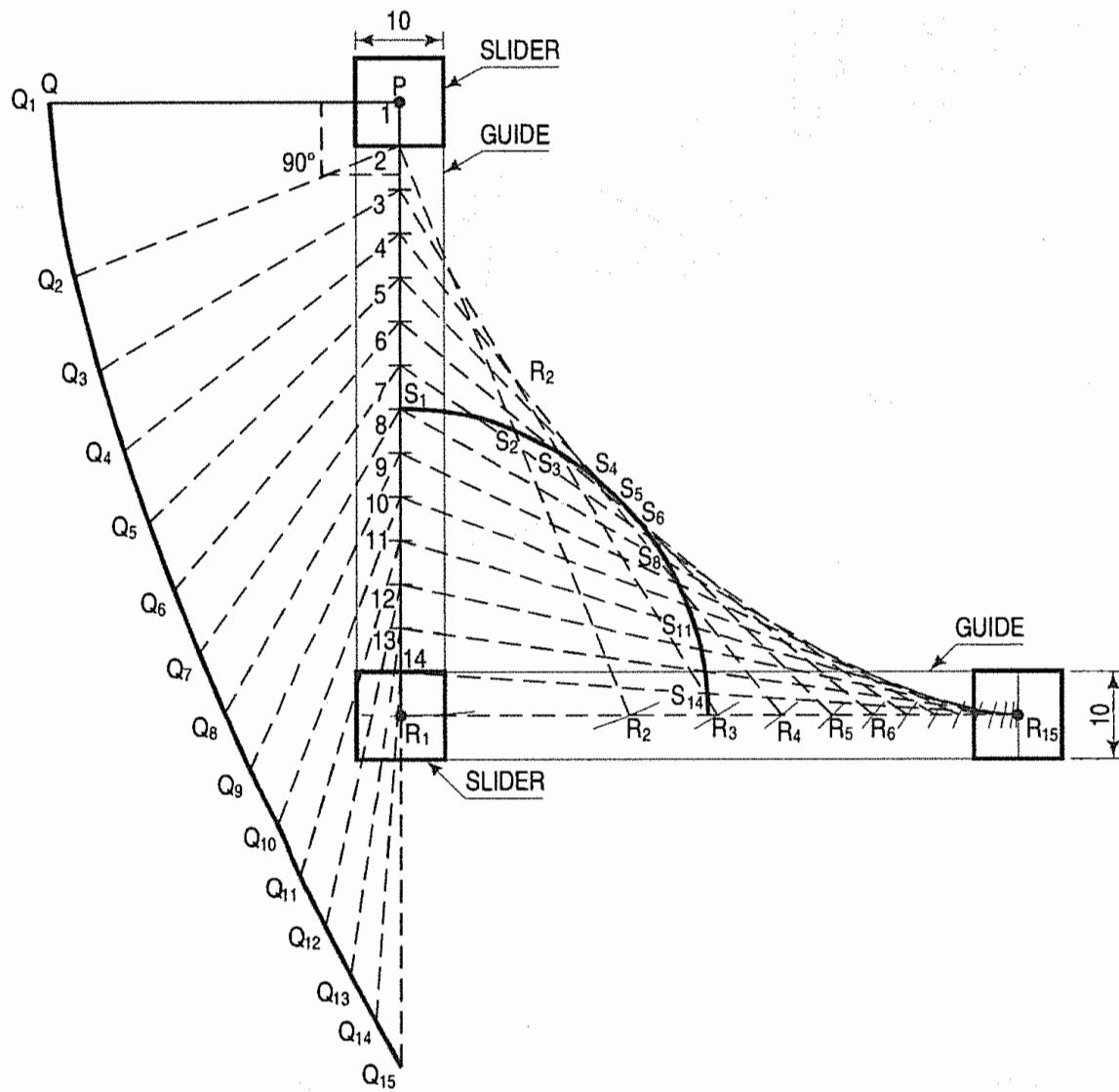


FIG. 7-24

Problem 7-21. The rod PQ as shown in fig. 7-25 is hinged (pinned) to the crank AO at A . OA rotates about O and the rod PQ is constrained to pass through the point R . Draw the loci of the ends P and Q for one complete revolution of OA . $PQ = 1100 \text{ mm}$, $OA = 220 \text{ mm}$, $AP = 300 \text{ mm}$ and $OR = 500 \text{ mm}$.

Scale 100 mm = 10 mm.

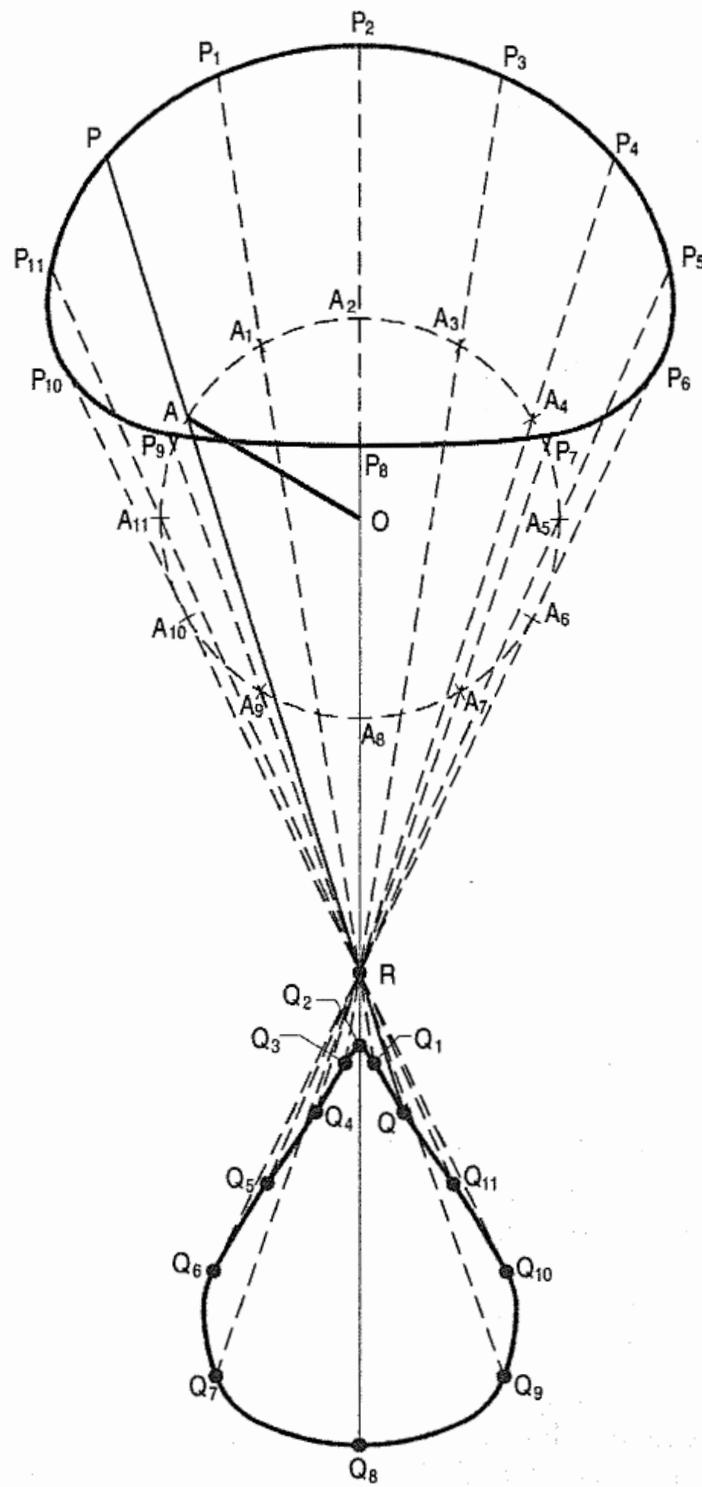


FIG. 7-25

- (i) Draw a mechanism to scale 100 mm = 10 mm for given dimensions.

- (ii) Divide circle described by the crank OA into 12 equal divisions. Say these division $A, A_1, A_2 \dots, A_{11}$.
- (iii) From each points of $A_1, A_2 \dots, A_{11}$, draw lines passing through R and mark length AP and RQ on respective lines. Say these points are $P, P_1, P_2 \dots, P_{11}$ and $Q, Q_1 \dots, Q_{11}$. Join respective points by smooth curves.

EXERCISES 7



- P, Q and R are the centres of three circles of diameters 75 mm, 45 mm and 30 mm respectively. $PQ = 95$ mm, $QR = 50$ mm and $PR = 75$ mm. Draw a circle touching the three circles.
- In a slider-crank mechanism, the crank OA is 450 mm long, and the connecting rod AB , 1050 mm long. Plot the locus of (i) the mid-point P of AB , and (ii) a point 600 mm from A on BA extended, for one revolution of the crank.
- The end P of a 100 mm long line PQ (fig. 7-26) slides vertically downwards. The end Q moves along the line AB towards A and then back to B . Plot the locus of the point O on PQ and 40 mm from P .

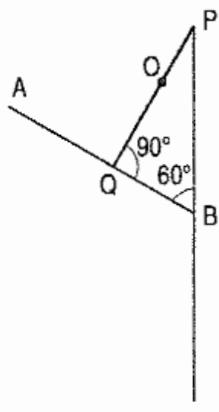


FIG. 7-26

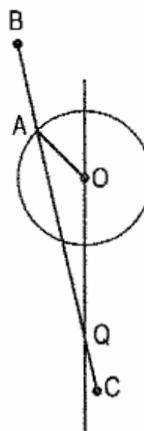


FIG. 7-27

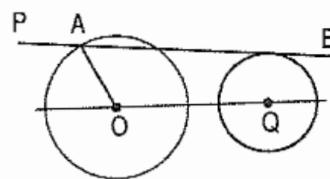


FIG. 7-28

- The rod BC (fig. 7-27) is attached to the crank AO at A . OA rotates about O and the rod BC is constrained to pass through the point Q . Draw the loci of the ends B and C for one complete revolution of OA . $BC = 1200$ mm, $OA = 225$ mm, $AB = 300$ mm and $OQ = 525$ mm.
- The cranks OA (fig. 7-28) rotates about O and the connecting rod AB slides in the same plane on the curved surface of a shaft (with centre Q) of 450 mm diameter. Trace the locus of (i) the end B and (ii) the point P beyond AB and 300 mm from A for one revolution of OA . $OA = 375$ mm, $AB = 1200$ mm and $OQ = 700$ mm.
- PO is a rod 50 mm long. It rotates about its end O with a speed of one revolution per second, while O moves along a straight line OB towards B with a speed of 120 mm per second. Draw to full-size scale the path traced out by the end P for one complete revolution of the rod. In the starting position assume the rod PO to be lying along the extension of the line BO . Show all construction lines.

7. Two equal cranks AB and CD (fig. 7-29) rotate in opposite directions about A and C and are connected by the rod BD . Plot the locus of the end P of the link PQ attached at right angles to BD at its mid-point Q for one complete revolution of the cranks. $AB = 300 \text{ mm}$; $BD = AC = 1050 \text{ mm}$; $PQ = 225 \text{ mm}$.

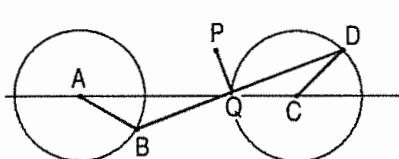


FIG. 7-29

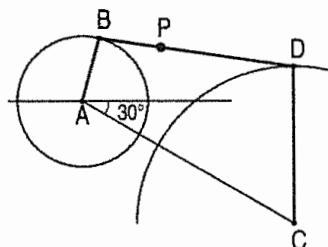


FIG. 7-30

8. Two cranks AB and CD (fig. 7-30) are connected by a link BD . The end B moves round the circumference of the circle with centre A , while the end D oscillates on an arc about C as centre. Plot the locus of the point P on BD , 450 mm from B , for one complete revolution of AB . $AB = 450 \text{ mm}$, $CD = 1050 \text{ mm}$, $BD = 1350 \text{ mm}$ and $AC = 1650 \text{ mm}$.
9. Two equal links AB and CD (fig. 7-31) connected by a rod BD , oscillate about their ends A and C . Plot the locus of (i) the mid-point P of BD and (ii) the point Q on BD . $AB = CD = 1200 \text{ mm}$; $BD = 900 \text{ mm}$; $BQ = 225 \text{ mm}$.

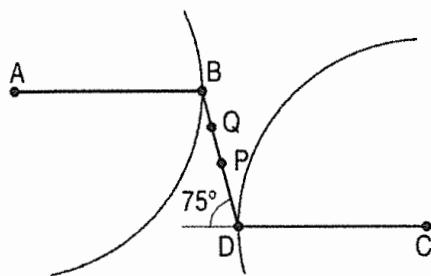


FIG. 7-31

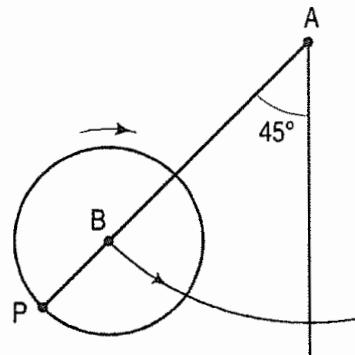


FIG. 7-32

10. The link AB (fig. 7-32) is 120 mm long and carries a circular disc of 40 mm radius. The end A is hinged while the disc can revolve about its centre B . The link turns uniformly to the right through 90° and at the same time the disc revolves uniformly in clockwise \rightarrow direction through one complete revolution. Plot the locus of the point P situated on the circumference of the disc.
11. A thin rod AB , 100 mm long revolves uniformly about its centre O . During its one revolution a point P moves along AB at uniform speed from A to B . Draw the locus of P .
12. Two cranks AB and CD are connected by a link BD . AB rotates about A , while CD oscillates about C . Trace the locus of the mid-point P of link BD during one complete revolution of the link AB . Assume that the points B and D are opposite sides of AC . $AB = 450 \text{ mm}$, $CD = 750 \text{ mm}$, $DB = 1050 \text{ mm}$, $AC = 900 \text{ mm}$. $\angle CAB = 120^\circ$.

13. In a slider-crank mechanism, the crank O_2P is 600 mm long and the connecting rod PQ , 1400 mm long. Plot the path of a point 600 mm from P on QP extended for one revolution of the crank.
14. A link AB 120 mm long rotates about fixed pivot A in an anti-clockwise direction. An ant is situated at 20 mm from the pivot point moves towards the end B with uniform velocity, while the link has rotated through $\frac{1}{6}$ of a revolution. Trace the path of an ant.

Chapter

8



ORTHOGRAPHIC PROJECTION

8-0. INTRODUCTION

Practical solid geometry or descriptive geometry deals with the representation of points, lines, planes and solids on a flat surface (such as a sheet of paper), in such a manner that their relative positions and true forms can be accurately determined.

8-1. PRINCIPLE OF PROJECTION

If straight lines are drawn from various points on the contour of an object to meet a plane, the object is said to be projected on that plane. The figure formed by joining, in correct sequence, the points at which these lines meet the plane, is called the *projection* of the object. The lines from the object to the plane are called *projectors*.

8-2. METHODS OF PROJECTION

In engineering drawing following four methods of projection are commonly used, they are:

- | | |
|-----------------------------|-----------------------------|
| (1) Orthographic projection | (3) Oblique projection |
| (2) Isometric projection | (4) Perspective projection. |

In the above methods (2), (3) and (4) represent the object by a pictorial view as eyes see it. In these methods of projection a three dimensional object is represented on a projection plane by one view only. While in the orthographic projection an object is represented by two or three views on the mutual perpendicular projection planes. Each projection view represents two dimensions of an object. For the complete description of the three dimensional object at least two or three views are required.

8-3. ORTHOGRAPHIC PROJECTION

When the projectors are parallel to each other and also perpendicular to the plane, the projection is called *orthographic projection*.

Step 1: Imagine that a person looks at the block [fig. 8-1(i)] from a theoretically infinite distance, so that the rays of sight from his eyes are parallel to one another and perpendicular to the front surface *F*. The view of this block will be the shaded figure, showing the front surface of the object in its true shape and proportion.

Step 2: If these rays of sight are extended further to meet perpendicularly at a vertical plane (marked V.P.) set up behind the block.

Step 3: The points at which they meet the plane are joined in proper sequence, the resulting figure (marked E) will also be exactly similar to the front surface and this is known as an *elevation* or *front-view*. This figure is the projection of the block. The lines from the block to the plane are the projectors. As the projectors are perpendicular to the plane on which the projection is obtained, it is the orthographic projection. The projection is shown separately in fig. 8-1(ii). It shows only two dimensions of the block viz. the height H and the width W . It does not show the thickness. Thus, we find that only one projection is insufficient for complete description of the block.

Let us further assume that another plane marked

H.P. (horizontal plane) [fig. 8-2(i)] is hinged at right angles to the first plane, so that the block is in front of the V.P. and above the H.P. The projection on the H.P. (figure P) shows the top surfaces of the block. If a person looks at the block from above, he will obtain the same view as the figure P and is known as a *plan* or *top-view*. It shows the width W and the thickness T of the block. It however does not show the height of the block.

One of the planes is now rotated or turned around on the hinges so that it lies in extension of the other plane. This can be done in two ways:

- (i) by turning the V.P. in direction of arrows A
- (ii) by turning the H.P. in direction of arrows B.

The H.P. when turned and brought in line with the V.P. is shown by dashed lines. The two projections can now be drawn on a flat sheet of paper, in correct relationship with each other, as shown in fig. 8-2(ii).

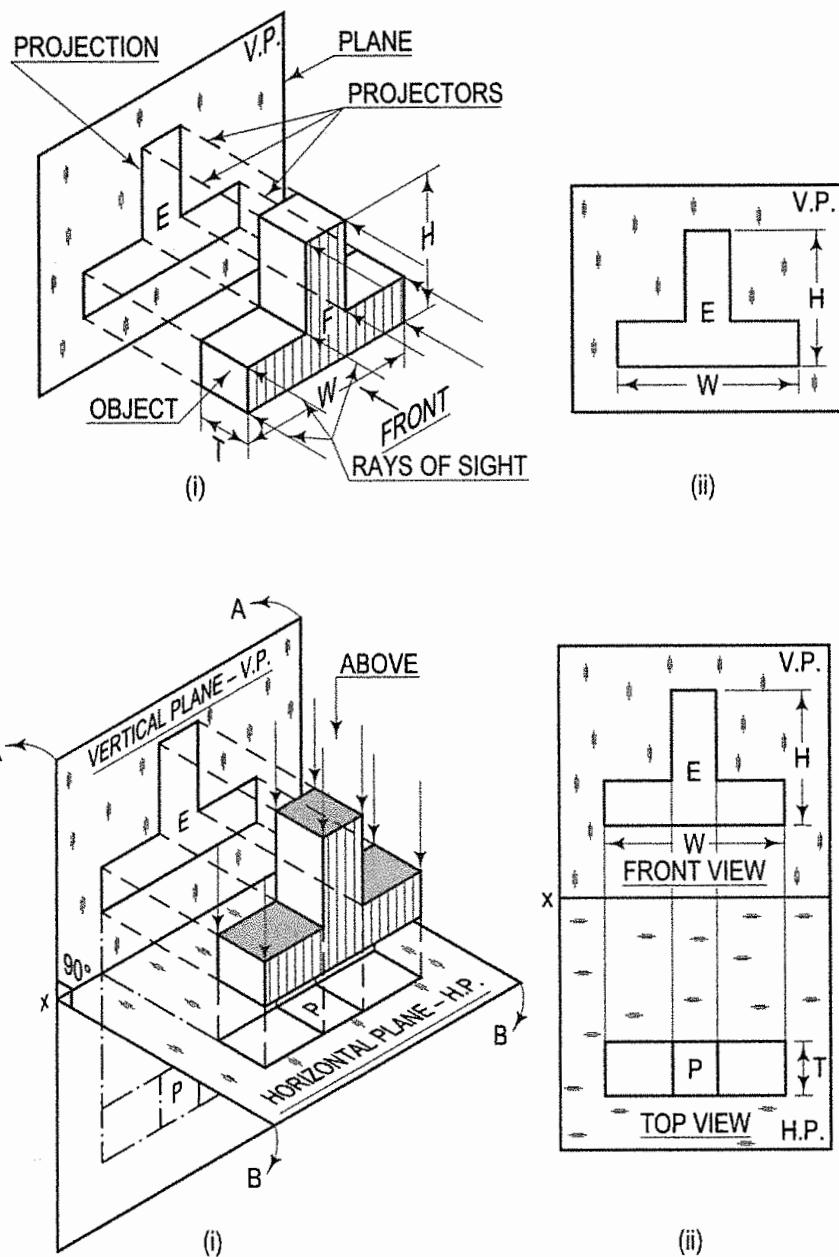


FIG. 8-2

When studied together, they supply all information regarding the shape and the size of the block. Any solid may thus be represented by means of orthographic projections or orthographic views.

8-4. PLANES OF PROJECTION

The two planes employed for the purpose of orthographic projections are called *reference planes* or *principal planes of projection*. They intersect each other at right angles. The *vertical plane* of projection (in front of the observer) is usually denoted by the letters V.P. It is often called the *frontal plane* and denoted by the letters F.P.

The other plane is the *horizontal plane* of projection known as the H.P. The line in which they intersect is termed the *reference line* and is denoted by the letters *xy*. The projection on the V.P. is called the *front view* or the *elevation* of the object. The projection on the H.P. is called the *top view* or the *plan*.

8-5. FOUR QUADRANTS

When the planes of projection are extended beyond the line of intersection, they form four quadrants or dihedral angles which may be numbered as in fig. 8-3. The object may be situated in any one of the quadrants, its position relative to the planes being described as "above or below the H.P." and "in front of or behind the V.P."

The planes are assumed to be transparent. The projections are obtained by drawing perpendiculars from the object to the planes, i.e. by looking from the front and from above. They are then shown on a flat surface by rotating one of the planes as already explained. *It should be remembered that the first and the third quadrants are always opened out while rotating the planes.* The positions of the views with respect to the reference line will change according to the quadrant in which the object may be situated. This has been explained in detail in the next chapter.

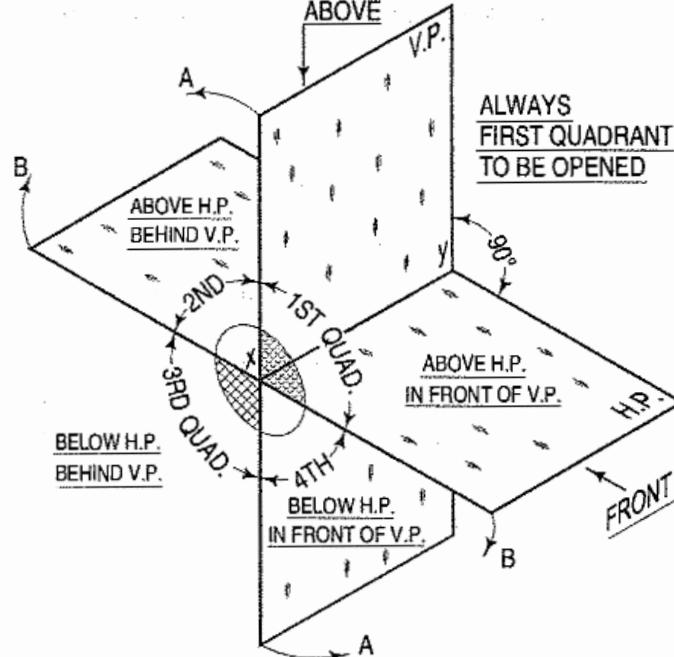


FIG. 8-3

8-6. FIRST-ANGLE PROJECTION

We have assumed the object to be situated in front of the V.P. and above the H.P. i.e. in the first quadrant and then projected it on these planes. This method of projection is known as *first-angle projection method*. The object lies between the observer and the plane of projection. In this method, when the views are drawn in their relative positions, the top view comes below the front view. In other words, the view seen from above is placed on the other side of (i.e. below) the front view. Each projection shows the view of that surface (of the object) which is remote from the plane on which it is projected and which is nearest to the observer.

TABLE 8-1
DIFFERENCE BETWEEN FIRST-ANGLE PROJECTION METHOD
AND THIRD-ANGLE PROJECTION METHOD

No.	First-angle projection method	Third-angle projection method
1.	The object is kept in the <i>first quadrant</i> .	The object is assumed to be kept in the <i>third quadrant</i> .
2.	The object lies between the observer and the plane of projection.	The plane of projection lies between the observer and the object.
3.	The plane of projection is assumed to be non-transparent.	The plane of projection is assumed to be transparent.
4.	In this method, when the views are drawn in their relative positions, the <i>plan</i> comes <i>below the elevation</i> , the view of the object as observed from the <i>left-side</i> is drawn to the <i>right of elevation</i> .	In this method, when the views are drawn in their relative positions, the <i>plan</i> , comes <i>above the elevation</i> , <i>left hand side view</i> is drawn to the <i>left hand side of the elevation</i> .
5.	This method of projection is now recommended by the "Bureau of Indian Standards" from 1991.	This method of projection is used in U.S.A. and also in other countries.

8-7. THIRD-ANGLE PROJECTION

In this method of projection, the object is assumed to be situated in the third quadrant [fig. 8-4(i)]. The planes of projection are assumed to be transparent. They lie between the object and the observer. When the observer views the object from the front, the rays of sight intersect the V.P. The figure formed by joining the points of intersection in correct sequence is the front view of the object. The top view is obtained in a similar manner by looking from above. When the two planes are brought in line with each other, the views will be seen as shown in fig. 8-4(ii). The top view in this case comes above the front view.

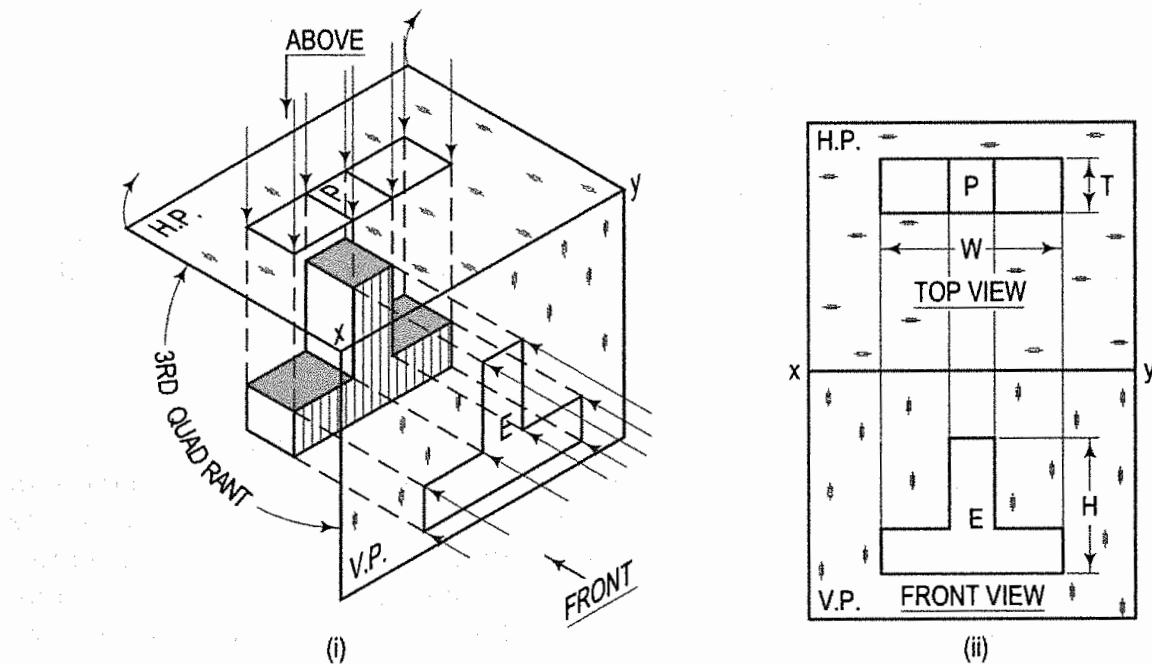


FIG. 8-4

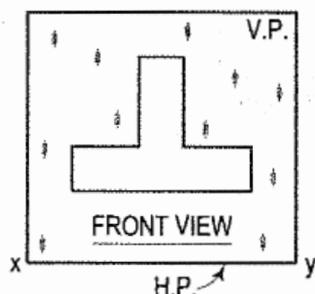
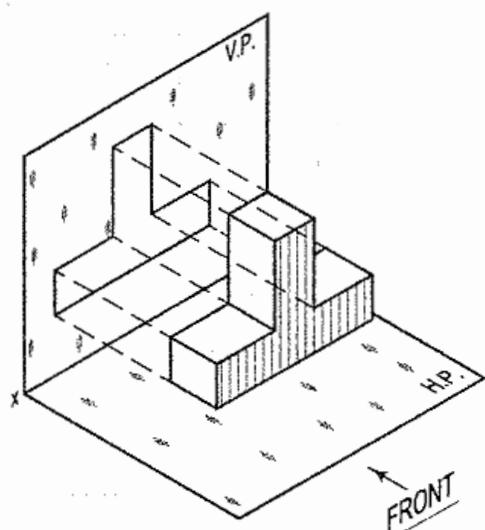
In other words, the view seen from above the object is placed on the same side of (i.e. above) the front view.

Each projection shows the view of that surface (of the object) which is nearest to the plane on which it is projected.

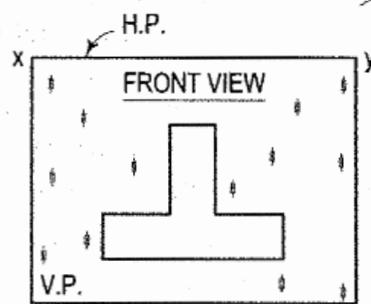
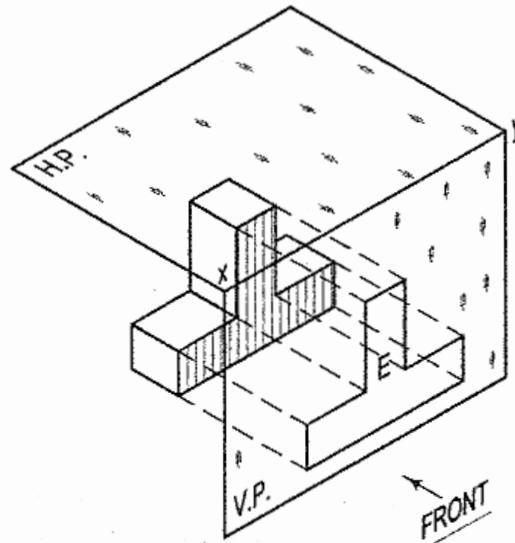
On comparison, it is quite evident that the views obtained by the two methods of projection are completely identical in shape, size and all other details. The difference lies in their relative positions only.

8-8. REFERENCE LINE

Studying the projections independently, it can be seen that while considering the front view (fig. 8-5 and fig. 8-6), which is the view as seen from the front, the H.P. coincides with the line xy . In other words, xy represents the H.P.



FIRST-ANGLE PROJECTION



THIRD-ANGLE PROJECTION

FIG. 8-5

FIG. 8-6

Similarly, while considering the top view (fig. 8-7 and fig 8-8), which is the view obtained by looking from above, the same line xy represents the V.P. Hence, when the two projections are drawn in correct relationship with each other (fig. 8-9), xy represents both the H.P. and the V.P. This line xy is called the *reference line*. The squares or rectangles for individual planes are thus unnecessary and are therefore discarded.

Further, in first-angle projection method, the H.P. is always assumed to be so placed as to coincide with the ground on or above which the object is situated. Hence, in this method, the line xy is also the line for the ground.

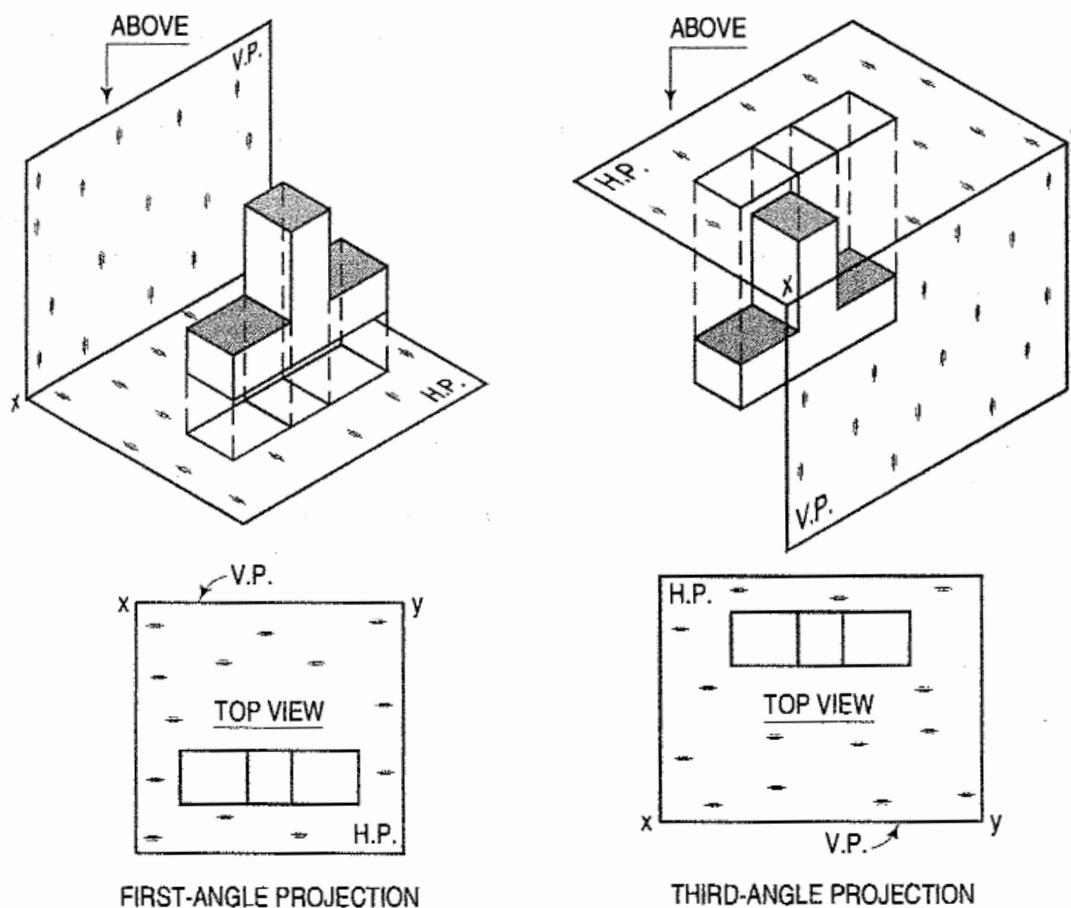


FIG. 8-7

In third-angle projection method, the H.P. is assumed to be placed above the object. The object may be situated on or above the ground. Hence, in this method, the line xy does not represent the ground. The line for the ground, denoted by letters GL, may be drawn parallel to xy and below the front view [fig. 8-9(ii)].

In brief, when an object is situated on the ground, in first-angle projection method, the bottom of its front view will coincide with xy ; in third-angle projection method, it will coincide with GL, while xy will be above the front view and parallel to Ground line.

Symbols for methods of projection: For every drawing it is absolutely essential to indicate the method of projection adopted. This is done by means of a symbolic figure drawn within the title block on the drawing sheet.

The symbolic figure for the first-angle projection method is shown in fig. 8-10, while that for the third-angle projection method is shown in fig. 8-11 which are self explanatory. These symbolic figures are actually the projections of a frustum of cone of convenient dimensions according to the size of drawing.

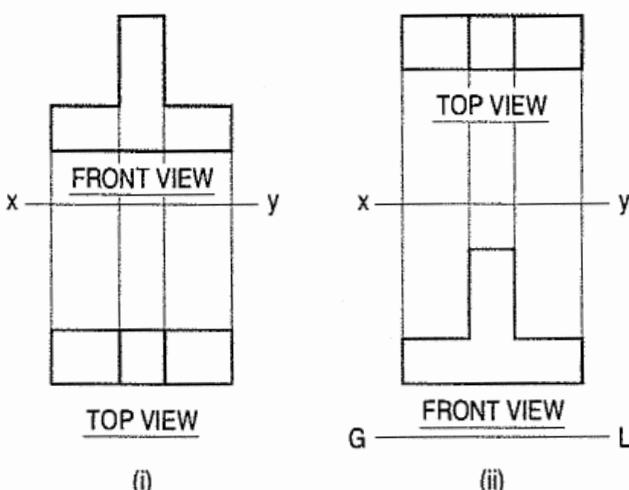


FIG. 8-9

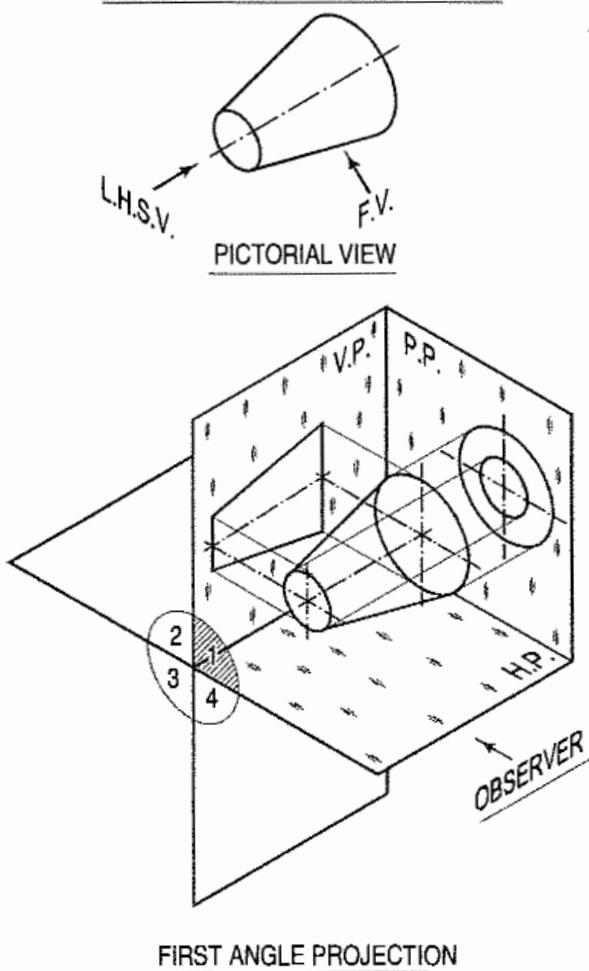
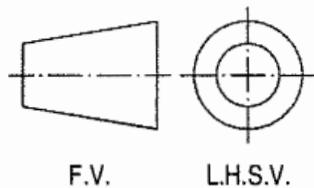
FIRST ANGLE PROJECTION METHODRELATION BETWEEN OBSERVER, OBJECT AND P.P.IDENTIFYING GRAPHICAL SYMBOL OF FIRST ANGLE PROJECTION

FIG. 8-10

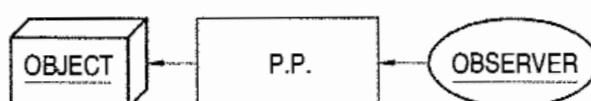
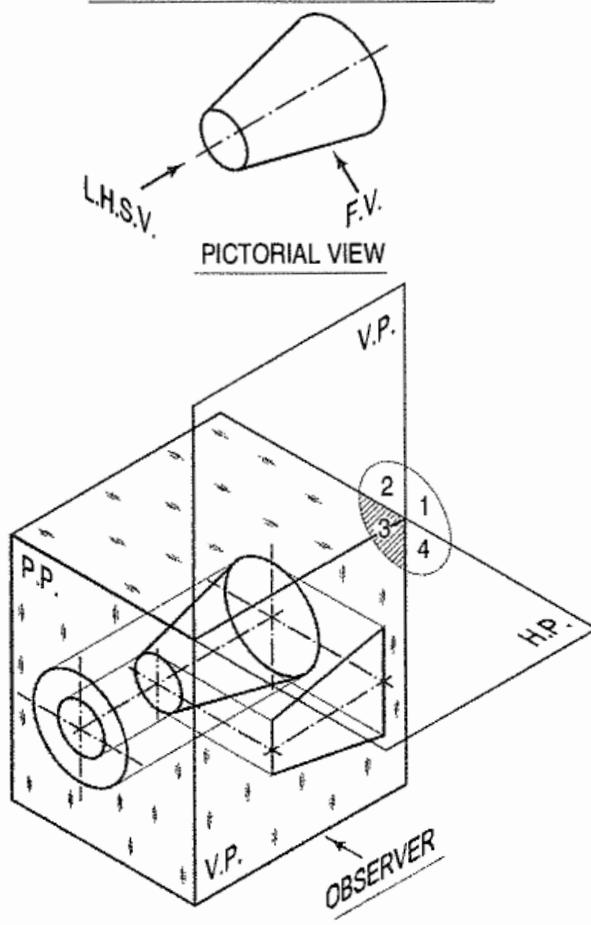
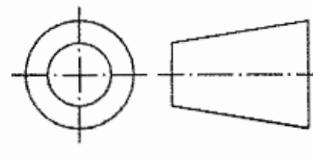
THIRD ANGLE PROJECTION METHODRELATION BETWEEN OBSERVER, OBJECT AND P.P.IDENTIFYING GRAPHICAL SYMBOL OF THIRD ANGLE PROJECTION

FIG. 8-11

Six views of an Object: There are three important elements of this projection system, namely

- an object
- plane of projection
- an observer.

Very often, two views are not sufficient to describe an object completely. The planes of projection being imaginary, following six views are obtained:

- | | |
|-------------------------|--------------------------|
| (1) Front view | (4) Right hand side view |
| (2) Top view | (5) Back view |
| (3) Left hand side view | (6) Bottom view |

These projections are shown projected on the respective planes, placed by the methods of first-angle projection and third-angle projection as shown in fig. 8-12 and fig. 8-13 respectively.

Ordinarily, two views — the front view and top view are shown. Two other views i.e. L.H.S.V. or R.H.S.V. may be required to describe an object completely. Only in exceptional cases, when an object is of a very complex nature, five or six views may be found necessary.

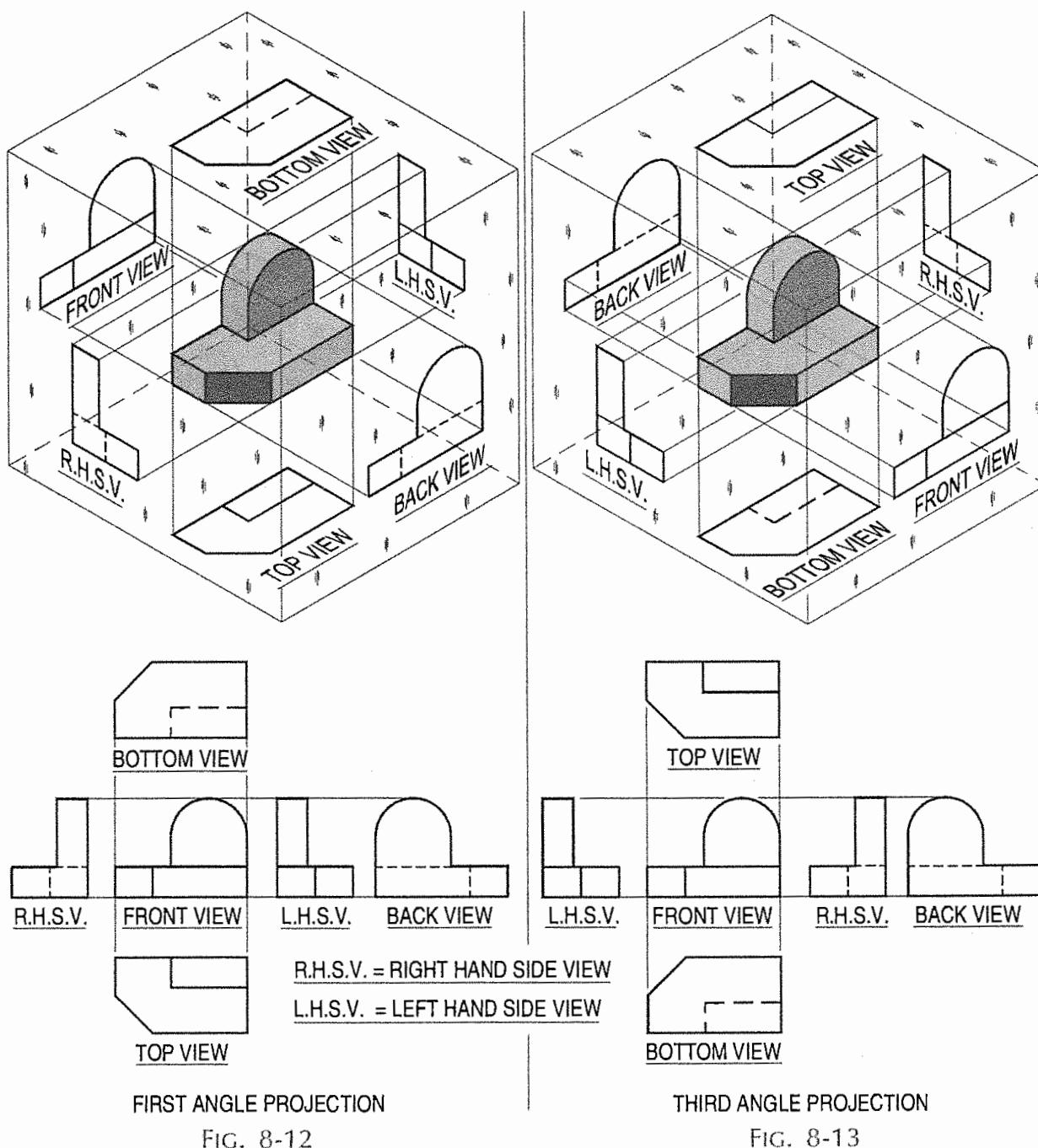


FIG. 8-12

FIG. 8-13

8-9. B.I.S. CODE OF PRACTICE



The method of first-angle projection is the British standard practice. The third-angle projection is the standard practice followed in America and in the continent of Europe.

In our country, the first-angle projection method was formerly in use. The Indian Standards Institution (I.S.I.) now Bureau of Indian Standards (B.I.S.), in its earlier versions of Indian Standard (IS:696) 'Code of Practice for General Engineering Drawing' published in 1955 and revised in 1960 had recommended the use of third-angle projection method.

In the second revised version of this standard published in December 1973, the committee responsible for its preparation left the option of selecting first-angle or third-angle projection method to the users.

The committee again reviewed the position and *finally recommended revised SP:46-1988 and SP:46-2003 for implementation of first-angle method of projection in our country*, by replacing earlier IS:696 drawing standard.

Persons engaged in engineering profession may come across drawings from industries and organizations following any one method. It is therefore necessary for them to be perfectly conversant with both the methods.

In this book, the method of first-angle projection has been generally followed. Third-angle projection method is also adequately treated in the form of illustrative problems and set exercises.

Conventions employed: In this book, actual points, ends of lines, corners of solids etc., in space are denoted by capital letters A, B, C etc. Their top views are marked by corresponding small letters a, b, c, etc., their front views by small letters with dashes a', b', c', their side views by a₁, b₁, c₁, and their auxiliary views by a'₁, b'₁, c'₁, etc. In pictorial views, the projectors from the points in space to the planes are shown by dashed lines.

The lines from the projections to the reference line xy (which are also called projectors, though they are the projections of the projectors) are shown as dash and dot lines. In orthographic views, the projectors and other construction lines are shown continuous, but thinner than the lines for actual projections.

8-10. TYPICAL PROBLEMS



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 18 for the following problem.

Problem 8-1. (fig. 8-14): A L-shaped solid object having dimensions of length (L), width (W) and height (H) in the fig. 8-14. Assuming that this object is lying in the first quadrant. Draw it's front view, top view and side view.

When the given object is in the first quadrant, its front view appears in the imaginary vertical plane V.P. behind it while its top view appears in an imaginary horizontal plane, H.P. below. The side view appears to the right or left of the front view depending on from which side the object is being viewed.

- (i) Mark the visible corners of the given block as shown

Drawing front orthogonal view:

Assume that you are viewing the object in the direction of the arrow towards the imaginary V.P. What you will see is a rectangle of height H and width W on V.P. This will be the front view. To draw this view:

- (ii) Draw a reference line xy , which represents the intersecting line of the planes V.P. and H.P. Draw a rectangle as shown W and H , above xy make sure that the width is parallel to the line xy . The rectangle is the front orthogonal view of the object.
- (iii) Draw a line parallel to and thickness of h to the line 1-2. The rectangle 1-2-4-3 is the front view of the horizontal L-shaped stem of the object.

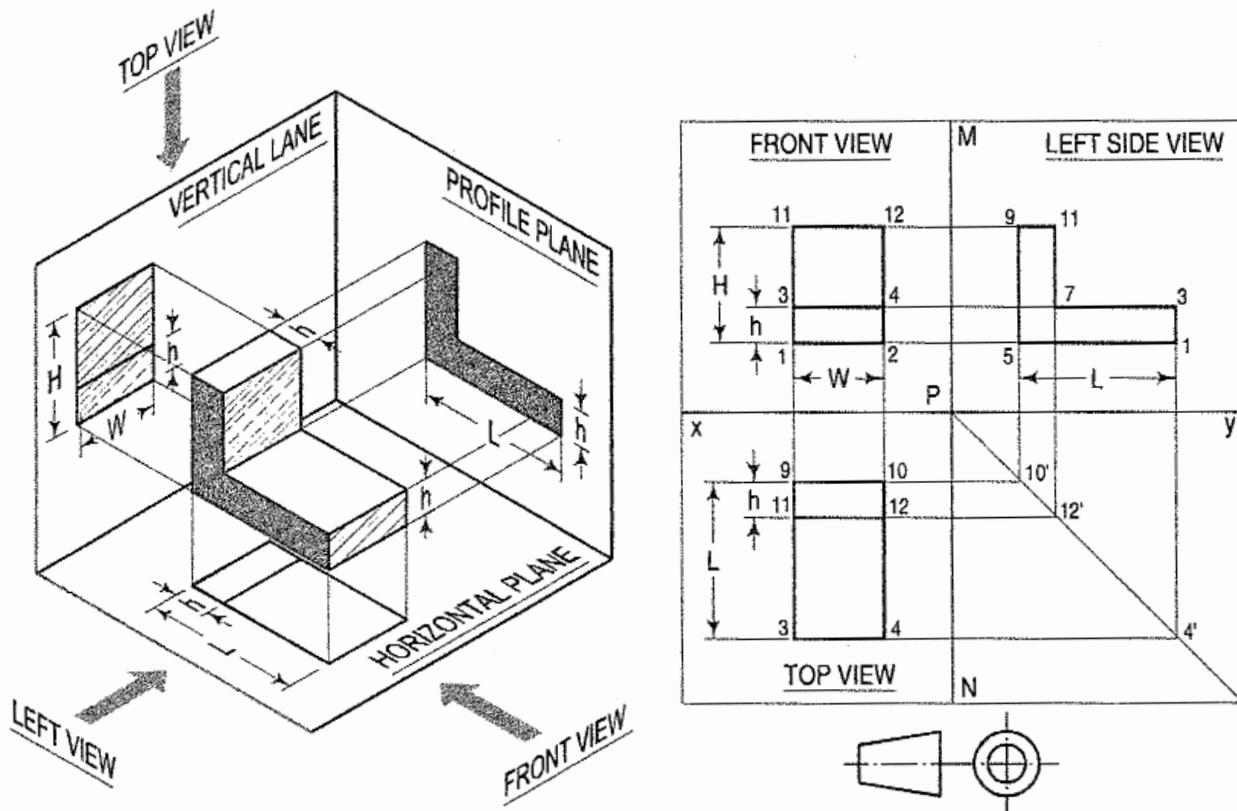


FIG. 8-14

Drawing top orthogonal view:

Now, if you look the object from the top, you will see a rectangle of Length L and width W on the horizontal plane. This is the top view of plan of the object. To draw this view:

- (iv) Draw vertical projectors from 1 and 2 and extend them beyond the line xy . Draw a line 9-10 below and parallel to the reference line xy . Draw the lines 9-3 and 10-4 equal to the length L of the object. Join line 3-4. The rectangle 9-10-3-4 is the top view of the object.
- (v) Draw a line 11-12 parallel to and below 9-10 of thickness h . The rectangle 9-11-12-10 is the view of the vertical stem of the object.

Drawing the side orthogonal view:

Now if you look at the object from the left side, what you will see is an L-shaped image having a length of L and height of H on the auxiliary plane, AP. This view appears adjacent and to the right of front view. To draw this view:

- (vi) Draw a reference vertical line MN at right angles to xy cutting it at P . From P draw a construction line at 45° in the fourth quadrant.
 - (vii) Project lines from the points 10, 12 and 4 of the top view to meet this inclined line at $10'$, $12'$ and $4'$.
 - (viii) Project lines from points 2, 4 and 12 from the front view parallel to line xy . From points $10'$, $12'$ and $4'$ project lines vertically upwards to meet these horizontal projections.
 - (ix) Join points 5-1-3-7-11-9. This will be the side view of the object.
 - (x) Finally draw the symbol of first angle projection at the right bottom corner of the drawing.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 19 for the following problem.

Problem 8-2. (fig. 8-15 and fig. 8-16): A pictorial view of a machine bracket is shown in the figure. Draw using the first angle projection method front view, top view and right end side view.

Assume that you are viewing the object in the direction of the arrow towards the imaginary V.P. What will you see? It is a rectangle of height 70 mm and width 100 mm on V.P. This will be the front view. To draw this view:

- (i) Draw a Reference line xy , which represents the intersecting line of the planes V.P. and H.P.
 - (ii) Draw rectangular block of size 100×70 and the thickness is 15 mm each on the two parallel portions of the slot.
 - (iii) Now on the top of this rectangular block draw a rectangle of size 60 mm \times 20 mm. This rectangle represents a boss of $\phi 60$ on the rectangular block.
 - (iv) Draw a hole of $\phi 50$ in the above boss. Note carefully that the lines are dashed lines as they are invisible from the front side. The $\phi 30$ hole is in the rectangular block.

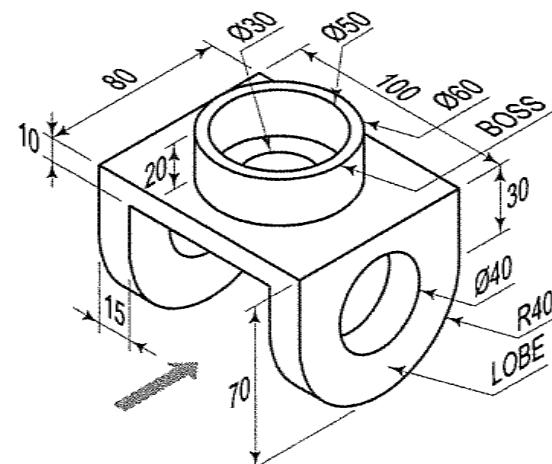


FIG. 8-15

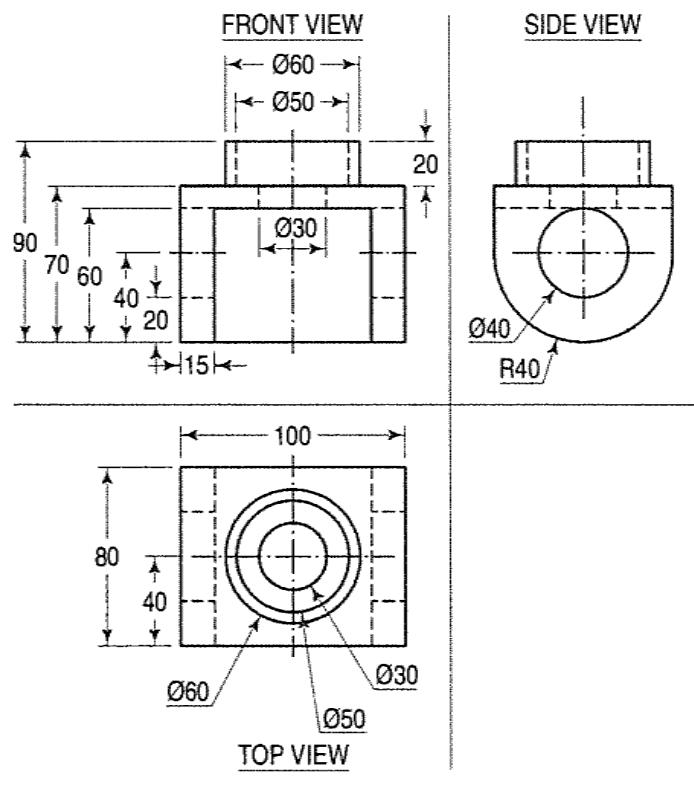


FIG. 8-16

- (v) Draw the two holes on the parallel edges of the rectangular block. These holes are represented by dotted lines in the front view as they are hidden. This completes the front view construction.
- (vi) To draw the top view, project all the details from front view.
- (vii) Draw a vertical centre line from the centre of the rectangular block.
- (viii) The top view of the rectangular block appears as a rectangle of size 100×80 .
- (ix) Draw the holes in the centre of the rectangle. These holes are drawn taking the common centre point on vertical centre line.
- (x) The two holes of $\phi 40$ on the lobes are not visible, so they are projected as dashed lines. This completes the top view.
- (xi) To draw the side view, draw projectors from all the points in front view and top view to side view. Join the intersection points. This completes the side view.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 20 for the following problem.

Problem 8-3. Fig. 8-17 shows the pictorial view of the object. Draw the front view, top view and left hand side view.

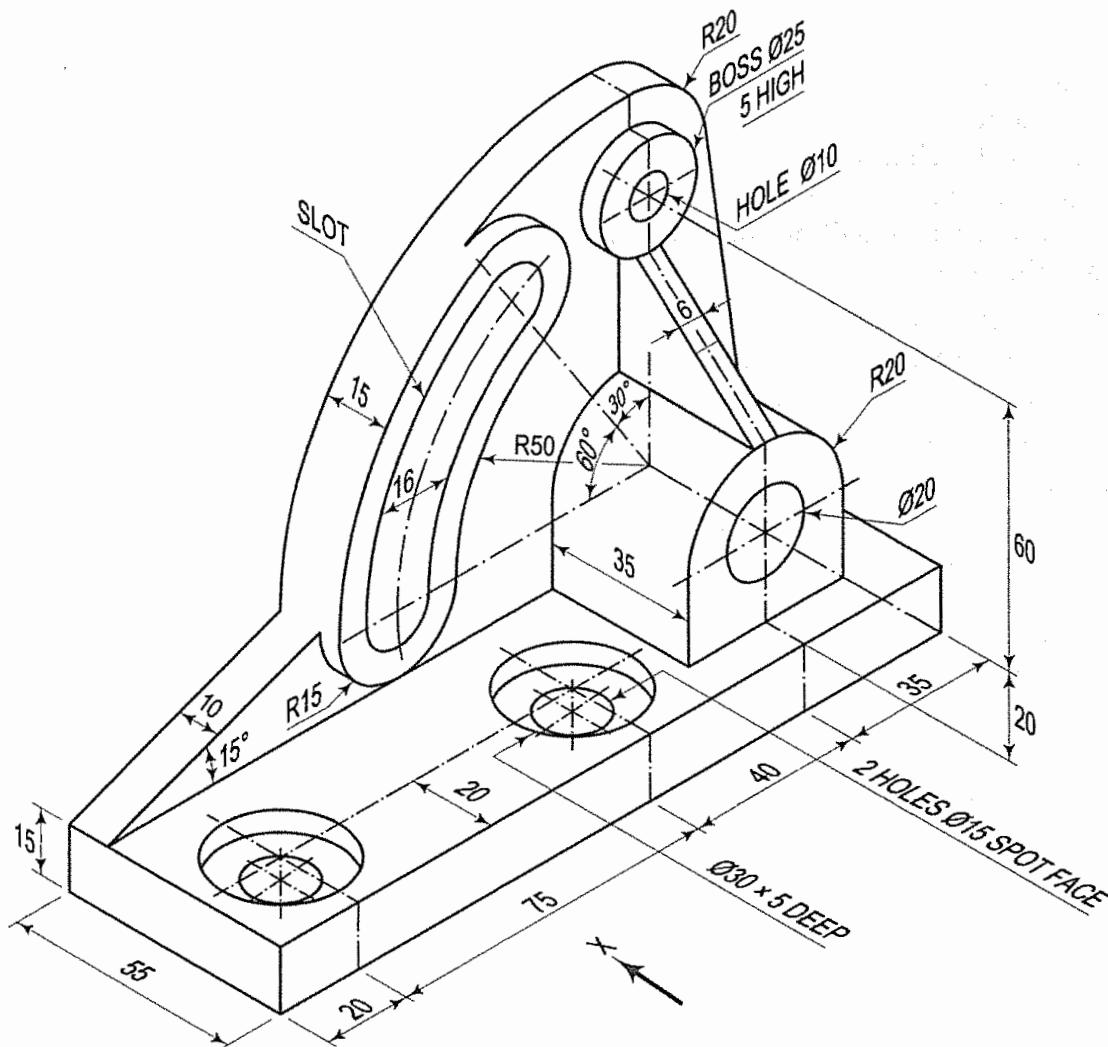
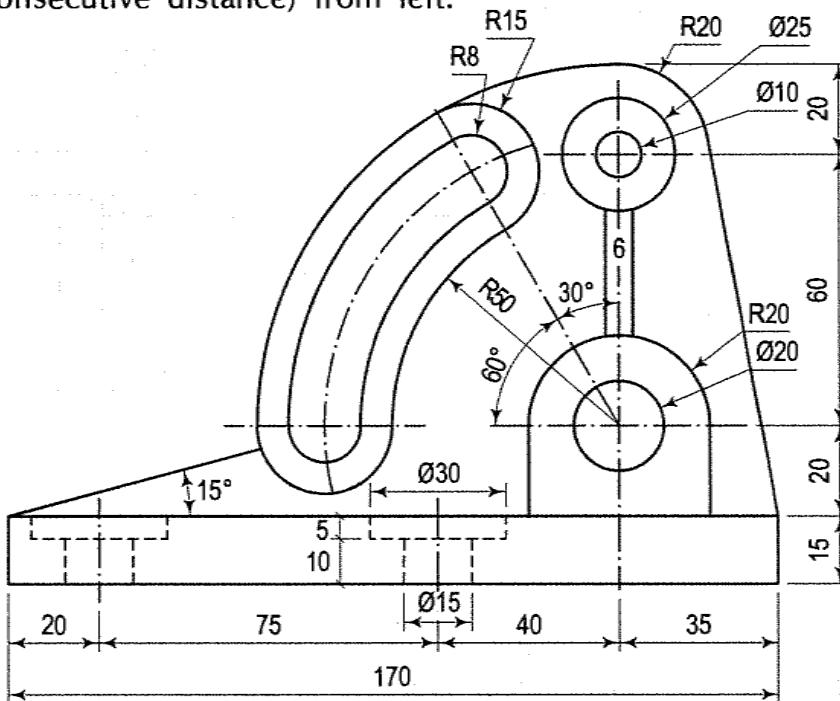


FIG. 8-17

- To draw F.V. in the direction of X, L.H.S.V. and T.V.
- As the L.H.S.V. is to be drawn, fix the position of xy at centre of page and x_1y_1 to the left side as L.H.S.V. will come to the right of F.V.
- Draw the rectangles for the F.V. (170×115), project it down to locate T.V. (170×55) and take the projections of F.V. to left to draw L.H.S.V., complete the rectangle of L.H.S.V. by taking the projections from T.V.
- First we will start with F.V. (fig. 8-18).
 - Draw the rectangle for base plate 170×15
 - Draw vertical centre lines at 20 mm and 75 mm and 40 mm (consecutive distance) from left.

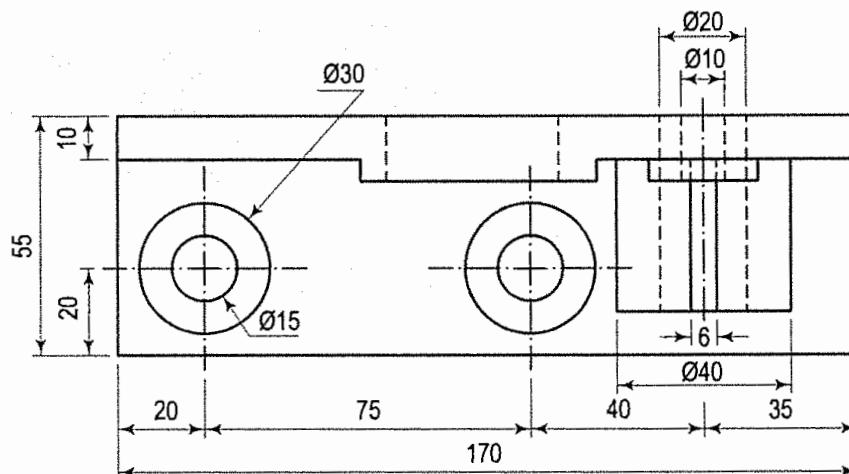


Front view

FIG. 8-18

- Draw hidden lines for 5 mm depth for two $\varnothing 30$ spot face holes, draw dotted lines for 2 holes of $\varnothing 15$ (Inside these holes).
- Draw horizontal centre line at 35 mm from base, in semi-circle.
- Referring third centre line and this horizontal line draw the circle of 10 mm radius and semicircle of 20 mm radius.
- Draw vertical lines from end of this semi-circle. Till it touches to base.
- Draw another horizontal centre line at 60 mm above previous horizontal centre line.
- With this as centre, draw circles of $\varnothing 10$ and $\varnothing 25$.
- Also draw the arc of radius 20 mm, join this to base with inclined line to represent rib.
- Draw two lines at 3 mm to both the sides, vertical centre lines of the circles to represent the rib.
- From third centre point, draw centre line at 30° .
- Draw arcs of 50 mm, 73 mm, 57 mm, 80 mm.

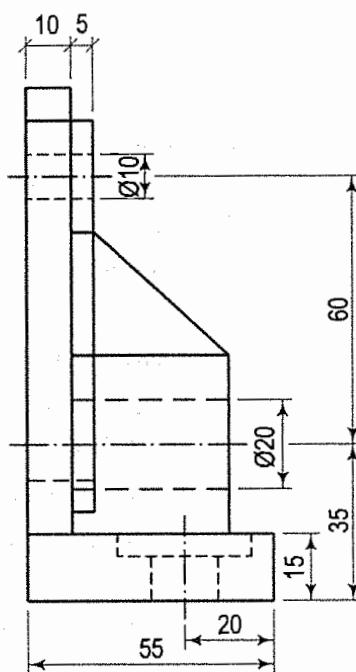
- (m) Draw an arc of centre line pattern of 65 mm radius. (Draw centre line for an arc)
- (n) Taking intersection of this as reference and horizontal centre line draw semi-circle of radius 15 mm and 8 mm.
- (o) Also draw same arcs, where this line intersects with 30° inclined line.
- (p) Erase unwanted arcs if any.
- (q) Draw inclined line at 15° from left end of block.
- (r) Show the required dimensions.
- (s) Now project all the required line to draw T.V.
- (v) Now draw the top view (fig. 8-19).



Top view

FIG. 8-19

- (a) Complete the rectangle of the top view (170×55)
- (b) Project the centre lines. (Vertical at 20, 75 and 40 consecutive distances from left and at 20 above base)
- (c) Draw the two circles of $\varnothing 30$ and $\varnothing 15$ to represent spot holes.
- (d) Draw projection of inclined rib 10 thick and circular slot 15 thick.
- (e) Draw hidden lines for inner circular slot.
- (f) Draw rectangle for semicircular projection from F.V. projections
- (g) Draw hidden lines for hole of $\varnothing 20$ in it.
- (h) Show the projections of bosse holes.
- (i) Draw hidden lines for hole of $\varnothing 10$.
- (j) Show the required dimensions.
- (vi) Take projections from F.V. and T.V. to complete L.H.S.V. (fig. 8-20).
- (a) Complete the base rectangle of 55 mm \times 15 mm. To represent the base.



L.H.S.V.

FIG. 8-20

- (b) Complete the rectangle at left side in L shape of 10 mm thick.
- (c) Show the projections of projected semi-circle.
- (d) Show hidden lines of hole in it.
- (e) Show projection of projected slot and show hidden lines in it.
- (f) Draw projection of boss and show hole in it by dotted line.
- (g) Show inclined line for the rib.
- (h) Draw front view, top view and L.H.S.V. in their relative position.

EXERCISES 8



1. Define orthographic projection. Describe briefly the method of obtaining an orthographic projection of an object.
2. Write short notes on:
Reference planes; Reference line; Projector; Front view; Ground line.
3. Sketch neatly the symbols used for indicating the method of projection adopted in a drawing. State where this symbol is drawn on a drawing sheet.
4. Explain briefly how the reference line represents both the principal planes of projection.
5. Explain clearly the difference between the first-angle projection method and the third-angle projection method.
6. Fill-up the blanks in the following with appropriate words selected from the list of words given below:
 - (a) In _____ projection, the _____ are perpendicular to the _____ of projection.
 - (b) In first-angle projection method,
 - (i) the _____ comes between the _____ and the _____.
 - (ii) the _____ view is always _____ the _____ view.
 - (c) In third-angle projection method,
 - (i) the _____ comes between the _____ and the _____.
 - (ii) the _____ view is always _____ the _____ view.

List of words for Exercise (6):

- | | | |
|----------|-----------------|---------------|
| 1. Above | 5. Object | 9. Projectors |
| 2. Below | 6. Orthographic | 10. Plane |
| 3. Front | 7. Observer | 11. Side |
| 4. Left | 8. Right | 12. Top. |

Answer to Exercise (6):

- (a) – 6, 9 and 10,
 - (b) (i) – 5, 7 and 10,
 - (b) (ii) – 12, 2 and 3,
 - (c) (i) – 10, 5 and 7,
 - (c) (ii) – 12, 1 and 3.
7. Why second and fourth quadrants are not used in practice?
 8. What is the convention of representing first-angle projection method?

9. The pictorial view of different types of objects are shown in fig. 8-21. Sketch, looking from arrow, elevation, plan and end-view using first-angle projection method.

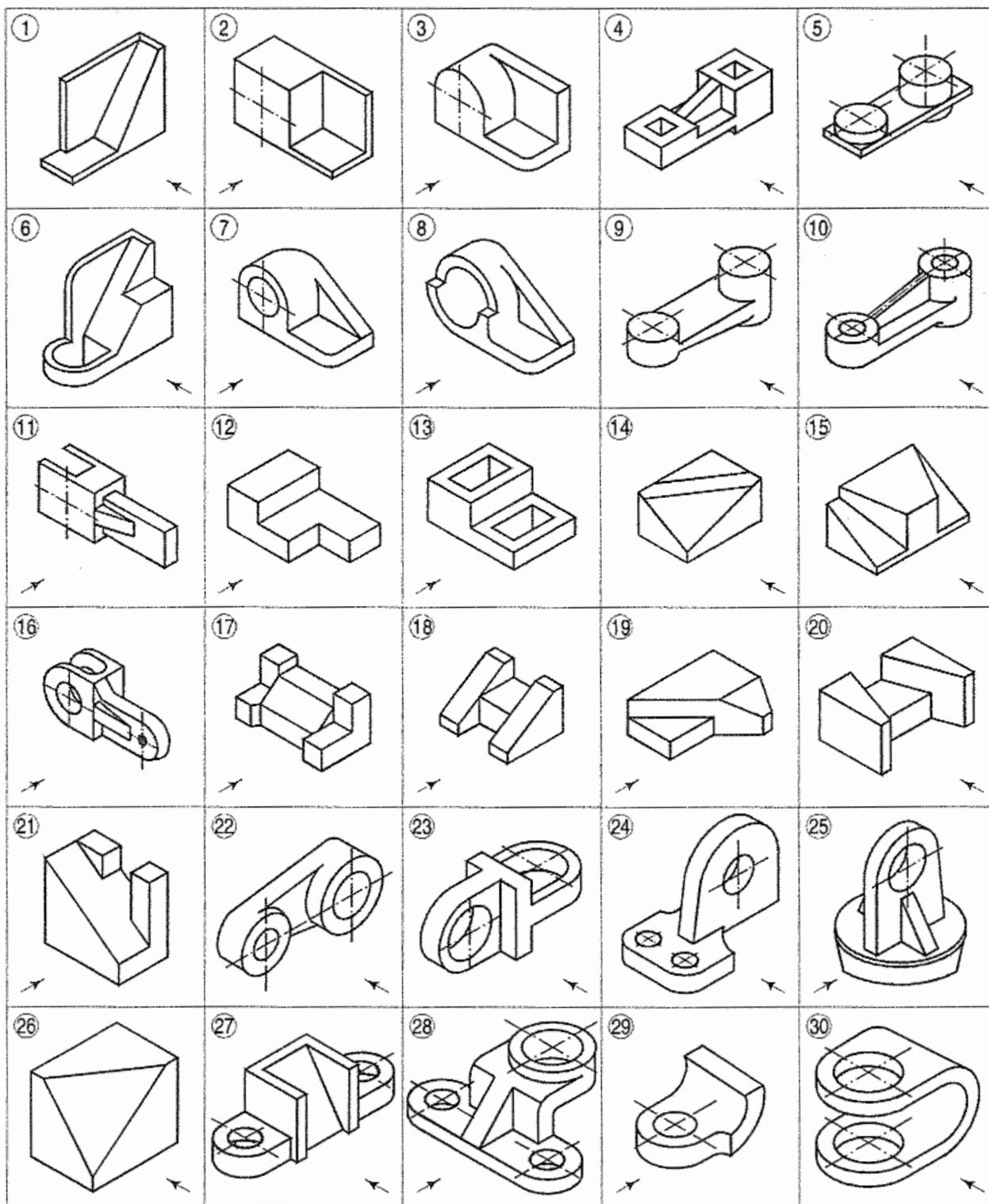


FIG. 8-21

[Draw orthographic projections of each object and then compare your answer with the solution given in fig. 8-26.]

10. What dimensions of an object are given by
 (i) Front view or elevation?
 (ii) Plan or top view?
 (iii) Left-hand side view and right-hand side view?
11. Fig. 8-22 and fig. 8-23 show the orthographic projections of the objects in the first-angle projection method.
 Draw them in the third-angle projection method.

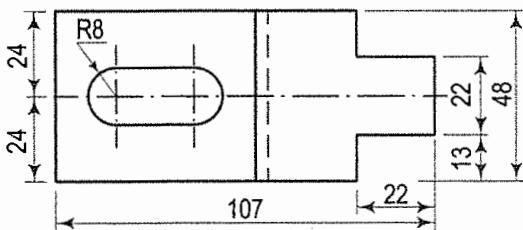
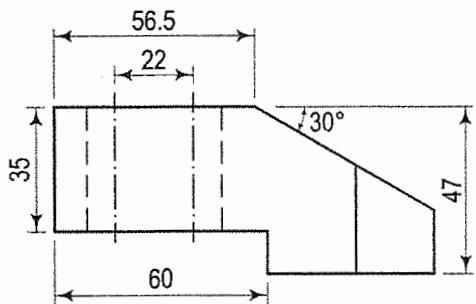


FIG. 8-22

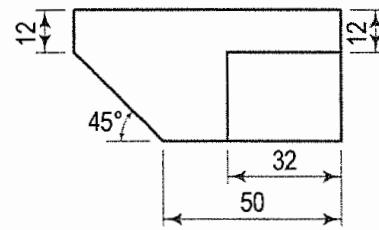
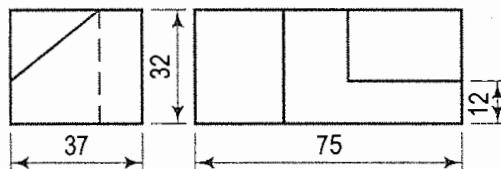


FIG. 8-23

12. Fig. 8-24 and fig. 8-25 show the orthographic projections of the objects in the third-angle projection method.
 Redraw them in the first-angle projection method.

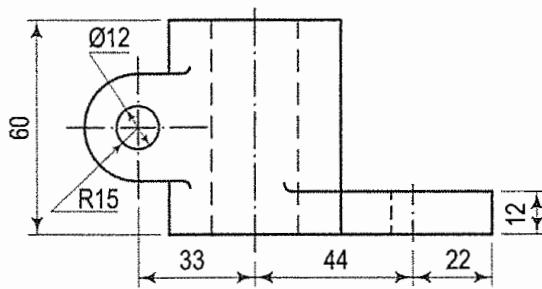
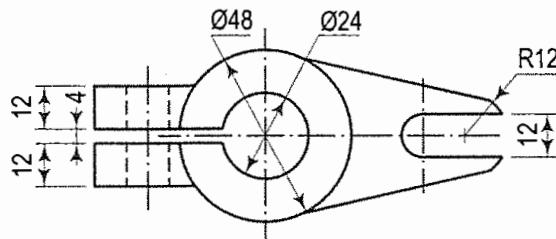


FIG. 8-24

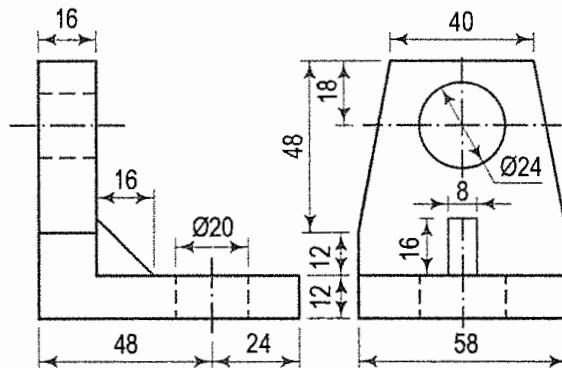
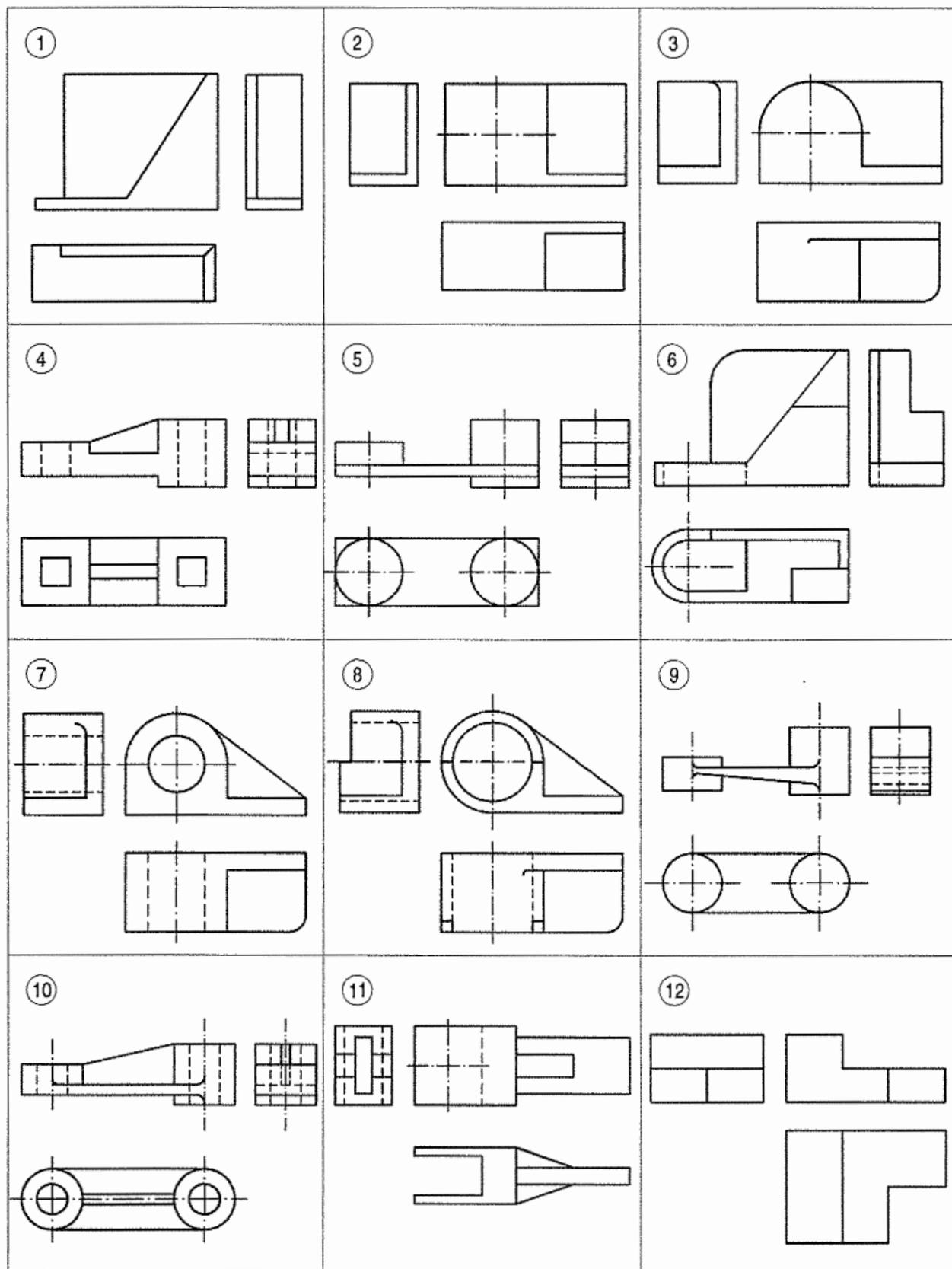


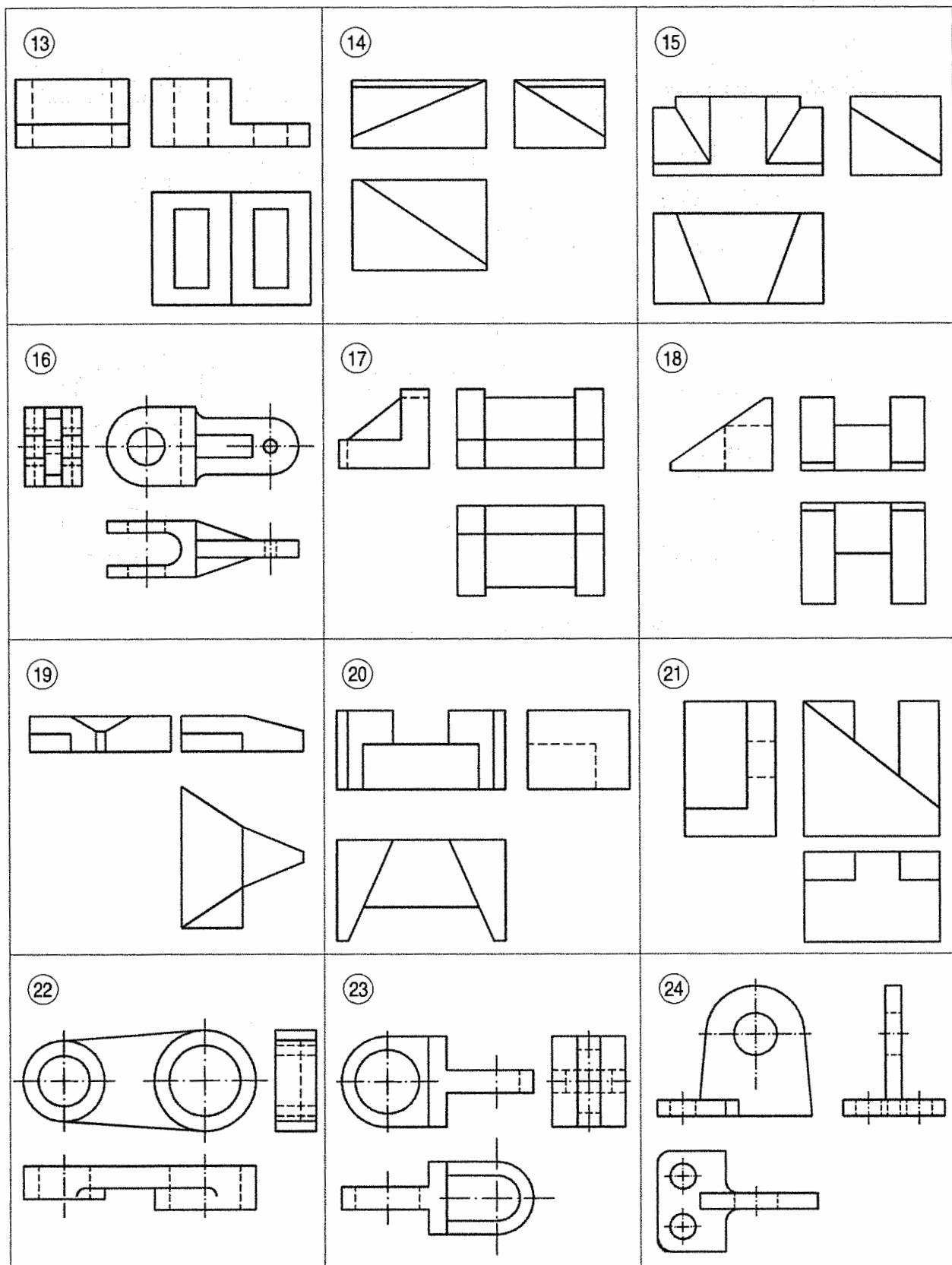
FIG. 8-25

13. What are the important elements of the projection system? Draw any simple object and obtain its six views, projected on imaginary planes by the methods of first angle projection and third angle projection.



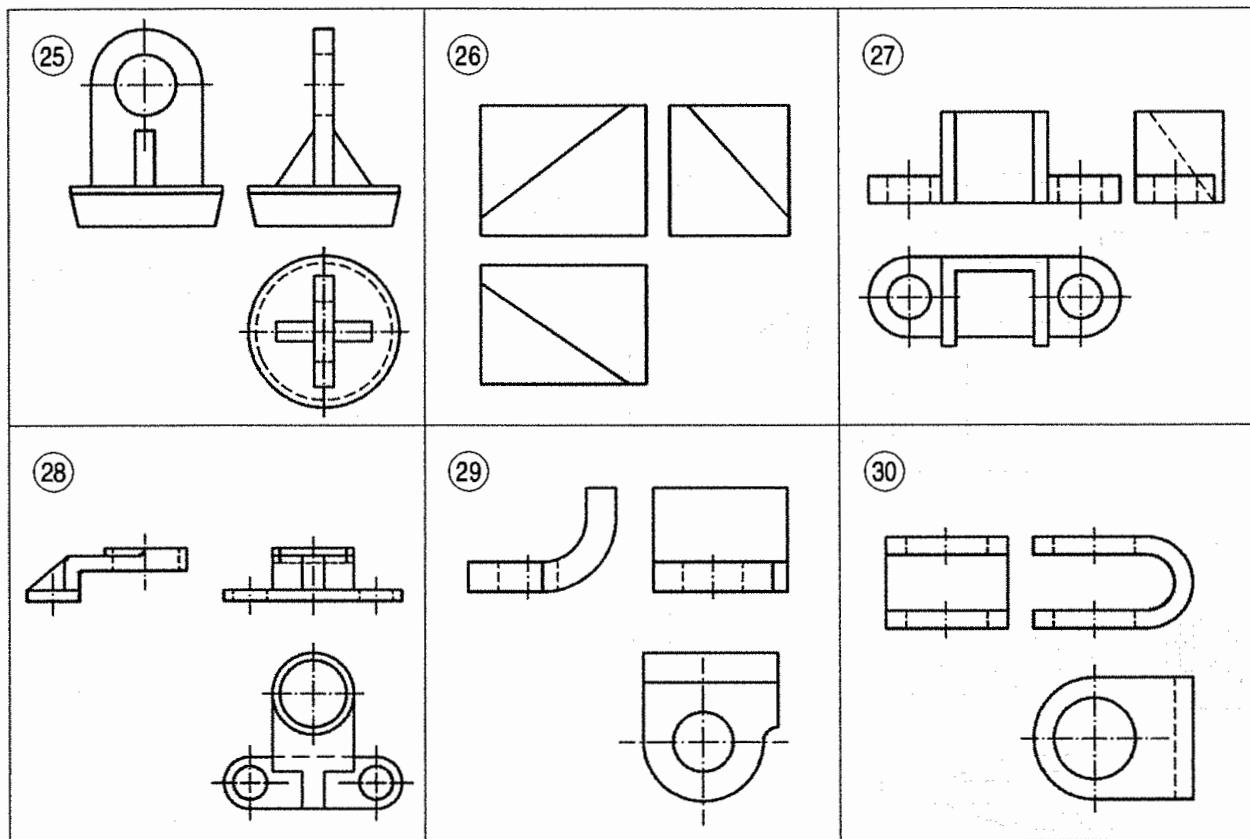
[Answer to Exercise (9), fig. 8-21] (Continued...)

FIG. 8-26



[Answer to Exercise (9), fig. 8-21] (Continued...)

FIG. 8-26



[Answer to Exercise (9), fig. 8-21]
FIG. 8-26



PROJECTIONS OF POINTS

9-0. INTRODUCTION

A point may be situated, in space, in any one of the four quadrants formed by the two principal planes of projection or may lie in any one or both of them. Its projections are obtained by extending projectors perpendicular to the planes.

One of the planes is then rotated so that the first and third quadrants are opened out. The projections are shown on a flat surface in their respective positions either above or below or in xy .



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 21 for the projections of points.

9-1. A POINT IS SITUATED IN THE FIRST QUADRANT

The pictorial view [fig. 9-1(i)] shows a point A situated above the H.P. and in front of the V.P., i.e. in the first quadrant. a' is its front view and a the top view. After rotation of the plane, these projections will be seen as shown in fig. 9-1(ii).

The front view a' is above xy and the top view a below it. The line joining a' and a (which also is called a projector), intersects xy at right angles at a point o . It is quite evident from the pictorial view that $a'o = Aa$, i.e. the distance of the front view from xy = the distance of A from the H.P. viz. h . Similarly, $ao = Aa'$, i.e. the distance of the top view from xy = the distance of A from the V.P. viz. d .

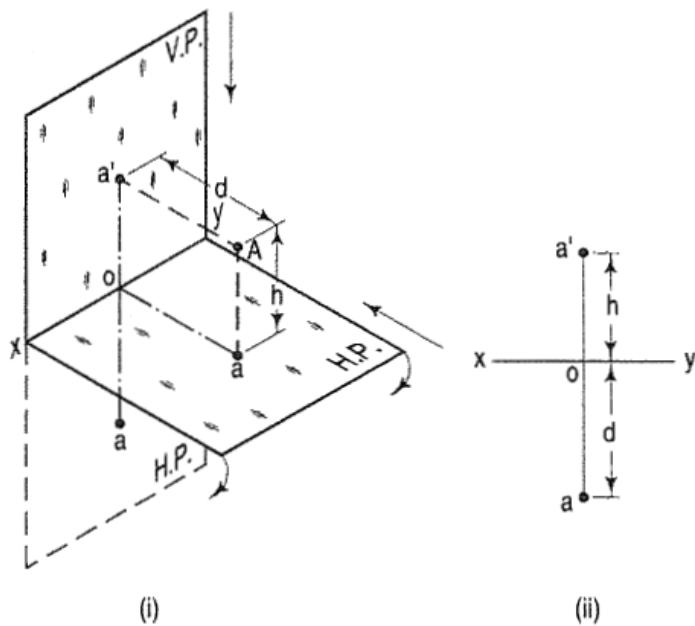


FIG. 9-1

9-2. A POINT IS SITUATED IN THE SECOND QUADRANT

A point B (fig. 9-2) is above the H.P. and behind the V.P., i.e. in the second quadrant. b' is the front view and b the top view.

When the planes are rotated, both the views are seen above xy . Note that $b'o = Bb$ and $bo = Bb'$.

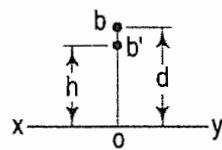
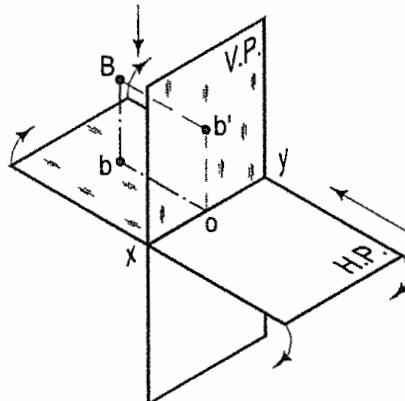


FIG. 9-2

9-3. A POINT IS SITUATED IN THE THIRD QUADRANT

A point C (fig. 9-3) is below the H.P. and behind the V.P., i.e. in the third quadrant. Its front view c' is below xy and the top view c above xy . Also $c'o = Cc$ and $co = Cc'$.

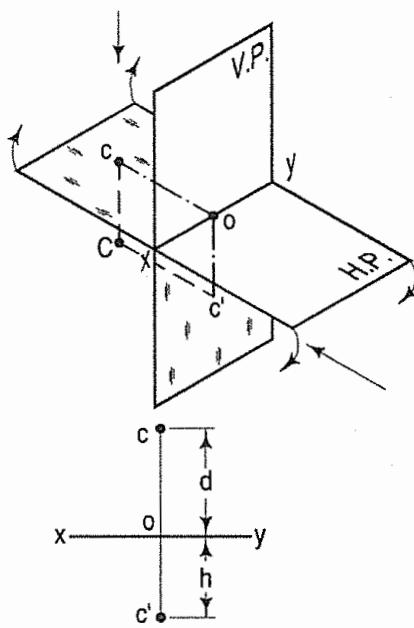


FIG. 9-3

9-4. A POINT IS SITUATED IN THE FOURTH QUADRANT

A point E (fig. 9-4) is below the H.P. and in front of the V.P., i.e. in the fourth quadrant. Both its projections are below xy , and $e'o = Ee$ and $eo = Ee'$.

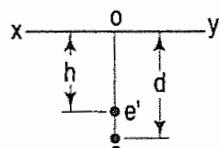
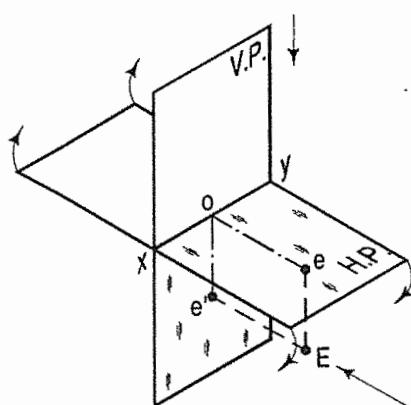


FIG. 9-4

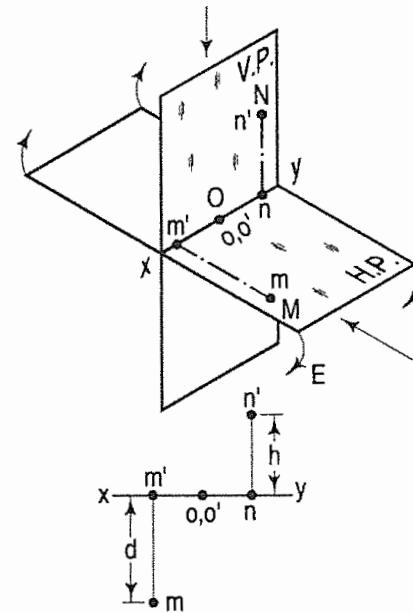


FIG. 9-5

Referring to fig. 9-5, we see that,

- A point M is in the H.P. and in front of the V.P. Its front view m' is in xy and the top view m below it.
- A point N is in the V.P. and above the H.P. Its top view n is in xy and the front view n' above it.
- A point O is in both the H.P. and the V.P. Its projection o and o' coincide with each other in xy .

9-5. GENERAL CONCLUSIONS

- The line joining the top view and the front view of a point is always perpendicular to xy . It is called a *projector*.
- When a point is above the H.P., its front view is above xy ; when it is below the H.P., the front view is below xy . The distance of a point from the H.P. is shown by the length of the projector from its front view to xy , e.g. $a'o$, $b'o$ etc.
- When a point is in front of the V.P., its top view is below xy ; when it is behind the V.P., the top view is above xy . The distance of a point from the V.P. is shown by the length of the projector from its top view to xy , e.g. ao , bo etc.
- When a point is in a reference plane, its projection on the other reference plane is in xy .

Problem 9-1. (fig. 9-1): A point A is 25 mm above the H.P. and 30 mm in front of the V.P. Draw its projections.

- Draw the reference line xy [fig. 9-1(ii)].

- (ii) Through any point o in it, draw a perpendicular.

As the point is above the H.P. and in front of the V.P. its front view will be above xy and the top view below xy .

- (iii) On the perpendicular, mark a point a' above xy , such that $a'o = 25$ mm. Similarly, mark a point a below xy , so that $ao = 30$ mm. a' and a are the required projections.

Problem 9-2. (fig. 9-6): A point A is 20 mm below the H.P. and 30 mm behind the V.P. Draw its projections.

As the point is below the H.P. and behind the V.P., its front view will be below xy and the top view above xy .

Draw the projections as explained in problem 9-1 and as shown in fig. 9-6.

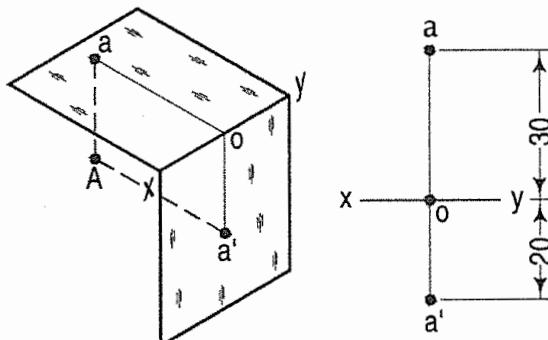


FIG. 9-6

Problem 9-3. (fig. 9-7): A point P is in the first quadrant. Its shortest distance from the intersection point of H.P., V.P. and Auxiliary vertical plane, perpendicular to the H.P. and V.P. is 70 mm and it is equidistant from principal planes (H.P. and V.P.). Draw the projections of the point and determine its distance from the H.P. and V.P.

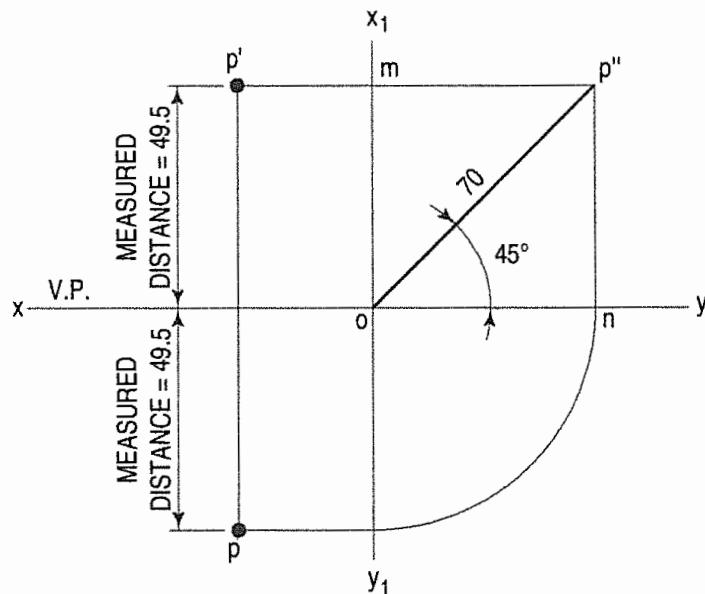


FIG. 9-7

Note: O represents intersection of H.P., V.P. and A.V.P.

- Draw xy and $x_1 y_1$ perpendicular reference lines.
- O represents intersection of H.P., V.P. and A.V.P.
- Draw from O a line inclined at 45° of 70 mm length.
- Project from P'' on xy line and $x_1 y_1$. The projections are n and m respectively as shown in figure. From O draw arc intersecting $x_1 y_1$.
- Draw a parallel line at convenient distance from $x_1 y_1$. Extend $P''m$ to intersect a parallel line at p' and p as shown.
- Measure distance from xy line, which is nearly 49.4974 mm say 49.5 mm.

Projections on auxiliary plane: Sometime projections of object on the principal (H.P. and V.P.) are insufficient. In such situation, another projection plane perpendicular to the principal planes is taken. This plane is known as auxiliary plane. The projection on the auxiliary plane is known as side view or side elevation. Refer fig. 9-8.

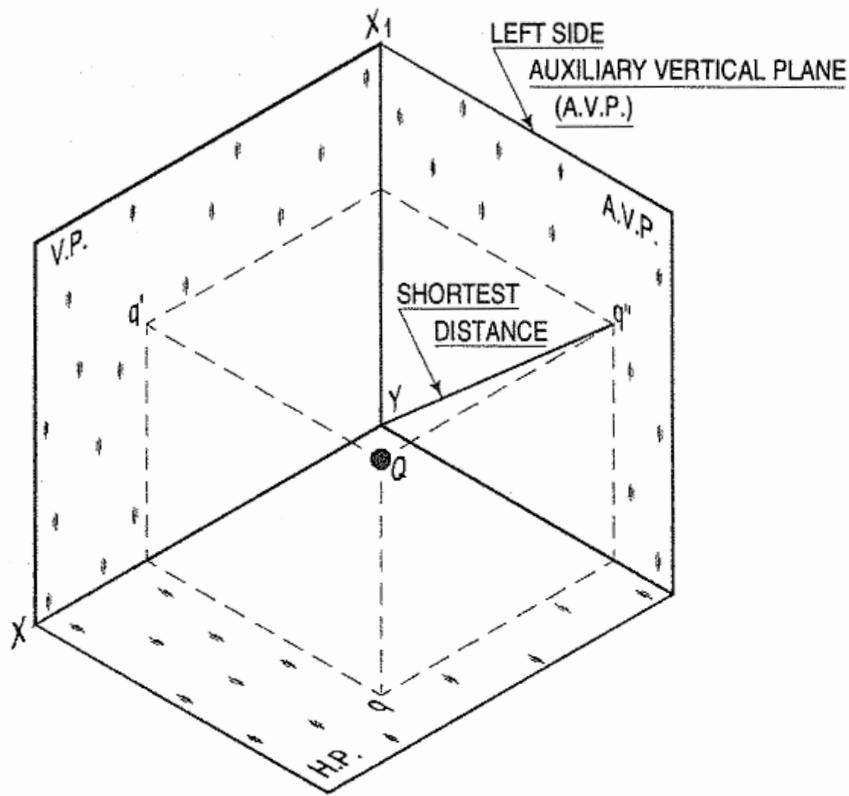


FIG. 9-8

The A.V.P. can be also taken right side also. For more details on projection on auxiliary plane, refer chapter 11.

EXERCISES 9

1. Draw the projections of the following points on the same ground line, keeping the projectors 25 mm apart.
 - A, in the H.P. and 20 mm behind the V.P.
 - B, 40 mm above the H.P. and 25 mm in front of the V.P.
 - C, in the V.P. and 40 mm above the H.P.
 - D, 25 mm below the H.P. and 25 mm behind the V.P.
 - E, 15 mm above the H.P. and 50 mm behind the V.P.
 - F, 40 mm below the H.P. and 25 mm in front of the V.P.
 - G, in both the H.P. and the V.P.
2. A point P is 50 mm from both the reference planes. Draw its projections in all possible positions.
3. State the quadrants in which the following points are situated:
 - (a) A point P; its top view is 40 mm above xy; the front view, 20 mm below the top view.
 - (b) A point Q, its projections coincide with each other 40 mm below xy.

4. A point P is 15 mm above the H.P. and 20 mm in front of the V.P. Another point Q is 25 mm behind the V.P. and 40 mm below the H.P. Draw projections of P and Q keeping the distance between their projectors equal to 90 mm. Draw straight lines joining (i) their top views and (ii) their front views.
5. Projections of various points are given in fig. 9-9. State the position of each point with respect to the planes of projection, giving the distances in centimetres.

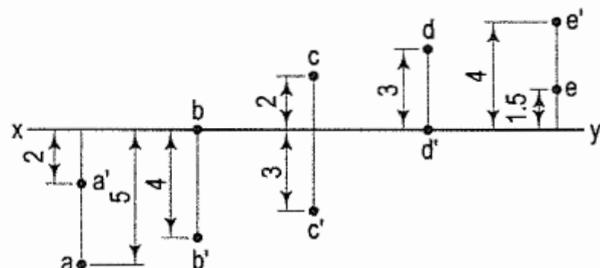


FIG. 9-9

6. Two points A and B are in the H.P. The point A is 30 mm in front of the V.P., while B is behind the V.P. The distance between their projectors is 75 mm and the line joining their top views makes an angle of 45° with xy . Find the distance of the point B from the V.P.
7. A point P is 20 mm below H.P. and lies in the third quadrant. Its shortest distance from xy is 40 mm. Draw its projections.
8. A point A is situated in the first quadrant. Its shortest distance from the intersection point of H.P., V.P. and auxiliary plane is 60 mm and it is equidistant from the principal planes. Draw the projections of the point and determine its distance from the principal planes.
9. A point 30 mm above xy line is the plan-view of two points P and Q . The elevation of P is 45 mm above the H.P. while that of the point Q is 35 mm below the H.P. Draw the projections of the points and state their position with reference to the principal planes and the quadrant in which they lie.
10. A point Q is situated in first quadrant. It is 40 mm above H.P. and 30 mm in front of V.P. Draw its projections and find its shortest distance from the intersection of H.P., V.P. and auxiliary plane.

Chapter

10



PROJECTIONS OF STRAIGHT LINES

10-0. INTRODUCTION



A straight line is the shortest distance between two points. Hence, the projections of a straight line may be drawn by joining the respective projections of its ends which are points.

The position of a straight line may also be described with respect to the two reference planes. It may be:

1. Parallel to one or both the planes.
2. Contained by one or both the planes.
3. Perpendicular to one of the planes.
4. Inclined to one plane and parallel to the other.
5. Inclined to both the planes.
6. Projections of lines inclined to both the planes.
7. Line contained by a plane perpendicular to both the reference planes.
8. True length of a straight line and its inclinations with the reference planes.
9. Traces of a line.
10. Methods of determining traces of a line.
11. Traces of a line, the projections of which are perpendicular to xy .
12. Positions of traces of a line.

10-1. LINE PARALLEL TO ONE OR BOTH THE PLANES

(FIG. 10-1)



- (a) Line AB is parallel to the H.P.

a and b are the top views of the ends A and B respectively. It can be clearly seen that the figure $ABba$ is a rectangle. Hence, the top view ab is equal to AB .

$a'b'$ is the front view of AB and is parallel to xy .

- (b) Line CD is parallel to the V.P.

The line $c'd'$ is the front view and is equal to CD ; the top view cd is parallel to xy .

- (c) Line EF is parallel to the H.P. and the V.P.

ef is the top view and $e'f'$ is the front view; both are equal to EF and parallel to xy .

Hence, when a line is parallel to a plane, its projection on that plane is equal to its true length; while its projection on the other plane is parallel to the reference line.

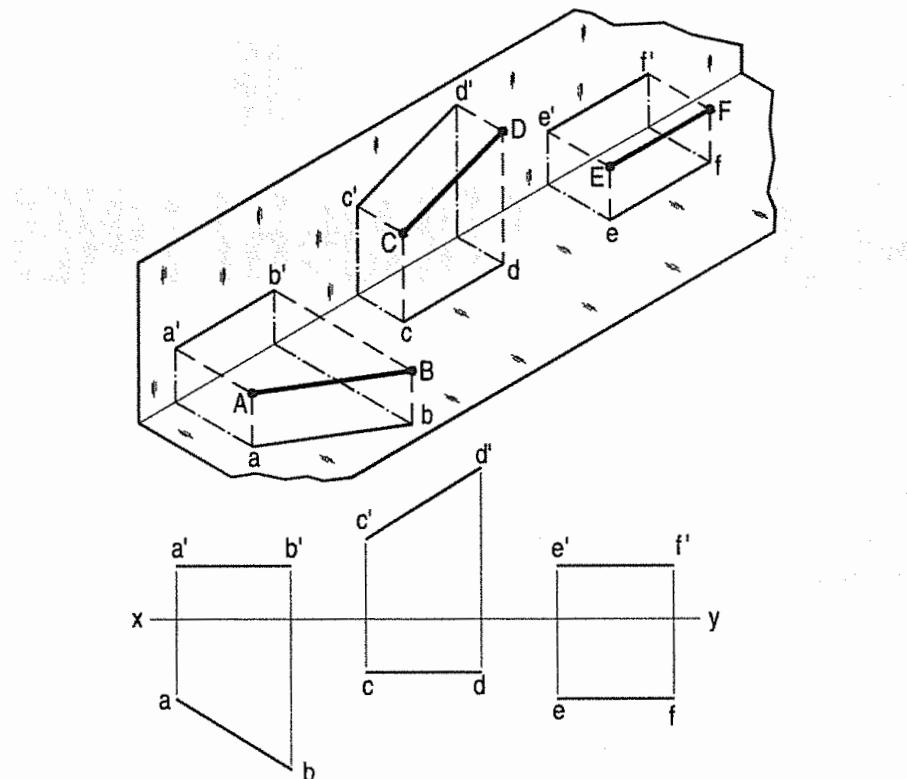


FIG. 10-1

10-2. LINE CONTAINED BY ONE OR BOTH THE PLANES

(FIG. 10-2)

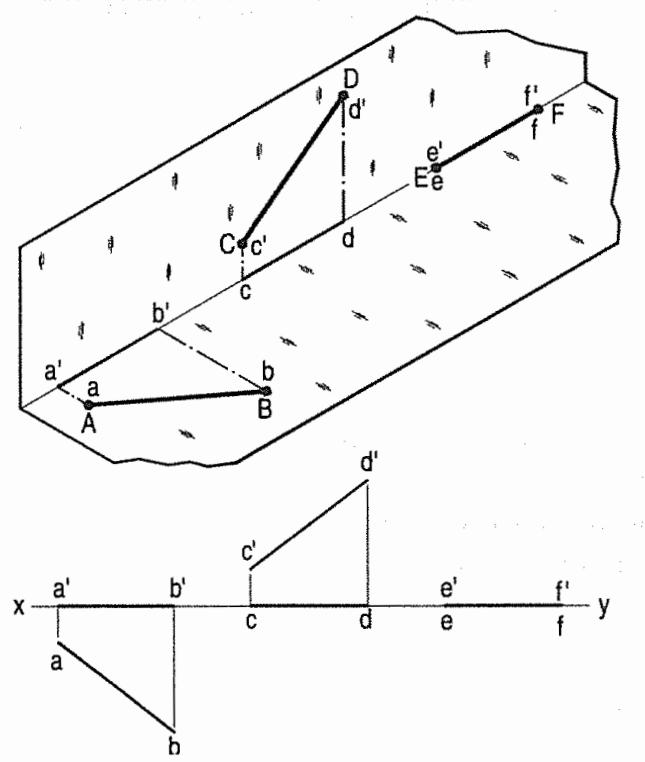


FIG. 10-2

Line AB is in the H.P. Its top view ab is equal to AB ; its front view $a' b'$ is in xy .

Line CD is in the V.P. Its front view $c'd'$ is equal to CD ; its top view cd is in xy .

Line EF is in both the planes. Its front view $e' f'$ and the top view ef coincide with each other in xy .

Hence, when a line is contained by a plane, its projection on that plane is equal to its true length; while its projection on the other plane is in the reference line.

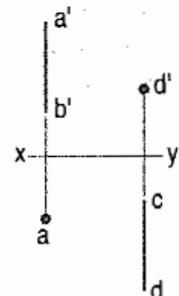
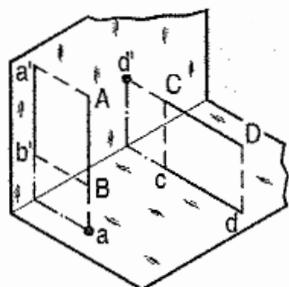
10-3. LINE PERPENDICULAR TO ONE OF THE PLANES

(FIG. 10-3)

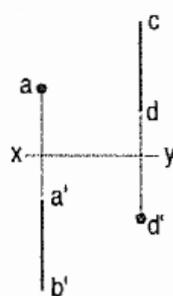


This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 22 for the line perpendicular to one of the planes.

When a line is perpendicular to one reference plane, it will be parallel to the other.



(FIRST-ANGLE PROJECTION)



(THIRD-ANGLE PROJECTION)

FIG. 10-3

- Line AB is perpendicular to the H.P. The top views of its ends coincide in the point a . Hence, the top view of the line AB is the point a . Its front view $a' b'$ is equal to AB and perpendicular to xy .
- Line CD is perpendicular to the V.P. The point d' is its front view and the line cd is the top view. cd is equal to CD and perpendicular to xy .

Hence, when a line is perpendicular to a plane its projection on that plane is a point; while its projection on the other plane is a line equal to its true length and perpendicular to the reference line.

In *first-angle projection method*, when top views of two or more points coincide, the point which is comparatively *farther* away from xy in the front view will be visible; and when their front views coincide, that which is *farther* away from xy in the top view will be visible.

In *third-angle projection method*, it is just the reverse. When top views of two or more points coincide the point which is comparatively *nearer* xy in the front view will be visible; and when their front views coincide, the point which is *nearer* xy in the top view will be visible.

10-4. LINE INCLINED TO ONE PLANE AND PARALLEL TO THE OTHER



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 23 for the line inclined to one plane and parallel to the other.

The inclination of a line to a plane is the angle which the line makes with its projection on that plane.

- (a) Line PQ_1 [fig. 10-4(i)] is inclined at an angle θ to the H.P. and is parallel to the V.P. The inclination is shown by the angle θ which PQ_1 makes with its own projection on the H.P., viz. the top view pq_1 .

The projections [fig. 10-4(ii)] may be drawn by first assuming the line to be parallel to both the H.P. and the V.P. Its front view $p'q'$ and the top view pq will both be parallel to xy and equal to the true length. When the line is turned about the end P to the position PQ_1 so that it makes the angle θ with the H.P. while remaining parallel to the V.P., in the front view the point q' will move along an arc drawn with p' as centre and $p'q'$ as radius to a point q'_1 so that $p'q'_1$ makes the angle θ with xy . In the top view, q will move towards p along pq to a point q_1 on the projector through q'_1 . $p'q'_1$ and pq_1 are the front view and the top view respectively of the line PQ_1 .

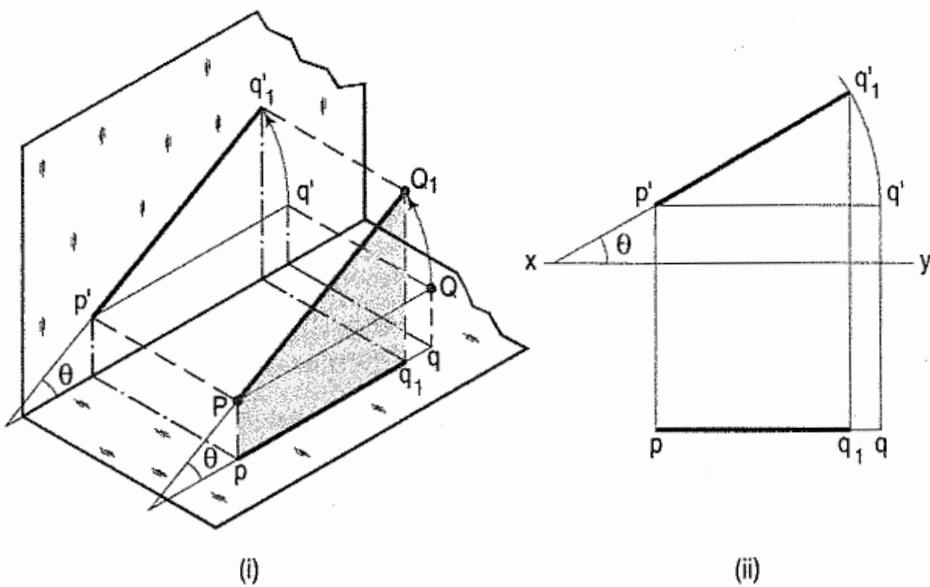


FIG. 10-4

- (b) Line RS_1 [fig. 10-5(i)] is inclined at an angle ϕ to the V.P. and is parallel to the H.P. The inclination is shown by the angle ϕ which RS_1 makes with its projection on the V.P., viz. the front view $r's'_1$. Assuming the line to be parallel to both the H.P. and the V.P., its projections $r's'$ and rs are drawn parallel to xy and equal to its true length [fig. 10-5(ii)].

When the line is turned about its end R to the position RS_1 so that it makes the angle ϕ with the V.P. while remaining parallel to the H.P., in

the top view the point s will move along an arc drawn with r as centre and rs as radius to a point s_1 so that rs_1 makes the angle ϕ with xy . In the front view, the point s' will move towards r' along the line $r's'$ to a point s'_1 on the projector through s_1 . rs_1 and $r's'_1$ are the projections of the line RS_1 .

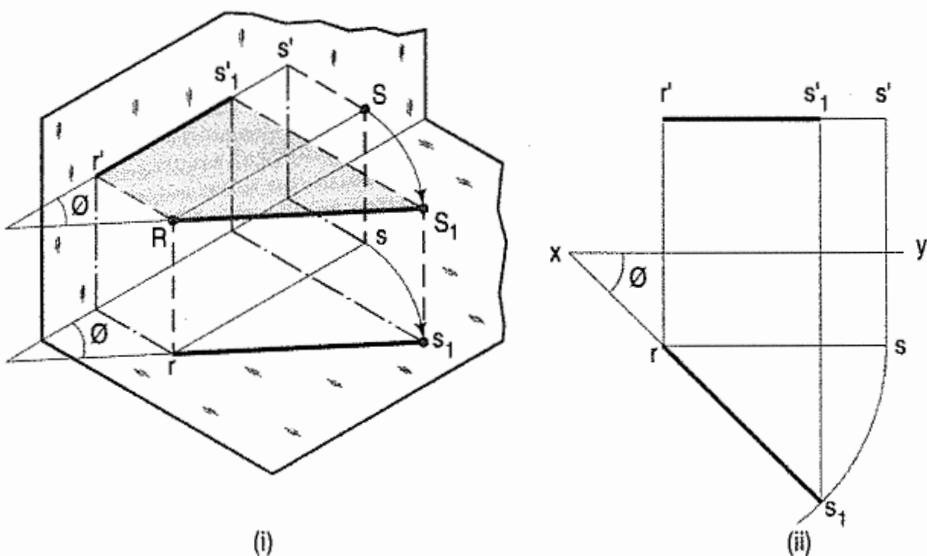


FIG. 10-5

Therefore, when the line is inclined to the H.P. and parallel to the V.P., its top view is shorter than its true length, but parallel to xy ; its front view is equal to its true length and is inclined to xy at its true inclination with the H.P. And when the line is inclined to the V.P. and parallel to the H.P., its front view is shorter than its true length but parallel to xy ; its top view is equal to its true length and is inclined to xy at its true inclination with the V.P.

Hence, when a line is inclined to one plane and parallel to the other, its projection on the plane to which it is inclined, is a line shorter than its true length but parallel to the reference line. Its projection on the plane to which it is parallel, is a line equal to its true length and inclined to the reference line at its true inclination.

In other words, the inclination of a line with the H.P. is seen in the front view and that with the V.P. is seen in the top view.

Problem 10-1. (fig. 10-6): A line PQ , 90 mm long, is in the H.P. and makes an angle of 30° with the V.P. Its end P is 25 mm in front of the V.P. Draw its projections.

As the line is in the H.P., its top view will show the true length and the true inclination with the V.P. Its front view will be in xy .

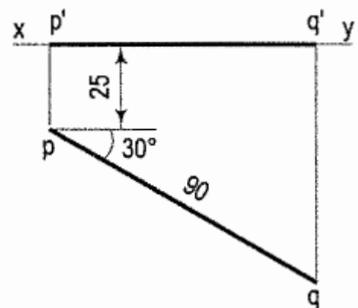


FIG. 10-6

- Mark a point p , the top view 25 mm below xy . Draw a line pq equal to 90 mm and inclined at 30° to xy .

- Project p to p' and q to q' on xy .

pq and $p'q'$ are the required top view and front view respectively.

Problem 10-2. (fig. 10-7): *The length of the top view of a line parallel to the V.P. and inclined at 45° to the H.P. is 50 mm. One end of the line is 12 mm above the H.P. and 25 mm in front of the V.P. Draw the projections of the line and determine its true length.*

As the line is parallel to the V.P., its top view will be parallel to xy and the front view will show its true length and the true inclination with the H.P.

- Mark a , the top view, 25 mm below xy and a' , the front view, 12 mm above xy .
- Draw the top view ab 50 mm long and parallel to xy and draw a projector through b .
- From a' draw a line making 45° angle with xy and cutting the projector through b at b' . Then $a'b'$ is the front view and also the true length of the line.

Problem 10-3. (fig. 10-8): *The front view of a 75 mm long line measures 55 mm. The line is parallel to the H.P. and one of its ends is in the V.P. and 25 mm above the H.P. Draw the projections of the line and determine its inclination with the V.P.*

As the line is parallel to the H.P., its front view will be parallel to xy .

- Mark a , the top view of one end in xy , and a' , its front view, 25 mm above xy .
- Draw the front view $a'b'$, 55 mm long and parallel to xy . With a as centre and radius equal to 75 mm, draw an arc cutting the projector through b' at b . Join a with b . ab is the top view of the line. Its inclination with xy , viz. ϕ is the inclination of the line with the V.P.

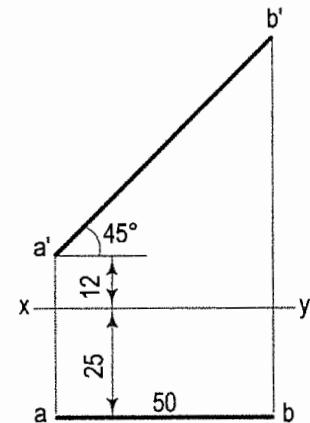


FIG. 10-7

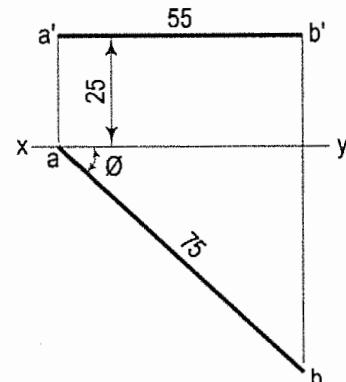


FIG. 10-8

EXERCISES 10(a)

- Draw the projections of a 75 mm long straight line, in the following positions:
 - (i) Parallel to both the H.P. and the V.P. and 25 mm from each.
(ii) Parallel to and 30 mm above the H.P. and in the V.P.
(iii) Parallel to and 40 mm in front of the V.P. and in the H.P.
 - (i) Perpendicular to the H.P., 20 mm in front of the V.P. and its one end 15 mm above the H.P.
(ii) Perpendicular to the V.P., 25 mm above the H.P. and its one end in the V.P.
(iii) Perpendicular to the H.P., in the V.P. and its one end in the H.P.
 - (i) Inclined at 45° to the V.P., in the H.P. and its one end in the V.P.
(ii) Inclined at 30° to the H.P. and its one end 20 mm above it; parallel to and 30 mm in front of the V.P.
(iii) Inclined at 60° to the V.P. and its one end 15 mm in front of it; parallel to and 25 mm above the H.P.

2. A 100 mm long line is parallel to and 40 mm above the H.P. Its two ends are 25 mm and 50 mm in front of the V.P. respectively. Draw its projections and find its inclination with the V.P.
3. A 90 mm long line is parallel to and 25 mm in front of the V.P. Its one end is in the H.P. while the other is 50 mm above the H.P. Draw its projections and find its inclination with the H.P.
4. The top view of a 75 mm long line measures 55 mm. The line is in the V.P., its one end being 25 mm above the H.P. Draw its projections.
5. The front view of a line, inclined at 30° to the V.P. is 65 mm long. Draw the projections of the line, when it is parallel to and 40 mm above the H.P., its one end being 30 mm in front of the V.P.
6. A vertical line AB , 75 mm long, has its end A in the H.P. and 25 mm in front of the V.P. A line AC , 100 mm long, is in the H.P. and parallel to the V.P. Draw the projections of the line joining B and C , and determine its inclination with the H.P.
7. Two pegs fixed on a wall are 4.5 metres apart. The distance between the pegs measured parallel to the floor is 3.6 metres. If one peg is 1.5 metres above the floor, find the height of the second peg and the inclination of the line joining the two pegs, with the floor.
8. Draw the projections of the lines in Exercises 1 to 6, assuming them to be in the third quadrant, taking the given positions to be below the H.P. instead of above the H.P., and behind the V.P., instead of in front of the V.P.

10-5. LINE INCLINED TO BOTH THE PLANES



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 24 for the line inclined to both the planes.

- (a) A line AB (fig. 10-9) is inclined at θ to the H.P. and is parallel to the V.P. The end A is in the H.P. AB is shown as the hypotenuse of a right-angled triangle, making the angle θ with the base.

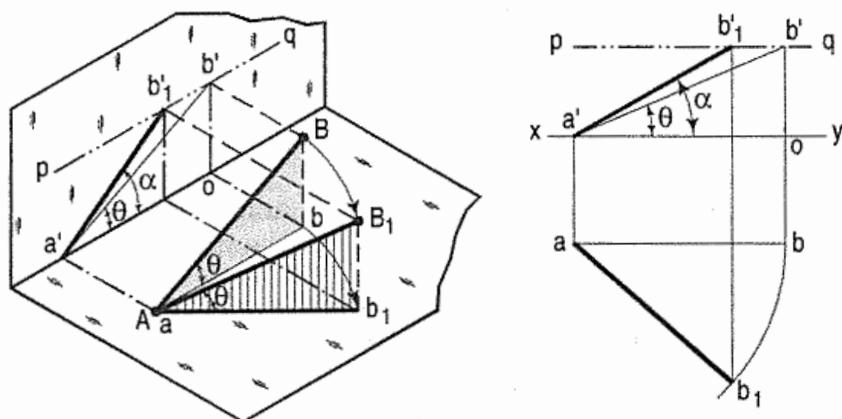


FIG. 10-9

The top view ab is shorter than AB and parallel to xy . The front view $a'b'$ is equal to AB and makes the angle θ with xy .

Keeping the end A fixed and the angle θ with the H.P. constant, if the end B is moved to any position, say B_1 , the line becomes inclined to the V.P. also.

In the top view, b will move along an arc, drawn with a as centre and ab as radius, to a position b_1 . The new top view ab_1 is equal to ab but shorter than AB .

In the front view, b' will move to a point b'_1 keeping its distance from xy constant and equal to $b'o$; i.e. it will move along the line pq , drawn through b' and parallel to xy . This line pq is the locus or path of the end B in the front view. b'_1 will lie on the projector through b_1 . The new front view $a'b'_1$ is shorter than $a'b'$ (i.e. AB) and makes an angle α with xy . α is greater than θ .

Thus, it can be seen that as long as the inclination θ of AB with the H.P. is constant, even when it is inclined to the V.P.

- (i) its length in the top view, viz. ab remains constant; and
 - (ii) the distance between the paths of its ends in the front view, viz. $b'o$ remains constant.
- (b) The same line AB (fig. 10-10) is inclined at ϕ to the V.P. and is parallel to the H.P. Its end A is in the V.P. AB is shown as the hypotenuse of a right-angled triangle making the angle ϕ with the base.

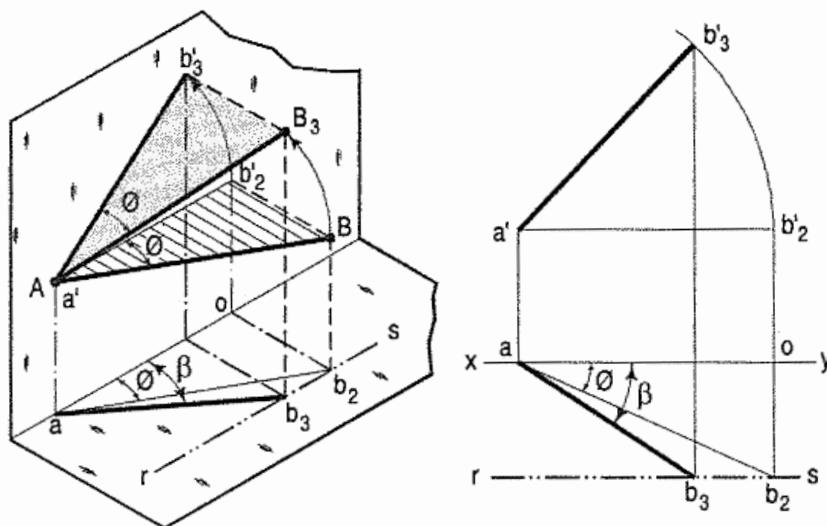


FIG. 10-10

The front view $a'b'_2$ is shorter than AB and parallel to xy . The top view ab_2 is equal to AB and makes an angle ϕ with xy .

Keeping the end A fixed and the angle ϕ with the V.P. constant, if B is moved to any position, say B_3 , the line will become inclined to the H.P. also.

In the front view, b'_2 , will move along the arc, drawn with a' as centre and $a'b'_2$ as radius, to a position b'_3 . The new front view $a'b'_3$ is equal to $a'b'_2$ but is shorter than AB .

In the top view, b_2 will move to a point b_3 along the line rs , drawn through b_2 and parallel to xy , thus keeping its distance from the path of a , viz. b_2o constant. rs is the locus or path of the end B in the top view. The point b_3 lies on the projector through b'_3 . The new top view ab_3 is shorter than ab_2 (i.e. AB) and makes an angle β with xy . β is greater than ϕ .

Here also we find that, as long as the inclination of AB with the V.P. does not change, even when it becomes inclined to the H.P.

- (i) its length in the front view, viz. $a'b'_2$ remains constant; and
- (ii) the distance between the paths of its ends in the top view, viz. b_2o remains constant.

Hence, when a line is inclined to both the planes, its projections are shorter than the true length and inclined to xy at angles greater than the true inclinations. These angles viz. α and β are called apparent angles of inclination.

10-6. PROJECTIONS OF LINES INCLINED TO BOTH THE PLANES



From Art. 10-5(a) above, we find that as long as the inclination of AB with the H.P. is constant

- (i) its length in the top view, viz. ab remains constant, and
- (ii) in the front view, the distance between the loci of its ends, viz. $b'o$ remains constant.

In other words if

- (i) its length in the top view is equal to ab , and
- (ii) the distance between the paths of its ends in the front view is equal to $b'o$, the inclination of AB with the H.P. will be equal to θ .

Similarly, from Art. 10-5(b) above, we find that as long as the inclination of AB with the V.P. is constant

- (i) its length in the front view, viz. $a'b'_2$ remains constant, and
- (ii) in the top view, the distance between the loci of its ends, viz. b_2o remains constant.

The reverse of this is also true, viz.

- (i) if its length in the front view is equal to $a'b'_2$, and
- (ii) the distance between the paths of its ends in the top view is equal to b_2o , the inclination of AB with the V.P. will be equal to ϕ .

Combining the above two findings, we conclude that when AB is inclined at θ to the H.P. and at ϕ to the V.P.

- (i) its lengths in the top view and the front view will be equal to ab_2 and $a'b'_2$ respectively, and
- (ii) the distances between the paths of its ends in the front view and the top view will be equal to b'_2o and b_2o respectively.

The two lengths when arranged with their ends in their respective paths and in projections with each other will be the projections of the line AB , as illustrated in problem 10-4.

Problem 10-4. Given the line AB , its inclinations θ with the H.P. and ϕ with the V.P. and the position of one end A . To draw its projections.

Mark the front view a' and the top view a according to the given position of A (fig. 10-12).

Let us first determine the lengths of AB in the top view and the front view and the paths of its ends in the front view and the top view.

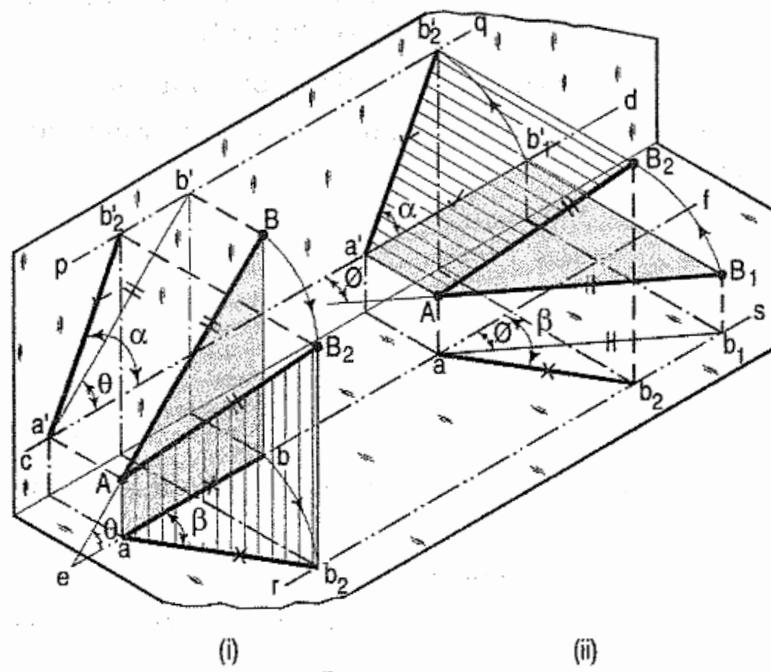


FIG. 10-11

(1) Assume AB to be parallel to the V.P. and inclined at θ to the H.P. AB is shown in the pictorial view as a side of the trapezoid $ABba$ [fig. 10-11(i)]. Draw the front view $a'b'$ equal to AB [fig. 10-12(i)] and inclined at θ to xy . Project the top view ab parallel to xy . Through a' and b' , draw lines cd and pq respectively parallel to xy . ab is the length of AB in the top view and, cd and pq are the paths of A and B respectively in the front view.

(2) Again, assume AB_1 (equal to AB) to be parallel to the H.P. and inclined at ϕ to the V.P. In the pictorial view [fig. 10-11(ii)], AB_1 is shown as a side of the trapezoid $AB_1b'_1a'$. Draw the top view ab_1 equal to AB [fig. 10-12(ii)] and inclined at ϕ to xy . Project the front view $a'b'_1$ parallel to xy . Through a and b_1 , draw lines ef and rs respectively parallel to xy . $a'b'_1$ is the length of AB in the front view and, ef and rs are the paths of A and B respectively in the top view.

We may now arrange

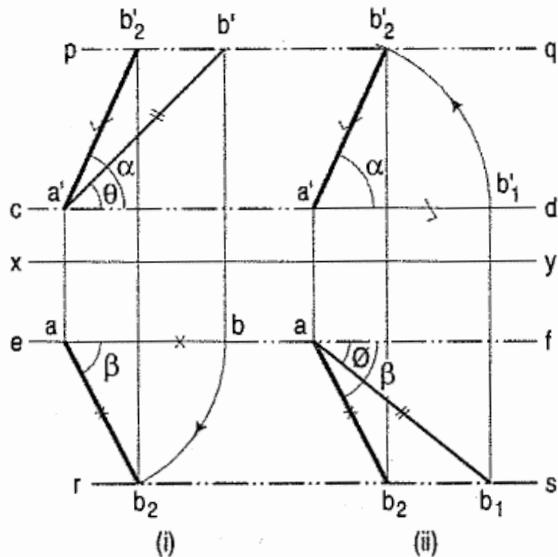


FIG. 10-12

(i) ab (the length in the top view) between its paths ef and rs , and
(ii) $a'b'_1$ (the length in the front view) between the paths cd and pq , keeping them in projection with each other, in one of the following two ways:

(a) In case (1) [fig. 10-11(i)], if the side Bb is turned about Aa , so that b comes on the path rs , the line AB will become inclined at ϕ to the V.P. Therefore, with a as centre [fig. 10-12(i)] and radius equal to ab , draw an arc cutting rs at a point b_2 . Project b_2 to b'_2 on the path pq .

Draw lines joining a with b_2 , and a' with b'_2 . ab_2 and $a'b'_2$ are the required projections. Check that $a'b'_2 = a'b'_1$.

- (b) Similarly, in case (2) [fig. 10-11(ii)], if the side $B_1b'_1$ is turned about Aa' till b'_1 is on the path pq , the line AB_1 will become inclined at θ to the H.P.

Hence, with a' as centre [fig. 10-12(ii)] and radius equal to $a'b'_1$, draw an arc cutting pq at a point b'_2 . Project b'_2 to b_2 in the top view on the path rs .

Draw lines joining a with b_2 , and a' with b'_2 . ab_2 and $a'b'_2$ are the required projections. Check that $ab_2 = ab$.

Fig. 10-13 shows (in pictorial and orthographic views) the projections obtained with both the above steps combined in one figure and as described below.

First, determine

- the length ab in the top view and the path pq in the front view and
- the length $a'b'_1$ in the front view and the path rs in the top view.

Then, with a as centre and radius equal to ab , draw an arc cutting rs at a point b_2 . With a' as centre and radius equal to $a'b'_1$, draw an arc cutting pq at a point b'_2 .

Draw lines joining a with b_2 and a' with b'_2 . ab_2 and $a'b'_2$ are the required projections. Check that b_2 and b'_2 lie on the same projector.

It is quite evident from the figure that the apparent angles of inclination α and β are greater than the true inclinations θ and ϕ respectively.

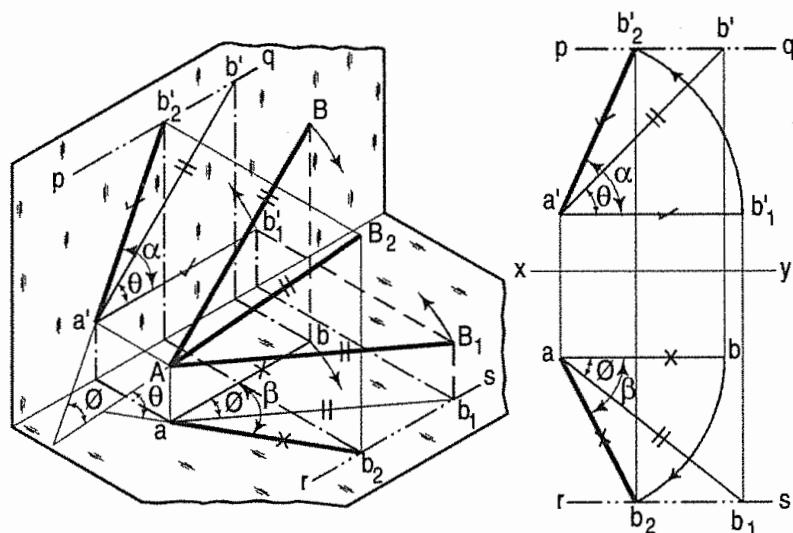


FIG. 10-13

10-7. LINE CONTAINED BY A PLANE PERPENDICULAR TO BOTH THE REFERENCE PLANES

As the two reference planes are at right angles to each other, the sum total of the inclinations of a line with the two planes, viz. θ and ϕ can never be more than 90° . When $\theta + \phi = 90^\circ$, the line will be contained by a third plane called the profile plane, perpendicular to both the H.P. and the V.P.

A line EF (fig. 10-14), is inclined at θ to the H.P. and at ϕ [equal to $(90^\circ - \theta)$] to the V.P. The line is thus contained by the profile plane marked P.P.

The front view $e'f'$ and the top view ef are both perpendicular to xy and shorter than EF .

Therefore, when a line is inclined to both the reference planes and contained by a plane perpendicular to them, i.e. when the sum of its inclinations with the H.P. and the V.P. is 90° , its projections are perpendicular to xy and shorter than the true length.

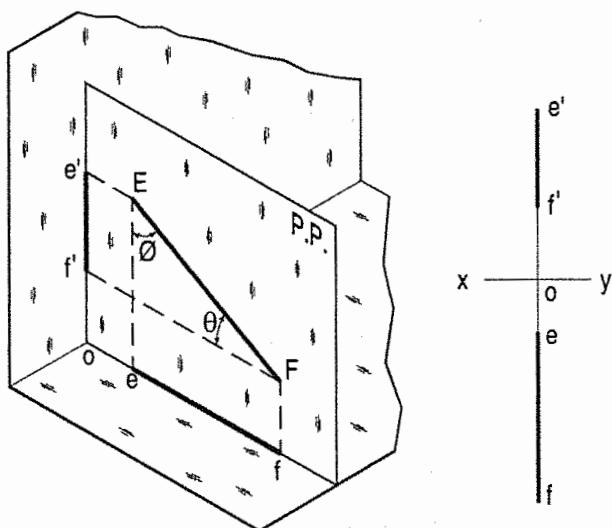
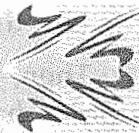


FIG. 10-14

10-8. TRUE LENGTH OF A STRAIGHT LINE AND ITS INCLINATIONS WITH THE REFERENCE PLANES



When projections of a line are given, its true length and inclinations with the planes are determined by the application of the following rule:

When a line is parallel to a plane, its projection on that plane will show its true length and the true inclination with the other plane.

The line may be made parallel to a plane, and its true length obtained by any one of the following *three* methods:

Method I:

Making each view parallel to the reference line and projecting the other view from it. This is the exact reversal of the processes adopted in Art. 10-5 for obtaining the projections.

Method II:

Rotating the line about its projections till it lies in the H.P. or in the V.P.

Method III:

Projecting the views on auxiliary planes parallel to each view.

(This method will be dealt with in chapter 11).

The following problem shows the application of the first two methods and problem 10-29 and problem 10-31 show application of third method.

Problem 10-5. The top view ab and the front view $a'b'$ of a line AB are given. To determine its true length and the inclinations with the H.P. and the V.P.

Method I:

Fig. 10-15(i) shows AB the line, $a'b'$ its front view and ab its top view. If the trapezoid $ABba$ is turned about Aa as axis, so that AB becomes parallel to the V.P., in the top view, b will move along an arc drawn with centre a and radius equal to ab , to b_1 , so that ab_1 is parallel to xy . In the front view, b' will move along its locus pq , to a point b'_1 on the projector through b_1 .

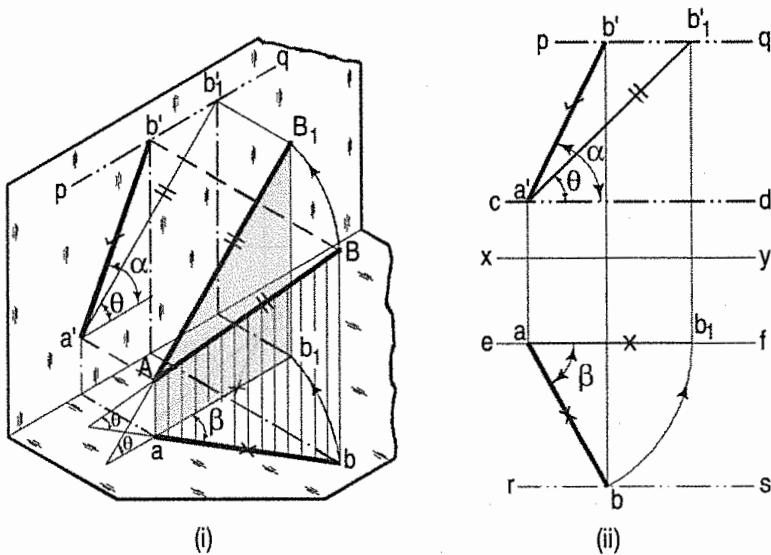


FIG. 10-15

- (i) Therefore, with centre a and radius equal to ab [fig. 10-15(ii)], draw an arc to cut ef at b_1 .
 - (ii) Draw a projector through b_1 to cut pq (the path of b') at b'_1 .
 - (iii) Draw the line $a'b'_1$ which is the true length of AB . The angle θ , which it makes with xy is the inclination of AB with the H.P.

Again, in fig. 10-16(i) AB is shown as a side of a trapezoid $ABb'a'$. If the trapezoid is turned about Aa' as axis so that AB is parallel to the H.P., the new top view will show its true length and true inclination with the V.P.

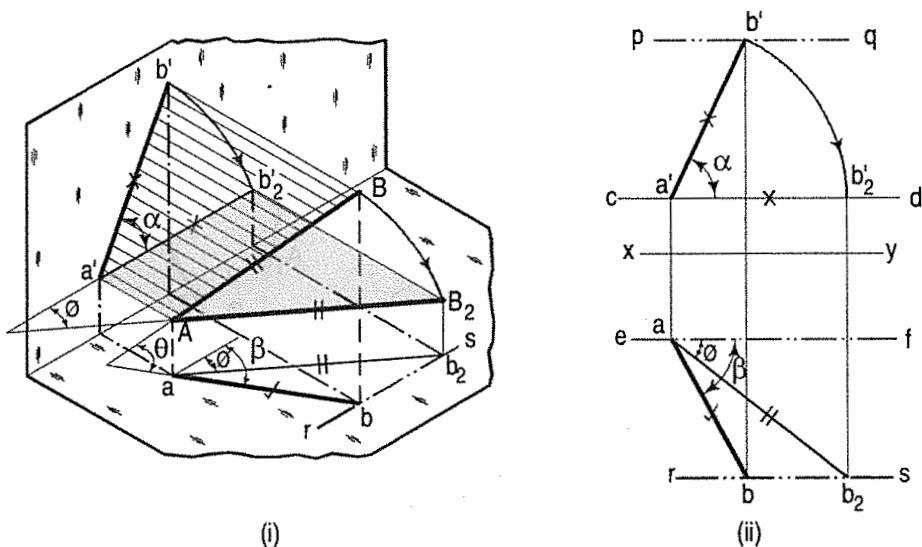


FIG. 10-16

- (i) With a' as centre and radius equal to $a'b'$ [fig. 10-16(ii)], draw an arc to cut cd at b'_2 .
 - (ii) Draw a projector through b'_2 to cut rs (the path of b) at b_2 .
 - (iii) Draw the line ab_2 , which is the true length of AB . The angle ϕ which it makes with xy is the inclination of AB with the V.P.

Fig. 10-17(i) shows the above two steps combined in one figure.

The same results will be obtained by keeping the end B fixed and turning the end A [fig. 10-17(ii)], as explained below.

- (i) With centre b and radius equal to ba , draw an arc cutting rs at a_1 (thus making ba parallel to xy).
- (ii) Project a_1 to a'_1 on cd (the path of a'). a'_1b' is the true length and θ is the true inclination of AB with the H.P.
- (iii) Similarly, with centre b' and radius equal to $b'a'$, draw an arc cutting pq at a_2' .
- (iv) Project a_2' to a_2 on ef (the path of a). a_2b is the true length and ϕ is the true inclination of AB with the V.P.

Method II:

Referring to the pictorial view in fig. 10-18(i) we find that AB is the line, ab its top view and $a'b'$ its front view.

In the trapezoid $ABb'a'$ (i) $a'A$ and $b'B$ are both perpendicular to $a'b'$ and are respectively equal to ao_1 and bo_2 (the distances of a and b from xy in the top view), and (ii) the angle between AB and $a'b'$ is the angle of inclination ϕ of AB with the V.P.

Assume that this trapezoid is rotated about $a'b'$, till it lies in the V.P.

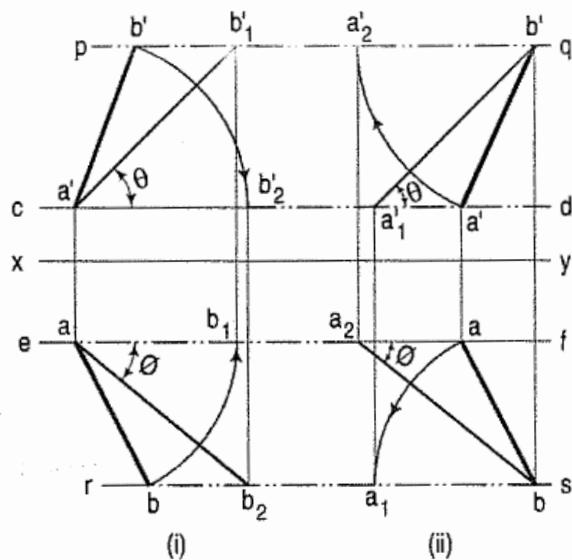


FIG. 10-17

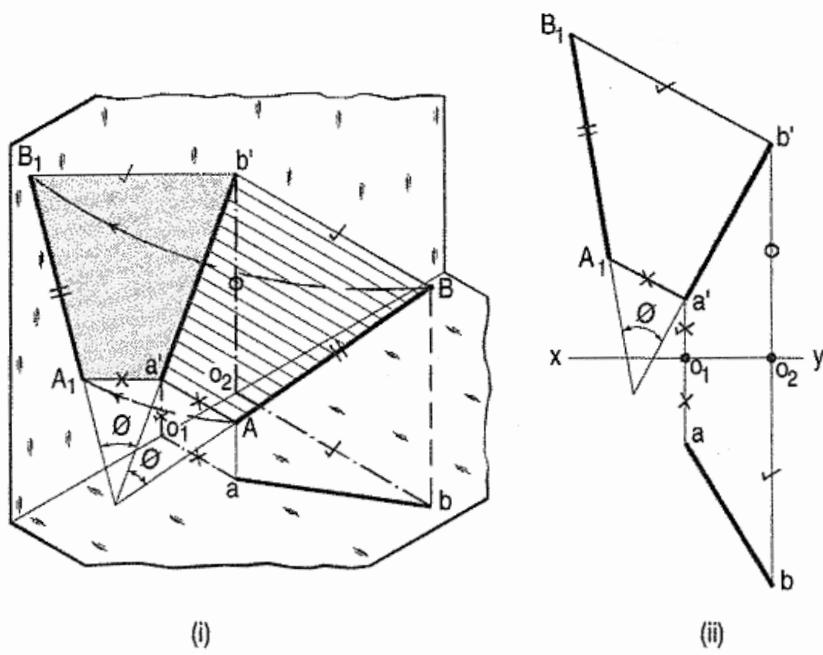


FIG. 10-18

In the orthographic view [fig. 10-18(ii)], this trapezoid is obtained by drawing perpendiculars to $a'b'$, viz. $a'A_1$ (equal to ao_1) and $b'B_1$ (equal to bo_2) and then joining A_1 with B_1 . The line A_1B_1 is the true length of AB and its inclination ϕ with $a'b'$ is the inclination of AB with the V.P.

Similarly, in trapezoid $ABba$ in fig. 10-19(i), AB is the line and ab its top view. Aa and Bb are both perpendicular to ab and are respectively equal to $a'o_1$ and $b'o_2$ (the distances of a' and b' from xy in the front view). The angle θ between AB and ab is the inclination of AB with the H.P.

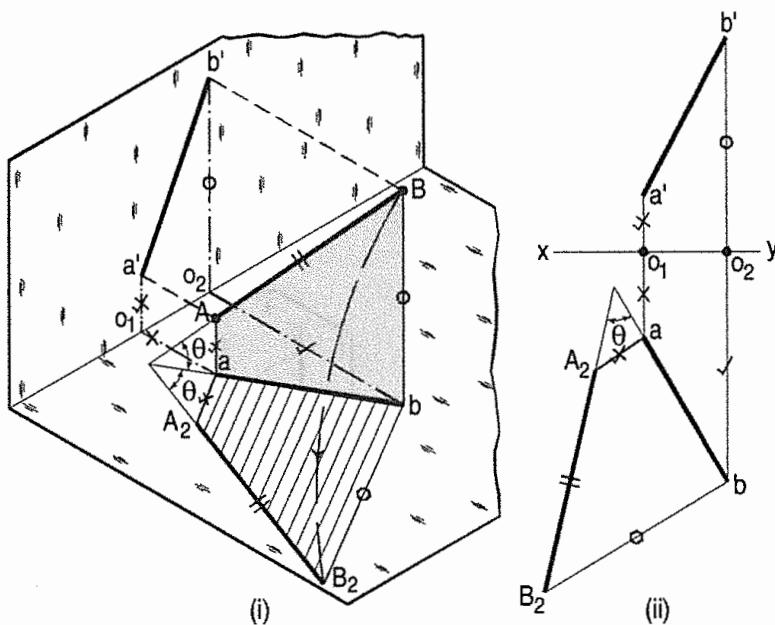


FIG. 10-19

This figure may now be assumed to be rotated about ab as axis, so that it lies in the H.P.

In the orthographic view [fig. 10-19(ii)], this trapezoid is obtained by erecting perpendiculars to ab , viz. aA_2 equal to $a'o_1$ and bB_2 equal to $b'o_2$ and joining A_2 with B_2 . The line A_2B_2 is the true length of AB and its inclination θ with ab is the inclination of AB with the H.P.

Note: The perpendiculars on ab or $a'b'$ can also be drawn on its other side assuming the trapezoid to be rotated in the opposite direction.

10-9. TRACES OF A LINE



When a line is inclined to a plane, it will meet that plane, produced if necessary. The point in which the line or line-produced meets the plane is called its *trace*.

The point of intersection of the line with the H.P. is called the *horizontal trace*, usually denoted as H.T. and that with the V.P. is called the *vertical trace* or V.T.

Refer to fig. 10-20.

- A line AB is parallel to the H.P. and the V.P. It has no trace.
- A line CD is inclined to the H.P. and parallel to the V.P. It has only the H.T. but no V.T.
- A line EF is inclined to the V.P. and parallel to the H.P. It has only the V.T. but no H.T.

Thus, when a line is parallel to a plane it has no trace upon that plane.

Refer to fig. 10-21.

- A line PQ is perpendicular to the H.P. Its H.T. coincides with its top view which is a point. It has no V.T.

- (ii) A line RS is perpendicular to the V.P. Its V.T. coincides with its front view which is a point. It has no H.T.

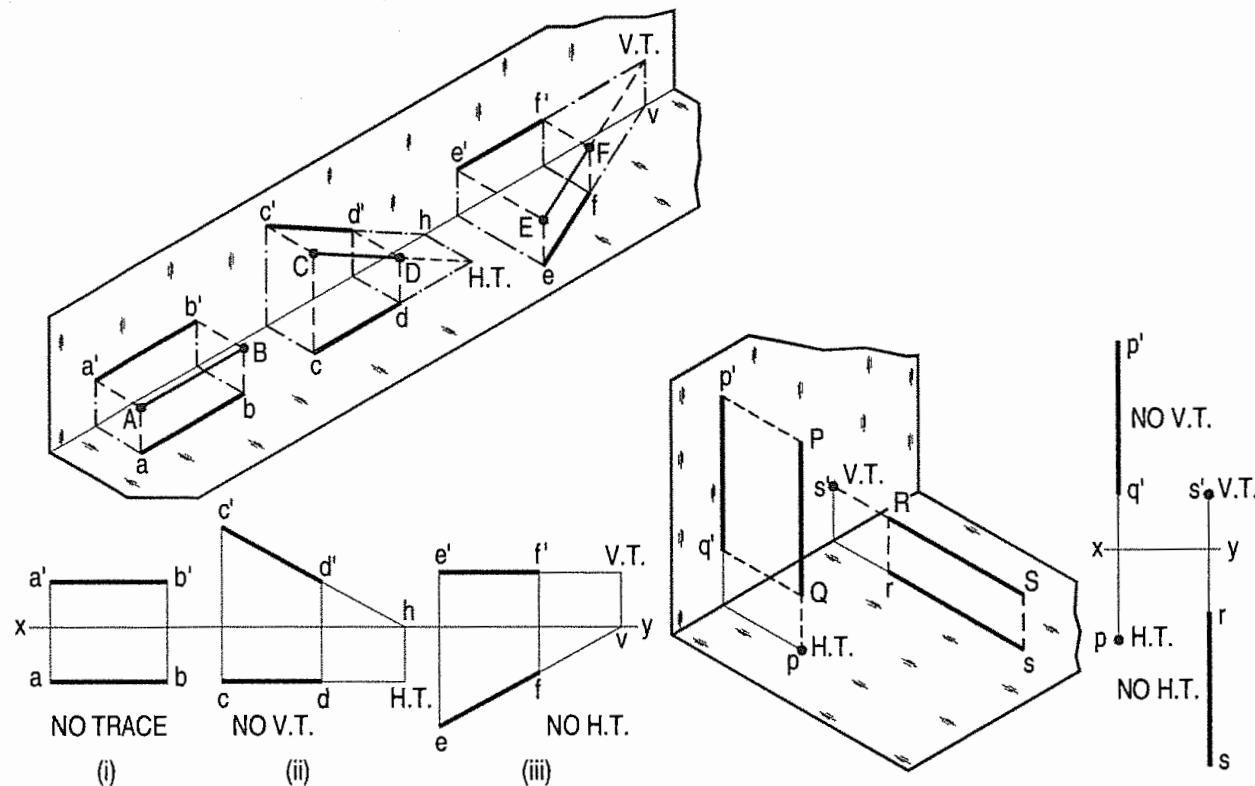


FIG. 10-20

FIG. 10-21

Hence, when a line is perpendicular to a plane, its trace on that plane coincides with its projection on that plane. It has no trace on the other plane.

Refer to fig. 10-22.

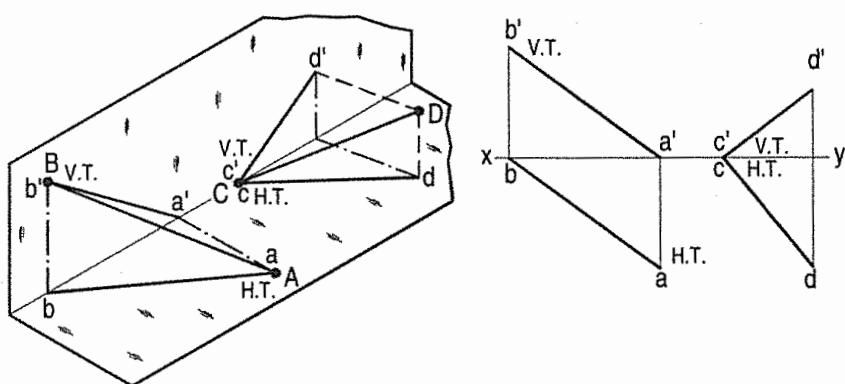


FIG. 10-22

- (i) A line AB has its end A in the H.P. and the end B in the V.P. Its H.T. coincides with the top view of A and the V.T. coincides with b' the front view of B .
 (ii) A line CD has its end C in both the H.P. and the V.P. Its H.T. and V.T. coincide with c and c' (the projections of C) in xy .

Hence, when a line has an end in a plane, its trace upon that plane coincides with the projection of that end on that plane.

10-10. METHODS OF DETERMINING TRACES OF A LINE

Method I:

Fig. 10-23(i) shows a line AB inclined to both the reference planes. Its end A is in the H.P. and B is in the V.P.

$a'b'$ and ab are the front view and the top view respectively [fig. 10-23(ii)].

The H.T. of the line is on the projector through a' and coincides with a . The V.T. is on the projector through b and coincides with b' .

Let us now assume that AB is shortened from both its ends, its inclination with the planes remaining constant. The H.T. and V.T. of the new line CD are still the same as can be seen clearly in fig. 10-24(i).

$c'd'$ and cd are the projections of CD [fig. 10-24(ii)]. Its traces may be determined as described below.

- Produce the front view $c'd'$ to meet xy at a point h .
- Through h , draw a projector to meet the top view cd -produced, at the H.T. of the line.
- Similarly, produce the top view cd to meet xy at a point v .
- Through v , draw a projector to meet the front view $c'd'$ -produced, at the V.T. of the line.

Method II:

$c'd'$ and cd are the projections of the line CD [fig. 10-25(ii)]. Determine the true length C_1D_1 from the front view $c'd'$ by trapezoid method. The point of intersection between $c'd'$ -produced and C_1D_1 -produced is the V.T. of the line.

Similarly, determine the true length C_2D_2 from the top view cd . Produce them to intersect at the H.T. of the line.

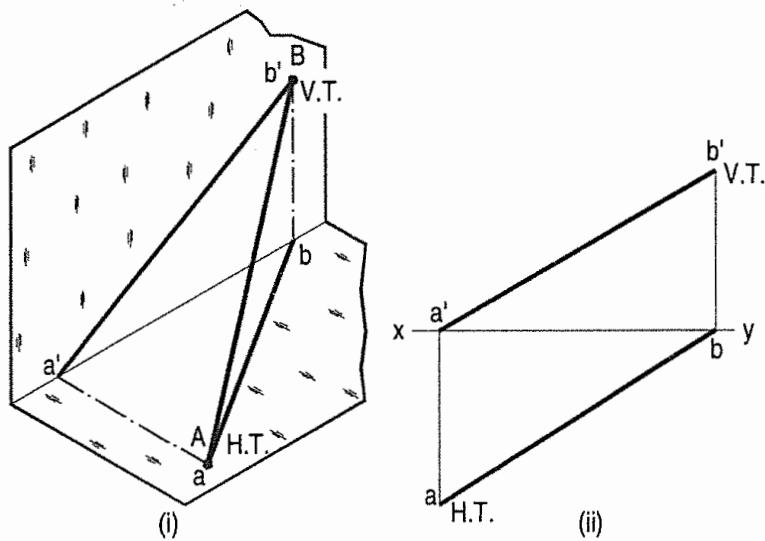


FIG. 10-23

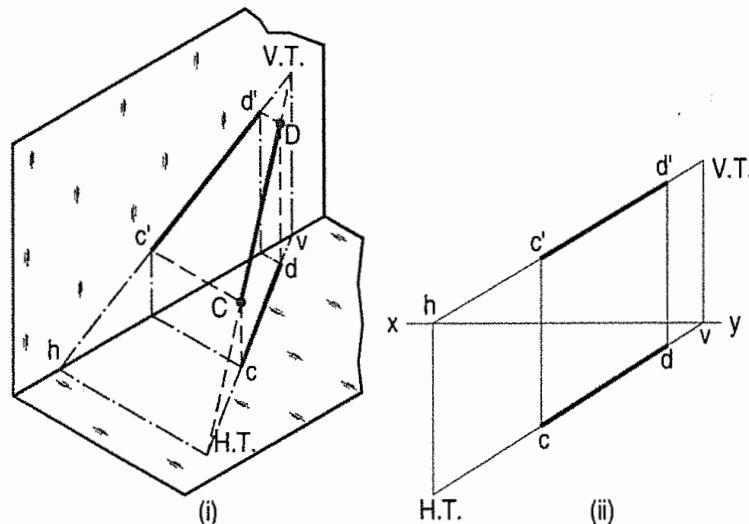


FIG. 10-24

The above is quite evident from the pictorial view shown in fig. 10-25(i).

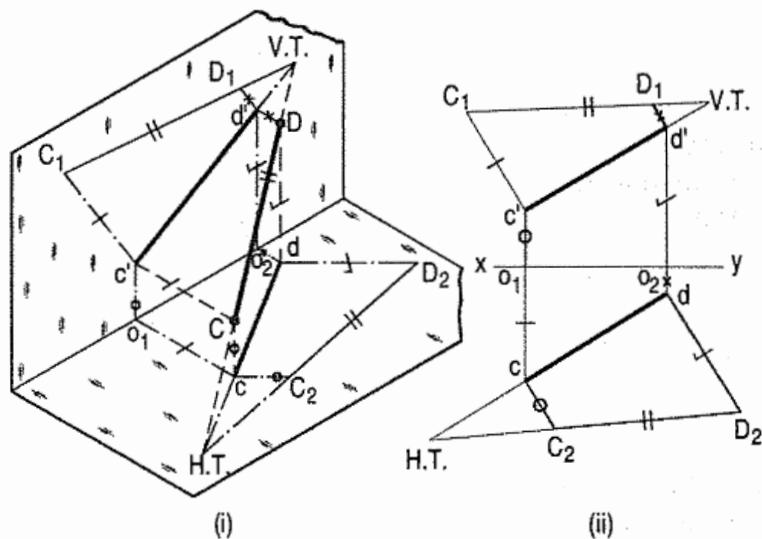


FIG. 10-25

10-11. TRACES OF A LINE, THE PROJECTIONS OF WHICH ARE PERPENDICULAR TO xy

When the projections of a line are perpendicular to xy , i.e. when the sum of its inclinations with the two principal planes of projection is 90° , it is not possible to find the traces by the first method. Method II must, therefore, be adopted as shown in fig. 10-26.

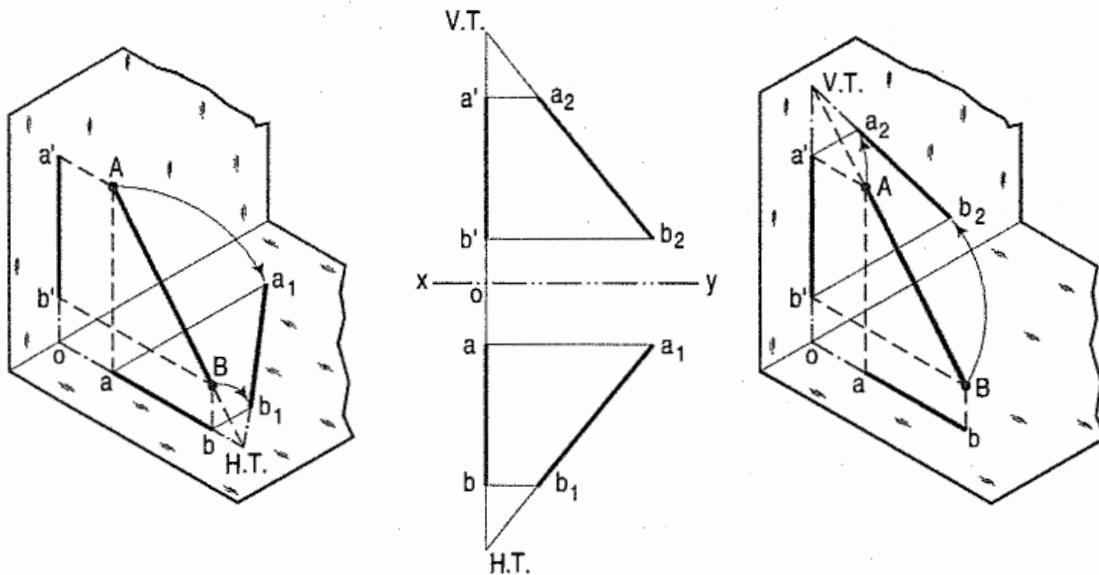


FIG. 10-26

10-12. POSITIONS OF TRACES OF A LINE

Although the line may be situated in the third quadrant, its both traces may be above or below xy , as shown in problem 10-6 and in fig. 10-27 and fig. 10-28. When a line intersects a plane, its traces on that plane will be contained by its projection on that plane as shown in problem 10-7.

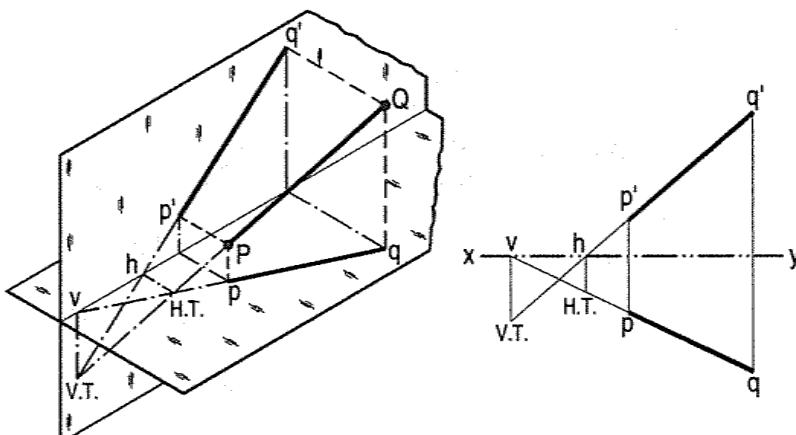


FIG. 10-27

Problem 10-6. Projections of a line PQ are given. Determine the positions of its traces.

Let pq and $p'q'$ be the projections of PQ (fig. 10-27 and fig. 10-28).

- Produce the top view pq to meet xy at v . Draw a projector through v to meet the front view $p'q'$ -produced at the V.T.
- Through h , the point of intersection between $p'q'$ -produced and xy , draw a projector to meet the top view pq -produced at the H.T.

Note that in fig. 10-27, the traces are below xy while in fig. 10-28 they are above it.

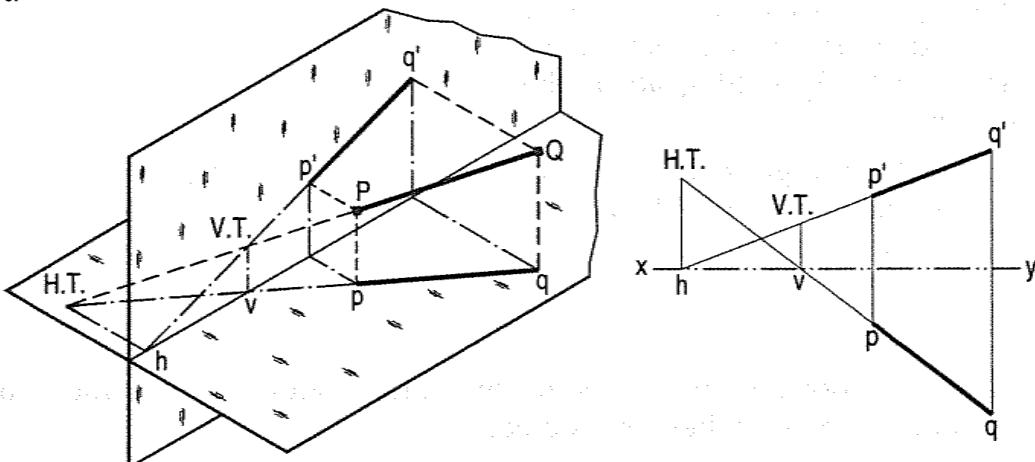


FIG. 10-28

Problem 10-7. A point A is 50 mm below the H.P. and 12 mm behind the V.P. A point B is 10 mm above the H.P. and 25 mm in front of the V.P. The distance between the projectors of A and B is 40 mm. Determine the traces of the line joining A and B .

Draw the projections ab and $a'b'$ of the line AB .

Method I: (fig. 10-29):

- Through v , the point of intersection between ab and xy , draw a projector to meet $a'b'$ at the V.T. of the line.
- Similarly, through h , the point of intersection between $a'b'$ and xy , draw a projector to cut ab at the H.T. of the line.

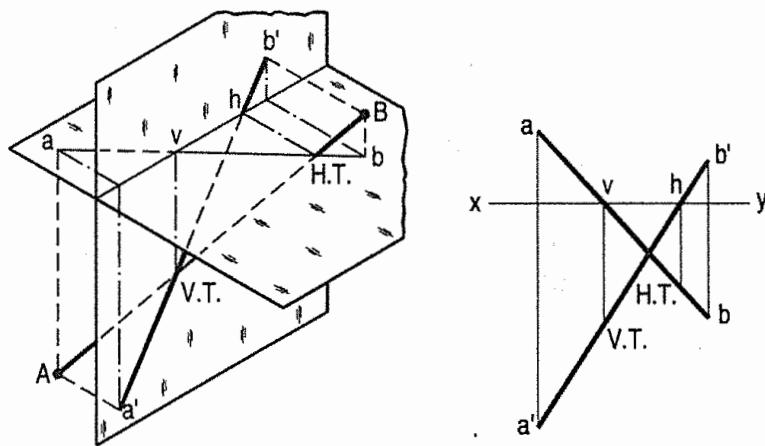


FIG. 10-29

Method II: (fig. 10-30):

At the ends a' and b' , draw perpendiculars to $a'b'$, viz. $a'A_1$ equal to $a'o_1$ and $b'B_1$ equal to $b'o_2$ on its opposite sides (as a and b are on opposite sides of xy).

Draw the line A_1B_1 intersecting $a'b'$ at the V.T. of the line.

Similarly, at the ends a and b , draw perpendiculars to ab , viz. aA_2 equal to $a'o_1$ and bB_2 equal to $b'o_2$, on its opposite sides (as a' and b' are on opposite sides of xy). Join A_2 with B_2 cutting ab at the H.T. of the line.

Note that $A_1B_1 = A_2B_2 = AB$ and that θ and ϕ are the inclinations of AB with the H.P. and the V.P. respectively.

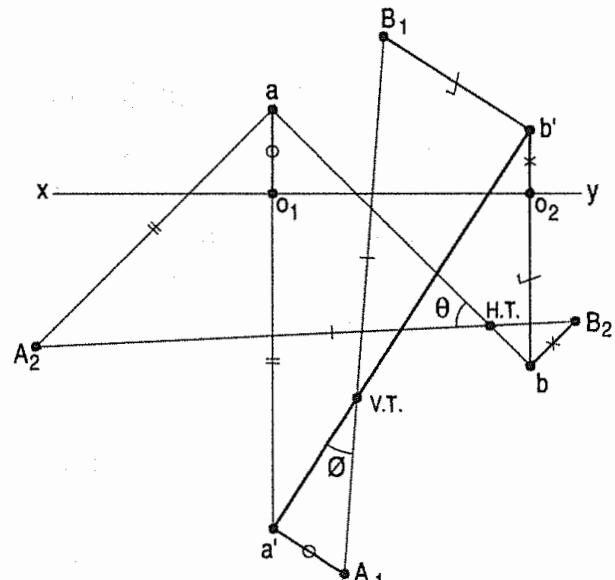


FIG. 10-30

10-13. ADDITIONAL ILLUSTRATIVE PROBLEMS

In the following problems, the ends of the lines should be assumed to be in the first quadrant, unless otherwise stated.

Problem 10-8. (fig. 10-31): A line AB , 50 mm long, has its end A in both the H.P. and the V.P. It is inclined at 30° to the H.P. and at 45° to the V.P. Draw its projections.

As the end A is in both the planes, its top view and the front view will coincide in xy .

- Assuming AB to be parallel to the V.P. and inclined at θ (equal to 30°) to the H.P., draw its front view ab' (equal to AB) and project the top view ab .

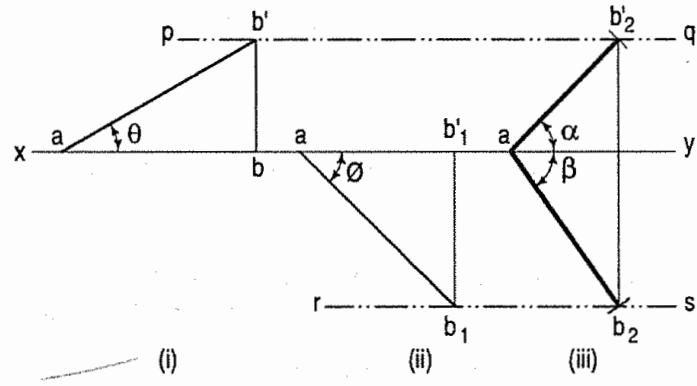


FIG. 10-31

(ii) Again assuming AB to be parallel to the H.P. and inclined at ϕ (equal to 45°) to the V.P., draw its top view ab_1 (equal to AB). Project the front view ab'_1 .

ab and ab'_1 are the lengths of AB in the top view and the front view respectively, and pq and rs are the loci of the end B in the front view and the top view respectively.

(iii) With a as centre and radius equal to ab'_1 , draw an arc cutting pq in b'_2 . With the same centre and radius equal to ab , draw an arc cutting rs in b_2 .

Draw lines joining a with b'_2 and b_2 . ab'_2 and ab_2 are the required projections.

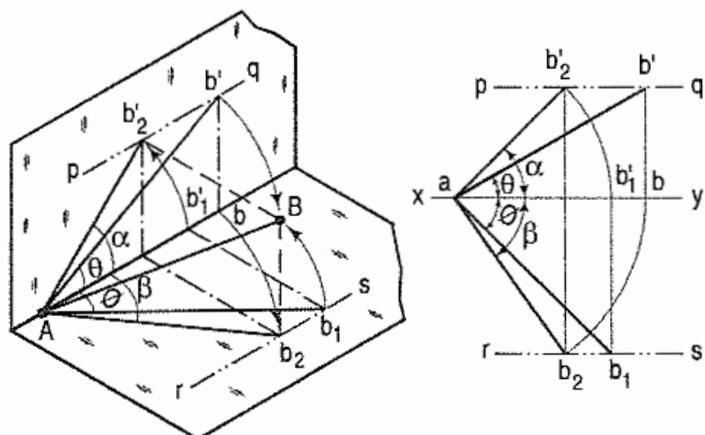


FIG. 10-32

Fig. 10-32 shows in pictorial and orthographic views, the solution obtained with all the above steps combined in one figure only.

Problem 10-9. (fig. 10-33): A line PQ 75 mm long, has its end P in the V.P. and the end Q in the H.P. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.

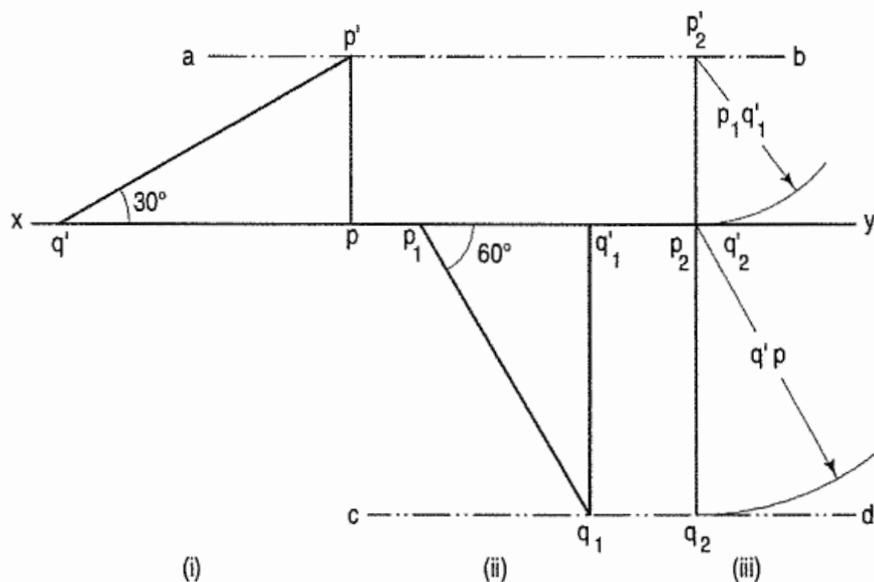


FIG. 10-33

The top view of P and the front view of Q will be in xy . As shown in the previous problem, determine

- the length of PQ in the top view, viz. $q'p$ and the path ab of the end P in the front view;
- the length $p_1q'_1$ in the front view and the path cd of the end Q in the top view.
- Mark any point p_2 (the top view of P) in xy and project its front view p'_2 in ab .
- With p'_2 as centre and radius equal to $p_1q'_1$, draw an arc cutting xy in q'_2 . It coincides with p_2 .
- With p_2 as centre and radius equal to $q'p$, draw an arc cutting cd in q_2 . p_2q_2 and $p'_2q'_2$ are the required projections. They lie in a line perpendicular to xy because the sum of the two inclinations is equal to 90° .

Problem 10-10. (fig. 10-34): A line PQ 100 mm long, is inclined at 30° to the H.P. and at 45° to the V.P. Its mid-point is in the V.P. and 20 mm above the H.P. Draw its projections, if its end P is in the third quadrant and Q in the first quadrant.

The front view and the top view of P will be below and above xy respectively, while those of Q will be above and below xy respectively.

- Mark m , the top view of the mid-point in xy and project its front view m' , 20 mm above xy .
- Through m' , draw a line making an angle θ (equal to 30°) with xy and with the same point as centre and radius equal to $\frac{1}{2} PQ$, cut it at P_1 below xy and at Q_1 above xy . Project P_1Q_1 to $p_1 q_1$ on xy . p_1q_1 is the length of PQ in the top view. ab and cd are the paths of P and Q respectively in the front view.
- Similarly, through m , draw a line making angle ϕ (equal to 45°) with xy and cut it with the same radius at P_2 above xy and at Q_2 below it.
- Project P_2Q_2 to $p'_2q'_2$ on the horizontal line through m' . $p'_2q'_2$ is the length of PQ in the front view and ef and gh are the paths of P and Q respectively in the top view.
- With m as centre and radius equal to mp_1 or mq_1 , draw arcs cutting ef at p_3 and gh at q_3 . With m' as centre and radius equal to $m'p'_2$ or $m'q'_2$, draw arcs cutting ab at p'_3 and cd at q'_3 . p_3q_3 and $p'_3q'_3$ are the required projections.

Problem 10-11. (fig. 10-35): The top view of a 75 mm long line AB measures 65 mm, while the length of its front view is 50 mm. Its one end A is in the H.P. and 12 mm in front of the V.P. Draw the projections of AB and determine its inclinations with the H.P. and the V.P.

- Mark the front view a' and the top view a of the given end A .

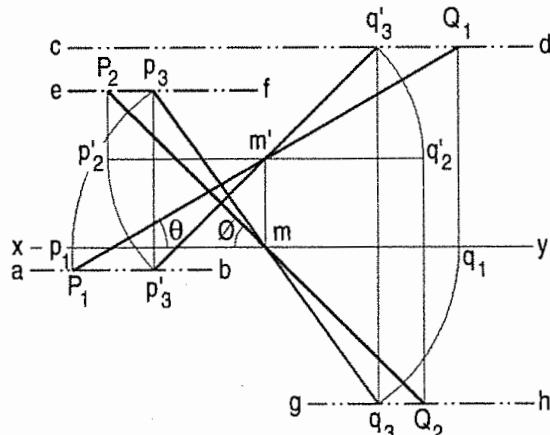


FIG. 10-34

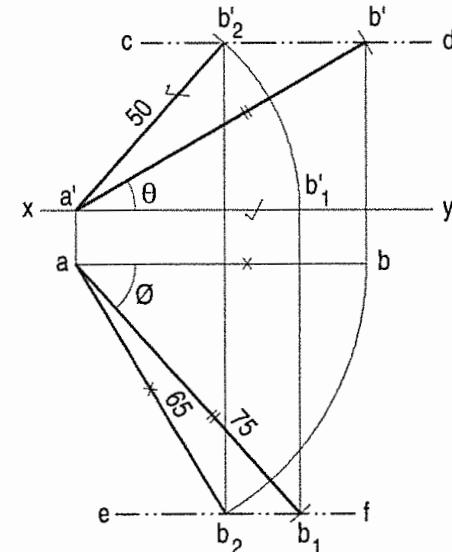


FIG. 10-35

- (ii) Assuming AB to be parallel to the V.P., draw a line ab equal to 65 mm and parallel to xy . With a' as centre and radius equal to 75 mm, draw an arc cutting the projector through b at b' . The line cd through b' and parallel to xy , is the locus of B in the front view and θ is the inclination of AB with the H.P.
- (iii) Similarly, draw a line $a'b'_1$ in xy and equal to 50 mm. With a as centre and radius equal to AB , draw an arc cutting the projector through b'_1 at b_1 . ef is the locus of B in the top view and ϕ is the inclination of AB with the V.P.
- (iv) With a' as centre and radius equal to $a'b'_1$, draw an arc cutting cd in b'_2 . With a as centre and radius equal to ab , draw an arc cutting ef in b_2 . $a'b'_2$ and ab_2 are the required projections.

Problem 10-12. (fig. 10-36): A line AB , 65 mm long, has its end A 20 mm above the H.P. and 25 mm in front of the V.P. The end B is 40 mm above the H.P. and 65 mm in front of the V.P. Draw the projections of AB and show its inclinations with the H.P. and the V.P.

- (i) As per given positions, draw the loci cd and gh of the end A , and ef and jk of the end B in the front view and the top view respectively.
- (ii) Mark any point a (the top view of A) in gh and project it to a' on cd . With a' as centre and radius equal to 65 mm, draw an arc cutting ef in b' . Join a' with b' . θ , the inclination of $a'b'$ with xy , is the inclination of AB with the H.P. Project b' to b on gh . ab is the length of AB in the top view.
- (iii) With a as centre and radius equal to 65 mm, draw an arc cutting jk in b_1 . Join a with b_1 . ϕ , the inclination of ab_1 with xy , is the inclination of AB with the V.P. Project b_1 to b'_1 on cd . $a'b'_1$ is the length of AB in the front view.

Arrange ab and $a'b'_1$ between their respective paths as shown. $a'b'_2$ and ab_2 are the required projections of AB .

Problem 10-13. (fig. 10-37 and fig. 10-38): The projectors of the ends of a line AB are 50 mm apart. The end A is 20 mm above the H.P. and 30 mm in front of the V.P. The end B is 10 mm below the H.P. and 40 mm behind the V.P. Determine the true length and traces of AB , and its inclinations with the two planes.

Draw two projectors 50 mm apart. On one projector, mark the top view a and the front view a' of the end A . On the other, mark the top view b and the front view b' of the end B , as per given distances. ab and $a'b'$ are the projections of AB .

Determine the true length, traces and inclinations by any one of the following two methods:

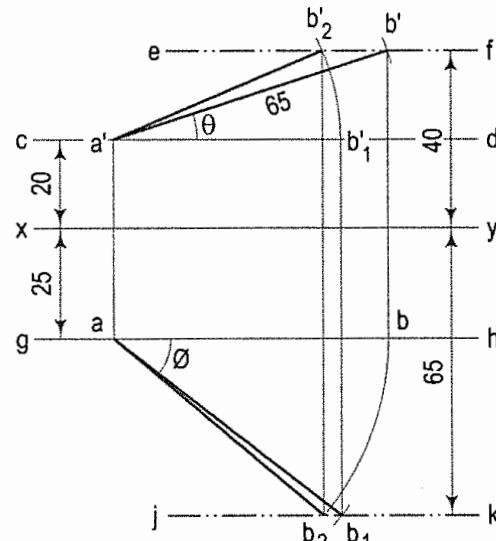


FIG. 10-36

Method I:

By making the line parallel to a plane (fig. 10-37):

- Keeping a fixed, turn ab to a position ab_1 , thus making it parallel to xy . Project b_1 to b'_1 on the locus of b' . $a'b'_1$ is the true length of AB and θ is its true inclination with the H.P.
- Similarly, turn $a'b'$ to the position a'_1b' and project a'_1 to a_1 on the path of a (because the end a has been moved). a_1b is the true length of AB and ϕ is its inclination with the V.P.

Traces:

- Through v the point of intersection of the top view ab with xy , draw a projector to cut $a'b'$ at the V.T.
- Through h the point of intersection of the front view $a'b'$ with xy , draw a projector to cut ab at the H.T. of the line.

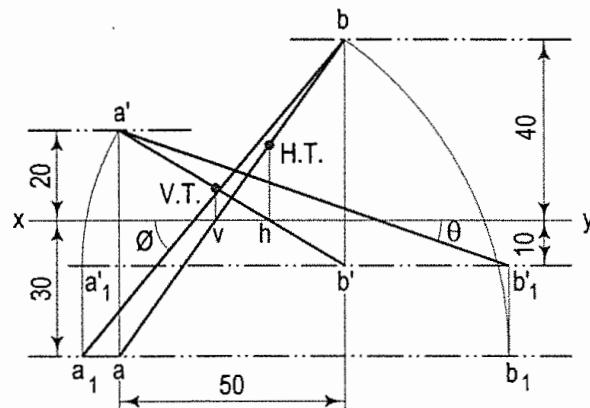


FIG. 10-37

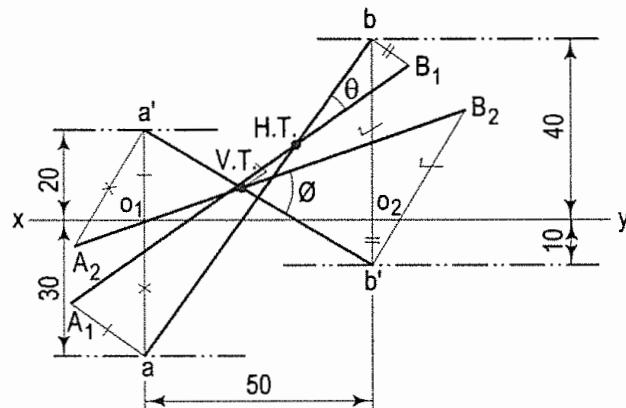


FIG. 10-38

Method II:

By rotating the line about its projections till it lies in H.P. or V.P. (fig. 10-38):

- At the ends a and b of the top view ab , draw perpendiculars to ab , viz. aA_1 equal to $a'o_1$ and bB_1 equal to $b'o_2$, on opposite sides of it (because a' and b' are on opposite sides of xy). A_1B_1 is the true length of AB . θ (its inclination with ab) is the inclination of AB with the H.P. and the point at which A_1B_1 intersects ab is the H.T. of AB .
- Similarly, at the ends a' and b' of the front view $a'b'$, draw perpendiculars to $a'b'$, viz. $a'A_2$ equal to $a'o_1$ and $b'B_2$ equal to $b'o_2$, on opposite sides of it. A_2B_2 is the true length of AB . ϕ (its inclination with $a'b'$) is the inclination of AB with the V.P. and the point at which A_2B_2 intersects $a'b'$ is the V.T. of AB .

Problem 10-14. (fig. 10-39): A line AB , 90 mm long, is inclined at 45° to the H.P. and its top view makes an angle of 60° with the V.P. The end A is in the H.P. and 12 mm in front of the V.P. Draw its front view and find its true inclination with the V.P.

- Mark a and a' , the projections of the end A .

- (ii) Assuming AB to be parallel to the V.P. and inclined at 45° to the H.P., draw its front view $a'b'$ equal to AB and making an angle of 45° with xy . Project b' to b so that ab the top view is parallel to xy . Keeping the end a fixed, turn the top view ab to a position ab_1 so that it makes an angle of 60° with xy . Project b_1 to b'_1 on the locus of b' . Join a' with b'_1 . $a'b'_1$ is the front view of AB .
- (iii) To find the true inclination with the V.P., draw an arc with a as centre and radius equal to AB , cutting the locus of b_1 in b_2 . Join a with b_2 . ϕ is the true inclination of AB with the V.P.

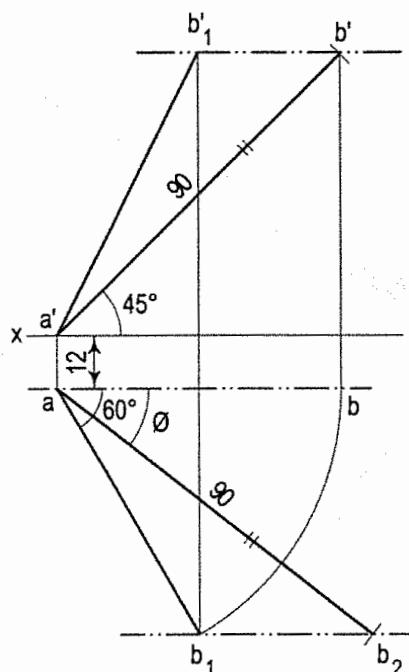


FIG. 10-39

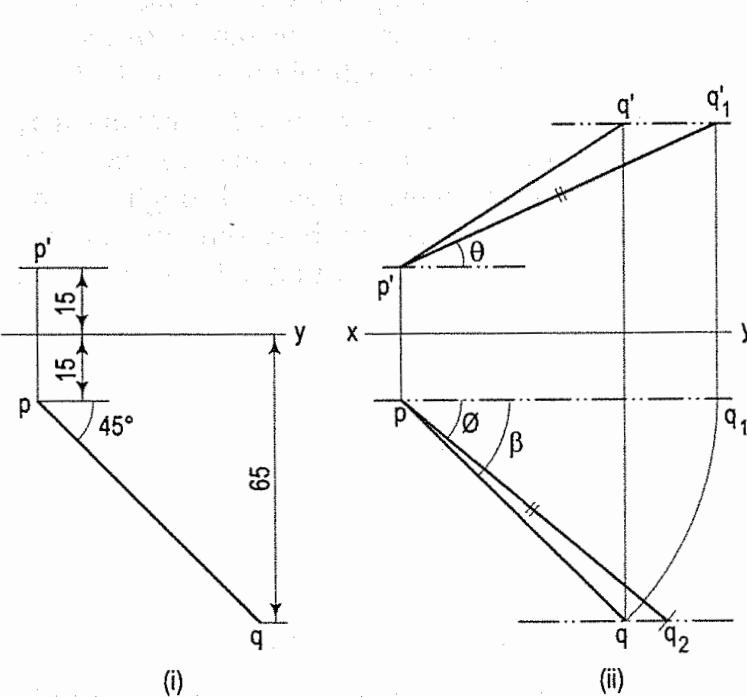


FIG. 10-40

Problem 10-15. (fig. 10-40): Incomplete projections of a line PQ , inclined at 30° to the H.P. are given in fig. 10-40(i). Complete the projections and determine the true length of PQ and its inclination with the V.P.

- Turn the top view pq [fig. 10-40(i)] to a position pq_1 , so that it is parallel to xy . Through p' , draw a line making an angle θ (equal to 30°) with xy and cutting the projector through q_1 at q'_1 . $p'q'_1$ is the front view of PQ .
- Through q'_1 , draw a line parallel to xy and cutting the projector through q at q' . $p'q'$ is the front view of PQ .
- With p as centre and radius equal to $p'q'_1$, draw an arc cutting the locus of q at q_2 .
- Join p with q_2 . ϕ is the inclination of PQ with the V.P.

Problem 10-16. (fig. 10-41): The end A of a line AB is 25 mm behind the V.P. and is below the H.P. The end B is 12 mm in front of the V.P. and is above the H.P. The distance between the projectors is 65 mm. The line is inclined at 40° to the H.P. and its H.T. is 20 mm behind the V.P. Draw the projections of the line and determine its true length and the V.T.

Draw the top view ab and mark the H.T. on it, 20 mm above xy .

We have seen that the line representing the true length obtained by the trapezoid method, intersects the top view or the top view-produced, at the H.T. at an angle equal to the true inclination of the line with the V.P.

- Hence, at the ends a and b , draw perpendiculars to ab on its opposite sides (as one end is below the H.P. and the other end above it). Through the H.T., draw a line making angle θ (equal to 40°) with ab and cutting the perpendiculars at A_1 and B_1 , as shown. A_1B_1 is the true length of AB . aA_1 and bB_1 are the distances of the ends A and B respectively from the H.P.
- Project a and b to a' and b' , making $a'o_1$ equal to aA_1 and $b'o_2$ equal to bB_1 . $a'b'$ is the front view of AB . Through v , the point of intersection between ab and xy , draw a projector cutting $a'b'$ at the V.T. of the line.

Problem 10-17. (fig. 10-42): A line AB , 90 mm long, is inclined at 30° to the H.P. Its end A is 12 mm above the H.P. and 20 mm in front of the V.P. Its front view measures 65 mm. Draw the top view of AB and determine its inclination with the V.P.

- Mark a and a' the projections of the end A . Through a' , draw a line ab' 90 mm long and making an angle of 30° with xy .
- With a' as centre and radius equal to 65 mm, draw an arc cutting the path of b' at b'_1 . $a'b'_1$ is the front view of AB .
- Project b' to b , so that ab is parallel to xy . ab is the length of AB in the top view.
- With a as centre and radius equal to ab , draw an arc cutting the projector through b'_1 at b_1 . Join a with b_1 . ab_1 is the required top view.

Determine ϕ as described in problem 10-14.

Problem 10-18. (fig. 10-43): The ends of a line PQ are on the same projector. The end P is 30 mm below the H.P. and 12 mm behind the V.P. The end Q is 55 mm above the H.P. and 45 mm in front of the V.P. Determine the true length and traces of PQ and its inclinations with the two planes.

Note: When the ends of a line are on the same projector or sum of angles of inclinations of a line with xy is 90° , use Method II only.

Draw the projections pq and $p'q'$ as per given positions of the ends P and Q . They will partly coincide with each other.

- At the ends p and q of the top view pq , erect perpendiculars, viz. pP_1 equal to $p'o$, and qQ_1 equal to $q'o$ and on opposite sides of pq . P_1Q_1 is the true length of PQ . θ is the inclination of PQ with the H.P. and the point of intersection between P_1Q_1 and pq is the H.T. of PQ .

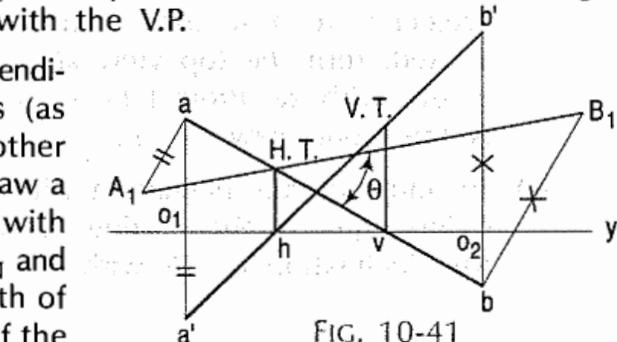


FIG. 10-41

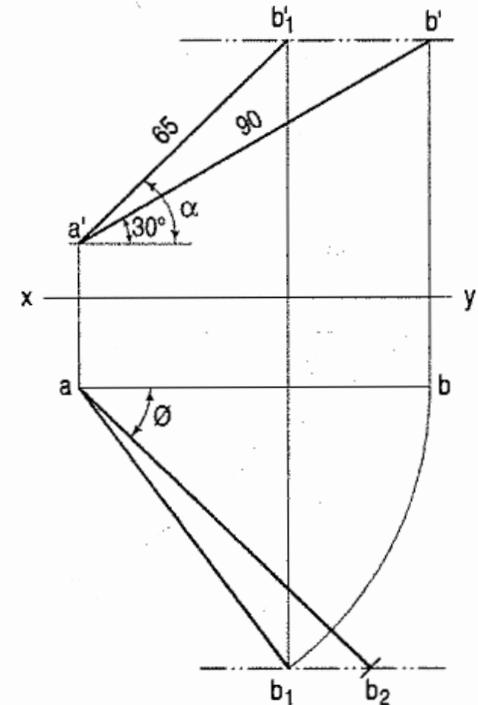


FIG. 10-42

- (ii) Similarly, draw perpendiculars to $p'q'$, viz. $p'P_2$ equal to po and $q'Q_2$ equal to qo and on opposite sides of $p'q'$. P_2Q_2 is the true length. ϕ is the true inclination of PQ with the V.P. and the point where P_2Q_2 cuts $p'q'$ is the V.T. of PQ .

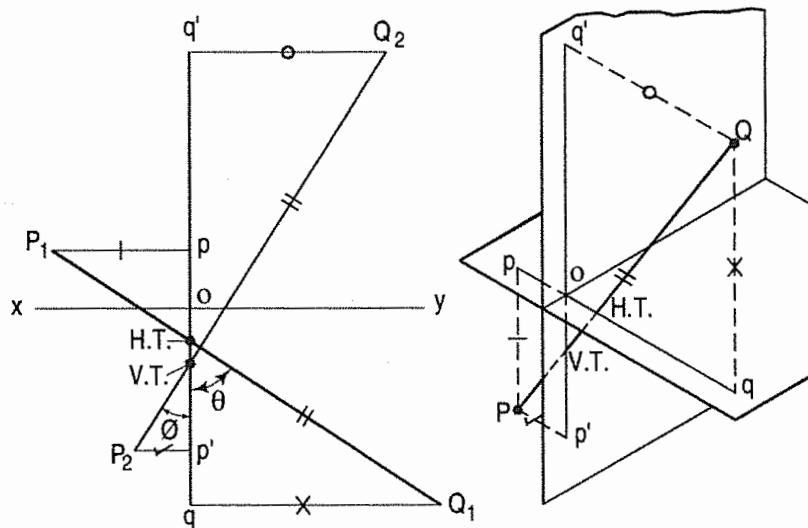


FIG. 10-43

Problem 10-19. (fig. 10-44): A line AB , inclined at 40° to the V.P., has its ends 50 mm and 20 mm above the H.P. The length of its front view is 65 mm and its V.T. is 10 mm above the H.P. Determine the true length of AB , its inclination with the H.P. and its H.T.

- Draw the front view $a'b'$ as per given positions of A and B and the given length.
- Draw a line parallel to and 10 mm above xy . This line will contain the V.T. Produce $a'b'$ to cut this line at the V.T. Draw a projector through V.T. to v on xy .
- Assuming a' V.T. to be the front view of a line which makes 40° angle with the V.P. and whose one end v is in the V.P., let us determine its true length.
- Keeping V.T. fixed, turn the end a' to a'_1 so that the line becomes parallel to xy . Through v , draw a line making an angle of 40° with xy and cutting the projector through a'_1 at a_1 . The line through a_1 , drawn parallel to xy , is the locus of A in the top view. Project a' to a on this line. av is the top view of the line, whose front view is a' V.T. and whose true length is equal to a_1v .
- But $a'b'$ is the given front view of AB . Therefore, project b' to b on av . ab is the top view of AB . Obtain the inclination θ with the H.P. by making the top view ab parallel to xy , as shown.

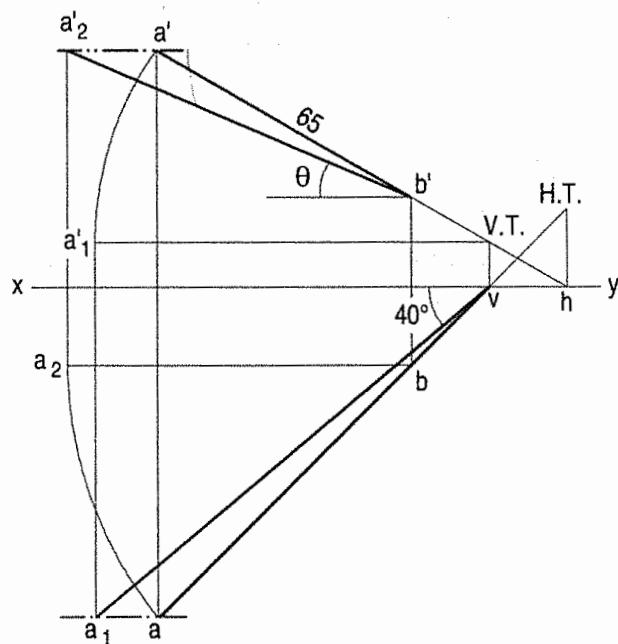


FIG. 10-44

Produce $a'b'$ to meet xy at h . Draw a projector through h to cut ab -produced, at the H.T. of the line.

Problem 10-20. (fig. 10-45): The front view $a'b'$ and the H.T. of a line AB , inclined at 23° to the H.P. are given in fig. 10-45(i). Determine the true length of AB , its inclination with the V.P. and its V.T.

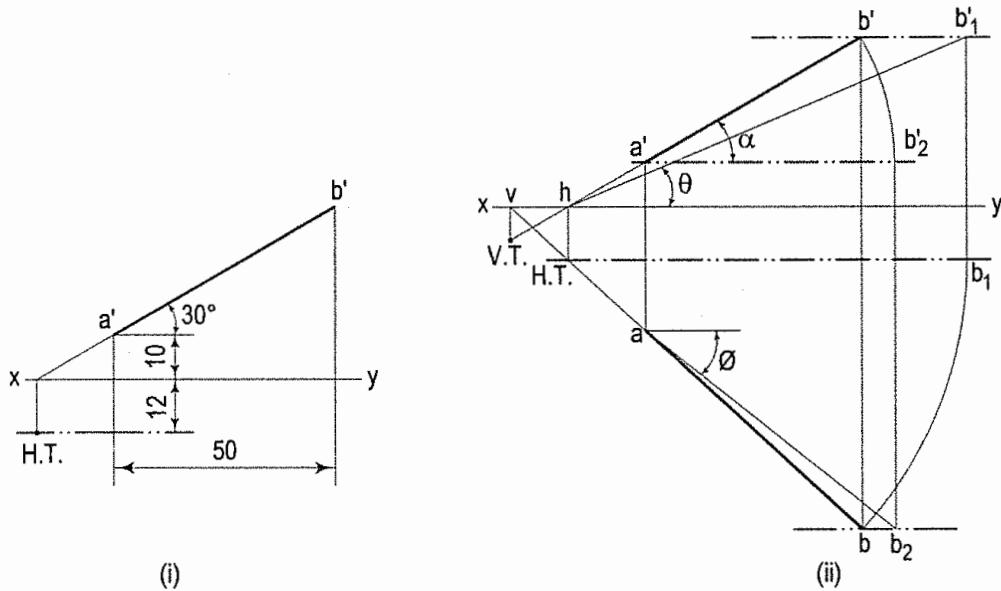


FIG. 10-45

Consider that hb' is the front view of a line inclined at 23° to the H.P. and the top view of whose one end is in H.T.

- Through h [fig. 10-45(ii)], draw a line making an angle of $\theta = 23^\circ$ with xy and cutting the locus of B in the front view in b'_1 . hb'_1 is the true length of the line whose length in the top view is H.T. b_1 .
 - With H.T. as centre and radius equal to H.T. b_1 , draw an arc cutting the projector through b' at b . H.T. b is the top view and hb' is the front view of a line which contains AB .
 - Therefore, through a' , draw a projector cutting H.T. b at a . ab is the top view of AB .
- Obtain the true length ab_2 (of AB) and its inclination ϕ with the V.P. by making $a'b'$ parallel to xy .
- Produce ba to meet xy in v . Draw a projector through v to cut $b'a'$ -produced, at the V.T. of the line.

Problem 10-21. (fig. 10-46): A tripod stand rests on the floor. One of its legs is 150 mm long and makes an angle of 70° with the floor. The other two legs are 163 mm and 175 mm long respectively. The upper ends of the legs are attached to the corners of a horizontal equilateral triangular frame of 50 mm side, one side of which is parallel to the V.P. In the top view, the legs appear as lines 120° apart, which if produced, would meet in a point. Draw the projections of the tripod and determine the angle which each of the other two legs makes with the floor. Assume the thickness of the frame and of the legs to be equal to that of the line.

- At any point P on xy , draw a line PA , 150 mm long and making 70° angle with xy . h is the height of the tripod and PA_1 is the length of the leg in the top view.

- (ii) Draw an equilateral triangle abc of 50 mm side with one side parallel to and below xy . Project the front view $a'b'c'$ at the height h above xy . Determine the lengths of the other two legs in the top view as described below.

- (iii) With b' as centre and radius equal to 163 mm, draw an arc cutting xy in q'_1 . Similarly with a' and c' as centre and radius equal to 150 mm and 175 mm, draw an arc cutting xy in p'_1 and r'_1 respectively. ap_1 , bq_1 and cr_1 are the lengths of the three legs in the top view and α and β respectively are their inclinations with the floor (H.P.).

- (iv) The legs in the top view are to be inclined at 120° to each other and to meet at a point, if produced. Therefore, draw lines bisecting the angles of the triangle, making ap equal to PA_1 , bq equal to bq_1 and cr equal to cr_1 , thus completing the top view.
(v) Project p, q and r to p', q' and r' respectively on xy . Complete the front view by drawing lines $a'p'$, $b'q'$ and $c'r'$.

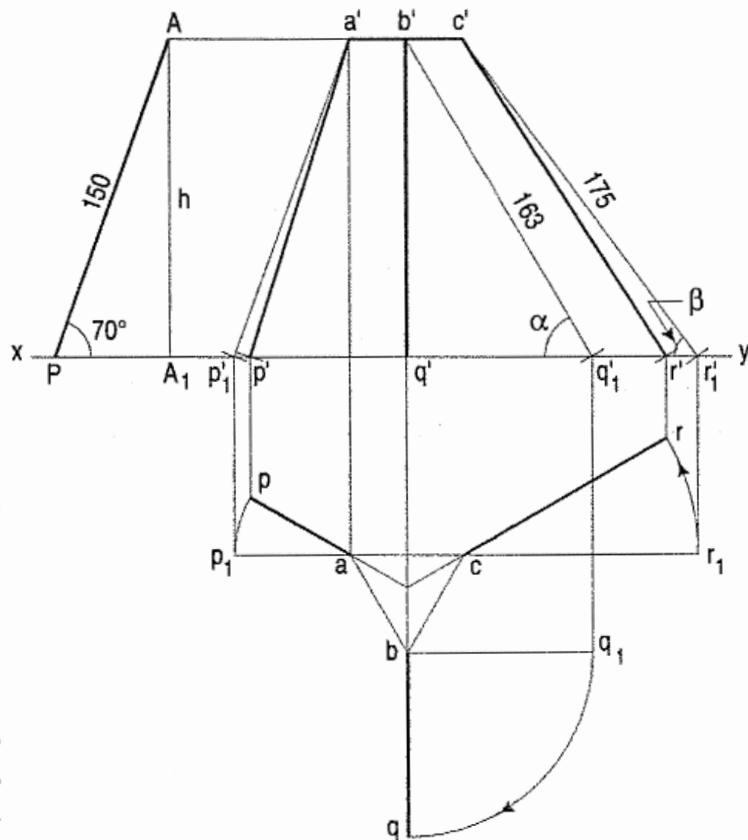


FIG. 10-46

Problem 10-22. (fig. 10-47): A straight road going uphill from a point A , due east to another point B , is 4 km long and has a slope of 15° . Another straight road from B , due 30° east of north, to a point C is also 4 km long but is on ground level. Determine the length and slope of the straight road joining the points A and C . Scale, $10 \text{ mm} = 0.4 \text{ km}$.

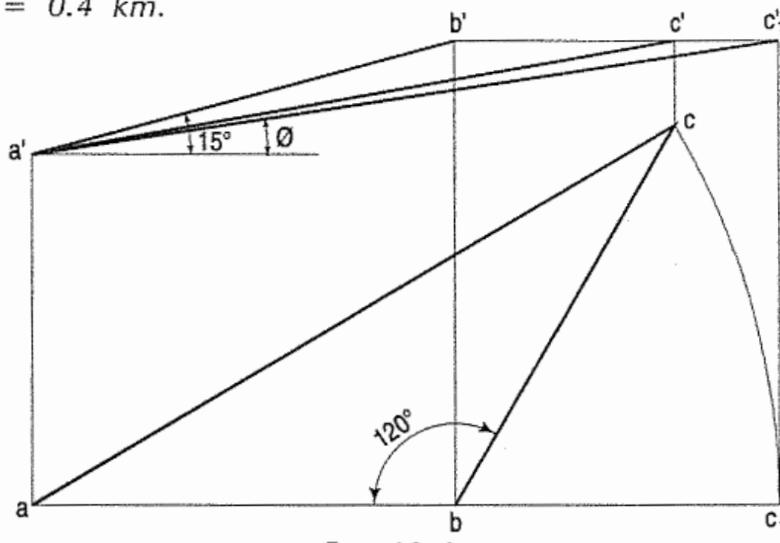


FIG. 10-47

- Mark any point a' . Draw a line $a'b'$, 100 mm long, to the right of a' and inclined upwards at 15° to the horizontal (to represent the road from A to B). Project its top view ab keeping it horizontal.
- As the road from B to C is on ground level, the top view bc will be equal to 100 mm and inclined at $(90^\circ + 30^\circ)$ i.e. 120° to ab .
- From b , draw a line bc , 100 mm long and making 120° angle with ab . Project c to c' making $b'c'$ horizontal. $a'c'$ and ac are the front view and the top view respectively of the road from A to C.

Determine the true length $a'c'_1$ and the angle θ as shown, which are respectively the length and slope of the road from A to C.

Problem 10-23. (fig. 10-48): Two lines AB and AC make an angle of 120° between them in their front view and top view. AB is parallel to both the H.P. and the V.P. Determine the real angle between AB and AC.

Draw any line $b'a'$ parallel to and above xy, and another line $a'c'$ of any length making 120° angle with $b'a'$. Join b' with c' .

- Project the top view ba parallel to xy and the top view ac , making 120° angle with ba . Join b with c . $b'a'$ or ba is the true length of AB. Determine the true lengths of AC and BC, viz. $a'c'_1$ and $b'c'_2$, as shown.
- Draw a triangle $a'b'c'_3$ making $a'c'_3$ equal to $a'c'_1$ and $b'c'_3$ equal to $b'c'_2$. $\angle b'a'c'_3$ is the real angle between AB and AC.

Problem 10-24. (fig. 10-49): An object O is placed 1.2 m above the ground and in the centre of a room $4.2 \text{ m} \times 3.6 \text{ m} \times 3.6 \text{ m}$ high. Determine graphically its distance from one of the corners between the roof and two adjacent walls. Scale, $10 \text{ mm} = 0.5 \text{ m}$.

- Draw the front view (of the room) $a'b'c'd'$ as seen from the front of, say 3.6 m wall. $a'b'$ is the width of the room and $a'd'$ is the height. The front view o' of the object will be seen 1.2 m above the mid-point of $a'b'$. c' and d' are the top corners of the room. $o'c'$ is the front view of the line joining the object with a top corner.
- Draw the top view of the room. It will be a rectangle of sides equal to 3.6 m and 4.2 m. The top view o of the object will be in the centre of the rectangle. oc is the top view of the line joining the object with the top corner.

Determine the true length $o'c'_1$, which will show the distance of the object from one of the top corners of the room.

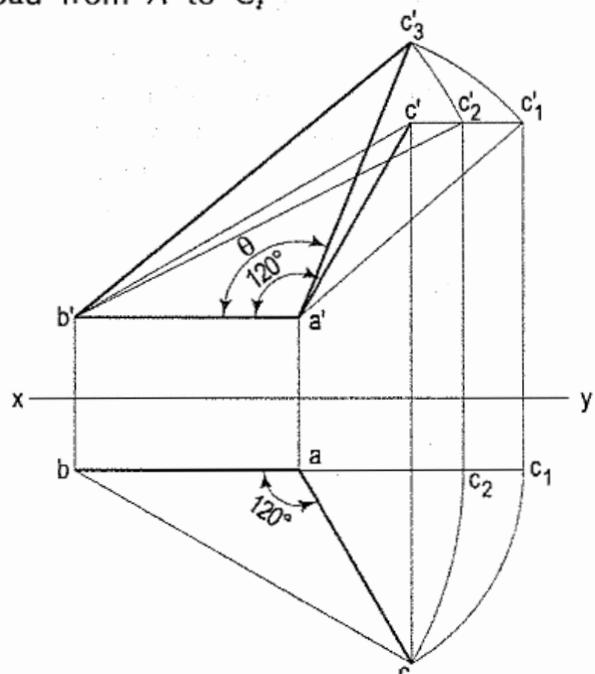


FIG. 10-48

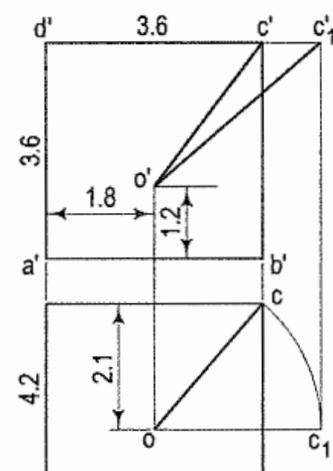
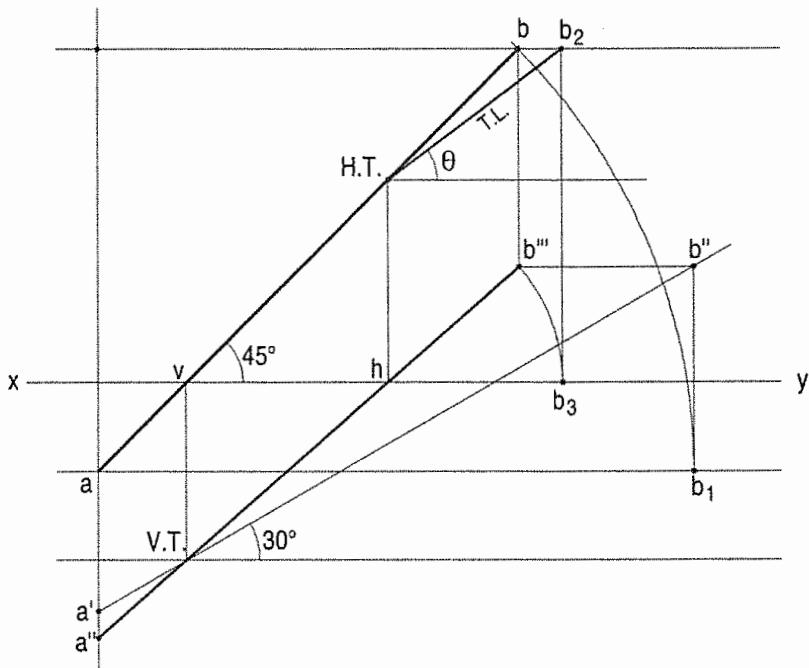


FIG. 10-49

Problem 10-25. The straight line AB is inclined at 30° to H.P., while its top view at 45° to a line xy. The end A is 20 mm in front of the V.P. and it is below the H.P. The end B is 75 mm behind the V.P. and it is above the H.P. Draw the projections of the line when its V.T. is 40 mm below. Find the true length of the portion of the straight line which is in the second quadrant and locate its H.T.

Refer to fig. 10-50.



$$\text{T.L.} = 50 \text{ mm. } \angle \text{V.P.}, \theta = 37^\circ$$

FIG. 10-50

- Mark the points a (top view of A) and b (top view of B) at the distances of 20 mm and 75 mm below and above xy respectively.
- Through the point a, draw a line at 45° intersecting xy and the path of b at v and b respectively as shown.
- Construct a line containing V.T. 40 mm below xy. Draw perpendicular from v to the line V.T.
- With a as centre and radius equal to ab, draw an arc which intersects at b₁ a line drawn from a parallel to xy. From V.T. draw a line at 30° intersecting the projector of b₁ at b''. From b'', draw b''b''' parallel to xy to intersect projector of b at b'''. Join V.T. b'''. Produce it to meet the projector from a at a''. a''b''' is the required projection. a''b''' intersects line xy at h. From h draw the perpendicular to meet ab. The intersection point represents H.T.
- With h as centre and radius equal to hb''', draw an arc intersecting at b₃. Draw projector from b₃ to cut the path of b at b₂. Join H.T. b₂. Measure the angle H.T. b₂ with xy. This is an angle made by the line with the V.P.

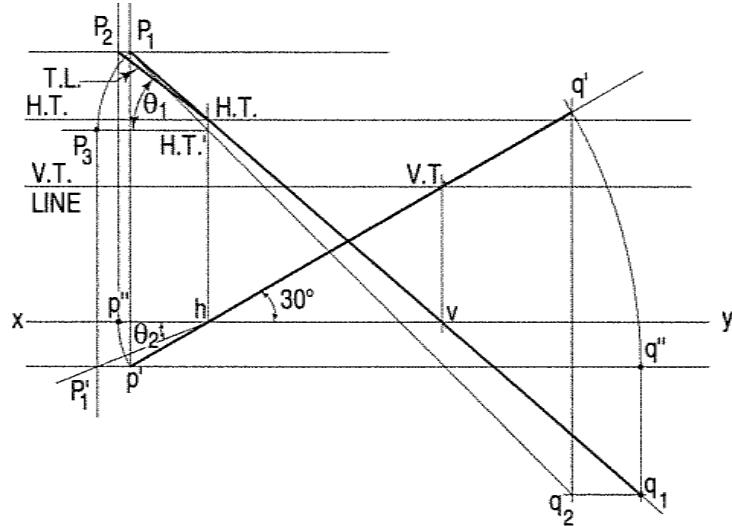
Problem 10-26. (fig. 10-51.): The front view of a line PQ makes an angle of 30° with xy. The H.T. of the line is 45 mm behind the V.P. While its V.T. is 30 mm above the H.P. The end P of the line is 10 mm below the H.P. and the end Q is in the first quadrant. The line is 150 mm long. Draw the projections of the line and determine the true-length of the portion of the line which is in the second quadrant. Also find the angle of the line with the H.P. and V.P.

- (i) Draw lines containing H.T. and V.T. at 45 mm and 30 mm above xy respectively. Mark point p' at 10 mm below line xy . Draw $p'q'$ at 30° from p' intersecting xy at h and line V.T. at V.T. From h , draw perpendicular to line H.T. to locate H.T.

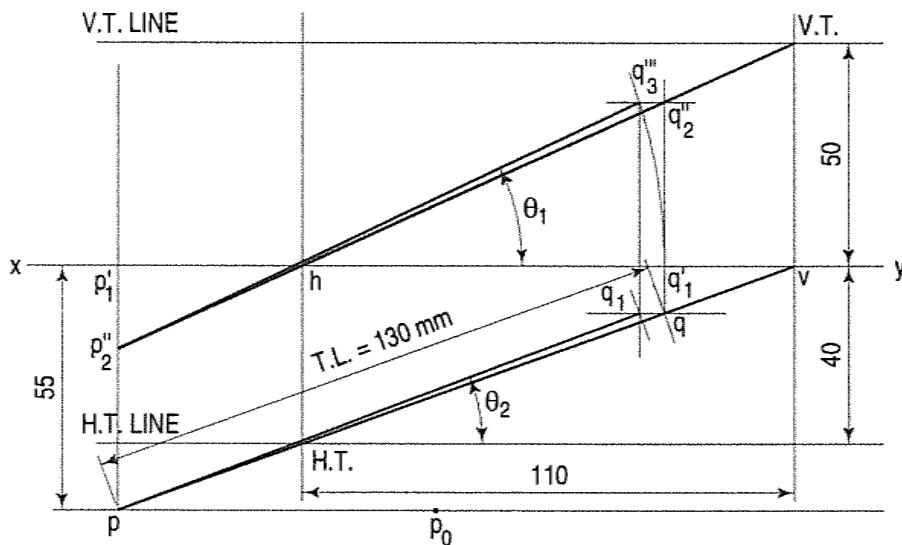
- (ii) Draw perpendicular from V.T. to intersect xy at v . Join v H.T. and produce to intersect the projector of p' at p_1 . Draw $p_1 q_1$ of 150 mm representing true length of the line in top view. From p' , draw line parallel to xy representing front view of the line PQ . Draw projector from q_1 to cut the line drawn from p' at q'' .

- (iii) Keeping p' fixed, turn $p'q''$ such that it cuts the line drawn from p' at q' . From q_1 draw line parallel to xy which intersects the vertical projector drawn from q' at q_2 . Join $P_1 q_2$. This is the required projection.
- (iv) Keeping h fixed, rotate hp' and make it parallel to xy . From P'' draw projector intersecting the path of p_1 at p_2 . H.T. p_2 is true-length of the line. Similarly keeping H.T. p_1 fixed, turn H.T. p_1 to make it parallel to xy as shown. From P_3 , draw projector to intersect the horizontal line drawn from $P'P_1$. Measure angle xhp'_1 . This is the required angle with H.P.

Problem 10-27. (fig. 10-52): The end P of a line PQ 130 mm long, is 55 mm in front of the V.P. The H.T. of the line is 40 mm in front of the V.P. and the V.T. is 50 mm above the H.P. The distance between H.T. and V.T. is 110 mm. Draw the projections of the line PQ and determine its angles with the H.P. and the V.P.



T.L. = 25 mm. \angle V.P., $\theta_1 = 37^\circ$ \angle V.P., $\theta_2 = 22^\circ$
FIG. 10-51



\angle H.P., $\theta_1 = 25.5^\circ$; \angle V.P., $\theta_2 = 21^\circ$
FIG. 10-52

- (i) Mark p below xy at a distance of 55 mm. Draw lines containing H.T. and V.T. at 40 mm below and 50 mm above xy respectively. Construct projectors through H.T. and V.T. 110 mm apart intersecting xy at h and v respectively.
- (ii) Draw perpendiculars from h and v intersecting the lines containing H.T. and V.T. Join H.T. v and produce to cut the line drawn from point P as shown. Join h V.T. and extend further to intersect the projector drawn from P at P''_2 .
- (iii) Mark true length 130 mm on H.T.v. Let it be pq . Draw projector from q cutting xy at q'_1 . With p'_1 as centre and radius equal to $p'_1q'_1$, draw an arc cutting a horizontal line drawn from q''_2 at q'''_3 . Join pq_1 and $p''_2q'''_3$, are the required projections. $\theta_1 = 25.5^\circ$ and $\theta_2 = 21^\circ$ are the measured angles.

Problem 10-28. (fig. 10-53): The distance between the end projectors of a line AB is 70 mm and the projectors through the traces are 110 mm apart. The end of a line is 10 mm above H.P. If the top view and the front view of the line make 30° and 60° with xy line respectively, draw the projections of the line and determine

- (i) the traces, (ii) the angles with the H.P. and the V.P., (iii) the true length of the line. Assume that the line is in the first quadrant.

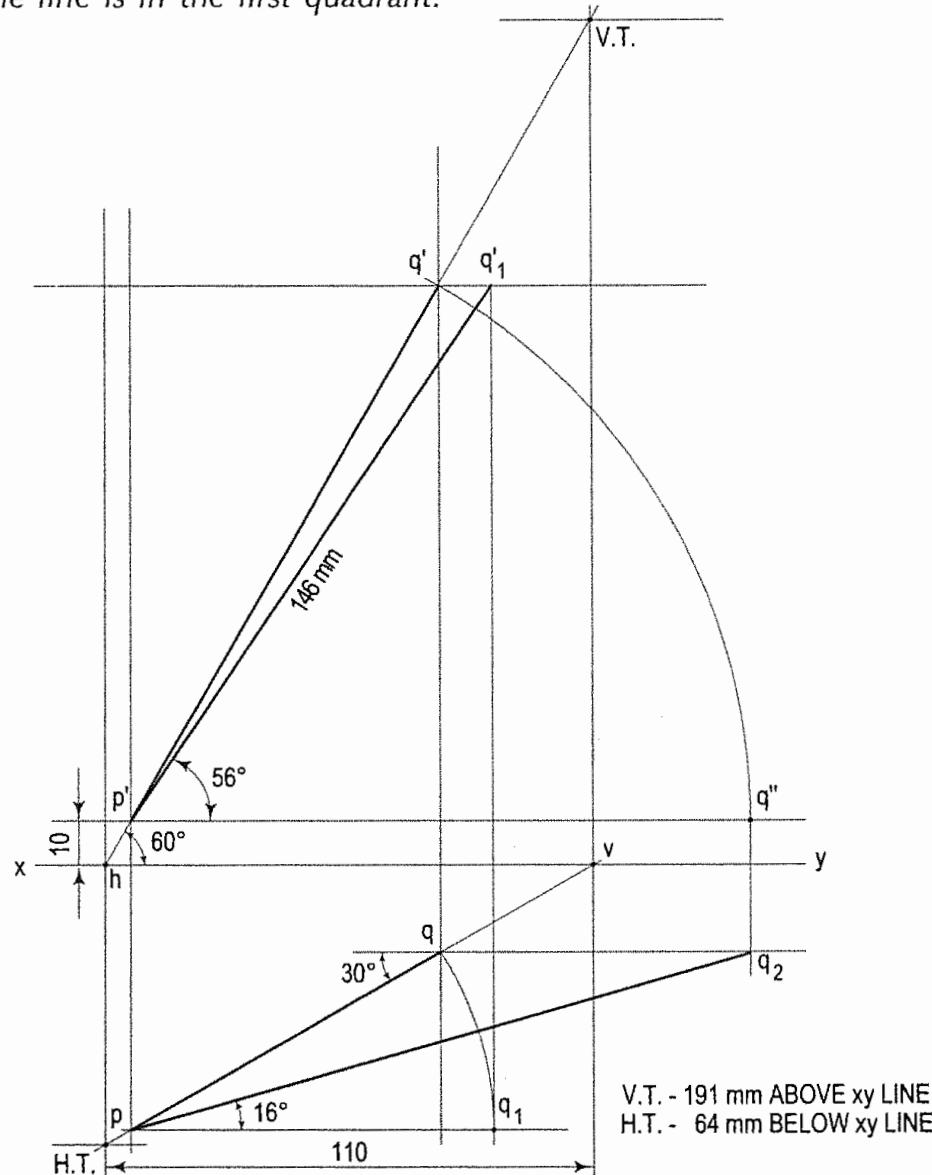


FIG. 10-53

- Draw two vertical lines 110 mm apart representing projectors through the traces of the line. Mark intersection of these projectors h and v on xy as shown.
- From v and h , draw lines at 30° and 60° .
- Mark p' , 10 mm above xy . Draw two vertical projectors at 70 mm apart keeping equal distance from the projectors through traces. $p'q'$ and pq represent front view and top view of the line as shown.
- Keeping p fixed, turn pq to position pq_1 . From q_1 , draw a vertical projector intersecting the path of q' at q'_1 . Join $p'q'_1$. This is the true length of the line. Measure angle $q''p'q'_1$ with the horizontal line as shown. This is the angle made by the line with the H.P. Similarly rotate $p'q'$ making it parallel to xy as shown.

Draw a vertical projector from q'' to intersect the path of q at q_2 . Measure the angle q_1pq_2 with the horizontal line. This is the angle made by the line with V.P.

Note: Problem 10-29 and problem 10-31 are solved by using auxiliary plane method.

Problem 10-29. Two pipes PQ and RS seem to intersect at a' and a in front view and top view as shown in fig. 10-54. The point A is 400 mm above H.P. and 300 mm in front of a wall.

Neglecting the thickness of the pipes, determine the clearance between the pipes.

Refer to fig. 10-54.

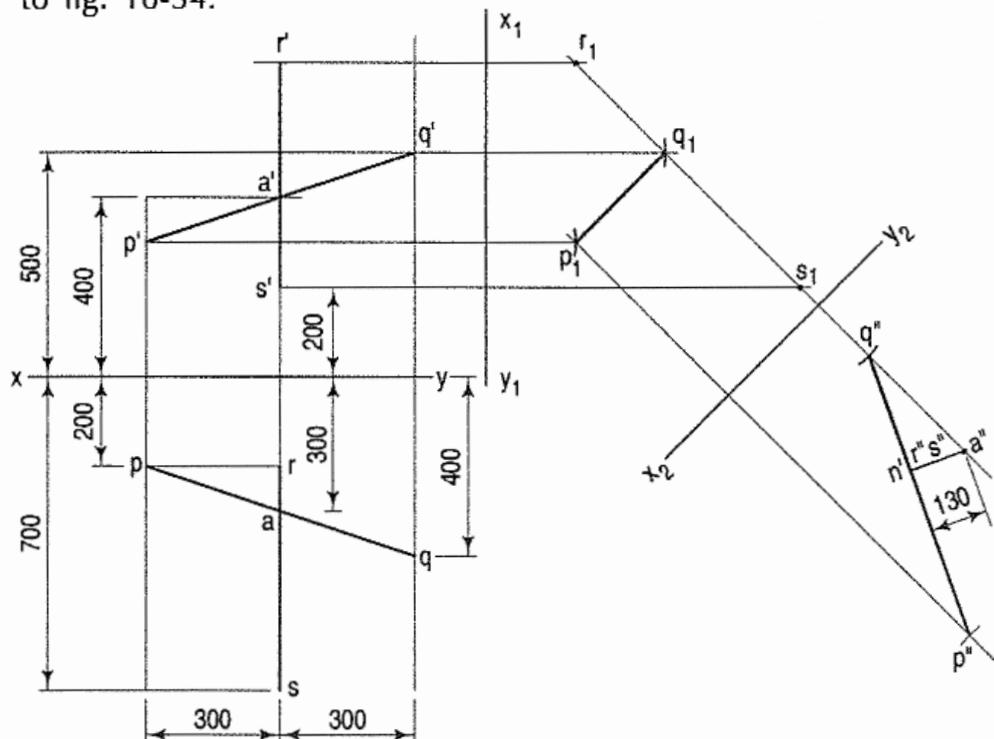


FIG. 10-54

- Draw the projections of pipes treating them as lines.
- Draw x_1y_1 perpendicular to the line xy and project auxiliary top view. Note that the distances of p_1 , q_1 , r_1 and s_1 from x_1y_1 are equal to the distances of p , q , r and s from xy .

(iii) Mark an auxiliary reference line x_2y_2 perpendicular to the line r_1s_1 for obtaining the point view of the line. Draw an auxiliary front view as shown. Note that the distances of p'' , q'' , r'' and s'' from x_2y_2 are equal to the distances of p' , q' , r' and s' from x_1y_1 .

(iv) $a'' n'$ represents clearance between two pipes which is approximately 130 mm.

Problem 10-30. The end projectors of line AB are 22 mm apart. A is 12 mm in front of the V.P. and 12 mm above the H.P. The point B 6 mm in front of the V.P. and 40 mm above the H.P. Locate the H.T. and the V.T. of the line and also determine its inclinations with the V.P. and the H.P.

If the line AB is shifted to II, III and IV quadrants as shown in fig. 10-55 (assume that the distances of A and B from the projection-planes are same as the first quadrant), draw the projections of line and locate the traces.

The solution of first part of problem is shown in pictorial view. For the second part of the problem, the locations of the line in the respective quadrants are shown in fig. 10-55.

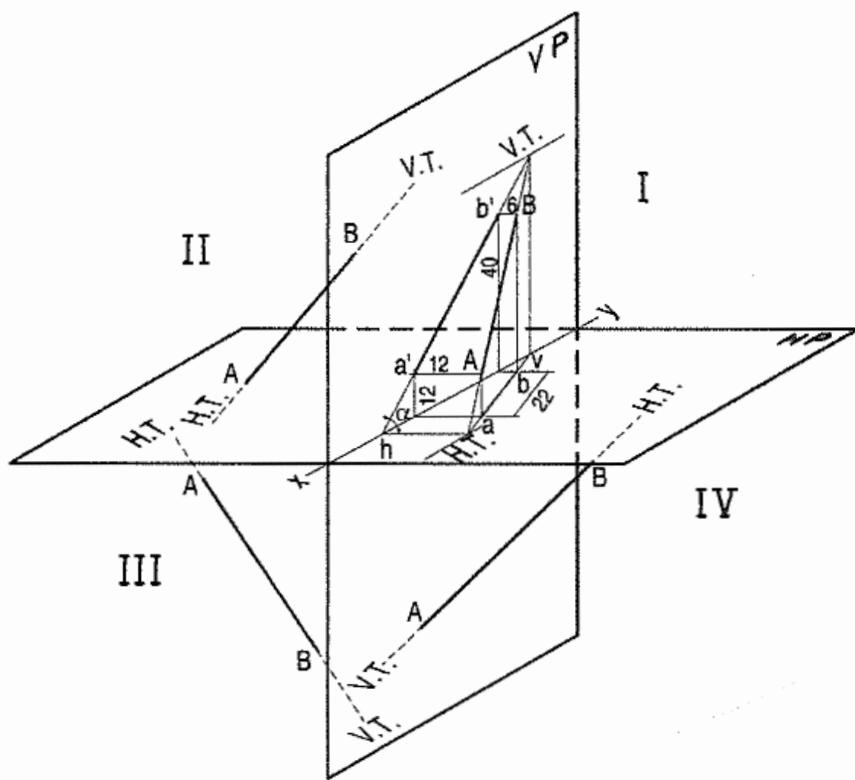


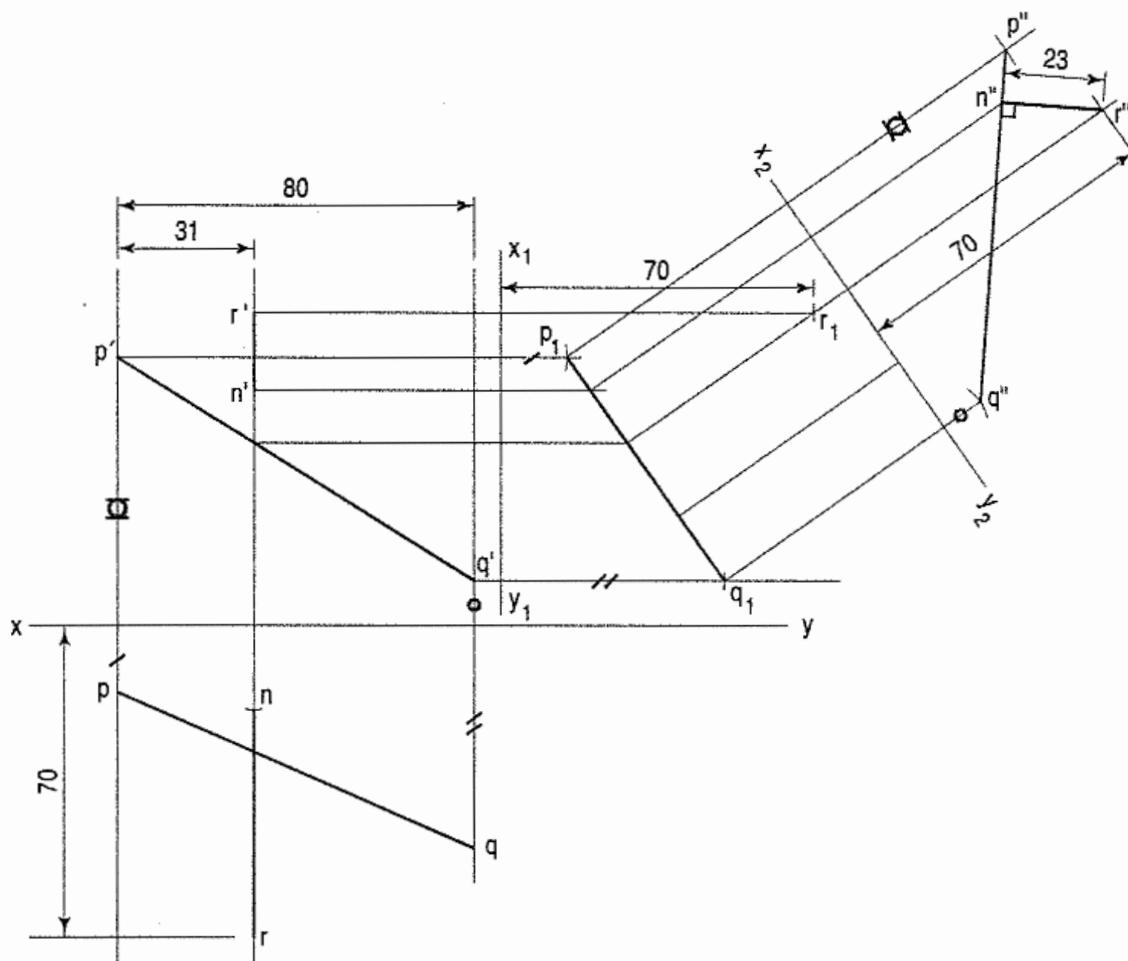
FIG. 10-55

Students are advised to draw the orthographic projections of the line for the respective quadrants.

Problem 10-31. The end projectors of line PQ are 80 mm apart. The end P is 15 mm in front of the V.P. and 60 mm above the H.P. While Q is 50 mm in front of the V.P. and 10 mm above the H.P. A point R is situated on the projector through P measured towards the projector of Q . The point R is 70 mm in front of the V.P. and above the H.P. A perpendicular is drawn from R on PQ . Draw its projections.

Refer to fig. 10-56.

- (i) Draw two end projectors of the line PQ at 80 mm apart.
- (ii) Mark the front view p' and the top view p at 60 mm and 15 mm from xy on the end projector of P . Similarly mark q' and q at 10 mm and 50 mm from xy on the end projector of Q .
- (iii) Draw a vertical line at 31 mm away from $p'p$ towards $q'q$. Mark the position of R in the top view r and the front view r' at 70 mm from xy as shown.
- (iv) Draw x_1y_1 perpendicular to xy as shown. Draw projectors from p' , q' and r' on x_1y_1 . Transfer the distances 15 mm, 50 mm and 70 mm of p , q and r from xy to the new top view p_1 , q_1 and r_1 from x_1y_1 .
- (v) Draw another reference line x_2y_2 for the new front view parallel to p_1q_1 . Transfer the distances 60 mm, 10 mm and 70 mm of p' , q' and r' from xy to the new front view p'' , q'' and r'' from x_2y_2 .
- (vi) From r'' , draw perpendicular to $p''q''$ intersecting at n'' as shown which is measured as 23 mm.



$$n''r'' = 23 \text{ mm}$$

FIG. 10-56

Problem 10-32. The end projectors of a line PQ are 65 mm apart. P is 25 mm behind the V.P. and 30 mm below the H.P. The point Q is 40 mm above the H.P. and 15 mm in front of the V.P. Find the third point C in the H.P. and in front of the V.P. such that its distance from a point P is 45 mm and that from Q is 60 mm. Determine inclinations of PQ with the H.P. and the V.P.

Refer to fig. 10-57.

- Draw the end projectors of the line PQ 65 mm apart. Mark the projection of ends P and Q according to given distances. $p'q'$ and pq are the front view and the top view respectively.
- Mark the front view of point c in the line xy because it is lying in the H.P. Extend the projection line from c' to c on the top view pq .
- Keeping c fixed, turn cp to cp_2 making parallel to xy .
- Project p_2 to p'' . Join $c'p''$. This is the true distance of the line cp . Similarly turn cq to cq_2 making it parallel to xy . Project q_2 to q'' join $c'q''$. This represents the true distance of the line cq .
- Measure angles θ and ϕ as shown.

Problem 10-33. (fig. 10-58): The distance between the end-projectors of a line PQ is 50 mm. A point P is 29 mm above H.P. and 20.71 mm behind V.P. While a point Q is 42 mm below H.P. and 30 mm in front of V.P.

Draw the projections of the line and determine the true length and the true inclinations of line with H.P. and V.P.

- Draw the end-projectors 50 mm apart. Mark p' and p , the front view and the top view of the end P at 29 mm and 20.71 mm respectively. Similarly mark q and q' at 42 mm and 30 mm on the end-projector Q as shown. Join $p'q'$ and pq . They are intersecting xy at r . Mark paths of p' , q' , p and q parallel to xy .
- With centre r and radius rp' , draw an arc intersecting xy at p'' . Through p'' , draw projector cutting the path of p at p_2 . Similarly with the same centre and radius rq' , draw an arc intersecting xy at q'' . Through q'' , draw projector cutting the path of q at q_2 . Join p_2q_2 which represents true length. Measure ϕ angle made by p_2q_2 with xy at r . This is an angle made by the line with V.P.
- Similarly centre as r and radii rp and rq , draw arcs intersecting xy at p_1 and q_1 respectively. Through p_1 and q_1 draw projectors to intersect the paths of p' and q' at p''' and q''' . Join $p'''q'''$. Measure θ angle made by it with xy . This is an angle by line with H.P.

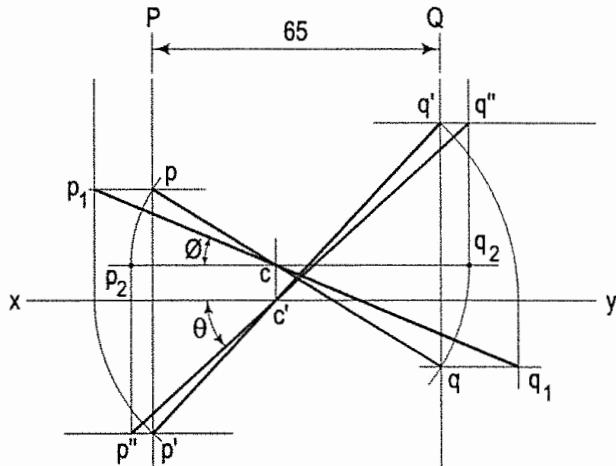
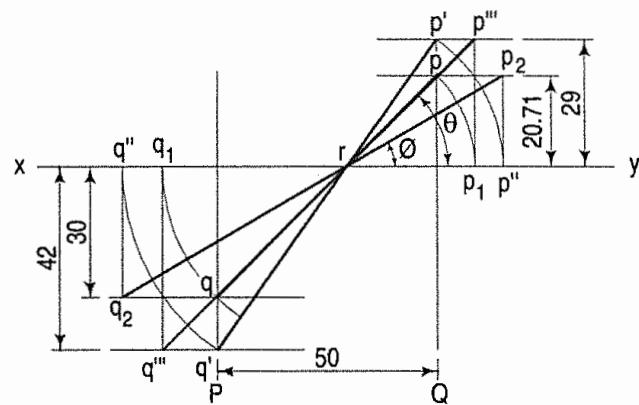


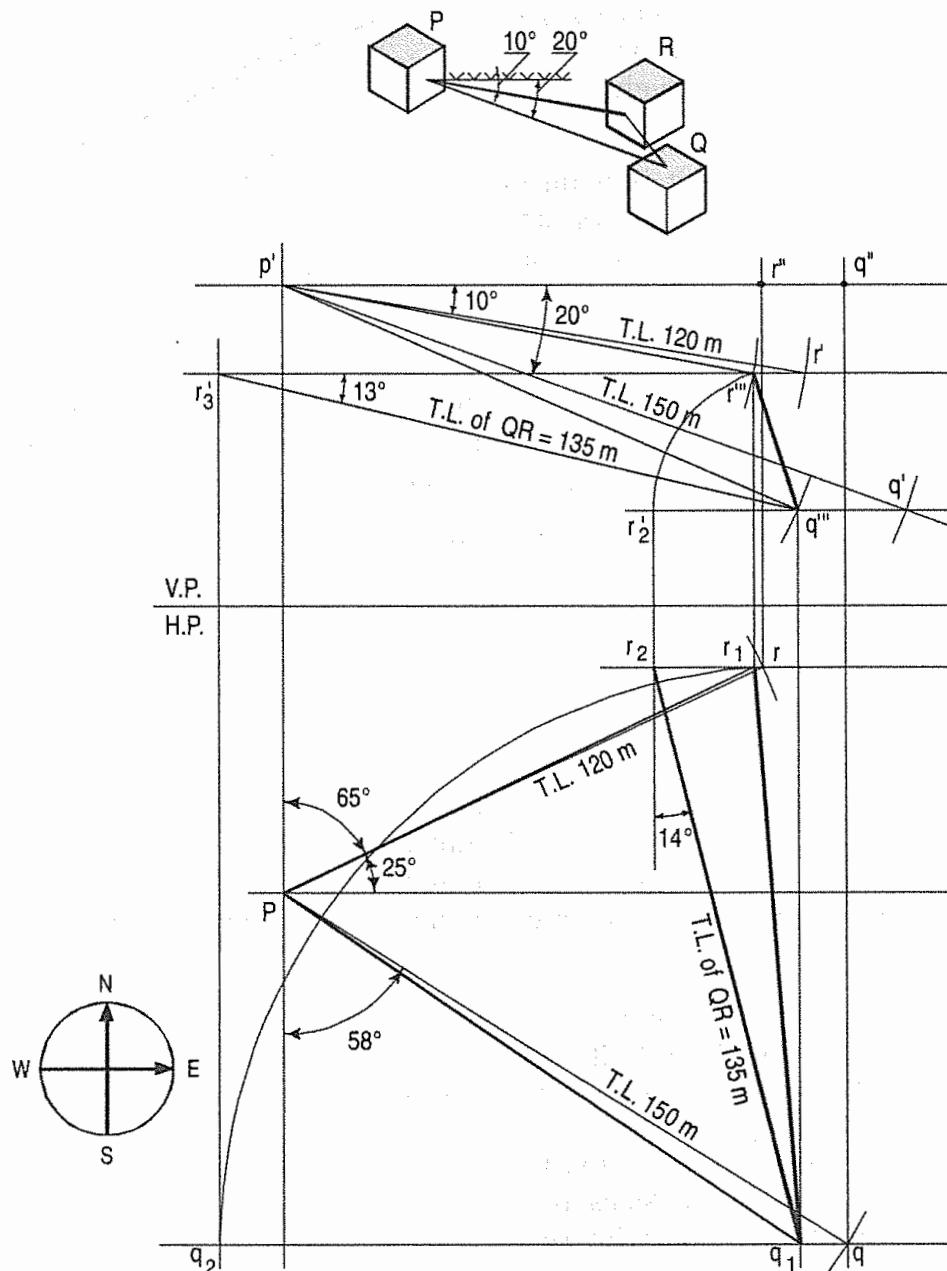
FIG. 10-57



$$\angle \theta = 45^\circ; \angle \phi = 30^\circ, \\ \text{true length} = 100 \text{ mm}$$

FIG. 10-58

Problem 10-34. (fig. 10-59): Two pipes emerge from a common tank. The pipe PQ is 150 metre long and bears $S\ 58^\circ E$ on a downward slope of 20° . The pipe PR is 120 metre long and bears $N\ 65^\circ E$ on a downward slope of 10° . Determine the length of pipe required to connect Q and R . Take scale 1 mm = 1 m.



T.L. of $QR = 135\text{ m}$. S 13 E at downward slope of 13°

FIG. 10-59

- Mark position of P in the top view and the front view as shown.
- At P , draw a line pq 150 mm at 58° with the vertical measuring anti-clockwise as the angle is required to measure from south to east. Similarly draw a line pr 120 mm at 25° measured from east to north.
- Draw the horizontal lines from q and r .
- From point p' , draw a line $p'q'$ 150 mm and $p'r'$ 120 mm at 20° and 10° with the horizontal lines. Draw horizontal lines from r' and q' .

- (v) Draw a projector from q to intersect the horizontal line drawn from p' at q'' . $p'q''$ is the front view of PQ . With p' as centre and radius equal to $p'q''$, draw an arc intersecting the path of q' at q''' . Join $p'q'''$ and draw a projector from q''' cutting the path of q at q_1 . Then $p'q'''$ and pq_1 are required projection. Similarly obtain the projection of $p'r'''$ and pr_1 for the line PR as shown. Join $r'''q'''$ and r_1q_1 . With centre q''' and a radius $q'''r'''$, draw an arc intersecting the path of q'' at r'_2 . Draw a projector from r'_2 cutting the path of r at r_2 . Join q_1r_2 and measure its true length and angle.
- (vi) With centre q_1 and radius q_1r_1 , draw an arc intersecting the path of q at q_2 . Draw a projector from q_2 cutting the path of r' at r'_3 . Join $q''r'_3$ and measure its true length and angle.
- (vii) $QR = 135$ m is the measured true length and $S 13 E$ at downward slope of 13° is the measured angle.

Note: Depression or front view angles are seen in front view while bearing angles are seen in top view.

Problem 10-35. (fig. 10-60): The projectors of two points P and Q are 70 mm apart. The point P is 25 mm behind the V.P. and 30 mm below the H.P. The point Q is 40 mm above the H.P. and 15 mm infront of the V.P. Find the third point S which is in the H.P. and infront of the V.P. such that its distance from point P is 90 mm and that from Q is 60 mm.

- Draw xy line.
- Draw the projectors of P and Q 70 mm apart.
- Mark on the projector of the point p the front view and top view of the point p at 25 mm and 30 mm from xy respectively, say p' and p . Similarly on the projector of the point Q , mark the front view q' and the top view q for given distance from the xy .
- Join $p'q'$ and pq . They are the front view and the top view of the line PQ .
- The front view $p'q'$ intersects xy line at m' . Taking m' as centre, $m'p'$ and $m'q'$ as radii rotate so that $p'q'$ becomes parallel to the xy line or intersect xy line at l and n .
- Draw the projectors from l and n to intersect path of the point p and the point q at p_1 and q_1 . Join p_1 and q_1 , it represents true length of the line PQ .
- Now p_1 as centre and 90 mm radius, draw the arc. Take q_1 as centre and 60 mm as radius, draw the another arc so that it intersects previous arc at s . From s draw the projector to intersect xy line at s' , which is front view of s .

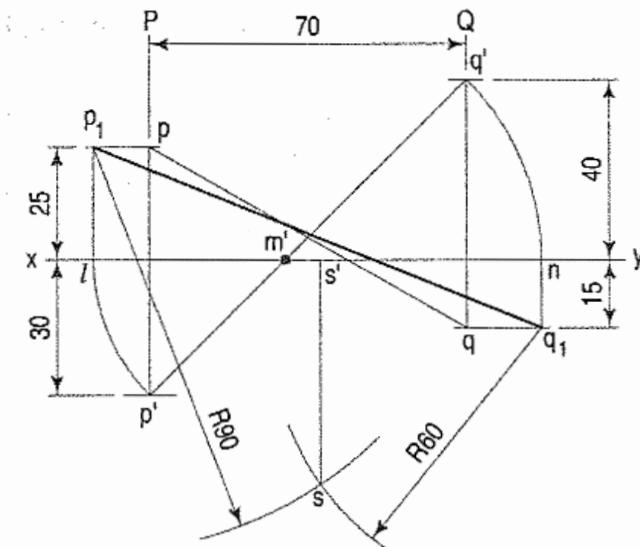


FIG. 10-60

Problem 10-36. (fig. 10-61): Two unequal lines PQ and PR meeting at P makes an angle of 130° between them in their front view and top view. Line PQ is parallel and 6 mm away from both the principal planes. Assume the front view length of PQ and PR 50 mm and 60 mm respectively. Determine real angle between them.

- Mark reference line xy . Draw at convenient distance above and below two parallel lines to the xy representing the front view and top view of PQ as shown. At the point P' construct angle of 130° with the length of $p'q'$ and $p'r'$ of 50 mm and 60 mm respectively.
- Draw projector from r' to intersect the line at angle of 130° at p in the top view of pq .
- Make pr and qr parallel to xy line and project above xy to intersect the path of r' at r'' and r''' respectively. Join $p'r'''$ and $q'r''$. They are true length of the line PR and the line QR .
- Construct triangle with sides $PQ = 50\text{ mm}$, $PR = p'r'''$ and $QR = q'r''$ as shown. Measure angle $\angle QPR$ equal and it is 120° approximately.

Problem 10-37. (fig. 10-62): The distance between end projectors of a straight line AB is 80 mm . The point A is 15 mm below the H.P. and 20 mm in front of the V.P. B is 60 mm behind the V.P. Draw projections of the line if it is inclined at 45° to V.P. Determine also true length and inclination with the H.P.

- Draw xy line and two vertical parallel lines 80 mm apart showing the end projectors of the line AB .
- Mark the point a' and the point a 15 mm and 20 mm below xy line on the projector of A .
- Mark the point b above xy line at distance of 60 mm on the projector of B . Join the point a and the point b .
- If we measure angle of ab with xy line it is 45° which is also inclination of the line AB with V.P.
- Therefore ab shows true length. From a' draw parallel line to xy to intersect the projector of B at b' . It is front view of the line AB . Measure angle of $a'b$ with xy line. It is 43° with xy line.

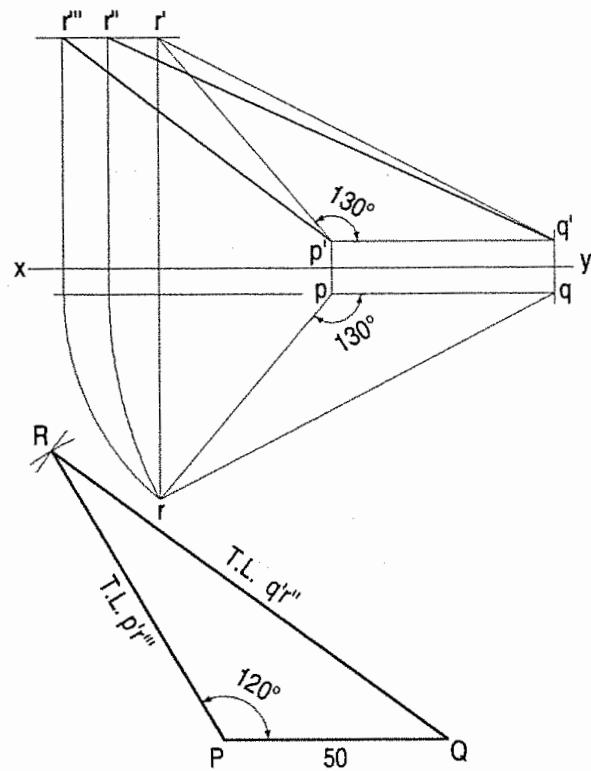


FIG. 10-61

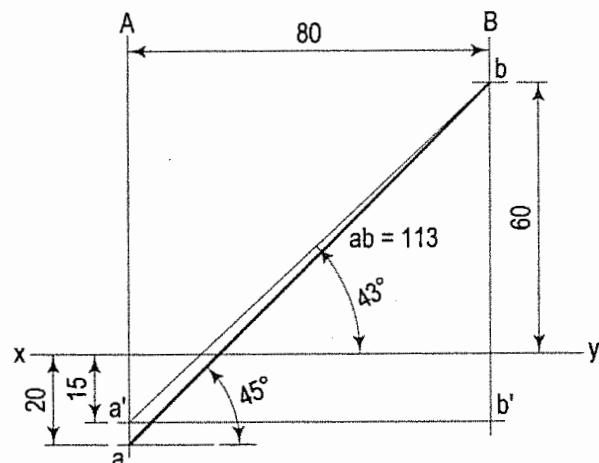


FIG. 10-62

Problem 10-38. (fig. 10-63): Two mangoes on a tree are respectively 2.0 m and 3.5 m above the ground and 1.5 m and 2.0 m away from 0.2 m thick compound wall, but on the opposite sides of it. The distance between the mangoes, measured along the ground and parallel to the wall is 2.7 m. Determine the real distance between the mangoes. Take scale 1 m = 10 mm.

- Pictorial view is shown for understanding purpose.
- Draw reference line (ground line) xy . Mark two parallel lines as end-projectors at 2.7 m (27 mm) apart.
- Let P be mango behind the wall and Q be in front of the wall.
- Mark P' and P along projector P to given distances. Similarly mark q' and q for given distances on the projector Q .
- Join the point p and the point q , the point p' and the point q' . Then pq and $p'q'$ are projections of PQ .
- Rotates pq taking q as centre, make it parallel to the ground line xy , intersecting at the point P_1 . Draw the projector from P_1 . Draw line parallel to the ground line xy from p' , intersecting at p'' .
- Join $p''q'$. The line $p''q'$ is true distance between mangoes P and Q . It is approximately 4.8 m.

Problem 10-39. (fig. 10-64): The front view of straight line AB is 60 mm long and is inclined at 60° to the reference line xy . The end point A is 15 mm above H.P. and 20 mm in front of V.P. Draw the projections of a line AB if it is inclined at 45° to the V.P. and is situated in the first quadrant (Dihedral angle). Determine its true length, and inclination with the H.P.

See fig. 10-64 which is self explanatory.

Problem 10-40. (fig. 10-65): A room $6 \text{ m} \times 5 \text{ m} \times 4 \text{ m}$ high has a light-bracket above the centre of the longer wall and 1 m below the ceiling. The light bulb is 0.3 m away from the wall. The switch for the light is on an adjacent wall, 1.5 m above the floor and 1 m from the other longer wall. Determine graphically the shortest distance between the bulb and the switch.

- Draw to scale 1 m = 10 mm front view and a top view of the room.
- Mark mid point of longer wall (i.e. 6 m), say 6 mm.

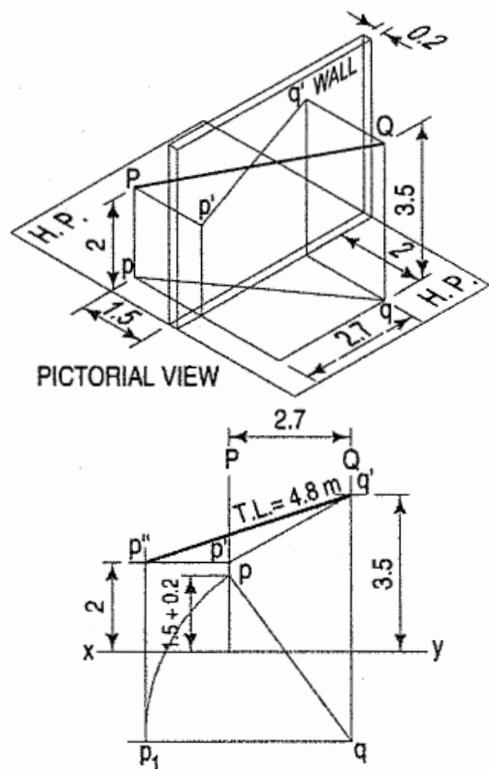


FIG. 10-63

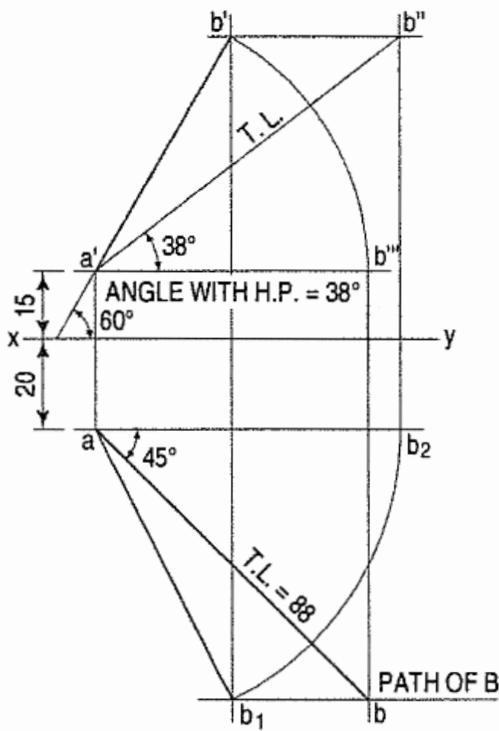


FIG. 10-64

- (iii) Mark the point in the V.P. at distance 1 m (= 10 mm) from midpoint m . It is the front view of the light bulb, say b' .
- (iv) From xy line at 0.3 m (i.e. 3 mm) on the projector passing through b' mark the point b (top view).
- (v) Consider adjacent wall right side. Mark s' front view of the switch at 1.5 m (i.e. 15 mm) above xy line and on the same line mark s (top view of switch) at 4 m away from xy .
- (vi) Join $b's'$ and bs which are the front view and the top view of the line.
- (vii) s as centre and bs as radius, draw the arc to intersect a parallel line passing through s at b_1 . From b_1 , draw projector to intersect a parallel line to xy drawn from b' at b'' . Join $s'b''$.
- (viii) Line $s'b''$ shows shortest distance between the switch and the bulb. Here it is approximately 5 m.

Problem 10-41. (fig. 10-66): The distance between end projectors of a line AB are 60 mm apart, while the projectors passing from H.T. and V.T. are 90 mm apart. The H.T. is 35 mm behind the V.P., the V.T. is 55 mm below the H.P. The point A is 7 mm behind the V.P. Find graphically true length of the line and inclinations with the H.P. and the V.P.

See fig. 10-66 which is self-explanatory.

Problem 10-42. (fig. 10-67): A line CD is inclined at 30° to the H.P. and it is in the first quadrant. The end C is 15 mm above the H.P. while the end D is in the V.P. The mid point M of the line is 40 mm above H.P. The distance between the end projectors of the line is 70 mm. Draw the projections of the line CD and the mid point M . Determine graphically the length of front view and top view and true length of the line. Also determine inclination of the line with the V.P.

- (i) Draw xy line.
- (ii) Draw two projectors 70 mm, apart.
- (iii) On the projector of C , mark c' at 15 mm above xy line.
- (iv) Draw a line parallel to xy at 40 mm, to represent the path of mid-point M .
- (v) From c' draw a 30° inclined line t cut the path of mid-point at m' . $c'm'$ is half true length. With m' as centre and radius equal to $c'm'$, draw an arc cutting the 30° inclined line at d' . $c'd'$ is true length. From d' , draw a line parallel to xy , to represent path of D in front view.

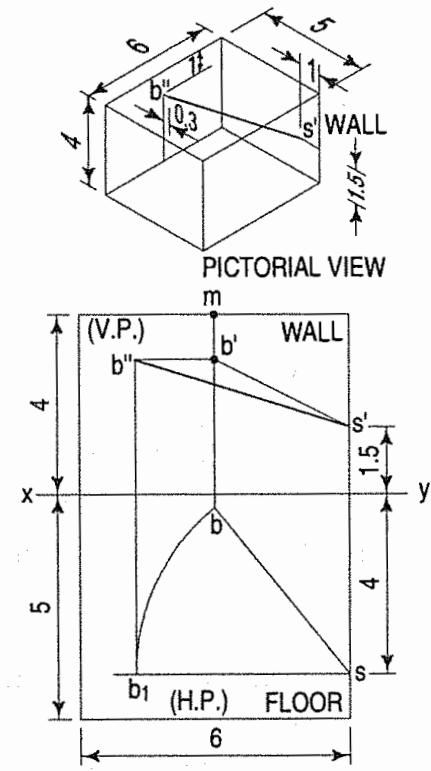
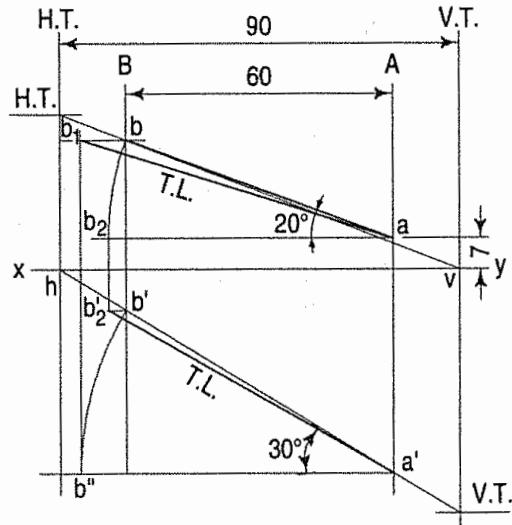


FIG. 10-65



True length = 74;
Angle with H.P. = 30° ;
Angle with V.P. = 20°

FIG. 10-66

- (vi) The path of D will intersect the projector of D at d'' . Join $c'd''$. It is front view of CD .

- (vii) With c' as centre and radius equal to $c'd''$, draw an arc cutting a line drawn parallel to xy from c' at d''' . Project d''' to cut a line drawn with c as centre and $c'd'$ (true length) as radius, at d_3 . From d_3 , draw a line parallel to xy , representing path of D in top view.

- (viii) Project d'' to cut path of D in top view at d_4 . Join cd_4 . It is top view of CD .

- (ix) The results are shown in fig. 10-67.

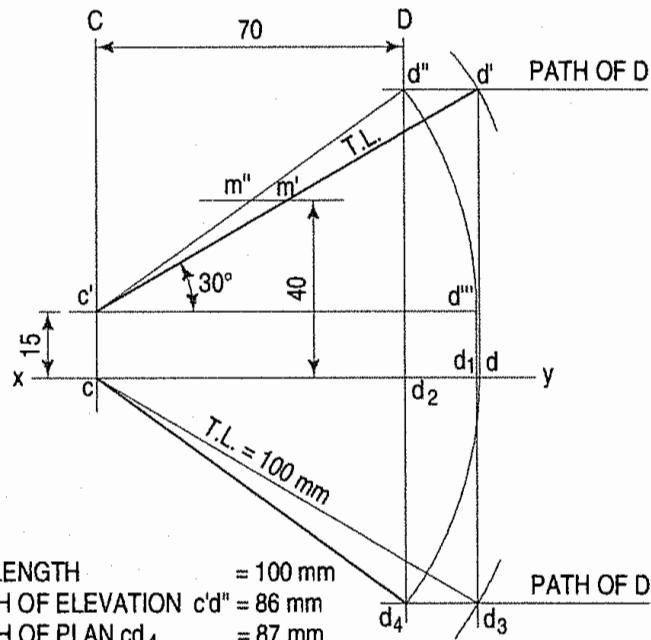


FIG. 10-67

EXERCISES 10(b)

- A line AB , 75 mm long, is inclined at 45° to the H.P. and 30° to the V.P. Its end B is in the H.P. and 40 mm in front of the V.P. Draw its projections and determine its traces.
- Draw the projections of a line AB , 90 mm long, its mid-point M being 50 mm above the H.P. and 40 mm in front of the V.P. The end A is 20 mm above the H.P. and 10 mm in front of the V.P. Show the traces and the inclinations of the line with the H.P. and the V.P.
- The front view of a 125 mm long line PQ measures 75 mm and its top view measures 100 mm. Its end Q and the mid-point M are in the first quadrant, M being 20 mm from both the planes. Draw the projections of the line PQ .
- A line AB , 75 mm long is in the second quadrant with the end A in the H.P. and the end B in the V.P. The line is inclined at 30° to the H.P. and at 45° to the V.P. Draw the projections of AB and determine its traces.
- The end A of a line AB is in the H.P. and 25 mm behind the V.P. The end B is in the V.P. and 50 mm above the H.P. The distance between the end projectors is 75 mm. Draw the projections of AB and determine its true length, traces and inclinations with the two planes.
- The top view of a 75 mm long line CD measures 50 mm. C is 50 mm in front of the V.P. and 15 mm below the H.P. D is 15 mm in front of the V.P. and is above the H.P. Draw the front view of CD and find its inclinations with the H.P. and the V.P. Show also its traces.
- A line PQ , 100 mm long, is inclined at 45° to the H.P. and at 30° to the V.P. Its end P is in the second quadrant and Q is in the fourth quadrant. A point R on PQ , 40 mm from P is in both the planes. Draw the projections of PQ .
- A line AB , 65 mm long, has its end A in the H.P. and 15 mm in front of the V.P. The end B is in the third quadrant. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.

9. The front view of a line AB measures 65 mm and makes an angle of 45° with xy . A is in the H.P. and the V.T. of the line is 15 mm below the H.P. The line is inclined at 30° to the V.P. Draw the projections of AB and find its true length and inclination with the H.P. Also locate its H.T.
10. A room is $4.8 \text{ m} \times 4.2 \text{ m} \times 3.6 \text{ m}$ high. Determine graphically the distance between a top corner and the bottom corner diagonally opposite to it.
11. A line AB is in the first quadrant. Its end A and B are 20 mm and 60 mm in front of the V.P. respectively. The distance between the end projectors is 75 mm. The line is inclined at 30° to the H.P. and its H.T. is 10 mm above xy . Draw the projections of AB and determine its true length and the V.T.
12. Two oranges on a tree are respectively 1.8 m and 3 m above the ground, and 1.2 m and 2.1 m from a 0.3 m thick wall, but on the opposite sides of it. The distance between the oranges, measured along the ground and parallel to the wall is 2.7 m. Determine the real distance between the oranges.
13. Draw an isosceles triangle abc of base ab 40 mm and altitude 75 mm with a in xy and ab inclined at 45° to xy . The figure is the top view of a triangle whose corners A , B and C are respectively 75 mm, 25 mm and 50 mm above the H.P. Determine the true shape of the triangle and the inclination of the side AB with the two planes.
14. Three points A , B and C are 7.5 m above the ground level, on the ground level and 9 m below the ground level respectively. They are connected by roads with each other and are seen at angles of depression of 10° , 15° and 30° respectively from a point O on a hill 30 m above the ground level. A is due north-east, B is due north and C is due south-east of O . Find the lengths of the connecting roads.
15. A pipe-line from a point A , running due north-east has a downward gradient of 1 in 5. Another point B is 12 m away from and due east of A and on the same level. Find the length and slope of a pipe-line from B which runs due 15° east of north and meets the pipe-line from A .
16. The guy ropes of two poles 12 m apart, are attached to a point 15 m above the ground on the corner of a building. The points of attachment on the poles are 7.5 m and 4.5 m above the ground and the ropes make 45° and 30° respectively with the ground. Draw the projections and find the distances of the poles from the building and the lengths of the guy ropes.
17. A plate chimney, 18 m high 0.9 m diameter is supported by two sets of three guy wires each, as shown in fig. 10-68. One set is attached at 3 m from the top and anchored 6 m above the ground level. The other set is fixed to the chimney at its mid-height and anchored on the ground. Determine the length and slope with the ground, of one of the wires from each set.
18. The projectors drawn from the H.T. and the V.T. of a straight line AB are 80 mm apart while those drawn from its ends are 50 mm apart. The H.T. is 35 mm in front of the V.P., the V.T. is 55 mm above the H.P. and the end A is 10 mm above the H.P. Draw the projections of AB and determine its length and inclinations with the reference planes.

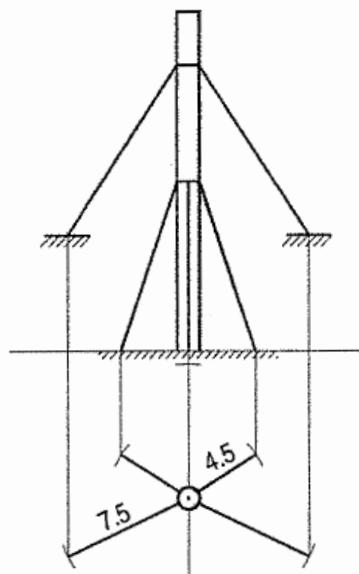


FIG. 10-68

19. Three guy ropes AB , CD and EF are tied at points A , C and E on a vertical post 15 m long at heights of 14 m, 12 m and 10 m respectively from the ground. The lower ends of the ropes are tied to hooks at points B , D and F on the ground level. If the points B , D and F lie at the corners of an equilateral triangle of 9 m long sides and if the post is situated at the centre of this triangle, determine graphically the length of each rope and its inclination with the ground. Assume the thickness of the post and the ropes to be equal to that of a line.
20. A line AB , 80 mm long, makes an angle of 60° with the H.P. and lies in an auxiliary vertical plane (A.V.P.), which makes an angle of 45° with the V.P. Its end A is 10 mm away from both the H.P. and the V.P.
Draw the projections of AB and determine (i) its true inclination with the V.P. and (ii) its traces.
21. A line PQ is 75 mm long and lies in an auxiliary inclined plane (A.I.P.) which makes an angle of 45° with the H.P. The front view of the line measures 55 mm and the end P is in the V.P. and 20 mm above the H.P.
Draw the projections of PQ and find (i) its inclinations with both the planes and (ii) its traces.
22. A line AB , 80 mm long, makes an angle of 30° with the V.P. and lies in a plane perpendicular to both the H.P. and the V.P. Its end A is in the H.P. and the end B is in the V.P. Draw its projections and show its traces.
23. The front view of a line makes an angle of 30° with xy . The H.T. of the line is 45 mm in front of the V.P., while its V.T. is 30 mm below the H.P. One end of the line is 10 mm above the H.P. and the other end is 100 mm in front of the V.P.
Draw the projections of the line and determine (i) its true length, and (ii) its inclinations with the H.P. and the V.P.
24. A room is $6 \text{ m} \times 5 \text{ m} \times 3.5 \text{ m}$ high. An electric bracket light is above the centre of the longer wall and 1 m below the ceiling. The bulb is 0.3 m away from the wall. The switch for the light is on an adjacent wall, 1.5 m above the floor and 1 m away from the other longer wall. Find graphically the shortest distance between the bulb and the switch.
25. Three lines oa , ob and oc are respectively 25 mm, 45 mm and 65 mm long, each making 120° angles with the other two and the shortest line being vertical. The figure is the top view of the three rods OA , OB and OC whose ends A , B and C are on the ground, while O is 100 mm above it. Draw the front view and determine the length of each rod and its inclination with the ground.
26. The projectors of the ends of a line PQ are 90 mm apart. P is 20 mm above the H.P. while Q is 45 mm behind the V.P. The H.T. and the V.T. of the line coincide with each other on xy , between the two end projectors and 35 mm away from the projector of the end P . Draw the projections of PQ and determine its true length and inclinations with the two planes.
27. A person on the top of a tower 30 m high, which rises from a horizontal plane, observes the angles of depression (below the horizon) of two objects H and K on the plane to be 15° and 25° , the direction of H and K from the tower being due north and due west respectively. Draw the top view to a scale of 1 mm = 0.5 m showing the relative positions of the person and the two objects. Measure and state in metres the distance between H and K .

28. Two pegs *A* and *B* are fixed in each of the two adjacent side walls (of a rectangular room) which meet in a corner. Peg *A* is 1.5 m above the floor, 1.2 m from the side wall and is protruding 0.3 m from the wall. Peg *B* is 2 m above the floor, 1 m from the other side wall and is also protruding 0.3 m from the wall. Find the distance between the ends of the pegs.
29. Two objects *A* and *B*, 10 m above and 7 m below the ground level respectively, are observed from the top of a tower 35 m high from the ground. Both the objects make an angle of depression of 45° with the horizon. The horizontal distance between *A* and *B* is 20 m. Draw to scale 1:250, the projections of the objects and the tower and find (a) the true distance between *A* and *B*, and (b) the angle of depression of another object *C* situated on the ground midway between *A* and *B*.
30. A room measures 8 m long, 5 m wide and 4 m high. An electric point hangs in the centre of the ceiling and 1 m below it. A thin straight wire connects the point to a switch kept in one of the corners of the room and 2 m above the floor. Draw the projections of the wire, and find the length of the wire and its slope-angle with the floor.
31. A rectangular tank 4 m high is strengthened by four stay rods one at each corner, connecting the top corner to a point in the bottom 0.7 m and 1.2 m from the sides of the tank. Find graphically the length of the rod required and the angle it makes with the surface of the tank.
32. Three vertical poles *AB*, *CD* and *EF* are respectively 5, 8 and 12 metres long. Their ends *B*, *D* and *F* are on the ground and lie at the corners of an equilateral triangle of 10 metres long sides. Determine graphically the distance between the top ends of the poles, viz. *AC*, *CE* and *EA*.
33. The front view of a line *AB* measures 70 mm and makes an angle of 45° with *xy*. *A* is in the H.P. and the V.T. of the line is 15 mm below the H.P. The line is inclined at 30° to the V.P. Draw the projections of *AB*, and find its true length, inclination with the H.P. and its H.T.
34. A line *AB* measures 100 mm. The projectors through its V.T. and the end *A* are 40 mm apart. The point *A* is 30 mm below the H.P. and 20 mm behind the V.P. The V.T. is 10 mm above the H.P. Draw the projections of the line and determine its H.T. and inclinations with the H.P. and the V.P.
35. A horizontal wooden platform is 3.5 m long and 2 m wide. It is suspended from a hook by means of chains attached at its four corners. The hook is situated vertically above the centre of the platform and at a distance of 5 m above it. Determine graphically the length of each chain and the angle which it makes with the platform. Assume the thickness of the platform and the chain to be equal to that of a line. Scale: 10 mm = 0.5 m.
36. A picture frame 2 m wide and 1 m high is to be fixed on a wall railing by two straight wires attached to the top corners. The frame is to make an angle of 40° with the wall and the wires are to be fixed to a hook on the wall on the centre line of the frame and 1.5 m above the railing. Find the length of the wires and the angle between them.
37. The top view of line *AB* measures 60 mm and inclined to reference line at 60° . The end point *A* is 15 mm above the H.P. and 30 mm in front of the V.P. Draw the projections of the line when it is inclined at 45° to the H.P. and is situated in the first quadrant. Find true length and inclination of the line with the V.P. and traces.

Chapter

11



PROJECTIONS ON AUXILIARY PLANES

Engineering Drawing & Design

11-0. INTRODUCTION



Two views of an object, viz. the front view and the top view (projected on the principal planes of projection), are sometimes not sufficient to convey all the information regarding the object. Additional views, called *auxiliary views*, are therefore, projected on other planes known as *auxiliary planes*. These views are often found necessary in technical drawings. Auxiliary views may also be used for determining

- (i) the true length of a line,
- (ii) the point-view of a line,
- (iii) the edge-view of a plane,
- (iv) the true size and form of a plane etc.

They are thus very useful in finding solutions of problems in practical solid geometry.

This chapter deals with the following topics:

1. Types of auxiliary planes and views.
2. Projection of a point on an auxiliary plane.
3. Projections of lines and planes by the use of auxiliary planes.
4. To determine true length of a line.
5. To obtain point-view of a line and edge-view of a plane.
6. To determine true shape of a plane figure.

11-1. TYPES OF AUXILIARY PLANES AND VIEWS



Auxiliary planes are of two types:

- (i) *auxiliary vertical plane* or A.V.P., and
 - (ii) *auxiliary inclined plane* or A.I.P.
- (i) Auxiliary vertical plane is perpendicular to the H.P. and inclined to the V.P. Projection on an A.V.P. is called *auxiliary front view*.
- (ii) Auxiliary inclined plane is perpendicular to the V.P. and inclined to the H.P. Projection on an A.I.P. is called *auxiliary top view*.

The orthographic views of the auxiliary projections are drawn by rotating the auxiliary plane about that principal plane to which it is perpendicular.

11-2. PROJECTION OF A POINT ON AN AUXILIARY PLANE



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 25 for the projection of a point on an auxiliary plane.



(1) Projection of a point on an auxiliary vertical plane:

A point A [fig. 11-1(i)] is situated in front of the V.P. and above the H.P. A.V.P. is the auxiliary vertical plane inclined at an angle α to the V.P. The H.P. and the A.V.P. meet at right angles in the line x_1y_1 .

a' and a are respectively, the front view and the top view of the point A. a'_1 is the auxiliary front view obtained by drawing a projector Aa'_1 perpendicular to the A.V.P. It can be clearly seen that a'_1o_1 (the distance of the auxiliary front view a'_1 from x_1y_1) = $a'o$ (the distance of the front view a' from xy) = Aa (the distance of the point A from the H.P.).

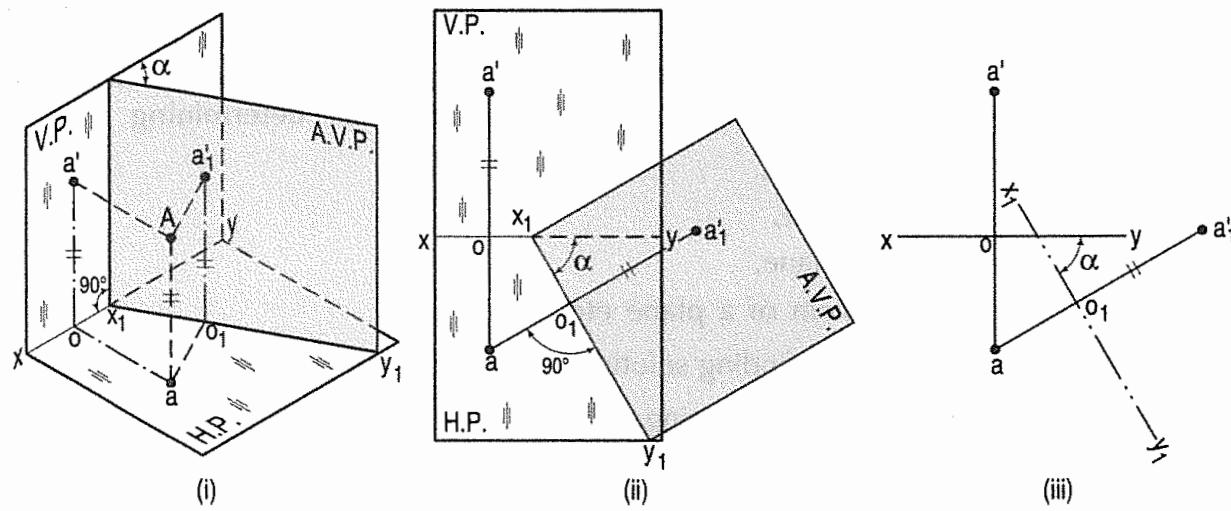


FIG. 11-1

Fig. 11-1(ii) shows the V.P. and the A.V.P. rotated about the H.P. to which they are perpendicular. The line of intersection x_1y_1 between the A.V.P. and the H.P., is inclined at the angles α to xy . The line joining the top view a with the auxiliary front view a'_1 , is at right angles to x_1y_1 and intersects it at o_1 . Note that $a'_1o_1 = a'o$.

To draw the orthographic views [fig. 11-1(iii)], start with the reference line xy and mark the front view a' and the top view a .

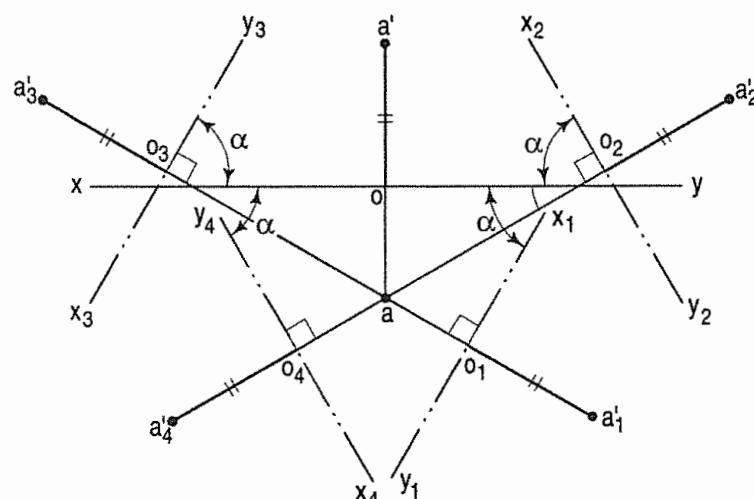


FIG. 11-2

Draw a new reference line x_1y_1 , making the angle α with xy . Through the top view a , draw a projector aa'_1 perpendicular to and intersecting x_1y_1 at o_1 and such that $a'_1o_1 = a'o$. a'_1 is the required auxiliary front view.

The new reference line making the angle α with xy , can be drawn in four different positions, as shown in fig. 11-2 by lines x_1y_1 , x_2y_2 etc. All the front views are projected from the top view a and their distances from their respective reference lines are equal, i.e. $a'_1o_1 = a'_2o_2 \dots = a'o$.

(2) Projection of a point on an auxiliary inclined plane:

A point P [fig. 11-3(i)] is situated above the H.P. and in front of the V.P. A.I.P. is an auxiliary inclined plane making an angle β with the H.P. It meets the V.P. at right angles and in a line x_1y_1 .

p' and p are respectively the front view and the top view of the point P . p_1 is the auxiliary top view obtained by drawing the projector Pp_1 perpendicular to the A.I.P. It can be seen that p_1o_1 (the distance of the auxiliary top view p_1 from x_1y_1) = po (the distance of the top view p from xy) = Pp' (the distance of the point P from the V.P.).

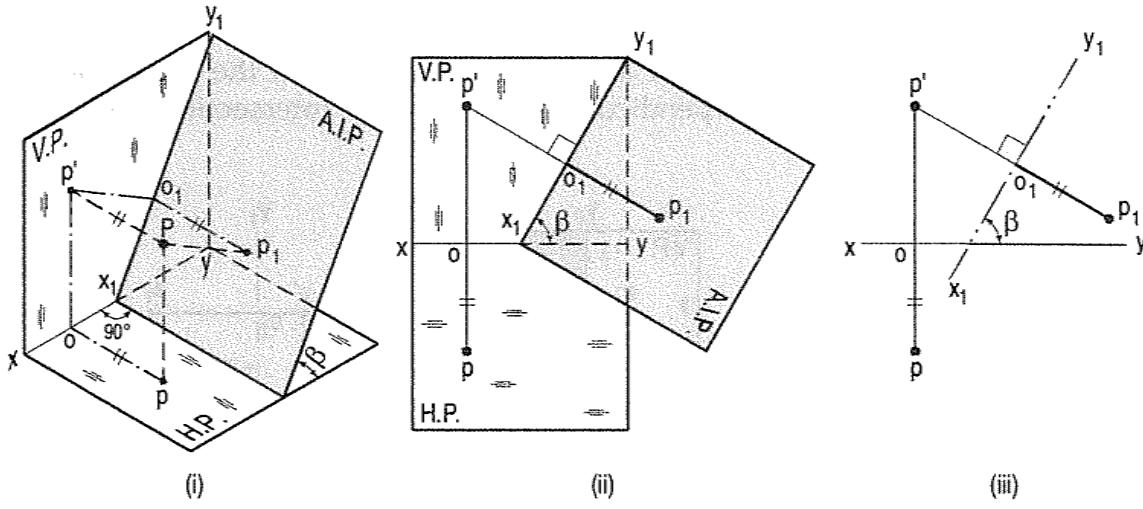


FIG. 11-3

The H.P. and the A.I.P. are then rotated about the V.P. to which they are perpendicular [fig. 11-3(ii)]. x_1y_1 , the line of intersection between the V.P. and the A.I.P. makes the angle β with xy . The line joining the front view p' and the auxiliary top view p_1 is at right angles to x_1y_1 and intersects it at o_1 . Note that $p_1o_1 = po$.

To draw the orthographic views [fig. 11-3(iii)], draw xy and mark p' and p . Draw x_1y_1 making the angle β with xy .

Through the front view p' , draw a projector p'_p_1 perpendicular to and intersecting x_1y_1 at o_1 and such that $p_1o_1 = po$. p_1 is the required auxiliary front view.

In this case also, the new reference line can be drawn in four different positions as shown in fig. 11-4 by lines x_1y_1 , x_2y_2 etc., each inclined at β to xy . All the top views are projected from the front view p' and their distances from their respective reference lines are equal, i.e. $p_1o_1 = p_2o_2 \dots = po$.

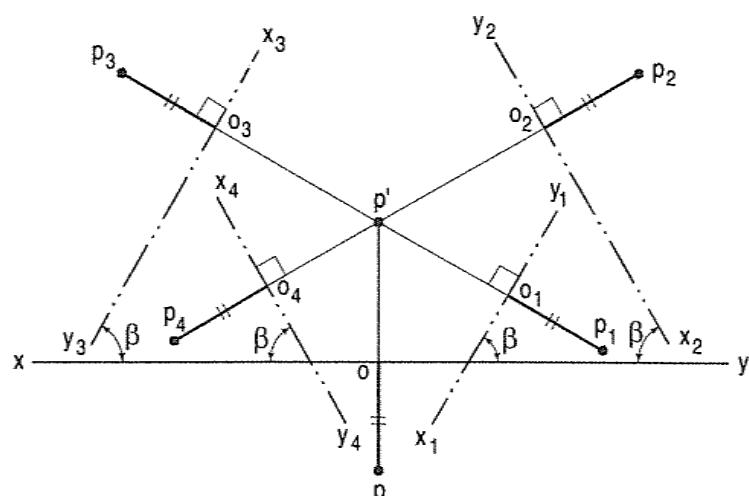


FIG. 11-4

If the inclination of the A.V.P. with the V.P. is increased so that $\alpha = 90^\circ$, the A.V.P. will be perpendicular to both the planes. Similarly, if the inclination of the A.I.P. with the H.P. is increased so that $\beta = 90^\circ$, it will also be perpendicular to both the H.P. and the V.P. This plane is called the profile plane (P.P.). It may be rotated about any one of the two principal planes. The view on this plane can, therefore, be projected from either the top view or the front view and named accordingly.

(3) Projection of a point on an auxiliary plane perpendicular to both the principal planes:

In fig. 11-5(i), A is a point. P.P. is an auxiliary plane perpendicular to the H.P. and the V.P. a_1 is the auxiliary view projected on the P.P. It can be seen that $a_1o_2 = a'o = Aa$ (the distance of A from the H.P.). Also $a_1o_1 = ao = Aa'$ (the distance of A from the V.P.).

Fig. 11-5(ii) shows the P.P. rotated about the V.P. a_1 lies on the projector drawn through the front view a' and perpendicular to the line of intersection between the V.P. and the P.P.

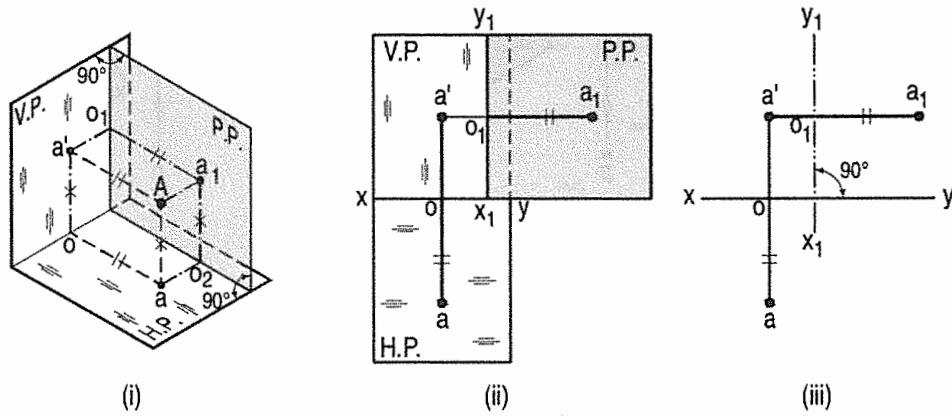


FIG. 11-5

It is thus projected from the front view and hence, called the auxiliary top view. In technical drawings, this view is generally termed as the *side view*, *end view* or *end front view*. Note that $a_1o_1 = ao$.

Note that, when seen from the left, the new reference line and the side view are placed to the right of the front view. When seen from the right, they would be placed to the left of the front view. Thus, the view seen from any side of the front view is placed on its other side.

Fig. 11-6 shows the P.P. rotated about the H.P. The view on the P.P. now lies on the projector drawn through the top view a . Hence, it is called the auxiliary front view. In this case, $a_1o_2 = a'o$.

The orthographic views [fig. 11-5(iii) and fig. 11-6(ii)] in both cases are self-explanatory.

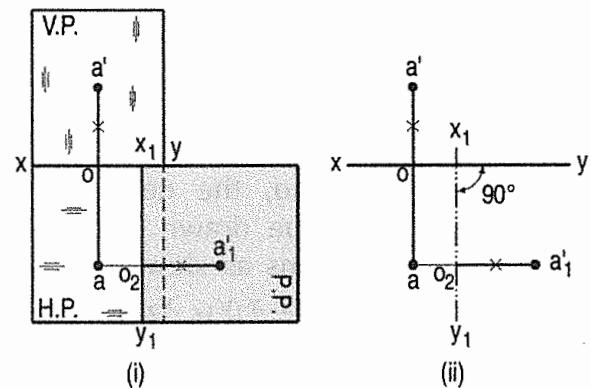


FIG. 11-6

General conclusions:

- (i) The auxiliary top view of a point lies on a line drawn through the front view, perpendicular to the new reference line (x_1y_1) and at a distance from it, equal to the distance of the first top view from its own reference line (xy).
- (ii) The auxiliary front view of a point lies on a line drawn through the top view, perpendicular to the new reference line (x_1y_1) and at a distance from it, equal to the distance of the first front view from its own reference line (xy).
- (iii) The distances of all the front views of the same point (projected from the same top view) from their respective reference lines are equal.
- (iv) The distances of all the top views of the same point (projected from the same front view) from their respective reference lines are equal.

Problem 11-1. (fig. 11-7 and fig. 11-8): The projections of a line AB are given. Draw (i) an auxiliary front view of the line on an A.V.P. inclined at 60° to the V.P. and (ii) an auxiliary top view on an A.I.P. making an angle of 75° with the H.P.

Let ab and $a'b'$ be the given projections.

- (i) Draw a new reference line x_1y_1 , inclined at 60° to xy to represent the A.V.P. (fig. 11-7).
- (ii) Project the auxiliary front view $a'_1b'_1$ from the top view ab , by making $a'_1o'_1$ equal to $a'o_1$, and $b'_1o'_2$ equal to $b'o_2$.
- (iii) Similarly, draw x_2y_2 for the A.I.P. inclined at 75° to xy (fig. 11-8).
- (iv) Project the auxiliary top view a_2b_2 from the front view $a'b'$, making $a_2o''_1$ equal to ao_1 and $b_2o''_2$ equal to bo_2 .

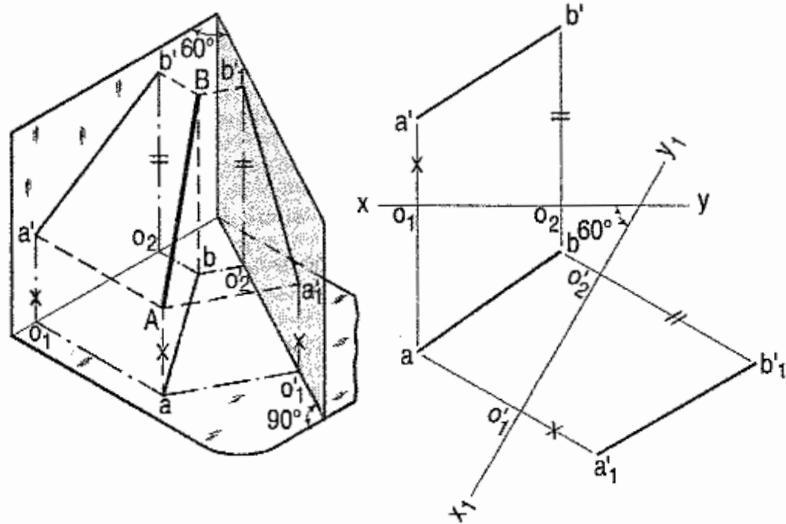


FIG. 11-7

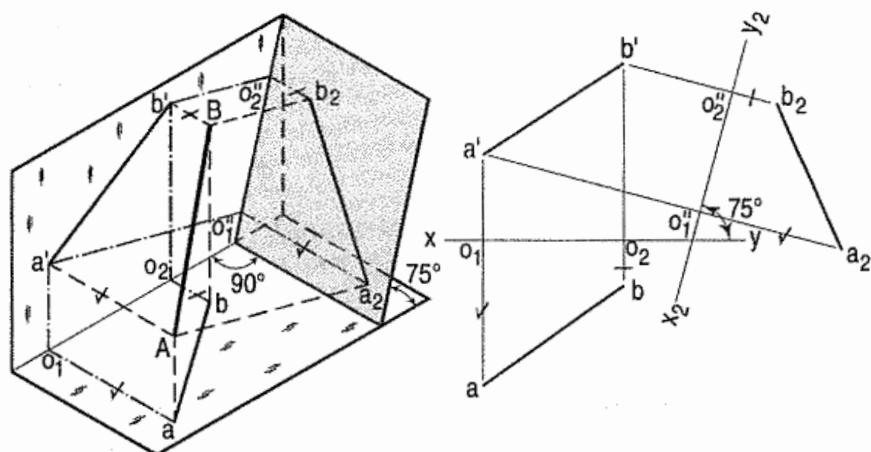


FIG. 11-8

11-3. PROJECTIONS OF LINES AND PLANES BY THE USE OF AUXILIARY PLANES

Projections of lines and planes at given inclinations to one or both the planes may also be obtained by the use of auxiliary planes. The method adopted is called the *alteration or change-of-reference-line method*.

The line, in its initial position, is assumed to be parallel to both the planes of projection. Then, instead of making the line inclined to one of the planes, an auxiliary plane inclined to the line is assumed, i.e. a new reference line is drawn and the view is projected on it.

In case of a plane, it is kept parallel to one of the planes of projection in the initial stage, the required views being obtained by projecting it on new reference lines.

Problem 11-2. (fig. 11-9): A line AB, 50 mm long, is inclined at 30° to the H.P. and its top view makes an angle of 60° with the V.P. Draw its projections.

- Draw the projections ab and $a'b'$, assuming AB to be parallel to both the planes.
- Project a new top view a_1b_1 on a new reference line x_1y_1 inclined at 30° to $a'b'$. a_1b_1 is still parallel to x_1y_1 .
- Draw the another reference line x_2y_2 to represent an A.V.P. inclined at 60° to the top view a_1b_1 .
- Project the required front view $a'_1b'_1$ on x_2y_2 . Note that $a_1o_1 = ao$, $a'_1o_2 = a'o_1$ etc.

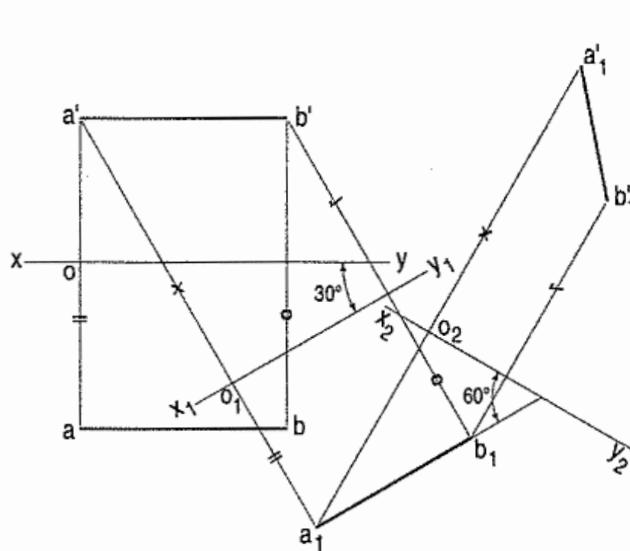


FIG. 11-9

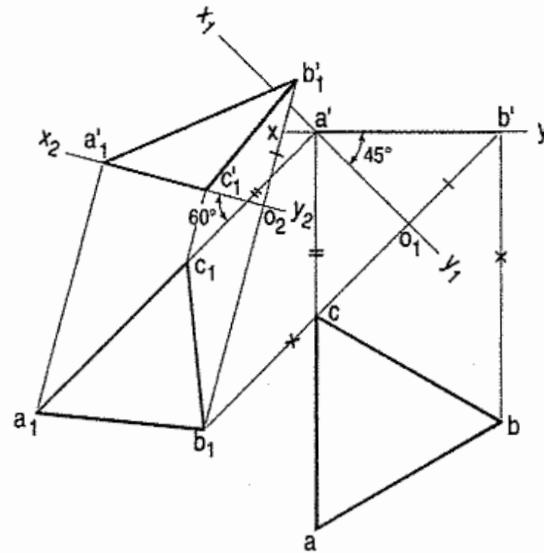


FIG. 11-10

Problem 11-3. (fig. 11-10): An equilateral triangle of 40 mm long sides has an edge on the ground and inclined at 60° to the V.P. Its plane makes an angle of 45° with the H.P. Draw its projections.

- Draw the top view abc and the front view $a'b'$, assuming the triangle to be lying on the ground and one side perpendicular to the V.P.
- Draw x_1y_1 inclined at 45° to $a'b'$ (the front view) and passing through a' (because the triangle has its edge on the ground).

- (iii) Project the new top view $a_1b_1c_1$. Again draw a new reference line x_2y_2 inclined at 60° to the line a_1c_1 (which is to make 60° angle with the V.P.).
- (iv) Project the final front view $a'_1b'_1c'_1$ on x_2y_2 . Note that $a'_1c'_1$ is in x_2y_2 .

11-4. TO DETERMINE TRUE LENGTH OF A LINE



We have seen that the true length of a line and its inclinations with the planes of projection can be determined by making each of its projections parallel to xy (chapter 10).

Instead of changing the position of the projection, that of the plane may be altered, i.e. a new reference line representing an auxiliary plane may be drawn parallel to the projection. The auxiliary projection on that reference line will show the true length and true inclination of the line with the other plane.

Problem 11-4. (fig. 11-11): *The projections of a line AB are given. To determine its true length and true inclinations with the reference planes.*

Let ab and $a'b'$ be the given projections of AB .

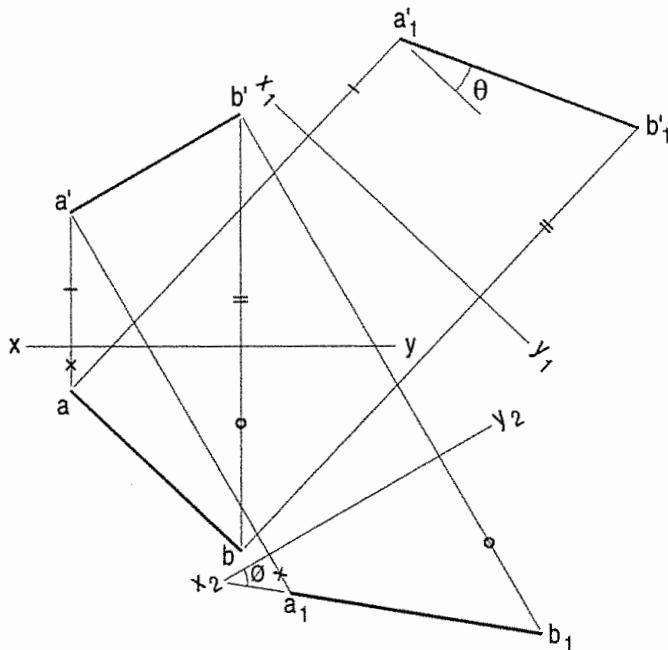


FIG. 11-11

- (i) Draw a reference line x_1y_1 to represent an A.V.P. parallel to ab , the top view.
- (ii) Project the auxiliary front view $a'_1b'_1$ which is the true length of AB . θ is its inclination with the H.P.
- (iii) Similarly, draw x_2y_2 parallel to the front view $a'b'$ and project the auxiliary top view a_1b_1 . It is the true length of AB and ϕ is its true inclination with the V.P.

Problem 11-5. (fig. 11-12): *The projections of a line AB viz. ab and $a'b'$ are on the same projector as shown in fig. 11-12(ii). Find the true length, inclinations with the H.P. and the V.P. and the traces of AB.*

- (i) Draw a reference line x_1y_1 parallel to the projections. It will be perpendicular to xy and will represent an auxiliary plane at right angles to both the H.P. and the V.P. as shown in fig. 11-12(i).
- (ii) Project the side view $a'_1b'_1$, which will be the true length of AB . θ and ϕ are its true inclinations with the H.P. and the V.P. respectively.

- (iii) Produce $a'_1b'_1$ to meet xy at v and x_1y_1 at h . The H.T. will be on the top view ab -produced, so that oH.T. = o_1h . The V.T. is on the front view $a'b'$ -produced, so that oV.T. = o_1v .

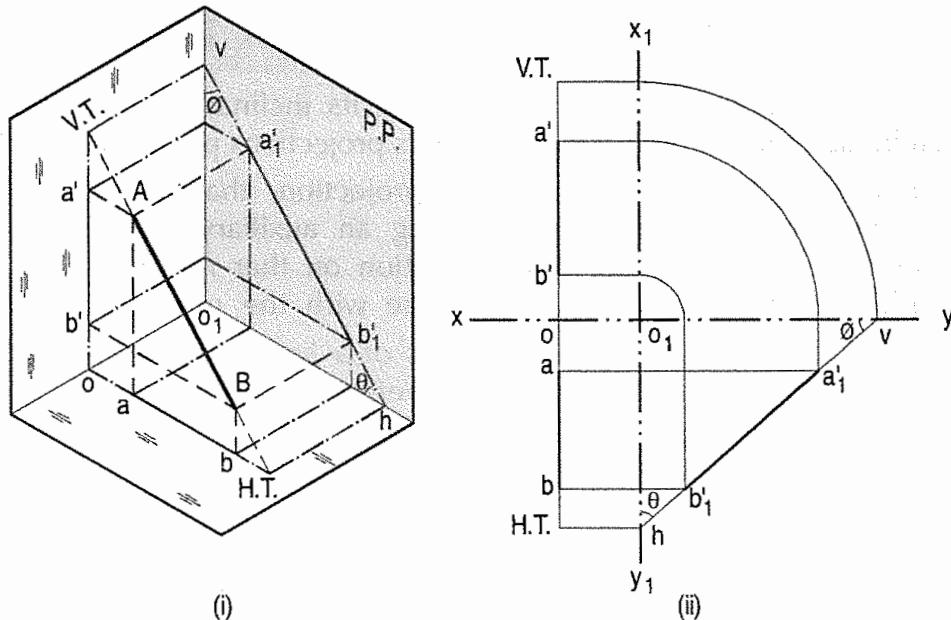


FIG. 11-12

The distances of the ends of the line from x_1y_1 are kept equal to their respective distances above xy by drawing lines inclined at 45° as shown.

11-5. TO OBTAIN POINT-VIEW OF A LINE AND EDGE-VIEW OF A PLANE

We have seen in chapter 10 that when a line is perpendicular to a reference plane, its projection on that plane is a point; while its projection on the other reference plane shows its true length. In other words, the projection of the view of a line showing its true length, on an auxiliary plane perpendicular to that view will be a point.

Similarly, a plane will be seen as a line, when it is projected on a plane, perpendicular to the true length of any one of its elements. The projection of the line-view or edge-view of a plane on an auxiliary plane parallel to it, will show the true shape and size of the plane.

Uses of the point-view of a line and edge-view of a plane on auxiliary planes are illustrated in problems 11-6 and 11-7 respectively.

Problem 11-6. (fig. 11-13): *The projections of a line PQ are given. Determine*

- (i) *the distance of its mid-point from xy, and*
- (ii) *the shortest distance of the line from xy.*

Let pq and $p'q'$ be the given projections of PQ .

- (i) Draw a new reference line x_1y_1 perpendicular to xy and project the side view $p'_1q'_1$ on it.

When considering the side view, x_1y_1 is the edge-view of the V.P. and xy is that of the H.P. The point o is the side view or the point-view of the line of intersection of the V.P. and the H.P., i.e. x_1y_1 and xy . The line joining any point on $p'_1q'_1$, with o will show the shortest distance of that point from xy .

- (ii) Find the mid-point a'_1 of $p'_1q'_1$ and join it with o . oa'_1 is the distance of the mid-point of PQ from xy .
- (iii) From o , draw a line ob perpendicular to $p'_1q'_1$. ob is the shortest distance of PQ from xy . It will be perpendicular to both PQ and xy .

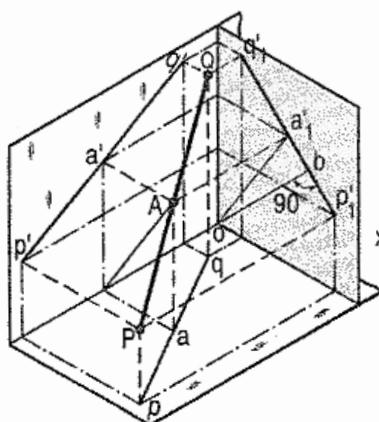


FIG. 11-13

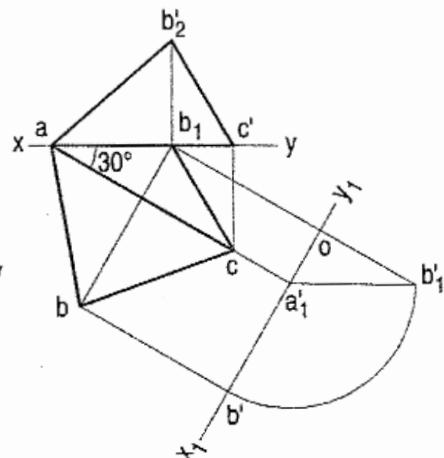
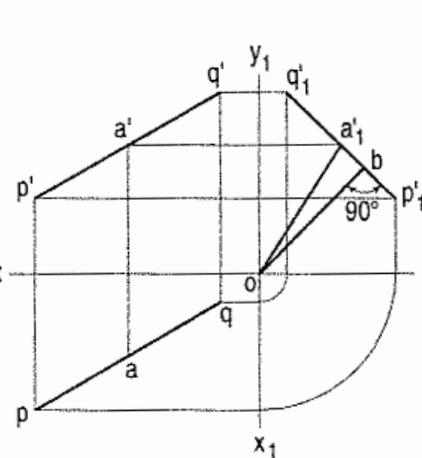


FIG. 11-14

Problem 11-7. (fig. 11-14): An isosceles triangle ABC , base 60 mm and altitude 40 mm has its base AC in the H.P. and inclined at 30° to the V.P. The corners A and B are in the V.P. Draw its projections.

Assume the triangle to be lying in the H.P. with the base AC inclined at 30° to the V.P. and A in the V.P. Its true shape will be seen in the top view.

- (i) Therefore, in the top view, draw ac 60 mm long and inclined at 30° to xy and complete the triangle abc .
- (ii) Project c to c' on xy . When the triangle is tilted about the edge AC , so that the corner B is in the V.P., in the top view the point b will move along a line perpendicular to ac , to a point b_1 in xy . The distance of the front view of B below xy may now be determined by means of an auxiliary plane.
- (iii) Draw a reference line x_1y_1 perpendicular to ac .
- (iv) Project an auxiliary front view of the triangle abc . It will be a line $b'a'_1$, showing the edge-view of the triangle. a'_1 is the point-view of the line ac . When b moves to b_1 , b' will move along the arc drawn with a'_1 as centre and radius equal to a'_1b' to b'_1 on the line through b_1 drawn perpendicular to x_1y_1 . ob'_1 is the required distance.
- (v) Therefore, through b_1 , draw a line perpendicular to xy and on it, mark a point b'_2 such that $b_1b'_2 = ob'_1$.
- (vi) Join a and c' with b'_2 . ab'_2c' and ab_1c are the required projections.

11-6. TO DETERMINE TRUE SHAPE OF A PLANE FIGURE

The true shape of any plane figure may be determined by means of its projections on auxiliary planes, as illustrated in problems 11-8 and 11-9.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 26 for the following problem.

Problem 11-8. (fig. 11-15):
Projections of a pentagon resting on the ground on one of its sides are given. Determine the true shape of the pentagon.

ae is the true length of the side because it is parallel to the H.P.

- (i) Draw a reference line x_1y_1 perpendicular to ae. Project an auxiliary front view on it. It will give a point-view a'_1 and e'_1 of ae and an edge-view $a'_1c'_1$ of the pentagon.
- (ii) Draw another reference line x_2y_2 parallel to $a'_1c'_1$ and project an auxiliary top view $a_1b_1c_1d_1e_1$, which will be the true shape of the pentagon.

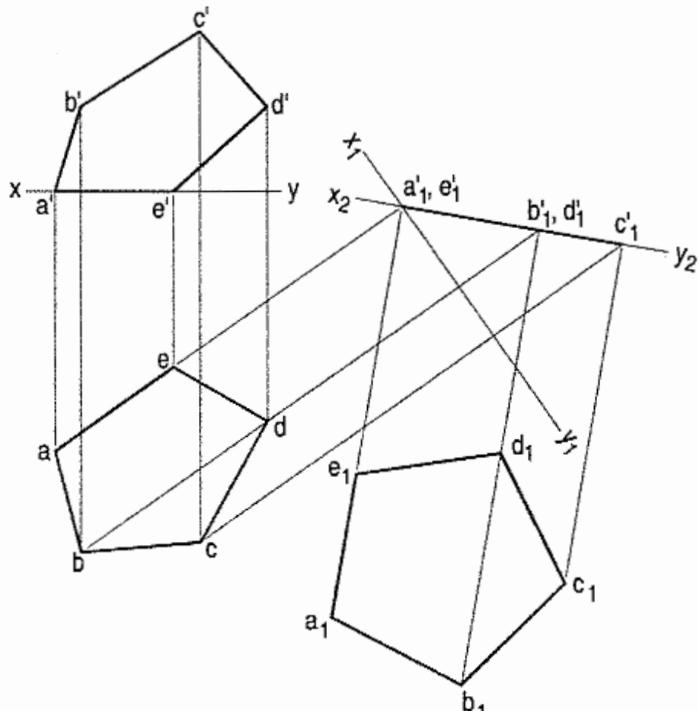


FIG. 11-15

Problem 11-9. Fig. 11-16 shows the top view abcd and front view $a'b'c'd'$ of a quadrilateral. Determine its true shape.

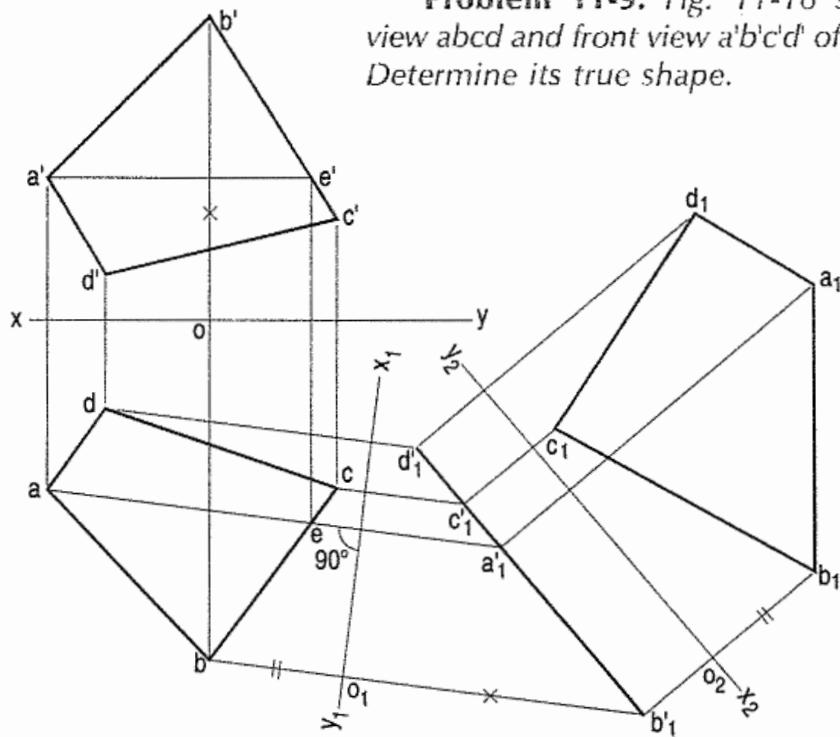


FIG. 11-16

- (i) Through any corner, say a' , draw a line parallel to xy and meeting $b'c'$ at e' .
- (ii) Project e' to e on the line bc in the top view. ae is the true length of the element AE .
- (iii) Draw a reference line x_1y_1 perpendicular to ae . Project a new front view from the top view. It is a line $d'_1b'_1$.
- (iv) Again, draw another reference line x_2y_2 parallel to the line-view $d'_1b'_1$ and project on it a new top view $a_1b_1c_1d_1$ which will show the true shape of the quadrilateral.

Note that $b'_1o_1 = b'o$, $b'_1o_2 = bo_1$ etc.

Problem 11-10. (fig. 11-17): The projections of a triangular plate PQR appear as under:

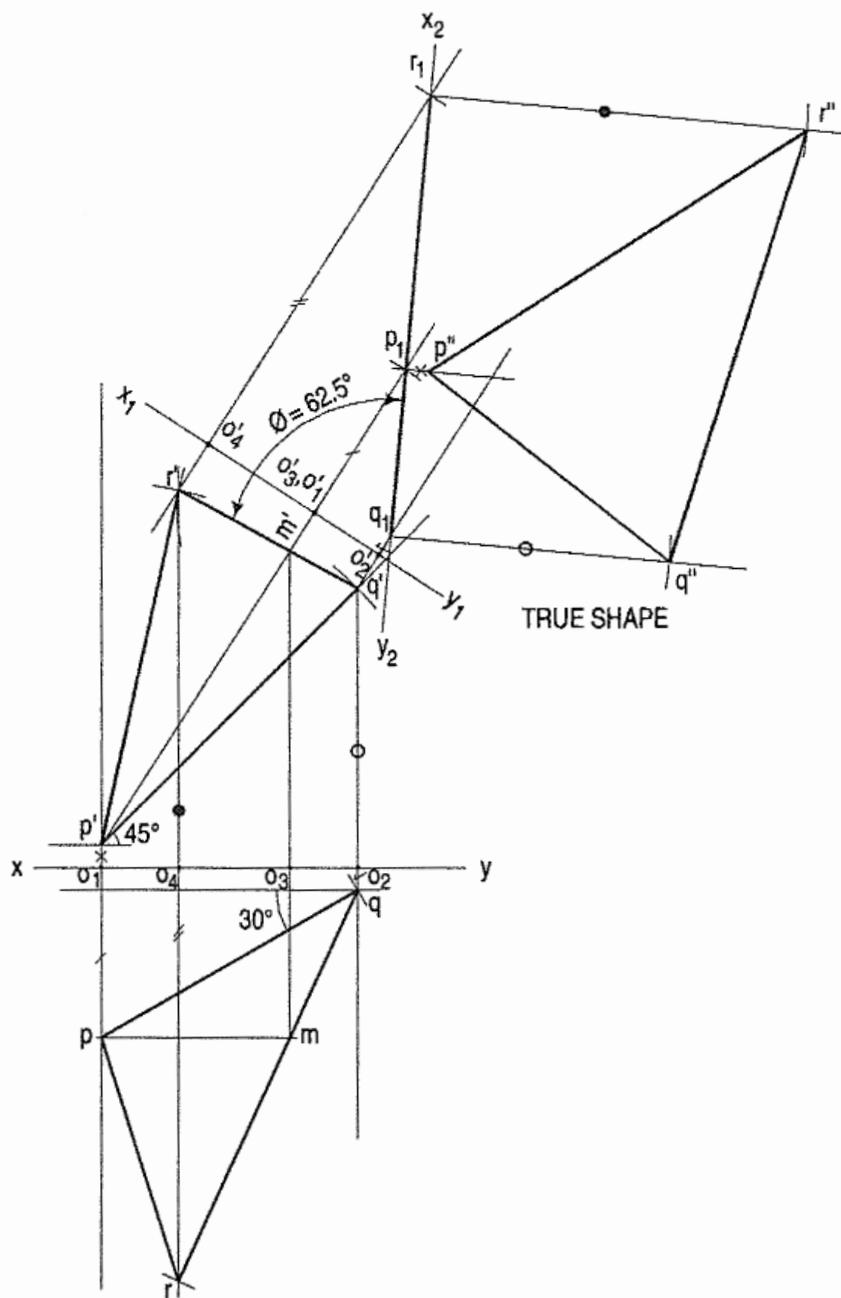


FIG. 11-17

(a) Top view : pq makes 30° with xy .

$qr = 95 \text{ mm}$. The corner q is 5 mm in front of the V.P.

(b) Front view : $p'q' = 80 \text{ mm}$; $q'r' = 45 \text{ mm}$ and $r'p' = 80 \text{ mm}$.

$p'q'$ makes an angle of 45° with xy . The corner P of the plate is 5 mm above the H.P.

Draw the projections of the plate and determine true shape of the triangular plater

- Mark xy . Draw top view and front view from the given dimensions.
- Mark a line pm in the top view parallel to xy . Draw the projection of pm in the front view $p'm'$ as shown.
- Select x_1y_1 perpendicular to $p'm'$ and project new top view taking the distances o_1p , o_4r and o_2q from the top view.
- Select x_2y_2 and draw new front view as shown in fig. 11-17.
- Measure angle ϕ .

Problem 11-11. A regular pentagon of 50 mm side is resting on one of its sides on the H.P. having that side parallel to and 25 mm in front of V.P. It is tilted about that side so that its highest corner rests in the V.P. Draw the projections of the pentagon.

- Keep the plane of pentagon parallel to V.P. Draw the front view and top view as shown in fig. 11-18.
- Project $a'b'c'd'e'$ on the line parallel to and 25 mm behind x_1y_1 . Taking a' as a center, tilt $a''d''$ such that d'' touches x_1y_1 at d''' . Complete the projections as shown in fig. 11-18.

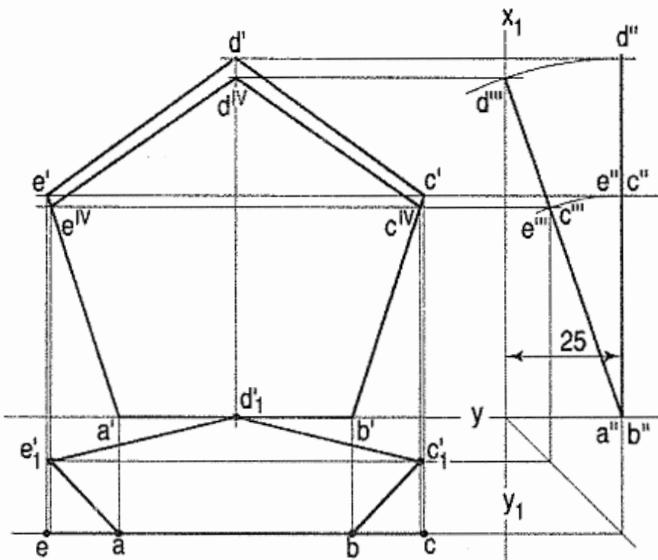


FIG. 11-18

Note that the points e''' and c''' should be transferred in the top view correctly as shown.

Problem 11-12. A thin square plate with 60 mm side stands on one of its corners in H.P. and the opposite corner is raised so that one of its diagonals is twice that of the others. If, one of the diagonals is parallel to both the planes, draw its projection and determine an inclination of the plane of the plate with H.P.

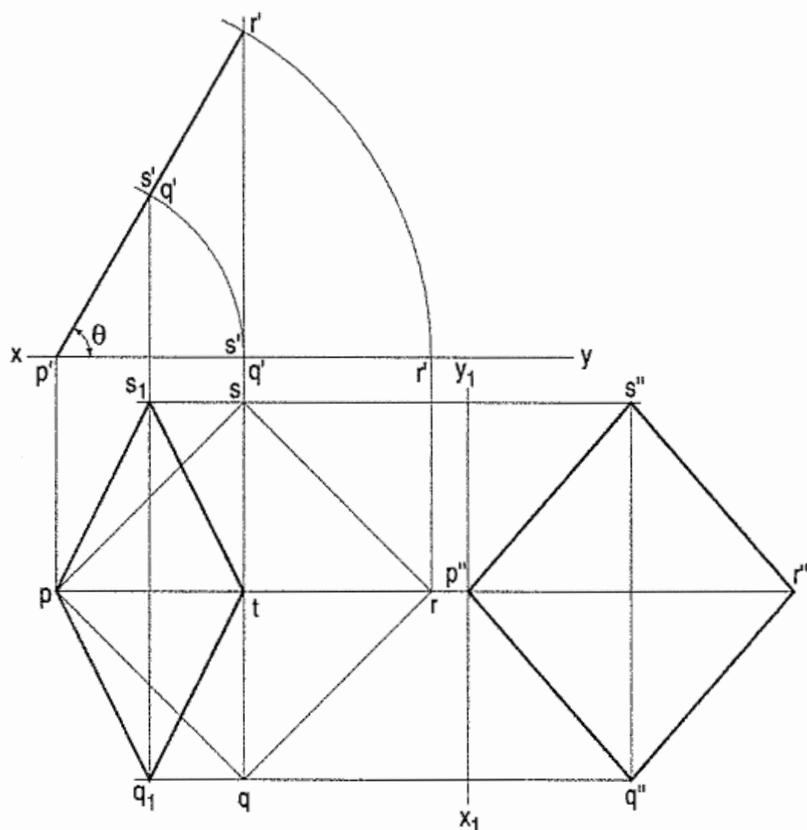


FIG. 11-19

- Assuming that the square plate is equally inclined with a vertical plane, draw its top view and front view as shown in fig. 11-19.
- Raise r' so that the diagonal qs becomes twice the diagonal pt . Draw the top view p, q_1, t and s_1 . Measure angle θ .
- Draw new reference line x_1y_1 parallel to s_1q_1 . Project new front view, keeping the same distances of points p', q', r' and s' from xy .
- Measure angle θ .

EXERCISES 11

Solve the following exercises by applying the method of projections on auxiliary planes:

- Determine the true length, inclinations with the H.P. and the V.P., and the traces of the line PQ in problem 10-15 of chapter 10.
- Find the true length and the distance of the mid-point from xy of the line whose projections are given in problem 10-13 of chapter 10.
- Draw the projections of the various plane figures as required in Exercises 1, 2 and 3 of chapter 12.
- abc is an equilateral triangle of altitude 50 mm with ab in xy and c below it. abc' is an isosceles triangle of altitude 75 mm and c' is above xy . Determine the true shape of the triangle ABC , of which abc is the top view and abc' is the front view.

5. Determine the true shape of the figure, the top view of which is a regular pentagon of 35 mm sides, having one side inclined at 30° to xy and whose front view is a straight line making an angle of 45° to xy .
6. An equilateral triangle ABC of sides 75 mm long has its side AB in the V.P. and inclined at 60° to the H.P. Its plane makes an angle of 45° with the V.P. Draw its projections.
7. An isosceles triangle PQR having the base PQ 50 mm long and altitude 75 mm has its corners P , Q , and R 25 mm, 50 mm and 75 mm respectively above the ground. Draw its projections.
8. A thin regular pentagonal plate of 60 mm long edges has one of its edges in the H.P. and perpendicular to the V.P. while its farthest corner is 60 mm above the H.P. Draw the projections of the plate. Project another front view on an A.V.P. making an angle of 45° with the V.P.
9. A thin composite plate consists of a square of 70 mm long sides with an additional semi-circle constructed on CD as diameter. The side AB is vertical and the surface of the plate makes an angle of 45° with the V.P. Draw its projections. Project another top view on an A.I.P. making an angle of 30° with the side AB .
10. A 60° set-square of 125 mm longest side is so kept that the longest side is in the H.P. making an angle of 30° with the V.P. and the set-square itself inclined at 45° to the H.P. Draw the projections of the set-square.
11. A plane figure is composed of an equilateral triangle ABC and a semi-circle on AC as diameter. The length of the side AB is 50 mm and is parallel to the V.P. The corner B is 20 mm behind the V.P. and 15 mm below the H.P. The plane of the figure is inclined at 45° to the H.P. Draw the projections of the plane figure.
12. An equilateral triangle ABC having side length as 50 mm is suspended from a point O on the side AB 15 mm from A in such a way that the plane of the triangle makes an angle of 60° with the V.P. The point O is 20 mm below the H.P. and 40 mm behind the V.P. Draw the projections of the triangle.
13. A hexagonal plate of side 40 mm, is resting on a corner in V.P. with its surface making an angle of 30° with the V.P. The front view of the diagonal passing through that corner is inclined at 45° to the line xy . Draw the projections of the hexagonal plate.
14. A rectangular plate 50 mm \times 70 mm stands on one of its shorter edges in H.P. and is raised about this edge so that the top view becomes a square of 50 mm. Determine an inclination of the plate with the horizontal plane.

Chapter

12



PROJECTIONS OF PLANES

12-0. INTRODUCTION



Plane figures or surfaces have only two dimensions, viz. length and breadth. They do not have thickness. A plane figure may be assumed to be contained by a plane, and its projections can be drawn, if the position of that plane with respect to the principal planes of projection is known.

In this chapter, we shall discuss the following topics:

1. Types of planes and their projections.
2. Traces of planes.

12-1. TYPES OF PLANES



Planes may be divided into two main types:

- (1) Perpendicular planes.
- (2) Oblique planes.

(1) Perpendicular planes: These planes can be divided into the following sub-types:

- (i) Perpendicular to both the reference planes.
- (ii) Perpendicular to one plane and parallel to the other.
- (iii) Perpendicular to one plane and inclined to the other.

(i) *Perpendicular to both the reference planes* (fig. 12-1): A square $ABCD$ is perpendicular to both the planes. Its H.T. and V.T. are in a straight line perpendicular to xy .

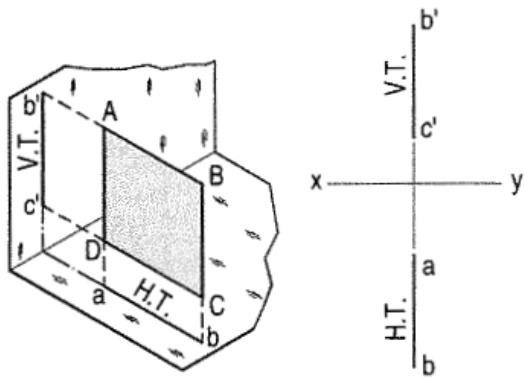


FIG. 12-1

The front view $b'c'$ and the top view ab of the square are both lines coinciding with the V.T. and the H.T. respectively.

(ii) *Perpendicular to one plane and parallel to the other plane:*

- (a) Plane, perpendicular to the H.P. and parallel to the V.P. [fig. 12-2(i)].

A triangle PQR is perpendicular to the H.P. and is parallel to the V.P. Its H.T. is parallel to xy . It has no V.T.

The front view $p'q'r'$ shows the exact shape and size of the triangle. The top view pqr is a line parallel to xy . It coincides with the H.T.

- (b) Plane, perpendicular to the V.P. and parallel to the H.P. [fig. 12-2(ii)].

A square $ABCD$ is perpendicular to the V.P. and parallel to the H.P. Its V.T. is parallel to xy . It has no H.T.

The top view $abcd$ shows the true shape and true size of the square. The front view $a'b'$ is a line, parallel to xy . It coincides with the V.T.

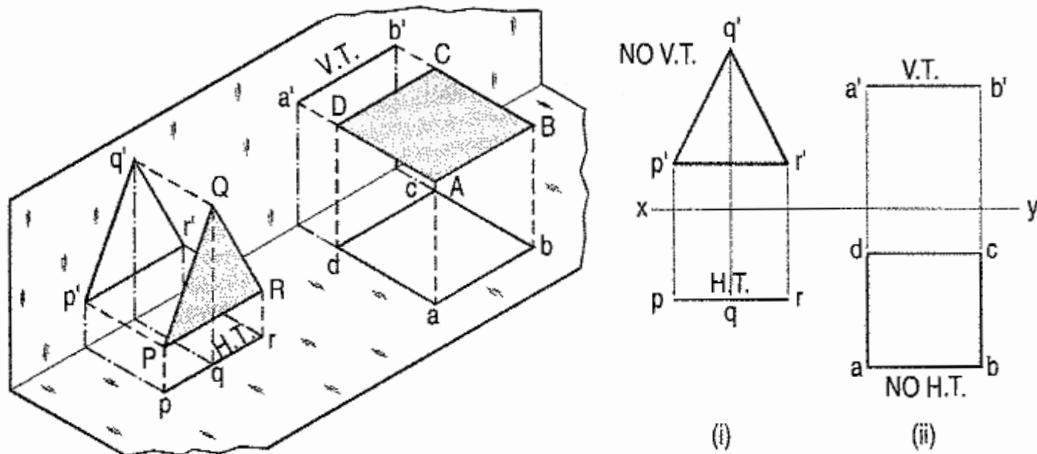


FIG. 12-2

(iii) *Perpendicular to one plane and inclined to the other plane:*

- (a) Plane, perpendicular to the H.P. and inclined to the V.P. (fig. 12-3).

A square $ABCD$ is perpendicular to the H.P. and inclined at an angle ϕ to the V.P. Its V.T. is perpendicular to xy . Its H.T. is inclined at ϕ to xy .

Its top view ab is a line inclined at ϕ to xy . The front view $a'b'c'd'$ is smaller than $ABCD$.

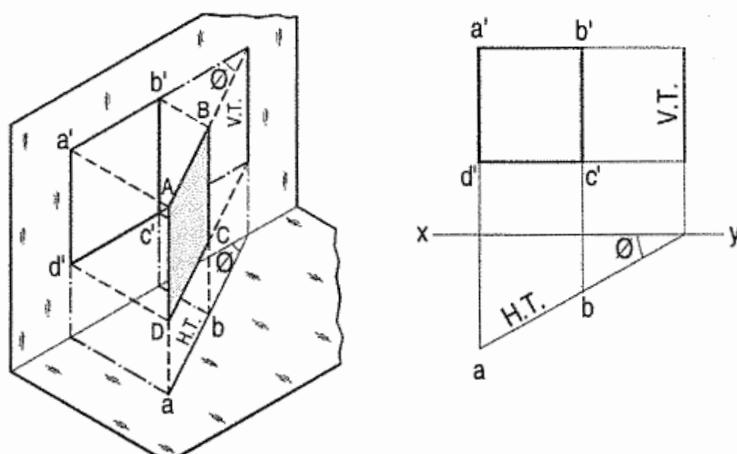


FIG. 12-3

(b) Plane, perpendicular to the V.P. and inclined to the H.P. (fig. 12-4).

A square $ABCD$ is perpendicular to the V.P. and inclined at an angle θ to the H.P. Its H.T. is perpendicular to xy . Its V.T. makes the angle θ with xy . Its front view $a'b'$ is a line inclined at θ to xy . The top view $abcd$ is a rectangle which is smaller than the square $ABCD$.

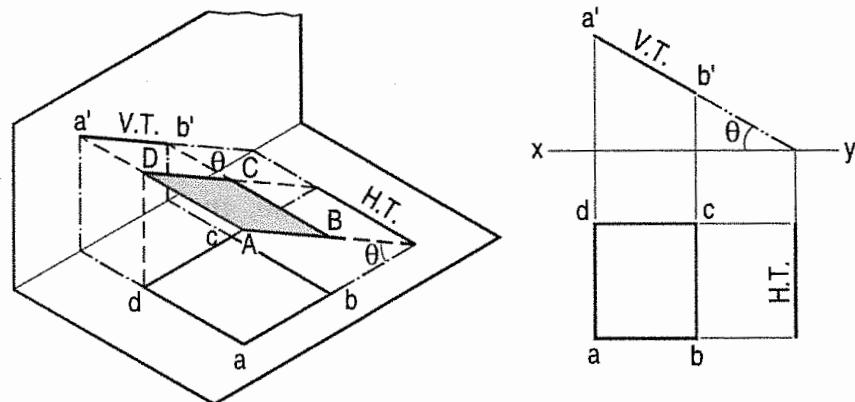


FIG. 12-4

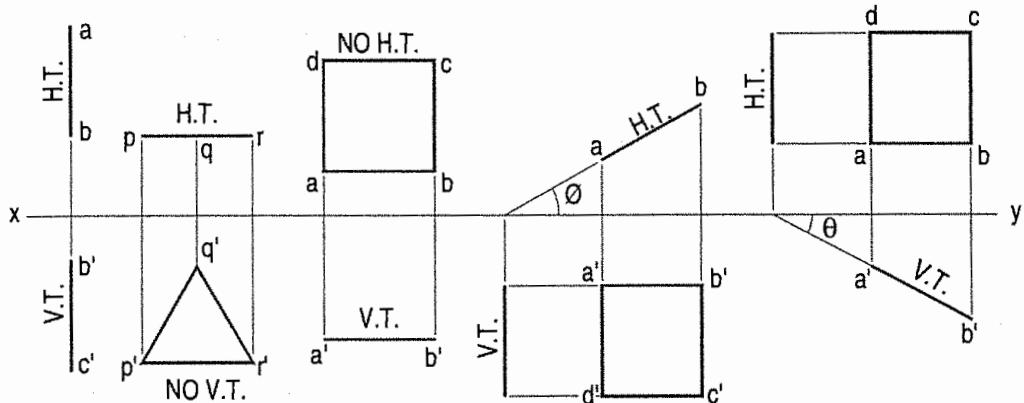


FIG. 12-5

Fig. 12-5 shows the projections and the traces of all these perpendicular planes by third-angle projection method.

(2) **Oblique planes:** Planes which are inclined to both the reference planes are called *oblique planes*. Representation of oblique planes by their traces is too advanced to be included in this book.

A few problems on the projections of plane figures inclined to both the reference planes are however, illustrated at the end of the chapter. They will prove to be of great use in dealing with the projections of solids.

12-2. TRACES OF PLANES

A plane, extended if necessary, will meet the reference planes in lines, unless it is parallel to any one of them.

These lines are called the *traces* of the plane. The line in which the plane meets the H.P. is called the *horizontal trace* or the H.T. of the plane. The line in which it meets the V.P. is called its *vertical trace* or the V.T. A plane is usually represented by its traces.

12-3. GENERAL CONCLUSIONS



(1) Traces:

- (a) When a plane is perpendicular to both the reference planes, its traces lie on a straight line perpendicular to xy .
- (b) When a plane is perpendicular to one of the reference planes, its trace upon the other plane is perpendicular to xy (except when it is parallel to the other plane).
- (c) When a plane is parallel to a reference plane, it has no trace on that plane. Its trace on the other reference plane, to which it is perpendicular, is parallel to xy .
- (d) When a plane is inclined to the H.P. and perpendicular to the V.P., its inclination is shown by the angle which its V.T. makes with xy . When it is inclined to the V.P. and perpendicular to the H.P., its inclination is shown by the angle which its H.T. makes with xy .
- (e) When a plane has two traces, they, produced if necessary, intersect in xy (except when both are parallel to xy as in case of some oblique planes).

(2) Projections:

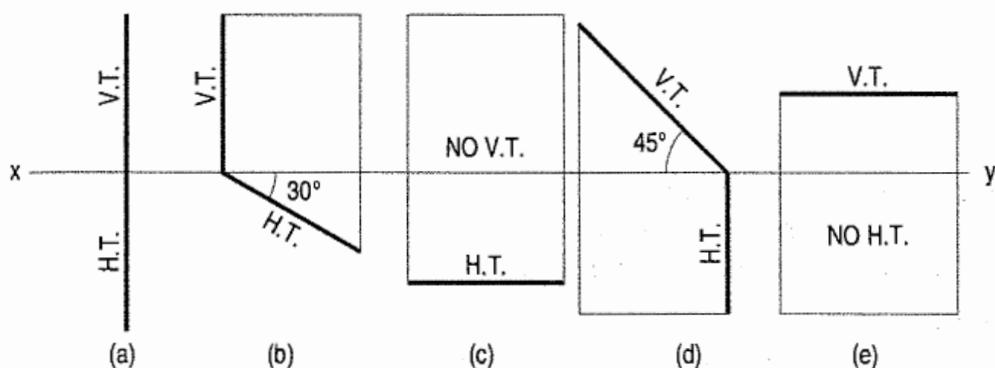
- (a) When a plane is perpendicular to a reference plane, its projection on that plane is a straight line.
- (b) When a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.
- (c) When a plane is perpendicular to one of the reference planes and inclined to the other, its inclination is shown by the angle which its projection on the plane to which it is perpendicular, makes with xy . Its projection on the plane to which it is inclined, is smaller than the plane itself.

Problem 12-1. Show by means of traces, each of the following planes:

- (a) Perpendicular to the H.P. and the V.P.
- (b) Perpendicular to the H.P. and inclined at 30° to the V.P.
- (c) Parallel to and 40 mm away from the V.P.
- (d) Inclined at 45° to the H.P. and perpendicular to the V.P.
- (e) Parallel to the H.P. and 25 mm away from it.

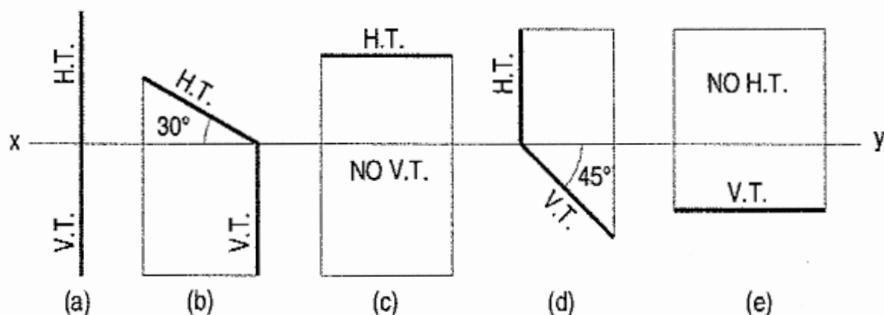
Fig. 12-6 and fig. 12-7 show the various traces.

- (a) The H.T. and the V.T. are in a line perpendicular to xy .
- (b) The H.T. is inclined at 30° to xy ; the V.T. is normal to xy ; both the traces intersect in xy .
- (c) The H.T. is parallel to and 40 mm away from xy . It has no V.T.
- (d) The H.T. is perpendicular to xy ; the V.T. makes 45° angle with xy ; both intersect in xy .
- (e) The V.T. is parallel to and 25 mm away from xy . It has no H.T.



(First-angle projection)

FIG. 12-6



(Third-angle projection)

FIG. 12-7

12-4. PROJECTIONS OF PLANES PARALLEL TO ONE OF THE REFERENCE PLANES



The projection of a plane on the reference plane parallel to it will show its true shape. Hence, beginning should be made by drawing that view. The other view which will be a line, should then be projected from it.

(1) When the plane is parallel to the H.P.: The top view should be drawn first and the front view projected from it.

Problem 12-2. (fig. 12-8): An equilateral triangle of 50 mm side has its V.T. parallel to and 25 mm above xy. It has no H.T. Draw its projections when one of its sides is inclined at 45° to the V.P.

As the V.T. is parallel to xy and as there is no H.T., the triangle is parallel to the H.P. Therefore, begin with the top view.

- Draw an equilateral triangle abc of 50 mm side, keeping one side, say ac , inclined at 45° to xy .
- Project the front view, parallel to and 25 mm above xy , as shown.

(2) When the plane is parallel to the V.P.: Beginning should be made with the front view and the top view projected from it.

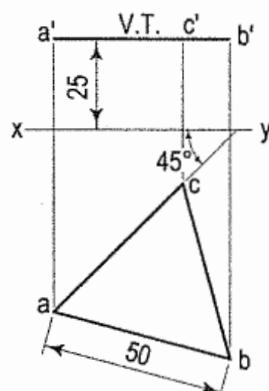


FIG. 12-8

Problem 12-3. (fig. 12-9): A square ABCD of 40 mm side has a corner on the H.P. and 20 mm in front of the V.P. All the sides of the square are equally inclined to the H.P. and parallel to the V.P. Draw its projections and show its traces.

As all the sides are parallel to the V.P., the surface of the square also is parallel to it. The front view will show the true shape and position of the square.

- Draw a square $a'b'c'd'$ in the front view with one corner in xy and all its sides inclined at 45° to xy .
- Project the top view keeping the line ac parallel to xy and 30 mm below it. The top view is its H.T. It has no V.T.

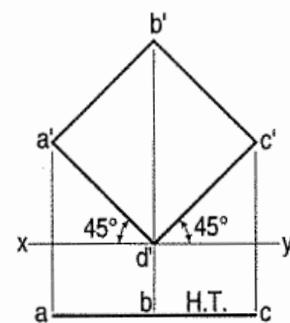
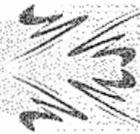


FIG. 12-9

12-5. PROJECTIONS OF PLANES INCLINED TO ONE REFERENCE PLANE AND PERPENDICULAR TO THE OTHER



When a plane is inclined to a reference plane, its projections may be obtained in two stages. In the initial stage, the plane is assumed to be parallel to that reference plane to which it has to be made inclined. It is then tilted to the required inclination in the second stage.

(1) Plane, inclined to the H.P. and perpendicular to the V.P.: When the plane is inclined to the H.P. and perpendicular to the V.P., in the initial stage, it is assumed to be parallel to the H.P. Its top view will show the true shape. The front view will be a line parallel to xy . The plane is then tilted so that it is inclined to the H.P. The new front view will be inclined to xy at the true inclination. In the top view the corners will move along their respective paths (parallel to xy).

Problem 12-4. (fig. 12-10): A regular pentagon of 25 mm side has one side on the ground. Its plane is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections and show its traces.

Assuming it to be parallel to the H.P.

- Draw the pentagon in the top view with one side perpendicular to xy [fig. 12-10(i)]. Project the front view. It will be the line $a'c'$ contained by xy .
- Tilt the front view about the point a' , so that it makes 45° angle with xy .
- Project the new top view $ab_1c_1d_1e$ upwards from this front view and horizontally from the first top view. It will be more convenient if the front view is reproduced in the new position separately and the top view projected from it, as shown in fig. 12-10(ii). The V.T. coincides with the front view and the H.T. is perpendicular to xy , through the point of intersection between xy and the front view-produced.

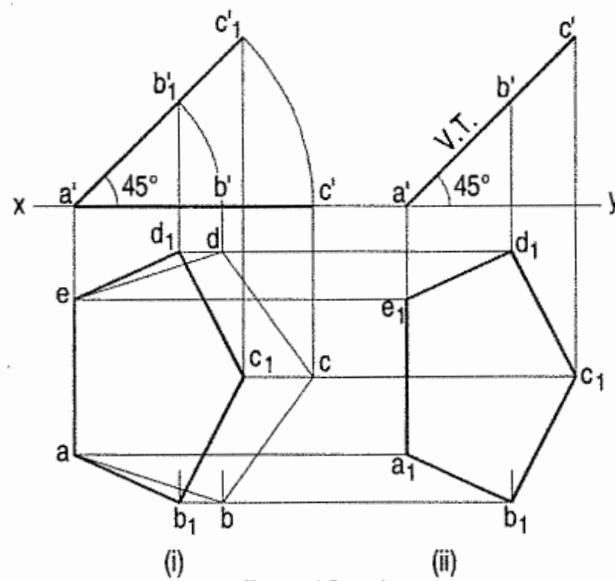


FIG. 12-10

(2) Plane, inclined to the V.P. and perpendicular to the H.P.: In the initial stage, the plane may be assumed to be parallel to the V.P. and then tilted to the required position in the next stage. The projections are drawn as illustrated in the next problem.

Problem 12-5. (fig. 12-11): Draw the projections of a circle of 50 mm diameter, having its plane vertical and inclined at 30° to the V.P. Its centre is 30 mm above the H.P. and 20 mm in front of the V.P. Show also its traces.

A circle has no corners to project one view from another. However, a number of points, say twelve, equal distances apart, may be marked on its circumference.

- Assuming the circle to be parallel to the V.P., draw its projections. The front view will be a circle [fig. 12-11(i)], having its centre 30 mm above xy. The top view will be a line, parallel to and 20 mm below xy.
- Divide the circumference into twelve equal parts (with a 30° - 60° set-square) and mark the points as shown. Project these points in the top view. The centre O will coincide with the point 4.
- When the circle is tilted, so as to make 30° angle with the V.P., its top view will become inclined at 30° to xy. In the front view all the points will move along their respective paths (parallel to xy). Reproduce the top view keeping the centre o at the same distance, viz. 20 mm from xy and inclined at 30° to xy [fig. 12-11(ii)].
- For the final front view, project all the points upwards from this top view and horizontally from the first front view. Draw a freehand curve through the twelve points $1'_1$, $2'_1$ etc. This curve will be an ellipse.

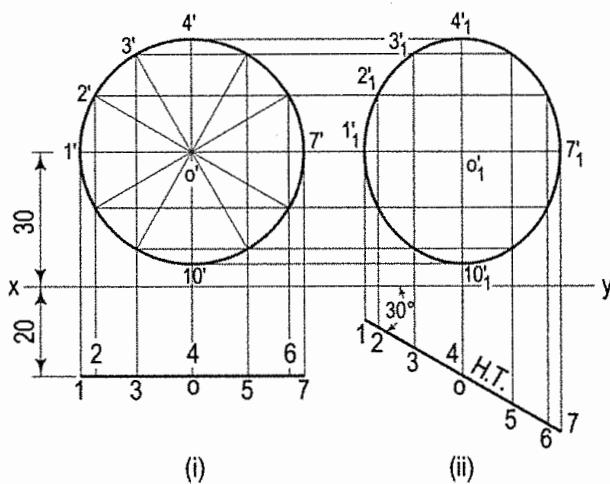


FIG. 12-11

12-6. PROJECTIONS OF OBLIQUE PLANES

When a plane has its surface inclined to one plane and an edge or a diameter or a diagonal parallel to that plane and inclined to the other plane, its projections are drawn in three stages.

- If the surface of the plane is inclined to the H.P. and an edge (or a diameter or a diagonal) is parallel to the H.P. and inclined to the V.P.,
 - in the initial position the plane is assumed to be parallel to the H.P. and an edge perpendicular to the V.P.
 - It is then tilted so as to make the required angle with the H.P. As already explained, its front view in this position will be a line, while its top view will be smaller in size.
 - In the final position, when the plane is turned to the required inclination with the V.P., only the position of the top view will change. Its shape and size will not be affected. In the final front view, the corresponding distances of all the corners from xy will remain the same as in the second front view.

If an edge is in the H.P. or on the ground, in the initial position, the plane is assumed to be lying in the H.P. or on the ground, with the edge perpendicular to the V.P. If a corner is in the H.P. or on the ground, the line joining that corner with the centre of the plane is kept parallel to the V.P.

- (2) Similarly, if the surface of the plane is inclined to the V.P. and an edge (or a diameter or a diagonal) is parallel to the V.P. and inclined to the H.P.,
- in the initial position, the plane is assumed to be parallel to the V.P. and an edge perpendicular to the H.P.
 - It is then tilted so as to make the required angle with the V.P. Its top view in this position will be a line, while its front view will be smaller in size.
 - When the plane is turned to the required inclination with the H.P., only the position of the front view will change. Its shape and size will not be affected. In the final top view, the corresponding distances of all the corners from xy will remain the same as in the second top view.

If an edge is in the V.P. in the initial position, the plane is assumed to be lying in the V.P. with an edge perpendicular to the H.P. If a corner is in the V.P., the line joining that corner with centre of the plane is kept parallel to the H.P.

Problem 12-6. (fig. 12-12): A square ABCD of 50 mm side has its corner A in the H.P., its diagonal AC inclined at 30° to the H.P. and the diagonal BD inclined at 45° to the V.P. and parallel to the H.P. Draw its projections.

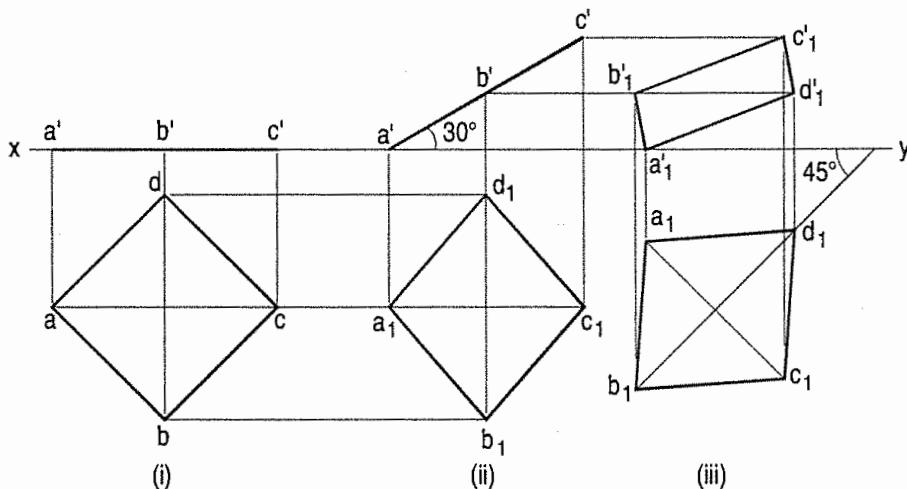


FIG. 12-12

In the initial stage, assume the square to be lying in the H.P. with AC parallel to the V.P.

- Draw the top view and the front view. When the square is tilted about the corner A so that AC makes 30° angle with the H.P., BD remains perpendicular to the V.P. and parallel to the H.P.
- Draw the second front view with $a'_1c'_1$ inclined at 30° to xy , keeping a'_1 or c'_1 in xy . Project the second top view. The square may now be turned so that BD makes 45° angle with the V.P. and remains parallel to the H.P. Only the position of the top view will change. Its shape and size will remain the same.

- (iii) Reproduce the top view so that b_1d_1 is inclined at 45° to xy . Project the final front view upwards from this top view and horizontally from the second front view.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 27 for the following problem.

Problem 12-7. (Fig. 12-13): A rectangular plane surface of size $L \times W$ is positioned in the first quadrant and is inclined at an angle of 60° with the H.P. and 30° with the V.P. Draw its projections.

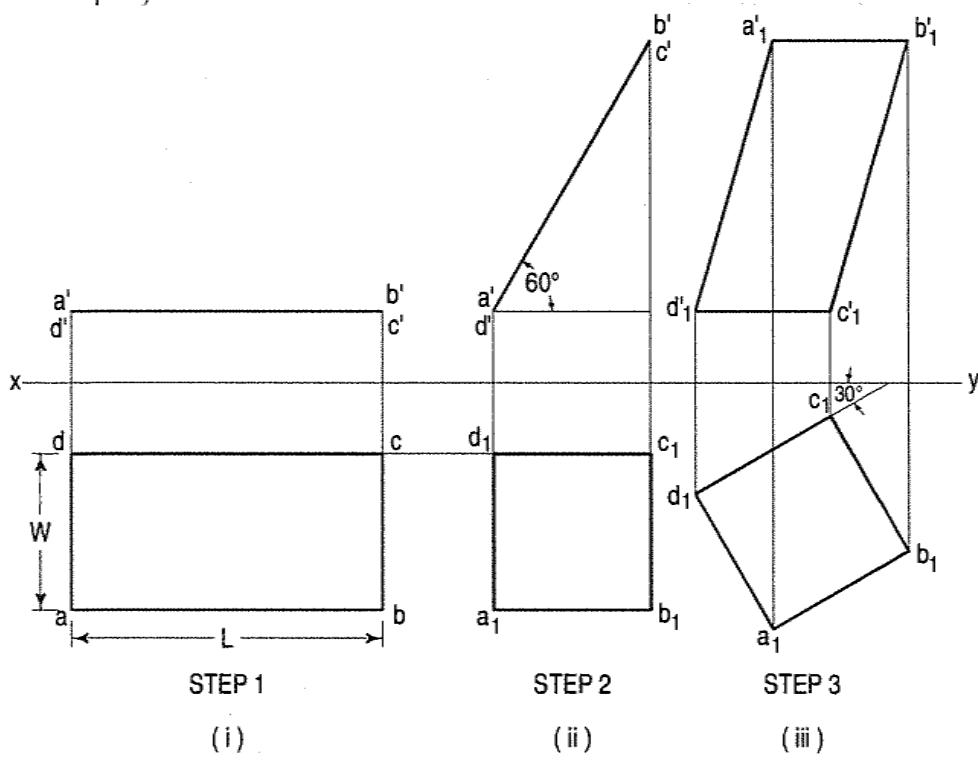


FIG. 12-13

- The plane is first assumed to be parallel to H.P. with its shorter edge perpendicular to V.P. In this position, true shape and size of the plane is given by its projection on H.P. The front view will be a true line parallel to the reference line xy .
- Rotate the front view projection by 60° (the angle of inclination of plane with H.P.) as shown in Step 2 of fig. 12-13(ii). Draw vertical lines from the ends of line $a'd'$ and $b'c'$ to intersect horizontal lines drawn from the top view $abcd$ (step 1) at points b_1 , c_1 , d_1 and a_1 . Join $a_1b_1c_1d_1$ to obtain the top view of the plane in this inclined position.
- Now rotate the edge d_1c_1 of the top view (step 2) by 30° (the angle of inclination of plane with V.P.) and reproduce it as shown in step 3 of the fig. 12-13(iii). Draw projections from a_1 , b_1 , c_1 and d_1 to intersect the horizontal projections from $a'd'$ and $b'c'$ to get the points a'_1 , b'_1 , c'_1 and d'_1 . Join the lines $a'_1b'_1c'_1d'_1$ to obtain the final front view of the given plane surface.

Problem 12-8. (fig. 12-14): Draw the projections of a regular hexagon of 25 mm side, having one of its sides in the H.P. and inclined at 60° to the V.P., and its surface making an angle of 45° with the H.P.

- Draw the hexagon in the top view with one side perpendicular to xy . Project the front view $a'c'$ in xy .
- Draw $a'c'$ inclined at 45° to xy keeping a' or c' in xy and project the second top view.
- Reproduce this top view making a_1f_1 inclined at 60° to xy and project the final front view.

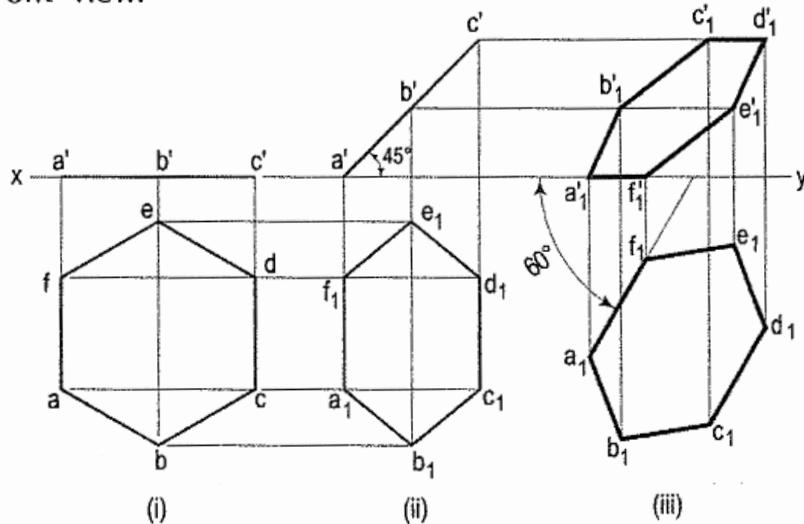


FIG. 12-14

Problem 12-9. (fig. 12-15): Draw the projections of a circle of 50 mm diameter resting in the H.P. on a point A on the circumference; its plane inclined at 45° to the H.P. and

- the top view of the diameter AB making 30° angle with the V.P.;
- the diameter AB making 30° angle with the V.P.

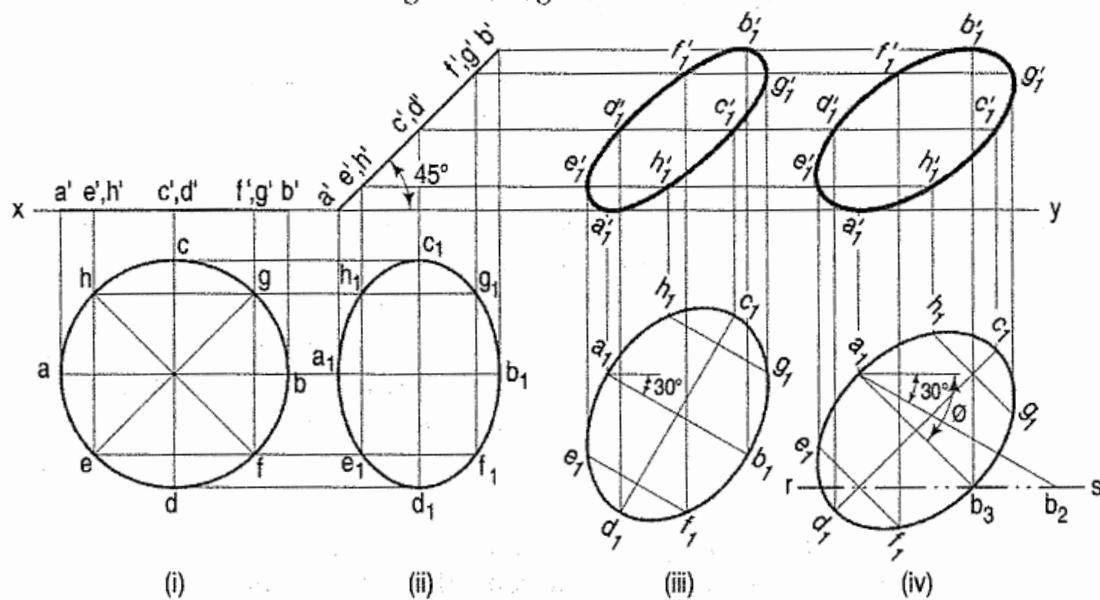


FIG. 12-15

Draw the projections of the circle with A in the H.P. and its plane inclined at 45° to the H.P. and perpendicular to the V.P. [fig. 12-15(i) and fig. 12-15(ii)].

- In the second top view, the line a_1b_1 is the top view of the diameter AB. Reproduce this top view so that a_1b_1 makes 30° angle with xy [fig. 12-15(iii)]. Project the required front view.

- (b) If the diameter AB , which makes 45° angle with the H.P., is inclined at 30° to the V.P. also, its top view a_1b_1 will make an angle greater than 30° with xy . This apparent angle of inclination is determined as described below.

Draw any line a_1b_2 equal to AB and inclined at 30° to xy [fig. 12-15(iv)]. With a_1 as centre and radius equal to the top view of AB , viz. a_1b_1 , draw an arc cutting rs (the path of B in the top view) at b_3 . Draw the line joining a_1 with b_3 , and around it, reproduce the second top view. Project the final front view. It is evident that a_1b_3 is inclined to xy at an angle ϕ which is greater than 30° .

Problem 12-10. (fig. 12-16): A thin 30° - 60° set-square has its longest edge in the V.P. and inclined at 30° to the H.P. Its surface makes an angle of 45° with the V.P. Draw its projections.

In the initial stage, assume the set-square to be in the V.P. with its hypotenuse perpendicular to the H.P.

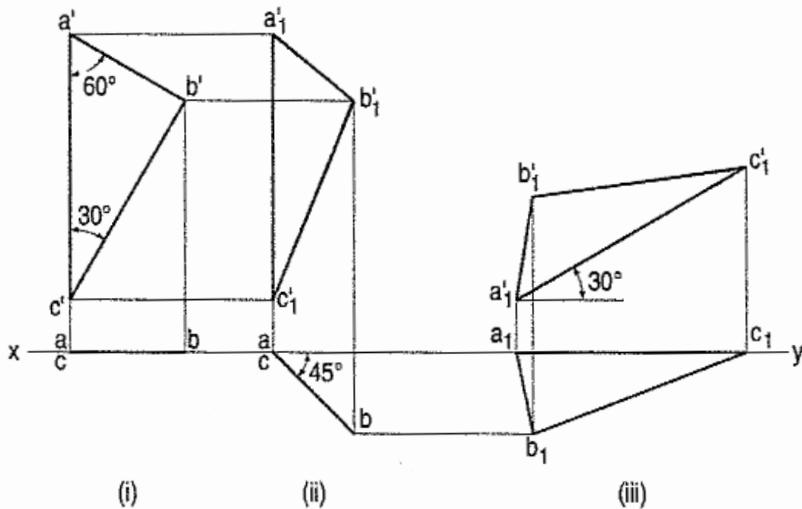


FIG. 12-16

- (i) Draw the front view $a'b'c'$ and project the top view ac in xy .
- (ii) Tilt ac around the end a so that it makes 45° angle with xy and project the front view $a'_1b'_1c'_1$.
- (iii) Reproduce the second front view $a'_1b'_1c'_1$ so that $a'_1b'_1$ makes an angle of 30° with xy . Project the final top view $a_1b_1c_1$.

Problem 12-11. (fig. 12-17): A thin rectangular plate of sides $60\text{ mm} \times 30\text{ mm}$ has its shorter side in the V.P. and inclined at 30° to the H.P. Project its top view if its front view is a square of 30 mm long sides.

As the front view of the plate is a square, its surface must be inclined to the V.P. Hence, assume the plate to be in the V.P. with its shorter edge perpendicular to the H.P.

- (i) Draw the front view $a'b'c'd'$ and project the top view ab in xy [fig. 12-17(i)].
- (ii) The line ab should be so inclined to xy that the front view becomes a square. Therefore, draw the square $a'_1b'_1c'_1d'_1$ of side equal to $a'd'$. With a as centre and radius equal to ab draw an arc cutting the projector through b'_1 at b . Then ab is the new top view.
- (iii) Reproduce the second front view in such a way that $a'_1d'_1$ makes 30° angle with xy . Project the final top view as shown.

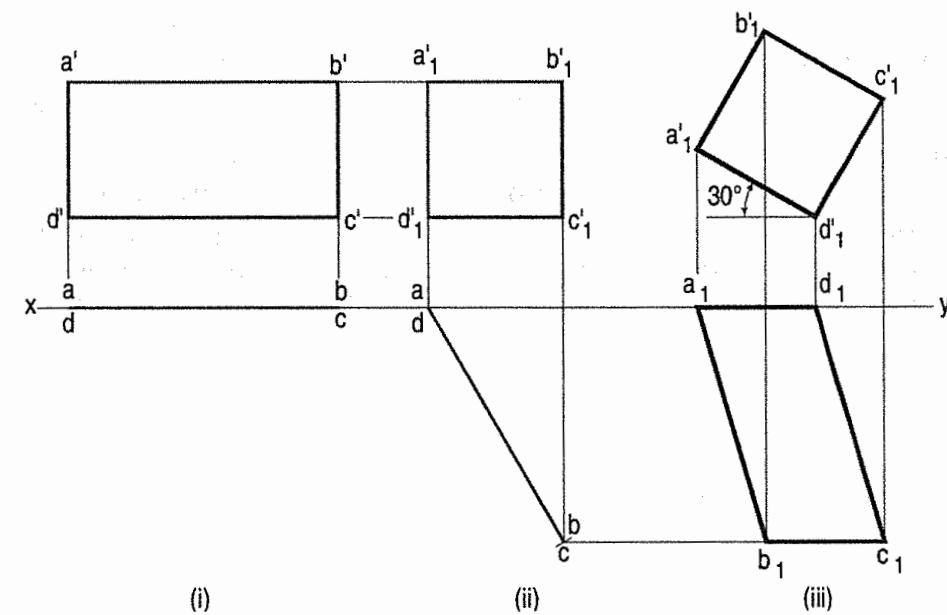
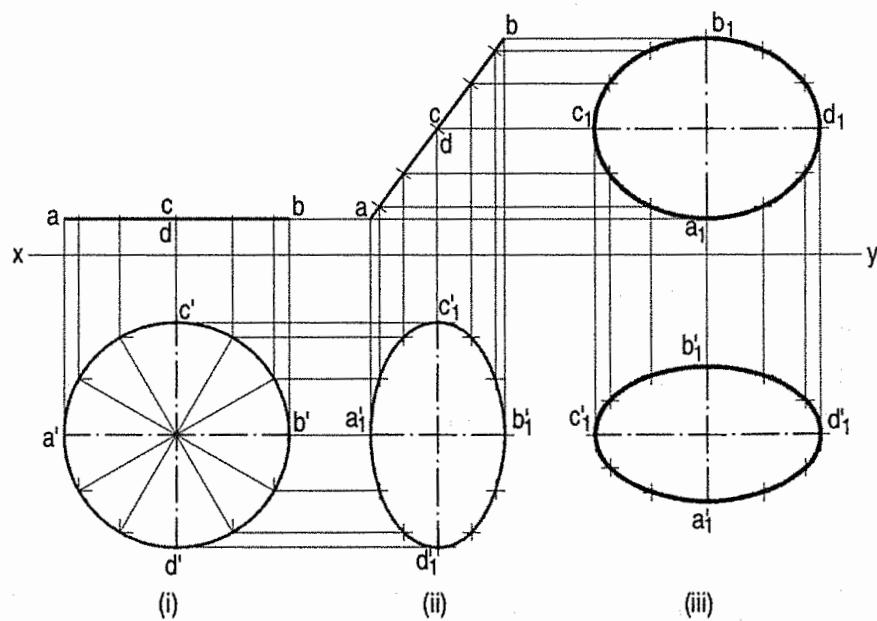


FIG. 12-17

Problem 12-12. (fig. 12-18): A circular plate of negligible thickness and 50 mm diameter appears as an ellipse in the front view, having its major axis 50 mm long and minor axis 30 mm long. Draw its top view when the major axis of the ellipse is horizontal.

As the plate is seen as an ellipse in the front view, its surface must be inclined to the V.P.



(Third-angle projection)

FIG. 12-18

- Therefore, assume it to be parallel to the V.P. and draw its front view and the top view.
- Turn the line ab so that its length in the front view becomes 30 mm, and project the front view. It will be an ellipse whose major axis is vertical.
- Reproduce this view so that the major axis $c'_1d'_1$ is horizontal, and project the required top view.

Problem 12-13. Fig. 12-19 shows a thin plate of negligible thickness. It rests on its PQ edge with its plane perpendicular to V.P. and inclined 40° to the H.P. Draw its projections.

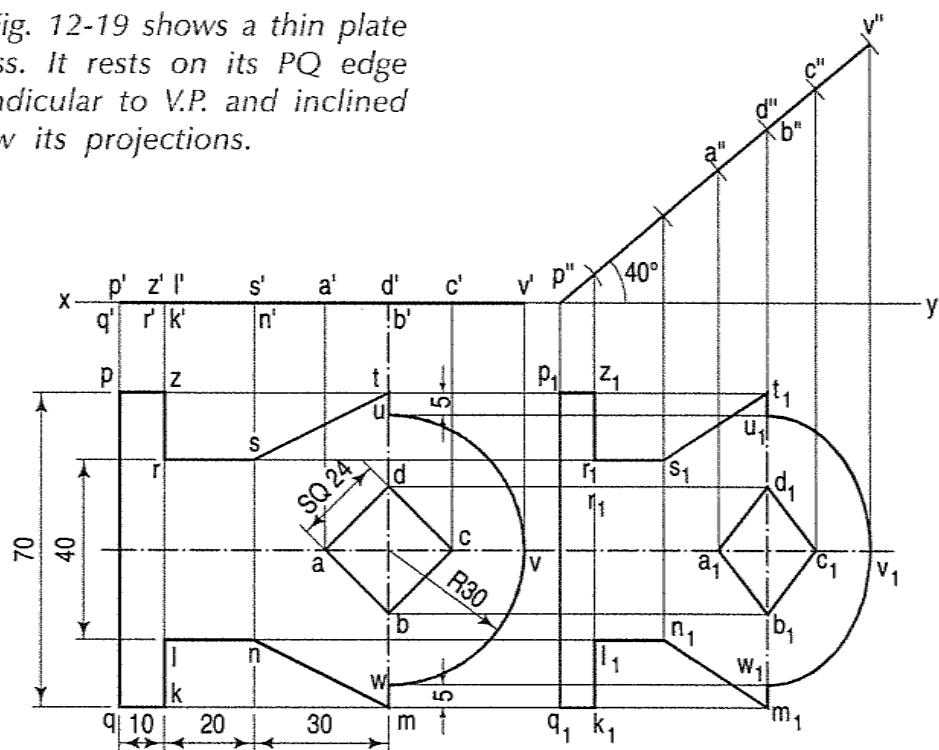


FIG. 12-19

- Keep the plane of plate in the H.P. and draw the projections as shown.
- Tilt front view at p'' making an angle of 40° .
- Project p'', a'', d'', c'', v'' etc. in the top view. Draw horizontal projectors intersecting previously drawn projectors from the front view. Join by smooth curve to complete the top view.

Problem 12-14. (fig. 12-20): A pentagonal plate of 45 mm side has a circular hole of 40 mm diameter in its centre. The plane stands on one of its sides on the H.P. with its plane perpendicular to V.P. and 45° inclined to the H.P. Draw the projections.

- Keep the plane of plate in the horizontal plane.
- Draw top view and front view as shown.
- Tilt the front view $a'' \dots d''$ at a'' making an angle of 45° . Draw the projectors from various points $a'' \dots d''$.
- Draw horizontal projectors from the top view $abcd$ as shown. Join the intersection points and complete new top view a_1, b_1, c_1, d_1, e_1 , as shown.

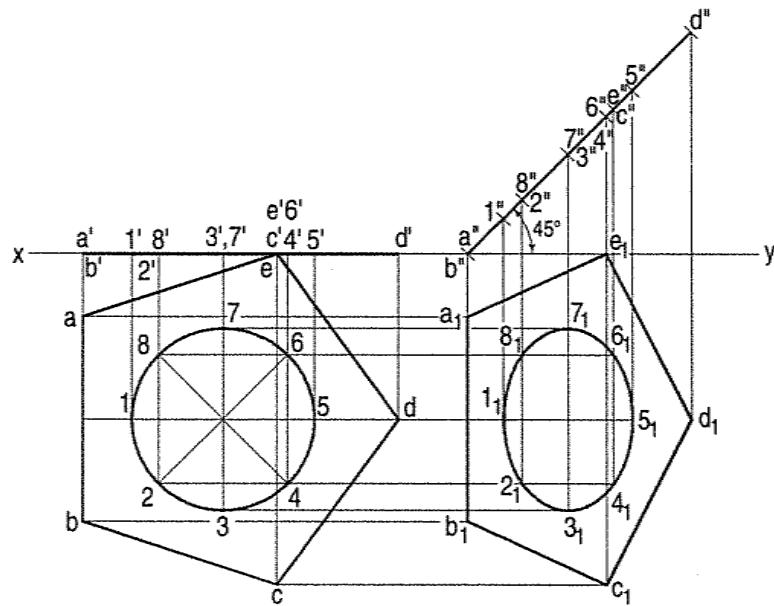


FIG. 12-20

Problem 12-15. (fig. 12-21): A thin circular plate of 70 mm diameter is resting on its circumference such that its plane is inclined 60° to the H.P. and 30° to the V.P. Draw the projections of the plate.

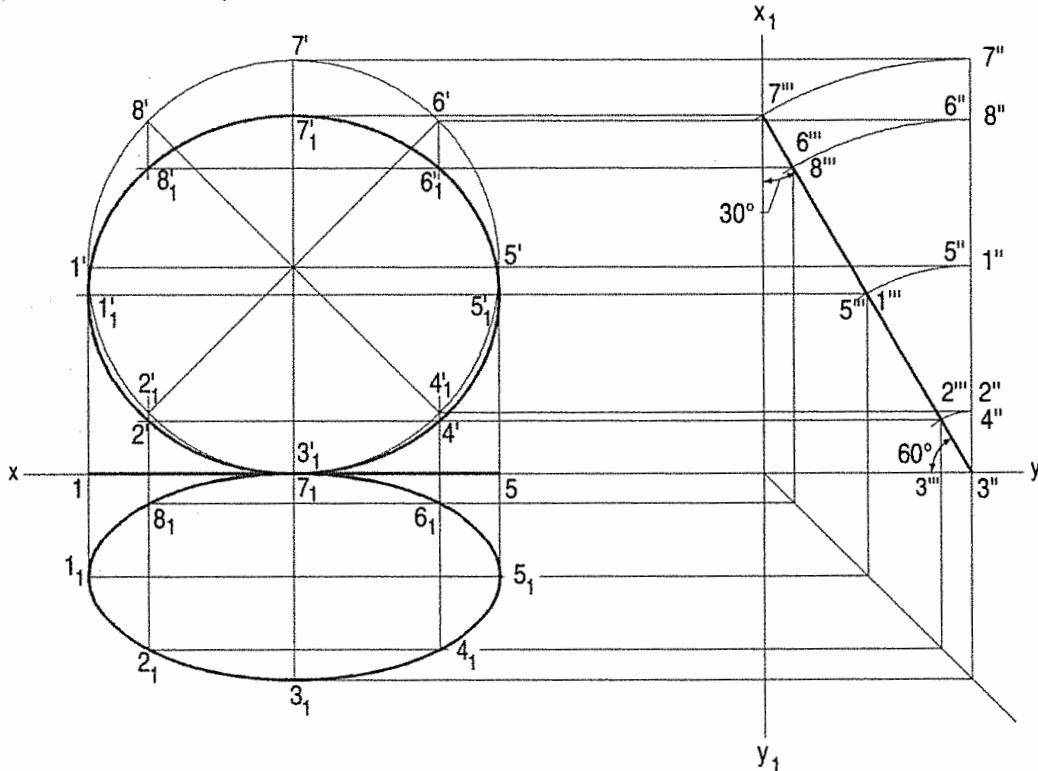


FIG. 12-21

- Draw the projection of the plate keeping its plane parallel to the V.P. as shown in fig. 12-21.
- Mark a reference line x_1y_1 perpendicular to xy line to represent the auxiliary plane which is at right angle to both the H.P. and the V.P.
- Divide the front view in eight parts and mark the points $1', 2', \dots, 8'$. Project these points on the side view as $1'', 2'', \dots, 8''$.
- Tilt the side view $3'' 7''$ such that it touches the x_1y_1 line and also makes 60° with the xy line.
- Complete the projection as shown in fig. 12-21.

Problem 12-16. (fig. 12-22): $PQRS$ is a rhombus having diagonal $PR = 60$ mm and $QS = 40$ mm and they are perpendicular to each other. The plane of the rhombus is inclined with H.P. such that its top view appears to be square. The top view of PR makes 30° with the V.P. Draw its projections and determine inclination of the plane with the H.P.

- Assume that the rhombus is lying in H.P. with its longest diagonal parallel to xy line.
- Draw the plans of diagonals $PR = 60$ mm and $QS = 40$ mm (true length) perpendicular each other as shown.
- Join points p, q, r and s . It is top view of the rhombus. Project the points p, q, r and s in the xy line. It is front view of the rhombus points p', q', r' and s' in the xy line as the plane of rhombus is perpendicular to the V.P.

- (iv) PR and QS are lying in H.P. pr and qs are true length. As the plane of the rhombus is inclined to H.P., the top view of the rhombus is going to be a square. But diagonal qs does not change in the length as it is perpendicular to V.P.
- (v) Draw the projectors from the points p , q and r , s parallel to the xy . From q_1 and s_1 draw square $p_1 q_1 r_1 s_1$ such that $s_1 q_1 = p_1 r_1$ as shown in fig. 12-22.
- (vi) Draw vertical projectors from p_1 , q_1 , r_1 and s_1 .
- (vii) Projector of p_1 intersects at p'' in xy line. Taking p'' as centre and the radius equal to 60 mm ($p'r'$), draw the arc to intersect the vertical projectors of r_1 at r'' . Join $p''r''$. Measure angle of $p''r''$ with xy line.
- (viii) Tilt diagonal p_1r_1 at 30° with xy and reproduce square $p_2q_2r_2s_2$. Draw vertical projectors from p_2 , q_2 , r_2 and s_2 to intersect the horizontal projectors from p'' , q'' , r'' and s'' at p''' , q''' , r''' and s''' . Join the points p''', q''', r''' and s''' as shown.

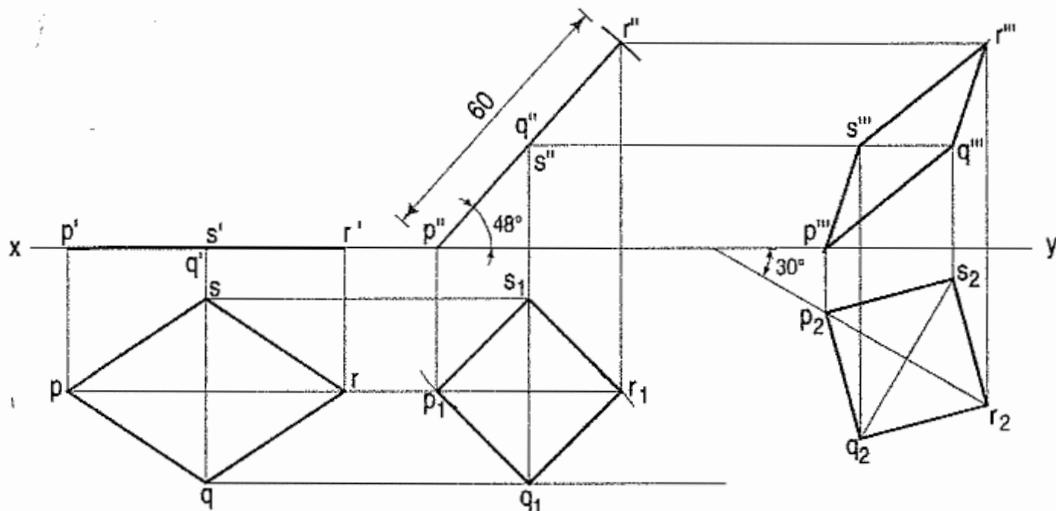


FIG. 12-22

EXERCISES 12

1. Draw an equilateral triangle of 75 mm side and inscribe a circle in it. Draw the projections of the figure, when its plane is vertical and inclined at 30° to the V.P. and one of the sides of the triangle is inclined at 45° to the H.P.
2. A regular hexagon of 40 mm side has a corner in the H.P. Its surface is inclined at 45° to the H.P. and the top view of the diagonal through the corner which is in the H.P. makes an angle of 60° with the V.P. Draw its projections.
3. Draw the projections of a regular pentagon of 40 mm side, having its surface inclined at 30° to the H.P. and a side parallel to the H.P. and inclined at an angle of 60° to the V.P.
4. Draw the projections of a rhombus having diagonals 125 mm and 50 mm long, the smaller diagonal of which is parallel to both the principal planes, while the other is inclined at 30° to the H.P.

5. Draw a regular hexagon of 40 mm side, with its two sides vertical. Draw a circle of 40 mm diameter in its centre. The figure represents a hexagonal plate with a hole in it and having its surface parallel to the V.P. Draw its projections when the surface is vertical and inclined at 30° to the V.P. Assume the thickness of the plate to be equal to that of a line.
6. Draw the projections of a circle of 75 mm diameter having the end A of the diameter AB in the H.P., the end B in the V.P., and the surface inclined at 30° to the H.P. and at 60° to the V.P.
7. A semi-circular plate of 80 mm diameter has its straight edge in the V.P. and inclined at 45° to the H.P. The surface of the plate makes an angle of 30° with the V.P. Draw its projections.
8. The top view of a plate, the surface of which is perpendicular to the V.P. and inclined at 60° to the H.P. is a circle of 60 mm diameter. Draw its three views.
9. A plate having shape of an isosceles triangle has base 50 mm long and altitude 70 mm. It is so placed that in the front view it is seen as an equilateral triangle of 50 mm sides and one side inclined at 45° to xy. Draw its top view.
10. Draw a rhombus of diagonals 100 mm and 60 mm long, with the longer diagonal horizontal. The figure is the top view of a square of 100 mm long diagonals, with a corner on the ground. Draw its front view and determine the angle which its surface makes with the ground.
11. A composite plate of negligible thickness is made-up of a rectangle 60 mm \times 40 mm, and a semi-circle on its longer side. Draw its projections when the longer side is parallel to the H.P. and inclined at 45° to the V.P., the surface of the plate making 30° angle with the H.P.
12. A 60° set-square of 125 mm longest side is so kept that the longest side is in the H.P. making an angle of 30° with the V.P. and the set-square itself inclined at 45° to the H.P. Draw the projections of the set-square.
13. A plane figure is composed of an equilateral triangle ABC and a semi-circle on AC as diameter. The length of the side AB is 50 mm and is parallel to the V.P. The corner B is 20 mm behind the V.P. and 15 mm below the H.P. The plane of the figure is inclined at 45° to the H.P. Draw the projections of the plane figure.
14. An equilateral triangle ABC having side length as 50 mm is suspended from a point O on the side AB 15 mm from A in such a way that the plane of the triangle makes an angle of 60° with the V.P. The point O is 20 mm below the H.P. and 40 mm behind the V.P. Draw the projections of the triangle.
15. PQRS and ABCD are two square thin plates with their diagonals measuring 30 mm and 60 mm. They are touching the H.P. with their corners P and A respectively, and touching each other with their corresponding opposite corners R and C. If the plates are perpendicular to each other and perpendicular to V.P. also, draw their projections and determine the length of their sides.

Chapter

13



PROJECTIONS OF SOLIDS

13-0. INTRODUCTION



A solid has three dimensions, viz. length, breadth and thickness. To represent a solid on a flat surface having only length and breadth, at least two orthographic views are necessary. Sometimes, additional views projected on auxiliary planes become necessary to make the description of a solid complete.

This chapter deals with the following topics:

1. Types of solids.
2. Projections of solids in simple positions.
 - (a) Axis perpendicular to the H.P.
 - (b) Axis perpendicular to the V.P.
 - (c) Axis parallel to both the H.P. and the V.P.
3. Projections of solids with axes inclined to one of the reference planes and parallel to the other.
 - (a) Axis inclined to the V.P. and parallel to the H.P.
 - (b) Axis inclined to the H.P. and parallel to the V.P.
4. Projections of solids with axes inclined to both the H.P. and the V.P.
5. Projections of spheres.

13-1. TYPES OF SOLIDS



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 28 for the types of solids.

Solids may be divided into two main groups:

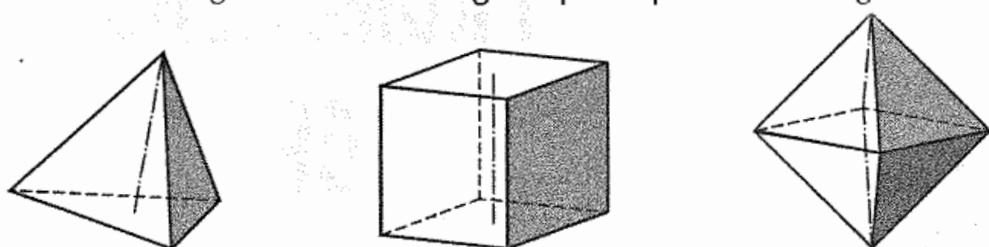
- (1) Polyhedra
- (2) Solids of revolution.

(1) **Polyhedra:** A polyhedron is defined as a solid bounded by planes called faces. When all the faces are equal and regular, the polyhedron is said to be regular.

There are seven regular polyhedra which may be defined as stated below:

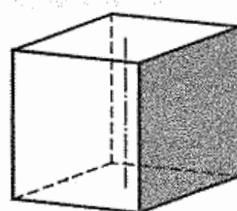
- (i) *Tetrahedron* (fig. 13-1): It has four equal faces, each an equilateral triangle.
- (ii) *Cube or hexahedron* (fig. 13-2): It has six faces, all equal squares.

(iii) *Octahedron* (fig. 13-3): It has eight equal equilateral triangles as faces.



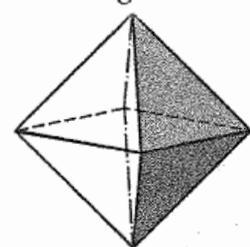
Tetrahedron

FIG. 13-1



Cube

FIG. 13-2

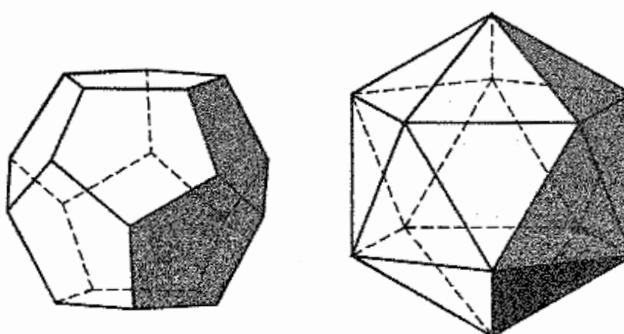


Octahedron

FIG. 13-3

(iv) *Dodecahedron* (fig. 13-4): It has twelve equal and regular pentagons as faces.

(v) *Icosahedron* (fig. 13-5): It has twenty faces, all equal equilateral triangles.



Dodecahedron

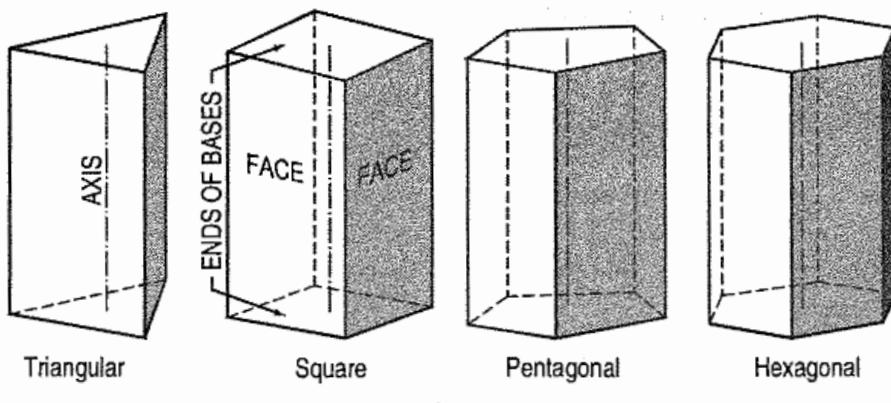
FIG. 13-4

Icosahedron

FIG. 13-5

(vi) *Prism*: This is a polyhedron having two equal and similar faces called its ends or bases, parallel to each other and joined by other faces which are parallelograms. The imaginary line joining the centres of the bases is called the axis.

A right and regular prism (fig. 13-6) has its axis perpendicular to the bases. All its faces are equal rectangles.



Prisms

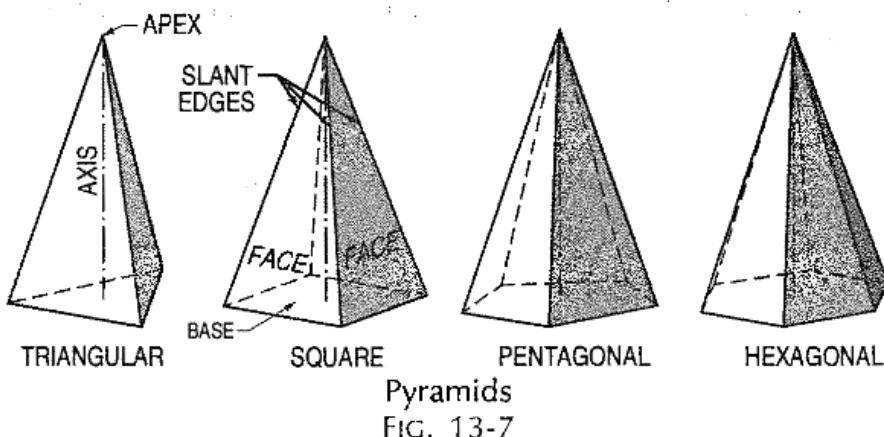
FIG. 13-6

(vii) *Pyramid*: This is a polyhedron having a plane figure as a base and a number of triangular faces meeting at a point called the vertex or apex. The imaginary line joining the apex with the centre of the base is its axis.

A right and regular pyramid (fig. 13-7) has its axis perpendicular to the base which is a regular plane figure. Its faces are all equal isosceles triangles.

Oblique prisms and pyramids have their axes inclined to their bases.

Prisms and pyramids are named according to the shape of their bases, as triangular, square, pentagonal, hexagonal etc.



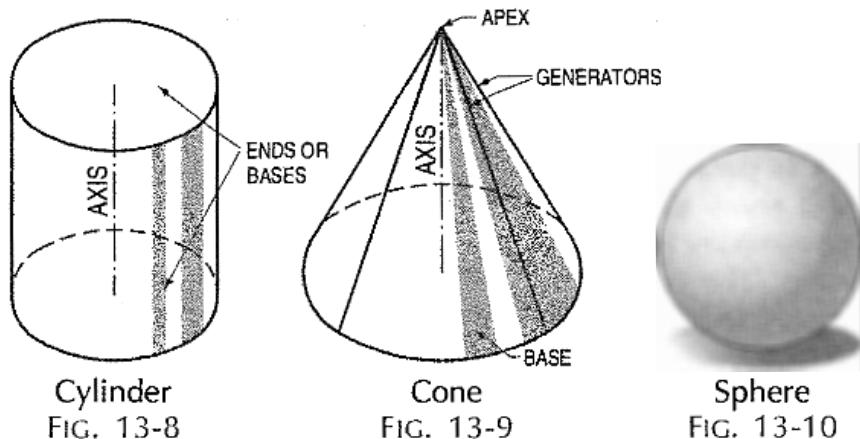
Pyramids

FIG. 13-7

(2) Solids of revolution:

- Cylinder* (fig. 13-8): A *right circular cylinder* is a solid generated by the revolution of a rectangle about one of its sides which remains fixed. It has two equal circular bases. The line joining the centres of the bases is the axis. It is perpendicular to the bases.
- Cone* (fig. 13-9): A *right circular cone* is a solid generated by the revolution of a right-angled triangle about one of its perpendicular sides which is fixed.

It has one circular base. Its axis joins the apex with the centre of the base to which it is perpendicular. Straight lines drawn from the apex to the circumference of the base-circle are all equal and are called *generators* of the cone. The length of the generator is the slant height of the cone.



Cylinder

FIG. 13-8

Cone

FIG. 13-9

Sphere

FIG. 13-10

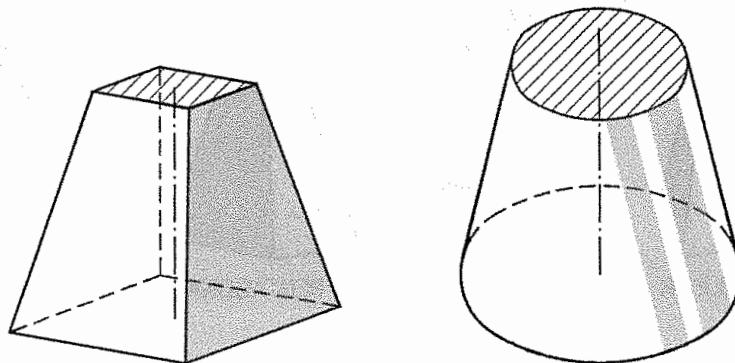
- Sphere* (fig. 13-10): A *sphere* is a solid generated by the revolution of a semi-circle about its diameter as the axis. The mid-point of the diameter is the centre of the sphere. All points on the surface of the sphere are equidistant from its centre.

Oblique cylinders and cones have their axes inclined to their bases.

- Frustum*: When a pyramid or a cone is cut by a plane parallel to its base, thus removing the top portion, the remaining portion is called its *frustum* (fig. 13-11).

- (v) *Truncated*: When a solid is cut by a plane inclined to the base it is said to be *truncated*.

In this book mostly right and regular solids are dealt with. Hence, when a solid is named without any qualification, it should be understood as being right and regular.



Frustums

FIG. 13-11

13-2. PROJECTIONS OF SOLIDS IN SIMPLE POSITIONS

A solid in simple position may have its axis perpendicular to one reference plane or parallel to both. When the axis is perpendicular to one reference plane, it is parallel to the other. Also, when the axis of a solid is perpendicular to a plane, its base will be parallel to that plane. We have already seen that when a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

Therefore, the projection of a solid on the plane to which its axis is perpendicular, will show the true shape and size of its base.

Hence, when the axis is perpendicular to the ground, i.e. to the H.P., the top view should be drawn first and the front view projected from it.

When the axis is perpendicular to the V.P., beginning should be made with the front view. The top view should then be projected from it.

When the axis is parallel to both the H.P. and the V.P., neither the top view nor the front view will show the actual shape of the base. In this case, the projection of the solid on an auxiliary plane perpendicular to both the planes, viz. the side view must be drawn first. The front view and the top view are then projected from the side view. The projections in such cases may also be drawn in two stages.

(1) Axis perpendicular to the H.P.:

Problem 13-1. (fig. 13-12): Draw the projections of a triangular prism, base 40 mm side and axis 50 mm long, resting on one of its bases on the H.P. with a vertical face perpendicular to the V.P.

- (i) As the axis is perpendicular to the ground i.e. the H.P. begin with the top view. It will be an equilateral triangle of sides 40 mm long, with one of its

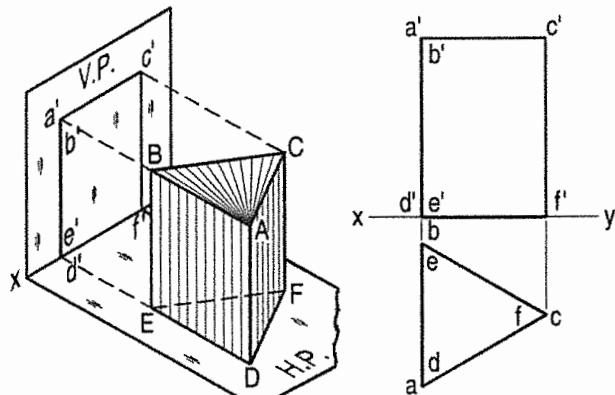


FIG. 13-12

sides perpendicular to xy . Name the corners as shown, thus completing the top view. The corners d , e and f are hidden and coincide with the top corners a , b and c respectively.

- (ii) Project the front view, which will be a rectangle. Name the corners. The line $b'e'$ coincides with $a'd'$.

Problem 13-2. (fig. 13-13): Draw the projections of a pentagonal pyramid, base 30 mm edge and axis 50 mm long, having its base on the H.P. and an edge of the base parallel to the V.P. Also draw its side view.

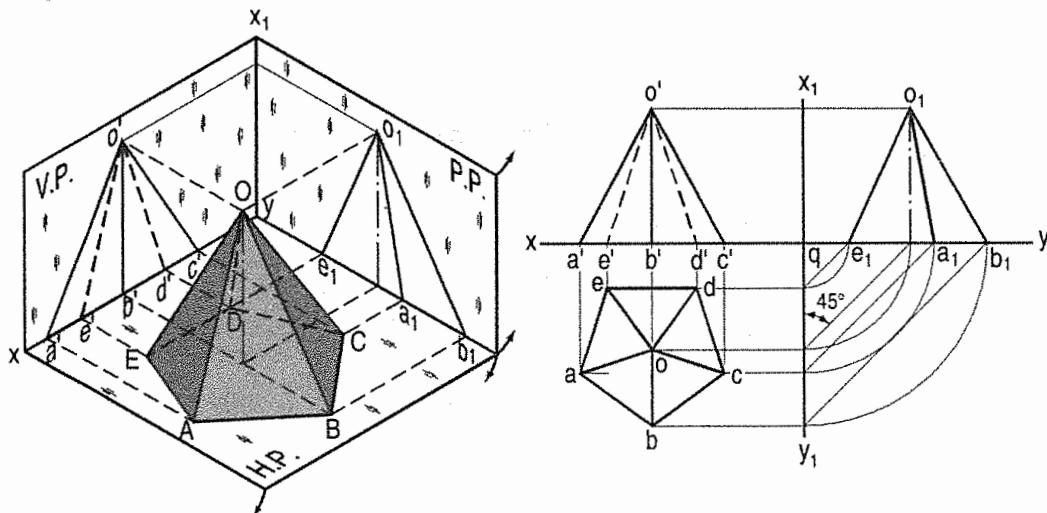


FIG. 13-13

- Assume the side DE which is nearer the V.P., to be parallel to the V.P. as shown in the pictorial view.
- In the top view, draw a regular pentagon $abcde$ with ed parallel to and nearer xy . Locate its centre o and join it with the corners to indicate the slant edges.
- Through o , project the axis in the front view and mark the apex o' , 50 mm above xy . Project all the corners of the base on xy . Draw lines $o'a'$, $o'b'$ and $o'c'$ to show the visible edges. Show $o'd'$ and $o'e'$ for the hidden edges as dashed lines.
- For the side view looking from the left, draw a new reference line x_1y_1 perpendicular to xy and to the right of the front view. Project the side view on it, horizontally from the front view as shown. The respective distances of all the points in the side view from x_1y_1 , should be equal to their distances in the top view from xy . This is done systematically as explained below:
- From each point in the top view, draw horizontal lines upto x_1y_1 . Then draw lines inclined at 45° to x_1y_1 (or xy) as shown. Or, with q , the point of intersection between xy and x_1y_1 as centre, draw quarter circles. Project up all the points to intersect the corresponding horizontal lines from the front view and complete the side view as shown in the figure. Lines o_1d_1 and o_1c_1 coincide with o_1e_1 and o_1a_1 respectively.

Problem 13-3. (fig. 13-14): Draw the projections of (i) a cylinder, base 40 mm diameter and axis 50 mm long, and (ii) a cone, base 40 mm diameter and axis 50 mm long, resting on the H.P. on their respective bases.

- (i) Draw a circle of 40 mm diameter in the top view and project the front view which will be a rectangle [fig. 13-14(ii)].
 - (ii) Draw the top view [fig. 13-14(iii)]. Through the centre o, project the apex o' , 50 mm above xy . Complete the triangle in the front view as shown.

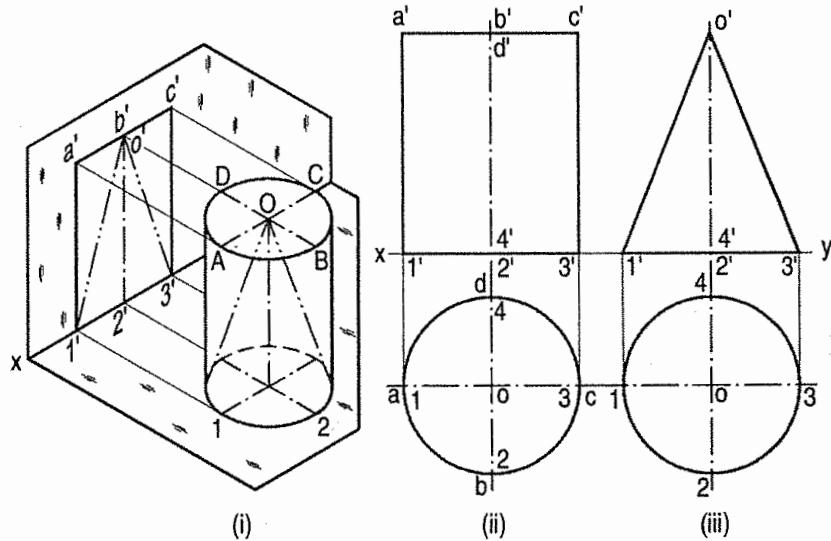


FIG. 13-14

In the pictorial view [fig. 13-14(i)], the cone is shown as contained by the cylinder.

Problem 13-4. (fig. 13-15): A cube of 50 mm long edges is resting on the H.P. with its vertical faces equally inclined to the V.P. Draw its projections.

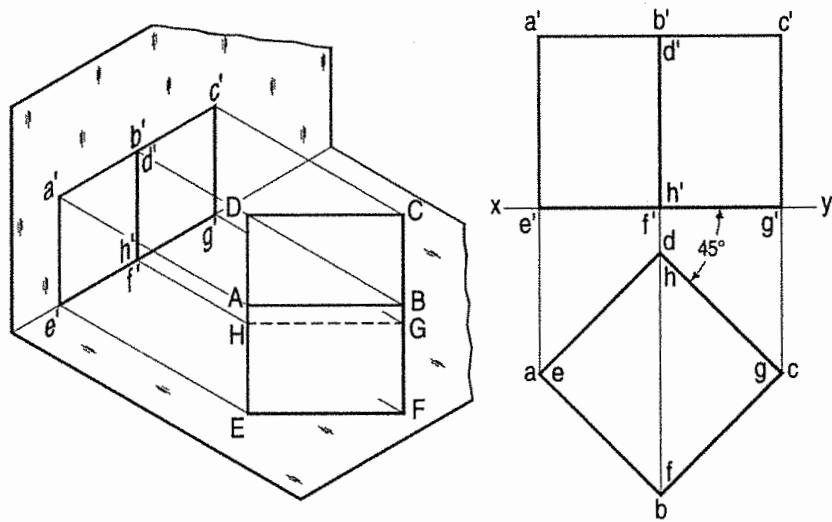


FIG. 13-15

Begin with the top view.

- (i) Draw a square $abcd$ with a side making 45° angle with xy .
 - (ii) Project up the front view. The line $d' h'$ will coincide with $b' f'$.

Problem 13-5. (fig. 13-16): Draw the projections of a hexagonal pyramid, base 30 mm side and axis 60 mm long, having its base on the H.P. and one of the edges of the base inclined at 45° to the V.P.

- In the top view, draw a line af 30 mm long and inclined at 45° to xy . Construct a regular hexagon on af . Mark its centre o and complete the top view by drawing lines joining it with the corners.
- Project up the front view as described in problem 13-2, showing the line $o'e'$ and $o'f'$ for hidden edges as dashed lines.

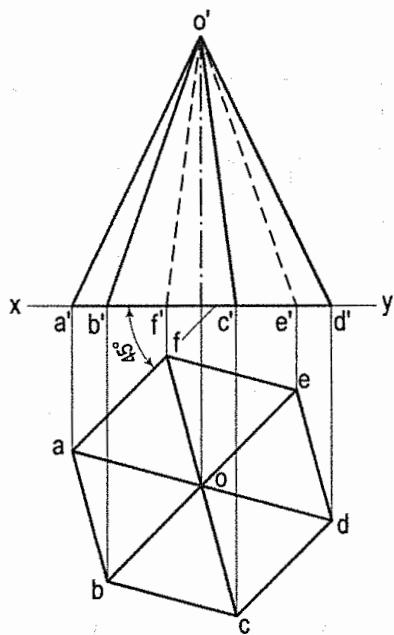


FIG. 13-16

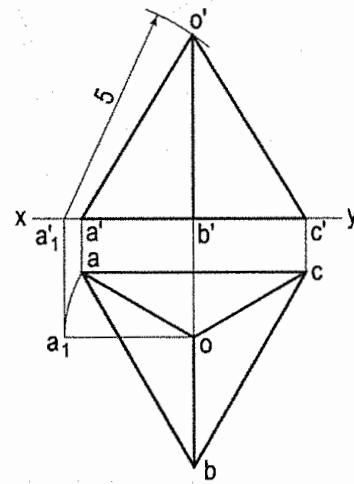


FIG. 13-17

Problem 13-6. (fig. 13-17): A tetrahedron of 5 cm long edges is resting on the H.P. on one of its faces, with an edge of that face parallel to the V.P. Draw its projections and measure the distance of its apex from the ground.

- All the four faces of the tetrahedron are equal equilateral triangles of 5 cm side.
- Draw an equilateral triangle abc in the top view with one side, say ac , parallel to xy . Locate its centre o and join it with the corners.
 - In the front view, the corners a' , b' and c' will be in xy . The apex o' will lie on the projector through o so that its true distance from the corners of the base is equal to 5 cm.
 - To locate o' , make oa (or ob or oc) parallel to xy . Project a_1 to a'_1 on xy . With a'_1 as centre and radius equal to 5 cm cut the projector through o in o' . Draw lines $o'a'$, $o'b'$ and $o'c'$ to complete the front view. $o'b'$ will be the distance of the apex from the ground.

(2) Axis perpendicular to the V.P.:

Problem 13-7. (fig. 13-18): A hexagonal prism has one of its rectangular faces parallel to the H.P. Its axis is perpendicular to the V.P. and 3.5 cm above the ground.

Draw its projections when the nearer end is 2 cm in front of the V.P. Side of base 2.5 cm long; axis 5 cm long.

- Begin with the front view. Construct a regular hexagon of 2.5 cm long sides with its centre 3.5 cm above xy and one side parallel to it.
- Project down the top view, keeping the line for nearer end, viz. 1-4, 2 cm below xy .

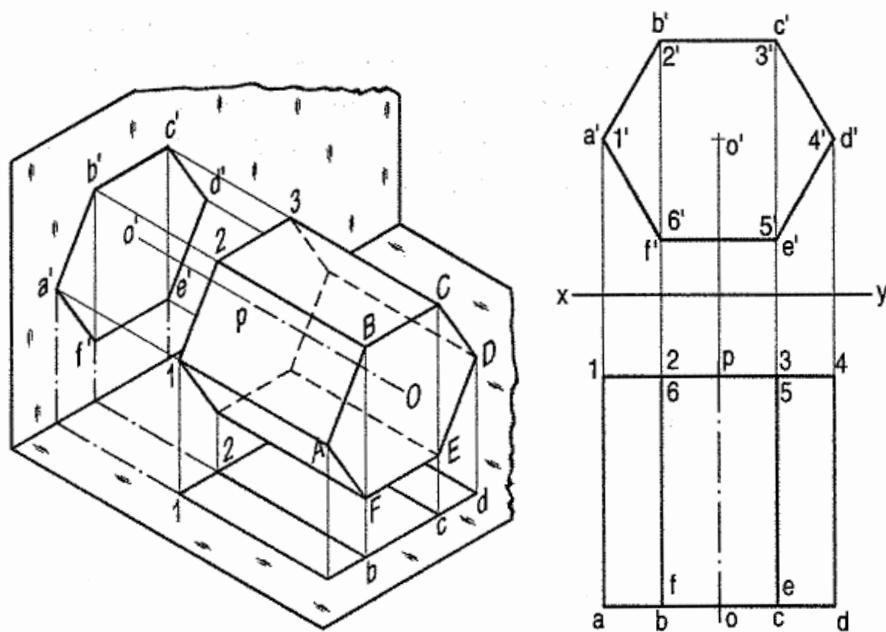


FIG. 13-18

Problem 13-8. (fig. 13-19): A square pyramid, base 40 mm side and axis 65 mm long, has its base in the V.P. One edge of the base is inclined at 30° to the H.P. and a corner contained by that edge is on the H.P. Draw its projections.

- Draw a square in the front view with the corner d' in xy and the side $d'c'$ inclined at 30° to it. Locate the centre o' and join it with the corners of the square.
- Project down all the corners in xy (because the base is in the V.P.). Mark the apex o on a projector through o' . Draw lines for the slant edges and complete the top view.

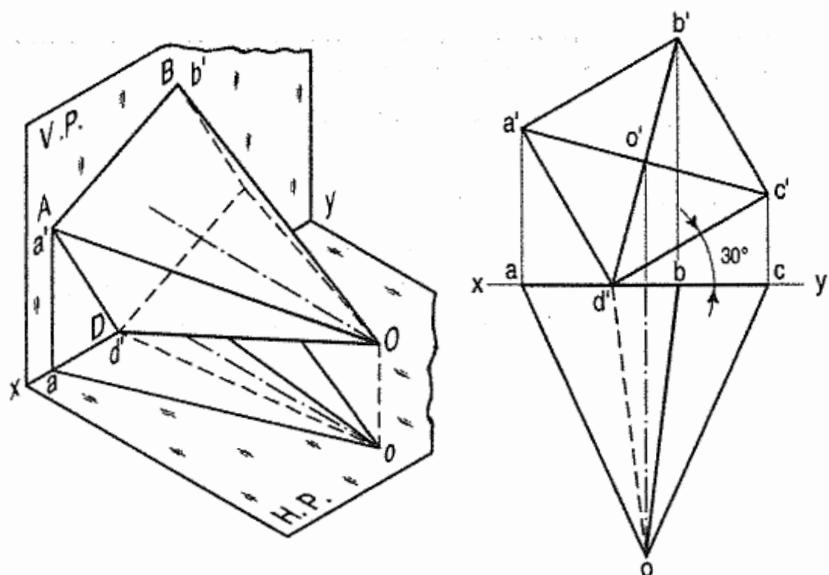


FIG. 13-19

(3) Axis parallel to both the H.P. and the V.P.:

Problem 13-9. (fig. 13-20): A triangular prism, base 40 mm side and height 65 mm is resting on the H.P. on one of its rectangular faces with the axis parallel to the V.P. Draw its projections.

As the axis is parallel to both the planes, begin with the side view.

- Draw an equilateral triangle representing the side view, with one side in xy .
- Project the front view horizontally from this triangle.
- Project down the top view from the front view and the side view, as shown.

This problem can also be solved in two stages as explained in the next article.

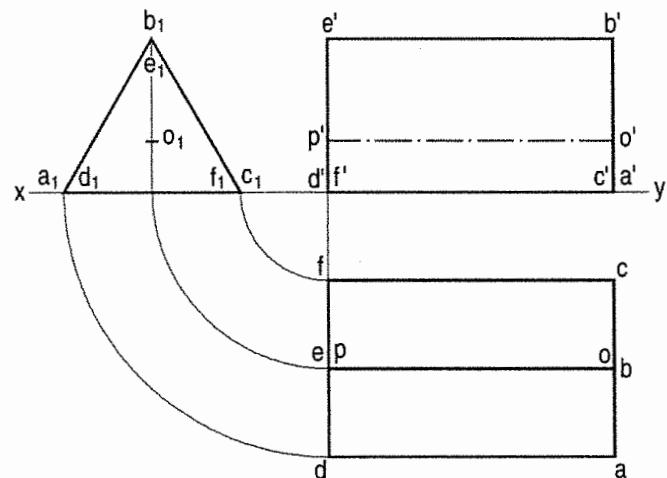


FIG. 13-20

EXERCISES 13(a)

Draw the projections of the following solids, situated in their respective positions, taking a side of the base 40 mm long or the diameter of the base 50 mm long and the axis 65 mm long.

- A hexagonal pyramid, base on the H.P. and a side of the base parallel to and 25 mm in front of the V.P.
- A square prism, base on the H.P., a side of the base inclined at 30° to the V.P. and the axis 50 mm in front of the V.P.
- A triangular pyramid, base on the H.P. and an edge of the base inclined at 45° to the V.P.; the apex 40 mm in front of the V.P.
- A cylinder, axis perpendicular to the V.P. and 40 mm above the H.P., one end 20 mm in front of the V.P.
- A pentagonal prism, a rectangular face parallel to and 10 mm above the H.P., axis perpendicular to the V.P. and one base in the V.P.
- A square pyramid, all edges of the base equally inclined to the H.P. and the axis parallel to and 50 mm away from both the H.P. and the V.P.
- A cone, apex in the H.P. axis vertical and 40 mm in front of the V.P.
- A pentagonal pyramid, base in the V.P. and an edge of the base in the H.P.

13-3. PROJECTIONS OF SOLIDS WITH AXES INCLINED TO ONE OF THE REFERENCE PLANES AND PARALLEL TO THE OTHER

When a solid has its axis inclined to one plane and parallel to the other, its projections are drawn in two stages.

- In the initial stage, the solid is assumed to be in simple position, i.e. its axis perpendicular to one of the planes.

If the axis is to be inclined to the ground, i.e. the H.P., it is assumed to be perpendicular to the H.P. in the initial stage. Similarly, if the axis is to be inclined to the V.P., it is kept perpendicular to the V.P. in the initial stage.

Moreover

- (i) if the solid has an edge of its base parallel to the H.P. or in the H.P. or on the ground, that edge should be kept perpendicular to the V.P.; if the edge of the base is parallel to the V.P. or in the V.P., it should be kept perpendicular to the H.P.
- (ii) If the solid has a corner of its base in the H.P. or on the ground, the sides of the base containing that corner should be kept equally inclined to the V.P.; if the corner is in the V.P., they should be kept equally inclined to the H.P.

(2) Having drawn the projections of the solid in its simple position, the final projections may be obtained by one of the following two methods:

(i) **Alteration of position:** The position of one of the views is altered as required and the other view projected from it.

(ii) **Alteration of reference line or auxiliary plane:** A new reference line is drawn according to the required conditions, to represent an auxiliary plane and the final view projected on it.

In the first method, the reproduction of a view accurately in the altered position is likely to take considerable time, specially, when the solid has curved surfaces or too many edges and corners. In such cases, it is easier and more convenient to adopt the second method. Sufficient care must however be taken in transferring the distances of various points from their respective reference lines.

After determining the positions of all the points for the corners in the final view, difficulty is often felt in completing the view correctly. The following sequence for joining the corners may be adopted:

- (a) Draw the lines for the edges of the visible base. The base, which (compared to the other base) is further away from xy in one view, will be fully visible in the other view.
- (b) Draw the lines for the longer edges. The lines which pass through the figure of the visible base should be dashed lines.
- (c) Draw the lines for the edges of the other base.

It should always be remembered that, when two lines representing the edges cross each other, one of them must be hidden and should therefore be drawn as a dashed line.

13-3-1. AXIS INCLINED TO THE V.P. AND PARALLEL TO THE H.P.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 29 for the following problem.

Problem 13-10. (fig. 13-21): *Draw the projections of a pentagonal prism, base 25 mm side and axis 50 mm long, resting on one of its rectangular faces on the H.P., with the axis inclined at 45° to the V.P.*

In the simple position, assume the prism to be on one of its faces on the ground with the axis perpendicular to the V.P.

Draw the pentagon in the front view with one side in xy and project the top view [fig. 13-21(i)].

The shape and size of the figure in the top view will not change, so long as the prism has its face on the H.P. The respective distances of all the corners in the front view from xy will also remain constant.

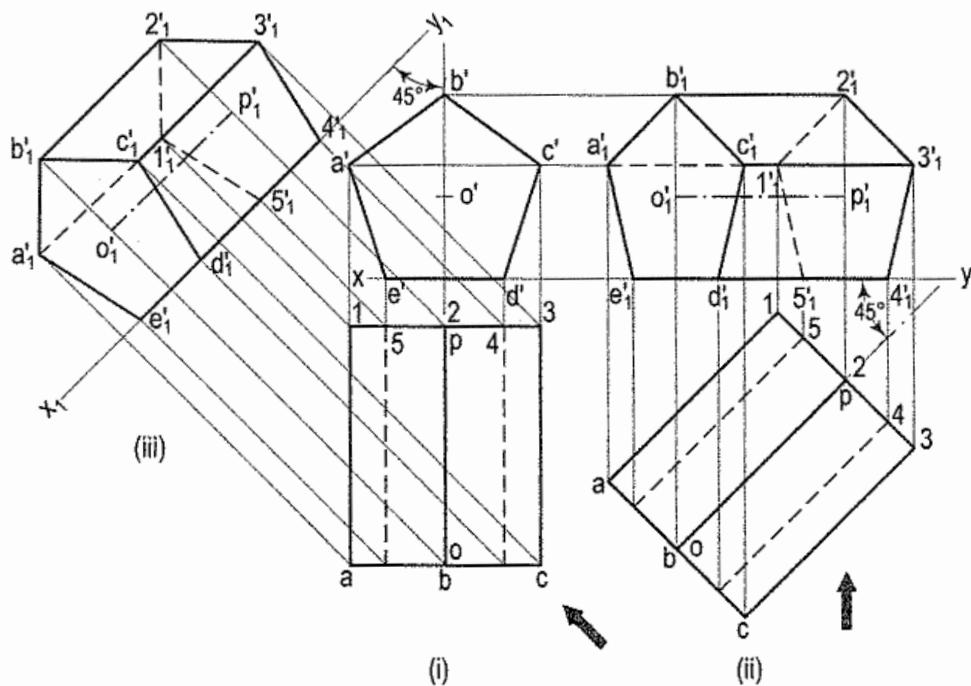


FIG. 13-21

Method I: [fig. 13-21(ii)]:

- Alter the position of the top view, i.e. reproduce it so that the axis is inclined at 45° to xy . Project all the points upwards from this top view and horizontally from the first front view, e.g. a vertical from a intersecting a horizontal from a' at a point a'_1 .
- Complete the pentagon $a'_1b'_1c'_1d'_1e'_1$ for the fully visible end of the prism. Next, draw the lines for the longer edges and finally, draw the lines for the edges of the other end. Note carefully that the lines $a'_11'_1$, $1'_12'_1$ and $1'_15'_1$ are dashed lines. $e'_15'_1$ is also hidden but it coincides with other visible lines.

Method II: [fig. 13-21(iii)]:

- Draw a new reference line x_1y_1 , making 45° angle with the top view of the axis, to represent an auxiliary vertical plane.
- Draw projectors from all the points in the top view perpendicular to x_1y_1 and on them, mark points keeping the distance of each point from x_1y_1 equal to its distance from xy in the front view. Join the points as already explained. The auxiliary front view and the top view are the required projections.

Problem 13-11. (fig. 13-22): Draw the projections of a cylinder 75 mm diameter and 100 mm long, lying on the ground with its axis inclined at 30° to the V.P. and parallel to the ground.

Adopt the same methods as in the previous problem. The ellipses for the ends should be joined by common tangents. Note that half of the ellipse for the hidden base will be drawn as dashed line.

Fig. 13-22(iii) shows the front view obtained by the method II.

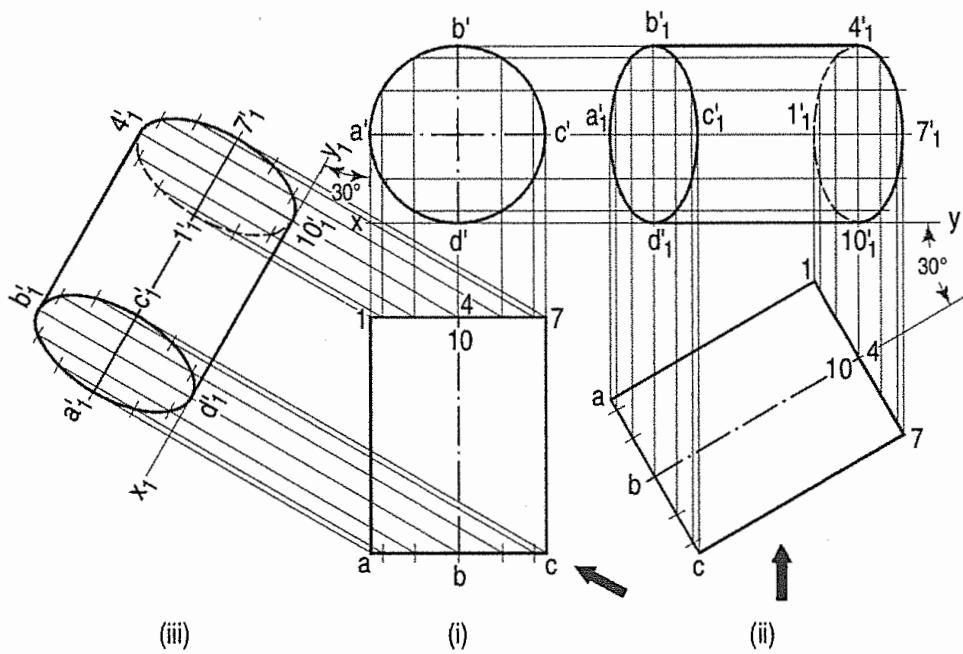


FIG. 13-22

13-3-2. AXIS INCLINED TO THE H.P. AND PARALLEL TO THE V.P.



Problem 13-12. (fig. 13-23): A hexagonal pyramid, base 25 mm side and axis 50 mm long, has an edge of its base on the ground. Its axis is inclined at 30° to the ground and parallel to the V.P. Draw its projections.

In the initial position assume the axis to be perpendicular to the H.P.

Draw the projections with the base in xy and its one edge perpendicular to the V.P. [fig. 13-23(i)].

If the pyramid is now tilted about the edge AF (or CD) the axis will become inclined to the H.P. but will remain parallel to the V.P. The distances of all the corners from the V.P. will remain constant.

The front view will not be affected except in its position in relation to xy . The new top view will have its corners at same distances from xy , as before.

Method I: [fig. 13-23(ii)]:

- Reproduce the front view so that the axis makes 30° angle with xy and the point a' remains in xy .
- Project all the points vertically from this front view and horizontally from the first top view. Complete the new top view by drawing (a) lines joining the apex o'_1 with the corners of the base and (b) lines for the edges of the base.

The base will be partly hidden as shown by dashed line a_1b_1 , e_1f_1 and f_1a_1 . Similarly o_1f_1 and o_1a_1 are also dashed lines.

Method II: [fig. 13-23(iii)]:

- Through a' draw a new reference line x_1y_1 inclined at 30° to the axis, to represent an auxiliary inclined plane.
- From the front view project the required top view on x_1y_1 , keeping the distance of each point from x_1y_1 equal to the distance of its first top view from xy , viz. $e_1q = eb'$ etc.

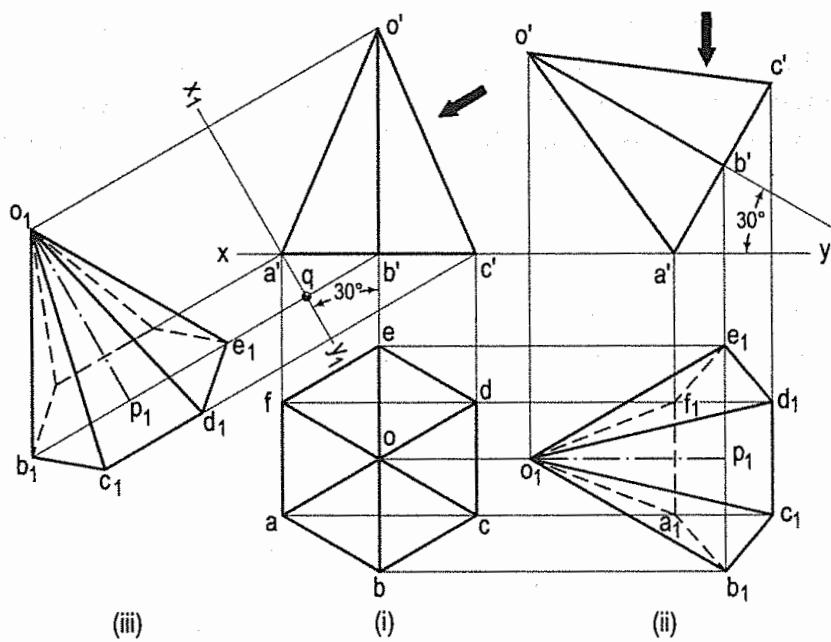


FIG. 13-23

Problem 13-13. (fig. 13-24): Draw the projections of a cone, base 75 mm diameter and axis 100 mm long, lying on the H.P. on one of its generators with the axis parallel to the V.P.

- Assuming the cone to be resting on its base on the ground, draw its projections.
- Re-draw the front view so that the line $o'7'$ (or $o'1'$) is in xy . Project the required top view as shown. The lines from o_1 should be tangents to the ellipse.

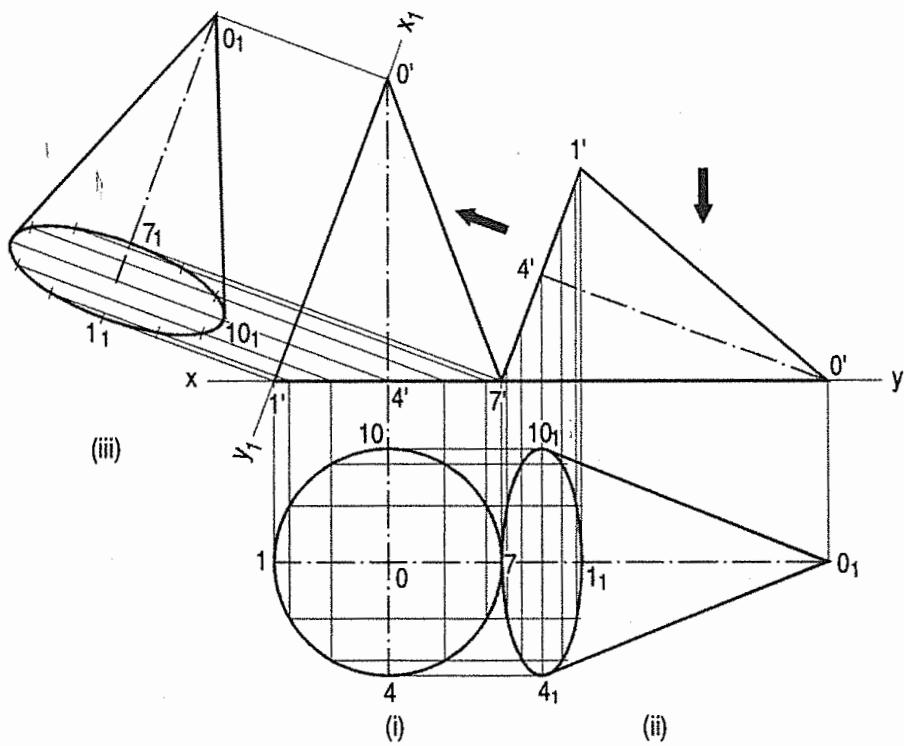


FIG. 13-24

The top view obtained by auxiliary-plane method is shown in fig. 13-24(iii). The new reference line $x'y_1$ is so drawn as to contain the generator $o'1'$ instead of $o'7'$ (for sake of convenience). The cone is thus lying on the generator $o'1'$. Note that $1'1_1 = 1'1$, $o'o_1 = 4'o$ etc. Also note that the base is fully visible in both the methods.

Problem 13-14. The projections of a cylinder resting centrally on a hexagonal prism are given in fig. 13-25(i). Draw its auxiliary front view on a reference line inclined at 60° to xy .

See fig. 13-25(ii) which is self-explanatory.

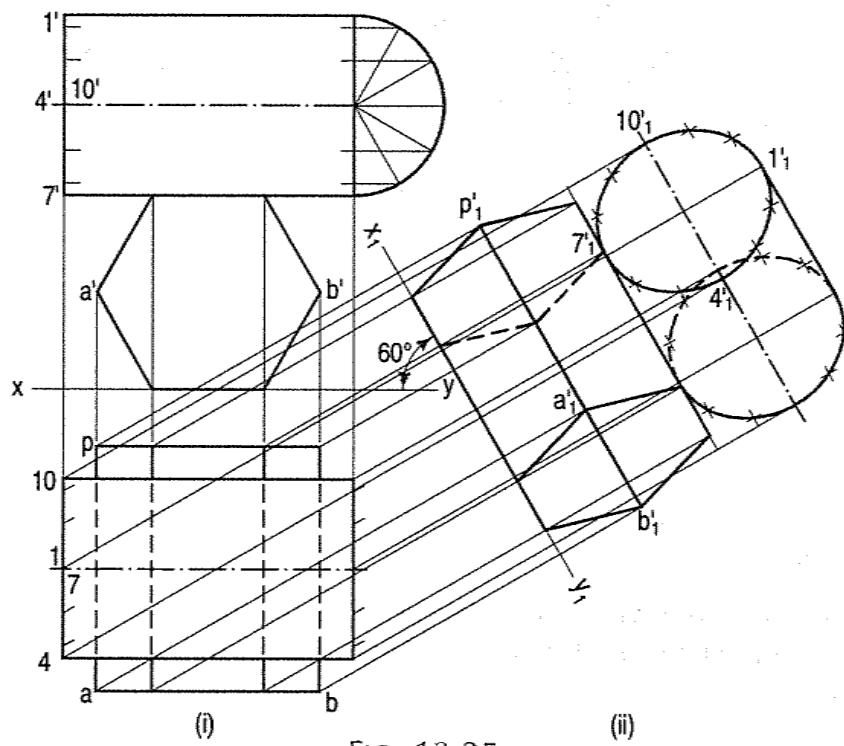


FIG. 13-25

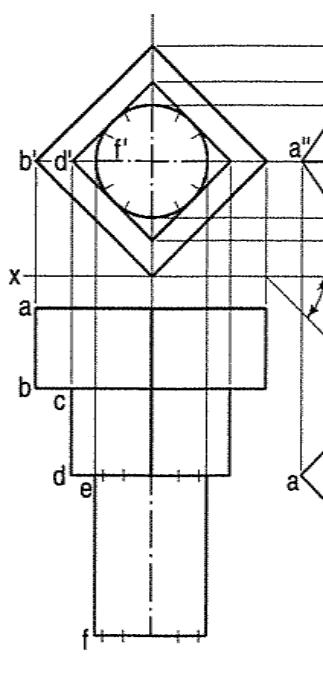


FIG. 13-26

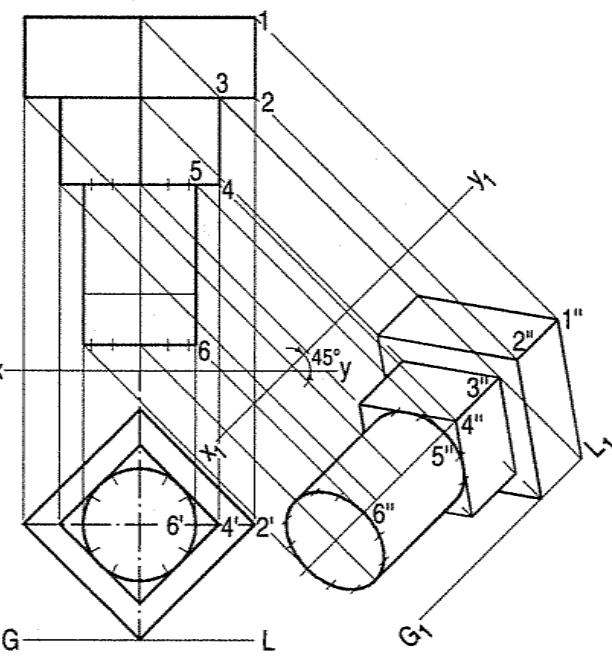


FIG. 13-27

Problem 13-15. (fig. 13-26 and fig. 13-27): A square-headed bolt 25 mm diameter, 125 mm long and having a square neck has its axis parallel to the H.P. and inclined at 45° to the V.P.

All the faces of the square head are equally inclined to the H.P. Draw its projections neglecting the threads and chamfer.

See fig. 13-26. The projections are obtained by the change-of-position method. The length of the bolt is taken shorter.

Fig. 13-27 shows the views in third-angle projection, obtained by the auxiliary-plane method.

Problem 13-16. (fig. 13-28): A hexagonal prism, base 40 mm side and height 40 mm has a hole of 40 mm diameter drilled centrally through its ends. Draw its projections when it is resting on one of its corners on the H.P. with its axis inclined at 60° to the H.P. and two of its faces parallel to the V.P.

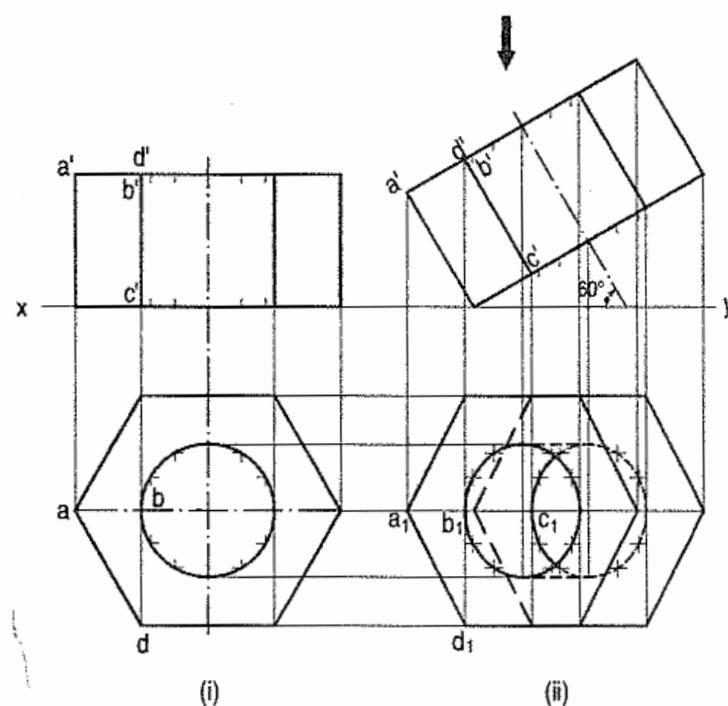


FIG. 13-28

- (i) Begin with the top view and project up the front view assuming the axis to be vertical.
- (ii) Tilt the front view, and project the required top view. Note that a part of the ellipse for the lower end of the hole will be visible.

Problem 13-17. (fig. 13-29): The projections of a hopper made of tin sheet are given. Project another top view on an auxiliary inclined plane making 45° angle with the H.P.

- (i) Draw a new reference line x_1y_1 inclined at 45° to xy and project the required top view on it, from the front view.
- (ii) Show carefully, the visible ellipses for the outer as well as the inner parts of the hopper rings.

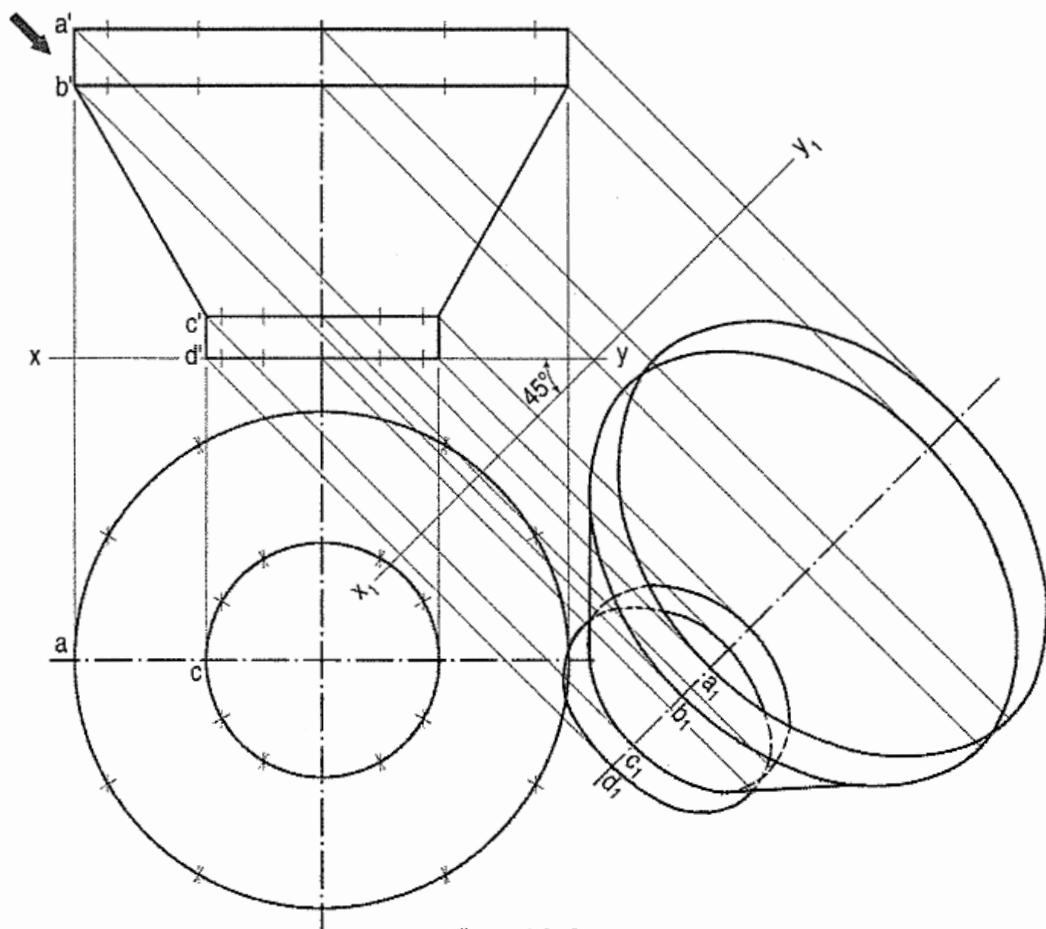


FIG. 13-29

13-4. PROJECTIONS OF SOLIDS WITH AXES INCLINED TO BOTH THE H.P. AND THE V.P.

The projections of a solid with its axis inclined to both the planes are drawn in three stages:

- (i) Simple position
- (ii) Axis inclined to one plane and parallel to the other
- (iii) Final position.

The second and final positions may be obtained either by the alteration of the positions of the solid, i.e. the views, or by the alteration of reference lines.

Problem 13-18. A square prism, base 40 mm side and height 65 mm, has its axis inclined at 45° to the H.P. and has an edge of its base, on the H.P. and inclined at 30° to the V.P. Draw its projections.

Method I: (fig. 13-30):

- (i) Assuming the prism to be resting on its base on the ground with an edge of the base perpendicular to the V.P., draw its projections.
Assume the prism to be tilted about the edge which is perpendicular to the V.P., so that the axis makes 45° angle with the H.P.
- (ii) Hence, change the position of the front view so that the axis is inclined at 45° to xy and f' (or e') is in xy . Project the second top view.

Again, assume the prism to be turned so that the edge on which it rests, makes an angle of 30° with the V.P., keeping the inclination of the axis with the ground constant. The shape and size of the second top view will remain the same; only its position will change. In the front view, the distances of all the corners from xy will remain the same as in the second front view.

- (iii) Therefore, reproduce the second top view making f_1g_1 inclined at 30° to xy . Project the final front view upwards from this top view and horizontally from the second front view, e.g. a vertical from a_1 and a horizontal from a' intersecting at a'_1 . As the top end is further away from xy in the top view it will be fully visible in the front view. Complete the front view showing the hidden edges by dashed lines.
- (iv) The second top view may be turned in the opposite direction as shown. In this position, the lower end of the prism, viz. $e'_1f'_1g'_1h'_1$ will be fully visible in the front view.

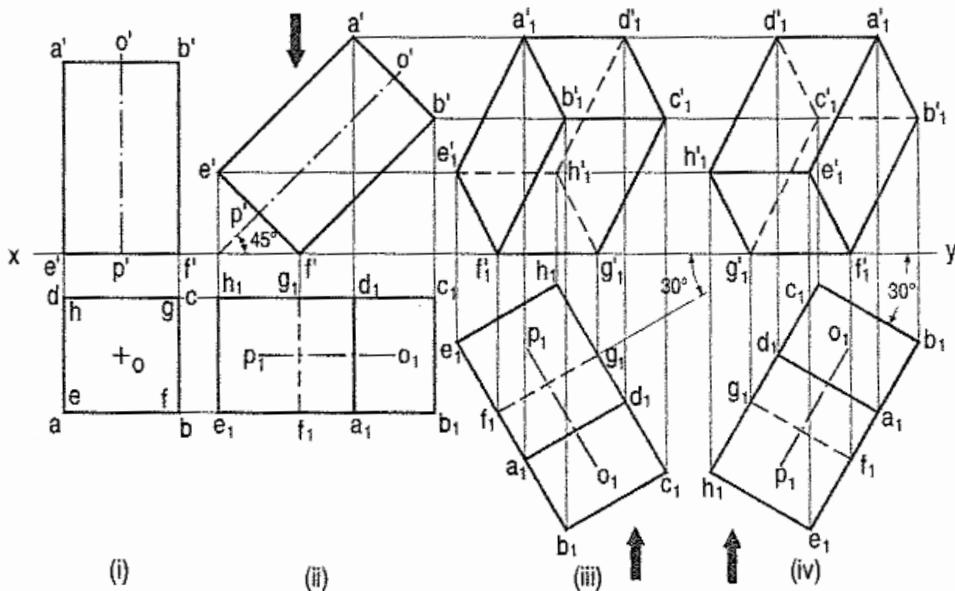


FIG. 13-30

Method II: (fig. 13-31):

- (i) Draw the top view and the front view in simple position.
- (ii) Through f' , draw a new reference line x_1y_1 making 45° angle with the axis. On it, project the auxiliary top view.
- (iii) Draw another reference line x_2y_2 inclined at 30° to the line f_1g_1 . From the auxiliary top view, project the required front view, keeping the distance of each point from x_2y_2 , equal to its distance (in the first front view) from x_1y_1 i.e. $a'_1q_1 = a'q$ etc. The problem is thus solved by change-of-reference line method only.

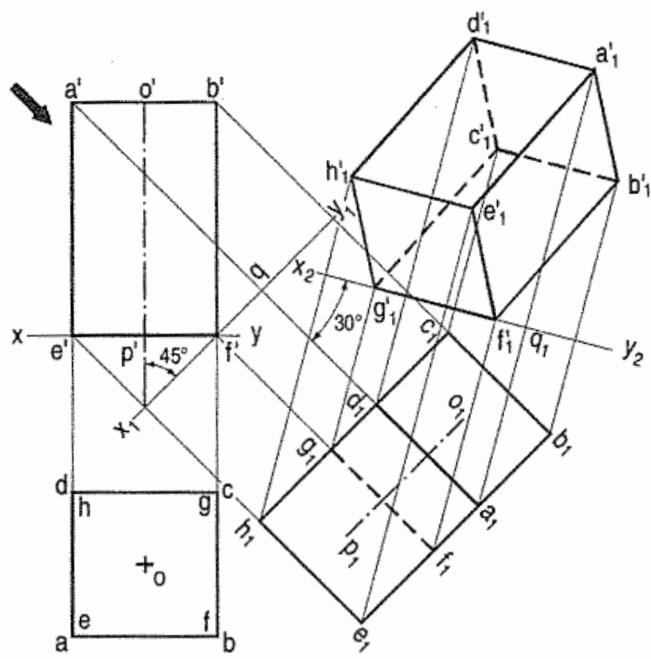


FIG. 13-31

Note: The new reference line satisfying the required conditions may be drawn in various positions, as explained in chapter 11.

Problem 13-19. (fig. 13-32): Draw the projections of a cone, base 45 mm diameter and axis 50 mm long, when it is resting on the ground on a point on its base circle with (a) the axis making an angle of 30° with the H.P. and 45° with the V.P.; (b) the axis making an angle of 30° with the H.P. and its top view making 45° with the V.P.

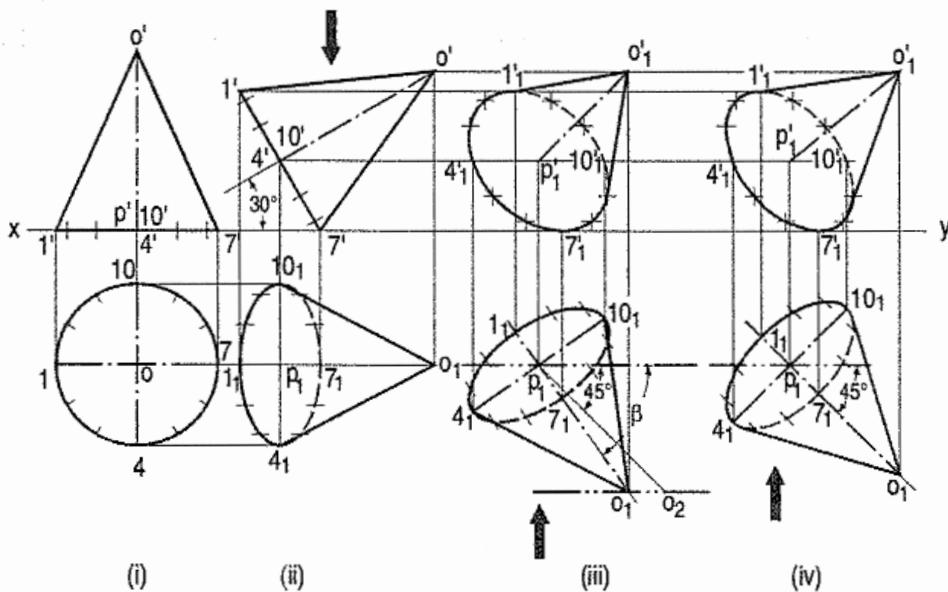


FIG. 13-32

- Draw the top view and the front view of the cone with the base on the ground.
- Tilt the front view so that the axis makes 30° angle with xy . Project the second top view.
 - In order that the axis may make an angle of 45° with the V.P., let us determine the apparent angle of inclination which the top view of the axis, viz. o_1p_1 should make with xy and which will be greater than 45° .
 - Mark any point p_1 below xy . Draw a line p_1o_2 equal to the true length of the axis, viz. $o'p'$, and inclined at 45° to xy . With p_1 as centre and radius equal to p_1o_1 (the length of the top view of the axis) draw an arc cutting the locus of o_2 at o_1 . Then β is the apparent angle of inclination and is greater than 45° . Around p_1o_1 as axis, reproduce the second top view and project the final front view as shown.

Note that the base of the cone is not visible in the front view because it is nearer xy in the top view.

- When the top view of the axis is to make 45° angle with the V.P., it is evident that p_1o_1 should be inclined at 45° to xy . Hence, reproduce the top view accordingly and project the required front view [fig. 13-32(iv)].

Problem 13-20. (fig. 13-33): A pentagonal pyramid, base 25 mm side and axis 50 mm long has one of its triangular faces in the V.P. and the edge of the base contained by that face makes an angle of 30° with the H.P. Draw its projections.

- In the initial position, assume the pyramid as having its base in the V.P. and an edge of the base perpendicular to the H.P. The front view will have to be drawn first and the top view projected from it.
- Change the position of the top view so that the line o_1 (for the face $o_1 5'$) is in xy . Project the second front view.
- Tilt this front view so that the line $1' 5'_1$ makes 30° angle with xy . Project the final top view. Note that the base is not visible in the top view as it is nearer xy in the front view.

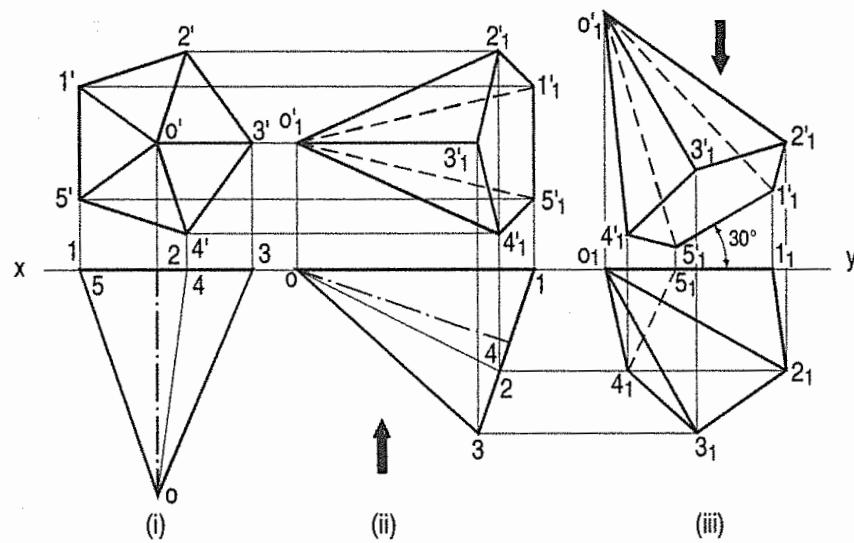


FIG. 13-33

Problem 13-21. (fig. 13-34): A square pyramid, base 38 mm side and axis 50 mm long, is freely suspended from one of the corners of its base. Draw its projections, when the axis as a vertical plane makes an angle of 45° with the V.P. When a pyramid is suspended freely from a corner of its base, the imaginary line joining that corner with the centre of gravity of the pyramid will be vertical.

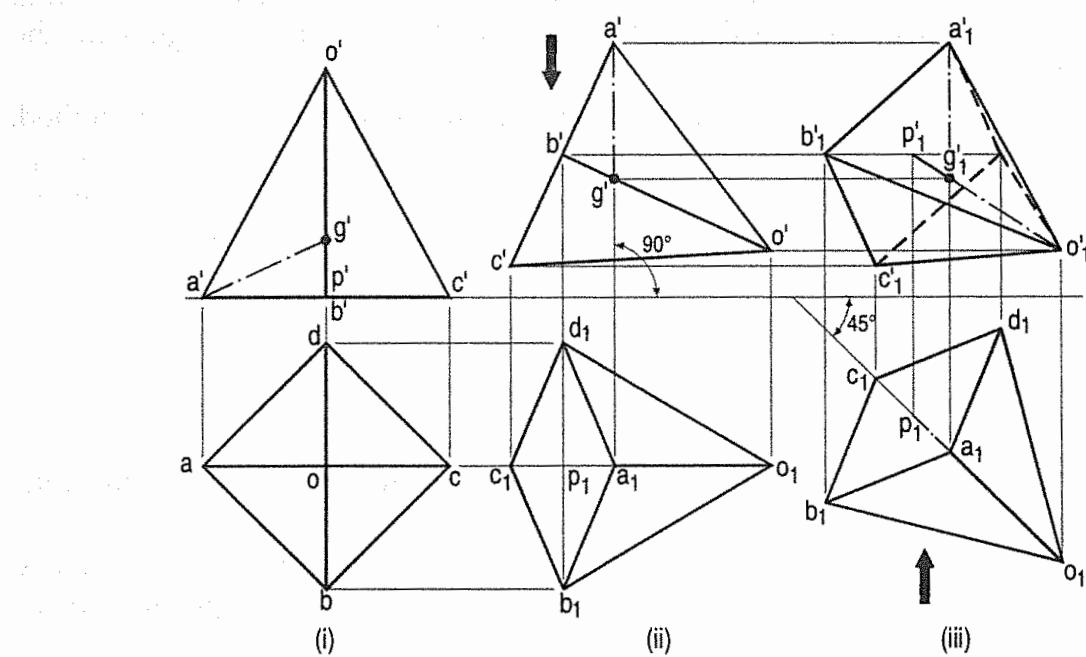


FIG. 13-34

The centre of gravity of a pyramid lies on its axis and at a distance equal to $\frac{1}{4}$ of the length of the axis from the base.

Assume the pyramid to be suspended from the corner A of the base.

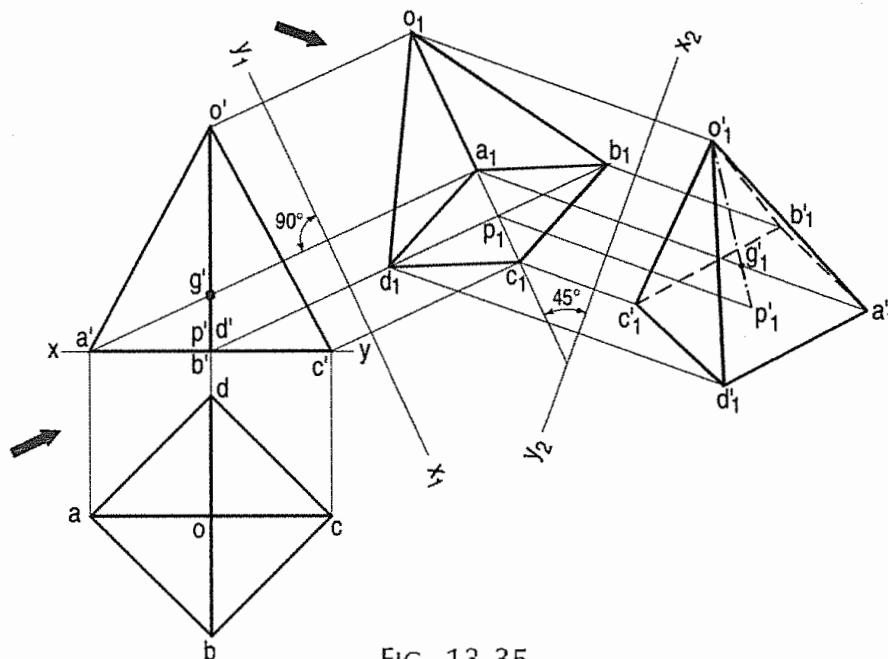


FIG. 13-35

In the initial position, the pyramid should be kept with its base on the ground and the line joining A with the centre of gravity G, parallel to the V.P. In the top view, g will coincide with o the top view of the axis.

- Draw a square abcd (in the top view) with ag, i.e. ao parallel to xy. Project the front view. Making g' at a distance equal to $\frac{1}{4}$ of the axis from xy. Join a' with g'.
- Tilt the front view so that a'g' is perpendicular to xy and project the top view. The axis will still remain parallel to the V.P.
- Reproduce this top view so that o_1p_1 (the top view of the axis) is inclined at 45° to xy. The axis as a vertical plane will thus be making 45° angle with the V.P. Project the final front view.

Fig. 13-35 shows the projections obtained by the change-of-reference-line method.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 30 for the following problem.

Problem 13-22. (fig. 13-36): A hexagonal pyramid, base 25 mm side and axis 55 mm long, has one of its slant edges on the ground. A plane containing that edge and the axis is perpendicular to the H.P. and inclined at 45° to the V.P. Draw its projections when the apex is nearer the V.P. than the base.

Assume the pyramid to be resting on the ground on its base with a slant edge parallel to the V.P.

- Draw the top view of the pyramid with a side of the hexagon parallel to xy. The lines ao and do for the slant edges will also be parallel to xy. Project the front view.
- Tilt this front view so that a'o' or d'o' is in xy. Project the second top view.

- (iii) Draw a new reference line x_1y_1 making 45° angle with o_1d_1 (the top view of the axis) and project the final front view.

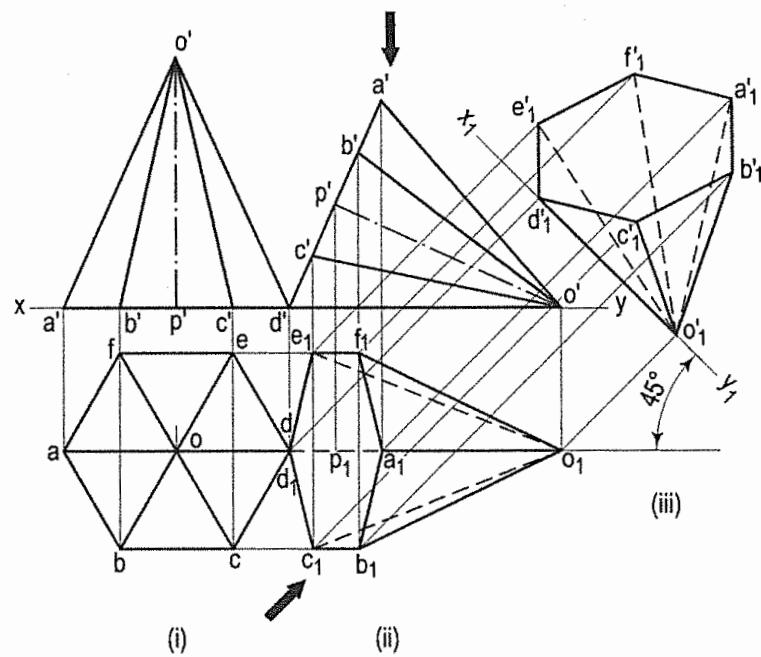


FIG. 13-36

The problem is thus solved by combination of the change-of-position and change-of-reference-line methods.

Problem 13-23. (fig. 13-37): Draw the projections of a cube of 25 mm long edges resting on the H.P on one of its corners with a solid diagonal perpendicular to the V.P.

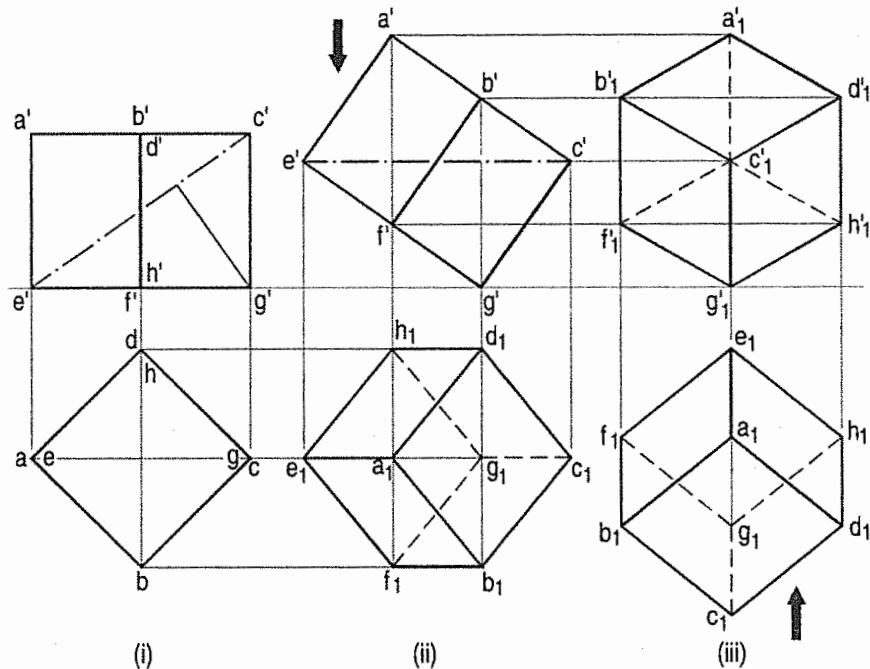


FIG. 13-37

Assume the cube to be resting on one of its faces on the H.P. with a solid diagonal parallel to the V.P.

- (i) Draw a square $abcd$ in the top view with its sides inclined at 45° to xy . The line ac representing the solid diagonals AG and CE is parallel to xy . Project the front view.

- (ii) Tilt the front view about the corner g' so that the line $e'c'$ becomes parallel to xy . Project the second top view. The solid diagonal CE is now parallel to both the H.P. and the V.P.
- (iii) Reproduce the second top view so that the top view of the solid diagonal, viz. e_1c_1 is perpendicular to xy . Project the required front view.

Problem 13-24. (fig. 13-38):

A triangular prism, base 40 mm side and axis 50 mm long, is lying on the H.P. on one of its rectangular faces with the axis perpendicular to the V.P. A cone, base 40 mm diameter and axis 50 mm long, is resting on the H.P. and is leaning centrally on a face of the prism, with its axis parallel to the V.P. Draw the projections of the solids and project another front view on a reference line making 60° angle with xy .

It will first be necessary to draw the cone with its base on the H.P. to determine the length of its generator and to project the top view.

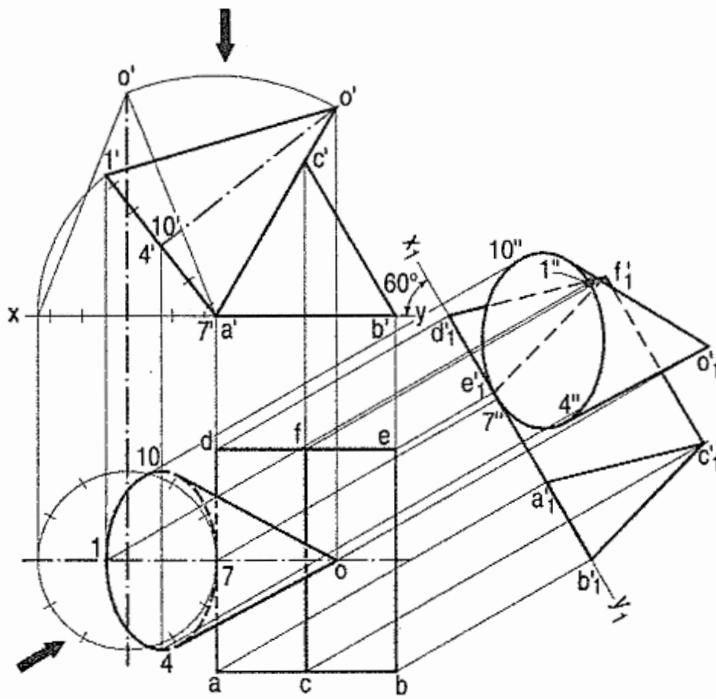


FIG. 13-38

Next, draw a triangle $a'b'c'$ for the prism and a triangle $o'1'7'$ for the cone as shown by the construction lines. Project the top view. Draw a reference line x_1y_1 and project the required front view as shown.

Problem 13-25. A pentagonal prism is resting on one of the corners of its base on the H.P. The longer edge containing that corner is inclined at 45° to the H.P. The axis of the prism makes an angle of 30° to the V.P. Draw the projections of the solid.

Also, draw the projections of the solid when the top view of axis is inclined at 30° to xy . Take the side of base 45 mm and height 70 mm.

- (i) Assuming the prism to be resting on its base on the horizontal plane, draw its projections keeping one of the sides of its base perpendicular to xy .
- (ii) Redraw the front view so that the edge $c'3'$ is inclined at 45° to xy . Project the required top view as shown in fig. 13-39(i).
- (iii) Determine the apparent angle of inclination which the top view of the axis should make with xy when the axis makes an angle of 30° with the V.P.
- (iv) Mark any point p_1 below xy . Draw a line p_1o_2 equal to the true length of the axis (70 mm) and inclined at 30° to xy . With p_1 as centre and radius equal to p_1o_1 (the length of the top view of the axis) draw an arc cutting the locus of o_2 at o_1 . Then β is the required apparent angle of inclination. Considering p_1o_1 as axis, reproduce the second top view and project the final front view as shown in fig. 13-39(i).

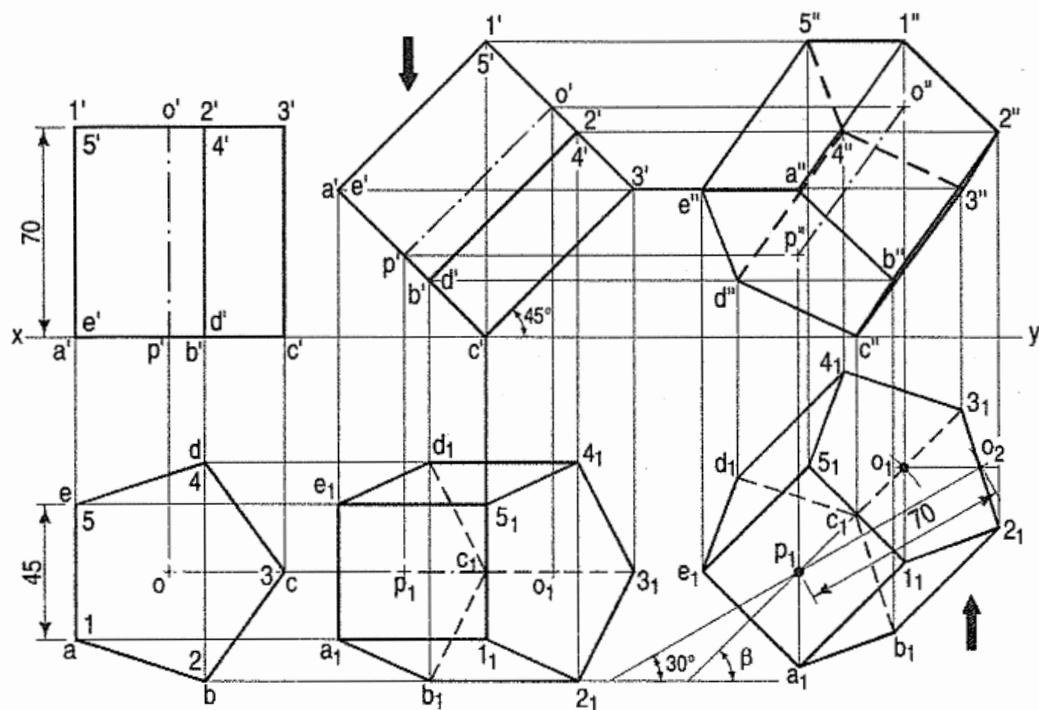


FIG. 13-39(i)

- (v) When the top view of axis makes an angle of 30° with the V.P., it is evident that $p_1 o_1$ is inclined at 30° to xy . Hence, reproduce the top view and the front view as shown in fig. 13-39(ii).

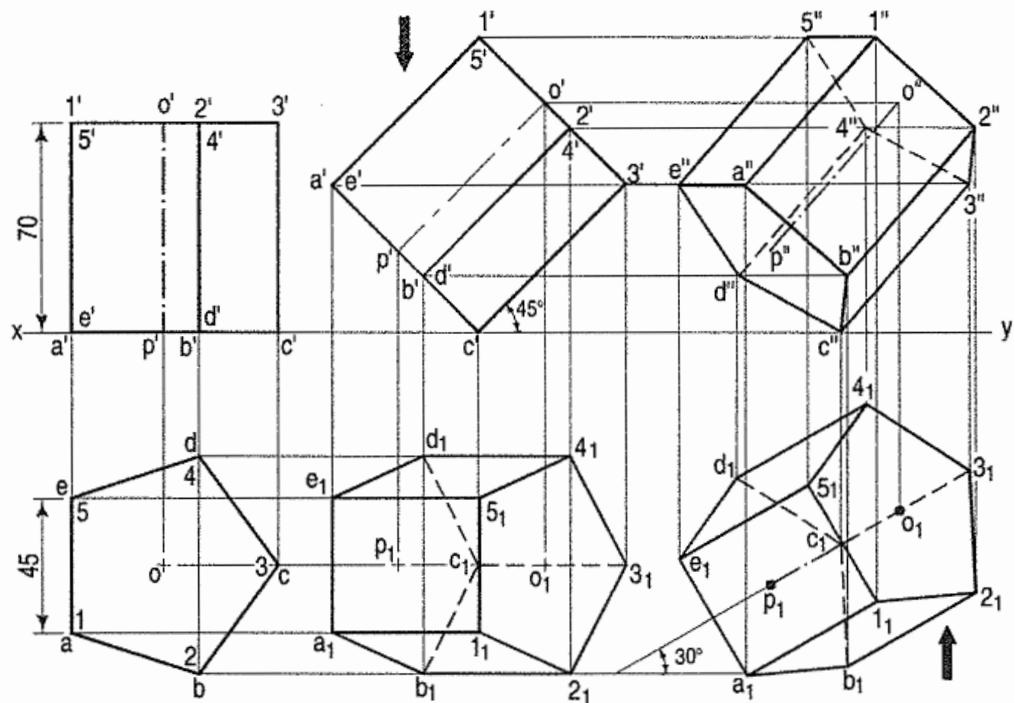


FIG. 13-39(ii)

Problem 13-26 (fig 13-40): A square prism, with the side of its base 40 mm and axis 70 mm long is lying on one of its base edges on the H.P. in such a way that this base edge makes an angle of 45° with the V.P. and the axis is inclined at 30° to the H.P. Draw the projections of the solid using the 'auxiliary plane method'.

- In the initial position assume the axis of the prism to be perpendicular to the H.P. Draw the projections as shown.
- Draw a new reference line x_1y_1 making an angle of 30° with the front view of the axis, to represent an auxiliary horizontal plane. Draw projectors from a' , b' , c' , d' and $1'$, $2'$, $3'$, $4'$ perpendicular to x_1y_1 and on them, mark these points keeping the distance of each point from x_1y_1 equal to its distance from xy in the top view. Join the points as shown.
- Draw another reference line x_2y_2 inclined at 45° to the line a_1 (or b_2). From the auxiliary top view, project the required new front view, keeping the distance of each point from x_2y_2 , equal to its distance from x_1y_1 , i.e. $q'1'' = q1'$ etc. Join the points as shown. Note that the view is obtained by observing the auxiliary top view from the top, along the projectors.

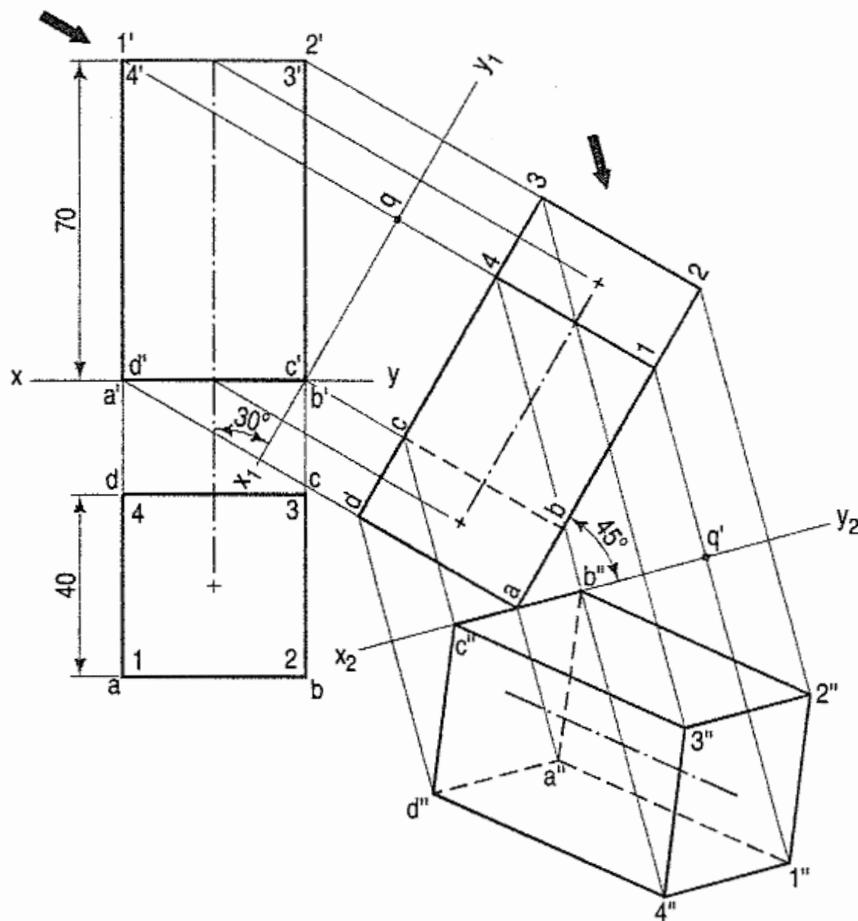


FIG. 13-40

Problem 13-27. A hexagonal prism, with the side of the hexagon 30 mm and height of 70 mm is resting on the H.P. on one of the edges of its hexagonal base in such a way that, the edge is at 60° to the V.P. and the base is at 30° to the H.P. Draw to scale 1:1, the view from the front and the view from the top.

Refer to fig. 13-41.

- Draw the top view and the front view in simple position keeping the axis perpendicular to the H.P.
- Draw a new reference line x_1y_1 making 60° angle with the axis. On it, project the auxiliary top view.

- (iii) Draw another reference line x_2y_2 inclined at 60° to the edge of base c_1d_1 . From the auxiliary top view, project the required new front view, keeping the distance of each point from x_2y_2 , equal to its distance from x_1y_1 i.e. $q'3' = q''3''$ etc. Join the points as shown. It should be noted that the edge of base away from x_2y_2 will be observed as full lines and nearest lines from x_2y_2 will be dotted lines. i.e. $c''d'', d''e''$ and $e''f''$ are full lines while $f''a'', a''b''$ and $b''c''$ are dotted lines. Note that the view is drawn by observing the auxiliary top view from the top along the projectors.

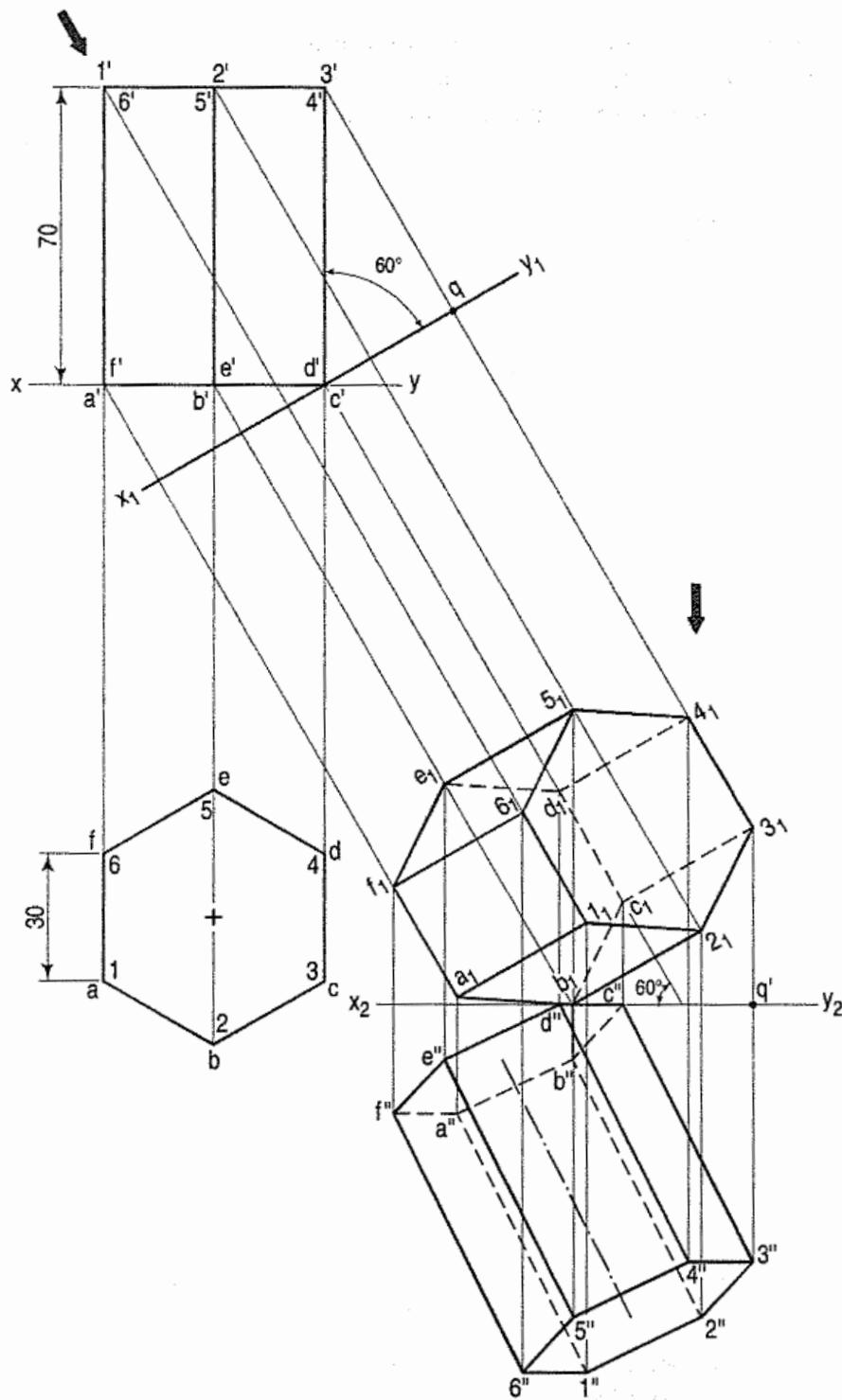


FIG. 13-41

Problem 13-28. A regular pentagonal prism lies with its axis inclined at 60° to the H.P. and 30° to the V.P. The prism is 60 mm long and has a face width of 25 mm. The nearest corner is 10 mm away from the V.P. and the farthest shorter edge is 100 mm from the H.P. Draw the projections of the solid.

- Draw initial position of the prism as shown in fig. 13-42.
- With $4'$ as centre and radius equal to 100 mm, draw an arc. Mark tangent to the arc making 60° with the axis as shown. This is a new reference line x_1y_1 . Project the required new top view.
- Draw another reference line x_2y_2 inclined at 30° angle to the axis of new top view. Project the various points to obtain new front view as shown in fig. 13-42. Observe the auxiliary top view from the top along the projectors.

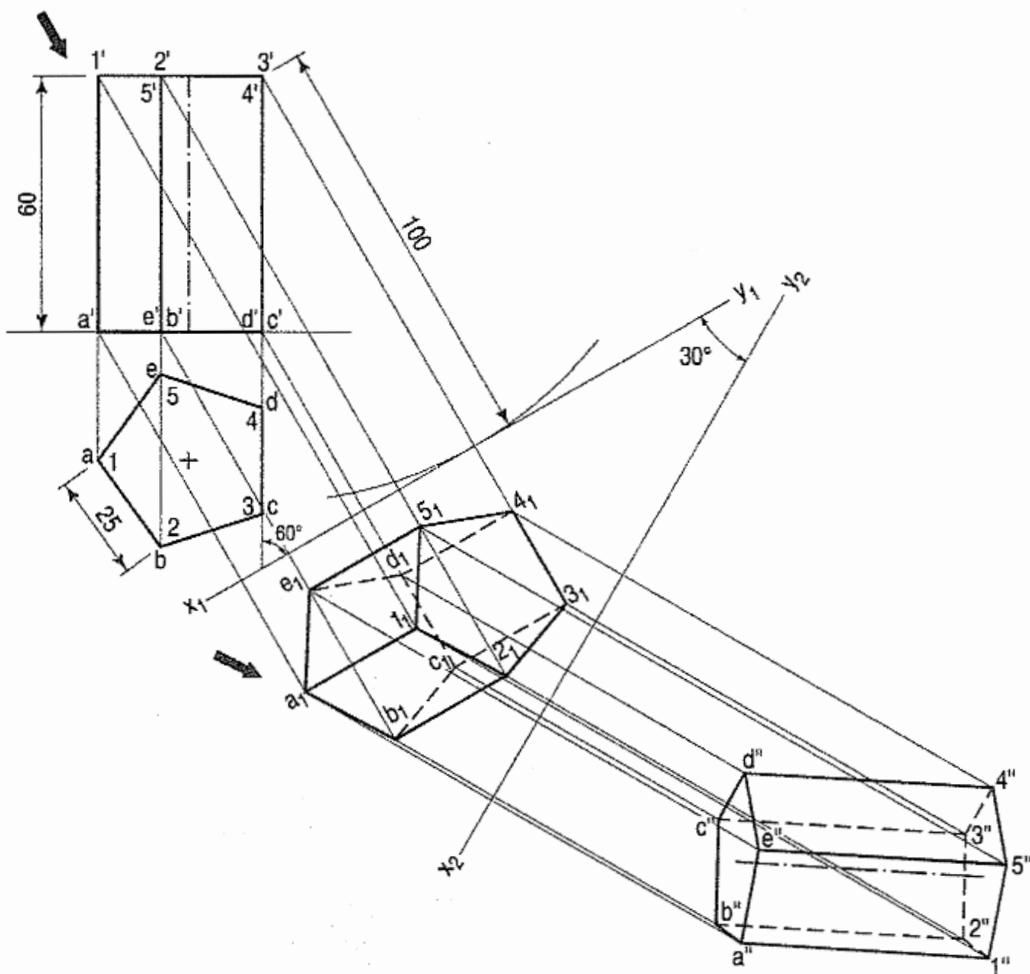


FIG. 13-42

Problem 13-29. A square pyramid of 50 mm side of base and 50 mm length of axis is resting on one of its triangular faces on the H.P. having a slant edge containing that face parallel to the V.P. Draw the projections of the pyramid.

- Assuming the axis of pyramid perpendicular to the H.P., draw the front view and the top view as shown in fig. 13-43.
- Draw new reference line x_1y_1 coinciding with $o'c'$ in the front view. Project new top view, keeping the distance of $a_1, b_1 \dots o_1$ from x_1y_1 equal to the distance of $a, b, \dots o$ from xy . Join these points.

- (iii) Draw another reference line x_2y_2 parallel to the slant edge o_1c_1 or o_1b_1 . Project new front view as shown. Observe auxiliary top view from the base a, b, c, d, o along projectors.

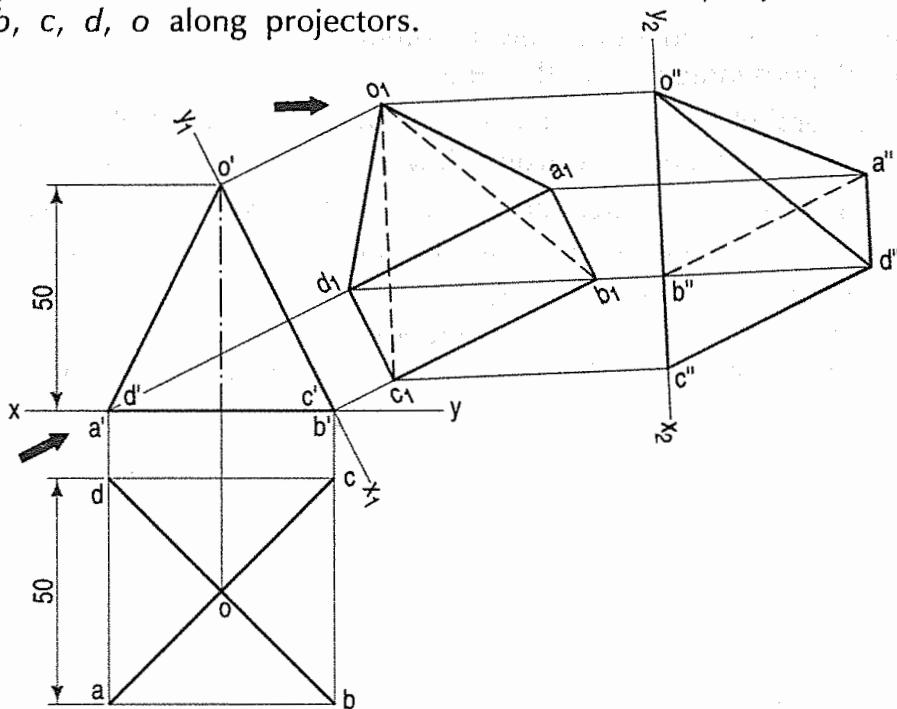


FIG. 13-43

Problem 13-30. A regular pentagonal pyramid, base 30 mm side and height 80 mm rests on one edge of its base on the ground so that the highest point in the base is 30 mm above the ground. Draw its projection when the axis is parallel to the V.P.

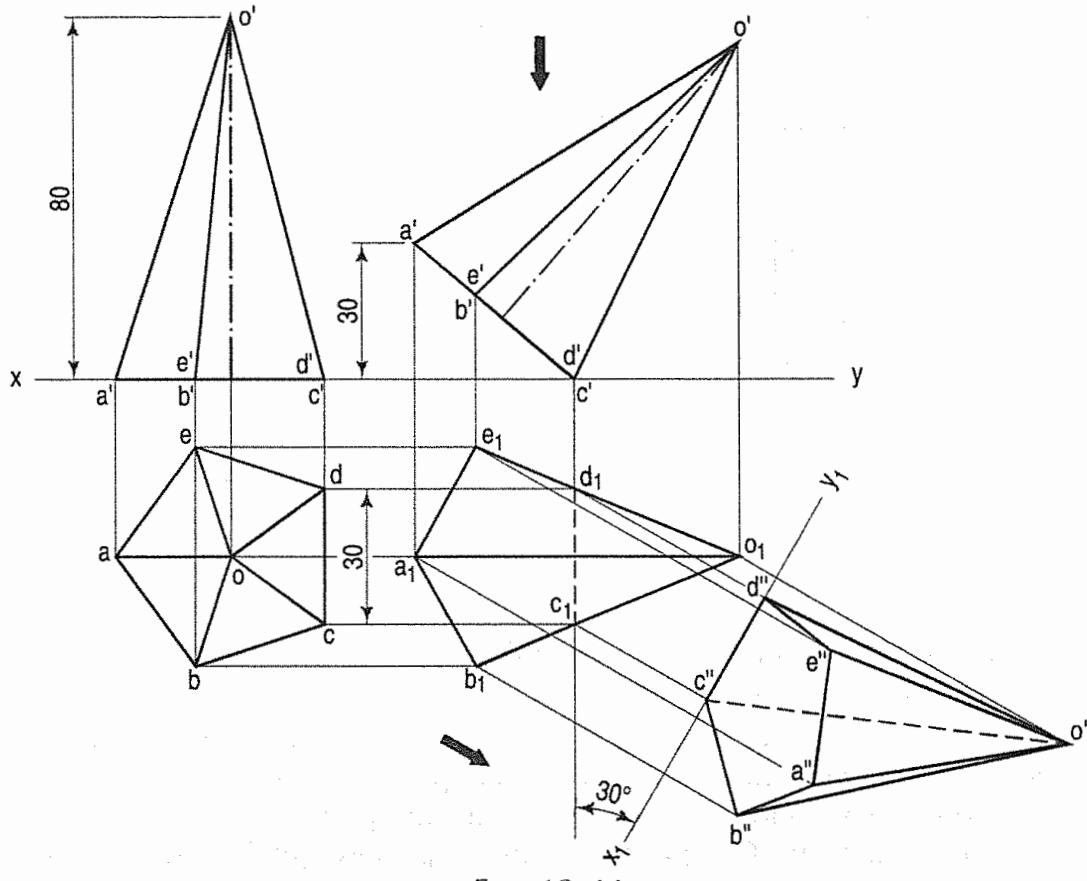


FIG. 13-44

Draw another front view on a reference line inclined at 30° to the edge on which it is resting so that the base is visible.

- Draw top view and front view in simple position assuming the axis of the pyramid perpendicular to the H.P.
- Draw a parallel line at a distance of 30 mm from xy. Mark the point c' on the line xy and reproduce the front view as shown fig. 13-44.
- Project points a', b', c' etc. and obtain new top view keeping distance of points a_1, b_1, c_1 etc. from xy equal to distance of a, b, c , etc. from the line xy .
- Draw another reference line x_1y_1 , making an angle of 30° with the side of base c_1d_1 and obtain a new front view as shown. Note that the base is visible. Observe from the base a, b, c, d, e along the projectors.

Problem 13-31. A regular pentagonal pyramid with the sides of its base 30 mm and height 80 mm rests on an edge of the base. The base is tilted until its apex is 50 mm above the level of the edge of the base on which it rests. Draw the projection of the pyramid when the edge on which it rests, is parallel to the V.P. and the apex of the pyramid points towards V.P.

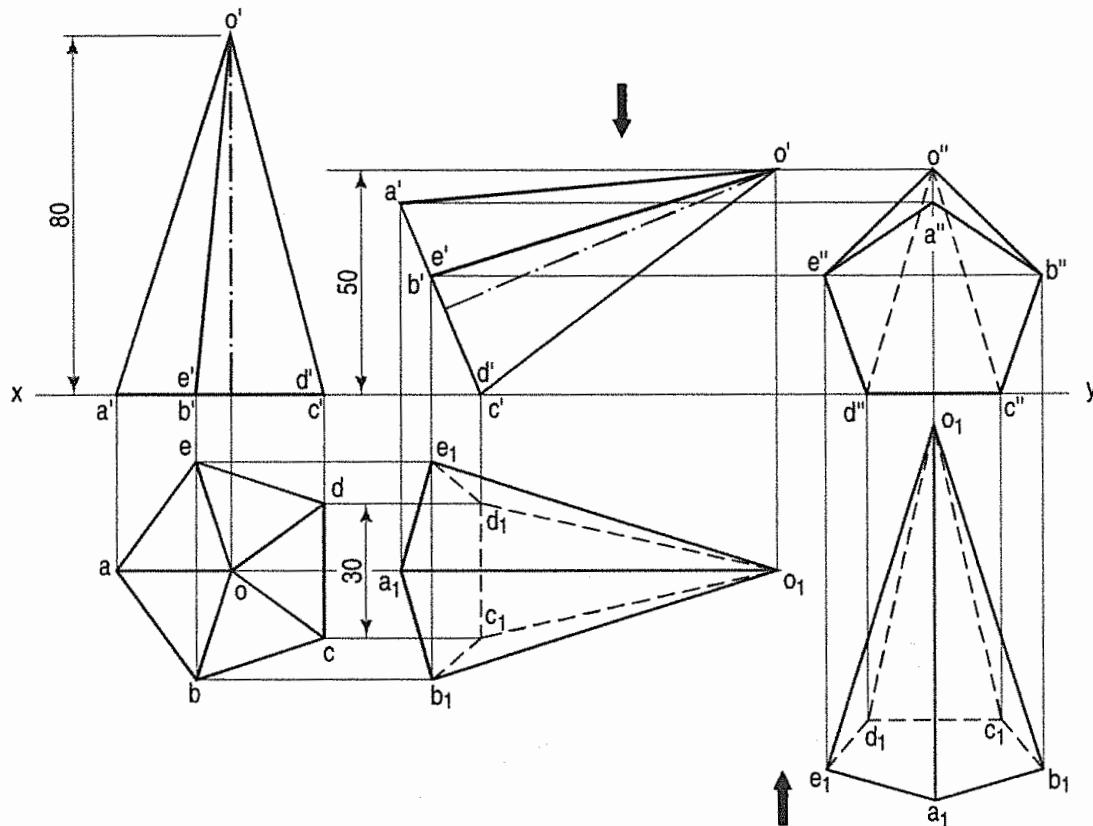


FIG. 13-45

- Draw top view and front view assuming the axis of the pyramid perpendicular to the H.P. as shown in fig. 13-45.
- Draw a parallel line at a distance of 50 mm from xy . Reproduce the front view as shown. Draw projectors from points a', b', c' etc. vertically from the front view and horizontally from the points a, b, c etc. from the previous top view. Complete the new top view, joining the intersection of the projectors in the correct sequence as shown.

- (iii) Redraw the top view keeping c_1d_1 parallel to xy . Project the points a_1, b_1, c_1 etc. vertically from the new top view and horizontal projectors from the points a', b', c' etc. of the front view. Join the intersection points of both the projectors in the correct sequence as shown.

Problem 13-32. A right regular pentagonal pyramid, with the sides of the base 30 mm and height 65 mm rests on the edge of its base on the horizontal plane, the base being tilted until the vertex is at 60 mm above the H.P. Draw the projections of the pyramid when the edge on which it rests, is made parallel to F.R.P. Assuming the pyramid to be resting on its base on the horizontal plane, draw its projections keeping one of the sides of the base perpendicular to xy .

Method I: Changing position of reference line [fig. 13-46(i)]:

- With o' as centre and radius equal to 60 mm, draw an arc. Draw the tangent to the arc passing through c' or d' . This is a new reference line x_1y_1 . Project the required top view.
- Draw another reference line x_2y_2 parallel to c_1d_1 . Project new front view as shown. Observe auxiliary top view from the base a, b, c, d, e along the projectors.

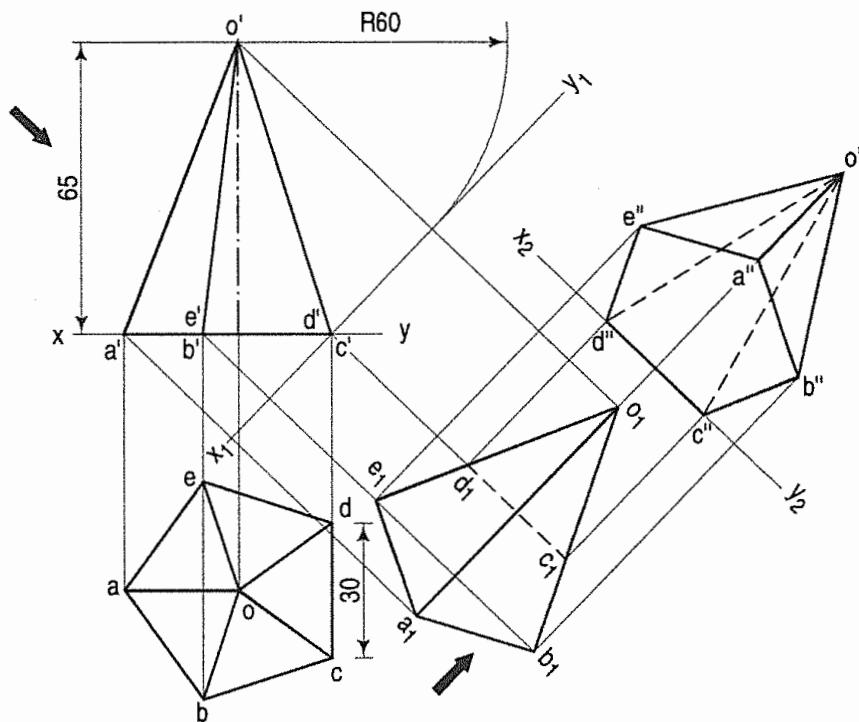


FIG. 13-46(i)

Method II: Changing positions of solid [(fig. 13-46(ii))]:

- Draw a line 60 mm parallel to xy . Mark point c' or d' on xy . With c' as centre and the radius equal to $o'c'$, draw an arc cutting the above line at o' . With o' and c' as centre and radius equal to $o'c'$ and $a'c'$ draw an arc cutting each other at the point a' . Join a', o' and c' as shown. Project the required top view as shown.
- Redraw the top view keeping side of base $c'd'$ parallel to xy . Project new front view as shown.

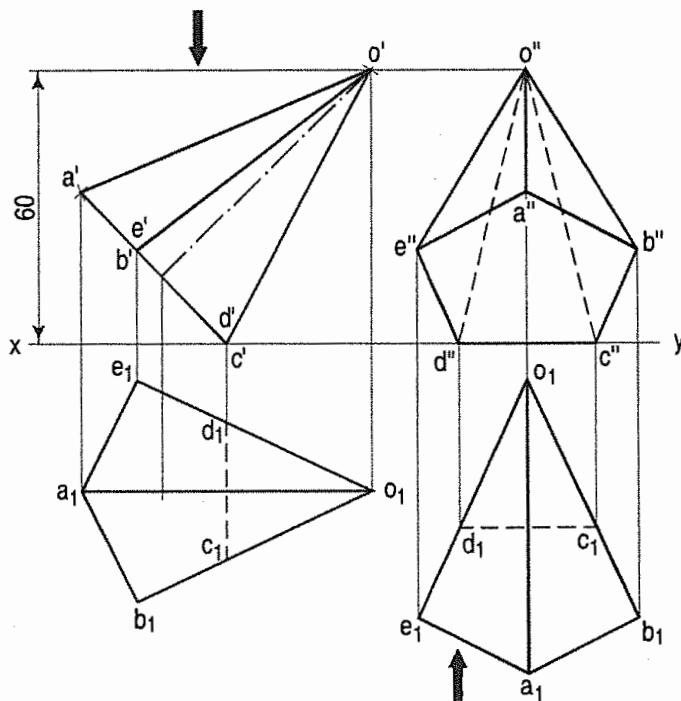


FIG. 13-46(ii)

Problem 13-33. The front view, part top view and part auxiliary view of a casting are given in fig. 13-47(i). Project its side view.

See fig. 13-47(ii).

The construction for the ellipse for 38 mm diameter circle has been shown in detail. Horizontal distances are taken from the auxiliary view. Other ellipses are drawn in the same manner.

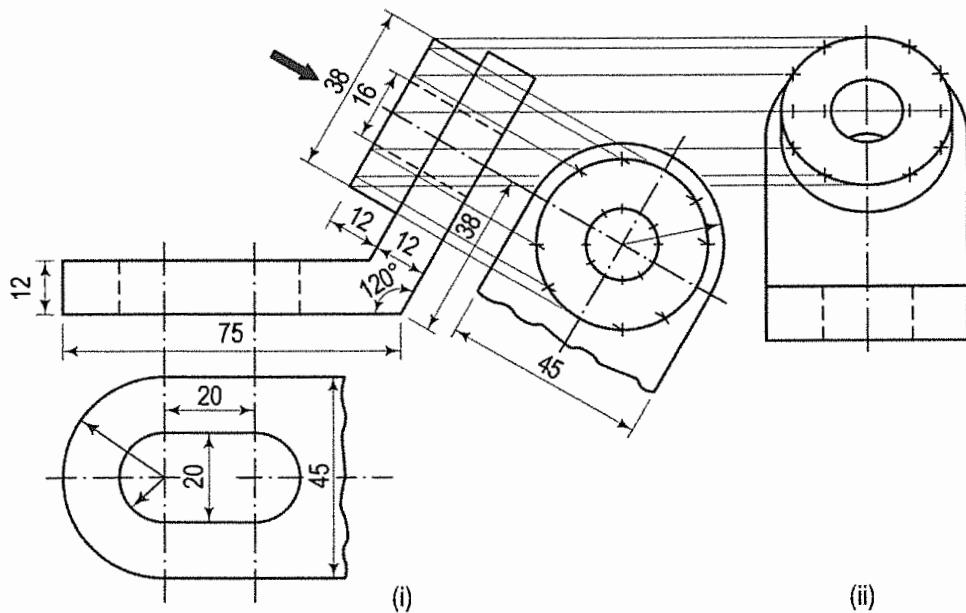


FIG. 13-47

13-5. PROJECTIONS OF SPHERES

The projection of a sphere in any position on any plane is always a circle whose diameter is equal to the diameter of the sphere (fig. 13-48). This circle represents the contour of the sphere.

A flat circular surface is formed when a sphere is cut by a plane. A hemisphere (i.e. a sphere cut by a plane passing through its centre) has a flat circular face of diameter equal to that of the sphere.

When it is placed on the ground on its flat face, its front view is a semi-circle, while its top view is a circle [fig. 13-49(i)].

When the flat face is inclined to the H.P. or the ground and is perpendicular to the V.P. it is seen as an ellipse (partly hidden) in the top view [fig. 13-49(ii)], while the contour of the hemisphere is shown by the arc of the circle drawn with radius equal to that of the sphere.

Fig. 13-50 shows the projections of a sphere, a small portion of which is cut off by a plane. Its flat face is perpendicular to the H.P. and inclined to the V.P. An ellipse is seen in the front view within the circle for the sphere.

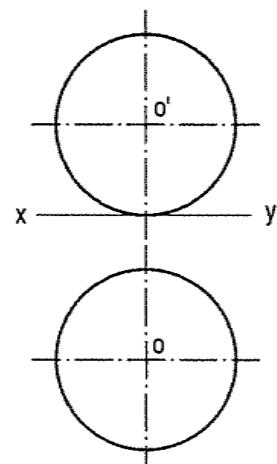


FIG. 13-48

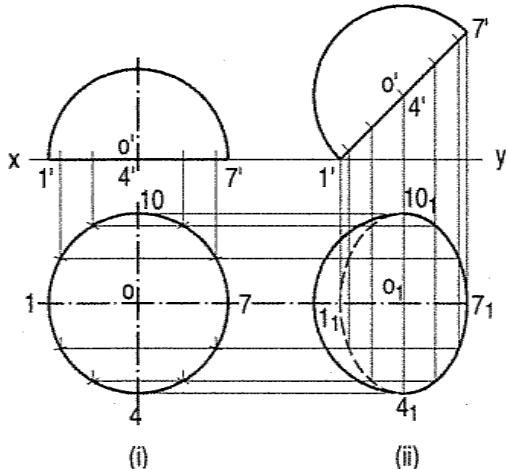


FIG. 13-49

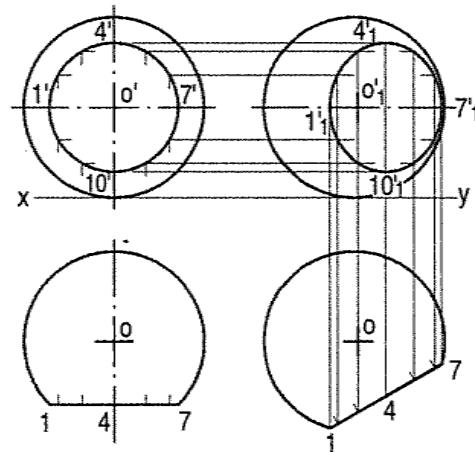


FIG. 13-50

When the flat face of a cut sphere is perpendicular to the V.P. and inclined to the H.P., its projections can be drawn as described in problem 13-34.

Problem 13-34. (fig. 13-51): A brass flower-vase is spherical in shape with flat, circular top 35 cm diameter and bottom 25 cm diameter and parallel to each other. The greatest diameter is 40 cm. Draw the projections of the vase when its axis is parallel to the V.P. and makes an angle of 60° with the ground.

- Draw the front view of the vase resting on its bottom with its axis vertical. Project the top view.
- Tilt the front view so that the axis makes 60° angle with xy and project the top view. Note that a part of the ellipse for the bottom is also visible.

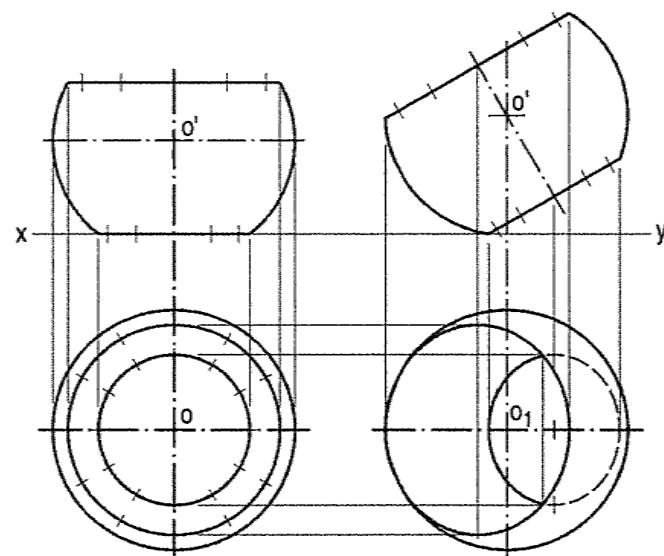


FIG. 13-51

(1) Spheres in contact with each other: Projections of two equal spheres resting on the ground and in contact with each other, with the line joining their centres parallel to the V.P., are shown in fig. 13-52.

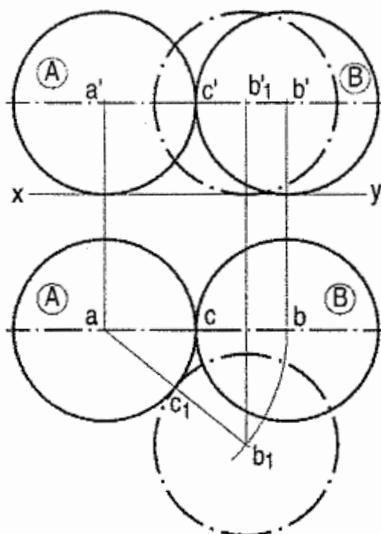


FIG. 13-52

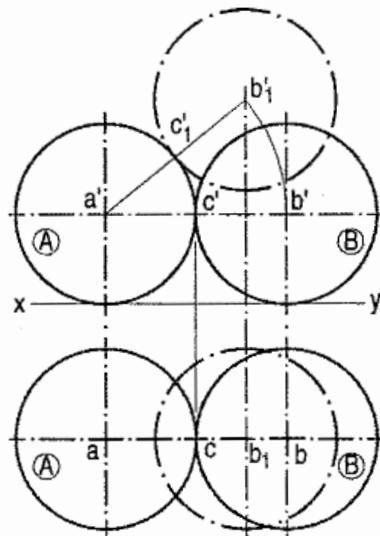


FIG. 13-53

As the spheres are equal in size, the line joining their centres is parallel to the ground also. Hence, both ab and $a'b'$ show the true length of that line (i.e. equal to the sum of the two radii or the diameter of the spheres). The point of contact between the two spheres is also visible in each view.

If the position of one of the spheres, say sphere B , is changed so that the line joining their centres is inclined to the V.P., in the front view, the centre b' will move along the line $a'b'$ to b'_1 . The true length of the line joining the centres and the point of contact are now seen in the top view only.

When the sphere B is so moved that it remains in contact with the sphere A and the line joining their centres is parallel to the V.P., but inclined to the ground (fig. 13-53), the true length of that line and the point of contact are visible in the front view only.

Problem 13-35. (fig. 13-54): Three equal spheres of 38 mm diameter are resting on the ground so that each touches the other two and the line joining the centres of two of them is parallel to the V.P.

A fourth sphere of 50 mm diameter is placed on top of the three spheres so as to form a pile. Draw three views of the arrangement and find the distance of the centre of the fourth sphere above the ground.

As the spheres are resting on the ground and are equal in size, the lines joining their centres will be parallel to the ground. In the top view, the centres will lie at the corners of an equilateral triangle of sides equal to the sum of the two radii, i.e. 40 mm.

Draw (in the top view) an equilateral triangle abc of 40 mm long sides with one side, say ab , parallel to xy . At its corners, draw three circles of 40 mm diameter. Project the front view. The centres will lie on a line parallel to and 20 mm above xy .

When the fourth sphere is placed on top, its centre d in the top view will be in the centre of the triangle. In the front view, it will lie on a projector through d .

The true distance between the centre of the top sphere and that of any one of the bottom spheres will be equal to the sum of the two radii, viz. 20 mm + 25 mm, i.e. 45 mm.

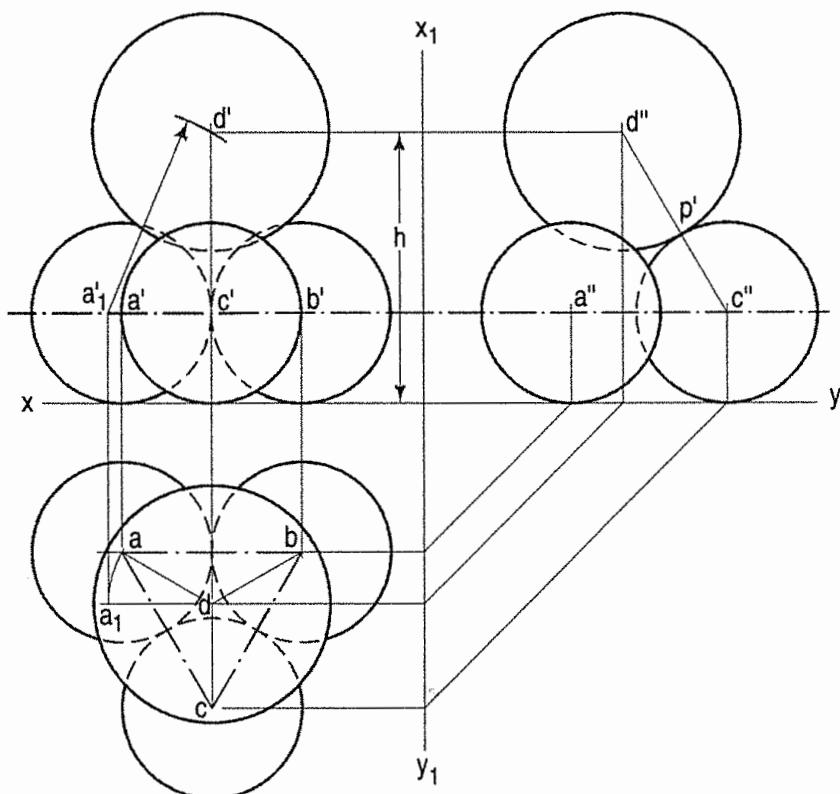


FIG. 13-54

But as none of the lines da , db or dc is parallel to xy , their front views will not show their true lengths. Therefore, to locate the position of the centre of the top sphere in the front view,

- make one of the lines, say da , parallel to xy ;
- project a_1 to a'_1 on the path of a' and
- with a'_1 as centre and radius equal to 45 mm, draw an arc cutting the projector through d at the required point d' . With d' as centre and radius equal to 25 mm, draw the required circle which will be partly hidden as shown. h is the distance of the centre of the sphere from the ground.
- Project the side view. As $c'd'$ is parallel to the new reference line, $c''d''$ will be equal to 45 mm and the point of contact p' between the spheres having centres c and d will be visible.

(2) **Unequal spheres:** When two unequal spheres are on the ground and are in contact with each other, their point of contact and the true length of the line joining their centres will be seen in the front view if that line is parallel to the V.P. In the top view, the length of the line will be shorter but will remain constant even when it is inclined to the V.P.

Problem 13-36. (fig. 13-55): Three spheres A, B and C of 75 mm, 50 mm and 30 mm diameters respectively, rest on the ground each touching the other two. Draw their projections and show the three points of contact when the line joining the centres of the spheres A and B is parallel to the V.P.

- With centre a' and radius equal to 37.5 mm, draw a circle of sphere A, mark a' at 37.5 mm above xy in front view. With $a'b'$ equal to 62.5 mm, mark point b' 25 mm above xy . With b' as centre and radius equal to 25 mm, draw a circle of sphere B in front view.

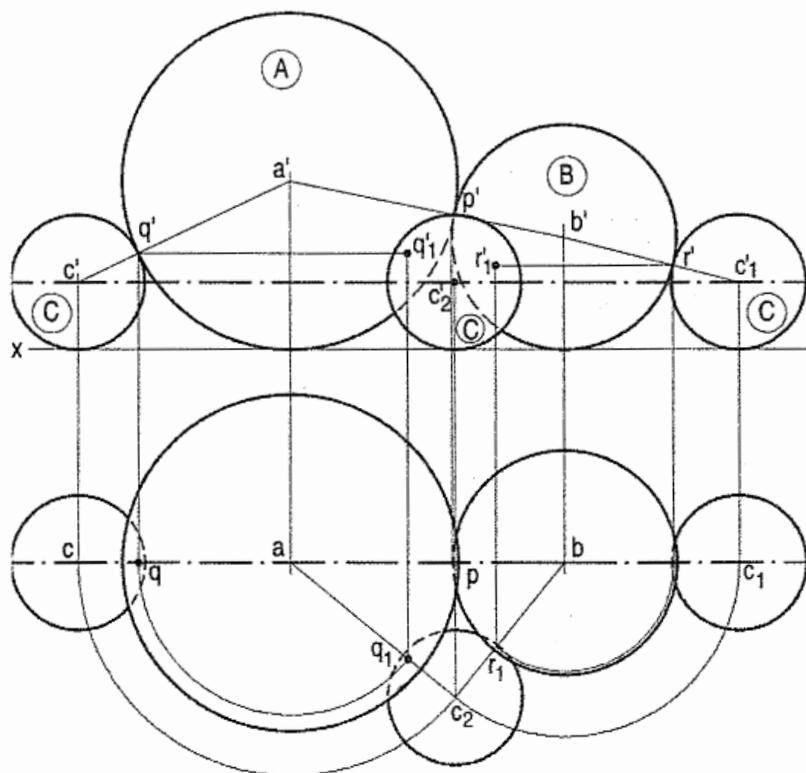


FIG. 13-55

- (ii) Project the centres and obtain points a and b on a line parallel to xy in top view. With a as centre and radius equal to 37.5 mm of sphere A , and with b as centre and radius equal to 25 mm of sphere B , draw circles in the top view.
- (iii) Similarly, draw the views of sphere C in contact with spheres A and B .
- (iv) With a as centre and radius equal to ac , and with b as centre and radius equal to bc_1 , draw arcs intersecting each other at c_2 . With c_2 centre draw top view of the sphere C .
- (v) Draw the projector through c_2 to cut the path of c' at c'_2 . Then c'_2 is the required centre of the sphere C in the front view. p , q_1 and r_1 , and p' , q'_1 and r'_1 are the points of contact in the top view and the front view respectively.

Problem 13-37. (fig. 13-56): A square prism, base 20 mm side and axis 50 mm long, is resting on its base on the ground with two faces perpendicular to the V.P. Determine the radius of four equal spheres resting on the ground, each touching a face of the prism and other two spheres. Draw the projections of the arrangement.

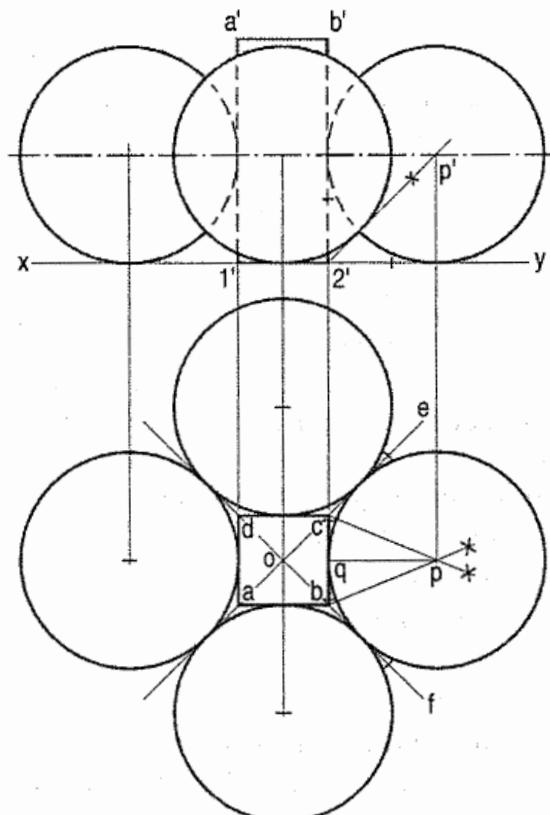


FIG. 13-56

- (i) Draw the front and top views of the prism. In the top view, draw diagonals of the square (intersecting each other at o) and produce them on both sides.
- (ii) Draw the bisectors of angles bce and cbf intersecting each other at p . From p , draw a perpendicular pq to bc . Then pq is the required radius of the sphere and p is the centre of the circle for the sphere.
- (iii) Obtain the other three centres in the same manner. Or, with o as centre and radius equal to op , draw a circle to cut the centre lines through o at the required centres. Draw the four circles.
- (iv) Draw a bisector of angle $b'2'y$ intersecting the projector through p at p' . Then p' is the centre of the sphere in the front view. The centres for the other circles will lie on the horizontal line through p' . Project their exact positions from the top view and draw the circles.

Problem 13-38. (fig. 13-57): Six equal spheres are resting on the ground, each touching other two spheres and a triangular face of a hexagonal pyramid resting on its base on the ground.

Draw the projections of the solids when a side of the base of the pyramid is perpendicular to the V.P.

Determine the diameter of each sphere. Base of the pyramid 20 mm side; axis 50 mm long.

- (i) Draw the projections of the pyramid in the required position. Assuming the solid to be a prism, locate the positions of the centre of one sphere (viz. p and p') in the two views.
- (ii) Draw a line joining p' with a' (the centre of the base) which coincides with $2'$. The centre of the required sphere will lie on this line. Draw a bisector of angle $o'3'y$ cutting $a'p'$ at c' . Draw a line $c'q'$ perpendicular to xy .
- (iii) With c' as centre and radius $c'q'$, draw one of the required circles. Project c' to c on op in the top view. Then c is the centre of the circle in the top view. Other centres may be located in the top view as shown and projected down in the front view.
- (iv) Draw the six circles in the top view and four in the front view as shown in the fig. 13-57.

Problem 13-39. The projections of a paper-weight with a spherical knob are given in fig. 13-58(i). Draw the two views and project another top view when its flat base makes an angle of 60° with the H.P.

See fig. 13-58(ii) which is self-explanatory.

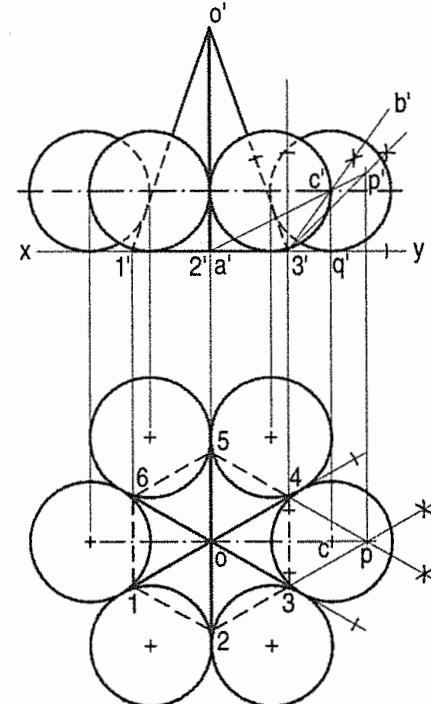


FIG. 13-57

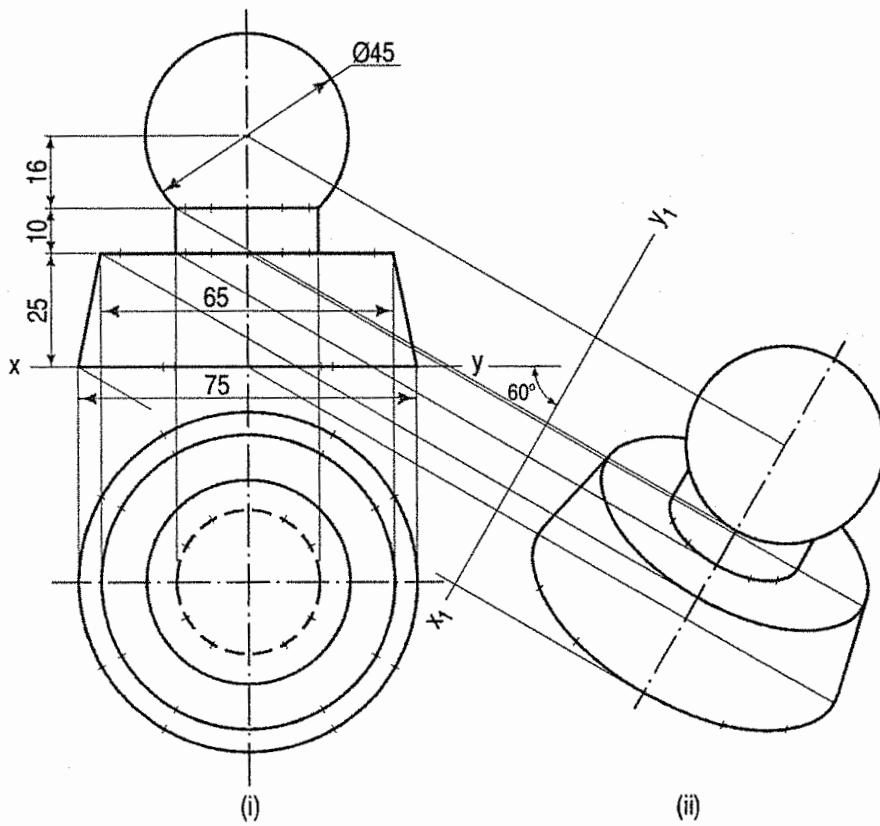


FIG. 13-58

Problem 13-40. (fig. 13-59): A vertical hexagonal prism of base side 20 mm and thickness 15 mm has one side of hexagon perpendicular to the V.P. A right cone of 34 mm diameter and height 40 mm is placed on the top face of prism such that the base of cone touches top surface of prism while the axes of both coincide. Draw the front view and top view of the combined object. Draw also projections when axes of combined solid is inclined at 35° with auxiliary plane.

- Draw the top view of hexagonal prism keeping one of sides perpendicular to xy . (i.e. ab or ed). Project above xy line and draw the front view of prism of height 15 mm.
- Inscribe circle in the top view touching sides of the prism. Project it in the front view and mark the height of the cone as shown.
- Draw auxiliary x_1y_1 inclined at 35° with the axes of the combined solids.
- Draw the projectors from the various points of combined solids in the front view.
- Taking distance of various points from the top view of combined solids from xy and mark same distances along the respective projectors.
- Complete auxiliary top view as shown.

Problem 13-41. (fig. 13-60): A vertical cylindrical disc of thickness 10 mm and diameter 50 mm is resting on the ground. A vertical frustum of pentagonal pyramid, having bottom of 20 mm sides, top face of 40 mm sides with 60 mm height is resting on the top surface of the disc so that axes of the both solids coincide. Take one of sides of the base of pentagon is perpendicular to V.P. Draw the projections of combined solid when the axis of combined solids is inclined to 30° with the H.P.

- Draw the top view of frustum of pyramid (pentagon) keeping one of the sides perpendicular to xy as shown.

- (ii) Project the front view marking height of cylindrical disc and frustum of the pyramid 10 mm and 60 mm respectively.
- (iii) Draw a line at angle of 30° with xy . (As axis is inclined with H.P., its inclination observed in the front view).
- (iv) Reproduce the front view considering inclined line as axes of the combined solids.
- (v) Draw the vertical projectors from various points of the front view.
- (vi) Draw horizontal projectors to intersect respective vertical projectors. Obtain the auxiliary top view as shown.

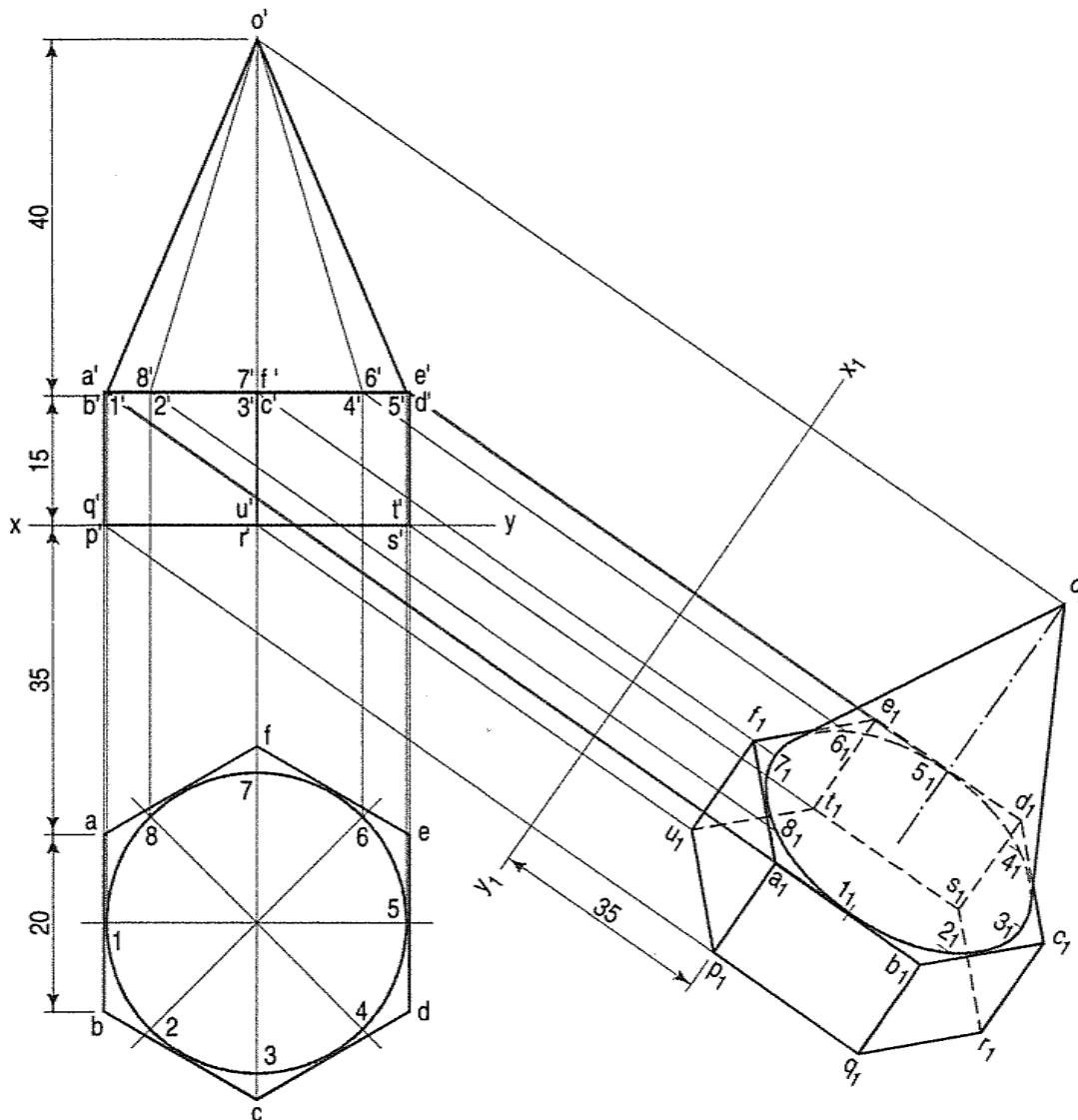


FIG. 13-59

Problem 13-42. (fig. 13-61): A right hexagonal prism of side 25 mm and 20 mm thick with one side of the base is perpendicular to the V.P. resting on the ground. A vertical frustum of square pyramid of base 20 mm sides and top face side 30 mm and height 50 mm is resting on the prism such that one side of square makes 45° with the V.P. Assume that axes of both solids are coinciding. Draw the projections of the combined solids when top corner of the square pyramid is 70 mm above the ground (H.P.). Determine angle of combined solids with the H.P.

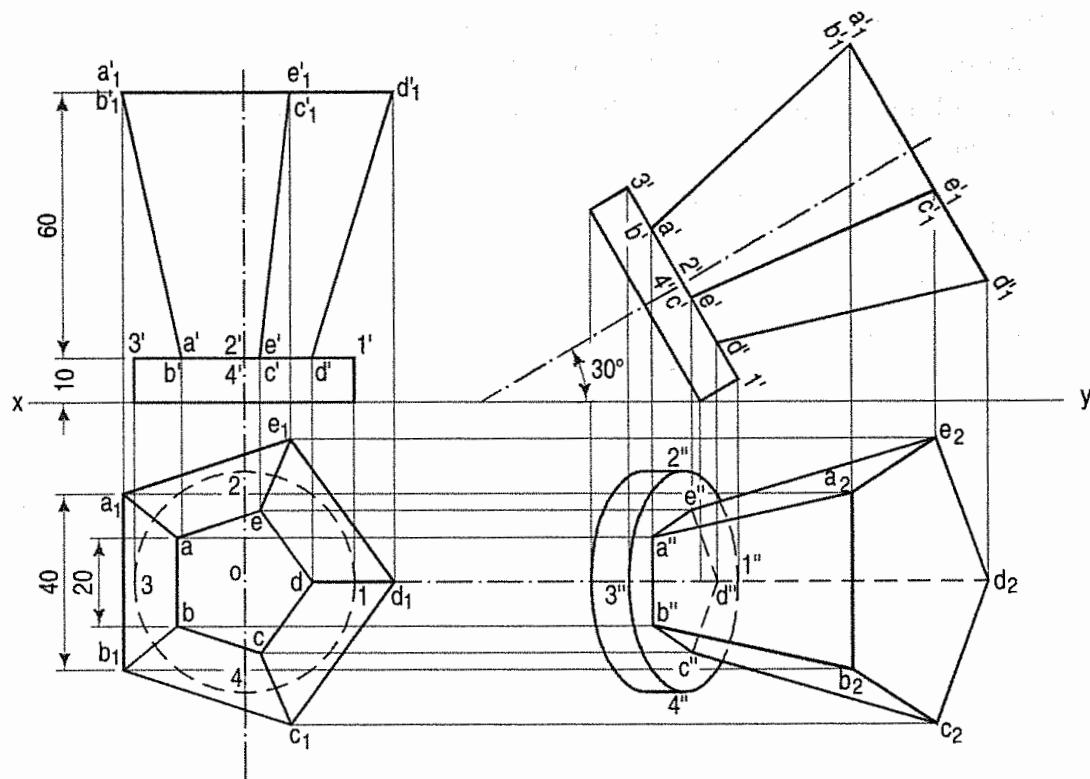


FIG. 13-60

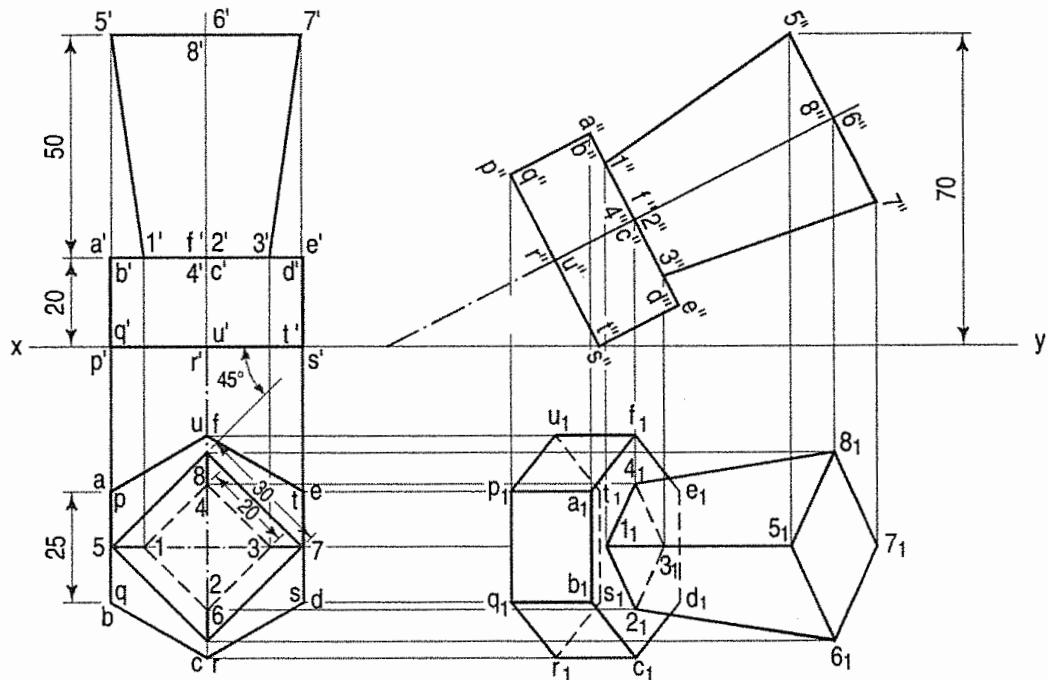


FIG. 13-61

- Draw the top view and front view as shown in figure. Keep one side of the hexagonal perpendicular to the xy .
- Project the front view as shown.
- Draw a parallel line at distance 70 mm away from xy . Reproduce the front view of the combined solids as shown.
- Draw the projectors from the new front view.

- (v) Draw from the top view horizontal projectors to intersect respective projectors drawn from the new front view.
- (vi) Complete the top view as shown.

EXERCISES 13(b)

1. A rectangular block 75 mm \times 50 mm \times 25 mm thick has a hole of 30 mm diameter drilled centrally through its largest faces. Draw the projections when the block has its 50 mm long edge parallel to the H.P. and perpendicular to the V.P. and has the axis of the hole inclined at 60° to the H.P.
2. Draw the projections of a square pyramid having one of its triangular faces in the V.P. and the axis parallel to and 40 mm above the H.P. Base 30 mm side; axis 75 mm long.
3. A cylindrical block, 75 mm diameter and 25 mm thick, has a hexagonal hole of 25 mm side, cut centrally through its flat faces. Draw three views of the block when it has its flat faces vertical and inclined at 30° to the V.P. and two faces of the hole parallel to the H.P.
4. Draw three views of an earthen flower pot, 25 cm diameter at the top, 15 cm diameter at the bottom, 30 cm high and 2.5 cm thick, when its axis makes an angle of 30° with the vertical.
5. A tetrahedron of 75 mm long edges has one edge parallel to the H.P. and inclined at 45° to the V.P. while a face containing that edge is vertical. Draw its projections.
6. A hexagonal prism, base 30 mm side and axis 75 mm long, has an edge of the base parallel to the H.P. and inclined at 45° to the V.P. Its axis makes an angle of 60° with the H.P. Draw its projections.
7. A pentagonal prism is resting on a corner of its base on the ground with a longer edge containing that corner inclined at 45° to the H.P. and the vertical plane containing that edge and the axis inclined at 30° to the V.P. Draw its projections. Base 40 mm side; height 65 mm.
8. Draw three views of a cone, base 50 mm diameter and axis 75 mm long, having one of its generators in the V.P. and inclined at 30° to the H.P., the apex being in the H.P.
9. A square pyramid, base 40 mm side and axis 90 mm long, has a triangular face on the ground and the vertical plane containing the axis makes an angle of 45° with the V.P. Draw its projections.
10. A frustum of a pentagonal pyramid, base 50 mm side, top 25 mm side and axis 75 mm long, is placed on its base on the ground with an edge of the base perpendicular to the V.P. Draw its projections. Project another top view on a reference line parallel to the line which shows the true length of the slant edge. From this top view, project a front view on an auxiliary vertical plane inclined at 45° to the top view of the axis.
11. Draw the projections of a cone, base 50 mm diameter and axis 75 mm long, lying on a generator on the ground with the top view of the axis making an angle of 45° with the V.P.
12. The front view, incomplete top view and incomplete auxiliary top view of a casting are given in fig. 13-47. Draw all the three views completely in the third-angle projection.

13. A line sketch (in two views) of a shed with a curved roof is given in fig. 13-62. Draw its front view on an auxiliary vertical plane inclined at 60° to the V.P. All dimensions are in metres. Scale, 10 mm = 0.5 m.

14. The front view of a hexagonal pyramid [base 25 mm side] having one of its triangular faces resting centrally on a triangular face of a square pyramid [base 50 mm side and axis 50 mm long] is given in fig. 13-63. The plane containing the two axes is parallel to the V.P. Draw the top view of the solids. From this top view, project a front view on a reference line x_1y_1 inclined at 30° to xy ; (ii) from the given front view, project another top view on a reference line x_2y_2 inclined at 45° to xy .

15. A cube of 50 mm long edges is resting on the ground with its vertical faces equally inclined to the V.P. A hexagonal pyramid, base 25 mm side and axis 50 mm long, is placed centrally on top of the cube so that their axes are in a straight line and two edges of its base parallel to the V.P. Draw the front and top views of the solids. Project another top view on an A.I.P. making an angle of 45° with the H.P. From this top view project another front view on an auxiliary vertical plane inclined at 30° to the top view of the combined axis.

16. Four equal spheres of 25 mm diameter are resting on the ground, each touching the other two spheres, so that a line joining the centres of two touching spheres is inclined at 30° to the V.P. A fifth sphere of 30 mm diameter is placed centrally on top of the four spheres, thus forming a pile. Draw the projections of the spheres and measure the height of the centre of the top sphere above the ground.

17. Three spheres of 25 mm, 50 mm and 75 mm diameter respectively are resting on the ground so that each touches the other two. Draw their projections when the top view of the line joining centres of any two of them is perpendicular to the V.P.
18. Three equal cones, base 50 mm diameter and axis 75 mm long, are placed on the ground on their bases, each touching the other two. A sphere of 40 mm diameter is placed centrally between them. Draw three views of the arrangement and determine the height of the centre of the sphere above the ground.
19. Five equal spheres are resting on the ground each touching the other two spheres and a vertical face of a pentagonal prism of 25 mm side. Determine the diameter of the spheres and draw the projections when a side of the base of the prism is perpendicular to the V.P.

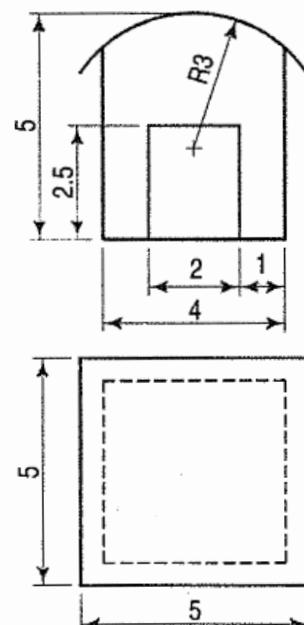


FIG. 13-62

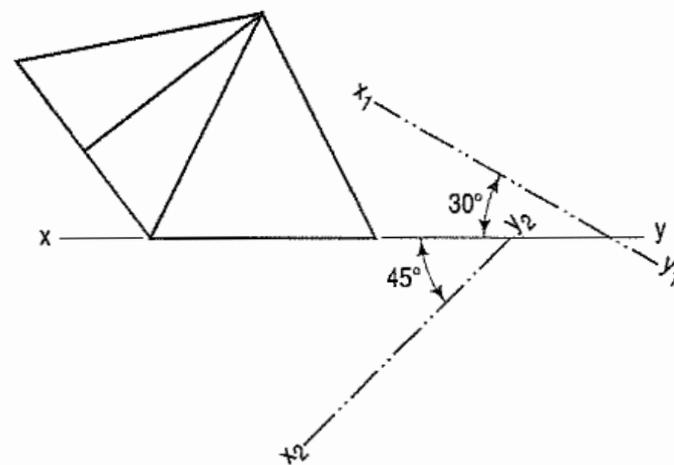


FIG. 13-63

20. Four equal spheres are resting on the ground, each touching the other two spheres and a triangular face of a square pyramid, having base 25 mm side and axis 50 mm long. Draw their projections and find the diameter of the spheres.
21. One of the body diagonals of a cube of 45 mm edge is parallel to the H.P. and inclined at 45° to the V.P. Draw the front view and the top view of the cube.
22. A pentagonal pyramid, base 40 mm side and height 75 mm rests on one edge of its base on the ground so that the highest point in the base is 25 mm above the ground. Draw its projections when the axis is parallel to the V.P. Draw another front view on a reference line inclined at 30° to the edge on which it is resting, and so that the base is visible.
23. A thin lamp shade in the form of a frustum of a cone has its larger end 200 mm diameter, smaller end 75 mm diameter and height 150 mm. Draw its three views when it is lying on its side on the ground and the axis parallel to the V.P.
24. A bucket made of tin sheet has its top 200 mm diameter and bottom 125 mm diameter with a circular ring 40 mm wide attached at the bottom. The total height of the bucket is 250 mm. Draw its projections when its axis makes an angle of 60° with the vertical.
25. A hexagonal pyramid, side of the base 25 mm long and height 70 mm, has one of its triangular faces perpendicular to the H.P. and inclined at 45° to the V.P. The base-side of this triangular face is parallel to the H.P. Draw its projections.
26. A pentagonal pyramid has an edge of the base in the V.P. and inclined at 30° to the H.P., while the triangular face containing that edge makes an angle of 45° with the V.P. Draw three views of the pyramid. Length of the side of the base is 30 mm, while that of the axis is 80 mm.
27. A square pyramid, base 40 mm side and axis 75 mm long is placed on the ground on one of its slant edges, so that the vertical plane passing through that edge and the axis makes an angle of 30° with the V.P. Draw its three views.
28. A hexagonal prism, side of base 40 mm and height 50 mm is lying on the ground on one of its bases with a vertical face perpendicular to the V.P. A tetrahedron is placed on the prism so that the corners of one of its faces coincide with the alternate corners of the top surface of the prism. Draw the projections of the solids. Project another top view on an auxiliary inclined plane making 45° with the H.P.
29. A square duct is in the form of a frustum of a square pyramid. The sides of top and bottom are 150 mm and 100 mm respectively and its length is 150 mm. It is situated in such a way that its axis is parallel to the H.P. and lies in a plane inclined at 60° to the V.P. Draw the projections of the duct, assuming the thickness of the duct-sheet to be negligible.
30. A pentagonal pyramid, base 30 mm edge and axis 75 mm long, stands upon a circular block, 75 mm diameter and 25 mm thick, so that their axes are in a straight line. Draw the projections of the solids when the base of the block is inclined at 30° to the ground, an edge of the base of the pyramid being parallel to the V.P.
31. The body diagonal of a cube is 75 mm long. The cube has a central 25 mm square hole. The faces of the hole make 45° with the side faces of the cube. Draw the projections of the cube when a body diagonal is perpendicular to the H.P.

32. A bucket, 300 mm diameter at the top and 225 mm diameter at the bottom has a circular ring 225 mm diameter and 50 mm wide attached at the bottom. The total height of the bucket is 300 mm. Draw the projections of the bucket when its axis is inclined at 60° to the H.P. and as a vertical plane makes an angle of 45° with the V.P. Assume the thickness of the plate of the bucket to be equal to that of a line.
33. The vertex-angle of the cone just touching the edges of a vertical hexagonal pyramid 125 mm in height is 45° . Draw the projections of the pyramid on a 45° inclined plane when the former is truncated by a plane making 45° with the axis and bisecting the axis.
34. A knob of a machine handle consists of 15 mm diameter \times 150 mm long cylindrical portion and 40 mm diameter spherical portion. The centre of the sphere lies on the axis of the cylindrical portion. Draw the projections if its axis is inclined at 45° to the horizontal plane.
35. Six equal spheres rest on the ground in contact with each other and also with the slanting faces of a regular upright hexagonal pyramid, 25 mm edge of base and 125 mm length of axis. Draw the projections and find the diameter of the sphere.
36. A cylinder, 100 mm diameter and 150 mm long, has a rectangular slot 50 mm \times 30 mm cut through it. The axis of the slot bisects the axis of the cylinder at right angles and the 50 mm side of the slot makes an angle of 60° with the base of the cylinder. Draw three views of the cylinder.
37. A very thin glass shade for a table lamp is the portion of a sphere 125 mm diameter included between two parallel planes at 15 mm and 55 mm from the centre, making the height 70 mm. If the axis of the shade is inclined at 30° to the vertical, obtain the projections of the shade.
38. A cone frustum, base 75 mm diameter, top 35 mm diameter and height 65 mm has a hole of 30 mm diameter drilled through it so that the axis of the hole coincides with that of the cone. It is resting on its base on the ground and is cut by a section plane perpendicular to the V.P., parallel to an end generator and passing through the top end of the axis. Draw sectional top view and sectional side view of the frustum.
39. Three vertical poles *AB*, *CD* and *EF* are respectively 5, 8 and 12 metres long. Their ends *B*, *D* and *F* are on the ground and lie at the corners of an equilateral triangle of 10 metres long sides. Determine graphically the distance between the top ends of the poles, viz. *AC*, *CE* and *EA*.
40. Two cylinders of 80 mm diameter each meet each other at right angles. The axis of one of the cylinders is parallel to both the reference planes and is 40 mm in front of the axis of the other cylinder. Draw three views of the cylinders showing lines of intersection in them. Take any suitable lengths of the cylinders.
41. A tetrahedron of side 40 mm rests on the top face of a hexagonal prism of base and height 25 mm such that their apex coincide. Draw the projections when the combination rests with one of the sides of the prism on the H.P., is perpendicular to the V.P., and the axis is inclined at 30° to the H.P.
42. A pentagonal pyramid, base 30 mm side and axis 70 mm long, has one of its slant edges in the H.P. and inclined at 30° to the V.P. Draw the projections of the solid when the apex is towards the observer.

Chapter

14



SECTIONS OF SOLIDS

14-0. INTRODUCTION

Invisible features of an object are shown by dotted lines in their projected views. But when such features are too many, these lines make the views more complicated and difficult to interpret. In such cases, it is customary to imagine the object as being cut through or *sectioned* by planes. The part of the object between the cutting plane and the observer is assumed to be removed and the view is then shown *in section*.

The imaginary plane is called a *section plane* or a *cutting plane*. The surface produced by cutting the object by the section plane is called the *section*. It is indicated by thin section lines uniformly spaced and inclined at 45°.

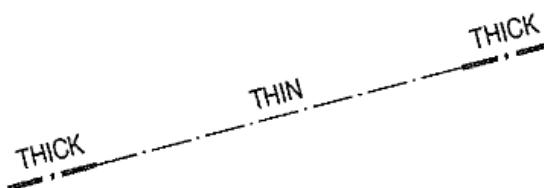
The projection of the section along with the remaining portion of the object is called a *sectional view*. Sometimes, only the word *section* is also used to denote a sectional view.

(1) **Section planes:** Section planes are generally perpendicular planes. They may be perpendicular to one of the reference planes and either perpendicular, parallel or inclined to the other plane. They are usually described by their traces. It is important to remember that the projection of a section plane, on the plane to which it is perpendicular, is a straight line. This line will be parallel, perpendicular or inclined to *xy*, depending upon the section plane being parallel, perpendicular or inclined respectively to the other reference plane.

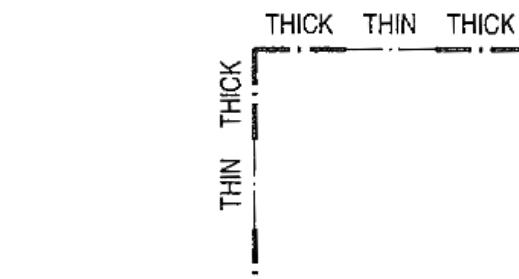
As per latest B.I.S. convention (SP: 46-2003), the cutting-plane line should be drawn as shown in fig. 3-2 which is reproduced here in fig. 14-1 for ready reference.



(a) PARALLEL CUTTING PLANE



(b) INCLINED CUTTING PLANE



(c) CUTTING PLANE AT CHANGING POSITION

FIG. 14-1

(2) Sections: The projection of the section on the reference plane to which the section plane is perpendicular, will be a straight line coinciding with the trace of the section plane on it. Its projection on the other plane to which it is inclined is called *apparent section*. This is obtained by

- projecting on the other plane, the points at which the trace of the section plane intersects the edges of the solid and
- drawing lines joining these points in proper sequence.

(3) True shape of a section: The projection of the section on a plane parallel to the section plane will show the true shape of the section. Thus, when the section plane is parallel to the H.P. or the ground, the true shape of the section will be seen in *sectional top view*. When it is parallel to the V.P., the true shape will be visible in the *sectional front view*.

But when the section plane is inclined, the section has to be projected on an auxiliary plane parallel to the section plane, to obtain its true shape. When the section plane is perpendicular to both the reference planes, the sectional side view will show the true shape of the section. In this chapter sections of different solids are explained in stages by means of typical problems as follows:

1. Sections of prisms
2. Sections of pyramids
3. Sections of cylinders
4. Sections of cones
5. Sections of spheres.

14-1. SECTIONS OF PRISMS

These are illustrated according to the position of the section plane with reference to the principal planes as follows:

- (1) Section plane parallel to the V.P.
- (2) Section plane parallel to the H.P.
- (3) Section plane perpendicular to the H.P. and inclined to the V.P.
- (4) Section plane perpendicular to the V.P. and inclined to the H.P.
- (1) Section plane parallel to the V.P.

Problem 14-1. (fig. 14-2): A cube of 35 mm long edges is resting on the H.P. on one of its faces with a vertical face inclined at 30° to the V.P. It is cut by a section plane parallel to the V.P. and 9 mm away from the axis and further away from the V.P. Draw its sectional front view and the top view.

In fig. 14-2(i), the section plane is assumed to be transparent and the cube is shown with the cut-portion removed. It can be seen that four edges of the cube are cut and hence, the section is a figure having four sides.

Draw the projections of the whole cube in the required position [fig. 14-2(ii)].

As the section plane is parallel to the V.P., it is perpendicular to the H.P.; hence, the section will be seen as a line in the top view coinciding with the H.T. of the section plane.

- (i) Draw a line H.T. in the top view (to represent the section plane) parallel to xy and 9 mm from o .

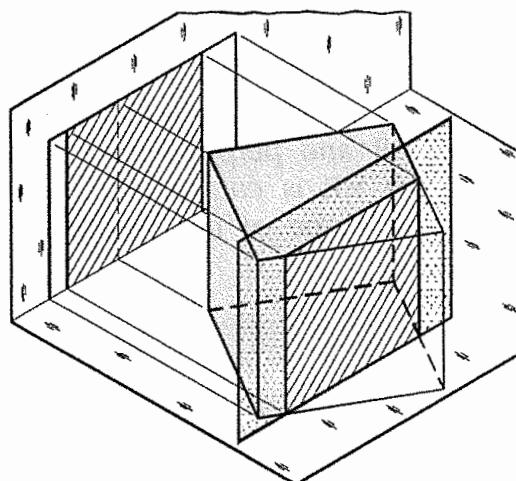
(ii) Name the points at which the edges are cut, viz. ab at 1, bc at 2, gf at 3 and fe at 4.

(iii) Project these points on the corresponding edges in the front view and join them in proper order.

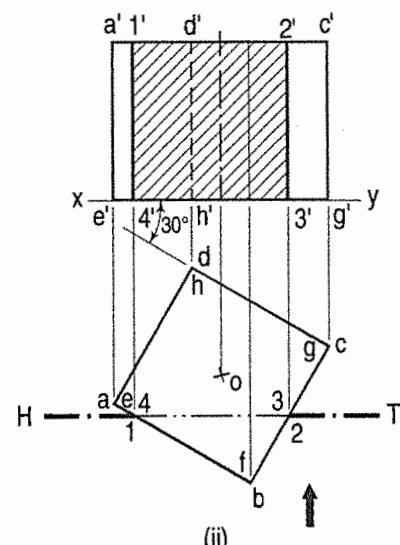
As the section plane is parallel to the V.P., figure $1' 2' 3' 4'$ in the front view, shows the true shape of the section.

Show the views by dark but thin lines, leaving the lines for the cut-portion fainter.

(iv) Draw section lines in the rectangle for the section.



(i)



(ii)

FIG. 14-2

(2) Section plane parallel to the H.P.

Problem 14-2. (fig. 14-3): A triangular prism, base 30 mm side and axis 50 mm long, is lying on the H.P. on one of its rectangular faces with its axis inclined at 30° to the V.P. It is cut by a horizontal section plane, at a distance of 12 mm above the ground. Draw its front view and sectional top view.

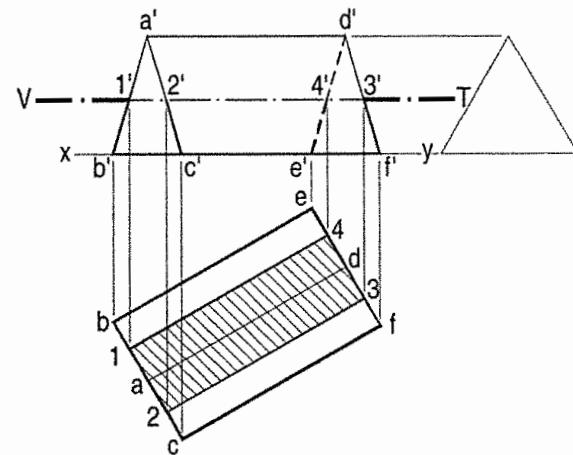
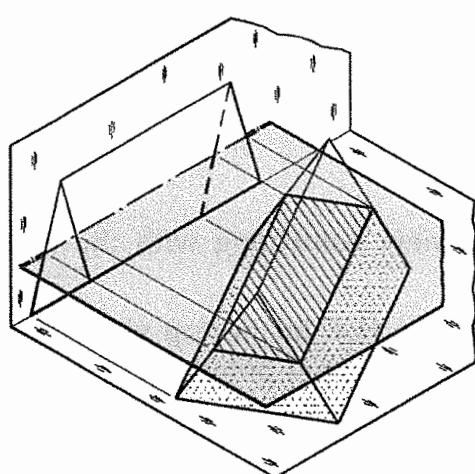


FIG. 14-3

Draw the projections of the prism in the required position.

As the section plane is horizontal, i.e. parallel to the H.P., it is perpendicular to the V.P. Hence, the section will be seen as a line in the front view, coinciding with the V.T. of the section plane.

- Therefore, draw a line V.T. in the front view to represent the section plane, parallel to xy and 12 mm above it.
- Name in correct sequence, points at which the edges are cut viz. $a'b'$ at 1', $a'c'$ at 2', $d'f'$ at 3' and $d'e'$ at 4'.
- Project these points on the corresponding lines in the top view and complete the sectional top view by joining them in proper order.

As the section plane is parallel to the H.P., the figure 1 2 3 4 (in the top view) is the true shape of the section.

(3) Section plane perpendicular to the H.P. and inclined to the V.P.

Problem 14-3. (fig. 14-4): A cube in the same position as in problem 14-1, is cut by a section plane, inclined at 60° to the V.P. and perpendicular to the H.P., so that the face which makes 60° angle with the V.P. is cut in two equal halves. Draw the sectional front view, top view and true shape of the section.

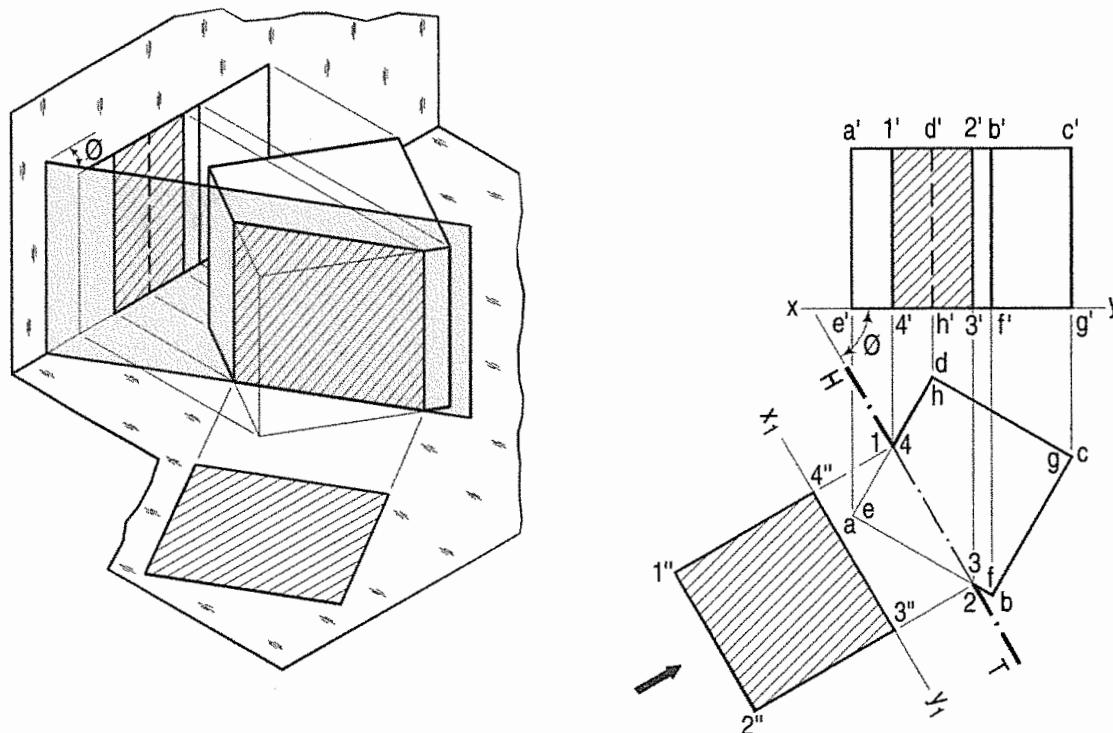


FIG. 14-4

The section will be seen as a line in the top view coinciding with the H.T. of the section plane.

- Draw the projections of the cube. Draw a line H.T. in the top view inclined at 60° to xy and cutting the line ad (or bc) at its mid-point.
- Name the corners at which the four edges are cut and project them in the front view. As the section plane is inclined to the V.P., the front view of the section viz. 1' 2' 3' 4' does not reveal its true shape. Only the vertical lines show true lengths, while the true lengths of the horizontal lines are seen in the top view.

The true shape of the section will be seen when it is projected on an auxiliary vertical plane, parallel to the section plane.

- (iii) Therefore, draw a new reference line x_1y_1 parallel to the H.T. and project the section on it. The distances of the points from x_1y_1 should be taken equal to their corresponding distances from xy in the front view. Thus 4" and 3" will be on x_1y_1 . 1" 4" and 2" 3" will be equal to 1' 4' and 2' 3' respectively. Complete the rectangle 1" 2" 3" 4" which is the true shape of the section and draw section lines in it.

(4) Section plane perpendicular to the V.P. and inclined to the H.P.

Problem 14-4. (fig. 14-5): A cube in the same position as in problem 14-1 is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and passing through the top end of the axis. (i) Draw its front view, sectional top view and true shape of the section. (ii) Project another top view on an auxiliary plane, parallel to the section plane.

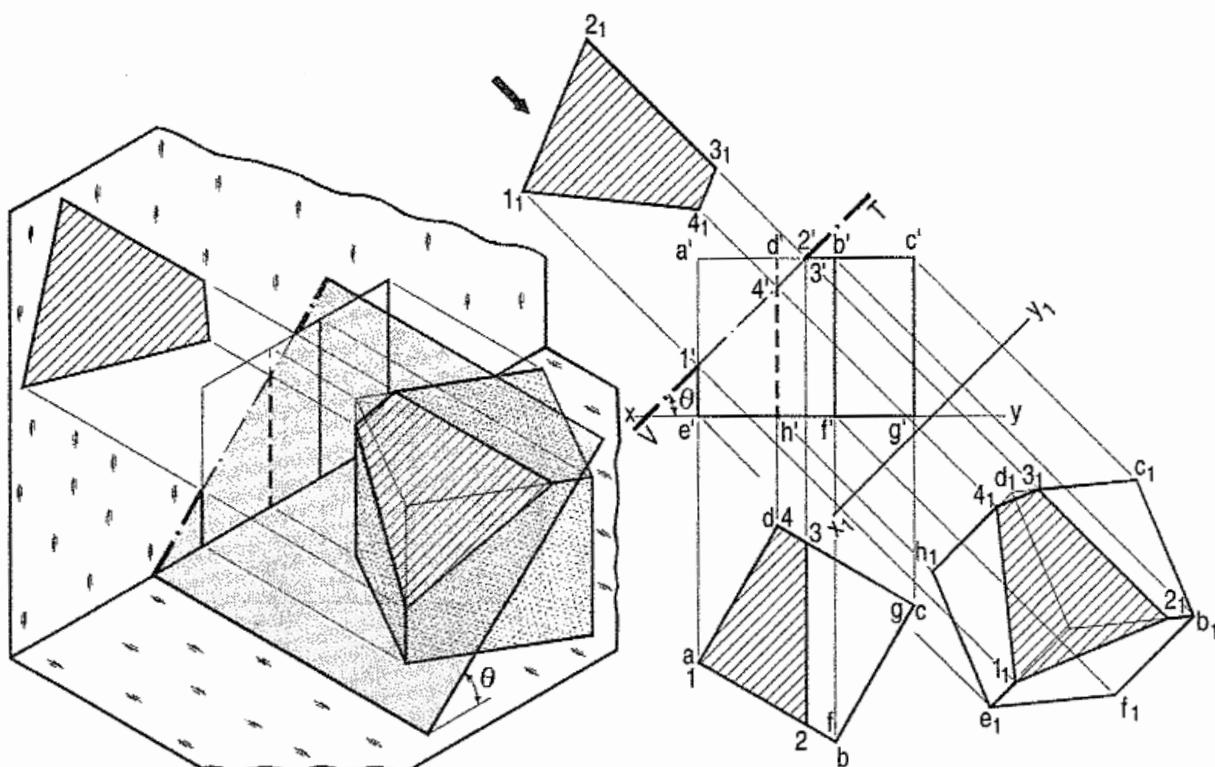


FIG. 14-5

The section will be seen as a line in the front view.

- (i) Draw a line V.T. in the front view, inclined at 45° to xy and passing through the top end of the axis. It cuts four edges, viz. $a' e'$ at 1', $a' b'$ at 2', $c' d'$ at 3' and $d' h'$ at 4'.
- (ii) Project the top view of the section, viz. the figure 1 2 3 4. It does not show the true shape of the section, as the section plane is inclined to the H.P. To determine the true shape, an auxiliary top view of the section should be projected on an A.I.P. parallel to the section plane.
- (iii) Assuming the new reference line for the A.I.P. to coincide with the V.T., project the true shape of the section as shown by quadrilateral 1₁ 2₁ 3₁ 4₁.

The distances of all the points from the V.T. should be taken equal to their corresponding distances from xy in the top view, e.g. $1_1 1' = e' 1$, $4_1 4' = h' 4$ etc.

- (iv) To project an auxiliary sectional top view of the cube, draw a new reference line $x_1 y_1$, parallel to the V.T. The whole cube may first be projected and the points for the section may then be projected on the corresponding lines for the edges. Join these points in correct sequence and obtain the required top view.
- (v) Draw section lines in the cut-surface, in the views where it is seen. Keep the lines for the removed edges thin and fainter.

Additional problems on sections of prisms:

Problem 14-5. (fig. 14-6): A square prism, base 40 mm side, axis 80 mm long, has its base on the H.P. and its faces equally inclined to the V.P. It is cut by a plane, perpendicular to the V.P., inclined at 60° to the H.P. and passing through a point on the axis, 55 mm above the H.P. Draw its front view, sectional top view and another top view on an A.I.P. parallel to the section plane.

The problem is similar to problem 14-4 and needs no further explanation. The true shape of the section is seen in the auxiliary top view.

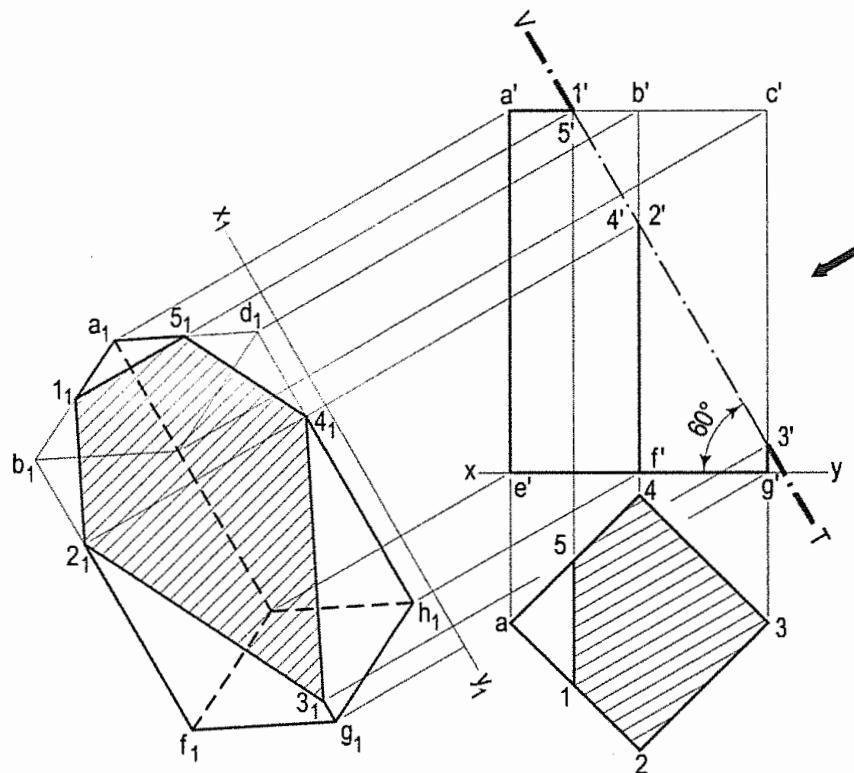


FIG. 14-6

Problem 14-6. (fig. 14-7): A hexagonal prism, has a face on the H.P. and the axis parallel to the V.P. It is cut by a vertical section plane, the H.T. of which makes an angle of 45° with xy and which cuts the axis at a point 20 mm from one of its ends. Draw its sectional front view and the true shape of the section. Side of base 25 mm long; height 65 mm.

- (i) Draw the front view and the top view of the prism and show the H.T. of the section plane in the top view. Name in proper sequence, the points at which the lines are cut.

- (ii) Project them on the corresponding lines in the front view. The positions of points 4 and 5 cannot be located directly. Hence, project them on the first top view to 4_1 on ef and 5_1 on ed . From this top view, obtain their positions $4'_1$ and $5'_1$ on the corresponding lines in the first front view. As the two front views are identical, these points can now be transferred to the second front view by making $e'4$ equal to $e'4'_1$ and $e'5$ equal to $e'5'_1$. $4'$ and $5'$ are the projections of points 4 and 5 respectively. Complete the sectional front view as shown.

- (iii) Obtain the true shape of the section on x_1y_1 as explained in problem 14-3, making $o''1''$ equal to $o'1'$, etc.

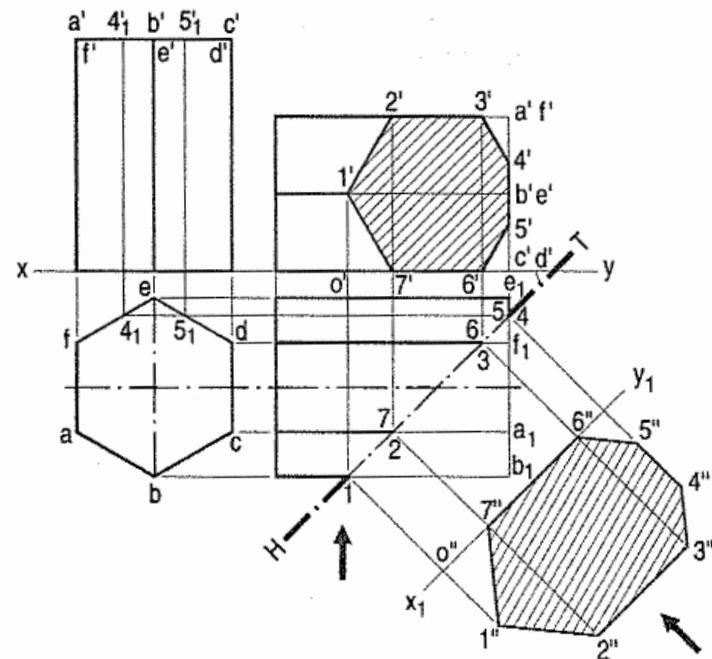


FIG. 14-7

Problem 14-7. (fig. 14-8): A pentagonal prism, base 28 mm side and height 65 mm has an edge of its base on the H.P. and the axis parallel to the V.P. and inclined at 60° to the H.P. A section plane, having its H.T. perpendicular to xy , and the V.T. inclined at 60° to xy and passing through the highest corner, cuts the prism. Draw the sectional top view and true shape of the section.

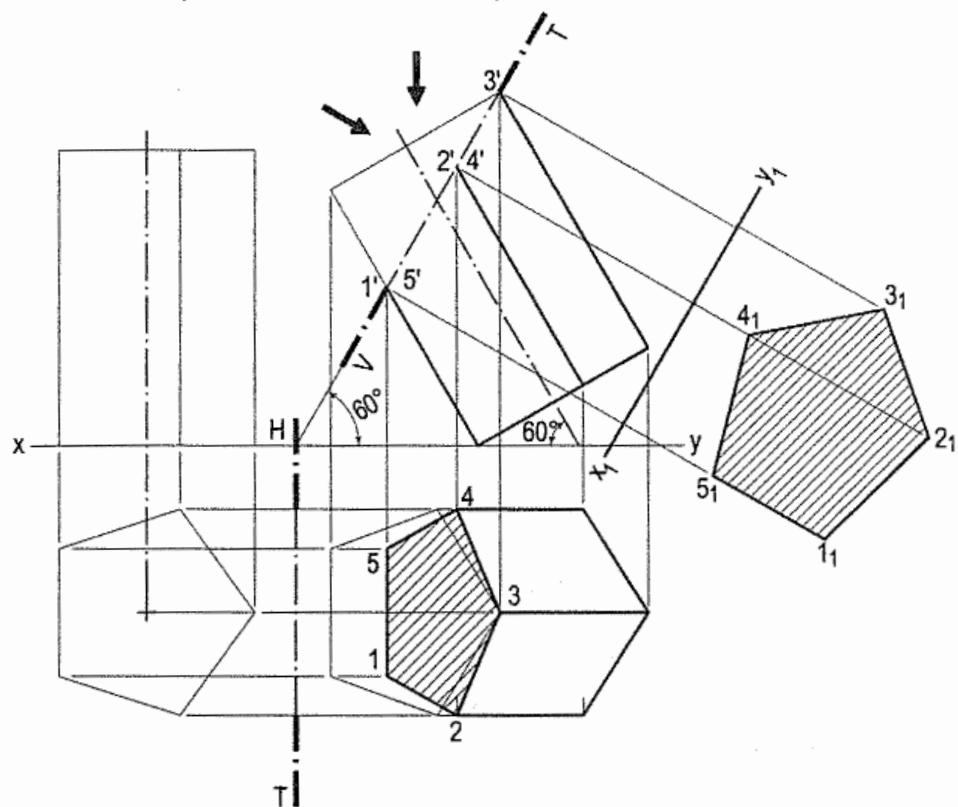


FIG. 14-8

- Draw the projections of the prism in the required position.
- Draw the line V.T. passing through the highest corner $3'$ and inclined at 60° to xy . A perpendicular to xy through V will be the H.T. of the section plane.
- Project the sectional top view and the true shape of the section, as shown in the figure.

Problem 14-8. (fig. 14-9): A hollow square prism, base 40 mm side (outside), height 65 mm and thickness 8 mm is resting on its base on the H.P. with a vertical face inclined at 30° to the V.P. A section plane, inclined at 30° to the H.P., perpendicular to the V.P. and passing through the axis at a point 12 mm from its top end, cuts the prism. Draw its sectional top view, sectional side view and true shape of the section.

- Draw the projections of the prism in the given position, showing the hidden edges by dashed lines.
- Draw a line V.T. for the cutting plane and mark points at which the inside and outside edges are cut.
- Project the sectional top view, true shape of the section and the sectional side view as shown.

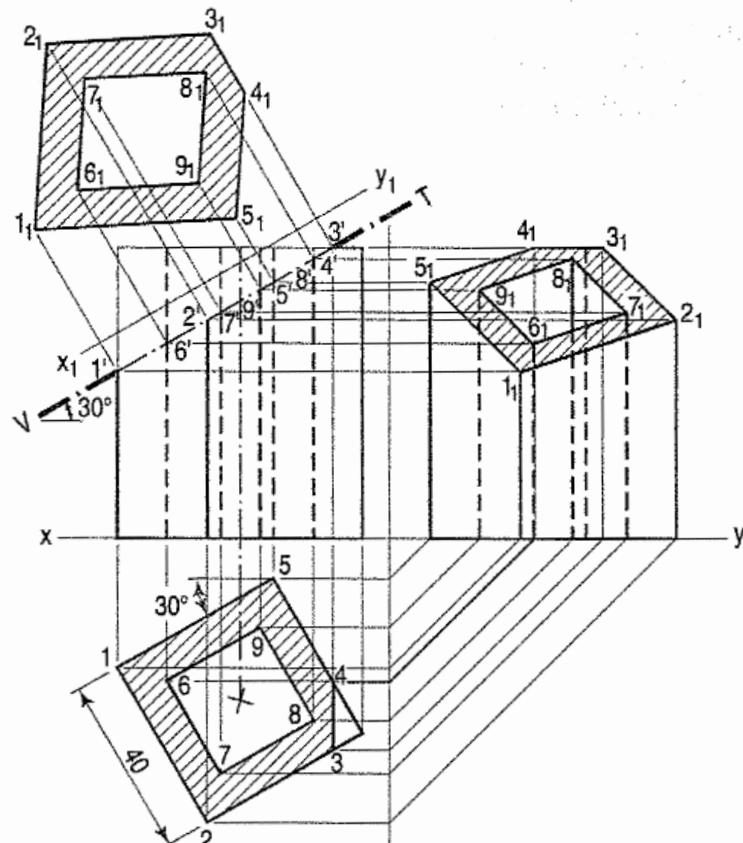


FIG. 14-9

14-2. SECTIONS OF PYRAMIDS



The following cases are discussed in details.

- Section plane parallel to the base of the pyramid.
- Section plane parallel to the V.P.
- Section plane perpendicular to the V.P. and inclined to the H.P.
- Section plane perpendicular to the H.P. and inclined to the V.P.

(1) Section plane parallel to the base of the pyramid.

Problem 14-9. (fig. 14-10): A pentagonal pyramid, base 30 mm side and axis 65 mm long, has its base horizontal and an edge of the base parallel to the V.P. A horizontal section plane cuts it at a distance of 25 mm above the base. Draw its front view and sectional top view.

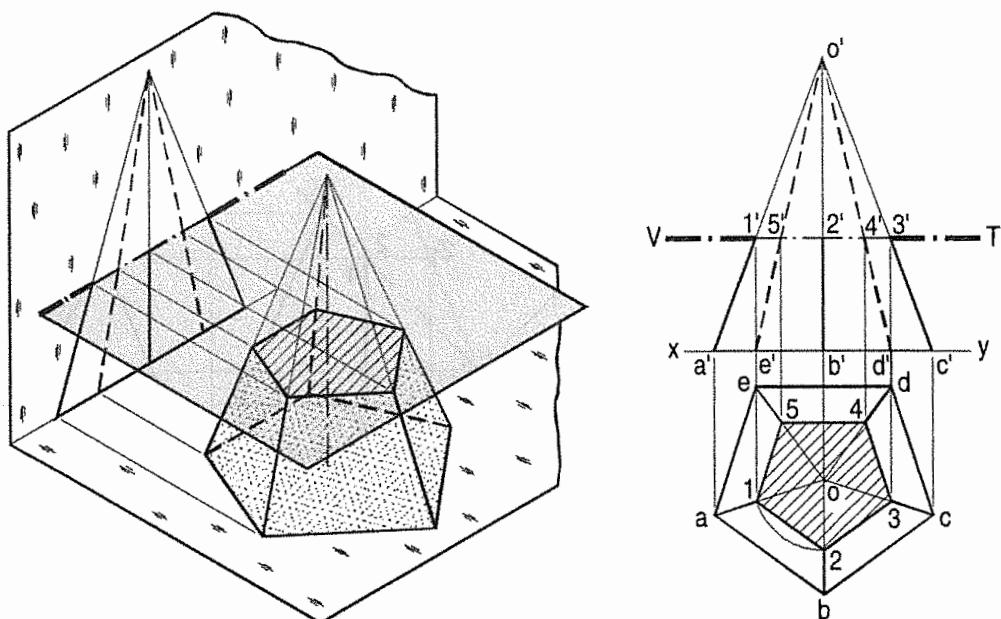


FIG. 14-10

- Draw the projections of the pyramid in the required position and show a line V.T. for the section plane, parallel to and 25 mm above the base. All the five slant edges are cut.
- Project the points at which they are cut, on the corresponding edges in the top view. The point 2' cannot be projected directly as the line ob is perpendicular to xy. But it is quite evident from the projections of other points that the lines of the section in the top view, viz. 3-4, 4-5 and 5-1 are parallel to the edges of the base in their respective faces and that the points 1, 3, 4 and 5 are equidistant from o.
- Hence, line 1-2 also will be parallel to ab and o2 will be equal to o1, o3 etc. Therefore, with o as centre and radius o1, draw an arc cutting ob at a point 2 which will be the projection of 2'. Complete the sectional top view in which the true shape of the section, viz. the pentagon 1, 2, 3, 4 and 5 is also seen.
- Hence, when a pyramid is cut by a plane parallel to its base, the true shape of the section will be a figure, similar to the base; the sides of the section will be parallel to the edges of the base in the respective faces and the corners of the section will be equidistant from the axis.

(2) Section plane parallel to the V.P.

Problem 14-10. (fig. 14-11): A triangular pyramid, having base 40 mm side and axis 50 mm long, is lying on one of its faces, with the axis parallel to the V.P. A section plane, parallel to the V.P. cuts the pyramid at a distance of 6 mm from the axis. Draw its sectional front view and the top view.

- Draw the projections of the pyramid in the required position and show a line H.T. (for the cutting plane) in the top view parallel to xy and 6 mm from the axis.
- Project points 1, 2 and 3 (at which the edges are cut) on corresponding edges in the front view and join them. Figure 1' 2' 3' shows the true shape of the section.

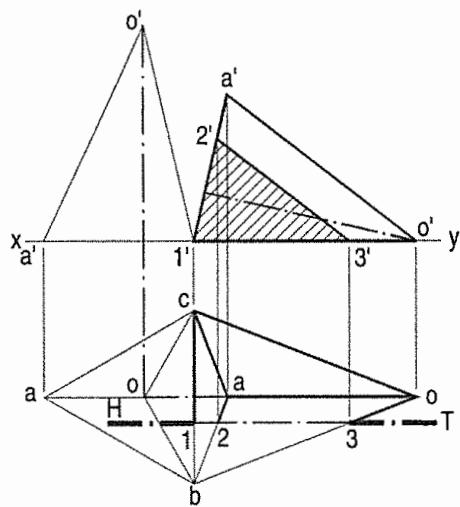


FIG. 14-11

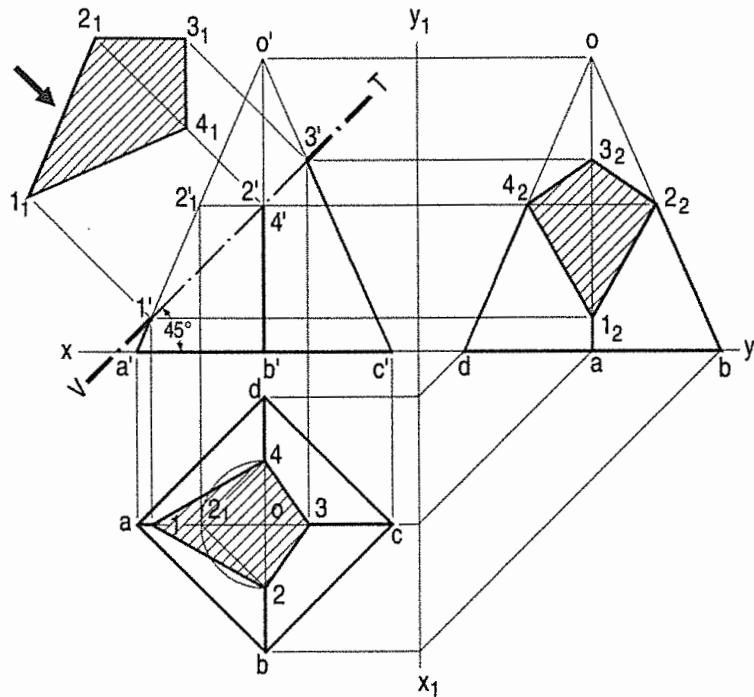


FIG. 14-12

(3) Section plane perpendicular to the V.P. and inclined to the H.P.

Problem 14-11. (fig. 14-12): A square pyramid, base 40 mm side and axis 65 mm long, has its base on the H.P. and all the edges of the base equally inclined to the V.P. It is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and bisecting the axis. Draw its sectional top view, sectional side view and true shape of the section.

- Draw the projections of the pyramid in the required position. The section plane will be seen as a line in the front view. Hence, draw a line V.T. through the mid-point of the axis and inclined at 45° to xy . Name in correct sequence the points at which the four edges are cut and project them in the top view. Here also, points 2' and 4' cannot be projected directly.

Hence, assume a horizontal section through 2' and draw a line parallel to the base, cutting $o' a'$ at $2'_1$. Project $2'_1$ to 2_1 on oa in the top view. From 2_1 draw a line parallel to ab and cutting ob at a point 2. Or, with o as centre and radius $o 2_1$, draw an arc cutting ob at 2 and ob at 4. Complete the section 1 2 3 4 by joining the points and draw section lines in it.

- Assuming the V.T. to be the new reference line, draw the true shape of the section. Project the side view from the two views. The removed portion of the pyramid may be shown by thin and faint lines.

(4) Section plane perpendicular to the H.P. and inclined to the V.P.

Problem 14-12. (fig. 14-13): A pentagonal pyramid has its base on the H.P. and the edge of the base nearer the V.P., parallel to it. A vertical section plane, inclined at 45° to the V.P., cuts the pyramid at a distance of 6 mm from the axis. Draw the top view, sectional front view and the auxiliary front view on an A.V.P. parallel to the section plane. Base of the pyramid 30 mm side; axis 50 mm long.

The section plane will be seen as a line in the top view. It is to be at a distance of 6 mm from the axis.

- (i) Hence, draw a circle with o as centre and radius equal to 6 mm.
- (ii) Draw a line H.T., tangent to this circle and inclined at 45° to xy . It can be drawn in four different positions, of which any one may be selected.
- (iii) Project points 1, 2 etc. from the top view to the corresponding edges in the front view. Here again, point 2 cannot be projected directly. The process shown in problem 14-11 must be reversed. With centre o and radius $o2$ draw an arc cutting any one of the slant edges, say oc at 2_1 . Project 2_1 to $2'_1$ on $o'c'$.
- (iv) Through $2'_1$, draw a line parallel to the base, cutting $o'b'$ at $2'$. Then $2'$ is the required point. Complete the view. It will show the apparent section.
- (v) Draw a reference line x_1y_1 parallel to the H.T. and project an auxiliary sectional front view which will show the true shape of the section also.

Additional problems on sections of pyramids:

Problem 14-13. (fig. 14-14): A hexagonal pyramid, base 30 mm side and axis 65 mm long, is resting on its base on the H.P. with two edges parallel to the V.P. It is cut by a section plane, perpendicular to the V.P. and intersecting the axis at a point 25 mm above the base. Draw the front view, sectional top view, sectional side view and true shape of the section.

This problem is similar to problem 14-11. In this case, the base is also cut and hence, the section is a heptagon. Care must be taken to name the points in proper sequence.

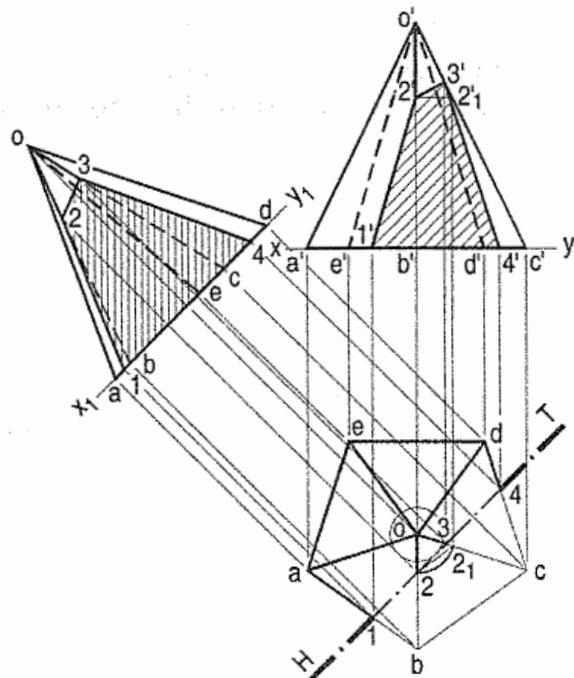


FIG. 14-13

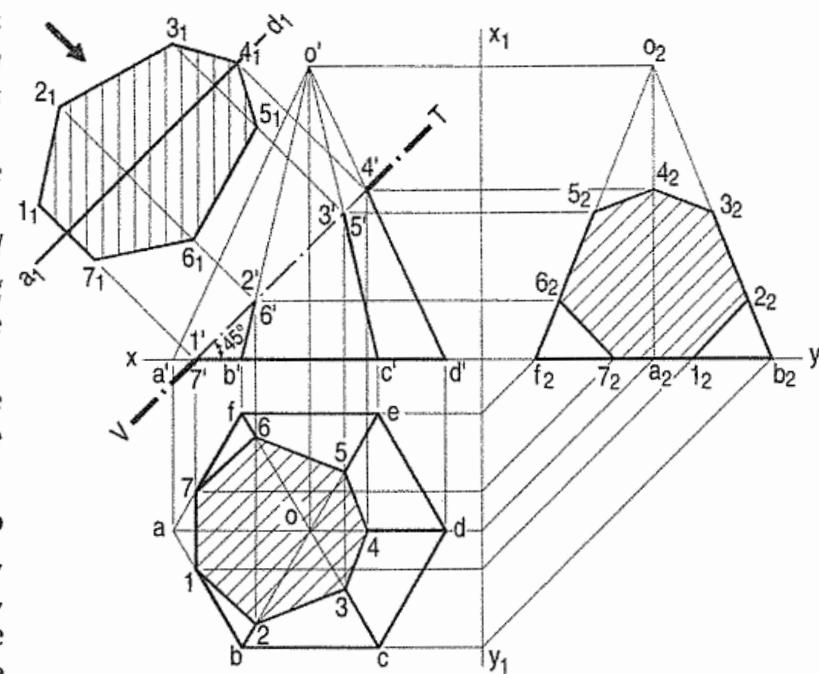


FIG. 14-14

The true shape may be drawn on the V.T. as a new reference line or around the centre line a_1d_1 , drawn parallel to the V.T. as shown.

The distances of the points $1_1, 2_1$ etc. from a_1d_1 are taken equal to the distances of points $1, 2$ etc. from the line ad (which is parallel to xy).

Problem 14-14. (fig. 14-15): A pentagonal pyramid, base 30 mm side and axis 60 mm long, is lying on one of its triangular faces on the H.P. with the axis parallel to the V.P. A vertical section plane, whose H.T. bisects the top view of the axis and makes an angle of 30° with the reference line, cuts the pyramid, removing its top part. Draw the top view, sectional front view, true shape of the section and development of the surface of the remaining portion of the pyramid.

- Draw the H.T. of the section plane and name the points at which the edges are cut, in correct sequence, i.e. mark the visible edges first and then the hidden edges.
- Project the sectional front view which will show the apparent section.

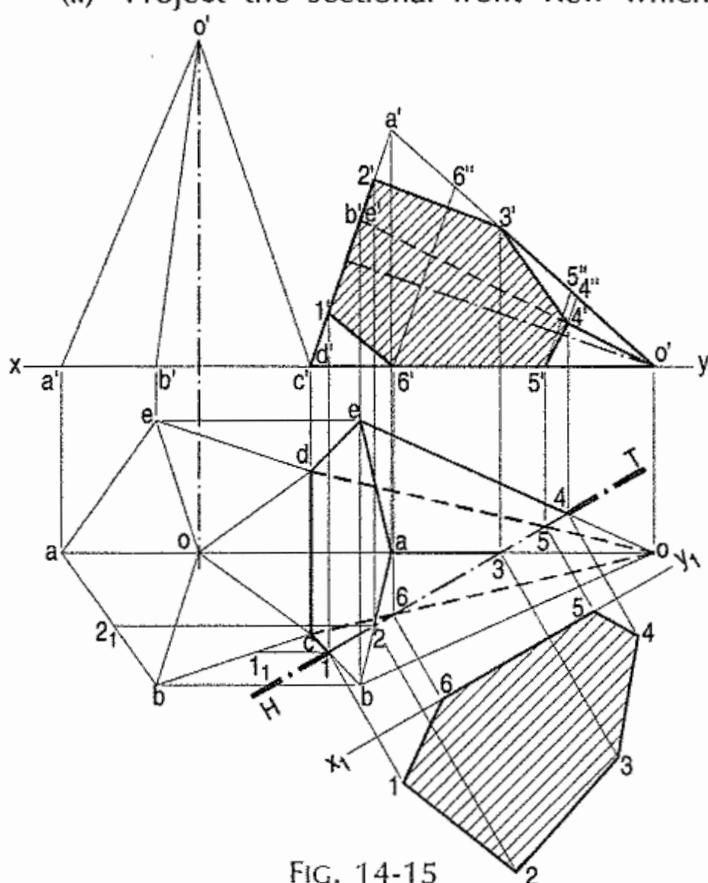


FIG. 14-15

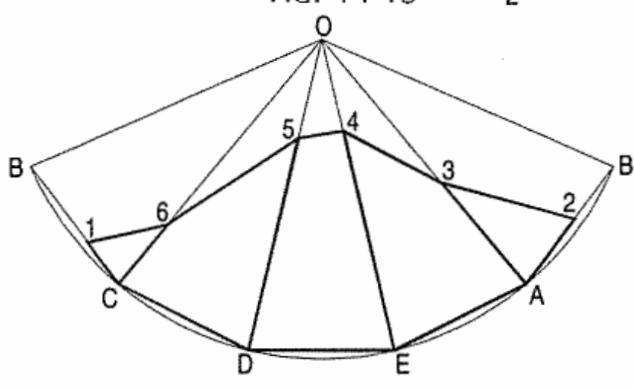


FIG. 14-16

- Obtain the true shape of the section on x_1y_1 as a new reference line drawn parallel to the H.T.

Development (fig. 14-16): The line $o'a'$ shows the true length of the slant edge.

- With any point O as centre and radius $o'a'$, draw an arc and construct the development of the whole pyramid. Mark points on it, taking the positions of 1 and 2 from the first top view and those of other points by projecting them on the true-length-line $o'a'$.
- Draw lines joining these points and complete the development as shown in the figure.

Problem 14-15. (fig. 14-17): A hexagonal pyramid, base 30 mm side and axis 60 mm long, has a triangular face on the H.P. and the axis parallel to the V.P. It is cut by a horizontal section plane which bisects the axis. Draw the front view and sectional top view and develop the surface of the cut-pyramid.

The V.T. cuts six edges. The sectional top view shows the true shape of the section also.

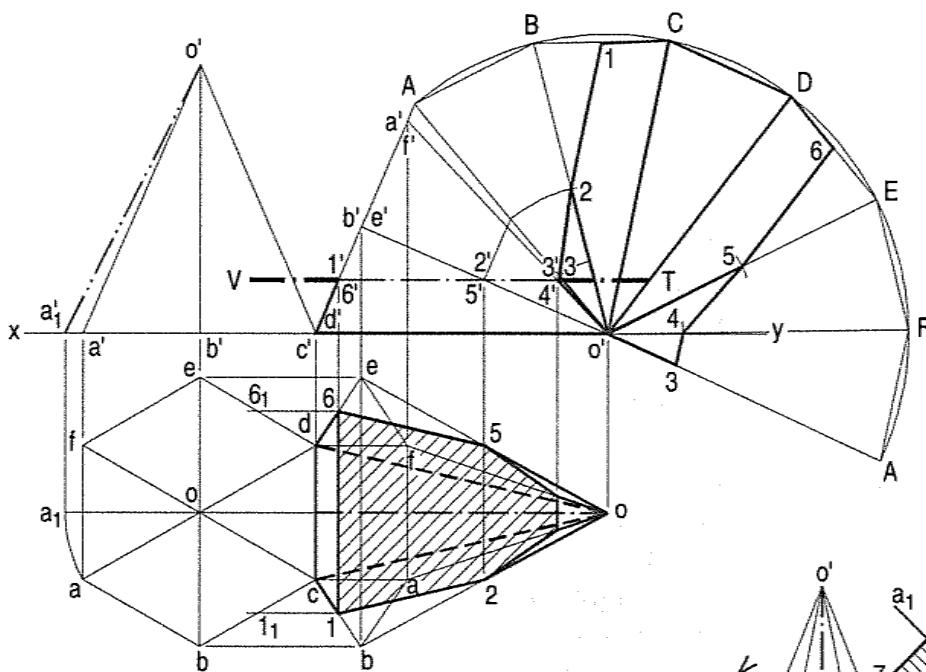


FIG. 14-17

Development: None of the edges shows the true length of the slant edge.

- Hence, determine the true length $o'a'_1$ and draw the development of the whole pyramid.
- Locate positions of the points 1 and 6 by projecting them on the first top view and positions of other points by drawing lines through them, parallel to the base and upto the true length line $o'A$.
- Mark these points on the development and complete it as shown.

Problem 14-16. (fig. 14-18): A hexagonal pyramid, base 30 mm side and axis 75 mm long, resting on its base on the H.P. with two of its edges parallel to the V.P. is cut by two section planes, both perpendicular to the V.P. The horizontal section plane cuts the axis at a point 35 mm from the apex. The other plane which makes an angle of 45° with the H.P., also intersects the axis at the same point. Draw the front view, sectional top view, true shape of the section and development of the surface of the remaining part of the pyramid.

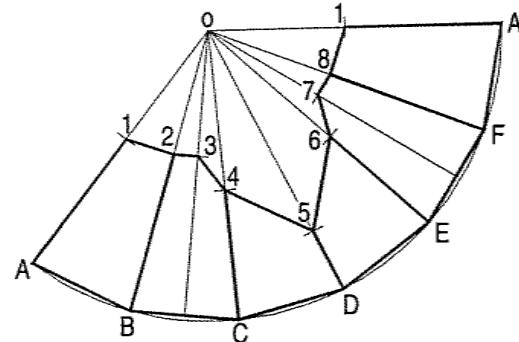
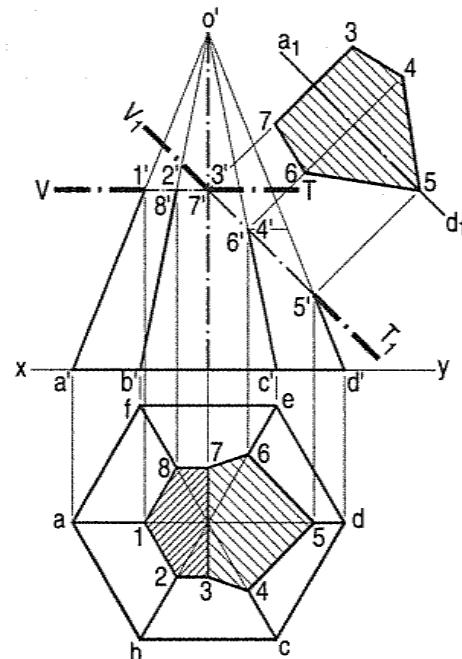


FIG. 14-18

- Draw lines $V.T$ and V_1T_1 for the two section planes. The top view will show the true shape of the horizontal section, the sides of which are parallel to the respective sides of the base. The true shape of the other section may be obtained on V_1T_1 as the reference line or around a_1d_1 .
- Draw the development with $o'a'$ or $o'd'$ as radius and locate the points on it, as shown in the figure.

14-3. SECTIONS OF CYLINDERS



We shall now learn the following three cases. They are

- (1) Section plane parallel to the base
- (2) Section plane parallel to the axis
- (3) Section plane inclined to the base.

(1) Section plane parallel to the base:

When a cylinder is cut by a section plane parallel to the base, the true shape of the section is a circle of the same diameter.

(2) Section plane parallel to the axis:

When a cylinder is cut by a section plane parallel to the axis, the true shape of the section is a rectangle, the sides of which are respectively equal to the length of the axis and the length of the section plane within the cylinder (fig. 14-19). When the section plane contains the axis, the rectangle will be of the maximum size.

(3) Section plane inclined to the base:

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 31 for the following problem.

Problem 14-17. (fig. 14-20): A cylinder of 40 mm diameter, 60 mm height and having its axis vertical, is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and intersecting the axis 32 mm above the base. Draw its front view, sectional top view, sectional side view and true shape of the section.

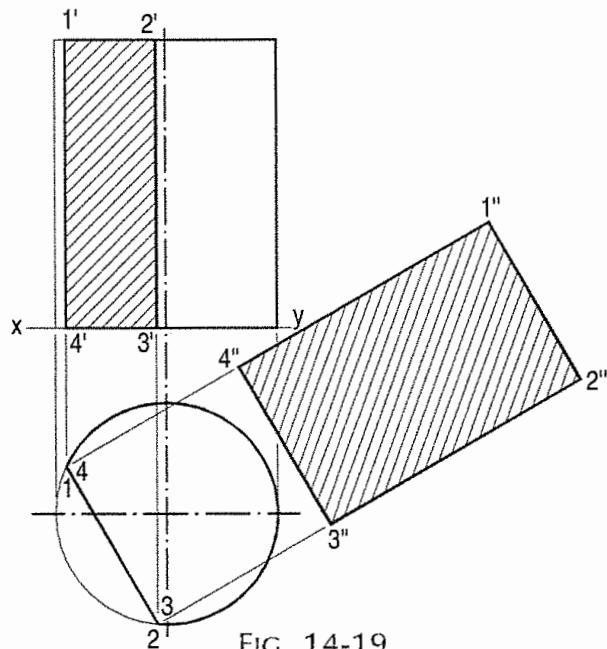
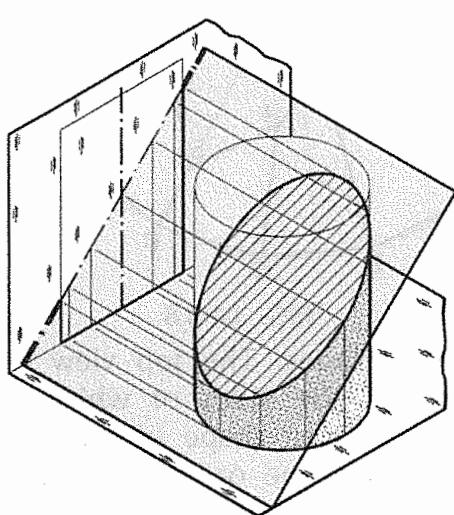


FIG. 14-19

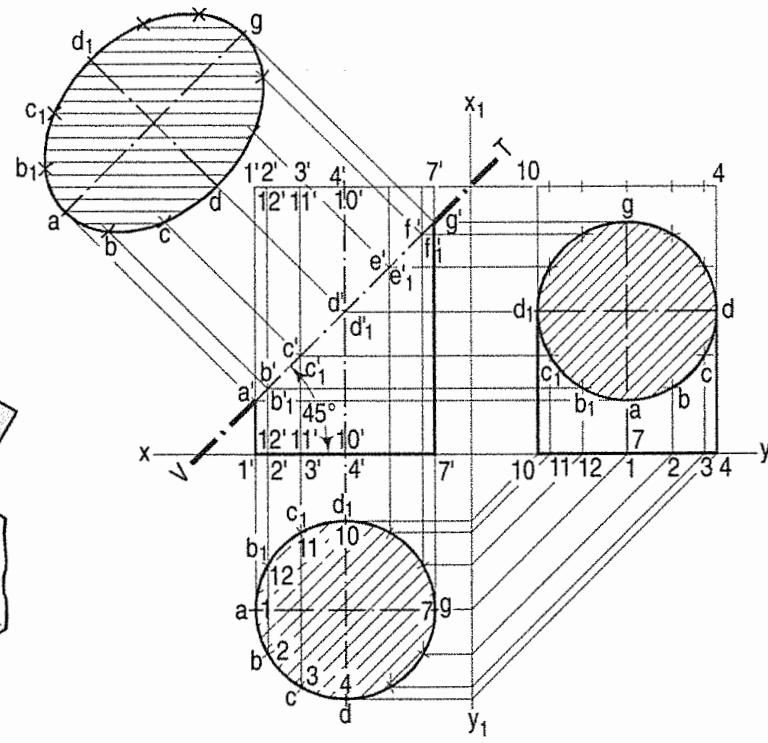


FIG. 14-20

As the cylinder has no edges, a number of lines representing the generators may be assumed on its curved surface by dividing the base-circle into, say 12 equal parts.

- (i) Name the points at which these lines are cut by the V.T. In the top view, these points lie on the circle and hence, the same circle is the top view of the section. The width of the section at any point, say c' , will be equal to the length of the chord cc_1 in the top view.
- (ii) The true shape of the section may be drawn around the centre line ag drawn parallel to V.T. as shown. It is an ellipse the major axis of which is equal to the length of the section plane viz. $a'g'$, and the minor axis equal to the diameter of the cylinder viz. dd_1 .
- (iii) Project the sectional side view as shown. The section will be seen as a circle because the section plane makes 45° angle with xy .

Additional problems on sections of cylinders:

Problem 14-18. (fig. 14-21): A cylinder 50 mm diameter and 60 mm long, is resting on its base on the ground. It is cut by a section plane perpendicular to the V.P., the V.T. of which cuts the axis at a point 40 mm from the base and makes an angle of 45° with the H.P. Draw its front view, sectional top view and another sectional top view on an A.I.P. parallel to the section plane.

In this case, the top end of the cylinder is also cut. Hence, the true shape of the section is a part of an ellipse as shown in the auxiliary top view.

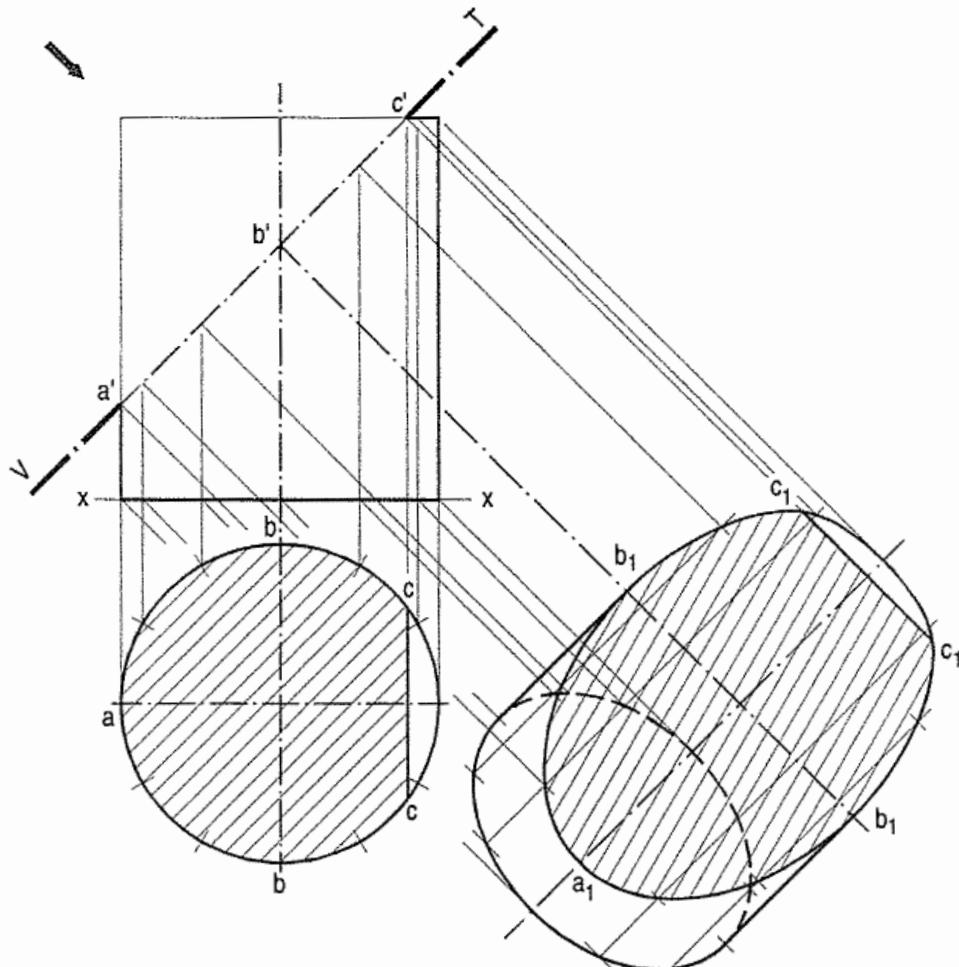


FIG. 14-21

Problem 14-19. (fig. 14-22): A cylinder, 55 mm diameter and 65 mm long, has its axis parallel to both the H.P. and the V.P. It is cut by a vertical section plane inclined at 30° to the V.P., so that the axis is cut at a point 30 mm from one of its ends and both the bases of the cylinder are partly cut. Draw its sectional front view and true shape of the section.

Draw the projections of the cylinder and a line H.T. for the section plane. Project the points at which the bases and the lines are cut. The points on the bases cannot be projected directly. Therefore, project them

- To the first top view i.e. a to a_1 and e to e_1 .
- Then to the first front view, i.e. a_1 to a'_1 and e_1 to e'_1 .
- Finally, transfer them to the second front view to a' and e' each, at two places as shown.
- Draw the true shape of the section either on a new reference line or symmetrically around the centre line and making aa equal to $a'a'$, cc equal to $c'c'$ etc.

Problem 14-20. (fig. 14-23): A hollow cylinder, 50 mm outside diameter, axis 70 mm long and thickness 8 mm has its axis parallel to the V.P. and inclined at 30° to the vertical. It is cut in two equal halves by a horizontal section plane. Draw its sectional top view.

The figure is self-explanatory. Note that a part of the ellipse for the inside bottom will also be visible.

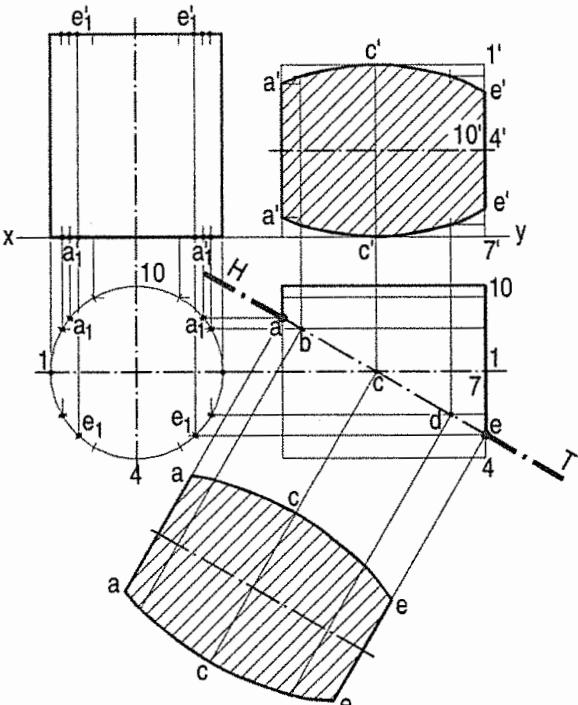


FIG. 14-22

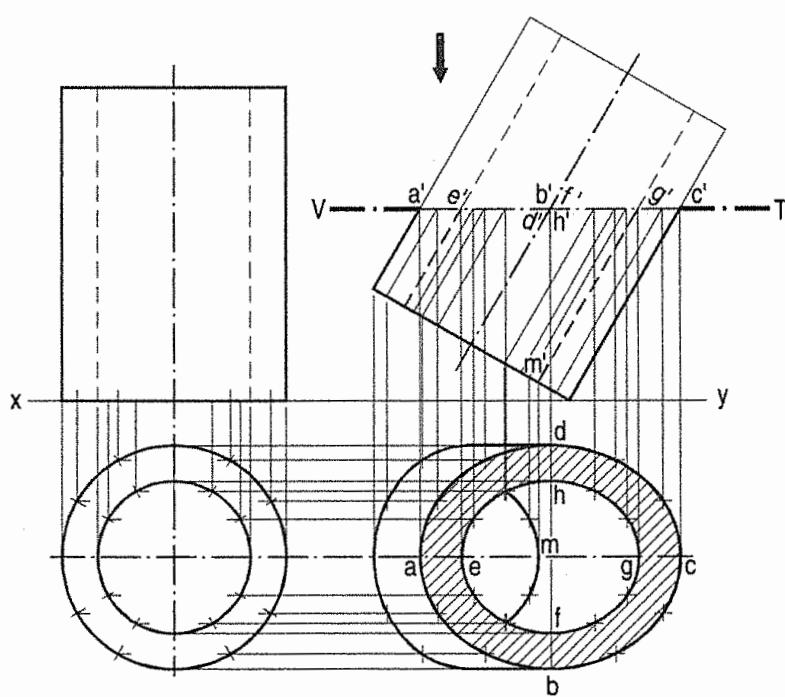


FIG. 14-23

14-4. SECTIONS OF CONES



This is discussed in details as follows:

- (1) Section plane parallel to the base of the cone.
- (2) Section plane passing through the apex of the cone.
- (3) Section plane inclined to the base of the cone at an angle smaller than the angle of inclination of the generators with the base.
- (4) Section plane parallel to a generator of the cone.
- (5) Section plane inclined to the base of the cone at an angle greater than the angle of inclination of the generators with the base.

(1) Section plane parallel to the base of the cone:

The cone resting on the H.P. on its base [fig. 14-24(i)] is cut by a section plane parallel to the base. The true shape of the section is shown by the circle in the top view, whose diameter is equal to the length of the section viz. $a'a'$. The width of the section at any point, say b' , is equal to the length of the chord bb_1 .

Problem 14-21. [fig. 14-24(ii)]: To locate the position in the top view of any given point p' in the front view of the above cone.

Method I:

- (i) Through p' , draw a line $r'r'$ parallel to the base.
- (ii) With o as centre and diameter equal to $r'r'$, draw a circle in the top view.
- (iii) Project p' to points p and p_1 on this circle. p is the top view of p' . p_1 is the top view of another point p'_1 on the back side of the cone and coinciding with p' . The chord pp_1 shows the width of the horizontal section of the cone at the point p' . This method may be called the *circle method*.

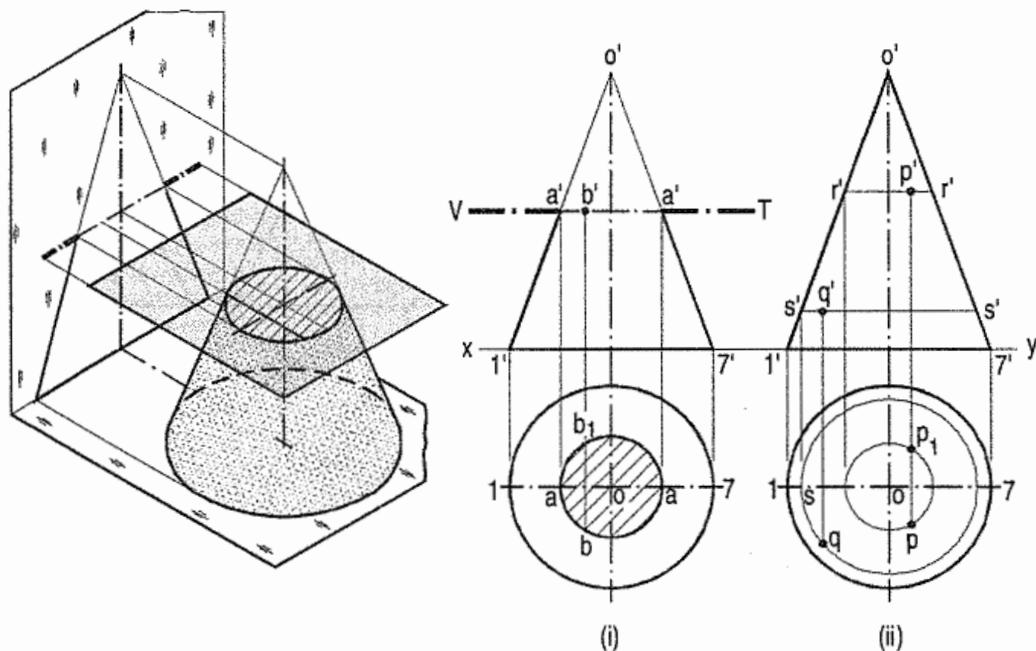


FIG. 14-24

Method II:

When the position of a point in the top view say q is given, its front view q' can be determined by reversing the above process.

- (i) With centre o and radius oq , draw a circle cutting the horizontal centre line at s .
- (ii) Through s , draw a projector cutting the slant side $o'1'$ at s' .
- (iii) Draw the line $s's'$ parallel to the base, intersecting a projector through q at the required point q' .

(2) Section plane passing through the apex of the cone:

Problem 14-22. [fig. 14-25(i)]: A cone, diameter of base 50 mm and axis 50 mm long is resting on its base on the H.P. It is cut by a section plane perpendicular to the V.P., inclined at 75° to the H.P. and passing through the apex. Draw its front view, sectional top view and true shape of the section.

Draw the projections of the cone and on it, show the line V.T. for the section plane.

Mark a number of points a' , b' etc. on the V.T. and project them to points a , b etc. in the top view by the circle method. It will be found that these points lie on a straight line through o .

Thus, od is the top view of the line or generator $o'd'$ and triangle odd_1 is the top view of the section. The width of the section at any point b' on the section is the line bb_1 , obtained by projecting b' on this triangle. This method is called the generator method.

Project the true shape of the section. It is an isosceles triangle, the base of which is equal to the length of the chord on the base-circle and the altitude is equal to the length of the section plane within the cone.

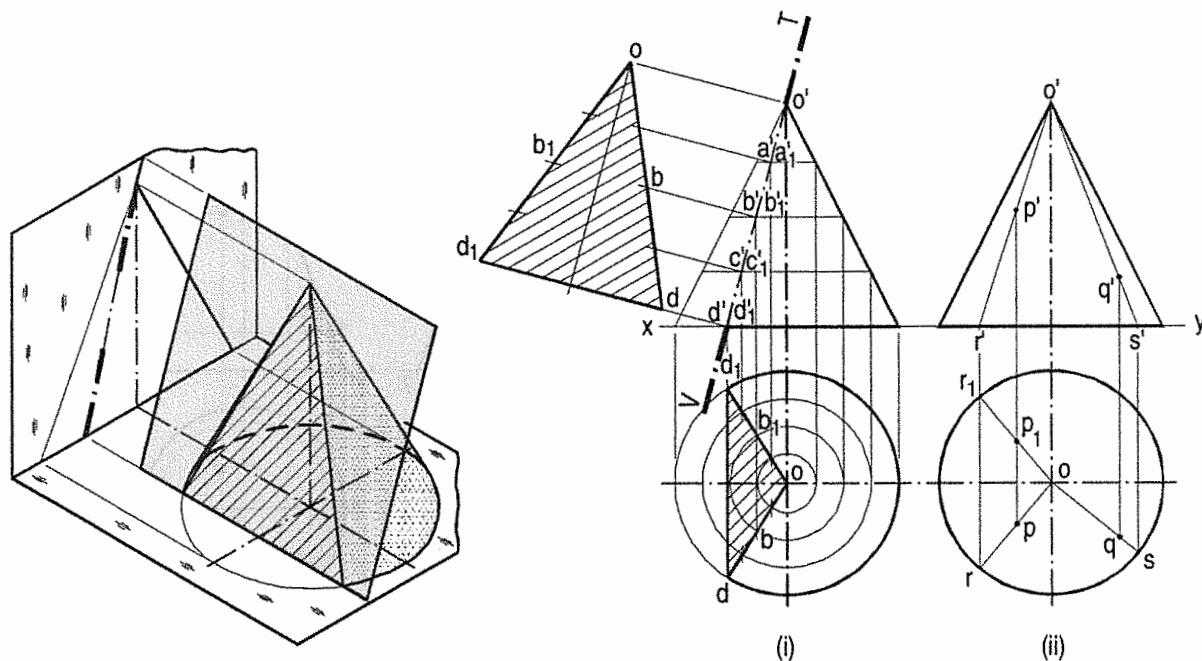


FIG. 14-25

Problem 14-23. [fig. 14-25(ii)]: To determine by generator method, the position in the top view of a given point p' in the front view of the above cone.

Draw the line $o'p'$ and produce it to cut the base at r' . Project r' to points r and r_1 on the base-circle in the top view. Draw lines or and or_1 . Thus, or is the top view of the generator $o'r'$, and or_1 that of the generator (at the back) which coincides with $o'r'$. Project p' to p and p_1 on or and or_1 respectively. Thus, p is the top view of p' , and p_1 is the top view of another point on the other side of the cone and coinciding with p' . The line pp_1 is the width of the horizon-section of the cone at p' .

The position in the front view of any point in the top view, say q , may be determined by reversing the process. Draw the line oq and produce it to cut the base-circle at s . Project s to s' on the base in the front view. Join o' with s' . Through, q , draw a projector to cut $o's'$ at the required point q' .

Sectional views of cones may be obtained by applying any one of the above two methods for locating the positions of points. The generator method is more suitable particularly when the cone is in inclined positions.

(3) Section plane inclined to the base of the cone at an angle smaller than the angle of inclination of the generators with the base:

Problem 14-24. A cone, base 75 mm diameter and axis 80 mm long is resting on its base on the H.P. It is cut by a section plane perpendicular to the V.P., inclined at 45° to the H.P. and cutting the axis at a point 35 mm from the apex. Draw its front view, sectional top view, sectional side view and true shape of the section.

Draw a line V.T. in the required position in the front view of the cone. The positions of points on this line and the width of section at each point can be determined by one of the methods explained in problem 14-21 and problem 14-23 and as described below.

- (i) **Generator method** [fig. 14-26(i) and fig. 14-26(ii)]:
- (a) Divide the base-circle into a number of equal parts, say 12. Draw lines (i.e. generators) joining these points with o . Project these points on the line representing the base in the front view.
- (b) Draw lines $o'2'$, $o'3'$ etc. cutting the line for the section at points b' , c' etc. Project these points on the corresponding lines in the top view. For example, point b' on $o'2'$, also represents point b'_1 on $o'-12'$ which coincides with $o'-12'$. Therefore, project b' to b on $o'2$ and to b_1 on $o'-12$. b and b_1 are the points on the section (in the top view).
- (c) Similarly, obtain other points. Point d' cannot be projected directly. Hence, the same method as in case of pyramids should be employed to determine the positions d and d_1 , as shown. In addition to these, two more points

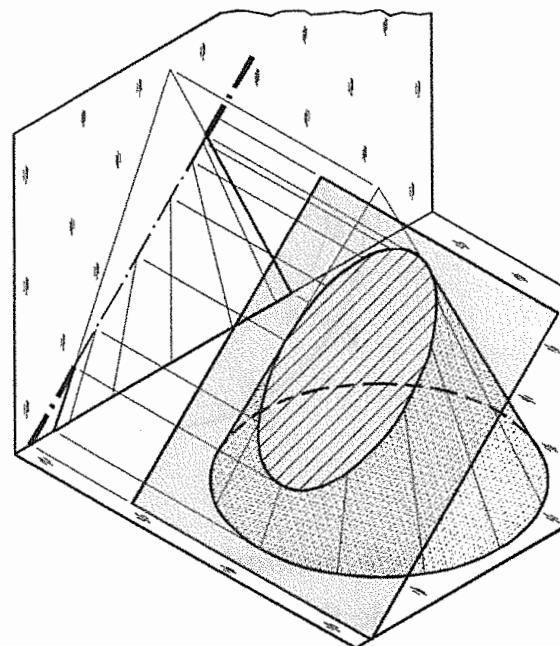


FIG. 14-26(i)

for the maximum width of the section at its centre should also be obtained. Mark m' , the mid-point of the section and obtain the points m and m_1 . Draw a smooth curve through these points.

- (d) The true shape of the section may be obtained on the V.T. as a new reference line or symmetrically around the centre line ag , drawn parallel to the V.T. as shown. It is an ellipse whose major axis is equal to the length of the section and minor axis equal to the width of the section at its centre.

Draw the sectional side view by projecting the points on corresponding generators, as shown.

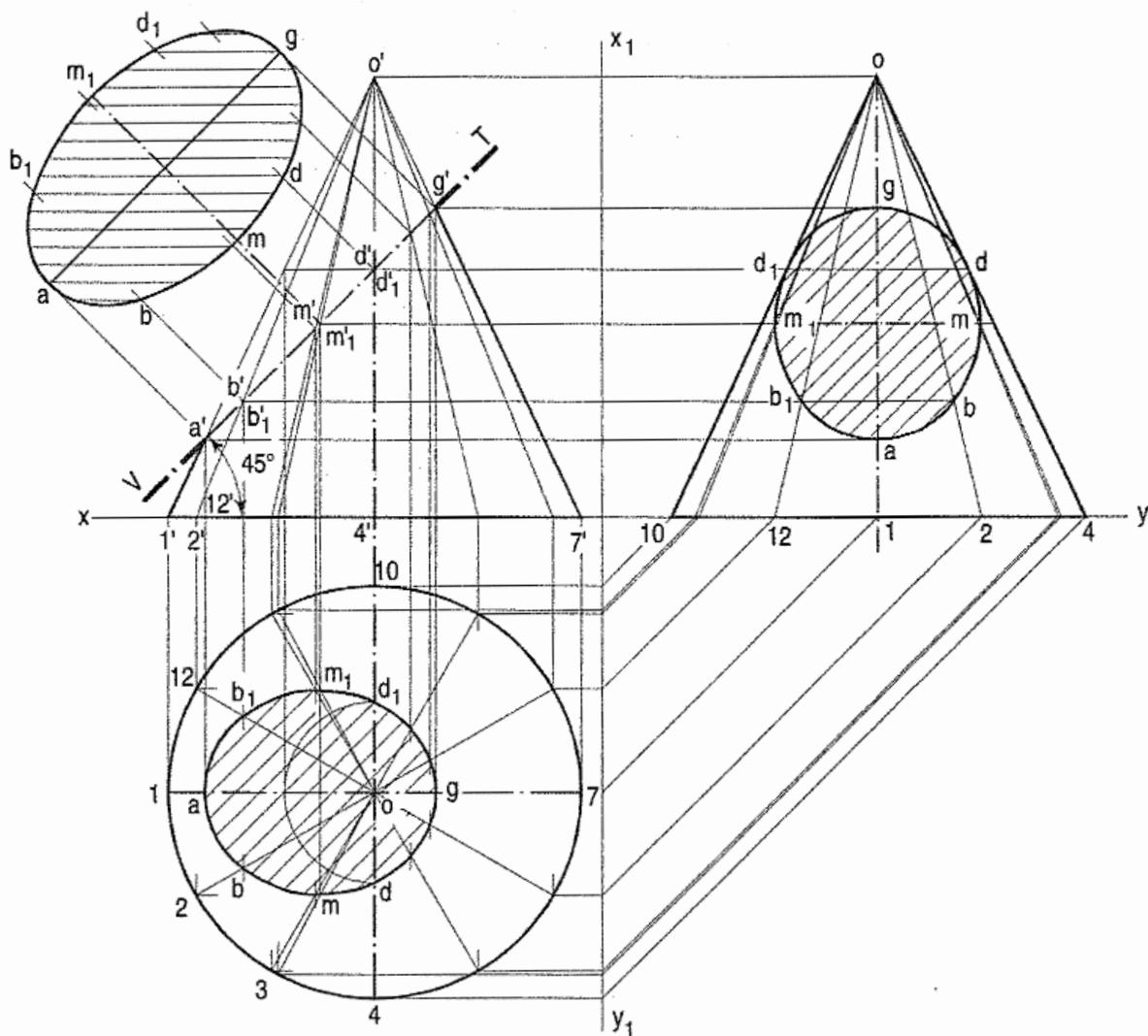


FIG. 14-26(ii)

(ii) **Circle method** (fig. 14-27):

- Divide the line of section into a number of equal parts. Determine the width of section at, and the position of each division-point in the top view by the circle method. For example, through c' , draw a line $c''c''$ parallel to the base.
- With o as centre and radius equal to half of $c''c''$, draw an arc. Project c' to c and c_1 on this arc. Then c and c_1 are the required points. The straight line joining c and c_1 will be the width of the section at c' .

- (c) Similarly, obtain all other points and draw a smooth curve through them. This curve will show the apparent section. The maximum width of the section will be at the mid-point e' . It is shown in the top view by the length of the chord joining e and e_1 .
- (d) Draw a reference line $x_1 y_1$ parallel to the V.T. and project the true shape of the section. In the figure, the auxiliary sectional top view of the truncated cone is shown. It shows the true shape of the section.

The sectional side view (not shown in the figure) may be obtained by projecting all the division-points horizontally and then marking the width of the section at each point, symmetrically around the axis of the cone.

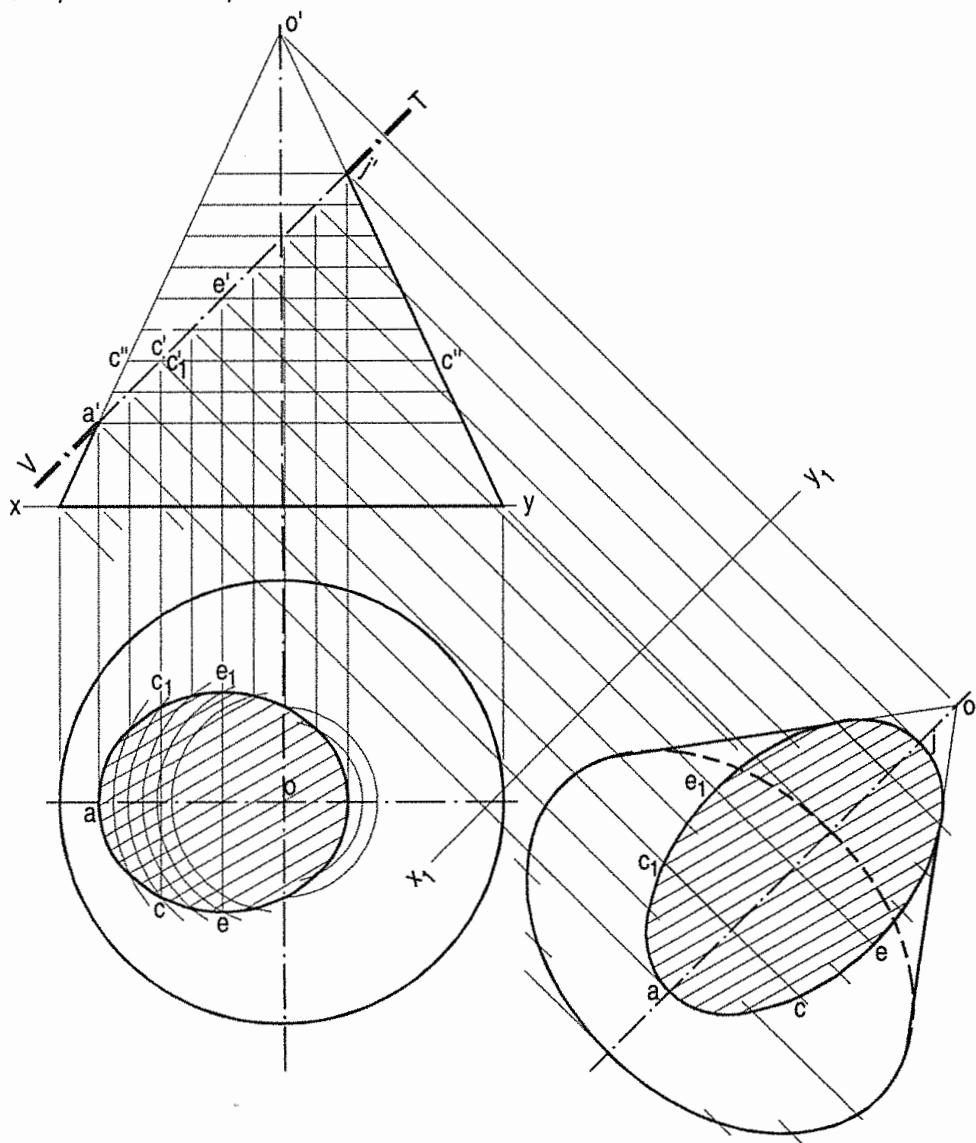


FIG. 14-27

(4) Section plane parallel to a generator of the cone:

Problem 14-25. (fig. 14-28): The cone in same position as in problem 14-24, is cut by a section plane perpendicular to the V.P. and parallel to and 12 mm away from one of its end generators. Draw its front view, sectional top view and true shape of the section.

- (i) Draw a line V.T. (for the section plane) parallel to and 12 mm away from the generator $o'1'$.

- (ii) Draw the twelve generators in the top view and project them to the front view. All the generators except $o'1'$, $o'2'$ and $o'12'$ are cut by the section plane. Project the points at which they are cut, to the corresponding generators in the top view. The width of the section at the point where the base is cut will be the chord aa_1 . Draw a curve through $a \dots f \dots a_1$. The figure enclosed between aa_1 and the curve is the apparent section.
- (iii) Obtain the true shape of the section as explained in the previous problem. It will be a parabola.

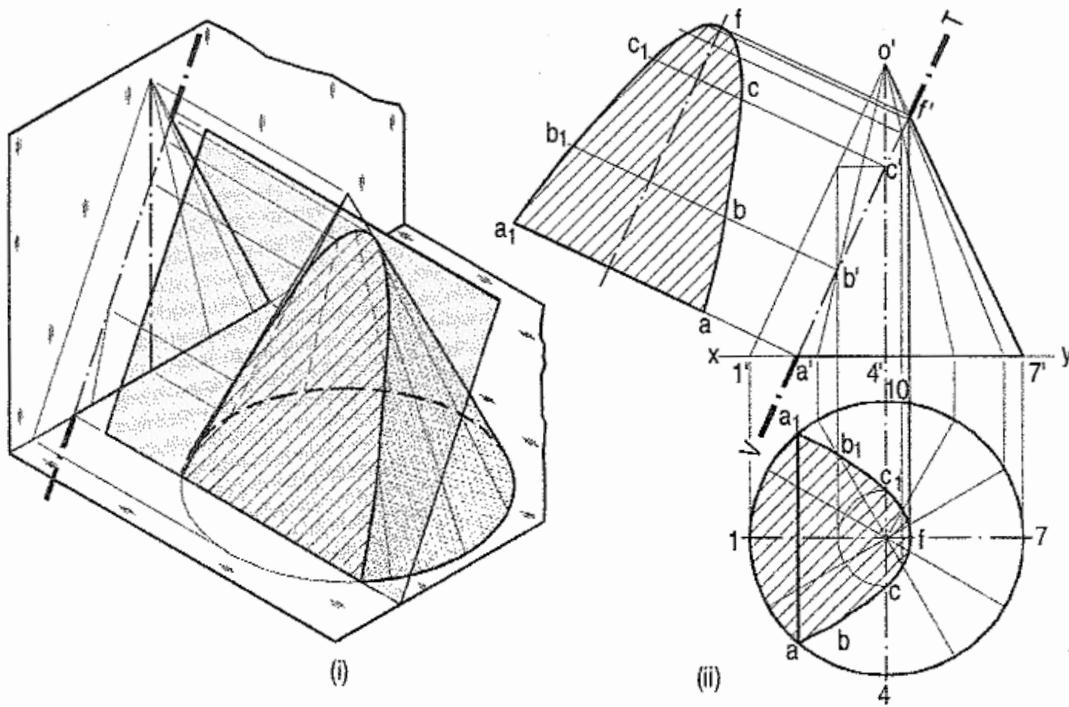


FIG. 14-28

- (5) Section plane inclined to the base of the cone at an angle greater than the angle of inclination of the generators with the base:



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 32 for the following problem.

Problem 14-26. [fig. 14-29(i) and fig. 14-29(ii)]: A cone, base 45 mm diameter and axis 55 mm long is resting on the H.P. on its base. It is cut by a section plane, perpendicular to both the H.P. and the V.P. and 6 mm away from the axis. Draw its front view, top view and sectional side view.

The section will be seen as a line, perpendicular to xy , in both the front view and the top view. The side view will show the true shape of the section. The width of the section at any point, say c' , will be equal to cc_1 obtained by the circle method [fig. 14-29(i)].

- Draw the side view of the cone.
- Project the points (on the section) in the side view taking the widths from the top view. For example, through c' draw a horizontal line. Mark on it points c'' and c''_1 equidistant from and on both sides of the axis so that $c''c''_1 = cc_1$.
- Draw a curve through the points thus obtained. It will be a hyperbola.

Fig. 14-29(ii) shows the views obtained by the generator method.

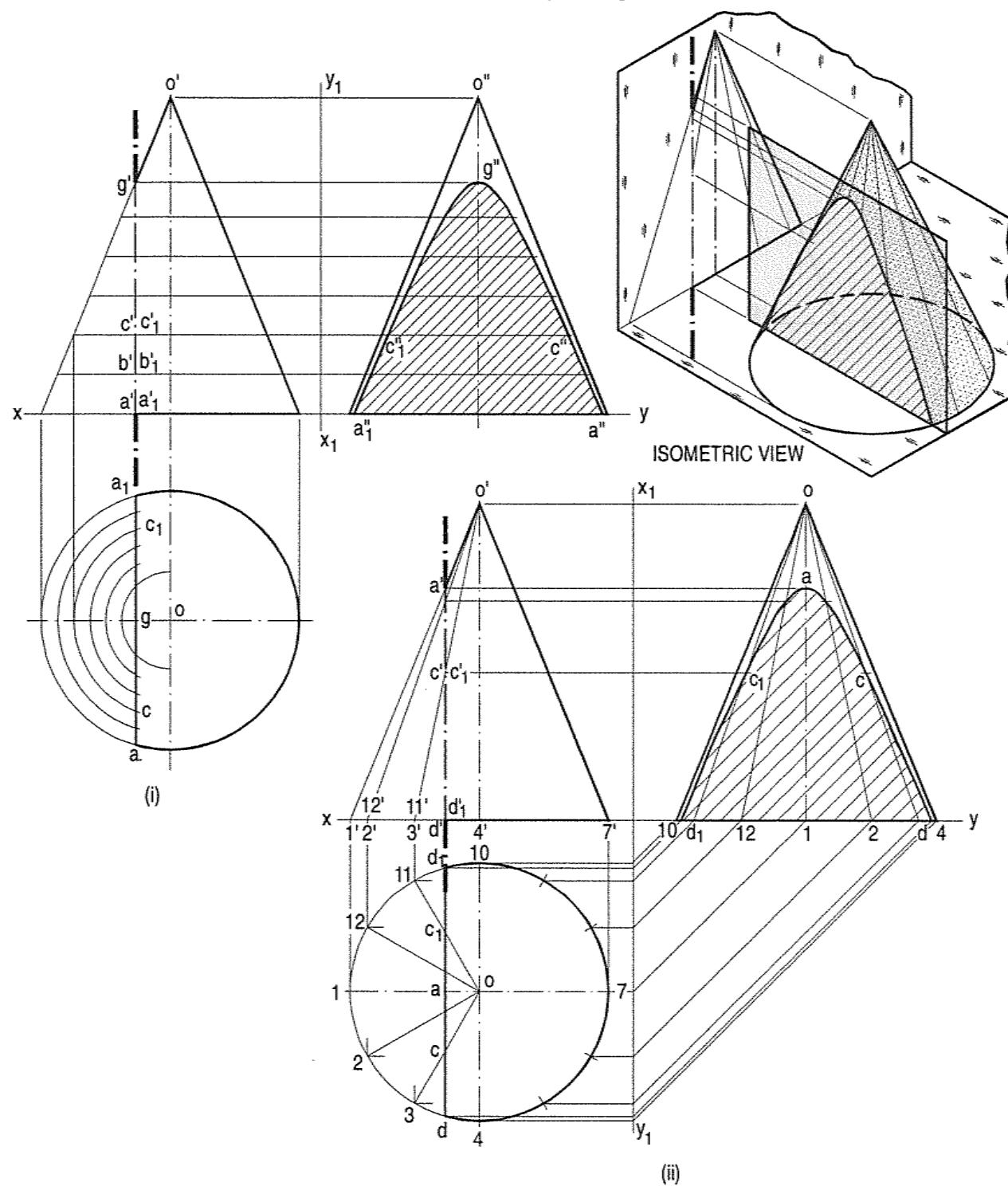


FIG. 14-29

Additional problems on sections of cones:

Problem 14-27. (fig. 14-30): A cone, diameter of base 50 mm and axis 65 mm long, is lying on the H.P. on one of its generators with the axis parallel to the V.P. It is cut by a horizontal section plane 12 mm above the ground. Draw its front view, sectional top view, and development of its surface.

Use the generator method and project the points in the top view. The curve will show the true shape of the section viz. a parabola.

For development, the true lengths of the cut-generators are obtained by drawing lines parallel to the base. Positions of points a and a_1 are determined by projecting them on the base-circle in the first top view.

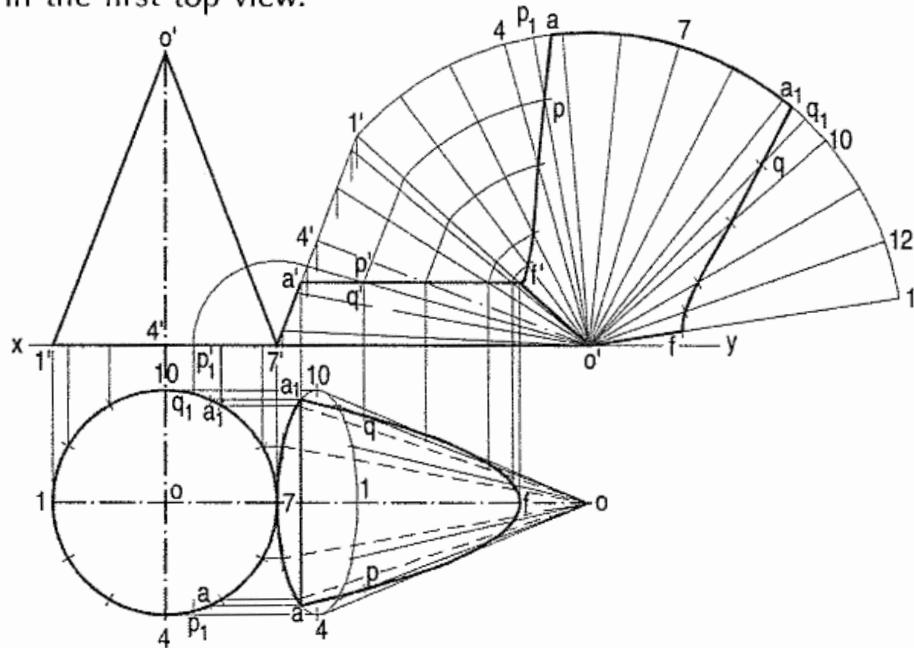


FIG. 14-30

Problem 14-28. (fig. 14-31): A cone, base 70 mm diameter, axis 75 mm long and resting on its base on the H.P., is cut by a vertical section plane, the H.T. of which makes an angle of 60° with the reference line and is 12 mm away from the top view of the axis. (i) Draw the sectional front view and the true shape of the section. (ii) Also draw the sectional front view and the top view when the same section plane is parallel to the V.P.

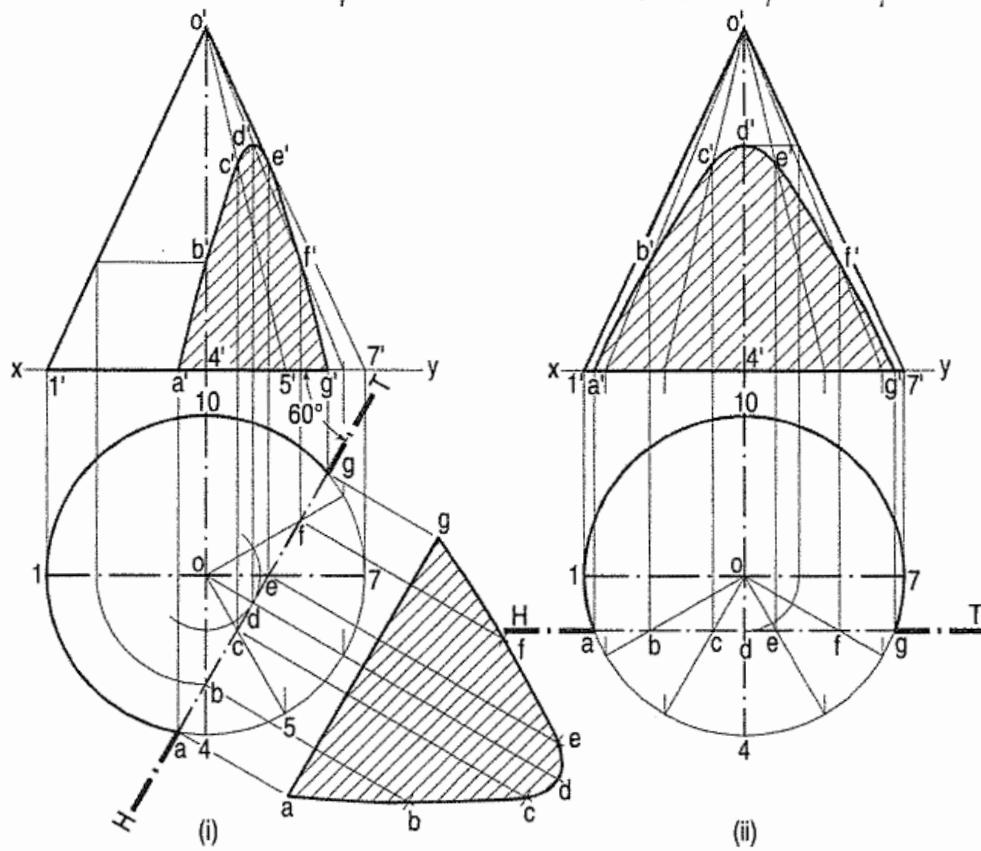


FIG. 14-31

- Draw a circle with centre o and radius equal to 12 mm.
- Draw a line for the section plane, tangent to this circle and inclined at 60° to xy [fig. 14-31(i)].
- Project the front view by the generator method as shown. Note how the point f' is obtained.

Fig. 14-31(ii) shows the two views when the section plane is parallel to the V.P.

Problem 14-29. (fig. 14-32): A cone, base 60 mm diameter and axis 60 mm long is lying on the H.P. on one of its generators with the axis parallel to the V.P. A vertical section plane parallel to the generator which is tangent to the ellipse (for the base) in the top view, cuts the cone bisecting the axis and removing a portion containing the apex. Draw its sectional front view and true shape of the section.

- Name in correct sequence the points at which the base and the generators are cut and project them in the front view.
- Project the true shape of the section on the new reference line $x_1 y_1$ drawn parallel to the H.T.

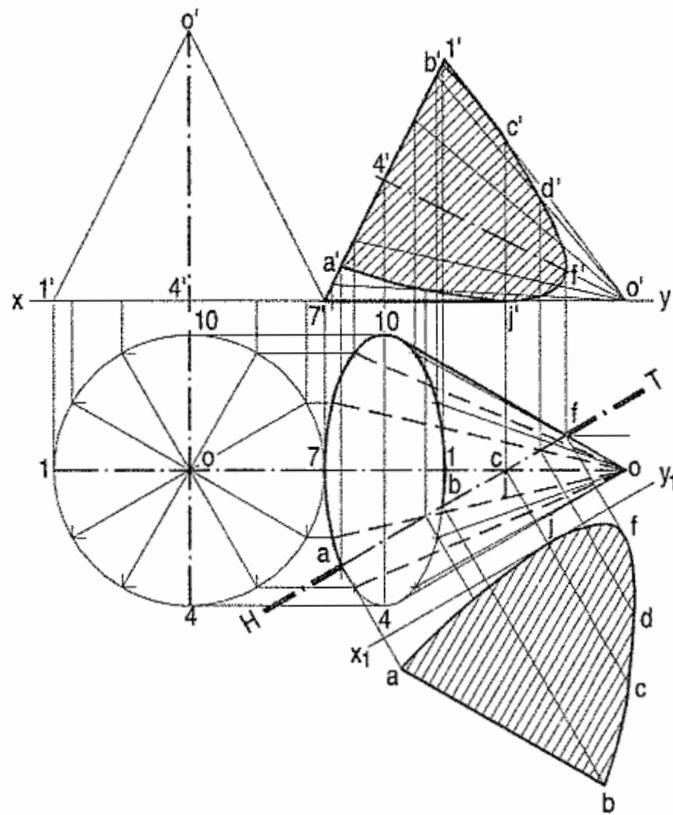


FIG. 14-32

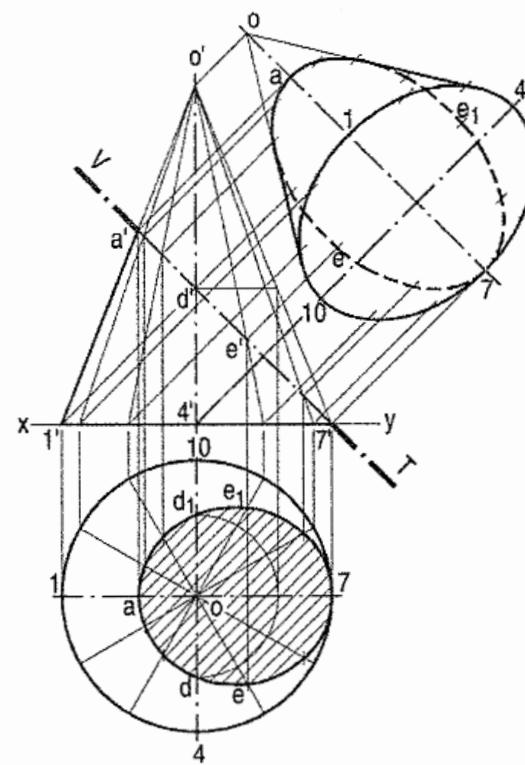


FIG. 14-33

Problem 14-30. (fig. 14-33): A cone, base 60 mm diameter and axis 75 mm long, is resting on the H.P. on its base. It is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and intersecting the axis 30 mm above the base. Draw its front view and sectional top view. Also draw its top view when it is lying on the ground on its cut-surface with the axis parallel to the V.P.

See the figure which is self-explanatory. Note that, when the cone is tilted so as to lie on the cut-surface, its base is fully visible, while the section is hidden in the top view.

14-5. SECTIONS OF SPHERES



These are discussed in details as under.

- (1) Section plane parallel to the H.P.
- (2) Section plane parallel to the V.P.
- (3) Section plane perpendicular to the V.P. and inclined to the H.P.
- (4) Section plane perpendicular to the H.P. and inclined to the V.P.

(1) Section plane parallel to the H.P.: When a sphere is cut by a plane, the true shape of the section is always a circle.

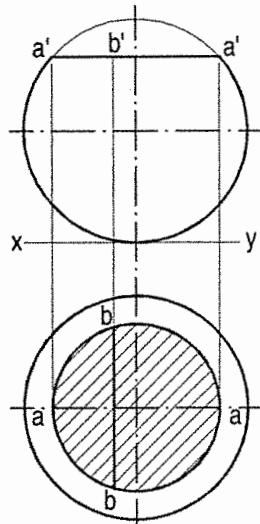


FIG. 14-34

The sphere in fig. 14-34 is cut by a horizontal section plane. The true shape of the section (seen in the top view) is a circle of diameter $a'a'$. The width of the section at any point say b' , is equal to the length of the chord bb' .

(2) Section plane parallel to the V.P.: When the sphere is cut by a section plane parallel to the V.P. (fig. 14-35), the true shape of the section, seen in the front view, is a circle of diameter cc' . The width of section at any point d is equal to the length of the chord $d'd'$.

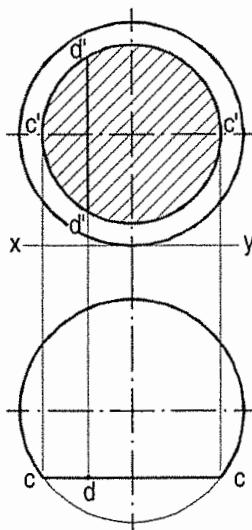


FIG. 14-35

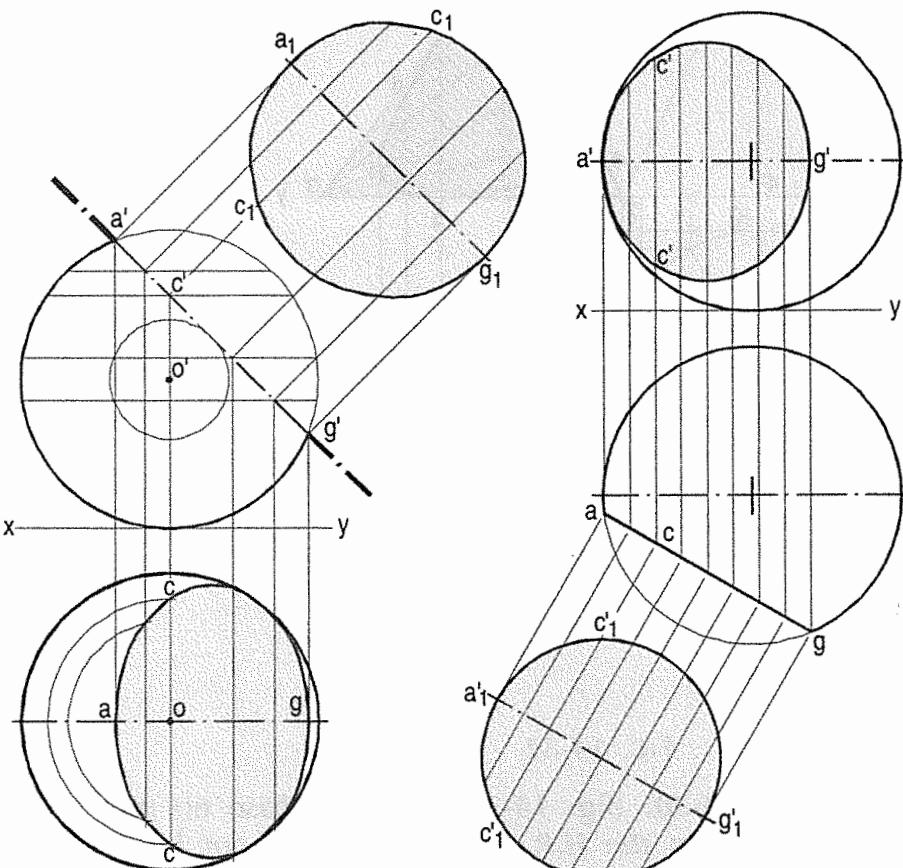


FIG. 14-36

FIG. 14-37

(3) Section plane perpendicular to the V.P. and inclined to the H.P.:

Problem 14-31. (fig. 14-36): A sphere of 50 mm diameter is cut by a section plane perpendicular to the V.P., inclined at 45° to the H.P. and at a distance of 10 mm from its centre. Draw the sectional top view and true shape of the section.

Draw a line (for the section plane) inclined at 45° to xy and tangent to the circle of 10 mm radius drawn with o' as centre. Mark a number of points on this line.

Method I:

- Find the width of section at each point in the top view as shown in fig. 14-34. For example, the chord cc' is the width of section at the point c' .
- Draw a curve through the points thus obtained. It will be an ellipse. The true shape of the section will be a circle of diameter $a'g'$.

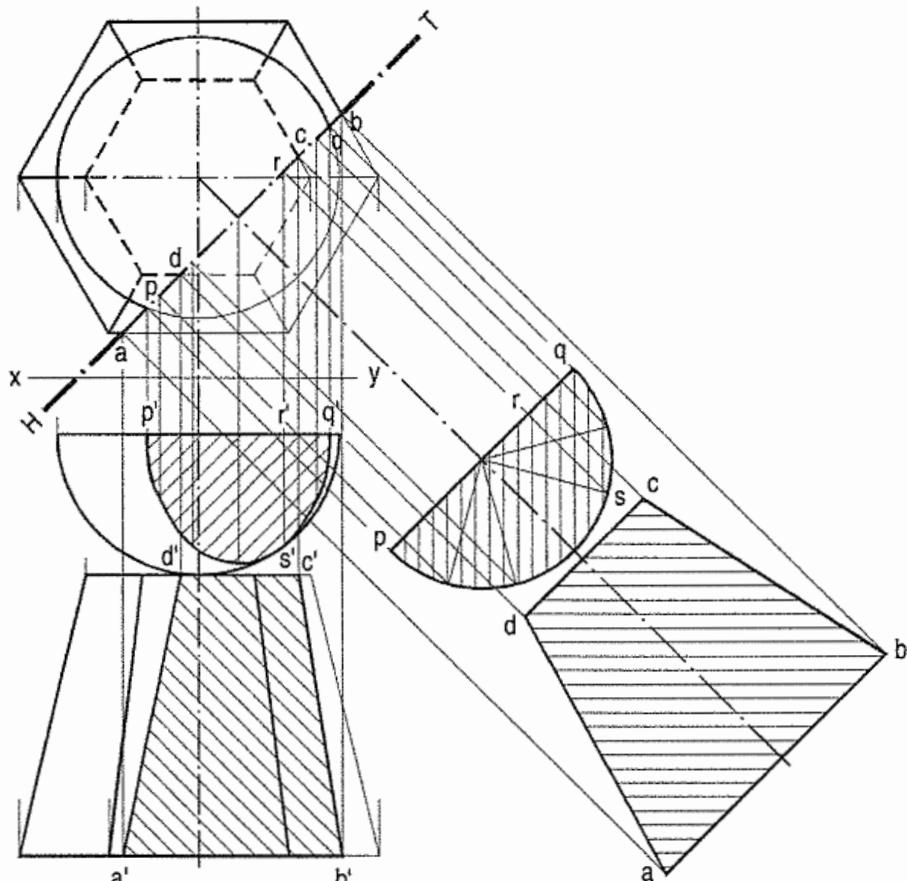
Method II:

It is known that the true shape of the section is a circle of diameter equal to $a'g'$. The width of section at any point say c' is equal to the chord c_1c_1' on this circle. Therefore, project c' to points c in the top view so that $cc = c_1c_1'$. Similarly, obtain other points and draw the ellipse through them.

Fig. 14-37 shows the sectional front view and true shape of the section when the section plane is vertical and inclined to the V.P.

(4) Section plane perpendicular to the H.P. and inclined to the V.P.:

Problem 14-32. (fig. 14-38): *The projections of a hemisphere 50 mm diameter, placed centrally on the top of a frustum of a hexagonal pyramid, base 32 mm side, top 20 mm side and axis 50 mm long are given. Draw the sectional front view when the vertical section plane H.T. inclined at 45° to the V.P. and 10 mm from the axis, cuts them. Also draw the true shapes of the sections of both the solids.*



(Third-angle projection)

FIG. 14-38

The widths of the section of the sphere at various points are obtained from the semi-circle drawn in the top view.

14-6. TYPICAL PROBLEMS OF SECTIONS OF SOLIDS

Problem 14-33. A solid composed of a half-cone and a half-hexagonal pyramid is shown in fig. 14-39. It is cut by a section plane, which makes an angle of 30° with the base, is perpendicular to the V.P. and contains an edge of the base of the pyramid. Draw its sectional top view, true shape of the section and development of the surface of the remaining portion. Base of cone 60 mm diameter; axis 70 mm long.

- Draw a line V.T. inclined at 30° to the base and passing through a.
- Project the sectional top view. Note how points b and b₁ are obtained. The true shape of the section will be partly elliptical.
- Draw the development of the half-cone and half-pyramid and show the lines for the section in it.

Problem 14-34. (fig. 14-40); A cylinder, base 50 mm diameter and axis 75 mm long, has a square hole of 25 mm side cut through it so that the axis of the hole coincides with that of the cylinder. The cylinder is lying on the H.P. with the axis perpendicular to the V.P. and the faces of the hole equally inclined to the H.P.

A vertical section plane, inclined at 60° to the V.P. cuts the cylinder in two equal halves. Project the front view of the cylinder on an A.V.P. parallel to the section plane.

- Assuming the cylinder to be whole, draw its auxiliary front view.
- Project the points at which the generators of the cylinder and the edges of the hole are cut. The section of the cylinder will be a part of an ellipse. Join the points at which the edges of the hole are cut. The back edges

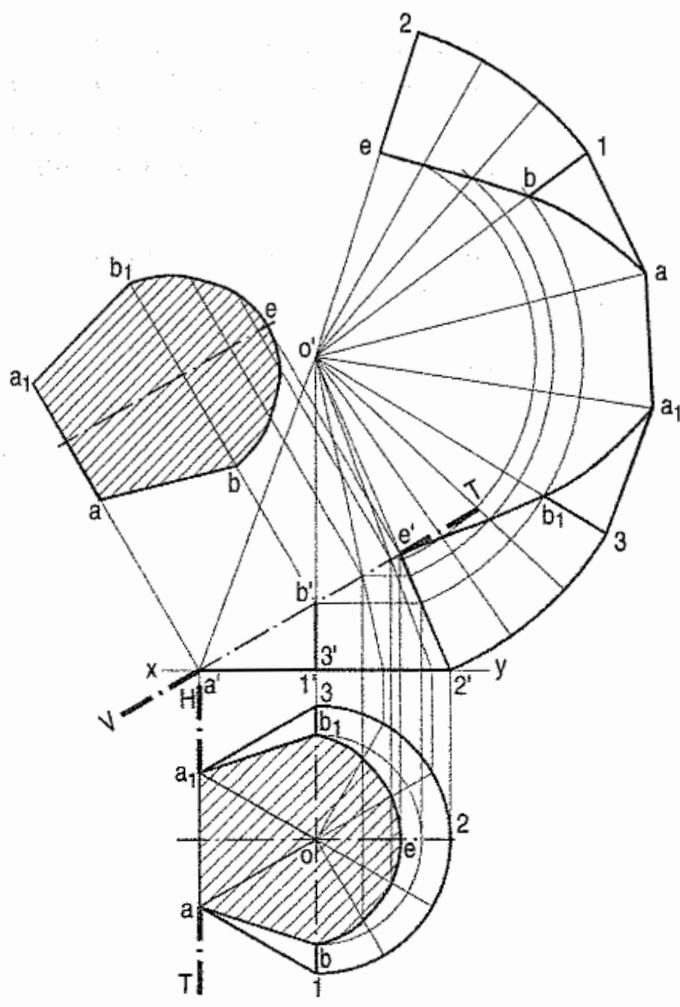


FIG. 14-39

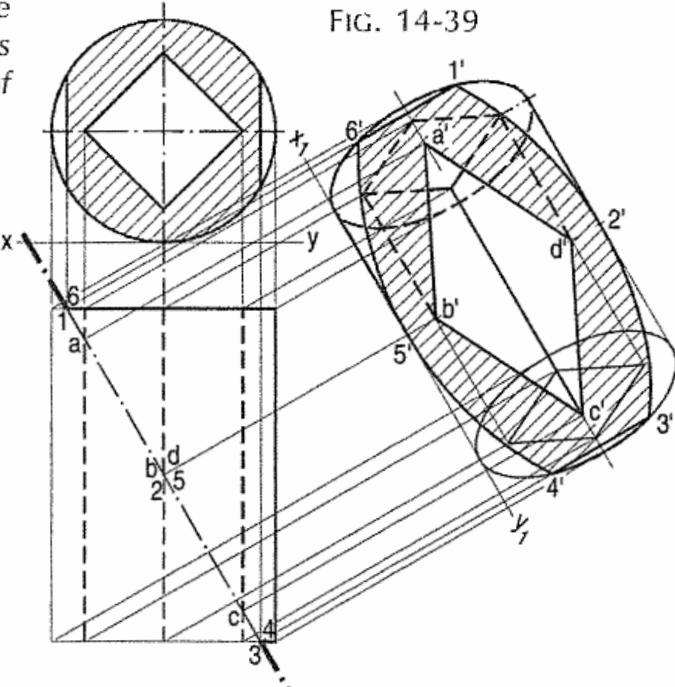


FIG. 14-40

of the hole will be visible within the section and hence, must be shown as full lines.

- (iii) Complete the view by showing the section and the remaining portion of the cylinder with dark lines.

Problem 14-35. (fig. 14-41):

A square prism axis 110 mm long is resting on its base in the H.P. The edges of the base are equally inclined to V.P.

The prism is cut by an A.I.P. passing through the mid-point of the axis in such a way that the true shape of the section is rhombus having diagonals of 100 mm and 50 mm. Draw the projections and determine the inclination of A.I.P. with the H.P.

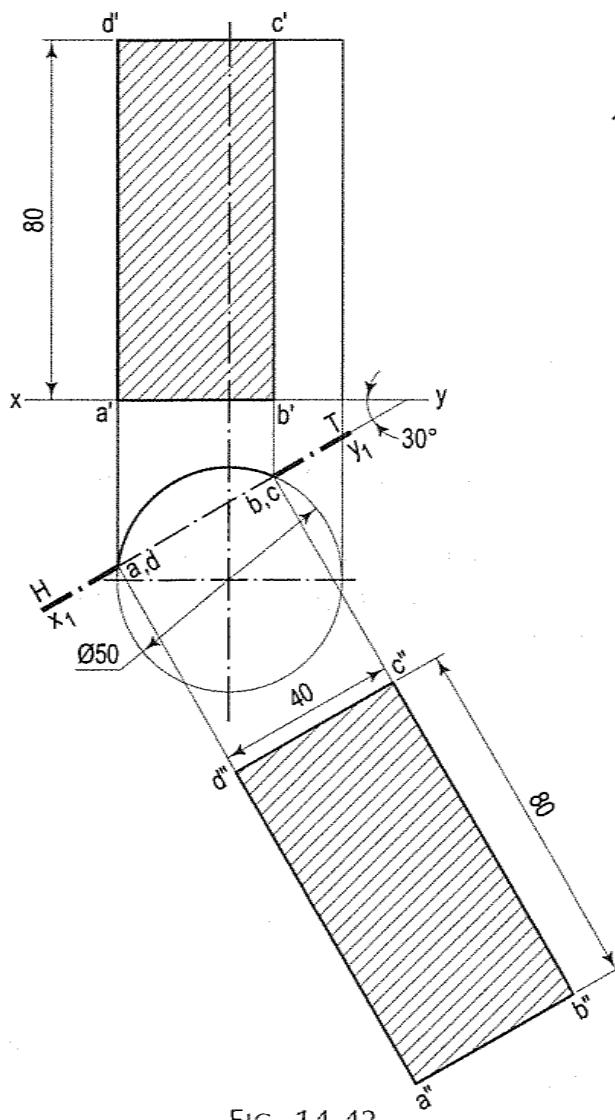


FIG. 14-42

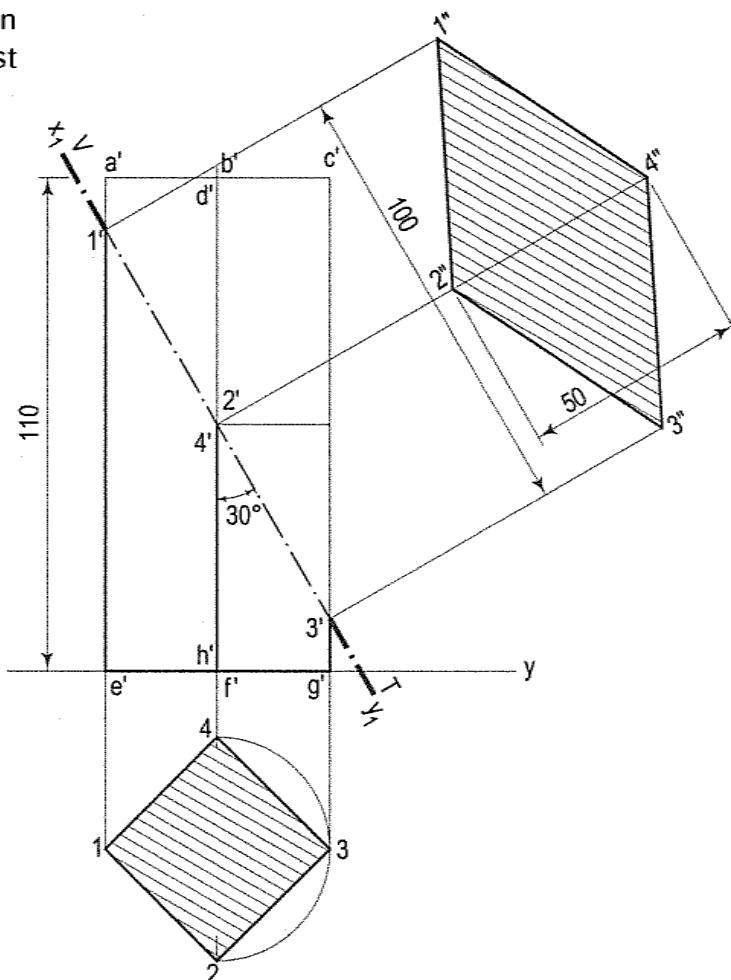


FIG. 14-41

- (i) Draw the top view and the front view as shown.
- (ii) Mark the mid-point of the axis in the front view.
- (iii) With the mid-point of the axis as centre and radius equal to 50 mm (half of the longer diagonal i.e. 100 mm), draw an arc cutting the two opposite vertical sides of the prism. Project the points and complete the true shape as shown.

Problem 14-36. (fig. 14-42): A vertical cylinder 50 mm diameter is cut by an A.V.P. making 30° to the V.P. in such a way that the true shape of the section is a rectangle of 40 mm × 80 mm sides. Draw the projections and the true shape of the section.

- Draw a top view and an front view of the cylinder.
- Draw x_1y_1 at 30° to xy in such a way that the chord length in the top view is 40 mm. Project points 1, 2, 3 and 4 and draw the rectangle of 40 mm \times 80 mm as shown.

Problem 14-37. (fig. 14-43): A hexagonal pyramid, base 30 mm side and axis 70 mm long is resting on its slant edge of the face on the horizontal plane.

A section plane, perpendicular to the V.P., inclined to the H.P. passes through the highest corner of the base and intersecting the axis at 25 mm from the base. Draw the projections of the solid and determine the inclination of the section plane with the H.P.

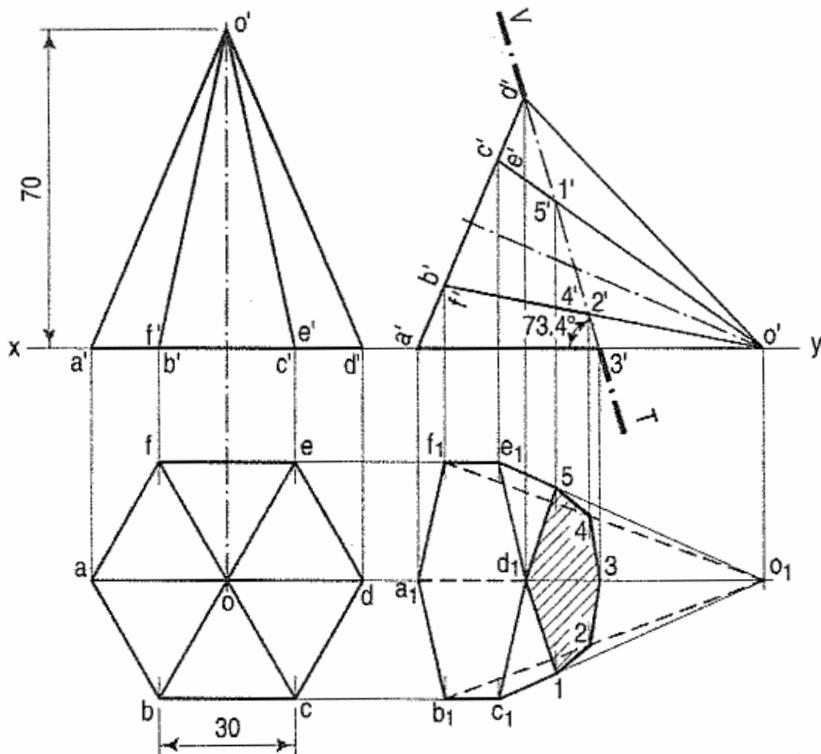


FIG. 14-43

- Draw the top view and the front view keeping one of the sides of the base parallel to xy .
- With a' and o' , as centres and radii equal to $a'd'$ and $o'd'$ draw arcs intersecting each other at point d' . Draw a section plane passing through d' and point 25 mm away from the base along the axis as shown.
- Measure the angle by V.T. with xy .

Problem 14-38. (fig. 14-44): A pentagonal pyramid, base side 30 mm, length of axis 80 mm is resting on a base edge on the H.P. with a triangular face containing that edge being perpendicular to the V.P. and inclined to the H.P. at 60° . It is cut by a horizontal section plane whose V.T. passes through the mid-point of the axis. Draw the front view, sectional top view and add a profile view.

- Draw the top view and the front view keeping one of the sides of the base perpendicular to xy .
- Tilt the front view on the points c' , d' as shown.
- Draw a line parallel to xy and passing through the mid-point of the axis representing V.T. of the section plane.
- Complete the projection as shown.

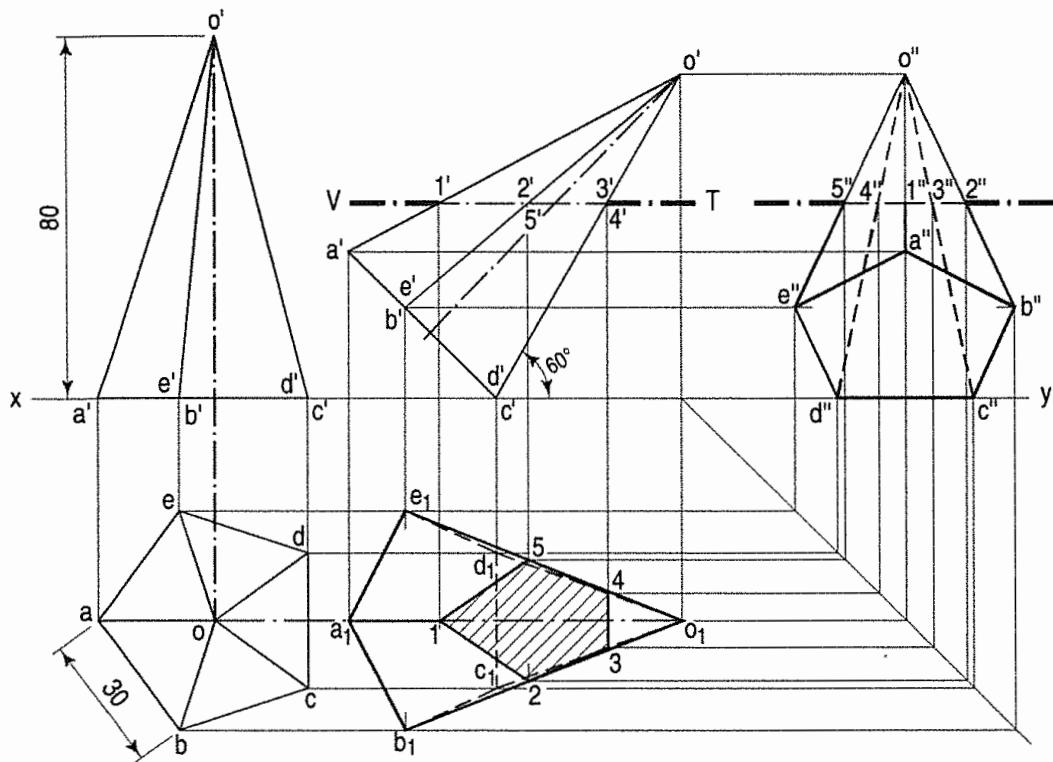


FIG. 14-44

Problem 14-39. (fig. 14-45): A cone, diameter of the base 60 mm and axis 70 mm long is resting on its base on the H.P. It is cut by an A.I.P. so that the true shape of the section is an isosceles triangle having 50 mm base. Draw the top view, the front view and the true shape of the section.

When a section plane passes through an apex of a cone and cuts the base of the cone, the true-shape of section is a triangle.

- Draw a top view and an front view as shown.
- Mark chord ab of 50 mm (the base of triangle) in the top view. Project points a and b in the front view intersecting base at a' or b' . Join points a' and o' .

This represents V.T. of the section plane.

- Considering line V.T. as new line $x_1 y_1$, draw the projectors from o' , a' and b' .
- Construct the true shape of triangle as shown.

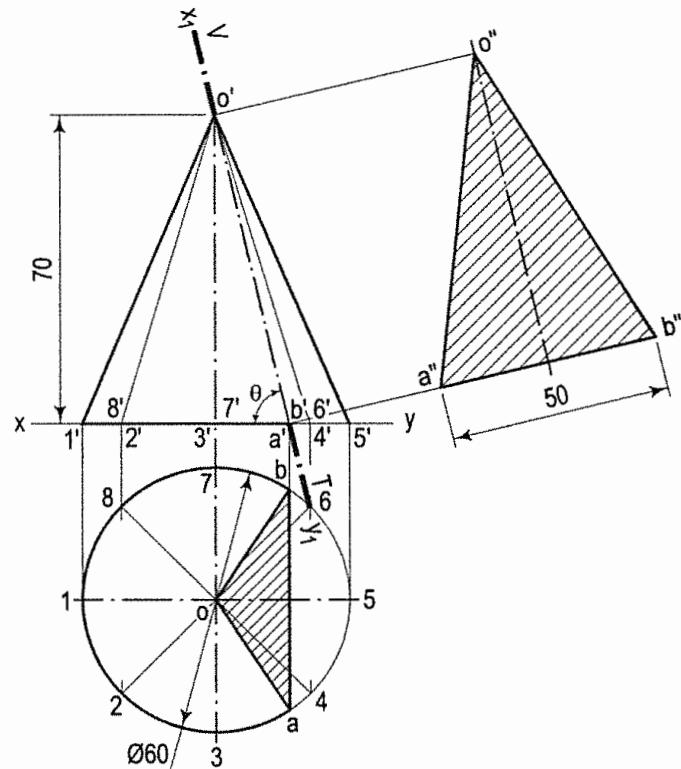


FIG. 14-45

Problem 14-40. (fig. 14-46): A square pyramid of 60 mm side of base and 70 mm length of axis is resting on its base on the H.P., having a side of base perpendicular to the V.P. It is cut by two cutting planes. One is parallel to its extreme right face and 10 mm away from it. While the other is parallel to the extreme left face.

Both the cutting planes intersect each other on the axis of the pyramid. Draw the sectional top view, front view and the left hand side view.

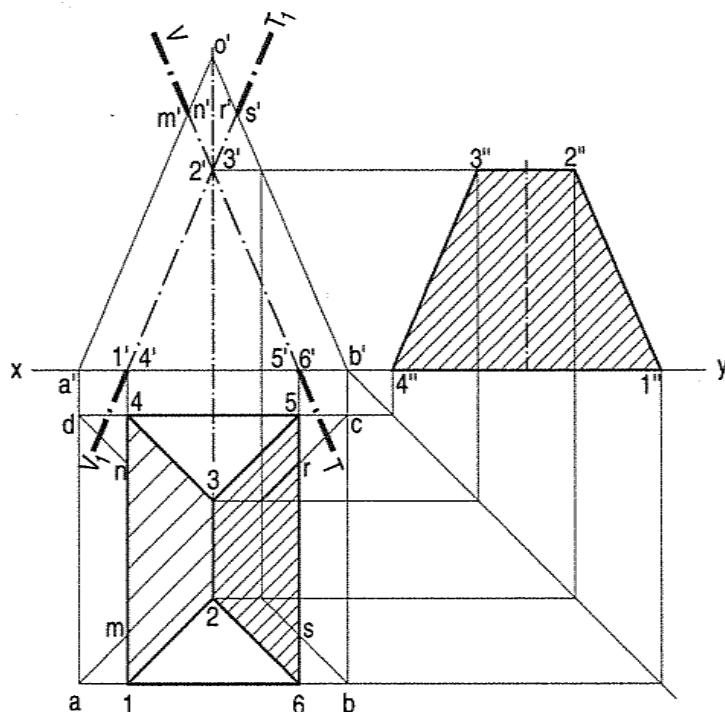


FIG. 14-46

- Draw the top view and the front view as shown.
- Draw lines V.T. and V_1T_1 representing section planes in the front view intersecting at 2' and 3'.
- Complete the projections. Note that the intersection points 2' and 3' are transferred on the slant edge $o'b'$ and then projected in the top view.

Problem 14-41. (fig. 14-47): A cylinder of diameter 50 mm and axial height 90 mm having 34 mm square hole centrally along the axis, rests on a point on the circular edge of the base remaining on the H.P. The axis of the cylinder is parallel to the V.P. and inclined at 30° to the H.P. and the rectangular faces of the square hole remain equally inclined to the V.P. A section plane perpendicular to the V.P. and inclined to the horizontal plane passing through the mid-point of the axis, such that the apparent section in the top view is a circle of 50 mm diameter cuts the cylinder. Draw the front view, section plane, true shape of the section and find the inclination of the section plane with H.P.

Fig. 14-47 shows the projections of the solid. Note that the generators 4 and 6 cut the apparent section at the points s , q , s_1 and q_1 . These points are transferred in the front view as shown. The required section plane must pass through the midpoint of the axis, and s , q (s_1 and q_1).

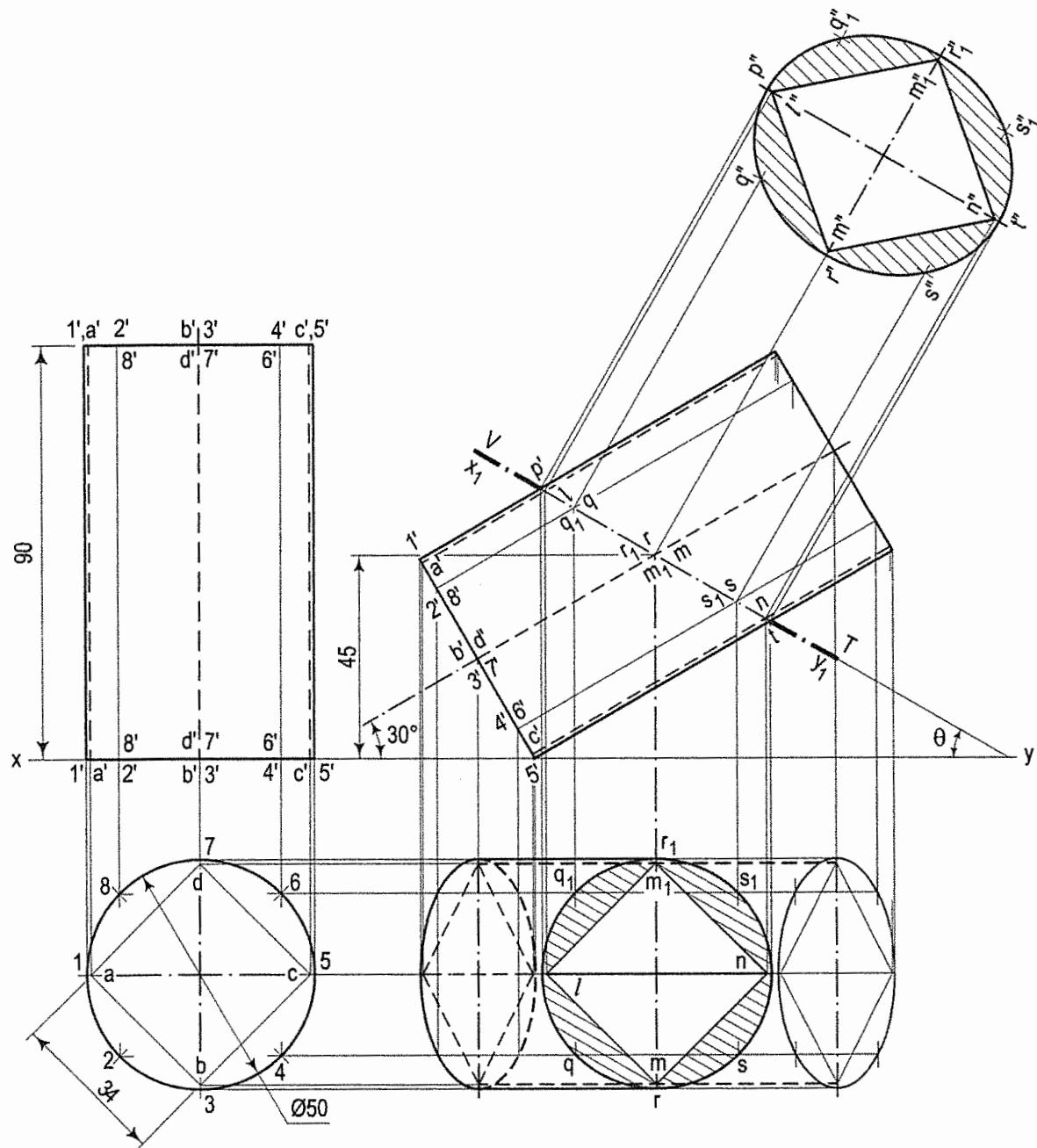


FIG. 14-47

Problem 14-42. (fig. 14-48): A cylindrical disc of 46 mm diameter is resting on the H.P. and has height of 20 mm. A hexagonal prism of side 23 mm and height of 30 mm is resting on the disc such that their axes are in one line and its faces are equally inclined with the V.P.. It is cut by the auxiliary plane, offset 10 mm in front of the centre of the disc and is inclined at 40° with xy. Draw the projection of combined solids and obtain true shape of the section.

- Draw the projections of the combined solids with given positions.
- Draw an offset circle of 10 mm radius and mark a cutting plane inclined at 40° , touching this circle.
- Project various points in the front view as shown.
- Draw x_1y_1 parallel to the cutting plane.
- Obtain true shape as shown.

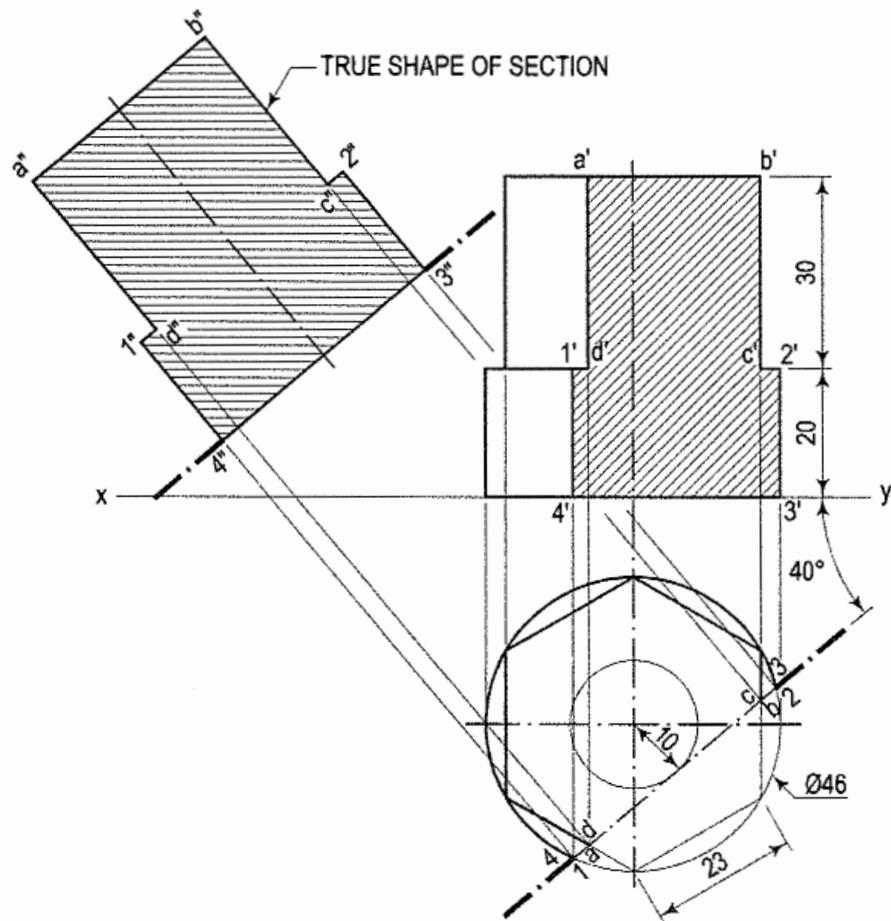


FIG. 14-48

Problem 14-43. (fig. 14-49): A square prism of 40 mm side is resting on H.P. and has height of 30 mm. Its faces are equally inclined with the V.P. A frustum of cone having base diameter 40 mm and top 30 mm diameter with height 30 mm is kept on the prism such that axes of the both solids are coinciding. A sectional plane cuts the combined axes and inclined at 55° with H.P. passes through left corners of the prism. Draw the front view and section top view. Draw also true-shape of the section.

- Draw the top view and front view of combined solid as shown.
- In front view, draw a section plane at angle of 55° with xy passing through the corner of square prism. Mark points 1', 2', 3', 4', 5' and 6' as shown.
- Project these points in the top view and draw section line as shown in fig. 14-49.

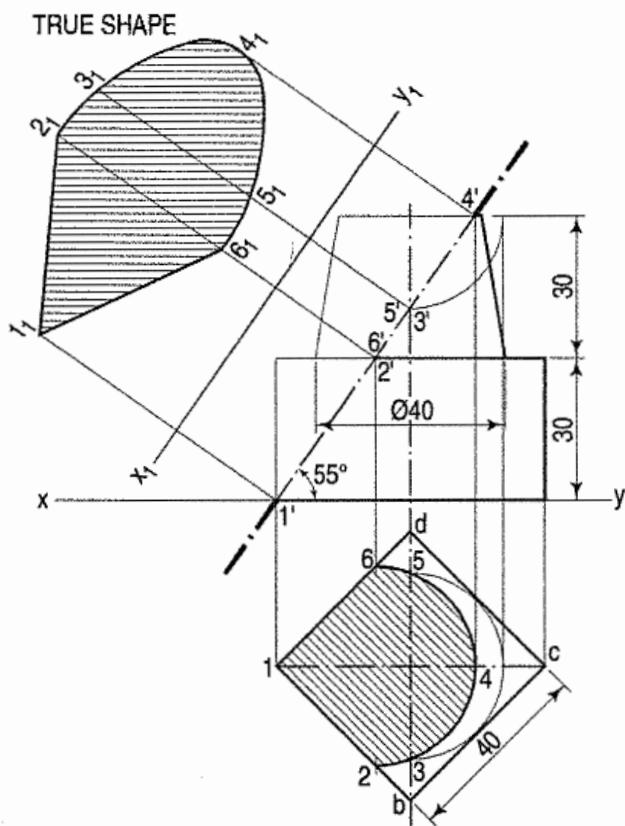


FIG. 14-49

- (iv) Draw a new reference line x_1y_1 parallel to the section plane and project the section on it.
- (v) The distances of the points from x_1y_1 should be taken equal to their corresponding distances from xy in the top view of combined solids.

Problem 14-44. (fig. 14-50): A horizontal frustum of square pyramid having front square of 20 mm side and back square of 30 mm at length of 60 mm has axis perpendicular to the V.P. and the side of square is inclined to 45° with H.P. It is cut by section plane making 40° with V.P. and passing through a point on axis 30 mm from the surface of large square side. Draw projections of the frustum.

- (i) Mark point b' and construct the square $a' b' c' d'$ with sides equally inclined to the xy . This is a front view of the frustum of square pyramid.
- (ii) Project the top view such that the axis remains perpendicular to xy as shown in fig. 14-50.
- (iii) Draw a section plane at distance of 20 mm along axis from large square at angle of 40° with xy .
- (iv) Mark the points p, q, r, s and project them in the front view. Draw section lines in the front view for the section and complete the view.

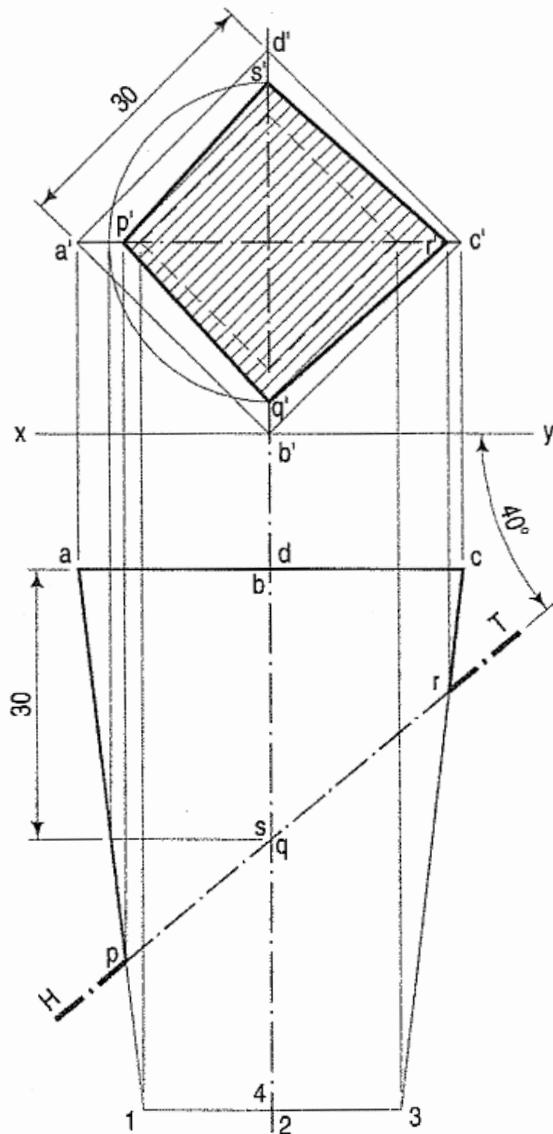


FIG. 14-50

EXERCISES 14

1. A cube of 50 mm long edges is resting on the H.P. with a vertical face inclined at 30° to the V.P. It is cut by a section plane, perpendicular to the V.P., inclined at 30° to the H.P. and passing through a point on the axis, 38 mm above the H.P. Draw the sectional top view, true shape of the section and development of the surface of the remaining portion of the cube.
2. A hexagonal prism, side of base 35 mm and height 75 mm is resting on one of its corners on the H.P. with a longer edge containing that corner inclined at 60° to the H.P. and a rectangular face parallel to the V.P. A horizontal section plane cuts the prism in two equal halves.
 - (i) Draw the front view and sectional top view of the cut prism.
 - (ii) Draw another top view on an auxiliary inclined plane which makes an angle of 45° with the H.P.
3. A pentagonal prism, side of base 50 mm and length 100 mm has a rectangular face on the H.P. and the axis parallel to the V.P. It is cut by a vertical section

plane, the H.T. of which makes an angle of 30° with xy and bisects the axis. Draw the sectional front view, top view and true shape of the section. Develop the surface of the remaining half of the prism.

4. A hollow square prism, base 50 mm side (outside), length 75 mm and thickness 9 mm is lying on the H.P. on one of its rectangular faces, with the axis inclined at 30° to the V.P. A section plane, parallel to the V.P. cuts the prism, intersecting the axis at a point 25 mm from one of its ends. Draw the top view and sectional front view of the prism.
5. A cylinder, 65 mm diameter and 90 mm long, has its axis parallel to the H.P. and inclined at 30° to the V.P. It is cut by a vertical section plane in such a way that the true shape of the section is an ellipse having the major axis 75 mm long. Draw its sectional front view and true shape of the section.
6. A cube of 65 mm long edges has its vertical faces equally inclined to the V.P. It is cut by a section plane, perpendicular to the V.P., so that the true shape of the section is a regular hexagon. Determine the inclination of the cutting plane with the H.P. and draw the sectional top view and true shape of the section.
7. A vertical hollow cylinder, outside diameter 60 mm, length 85 mm and thickness 9 mm is cut by two section planes which are normal to the V.P. and which intersect each other at the top end of the axis. The planes cut the cylinder on opposite sides of the axis and are inclined at 30° and 45° respectively to it. Draw the front view, sectional top view and auxiliary sectional top views on planes parallel to the respective section planes.
8. A square pyramid, base 50 mm side and axis 75 mm long, is resting on the H.P. on one of its triangular faces, the top view of the axis making an angle of 30° with the V.P. It is cut by a horizontal section plane, the V.T. of which intersects the axis at a point 6 mm from the base. Draw the front view, sectional top view and the development of the sectioned pyramid.
9. A pentagonal pyramid, base 30 mm side and axis 75 mm long, has its base horizontal and an edge of the base parallel to the V.P. It is cut by a section plane, perpendicular to the V.P., inclined at 60° to the H.P. and bisecting the axis. Draw the front view and the top view when the pyramid is tilted so that it lies on its cut-face on the ground with the axis parallel to the V.P. Show the shape of the section by dotted lines. Develop the surface of the truncated pyramid.
10. A tetrahedron of 65 mm long edges is lying on the H.P. on one of its faces, with an edge perpendicular to the V.P. It is cut by a section plane which is perpendicular to the V.P. so that the true shape of the section is an isosceles triangle of base 50 mm long and altitude 40 mm. Find the inclination of the section plane with the H.P. and draw the front view, sectional top view and the true shape of the section.
11. A hexagonal pyramid, base 50 mm side and axis 100 mm long, is lying on the H.P. on one of its triangular faces with the axis parallel to the V.P. A vertical section plane the H.T. of which makes an angle of 30° with the reference line, passes through the centre of the base and cuts the pyramid, the apex being retained. Draw the top view, sectional front view, true shape of the section and the development of the surface of the cut-pyramid.
12. A cone, base 75 mm diameter and axis 75 mm long, has its axis parallel to the V.P. and inclined at 45° to the H.P. A horizontal section plane cuts the

- cone through the mid-point of the axis. Draw the front view, sectional top view and an auxiliary top view on a plane parallel to the axis.
13. A cone, base 65 mm diameter and axis 75 mm long, is lying on the H.P. on one of its generators with the axis parallel to the V.P. A section plane which is parallel to the V.P. cuts the cone 6 mm away from the axis. Draw the sectional front view and development of the surface of the remaining portion of the cone.
14. The cone in above problem 13 is cut by a horizontal section plane passing through the centre of the base. Draw the sectional top view and another top view on an auxiliary plane parallel to the axis of the cone.
15. A hemisphere of 65 mm diameter, lying on the H.P. on its flat face, is cut by a vertical section plane inclined to the V.P. so that the semi-ellipse seen in the front view has its minor axis 45 mm long and half major axis 25 mm long. Draw the top view, sectional front view and true shape of the section.
16. The top view of a cylinder 75 mm diameter, 125 mm long, placed on top of the frustum of a cone, base 100 mm diameter, top 50 mm diameter and axis 125 mm long is shown in fig. 14-51. Both the solids are cut by a vertical section plane, the H.T. of which is 12 mm from the axis of the frustum and makes 30° angle with xy . Draw the sectional front view and true shape of the sections.
17. A sphere of 75 mm diameter is cut by a section plane, perpendicular to the V.P. and inclined at 30° to the H.P. in such a way that the true shape of the section is a circle of 50 mm diameter. Draw its front view, sectional top view and sectional side view.
18. A frustum of a cone, base 75 mm diameter, top 50 mm diameter and axis 75 mm long, has a hole of 30 mm diameter drilled centrally through its flat faces. It is resting on its base on the H.P. and is cut by a section plane, the V.T. of which makes an angle of 60° with xy and bisects the axis. Draw its sectional top view and an auxiliary top view on a reference line parallel to the V.T., showing clearly the shape of the section.
19. A hexagonal prism, side of the base 25 mm long and axis 65 mm long is resting on an edge of the base on the H.P., its axis being inclined at 60° to the H.P. and parallel to the V.P. A section plane, inclined at 45° to the V.P. and normal to the H.P., cuts the prism and passes through a point on the axis at a distance of 20 mm from the top end of the axis. Draw its sectional front view and true shape of the section.
20. A pentagonal pyramid, edge of base 25 mm long and height 50 mm is resting on the H.P. on a corner of its base in such a way that the slant edge containing that corner makes an angle of 60° with the H.P. and is parallel to the V.P. It is cut by a section plane making an angle of 30° with the V.P., perpendicular to the H.P. and passing through a point on the axis at a distance of 6 mm from its base. Draw its sectional front view and true shape of the section.
21. The distance between the opposite parallel faces of a 50 mm thick hexagonal block is 75 mm. The block has one of its rectangular faces parallel to the H.P. and its axis makes an angle of 30° with the V.P. It is cut by a section plane making

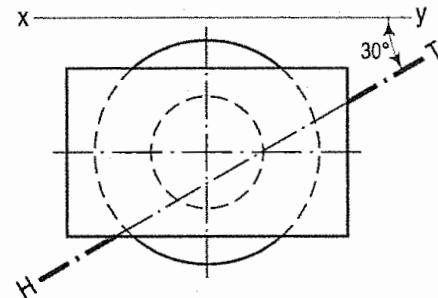


FIG. 14-51

- an angle of 30° with the H.P., normal to the V.P. and bisecting the axis. Draw its sectional top view and another top view on a plane parallel to the section.
22. *PQR* is an isosceles triangle having base *PR* horizontal and 50 mm long, and altitude 50 mm. A point *A* is taken on *PR* at a distance of 15 mm from *P* and a straight line *AB* is drawn parallel to *PQ* cutting *QR* at *B*. If *AB* is regarded as the V.T. of an inclined plane perpendicular to the V.P., cutting a cone of which *PQR* is the front view, draw the sectional top view, sectional side view and true shape of the section.
23. A cone of 55 mm diameter and 75 mm height is resting on the H.P. on one of its generators in such a way that, the generator is parallel to the V.P. It is cut by a plane parallel to the V.P. and inclined at 90° to the H.P. and passing through a point 15 mm in front of its axis. Draw the sectional front view and the top view of the cone.
24. The true section of a vertical square prism cut by an inclined plane is a rectangle of 75 mm \times 40 mm. The plane cuts one of the side faces at a height of 40 mm from the base. Draw three views of the cut prism when it rests on the cut face on the H.P. with its axis remaining parallel to the V.P.
25. An equilateral triangular prism, base 50 mm side and height 100 mm is standing on the H.P. on its triangular face with one of the sides of that face inclined at 90° to the V.P. It is cut by an inclined plane in such a way that the true shape of the section is a trapezium of 50 mm and 12 mm parallel sides. Draw the projections and true shape of the section and find the angle which the cutting plane makes with the H.P.
26. A horizontal cylinder, 30 mm diameter and length 60 mm, is placed centrally on the top of a frustum of a cone, diameter of the base 45 mm, diameter of the top 25 mm and height 45 mm. Draw a sectional front view of the two solids on a vertical plane, distance 12 mm from the axis of the cone and making an angle of 60° with the axis of the cylinder.
27. A cone, base 75 mm diameter and axis 100 mm long, has its base on the H.P. A section plane, parallel to one of the end generators and perpendicular to the V.P., cuts the cone intersecting the axis at a point 75 mm from the base. Draw the sectional top view and project another top view on a plane parallel to the section plane, showing the shape of the section clearly.
28. A solid is made up of a cylinder, 30 mm diameter and 75 mm long, which joins another cylinder, 75 mm diameter and 25 mm long, by a fillet of 20 mm radius, the axes of the two cylinders being in a straight line. Draw the top view of a horizontal section of the solid made by a plane parallel to and 15 mm above the axis.
29. A cone frustum, base 75 mm diameter, top 35 mm diameter and height 65 mm has a hole of 30 mm diameter drilled through it so that the axis of the hole coincides with that of the cone. It is resting on its base on the H.P. and is cut by a section plane perpendicular to the V.P., parallel to an end generator and passing through the top end of the axis. Draw sectional top view and sectional side view of the frustum.
30. A cube of 25 mm edge rests on one of its corners on the H.P. so that a solid diagonal is vertical and two of its faces are perpendicular to the V.P. A vertical section plane parallel to the V.P. cuts the cube at a distance of 8 mm from the solid diagonal and nearer to the V.P. Draw its sectional front view.

Chapter 15



DEVELOPMENT OF SURFACES

15-0. INTRODUCTION



Imagine that a solid is enclosed in a wrapper of thin material, such as paper. If this covering is opened out and laid on a flat plane, the flattened-out paper is the development of the solid. Thus, when surfaces of a solid are laid out on a plane, the figure obtained is called its development.

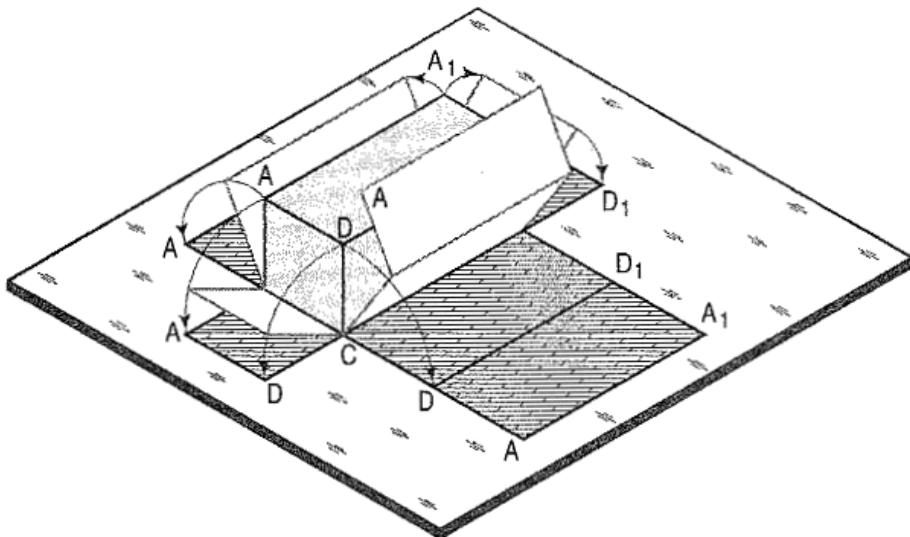


FIG. 15-1

Fig. 15-1 shows a square prism covered with paper in process of being opened out. Its development (fig. 15-2) consists of four equal rectangles for the faces and two similar squares for its ends. Each figure shows the true size and shape of the corresponding surface of the prism. The development of a solid, thus represents the actual shape of all its surfaces which, when bent or folded at the edges, would form the solid.

Hence, it is very important to note that every line on the development must be the true length of the corresponding edge on the surface.

The knowledge of development of surfaces is essential in many industries such as automobile, aircraft, ship building, packaging and sheet-metal work. In construction of boilers, bins, process-vessels, hoppers, funnels, chimneys etc., the plates are marked and cut according to the developments which, when folded, form the desired objects. The form of the sheet obtained by laying all the outer surfaces of the solid with suitable allowances for the joints is known as pattern.

Only the surfaces of polyhedra (such as prisms and pyramids) and single-curved surfaces (as of cones and cylinders) can be accurately developed. Warped and double-curved surfaces are undevelopable. These can however be approximately developed by dividing them up into a number of parts.

This chapter deals with the following topics:

1. Methods of development.
2. Developments of lateral surfaces of right solids.
3. Development of transition pieces.
4. Spheres (approximate method).

15-1. METHODS OF DEVELOPMENT



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 33 for the methods of development of surfaces.

The following are the principal methods of development:

(1) **Parallel-line development:**

It is employed in case of prisms and cylinders in which stretch-out-line principle is used. Lines A-A and A_1-A_1 in fig. 15-2 are called the stretch-out lines.

(2) **Radial-line development:** It is used for pyramids and cones in which the true length of the slant edge or the generator is used as radius.

(3) **Triangulation development:**

This is used to develop transition pieces. This is simply a method of dividing a surface into a number of triangles and transferring them into the development.

(4) **Approximate method:** It is used to develop objects of double curved or warped surfaces as sphere, paraboloid, ellipsoid, hyperboloid and helicoid.

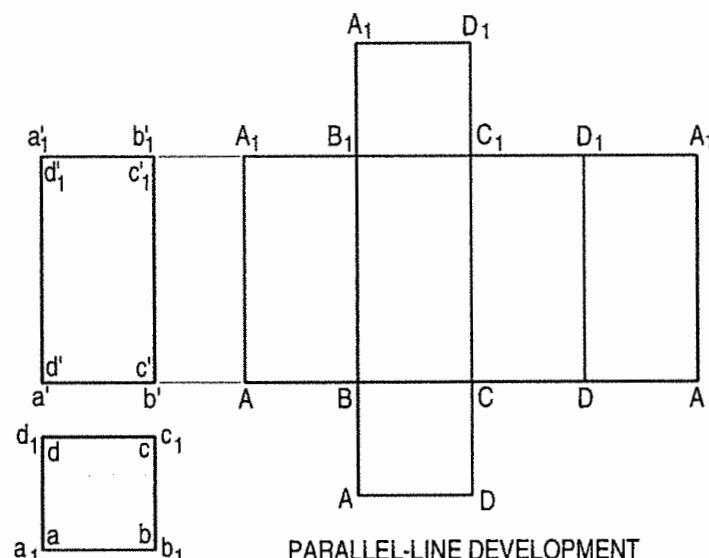


FIG. 15-2

15-2. DEVELOPMENTS OF LATERAL SURFACES OF RIGHT SOLIDS



The methods of drawing developments of surfaces of various solids are explained by means of the following typical problems. Only the lateral surfaces of the solids (except the cube) have been developed. The ends or bases have been omitted. They can be easily incorporated if required.

15-2-1. CUBE



The development of the surface of a cube consists of six equal squares, the length of the side of the squares being equal to the length of the edge of the cube.

Problem 15-1. Draw the development of the surface of the part P of the cube, the front view of which is shown in fig. 15-3(i).

Name all the corners of the cube and also the points at which the edges are cut.

- (i) Draw the stretch-out lines A-A and E-E directly in line with the front view, and assuming the cube to be whole, draw four squares for the vertical faces, one square for the top and another for the bottom as shown in fig. 15-3(ii).

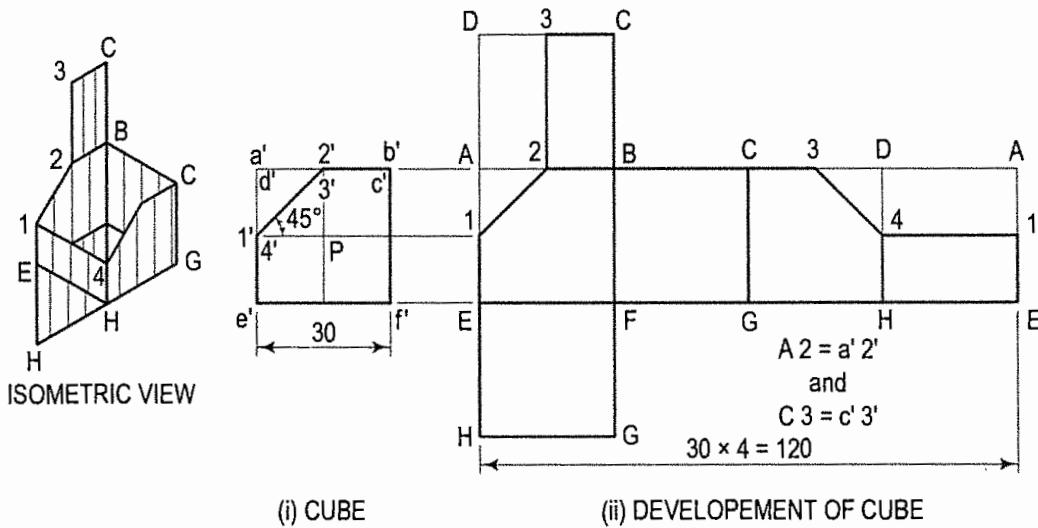


FIG. 15-3

- (ii) Name all the corners. Draw a horizontal line through 1' to cut AE at 1 and DH at 4. a' b' is the true length of the edge. Hence, mark a point 2 on AB and 3 on CD such that $A2 = a' 2'$ and $C3 = c' 3'$. Mark the point 3 on CD in the top square also.
 (iii) Draw lines 1-2, 2-3, 3-4 and 4-1, and complete the development as shown. Keep lines for the removed portion, viz. A1, A2, 3D, D4 and DA thin and fainter.

Problem 15-2. Draw the development of the surface of the part P of the cube shown in two views in fig. 15-4(i).

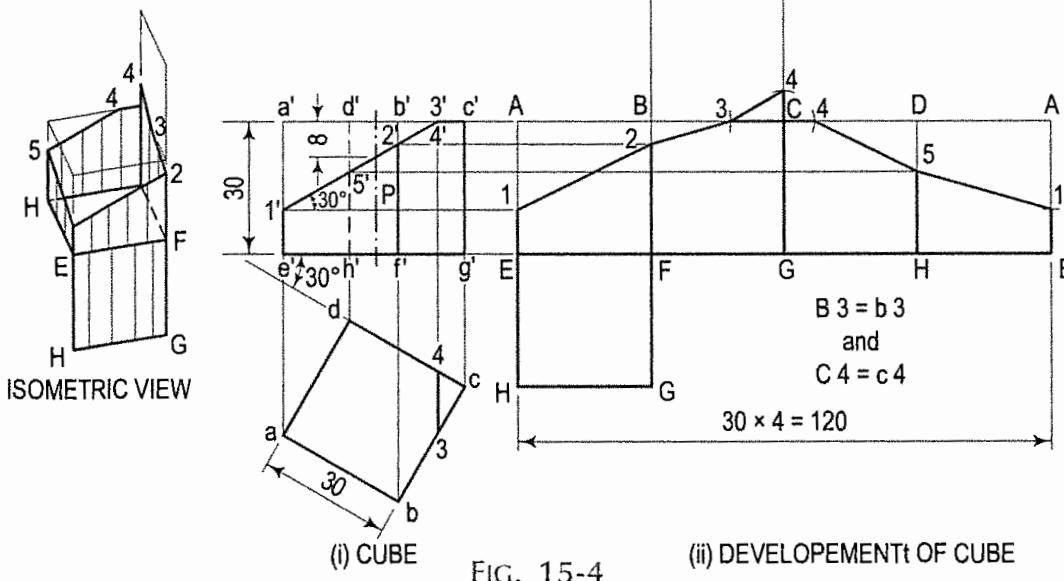


FIG. 15-4

Name all the corners of the cube and also the points at which the edges are cut. Draw the development assuming the cube to be whole [fig. 15-4(ii)] as explained in problem 15-1.

- Draw horizontal lines through points $1'$, $2'$ and $5'$ to cut AE in 1, BF in 2 and DH in 5 respectively. Lines $b'c'$ and $c'd'$ do not show the true lengths of the edges. The sides of the square in the top view show the true length. Therefore, mark points 3 in BC and 4 in CD such that $B3 = b3$ and $C4 = c4$.
- Draw lines joining 1, 2, 3 etc. in correct sequence and complete the required development. Keep the lines for the removed part fainter.

15-2-2. PRISMS



Development of the lateral surface of a prism consists of the same number of rectangles in contact as the number of the sides of the base of the prism. One side of the rectangle is equal to the length of the axis and the other side equal to the length of the side of the base.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 34 for the following problem.

Problem 15-3. Draw the development of the lateral surface of the part P of the pentagonal prism shown in fig. 15-5(i).

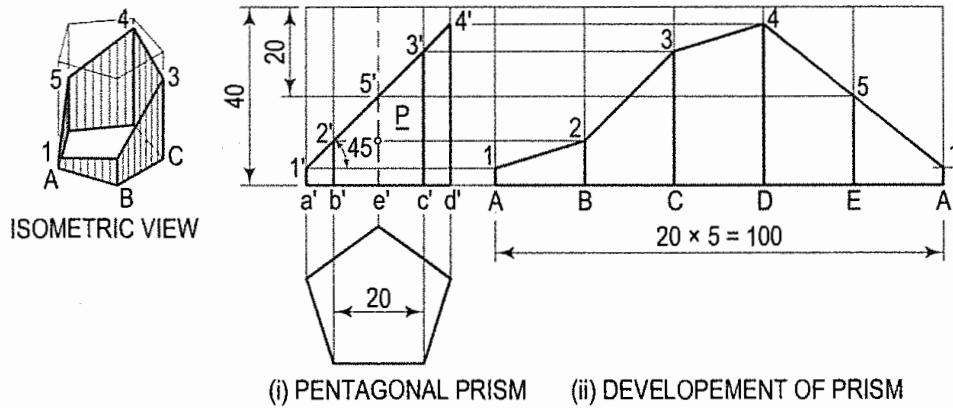
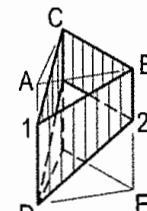


FIG. 15-5

Name the corners of the prism and the points at which the edges are cut.

- Draw the development assuming the prism to be whole [fig. 15-5(ii)]. It is made up of five equal rectangles.
- Draw horizontal lines through points $1'$, $2'$ etc. to cut the lines for the corresponding edges in the development at points 1, 2 etc.
- Draw lines joining these points and complete the development as shown.



ISOMETRIC VIEW

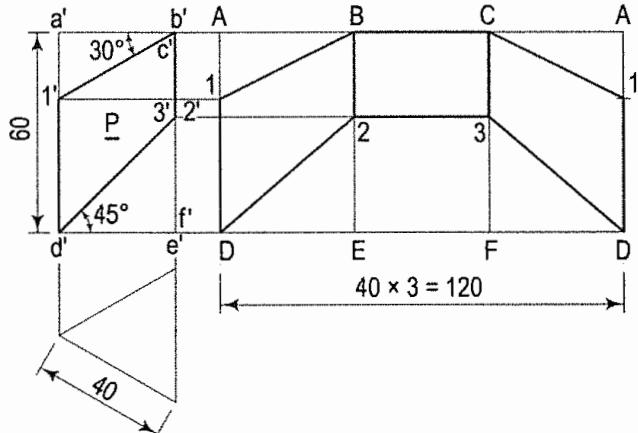


FIG. 15-6

Problem 15-4. Draw the development of the lateral surface of the part P of the triangular prism shown in fig. 15-6(i).

Draw the development of the lateral surface of the whole prism [fig. 15-6(ii)] and obtain points 1, 2 and 3 on it. Draw lines $1B$, $C1$, $D2$, $2-3$ and $3D$, and complete the development as shown.

Problem 15-5. Draw the development of the lateral surface of the part P of the hexagonal prism shown in fig. 15-7(i).

Name the points at which the edges are cut and draw the development assuming the prism to be whole [fig. 15-7(ii)].

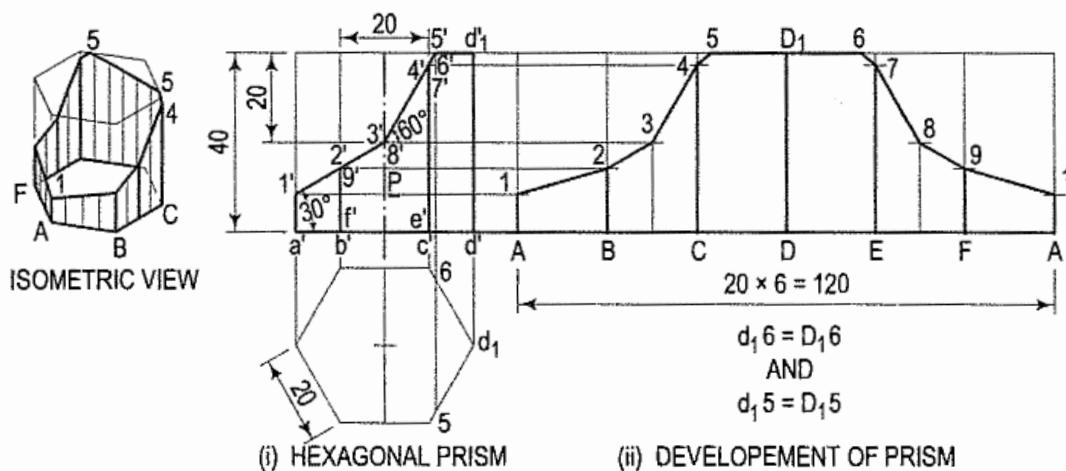


FIG. 15-7

- Obtain all the points except 5 and 6 by drawing horizontal lines. Note that points 3 and 8 lie on vertical lines drawn through the mid-points of BC and EF.
- Mark points 5 and 6 such that $5D_1 = 5d_1$ and $D_16 = d_16$.
- Draw lines joining points 1, 2, 3 etc. in correct sequence and complete the required development as shown.

Problem 15-6. Draw the development of the lateral surface of the truncated prism shown in fig. 15-8(i). Also, draw the front view of the line joining the points P and Q (whose projections p , p' and q , q' are given) along the surface of the prism by the shortest distance.

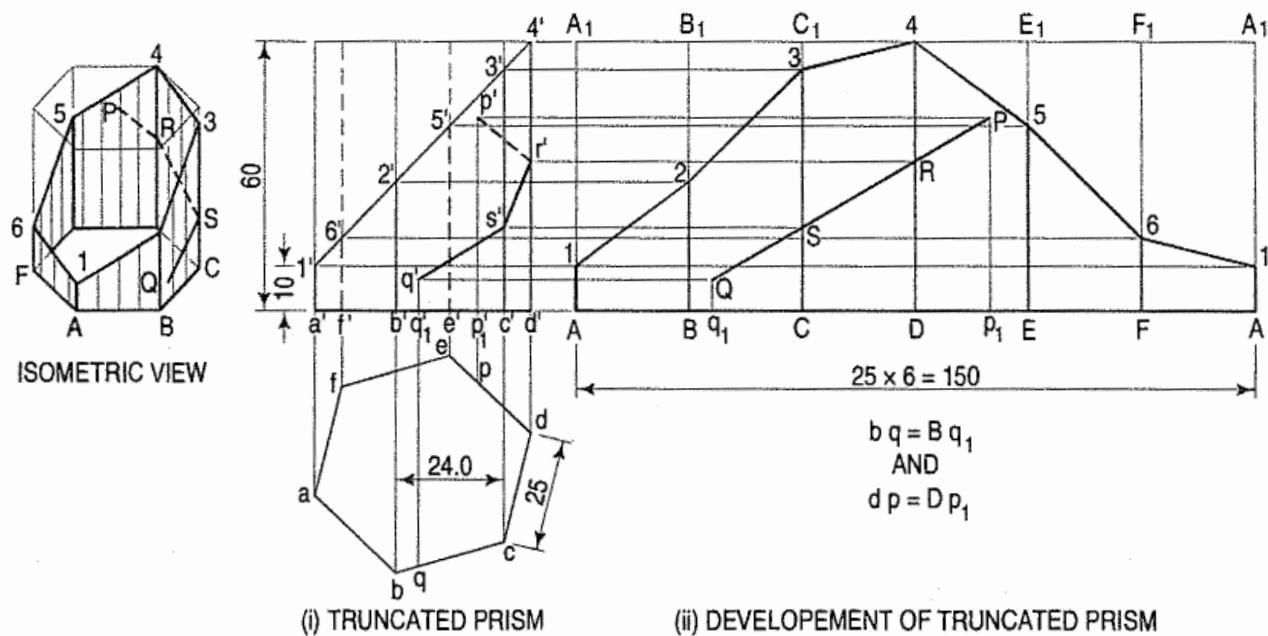


FIG. 15-8

Draw the required development [fig. 15-8(ii)] as explained in problem 15-3.

- Mark a point p_1 on DE and q_1 on BC such that $Dp_1 = dp$ and $Bq_1 = bq$. Draw verticals through points p_1 and q_1 , and on them, obtain points P and Q by drawing horizontal lines through points p' and q' .
- Draw a straight line joining P with Q . Then PQ shows the shortest distance between them. To draw this line in the front view, the process must be reversed. Let PQ cut the line $D4$ at R and $C3$ at S .
- Draw horizontals through R and S cutting $d'4'$ at r' and $c'3'$ at s' . Draw lines $p'r'$, $r's'$ and $s'q'$ which show the front view of the line PQ . Note that $p'r'$ is a hidden line.

Problem 15-7. The projections of a square prism with a hole drilled in it are given in fig. 15-9(i). Draw the development of the lateral surface of the prism.

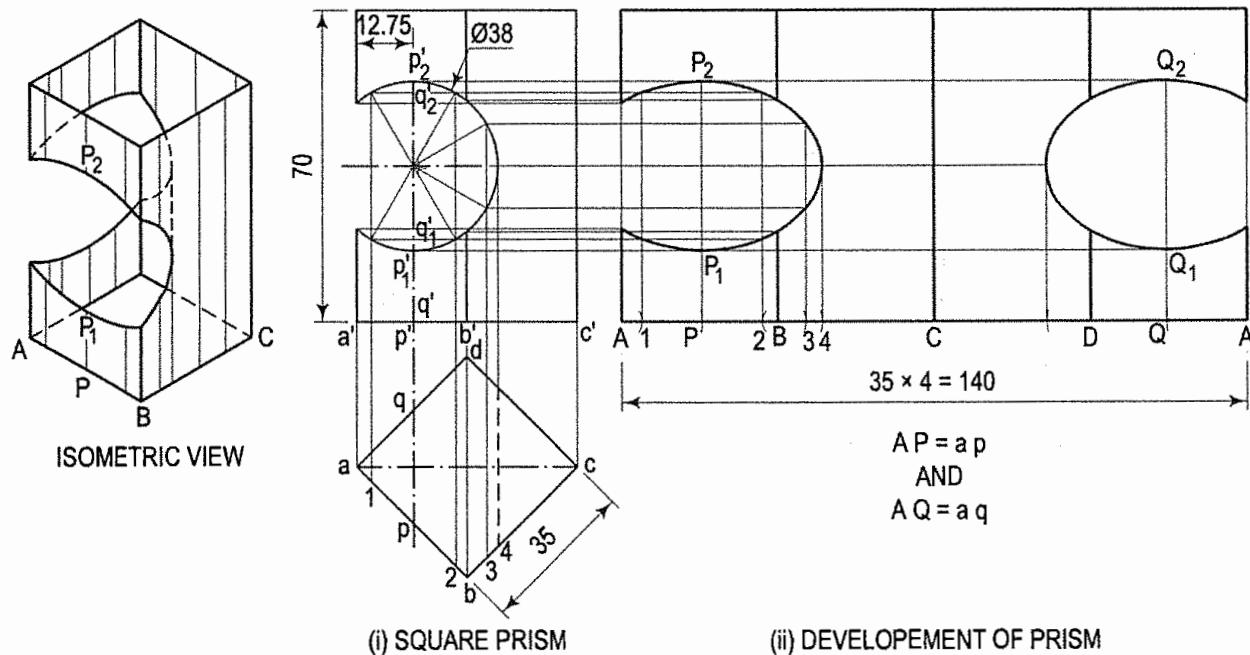


FIG. 15-9

- Mark a number of points on the circle for the hole. Draw the development of the whole prism [fig. 15-9(ii)] and locate the positions of these points on it.
- For example, to locate points p'_1 and p'_2 and points q'_1 and q'_2 which coincide with them in the front view, draw a perpendicular through them cutting the base at p' . Project p' to p on ab and to q on ad . Mark points P and Q on AB and DA respectively such that $AP = ap$ and $AQ = aq$.
- Draw verticals at points P and Q . Draw horizontal lines through p'_1 and p'_2 cutting these verticals at P_1 , Q_1 , P_2 and Q_2 . Locate all points in the same manner and draw smooth curves through them, thus completing the development.

15-2-3. CYLINDERS



The development of the lateral surface of a cylinder is a rectangle having one side equal to the circumference of its base-circle and the other equal to its length.

Problem 15-8. Develop the lateral surface of the truncated cylinder shown in fig. 15-10(i).

- Divide the circle in the top view into twelve equal parts. Project the division points to the front view and draw the generators. Mark points a' , b' and b'_1 , c' and c'_1 etc. in which the generators are cut.
- Draw the development of the lateral surface of the whole cylinder along with the generators [fig. 15-10(ii)]. The length of the line 1-1 is equal to $\pi \times D$ (circumference of the circle). This length can also be marked approximately by stepping off with a bow divider, twelve divisions, each equal to the chord-length ab . (The length thus obtained is about 1% shorter than the exact length; but this is permitted in drawing work.)

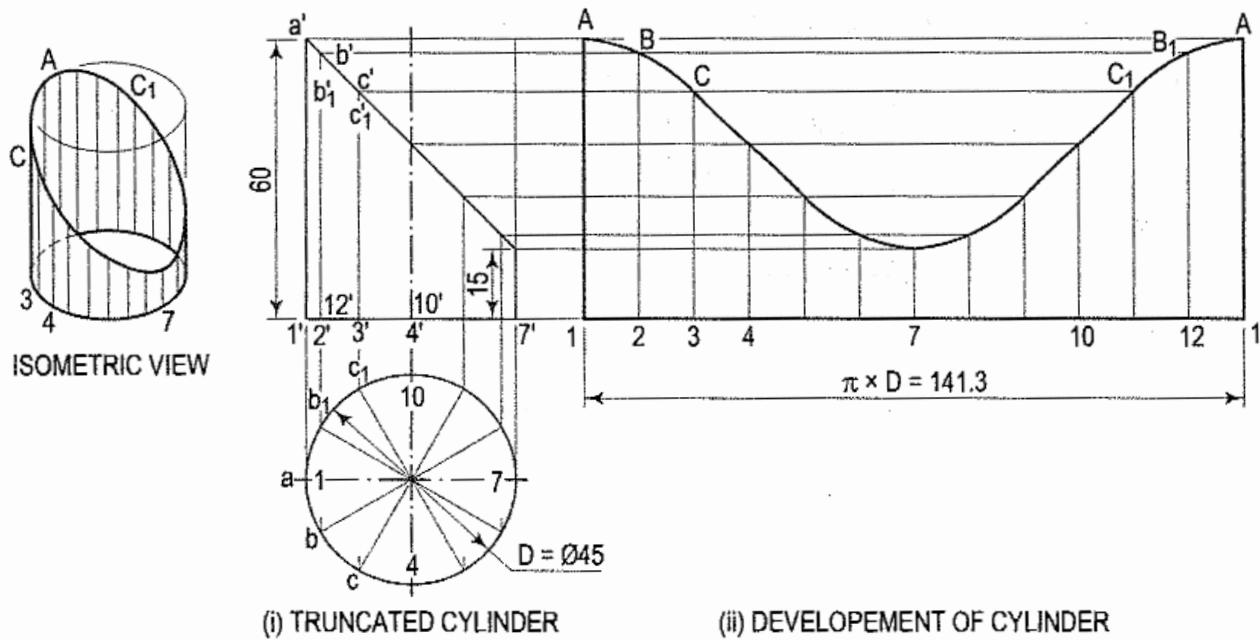


FIG. 15-10

- Draw horizontal lines through points a' , b' and b'_1 etc. to cut the corresponding generators in points A , B and B_1 etc. Draw a smooth curve through the points thus obtained. The figure 1-A-A-1 is the required development.

Problem 15-9. Draw the development of the lateral surface of the part P of the cylinder shown in fig. 15-11(i).

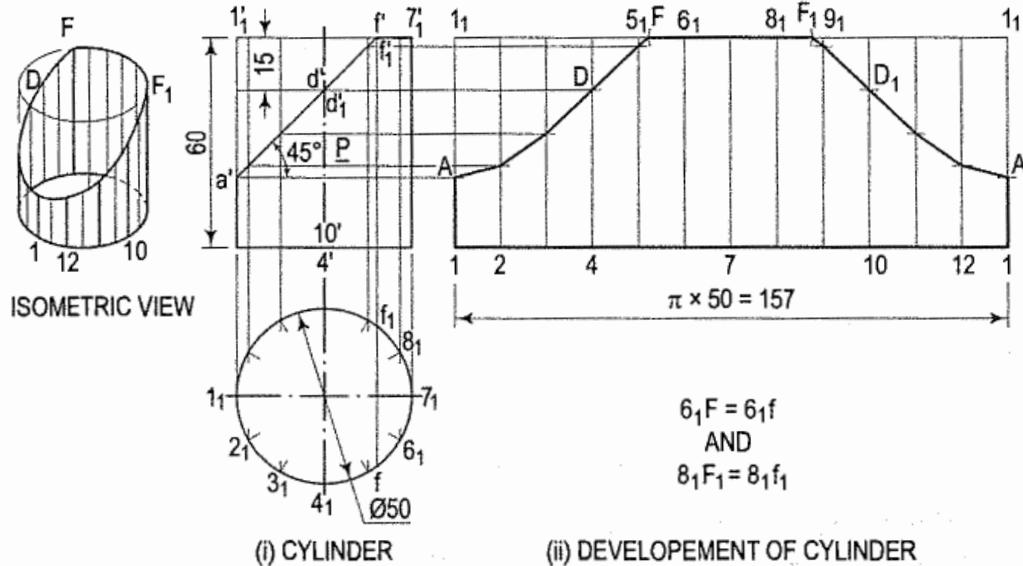


FIG. 15-11

Draw the development as explained in problem 15-8. Positions of the points at which the upper end of the cylinder is cut should be obtained from the top view. Mark these points, viz. F and F_1 on the line 1_1-1_1 between points 5_1 and 6_1 and between 8_1 and 9_1 in such a way that $6_1F = 6_1f$ and $8_1F_1 = 8_1f_1$. Draw curves FA and F_1A passing through these points and complete the required development as shown.

Problem 15-10. Draw the development of the lateral surface of the cylinder cut by three planes as shown in fig. 15-12(i).

See fig. 15-12(ii).

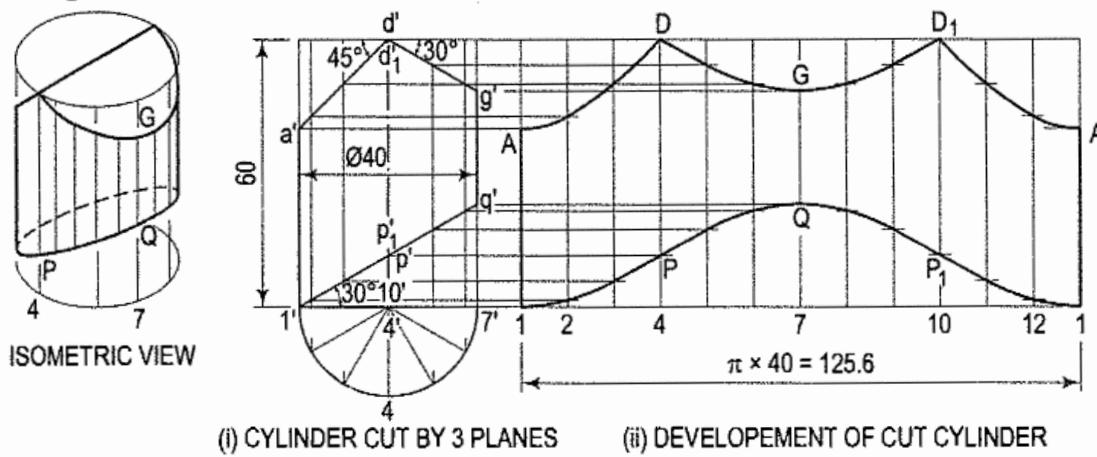


FIG. 15-12

Problem 15-11. (fig. 15-13(ii)): Draw the development of the lateral surface of the cylinder having a square hole in it as shown in fig. 15-13(i).

See fig. 15-13(ii).

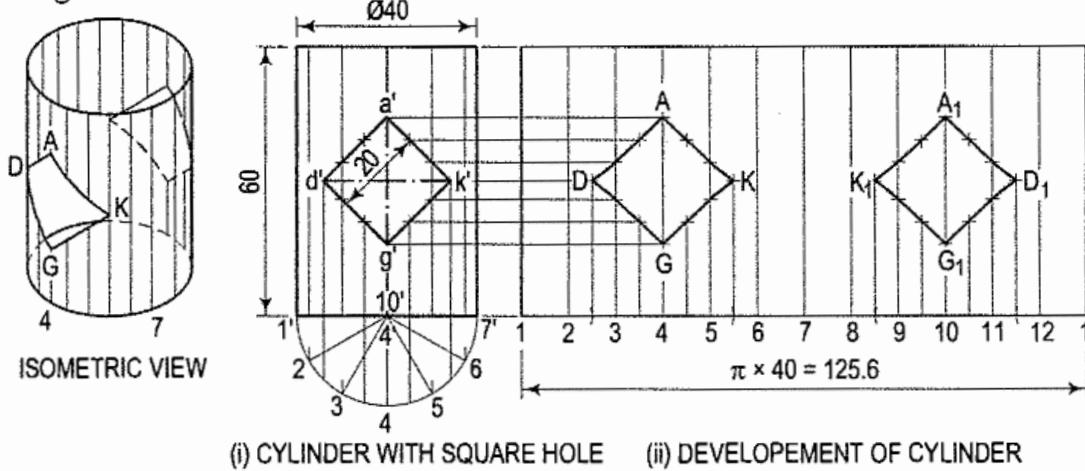


FIG. 15-13

Problem 15-12. Draw the development of the lateral surface of the cylinder cut as shown in fig. 15-14(i).

See fig. 15-14(ii).

Problem 15-13. Develop the surface of the three-piece cylindrical pipe elbow shown in fig. 15-15(i).

The front view is drawn as shown below:

Draw a square $efgh$ of sides equal to the diameter of the pipes (i.e. 50 mm). Bisect angles ehf and ghf by lines Ph and Qh to intersect ef and fg at x and y respectively. Then xy is the axis of the middle piece B .

Parts A and C are similar and are in the form of cylinders truncated at one end only. The parts A and C are as shown in fig 15-15(ii) and fig. 15-15(iv) are developed on a stretch-out line drawn directly in line with its base. The twelve divisions are obtained from the semi-circle drawn on the base as diameter. The part B is truncated at both the ends. It is developed on a stretch-out line drawn through h and at right angles to its axis xy as shown in fig 15-15(iii).

As the curves of the part B are exactly similar to those of A and C, the three developments may also be drawn combined to minimize wastage on plate as shown in fig. 15-15(v).

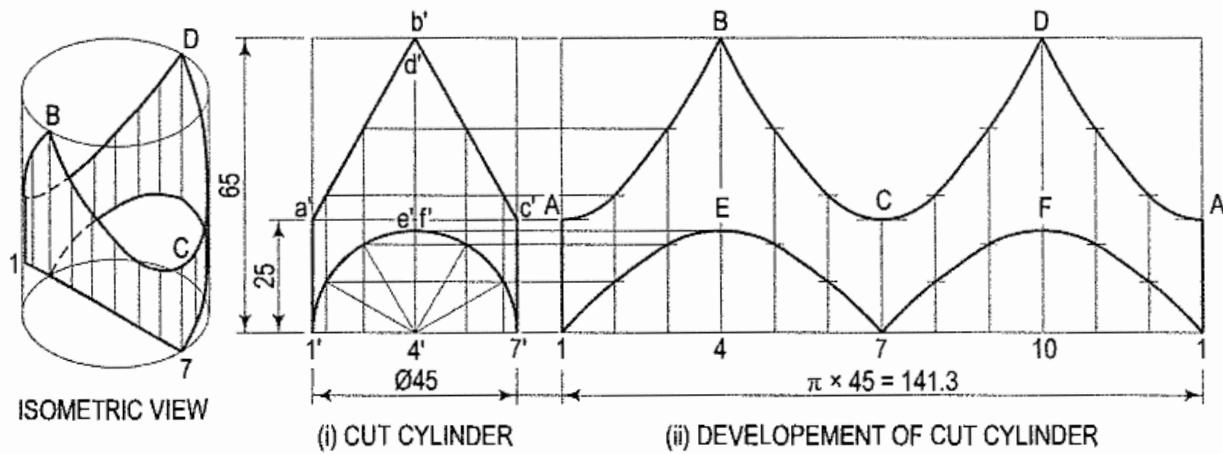


FIG. 15-14

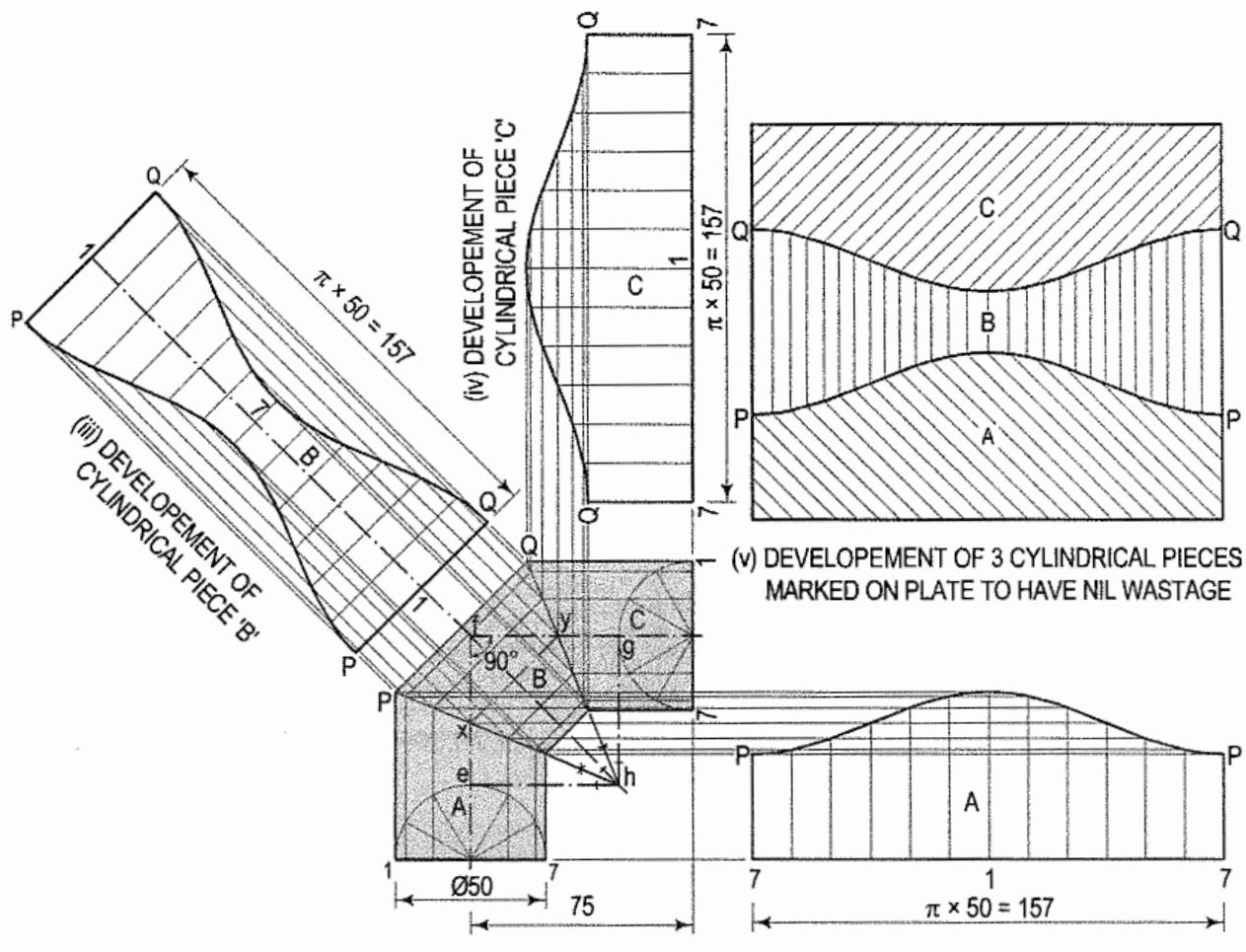


FIG. 15-15

Problem 15-14. Three cylindrical pipes of 50 mm diameter form a Y-piece as shown in the front view in fig. 15-16(i). Draw the development of the surface of each pipe.

- Draw a semi-circle on the base of pipe A as diameter and obtain twelve divisions. Draw the development [fig. 15-16(ii)] as in the previous problem.
- Draw any convenient line at right angles to the axis of the pipe B [fig. 15-16(iii)]. On this line as a stretch-out line, draw the development as shown. Pipes B and C are similar and hence, their developments will also be similar.

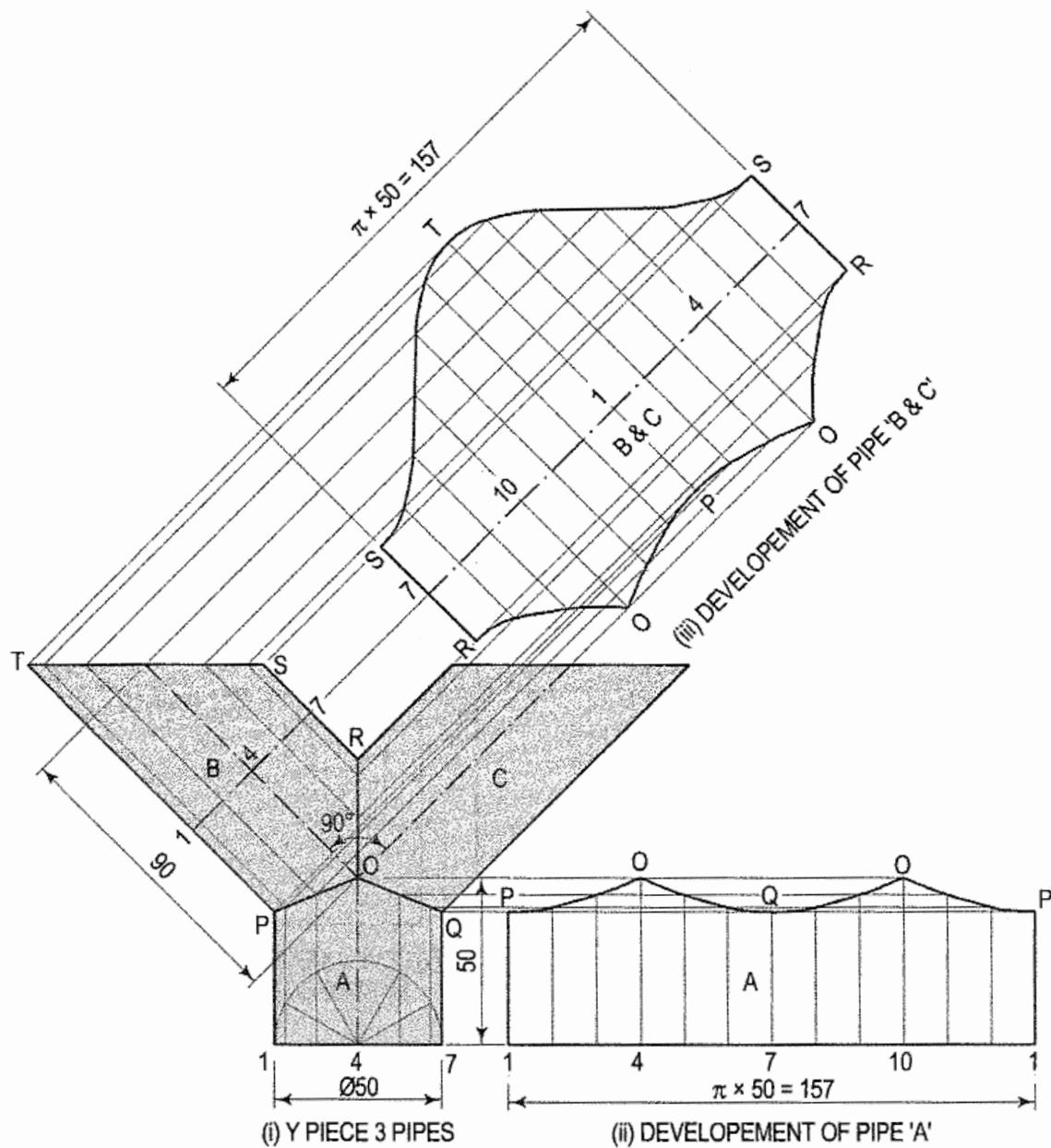


FIG. 15-16

15-2-4. PYRAMIDS

The development of the lateral surface of a pyramid consists of a number of equal isosceles triangles in contact. The base and the sides of each triangle are respectively equal to the edge of the base and the slant edge of the pyramid

Note: The true length of a slant edge of pyramid can be measured from the front view, if the top view of that edge is parallel to xy ; and it can be measured from the top view, if the slant edge is parallel to xy in the front view.

Method of drawing the development of the lateral surface of a pyramid:

- With any point O as centre and radius equal to the true length of the slant edge of the pyramid, draw an arc of the circle. With radius equal to the true length of the side of the base, step-off (on this arc) the same number of divisions as the number of sides of the base.
- Draw lines joining the division-points with each other in correct sequence and also with the centre for the arc. The figure thus formed (excluding the arc) is the development of the lateral surface of the pyramid.

Problem 15-15. Draw the development of the lateral surface of the part P of the triangular pyramid shown in fig. 15-17(i). The line $o'1'$ in the front view is the true length of the slant edge because it is parallel to xy in the top view. The true length of the side of the base is seen in the top view.

- Draw the development of the lateral surface of the whole pyramid [fig. 15-17(ii)] as explained above. On O_1 mark a point A such that $OA = o'a'$. $o'2'$ (with which $o'3'$ coincides) is not the true length of the slant edge.
- Hence, through b' , draw a line parallel to the base and cutting $o'a'$ at b'' . $o'b''$ is the true length of $o'b'$ as well as $o'c'$. Mark a point B in O_2 and C in O_3 such that $OB = OC = o'b''$.
- Draw lines AB , BC and CA and complete the required development as shown. Keep the arc and the lines for the removed part fainter.

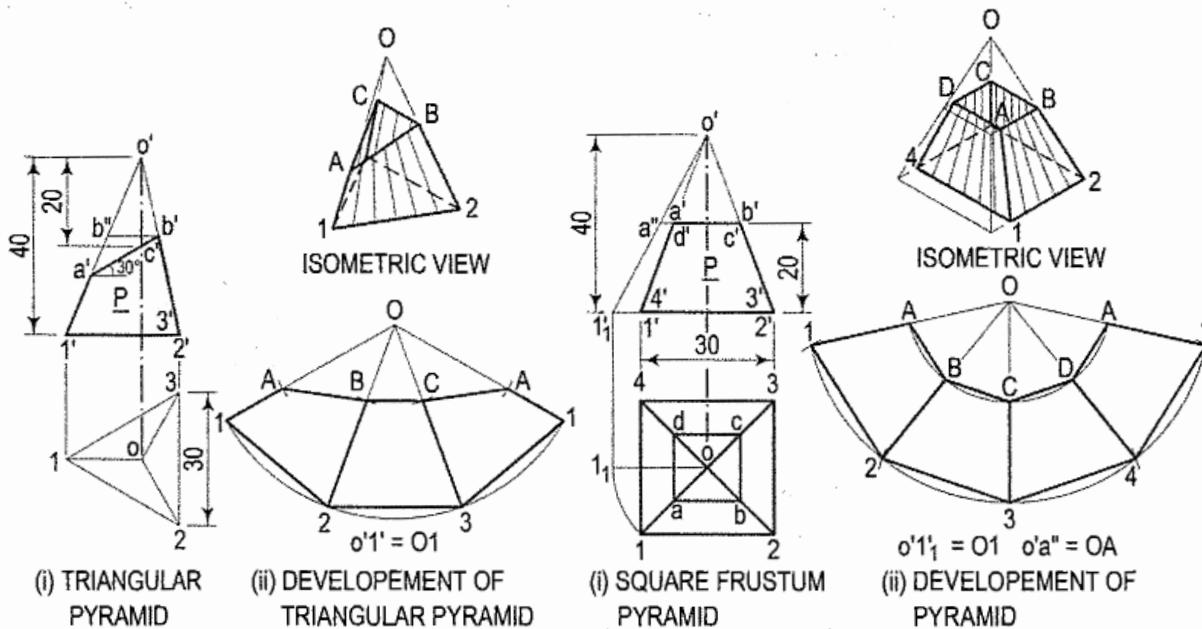


FIG. 15-17

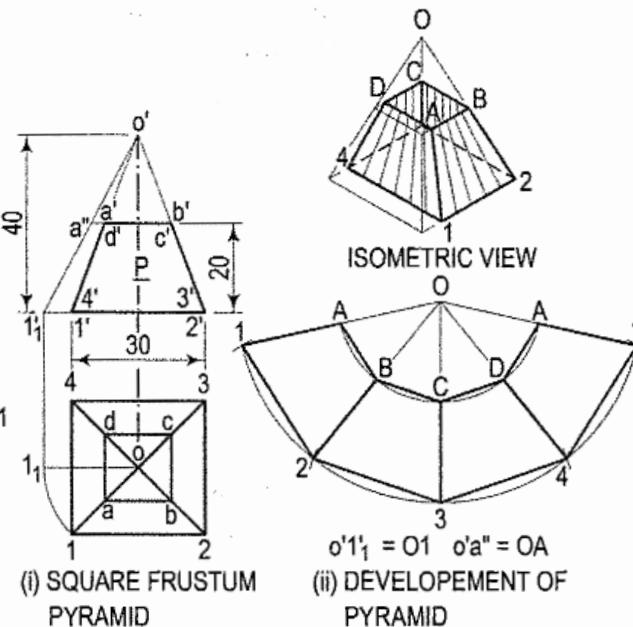


FIG. 15-18

Problem 15-16. Draw the development of the lateral surface of the frustum of the square pyramid shown in fig. 15-18(i).

- Determine the position of the apex. None of the lines in the front view shows the true length of the slant edge. Therefore, draw the top view and make any one line (for the slant edge) horizontal, i.e. parallel to xy and determine the true length $o'1'1$. Through a' , draw a line parallel to the base and obtain the true length $o'a''$.

- (ii) With O as centre and radius $o'1'$, draw an arc and obtain the development of the lateral surface of the whole pyramid [fig. 15-18(ii)].
- (iii) With centre O and radius $o'a''$, draw an arc cutting $O1$, $O2$ etc. at points A , B etc. respectively.
- (iv) Draw lines AB , BC , CD and DA and complete the required development. Note that these lines are respectively parallel to lines 1-2, 2-3 etc.

Problem 15-17. Draw the development of the lateral surface of the part P of the square pyramid shown in fig. 15-19(i).

Draw the top view of the pyramid and determine the true length $o'1'$ of the slant edges as explained in problem 15-16. On this line, obtain the true lengths $o'a''$ and $o'b''$.

- (i) Draw the development of the lateral surface of the whole pyramid [fig. 15-19(ii)].
- (ii) Mark a point A in $O1$ and D in $O4$ such that $OA = OD = o'a''$. Similarly, mark B in $O2$ and C in $O3$ such that $OB = OC = o'b''$. Draw lines AB , BC etc. and complete the required development as shown.

Problem 15-18. Draw the development of the lateral surface of the pentagonal pyramid, the lower part of which is removed as shown in fig. 15-20(i).

None of the lines in the front view shows the true length of the slant edges. Hence, draw the top view and determine the true length $o'c'_1$. Through points $1'$, $2'$ etc., draw lines parallel to the base and obtain the true lengths $o'1''$, $o'2''$ etc.

- (i) Draw the development of the lateral surface of the whole pyramid [fig. 15-20(ii)]. Mark points 1, 2, 3 etc. on lines OB , OC , OD etc. such that $O1 = o'1''$, $O2 = o'2''$, $O3 = o'3''$ etc.
- (ii) Draw lines joining points A , 1, 2 etc. in correct sequence and complete the required development. Note that the lines for the lower part should be fainter.

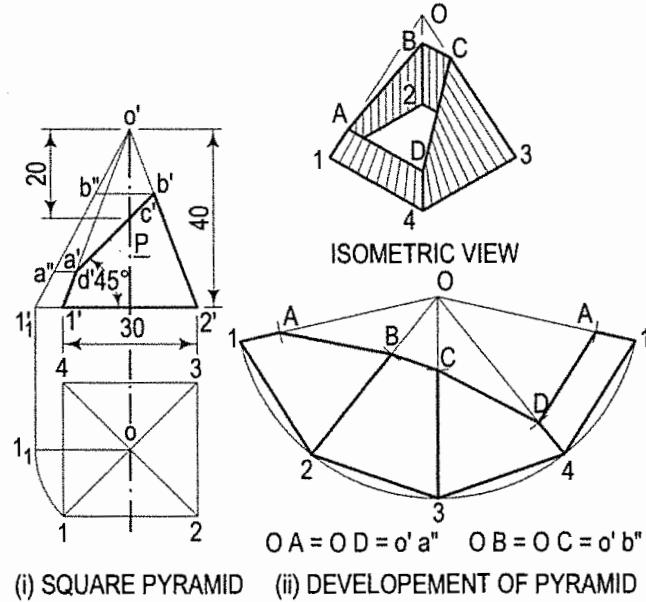


FIG. 15-19

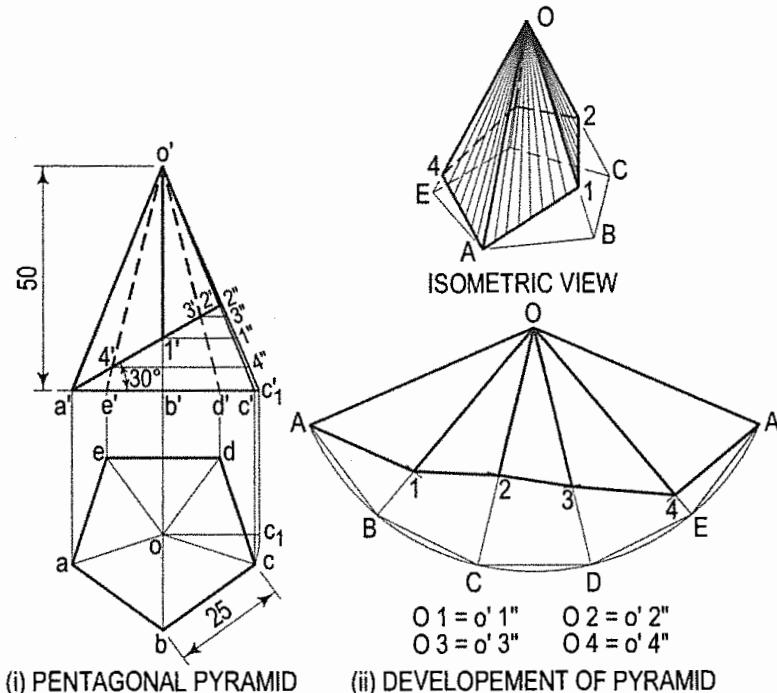


FIG. 15-20

Problem 15-19. Draw the development of the lateral surface of the part P of the pentagonal pyramid shown in fig. 15-21(i).

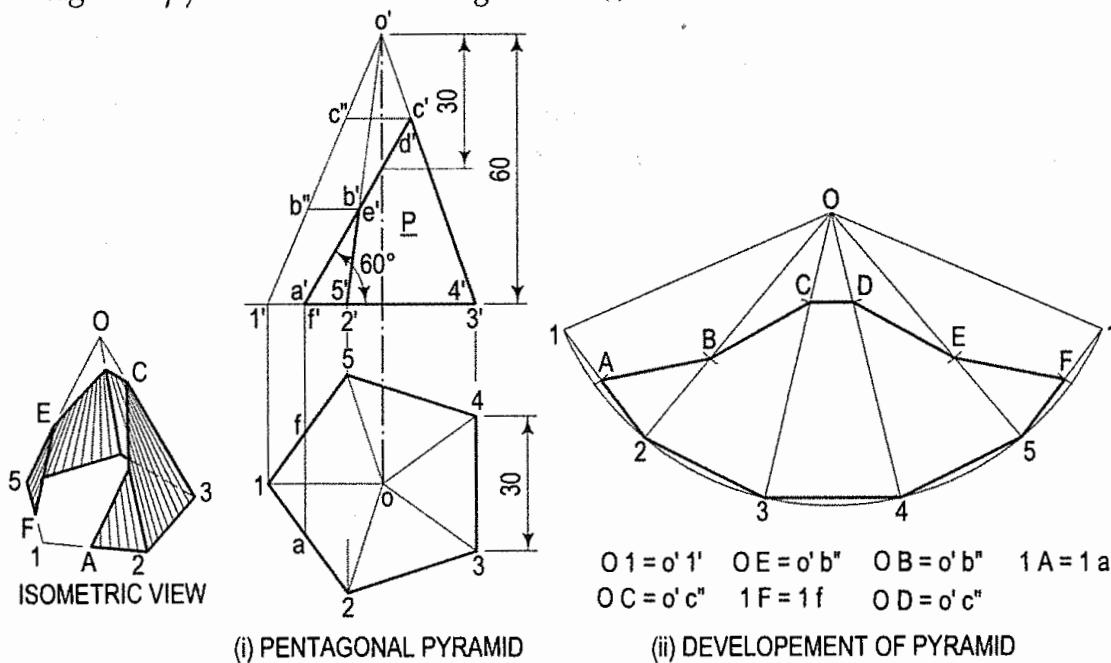


FIG. 15-21

The line $o'1'$ shows the true length of the slant edges. On it, obtain true lengths $o'b''$ and $o'c''$. Two edges of the base are also cut.

- Draw the top view and obtain the true lengths $1a$ and $1f$ as shown. Draw the development of the lateral surface of the whole pyramid.
- Obtain points B , C , D and E as explained in problem 15-18. Mark a point A in $1-2$ and F in $5-1$ such that $1A = 1a$ and $1F = 1f$.
- Draw lines joining points A , B , C etc. and complete the required development.

Problem 15-20. Draw the development of the lateral surface of the part P of the hexagonal pyramid shown in fig. 15-22(i).

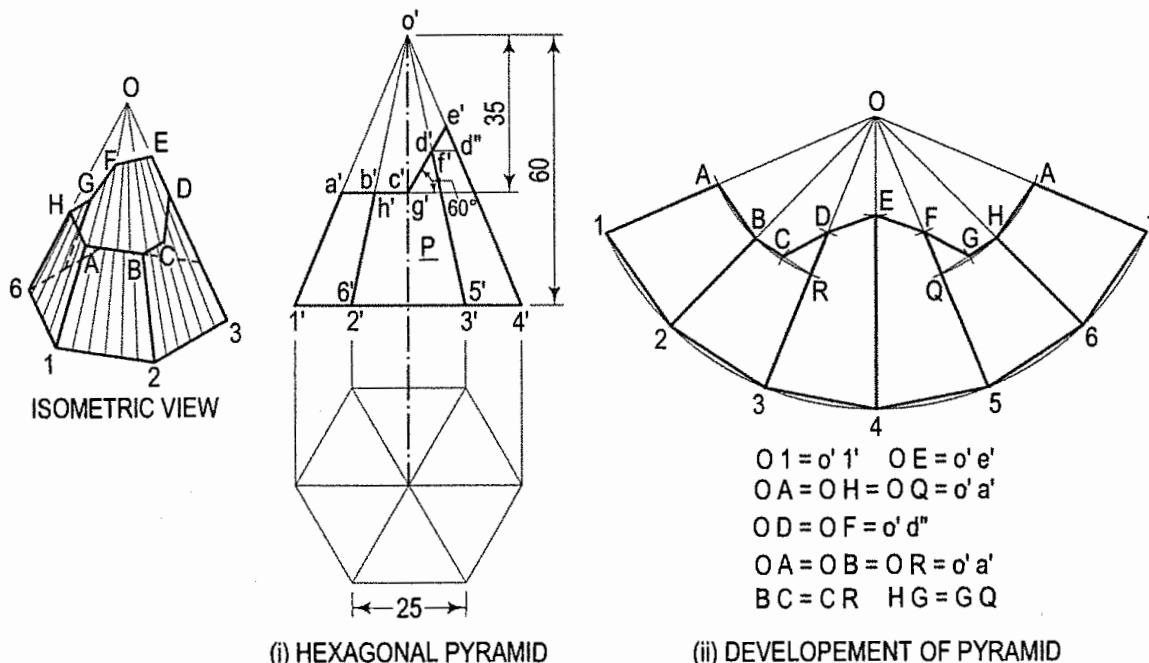


FIG. 15-22

Lines $o'1'$ and $o'4'$ show the true length of the slant edges. Draw the development of the lateral surface of the whole pyramid [fig. 15-22(ii)]. Obtain the development of the left-half of the pyramid as explained in problem 15-16 and that of the right-half as explained in problem 15-15.

Note that the points C and G are the mid-points of the lines BR and HQ respectively.

Problem 15-21. (fig. 15-23): A frustum of a square pyramid has its base 50 mm side, top 25 mm side and height 75 mm. Draw the development of its lateral surface.

Also, draw the projections of the frustum (when its axis is vertical and a side of its base is parallel to the V.P.), showing the line joining the mid-point of a top edge of one face with the mid-point of the bottom edge of the opposite face, by the shortest distance.

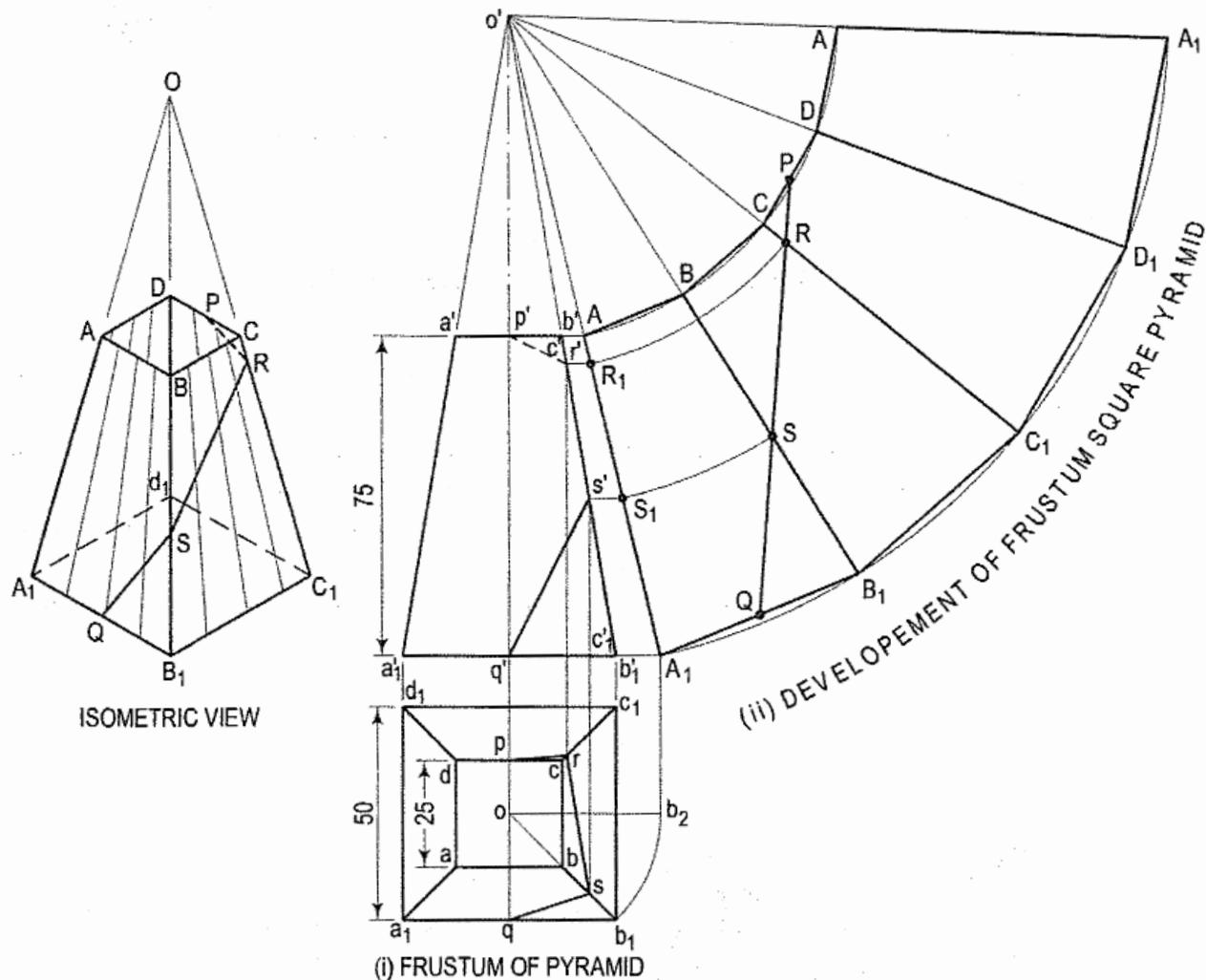


FIG. 15-23

Draw the development as explained in problem 15-16.

- Mark the mid-point P of CD and Q of A_1B_1 . Draw a line joining P and Q and cutting CC_1 at R and BB_1 at S . Transfer these points to the front view and the top view. For example, with o' as centre and radius $o'R$, draw an arc cutting $o'A_1$ at R_1 . Through R_1 , draw a line parallel to the base and cutting $c'c'_1$ at r' . Project r' to r on cc_1 in the top view. r' and r are the projections of R .
- Similarly, obtain s' and s on $b'b'_1$ and bb_1 respectively. Draw lines pr , rs and sq which will show the top view of the line PQ . $p'r's'q'$ will be the path of the line PQ in the front view.

15-2-5. CONE

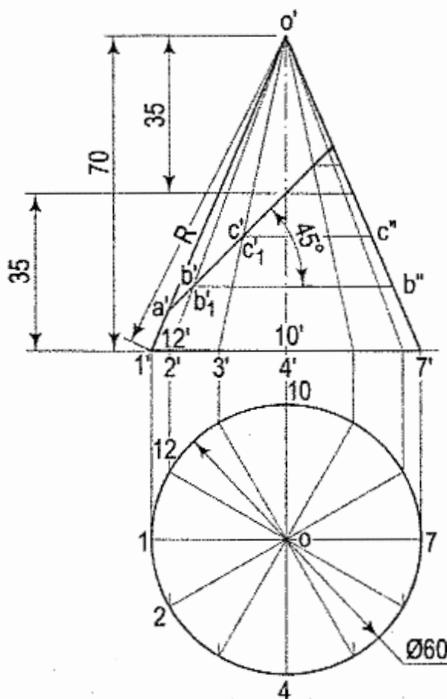


The development of the curved surface of a cone is a sector of a circle, the radius and the length of the arc of which are respectively equal to the slant height and the circumference of the base-circle of the cone.

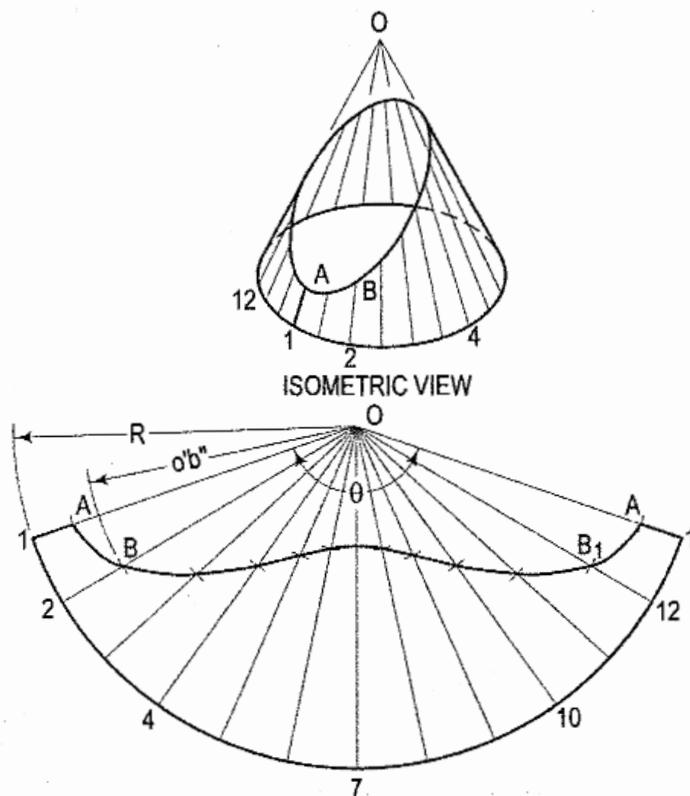


This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 35 for the following problem.

Problem 15-22. Draw the development of the lateral surface of the truncated cone shown in fig. 15-24(i).



(i) TRUNCATED CONE



(ii) DEVELOPMENT OF TRUNCATED CONE

FIG. 15-24

Assuming the cone to be whole, let us draw its development.

- Draw the base-circle in the top view and divide it into twelve equal parts.
- With any point O as centre and radius equal to $o'1'$ or $o'7'$, draw an arc of the circle [fig. 15-24(ii)]. The length of this arc should be equal to the circumference of the base circle. This can be determined in two ways.
- Calculate the subtended angle θ by the formula,

$$\theta = 360^\circ \times \frac{\text{radius of the base circle}}{\text{slant height}}$$

Cut-off the arc so that it subtends the angle θ at the centre and divide it into twelve equal parts.

- Step-off with a bow-divider, twelve equal divisions on the arc, each equal to one of the divisions of the base-circle.

(This will give an approximate length of the circumference. Note that the base-circle should not be divided into less than twelve equal parts.)

- (v) Join the division-points with O, thus completing the development of the whole cone with twelve generators shown in it [fig. 15-24(ii)].
- (vi) The truncated portion of the cone may be deducted from this development by marking the positions of points at which generators are cut and then drawing a curve through them. For example, generators $o'2'$ and $o'12'$ in the front view are cut at points b' and b'_1 which coincide with each other. The true length of $o'b'$ may be obtained by drawing a line through b' , parallel to the base and cutting $o'7'$ at b'' . Then $o'b''$ is the true length of $o'b'$.
- (vii) Mark points B and B_1 on generators $O2$ and $O-12$ respectively, such that $OB = OB_1 = o'b''$. Locate all points in the same way and draw a smooth curve through them. The figure enclosed this curve and the arc is the development of the truncated cone.

Problem 15-23. Draw the development of the lateral surface of the part P of the cone shown in fig. 15-25(i).

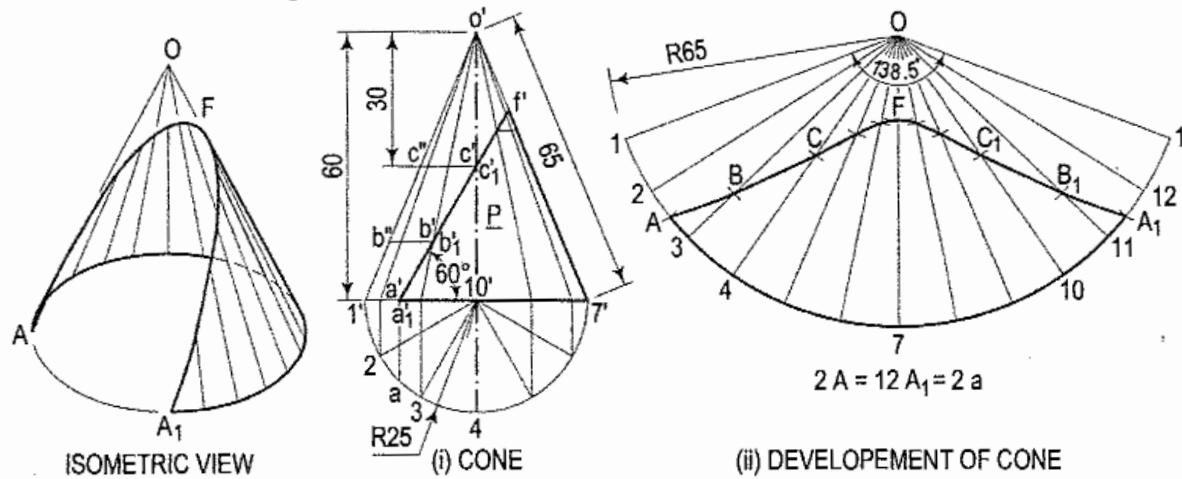


FIG. 15-25

Draw the development as explained in problem 15-22 [fig. 15-25(ii)]. For the points at which the base of the cone is cut, mark points A and A_1 on the arcs 2-3 and 11-12 respectively, such that $A_2 = A_1-12 = a_2$. Draw the curve passing through the points A, B, C etc. The figure enclosed between this curve and the arc $A-A_1$ is the required development.

Problem 15-24. Draw the development of the lateral surface of the part P of the cone shown in fig. 15-26(i).

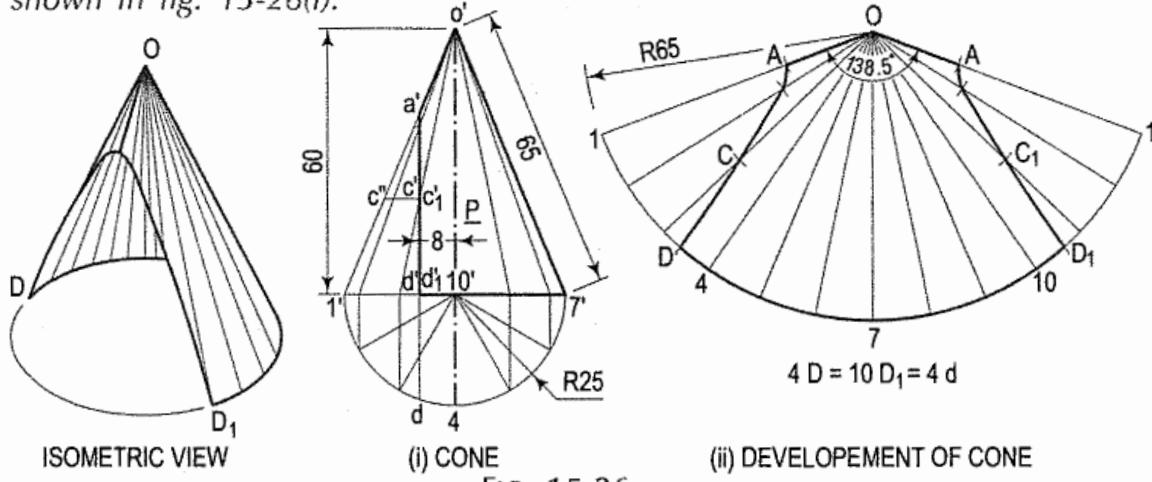


FIG. 15-26

Draw the development as explained in problem 15-23 and as shown in fig. 15-26(ii).

Problem 15-25. Draw the development of the lateral surface of the part P of the cone shown in fig. 15-27(i).

Draw the development of the lateral surface of the whole cone [fig. 15-27(ii)]. With O as centre and radius $o'q'$, draw the arc QQ cutting O_1 at Q.

Obtain the curve for the lower part as explained in problem 15-22. The figure enclosed between this curve and the arc QQ is the required development.

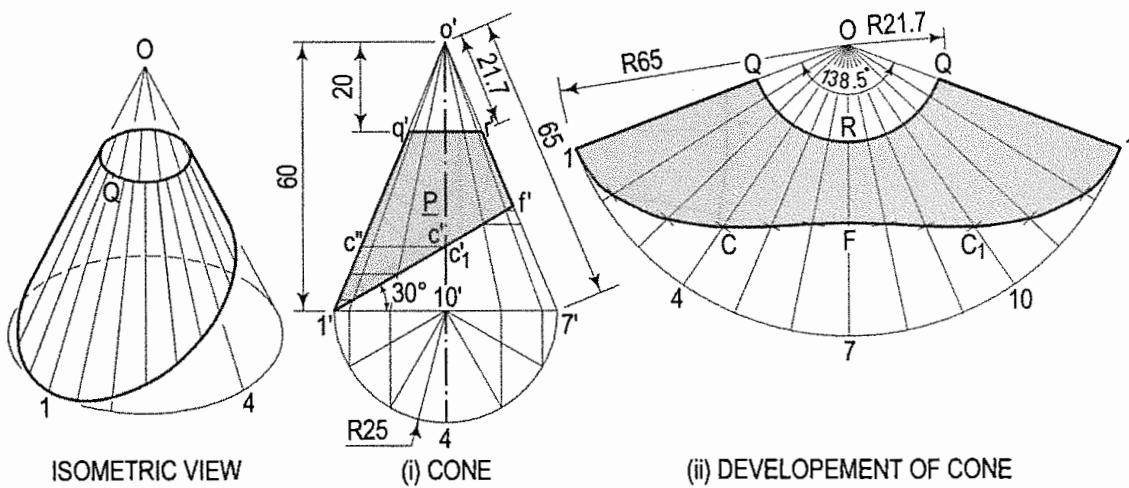


FIG. 15-27

Problem 15-26. Draw the projections of a cone resting on the ground on its base and show on them, the shortest path by which a point P, starting from a point on the circumference of the base and moving around the cone will return to the same point. Base of cone 61 mm diameter; axis 75 mm long.

- (i) Draw the projections and the development of the surface of the cone showing all twelve generators (fig. 15-28). The development may be drawn attached to $o'1'$.
- (ii) Assume that P starts from the point 1 (i.e. point $1'$ in the front view). Draw a straight line $1'1'$ on the development. This line shows the required shortest path.

To transfer this line to the front view the process adopted in problem 15-22 must be reversed. Let us take a point P_4 at which the path cuts the generator $o'4$. Mark a point P''_4 on $o'1'$ such that $o'P''_4 = o'P_4$. This can be done by drawing an arc with o' as centre and radius equal to $o'P_4$ cutting $o'1'$ at P''_4 . Through P''_4 , draw a line parallel to the base cutting $o'4'$ at P'_4 . Then p'_4 is the position of the point p_4 in the front view.

Similarly, transfer all the points to the front view and draw the required curve through them. The curve at the back will coincide with the front curve.

- (iii) Project these points to the top view on the respective generators. p'_4 and p'_{10} cannot be projected directly. Hence, project p''_4 to a point q on o_1 . With o as centre and radius equal to oq , draw an arc cutting o_4 at p_4 and o_{10} at p_{10} . Thus $op_4 = op_{10} = oq$. A curve drawn through the points thus obtained will show the path in the top view.

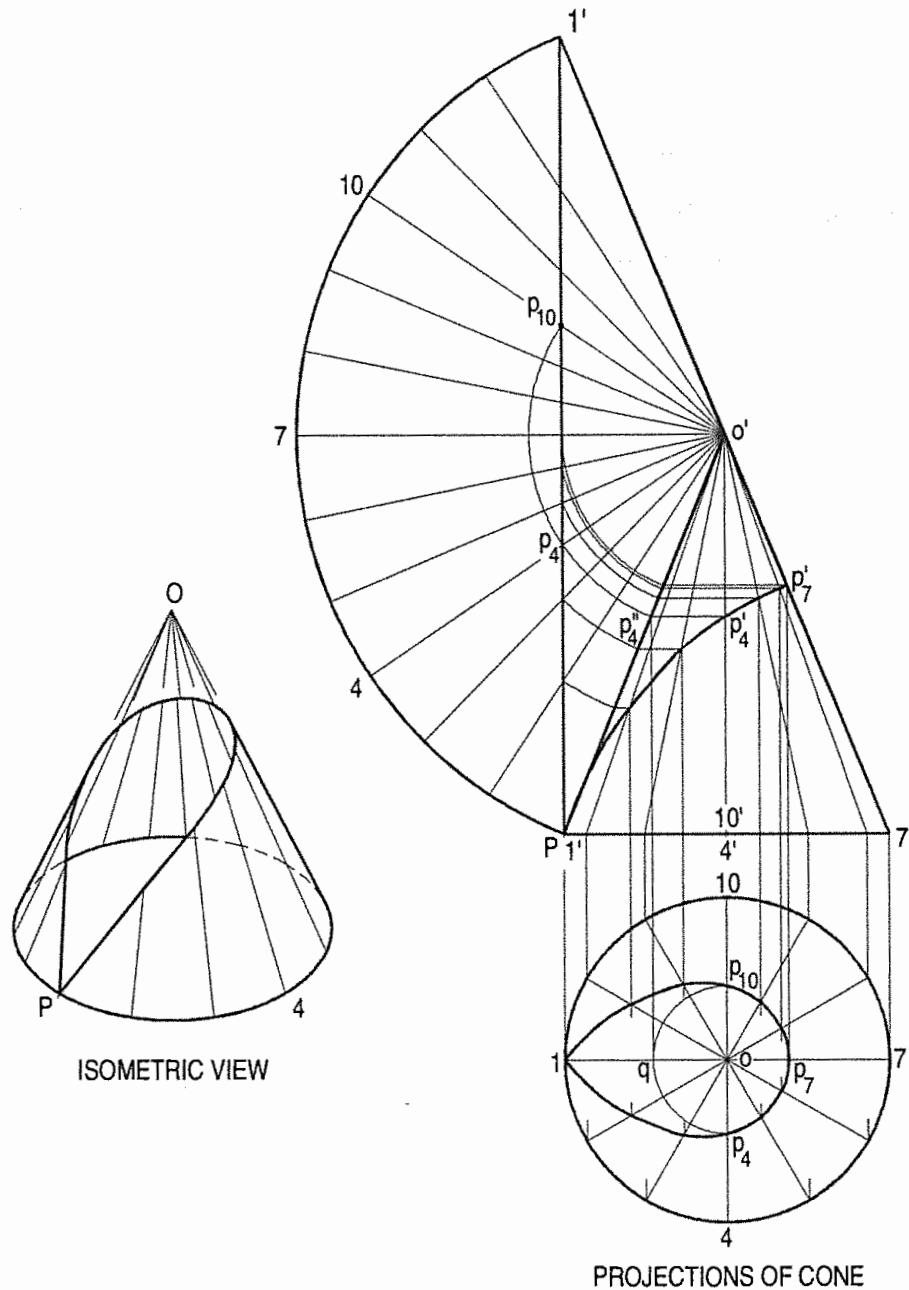


FIG. 15-28

Problem 15-27. (fig. 15-29): Draw the development of the lateral surface of the part of the cone, the front view of which is shown in fig. 15-29.

- Draw a semi-circle on the base as a diameter and divide it into six equal parts for positions of generators.
- Draw the development assuming the cone to be whole. Obtain points on the generators in the development as explained in problem 15-22. Additional points such as \$a'\$ may also be marked to determine the correct shape of the curve.
- Draw curves through these points and complete the development as shown.

Problem 15-28. (fig. 15-30): The development of the conical surface is a sector of the semi-circular plane. The radius of the sector is 75 mm and the angle subtended by the arc at the centre is \$120^\circ\$. Construct the cone and draw its projections when the apex is 35 mm above the H.P. and its axis is parallel to the V.P.

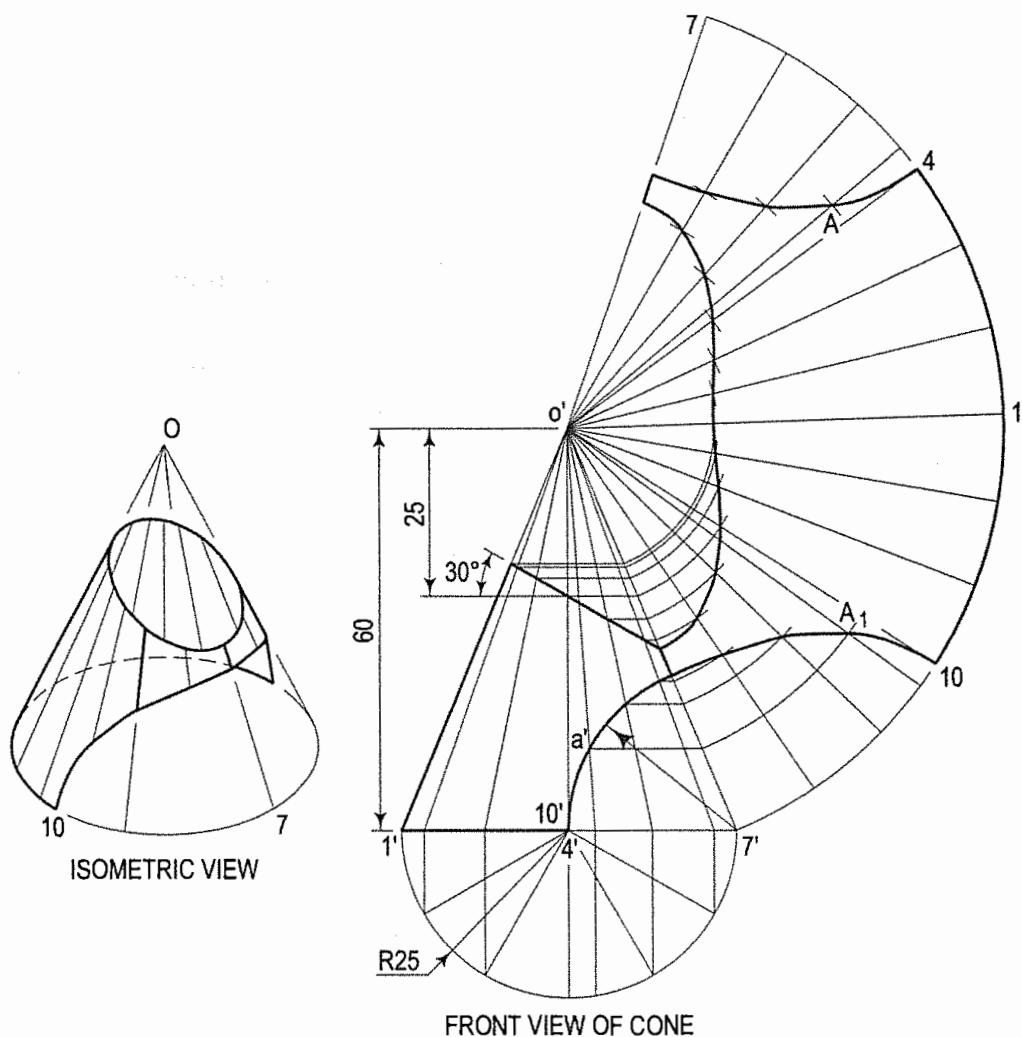
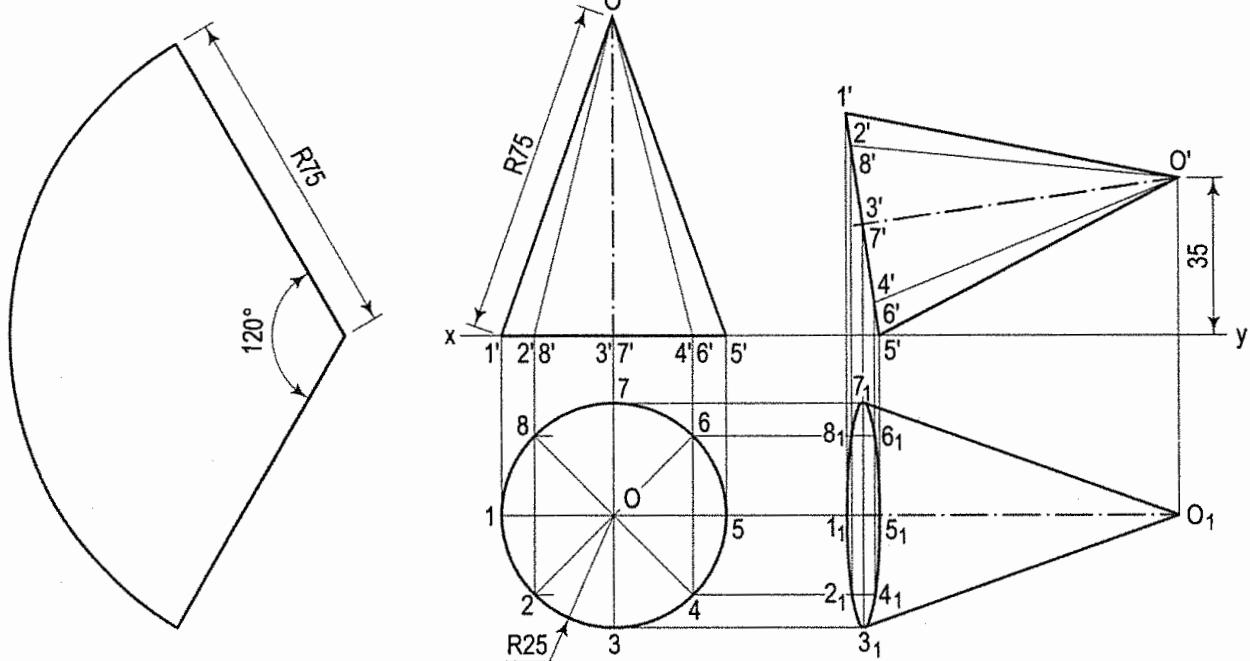


FIG. 15-29



(ii) CONE AND PROJECTIONS GENERATED FROM DEVELOPMENT OF CONE

FIG. 15-30

- (ii) Determine the radius of the base circle of the cone using following formula:

where S = The arc of circle

R – Radius of the arc

θ = Angle subtended

r = Radius of base circle of the cone

r = Radius of base circle of the cone.

Substitute $R = 75 \text{ mm}$, $\theta = \frac{\pi \times 120}{180} \text{ radian}$, we obtain

- (ii) Draw the circle of radius of 25 mm representing the top view of the cone.
 - (iii) Project the front view with the height equal to 75 mm.
 - (iv) Tilt the front view such that the apex is 35 mm above xy line.
 - (v) Complete the projection as shown in fig. 15-30.

Problem 15-29. Draw the shape of the sheet metal required for the funnel shown in fig. 15-31.

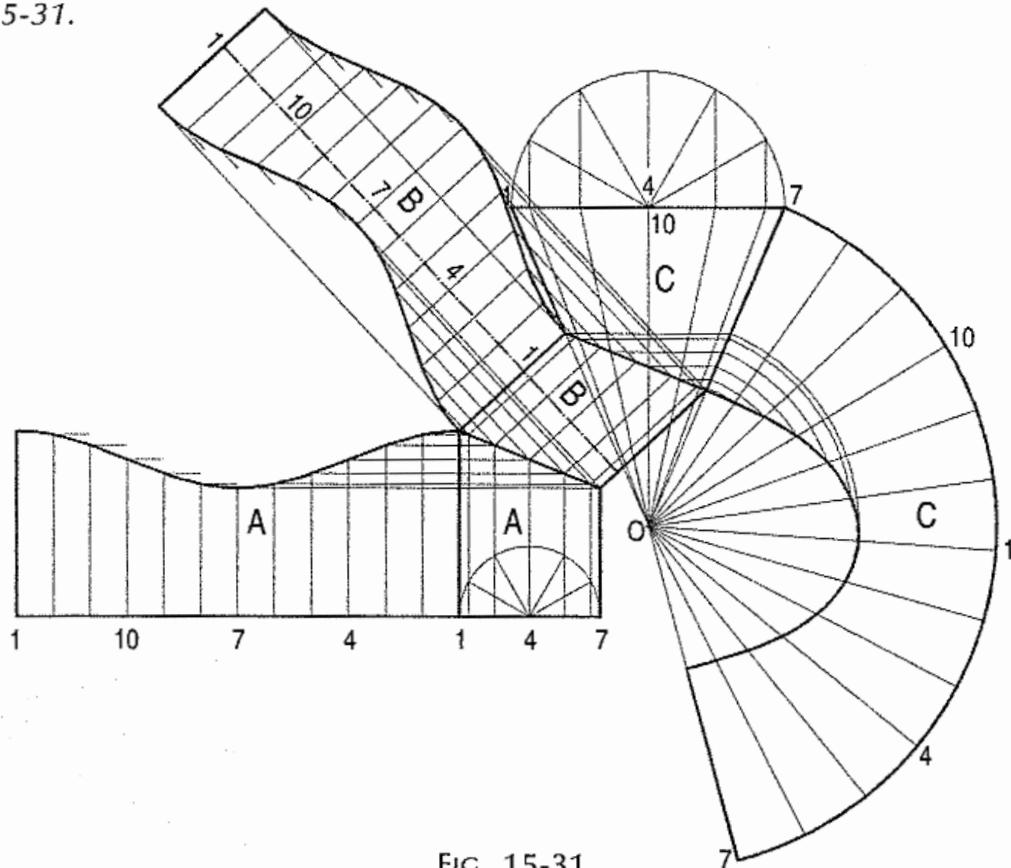


FIG. 15-31

The funnel consists of a conical part C and two cylindrical pipes A and B, all truncated. Draw the developments of the pipes as shown in problem 15-8, and that of the conical piece, as shown in problem 15-22. These are the required shapes.

Problem 15-30. The projections of a solid composed of a truncated half-cylinder and a cut half-prism are given in fig. 15-32(i). Draw the development of its lateral surface.

Assuming the solid to be whole; draw the development of its surface [fig. 15-32(ii)].

Draw a stretch-out line and on it, step-off

- (i) 1A equal to the arc 1a

- (ii) AB , BC and CD each equal to the edge ab of the base, and

- (iii) D_1 equal to the arc d_1 . Complete the rectangle.

Draw perpendiculars at A , B etc. and at other intermediate points. Locate on them, positions of points at which they are cut and draw the curves and straight lines as shown.

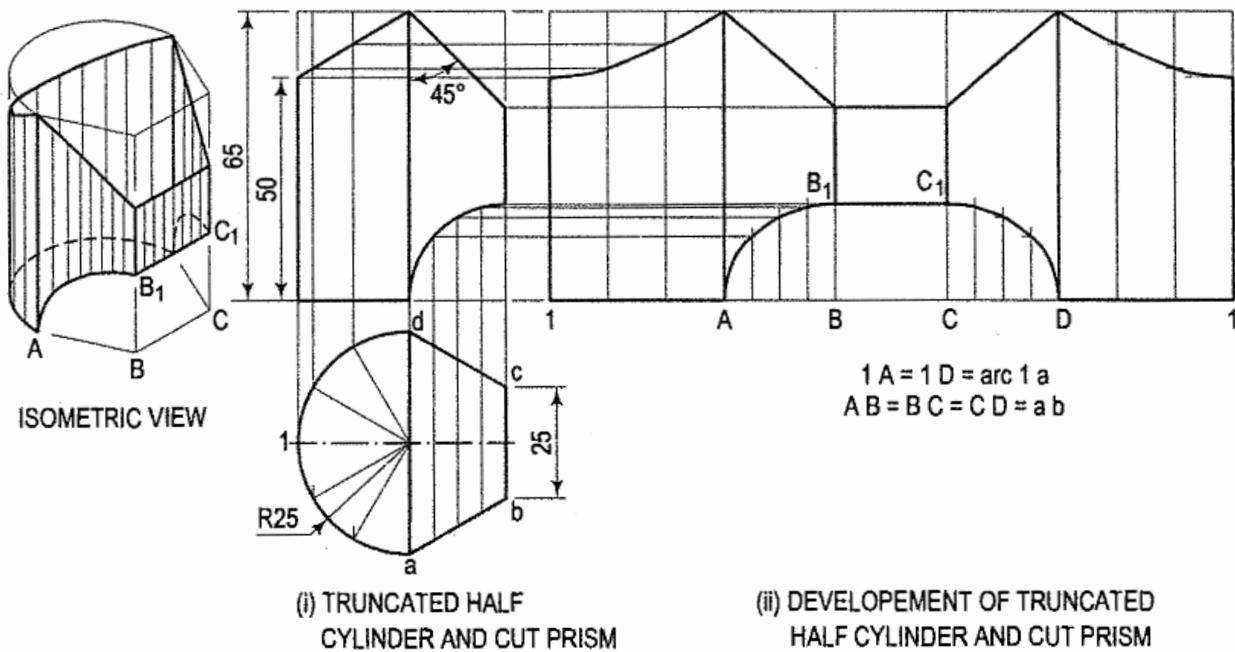


FIG. 15-32

Problem 15-31. Draw the development of the lateral surface of the solid shown in two views (drawn in first-angle projection) in fig. 15-33(i).

The solid is made-up of portions of frusta of a cone and a hexagonal pyramid.

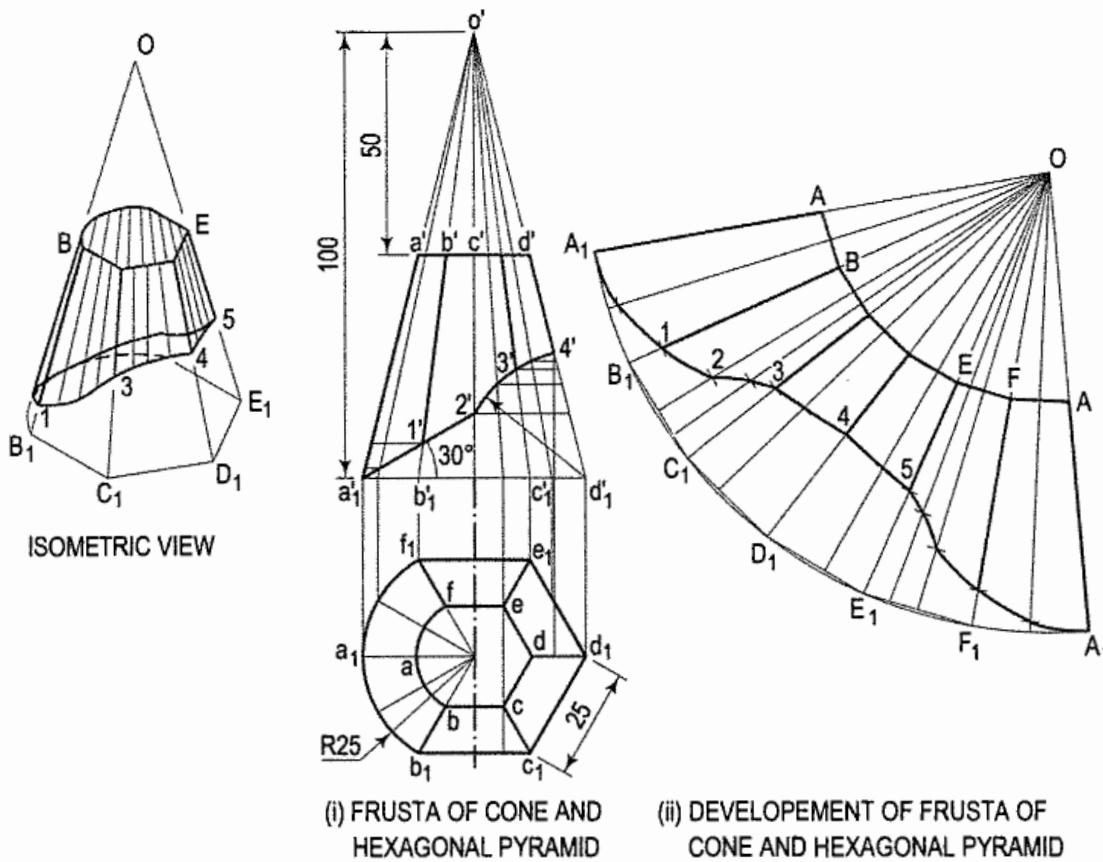


FIG. 15-33

The slant height of the cone is equal to the slant edge of the pyramid.

- Therefore, with any point O as centre and radii $o'a'$ and $o'a'_1$, draw arcs [fig. 15-33(ii)].
- On the outer arc, step-off distances (i) A_1B_1 equal to the arc a_1b_1 , (ii) B_1C_1 , C_1D_1 , D_1E_1 and E_1F_1 , each equal to the side of the base, viz. b_1c_1 and (iii) F_1A_1 equal to the arc f_1a_1 .
- Join these points with O , cutting the inner arc at points A , B etc. Locate positions of various points as explained in problems 15-18 and 15-22 and complete the development as shown.

15-3. DEVELOPMENT OF TRANSITION PIECES



Pipes are used in many industries to convey hot or cold fluids. When two different sizes and shapes of pipes are joined using special pipe joint which is known as transition piece. In most cases, transition pieces are composed of plane surfaces and conical surfaces, the latter being developed by triangulation. The procedure of development of few transition pieces is illustrated in the following problems.

Problem 15-32. In air-conditioning system a rectangular duct of $100 \text{ mm} \times 50 \text{ mm}$ connects another rectangular duct of $50 \text{ mm} \times 25 \text{ mm}$ through the transition piece as shown in fig. 15-34(i). Neglecting thickness of a metal sheet, develop the lateral surface of the transition piece as shown in fig. 15-34(ii).

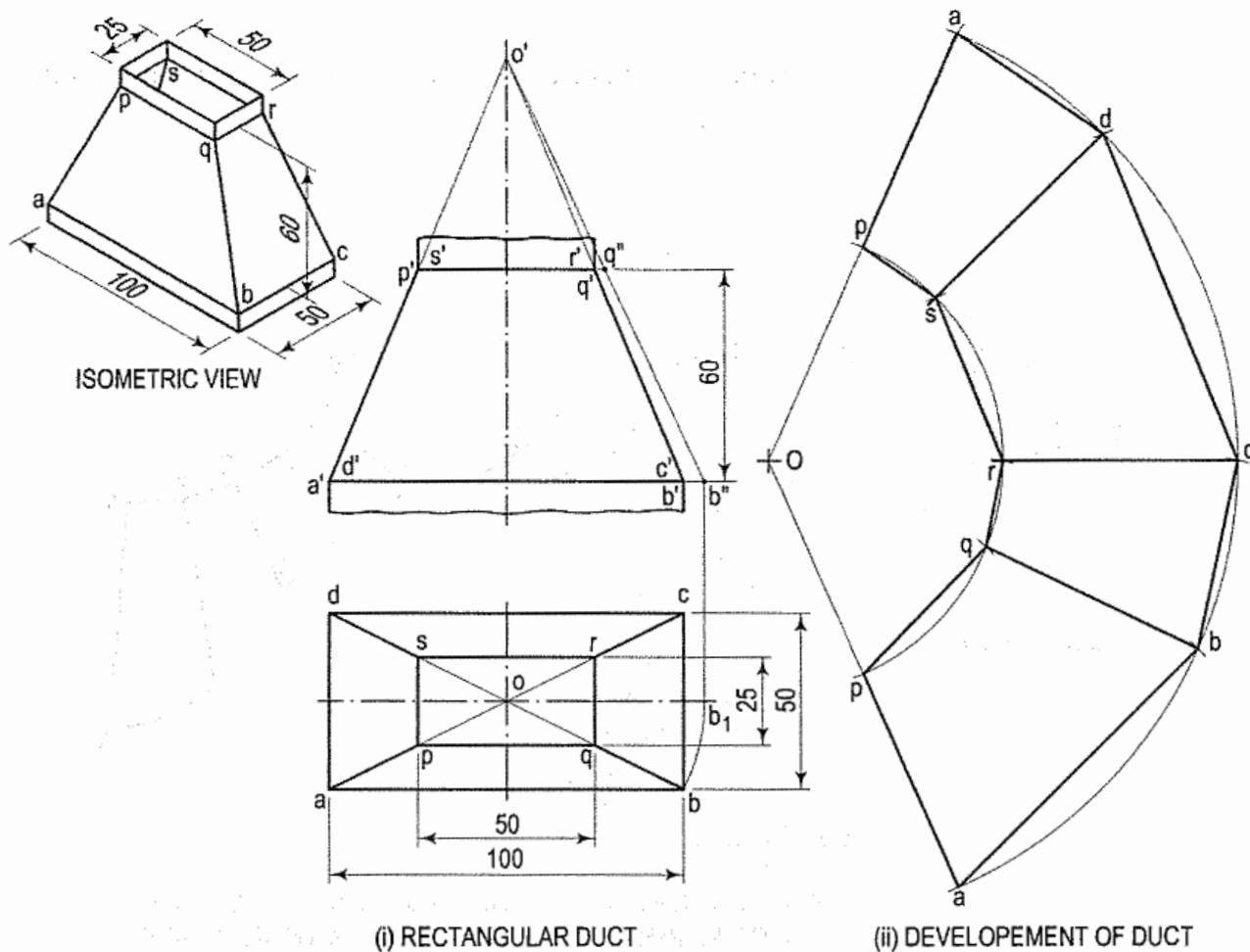


FIG. 15-34

The transition piece is a frustum of a rectangular pyramid.

- Determine the position of the apex of the pyramid by extending $a'p'$ and $b'q'$ as shown. None of the lines in the front view shows the true length of the slant edge. Therefore, draw the top view and make any slant line parallel to xy and determine its true length $o'b''$.
- With O as centre and radius $o'b''$, draw an arc and obtain the development of a whole pyramid as shown.
- With O as centre and radius $o'q''$, draw an arc cutting oa , ob , oc at points p , q , etc. respectively. Join them in sequence and complete the development as shown.

Problem 15-33. (fig. 15-35): An air-conditioning duct of a square cross-section 70 mm \times 70 mm connects a circular pipe of 40 mm diameter through the transition piece. Draw the projections and develop the lateral surface of the transition piece.

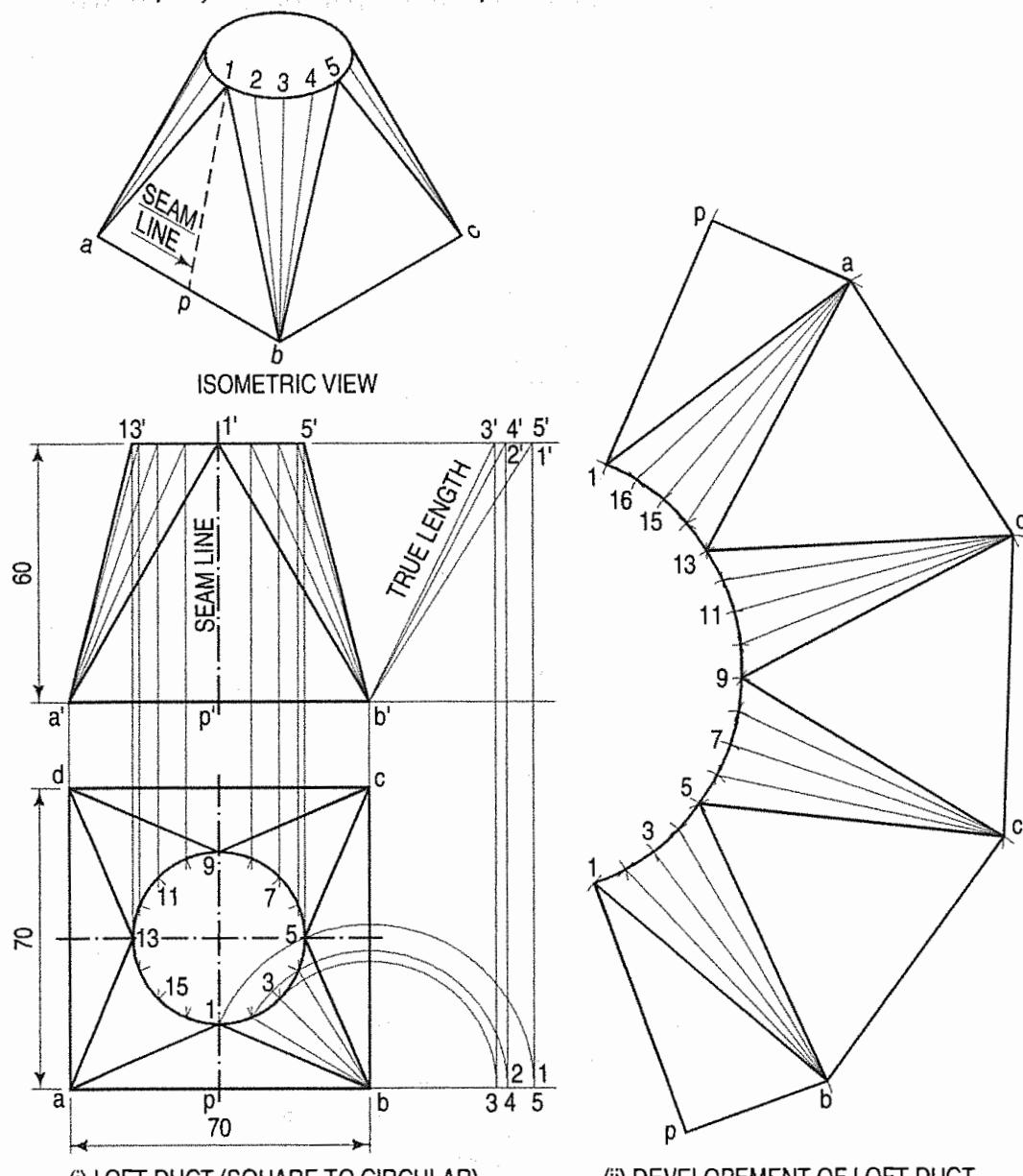


FIG. 15-35

- Draw the front view and the top view as shown in fig. 15-35(i).
- Divide the top view of circle into some convenient divisions, say 16 parts as shown.

- (iii) Note that the transition piece is composed of four isosceles triangles and four conical surfaces. The seam is along line 1-P.
- (iv) Begin the development from the seam line 1-P (1'-P'). As shown in fig. 15-35(ii) draw the right angle triangle 1-P-b, whose base pb is equal to half the side ab and whose hypotenuse 1-b is equal to the true length 1'-b' of side 1-b.
- (v) The conical surfaces are developed by the triangulation method as follows.
- (vi) In the top view, join division of the circle 1, 2, 3 etc. with the corner a, b, c and d. Project them in the front view as shown. Obtain the true length of sides of each triangle as shown.
- (vii) With b as centre and 2'b' (true length) radius draw an arc, cutting the arc drawn with 1' as centre and 1'2' as radius. Similarly, obtain the points 3', 4', 5' etc. Join them in the proper order as shown.

Problem 15-34. A steam pipe bends at certain angle is connected by a transition piece as shown in fig. 15-36(i). Draw the development of lateral surface of the transition piece A.

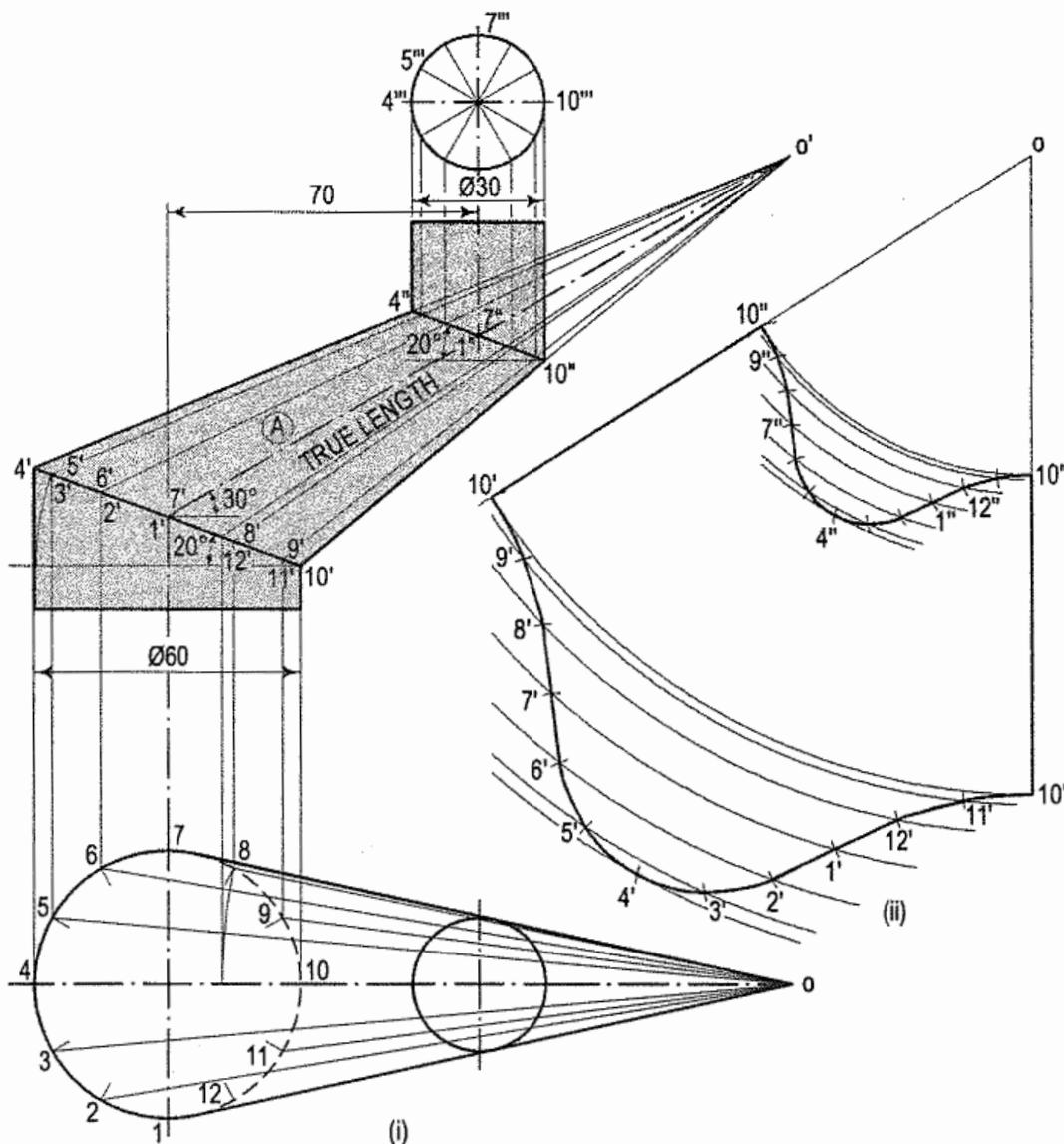


FIG. 15-36

- (i) Draw the top view and the front view. The transition piece is a truncated cone. The apex of the cone is determined by extending extreme generators 4' 4" and 10' 10". The projections of the base and the top of truncated cone are inclined to xy , therefore in the top view their projections do not show the true shapes.
- (The true-shape is an ellipse.) For approximate method of development, these projections in the top view can be taken as the true shape.
- (ii) Draw the projections and the development of the cone showing all generators. Obtain the true length of generators.
- (iii) With O as centre and radius O10', draw an arc of circle. Similarly draw the arcs of circle taking radii O9', O8' etc.
- (iv) With 10' as centre and radius equal to 4'-3" (a division of base circle in the top view), draw an arc cutting the previously drawn arc. Similarly obtain points 8', 7', 6' etc. Join them by smooth curve. Obtain the development of the top circle similar way by taking 4"-5" radius (a division of top circle).

Problem 15-35. The orthographic projections of an exhaust pipe required for an engine is shown in fig. 15-37. Draw development of the transition connector by triangulation method.

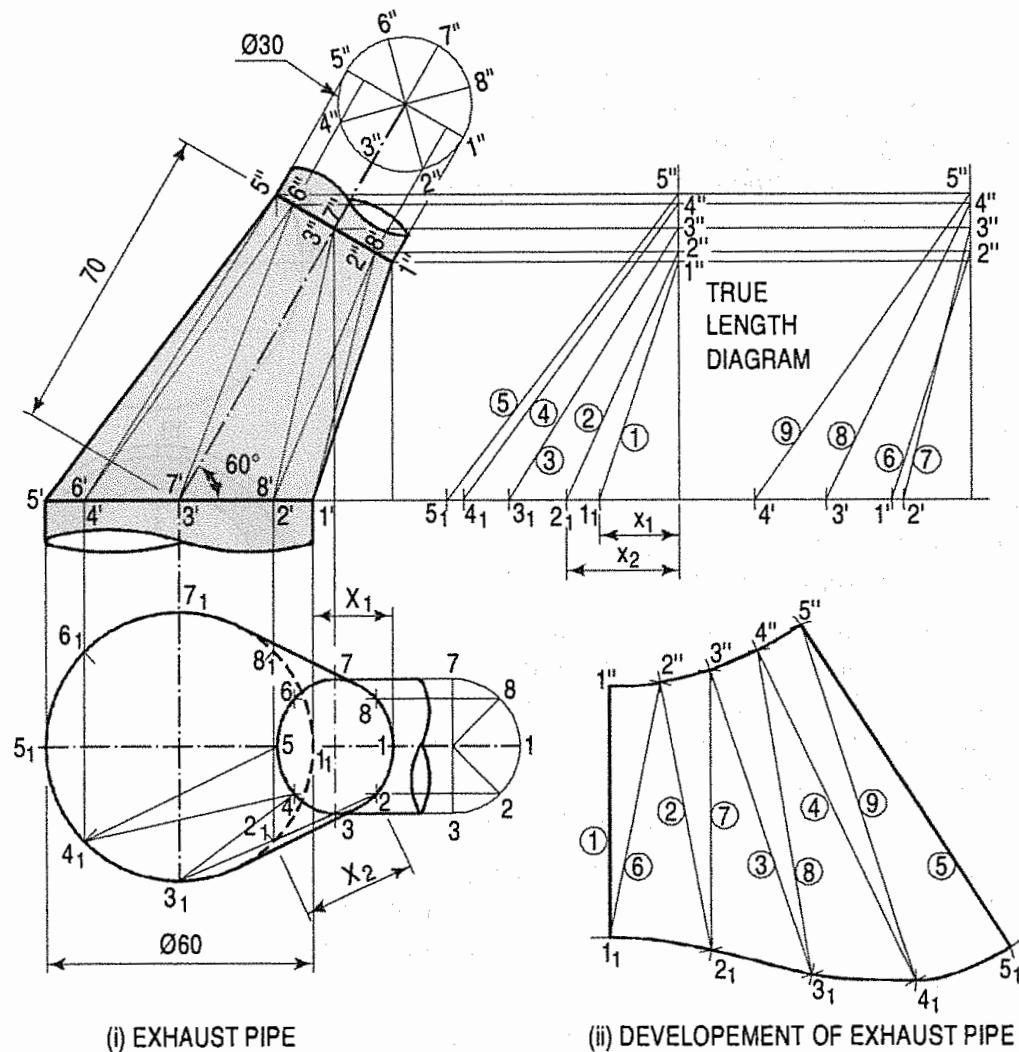


FIG. 15-37

- (i) Draw the given projections. Divide the circle of base in the top view into eight equal parts, say $1_1, 2_1 \dots$ etc. Project them in the front view $1', 2' \dots$ etc. as shown.
- (ii) Draw an auxiliary view of the top pipe as shown and divide it into eight equal parts. Number them $1'', 2'' \dots$ etc.
- (iii) Form the triangles on the lateral surface as shown.
- (iv) Determine true length of each side of a triangle as shown.
- (v) Mark the length of $1''1_1$. With 1 as centre and radius equal to $1'2''$ [line-(6)] draw an arc cutting the arc drawn with radius $1''2''$ (division of auxiliarly circle) and the centre as $1''$. A half development is shown.

15-4. SPHERES (APPROXIMATE METHOD)

The surface of a sphere can be approximately developed by dividing it into a number of parts. The divisions may be made in two different ways:

- (i) in zones (ii) in lunes.

A *zone* is a portion of the sphere enclosed between two planes perpendicular to the axis. A *lune* is the portion between two planes which contain the axis of the sphere.

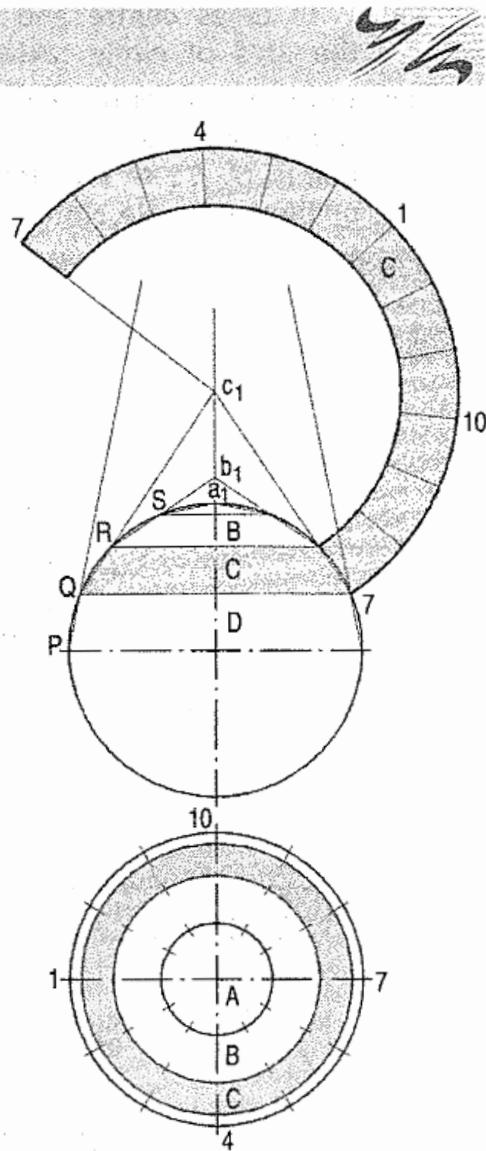
(1) **Zone method:** Fig. 15-38 shows the top half of a sphere divided into four zones of equal width. By joining the points P, Q, R etc. by straight lines, each zone becomes a cone frustum, except the upper-most zone which becomes a cone of small altitude.

Developments of these cone frusta and the upper cone will give the development of the half sphere. For example, take the zone C . It is a frustum of a cone whose vertex is at c_1 . The surface of this frustum is shown developed in the front view. The length of the divisions on the arc is obtained from the top view. All the zones can be developed in the same manner.

(2) **Lune method:** A sphere may be divided into twelve lunes, one of which is shown in the front view in fig. 15-39. The semi-circle qr is the top view of the centre line of that lune. It is evident that the length of the lune is equal to the length of the arc qr and its maximum width is equal to gh .

Divide the semi-circle into a number of equal parts say 8 and project the division-points on the front view to points $1', 2'$ etc. With q' as centre and radii equal to $q'1', q'2'$ and $q'3'$, draw arcs ab, cd and ef which will show the widths of the lune at points 1 and 7, 2 and 6, and 3 and 5 respectively.

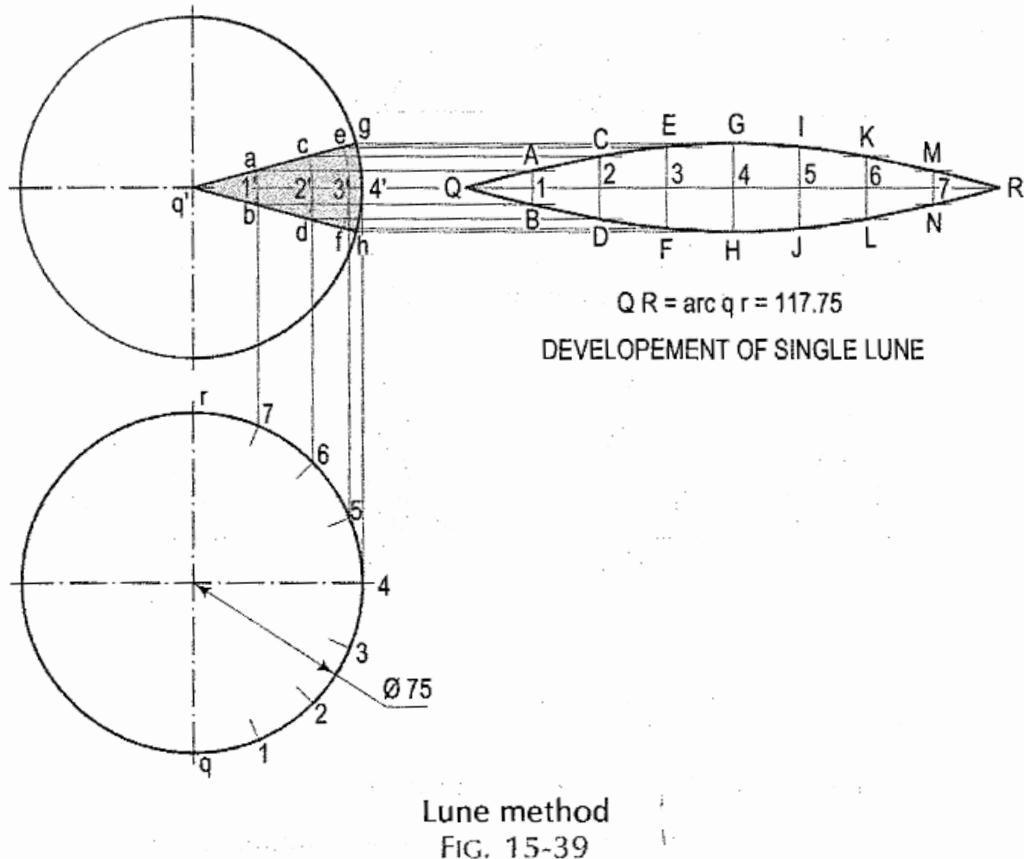
Draw a line QR equal to the length of the arc qr . This may be obtained by stepping-off eight divisions, each equal to the chord-length $q1$.



Zone method

FIG. 15-38

Draw perpendiculars at each division-point and make AB and MN equal to ab at points 1 and 7, CD and KL equal to cd at points 2 and 6 etc. Draw smooth curves through points Q, A, C etc. The figure thus obtained will be the approximate development of one-twelfth of the surface of the sphere. Development of surfaces of some more solids cut by different planes, and solids with holes cut or drilled through them are treated in chapters 14 and 16.



EXERCISES 15

1. Draw the development of the lateral surface of the part P of each of the solids, the front views of which are shown in fig. 15-40 and described below.

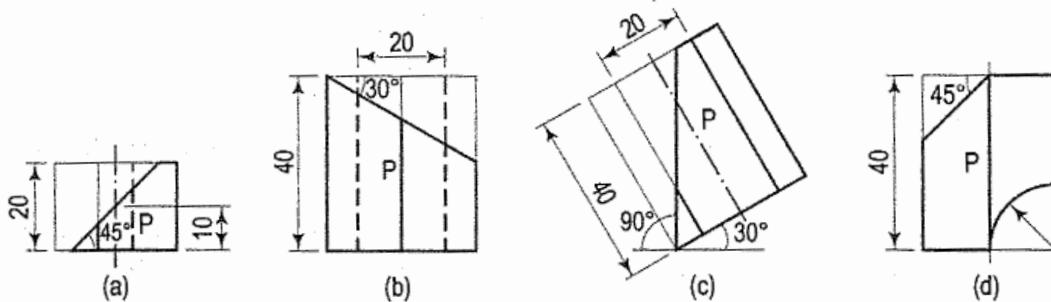


FIG. 15-40

- (a) A cube, one vertical face inclined at 30° to the V.P.
- (b) A pentagonal prism, a side of the base parallel to the V.P.
- (c) A hexagonal prism, two faces parallel to the V.P.
- (d) A square prism, length of the side of the base 20 mm and all faces equally inclined to the V.P.

2. Draw the development of the lateral surface of the part P of each of the cylinders, the front views of which are shown in fig. 15-41.

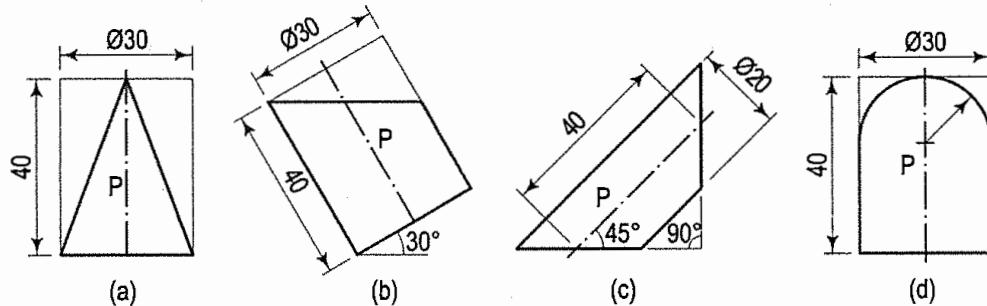


FIG. 15-41

3. Draw the development of the lateral surface of the part P of each of the pyramids, the front views of which are shown in fig. 15-42, and described below.

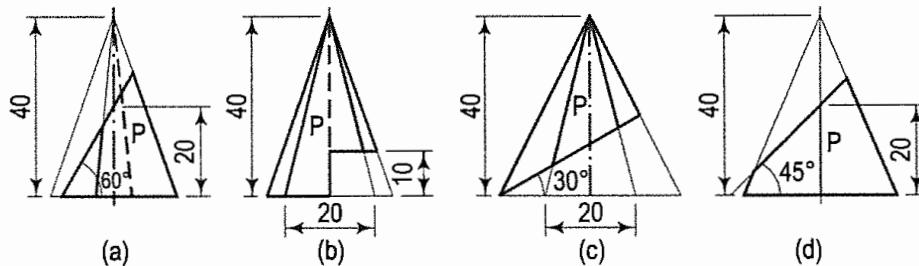


FIG. 15-42

- (a) A square pyramid, side of the base 20 mm long and one side of the base inclined at 30° to the V.P.
 - (b) A pentagonal pyramid, one side of the base parallel to the V.P.
 - (c) A hexagonal pyramid, two sides of the base parallel to the V.P.
 - (d) A square pyramid, side of the base 20 mm long and all the sides of the base equally inclined to the V.P.
4. Draw the development of the lateral surface of the part P of each of the cones, front views of which are shown in fig. 15-43.

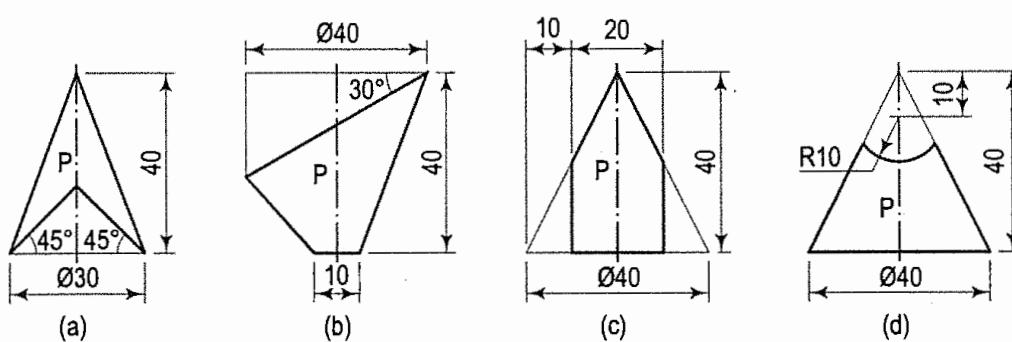


FIG. 15-43

Refer to fig. 15-44 for the following exercises and for dimensions, assume each square to be of 10 mm side.

5. Draw the development of the surfaces of the portions of the following prisms, front views of which are shown in the top row:

- (a) A hexagonal prism having a face parallel to the V.P.
- (b) A square prism, all faces equally inclined to the V.P.
- (c) A pentagonal prism having a vertical face parallel to the V.P.
- (d) A triangular prism having a vertical face parallel to the V.P.
- (e) A hexagonal prism having two faces perpendicular to the V.P.

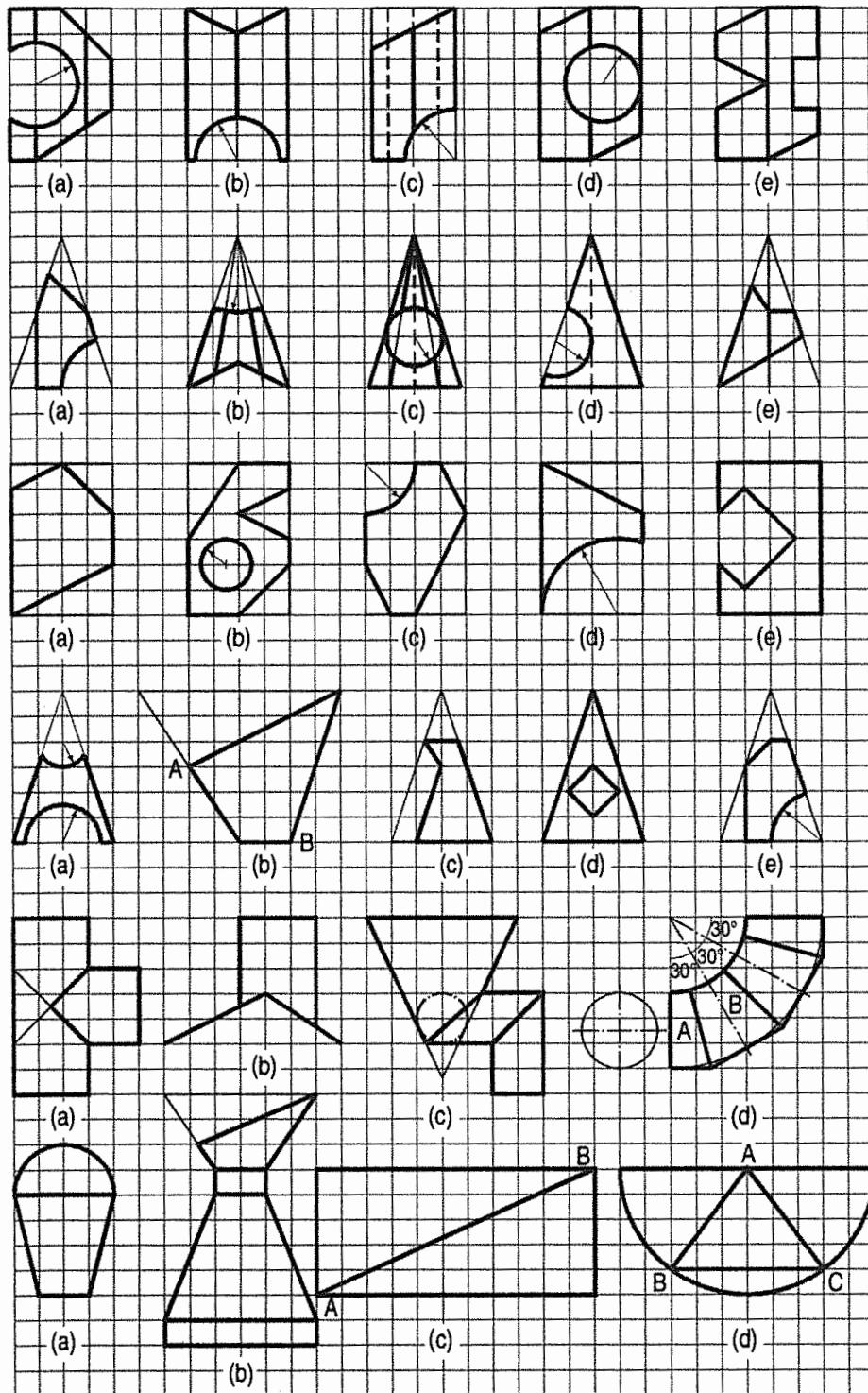


FIG. 15-44

6. Draw the development of the surfaces of the portions of the following pyramids, front views of which are shown in the second row:

- (a) A square pyramid having a side of base perpendicular to the V.P.
 (b) A hexagonal pyramid having a side of base parallel to the V.P.
 (c) A pentagonal pyramid having a side of base parallel to the V.P.
 (d) A triangular pyramid having a side of base parallel to the V.P.
 (e) A square pyramid having all sides of the base equally inclined to the V.P.
7. Draw the development of the surfaces of the portions of the cylinders shown in the third row.
8. Draw the development of the surfaces of the portions of the cones shown in the fourth row.
9. Refer to the fifth row, and
- Draw the development of the pipes forming a Tee shown at (a).
 - Draw the development of the cylindrical steel chimney erected on a roof [shown at (b)], assuming the squares to be of 30 cm side.
 - Draw the development of the three parts of the funnel shown at (c).
 - Develop parts A and B of the transition piece shown at (d).
10. Refer to the last row, and
- Develop the surface of the conical buoy with a hemispherical top shown at (a).
 - Determine the shape of the tin sheet required to prepare the can shown at (b).
 - The development of the surface of a cylinder is given at (c). Draw the front view of the cylinder showing the line AB in it.
 - The development of the surface of a cone is shown at (d). Draw the projections of the cone showing lines AB, BC and CA in each view.
11. A pipe 40 mm diameter and 120 mm long (along the axis) is welded to the vertical side of a tank. Show the development of the pipe, if it makes an angle of 60° with the side to which it is welded, the other end of the pipe making an angle of 30° with its own axis. Neglect thickness of the pipe.
12. The inside of a hopper of a floor mill is to be lined with tin sheet. The top and bottom of the hopper are regular pentagons with each side equal to 450 mm and 300 mm respectively (internally). The height of the hopper is 450 mm. Draw the shape to which the tin sheet is to be cut so as to fit in the hopper. Scale, 1:10.
13. A 50 mm cylindrical pipe branches off at 90° from a 75 mm cylindrical main pipe as shown in fig. 15-45. Draw the developments of both the pipes at the joint. Assume suitable lengths for the main pipe as well as for the branch pipe.
14. A cone of 90 mm diameter of base and 90 mm height stands on its base on the ground. A semi-circular hole of 50 mm diameter is cut through the cone. The axis of the hole is horizontal and intersects the axis of the cone. It is 30 mm above the base of the cone. The flat surface of the hole contains the axis of the cone and is perpendicular to the V.P. Draw three views of the cone and also develop the surface of the cone.

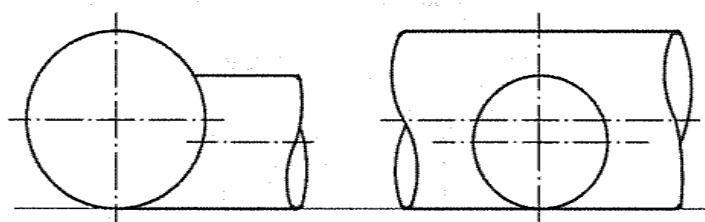


FIG. 15-45

Chapter

16



INTERSECTION OF SURFACES

16-0. INTRODUCTION

The intersecting surfaces may be two plane surfaces or two curved surfaces of solids. The lateral surface of every solid taken as a whole is a curved surface. This surface may be made of only curved surface as in case of cylinders, cones etc. or of plane surfaces as in case of prisms, pyramids etc. In the former case, the problem is said to be on the intersection of surfaces and in the latter case, it is commonly known as the problem on interpenetration of solids. It may, however, be noted that when two solids meet or join or interpenetrate, it is the curved surfaces of the two that intersect each other. The latter problem also is, therefore, on the intersection of surfaces.

In this chapter, we shall learn about the intersection of surfaces as shown below:

1. Line of intersection
2. Methods of determining the line of intersection between surfaces of two interpenetrating solids
3. Intersection of two prisms
4. Intersection of cylinder and cylinder
5. Intersection of cylinder and prism
6. Intersection of cone and cylinder
7. Intersection of cone and prism
8. Intersection of cone and cone
9. Intersection of sphere and cylinder or prism.

16-1. LINE OF INTERSECTION



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 36 for the line of intersection.

In engineering practice, objects constructed may have constituent parts, the surfaces of which intersect one another in lines which are called *lines of intersection*. A dome fitted on a boiler is one such example. The surface of the dome extends upto the line of intersection only. For accurate development of the surface of the dome, this line of intersection must be accurately located and shown in two orthographic views. The shape of the hole to be cut in the boiler-shell is also determined from the shape of the same line of intersection.

Thus, the line of intersection of the two surfaces is a line common to both. It is composed of points at which the lines of one surface intersect those on the other surface. The line of intersection may be straight or curved, depending upon the nature of intersecting surfaces.

Two plane surfaces (e.g. faces of prisms and pyramids) intersect in a straight line. The line of intersection between two curved surfaces (e.g. of cylinders and cones) or between a plane surface and a curved surface is a curve.

When a solid completely penetrates another solid, there will be two lines of intersection. These lines are, sometimes, called *the lines or curves of interpenetration*. The portion of the penetrating solid which lies hidden within the other solid is shown by dotted lines.

16-2. METHODS OF DETERMINING THE LINE OF INTERSECTION BETWEEN SURFACES OF TWO INTERPENETRATING SOLIDS



(1) **Line method:** A number of lines are drawn on the lateral surface of one of the solids and in the region of the line of intersection. Points of intersection of these lines with the surface of the other solid are then located. These points will obviously lie on the required line of intersection. They are more easily located from the view in which the lateral surface of the second solid appears edgewise (i.e. as a line). The curve drawn through these points will be the line of intersection.

(2) **Cutting-plane method:** The two solids are assumed to be cut by a series of cutting planes. The cutting planes may be vertical (i.e. perpendicular to the H.P.), edgewise (i.e. perpendicular to the V.P.) or oblique. The cutting planes are so selected as to cut the surface of one of the solids in straight lines and that of the other in straight lines or circles.

Each method is explained in detail while solving illustrative problems. Sound knowledge of projections of solids in various positions is quite essential while dealing with these problems.

16-3. INTERSECTION OF TWO PRISMS

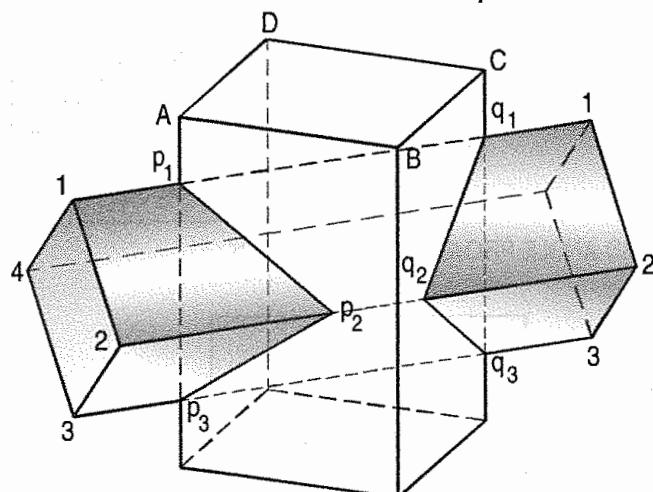


Prisms have plane surfaces as their faces. The line of intersection between two plane surfaces is obtained by locating the positions of points at which the edges of one surface intersect the other surface and then joining the points by a straight line. These points are called *vertices* (plural of vertex). The line of intersection between two prisms is therefore a closed figure composed of a number of such lines meeting at the vertices.

It is determined by locating the points at which edges of one prism intersect edges or faces of the other prism and then joining them in correct sequence.

Problem 16-1. (fig. 16-1): A vertical square prism, base 50 mm side, is completely penetrated by a horizontal square prism, base 35 mm side, so that their axes intersect. The axis of the horizontal prism is parallel to the V.P., while the faces of the two prisms are equally inclined to the V.P. Draw the projections of the solids, showing lines of intersection. (Assume suitable lengths for the prisms.)

- (i) Draw the projections of the prisms in the required position. The faces of the vertical prism are seen as lines in the top view. Hence, let us first locate the points of intersection in that view.
- (ii) Lines 1-1 and 3-3 intersect the edge of the vertical prism in points p_1 and p_3 (coinciding with a). Lines 2-2 and 4-4 intersect the faces at p_2 and p_4 respectively. The exact positions of these points along the length of the prism may now be determined by projecting them on corresponding lines in the front view. For example, p_2 is projected to p'_2 on the line 2'2'. Note that p'_4 coincides with p'_2 .
- (iii) Draw lines $p'_1 p'_2$ and $p'_2 p'_3$. Lines $p'_1 p'_4$ and $p'_3 p'_4$ coincide with the front lines. These lines show the line of intersection. Lines $q'_1 q'_2$ and $q'_2 q'_3$ on the other side are obtained in the same manner. Note that the lines for the hidden portion of the edges are shown as dashed lines. The portions $p'_1 p'_3$ and $q'_1 q'_3$ of vertical edges $a'a'$ and $c'c'$ do not exist and hence, must be removed or kept fainter.



(i)

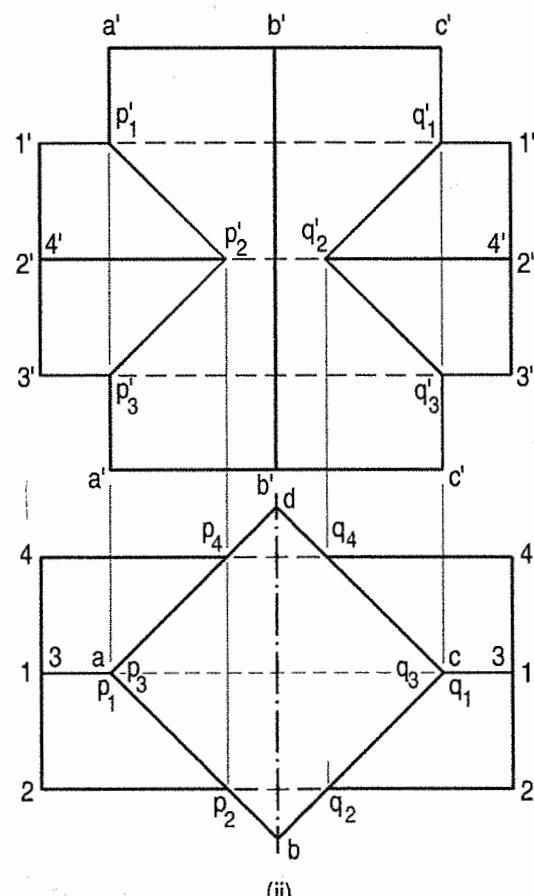


FIG. 16-1

Problem 16-2. (fig. 16-2): A vertical square prism, base 50 mm side is completely penetrated by a horizontal square prism, base 35 mm side so that their axes are 6 mm apart. The axis of the horizontal prism is parallel to the V.P., while the faces of both prisms are equally inclined to the V.P. Draw the projections of the prisms showing lines of intersection.

Points $p'_1 \dots p'_4$ at which edges of the horizontal prism intersect faces of the vertical prism may be located from the top view. In addition to these points, it will be necessary to find points at which edges of the vertical prism are cut. They will be the points at which these edges intersect the faces of the horizontal prism.

- (i) For this purpose, draw the side view. In this view, all faces of the horizontal prism are seen as lines. Mark points e and f at which the line $a''a'$ intersects the faces. Project these two points to e' and f' on the line $a'a'$ in the front view. Join all the points of intersection in correct sequence. Care must be taken to determine visible and hidden lines. Only two lines viz. $p'_1 p'_2$ and $p'_2 p'_3$ are visible.

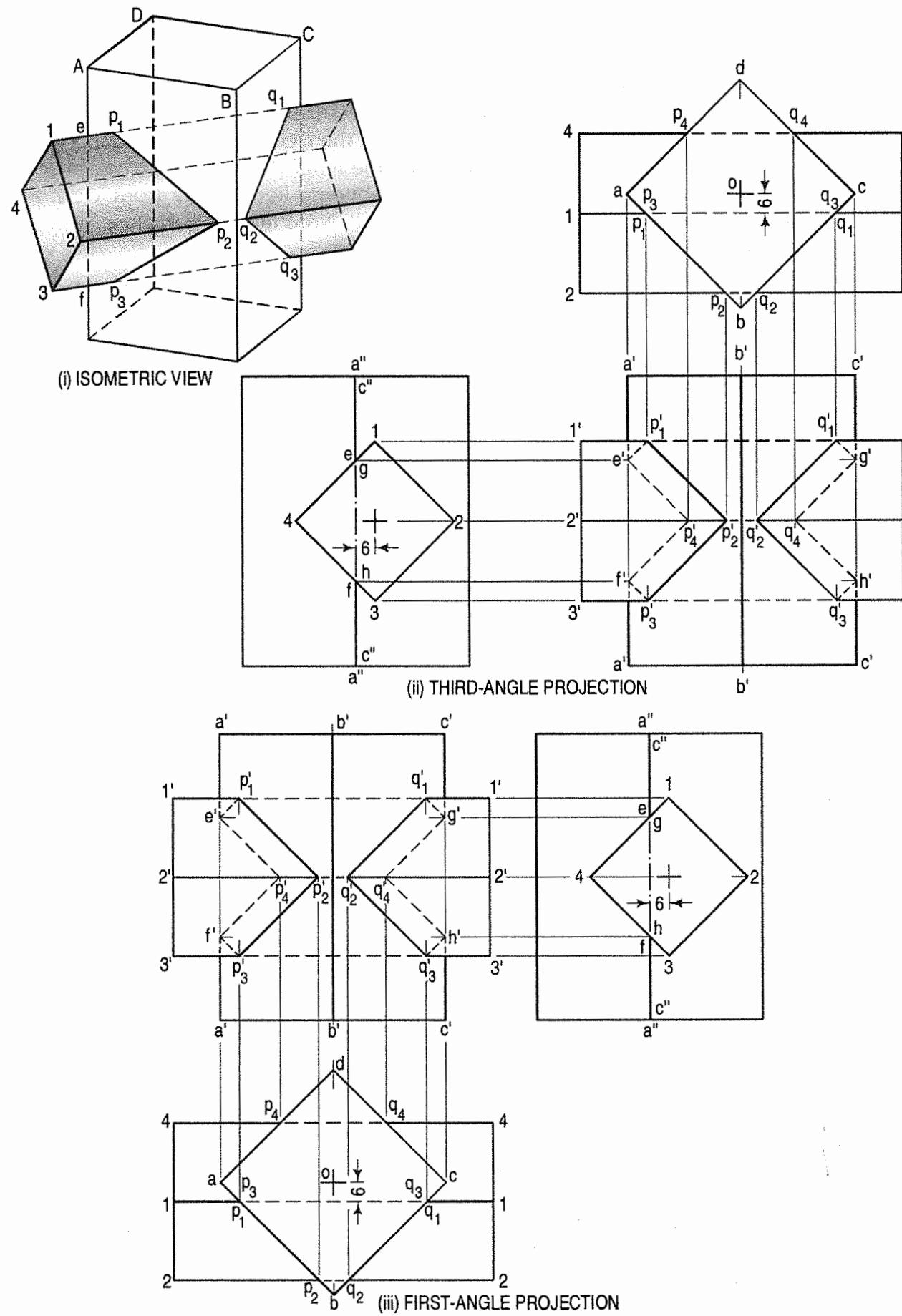


FIG. 16-2

- (ii) Locate points (on the other side) at which the edges come out and also the two points g' and h' at which the edge $c'c'$ is cut.
- (iii) Draw lines joining these points. They will be exactly similar to lines $p'_1 p'_2$ etc. on the left-hand side.

Problem 16-3. (fig. 16-3): A vertical square prism, base 50 mm side and height 90 mm has a face inclined at 30° to the V.P. It is completely penetrated by another square prism, base 38 mm side and axis 100 mm long, faces of which are equally inclined to the V.P. The axes of the two prisms are parallel to the V.P. and bisect each other at right angles. Draw the projections showing lines of intersection.

Adopt the same method as explained in problem 16-2.

- (i) The edges 1-1 and 3-3 enter one face of the vertical prism and come out of its opposite face.
- (ii) Obtain the points (from the top view) at which all edges intersect the faces and also the four key points (from the side view).

Note carefully the lines for visible and hidden edges, shown as full lines and dotted lines respectively.

Although the two axes are intersecting, the visible portions of the lines of intersection, when the penetrating prism enters and comes out differ because the penetrated prism has its faces inclined to the V.P.

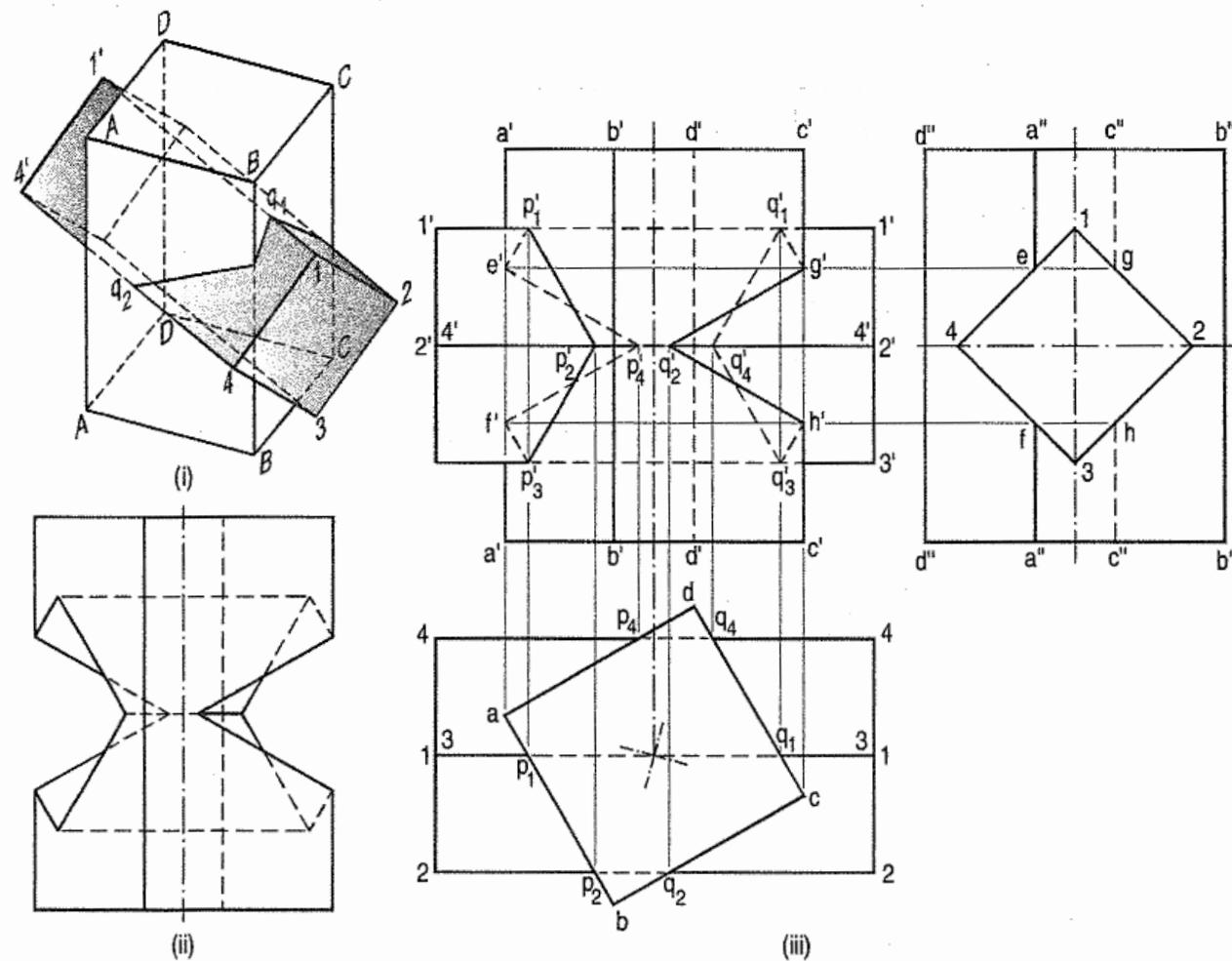


FIG. 16-3

Lines $p'_1 p'_2$ and $p'_2 p'_3$ are visible on the left side while $q'_2 g'$ and $q'_2 h'$ are visible on the right side. Edges $a'e'$ and $a'f'$ are partly hidden, while $c'g'$ and $h'c'$ are fully visible.

Fig. 16-3(b) shows the front view of the vertical prism, when the penetrating prism has been removed. Note that the edges of the back portions of the hole are partly visible.

Problem 16-4. (fig. 16-4): A square pipe of 50 mm side has a similar branch of 40 mm side. The axis of the main pipe is vertical and is intersected by the axis of the branch at an angle of 45° . All the faces of both the pipes are equally inclined to the V.P. Draw the projections of the pipes, showing lines of intersection. Also develop the surfaces of both the pipes.

The line of intersection between the two pipes is obtained [fig. 16-4(i)] in the same manner as shown in problem 16-1. As the axes are intersecting, the edge $a'a'$ is cut by the two edges of the branch at points p'_1 and p'_3 . The other two edges of the branch enter the faces of the main pipe at points p'_2 and p'_4 .

Developments of the surfaces of the two pipes are shown in fig. 16-4(ii).

- Heights of all the points for fig. 16-4(ii) are obtained from the front view, e.g. $P_1A = p'_1 a'$, $P_11 = p'_1 1'$ etc.
- The exact position of the point P_2 is located from the top view by making $AE = ap_2$ and then erecting a perpendicular at E . The point P_4 is similarly located.

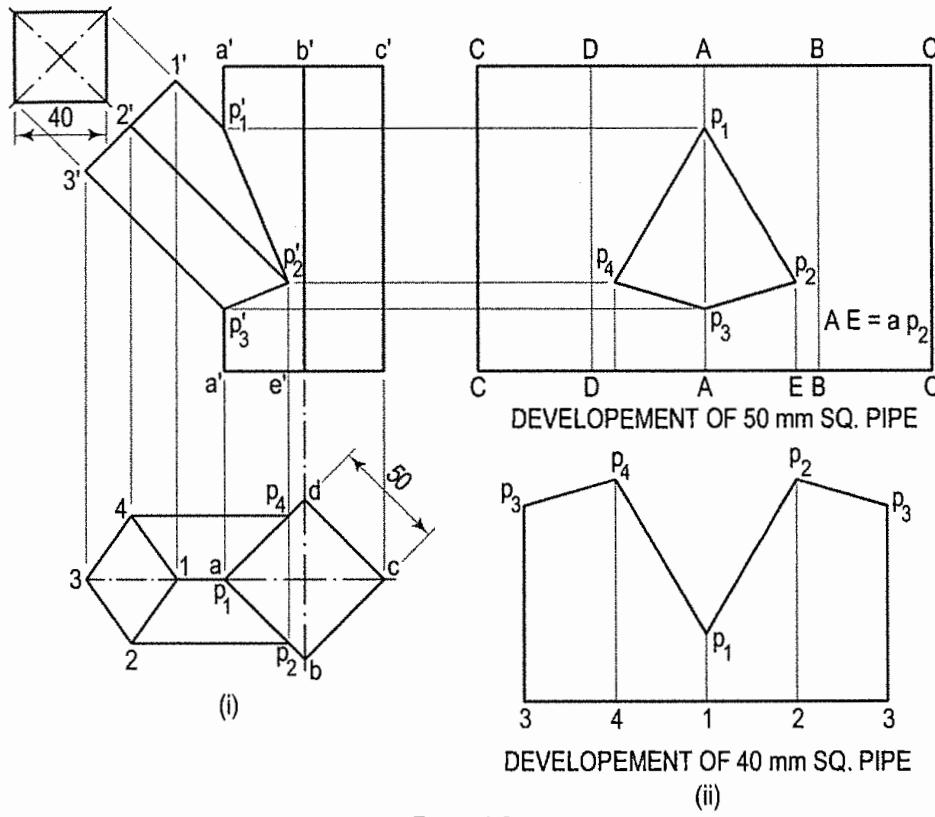


FIG. 16-4

Problem 16-5. (fig. 16-5): A vertical square prism, base 50 mm side, is intersected by another square prism, base 35 mm side, the axis of which is parallel to the V.P. and inclined at 30° to the H.P. The axes of the two prisms are 6 mm apart and their faces are equally inclined to the V.P. Draw the projections showing the line of intersection.

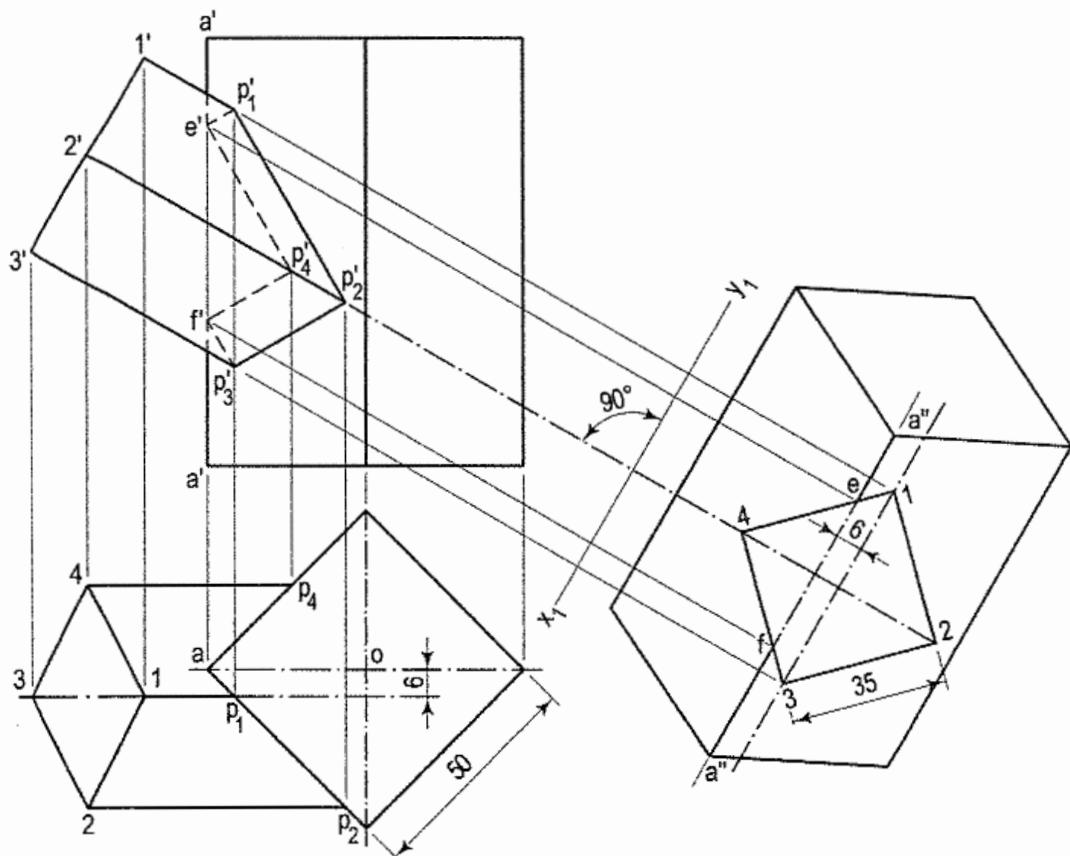


FIG. 16-5

- Obtain points of intersection of the edges of the inclined prism from the top view. For the points at which the edge $a'a'$ of the vertical prism is cut, it will be necessary to project a view in which faces of the inclined prism will be seen as lines.
- Therefore, project an auxiliary top view on a reference line x_1y_1 drawn perpendicular to the axis of the inclined prism.
- Mark points e and f at which $a''a''$ is pierced by the faces and project them to points e' and f' on the corresponding line $a'a'$ in the front view. Draw straight lines joining the six points in correct sequence.

Problem 16-6. (fig. 16-6): A square pyramid is of base sides 40 mm and height 55 mm. The sides of the base are equally inclined with the V.P. It is penetrated by a horizontal triangular prism of sides 40 mm and 70 mm axis long. The axis is perpendicular to the V.P. 25 mm above the base of the pyramid and 8 mm away from the axis of the pyramid. Assume that one of the faces of the prism is vertical and passes through the pyramid. Draw the front view, top view and side-view showing the intersection curve.

- Draw the projections of the square pyramid and triangular prism in the required position.
- In the front view, the edges $1'-1'$, $2'-2'$ and $3'-3'$ of horizontal prism, intersects the faces of the vertical pyramid at P'_1 , P'_2 , q'_2 , P'_3 , P'_4 , q'_4 , P'_5 , q'_5 , and P'_6 , respectively.
- Project points P'_1 , to P'_6 , in the top view and obtain intersection point P_1 , P_2 , q_2 , P_3 , P_4 , q_4 , P_5 , q_5 , and P_6 .
- Similarly take projections from top view and front view and complete the side view.

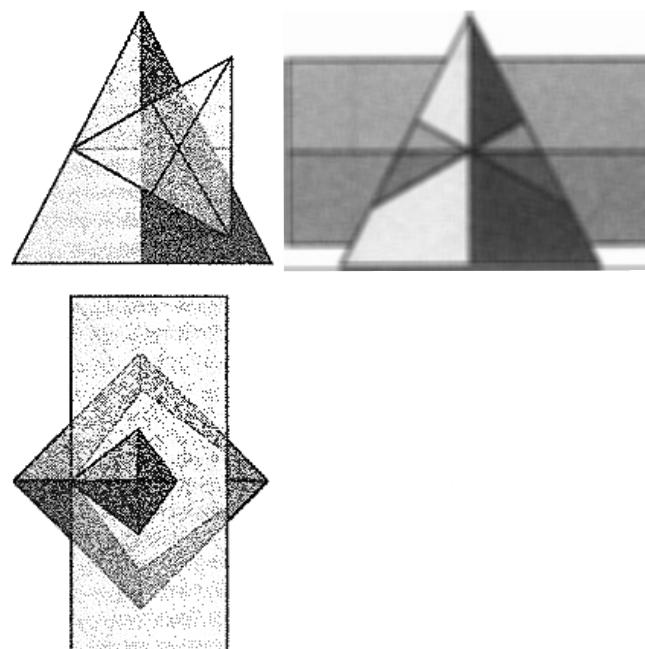


FIG. 16-6(i)

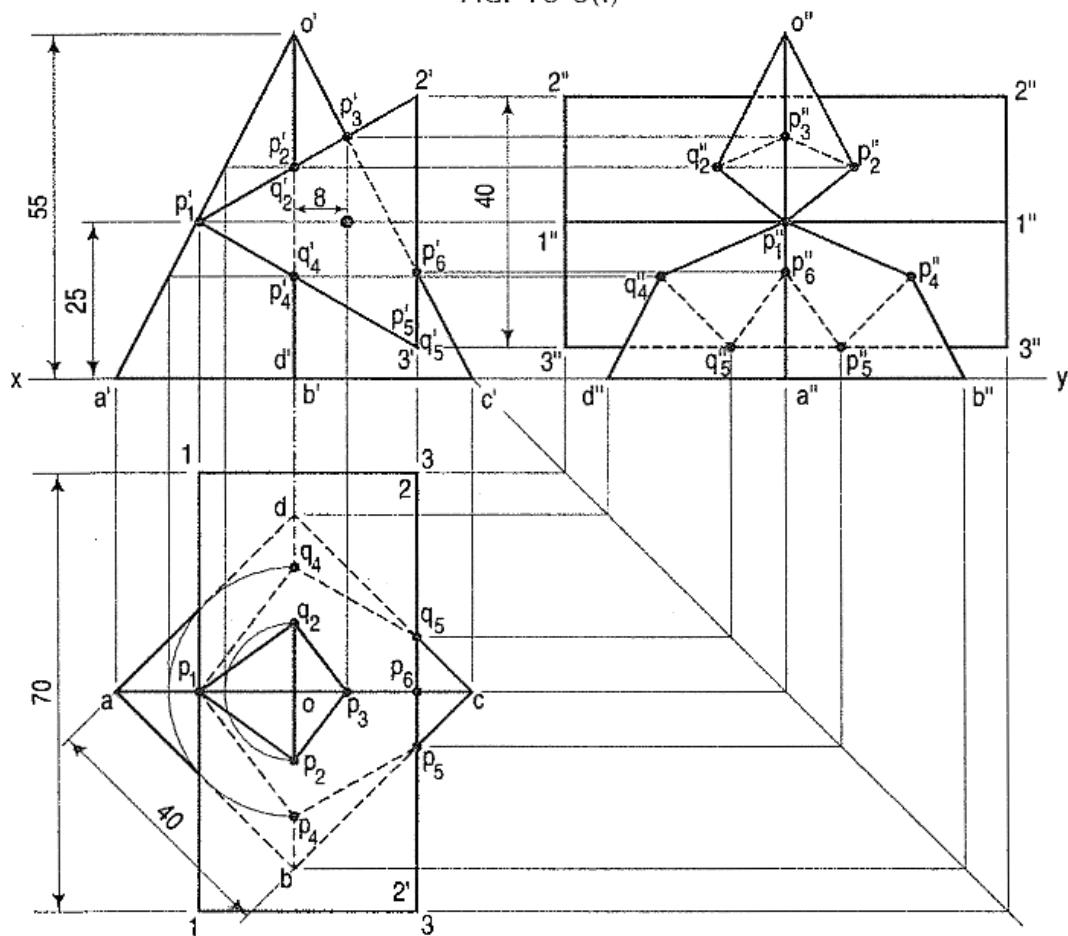


FIG. 16-6(ii)

Problem 16-7. (fig. 16-7): A square pyramid of base sides 50 mm and height 60 mm. The sides of base are equally inclined with the V.P. It is penetrated by a horizontal triangular prism of sides 30 mm and 80 mm axis long. The axes of both solids are intersecting each other. The axis of triangular prism is 22 mm above H.P. and perpendicular to the V.P. One of faces of triangular prism is perpendicular to the V.P. Draw the top view, front view and side view showing the curve of the penetration.

- Draw the projections of the square pyramid and triangular prism in the required position.
- In the front view, draw lines $o'e'$, $o'b'$, $o'g'$ and $o'h'$ passing through $1'$, $2'$, $3'$ and intersecting points respectively.
- As the axes are intersecting, the edges $o'b'$ and $o'd'$ are cut by the two edges of the horizontal prism at P'_2 , q'_2 , P'_4 and q'_4 .
- The third edge of the horizontal prism enters the faces of vertical prism at P'_3 , q'_3 , P'_5 and q'_5 .
- Project these points in the top view and obtain the intersecting points P_1 , P_2 , P_3 , P_4 , P_5 , q_1 , q_2 , q_3 , q_4 and q_5 .
- Take projections from front view and top view and complete side view.

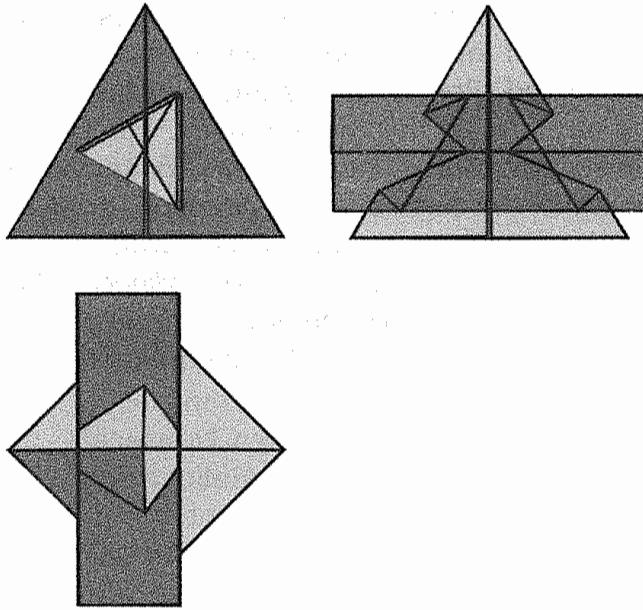


FIG. 16-7(i)

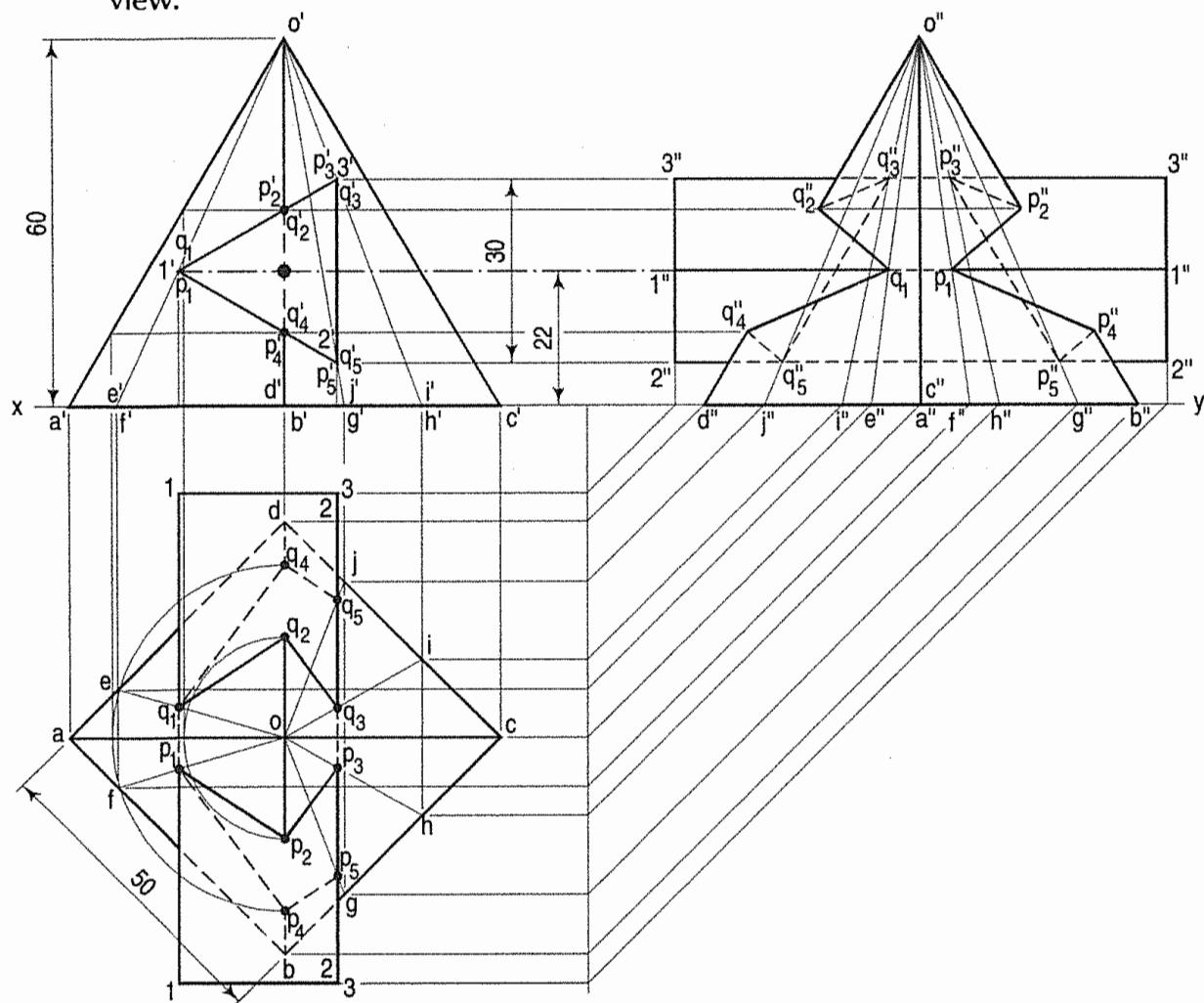


FIG. 16-7(ii)

16-4. INTERSECTION OF CYLINDER AND CYLINDER

As cylinders have their lateral surfaces curved, the line of intersection between them will also be curved. Points on this line may be located by any one of the two methods. For plotting an accurate curve, certain *critical or key points*, at which the curve changes direction, must also be located. These are the points at which outermost or extreme lines of each cylinder pierce the surface of the other cylinder. In prisms, vertices are the key points.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 37 for the line of intersection.

Problem 16-8. (fig. 16-8): A vertical cylinder of 80 mm diameter is completely penetrated by another cylinder of 60 mm diameter, their axes bisecting each other at right angles. Draw their projections showing curves of penetration, assuming the axis of the penetrating cylinder to be parallel to the V.P.

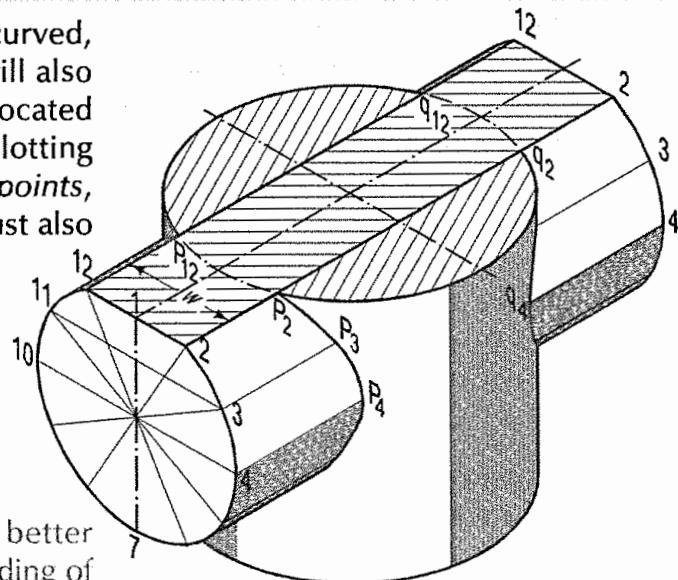


FIG. 16-8(i)

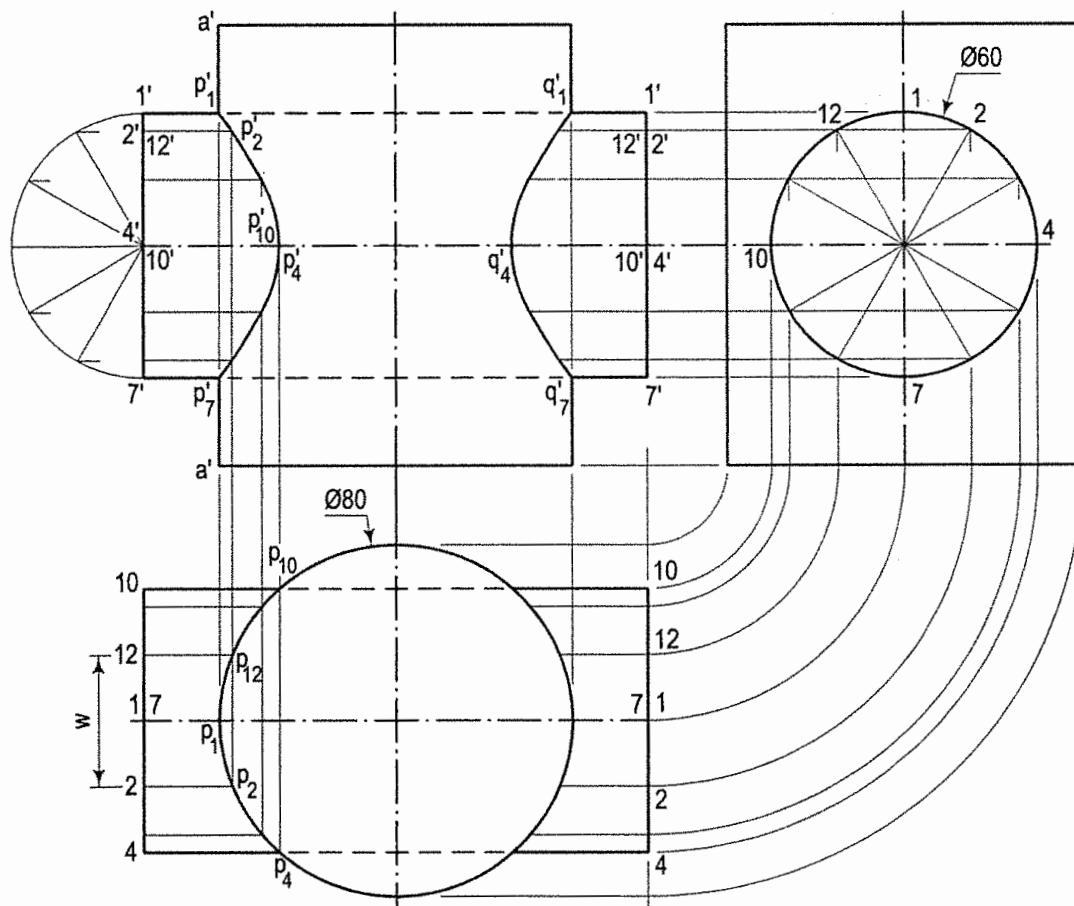


FIG. 16-8(ii)

Draw the front view and the top view and show lines for twelve generators in the horizontal cylinder in both views.

(a) *Line method:*

- Mark points p_1, p_2 etc. at which lines 1-1, 2-2 etc. intersect the circle (showing the surface of the vertical cylinder) in the top view and project them to p'_1, p'_2 etc. on corresponding lines $1'1'$, $2'2'$ etc. in the front view.
- Draw the required curves on both sides of the axis through points thus located. Hidden portions of the curves coincide with the visible portions. Points p'_1, p'_4, p'_7 and p'_{10} are the key points where the curve changes direction.

(b) *Cutting-plane method:* It will be seen that in this problem, there is practically no difference between the line method and the cutting-plane method. But the latter method proves more useful in solving problems in which none of the projections shows a line-view of the surface of a solid. Assume a series of horizontal cutting planes passing through the lines on the horizontal cylinder and cutting both cylinders.

Sections of the horizontal cylinder will be rectangles, while those of the vertical cylinder will always be circles of the same diameter as its own. Points at which sides of the rectangles intersect the circle will lie on the curve of intersection. For example, let a horizontal section plane through points 2 and 12 [fig. 16-8(i)].

In the front view, it will be seen as a line coinciding with the line $2'2'$. The section of the horizontal cylinder will be a rectangle of width w (i.e. the line 2-12). The section of the vertical cylinder will be a circle. Points p_2 and p_{12} at which the sides (2-2 and 12-12) of the rectangle cut the circle, lie on the curve. These points are first marked in the top view [fig. 16-8(ii)] and then projected to points p'_2 and p'_{12} on lines $2'2'$ and $12'12'$ in the front view. Points on the other side of the vertical axis are located in the same manner.

The problem may also be solved by assuming cutting planes to be vertical and parallel to both axes. They will be seen as lines in the top view and the side view. Sections of both cylinders will be rectangles and will be seen in their true sizes in the front view. Points at which sides of sections of one cylinder intersect sides of corresponding sections of the other, will lie on the curve of intersection.

Problem 16-9. (fig. 16-9): A cylindrical pipe of 30 mm diameter has a similar branch of the same size. The axis of the main pipe is vertical and is intersected by that of the branch at right-angles. Draw the projections of the pipes, assuming suitable lengths, when the two axes lie in a plane parallel to the V.P. Develop the surfaces of the two pipes.

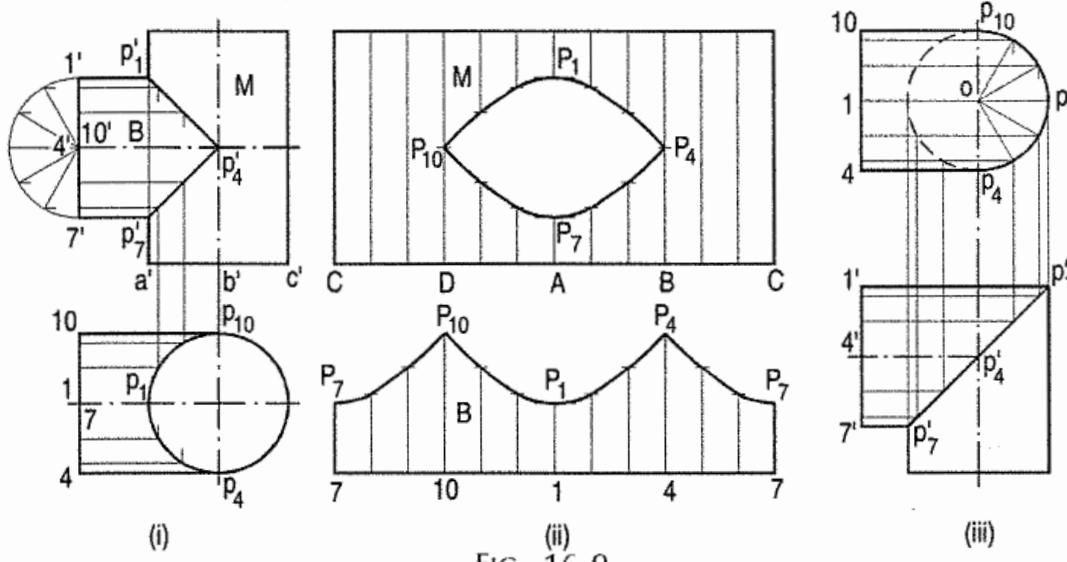


FIG. 16-9

Adopt the same method as explained in problem 16-8. The curve of intersection in the front view is seen as two straight lines meeting at right-angles [fig. 16-9(i)]. Developments of surfaces of the pipes are shown in fig. 16-9(ii).

Fig. 16-9(iii) shows in third-angle projection method, projections of the pipes of the same size joining at right-angles and forming an elbow. Note that the curve of intersection is seen as a straight line joining the two corners.

Problem 16-10. (fig. 16-10): A vertical cylinder of 80 mm diameter is penetrated by another cylinder of 60 mm diameter, the axis of which is parallel to both the H.P. and the V.P. The two axes are 8 mm apart. Draw the projections showing curves of intersection.

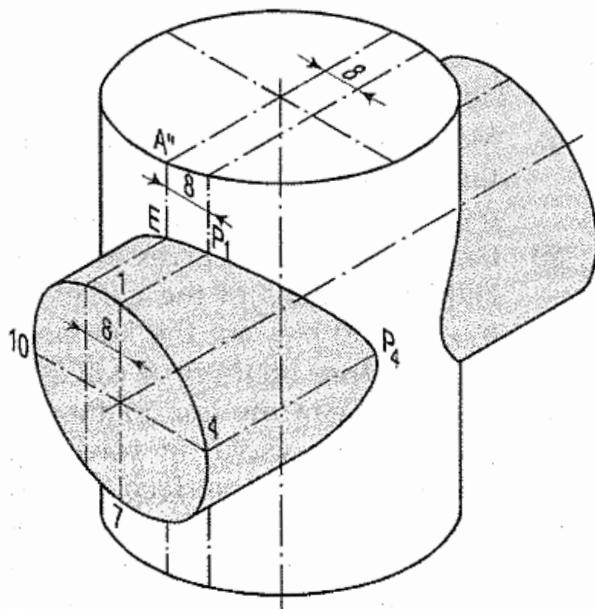


FIG. 16-10(i)

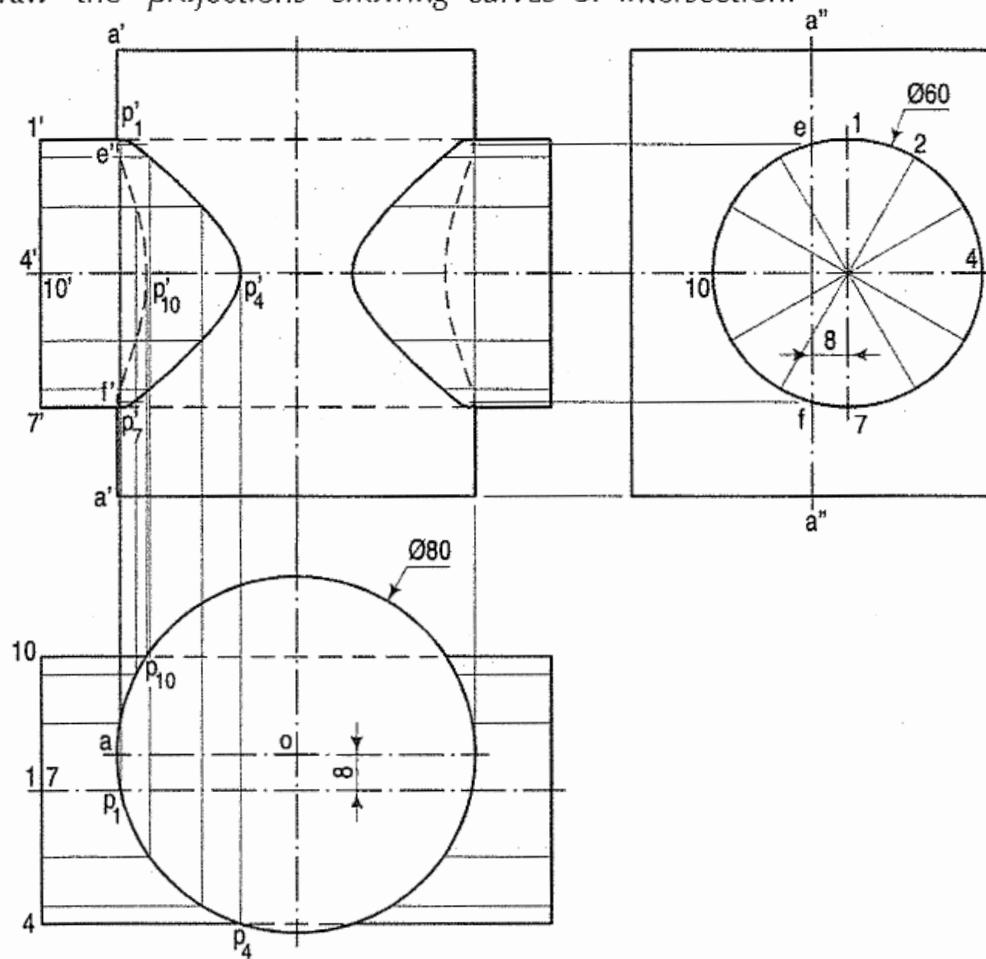


FIG. 16-10(ii)

- Obtain the twelve points as in problem 16-8. Two more key points at which the extreme line $a'a'$ of the vertical cylinder is cut, must be located. These are found from the side view.
- Mark points e and f at which the line $a''a''$ intersects the circle.

- (iii) Project these points to e' and f' on the line $a'a'$.
- (iv) Draw the curve passing through all the points in correct sequence, showing the hidden portion by dotted lines. Plot a similar curve on the other side of the axis.

Problem 16-11. (fig. 16-11): A vertical cylinder of 75 mm diameter is penetrated by another cylinder of the same size. The axis of the penetrating cylinder is parallel to both the H.P. and the V.P. and is 9 mm away from the axis of the vertical cylinder. Draw the projections showing curves of intersection.

As the cylinders are of the same size and their axes are apart, a portion of the surface of the penetrating cylinder will be outside the vertical cylinder.

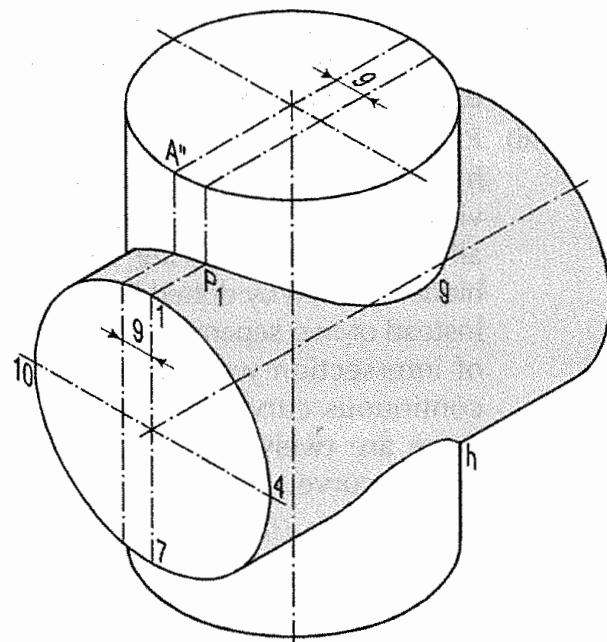


FIG. 16-11(i)

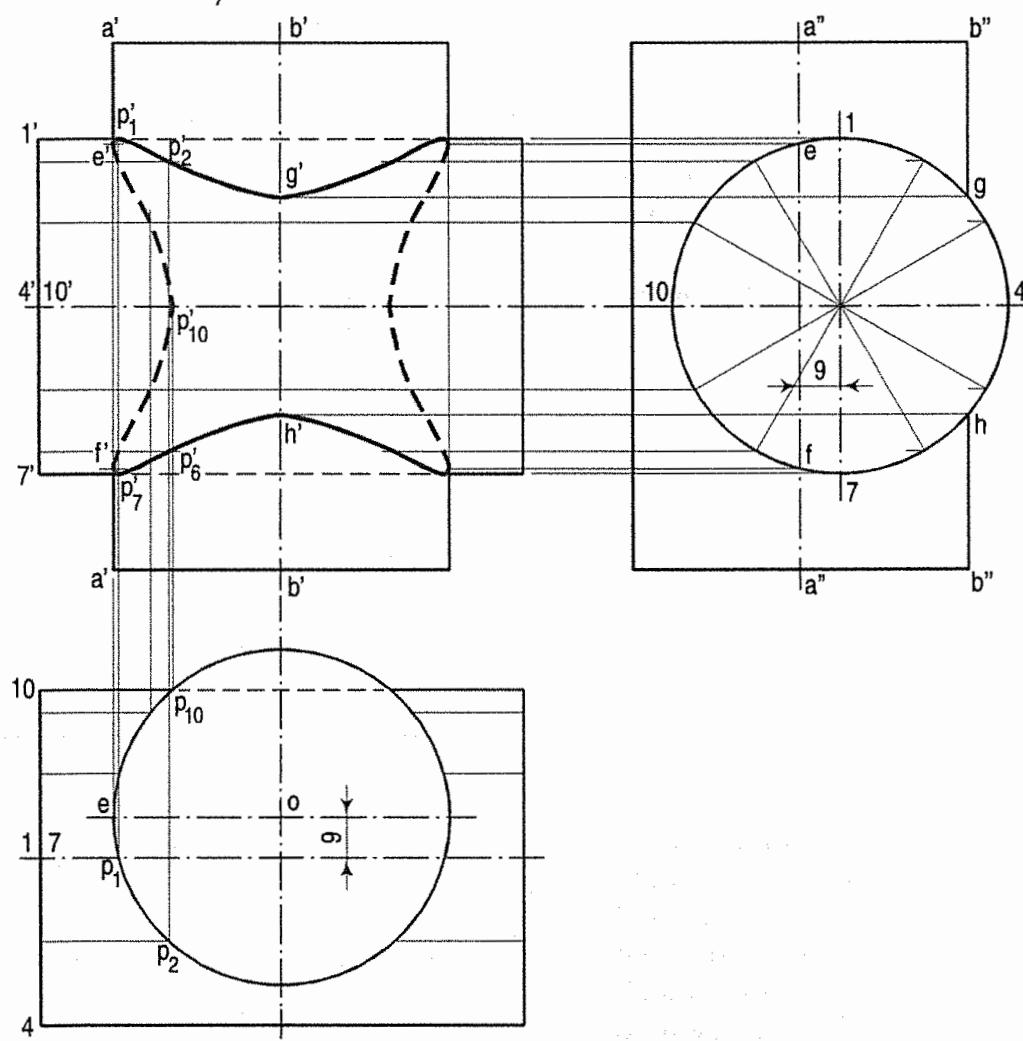


FIG. 16-11(ii)

- (i) Draw the three views and project points of intersection of lines which lie within the vertical cylinder.

- (ii) Locate key points e' and f' as shown in problem 16-10. In addition to these, mark two more key points g and h (in the side view) where the circle cuts the extreme line $b''b''$ of the vertical cylinder.
- (iii) Project these points to g' and h' on the line $b'b'$ in the front view. Draw a curve through all the points, showing the hidden portion by dotted lines. Instead of two separate curves of intersection we have one continuous curve. Note that there are twelve key points in this curve.

Problem 16-12. (fig. 16-12): A cylinder of 60 mm diameter, having its axis vertical is penetrated by another cylinder of 40 mm diameter. The axis of the penetrating cylinder is parallel to the V.P. and bisects the axis of the vertical cylinder, making an angle of 60° with it. Draw the projections showing curves of intersection.

Draw the projections of the cylinders in the required position and proceed to locate the points on curves of intersection as shown in problem 16-8. The back and front curves will coincide with each other.

Problem 16-13. (fig. 16-13): A vertical pipe of 60 mm diameter has a branch of 30 mm diameter. The axis of the branch is inclined at 45° to the ground, parallel to the V.P. and is 15 mm away from the axis of the main pipe. Draw the projections of the pipes showing the curve of intersection. Also, develop the surfaces of the two pipes, assuming any lengths.

Apply the same method as in problem 16-9. As the diameter of the branch and the distance between the two axes are respectively one-half and one-fourth of the diameter of the main pipe, one extreme line of the branch (in the top view) will be tangent to the circle. In the front view, the visible part of the curve will extend upto the centre line of the main pipe, while the hidden part will just touch the line $a'a'$.

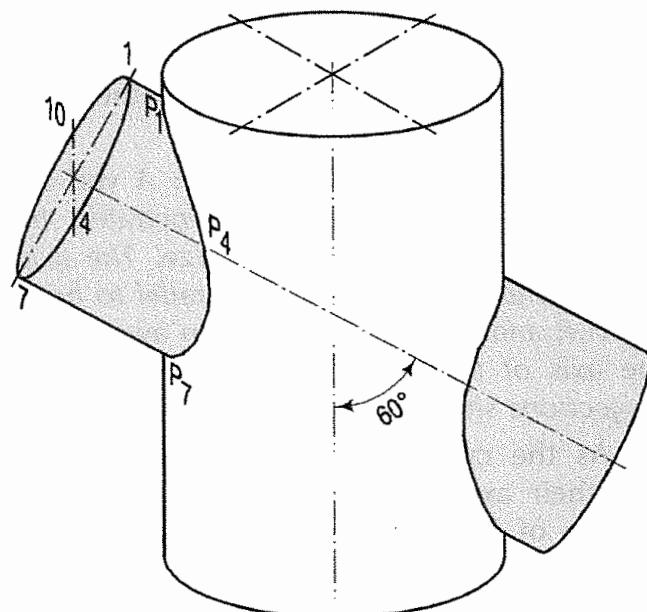


FIG. 16-12(i)

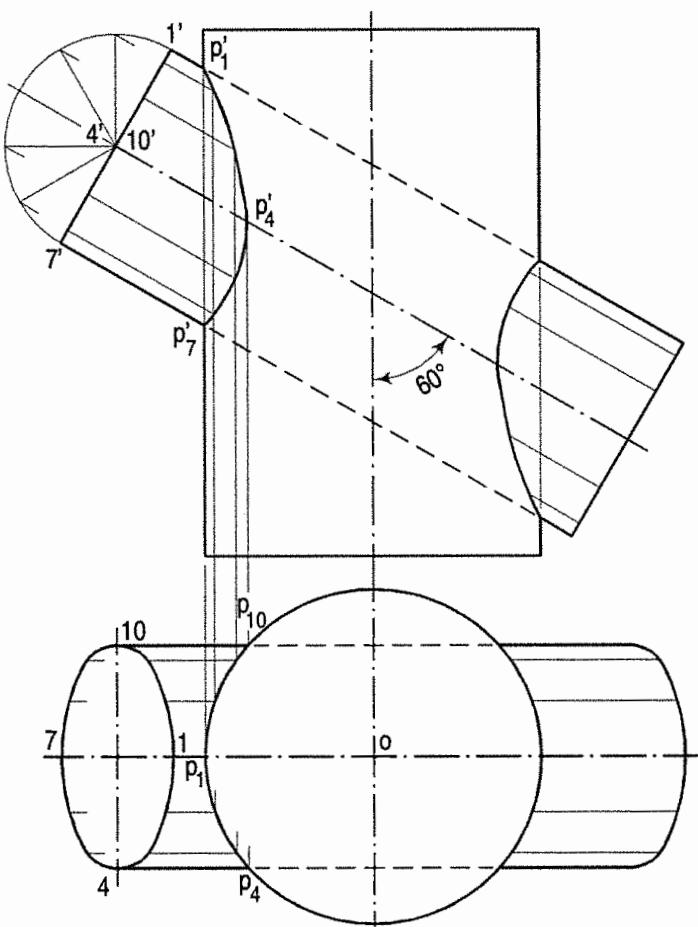


FIG. 16-12(ii)

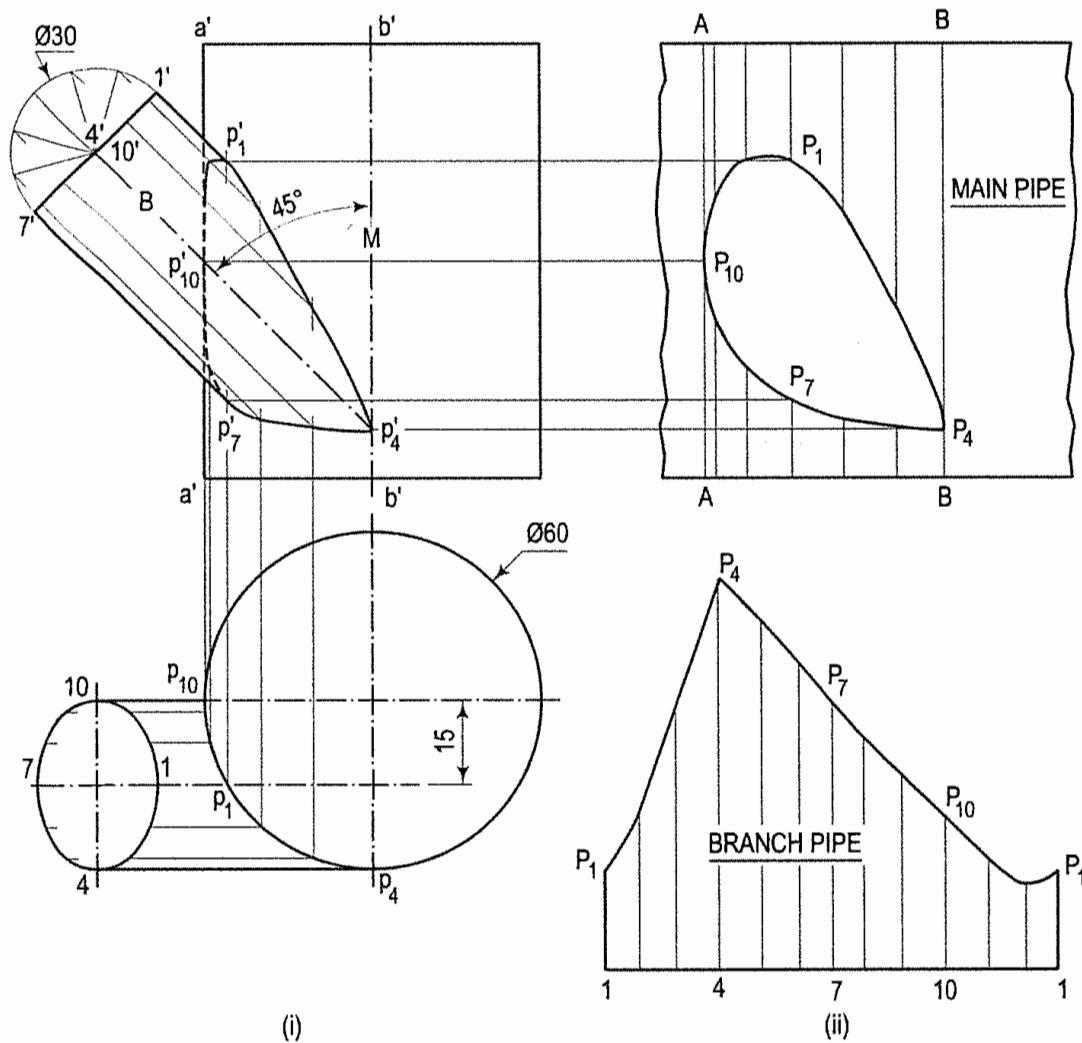


FIG. 16-13

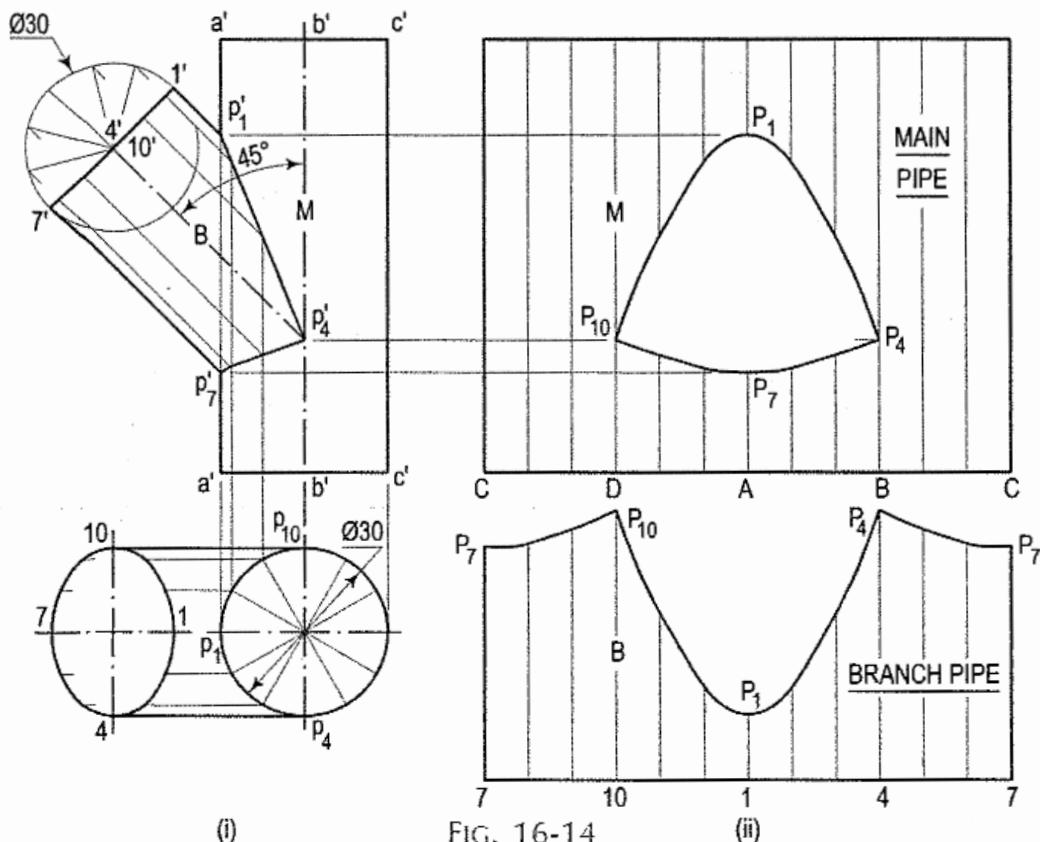
Only a part of the development of the main pipe, just sufficient to show the shape of the hole in it, is shown in fig. 16-13(ii). Distances along the length of the stretch-out line are taken from the top view and positions of points are projected from the front view. In the development of the branch, the stretch-out line is divided into twelve equal parts and heights of points are taken from the front view, e.g. $4p_4 = 4'p'_4$ etc.

Problem 16-14. (fig. 16-14): A cylindrical pipe of 30 mm diameter has a similar branch of the same size. The axis of the branch intersects that of the main pipe at an angle of 45°. Draw the projections, when the two axes lie in a plane parallel to the V.P., and the axis of the main pipe is vertical. Also, develop the surfaces of the two pipes, assuming suitable lengths.

The curve of intersection in the front view is made up of two straight lines meeting at right-angles [fig. 16-14(i)].

Developments of surfaces of the pipes are shown in fig. 16-14(ii).

If instead of the branch, two similar pipes interpenetrate, the curves of intersection in the front view would be seen as two straight lines joining the opposite corners and bisecting each other at right-angles.

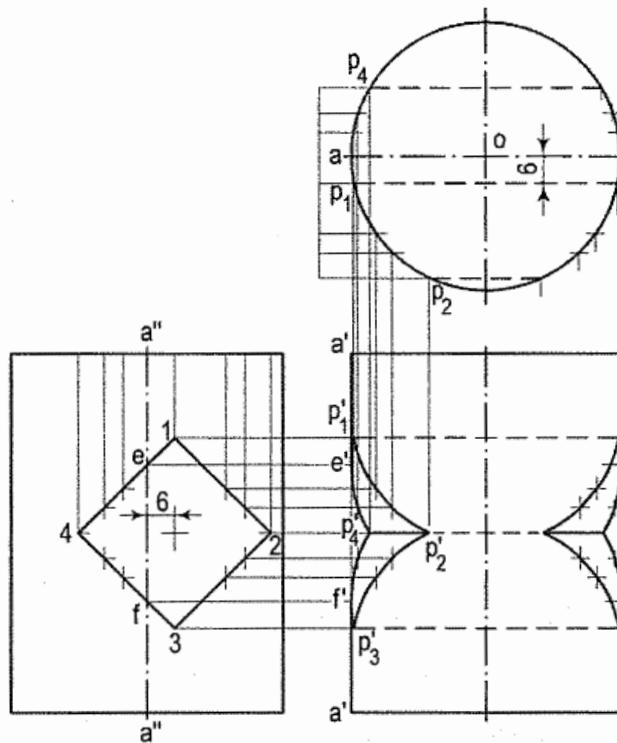


16-5. INTERSECTION OF CYLINDER AND PRISM

Problem 16-15. (fig. 16-15): A vertical cylinder of 60 mm diameter has a square hole of 30 mm sides cut through it. The axis of the hole is horizontal, parallel to the V.P. and 6 mm away from the axis of the cylinder. The faces of the hole are equally inclined to the H.P. and the V.P. Draw the projections of the cylinder showing the hole in it.

- (i) Draw three views of the cylinder showing the lines for the hole in given position. Project all the key points, i.e. points of intersection of the edges of the hole with the surface of the cylinder, viz. p'_1 , p'_2 etc. and those of the extreme lines of the cylinder with the surfaces of the hole, viz. e' , f' etc. Project a few intermediate points also.

(ii) Draw the required curve through these points on both sides of the axis. Note that the back portion of the hole is also visible. The curves from p'_1 to e' and p'_3 to f' bend in opposite directions.



(Third-angle projection)

FIG. 16-15

Problem 16-16. (fig. 16-16): A vertical cylinder of 60 mm diameter is penetrated by a horizontal square prism, base 40 mm side, the axis of which is parallel to the V.P. and 10 mm away from the axis of the cylinder. A face of the prism makes an angle of 30° with the H.P. Draw their projections, showing curves of intersection.

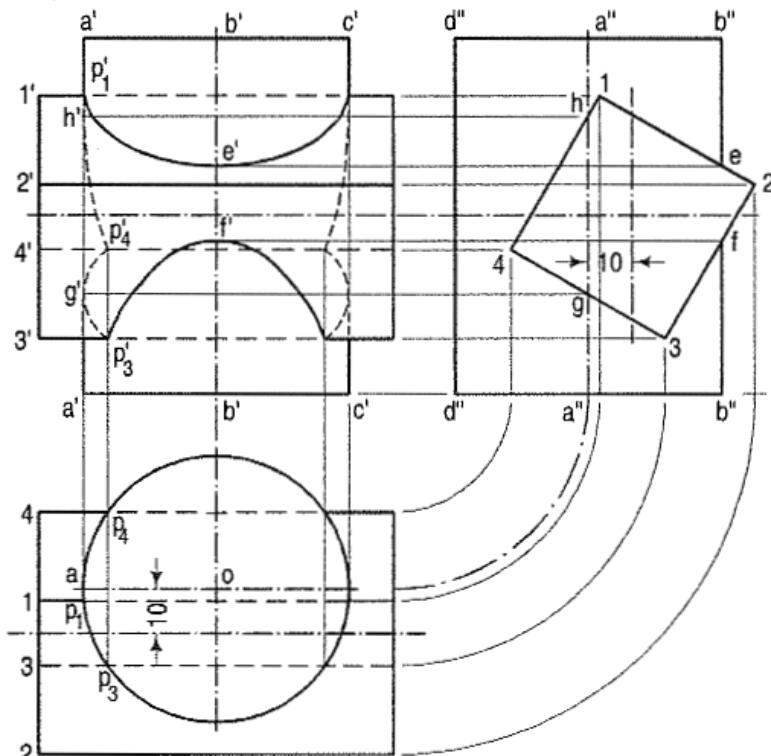


FIG. 16-16

- (i) Draw the views in the required position. One longer edge of the prism will remain outside the cylinder. Project all the key points, viz. points of intersection of the edges of the prism with the cylinder and those of the extreme lines of the cylinder with the surface of the prism, as shown in the figure.
- (ii) To obtain accurate shape of the curves, project few more intermediate points also. These are omitted from the figure. Draw the required curve of intersection through these points, taking precaution to show the hidden part by dashed lines.

Problem 16-17. (fig. 16-17):

A vertical square prism having its faces equally inclined to the V.P. is completely penetrated by a horizontal cylinder, the axis of which is parallel to the V.P. and 6 mm away from that of the prism. Draw the projections of the solids showing curves of intersection. The length of the sides of the base of the prism is 50 mm and the diameter of the cylinder is 40 mm.

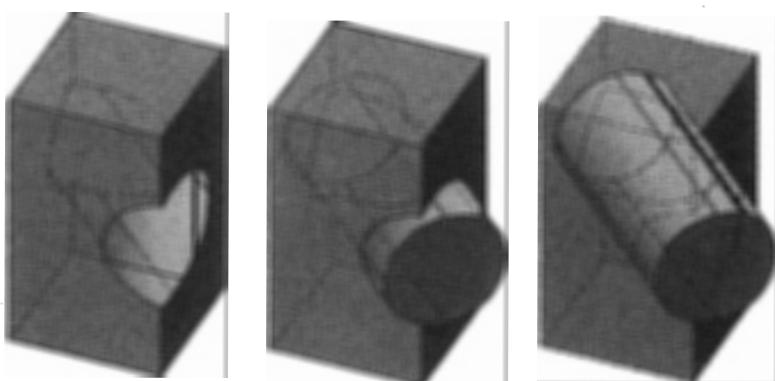


FIG. 16-17(i)

Obtain the twelve points as explained in problem 16-10. In addition, project the key points e' and f' and draw the curves as shown in the figure.

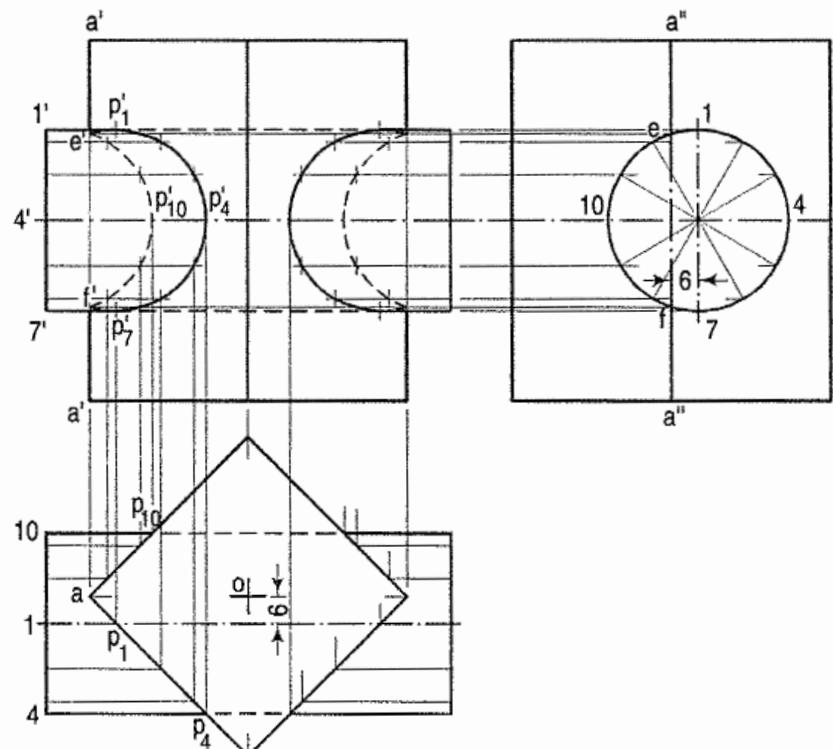
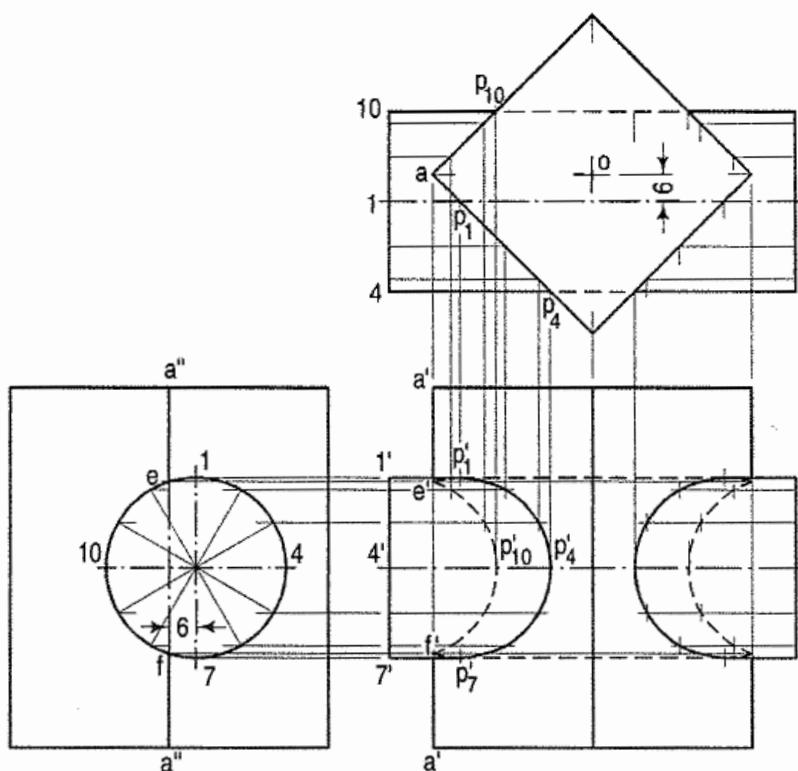


FIG. 16-17(ii)



(Third-angle projection)

FIG. 16-17(iii)

Problem 16-18. (fig. 16-18): A vertical square prism, base \$50\$ mm side, has a face inclined at \$30^\circ\$ to the V.P. It has a hole of \$65\$ mm diameter drilled through it. The centre line of the hole is parallel to both the H.P. and the V.P. and is \$5\$ mm away from the axis of the prism. Draw the projections of the prism.

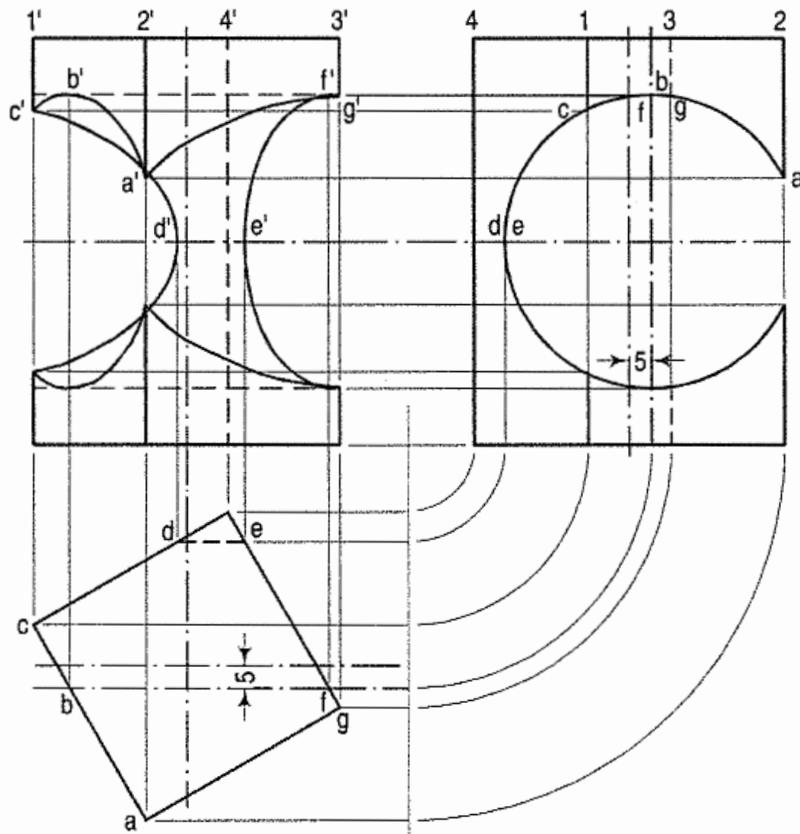


FIG. 16-18

- Draw the views and project all the key points as shown. Plot few intermediate points (between the key points) also.
- Draw a curve joining the points in correct sequence. As it is a hole cut in the prism, the curve at the back will also be visible. Only a small portion of the curve will not be seen.

Problem 16-19. (fig. 16-19): A connecting rod, 50 mm diameter, has a rectangular block 65 mm wide and 25 mm thick, forged at its end. The rod joins the block with a turned radius of 25 mm. Draw the projections of the rod showing curve of intersection.

The rod increases in diameter as it approaches the block. This forms what is called a fillet of 25 mm radius. As the width of the block is not as large as the biggest diameter of the rod, a curve of intersection is formed. Points on the curve are found as shown below.

- Assume a horizontal cutting plane passing through a line, say $a'a'$. The section of the rod is a circle (see the top view) which cuts the sides of the block at points b .
- Project these points to points b' on the line $a'a'$. Then the points b' lie on the curve. Obtain more points by assuming additional sections. The highest point on the curve will be on the line where the diameter of the section of the rod is equal to the width of the block.

Fig. 16-20 shows the shape of the curve when the diameter of the rod is equal to the width of the block. In this case, the highest point on the curve is on the horizontal line through the centre of the arc for the fillet.

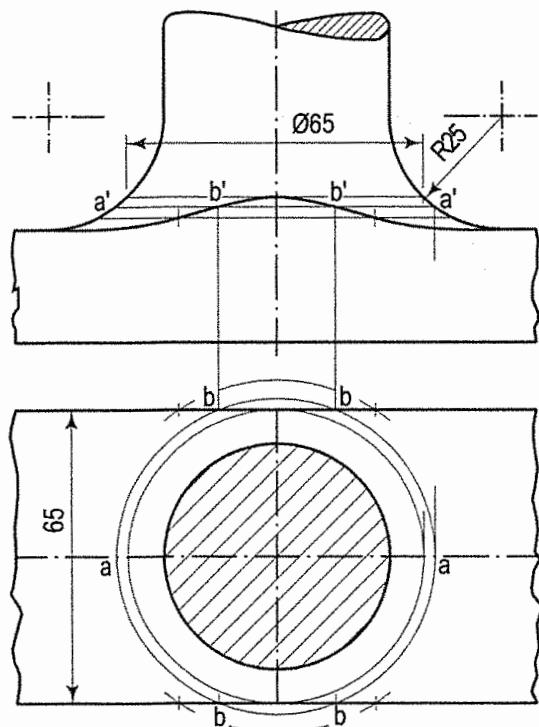


FIG. 16-19

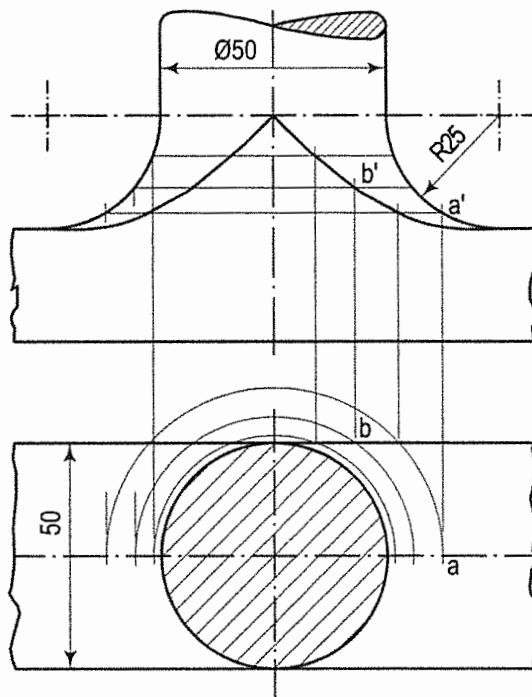


FIG. 16-20

Problem 16-20. Draw the front view and the side view of the forked end of the connecting rod shown in fig. 16-21, showing the curve of intersection.

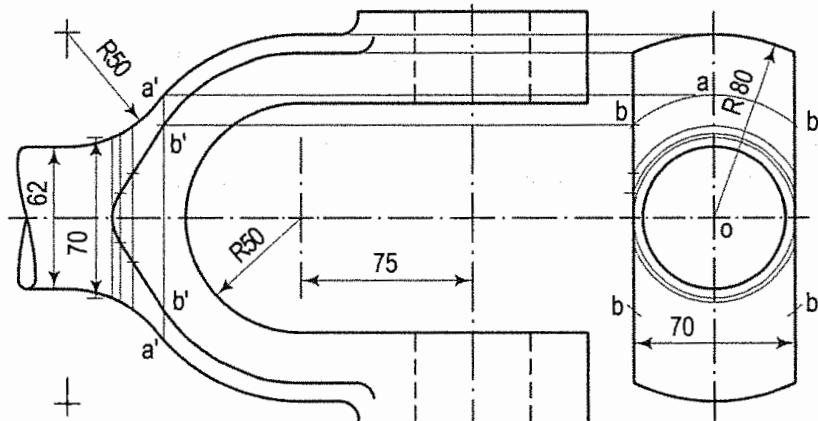
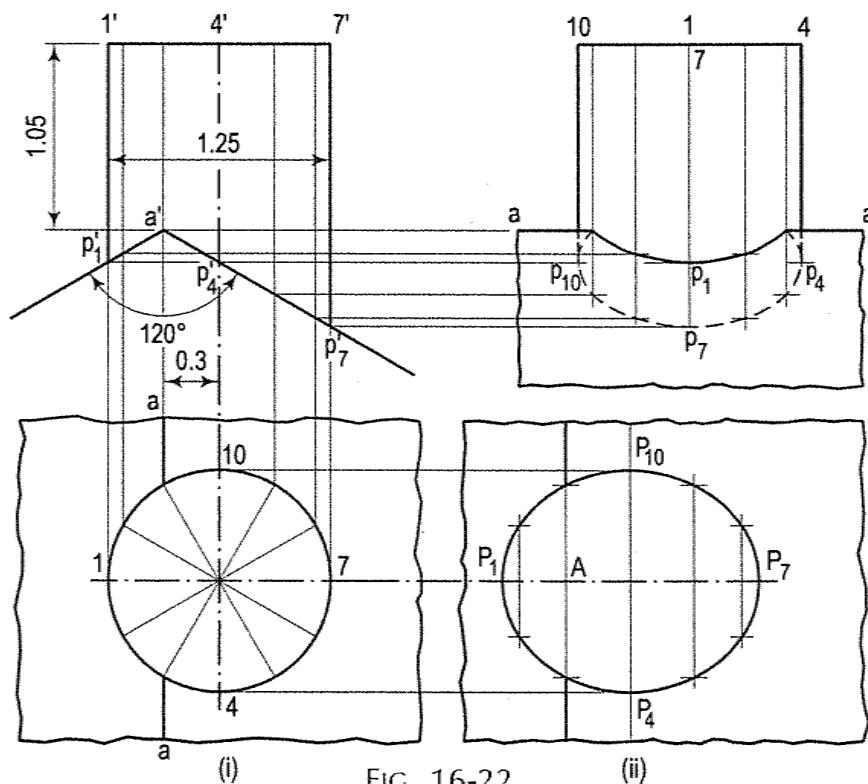


FIG. 16-21

In this case, a cylindrical rod meets a forked piece, the sides of which are flat surfaces, while the top and bottom surfaces are cylindrical.

- Assume a series of section planes perpendicular to the axis of the rod. Points at which circles for the sections intersect the flat faces of the fork, lie on the curve.
- Take a section plane say at $a'a'$.
- With centre o and radius equal to one-half of $a'a'$, draw arcs cutting the sides of the fork at points b (in the side view). Project these points to points b' on $a'a'$. Obtain other points in the same manner and draw the required curve through them.

Problem 16-21. Projections of a steel chimney erected on a roof, are given in fig. 16-22(i). Project its side view showing the curve of intersection. Determine the real shape of the hole in the roof. (All dimensions are in metres.)



- Draw the given views and project the side view.
- To obtain points on the curve, draw lines on the surface of the chimney by dividing the circle (in the top view) into twelve equal parts. Project points at which these lines intersect the two lines for the roof in the front view to corresponding lines in the side view.
- Draw a curve through the points thus obtained. The upper part of the curve will be hidden.

The real shape of the hole in the roof is shown above the side view. Horizontal distances are taken from the front view and widths are projected from the top view.

16-6. INTERSECTION OF CONE AND CYLINDER

Problem 16-22. A vertical cone, diameter of base 75 mm and axis 100 mm long, is completely penetrated by a cylinder of 45 mm diameter. The axis of the cylinder is parallel to the H.P. and the V.P. and intersects the axis of the cone at a point 28 mm above the base. Draw the projections of the solids showing curves of intersection.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 38 for the cutting-plane method.

- Cutting-plane method** (fig. 16-23 and fig. 16-24):

- Draw lines dividing the surface of the cylinder into twelve equal parts.
- Assume a horizontal cutting plane passing through say, point 2 (fig. 16-23). The section of the cylinder will be a rectangle of width w (i.e. the line 2-12), while that of the cone will be a circle of diameter ee. These two sections intersect at points p_2 and p_{12} . These sections are clearly indicated in the top view by the rectangle 2-2-12-12 and the circle of diameter ee (fig. 16-24).

In the front view, the cutting plane is seen as a line coinciding with $2'2'$. Points p_2 and p_{12} when projected on the line $2'2'$ (with which the line $12'-12'$ coincides) will give a point p'_2 (with which p'_{12} will coincide). Then p'_2 and p'_{12} are the points on the curve of intersection.

- (iii) To obtain the points systematically, draw circles with centre o and diameters dd , ee , ff etc. cutting lines through 1, 2 and 12 , 3 and 11 etc. at points p_1 , p_2 and p_{12} , p_3 and p_{11} etc.
- (iv) Project these points to the corresponding lines in the front view. Two more key points at which the curve changes direction must also be located. Their positions are determined from the side view. They are the points of nearest approach viz. m'' and n'' at which, lines drawn from the centre of the circle (i.e. the axis of the cylinder) and perpendicular to the extreme generators of the cone, cut the circle.

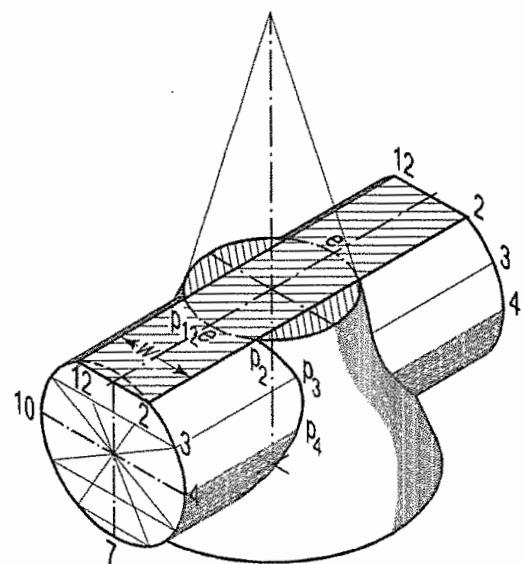


FIG. 16-23

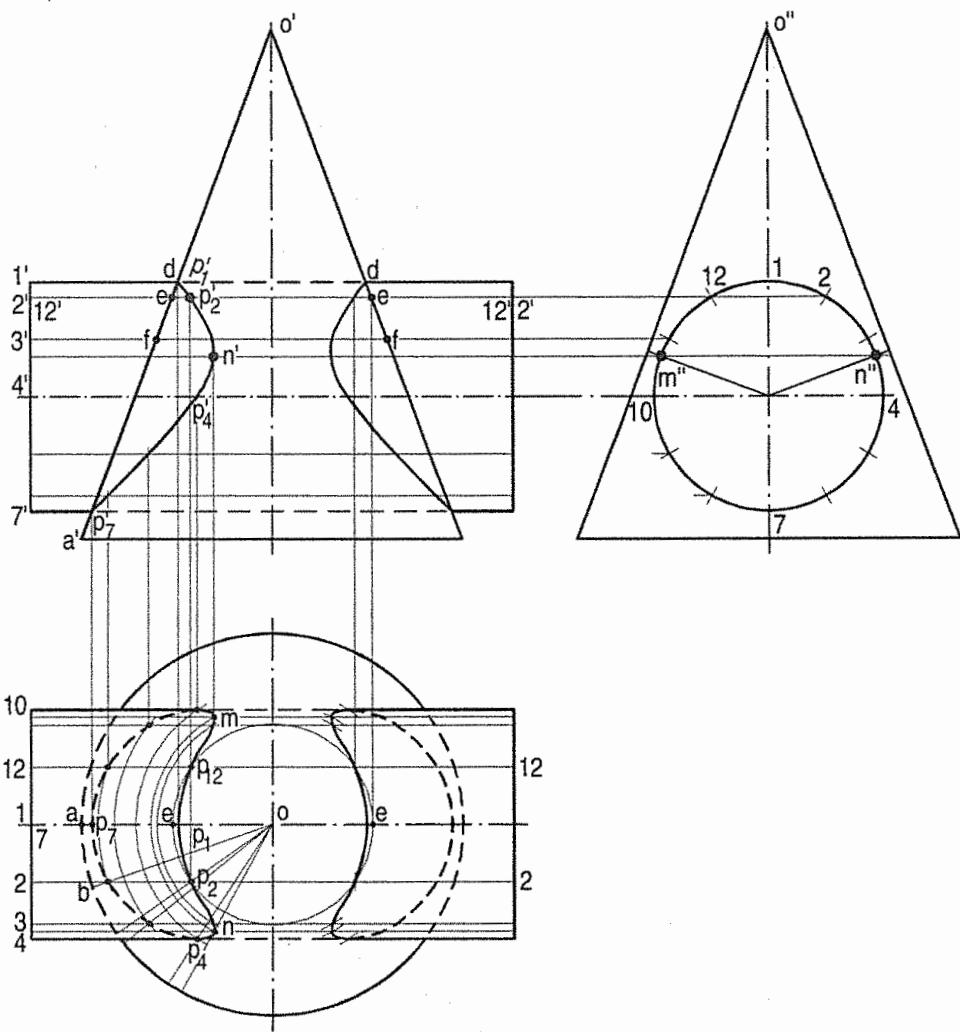


FIG. 16-24

- (v) Project these points to m' and n' in the front view and to m and n in the top view on the corresponding lines. Draw curves through these points in both the views. The back curve in the front view will coincide with the front curve. In the top view a part of the curve will lie hidden and hence, it will be dotted. Draw similar curves on the right-hand side of the axis of the cone.

(b) *Line method* (fig. 16-25): The surface of the cylinder is seen as a circle in the side view.

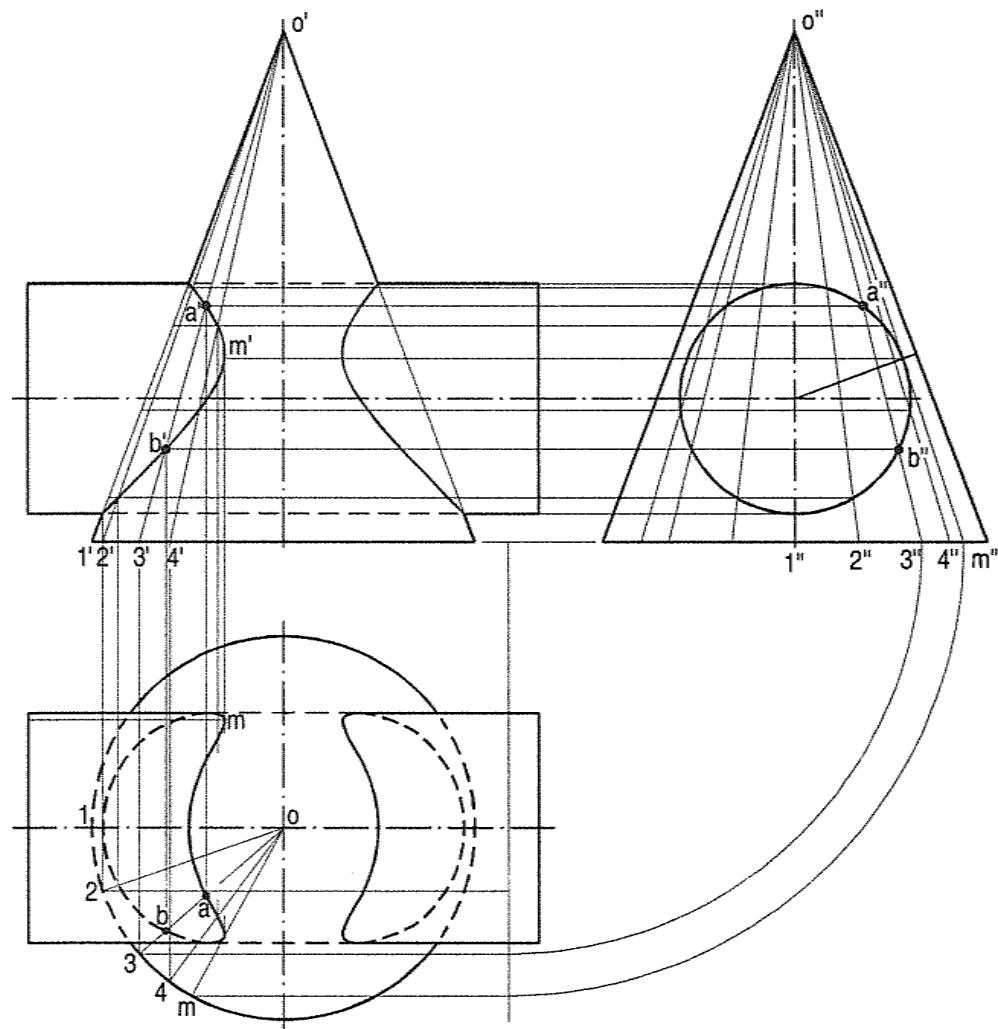


FIG. 16-25

- Hence, draw a number of lines (representing generators of the cone) $o''1'', o''2''$ etc. in the region of the circle and symmetrical on both sides of the axis. Points where these lines intersect the circle, lie on the curve of intersection.
- To project them in the front and top views, first project the lines in both the views and then, locate the positions of these points on them. Let us take the line $o''3''$ in the side view. Locate its position o_3 in the top view as shown and project its front view $o'3'$. Project points a'' and b'' to points a' and b' on $o'3'$ and from there, to a and b on o_3 .
- Project all points in the same manner and draw the required curves through them. This method is a type of cutting-plane method in which cutting planes pass through the apex and are parallel to the axis of the cylinder.

Problem 16-23. (fig. 16-26): A vertical cone, base 80 mm diameter, axis 100 mm long, is penetrated by a horizontal cylinder of 40 mm diameter, the axis of which is 25 mm above the base of the cone, parallel to the V.P. and 6 mm away from the axis of the cone. Draw the projections, showing curves of intersection.

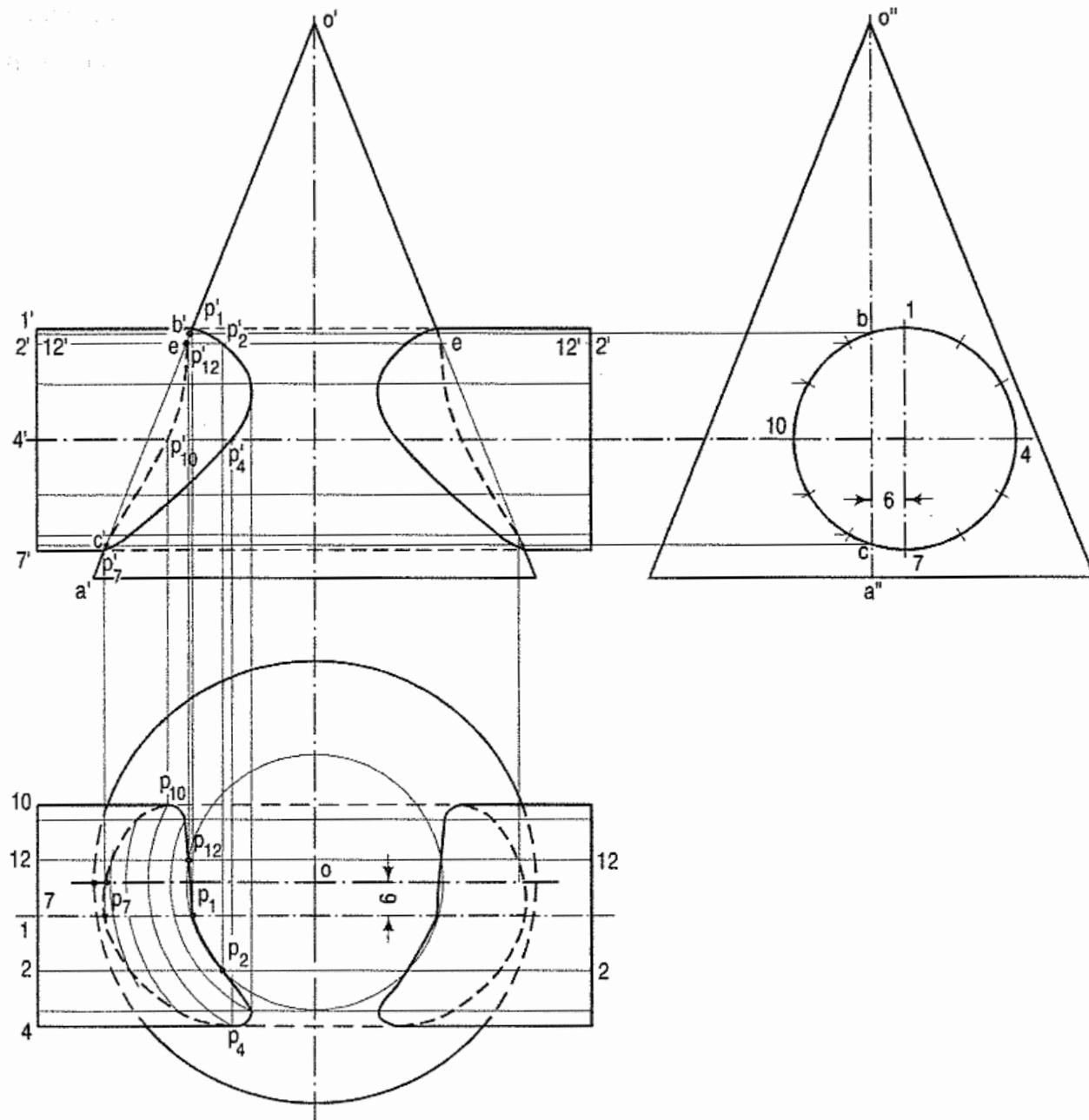


FIG. 16-26

Draw the three views of the solids.

- Assuming a horizontal cutting plane through the line 2'2', draw a circle in the top view with centre \$o\$ and diameter ee, cutting lines 2-2 and 12-12 at points \$p_2\$ and \$p_{12}\$.
- Project these points to \$p'_2\$ and \$p'_{12}\$ on the line 2'2' (with which 12'-12' coincides). Obtain other points and the key points \$b'\$ and \$c'\$ in the same manner and draw the curves as shown. As the axes do not intersect, the back curves in the front view are different from the front curves.

Problem 16-24. (fig. 16-27): A vertical cone, base 80 mm diameter and axis 110 mm long is penetrated by a horizontal cylinder, 45 mm diameter. The axis of the cylinder is 25 mm above the base of the cone, is parallel to the V.P. and is 10 mm away from the axis of the cone. Draw the projections of the solids showing curves of intersection.

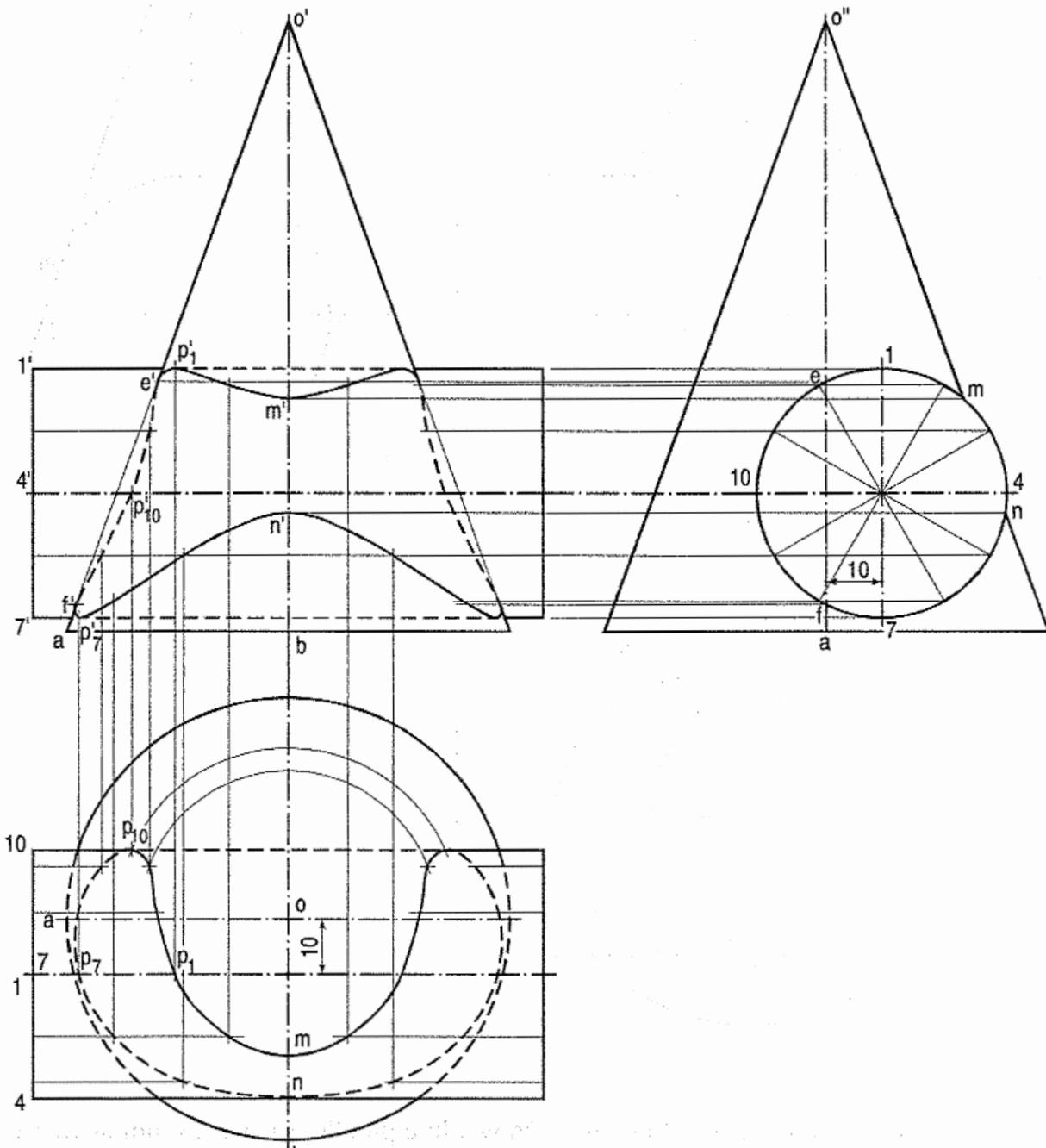


FIG. 16-27 (Continued)

The axis of the penetrating cylinder is so displaced that a portion of the cylinder remains outside the cone. The curve changes direction at key points m' and n' and becomes a single continuous curve.

Problem 16-25. (fig. 16-28): A vertical cone, base 75 mm diameter and axis 110 mm long is penetrated by a horizontal cylinder of 50 mm diameter in such a way that both the solids envelope an imaginary common sphere and their axes intersect each other. Draw the projections of the solids when their axes lie in a plane parallel to the V.P.

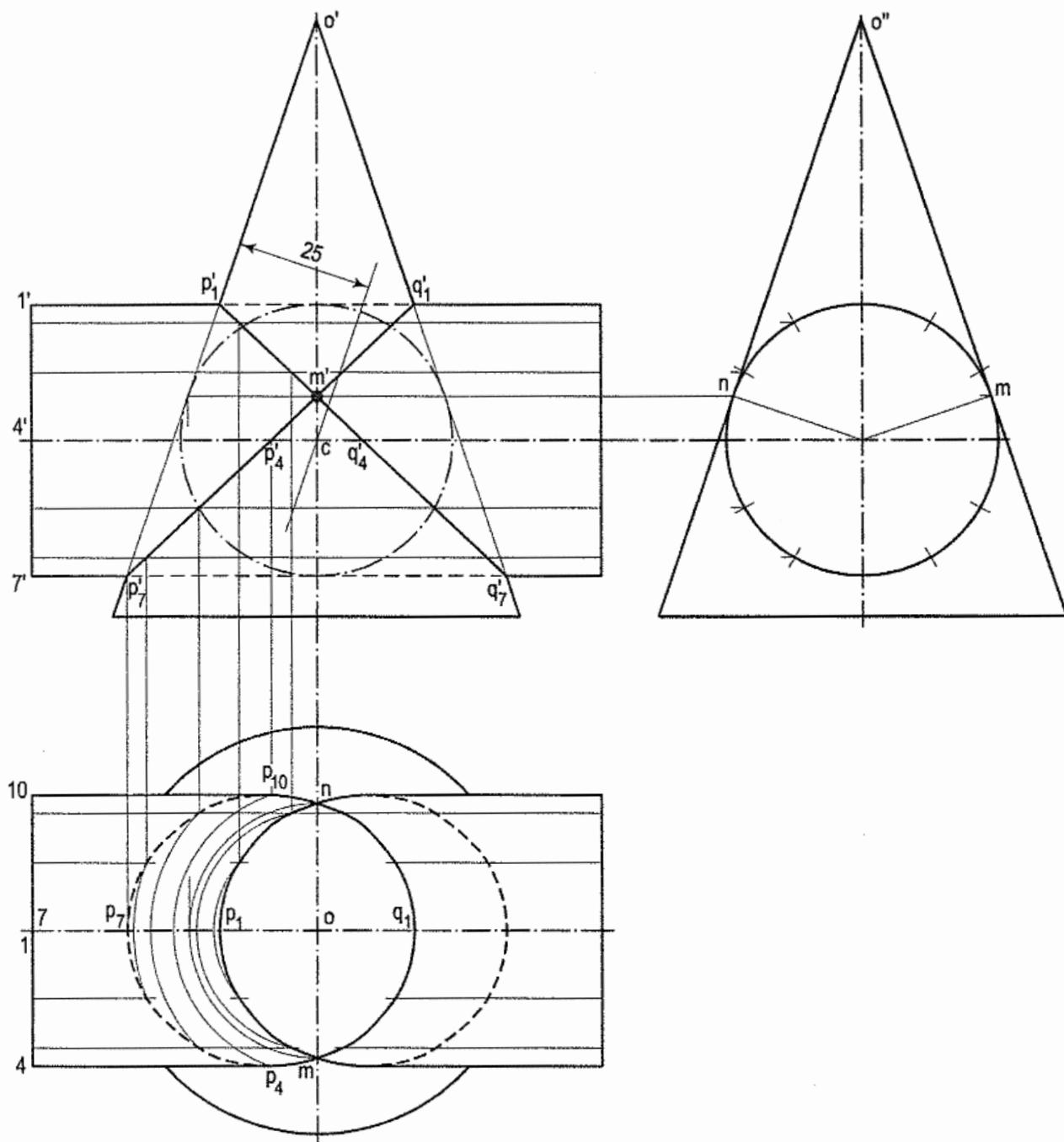


FIG. 16-28

- Draw the front view of the cone. Draw a line parallel to and 25 mm away from an end generator, intersecting the axis of the cone at a point c. Then c is the centre of the circle for the common sphere. Through c, draw the horizontal axis and complete the front view of the cylinder.
- Draw the top and side views and determine the lines of intersection as explained in problem 16-22. In the front view they appear as straight lines intersecting each other at right-angles.

Problem 16-26. (fig. 16-29): A funnel is made of two constituent parts: (i) a cylindrical pipe and (ii) a conical part, both enveloping a common sphere of 40 mm diameter, with their axes intersecting at right-angles. The diameter at the mouth of the funnel is 80 mm and

the total height of the funnel is 60 mm. Draw projections showing the curve of intersection when it is placed with its mouth on the ground and the two axes parallel to the V.P.

Draw the projection of the two parts. To form the funnel, the pipe must be extended beyond the centre line of the conical part. The points of intersection of the upper part of the pipe with the cone will therefore be on the right-hand side of the centre line.

The curve of intersection will be seen as a straight line in the front view and will be elliptical in the top view.

Problem 16-27. A vertical cone, diameter of base 75 mm and axis 90 mm long is penetrated by a cylinder of 50 mm diameter, the axis of which is parallel to and 10 mm away from that of the cone. Draw the projections showing curves of intersection, when (i) the plane containing the two axes is parallel to the V.P.; (ii) the plane containing the two axes is inclined at 45° to the V.P.

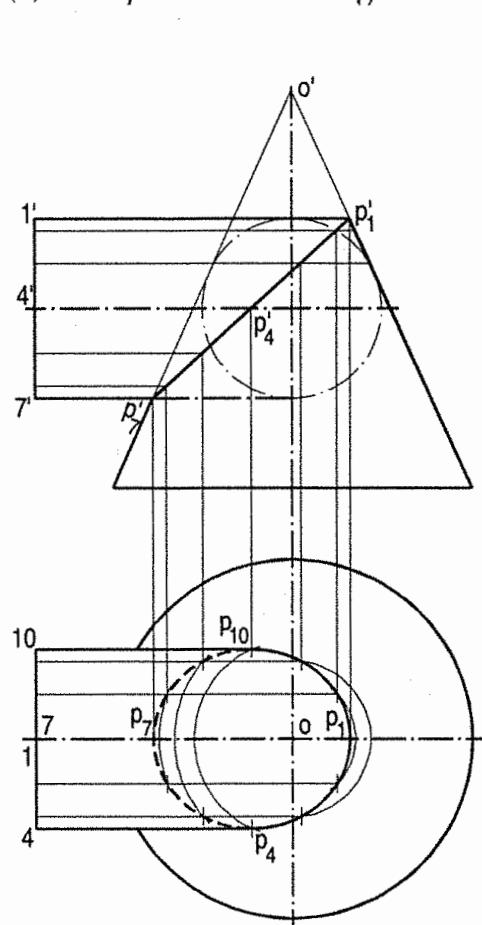


FIG. 16-29

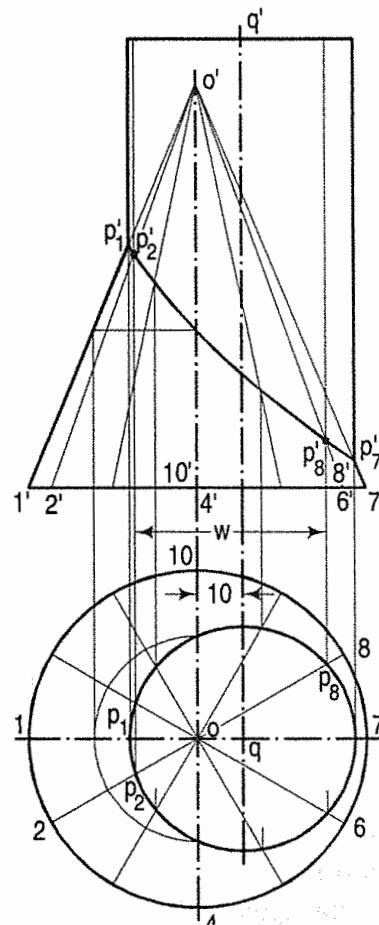


FIG. 16-30

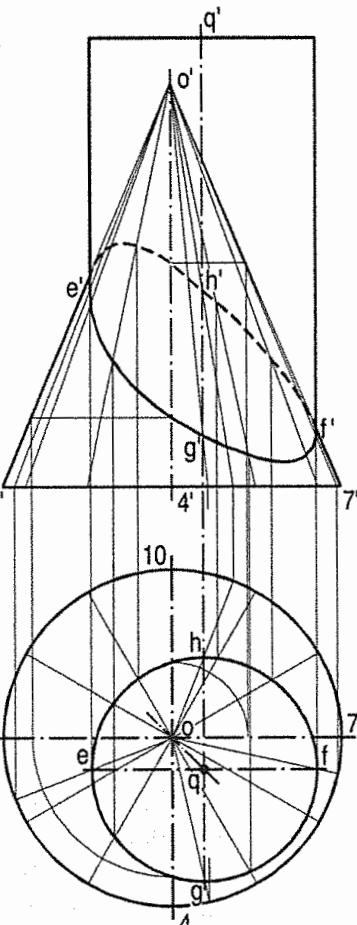


FIG. 16-31

- Draw the projections (fig. 16-30). The centre of the circle for the cylinder (in the top view) will lie on the horizontal centre line and 10 mm away from the centre of the circle for the cone. Divide the base-circle of the cone into twelve equal parts and draw twelve generators in both the views. The surface of the cylinder is seen as a circle in the top view. It cuts the lines on the surface of the cone at points p_1 , p_2 etc. Project these points to p'_1 , p'_2 etc. on the corresponding lines in the front view. Draw the required curve of intersection through these points. The curve at the back will coincide with the front curve.

The cutting-plane method is exactly similar. Assume a series of vertical cutting planes passing through the apex. The sections of the cone will be triangles and those of the cylinder will be rectangles. Points of intersection between these sections will lie on the curve. For example, take a cutting plane coinciding with the line 2-8. In the front view, the section of the cone will be shown by triangle $o'-2'-8'$, while that of the cylinder will be a rectangle of width w . Points p'_2 and p'_8 at which lines $o'2'$ and $o'8'$ cut the sides of the rectangle, lie on the curve of intersection.

- (ii) As the plane containing the two axes makes an angle of 45° with the V.P., the centre q of the circle for the cylinder will lie on a line drawn through o , inclined at 45° to xy and 10 mm away from o (fig. 16-31).

Draw the projections and adopt the same method as in the case (i). In addition to the twelve points, four key points e , f , g and h (where the extreme lines of the cylinder intersect the cone) must be located. Plot the curve which will be partly hidden. If a hole is drilled through the cone instead of a cylinder penetrating it, the curve will be fully visible.

Problem 16-28. (fig. 16-32): A vertical cylinder of 75 mm diameter is penetrated by a cone, base 75 mm diameter and axis 110 mm long, the two axes bisecting each other at right-angles. Draw the front view showing lines of intersections.

- (i) Divide the base circle of the cone into twelve equal parts and draw lines for the generators in both views. Assume a series of cutting planes perpendicular to the V.P. and passing through the apex. They will be seen as lines in the front view. The sections of the cone will be triangles and those of the cylinder will be ellipses.

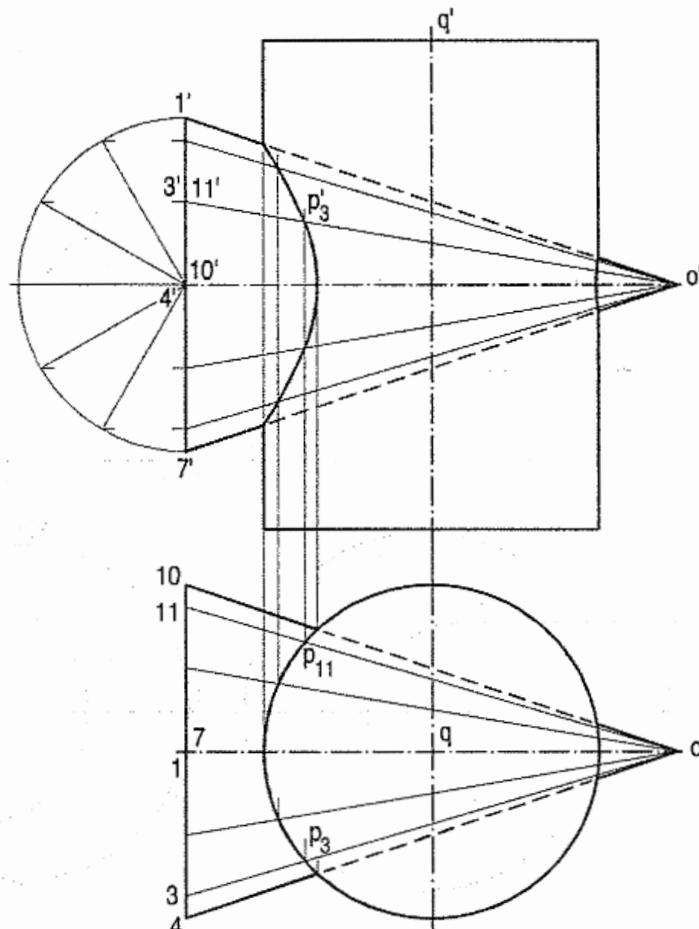


FIG. 16-32

Take a cutting plane coinciding with a line $3'o'$. In the top view, the section of the cone is shown by the triangle $o-3-11$ and that of the cylinder by the circle (the top view of the cylinder). The sides of the triangle intersect the circle at points p_3 and p_{11} .

- (ii) Project these points in the front view to p_3 on the line $3'o'$, with which $11'o'$ coincides. Obtain other points in the same manner and draw a curve through them. Points on the curve on the right-hand side of the axis of the cylinder are obtained simultaneously and in the same manner.

16-7. INTERSECTION OF CONE AND PRISM



Problem 16-29. (fig. 16-33): Draw an equilateral triangle of 100 mm side with one side horizontal. Draw a square of 35 mm side in its centre with its sides inclined at 45° to the base of the triangle. The figure shows the front view of a cone standing on its base on the ground and having a square hole cut through it. Draw three views of the cone.

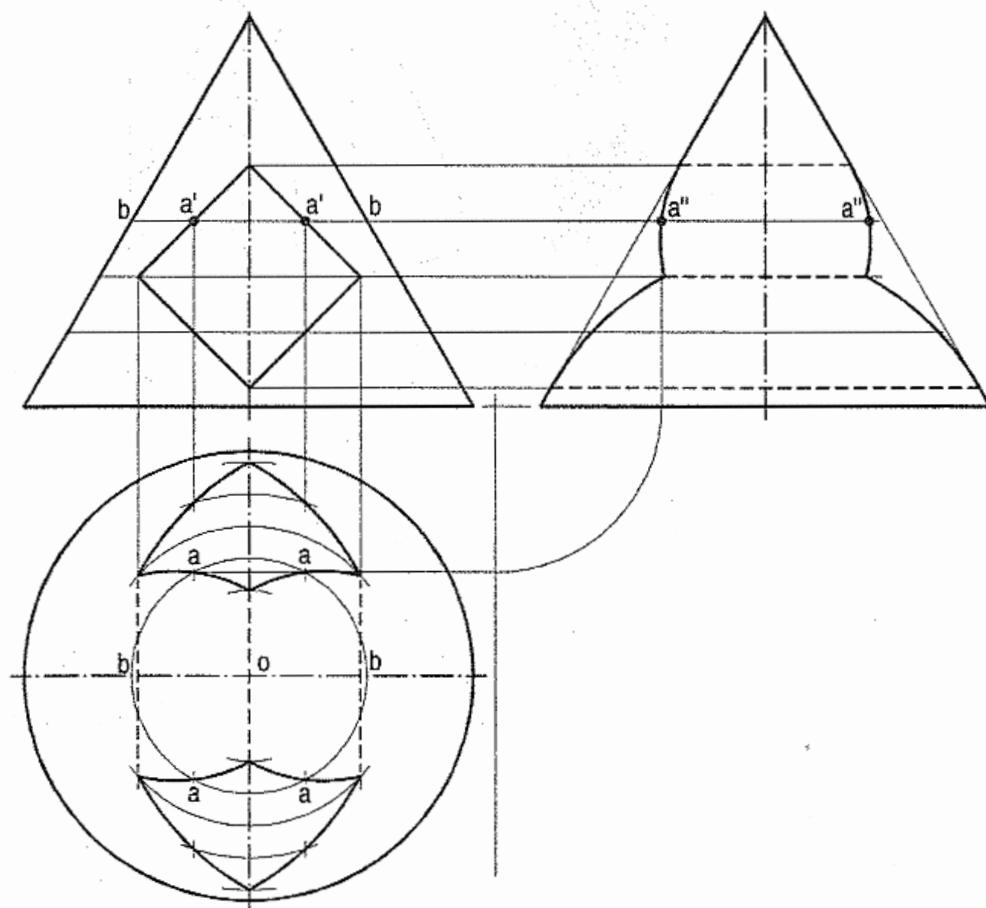


FIG. 16-33

- Project the top and side views from the given view. Assume the cone as cut by a horizontal cutting plane passing through points a' . The section of the cone will be a circle of diameter bb . The hole will be cut in two straight lines through points a' and perpendicular to the V.P.
- Therefore, with centre o (the apex in the top view) and diameter bb , draw a circle cutting the projectors through a' at points a . Assume additional cutting planes, particularly those which pass through corners of the square and find other points. Draw curves through these points.

The method of locating points a'' in the side view is clearly indicated by construction lines. Obtain all points in the same manner.

Problem 16-30. (fig. 16-34): A vertical cone, base 80 mm diameter and axis 90 mm long, is penetrated by a square prism of base 35 mm side. The axis of the prism is parallel to and 12 mm away from that of cone. Draw the projections when the plane containing the two axes is parallel to the V.P. and the faces of the prism are equally inclined to the V.P.

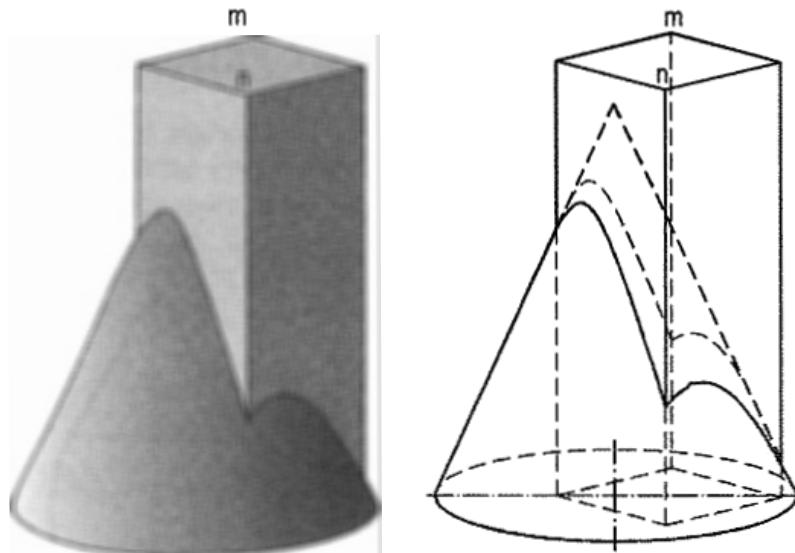


FIG. 16-34(i)

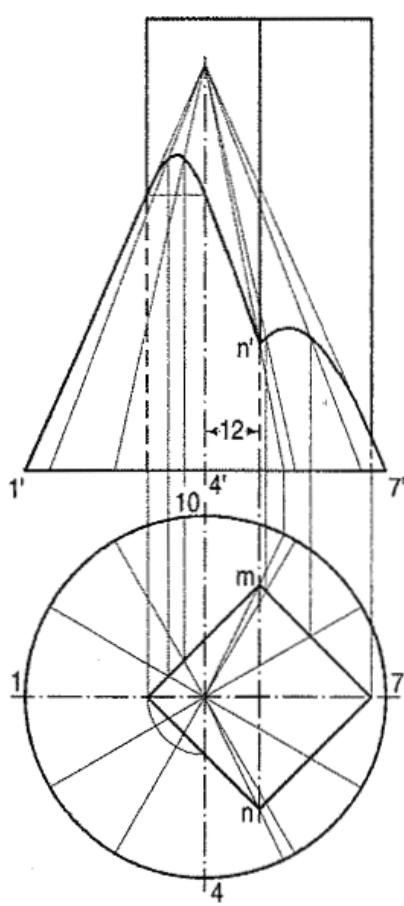
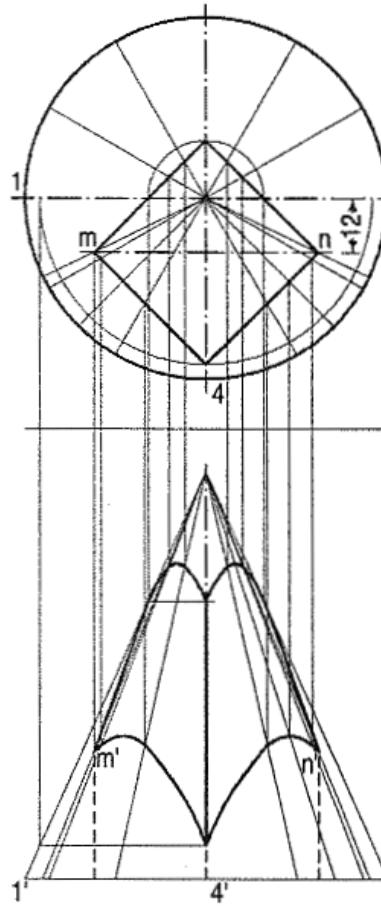
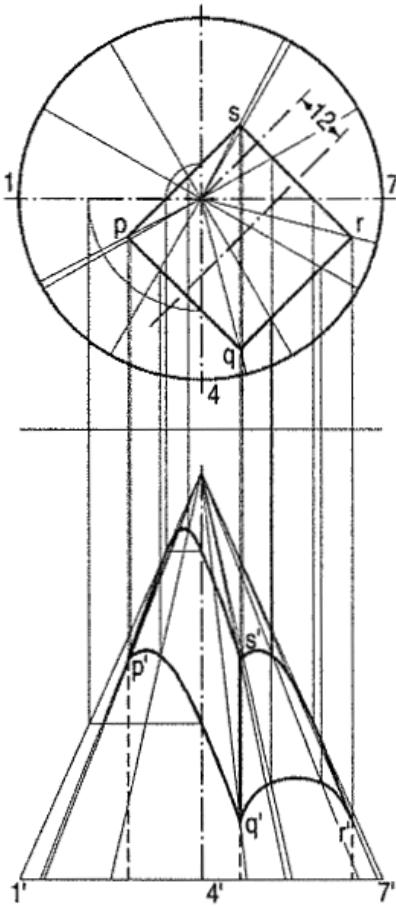


FIG. 16-34(ii)



(Third-angle projection)

FIG. 16-35



(Third-angle projection)

FIG. 16-36

Adopt the same method as shown in problem 16-27.

Fig. 16-35 and fig. 16-36 show two views of a cone having a square hole cut through it. The axes of the cone and the hole are vertical and 12 mm apart. The faces of the hole are equally inclined to the V.P. In fig. 16-35, the vertical plane containing the two axes is perpendicular to the V.P., while in fig. 16-36 it is inclined at 45° to the V.P.

16-8. INTERSECTION OF CONE AND CONE



Problem 16-31. (fig. 16-37): A vertical cone, base 90 mm diameter and axis 90 mm long, is penetrated by another cone of base 75 mm diameter and height 100 mm. The axes of the two cones bisect each other at right-angles. Draw the front view showing curves of intersection, when the axis of the penetrating cone is parallel to the V.P.

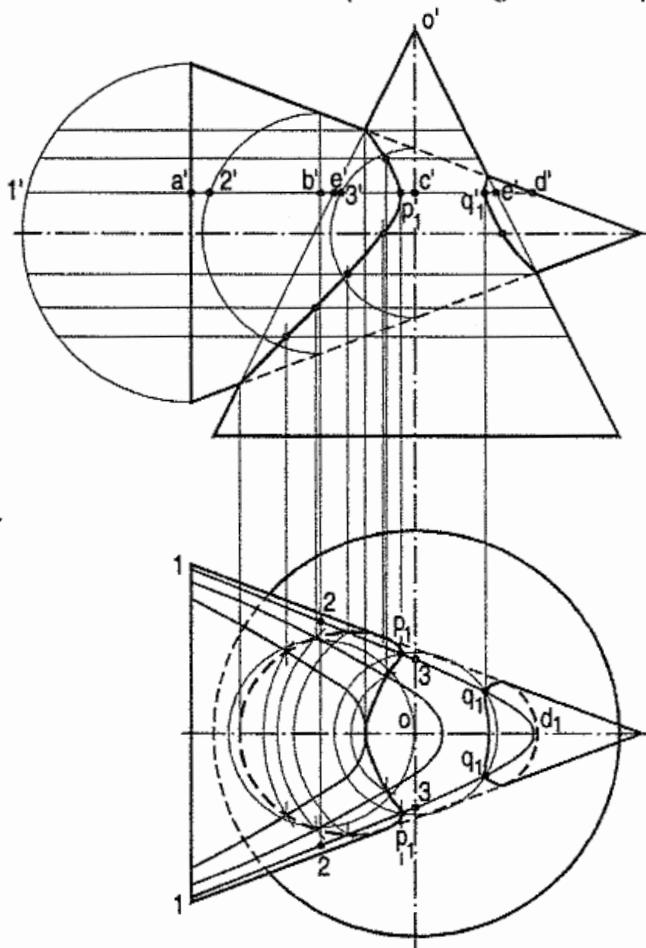


FIG. 16-37

- Assume a horizontal cutting plane coinciding with a line $a'd'$. The section of the vertical cone will be a circle of diameter ee' . The section of the horizontal cone will be a hyperbola. The width of the hyperbola at the point a' will be equal to twice the length of the line $a'1'$; that at the point b' will be twice $b'2'$ etc.
- In the top view, mark points on the projector through a' and symmetrical on both sides of the horizontal axis, so that $1-1 =$ twice $a'1'$. Similarly, mark points on the projector through b' so that $2-2 =$ twice $b'2'$ etc. Draw the hyperbola through the points thus obtained.
- Draw a circle with centre o and radius $c'e'$, cutting the hyperbola at points p_1 and q_1 . Project p_1 to p'_1 and q_1 to q'_1 on the cutting-plane line $a'd'$.

Then p_1 , q_1 , p'_1 and q'_1 are the points on the curves in the two views. Similarly, assume additional sections, preferably at equal distances on both sides of the axis (so that the sections will be the same). Draw the hyperbolas and circles in the top view and determine the points of intersection. Draw curves in both the views and on both sides of the vertical axis. The curves can be determined in the same manner even when the axes do not intersect.

16-9. INTERSECTION OF SPHERE AND CYLINDER OR PRISM



Problem 16-32. (fig. 16-38): A hole of 50 mm diameter is drilled through a sphere of 75 mm diameter. The axis of the hole is 10 mm away from the centre of the sphere. Draw three views of the sphere when a vertical plane containing the centre of the sphere and the axis of the hole is inclined at 60° to the V.P.

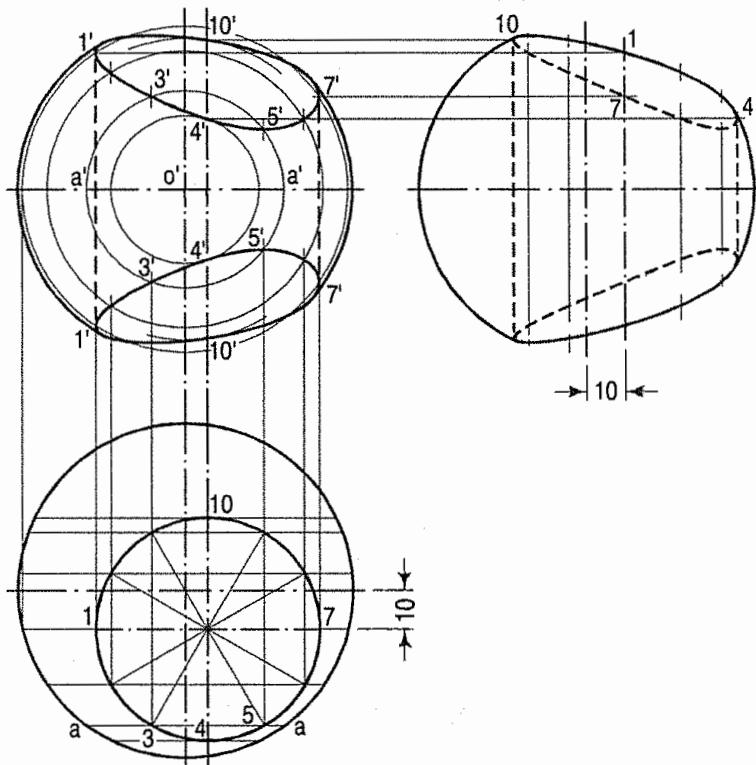


FIG. 16-38

- Draw the top view of the sphere and show the circle for the hole in the required position.
- Project the front view and the side view. Divide the circle for the hole into twelve equal parts.
- Assume a number of cutting planes parallel to the V.P. The sections of the sphere will be circles, while the hole will be cut in straight lines. Intersection of the circles with corresponding lines will give points on the curves.
- Let us take a section plane passing through points 3 and 5. The section of the sphere will be a circle of diameter aa. The lines through 3 and 5 intersect this circle at points 3' and 5' which lie on the curve. Obtain the twelve points in the same manner. Also plot the key points at which the section through the centre of the sphere is cut by the lines of the hole. Draw curves through all the points. Project the points on the side view horizontally from the front view on corresponding lines of the hole and draw curves through them.

Problem 16-33. (fig. 16-39): A sphere of 80 mm diameter is penetrated by a square prism, base 45 mm side, the axis of which passes through its centre. Draw the front view of the solids showing curves of intersection when the axis of the prism is parallel to both the planes and the faces are equally inclined to the V.P.

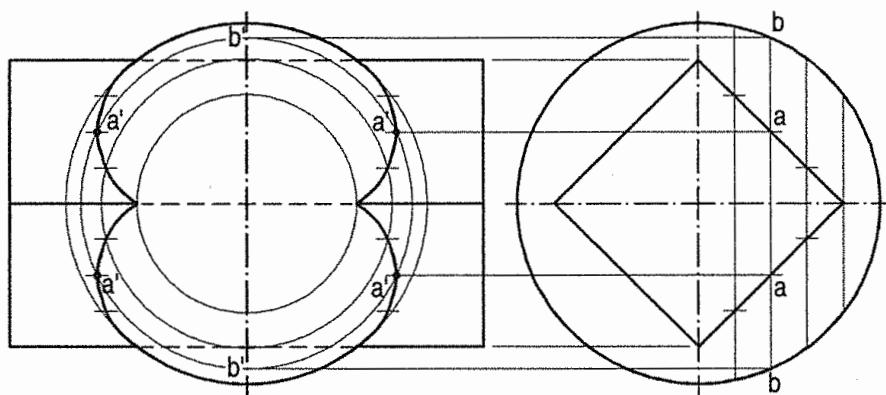


FIG. 16-39

- Draw the front view and project the side view. Assume a vertical cutting plane passing through, say points a . The section of the sphere will be a circle of diameter bb' . It is cut by the section of the prism (a rectangle of width aa') at points a' as shown in the front view. Then points a' are on the curve of intersection.
- Obtain more points together with the key points in the same manner and draw curves through them.

Problem 16-34. (fig. 16-40): Draw the development of a portion of the surface of the cone given in problem 16-22, showing the hole in it.

In fig. 16-40(i), the positions of points are taken from fig. 16-24, while in fig. 16-40(ii) they are obtained from fig. 16-25. In each case, distances along the arc are measured from the top view, while the distances of points (on the generators) from o are taken from the front view, after projecting them on the true length line.

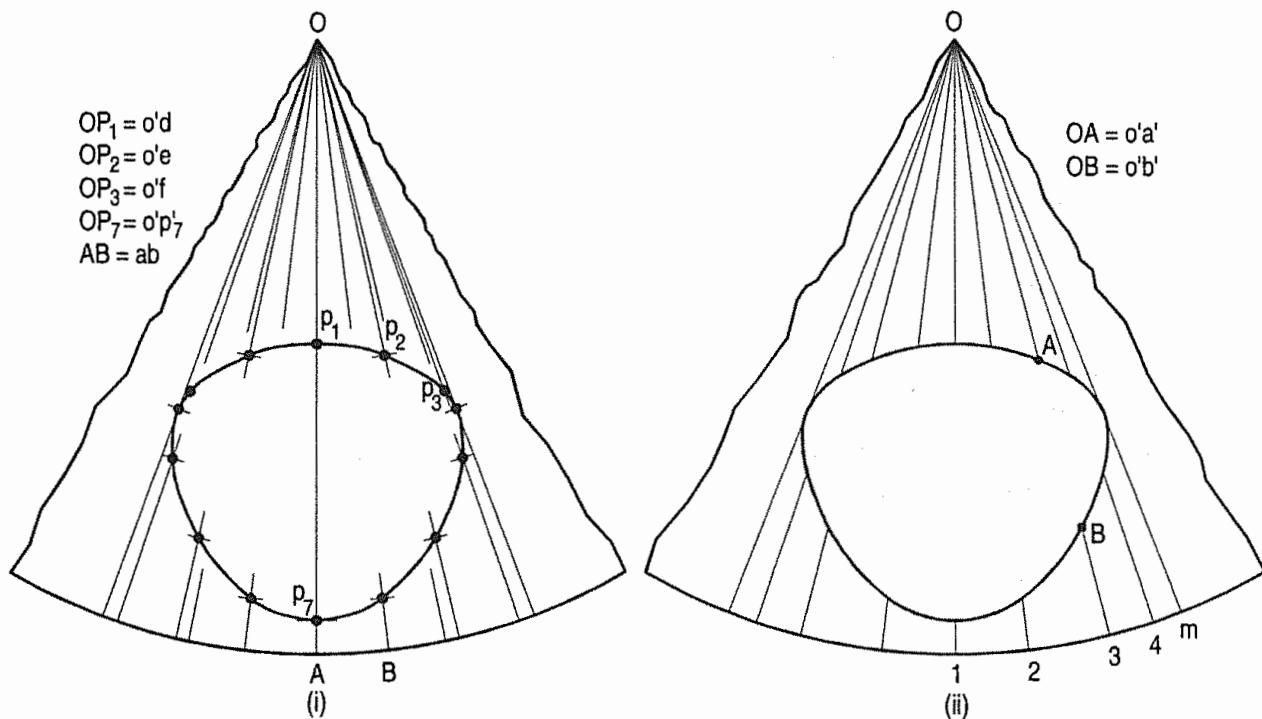


FIG. 16-40

EXERCISES 16



1. A vertical square prism, base 50 mm side has its faces equally inclined to the V.P. It is completely penetrated by another square prism of base 30 mm side, the axis of which is parallel to both the planes and is 6 mm away from the axis of the vertical prism. The faces of the horizontal prism also are equally inclined to the V.P. Draw the projections of the solids showing lines of intersection.
2. Two equal prisms whose ends are equilateral triangles of 40 mm side and axes 100 mm long, intersect at right angles. One face of each prism is on the ground. The axis of one of the prisms makes 30° with the V.P. Draw three views of the solids.
3. A square prism of base 50 mm side and height 125 mm stands on the ground with a side of the base inclined at 30° to the V.P. It is penetrated by a cylinder, 50 mm diameter and 125 mm long, whose axis is parallel to both the H.P. and the V.P. and bisects the axis of the prism. Draw the projections showing fully the curves of intersection.
4. A cylindrical boiler is 2 m in diameter and has a cylindrical dome 0.8 m diameter and 0.6 m high. The axis of the dome intersects the axis of the boiler. Draw three views of the arrangement. Develop the surface of the dome. Scale 1 cm = 0.2 m.
5. A vertical pipe, 75 mm diameter and 150 mm long, has two branches, one on each side. The horizontal branch is of 60 mm diameter while the other is of 50 mm diameter and inclined at 45° to the vertical. Assume the axis of 50 mm branch and the main pipe to be in the same plane, and that of 60 mm branch at 6 mm from the axis of the main pipe and parallel to the V.P. Draw the views of the pipe showing curves of intersection. Draw the developments of three pipes, assuming suitable lengths.
6. A cylinder of 75 mm diameter and 125 mm height, stands on its base on the ground. It is penetrated centrally by a cylinder, 50 mm diameter and 125 mm long, whose axis is parallel to the H.P. but inclined at 30° to the V.P. Draw the projections showing curves of intersection. Draw also the development of the penetrated cylinder.
7. A right circular cylinder of 75 mm diameter penetrates another of 100 mm diameter, their axes being at right angles to each other but 10 mm apart. Draw the projections of the curves of intersection on a plane parallel to the axes of the cylinders.
8. Two circular pipes of 75 mm and 50 mm diameters (inside) meet at 30° . The axes of both the pipes are in one plane and the 75 mm pipe is vertical. The thickness of the pipes is 6 mm in both cases. Draw the projections showing curves of intersection.
9. A square hole of 35 mm side is cut in a cylindrical shaft 75 mm diameter and 125 mm long. The axis of the hole intersects that of the shaft at right angles. All faces of the hole are inclined at 45° to the H.P. Draw three views of the shaft when the plane of the two axes is parallel to the V.P.
10. A cylinder of 50 mm diameter, branches off another cylinder of 75 mm diameter. The axis of the smaller cylinder is vertical and that of the other is horizontal. If the distance between the two axes is 10 mm, draw three views of the cylinders.

11. A cone frustum is 125 mm high, 85 mm diameter at the top and 115 mm diameter at the bottom. It is vertically placed and is completely penetrated by a horizontal cylinder 75 mm diameter and 125 mm long, the axis of which bisects the axis of the frustum. Draw the projections of the solids showing curves of intersection.
12. A conical funnel, 150 mm diameter at the mouth and 125 mm high has a pipe of 40 mm diameter attached at its apex. The length of the pipe is 100 mm and its axis is at right-angles to the axis of the funnel and in the same plane, with the outer side of the pipe in line with the apex of the funnel. Draw full size, the arrangement with sufficient number of views to show the joint.
13. A cone, 90 mm diameter of base, axis 110 mm long, stands on the ground and is completely penetrated by a cylinder, 50 mm diameter and 110 mm long. The axis of the cylinder is horizontal, parallel to the V.P. and passes through the axis of the cone, 75 mm from the apex. Draw the projections of both curves of intersection. Develop the surface of the cone.
14. A hole of 25 mm diameter is drilled in a cone having 75 mm diameter of base and 60 mm height. The axis of the hole is parallel to that of the cone and 6 mm away from it. Draw three views of the cone when a vertical plane containing the two axes is perpendicular to the V.P.
15. A conical hopper is fitted in vertical position on a horizontal pipe, 75 mm in diameter and 150 mm in length. The diameter of the hopper at the top is 110 mm and changes by 25 mm in every 25 mm of vertical distance. Draw the projections showing the curves of intersection if the top of the hopper is 70 mm above the axis of the pipe. Develop the surfaces of the hopper and the pipe.
16. A cylinder, 50 mm diameter of base and 100 mm height is centrally penetrated by a cone, 50 mm diameter of base and 75 mm height. The axis of the cylinder which is vertical, cuts the axis of the cone which is horizontal at 30 mm from the base of the cone. Draw the front view and the side view, showing curves of penetration.
17. A cone, base 75 mm diameter and axis 100 mm long, has an equilateral triangular hole of 40 mm side cut through it. The axis of the hole coincides with the axis of the cone. Draw three views of the cone when it is resting on its base on the ground with a face of the hole parallel to the V.P. Develop the surface of the cone.
18. A vertical cone, base 100 mm diameter and height 125 mm is penetrated by another cone, 50 mm diameter and axis 100 mm long. The axis of the penetrating cone is parallel to the H.P. and the V.P., 40 mm above the base and 10 mm from the axis of the vertical cone. It comes out equally on both sides of the cone. Draw the projections showing curves of intersection.
19. A sphere of 100 mm diameter is penetrated vertically by a triangular prism of base 40 mm side. Their axes coincide with each other. Draw the curves of penetration when two faces of the prism are equally seen in the front view.
20. A cylinder, 50 mm diameter and axis vertical, penetrates a pentagonal pyramid having its base on the ground and one side of the base parallel to the V.P. The diameter of the circumscribing circle of the base of the pyramid is 75 mm and its height is 100 mm. The two axes are 10 mm apart. Draw the projections showing curves of intersection, when the plane containing the two axes is parallel to the V.P. Develop the surface of the pyramid.

21. A rectangular slot for a cotter is cut in the tapering portion of a piston rod as shown in fig. 16-41. Draw three views of the rod, showing the curves of penetration in each view.

22. A frustum of a cone appears in the front view as a trapezium of 50 mm and 75 mm parallel sides, 90 mm apart. A cylindrical pipe joins the frustum and in the side view, the circle representing the pipe is found to touch the 50 mm side and the sides of the frustum. Draw three views of the frustum and the portion of the pipe joined as stated.

23. In a circle of 50 mm radius, draw a square of 30 mm side symmetrically around its centre. The figure is the top view of a sphere completely penetrated by a 75 mm long square prism. From this top view, project a front view on a reference line xy parallel to a diagonal of the square. From this front view, project another top view on a reference line making 45° angle with the axis of the prism.

24. A square prism of 50 mm side of a base and 100 mm length of axis is resting on its base on the ground having a side of base at 30° to the V.P. It is completely penetrated by a horizontal square prism of 40 mm side of a base and 100 mm length of axis; the axis of which is parallel to the V.P. and bisects the axis of the vertical prism at right-angle. Draw the projections of the solids showing the lines of intersection when a face of the penetrating solid is at 30° to the H.P.

25. A vertical cylinder of 60 mm diameter is penetrated by a square prism of 35 mm side, the axis of which is inclined at 30° to the ground but parallel to the V.P. The faces of the prism are equally inclined to the V.P. and the axis of the prism is 10 mm in front of the axis of the cylinder. Draw the projections of the solids showing the curves of interpenetration.

26. A cone, diameter of base 70 mm and axis 70 mm long is kept on the ground on its base. A vertical hole of 50 mm is drilled through the cone in such a way that the axis of the hole is 10 mm away from that of the cone. The plane containing both axes is parallel to the V.P. Draw three views of the solids.

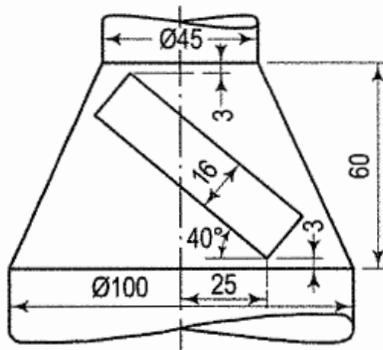


FIG. 16-41

Chapter

17



ISOMETRIC PROJECTION

17-1. INTRODUCTION

Isometric projection is a type of pictorial projection in which the three dimensions of a solid are not only shown in one view, but their actual sizes can be measured directly from it.

If a cube is placed on one of its corners on the ground with a solid diagonal perpendicular to the V.P., the front view is the *isometric projection* of the cube. The step-by-step construction is shown in fig. 17-1.

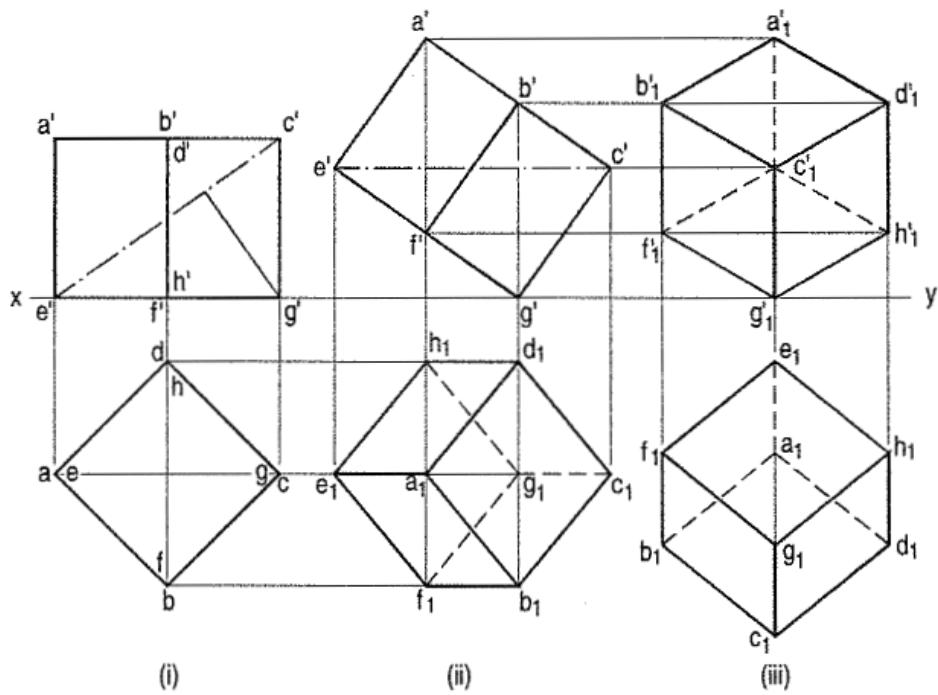


FIG. 17-1

To draw the projections of a cube of 25 mm long edges resting on the ground on one of its corners with a solid diagonal perpendicular to the V.P., assume the cube to be resting on one of its faces on the ground with a solid diagonal parallel to the V.P.

- (i) Draw a square $abcd$ in the top view with its sides inclined at 45° to xy . The line ac representing the solid diagonals AG and CE is parallel to xy . Project the front view.

- (ii) Tilt the front view about the corner g' so that the line $e'c'$ becomes parallel to xy . Project the second top view. The solid diagonal CE is now parallel to both the H.P. and the V.P.
- (iii) Reproduce the second top view so that the top view of the solid diagonal, viz. $e_1 c_1$ is perpendicular to xy . Project the required front view.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 39 for the introduction.

Fig. 17-2 shows the front view of the cube in the above position, with the corners named in capital letters. Its careful study will show that:

- (a) All the faces of the cube are equally inclined to the V.P. and hence, they are seen as similar and equal rhombuses instead of squares.
- (b) The three lines CB , CD and CG meeting at C and representing the three edges of the solid right-angle are also equally inclined to the V.P. and are therefore, equally foreshortened. They make equal angles of 120° with each other. The line CG being vertical, the other two lines CB and CD make 30° angle each, with the horizontal.
- (c) All the other lines representing the edges of the cube are parallel to one or the other of the above three lines and are also equally foreshortened.
- (d) The diagonal BD of the top face is parallel to the V.P. and hence, retains its true length.

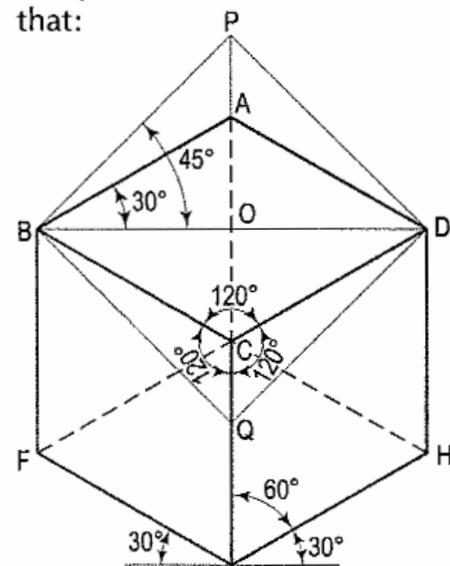


FIG. 17-2

This chapter deals with various topics of isometric projection as shown below:

1. Isometric axes, lines and planes
2. Isometric scale
3. Isometric drawing or isometric view
4. Isometric graph.

17-2. ISOMETRIC AXES, LINES AND PLANES

The three lines CB , CD and CG meeting at the point C and making 120° angles with each other are termed *isometric axes*. The lines parallel to these axes are called *isometric lines*. The planes representing the faces of the cube as well as other planes parallel to these planes are called *isometric planes*.

17-3. ISOMETRIC SCALE

As all the edges of the cube are equally foreshortened, the square faces are seen as rhombuses. The rhombus $ABCD$ (fig. 17-2) shows the isometric projection of the top square face of the cube in which BD is the true length of the diagonal.

Construct a square $BQDP$ around BD as a diagonal. Then BP shows the true length of BA .

$$\text{In triangle } ABO, \frac{BA}{BO} = \frac{1}{\cos 30^\circ} = \frac{2}{\sqrt{3}}$$

$$\text{In triangle } PBO, \frac{BP}{BO} = \frac{1}{\cos 45^\circ} = \frac{\sqrt{2}}{1}$$

$$\frac{BA}{BP} = \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}} = 0.815$$

$$\text{The ratio, } \frac{\text{isometric length}}{\text{true length}} = \frac{BA}{BP} = \frac{\sqrt{2}}{\sqrt{3}} = 0.815 \text{ or } \frac{9}{11} \text{ (approx.)}.$$

Thus, the isometric projection is reduced in the ratio $\sqrt{2} : \sqrt{3}$, i.e. the isometric lengths are 0.815 of the true lengths.

Therefore, while drawing an isometric projection, it is necessary to convert true lengths into isometric lengths for measuring and marking the sizes. This is conveniently done by constructing and making use of an isometric scale as shown below.

- (a) Draw a horizontal line BD of any length (fig. 17-3). At the end B , draw lines BA and BP , such that $\angle DBA = 30^\circ$ and $\angle DBP = 45^\circ$. Mark divisions of true length on the line BP and from each division-point, draw verticals to BD meeting BA at respective points. The divisions thus obtained on BA give lengths on isometric scale.

- (b) The same scale may also be drawn with divisions of natural scale on a horizontal line AB (fig. 17-4). At the ends A and B , draw lines AC and BC making 15° and 45° angles with AB respectively, and intersecting each other at C .

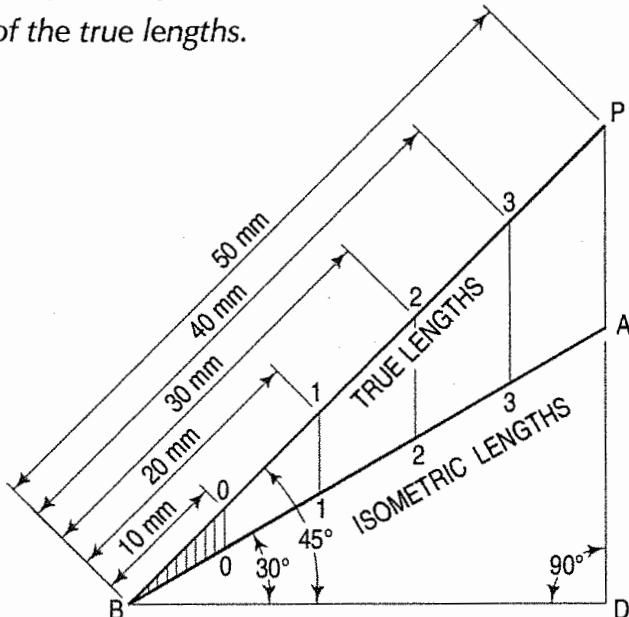


FIG. 17-3

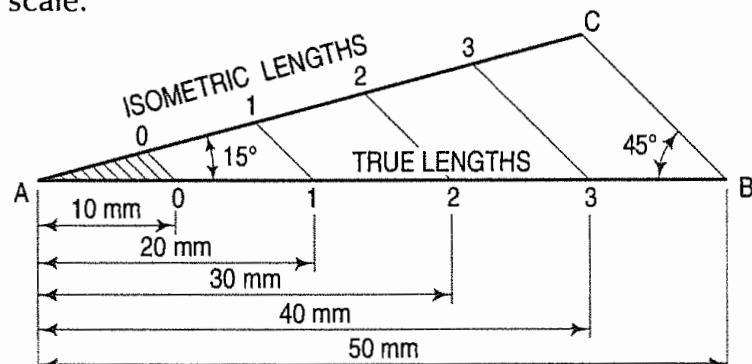


FIG. 17-4

From division-points of true lengths on AB , draw lines parallel to BC and meeting AC at respective points. The divisions along AC give lengths to isometric scale.

The lines BD and AC (fig. 17-2) represent equal diagonals of a square face of the cube, but are not equally shortened in isometric projection. BD retains its true length, while AC is considerably shortened. Thus, it is seen that lines which are not parallel to the isometric axes are not reduced according to any fixed ratio. Such lines are called non-isometric lines. The measurements should, therefore, be made on isometric axes and isometric lines only. The non-isometric lines are drawn by locating positions of their ends on isometric planes and then joining them.

17-4. ISOMETRIC DRAWING OR ISOMETRIC VIEW

If the foreshortening of the isometric lines in an isometric projection is disregarded and instead, the true lengths are marked, the view obtained [fig. 17-5(iii)] will be exactly of the same shape but larger in proportion (about 22.5%) than that obtained by the use of the isometric scale [fig. 17-5(ii)]. Due to the ease in construction and the advantage of measuring the dimensions directly from the drawing, it has become a general practice to use the true scale instead of the isometric scale.

To avoid confusion, the view drawn with the true scale is called *isometric drawing* or *isometric view*, while that drawn with the use of isometric scale is called *isometric projection*.

Referring again to fig. 17-2, the axes *BC* and *CD* represent the sides of a right angle in horizontal position. Each of them together with the vertical axis *CG*, represents the right angle in vertical position. Hence, in isometric view of any rectangular solid resting on a face on the ground, each horizontal face will have its sides parallel to the two sloping axes; each vertical face will have its vertical sides parallel to the vertical axis and the other sides parallel to one of the sloping axes.

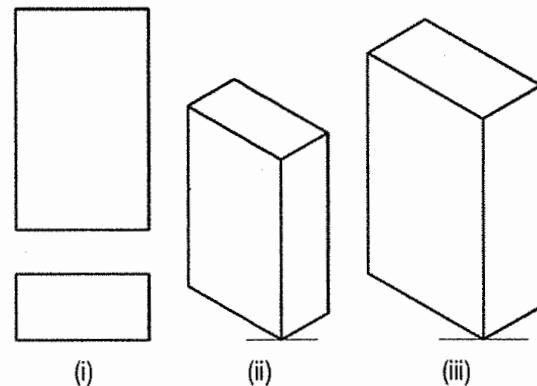


FIG. 17-5

In other words, the vertical edges are shown by vertical lines, while the horizontal edges are represented by lines, making 30° angles with the horizontal. These lines are very conveniently drawn with the T-square and a 30° - 60° set-square or drafter.

17-5. ISOMETRIC GRAPH

An isometric graph as shown in fig. 17-6 facilitates the drawing of isometric view of an object. Students are advised to make practice for drawing of isometric view using such graphs. See fig. 17-55 and fig. 17-56 of problem 17-33.

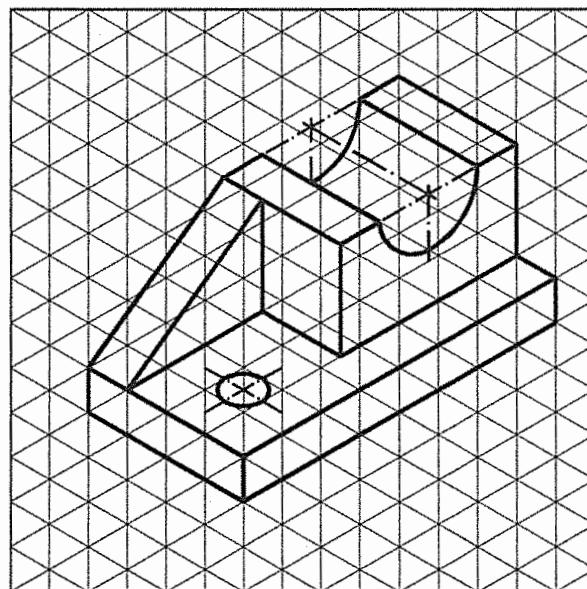


FIG. 17-6

17-6. ILLUSTRATIVE PROBLEMS

The procedure for drawing isometric views of planes, solids and objects of various shapes is explained in stages by means of illustrative problems.

In order that the construction of the view may be clearly understood, construction lines have not been erased. They are, however, drawn fainter than the outlines.

In an isometric view, lines for the hidden edges are generally not shown. In the solutions accompanying the problems, one or two arrows have been shown. They indicate the directions from which if the drawing is viewed, the given orthographic views would be obtained. Students need not show these arrows in their solutions.

17-6-1. ISOMETRIC DRAWING OF PLANES OR PLANE FIGURES



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 40 for the following problem.

Problem 17-1. The front view of a square is given in fig. 17-7(i). Draw its isometric view.

As the front view is a square, the surface of the square is vertical. In isometric view, vertical lines will be drawn vertical, while horizontal lines will be drawn inclined at 30° to the horizontal.

- Through any point d , draw a vertical line $da = DA$ [fig. 17-7(ii)].
- Again through d , draw a line $dc = DC$ inclined at 30° to the horizontal and at 60° to da .
- Complete the rhombus $abcd$ which is the required isometric view. The view can also be drawn in direction of the other sloping axis as shown in fig. 17-7(iii).

Problem 17-2. If fig. 17-7(i) is the top view of a square, draw its isometric view.

As the top view is a square, the surface of the square is horizontal. In isometric view, all the sides will be drawn inclined at 30° to the horizontal.

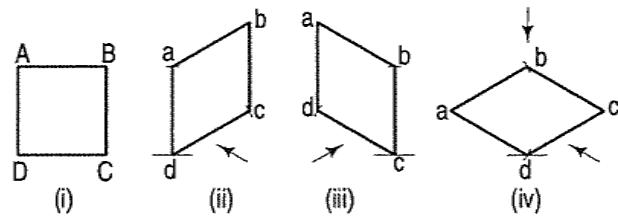


FIG. 17-7

- From any point d [fig. 17-7(iv)], draw two lines da and dc inclined at 30° to the horizontal and making 120° angle between themselves.
- Complete the rhombus $abcd$ which is the required isometric view.

Problem 17-3. The top view of a rectangle, the surface of which is horizontal is shown in fig. 17-8(i). Draw its isometric view.

Draw the required view as explained in problem 17-2 and as shown in either fig. 17-8(ii) or fig. 17-8(iii).

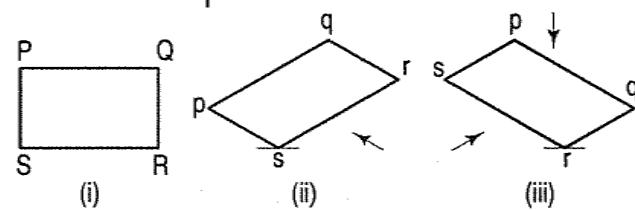


FIG. 17-8

Problem 17-4. The front view of a triangle having its surface parallel to the V.P. is shown in fig. 17-9(i). Draw its isometric view.

The surface of the triangle is vertical and the base ab is horizontal. ab will be drawn parallel to a sloping axis. The two sides of the triangle are inclined.

Hence they will not be drawn parallel to any isometric axis. In an isometric view, angles do not increase or decrease in any fixed proportion. They are drawn after determining the positions of the ends of the arms on isometric lines.

Therefore, enclose the triangle in the rectangle $ABQP$. Draw the isometric view $abqp$ of the rectangle [fig. 17-9(ii)].

Mark a point c in pq such that $pc = PC$. Draw the triangle abc which is the required isometric view. It can also be drawn in the other direction as shown in fig. 17-9(iii).

Problem 17-5. The front view of a quadrilateral whose surface is parallel to the V.P. is shown in fig. 17-10(i). Draw its isometric view.

Enclose the quadrilateral in a rectangle $ABEF$.

- Draw lines CP and CQ parallel to the sides FE and BE respectively.
- Draw the isometric view of the rectangle [fig. 17-10(ii)] and obtain the point d in fe as explained in problem 17-4. Draw the isometric view $cpeq$ of the rectangle $CPEQ$.
- Draw the quadrilateral $abcd$ which is the required isometric view.

If the given view is the top view of a quadrilateral whose surface is horizontal, i.e. parallel to the H.P., its isometric view will be as shown in fig. 17-10(iii).

Problem 17-6. If the view given in fig. 17-11(i) is

- the front view of a hexagon whose surface is parallel to the V.P. or
- the top view of the hexagon whose surface is horizontal, draw its isometric views.
- (a) fig 17-11(ii), (b) fig 17-11(iii).

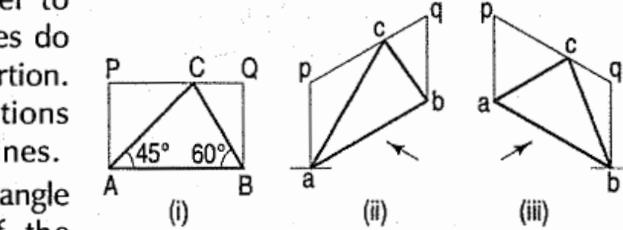


FIG. 17-9

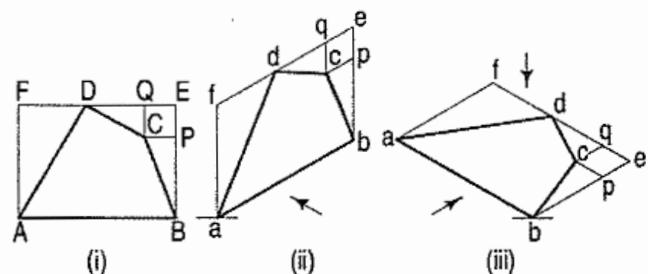


FIG. 17-10

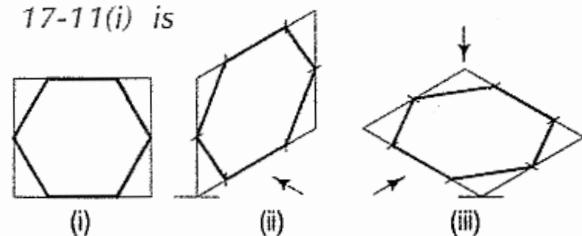


FIG. 17-11

In both cases, the views can be drawn in the other direction also.

Problem 17-7. Fig. 17-12(i) shows the front view of a circle whose surface is parallel to the V.P. Draw the isometric view of the circle.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 41 for the following problem.

I. Method of points:

- Enclose the circle in a square, touching it in points 1, 2, 3 and 4. Draw the diagonals of the square cutting the circle in points 5, 6, 7 and 8.
- Draw the isometric view of the square [fig. 17-12(ii) and [fig. 17-12(iii)]] and on it mark the mid-points 1, 2, 3 and 4 of its sides. Obtain points 5, 6, 7 and 8 on the diagonals as explained in problem 17-5.

Or, after determining the position of one point, draw through it, lines parallel to the sides of the rhombus and obtain the other three points. Draw a neat and smooth curve passing through the eight points viz. 1, 6, 2, 7 etc. The curve is the required isometric view. It is an ellipse.

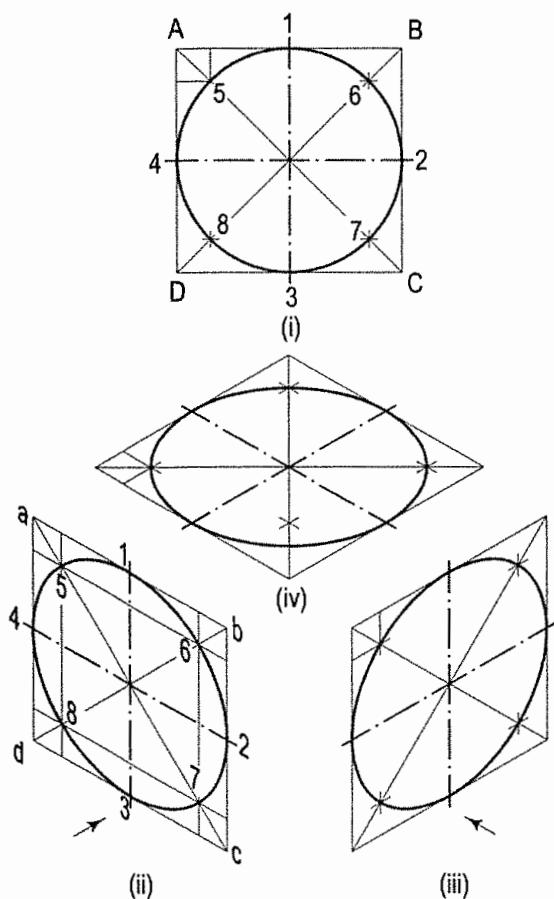


FIG. 17-12

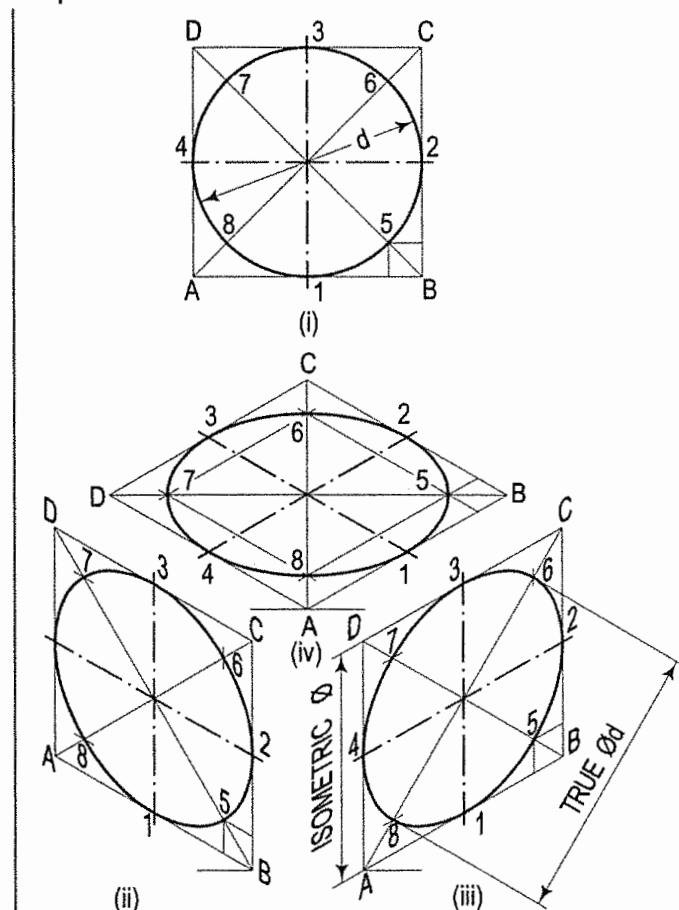


FIG. 17-13

If the view given in fig. 17-12(i) is the top view of a circle whose surface is horizontal, its isometric view will be as shown in fig. 17-12(iv).

As the isometric views have been drawn with the true scale, the major axis of the ellipse is longer than the diameter of the circle.

Fig. 17-13(ii), fig. 17-13(iii) and fig. 17-13(iv) show the isometric projection of the circle drawn with isometric scale. Note that when the length of the side of the rhombus is equal to the isometric diameter of the circle, the length of the major axis of the ellipse is equal to the true diameter of the circle.

II. Four-centre method:

Draw the isometric view of the square [fig. 17-14(i)]. Draw perpendicular bisectors of the sides of the rhombus, intersecting each other on the longer diagonal at points p and q , and which meet at the 120° -angles b and d .

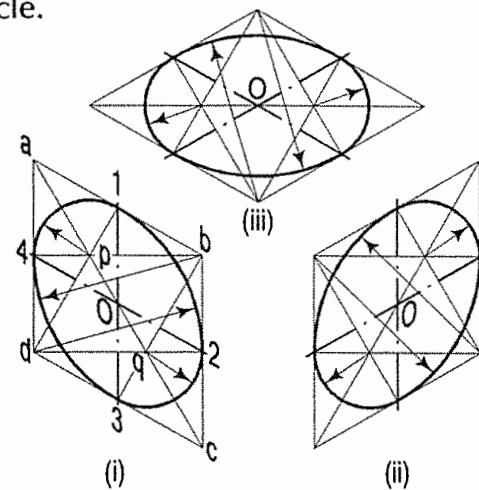


FIG. 17-14

Or, draw lines joining the 120° -angles b and d with the mid-points of the opposite sides and intersecting each other on the longer diagonal at points p and q . Two of these

lines will be drawn horizontal, while the other two will make 60° -angles with the horizontal. With centres b and d , draw arcs 3-4 and 1-2 respectively. With centres p and q , draw arcs 1-4 and 2-3 respectively and complete the required ellipse. Fig. 17-14(ii) shows the ellipse obtained in the rhombus drawn in the direction of the other sloping axis. Fig. 17-14(iii) shows the isometric view of the circle when its surface is horizontal.

The ellipse obtained by the four-centre method is not a true ellipse and differs considerably in size and shape from the ellipse plotted through points. But owing to the ease in construction and to avoid the labour of drawing freehand neat curves, this method is generally employed.

Problem 17-8. To draw the isometric view of a circle of a given diameter, around a given point.

Let O be the given point and D the diameter of the circle.

(a) When the surface of the circle is vertical [fig. 17-14(i)].

- Through O , draw a vertical centre line and another centre line inclined at 30° to the horizontal, i.e. parallel to a sloping isometric axis. On these lines, mark points 1, 2, 3 and 4 at a distance equal to $0.5D$ from O .
- Through these points, draw lines parallel to the centre lines and obtain the rhombus $abcd$ of sides equal to D .
- Draw the required ellipse in this rhombus by the four-centre method.

By drawing the second centre line parallel to the other sloping axis, the isometric view is obtained in another position as shown in fig. 17-14(ii).

(b) When the surface of the circle is horizontal [fig. 17-14(iii)].

Through O , draw the two centre lines parallel to the two sloping isometric axes, i.e. inclined at 30° to the horizontal. Draw the required ellipse as explained in (a) above.

Note: This construction is very useful in drawing isometric views of circular holes in solids.

Problem 17-9. Fig. 17-15(i) shows the front view of a semi-circle whose surface is parallel to the V.P. Draw its isometric view.

- Enclose the semi-circle in a rectangle. Draw the isometric view of the rectangle [fig. 17-15(ii) and [fig. 17-15(iii)].
- Using the four-centre method, draw the half-ellipse in it which is the required view. The centre for the longer arc may be obtained as shown or by completing the rhombus.

If the view given in fig. 17-15(i) is the top view of a horizontal semi-circle, its isometric view would be drawn as shown in fig. 17-16(i) and fig. 17-16(ii).

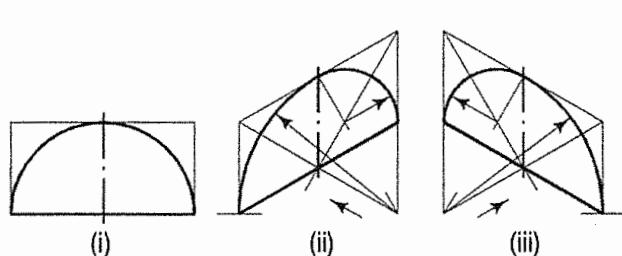


FIG. 17-15

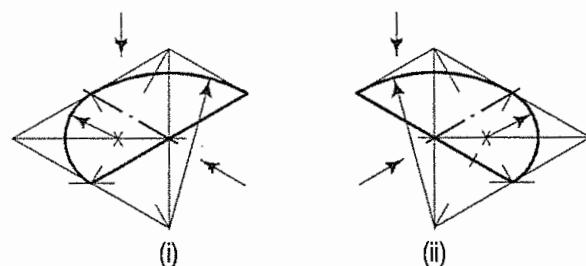


FIG. 17-16

Problem 17-10. Fig. 17-17(i) shows the front view of a semi-circle whose surface is parallel to the V.P. Draw the isometric view of the semi-circle.

See fig. 17-17(ii) and fig. 17-17(iii).

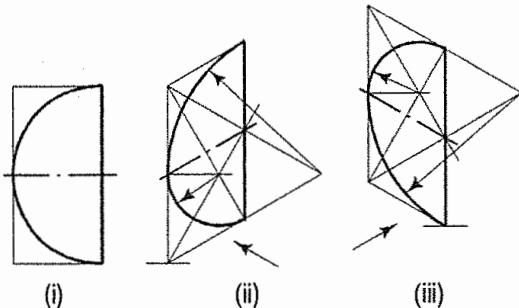


FIG. 17-17

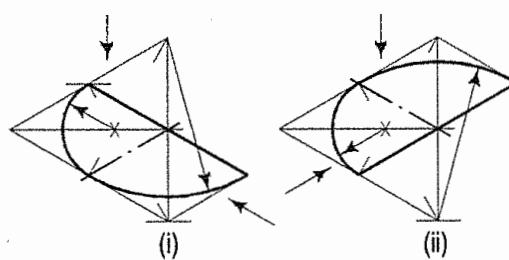
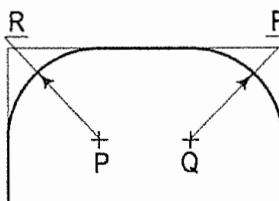


FIG. 17-18

If the view shown in fig. 17-17(i) is the top view of a semi-circle whose surface is horizontal, its isometric view will be as shown in fig. 17-18(i) or fig. 17-18(ii).

Problem 17-11. Fig. 17-19(i) shows the front view of a plane parallel to the V.P. Draw its isometric view.

- The upper two corners of the plane are rounded with quarter circles. Enclose the plane in a rectangle.



(i)

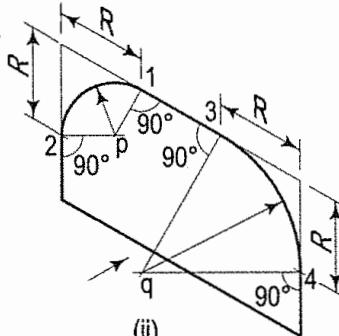


FIG. 17-19

- Draw the isometric view of the rectangle. From the upper two corners of the parallelogram, mark points on the sides at a distance equal to R , the radius of the arcs. At these points erect perpendiculars to the respective sides to intersect each other at points p and q . With p and q as centres, and radii p_1 and q_3 , draw the arcs and complete the required view.

It is interesting to note that although the arcs are of the same radius, they are drawn with different radii in their isometric views.

17-6-2. ISOMETRIC DRAWING OF PRISMS AND PYRAMIDS



We have seen that the isometric view of a cube is determined from its orthographic view in a particular position. The three edges of the solid right-angle of the cube are shown by lines parallel to the three isometric axes. A square prism or a rectangular prism also has solid right-angles. Hence, lines for its edges are also drawn parallel to the three isometric axes.

While drawing the isometric view of any solid, the following important points should be carefully noted:

- The isometric view should be drawn according to the given views and in such a way that maximum possible details are visible.
- At every point for the corner of a solid, at least three lines for the edges must converge. Of these, at least two must be for visible edges. Lines for the hidden edges need not be shown, but it is advisable to check up every corner so that no line for a visible edge is left out.
- Two lines (for visible edges) will never cross each other.

Problem 17-12. Draw the isometric view of a square prism, side of the base 20 mm long and the axis 40 mm long, when its axis is (i) vertical and (ii) horizontal.

- (i) When the axis is vertical, the ends of the prism will be horizontal. Draw the isometric view (the rhombus 1-2-3-4) of the top end [fig. 17-20(i)]. Its sides will make 30° -angles with the horizontal. The length of the prism will be drawn in the third direction, i.e. vertical. Hence, from the corners of the rhombus, draw vertical lines 1-5, 2-6 and 3-7 of length equal to the length of the axis. The line 4-8 should not be drawn, as that edge will not be visible. Draw lines 5-6 and 6-7, thus completing the required isometric view. Lines 7-8 and 8-5 also should not be drawn. Beginning may also be made by drawing lines from the point 6 on the horizontal line and then proceeding upwards.
- (ii) When the axis is horizontal, the ends will be vertical. The ends can be drawn in two ways as shown in fig. 17-20(ii) and fig. 17-20(iii). In each case, the length is shown in the direction of the third isometric axis.

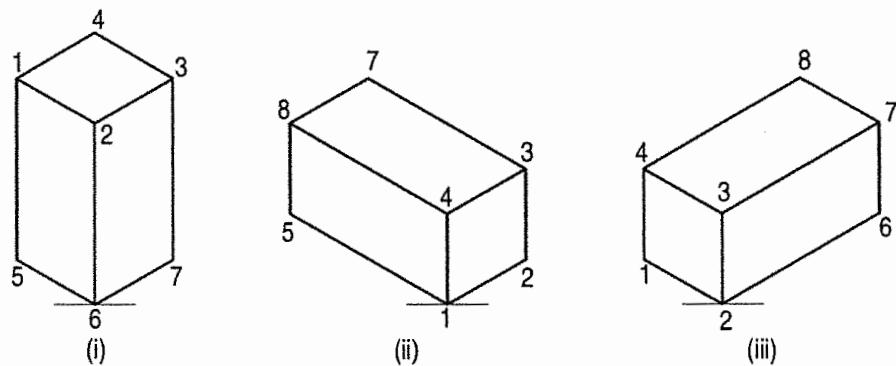
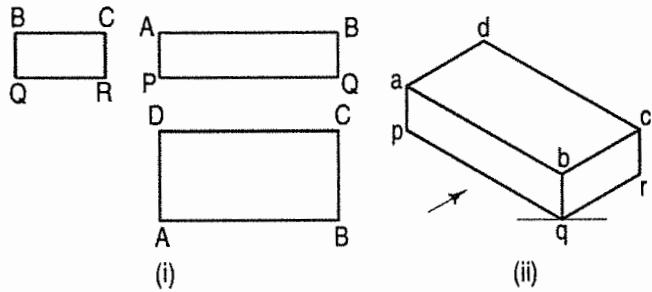


FIG. 17-20

Problem 17-13. Three views of a block are given in fig. 17-21(i). Draw its isometric view.

The block is in the form of a rectangular prism. Its shortest edges are vertical. Lines for these edges will be drawn vertical. Lines for all other edges which are horizontal, will be drawn inclined at 30° to the horizontal in direction of the two sloping axes as shown in fig. 17-21(ii).



Methods of drawing non-isometric lines.

FIG. 17-21

When an object contains inclined edges which in the isometric view would be shown by non-isometric lines, the view may be drawn by using any one of the following methods:

- (i) box method or
- (ii) co-ordinate or offset method.

(i) **Box method:** This method is used when the non-isometric lines or their ends lie in isometric planes. The object is assumed to be enclosed in a rectangular box. Initially, the box is drawn in isometric. The ends of the lines for the inclined edges are then located by measuring on or from the outlines of the box.

Problem 17-14. Three views of a block are given in fig. 17-22(i). Draw its isometric view.

- Enclose the block in a rectangular box.
- Draw the isometric view of the box [fig. 17-22(ii)].
- Mark points e and f on the line bc such that $be = BE$ and $fc = FC$.
- Complete the required view as shown.

Problem 17-15. Draw the isometric view of the plate shown in three views in fig. 17-23(i).

Obtain the required view as explained in problem 17-14 and as shown in fig. 17-23(ii).



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 42 for the following problem.

Problem 17-16. Draw the isometric view of the frustum of the hexagonal pyramid shown in fig. 17-24(i).

- Enclose the front view and the top view in rectangles.
- Draw the isometric view of the rectangular box [fig. 17-24(ii)]. Locate the six points of the base of the frustum on the sides of the bottom of the box. The upper six points on the top surface of the box are located by drawing isometric lines, e.g. P_1 and Q_1 intersecting at a point 1.
- Join the corners and complete the isometric view as shown.

(ii) **Co-ordinate or Offset method:** This method is adopted for objects in which neither non-isometric lines nor their ends lie in isometric planes.

Perpendiculars are dropped from each end of the edge to a horizontal or a vertical reference plane. The points at which the perpendiculars meet the plane, are located by drawing co-ordinates or offsets to the edges of the plane.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 43 for the following problem.

Problem 17-17. Draw the isometric view of the pentagonal pyramid, the projections of which are given in fig. 17-25(i).

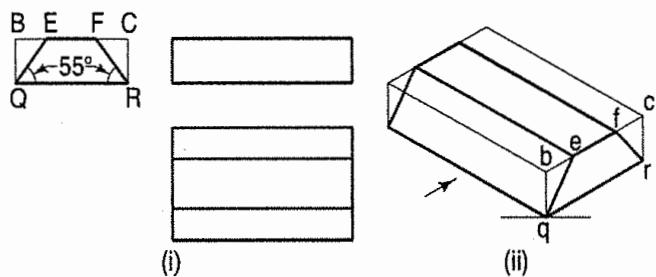


FIG. 17-22

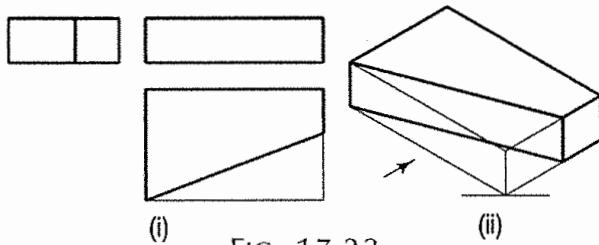


FIG. 17-23

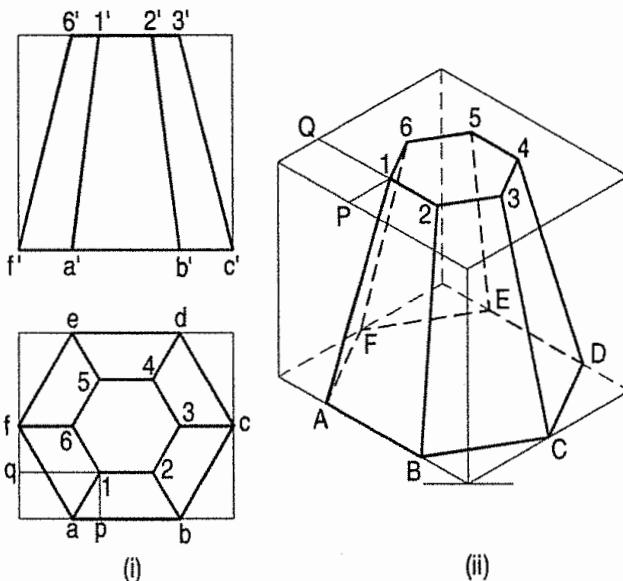


FIG. 17-24

- Enclose the base (in the top view) in an oblong.
- Draw an offset oq (i.e. pq) on the line ab .
- Draw the isometric view of the oblong and locate the corners of the base in it [fig. 17-25(ii)].
- Mark a point Q on the line AB such that $AQ = aq$. From Q , draw a line QP equal to qo and parallel to $2C$. At P , draw a vertical OP equal to $o'p'$.
- Join O with the corners of the base, thus completing the isometric view of the pyramid.

Fig. 17-25(iii) shows the isometric view of the same pyramid with its axis in horizontal position.

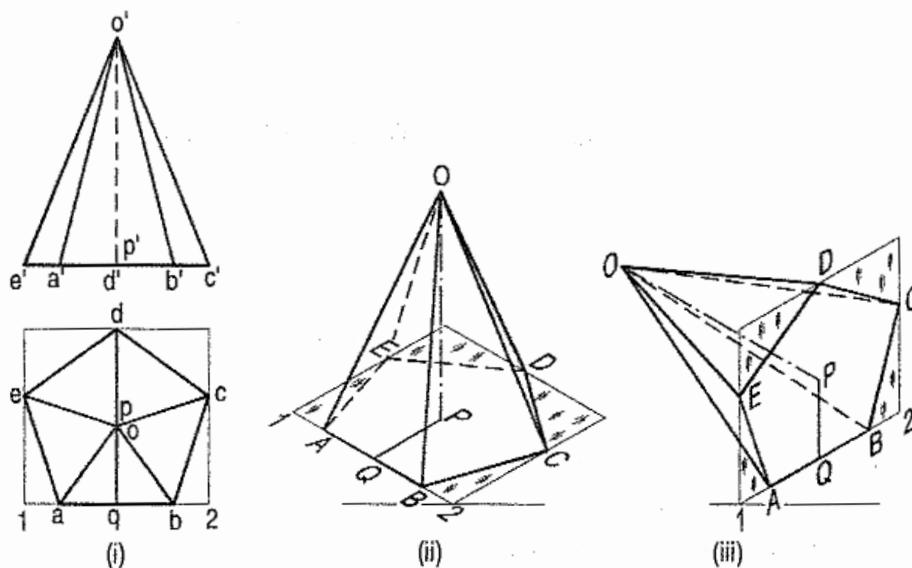


FIG. 17-25

Problem 17-18. Draw the isometric view of the truncated triangular pyramid shown in fig. 17-26(i).

- Draw the perpendiculars $d'x'$, $e'y'$ and $f'z'$ the front view and the offsets dq , er and fc in the top view.
- Draw the isometric view of the whole pyramid [fig. 17-26(ii)].
- Transfer the offsets and the verticals to this view and obtain points D , E and F on the lines OA , OB and OC respectively.
- Draw lines DE , EF and FD and complete the required isometric view.

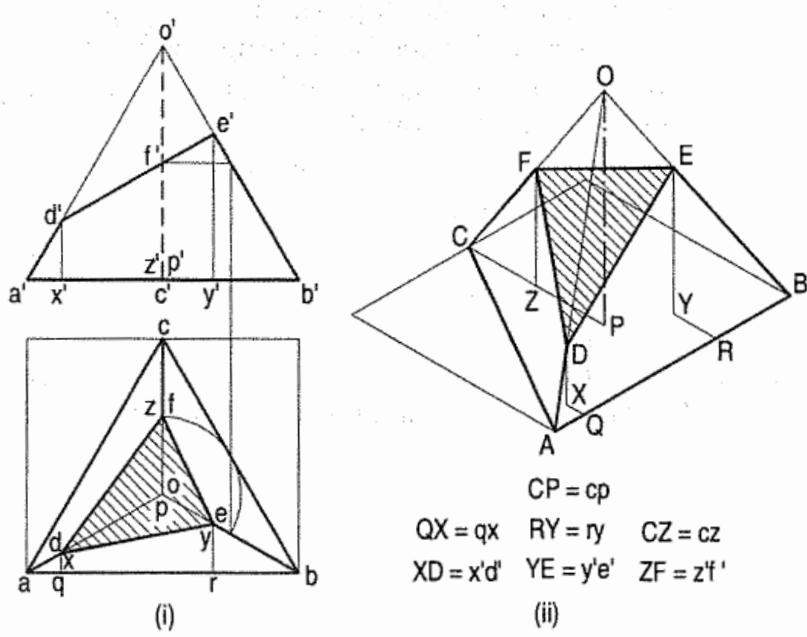


FIG. 17-26

17-6-3. ISOMETRIC DRAWING OF CYLINDERS

Problem 17-19. Draw the isometric view of the cylinder shown in fig. 17-27(i).

The axis of the cylinder is vertical, hence its ends are horizontal. Enclose the cylinder in a square prism.

Method I:

Draw the isometric view of the prism [fig. 17-27(ii)]. In the two rhombuses, draw the ellipses by the four-centre method. Draw two common tangents to the two ellipses. Erase the inner half of the lower ellipse and complete the required view.

Method II:

Draw the rhombus for the upper end of the prism [fig. 17-27(iii)] and in it, draw the ellipse by the four-centre method. From the centres for the arcs, draw vertical lines of length equal to the length of the axis, thus determining the centres for the lower ellipse. Draw the arcs for the half ellipse. Draw common tangents, thus completing the required view.

When the axis of the cylinder is horizontal, its isometric view is drawn by method I as shown in fig. 17-28(i).

Fig. 17-28(ii) shows the view drawn by method II, but the axis is shown sloping in the other direction.

Fig. 17-29 and fig. 17-30 respectively show the isometric views (drawn by method II) of a half-cylindrical disc with its axis in vertical and horizontal positions.

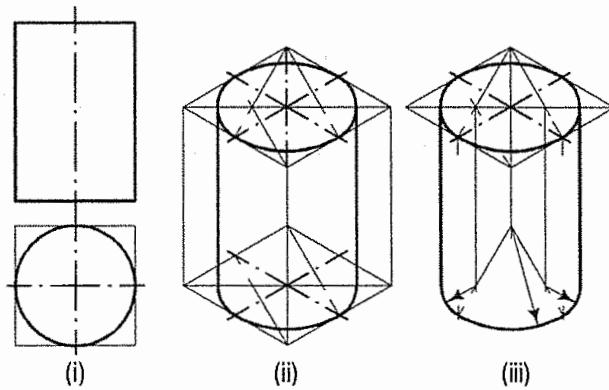


FIG. 17-27

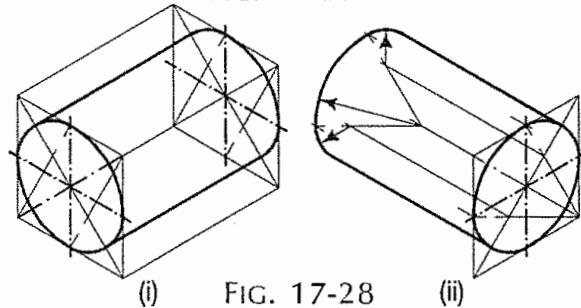


FIG. 17-28

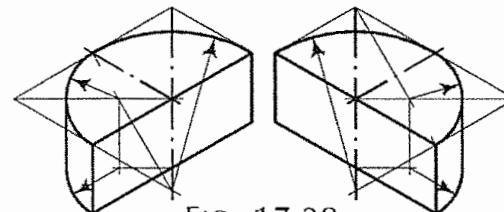


FIG. 17-29

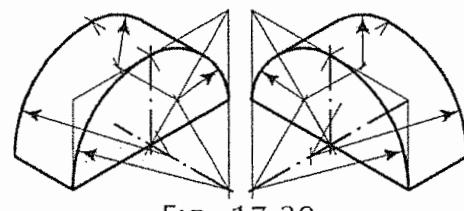


FIG. 17-30

17-6-4. ISOMETRIC DRAWING OF CONES

Problem 17-20. Draw the isometric view of a cone, base 40 mm diameter and axis 55 mm long (i) when its axis is vertical and (ii) when its axis is horizontal.

- Draw the ellipse for the base [fig. 17-31(i)]. Determine the position of the apex by the offset method.
- Draw tangents to the ellipse from the apex. Erase the part of the ellipse between the tangents and complete the view as shown.
- See fig. 17-31(ii) which is self-explanatory.

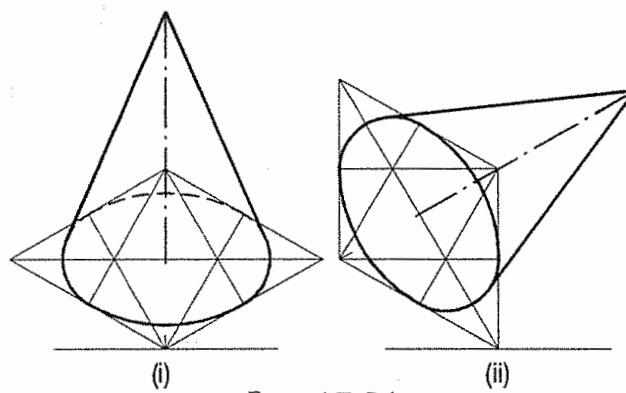


FIG. 17-31

Problem 17-21. Draw the isometric view of the frustum of a cone shown in fig. 17-32(i).

- Draw the ellipse for the base [fig. 17-32(ii)]. Draw the axis.
- Around the top end of the axis, draw the ellipse for the top.
- Draw common tangents, erase the unwanted part of the ellipse and complete the view as shown.

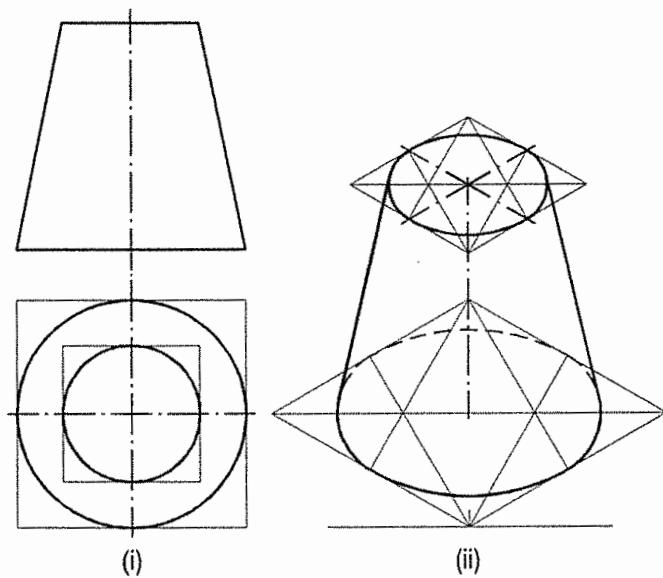


FIG. 17-32

17-6-5. ISOMETRIC DRAWING OF SPHERE

The orthographic view of a sphere seen from any direction is a circle of diameter equal to the diameter of the sphere. Hence, the isometric projection of a sphere is also a circle of the same diameter as explained below.

The front view and the top view of a sphere resting on the ground are shown in fig. 17-33(i). C is its centre, D is the diameter and P is the point of its contact with the ground.

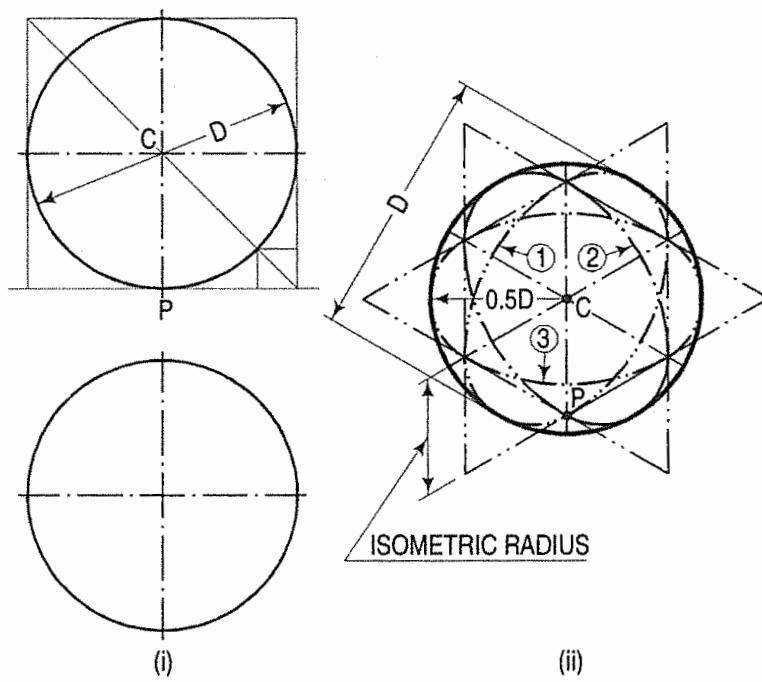


FIG. 17-33

Assume a vertical section through the centre of the sphere. Its shape will be a circle of diameter D . The isometric projection of this circle is shown in fig. 17-33(ii) by ellipses 1 and 2, drawn in two different vertical positions around the same centre C. The length of the major axis in each case is equal to D . The distance of the point P from the centre C is equal to the isometric radius of the sphere.

Again, assume a horizontal section through the centre of the sphere. The isometric projection of this circle is shown by the ellipse 3, drawn in a horizontal position around the same centre C. In this case also, the distance of the outermost points on the ellipse from the centre C is equal to $0.5D$.

Thus, it can be seen that in an isometric projection, the distances of all the points on the surface of a sphere from its centre, are equal to the radius of the sphere.

Hence, the isometric projection of a sphere is a circle whose diameter is equal to the true diameter of the sphere.

Also, the distance of the centre of the sphere from its point of contact with the ground is equal to the isometric radius of the sphere, viz. CP.

It is, therefore, of the utmost importance to note that, *isometric scale must invariably be used, while drawing isometric projections of solids in conjunction with spheres or having spherical parts.*

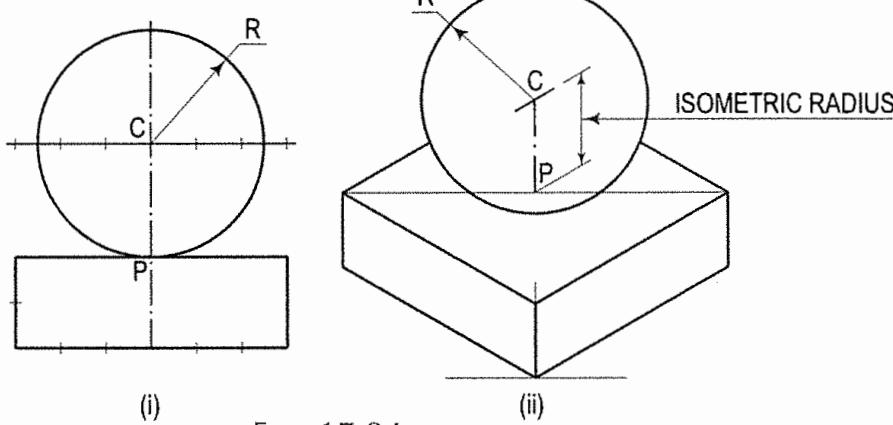


FIG. 17-34

Problem 17-22. Draw the isometric projection of a sphere resting centrally on the top of a square prism, the front view of which is shown in fig. 17-34(i).

- Draw the isometric projection (using isometric scale) of the square prism and locate the centre P of its top surface [fig. 17-34(ii)].
- Draw a vertical at P and mark a point C on it, such that $PC =$ the isometric radius of the sphere.
- With C as centre and radius equal to the radius of the sphere, draw a circle which will be the isometric projection of the sphere.

17-7. TYPICAL PROBLEMS OF ISOMETRIC DRAWING



The solutions given in the following typical problems are mostly self-explanatory. Explanations are however given where deemed necessary. Construction lines are left intact for guidance. Dotted lines for hidden edges have been shown in some views to make the construction more clear. Unless otherwise stated, all dimensions are given in millimetres.

Problem 17-23. A hexagonal prism having the side of base 26 mm and the height of 60 mm is resting on one of the corner of the base and its axis is inclined to 30° to the H.P. Draw its projections and also prepare the isometric view of the prism in the above stated condition.

- Draw the projections of the prism as shown in figure 17-35.
- Construct the isometric view as shown in fig. 17-36.

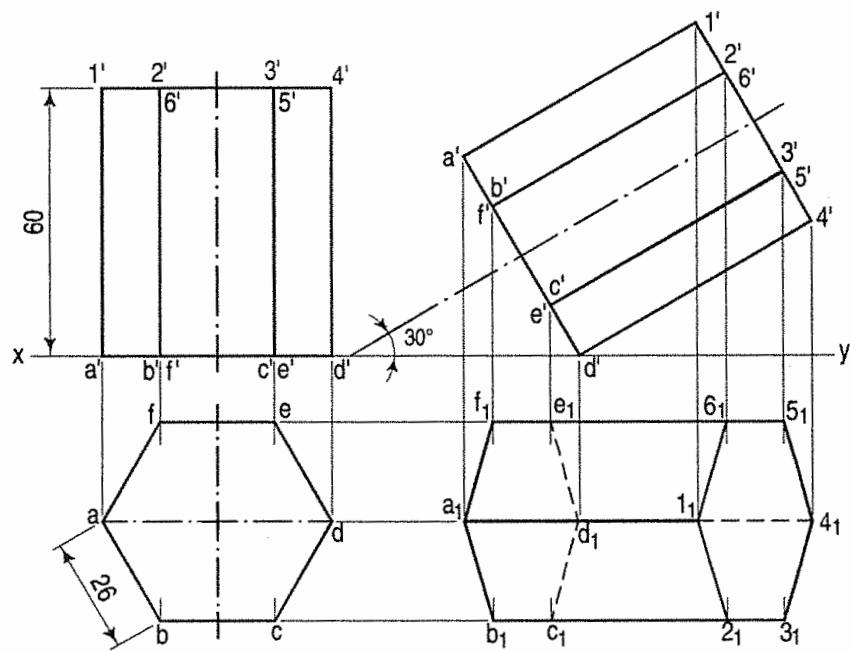


FIG. 17-35

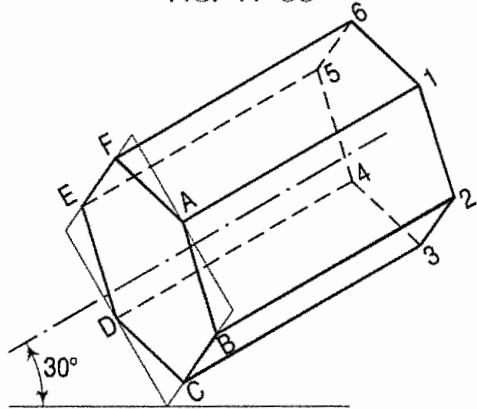


FIG. 17-36

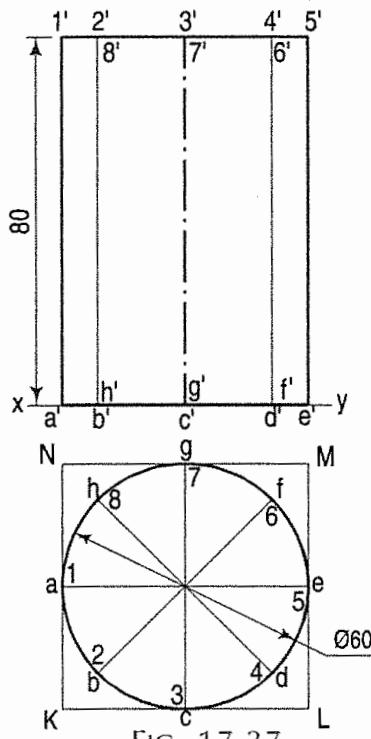


FIG. 17-37

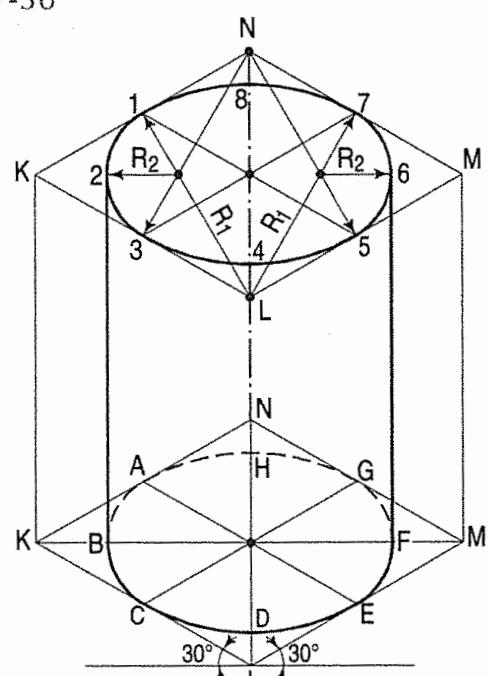


FIG. 17-38

Problem 17-24. (fig. 17-37): A cylindrical block of base, 60 mm diameter and height 80 mm, standing on the H.P. with its axis perpendicular to the H.P. Draw its isometric view.

The method shown in fig. 17-38 is self-explanatory.

Problem 17-25. The projection of pentagonal pyramid is shown in fig. 17-39. Draw its isometric view.

See fig. 17-40.

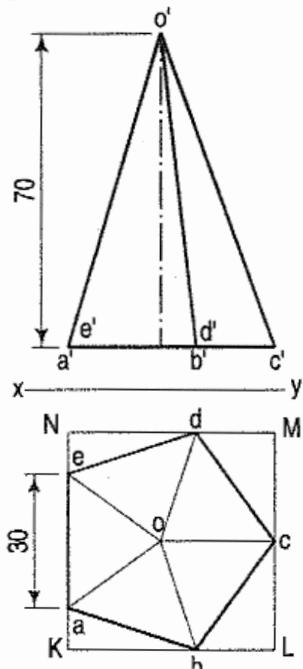


FIG. 17-39

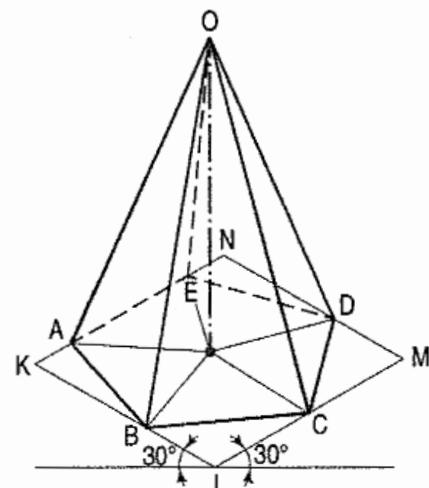


FIG. 17-40

Problem 17-26. The projection of the frustum of the cone is shown in fig. 17-41. Draw its isometric view.

See fig. 17-42.

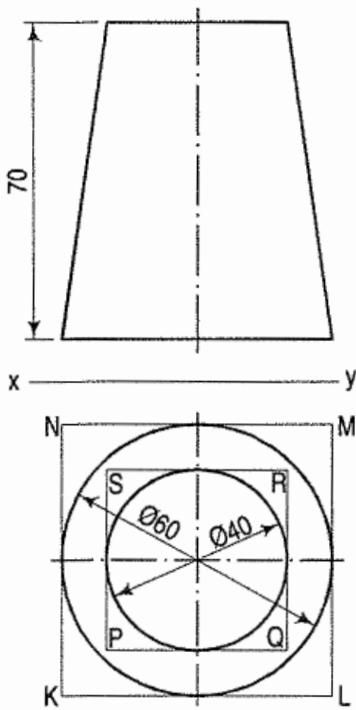


FIG. 17-41

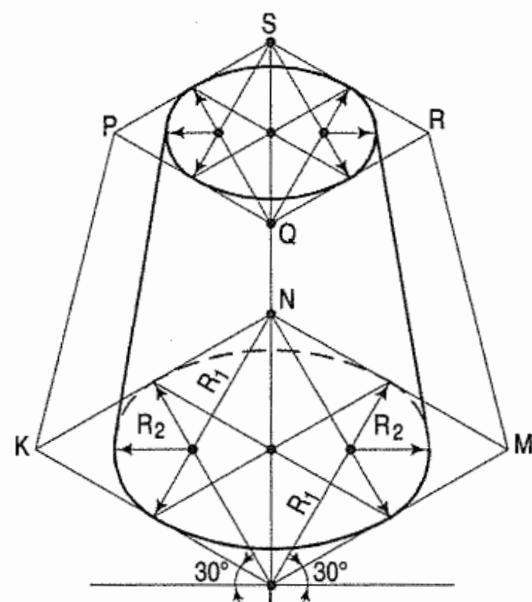


FIG. 17-42

Problem 17-27. The orthographic projections of the object is shown in fig. 17-43. Draw the isometric view of the object.

See fig. 17-44.

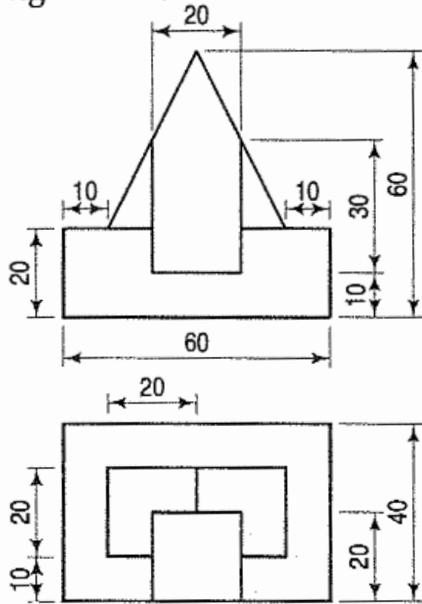


FIG. 17-43

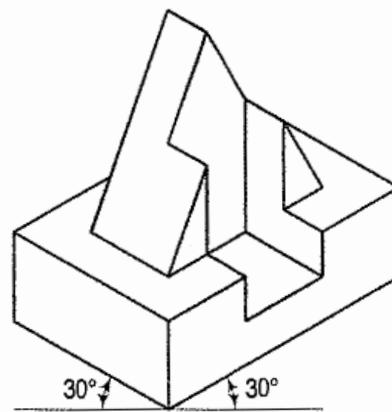


FIG. 17-44

Problem 17-28. Draw the isometric view of the casting shown in two views in fig. 17-45.

See fig. 17-46. Lines for the visible lower edges of the rectangular hole should be shown.

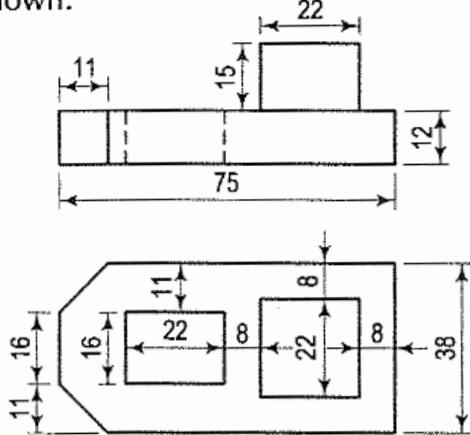


FIG. 17-45

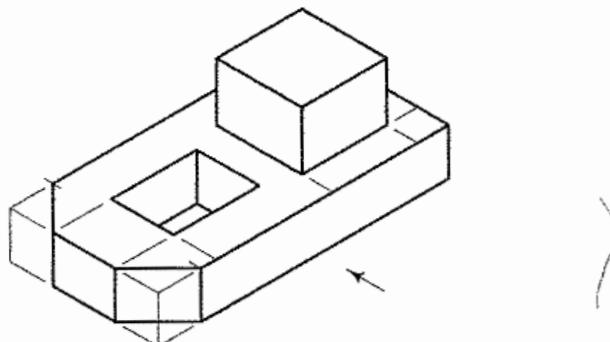


FIG. 17-46

Problem 17-29. Draw the isometric view of the block, two views of which are shown in fig. 17-47.

See fig. 17-48. Centres for the arcs for lower circular edges are obtained by drawing vertical lines from the centres for the upper arcs.

Problem 17-30. Draw the isometric view of the casting, shown in three views in fig. 17-49.

See fig. 17-50.

Problem 17-31. Draw the isometric view of the casting, two views of which are shown in fig. 17-51.

See fig. 17-52.

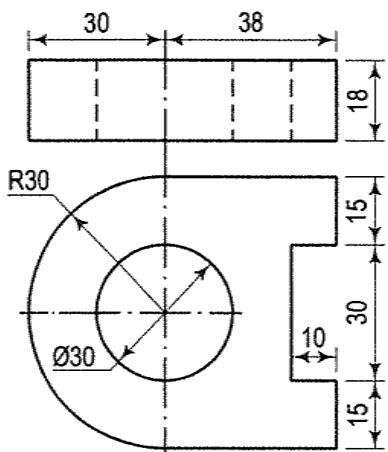


FIG. 17-47

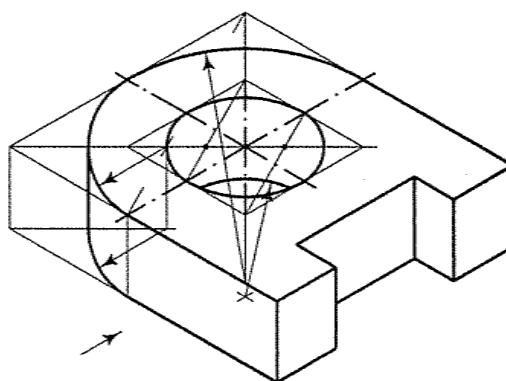
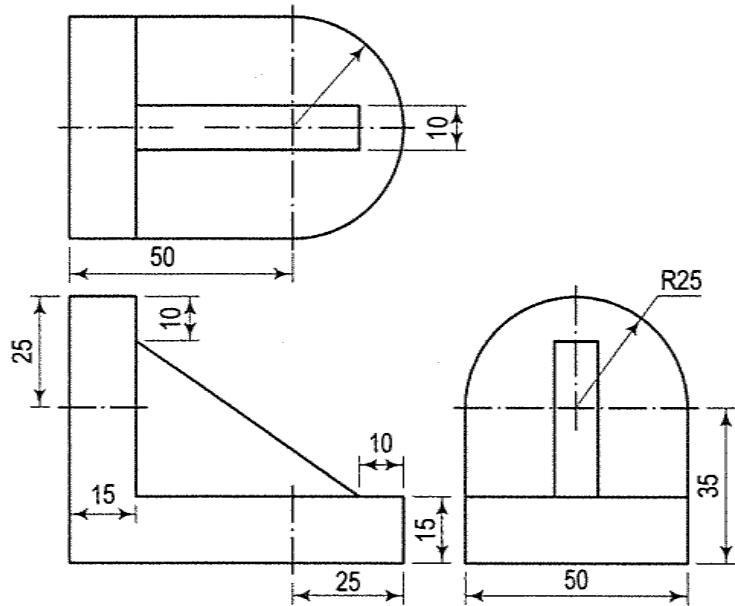


FIG. 17-48



(Third-angle projection)

FIG. 17-49

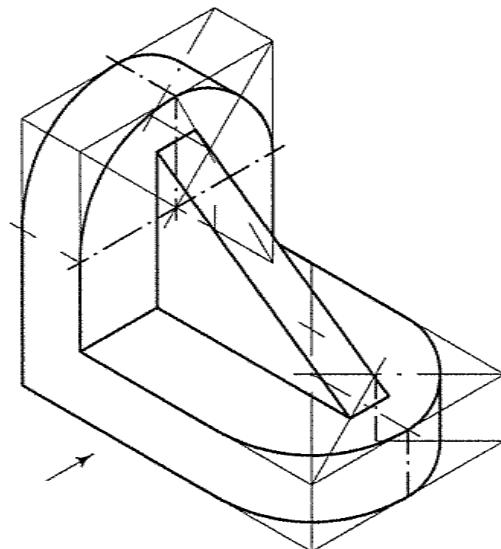
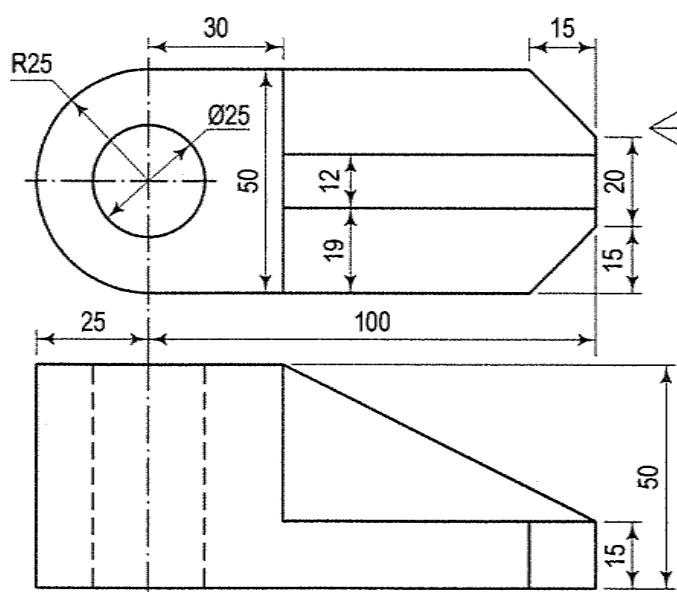


FIG. 17-50



(Third-angle projection)

FIG. 17-51

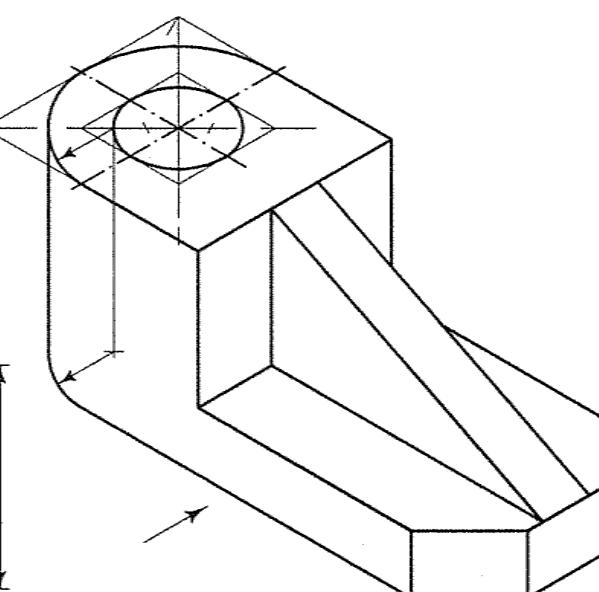


FIG. 17-52

Problem 17-32. The front view of a board fitted with a letter H and mounted on a wooden post is given in fig. 17-53. Draw its isometric view, assuming the thickness of the board and of the letter to be equal to 3 cm. Scale, half full size. (All dimensions are given in centimeters.)

See fig. 17-54.

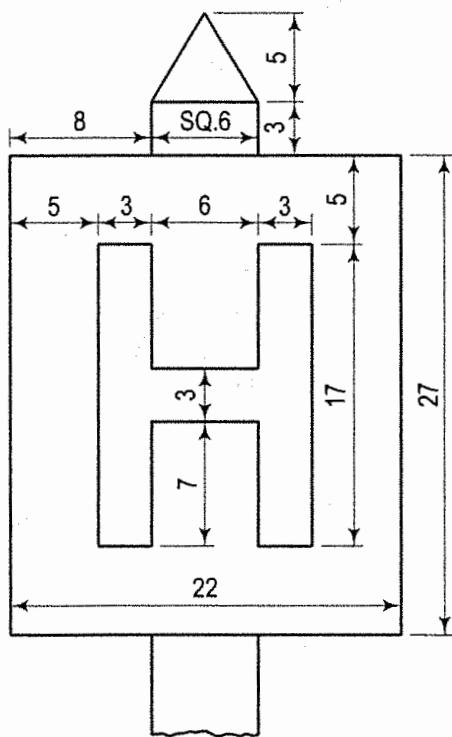


FIG. 17-53

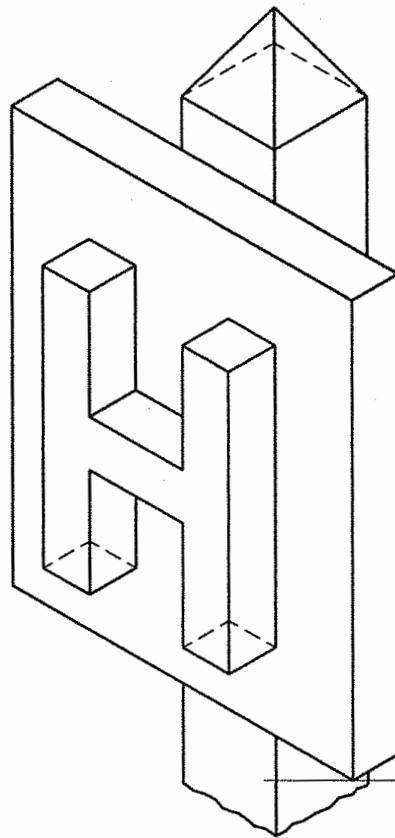


FIG. 17-54

Problem 17-33. Draw the isometric view of the casting shown in two views in fig. 17-55.

See fig. 17-56.

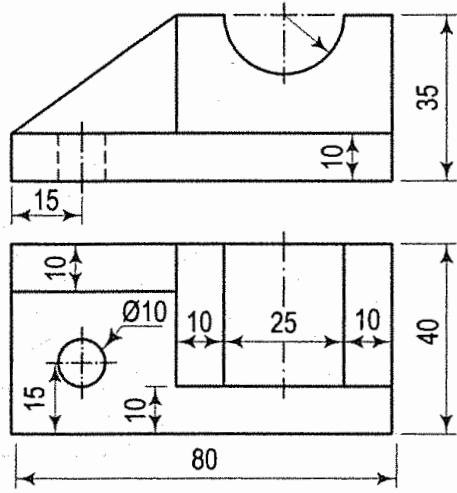


FIG. 17-55

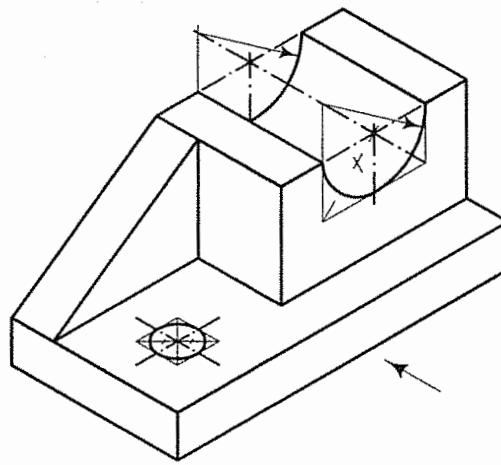


FIG. 17-56

Problem 17-34. Draw the isometric view of the model of steps, two views of which are shown in fig. 17-57. The top plate of the steps is 16 mm thick. See fig. 17-58.

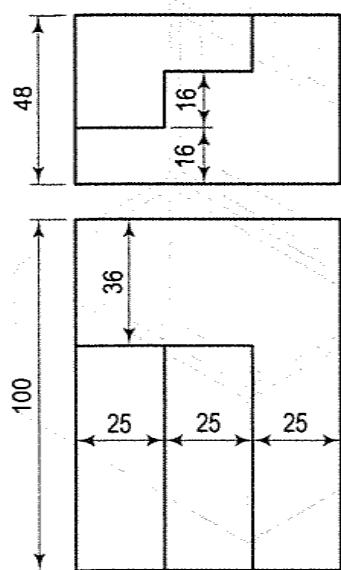


FIG. 17-57

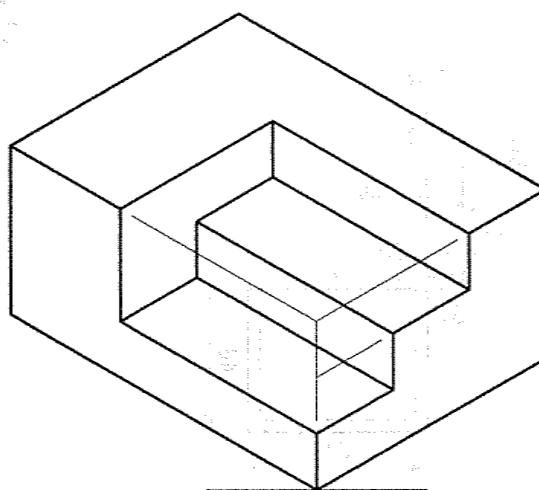


FIG. 17-58

Problem 17-35. Two pieces of wood joined together by a dovetail joint are shown in two views in fig. 17-59. Draw the isometric view of the two pieces separated but in a position ready for fitting.

See fig. 17-60.

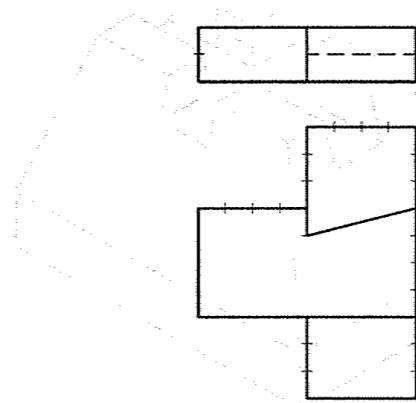


FIG. 17-59

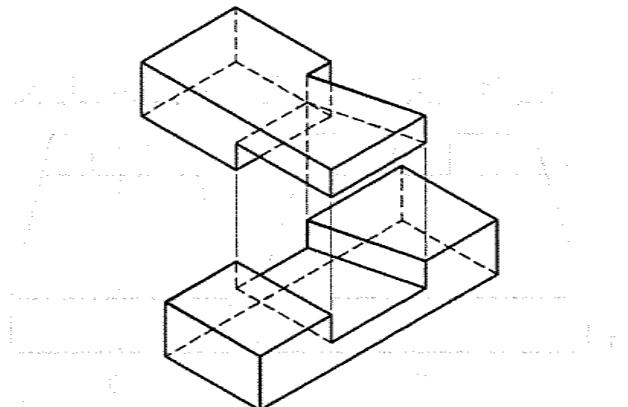


FIG. 17-60

Problem 17-36. The outside dimensions of a box made of 4 cm thick planks are 90 cm \times 60 cm \times 60 cm height. The depth of the lid on the outside is 12 cm. Draw the isometric view of the box when the lid is (a) 90° open and (b) 120° open.

Draw the orthographic view of the box with the lid in required positions as shown in fig. 17-61.

- This position is simple to draw in isometric view. Care must, however, be taken to deduct the thickness of the wood for the bottom and the top, when showing in the lines for the inside of the box and the lid (fig. 17-62).

- (b) In this position, points *P*, *Q*, *R* etc. for the lid are located by enclosing the lid in the oblong and transferring the same on the isometric view as shown in fig. 17-62. The view is left incomplete to avoid congestion.

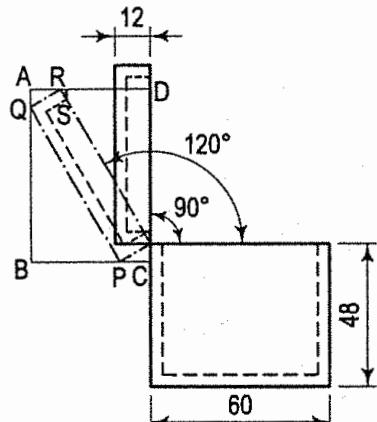


FIG. 17-61

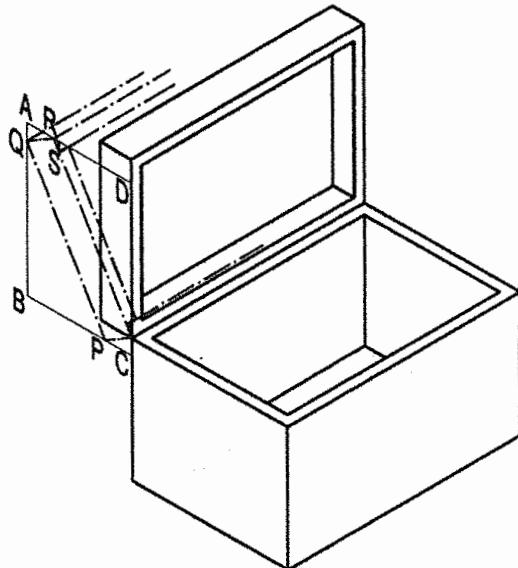


FIG. 17-62

Problem 17-37. Two views of a cast-iron block are shown in fig. 17-63. Draw its isometric view.

See fig. 17-64.

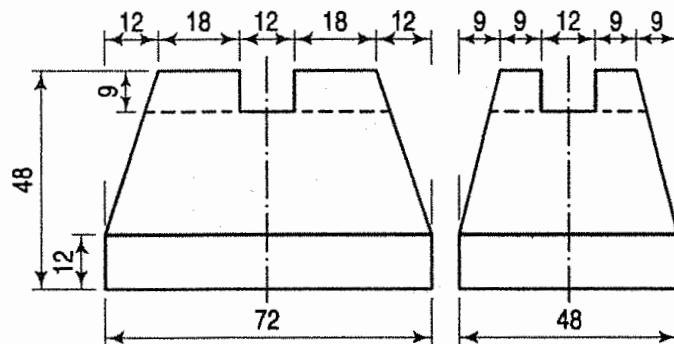


FIG. 17-63

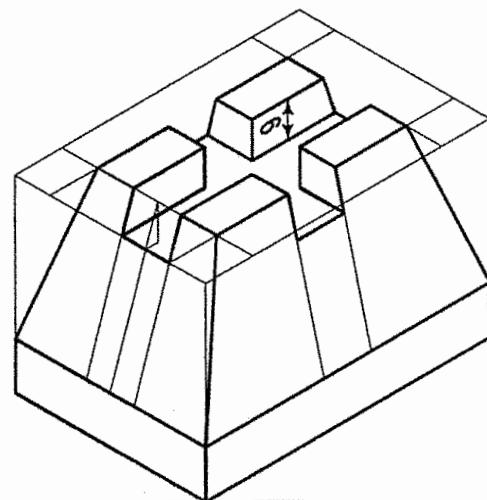


FIG. 17-64

The slope of the lines for the grooves on the outer surface on all the four sides is different and is obtained as shown by construction lines. The depth is measured along vertical lines.

Problem 17-38. Draw the isometric view of the casting shown in two views in fig. 17-65.

See fig. 17-66.

Problem 17-39. Draw the isometric view of the simple moulding shown in fig. 17-67.

See fig. 17-68.

The points on the curve are located by co-ordinate method. The parallel curve is obtained by drawing lines in the third direction and equal to the thickness of the moulding.

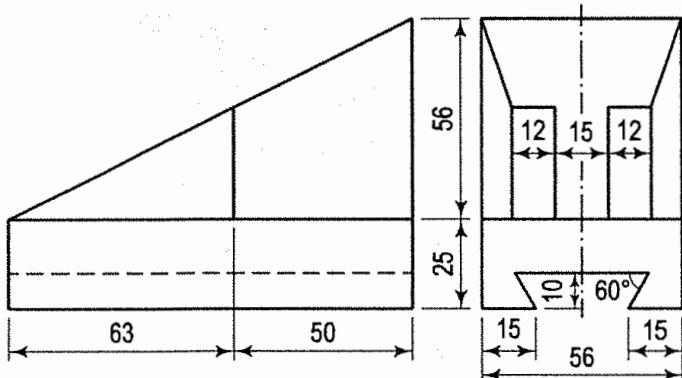


FIG. 17-65

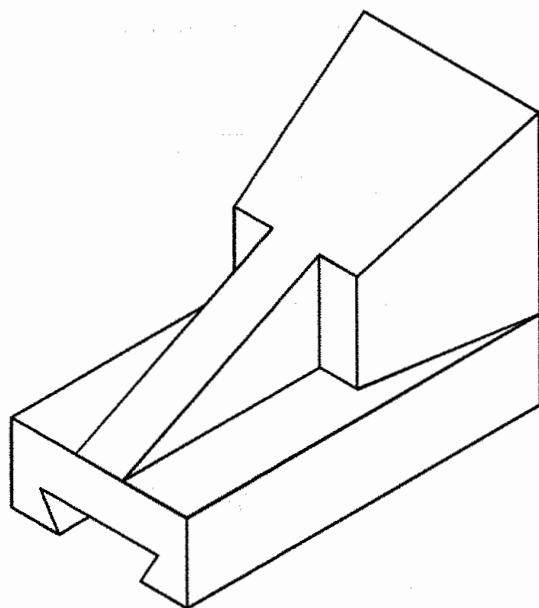


FIG. 17-66

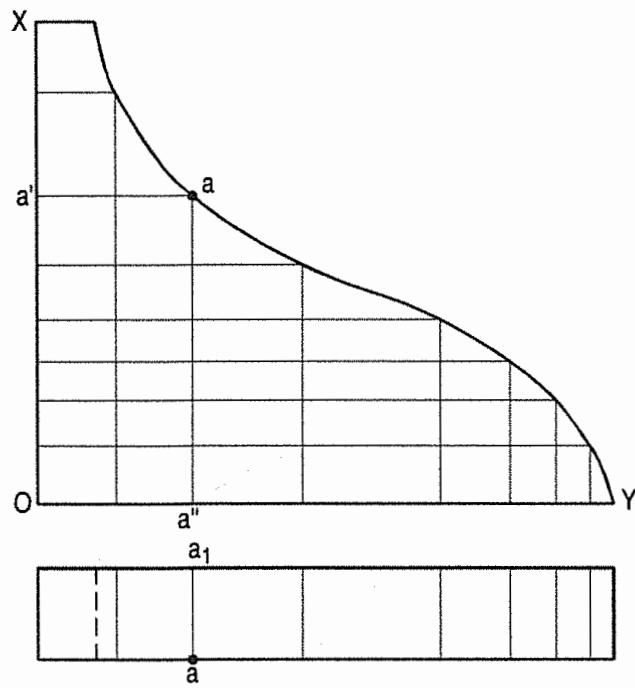


FIG. 17-67

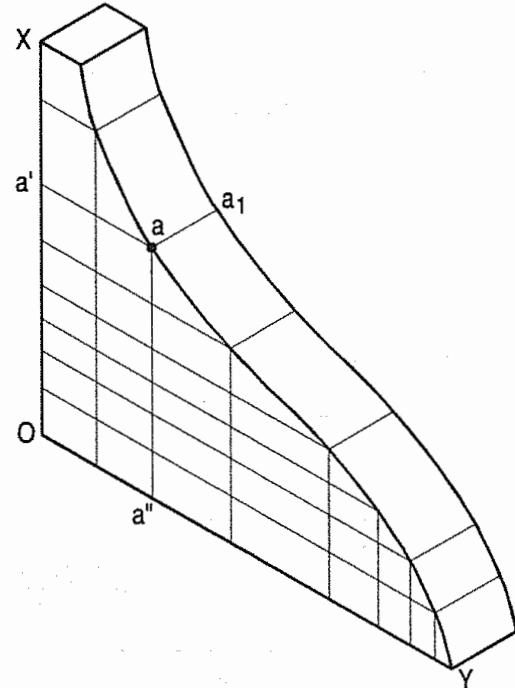


FIG. 17-68

Problem 17-40. The front view of three solids placed one above the other, with their axes in a straight line is shown in fig. 17-69. Draw the isometric view of the arrangement.

See fig. 17-70.

In this problem, isometric lengths must be taken for all dimensions except for the radius of the circle for the sphere.

The centre C of the sphere is at a distance equal to the isometric radius from the centre P of the top face of the cone frustum. The circle for the sphere is drawn with the true radius.

The ellipse for the section of the sphere is drawn within the rhombus constructed around the point Q on the axis.

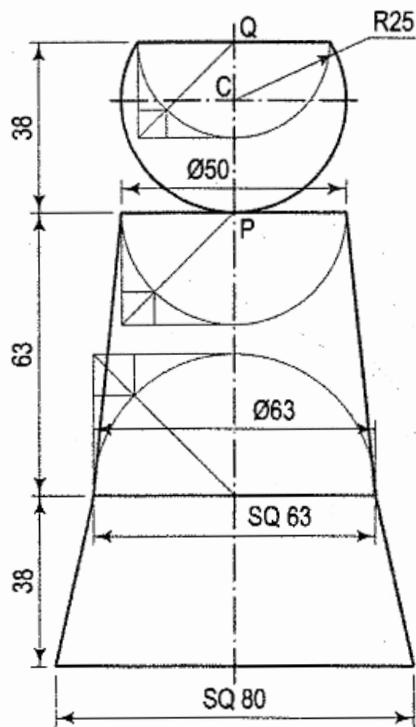


FIG. 17-69

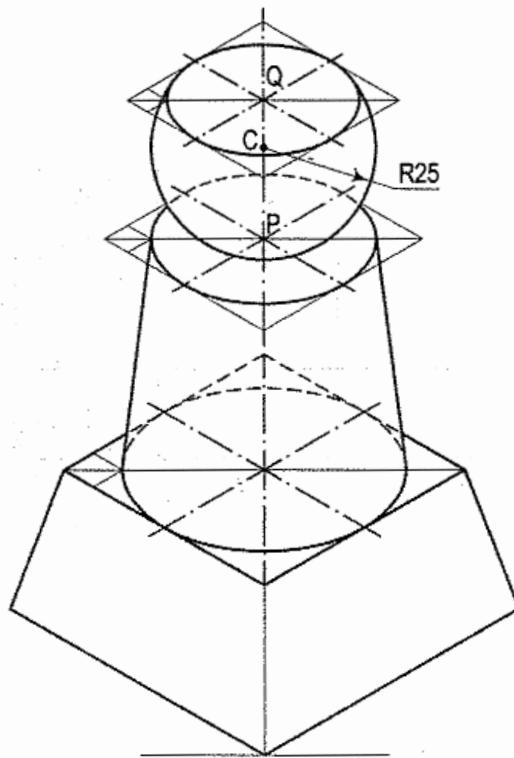


FIG. 17-70

Problem 17-41. Draw the isometric view of the clamping piece shown in fig. 17-71.

See fig. 17-72.

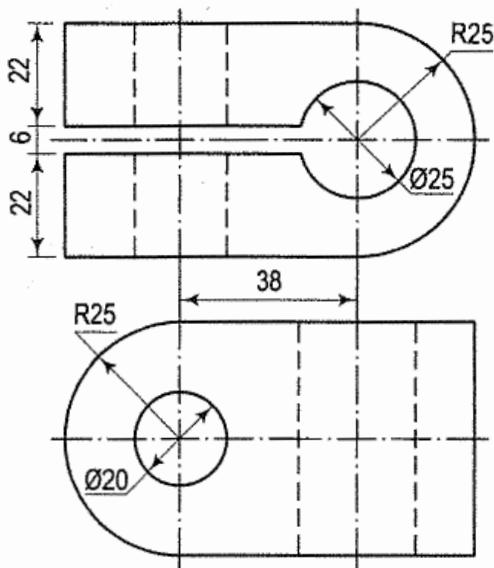


FIG. 17-71

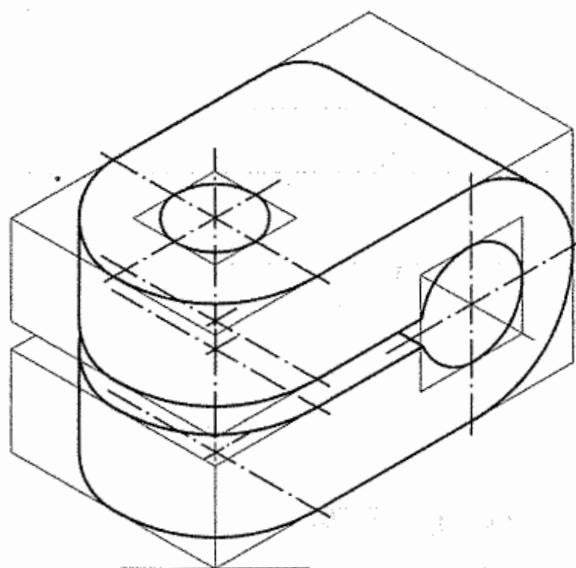


FIG. 17-72

Problem 17-42. (fig. 17-73): Draw the isometric view of a hexagonal nut for a 24 mm diameter bolt, assuming approximate dimensions. The threads may be neglected but chamfer must be shown.

See fig. 17-74.

Problem 17-43. Draw the isometric view of the paper-weight with spherical knob shown in fig. 17-75.

See fig. 17-76. Isometric scale should be used for all dimensions except for the radius of the circle for the sphere.

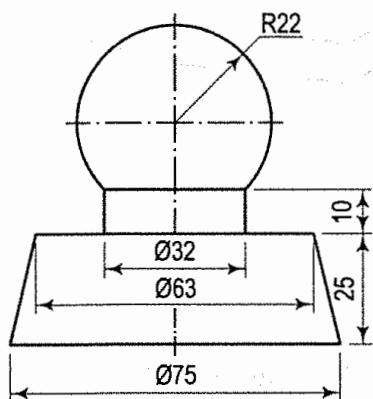


FIG. 17-75

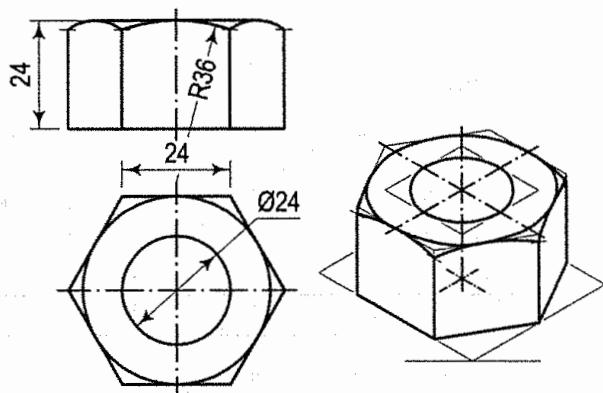


FIG. 17-73

FIG. 17-74

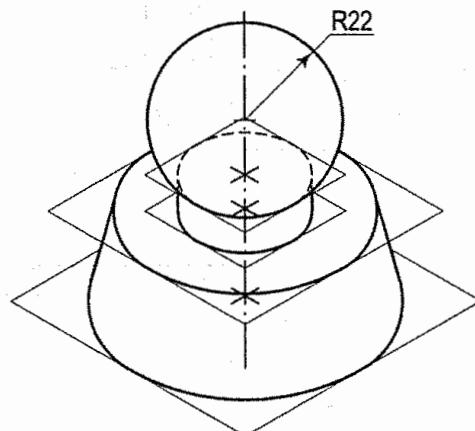


FIG. 17-76

Problem 17-44. (fig. 17-77): Draw the isometric view of a square-headed bolt 24 mm diameter and 70 mm long, with a square neck 18 mm thick and a head, 40 mm square and 18 mm thick.

See fig. 17-78.

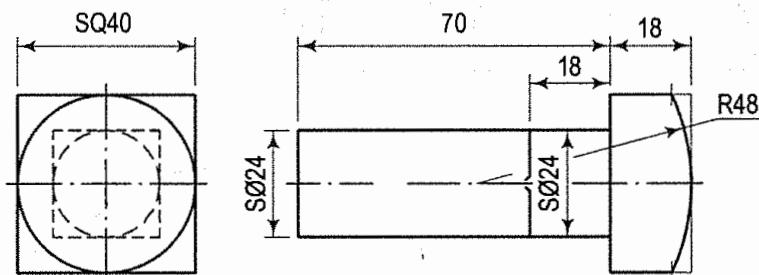


FIG. 17-77

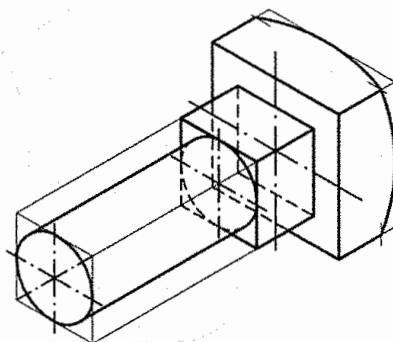


FIG. 17-78

Problem 17-45. Draw the isometric view of the casting shown in fig. 17-79.
See fig. 17-80.

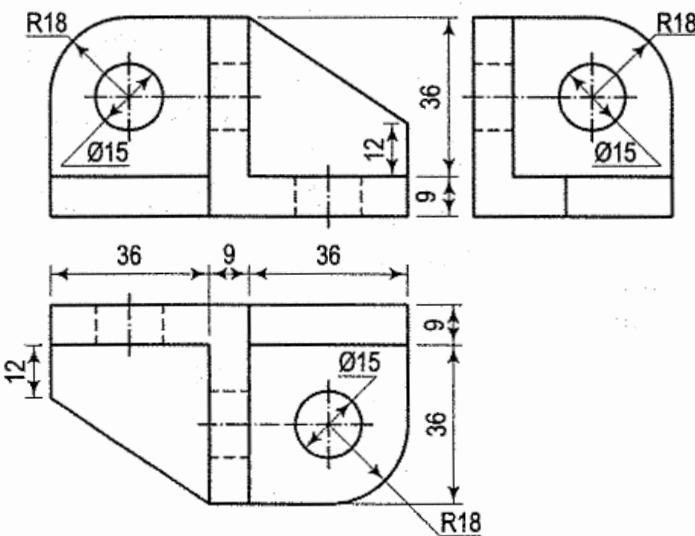


FIG. 17-79

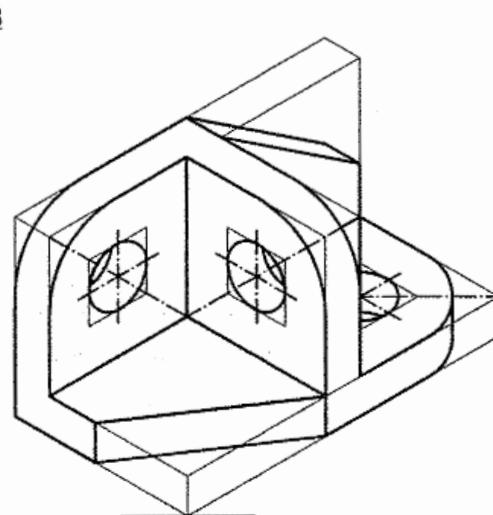


FIG. 17-80

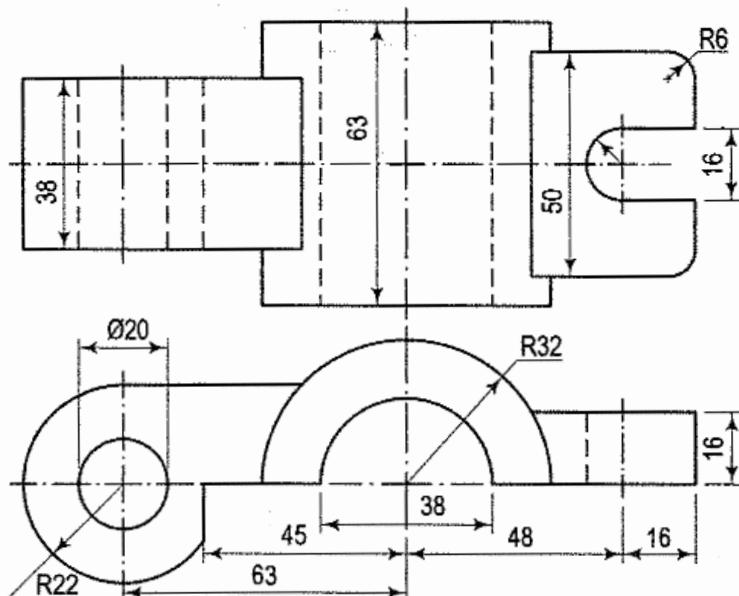


FIG. 17-81

Problem 17-46. The projections of a casting are shown in fig. 17-81. Draw its isometric view.

See fig. 17-82.

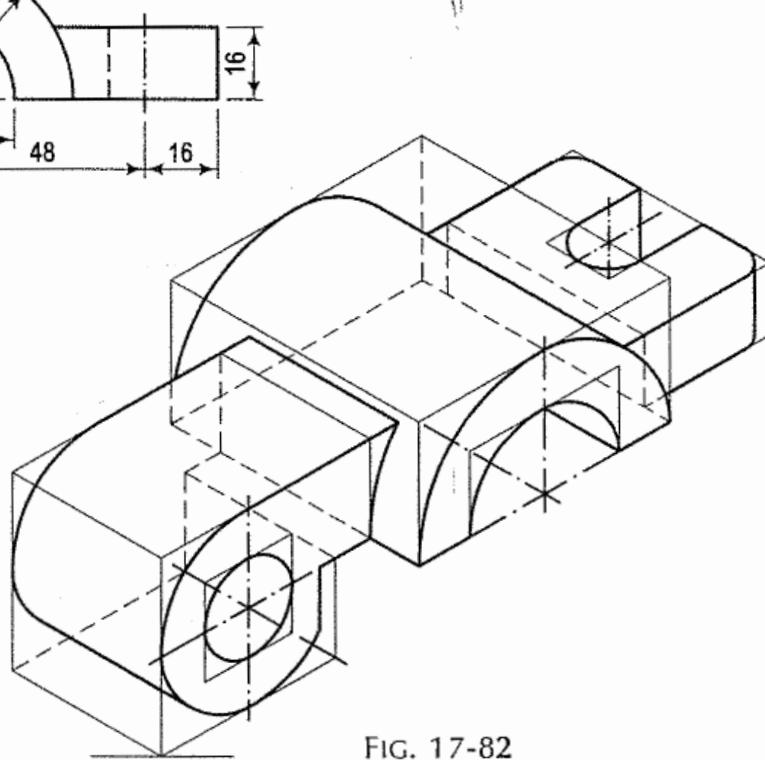


FIG. 17-82

Problem 17-47. Draw the isometric view of the bracket shown in two views in fig. 17-83.

See fig. 17-84.

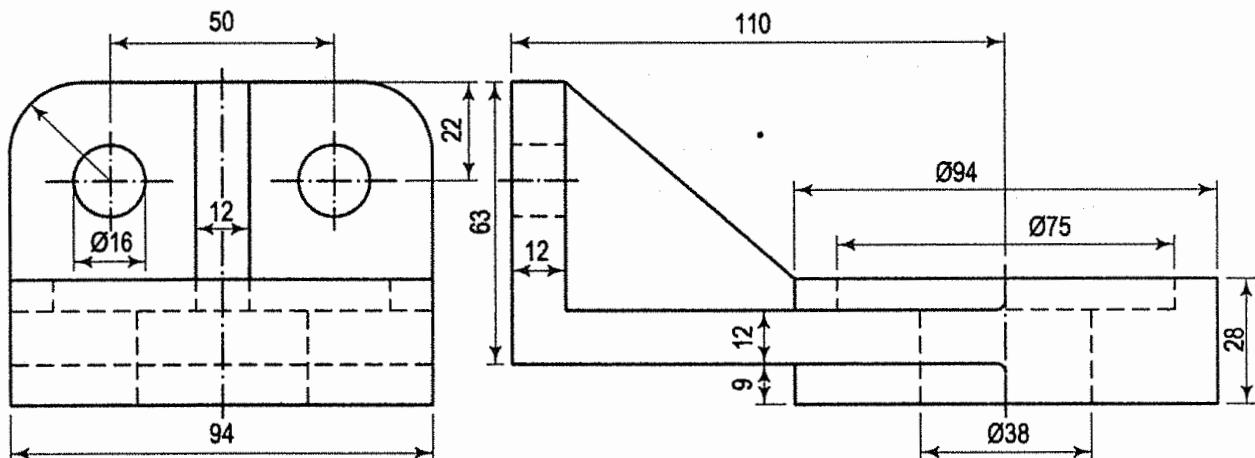


FIG. 17-83

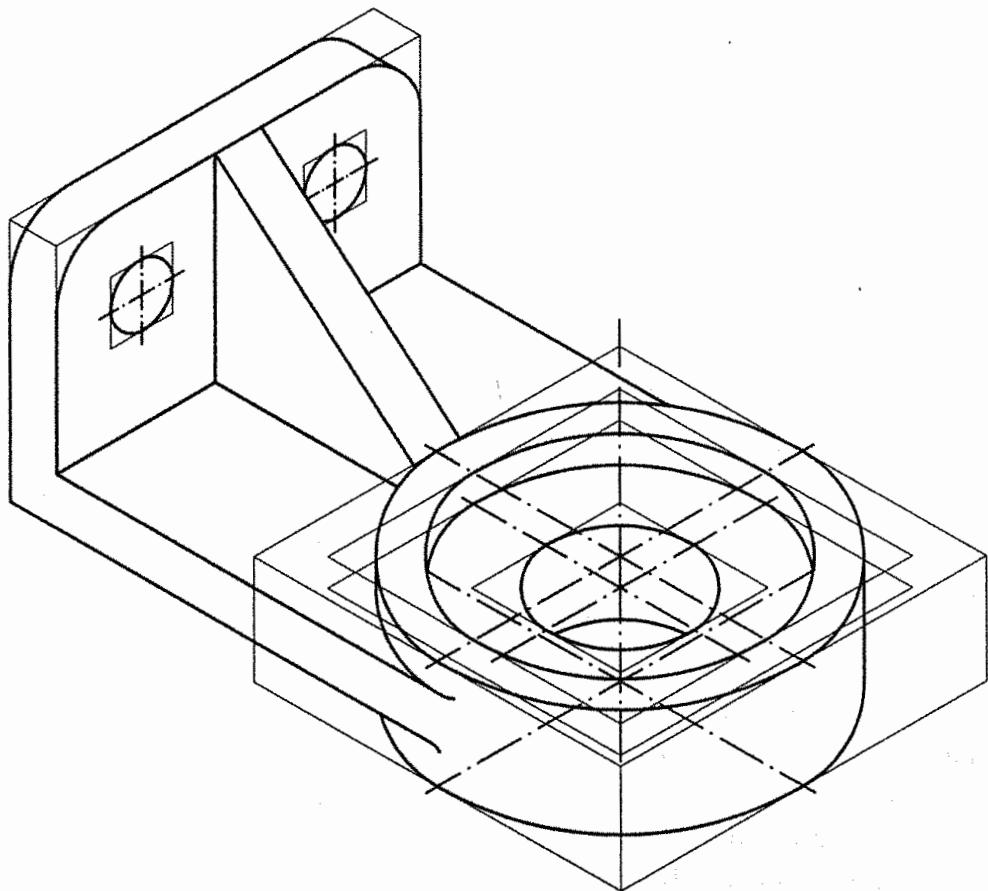


FIG. 17-84

Problem 17-48. Draw the isometric view of the machine-handle shown in fig. 17-85.

See fig. 17-86.

All measurements must be in isometric lengths except those for the diameters of spherical parts.

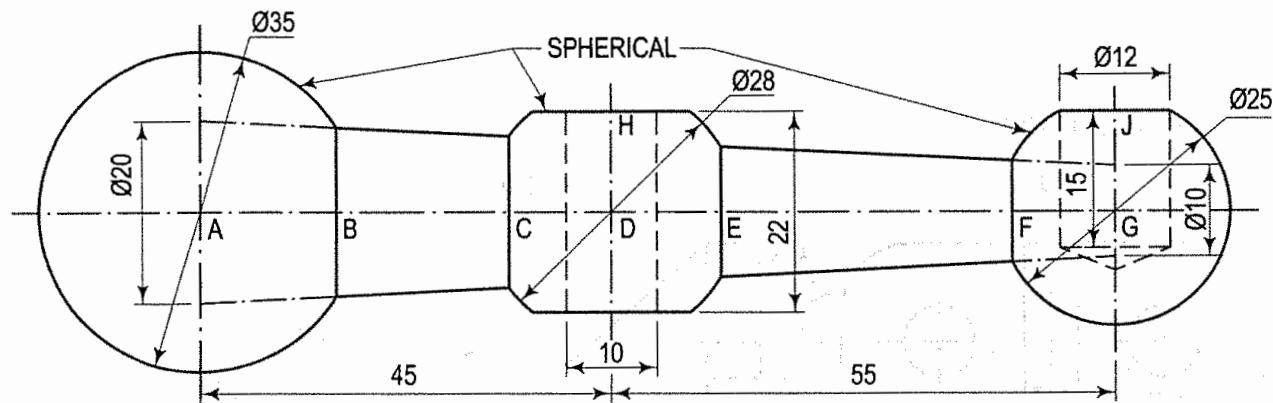


FIG. 17-85

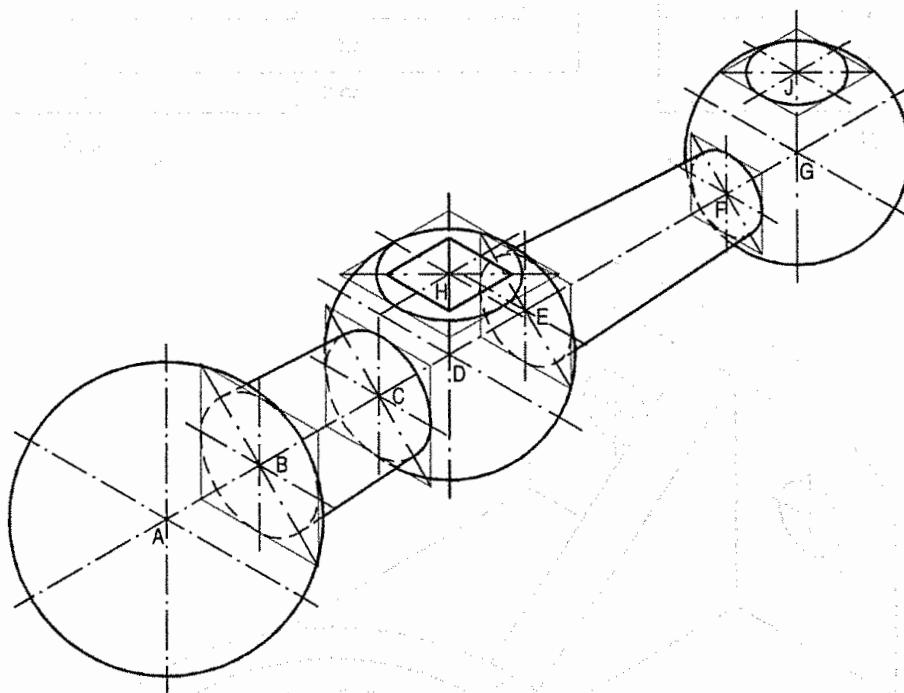


FIG. 17-86

- Draw an axis and mark on it the positions of points A, B etc.
- At points B, C, E and F, draw ellipses for circular sections of the conical handle. Ellipses at B and E will be completely hidden.
- With points A, D and G as centres, draw circles for the spheres, with their respective true radii.
- Mark points H and J on the vertical axes through D and G respectively and draw ellipses for the respective sections of the spheres.
- Around H, draw a rhombus for the square hole.
- The dotted lines for the depth of the holes are omitted.

Problem 17-49. The front view of a stool having a square top and four legs is shown in fig. 17-87. Draw its isometric view.

The legs lie along the slant edges of a frustum of a square pyramid (fig. 17-88).

Positions of the connecting horizontal strips between the legs at the top and at the bottom are determined by marking the heights along the axis and then drawing isometric lines upto the line AB, which shows the slope of the face of the frustum.

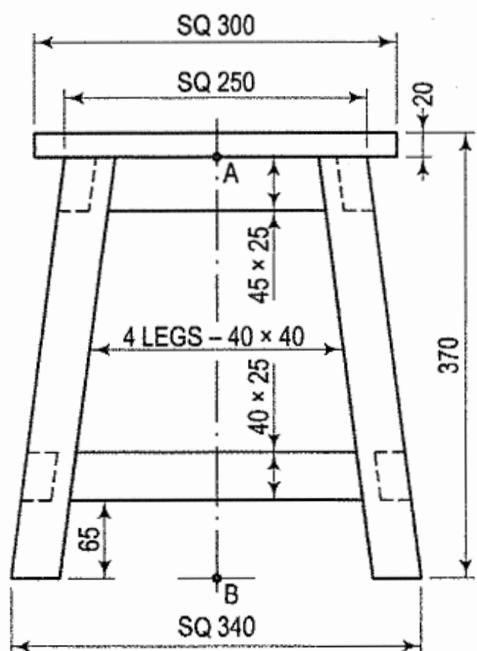


FIG. 17-87

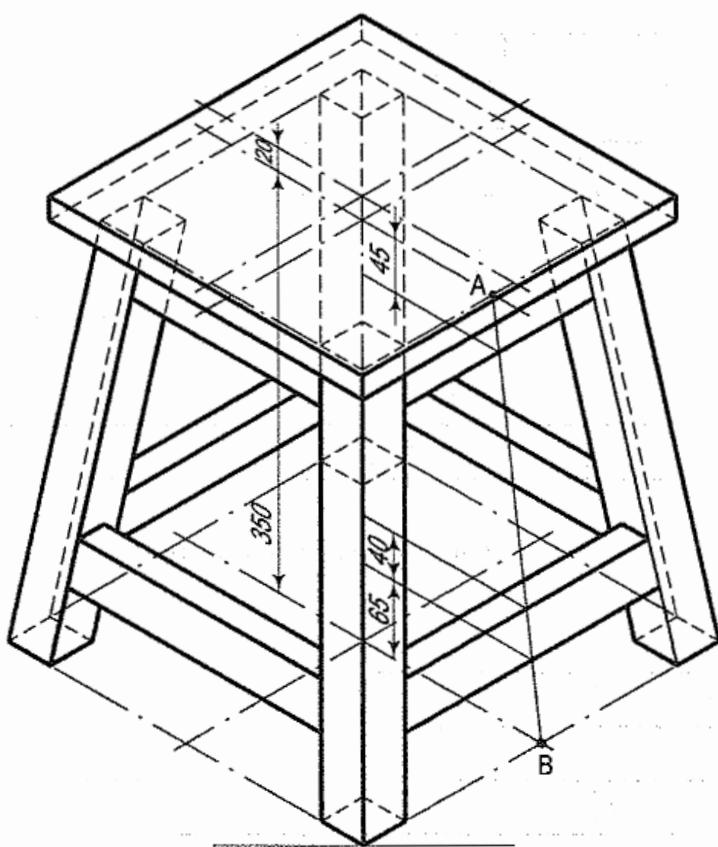


FIG. 17-88

EXERCISES 17

1. Projections of castings of various shapes are given in figs. 17-89 to 17-115. Draw the isometric view of each casting.

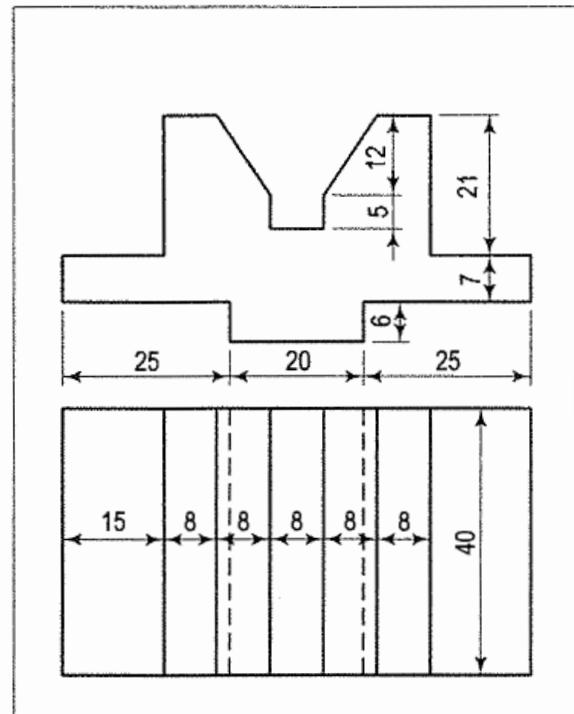


FIG. 17-89

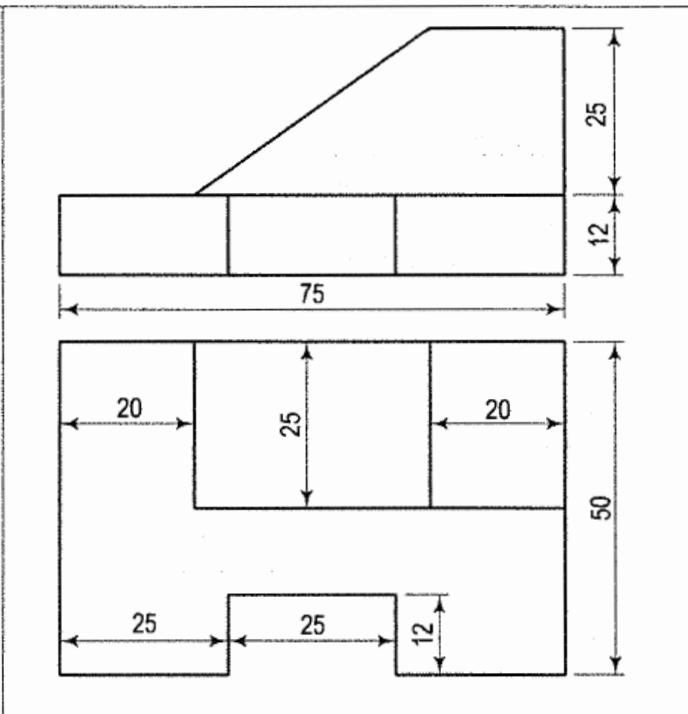
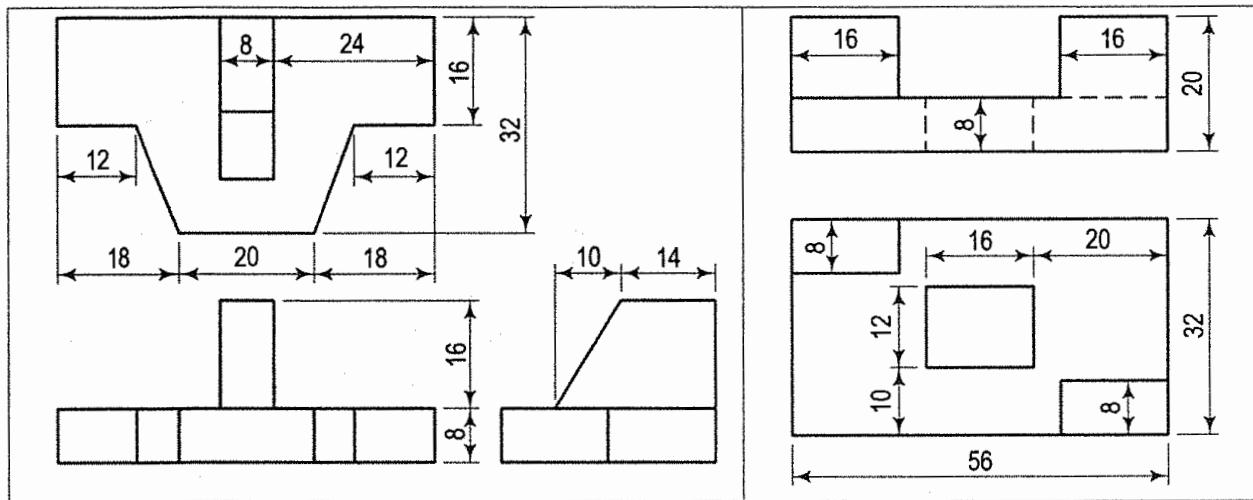


FIG. 17-90



(Third-angle projection)
FIG. 17-91

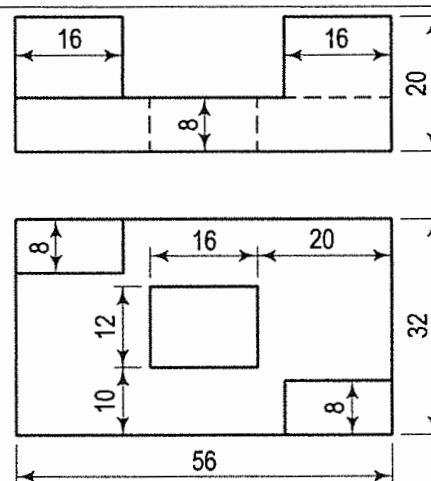


FIG. 17-92

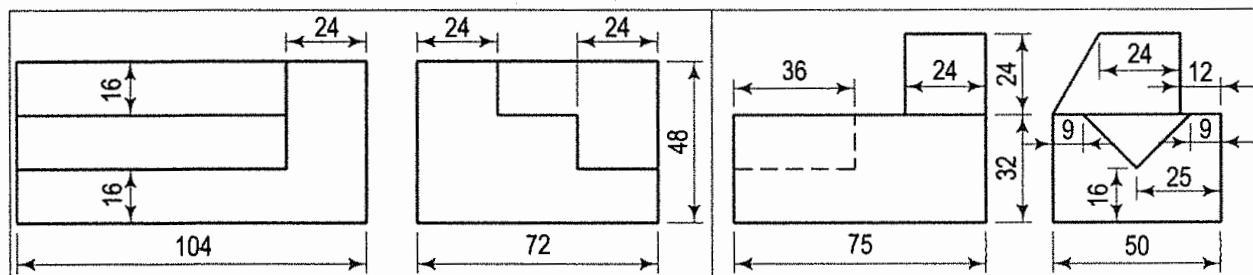
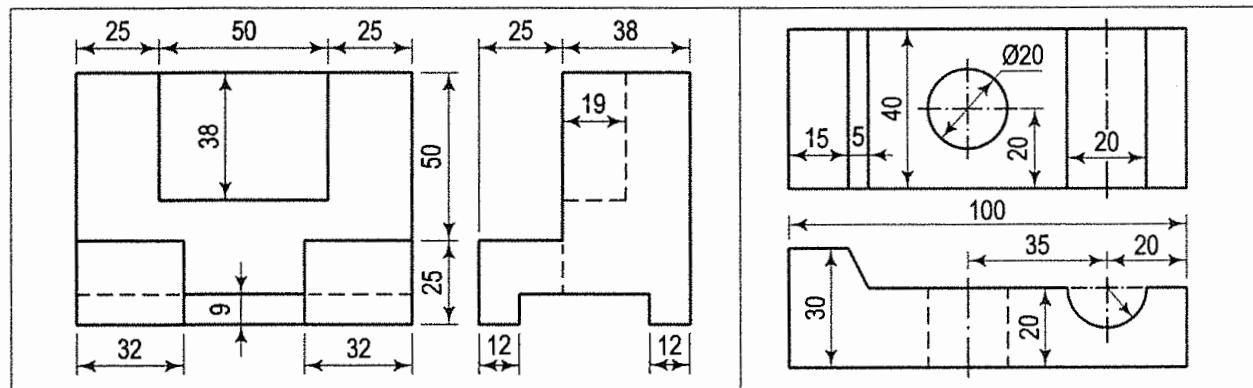


FIG. 17-93

FIG. 17-94



(Third-angle projection)
FIG. 17-95

(Third-angle projection)
FIG. 17-96

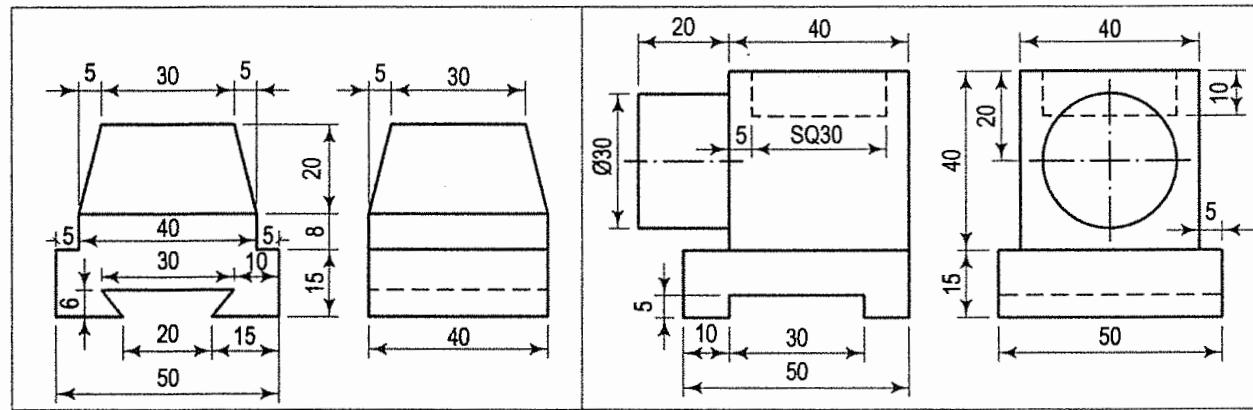


FIG. 17-97

FIG. 17-98

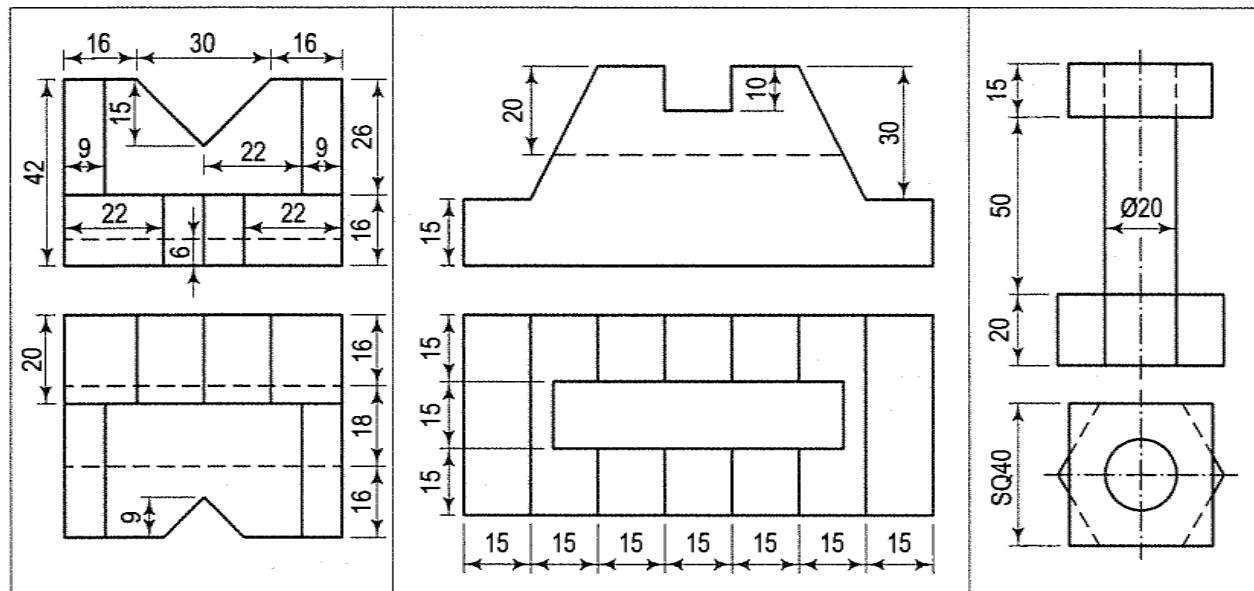


FIG. 17-99

FIG. 17-100

FIG. 17-101

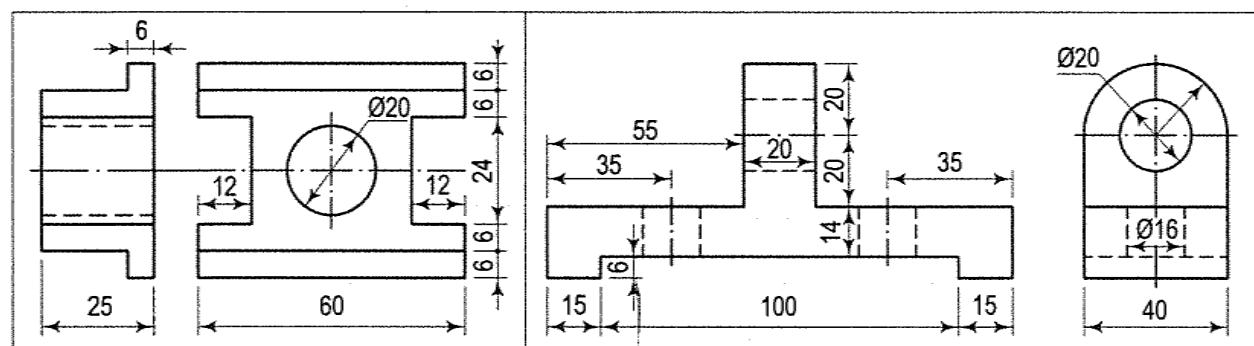


FIG. 17-102

FIG. 17-103

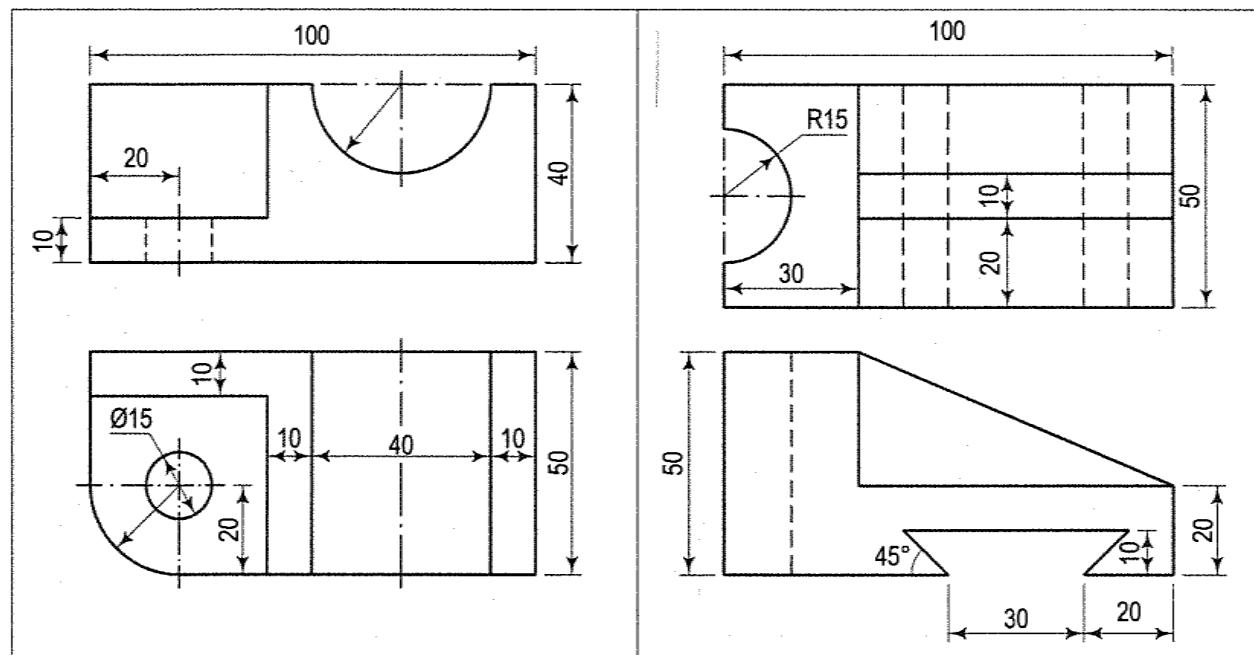
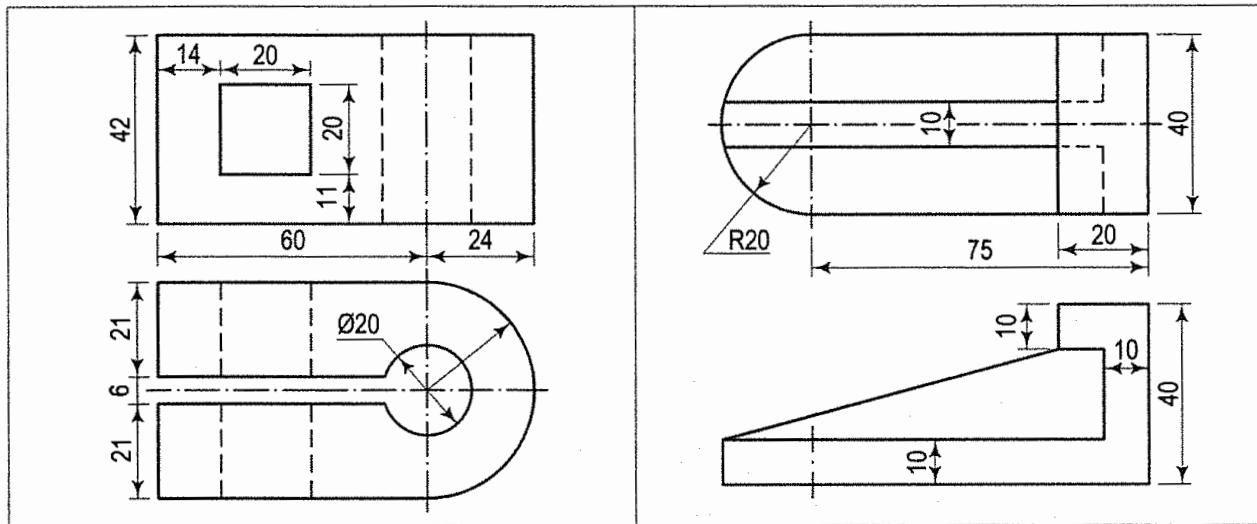


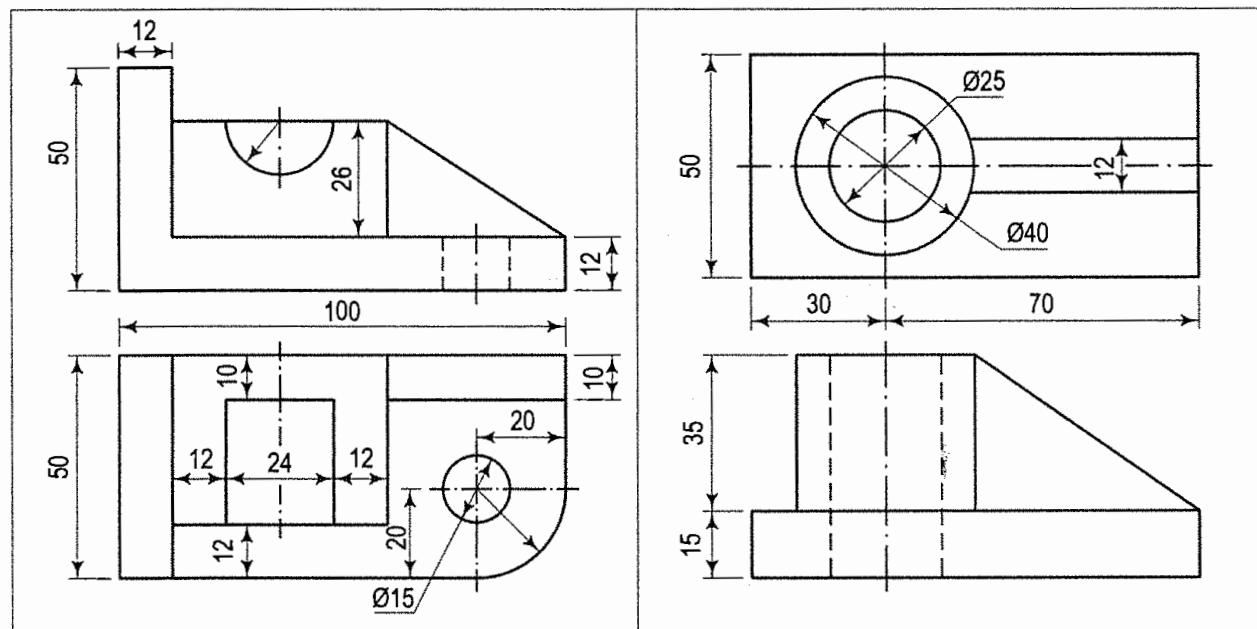
FIG. 17-104

(Third-angle projection)
FIG. 17-105



(Third-angle projection)

FIG. 17-107



(Third-angle projection)

FIG. 17-109

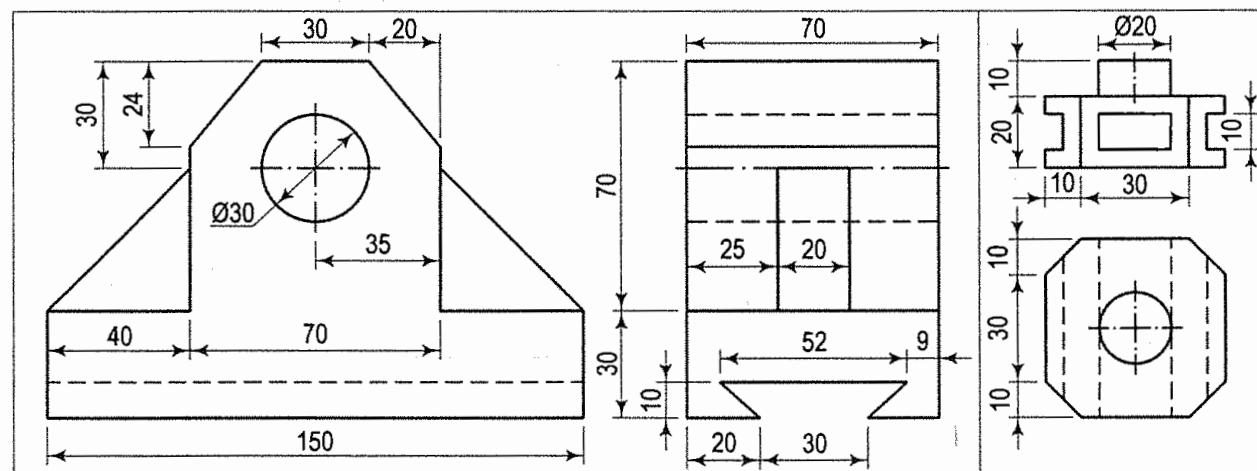
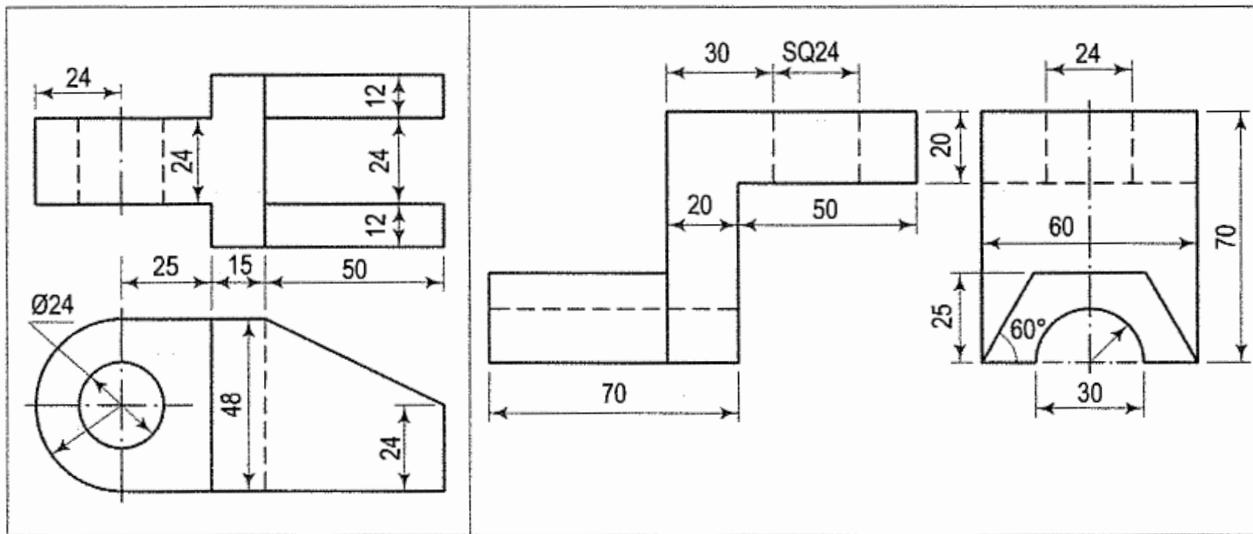


FIG. 17-110

FIG. 17-111



(Third-angle projection)

FIG. 17-112

FIG. 17-113

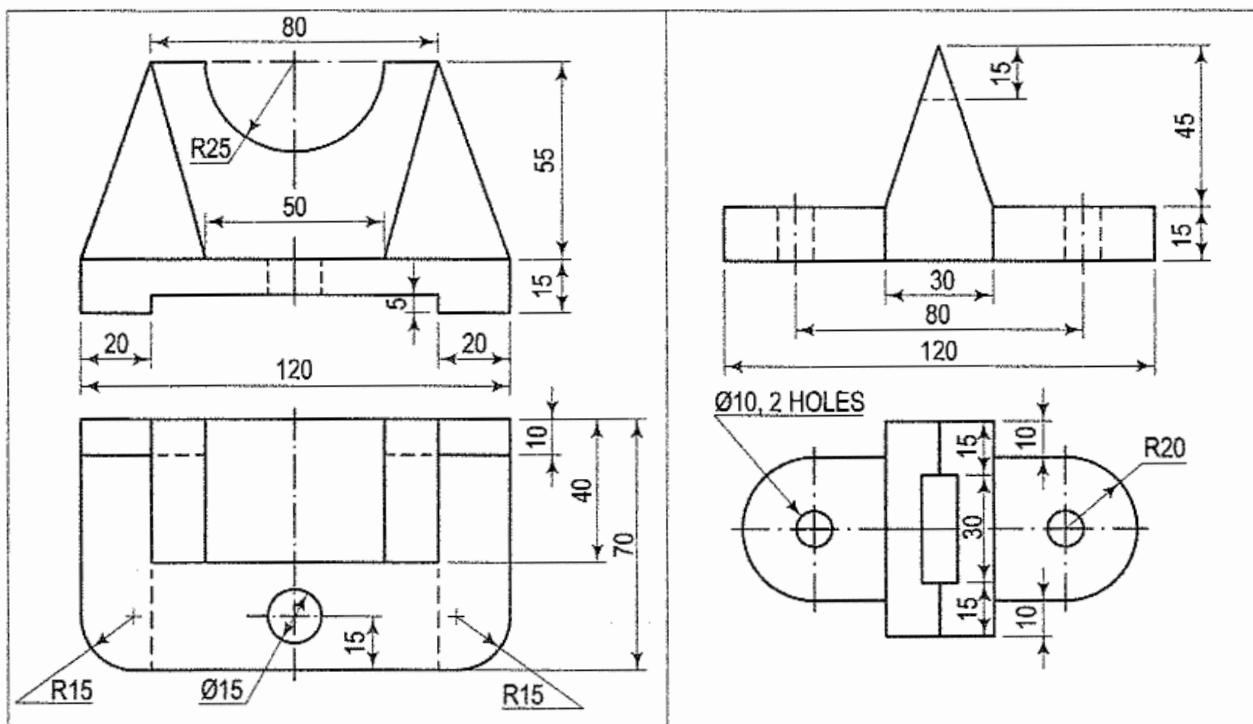
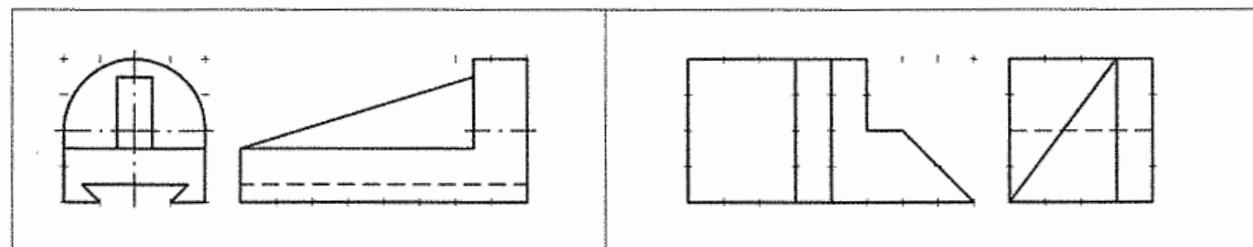


FIG. 17-114

FIG. 17-115

2. Assuming unit length to be equal to 10 mm, draw the isometric views of objects shown in figs. 17-116 to 17-125.



(Third-angle projection)

FIG. 17-116

FIG. 17-117

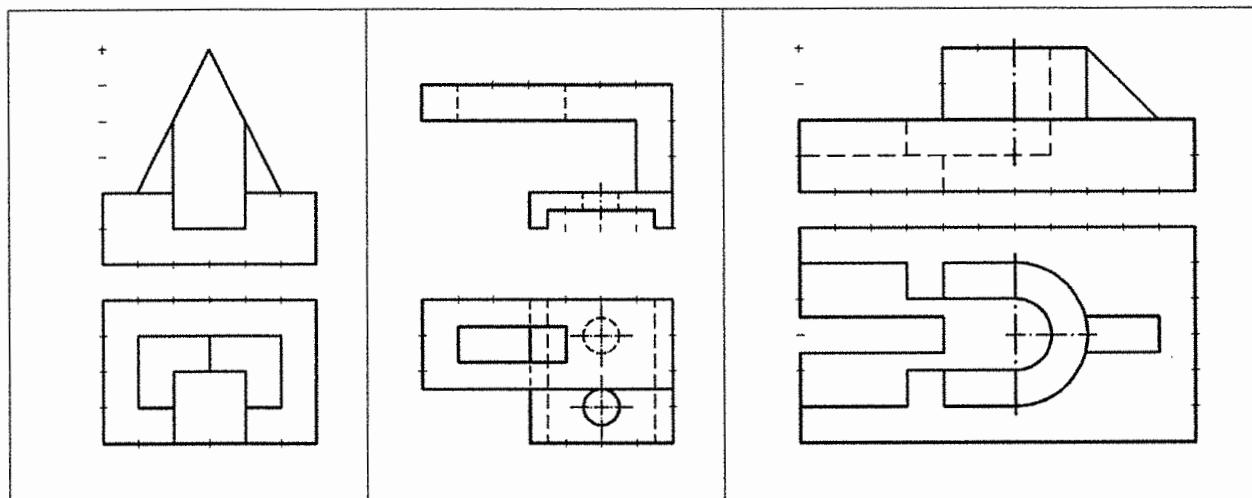
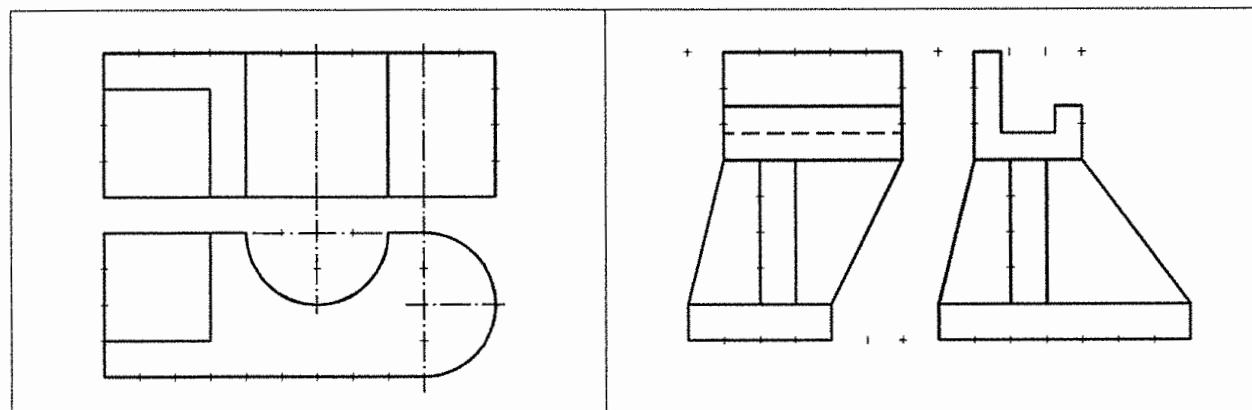


FIG. 17-118

FIG. 17-119

FIG. 17-120



(Third-angle projection)

FIG. 17-121

FIG. 17-122

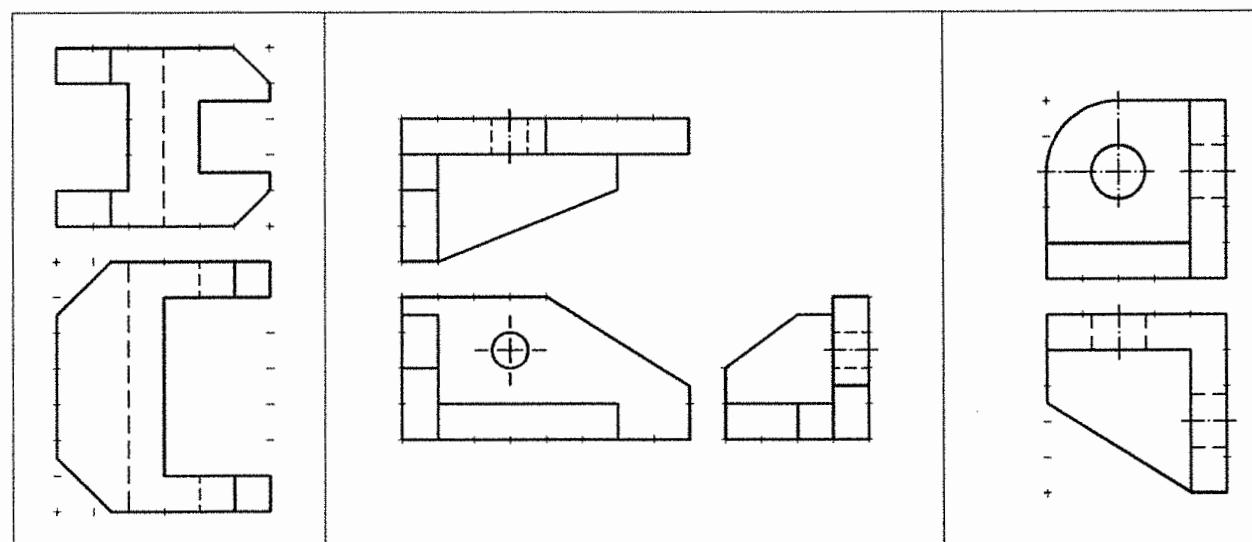


FIG. 17-123

(Third-angle projection)

FIG. 17-124

FIG. 17-125

3. Assuming simple graph of 10 mm grid, draw the isometric views of objects shown in fig. 17-126 and fig. 17-127.

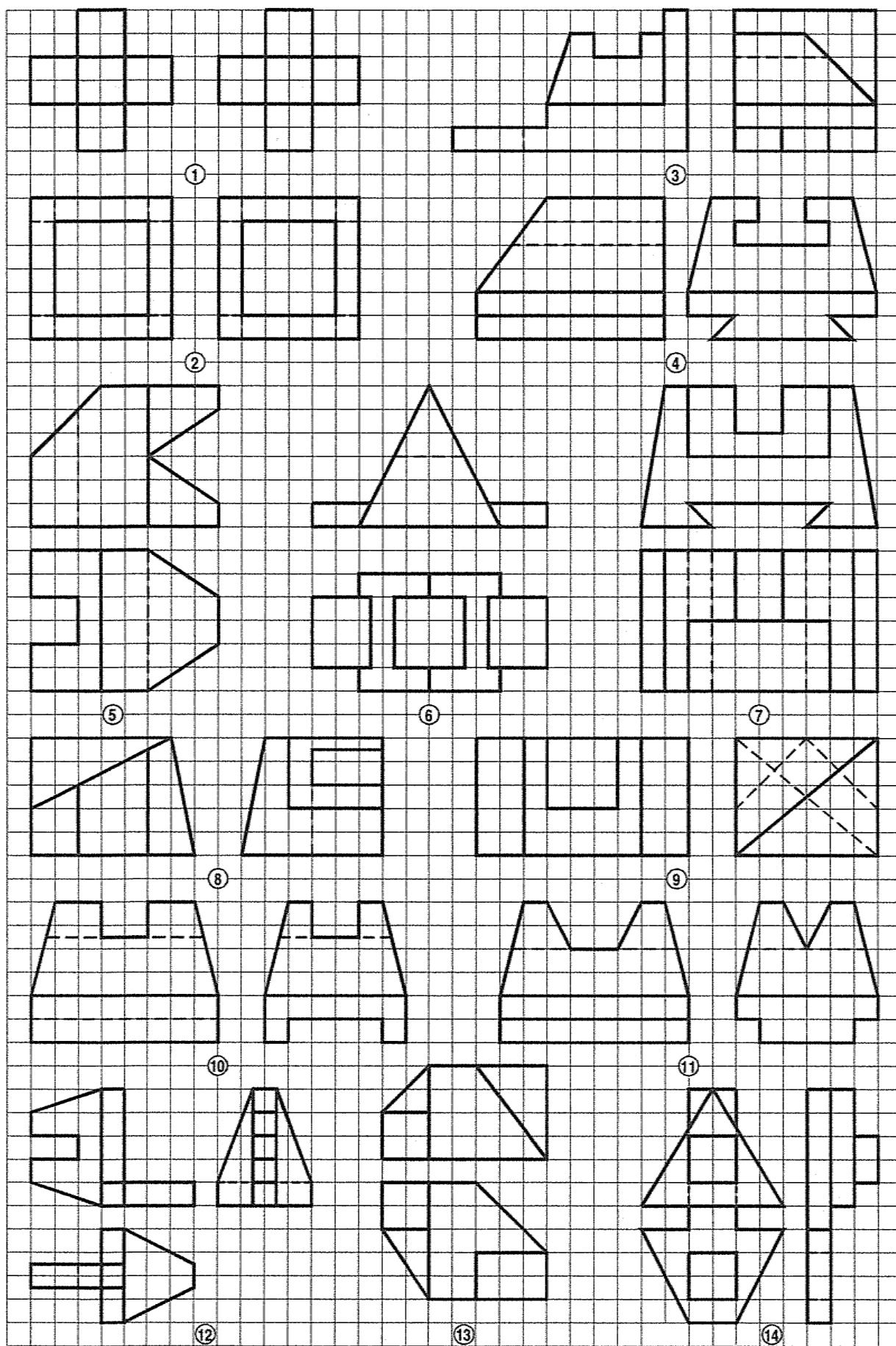


FIG. 17-126

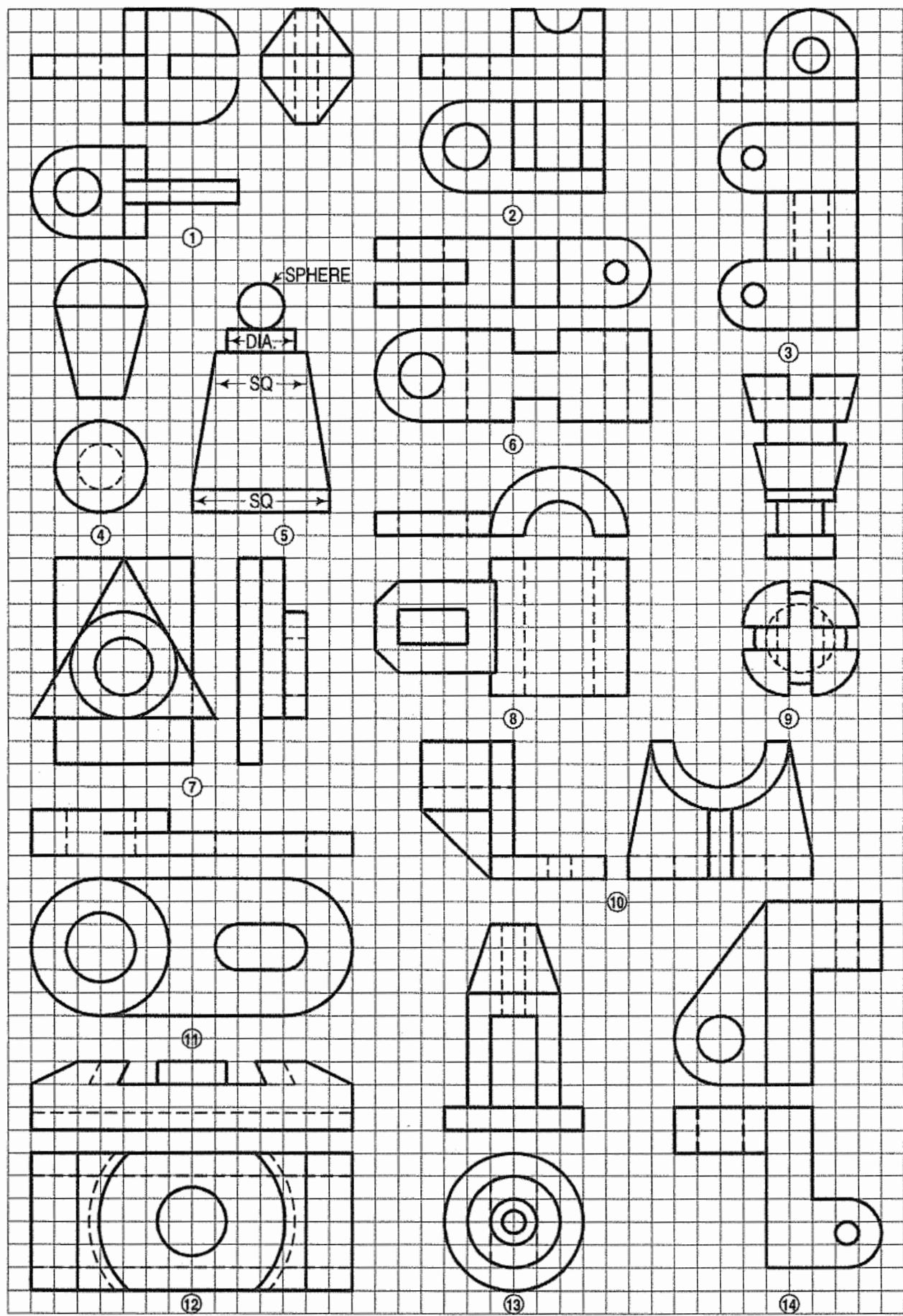


FIG. 17-127

4. Orthographic views of 35 objects with either (i) a line or (ii) lines or (iii) a view missing are given in fig. 17-128. Complete the given views. Also draw freehand, the isometric view of each object.

[For answer see fig. 17-194.]

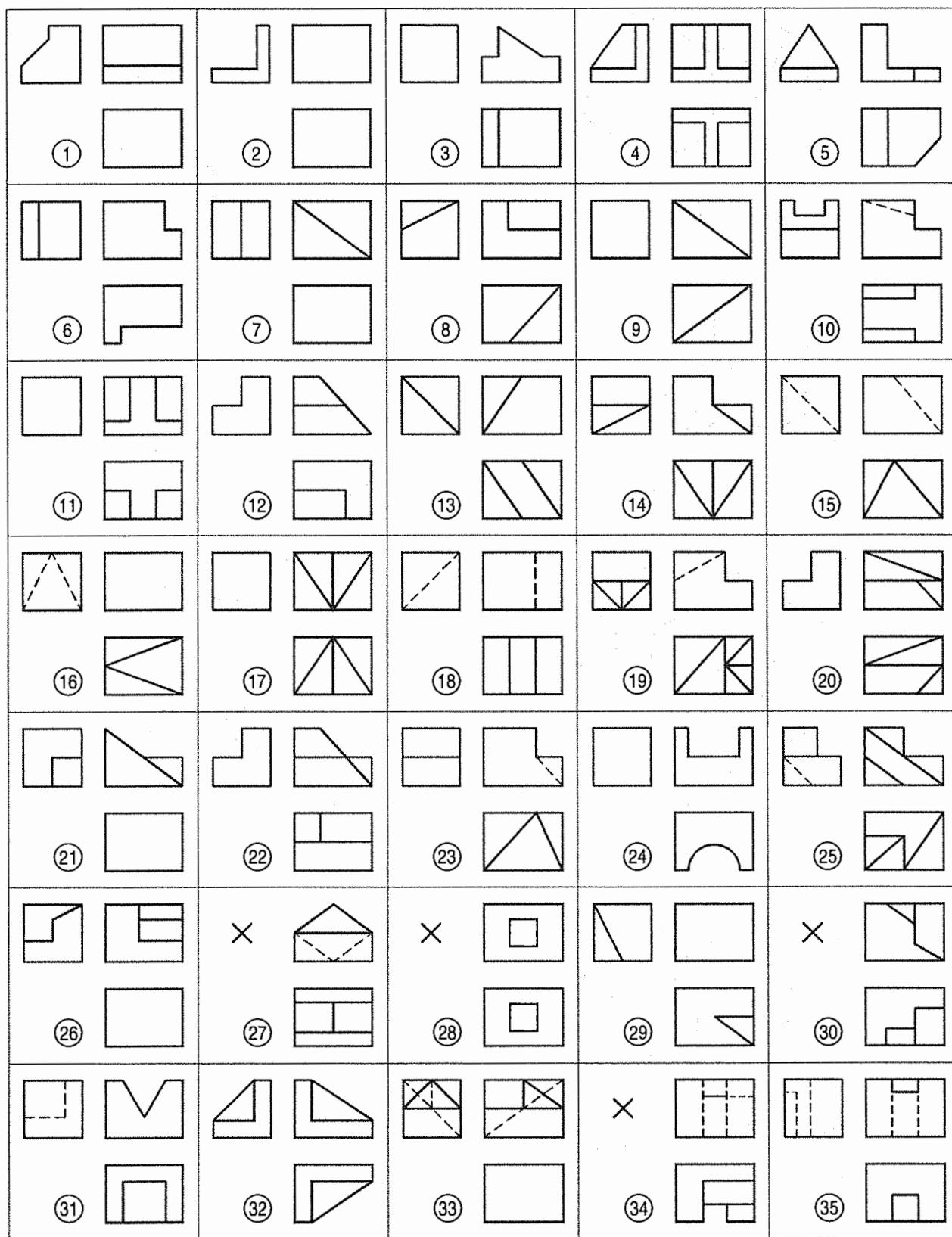
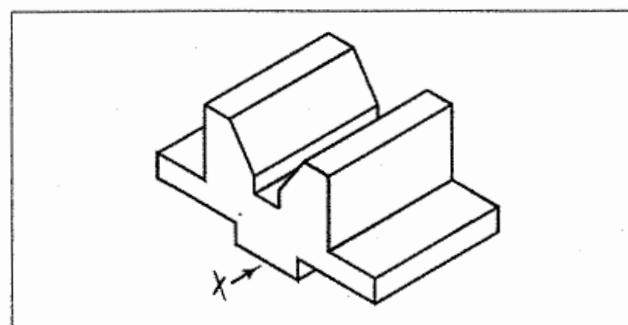
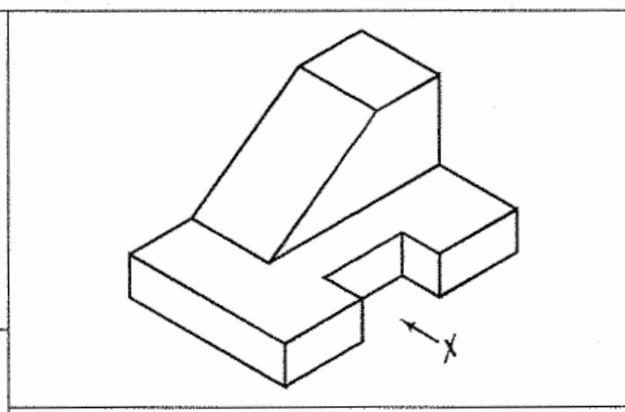


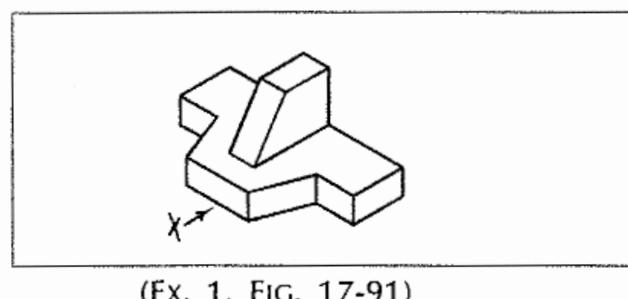
FIG. 17-128

SOLUTIONS TO EXERCISES 17

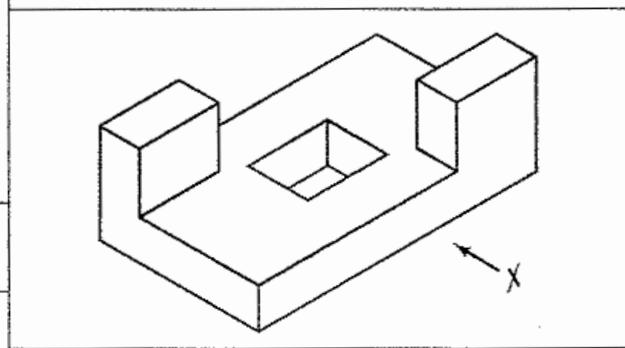
(Ex. 1. FIG. 17-89)
FIG. 17-129



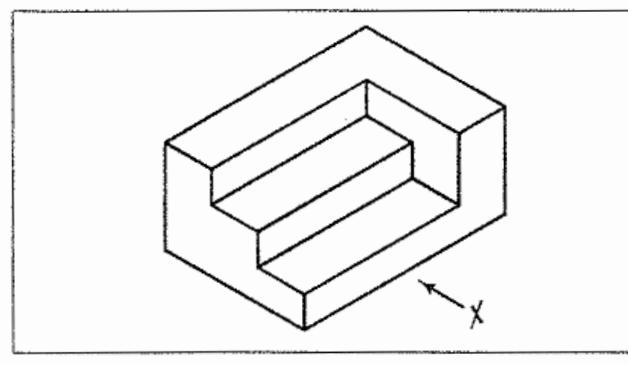
(Ex. 1. FIG. 17-90)
FIG. 17-130



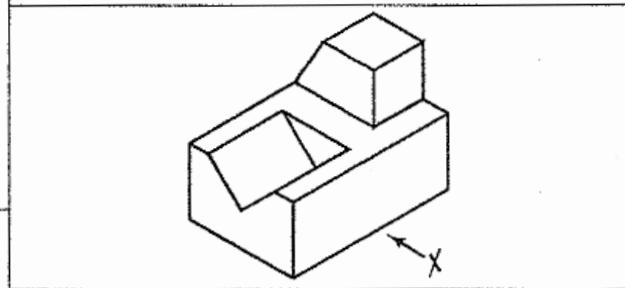
(Ex. 1. FIG. 17-91)
FIG. 17-131



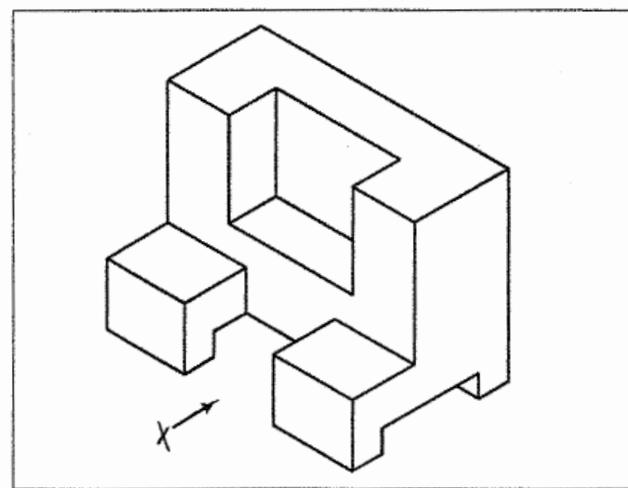
(Ex. 1. FIG. 17-92)
FIG. 17-132



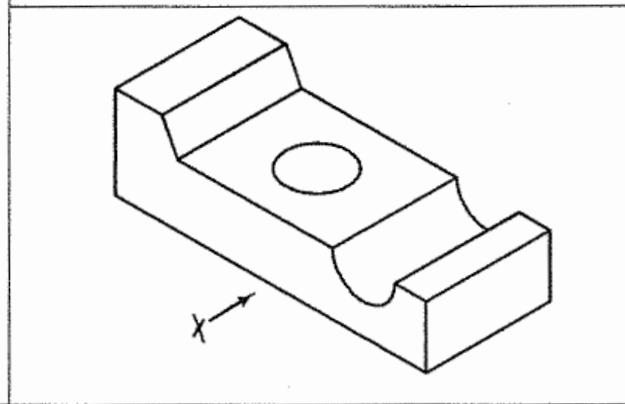
(Ex. 1. FIG. 17-93)
FIG. 17-133



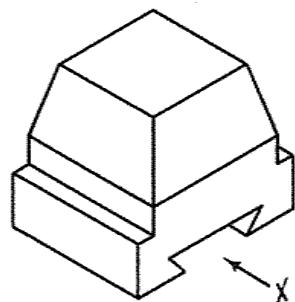
(Ex. 1. FIG. 17-94)
FIG. 17-134



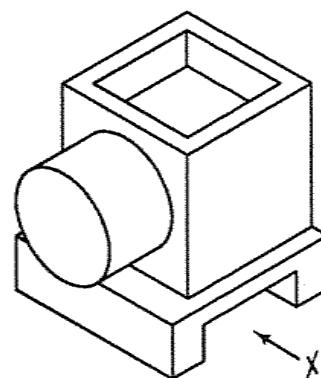
(Ex. 1. FIG. 17-95)
FIG. 17-135



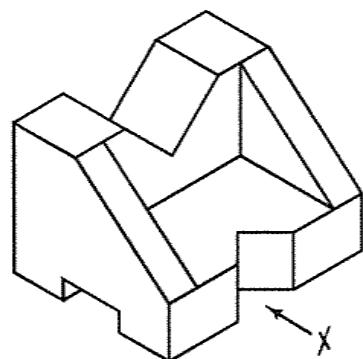
(Ex. 1. FIG. 17-96)
FIG. 17-136



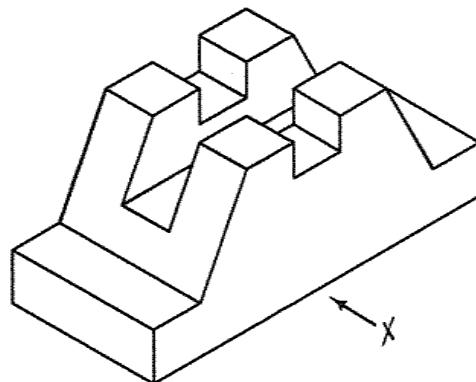
(Ex. 1. FIG. 17-97)
FIG. 17-137



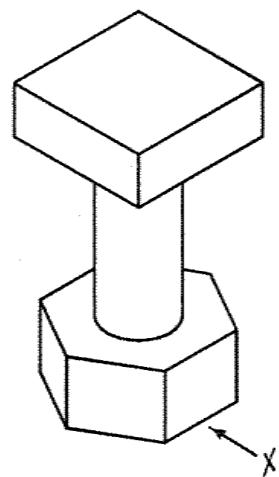
(Ex. 1. FIG. 17-98)
FIG. 17-138



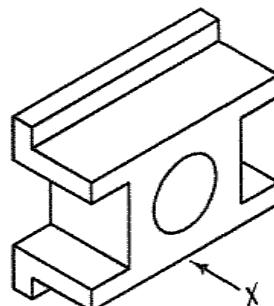
(Ex. 1. FIG. 17-99)
FIG. 17-139



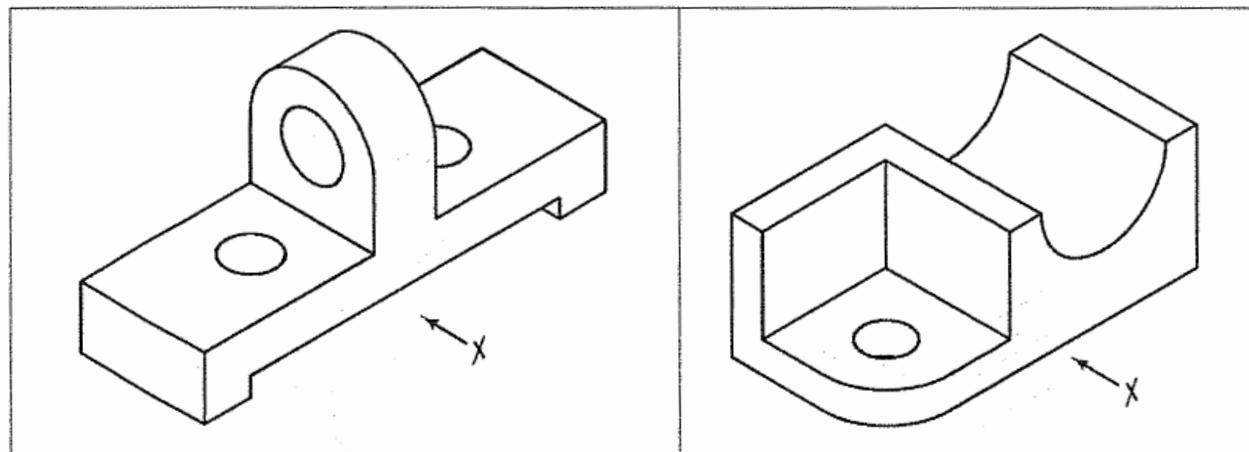
(Ex. 1. FIG. 17-100)
FIG. 17-140



(Ex. 1. FIG. 17-101)
FIG. 17-141

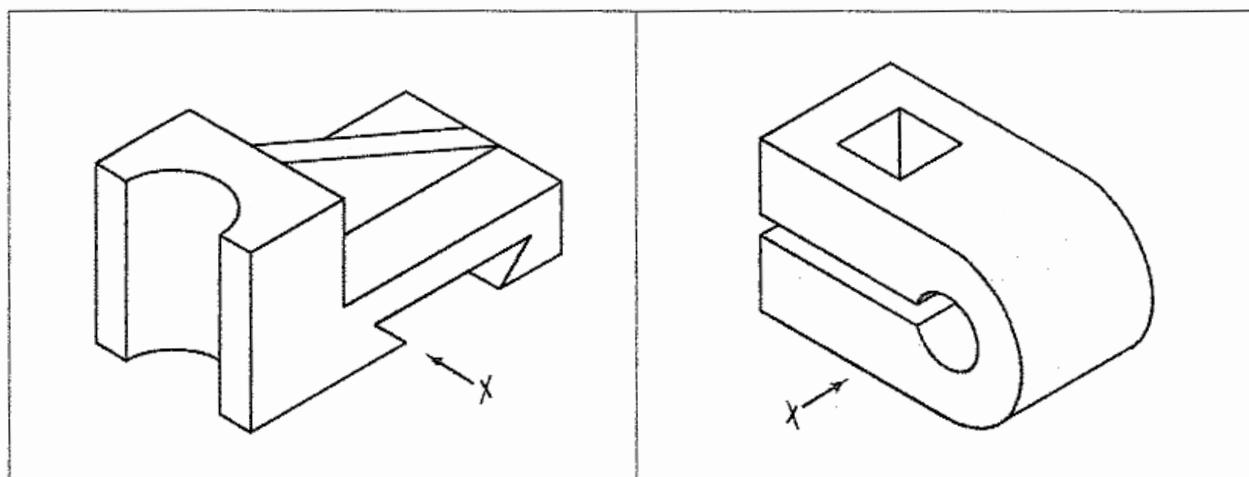


(Ex. 1. FIG. 17-102)
FIG. 17-142



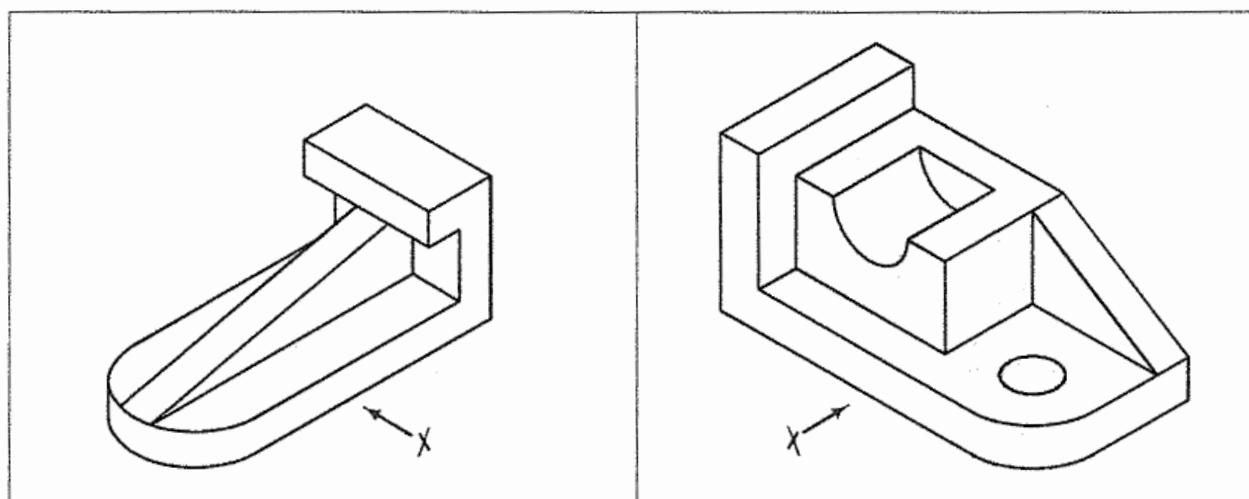
(Ex. 1, FIG. 17-103)
FIG. 17-143

(Ex. 1, FIG. 17-104)
FIG. 17-144



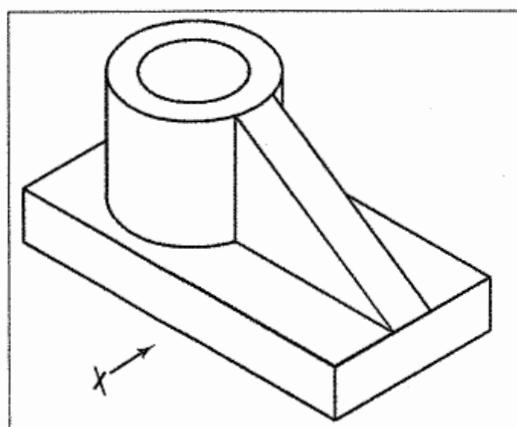
(Ex. 1, FIG. 17-105)
FIG. 17-145

(Ex. 1, FIG. 17-106)
FIG. 17-146

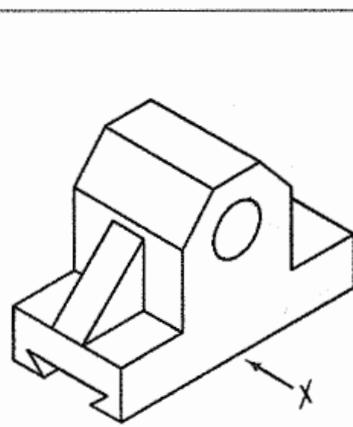


(Ex. 1, FIG. 17-107)
FIG. 17-147

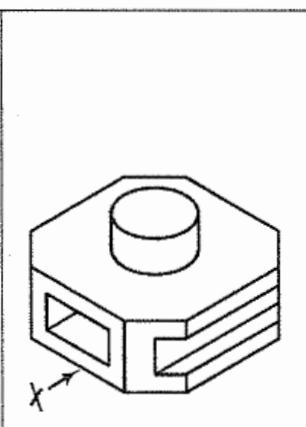
(Ex. 1, FIG. 17-108)
FIG. 17-148



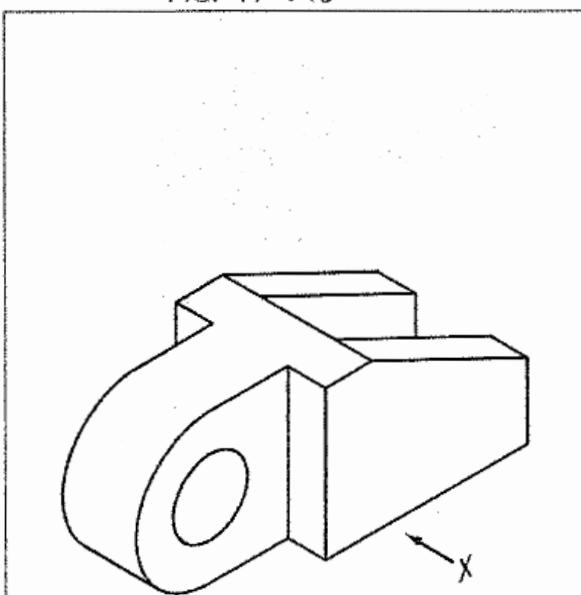
(Ex. 1. FIG. 17-109)
FIG. 17-149



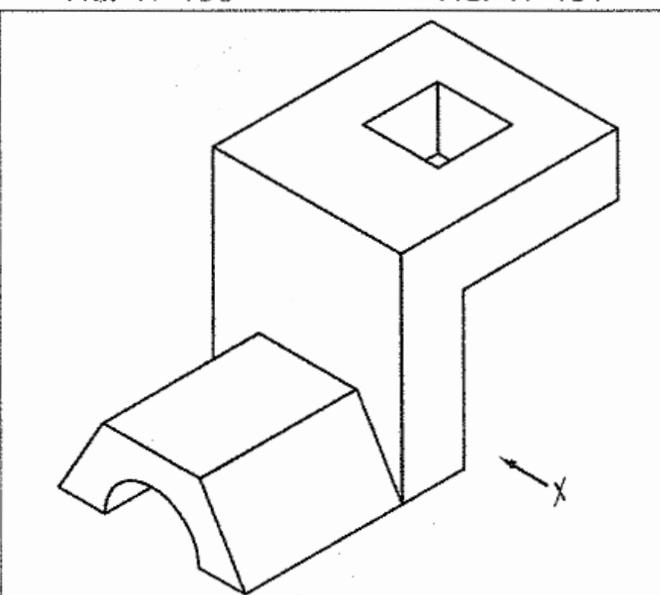
(Ex. 1. FIG. 17-110)
FIG. 17-150



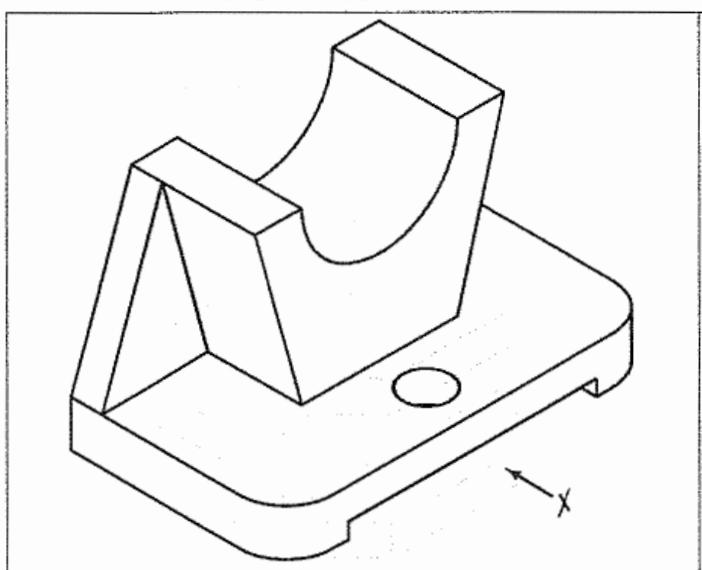
(Ex. 1. FIG. 17-111)
FIG. 17-151



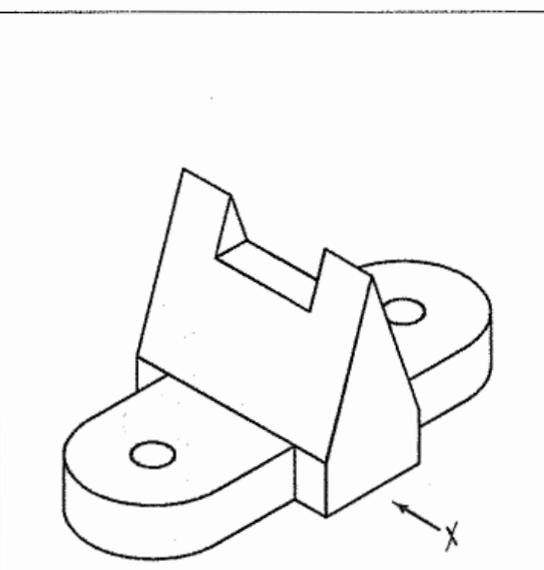
(Ex. 1. FIG. 17-112)
FIG. 17-152



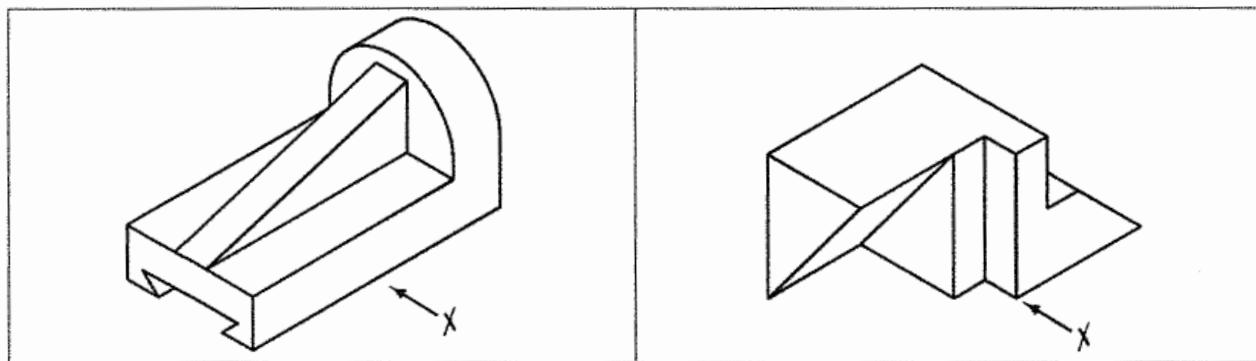
(Ex. 1. FIG. 17-113)
FIG. 17-153



(Ex. 1. FIG. 17-114)
FIG. 17-154

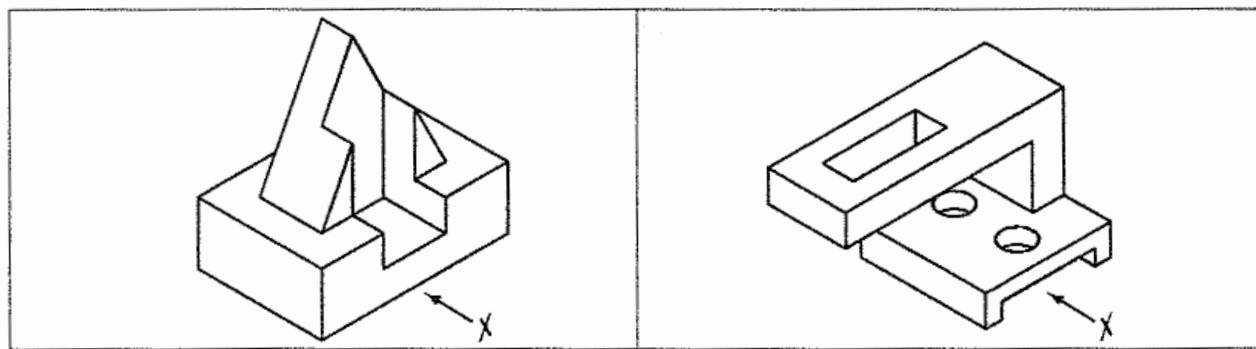


(Ex. 1. FIG. 17-115)
FIG. 17-155



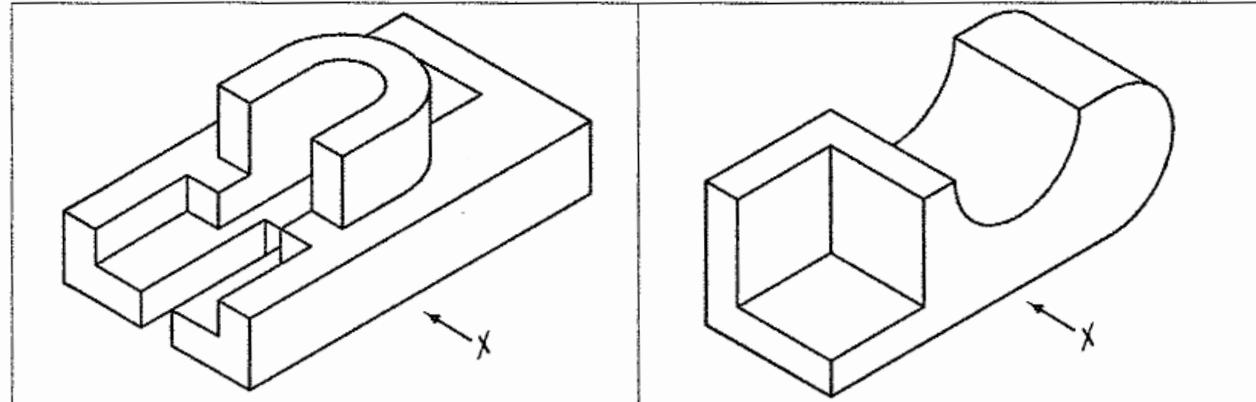
(Ex. 2. FIG. 17-116)
FIG. 17-156

(Ex. 2. FIG. 17-117)
FIG. 17-157



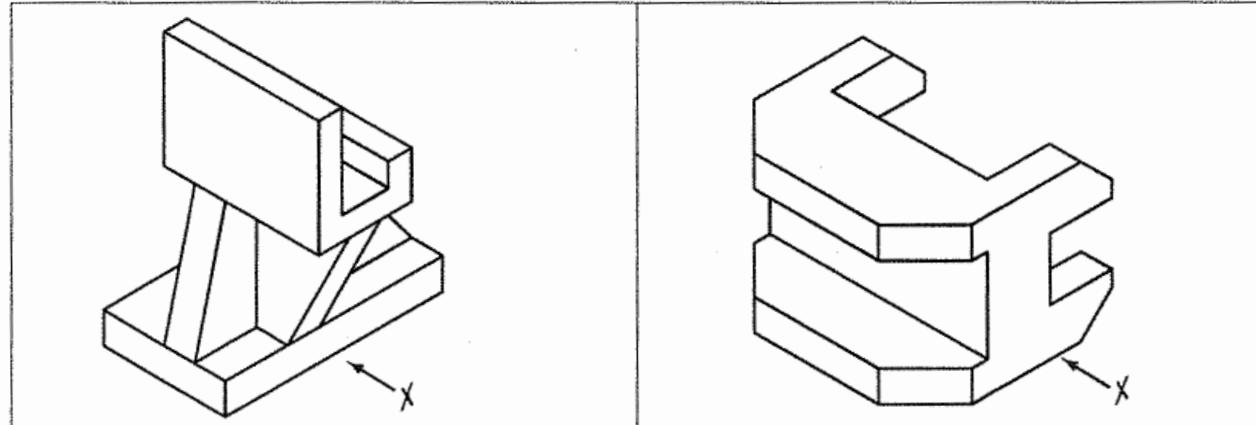
(Ex. 2. FIG. 17-118)
FIG. 17-158

(Ex. 2. FIG. 17-119)
FIG. 17-159



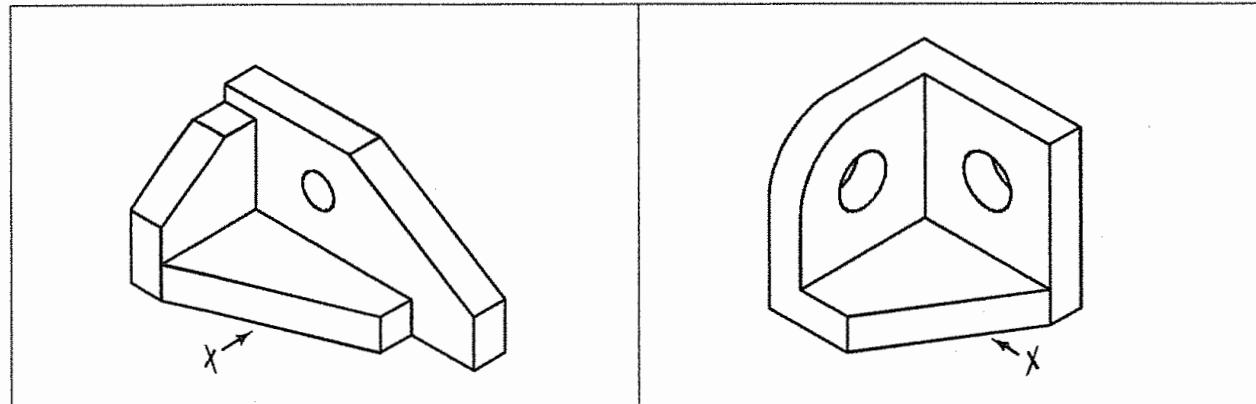
(Ex. 2. FIG. 17-120)
FIG. 17-160

(Ex. 2. FIG. 17-121)
FIG. 17-161



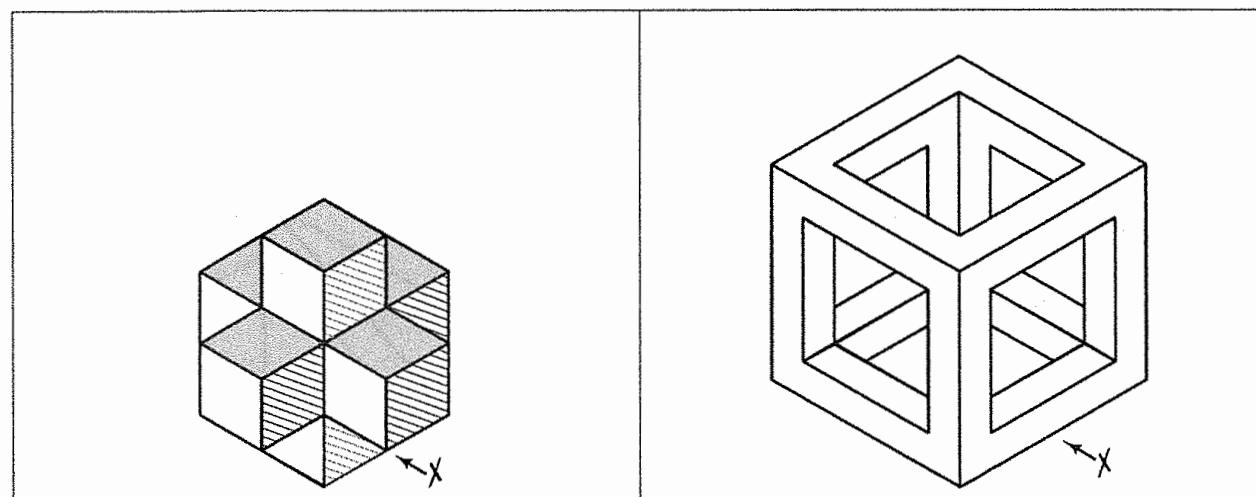
(Ex. 2. FIG. 17-122)
FIG. 17-162

(Ex. 2. FIG. 17-123)
FIG. 17-163



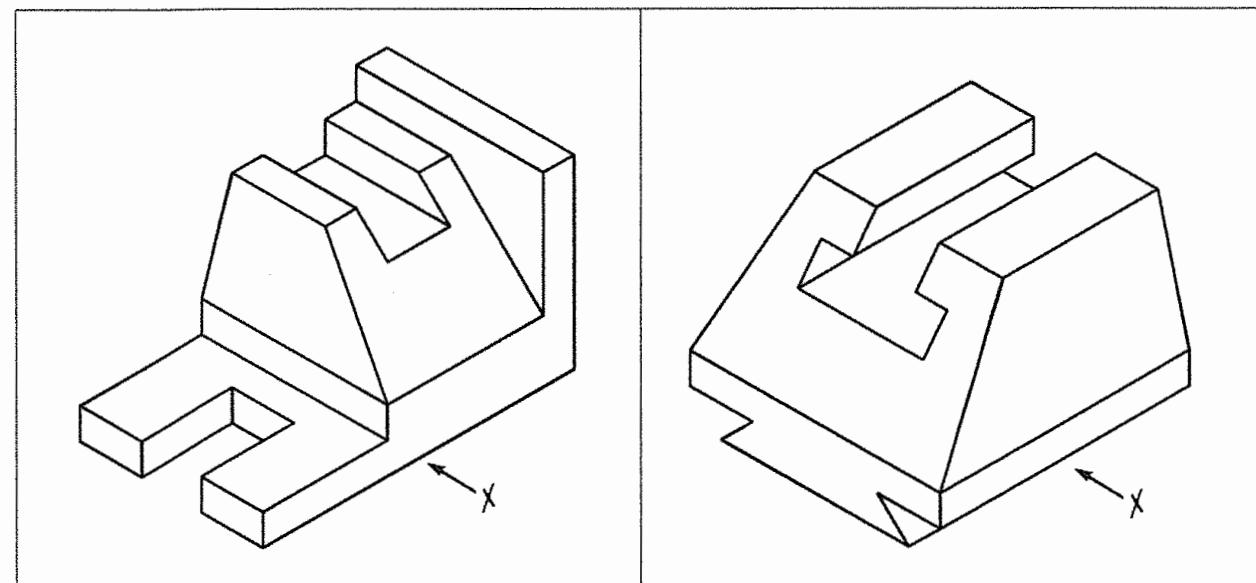
(Ex. 2. FIG. 17-124)
FIG. 17-164

(Ex. 2. FIG. 17-125)
FIG. 17-165



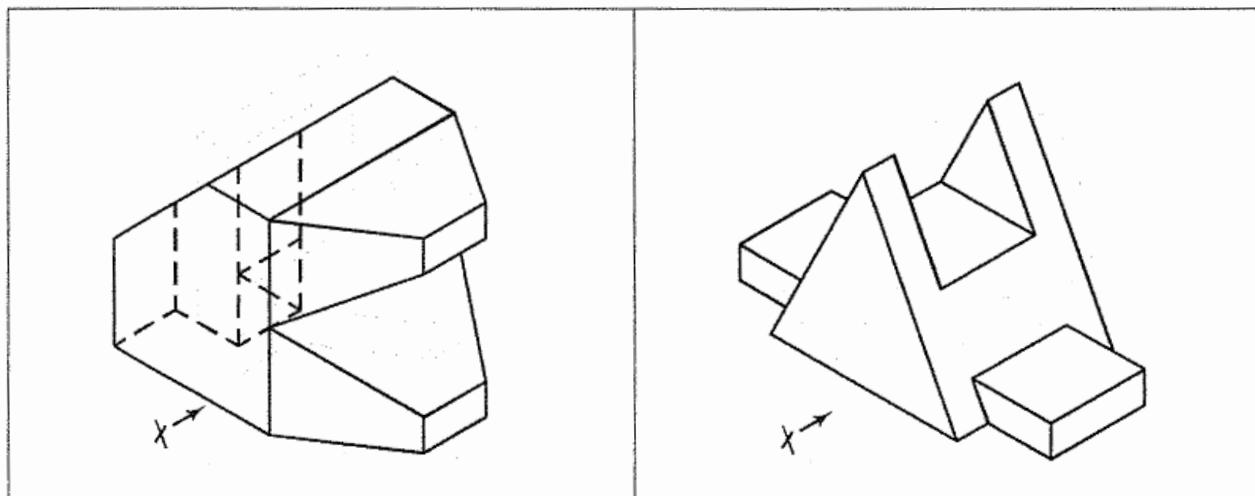
[Ex. 3. FIG. 17-126(1)]
FIG. 17-166

[Ex. 3. FIG. 17-126(2)]
FIG. 17-167

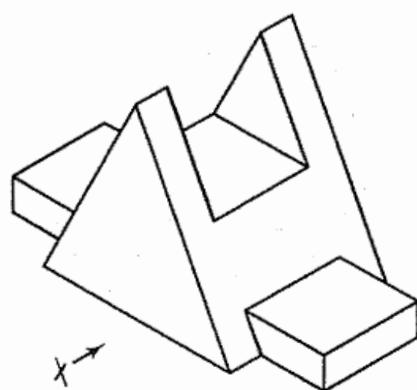


[Ex. 3. FIG. 17-126(3)]
FIG. 17-168

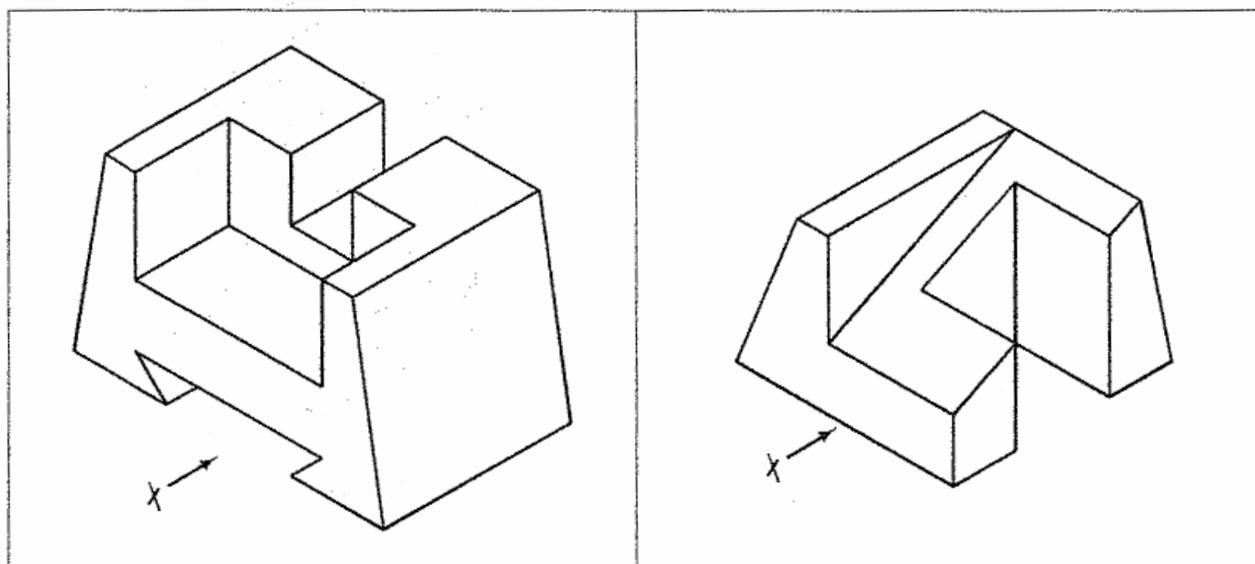
[Ex. 3. FIG. 17-126(4)]
FIG. 17-169



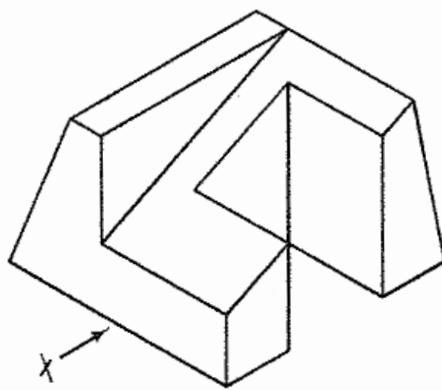
[Ex. 3. FIG. 17-126(5)]
FIG. 17-170



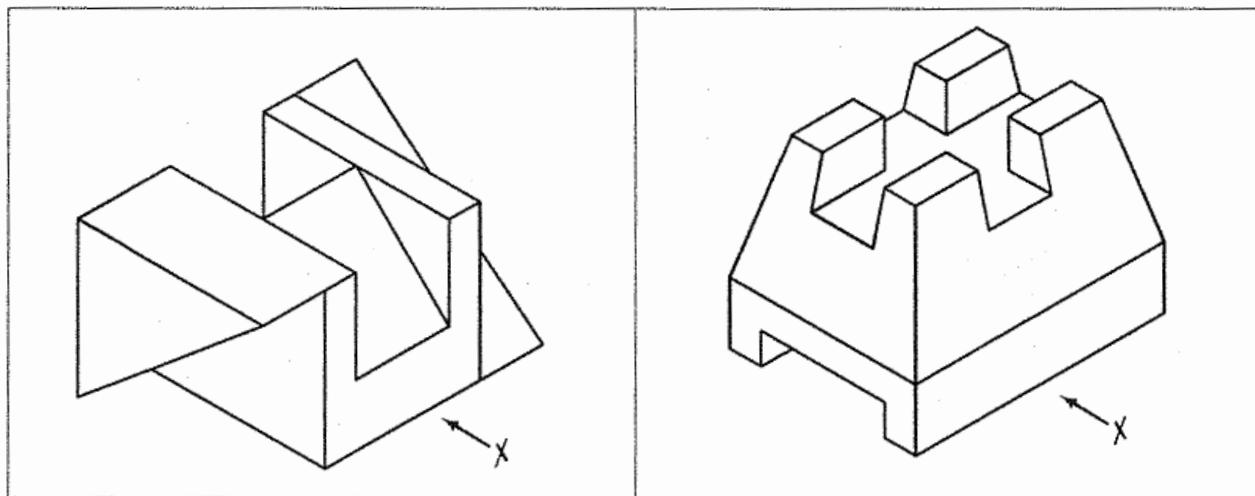
[Ex. 3. FIG. 17-126(6)]
FIG. 17-171



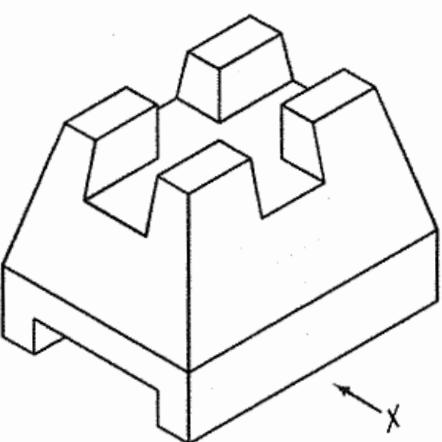
[Ex. 3. FIG. 17-126(7)]
FIG. 17-172



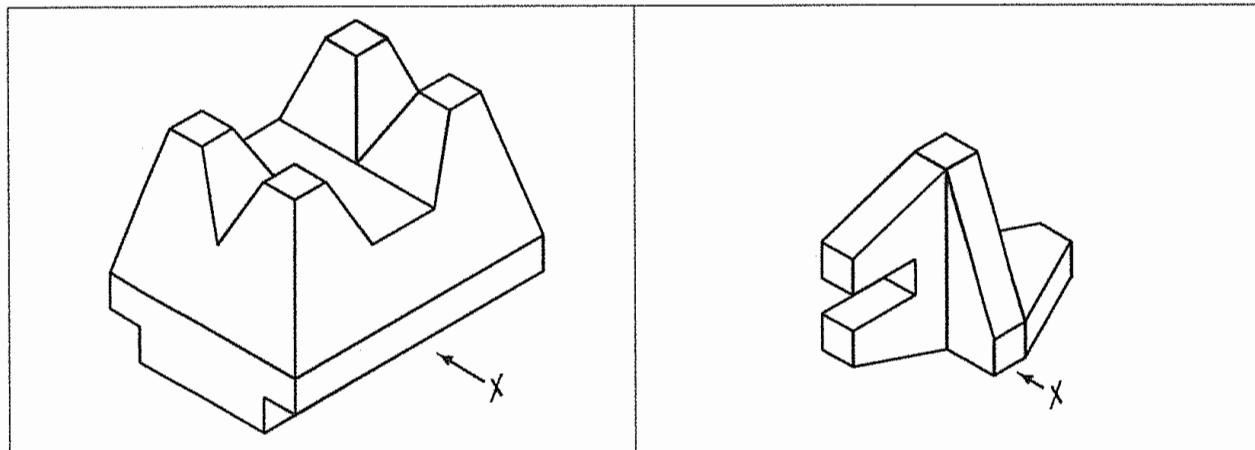
[Ex. 3. FIG. 17-126(8)]
FIG. 17-173



[Ex. 3. FIG. 17-126(9)]
FIG. 17-174

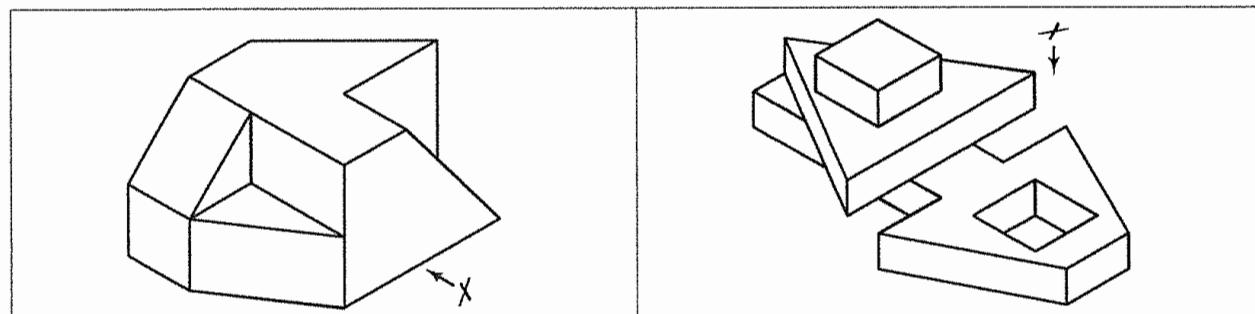


[Ex. 3. FIG. 17-126(10)]
FIG. 17-175



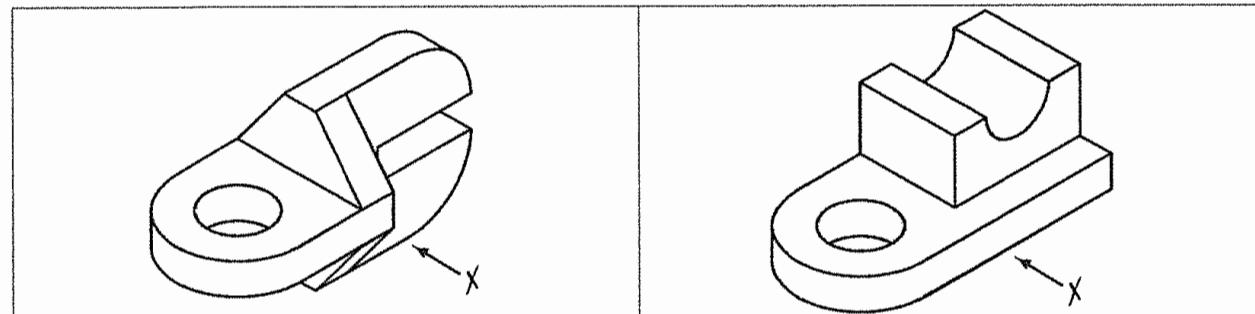
[Ex. 3. FIG. 17-126(11)]
FIG. 17-176

[Ex. 3. FIG. 17-126(12)]
FIG. 17-177



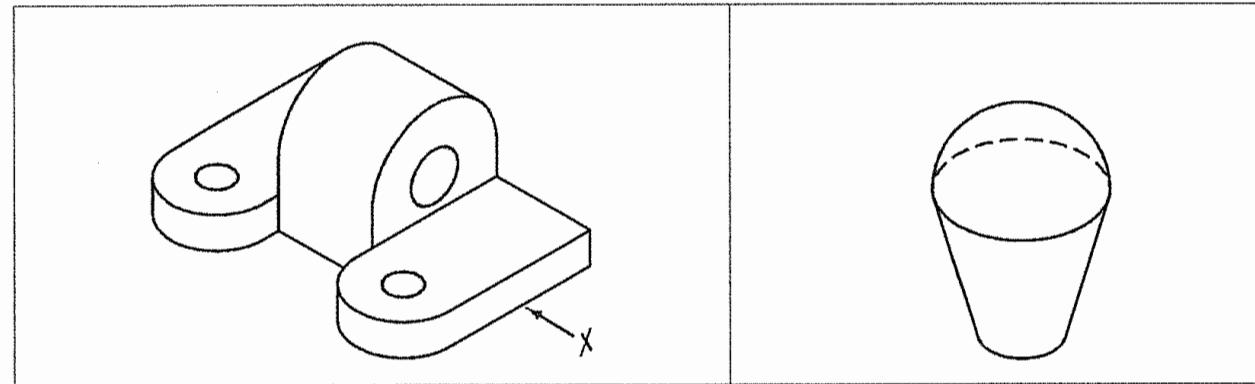
[Ex. 3. FIG. 17-126(13)]
FIG. 17-178

[Ex. 3. FIG. 17-126(14)]
FIG. 17-179



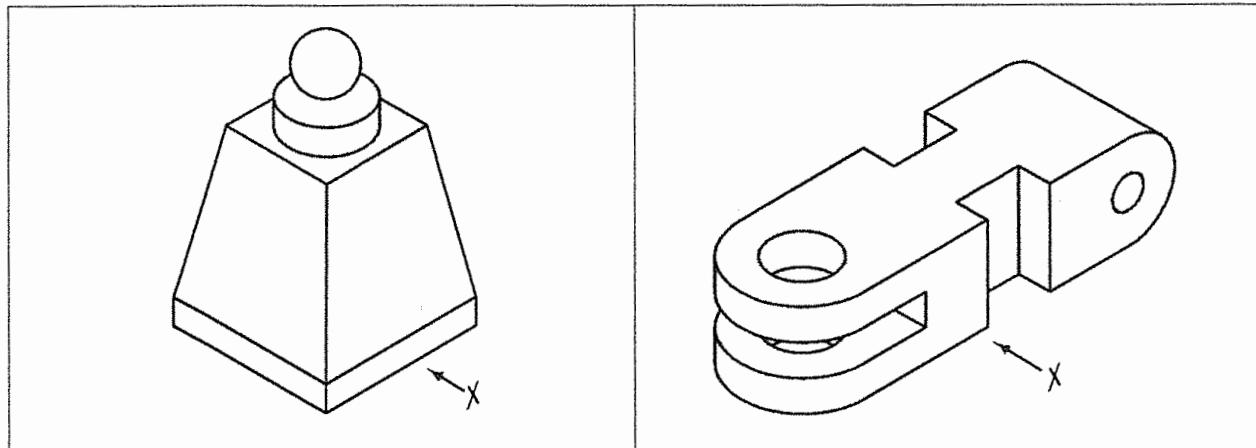
[Ex. 3. FIG. 17-127(1)]
FIG. 17-180

[Ex. 3. FIG. 17-127(2)]
FIG. 17-181



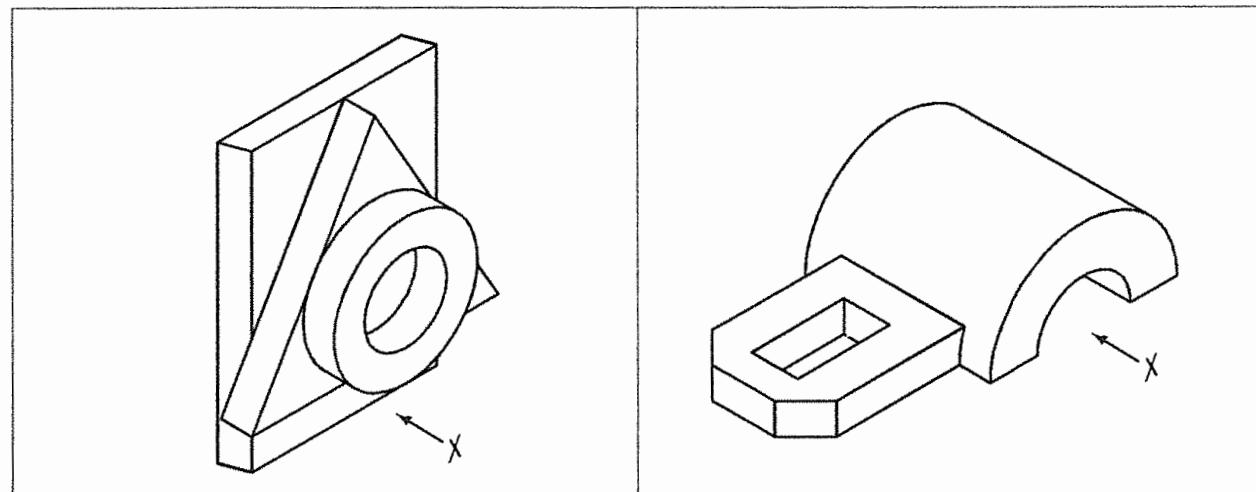
[Ex. 3. FIG. 17-127(3)]
FIG. 17-182

[Ex. 3. FIG. 17-127(4)]
FIG. 17-183



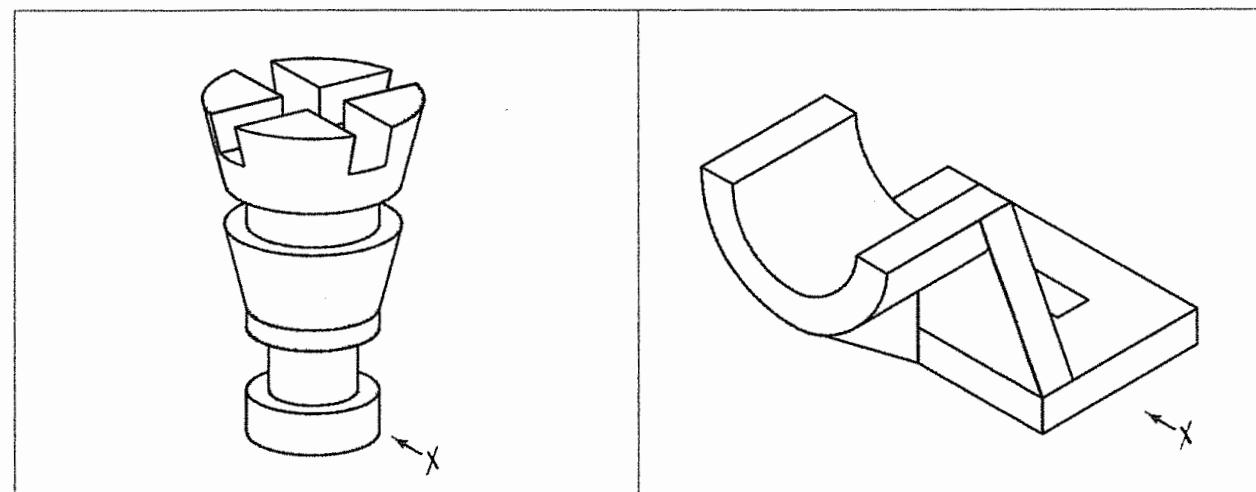
[Ex. 3. FIG. 17-127(5)]
FIG. 17-184

[Ex. 3. FIG. 17-127(6)]
FIG. 17-185



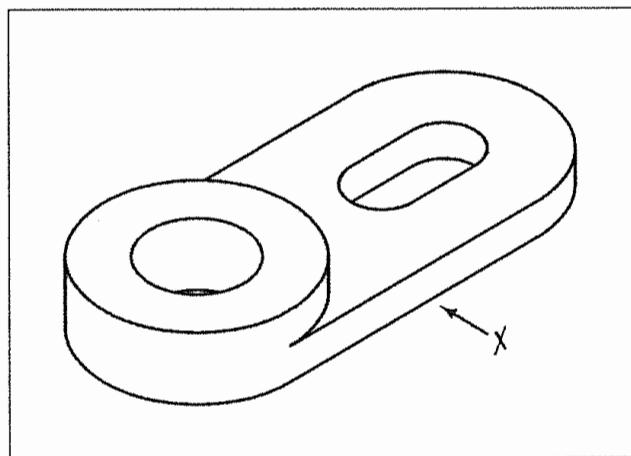
[Ex. 3. FIG. 17-127(7)]
FIG. 17-186

[Ex. 3. FIG. 17-127(8)]
FIG. 17-187

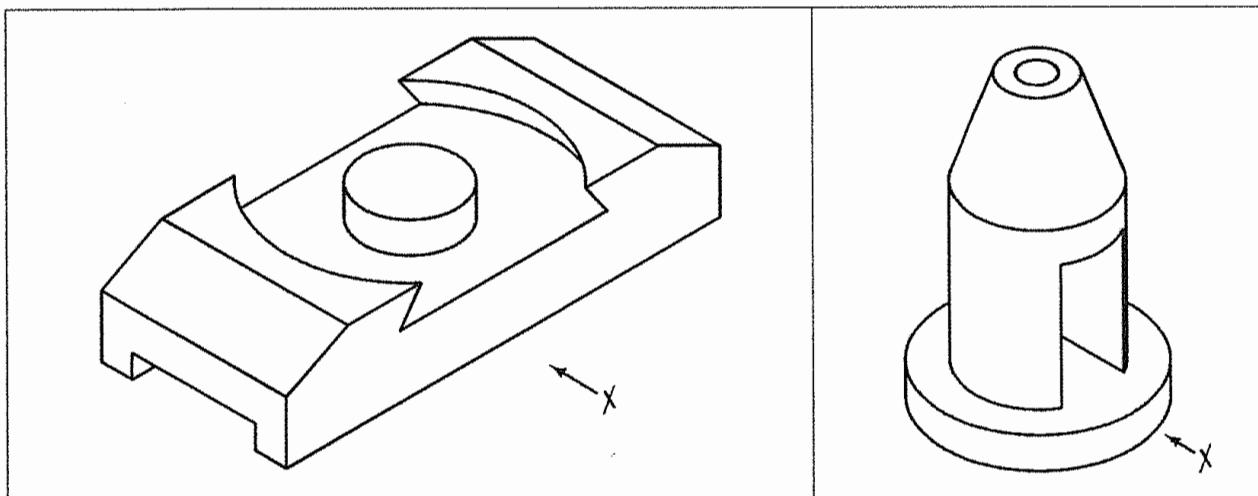


[Ex. 3. FIG. 17-127(9)]
FIG. 17-188

[Ex. 3. FIG. 17-127(10)]
FIG. 17-189

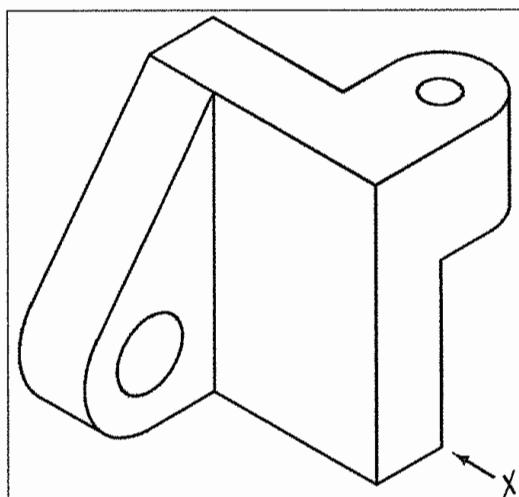


[Ex. 3. FIG. 17-127(11)]
FIG. 17-190

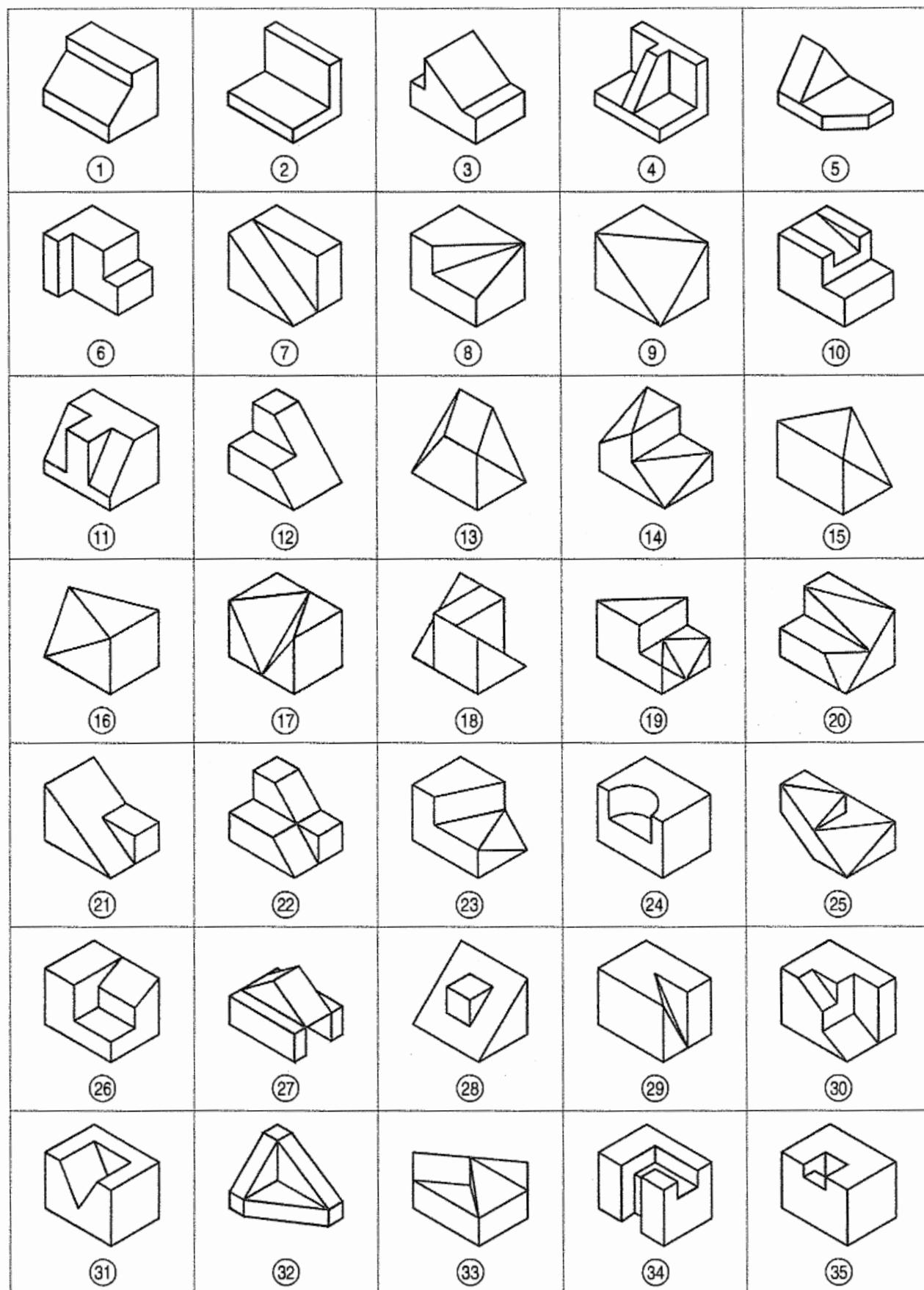


[Ex. 3. FIG. 17-127(12)]
FIG. 17-191

[Ex. 3. FIG. 17-127(13)]
FIG. 17-192



[Ex. 3. FIG. 17-127(14)]
FIG. 17-193



(Ex. 4. FIG. 17-128)

FIG. 17-194



18-1. INTRODUCTION

Pictorial projections are becoming popular due to the use of a computer in a modern drawing office. An oblique projection like isometric projection is another method of pictorial projection. The oblique projection represents three dimensional object on the projection plane by one view only. This type of drawing is useful for making an assembly of an object and provides directly a production drawing (working drawing) of the object for the manufacturing purpose.

This chapter deals with the following topics of oblique projection:

1. Principle of the oblique projection
2. The oblique projection and the isometric projection
3. Receding lines and receding angles
4. Types of the oblique projection
5. Rules for the choice of position of an object
6. Steps for drawing the oblique projection
7. Oblique projection of pyramid
8. Oblique projection of circle
9. Oblique projection of cylinder
10. Oblique projection of prism.

18-2. PRINCIPLE OF THE OBLIQUE PROJECTION

When an observer looks towards an object from infinity, the lines of sights (projectors) will be parallel to each other and inclined to the plane of projection, the resulting projection is known as the *oblique projection*.

Imagine that a cuboid is placed in front of a vertical plane as shown in fig. 18-1. In orthographic projection, the visual rays (edges) are perpendicular to the plane of projection and in oblique projection, the visual rays (edges) makes an angle of 60° with the plane of projection.

The face $PQRS$ which is parallel to the plane of projection, is projected in its true shape and size. The faces which are perpendicular to the plane of projection, are distorted in shape. The edges of perpendicular faces are inclined with the horizontal and projected to their true length.

The principal difference between the *oblique projection* and the *isometric projection* is that in case of the isometric projection all the three axes are inclined at an angle of 120° to each other whereas in the oblique projection third axis is inclined at an angle of 30° or 45° or 60° with respect to two perpendicular axes.

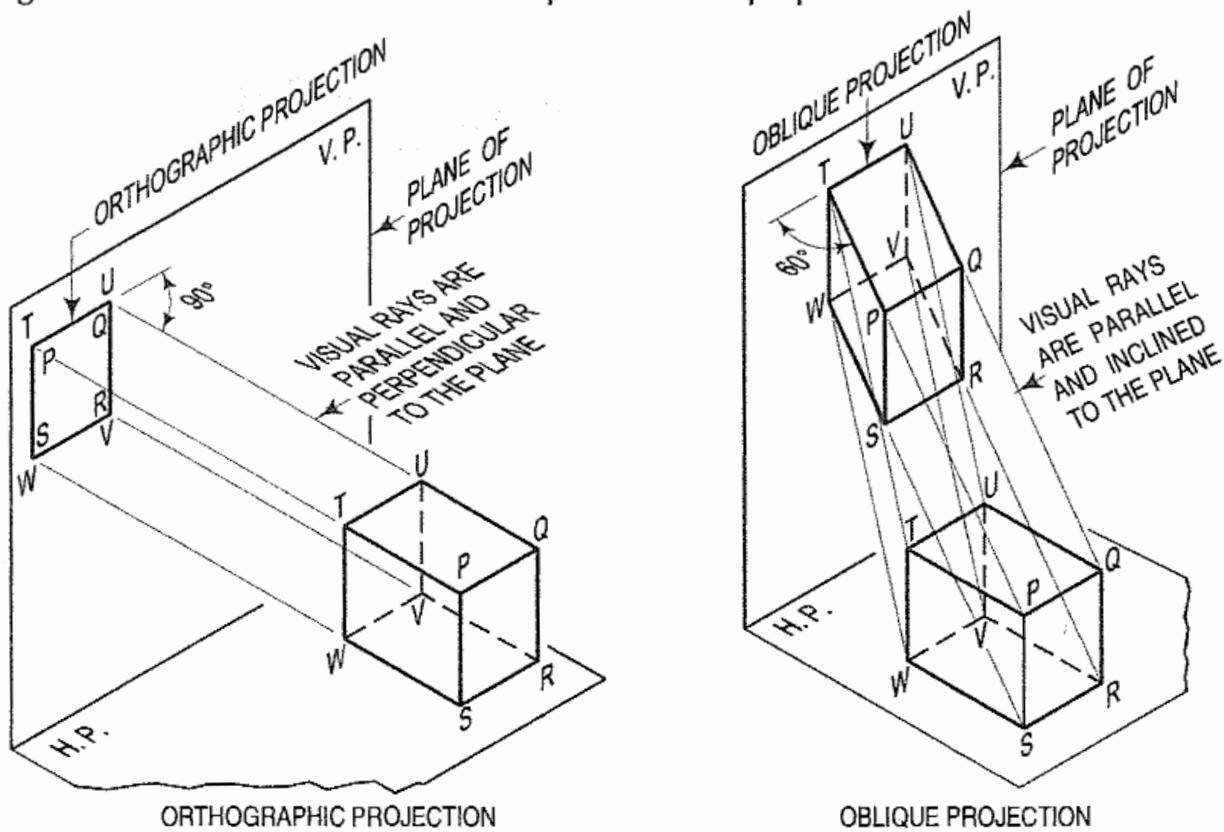


FIG. 18-1

18-3. THE OBLIQUE PROJECTION AND THE ISOMETRIC PROJECTION

The difference between the oblique projection and the isometric projection is given in table 18-1.

TABLE 18-1

Oblique projection		Isometric projection	
(i)	Projectors from an object are parallel to each other and inclined to the plane of projection.	(i)	Projectors from an object are parallel to each other and perpendicular to the plane of picture.
(ii)	The object is placed in such a way that one of its prominent faces remains parallel to the plane of projection. The object rests on one of its faces.	(ii)	The object is kept in such a way that its three mutual perpendicular edges (axes) make equal angles with the plane of projection. The object stands on one of its corners.
(iii)	The object is drawn with the actual dimensions.	(iii)	The object is drawn with the reduced (about 82%) dimensions.
(iv)	The faces of object which are perpendicular to the plane of projection will be distorted in the shape and size.	(iv)	All the faces of the object are distorted in the shape and size.
(v)	The choice of position of the object depends upon the shape and size.	(v)	The choice of position of the object depends upon the shape and size.

18-4. RECEDING LINES AND RECEDING ANGLES

In oblique projection, the faces of object which are perpendicular to the plane of projection are projected in the distorted shapes. The perpendicular edges of such planes are drawn at an angle of 30° or 45° or 60° with the horizontal. These inclined lines are known as the *receding lines* and their inclinations to the horizontal is known as *receding angles*.

The appearance of distortion of an object can be improved by shortening the length of the receding lines. Refer to fig. 18-3.

The receding lines may be inclined either upwards or downwards, or to the left or right depending upon the necessity to show the details of an object effectively. Refer fig. 18-2.

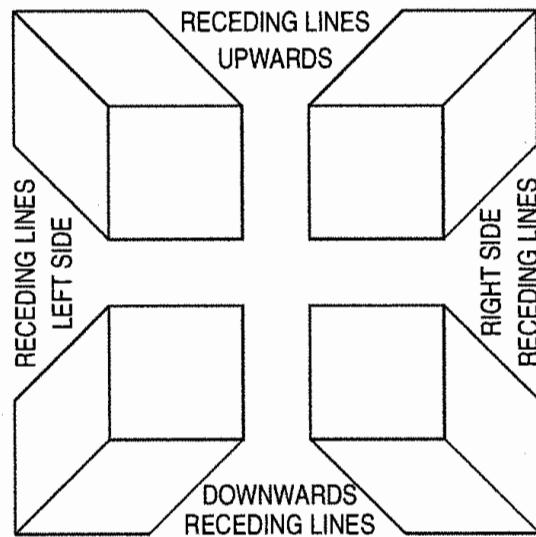


FIG. 18-2

18-5. TYPES OF THE OBLIQUE PROJECTION

The oblique projection is based on scales by which the receding lines are drawn.

(1) **Cavalier projection:** When the receding lines are drawn to full size scale and the projectors inclined at an angle of 30° or 45° or 60° to the plane of projection, such oblique projection is known as *cavalier projection*. Refer to fig. 18-3(i).

(2) **Cabinet projection:** If the receding lines are drawn to half size scale such oblique projection is known as the *cabinet projection*. Refer to fig. 18-3(ii).

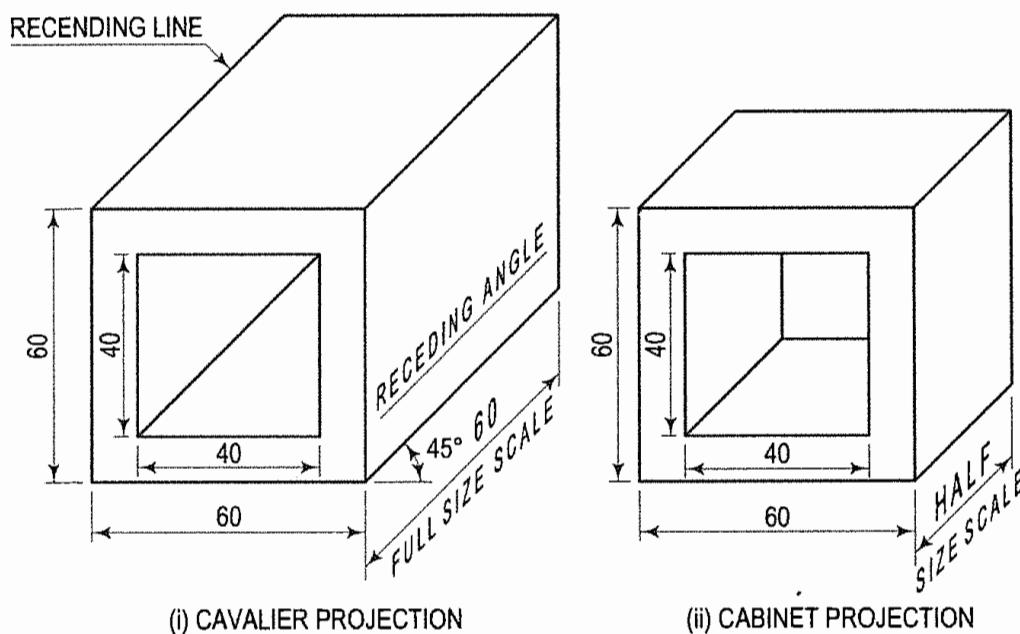


FIG. 18-3

18-6. RULES FOR THE CHOICE OF POSITION OF AN OBJECT

In oblique projection, the object is assumed to be placed with one face parallel to the plane of projection. Hence, that face appears in its true shape and size (fig. 18-4).

This gives two main dimensions of the object. The third dimension is shown by lines drawn at a convenient angle, generally 30° or 45° with the horizontal. To give a natural appearance, these lines are drawn $\frac{3}{4}$ or $\frac{1}{2}$ the actual lengths. Thus, in an oblique projection also, there are three axes — a vertical, a horizontal and a third, inclined at an angle of 30° or 45° with the horizontal.

Rectangular surfaces and circles parallel to the third axis are shown parallelograms and ellipses respectively. When an object has curved surfaces or long edges, the face containing such surfaces or edges is usually so placed that it may appear in its true shape. By doing so, the drawing is simplified and the amount of distortion is considerably reduced. Fig. 18-5 shows the guide with its longer edges parallel to the inclined axis. Comparing fig. 18-4 with this view, it can be seen that the former gives a clear idea of the shape of the guide. The choice of the position of an object should be such that minimum distortions of the object can occur. This can be achieved by observing the following rules:

Rule I. Keep the longest dimension parallel to the plane of projection. This may reduce the distortion effect of the object (fig. 18-6).

Rule II. The face of an object containing essential contours (i.e. circles and irregular shapes etc.) must be kept parallel to the plane of projection (fig. 18-7).

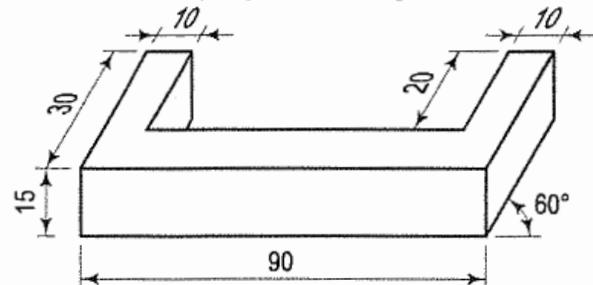


FIG. 18-6

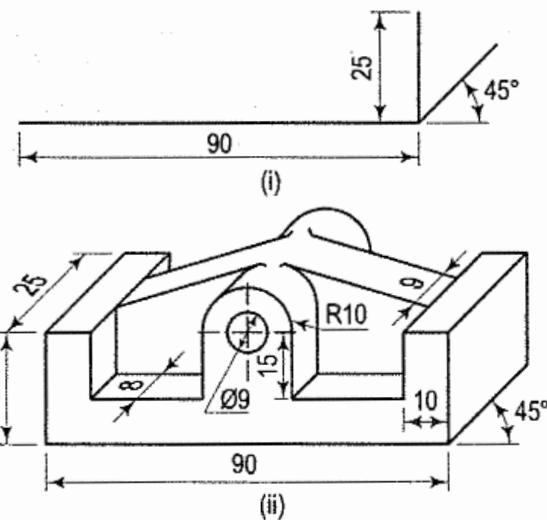


FIG. 18-4

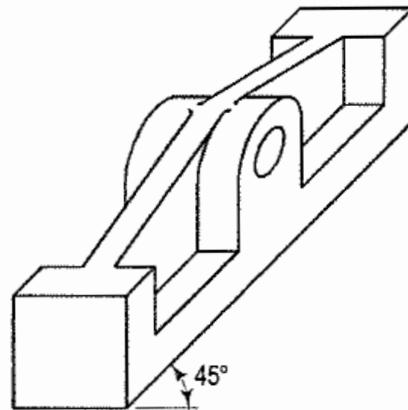


FIG. 18-5

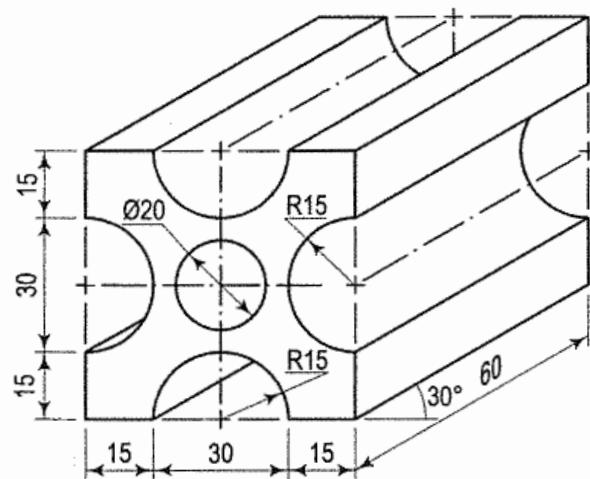


FIG. 18-7

18-7. STEPS FOR DRAWING THE OBLIQUE PROJECTION

The steps for preparing an oblique projection from orthographic projections are illustrated in the following problem.

Problem 18-1. (fig. 18-8 and fig. 18-9): *The orthographic projections of a casting are given in fig. 18-8. Draw the oblique projection when the receding axis is inclined at an angle of 30° to the horizontal.*

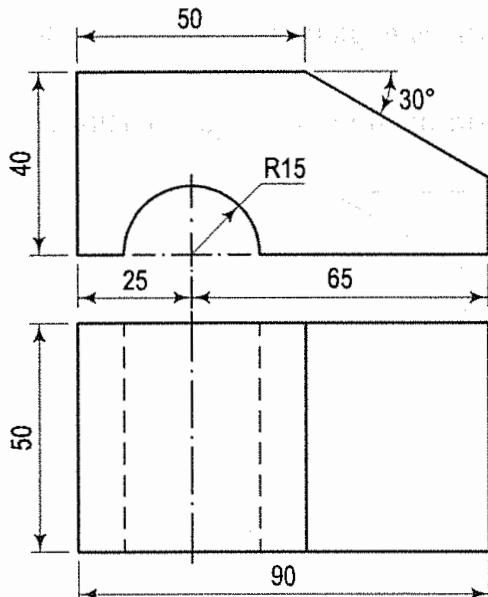
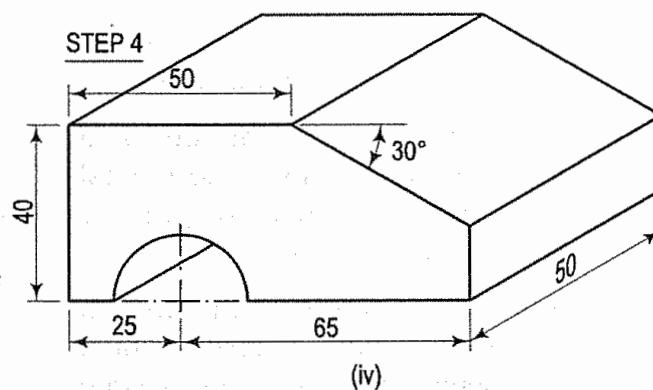
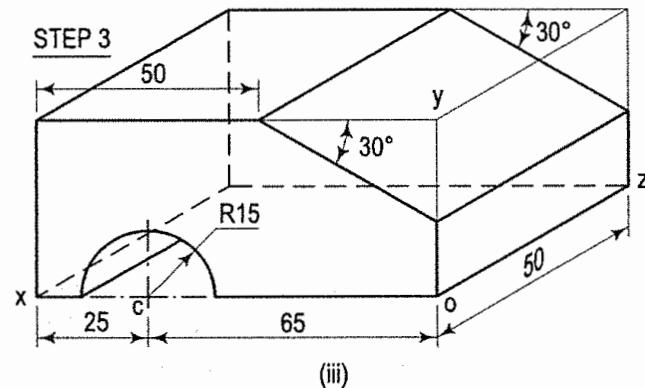
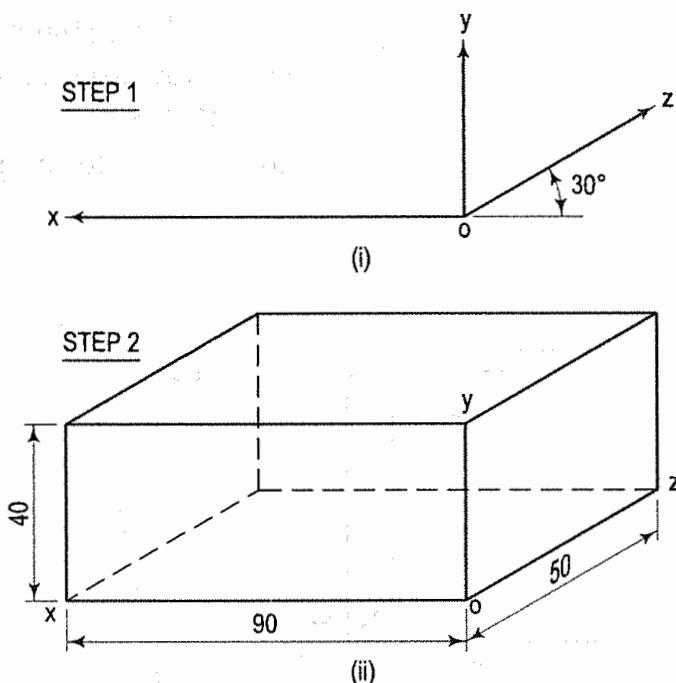


FIG. 18-8

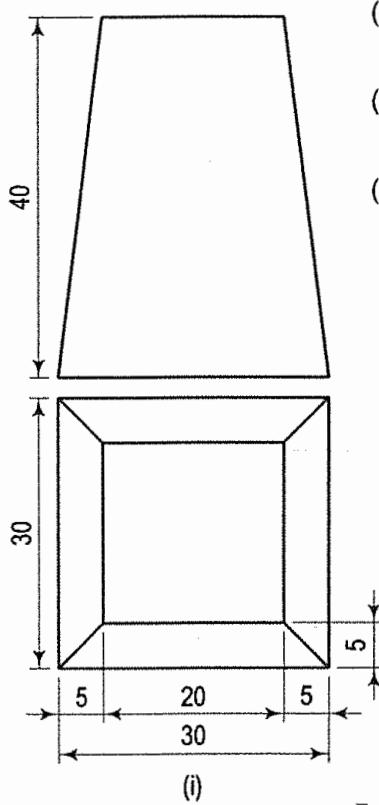


- Mark the point o and draw ox and oy mutually perpendicular axes. Draw receding axis oz at an angle of 30° with the axis ox (horizontal). Fig. 18-9(i).
- Construct a box by marking distances of 90 mm, 40 mm and 50 mm along the axes ox , oy and oz respectively as shown in fig. 18-9(ii).
- Mark a centre c of semi-circle along the axis ox at a distance of 65 mm from the point o . With the point c as centre and radius equal to 15 mm, draw a semi-circle. Mark a distance of 50 mm on the line parallel to ox and draw an angle of 30° as shown in fig. 18-9(iii).
- Complete the oblique view as shown in fig. 18-9(iv).

FIG. 18-9

18-8. OBLIQUE DRAWING OF PYRAMID

Problem 18-2. (fig. 18-10): A frustum of a square pyramid has its base 30 mm, top 20 mm, side and height 40 mm. Draw the oblique projection of the pyramid when it rests on its base on the H.P. with one of the sides of the base perpendicular to the V.P.



- Draw the front view and the top view of the frustum of the pyramid as per given conditions.
- Draw axes ox , oy and oz as explained in problem 18-1 [steps (i) and (ii)].
- Complete the projection as shown in fig. 18-10(ii).

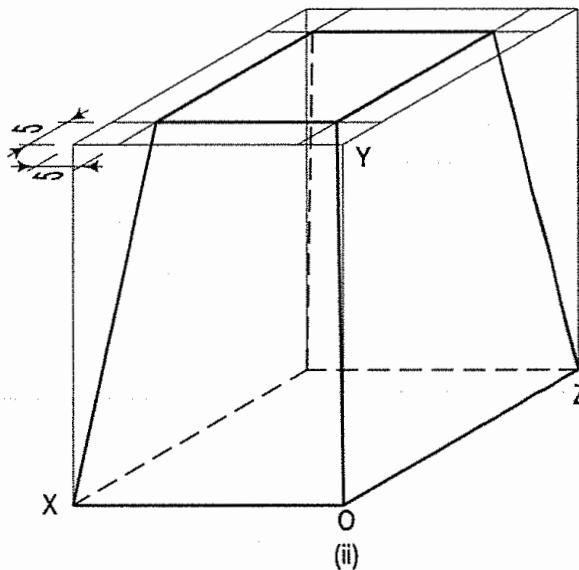


FIG. 18-10

18-9. OBLIQUE DRAWING OF CIRCLE

(1) Offset method (fig. 18-11):

- As shown in fig. 18-11, draw square to enclose circle in the front plane and divide the circle into 12 divisions.
- Through division points draw vertical and horizontal lines.
- Create rhombus in the side and top plane and transfer horizontal and vertical lines to get 12 intersection points.
- Join these points with smooth curve to get ellipse which represents the circle in the oblique projection.

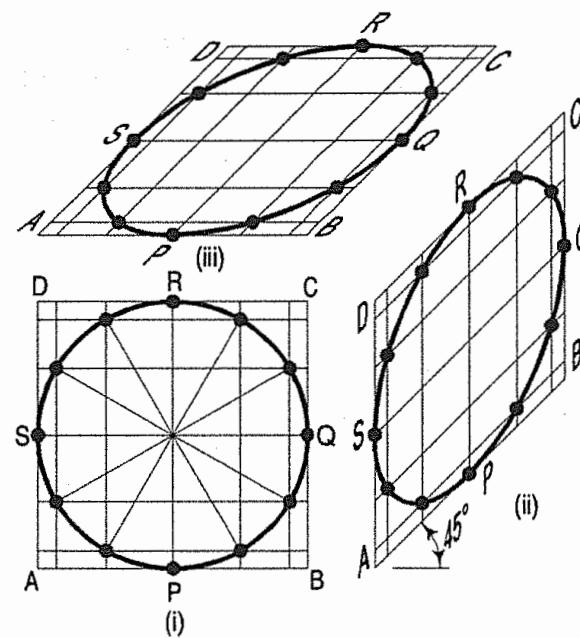


FIG. 18-11

(2) Four centre approximate method (fig. 18-12):

- Draw the rhombus $ABCD$ in top and side plane with the length of side equal to the diameter of a circle.
- Mark P, Q, R and S as the mid points of the respective sides.
- From P and Q , draw line perpendicular to AB and BC respectively, such that they intersect at C_1 .
- Similarly draw perpendiculars from R and S to get C_2 .
- Also mark C_3 and C_4 as the intersection points of perpendicular lines.
- With C_1 and C_2 as centre and C_1P and C_2R as radius draw arc PQ and arc RS respectively.

(vii) Similarly with C_3 and C_4 as centre and C_3P and C_4R as radius draw arc PS and arc RQ respectively.

(viii) These four arcs forms the ellipse, which represents the circle in the oblique projection.

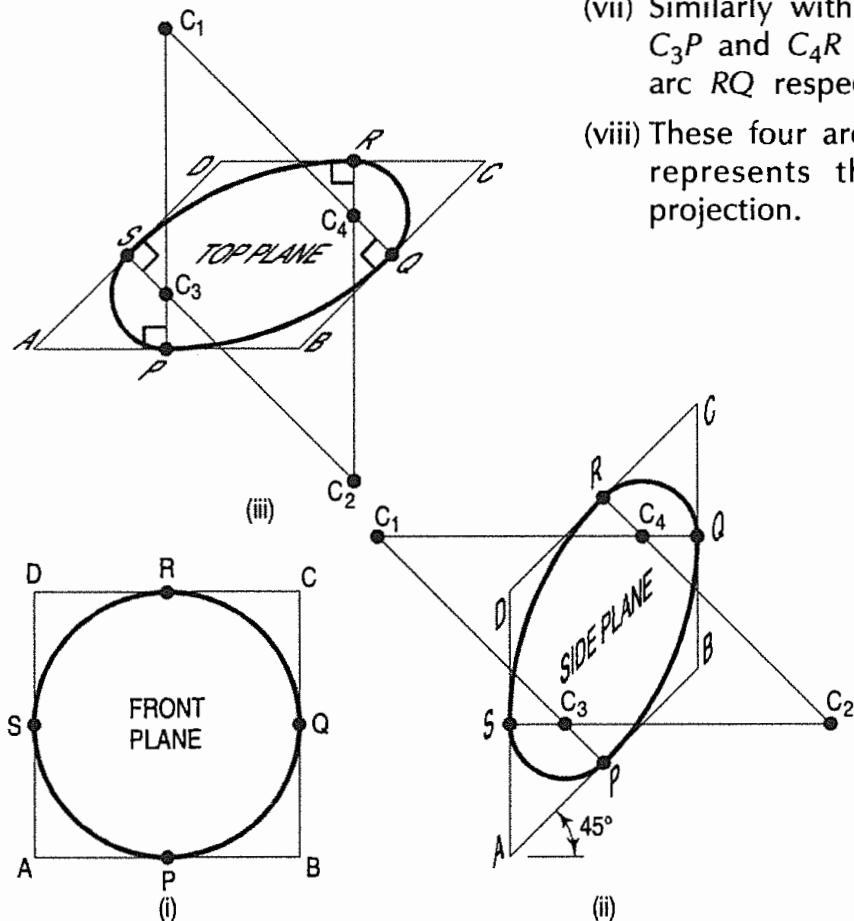


FIG. 18-12

18-10. OBLIQUE DRAWING OF CYLINDER

Problem 18-3. (fig. 18-13): A cylinder of diameter of base 40 mm and height 50 mm rests with its base on the H.P. Draw the oblique projection using the cavalier method.

- Construct the box of 40 mm \times 40 mm and height 50 mm.
- Using four centre approximate method, construct oblique circle on top and bottom face of box.
- Complete the projection as shown in fig. 18-13(ii).

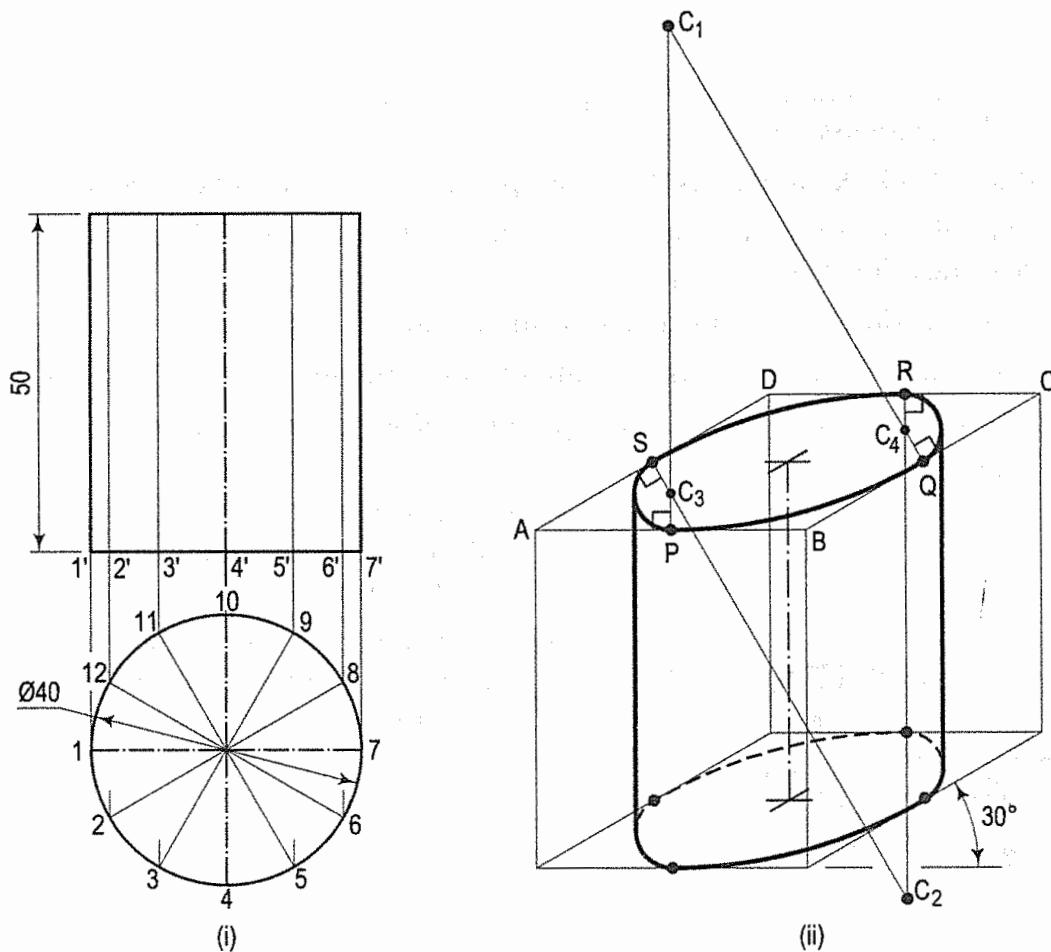


FIG. 18-13

18-11. OBLIQUE DRAWING OF PRISM



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 44 for the following problem.

Problem 18-4. A hexagonal prism, base 50 mm side and axis 100 mm long, resting on its base on H.P. is cut by a section plane perpendicular to V.P. and makes the angle of 45° with the H.P., passes through a point on the axis 20 mm from its top. Draw the oblique projection of the prism by the cavalier method with receding axis inclined at 45° with the horizontal when, (a) Two rectangular faces of the prism are equally inclined with V.P. (b) One rectangular face of the prism is parallel to V.P.

- Draw the front view and top view of the hexagonal prism as shown in fig. 18-14 and fig. 18-17.
- Enclose the top views in rectangular box.
- Draw oblique projection as shown in fig. 18-15 and fig. 18-18 using box method as explained in problem 17-14, problem 17-15 and problem 17-16.
- Similarly using the Box method, draw sectional oblique projection as shown in fig. 18-16 and fig. 18-19.

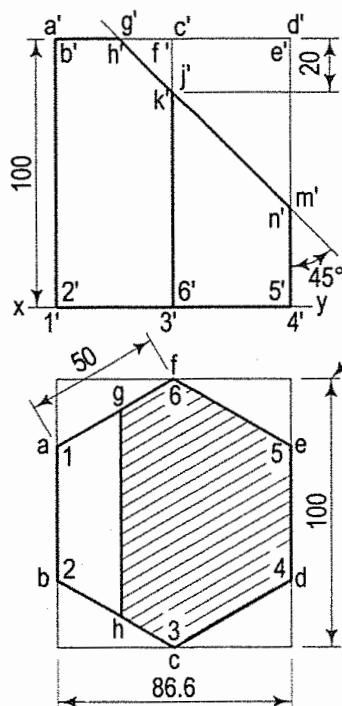


FIG. 18-14

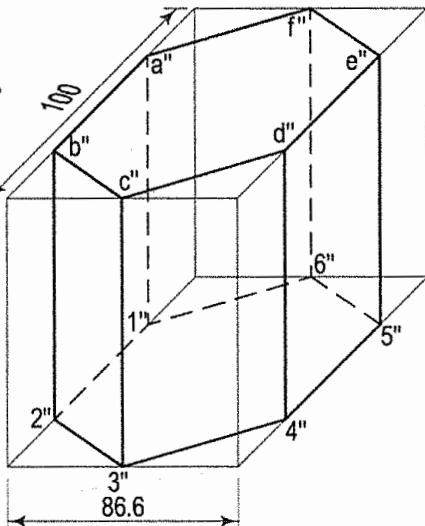


FIG. 18-15

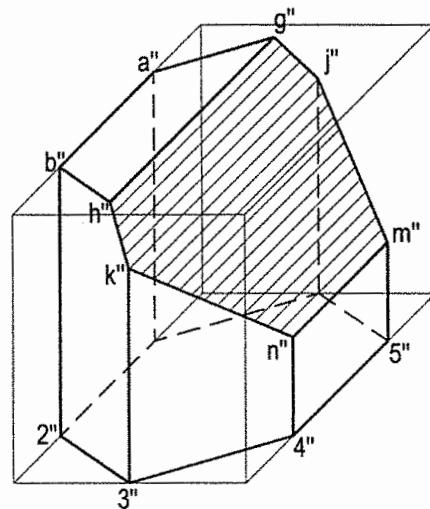


FIG. 18-16

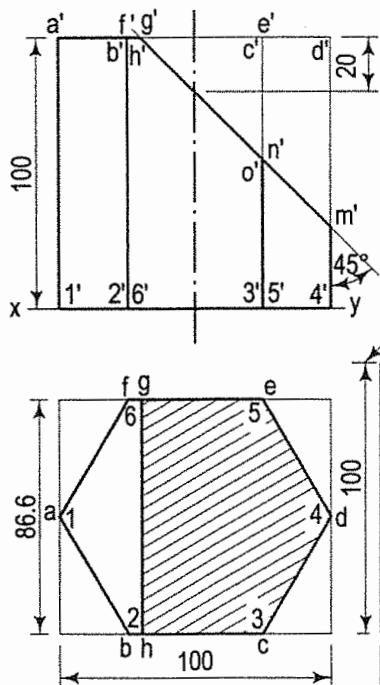


FIG. 18-17

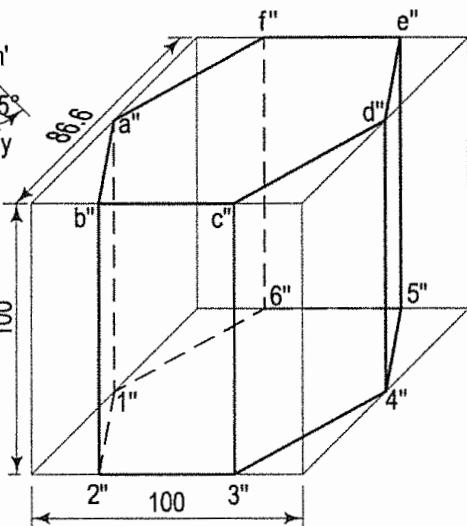


FIG. 18-18

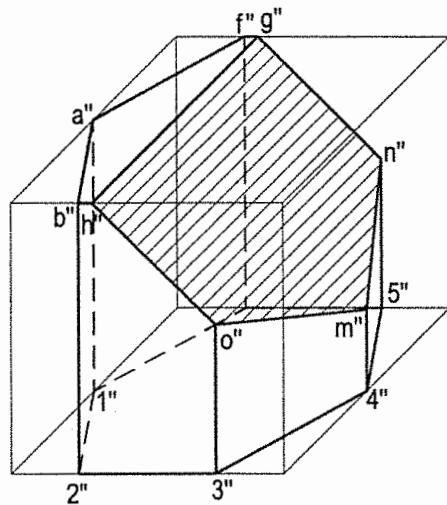


FIG. 18-19

Problem 18-5. (fig. 18-20): Draw the development of an oblique square prism whose cross-section is a regular square of 20 mm side and the top and the bottom bases are parallel to each other. The axis is 50 mm long. It is inclined at an angle of 60° to the base. The edges of base are equally inclined with the V.P.

- Draw the top view and the front view of the oblique prism as shown in fig. 18-20(i).
- From the points a' , b' , c' and d' and $1'$, $2'$, $3'$ and $4'$, draw the projectors. Mark 80 mm (the perimeter length of the square) along the projector of the point a' . Divide this length (80 mm) into four equal parts. Draw the edges of prism $a1$, $b2$, $c3$ and $d4$.

(iii) Complete the development as shown in fig. 18-20(ii).

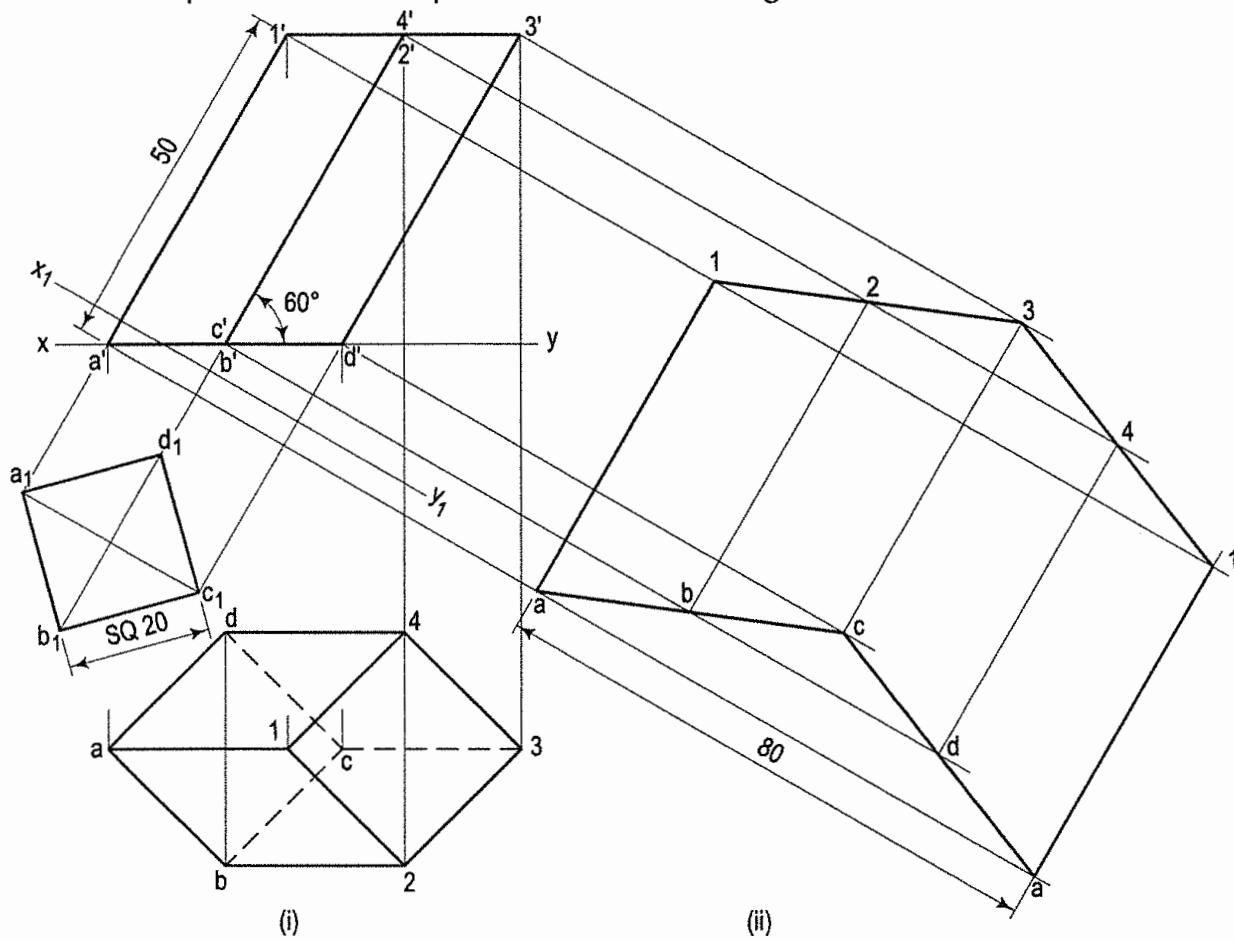


FIG. 18-20

18-12. TYPICAL PROBLEMS OF OBLIQUE PROJECTION

Problem 18-6. (fig. 18-21 and fig. 18-22): The orthographic projections of a casting are shown in fig. 18-21. Draw the oblique projection when the receding axis is inclined at an angle of 30° with the horizontal.

See fig. 18-22.

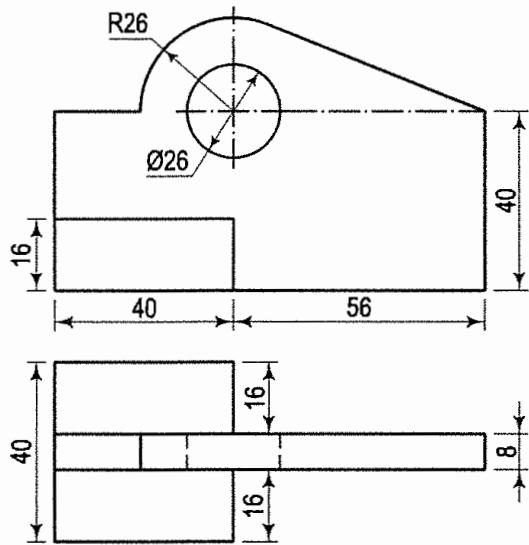


FIG. 18-21

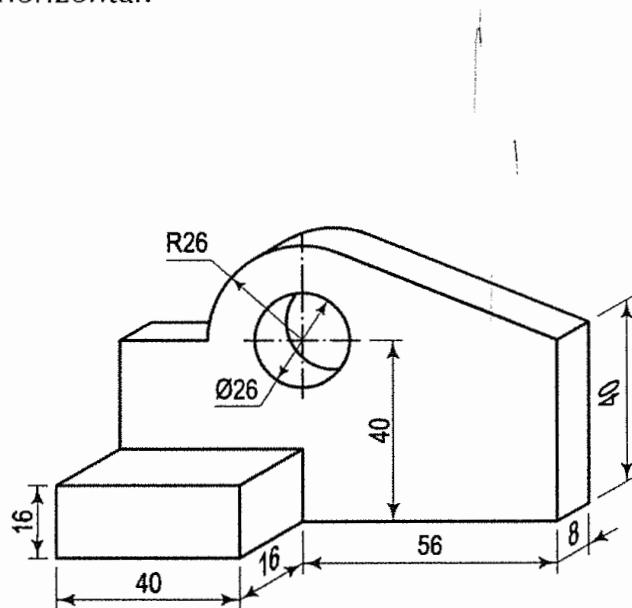


FIG. 18-22

Problem 18-7. (fig. 18-23 and fig. 18-24): The front view and sectional side view of a casting is shown in fig. 18-23. Draw the oblique projection when the receding axis is inclined at an angle of 45° with the horizontal.

See fig. 18-24.

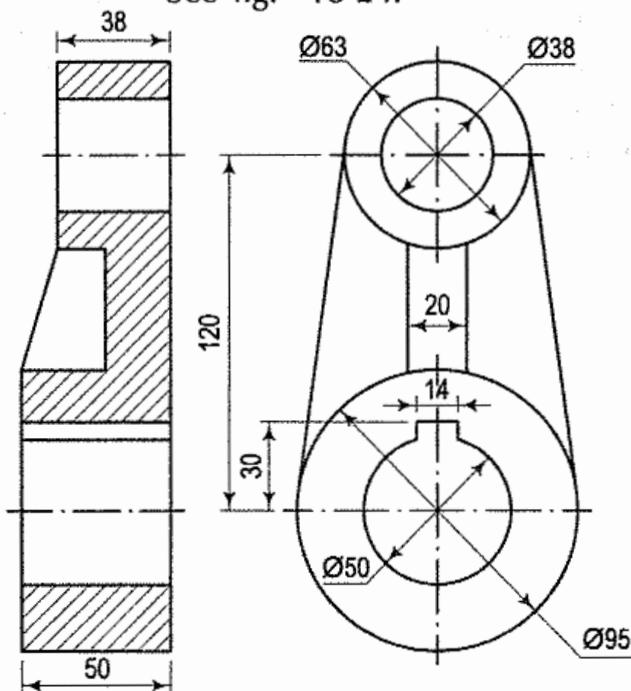


FIG. 18-23

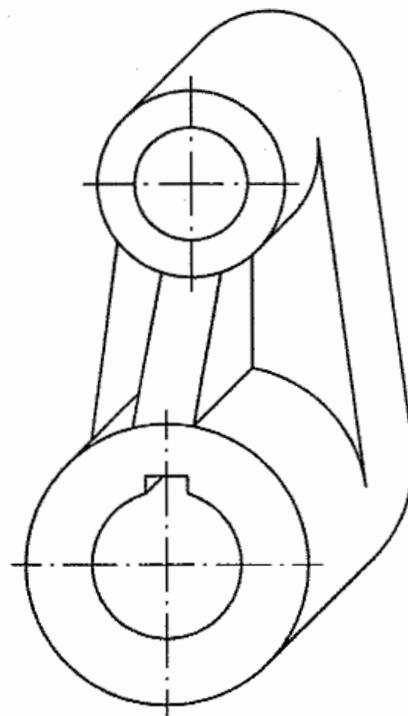


FIG. 18-24

EXERCISES 18

1. Differentiate clearly between the oblique projection and the isometric projection.
2. Explain the terms receding lines and receding angles.
3. Explain the rules pertaining to the choice of position for drawing oblique projection with minimum distortions.
4. Draw the oblique projection of a regular hexagonal prism, 25 mm side of the base and height 60 mm. When it rests on its base in the horizontal plane, the receding axis is inclined at an angle of 45° .
5. A right cylinder of 60 mm diameter and an axis 70 mm long has a co-axial through square hole of 20 mm side. Draw the oblique view when the cylinder rests on its base on the horizontal plane.
6. A cone of base 50 mm diameter and 60 mm long rests on its base on the H.P. A section plane perpendicular to the V.P. and inclined at 75° to the H.P., cuts the axis at a point 20 mm above the base. Draw the oblique projection of the cone. Assume that the receding axis is inclined at an angle of 30° with the horizontal.
7. Draw the oblique projection of two pieces of a mitre faced bridle joint made from the two wooden pieces of $160 \text{ mm} \times 65 \text{ mm} \times 75 \text{ mm}$ by the cabinet method. The thickness of grooves is 18 mm.
8. Two pieces of wood joined together by a dovetail joint are shown in two views in fig. 17-48. Draw the oblique projection of the two pieces separated but in a position ready for fitting.

9. Draw the oblique view of a square with a square pyramid placed on its top.
10. A frustum of a hexagonal pyramid of 25 mm side of the top and 30 mm side of the base, and height 60 mm rests with one of its base corners on the H.P. such that two of the base edges passing through the corner on which it rests make equal angle with the H.P. The axis of the pyramid is parallel to the H.P. and perpendicular to V.P. Draw the oblique projection by the cabinet method.
11. The orthographic projections of few objects are shown in fig. 18-25, fig. 18-26 and fig. 18-27. Draw the oblique projections.

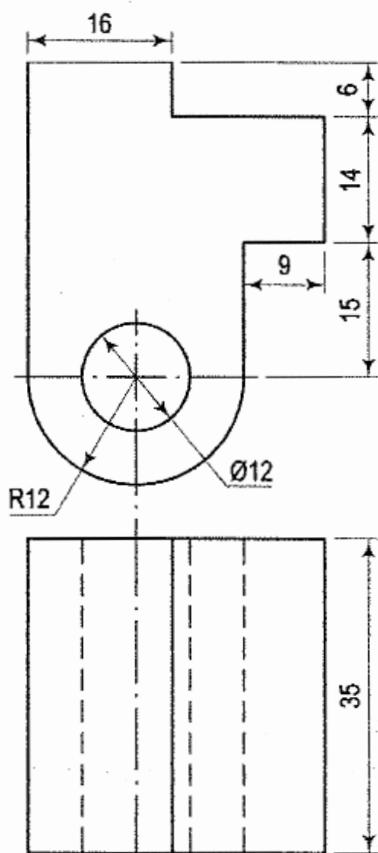


FIG. 18-25

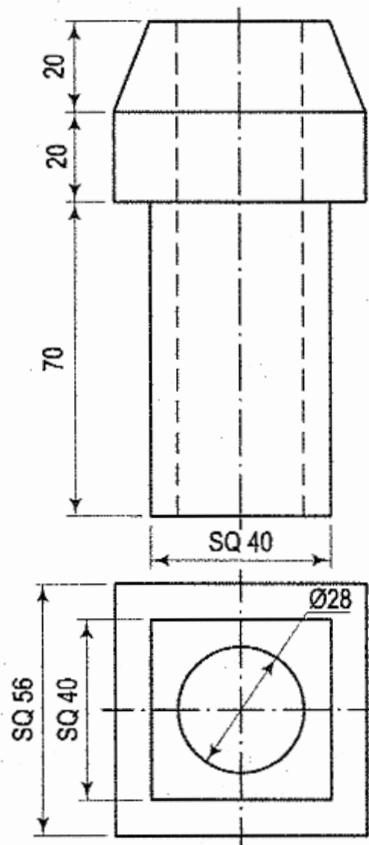


FIG. 18-26

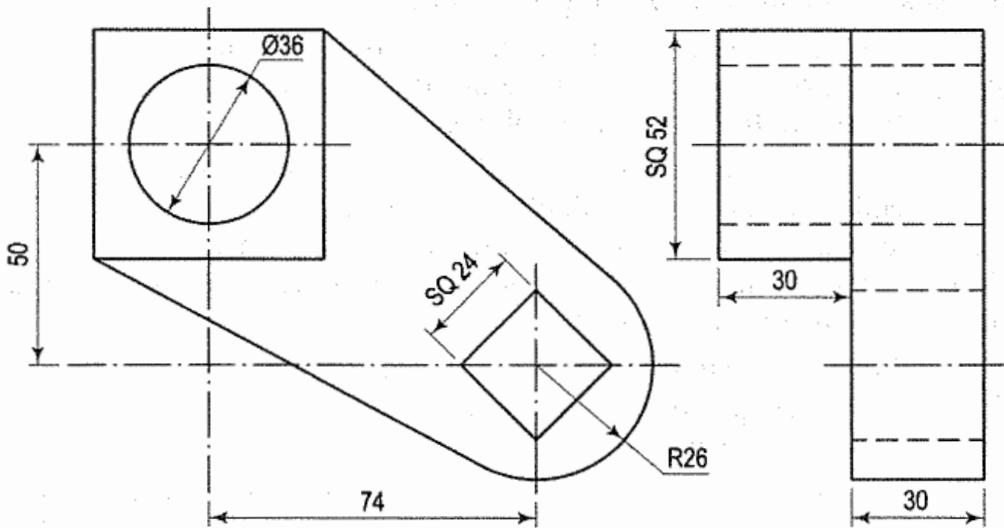


FIG. 18-27



19-1. INTRODUCTION



Perspective projection or perspective drawing is the representation of an object on a plane surface, called the *picture plane*, as it would appear to the eye, when viewed from a fixed position.

It may also be defined as the figure formed on the picture plane when visual rays from the eye to the object cut the picture plane. Perspective is mainly used in architecture. By means of perspective, the architect is able to show how an object would appear when constructed.

It is essential to have full knowledge of the principles of orthographic projection (third-angle method) before the theory of perspective drawing can be studied.

In this chapter, we shall deal with the topics of perspective drawing as follows:

- | | |
|--|---|
| 1. Principle of perspective projection | 6. Methods of drawing perspective view |
| 2. Definitions of perspective elements | 7. Types of perspective |
| 3. Station point | 8. Distance points |
| 4. Angle of vision | 9. Measuring line or line of heights |
| 5. Picture plane | 10. Perspectives of circles and solids. |

19-2. PRINCIPLE OF PERSPECTIVE PROJECTION



In perspective projection, the eye is assumed to be situated at a definite position relative to the object. The vertical plane, which (in perspective) is called the picture plane, is placed between the object and the eye.

Visual rays from the eye to the object converge to a point in the eye and are, therefore, inclined to the picture plane. The rays pierce the picture plane and form an image on it. This image is the perspective of the object.

19-3. DEFINITIONS OF PERSPECTIVE ELEMENTS



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 45 for the definitions of perspective elements.

Various elements used in obtaining the perspective view are defined below. Refer fig. 19-1.

(1) **Ground plane (GP):** It is a horizontal plane on which the object is assumed to be situated.

(2) **Station point (S):** It is the point where the eye of the observer is located while viewing the object.

(3) **Picture plane (PP):** It is a vertical transparent plane located between the station point and the object which is to be viewed. It is the plane on which the perspective is formed. The front view of perspective elements and of the object (if necessary) is also projected on this plane.

(4) **Horizontal plane (HP):** This imaginary plane is at the level of the eye, i.e. the station point. It is a horizontal plane, above the ground plane and at right angles to the picture plane.

(5) **Auxiliary ground plane (AGP):** It is a horizontal plane placed above the horizon plane. The top view of the object and of the perspective elements is projected on this plane.

(6) **Ground line (GL):** The line of intersection of the picture plane with the ground plane is called the ground line.

(7) **Horizon line (HL):** It is the line in which the horizon plane intersects the picture plane. It is parallel to the ground line.

(8) **Perpendicular axis (PA):** It is the line drawn through the station point, perpendicular to the picture plane. It is, sometimes called the *Line of sight or Axis of vision*.

(9) **Centre of vision (C):** The point in which the perpendicular axis pierces the picture plane is called the centre of vision. It lies on the horizon line.

(10) **Central plane (CP):** It is an imaginary vertical plane, which passes through the station point and the centre of vision. It contains the perpendicular axis. It is perpendicular to both, the picture plane and the ground plane.

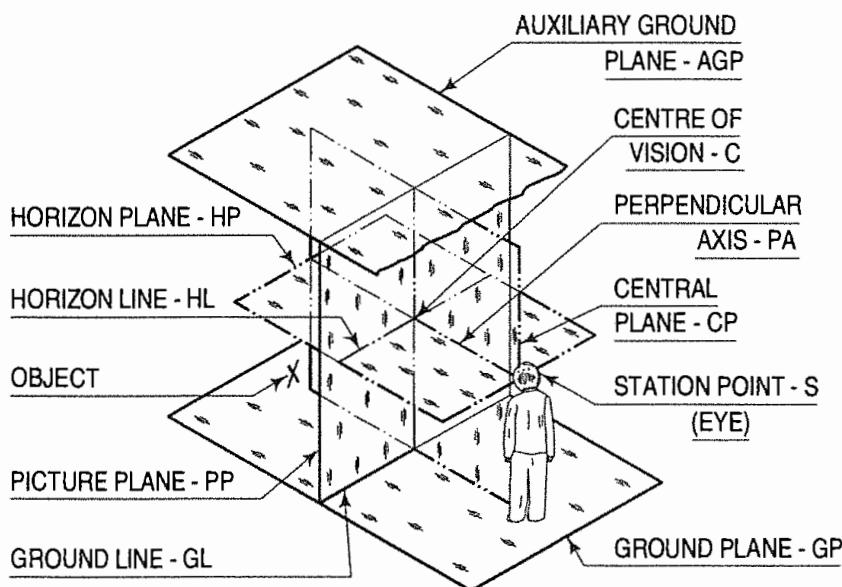


FIG. 19-1

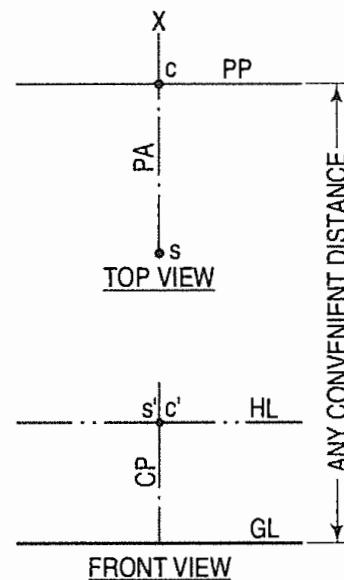
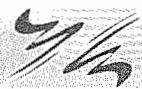


FIG. 19-2

Fig. 19-2 shows the projections of the perspective elements. In the top view, the picture plane PP is seen as a horizontal line. The object X is above PP, while S the station point is below PP. The line SC is the perpendicular axis and represents the central plane also. The ground plane, the horizon plane and the auxiliary ground plane will be seen as rectangles, but are not shown. In the front view, lines GL (ground line) and HL (horizon line) represent respectively the ground plane and the horizon plane. The station point s' and the centre of vision c' coincide with each other on HL. The central plane CP is seen as a vertical line through s' . The picture plane will be seen as a rectangle, but is not shown. The perspective view (when drawn) will be seen above or around GL.

19-4. STATION POINT



The position of the station point is of great importance. Upon its position, the general appearance of the perspective depends. Hence, it should be so located as to view the object in the best manner.

For large objects such as buildings, the station point is usually taken at the eye level of a person of normal height as shown in fig. 19-1 i.e. about 1.8 metres. For small objects, the station point should be fixed at such a height as would give a good view of the top surface as well as side surfaces.

The distance of the station point from the picture plane, when taken equal to about twice the greatest dimension of the object, usually gives good view in the perspective.

For objects having heights and widths more or less equal, the location of the station point may be so fixed that the angle between the visual rays from the station point to the outer-most boundaries of the object is approximately 30° .

The station point should be so situated in front of the object that the central plane passes through the centre of interest of the object. It may not, necessarily, be placed in front of the exact middle of the object. Refer fig. 19-1.

19-5. ANGLE OF VISION



Angle of vision is angle subtended by eye in horizontal or vertical direction in which one can visualize the things clearly. Horizontal and vertical angle of vision is generally 60° and 45° respectively. Refer fig. 19-3.

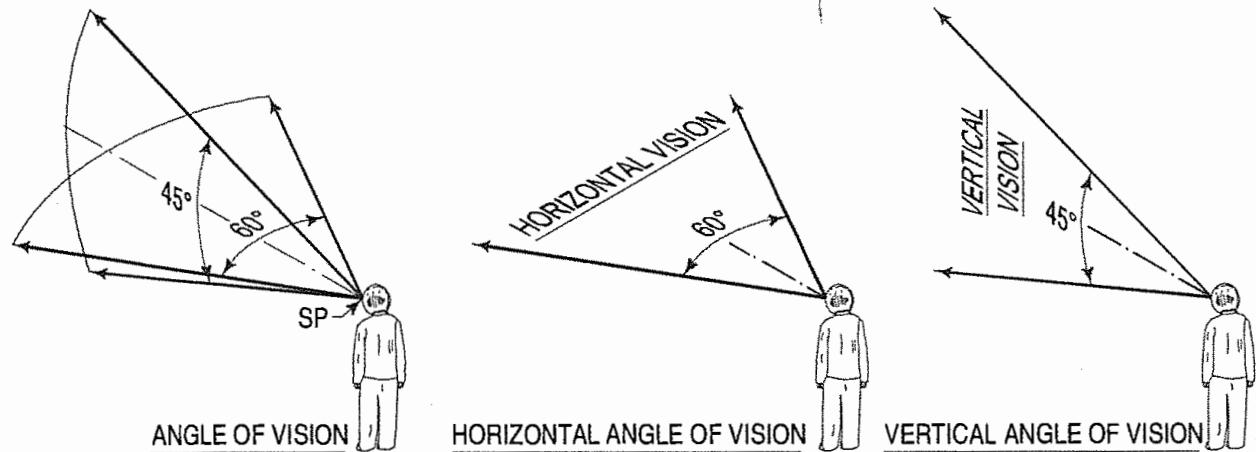


FIG. 19-3

19-6. PICTURE PLANE

The position of the picture plane relative to the object, determines the size of the perspective view. The perspective will show the object reduced in size when it is placed behind the picture plane. If the object is moved nearer the picture plane, the size of the perspective will increase. When the picture plane coincides with the object, the perspective of the object will be of its exact size. When the object is placed in front of the picture plane, its perspective, when projected back, will show the object enlarged in size.

In fig. 19-4(i), the line AB is behind the picture plane. Its perspective $A'B'$ is shorter than AB . In fig. 19-4(ii), AB is in the picture plane; its perspective $A'B'$ is equal to AB and coincides with it. Fig. 19-4(iii) shows the line AB placed in front of the picture plane; when projected back on the picture plane, its perspective $A'B'$ is longer than AB .

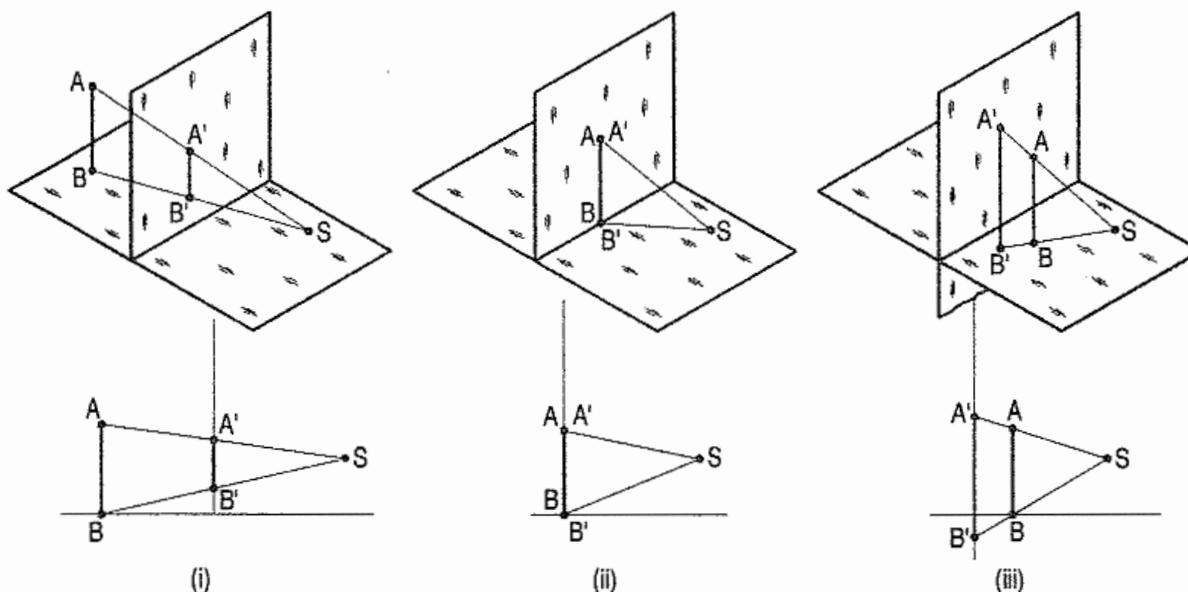


FIG. 19-4

19-7. METHODS OF DRAWING PERSPECTIVE VIEW

The perspective view of an object may be obtained by either

- (1) Visual-ray method
- (2) Vanishing-point method.

In the visual-ray method, points on the perspective are obtained by projecting

- (i) the top view and
- (ii) either the front view or the side view of the visual rays.

Vanishing-point method is comparatively simple. In addition to the top view of the visual rays, use of vanishing points of straight lines is made in this method. A front view or a side view of the object is also required to be drawn, for determining the heights.

19-7-1. VISUAL-RAY METHOD



This method is explained by means of the following three illustrative problems:

Problem 19-1. (fig. 19-5): A point A is situated 15 mm behind the picture plane and 10 mm above the ground plane. The station point S is 25 mm in front of the picture plane, 20 mm above the ground plane and lies in a central plane 10 mm to the right of the point. Draw the perspective view of the point A.

The pictorial view of the ground plane, the picture plane, the given point and the station point in their respective positions is given in fig. 19-5(i). The visual ray AS from the station point S to the point A is also shown. It passes through the picture plane. To mark the perspective of A, the point A' at which AS pierces the picture plane should be located.

In fig. 19-5(ii), an auxiliary ground plane (AGP) is shown placed above the point A, and the visual ray AS is shown projected on it, as is the top view of AS and a_1 is the top view of the point A' at which AS pierces the picture plane. a_1 shows the position of the point A' along the length of the picture plane. When the auxiliary ground plane is revolved and brought in the same plane as that of the picture plane, the view will be as shown in fig. 19-5(iii).

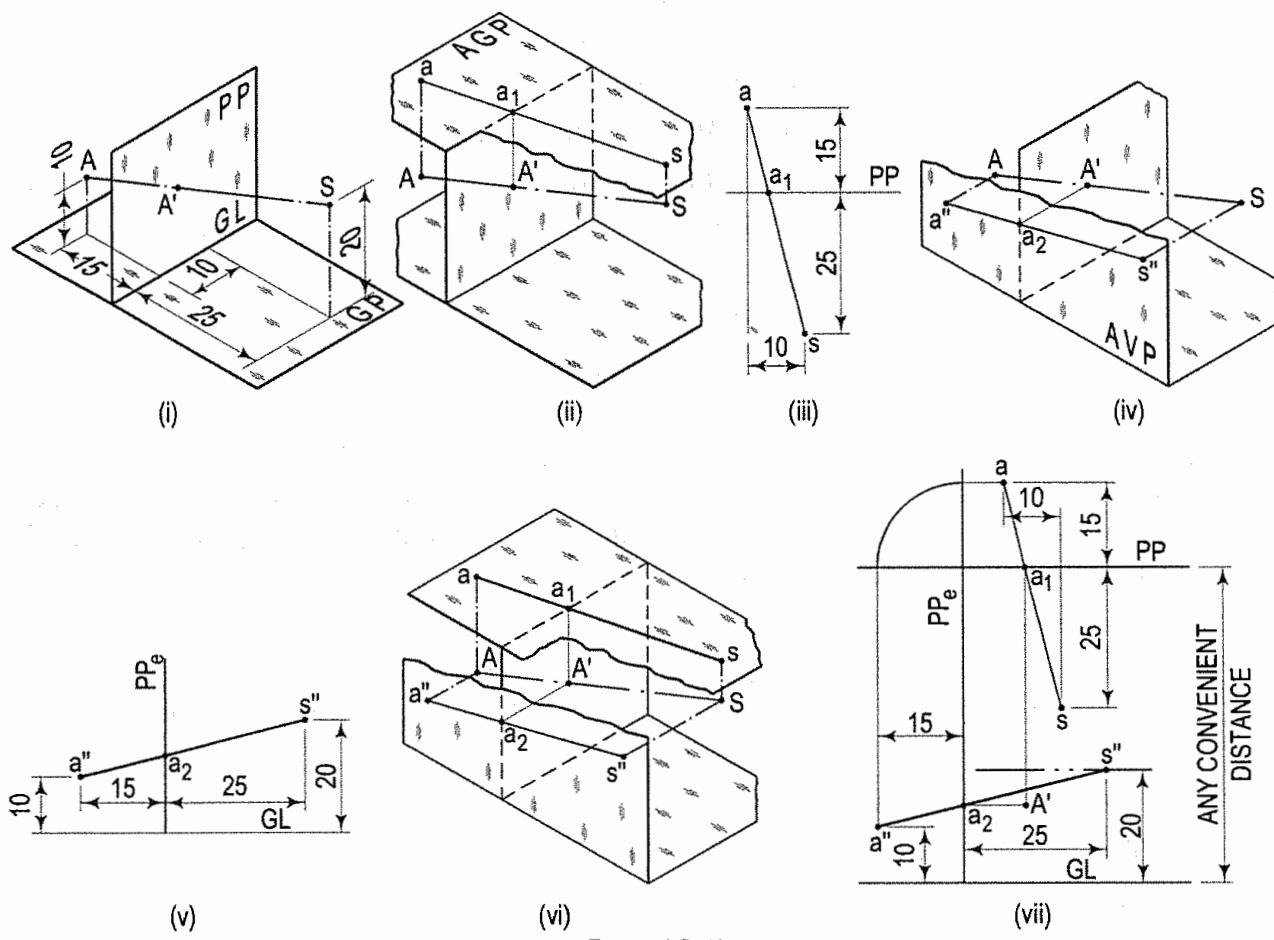


FIG. 19-5

To obtain the height of A' above the ground plane, an auxiliary vertical plane (AVP) perpendicular to both the picture plane and the ground plane is placed to the left of A and the side view of AS is projected on it [fig. 19-5(iv)]. $a''s''$ is the side

view of AS and a_2 is the side view of A' . It shows the height of A' above the ground plane. Fig. 19-5(v) shows the orthographic view (side view) when AVP is revolved and brought in the same plane as that of the picture plane.

Fig. 19-5(vii) shows the top view [fig. 19-5(iii)] and the side view [fig. 19-5(v)] combined together. A horizontal line drawn through a_2 and intersecting the vertical line through a_1 gives the point A' which is the perspective view of the point A . It is quite clear from the pictorial view [fig. 19-5(vi)] that A' lies in the picture plane on the line AS .

Steps in drawing the perspective view of the point A [fig. 19-5(vii)]:

- (i) Draw a horizontal line PP representing the picture plane in the top view.
- (ii) Mark a , the top view of A , 15 mm above PP .
- (iii) Draw a line (representing the central plane) perpendicular to PP and 10 mm to the right of a . On this line, mark s , the top view of the station point, 25 mm below PP .
- (iv) Draw a line joining a with s and intersecting PP at a point a_1 .
- (v) At any convenient distance below PP , draw a horizontal line GL . It is the ground line and also represents the ground plane in the front view.
- (vi) Draw a line HL parallel to and 20 mm above GL . It is the horizon line and also represents the horizon plane in the front view.
- (vii) At any point on GL and to the left of a , draw a vertical line PP_e (representing the picture plane in the side view).
- (viii) Mark a'' , the side view of A , 10 mm above GL and 15 mm to the left of PP_e .
- (ix) Mark s'' , the side view of the station point, on HL and 25 mm to the right of PP_e .
- (x) Draw a line joining a'' with s'' and intersecting PP_e at a point a_2 .
- (xi) Through a_1 , draw a vertical line. Through a_2 , draw a horizontal line intersecting the vertical line at a point A' .

Then A' is the perspective view of the point A .

Alternative method: Instead of the side view of AS , its front view $a's'$ may be projected on the picture plane (considering it as a vertical plane of projection) as shown in fig 19-6(i). The point A' must lie on this line $a's'$. It can be located by combining the top view and the front view as shown in fig. 19-6(ii) and fig. 19-6(iii), and as described below.

- (i) Draw the line as in the top view [steps (i) to (iv)].
- (ii) Draw the ground line GL at any convenient distance below PP and mark a' , the front view of A , 10 mm above GL and in projection with a .
- (iii) Draw the horizon line HL , 20 mm above GL and on it, mark s' , the front view of S , in projection with s .
- (iv) Draw a line joining a' with s' .
- (v) Draw a vertical line through a_1 intersecting $a's'$ at a point A' .

Then A' is the perspective view of the point A .

This method is comparatively simple and is generally adopted. In case of large objects, the perspective view often partly overlaps the front view. This, sometimes, causes confusion.

Perspective view of a straight line by the visual-ray method is drawn by first marking the perspectives of its ends which are points and then joining them.

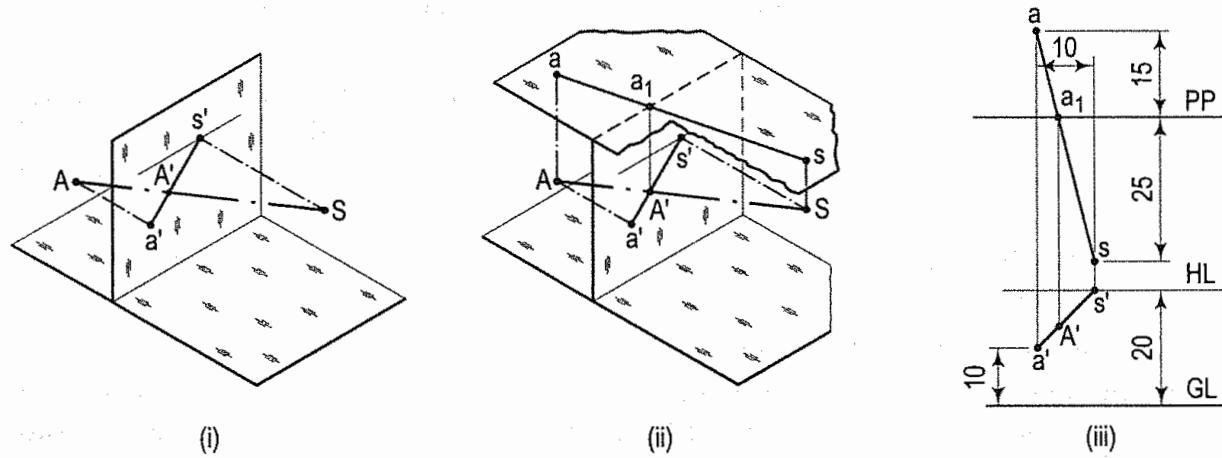


FIG. 19-6

Problem 19-2. (fig. 19-7): A straight line AB , 40 mm long, is parallel to and 15 mm above the ground plane, and inclined at 30° to the picture plane. The end A is 20 mm behind the picture plane. The station point is 40 mm above the ground plane, 50 mm in front of the picture plane and lies in a central plane which passes through the mid-point of AB . Draw its perspective view.

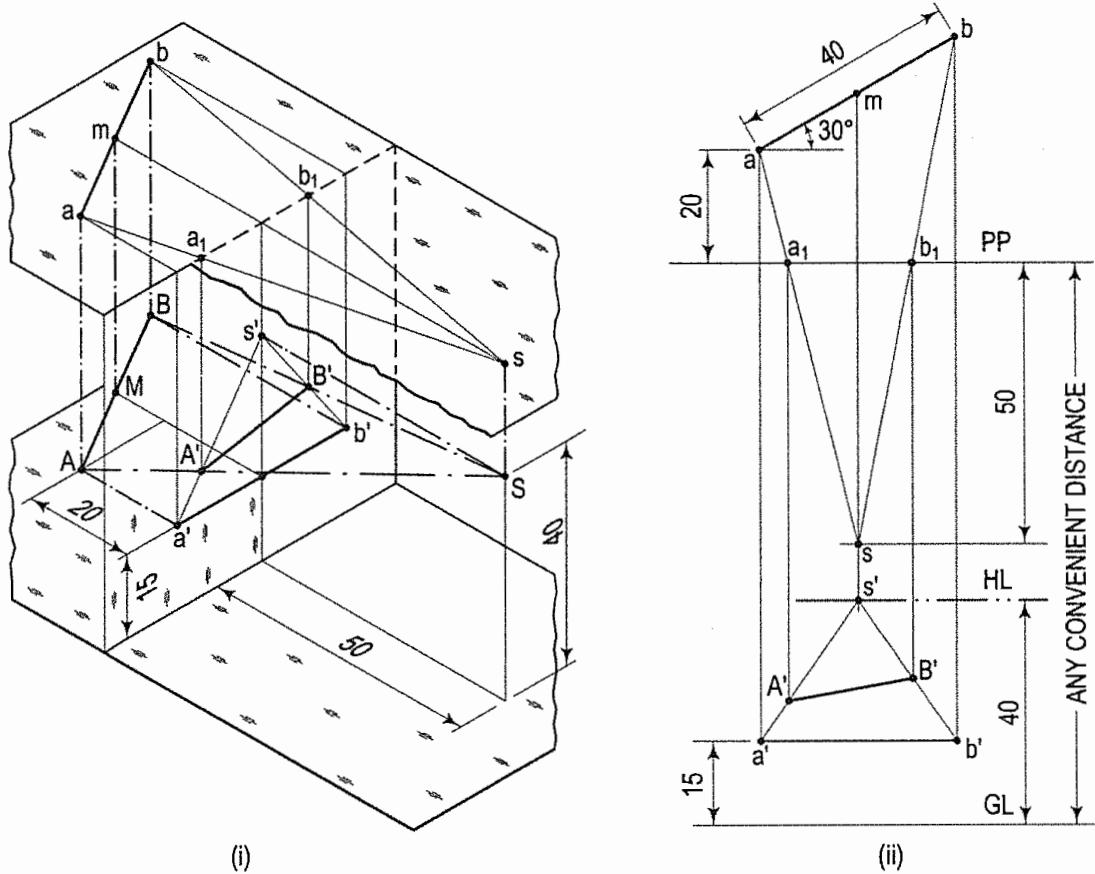


FIG. 19-7

- Draw a horizontal line PP . As AB is parallel to the ground plane, its top view will show its true length. Therefore, draw a line ab , 40 mm long, inclined at 30° to PP and the end a , 20 mm above PP .

- (ii) Draw a vertical line through m , the mid-point of ab and on it mark s , the top view of the station point, 50 mm below PP.
- (iii) Draw lines joining s with a and b , and intersecting PP at points a_1 and b_1 respectively.
- (iv) Draw the ground line GL at any convenient distance below PP. Draw the horizon line HL parallel to and 40 mm above GL. Project s' , the front view on HL.
- (v) From ab , project the front view $a'b'$, parallel to and 15 mm above GL. Draw lines joining s' with a' and b' .
- (vi) Through a_1 and b_1 , draw verticals to intersect $a's'$ and $b's'$ at points A' and B' respectively.
- (vii) Join A' with B' . Then $A'B'$ is the required perspective view of AB .

The perspective can also be obtained with the aid of the side view instead of the front view. Perspective view of any solid (by visual-ray method) can similarly be drawn by first obtaining the perspective of each corner and then joining them in correct sequence, taking care to show the hidden edges by dashed lines.

Problem 19-3. (fig. 19-8 and fig. 19-9): A rectangular pyramid, base 30 mm \times 20 mm and axis 35 mm long, is placed on the ground plane on its base, with the longer edge of the base parallel to and 30 mm behind the picture plane. The central plane is 30 mm to the left of the apex and the station point is 50 mm in front of the picture plane and 25 mm above the ground plane. Draw the perspective view of the pyramid.

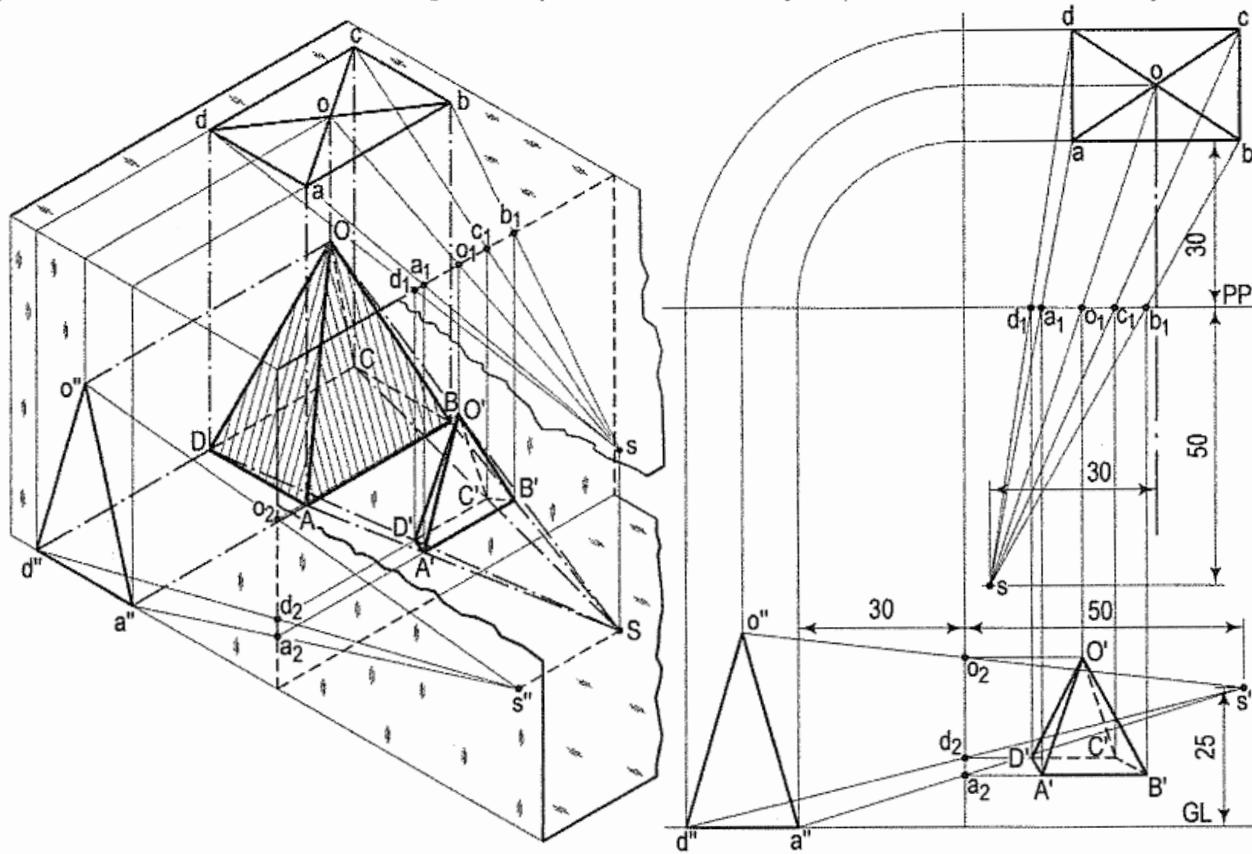


FIG. 19-8

Fig. 19-8 shows the perspective view of the pyramid obtained by means of its top view and the side view. The pictorial view shows clearly that points on the perspective lie in the picture plane on respective visual rays.

In fig. 19-9, the perspective view is drawn by means of its top view and the front view. It partly overlaps the front view.

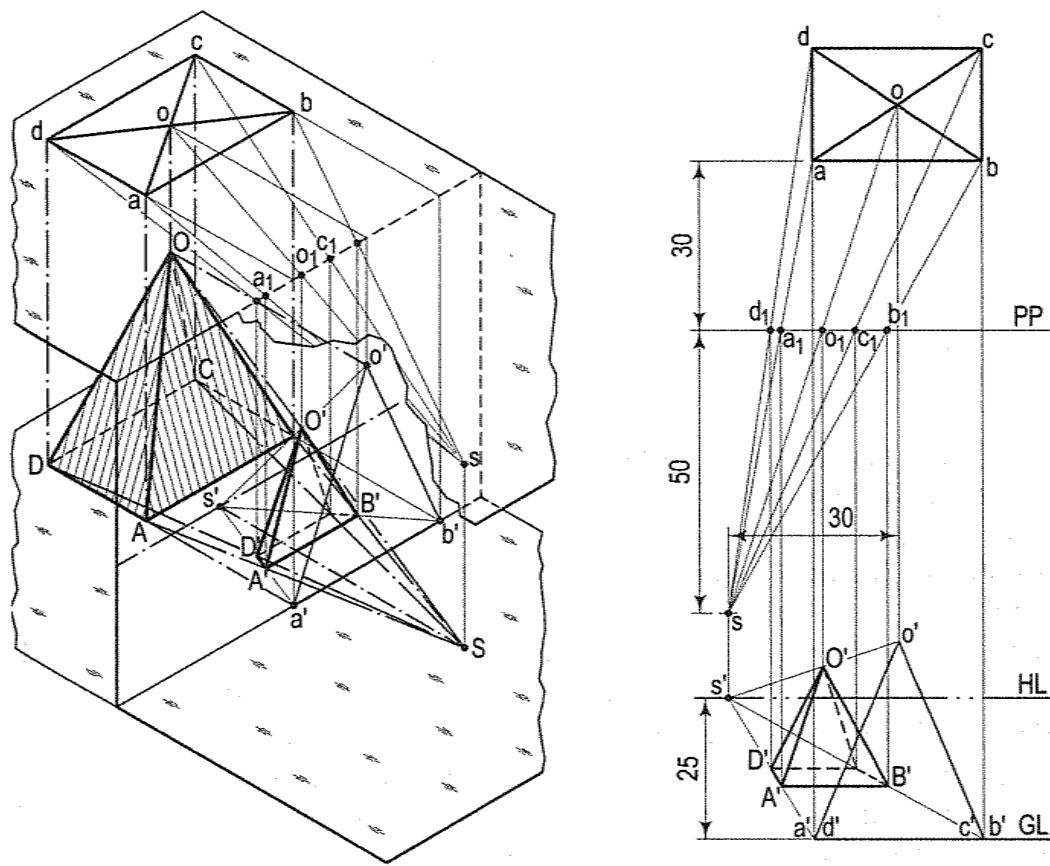


FIG. 19-9

19-7-2. VANISHING-POINT METHOD

Vanishing points: These are imaginary points infinite distance away from the station point. In practice, the point at which the visual ray from the eye to that infinitely distant vanishing point pierces the picture plane is referred to as the vanishing point.

If we stand between the rails of a long straight stretch of a railway track, it would appear as if the rails meet very far away at a point just at the level of the eye, i.e. on the horizon line. Even the telegraph and telephone wires running along the track at the sides of the track appear to meet at the same point. This point is a vanishing point.

In fig. 19-10, ab is the top view of a line AB lying on the ground plane and inclined at angle θ to the picture plane. When viewed from the station point s , its intercept on PP is a_1b_1 .

If the line is moved along the ground to the right, keeping the same inclination θ with the picture plane, its intercept will go on decreasing. The intercept becomes zero, or the line vanishes in a point at v when ab and the visual ray fall in a straight line. The point v is the top view position of the vanishing point for the horizontal line AB and for all lines parallel to AB , irrespective of their positions. The front-view position V of the vanishing point is obtained by projecting v , vertically on the horizon line.

Therefore, the vanishing point for any horizontal line is found by drawing a line parallel to the top view of that line from the top view of the station point. The point at which this line intersects the top view of the picture plane is then projected

on the horizon line. This point on the horizon line is the front-view position of the vanishing point.

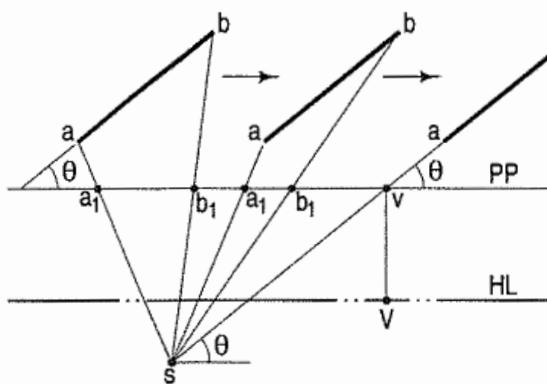


FIG. 19-10

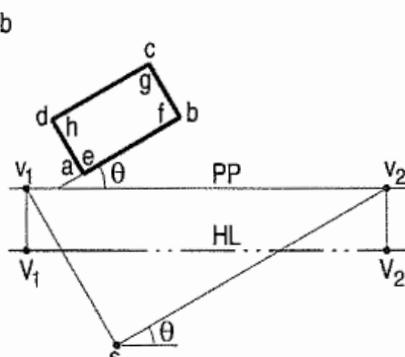


FIG. 19-11

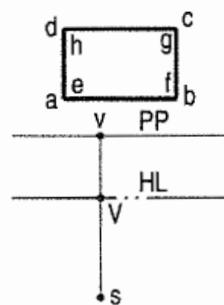


FIG. 19-12

In fig. 19-11, $abcd$ is the top view of a rectangular block placed on the ground plane so that a vertical face is inclined at angle θ to the picture plane.

The vanishing point for the line ab and for lines cd , ef and gh (which are parallel to ab) is obtained by drawing a line through s , parallel to ab and intersecting PP at a point v_2 . Through v_2 , a vertical line is drawn to meet HL at a point V_2 .

Then V_2 is the front-view position of the vanishing point. In perspective view of the block, edges AB , CD , EF and GH will converge to this point V_2 .

Similarly, V_1 is the vanishing point to which edges AD , BC , EH and FG will converge.

Thus, perspectives of all horizontal lines, if produced, pass through their respective vanishing points on the horizon line. Perspectives of all horizontal parallel lines converge to a vanishing point on the horizon line.

Vanishing point for lines perpendicular to the picture plane is obtained by drawing a line through the top view of the station point, and perpendicular to the picture plane. It lies on the horizon line and coincides with the centre of vision. It is the front-view position of the station point.

In fig. 19-12, V is the front-view position of the station point and the vanishing point, at which perspectives of lines AD , BC , EH and FG will converge. Thus, perspectives of all lines perpendicular to the picture plane converge to the centre of vision on the horizon line.

Lines which are parallel to the picture plane will have no vanishing points. They vanish at infinity. Therefore, perspectives of vertical lines are vertical; perspectives of horizontal lines which are parallel to the picture plane, remain horizontal; and perspectives of lines inclined to the ground plane and parallel to the picture plane will be inclined in the same direction (see fig. 19-13).

19-8. TYPES OF PERSPECTIVE



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 46 for the parallel or one point perspective.

(1) **Parallel perspective or one point perspective:** When an object has its one or more faces parallel to the picture plane, its perspective is called *parallel perspective* also called *one point perspective* as the edges converge to a single vanishing point of the parallel faces.

Fig. 19-13 shows the perspective view of a hut having its front face in the picture plane. The front face is seen in its true size and shape, while the back parallel face is of the same shape but reduced in size. As the lines AF , BG , CH , DJ and EK are perpendicular to the picture plane, their perspectives $A'F'$, $B'G'$ etc. converge to the centre of vision c' on HL . Note that vertical lines AE , CD etc. remain vertical in perspective. Similarly, horizontal lines ED and KJ , and sloping lines AB , BC , FG and GH (which are all parallel to the picture plane) remain respectively horizontal and sloping in perspective.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 47 for the angular or two point perspective.

(2) Angular perspective or two point perspective: When an object has its two faces inclined to the picture plane, its perspective is called *angular perspective* also called *two point perspective* as the edges of the object converge to two vanishing points.

Problem 19-4. (fig. 19-14): A rectangular block, $30\text{ mm} \times 20\text{ mm} \times 15\text{ mm}$, is lying on the ground plane on one of its largest faces. A vertical edge is in the picture plane and the longer face containing that edge makes an angle of 30° with the picture plane.

The station point is 50 mm in front of the picture plane, 30 mm above the ground plane and lies in a central plane which passes through the centre of the block. Draw the perspective view of the block.

- Draw the top view $abcd$ with a in PP and the longer edge ab inclined at 30° to PP . Mark its centre o . Mark s , the top view of the station point, on a vertical line through o and 50 mm below PP .
- Draw lines joining s with corners b , c and d , and intersecting PP at points b_1 , c_1 and d_1 .
- Draw the ground line GL at any distance below PP and the horizon line HL , 30 mm above GL .

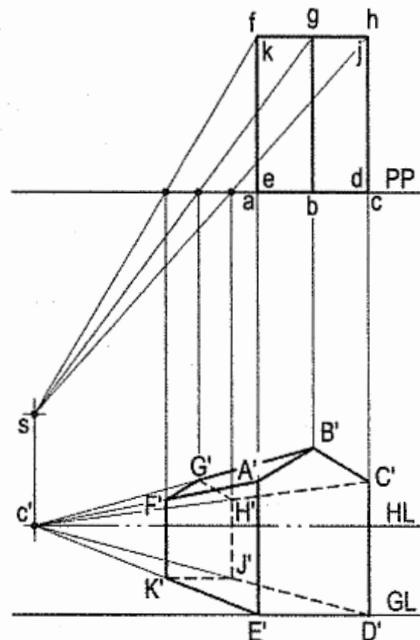


FIG. 19-13

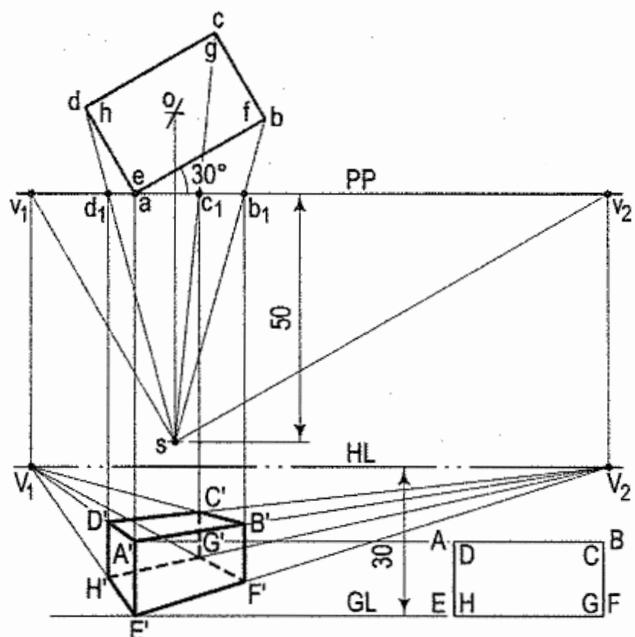


FIG. 19-14

- (iv) Through s , draw lines parallel to ad and ab cutting PP at points v_1 and v_2 respectively. Project v_1 to V_1 and v_2 to V_2 on HL. V_1 and V_2 are the vanishing points. Perspectives of edges AD , EH , BC and FG will converge to V_1 and those of edges AB , CD , EF and GH will converge to V_2 . Perspectives of vertical edges AE , BF , CG and DH will remain vertical.
- (v) As AE is in the picture plane, its perspective will be equal to the true length and the end E will lie on GL. Therefore, through a , draw a vertical line to a point E' on GL and on it, mark A' so that $A'E' = AE$.
(This length may be measured directly or may be projected from the front view as shown.)
- (vi) Draw lines joining A' and E' with V_1 and V_2 . Through b_1 , draw a vertical line to intersect $A'V_2$ at B' and $E'V_2$ at F' . Similarly, draw a vertical through d_1 and obtain points D' and H' .
- (vii) Draw lines joining B' and F' with V_1 and D' and H' with V_2 , intersecting at points C' and G' respectively. They must lie on the vertical line through c_1 . Note that lines meeting at C' are all hidden and therefore, shown dotted.

In fig. 19-15, the ground line GL has been so drawn that HL coincides with PP. Hence, V_1 and V_2 coincide with v_1 and v_2 respectively on PP. The perspective view is obtained in the same manner as described above.



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 48 for the oblique or three point perspective.

(3) Oblique perspective or three point perspective: When an object has its three faces inclined to the picture plane, its perspective is called *oblique perspective* also called *three point perspective* as edges of the object converge to three vanishing points, as shown in fig. 19-16.

Problem 19-5: Draw the perspective view of a cube of 80 mm side having its one corner of the edge on the ground plane and the other corner of the edge resting on the picture plane such that the edge is inclined at 30° to the picture plane. The other two edges of the corner are equally inclined with the picture plane. The station point is 100 mm in front of the picture plane, 150 mm above the ground plane and lies in a central plane which passes through the centre of the cube.

Refer fig. 19-16.

- (i) Draw projection plane lines PP and PP_e , perpendicular to each other.
- (ii) Draw side view on PP_e from auxiliary top view.
- (iii) Draw top view on PP by taking projection from side view.
- (iv) Mark the station point s for F.V. and s_1 for S.V. at a distance of 100 from PP and PP_e respectively and 150 from G.L.
- (v) Obtain the three vanishing points V_1 , V_2 and V_3 and complete the perspective view as shown.

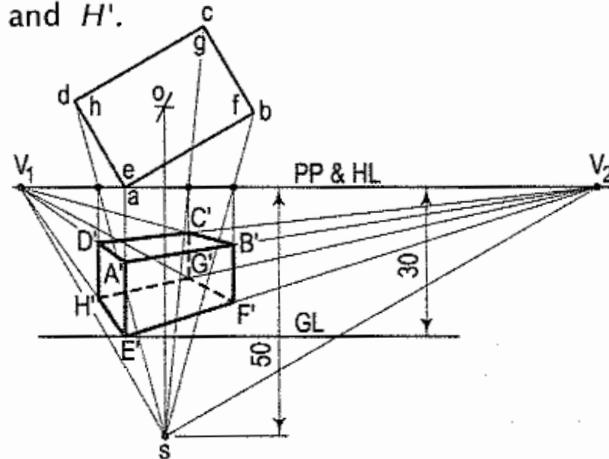


FIG. 19-15

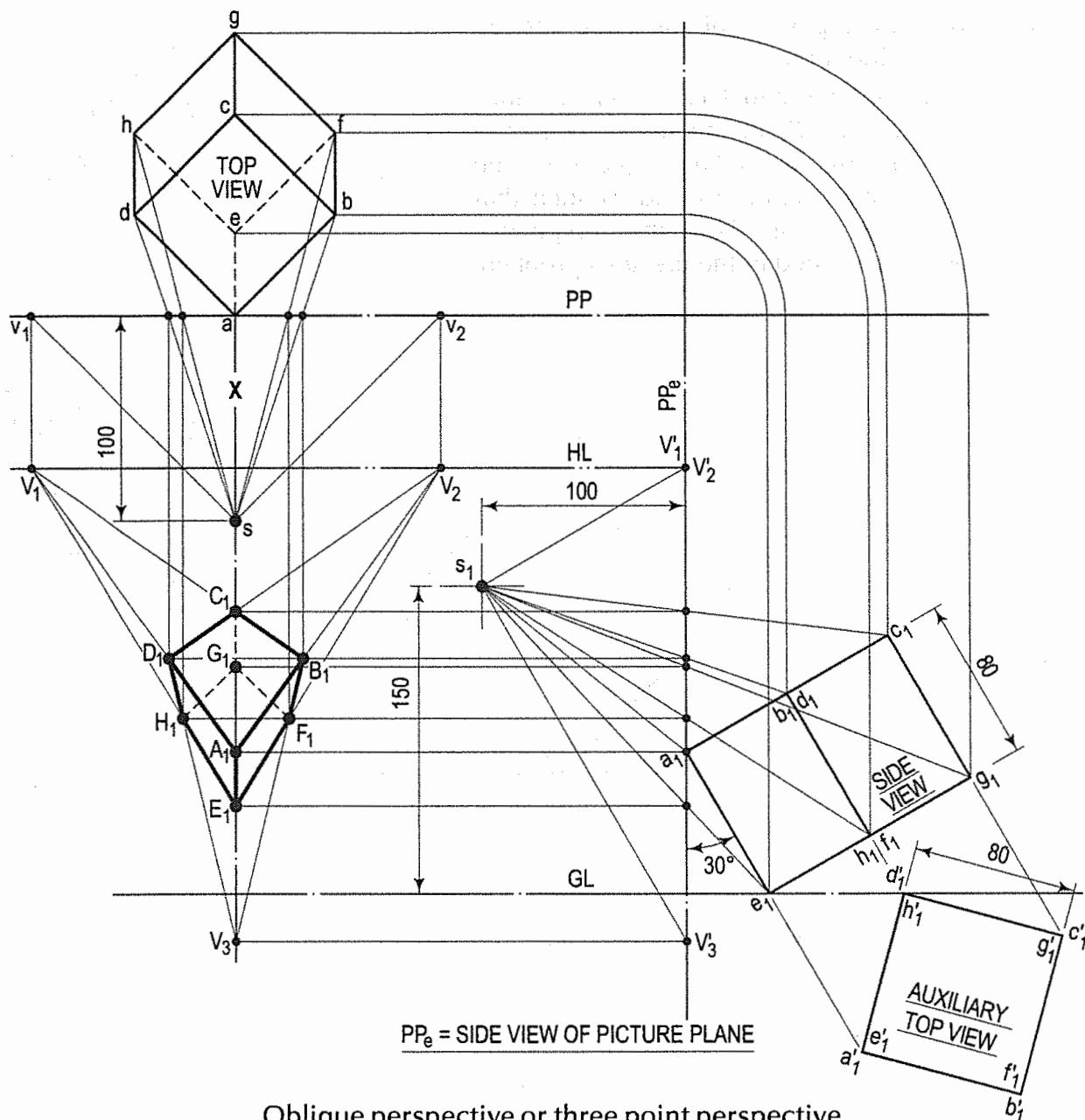


FIG. 19-16

19-9. DISTANCE POINTS

Vanishing points for all horizontal lines inclined at 45° to the picture plane are given special name of *distance points* on account of their definite positions.

They are equidistant from the centre of vision, the distance of each from the centre of vision being equal to the distance of the station point from the picture plane.

In fig. 19-17, $p_1c' = p_2c' = sc$. Thus, perspectives of all horizontal lines inclined at 45° to the picture plane converge to distance points on the horizon line.

Problem 19-6. (fig. 19-17): Draw the perspective view of a cube of 25 mm edge, lying on a face on the ground plane, with an edge in the picture plane and all vertical faces equally inclined to the picture plane. The station point is 50 mm in front of the picture plane, 35 mm above the ground plane and lies in a central plane which is 10 mm to the left of the centre of the cube.

- Draw the top view of the cube and of the station point.
- Draw lines GL and HL. Obtain distance points P_1 and P_2 in the same manner as the vanishing points. Or, project s to c' on HL and mark points P_1 and P_2 such that $P_1c' = P_2c' = sc$. Draw the perspective view as described in the previous problem.

Problem 19-7: Fig. 19-18 shows the two point perspective of model of steps with the base on ground plane and vertical edge in picture plane and inclined at 45° to picture plane. The station point is 2 m in front of vertical edge and 1 m from ground plane.

Observe that $v_1-a = v_2-a = s-a = 2.0$.

See fig. 19-18.

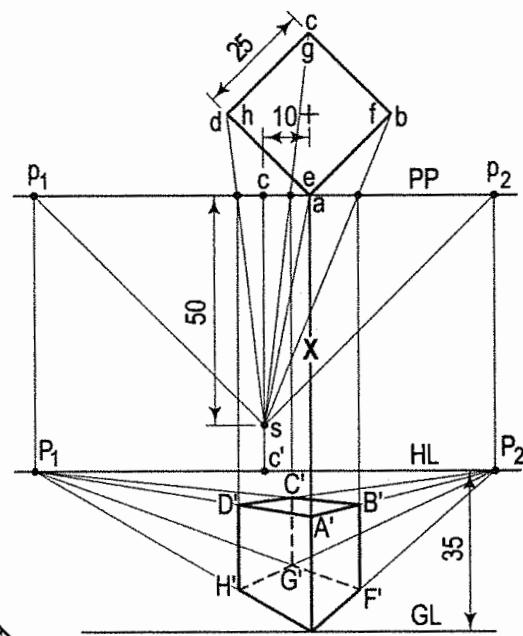


FIG. 19-17

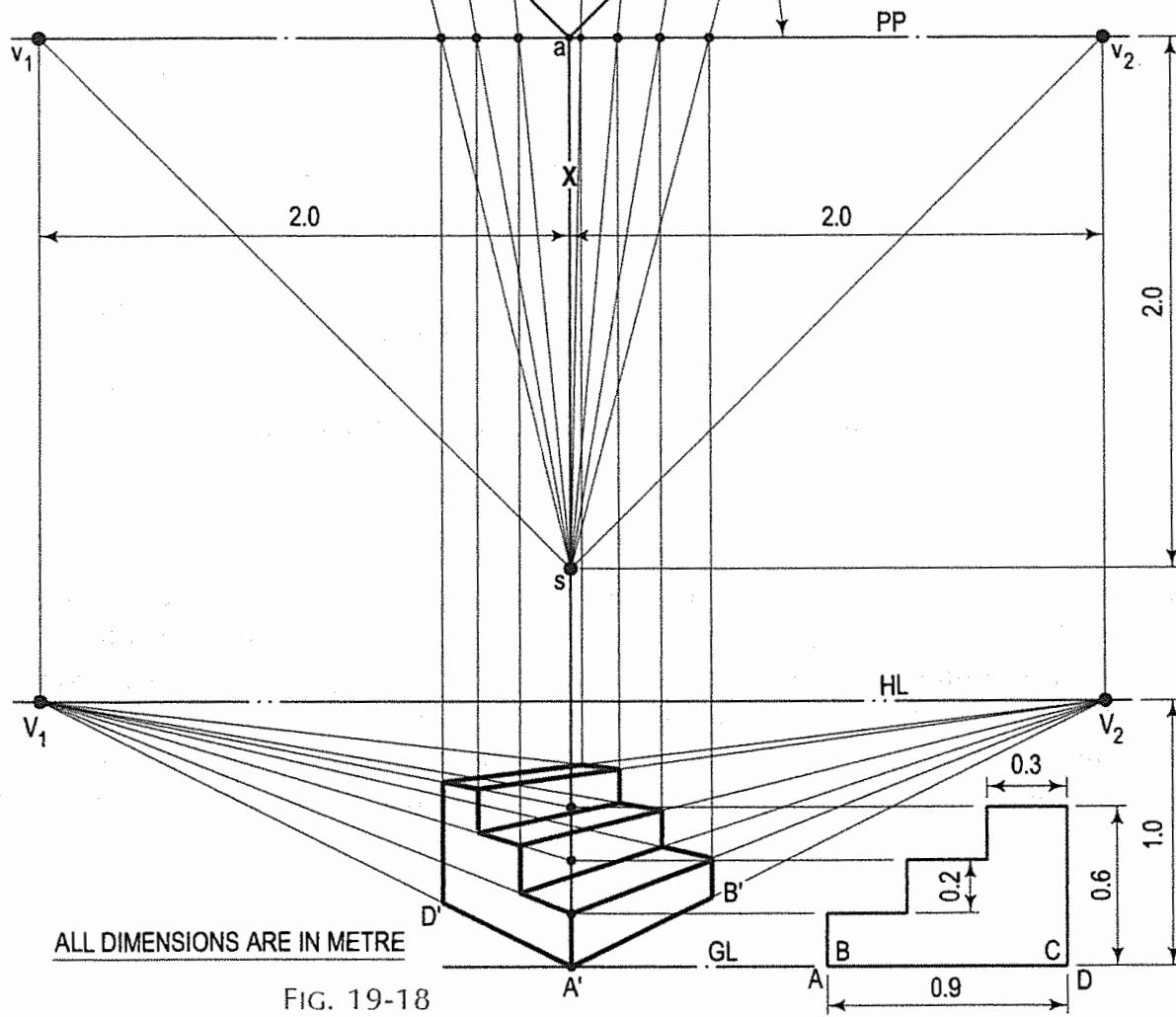


FIG. 19-18

19-10. MEASURING LINE OR LINE OF HEIGHTS

We have seen that when a line is in the picture plane, it is seen in its true length in perspective. When a line is behind the picture plane, it is foreshortened in its perspective view.

In fig. 19-19, ab is the top view of a rectangle $ABCD$ whose surface is vertical and inclined to the picture plane, the edge DC is on the ground plane and the edge AD is in the picture plane.

In its perspective view $A'B'C'D'$, $A'D'$ is equal to the true length AD , while $B'C'$ is shorter. $A'D'$ lies on a vertical line drawn through a . The length $B'C'$ is derived from $A'D'$. Let us now consider the rectangle $EBCF$ within the rectangle $ABCD$. $E'B'C'F'$ is its perspective view. $E'F'$ is shorter than EF or AD . Its length has been derived from $A'D'$.

Thus, aD' is the measuring line or the line of heights for points E , F , B and C . It is obtained by producing be to intersect PP at the point a ; through a , a perpendicular is dropped to meet GL at a point D' . In other words, if we imagine that the rectangle $EBCF$ is moved along the line eb to meet the picture plane, its edge EF will strike it at the line $A'D'$ showing its true length.

Thus, the measuring line or the line of heights is the trace or the line of intersection with the picture plane, of the vertical plane containing the point or points whose heights are to be determined. Heights of points lying in different vertical planes can be measured from their respective lines of heights. Heights on this line may be measured directly with a scale or may be projected from the front view.

Problem 19-8. (fig. 19-20): Determine the line of heights for points lying in the line ac which is the top view of a regular hexagon $ABCDEF$ (the front view of which is shown on GL) and then, draw its perspective view from the given position of the station point.

As ac is behind the picture plane, all the sides of the hexagon will be foreshortened.

- To determine the measuring line, produce ca to meet PP at a point h ; through h , draw a vertical to meet GL at a point H . Then hH is the measuring line for heights of all points in the hexagon.
- To draw the perspective view, determine the vanishing point V . Project horizontally, points A , B etc. from the front view to points a' , b' etc. on hH . Draw lines joining these points with V . Draw lines joining s with a , b and c . Through the points of intersection of these lines with PP , draw verticals to meet the corresponding lines converging to V at points A' , B' F' . Join these points in correct sequence. The resulting figure is the perspective view of the hexagon.

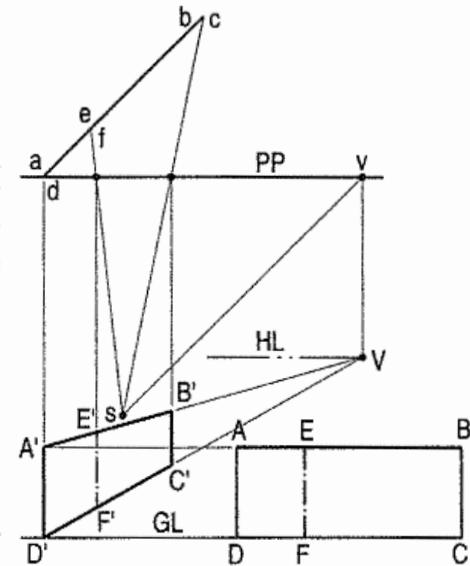


FIG. 19-19

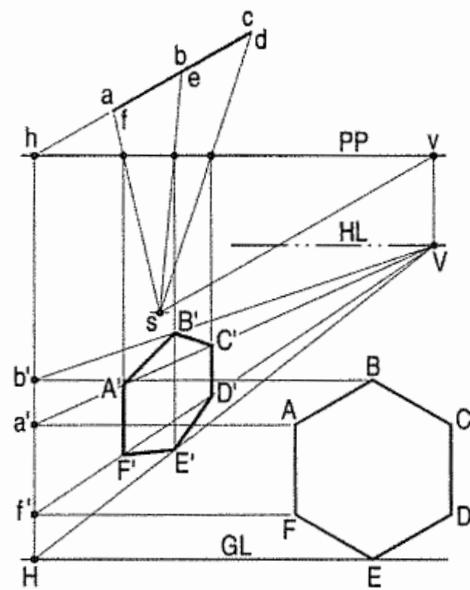


FIG. 19-20

19-11. PERSPECTIVES OF CIRCLES AND SOLIDS



A circle will appear as a circle in its perspective view when it is parallel to the picture plane. In other positions its perspective will be an ellipse (except when it lies in the central plane).

To obtain points for the ellipse, the circle is enclosed or crated in a square. The mid-points of the sides of the square are the four points for the ellipse. Points of intersection of the diagonals with the circle are the other four points. Lines are drawn through these points, parallel to the sides of the square. Perspective of the square along with the parallel lines is then drawn, locating the eight points for drawing the ellipse.

Problem 19-9. (fig. 19-21): Draw the perspective view of a circle of 50 mm diameter, lying on the ground plane and touching the picture plane. The station point is 80 mm in front of the picture plane and 60 mm above the ground plane. The central plane passes through the centre of the circle.

- (i) With o as centre, draw the circle touching PP. Enclose it in a square abcd with ab in PP. Draw the horizontal and vertical diameters and the diagonals of the square. Mark points 1 to 8 as shown. Through the points on the diagonals, draw lines ef and gh parallel to ad.
- (ii) Draw lines GL and HL, and mark s, the top view of the station point. Project s to c' on HL. c' is the centre of vision to which the perspectives of lines perpendicular to PP will converge.
- (iii) Draw the perspective view of the square. Draw the diagonals, intersecting each other at O'. Through O', draw horizontal and vertical lines, intersecting the sides at points 1', 2', 3' and 4'. Draw perspectives of lines ef and gh, cutting the diagonals at points 5', 6', 7' and 8'. Draw the ellipse through points 1', 5', 2'..... 8'. The ellipse is the required perspective view of the circle.
- (iv) It may be noted that the centre of the ellipse does not coincide with O', the centre of the perspective of the square.

Perspectives of concentric circles are not concentric ellipses as can be seen from fig. 19-22. The circles are on the ground plane. The station point is away to the left of the centre of the circles.

Fig. 19-23 shows perspectives of circles inscribed in the faces of a cube resting on the ground plane with an edge in the picture plane and all vertical faces equally inclined to the picture plane. The central plane passes through the centre of the cube. The ground line GL has been so placed that HL coincides with PP. Hence, distance points P_1 and P_2 also lie on PP.

Curve of any shape can similarly be drawn in perspective by enclosing it in a rectangle and then drawing horizontal and vertical lines through a number of points on the curve. Fig. 19-24 shows the perspective view of a moulding, the front view of which is shown on the right-hand side. aH is the line of heights.

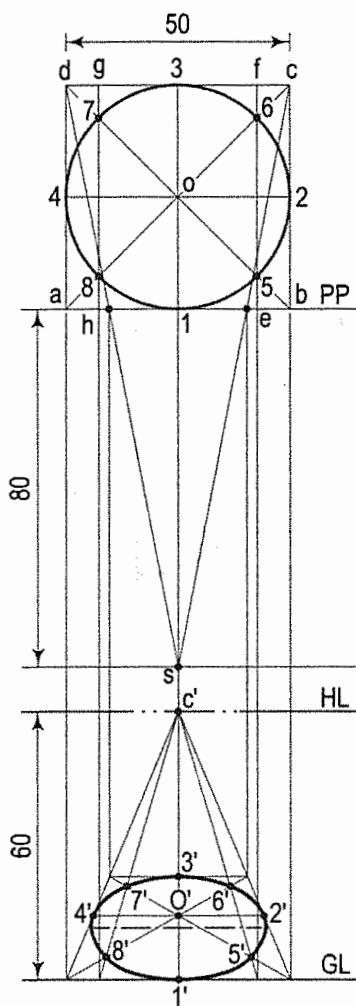


FIG. 19-21

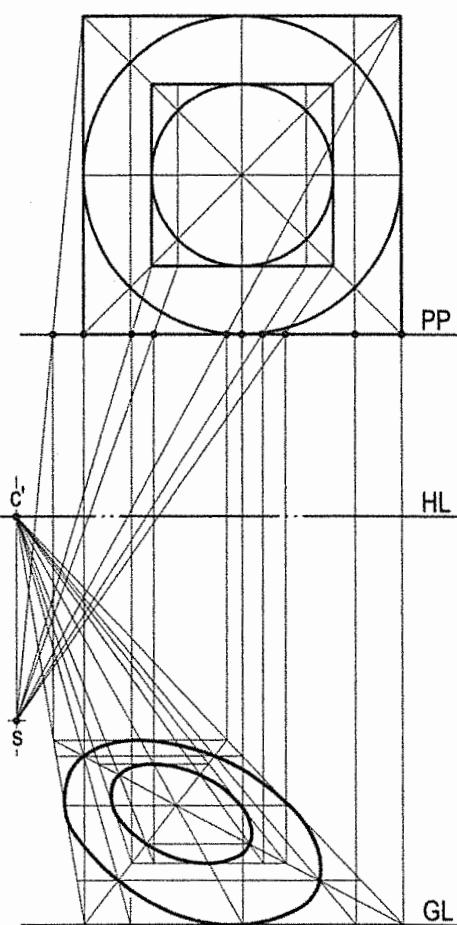


FIG. 19-22

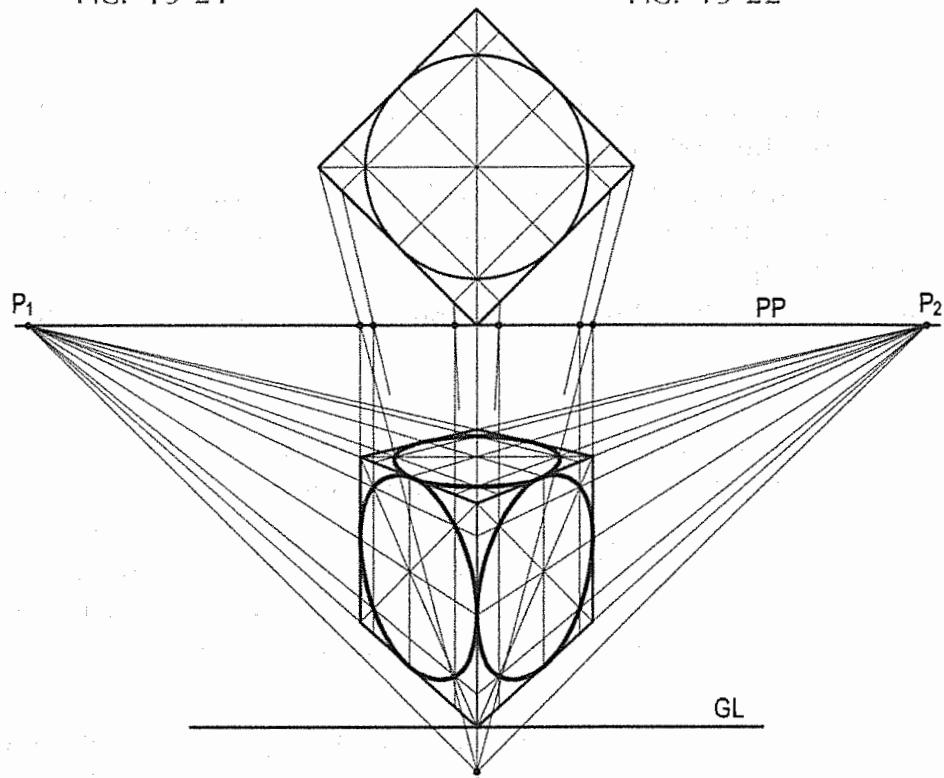


FIG. 19-23

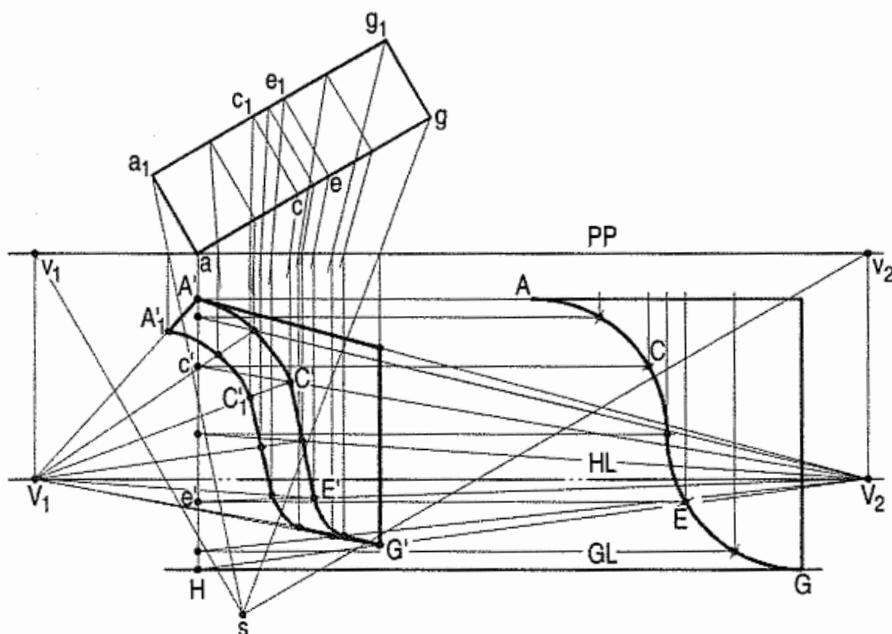


FIG. 19-24

19-12. TYPICAL PROBLEMS OF PERSPECTIVE PROJECTION

Problems 19-10 to 19-15: Draw perspective views of a straight line AB , 30 mm long, in the given positions. The station point is 40 mm in front of the picture plane (PP), 30 mm above the ground plane (GP) and lies in a central plane (CP) in the given positions.

Problem 19-10. (fig. 19-25): AB in CP and inclined at θ to PP; A in PP; CP, 10 mm to the right of A.

- (1) *Visual-ray method – by means of the top view and the front view* [fig. 19-25(i)]:
 - (i) Draw the top view ab with a in PP.
 - (ii) Draw the front view $a'b'$ in GL. Mark s , the top view and s' , the front view of the station point.
 - (iii) Join s with a and b , and s' with a' and b' . As A is in PP and on GP, its perspective A' will coincide with a' . B' will lie on $s'b'$ on a vertical through b_1 .
 - (iv) Join A' with B' . Then $A'B'$ is the perspective view of AB .
- (2) *Visual-ray method – by means of the top view and the side view* [fig. 19-25(ii)]:
 - (i) Draw the top view ab and the side view $a''b''$. Mark s , the top view and s'' the side view of the station point.
 - (ii) Join s with a and b , and s'' with a'' and b'' . As A is in PP and on GP, its perspective A' will lie on GL on the vertical through a . B' will lie at the point of intersection of the vertical through b_1 and the horizontal through b_2 . $A'B'$ is the required perspective view.
- (3) *Vanishing-point method* [fig. 19-25(iii)]:
 - (i) Draw the top view ab and mark s , the top view of the station point.
 - (ii) Draw sv parallel to ab . Through v , draw a vertical and mark V , the vanishing point on HL. A' will lie on GL on a vertical through a . Join A' with V .

- (iii) Draw a vertical through b_1 and obtain B' on $A'V$. $A'B'$ is the required perspective view.

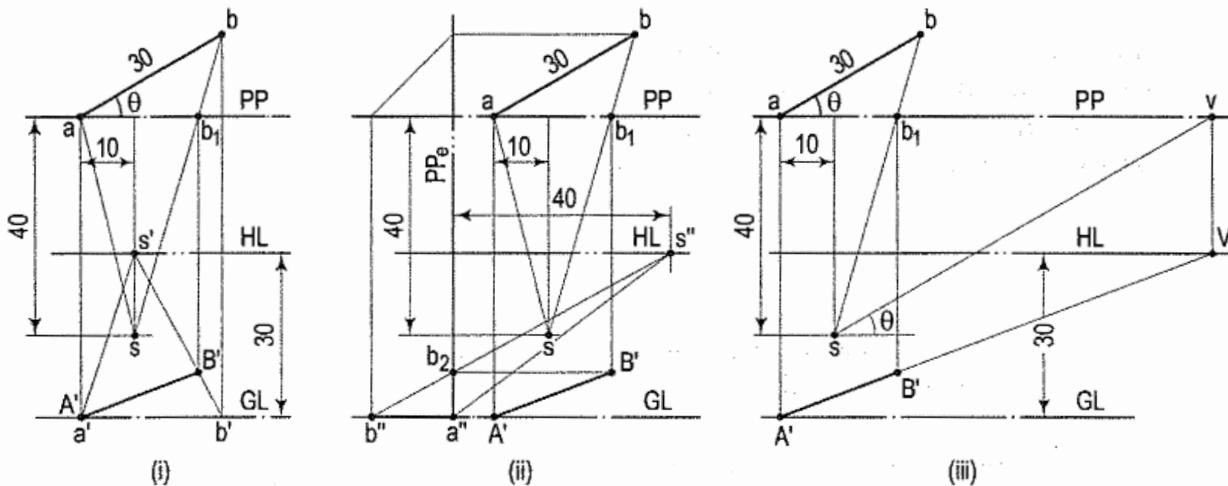


FIG. 19-25

Problem 19-11. (fig. 19-26): AB parallel to GP and 5 mm above it; inclined at 30° to PP ; A , 10 mm behind PP ; CP , 10 mm to the right of A .

- Draw the top view ab , mark s and obtain the vanishing point V .
- Join s with a and b . As A is behind PP , the line of heights will be necessary.
- Produce ba to h on PP . Draw the line of heights hH . Mark a point a' on hH , 5 mm above GL . Join a' with V . Obtain points A' and B' on the line $a'V$ as shown $A'B'$ is the required perspective view.

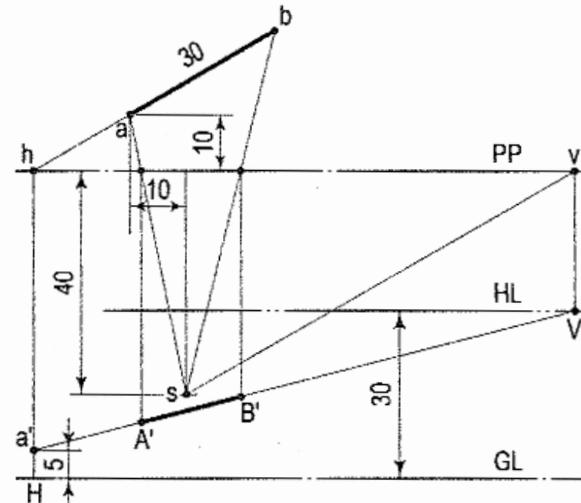


FIG. 19-26

Problem 19-12. (fig. 19-27): AB , perpendicular to PP ; A , 10 mm behind PP and 5 mm above GP ; CP , 20 mm to the right of AB .

- Draw the top view ab , and the front view a' . Mark s and s' .
- Join s with a and b , and s' with a' .
- Obtain the perspective $A'B'$ as shown. This is the visual-ray method.

The same figure can be interpreted as the solution by the vanishing-point method. As the line is perpendicular to PP , its vanishing point will be at the centre of vision c' , i.e. at s' on HL . The vertical through a is the line of heights. a' , marked 5 mm above GL , shows the height of A and B both. The perspective $A'B'$ lies on $a's'$, i.e. on $a'c'$.

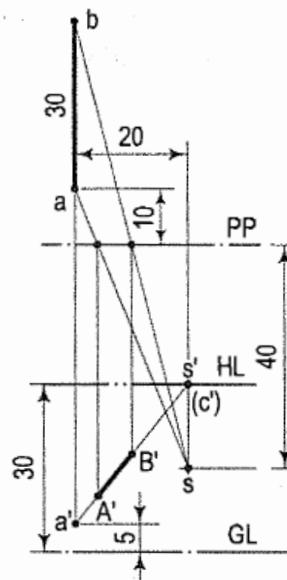


FIG. 19-27

Problem 19-13. (fig. 19-28): *AB, parallel to both PP and GP; 5 mm above GP and 15 mm behind PP; CP, 15 mm to the left of A.*

As the line is parallel to PP, it will have no vanishing point. The perspective view $A'B'$ is drawn by means of the top view and the front view (visual-ray method). It is parallel to $a'b'$. (See note at the end of problem 19-15.)

Problem 19-14. (fig. 19-29): *AB, parallel to and 15 mm behind PP; inclined at 30° to GP; A, 5 mm above GP; CP, 15 mm to the left of A.*

As AB is parallel to PP, it will have no vanishing point. The perspective $A'B'$ is obtained by means of the top view and the front view (visual-ray method). It is parallel to the front view $a'b'$. (See note at the end of problem 19-15.)

Problem 19-15. (fig. 19-30): *AB, perpendicular to GP and 10 mm behind PP; A, 5 mm above GP; CP, 20 mm to the right of AB.*

As AB is parallel to PP, it will have no vanishing point. a is the top view and $a'b'$ is the front view. The perspective $A'B'$ is obtained by visual-ray method. It is perpendicular to GL.

Note : Problems 19-13, 19-14 and 19-15 may also be interpreted as solved by vanishing-point method. The ends A and B of the line may each be assumed to be the ends of two separate lines perpendicular to the PP and which in perspective view would vanish at the centre of vision c' , i.e. at s' .

These problems may also be termed as cases of parallel perspective. Horizontal, inclined and vertical lines are each parallel to the PP. In their perspective views they remain respectively horizontal, inclined and vertical.

Problem 19-16. (fig. 19-31): *A rectangle ABCD, 30 mm \times 20 mm, has its surface parallel to and 10 mm above the ground plane. Its shorter edge AD is inclined at 60° to the picture plane which passes through the rectangle so that the corner A is 10 mm in front of it. The station point is 70 mm in front of the picture plane, 40 mm above the ground plane and lies in a central plane which passes through A. Draw the perspective view of the rectangle.*

- Draw the line PP and the top view $abcd$ in the given positions. Mark s and join it with the corners a , b , c and d .
- Obtain the vanishing points V_1 and V_2 . Through h (the point at which PP cuts ad), draw the line of heights hH .
- Draw the front view of the rectangle. It is the line AB , parallel to and 10 mm above GL. Project the line to a point H' on hH .

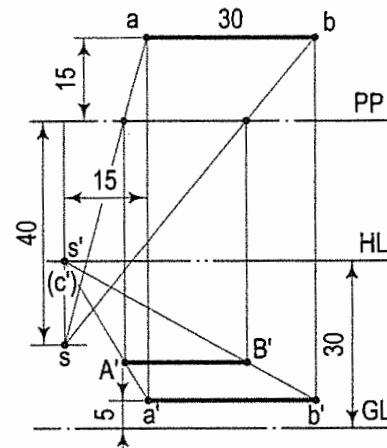


FIG. 19-28

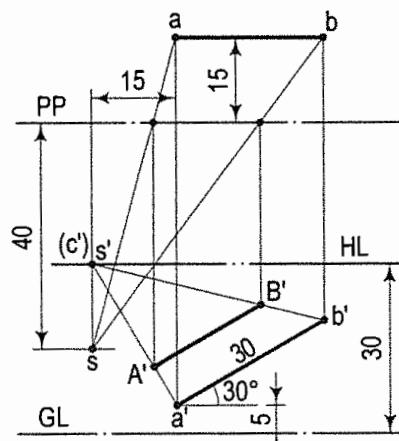


FIG. 19-29

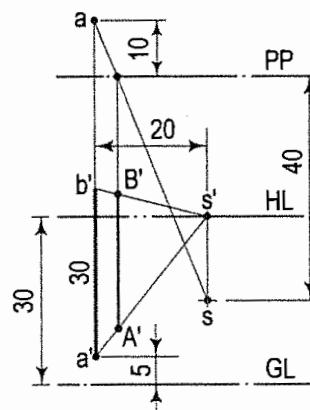


FIG. 19-30

- (iv) Draw a line through H' and V_1 and on it, obtain points A' and D' as shown.
- (v) Draw lines joining V_2 with A' and D' , and on them, obtain points B' and C' .
- (vi) Join B' with C' . Then $A'B'C'D'$ is the perspective view of the rectangle. Note that $B'C'$, if produced, will pass through V_1 .

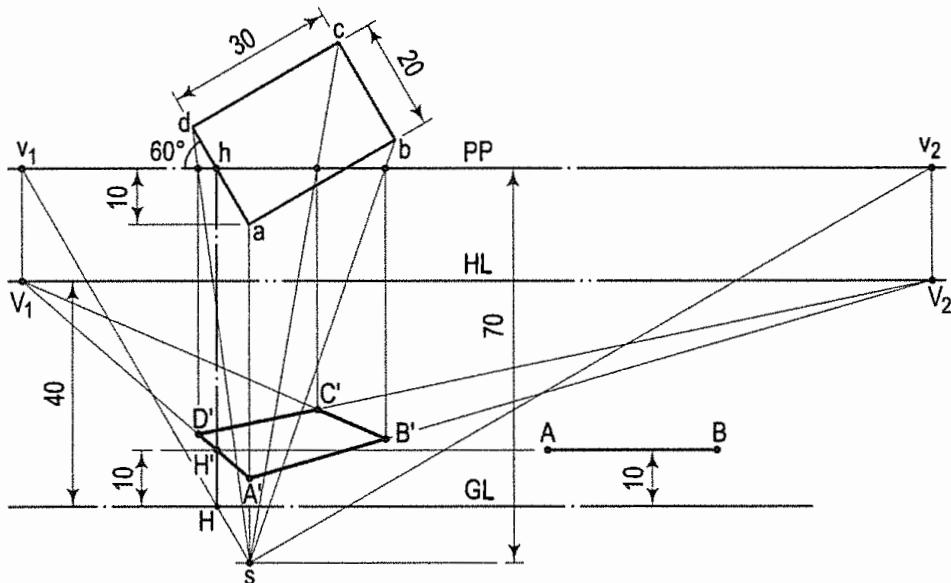


FIG. 19-31

It may also be noted that H' can be marked directly 10 mm above H . Hence, it is not absolutely essential to draw the front view in this particular case. (A line of heights may also be drawn through the point of intersection of ab with PP. In that case, H' will be joined with V_2 and the point B' obtained on $H'V_2$.)

Problem 19-17. (fig. 19-32): *Draw the perspective view of a circle of 40 mm diameter, having its surface vertical and inclined at 45° to the picture plane. The centre of the circle is 25 mm above the ground plane. The positions of the station point and the horizon level are as shown in the figure.*

- (i) Draw the top view of the circle. Obtain the vanishing point V .
- (ii) Draw the front view of the circle (on the right-hand side) with its centre 25 mm above GL and mark eight points on it. Mark these points on the top view also.
- (iii) Draw the line of heights hH and on it, project horizontally, the eight points from the front view. Join s with the points in the top view, and join V with the points on hH .
- (iv) Draw verticals through the points of intersection of $s1$, $s5$, etc. with PP, and obtain points $1'$, $5'$ etc.

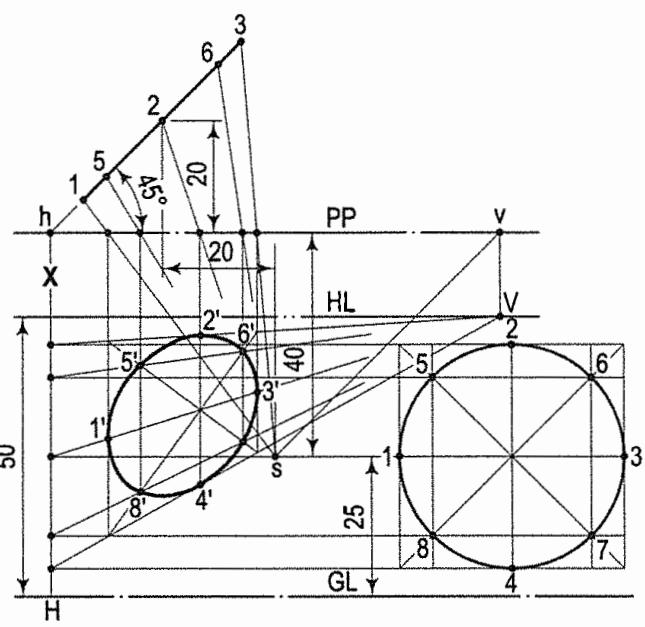


FIG. 19-32

- (v) Draw the ellipse through these points. It is the perspective view of the circle. Note that points 5', 6', 7' and 8' lie on the diagonals of the perspective view of the square in which the circle is enclosed.

Problem 19-18. (fig. 19-33): *Draw the perspective view of a pentagonal prism, lying on the ground plane on one of its rectangular faces, the axis being inclined at 30° to the picture plane, and a corner of the base touching the picture plane. The station point is 65 mm in front of the picture plane, and lies in a central plane which bisects the axis. The horizon is at the level of the top edge of the prism.*

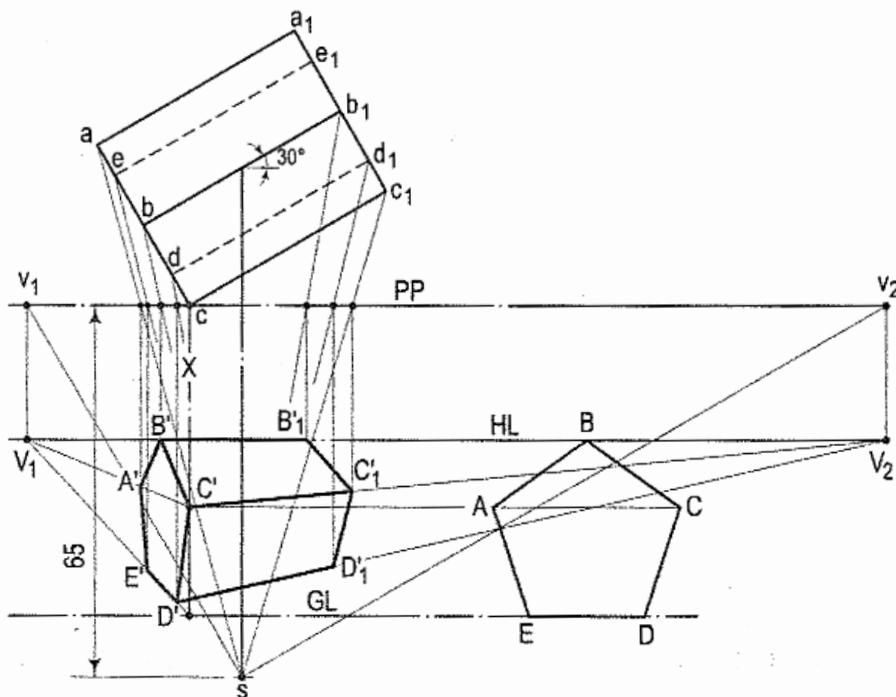


FIG. 19-33

- Draw the top view and the front view of the prism. Mark s and obtain the two vanishing points V_1 and V_2 . Through c , draw the line X which is the line of heights for points $A, B \dots E$. Join s with all the points in the top view.
- Project horizontally, all the corners in the front view to points on the line X and join these points with V_1 . Obtain points A', B', \dots, E' on these lines. Draw lines joining B', C' and D' with V_2 and obtain points B'_1, C'_1 and D'_1 on these lines. Complete the perspective view as shown. The vanishing point V_2 is not shown in the figure. Hidden edges of the prism are also not shown in the perspective view.

Problem 19-19. (fig. 19-34): *Draw the perspective view of the frustum of a hexagonal pyramid (top 15 mm side, base 25 mm side and axis 40 mm long), the top view of which is given in the figure, along with the positions of the station point and the horizon.*

- Enclose the two hexagons in two rectangles as shown in the figure. (They can be enclosed in different ways also.)
- Determine the vanishing points V_1 and V_2 . Draw the perspective of the larger rectangle. Note that the corner a is projected back on PP .
- Draw hH the line of heights for points p and q . On this line, mark the length of the axis viz. $h'H$. Join h' with V_2 and obtain the perspective of the line pq and the whole (small) rectangle.

- (iv) Mark the corners of the top of the frustum and complete the perspective view as shown. Hidden edges have not been shown.

The data in the problem on perspective view is generally given in the form of a figure showing the top view and the front view of the object together with the position of the station point or the observer.

In the following more advanced problems, the data of each problem along with the solution is given in the same figure. All construction lines are shown to make the solutions self-explanatory. Hints are given only where deemed necessary.

Students are advised to copy only the data of each object, draw its perspective view and then compare with the given solution. Lines of heights are marked with letter X.

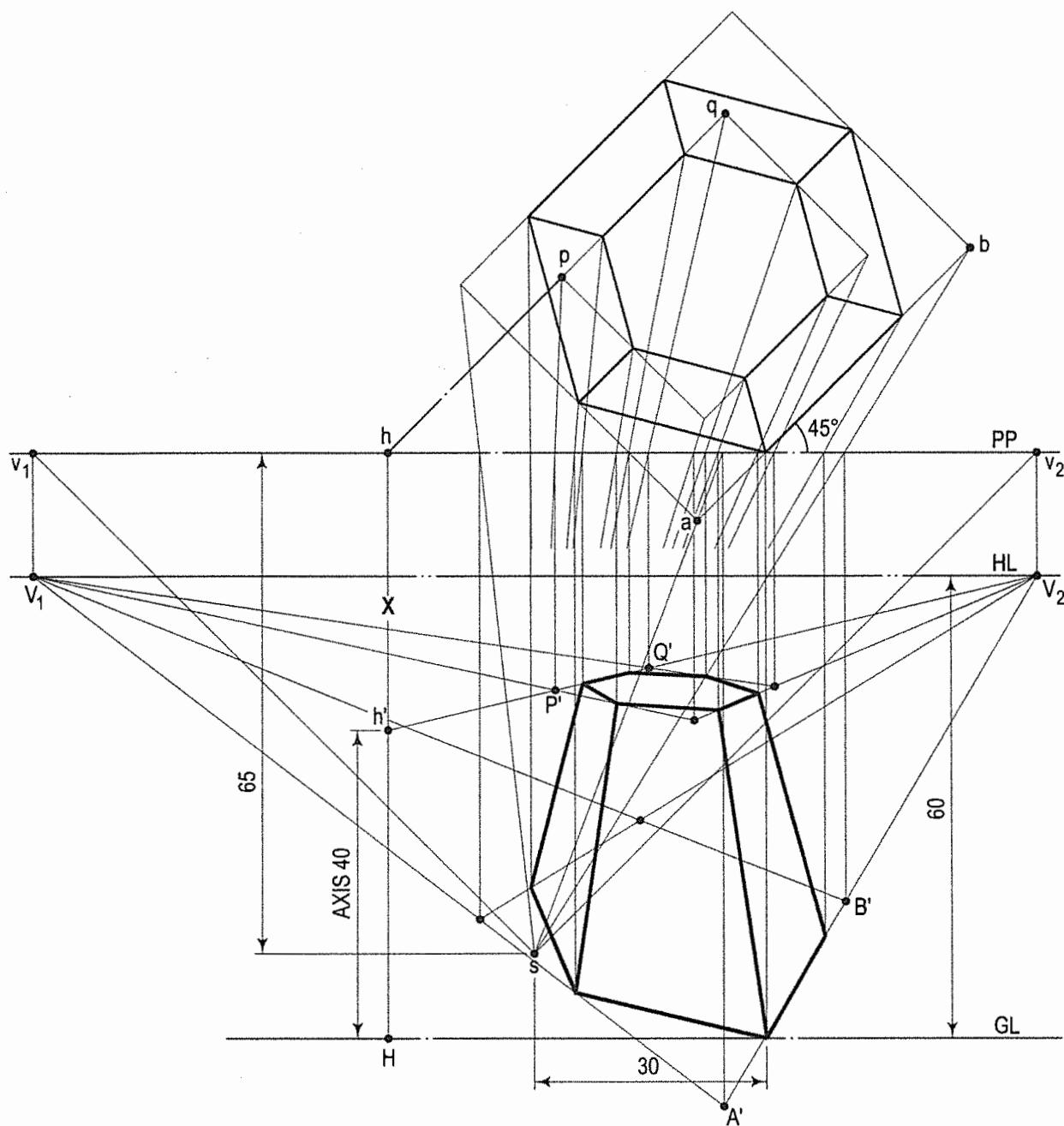


FIG. 19-34

Problem 19-20. (fig. 19-35): Letter P.

Perspectives of semi-circles are drawn by enclosing the outer semi-circle in a rectangle and then marking points at which the diagonals cut the semi-circles.

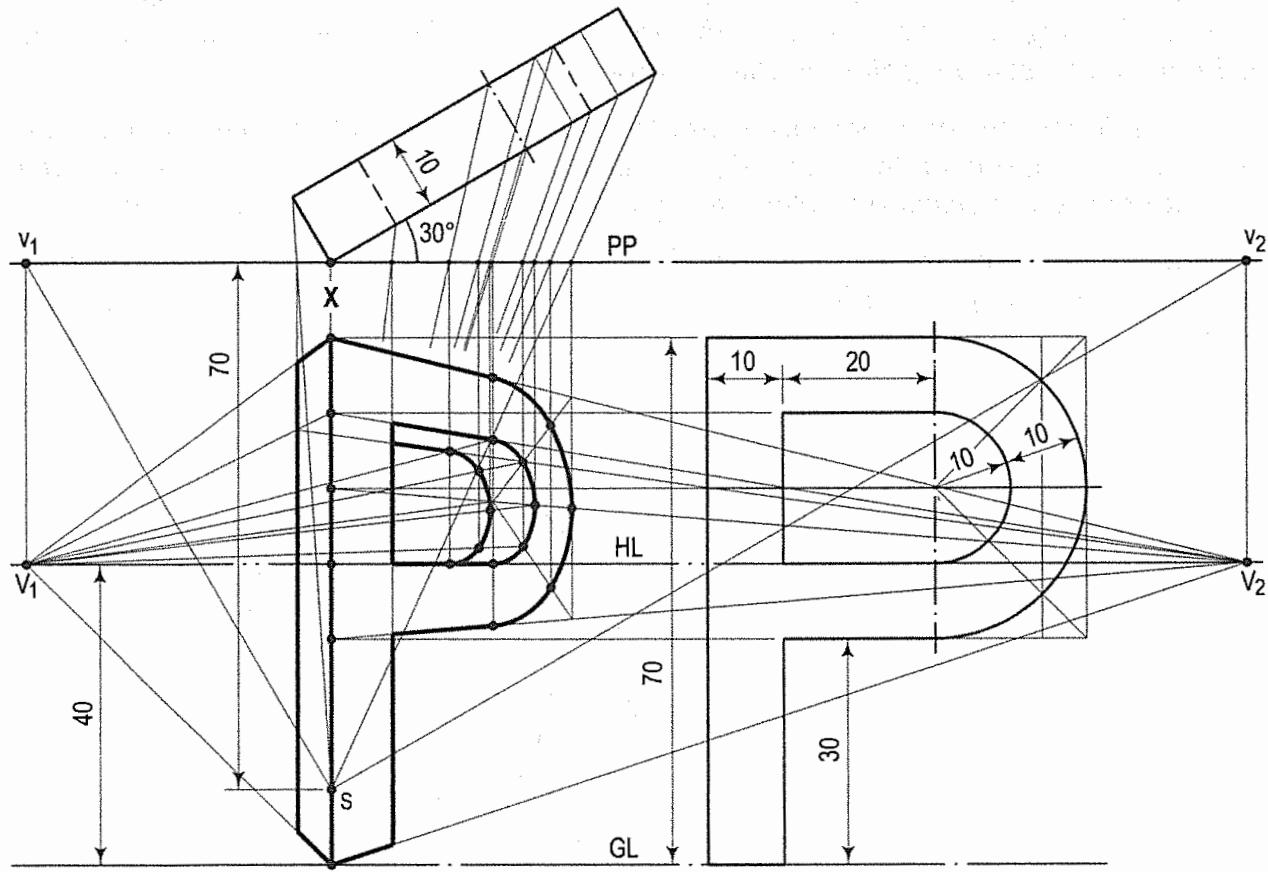


FIG. 19-35

Problem 19-21. (fig. 19-36): Guide-block.

This is a problem on parallel perspective. The front face is in PP and hence, it is seen in its true shape and size. It is above the ground plane and therefore above GL. Circles appear as circles. Lines perpendicular to PP converge to the centre of vision c'.

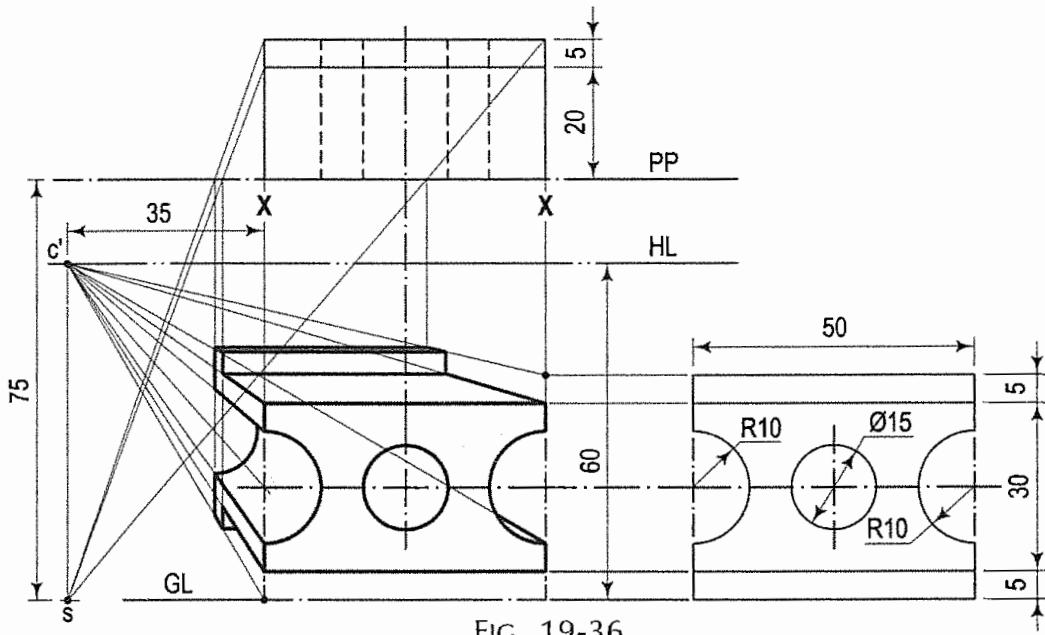


FIG. 19-36

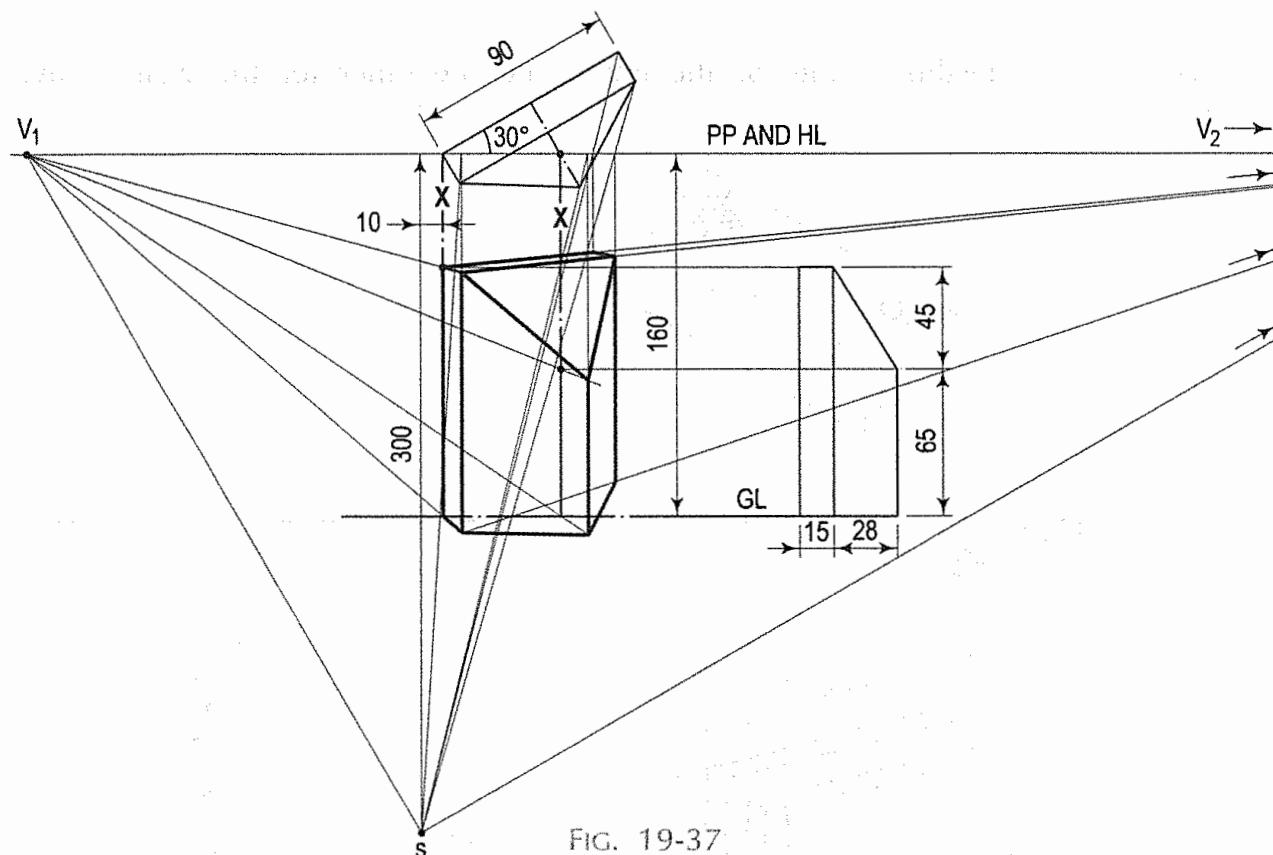


FIG. 19-37

Problem 19-22. (fig. 19-37): Mile-stone.

PP passes through the stone.

Hence, the perspective of the front part will appear bigger. A separate line of heights is needed for the front vertical edge.

The station point and vanishing points are not shown in the figure. HL is shown coinciding with PP.

Problem 19-23. (fig. 19-38): A square prism with a square pyramid placed centrally on its top.

This is also a case of parallel perspective, but some portions of the solids are in front of PP. The front corners are projected back on PP. Hence, these portions appear bigger in size.

Two lines of heights have been used for the prism (though even one is sufficient). Corners of the base of the pyramid are at the mid-points of the sides of the top of the prism. A separate line of heights is needed for the apex.

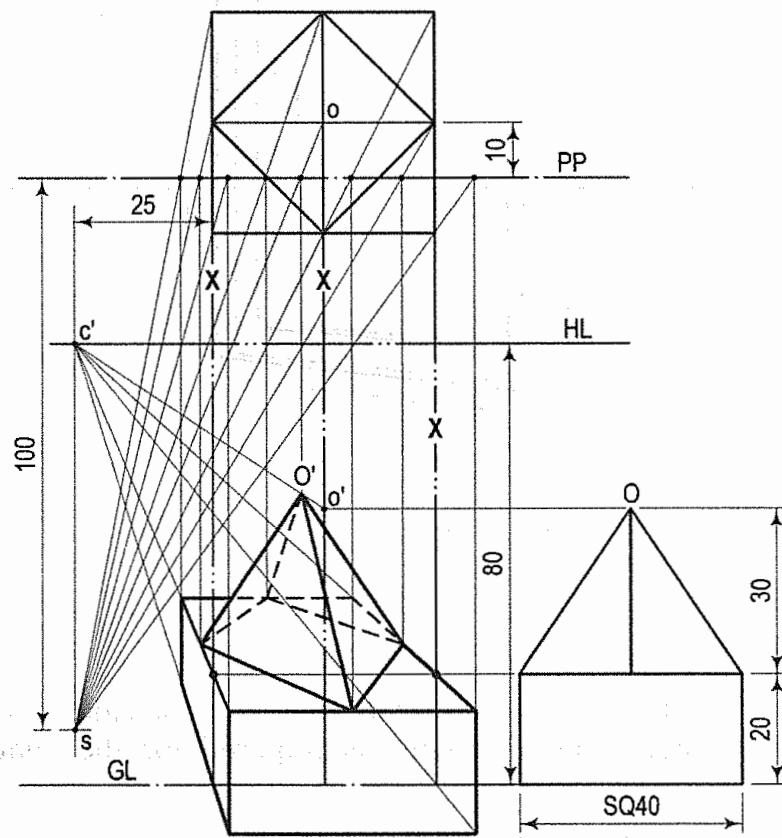


FIG. 19-38

Problem 19-24. (fig. 19-39): Window-frame placed in a wall.

Two lines of heights – one for the frame and the other for the wall – have been used.

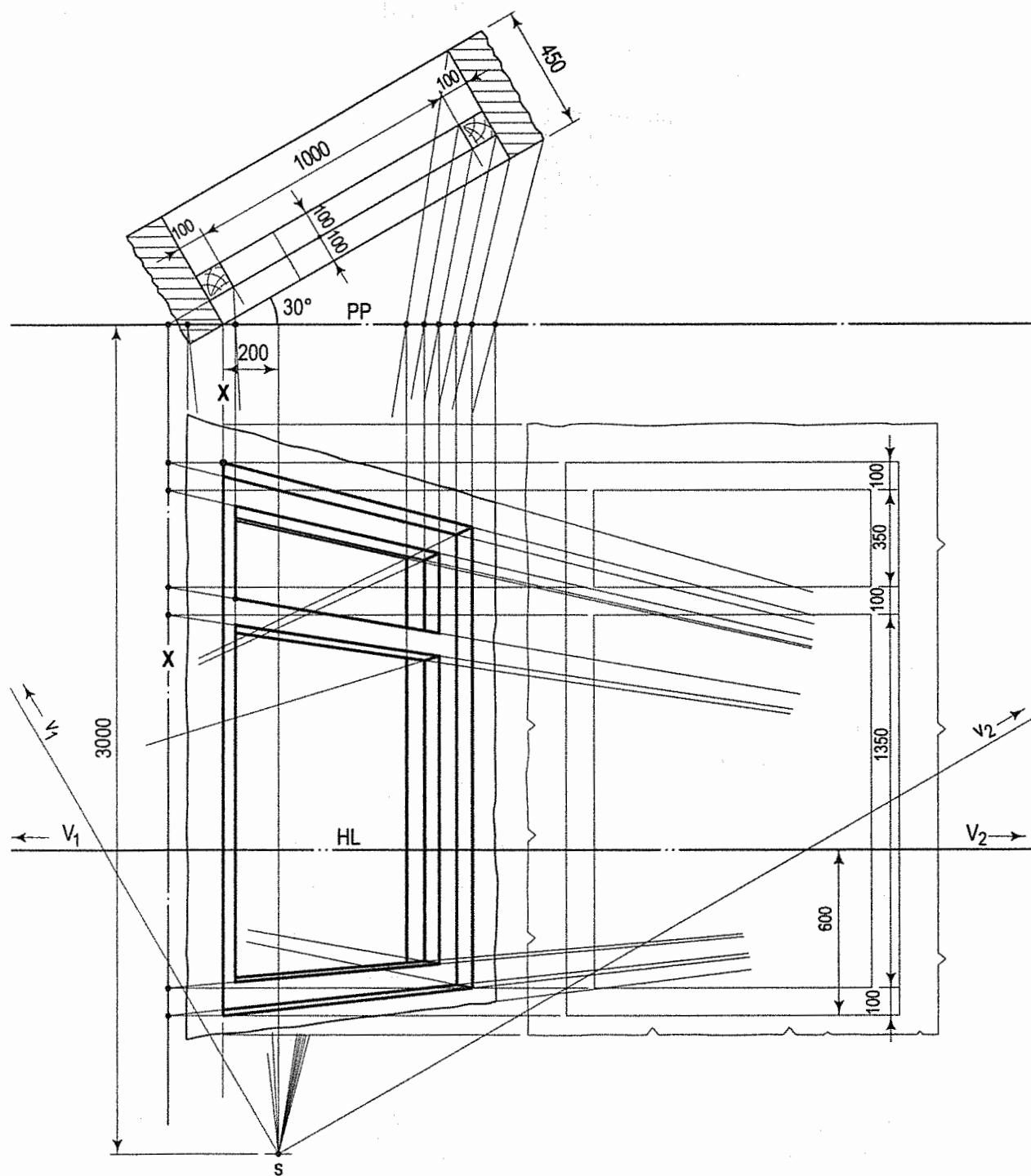


FIG. 19-39

Problem 19-25. (fig. 19-40): Casting.

GL has been so placed that HL coincides with PP. It may be taken lower. A separate line of heights is required for the central rib.

Problem 19-26. (fig. 19-41): Ogee arch in a wall.

A number of points are marked on the arch and the curves are drawn through the perspectives of these points.

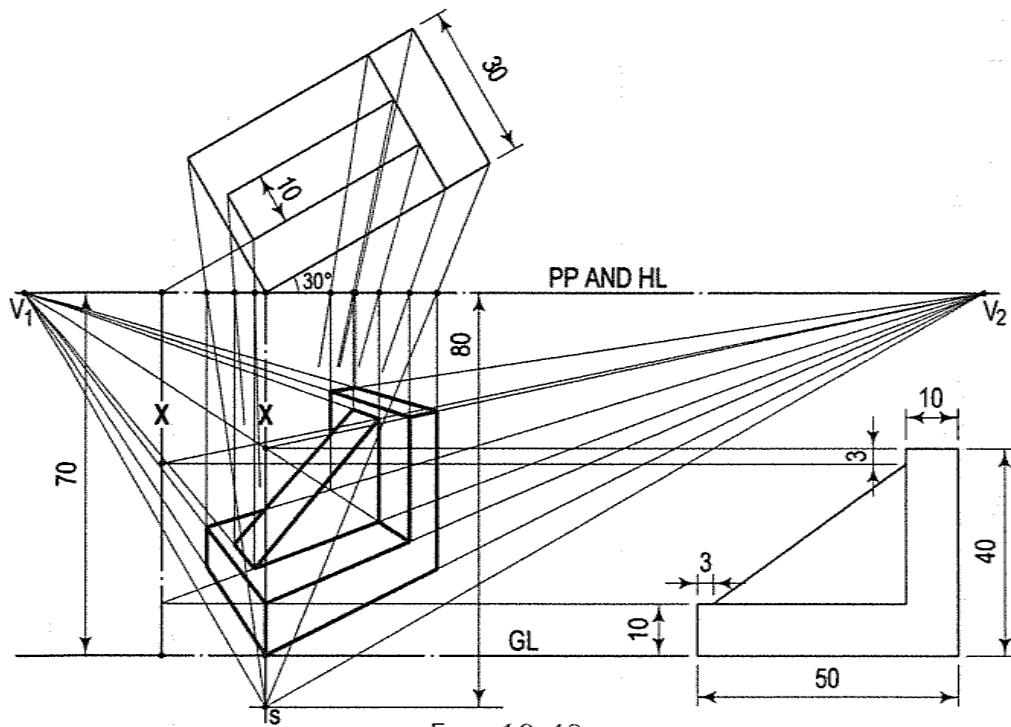


FIG. 19-40

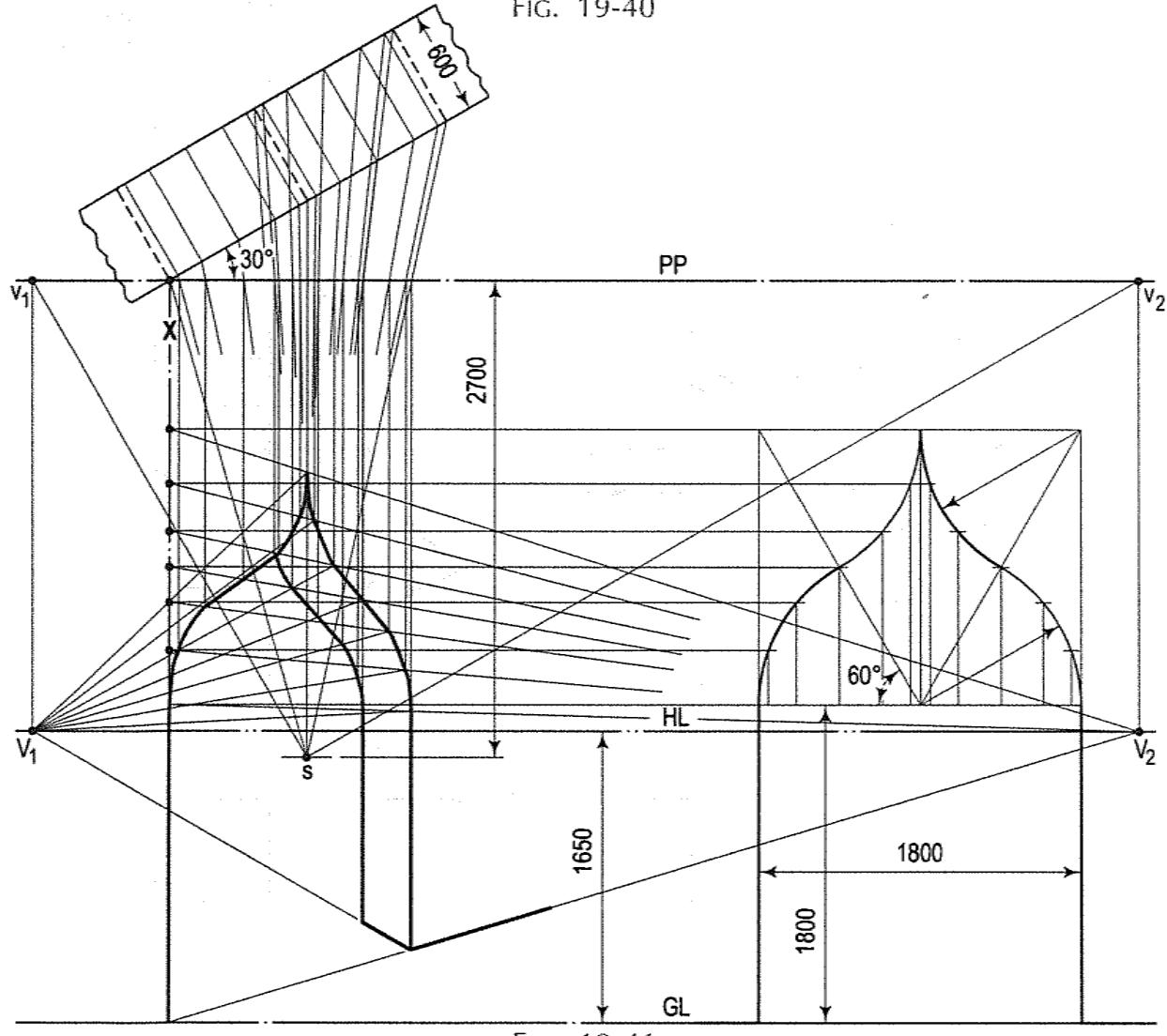
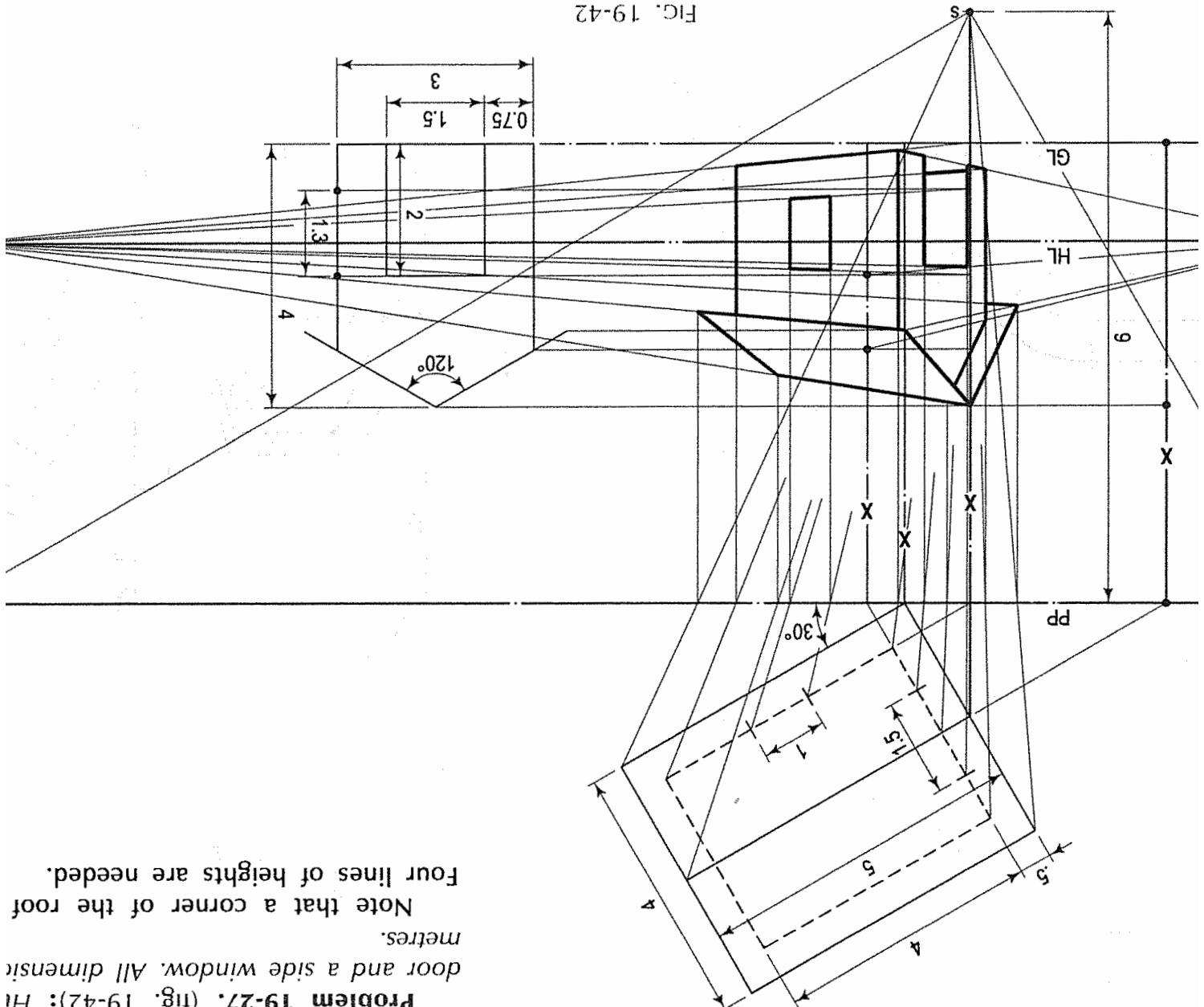


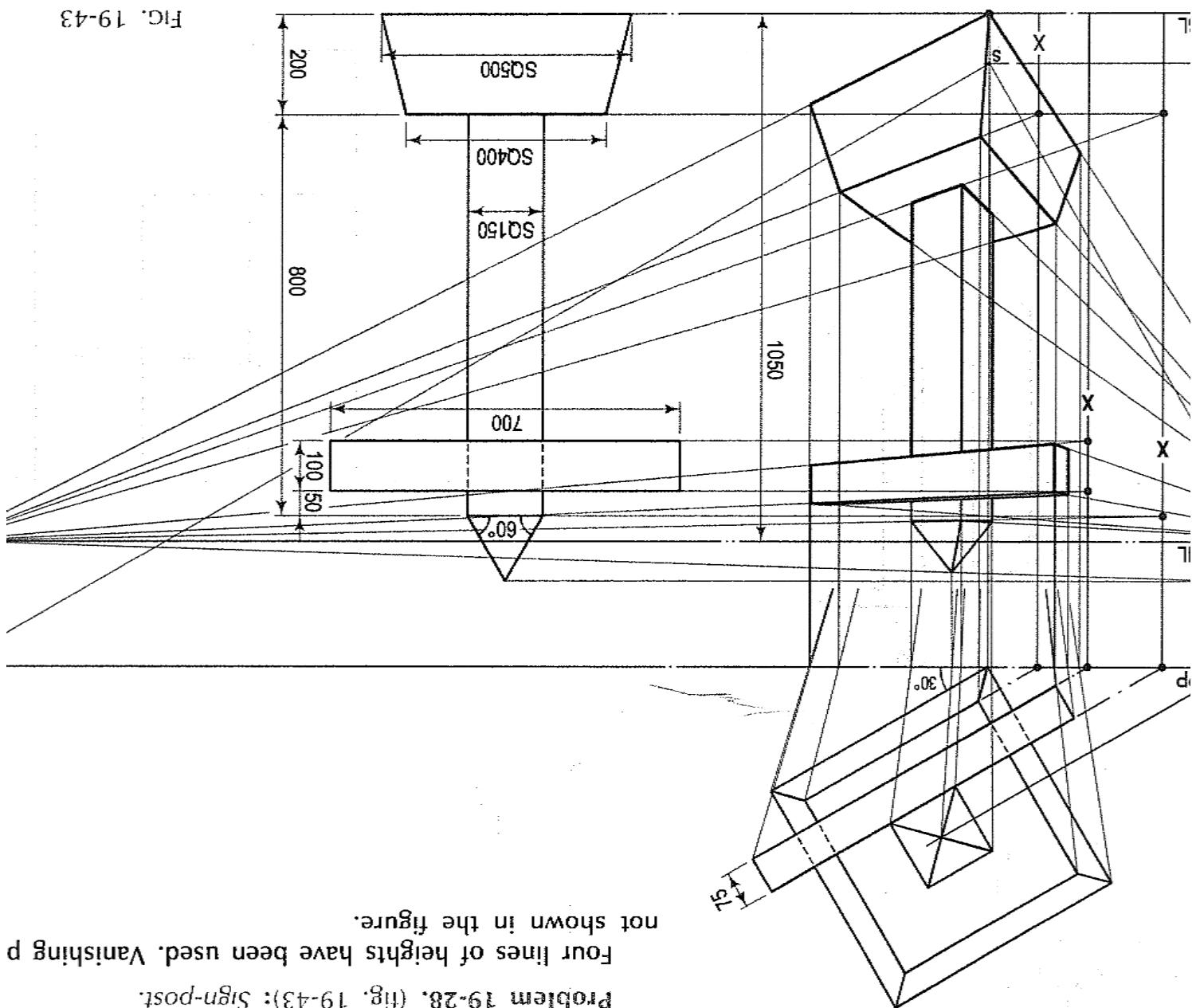
FIG. 19-41

FIG. 19-42



Problem 19-27. (Fig. 19-42): Hi
door and a side window. All dimensions
metres.

FIG. 19-43



not shown in the figure.

Four lines of heights have been used. Vanishing p

Problem 19-28. (fig. 19-43): Sign-post.

Problem 19-29. (fig. 19-44): *Model of steps.*

This is also a problem on parallel perspective, but the front face is behind PP. Hence, that face will be smaller in size.

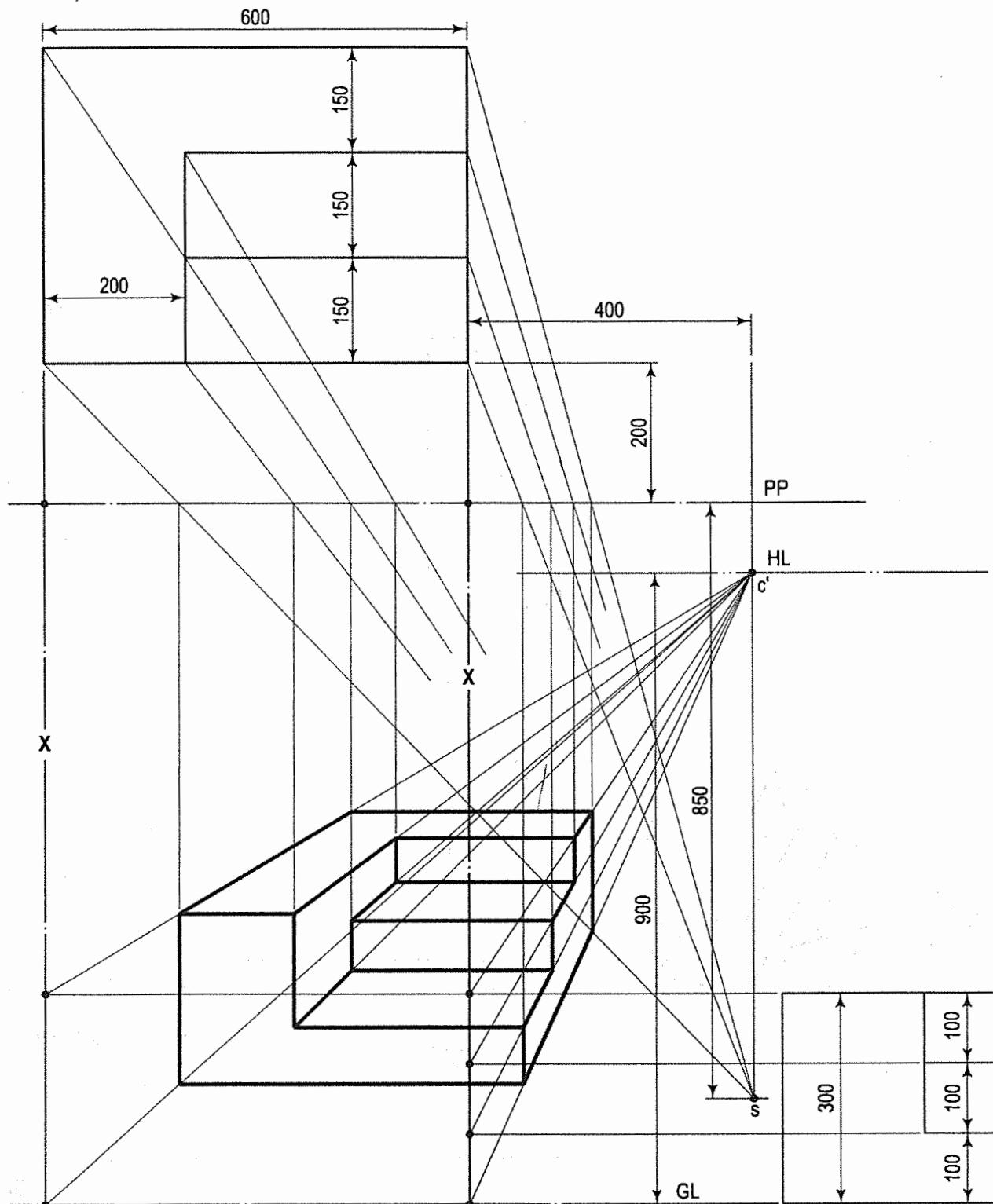


FIG. 19-44

Problem 19-30. (fig. 19-45): *Lamp-post.*

All faces of the lamp casing are of glass (except the bottom). The base is cylindrical.

The back edges (of the lamp casing) which are visible through the glass are shown thinner.

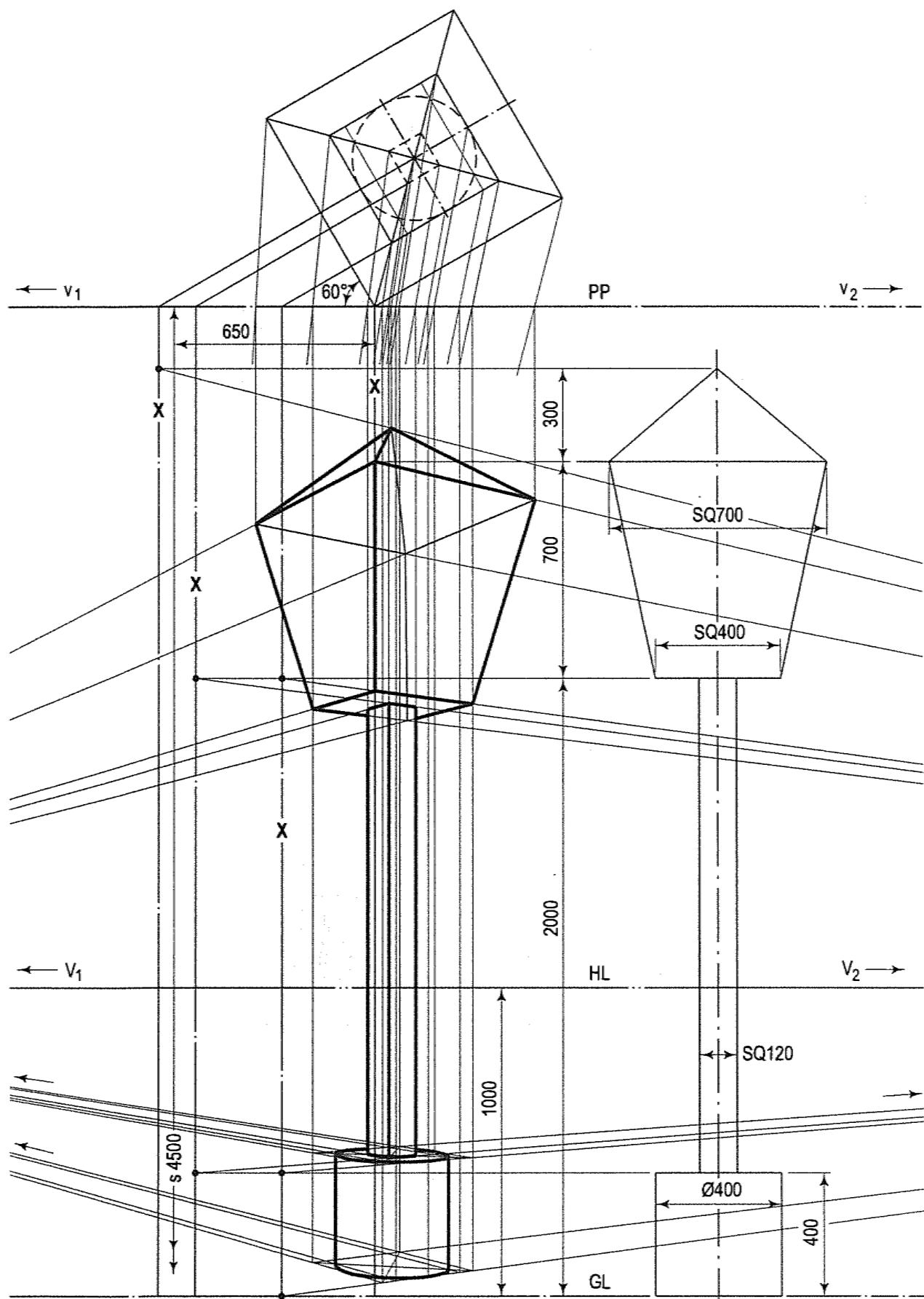
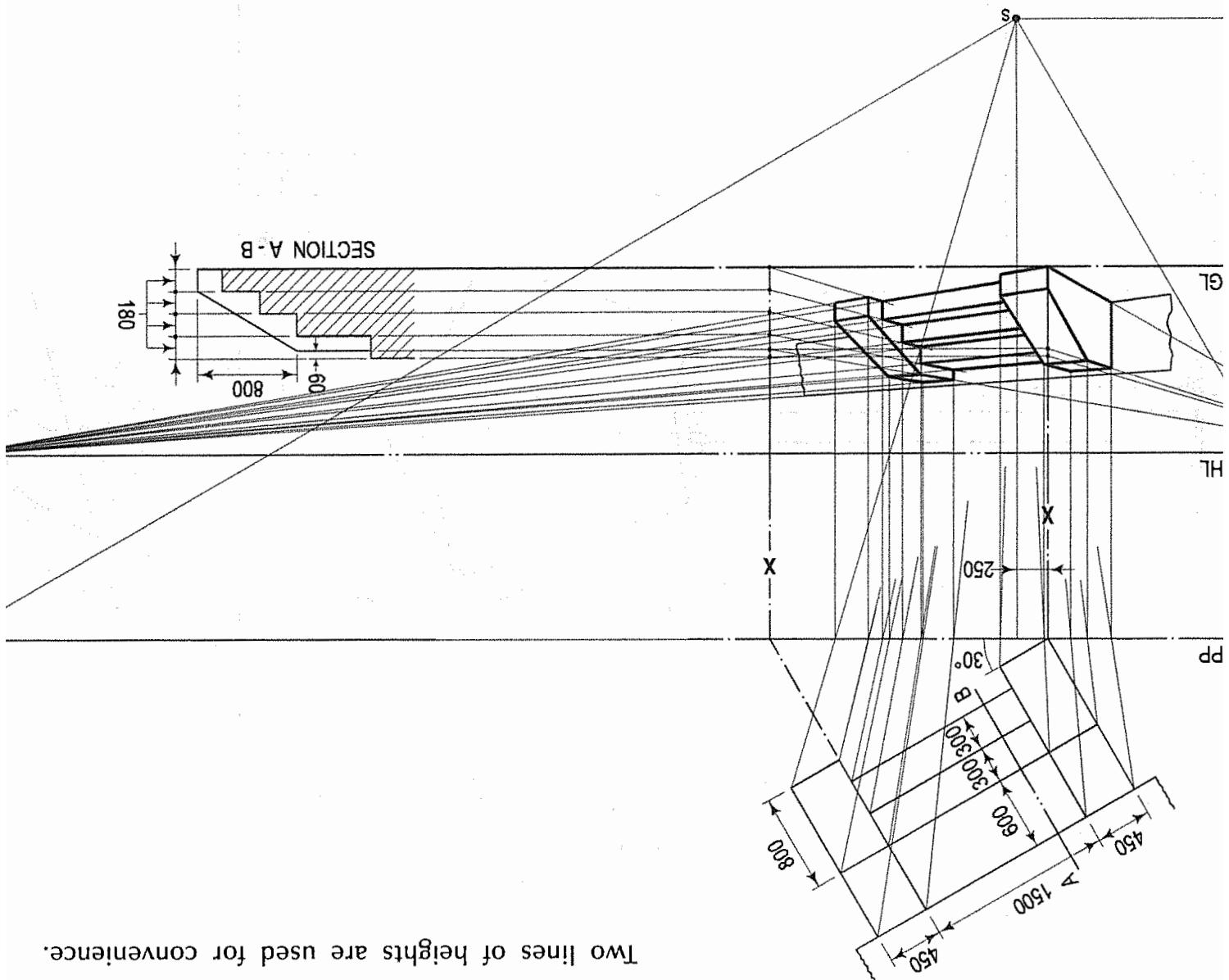


FIG. 19-45

FIG. 19-46



Problem 19-31. (fig. 19-46): Steps with guards at
Two lines of heights are used for convenience.

EXERCISES 19



1. A man stands at a distance of 5 m from a flight of four stone steps having a width of 2 m, tread 0.3 m and rise 0.2 m. The flight makes an angle of 45° with the picture plane and touches the same at a distance of 2 m to the right of the centre of vision. Draw the perspective view of the flight.
2. Draw the perspective view of a square pyramid of base 100 mm side and height of the apex 120 mm. The nearest edge of the base is parallel to and 30 mm behind the picture plane. The station point is situated at a distance of 300 mm from the picture plane, 60 mm above the ground plane and 200 mm to the right of the apex.
3. Draw the perspective view of the model of steps shown in fig. 17-57. The position of the steps relative to the picture plane is shown in fig. 19-47. The station point is 200 mm from the picture plane. Take horizon level to be 100 mm above the ground level.
4. Draw the perspective view of the memorial shown in fig. 19-48. The horizon level is 8 m above the ground plane, while the observer is stationed at a distance of 12 m from the picture plane. (All dimensions are in metres.)

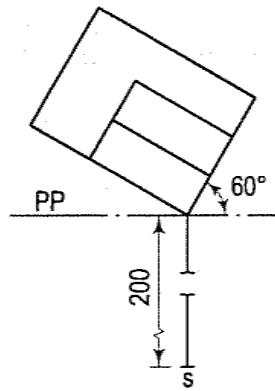


FIG. 19-47

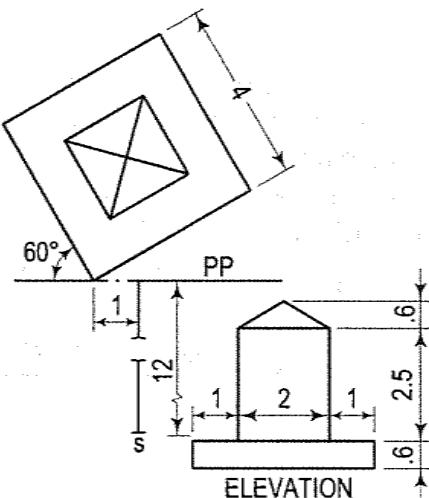


FIG. 19-48

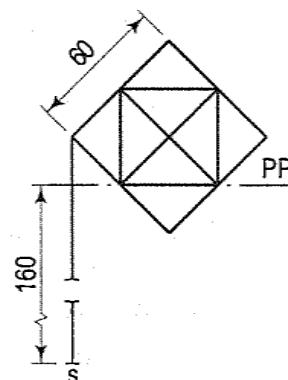


FIG. 19-49

5. Fig. 19-49 shows the top view of a square prism of 40 mm thickness with a square pyramid of 60 mm long axis, placed centrally on it. The station point is 160 mm from the picture plane and 100 mm above the ground plane. Draw the perspective view of the solids.
6. Draw the perspective view of the box shown in fig. 17-61. The top view of the box with the lid 90° open, along with the picture plane and the station point is shown in fig. 19-50. Assume horizon level to be 200 mm above the ground level.
7. Draw the perspective view of a semi-circular arched opening in a wall having the length of 4 m, thickness 0.5 m and height 3.5 m. The opening is 1.5 m wide and the springing points are at a height of 2 m. The wall makes an angle of 45° with the picture plane. Select a suitable position of the spector.

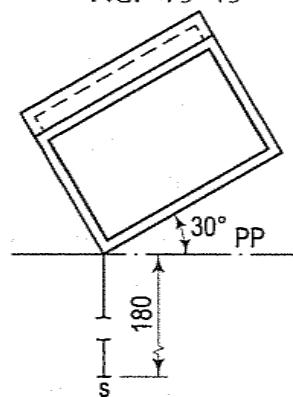


FIG. 19-50

8. Draw the perspective view of the wooden cabinet shown in fig. 19-51. The thickness of wood is 20 mm. The cabinet is placed with one edge touching the picture plane and 1000 mm to the left of the centre of vision. The station point is 2500 mm away from the picture plane and 1500 mm above the ground plane.

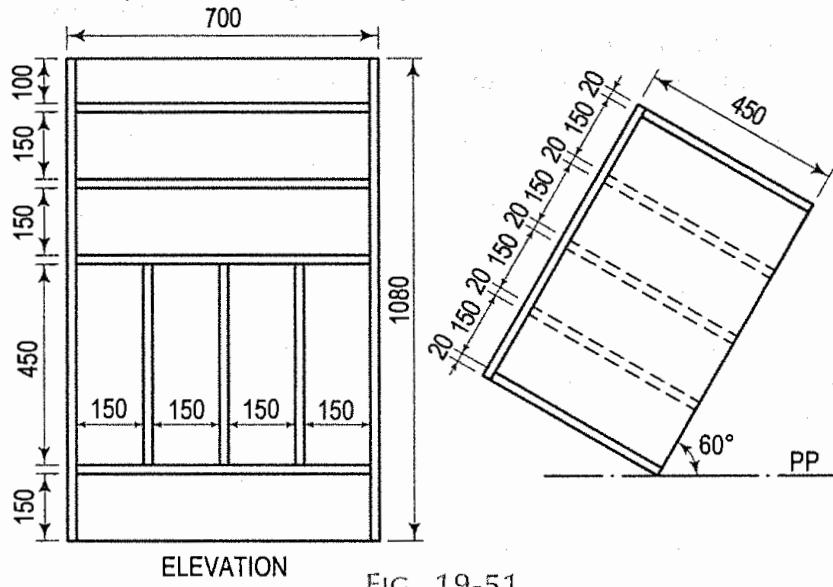


FIG. 19-51

9. Draw the perspective view of the pedestal for a statue (neglecting the sphere) shown in fig. 17-127(5). The edges of the base, as shown in fig. 19-52, are equally inclined to the picture plane. Assume the horizon plane to be 150 mm above the ground plane.
10. The casting shown in fig. 17-127(2) is placed behind the picture plane as shown in fig. 19-53. Draw its perspective view, assuming the horizon plane to be 80 mm above the ground plane.
11. Draw the perspective view of the model of a memorial shown in fig. 19-54. The station point is 200 mm in front of the picture plane and 160 mm above the ground plane.
12. Draw the perspective view of a vertical steel cup-board 1 m × 2 m × 0.35 m deep, having four shelves. One shutter is open. The front of the cup-board makes an angle of 30° with the picture plane, with a vertical edge touching it. Assume suitable position of the spectator at a height of 4 m above the ground.

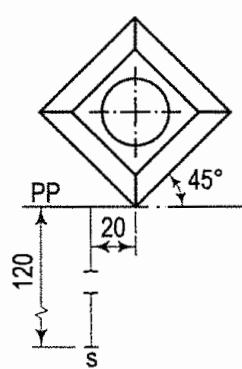


FIG. 19-52

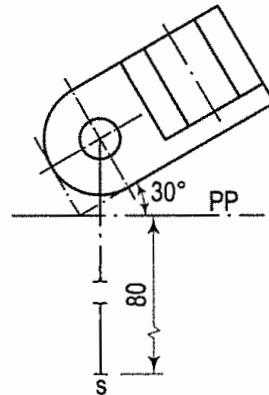


FIG. 19-53

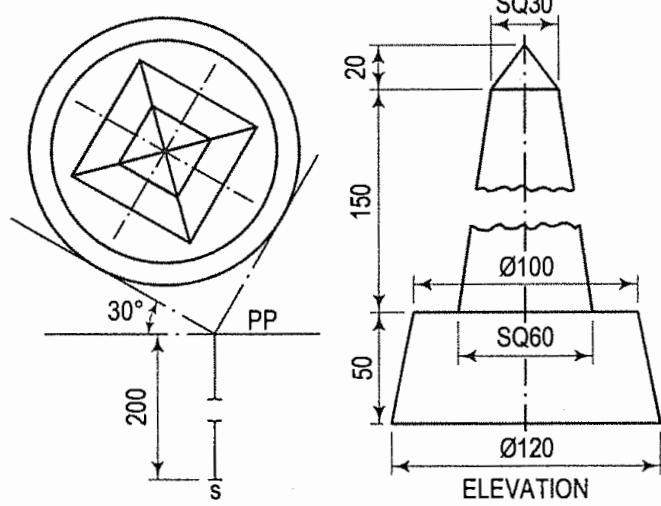
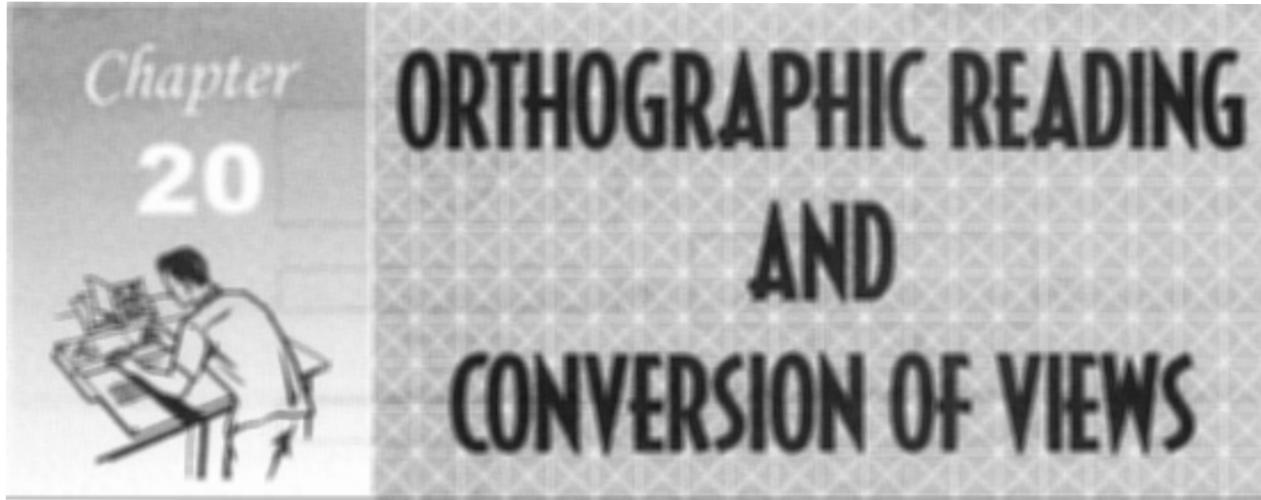


FIG. 19-54



20-1. INTRODUCTION

Orthographic reading is the ability to visualize the shape of an object from its drawing in orthographic views. Every engineer or technician connected with the work of construction should possess this ability. Without it, it would be difficult for him to execute, independently, any work according to a given drawing.

An engineering drawing is not read aloud. It is read mentally. The whole drawing cannot be read or interpreted at a glance. It should be read systematically and patiently. The easiest way to learn reading such a drawing is to learn how to prepare one.

However, it is not impossible to know how to read it without learning how to draw. In either case, a sound knowledge of the principles of orthographic projection is quite essential for reading the drawing without hesitation.

We studied in chapter 8 that *in orthographic projection, any one view shows only two dimensions of a three dimensioned object. Hence, it is impossible to visualize the shape of the object from a single view. The second view shows the third dimension. Thus, at least two views are necessary to determine its shape.* Sometimes, a third view is also necessary to completely visualize an object.

20-2. READING OF ORTHOGRAPHIC VIEWS (BLUE-PRINT READING)

Every object may be imagined as consisting of a number of components having forms of simple solids such as prisms, cylinders, cones, etc. with some additions or subtractions or both. The additions may be in the form of projections, while the subtractions may be in the form of holes, grooves, cavities, etc. It is not possible to determine from only one view whether there is an addition or subtraction. The other view or views must be referred to.

For example in fig. 20-1(i), the meaning of lines AB and CD can be determined only after referring to the other view. They might represent a projection as shown in figs. 20-1(ii) and (iii) or a cavity as shown in figs. 20-1(iv) and (v) or a cavity made by projections as shown in fig. 20-1(vi).

Note that one *front-view* represents six different types of objects whose *top-view* are different as shown in fig. 20-1.

Fig. 20-1 shows seven orthographic views of an object. The views are labeled (i) through (vi).

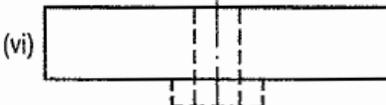
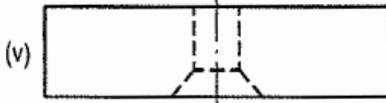
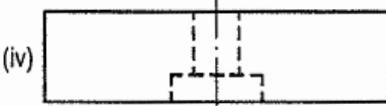
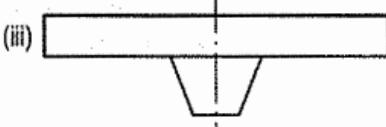
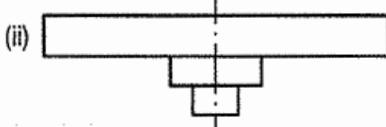
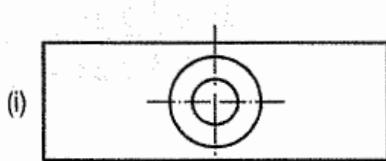
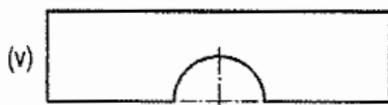
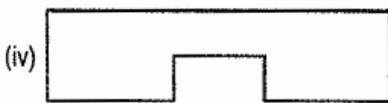
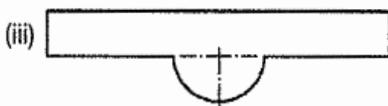
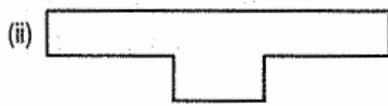
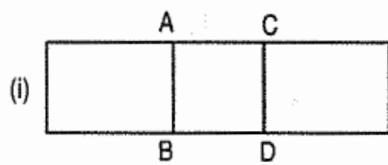


FIG. 20-1

FIG. 20-2

Similarly, the two circles in fig. 20-2(i) may either represent additions [figs. 20-2(ii) and (iii)] or holes [figs. 20-2(iv) and (v)] or both [figs. 20-2(vi) and (vii)]. Thus, every point and every line in an orthographic view has a meaning. A point may represent a corner or an edge. A line may represent an edge or a surface. The meaning of each point or line should be interpreted by systematically referring back and forth from one view to the other. Simultaneously, the shape of the object as a whole should be visualized.

Practice in reading a drawing can be had by either projecting one or more views from the given views, or by converting one or both the given views into sectional views.

20-3. MISSING LINES AND MISSING VIEWS

Most of the students in Engineering Drawing course find it difficult to visualize an object from two orthographic views. To overcome this difficulty, assume the object as basic geometrical solids such as prisms, cylinders, pyramids, cones and so on. It is impossible to determine whether the shape is addition or subtraction by observing only one view. Therefore, the each orthographic view of an object must be referred systematically back and forth. Sound knowledge of projections of points, lines, planes and solids is required in solving the problems of missing lines

or missing views. In orthographic views, if a line or lines are not drawn in order to test the understanding of students, such lines are known as *missing lines*. Similarly, if amongst the three views, only two views are given then third view will be known as *missing view*.

Following procedure can be adopted in order to identify missing lines or missing views.

- (i) From the given orthographic views, try to visualize an object and prepare a pictorial view.
- (ii) From pictorial view, prepare orthographic views and compare with given views. Read carefully each line in each view and find out corresponding projection from another view.

Following conclusion from the projections of lines and planes will be useful for identification of lines and planes in the given views.

- (a) When a line is perpendicular to a plane, its projection on that plane is point, while its projection on the other plane is a line equal to its true length.
- (b) When a line is inclined to both the planes (H.P. and V.P.), its projections are shorter than the true length in both the plane and inclinations to *xy* line are greater than true inclinations.
- (c) When a line is parallel to a plane, its projection on that plane will show its true length and true inclination with the other plane.
- (d) When a plane is perpendicular to a reference (principal planes H.P. and V.P.) plane, its projection on that plane is a straight line.
- (e) When a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.
- (f) When a plane is perpendicular to one of the reference plane and inclined to the other, its inclination is shown by the angle which its projection on the plane to which it is perpendicular, makes with *xy* line. Its projection on the plane to which it is inclined, is smaller than the plane it self.

- (iii) Remember that each line in one view, represents a plane in another view depending upon the position of the plane with reference to the plane of projection. When a line is observed in any two views, it will be plane in third view.

Study carefully following problems of missing lines or missing views.

Problem 20-1. In fig. 20-3(i) two views are given. Draw its missing view and show missing lines.

See fig 20-3(ii).

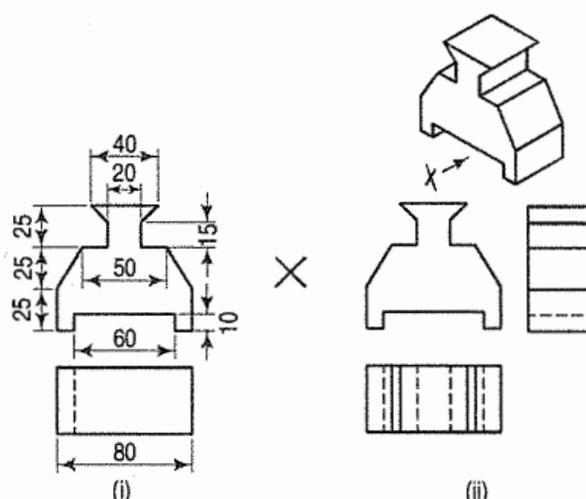


FIG. 20-3

Problem 20-2. In fig. 20-4(i) two views are given.
Draw its pictorial view and missing view.

See fig 20-4(ii).

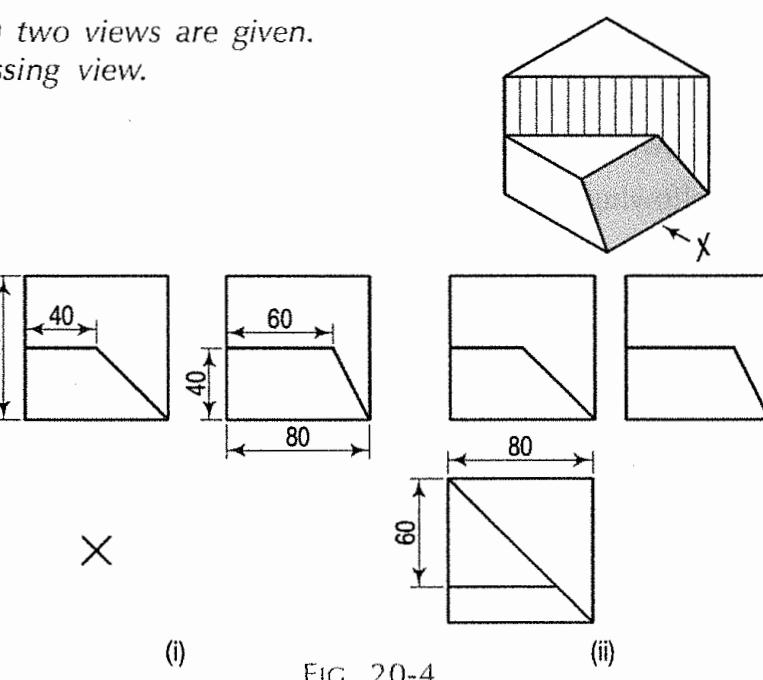


FIG. 20-4

Problem 20-3. In fig. 20-5(i) two views are given. Draw its missing view and show missing lines alongwith its pictorial view.

See fig 20-5(ii)

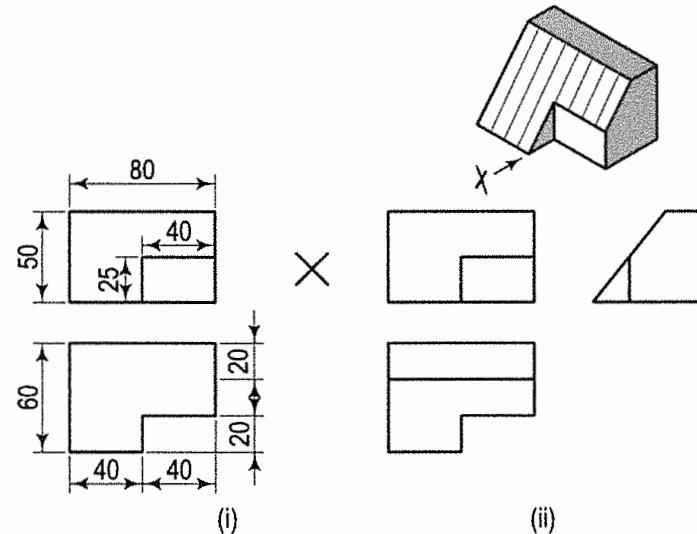


FIG. 20-5

Problem 20-4. In fig. 20-6(i) two views are given. Draw its pictorial view and missing view.

See fig 20-6(ii).

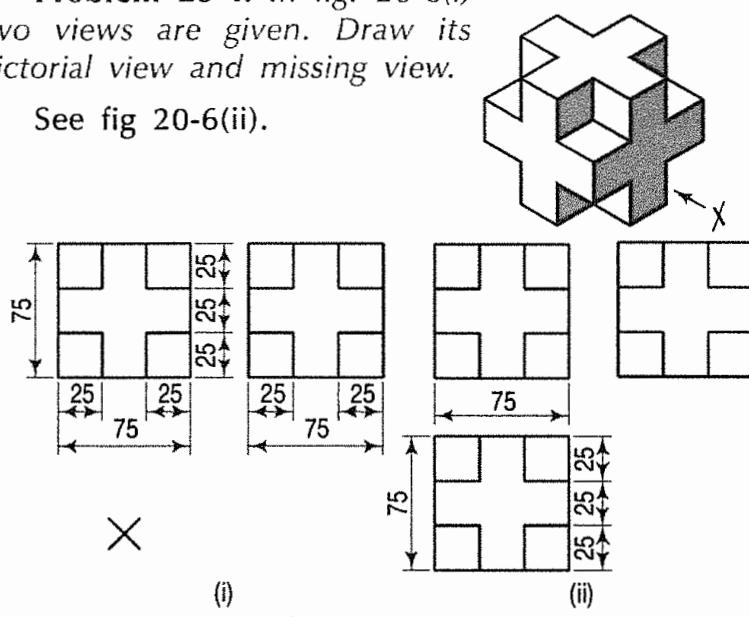


FIG. 20-6

20-4. IDENTIFICATION OF PLANES



Problem 20-5. Two views (front view and top view) are given in fig. 20-7(i). Draw third view (side view) and identify each plane.

See fig. 20-7(ii) along with pictorial view. Read each line and the plane in different views.

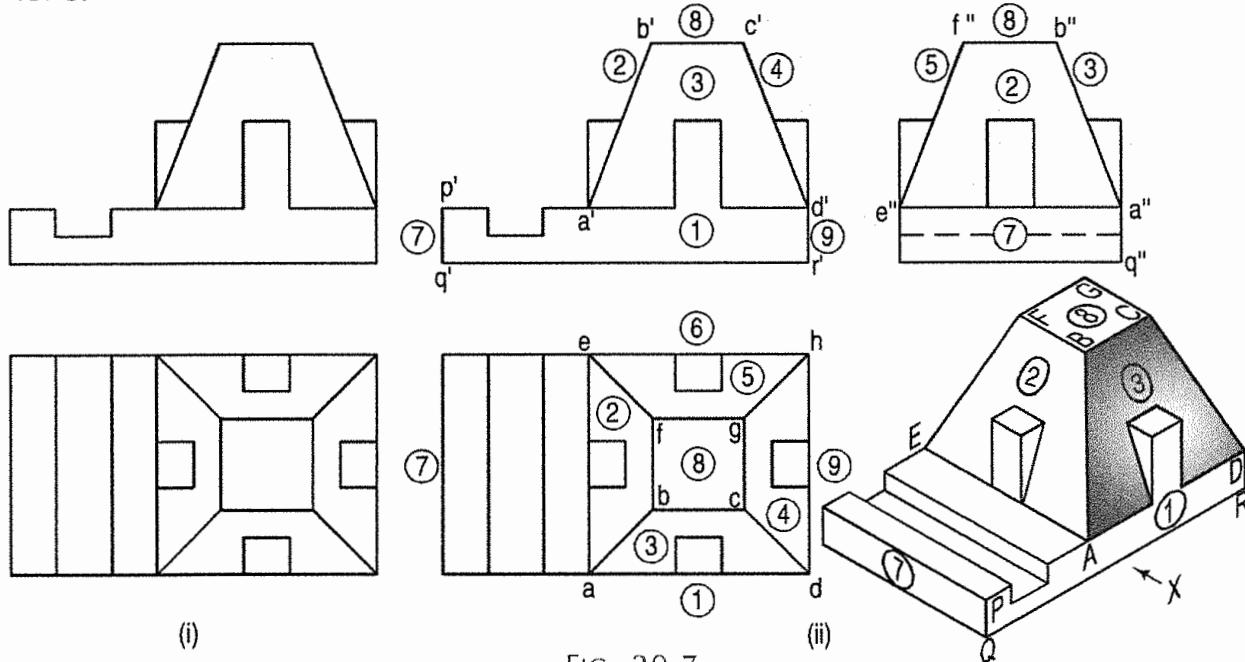


FIG. 20-7

Problem 20-6. Two views of a casting are shown in fig. 20-8. Using both first angle projection method and third angle projection method, draw the given views and add following views

- (i) the side view looking from the right and
- (ii) a sectional side view, section A-A.

Step 1: The shape of the casting may be visualized by imagining it to be broken up in three components *B*, *C* and *D*.

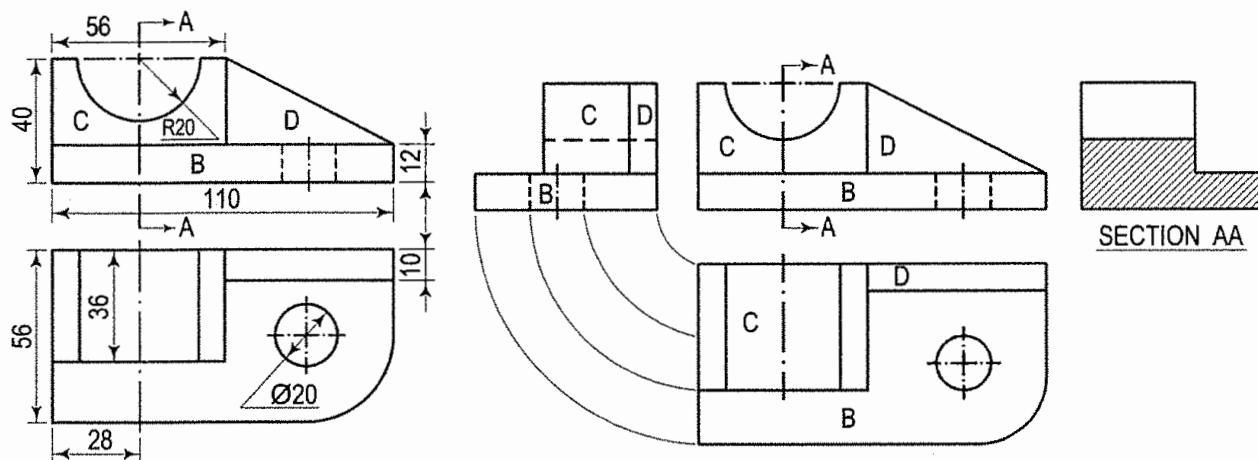


FIG. 20-8

FIG. 20-9

Step 2: After referring to both the views, it can be visualized that the part *B* is in the form of a rectangular block with one corner rounded, and having a circular hole.

The component C has a semi-circular cavity at its top, while the part D is a wedge-shaped piece of uniform thickness. The side view of each part may be imagined separately and then projected from the two views as shown in fig. 20-9.

Step 3: The sectional side view is obtained by imagining the block to be cut in two parts through the line A-A. The portion that is cut is shown in section by means of section lines. Note that the curved surface of the cavity behind the cutting-plane line is also shown as a rectangle in the sectional view. Dashed lines for hidden features may not be shown in the sectional view.

Fig. 20-10 shows the four views of the casting drawn according to the third-angle projection method. There is no change in shape or size of the views. Only their positions are changed.

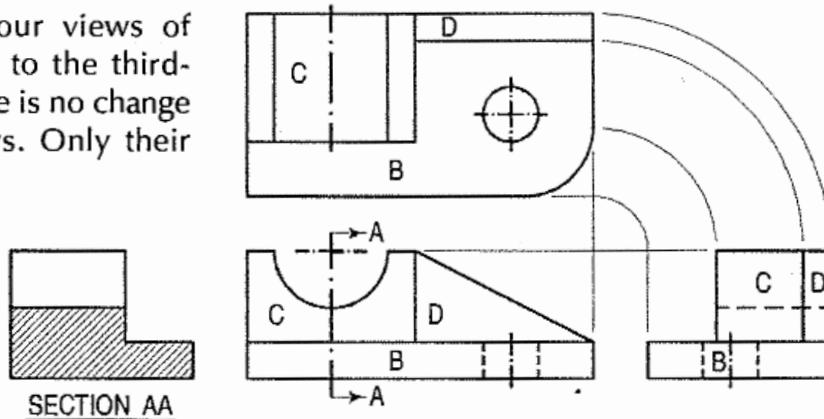


FIG. 20-10

Problem 20-7. Two views of a shaft support are given in fig. 20-11. Draw, scale full size,

- Sectional front view on A-A.
- Sectional top view on B-B.
- Side view from the left.

The object is composed of two cylinders and two ribs. Follow step 1, step 2 and step 3 of problem 20-6.

See fig. 20-12. Note that, although the cutting-plane line A-A passes through the rib, it is not shown cut in the front view according to the convention.

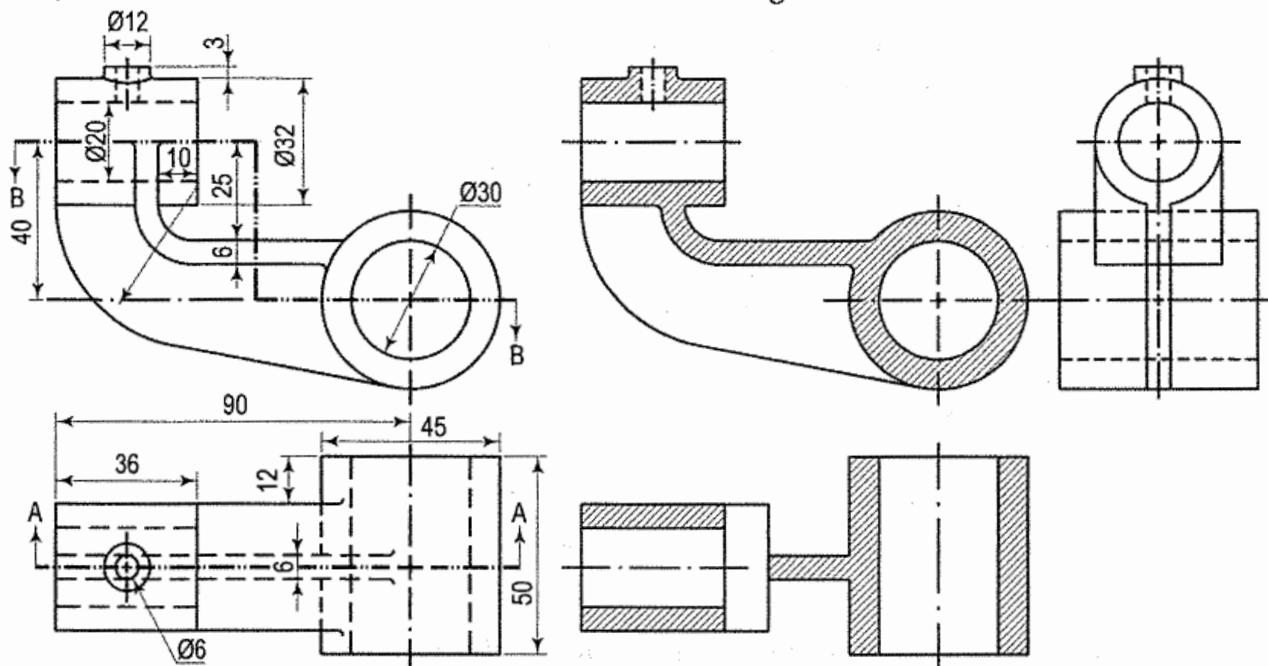


FIG. 20-11

FIG. 20-12

FIG. 20-12

20-5. CONVERSION OF PICTORIAL VIEWS INTO ORTHOGRAPHIC VIEWS



Having grasped the principles of orthographic and isometric projection, we proceed to deal with the application of the same to conversion of pictorial views of objects, for preparing the orthographic views.

Conversion of a pictorial view into orthographic views requires sound knowledge of the principles of pictorial projection and some imagination. A pictorial view may have been drawn according to the principles of isometric or oblique projection. In either case, it shows the object as it appears to the eye from one direction only. It does not show the real shapes of its surfaces or the contour. Hidden parts and constructional details are also not clearly shown. All these have to be imagined.

For converting a pictorial view of an object into orthographic views, the direction from which the object is to be viewed for its front view is generally indicated by means of an arrow. When this is not done, the arrow may be assumed to be parallel to a sloping axis. Other views are obtained by looking in directions parallel to each of the other two axes and placed in correct relationship with the front view.

When looking at the object in the direction of any one of the three axes, only two of the three overall dimensions (viz. length, height and depth or thickness) will be visible. Dimensions which are parallel to the direction of vision will not be seen. *Lines which are parallel to the direction of vision will be seen as points, while surfaces which are parallel to it will be seen as lines.*

While studying a pictorial view, it should be remembered that, *unless otherwise specified:*

- (i) A hidden part of a symmetrical object should be assumed to be similar to the corresponding visible part. For example, in fig. 20-49, the back rib is assumed to be similar as the front rib.
- (ii) All holes, grooves etc. should be assumed to be drilled or cut right through. Refer fig. 20-18 and fig. 20-19.
- (iii) Suitable radii should be assumed for small curves of fillets etc.

An object in its pictorial view may sometimes be shown with a portion cut and removed, to clarify some internal constructional details. While preparing its orthographic views, such object should be assumed to be whole, and the views should then be drawn as required.

20-6. ORTHOGRAPHIC PROJECTION



- (i) It is comparatively easy to prepare a drawing from an actual object. The object is carefully examined and then placed in a suitable position for the front view. The front view of the object is selected in such a way that the maximum details of the object are visible so that the minimum dotted lines are required.
- (ii) All the necessary views are then sketched freehand in a sketch book or on a pad.

- (iii) Measurements of all its details and overall sizes are taken and inserted in the views, along with important notes and instructions.
- (iv) Finally, a scale-drawing is prepared from these sketches.

A pictorial view of a rectangular plate is given in fig. 20-13(i). Its front view when seen in the direction of the arrow X, side view from the left, i.e. in the direction of the arrow Y and the top view in the direction of arrow Z, are shown in fig. 20-13(ii).

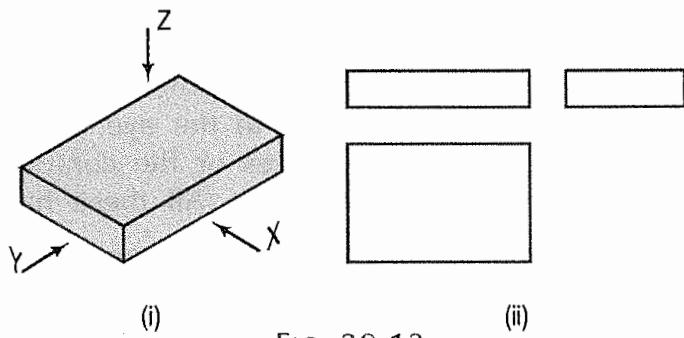


FIG. 20-13

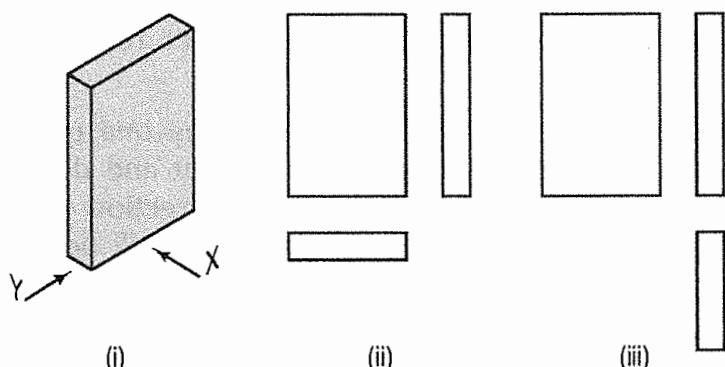


FIG. 20-14

The same plate is shown in fig. 20-14(i) with its longer edges vertical. Its front view looking in the direction of the arrow X, side view from the left and the top view are shown in fig. 20-14(ii). These three views are similar in shape and size to the views shown in fig. 20-13(ii). Only their positions and conditions have changed. Even when the front view is drawn looking in the direction of the arrow Y [fig. 20-14(iii)], the three views remain similar in shape and size.

The same plate is shown cut in various shapes in fig. 20-15. The front view and the side view in each case will be the same as in fig. 20-13(ii). The changed shapes will be seen in the top view of each plate. If these plates are kept in the position shown in fig. 20-14(i), the front view in each case (looking in direction of the arrow X) will show a different shape, while the side view and the top view will be rectangles.

A plate cut in three different ways is shown in fig. 20-16. The front view in each case is the same, viz. a rectangle with a vertical line ab for the edge AB. The side view from the right in each case will be a rectangle with a vertical line cd for the edge CD. The top view in each case shows the shape of the cut.

In the front view of the plate having different shapes of grooves (fig. 20-17), two vertical lines are drawn for the edges of rectangular as well as semi-circular grooves.

In case of the triangular groove, three vertical lines are required. Although edges *AB* and *CD* are cut, they are seen as continuous lines *ab* and *cd*. In the top view, shapes of the grooves are seen. The grooves are not visible in the side view and hence, they are shown by a hidden line.

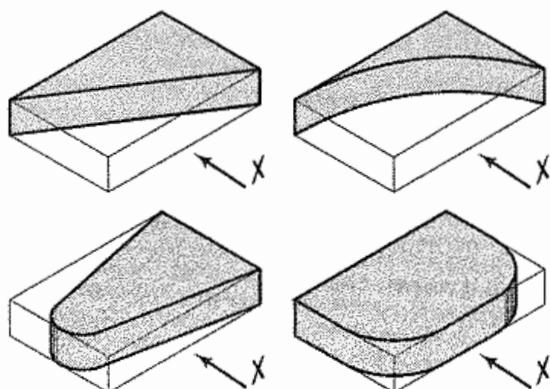


FIG. 20-15

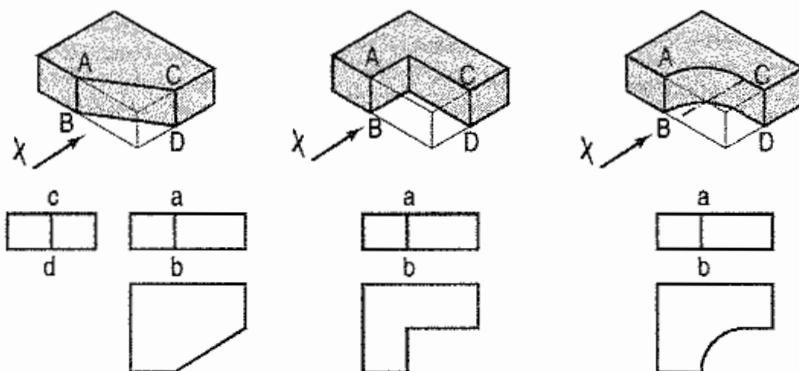


FIG. 20-16

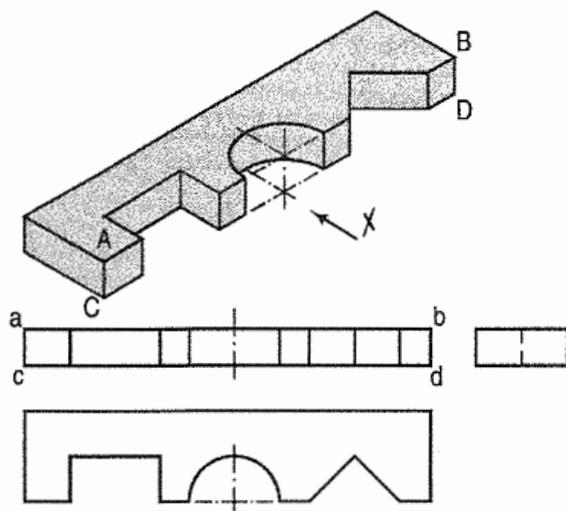


FIG. 20-17

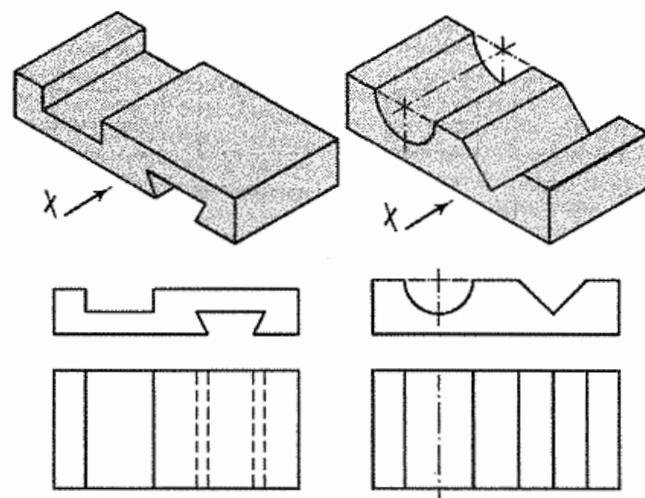


FIG. 20-18

Two plates having grooves in upper and lower surfaces are shown in fig. 20-18. The shapes of the grooves are seen in the front views. In top views, two lines for

the edges of rectangular as well as semi-circular grooves are drawn. For the triangular grooves, three lines are required. The tapered groove in the bottom surface is assumed to be cut throughout the width of the plate. It is not visible from above and hence, its four edges are shown by four hidden lines in the top view. In their side views, each groove should be shown by a horizontal hidden line. Therefore the side view in this case is not necessary.

A plate having holes of different shapes is shown in fig. 20-19. These shapes are seen in the top view. Holes are assumed to be cut or drilled right through. They are shown by hidden lines in the front and the side views. Here also the right side view is not necessary.

The semi-circular groove in the block shown in fig. 20-20 does not extend upto the opposite surface. Therefore, although the edge AB is broken, ab is seen as a continuous line in the front view. The groove is shown by hidden lines in the side view. In the top view it is drawn as a rectangle.

Fig. 20-21 shows a block in the shape of steps, along with its three views. In fig. 20-22, its face ABCD is inclined. Hence, in the top view, two lines ab and cd are drawn for that face.

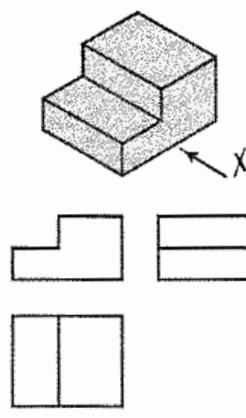


FIG. 20-21

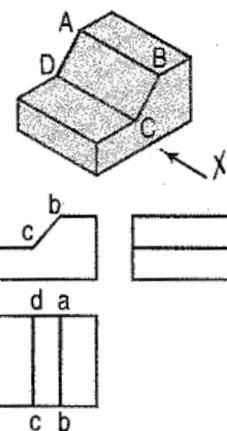


FIG. 20-22

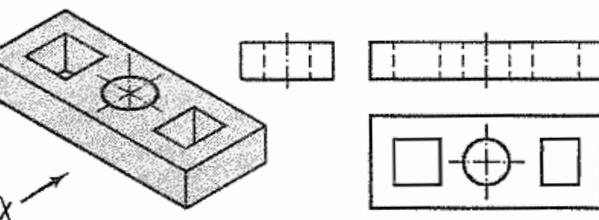


FIG. 20-19

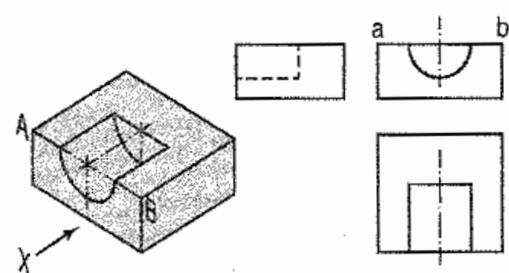


FIG. 20-20

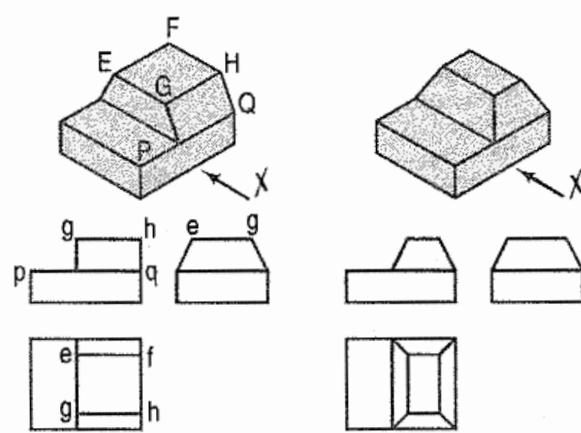


FIG. 20-23

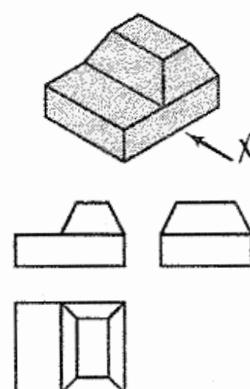


FIG. 20-24

The front and back faces of the upper step are shown inclined in fig. 20-23. Their inclinations are shown in the side view. In the front view, the line pq for the edge PQ is drawn. In the top view, lines ef and gh for edges EF and GH are shown.

In fig. 20-24, all the four side-faces of the upper step are inclined. Their inclinations are seen in the front view and in the side view. In the top view, two rectangles are seen for the upper step. Lines (for the four sloping edges) joining the corners of the rectangles should also be shown.

A casting having a hollow cylinder supported by a vertical rib is shown in fig. 20-25. The width of the rib is equal to the diameter of the cylinder. Hence, in the front view, vertical lines for the rib are tangential to the circle for the cylinder.

In the side view, the line showing the thickness of the rib vanishes just at the centre line. The rib is not visible when seen from top. Hence, it is shown by a hidden line in the top view. Note that the third view (top view) is a 'necessary' view to show the shape of rectangular slot.

In the bearing block shown in fig. 20-26, the line for the rib is inclined and tangent to the semi-circle in the front view. Vanishing points for the lines for the rib in the side view and the top view are obtained by projecting the tangent-point from the front view.

From the above discussion, the following important points are to be noted:

- (i) An object can be observed from six sides such as
 - (a) front side
 - (b) back side
 - (c) left-hand side
 - (d) right-hand side
 - (e) top side and
 - (f) bottom side.

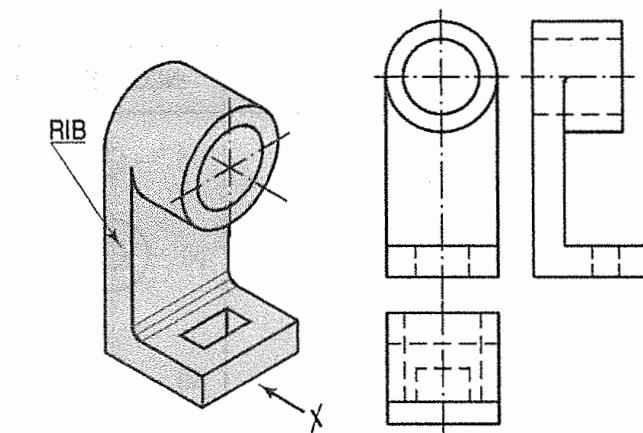


FIG. 20-25

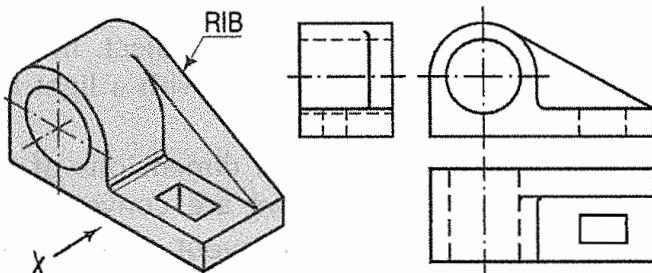


FIG. 20-26

It is not necessary to draw all the possible six views to describe completely the shape of the object. In practice, only those views which are *necessary to describe the shape of the object should be drawn*.

The view should be selected in such a way that minimum dotted lines are necessary to show internal details. The object which has both the right-hand side and left-hand side symmetrical shape requires only two views.

One of the views must be such that it completely describes the shape of the object. Refer fig. 20-16, fig. 20-32, fig. 20-47 and fig. 20-48.

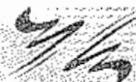
- (ii) Three views are necessary for the object which is not symmetrical. Refer fig. 20-22 and fig. 20-24.
- (iii) The view should not be drawn out of place. That is the side view and top view are not aligned with the front view. This should be avoided.
- (iv) Dotted lines must be drawn correctly as shown in table 20-1. Incorrect points are indicated by circle.

TABLE 20-1

INCORRECT	CORRECT	INCORRECT	CORRECT	INCORRECT	CORRECT
(i)		(iv)		(vii)	
(ii)		(v)		(viii)	
(iii)		(vi)		(ix)	

- (v) (a) When a visible line coincides with either a dotted line or a centre line, *the visible line is shown* and a centre line is extended beyond the outlines of the view.
 (b) When a section-plane line coincides with a centre line, the *centre line is shown* and the section-plane line is drawn outside the out-lines of the object at the ends of centre line by thick dashes.
 (c) When a dotted line coincides with the centre line, *the dotted line should be shown.*

20-7. PROCEDURE FOR PREPARING A SCALE-DRAWING



A scale-drawing must always be prepared from freehand sketches initially prepared from a pictorial view or a real object. In the initial stages of a drawing, always use a soft pencil viz. HB, and work with a light hand, so that lines are thin, faint and easy to erase, if necessary.

1. Determine overall dimensions of the required views. Select a suitable scale so that the views are conveniently accommodated in the drawing sheet.
2. Prepare the sheet layout as described in illustrative problem 20-8. Draw rectangles for the views, keeping sufficient space between them and from the borders of the sheet.
3. Draw centre lines in all the views. When a cylindrical part or a hole is seen as a rectangle, draw only one centre line for its axis. When it is seen as a circle, draw two centre lines intersecting each other at right angles at its centre.
4. Draw details simultaneously in all the views in the following order:
 - (i) Circles and arcs of circles.
 - (ii) Straight lines for the general shape of the subject.
 - (iii) Straight lines, small curves etc. for minor details.
5. After the views have been completed in all the details, erase all unnecessary lines completely. Make the outlines so faint that only their impressions exist.
6. Fair the views with 2H or 3H pencil, making the outlines uniform and intensely black (but not too thick). For doing this, adopt the same working order as stated in step 4 above.
7. Dimension the views completely. Keep all centre lines.
8. Draw section lines in the view or views which are shown in section.
9. Fill up the title block and furnish all other necessary particulars.
10. Check the drawing carefully and see that it is complete in all respects.

20-8. ILLUSTRATIVE PROBLEMS



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 49 for the following problem.

Problem 20-8 [fig. 20-27(i)]. A pictorial view of bearing block is shown in fig. 20-27(i). Draw the front view, left-hand side view and top view according to the First-Angle projection method.

The procedure of preparing the orthographic views is illustrated in the following steps:

Step 1 [fig. 20-27(ii)]:

- Take a half imperial size drawing paper of 560 mm × 380 mm.
- Draw the border lines taking $A = 30$ mm and $B = 10$ mm as shown in fig. 20-27(ii).

Now clear space in the drawing paper is 520 mm × 360 mm.

- The scale of drawing is decided from the size of object and number of views required to draw. Let L , W and H be the length, width and height of the object. The spacing between two views can be calculated as under:

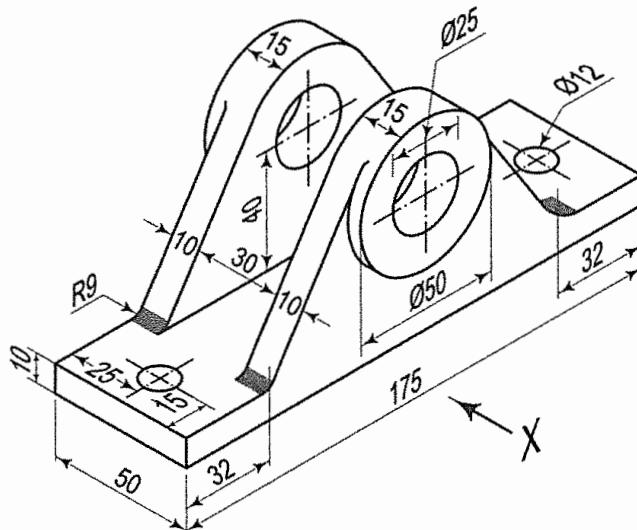


FIG. 20-27(i)

Views	Horizontal distance (E)	Vertical distance (F)
(a) For four views (front view, top view and two side views):	$E = \frac{520 - (L + 2W)}{4}$ mm	$F = \frac{360 - (H + W)}{3}$ mm
(b) For three views (front view, top view and side view):	$E = \frac{520 - (L + W)}{3}$ mm	$F = \frac{360 - (H + W)}{3}$ mm

If these distances E and F are less than 20 mm, adopt the suitable standard scale. In the above illustrated problem E and F are 95 mm and 75 mm respectively.

Select suitable scale approximately and draw required blocks depending upon the number of views (three in this problem).

Step 2 [fig. 20-27(iii)]:

- Mark centres for circles.
- Draw centre lines passing through these centres and extend in other views also.

Step 3 [fig. 20-27(iv)]:

- Draw circle of required diameter in the front view as well as in the top view.
- Draw the other lines in the front view and project them in the side view as well as in the top view.
- Erase construction lines. Refer fig. 20-27(iv).

Step 4 [fig. 20-27(v)]:

- (i) Clean the drawing paper. Fair first circles. Complete the views by fairing various lines. The thickness of different types of lines should be as suggested in fig. 3-1, table 3-2 and table 3-3. The lines should be clean, dense and uniform.

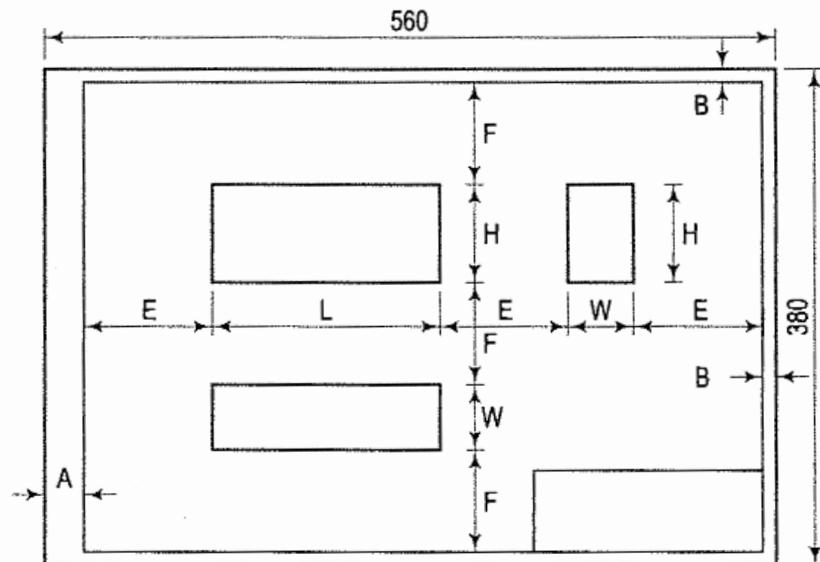


FIG. 20-27(ii)

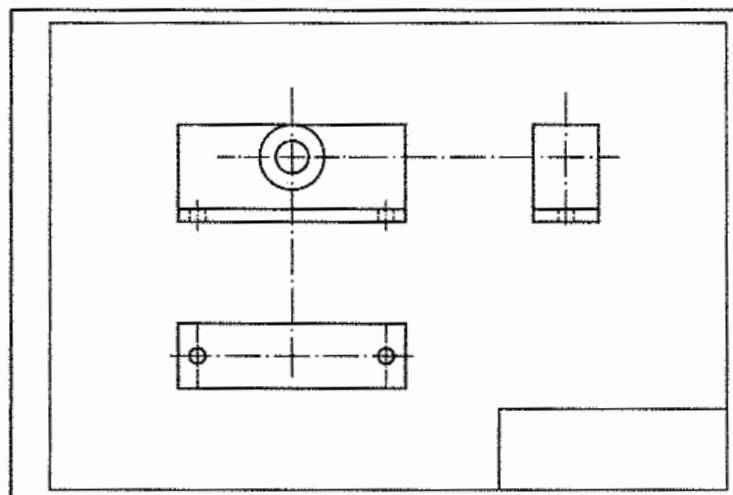


FIG. 20-27(iii)

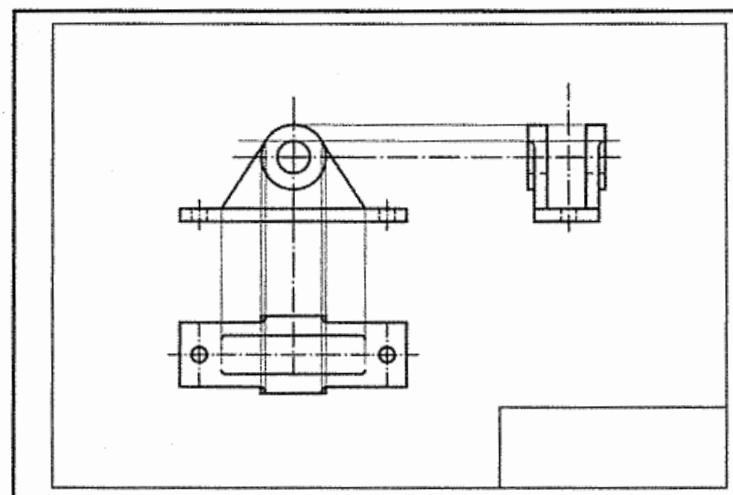


FIG. 20-27(iv)

- (ii) Draw extension lines.
- (iii) Draw dimension lines. Insert dimensions. Observe that the height of numerals should not be more than 3 mm. It should be written freehand. Aligned method of writing dimensions is recommended. Follow IS:11669 (1986) for dimensioning.
- (iv) Complete title block by specifying unit of dimensions, symbol for method of projection and scale adopted.
- (v) Fair boundary lines by 2H pencil.

Problem 20-9. Draw the front view and a side view of the wedge-shaped piece shown in pictorial view in fig. 20-28(i).

The edge AB is parallel to the isometric axis oz and hence, it will be seen as a horizontal line in the front view [fig. 20-28(ii)]. The edge CD will be seen as an inclined line.

Problem 20-10. Draw the front view and a side view of the wedge-shaped piece shown pictorially in fig. 20-29(i).

It can be seen from the shape of the circular hole that the pictorial view is drawn according to *oblique projection*. Hence, the front view [fig. 20-29(ii)] will be similar to the front face shown in the pictorial view.

It is interesting to note that the two pictorial views [figs. 20-28(i) and 20-29(i)] look very much similar. But from their orthographic views we find that in fig. 20-29, the piece is in an inverted (upside-down) position.

Problem 20-11. Draw the front view and a side view of the wedge-shaped piece shown pictorially in fig. 20-30(i).

See fig. 20-30(ii). Note that the bottom edge is horizontal.

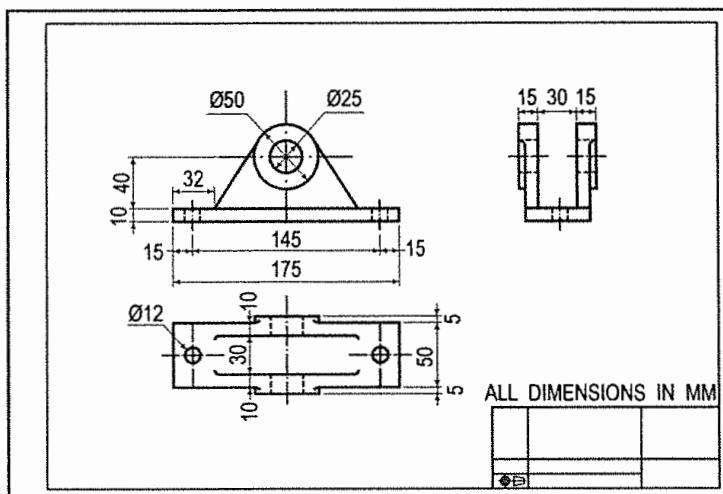


FIG. 20-27(v)

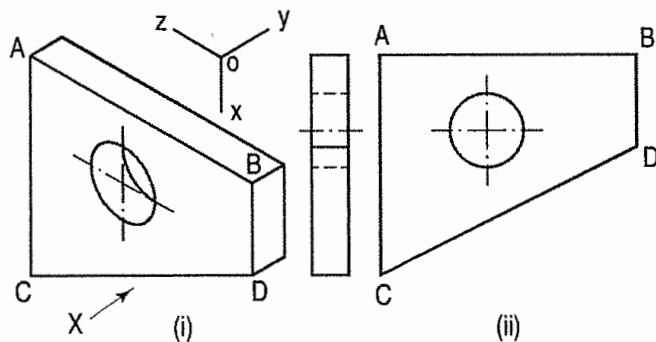


FIG. 20-28

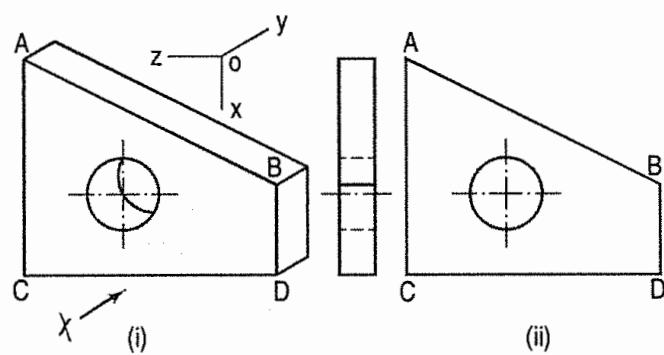


FIG. 20-29

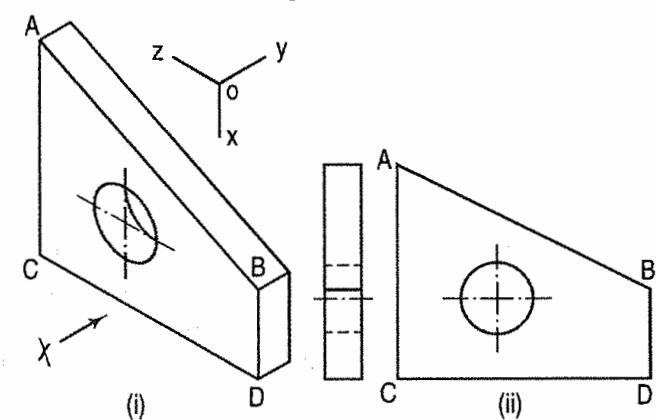


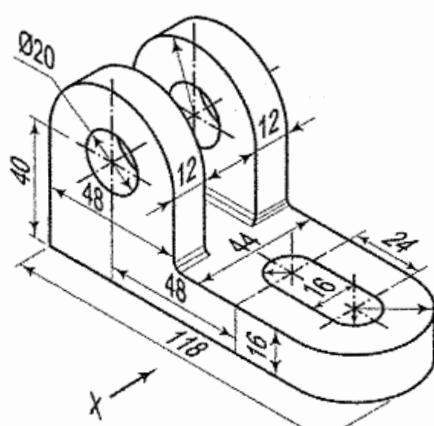
FIG. 20-30

Note: The three axes are shown along with the pictorial views, only for the purpose of explanation of the above problems No. 20-9, 20-10 and 20-11.

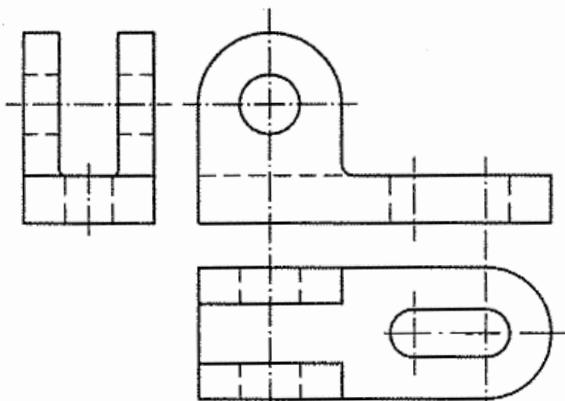
Problem 20-12. Draw the following views of the object shown pictorially in fig. 20-31(i).

- (i) Front view. (ii) Top view. (iii) Side view from the right.

See fig. 20-31(ii).



(i)

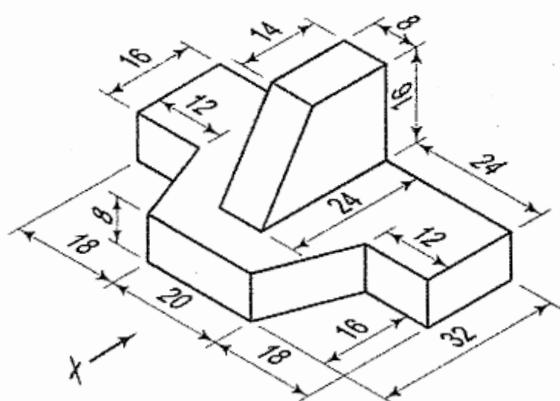


(ii)

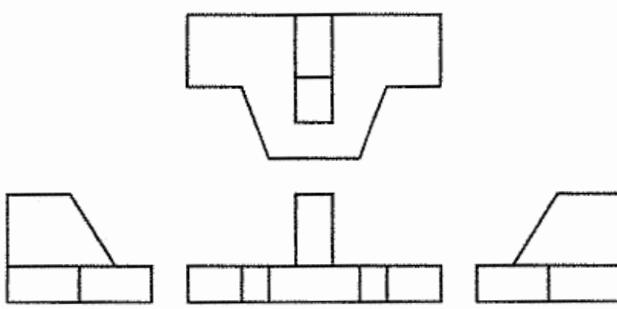
FIG. 20-31

Problem 20-13. Draw the following views of the block shown pictorially in fig. 20-32(i). Use third-angle projection method. (i) Front view. (ii) Top view. (iii) Both side views.

See fig. 20-32(ii).



(i)



(ii)

FIG. 20-32

EXERCISES 20

Pictorial views of objects are shown in fig. 20-33 to fig. 20-70. Draw, scale full size, views of each object as stated below. The front view in each case, should be drawn as seen in the direction of the arrow X. Unless otherwise specified, use first-angle projection method. Insert all dimensions in the views.

Solve each exercise independently and then, compare your answer with the given solution.

1. Fig. 20-33: (i) Front view. (ii) Side view. (iii) Top view.

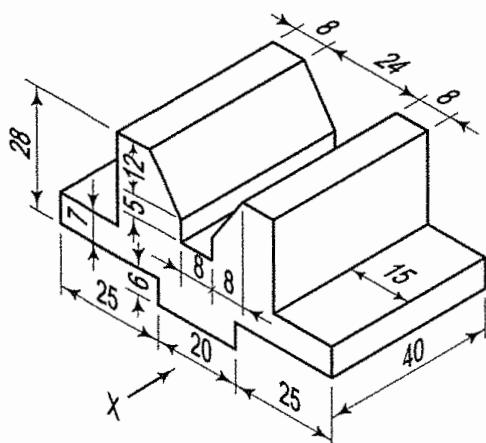


FIG. 20-33

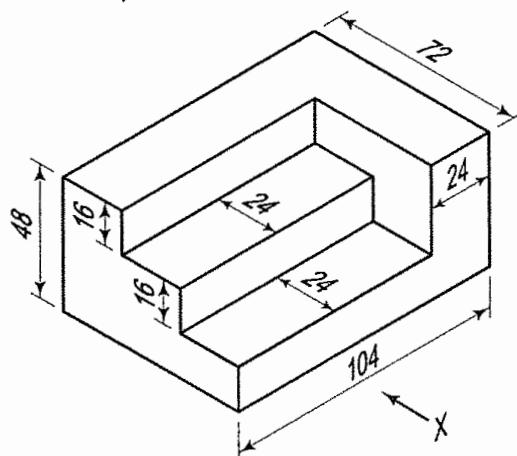


FIG. 20-34

2. Fig. 20-34: (i) Front view. (ii) Side view from the left. (iii) Top view. Use third-angle projection method.

3. Fig. 20-35: (i) Front view. (ii) Side view. (iii) Top view.

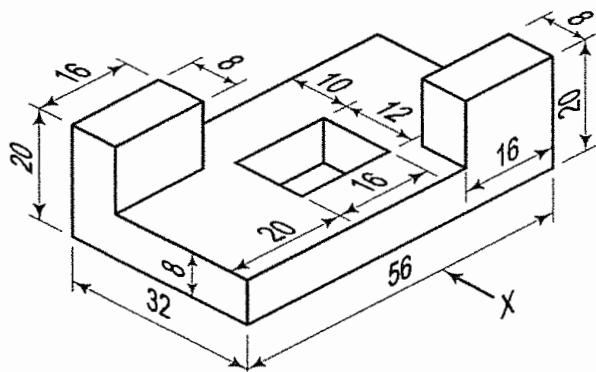


FIG. 20-35

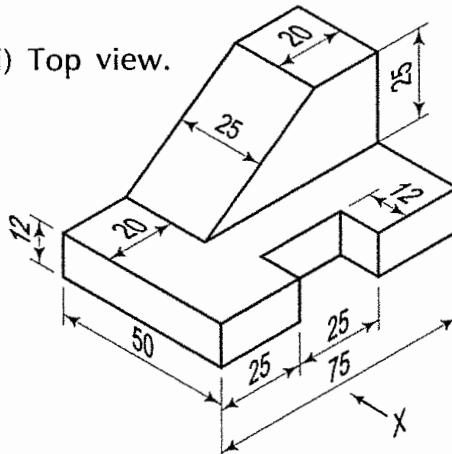


FIG. 20-36

4. Fig. 20-36: (i) Front view. (ii) Side view from the left. (iii) Top view.

5. Fig. 20-37: (i) Front view. (ii) Both side views. (iii) Top view. Use third-angle projection method.

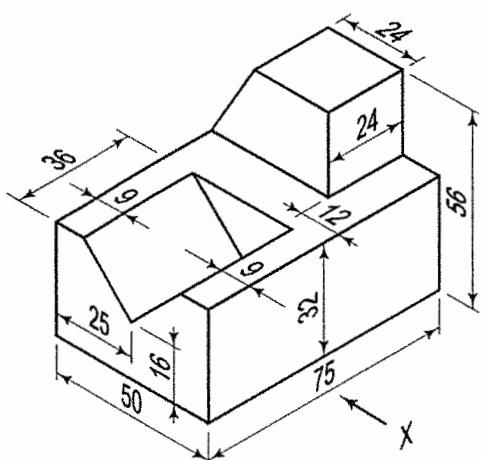


FIG. 20-37

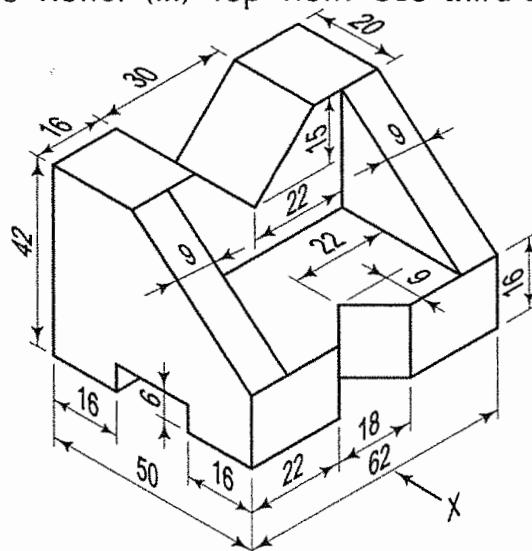


FIG. 20-38

6. Fig. 20-38: (i) Front view. (ii) Side view. (iii) Top view.

7. Fig. 20-39: (i) Front view. (ii) Side view from the left. (iii) Top view.

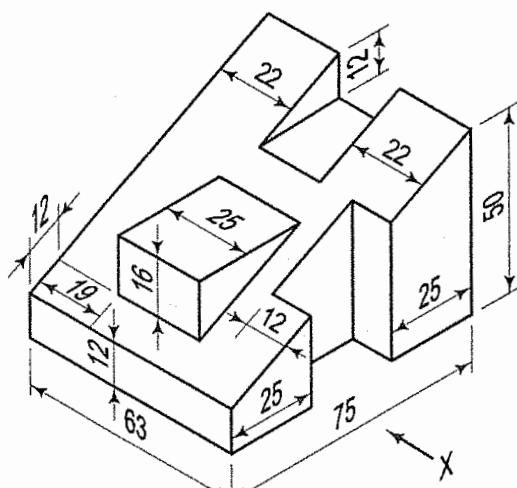


FIG. 20-39

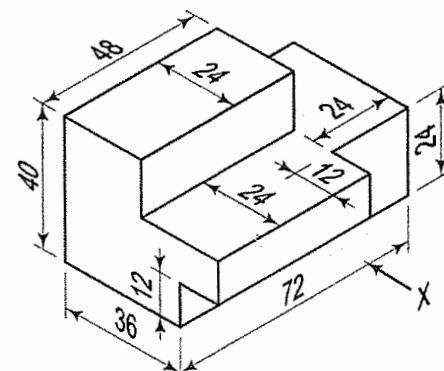


FIG. 20-40

8. Fig. 20-40: (i) Front view. (ii) Both side views. (iii) Top view.

9. Fig. 20-41: (i) Front view. (ii) Both side views. (iii) Top view.

Use third-angle projection method.

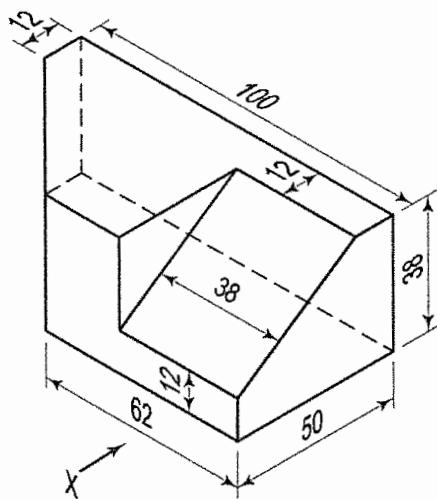


FIG. 20-41

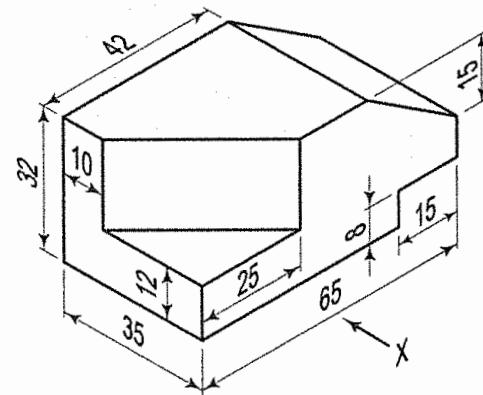


FIG. 20-42

10. Fig. 20-42: (i) Front view. (ii) Both side views. (iii) Top view.

11. Fig. 20-43: (i) Front view. (ii) Both side views. (iii) Top view. Use third-angle projection method.

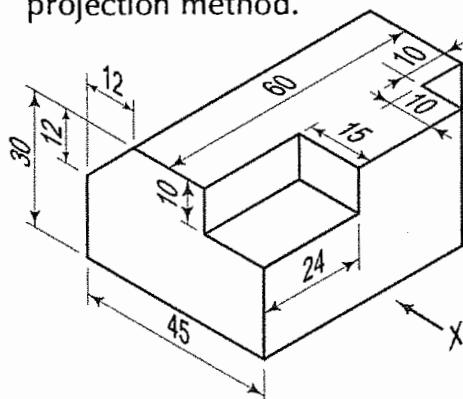


FIG. 20-43

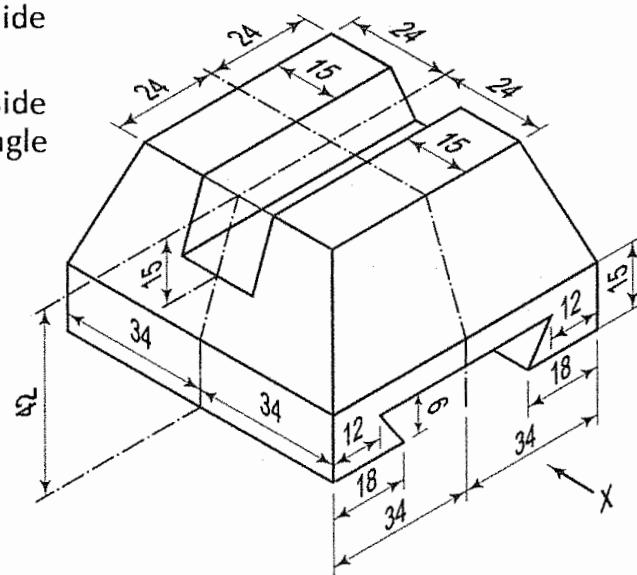


FIG. 20-44

12. Fig. 20-44: (i) Front view. (ii) Side view from left. (iii) Top view.

13. Fig. 20-45:

- (i) Front view.
- (ii) Side view.
- (iii) Top view.

14. Fig. 20-46:

All the six views.
Use third-angle projection method.

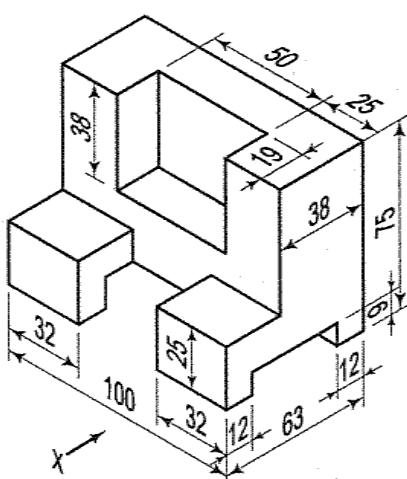


FIG. 20-45

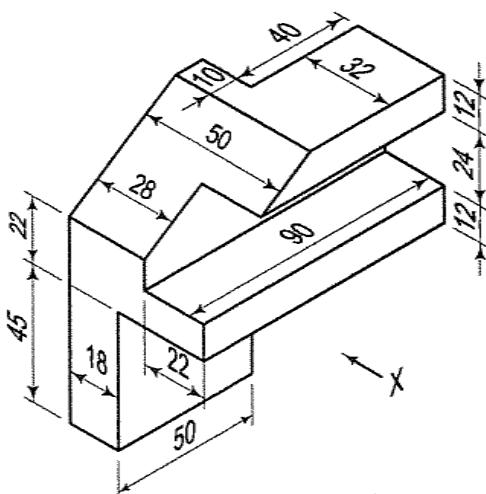


FIG. 20-46

15. Fig. 20-47: (i) Front view. (ii) Side view from the right. (iii) Top view.

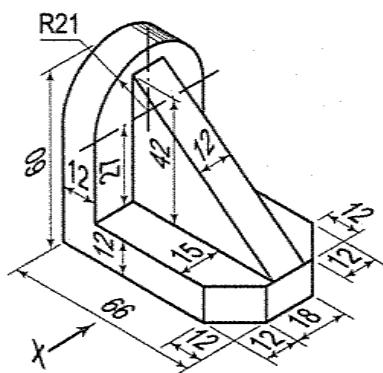


FIG. 20-47

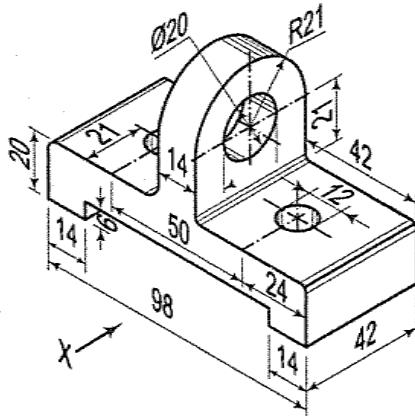


FIG. 20-48

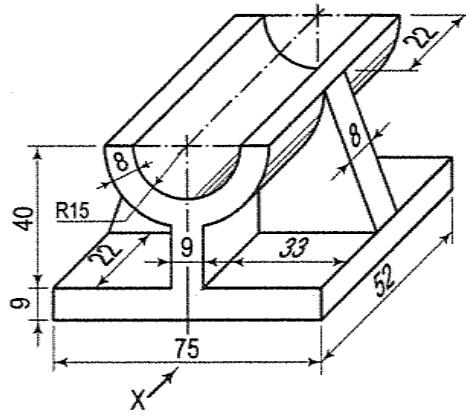


FIG. 20-49

16. Fig. 20-48: (i) Front view. (ii) Side view. (iii) Top view.

17. Fig. 20-49: (i) Front view. (ii) Side view. (iii) Top view.

Use third-angle projection method.

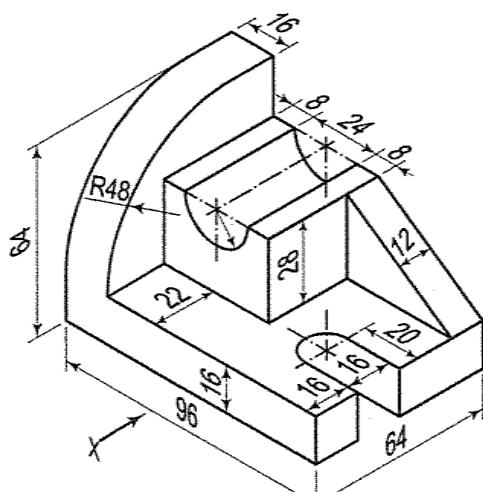


FIG. 20-50

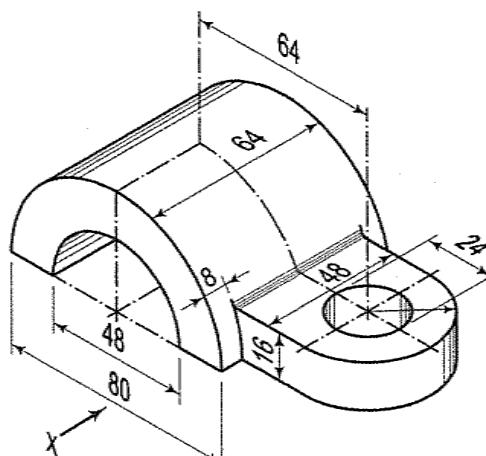


FIG. 20-51

18. and 19. Fig. 20-50 and fig. 20-51:

- (i) Front view. (ii) Side view from the right. (iii) Top view.

20. Fig. 20-52:
(i) Front view.
(ii) Both side views.
(iii) Top view.

21. Fig. 20-53:
(i) Front view.
(ii) Both side views.
(iii) Top view.

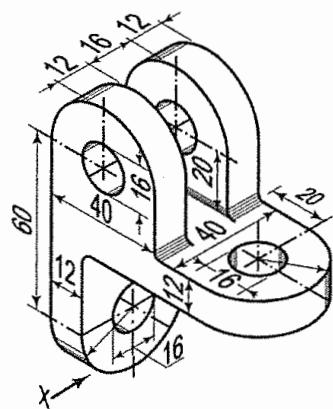


FIG. 20-52

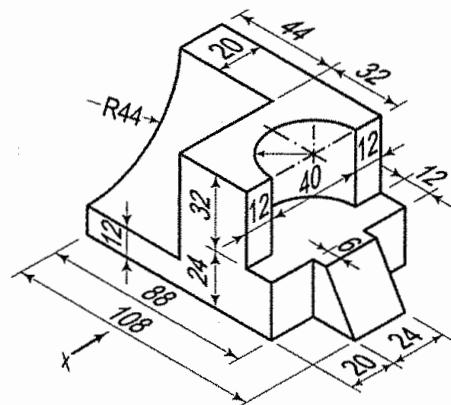


FIG. 20-53

22. Fig. 20-54: (i) Front view. (ii) Both side views. (iii) Top view.

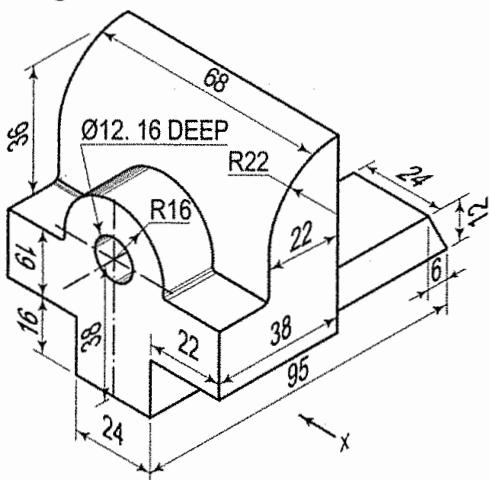


FIG. 20-54

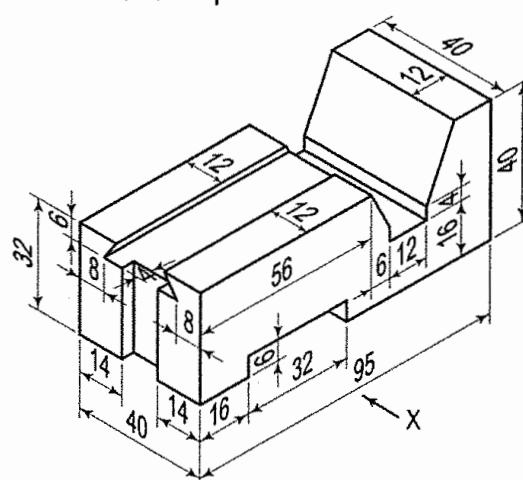


FIG. 20-55

23. Fig. 20-55: (i) Front view. (ii) Side view from the left. (iii) Top view. Use third-angle projection method.

24. Fig. 20-56: All the six views.

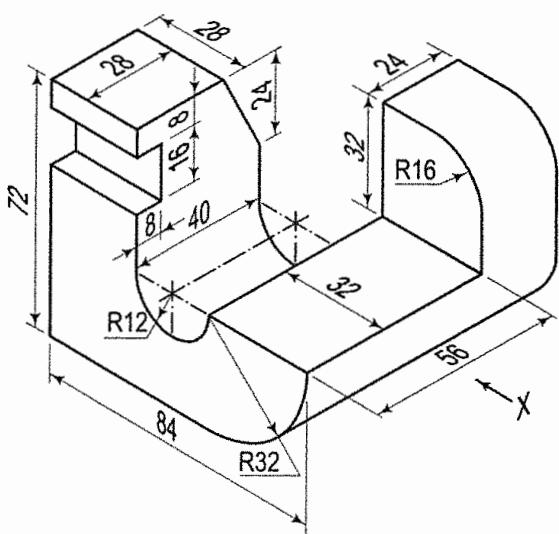


FIG. 20-56

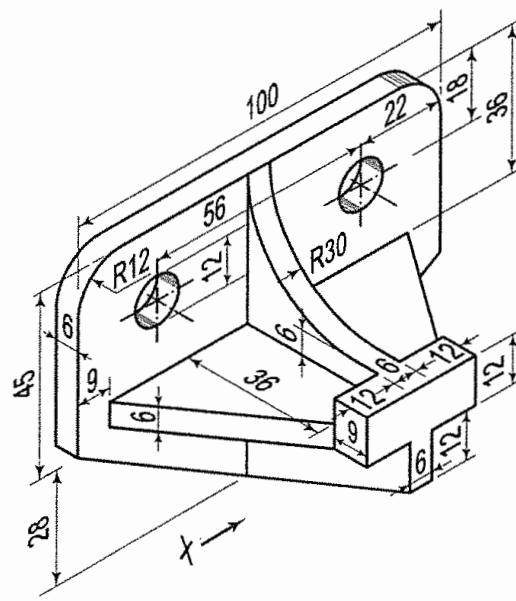


FIG. 20-57

25. Fig. 20-57: (i) Front view. (ii) Side view from the right. (iii) Top view.

26. Fig. 20-58 (i) Front view. (ii) Side view from the left. (iii) Top view.

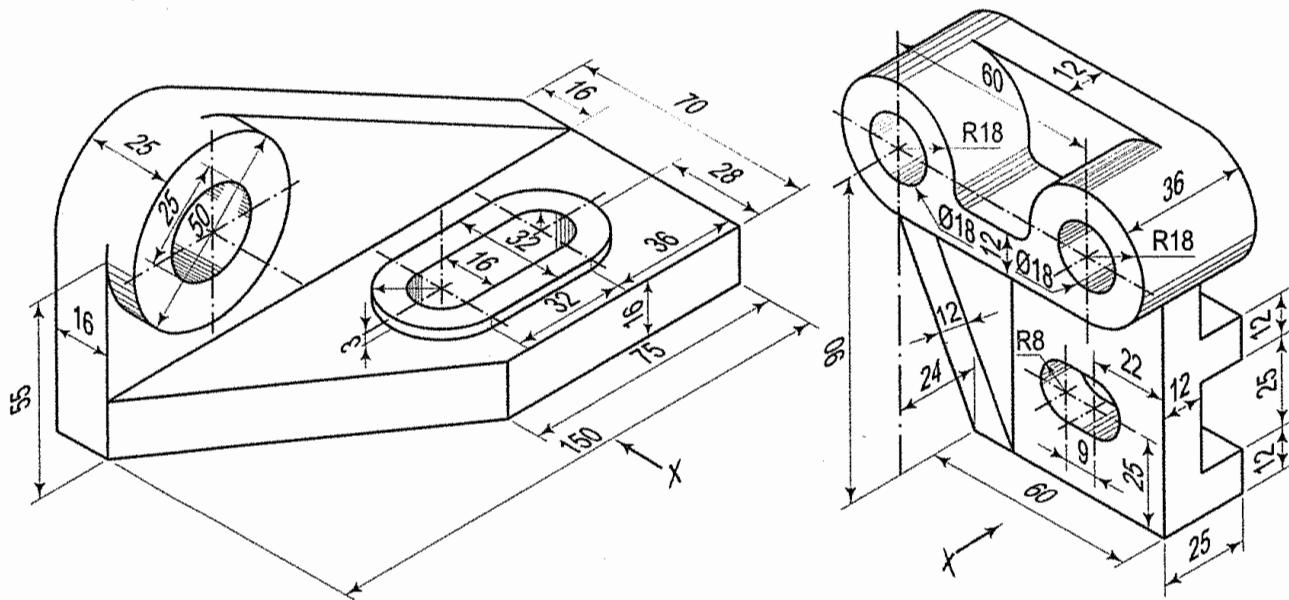


FIG. 20-58

FIG. 20-59

27. Fig. 20-59:

- (i) Front view.
- (ii) Side view from the right.
- (iii) Top view.

Use third-angle projection method.

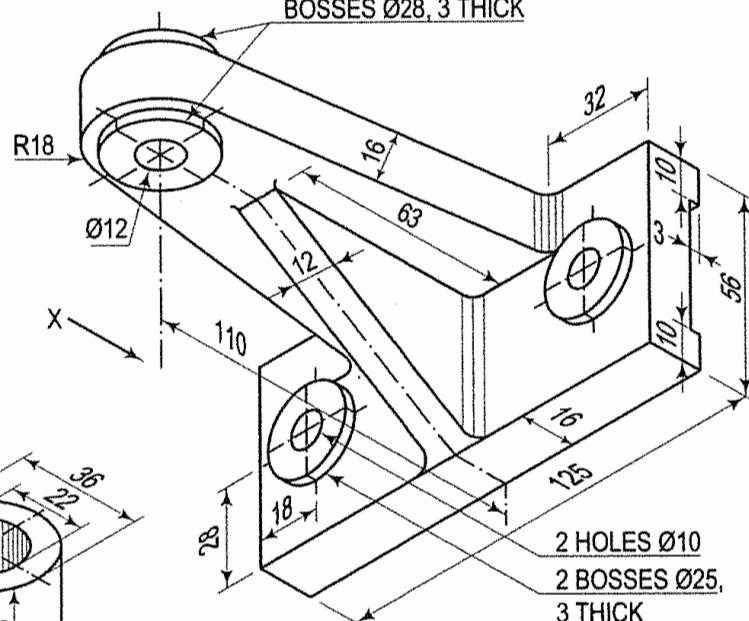


FIG. 20-60

28. Fig. 20-60:

- (i) Front view.
- (ii) Side view.
- (iii) Top view.

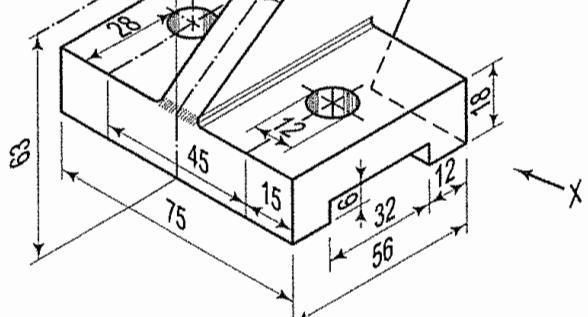


FIG. 20-61

29. Fig. 20-61:

- (i) Front view.
- (ii) Side view from the right.
- (iii) Top view.

30. Fig. 20-62: (i) Front view. (ii) Side view from the right. (iii) Top view.
Use third-angle projection method.

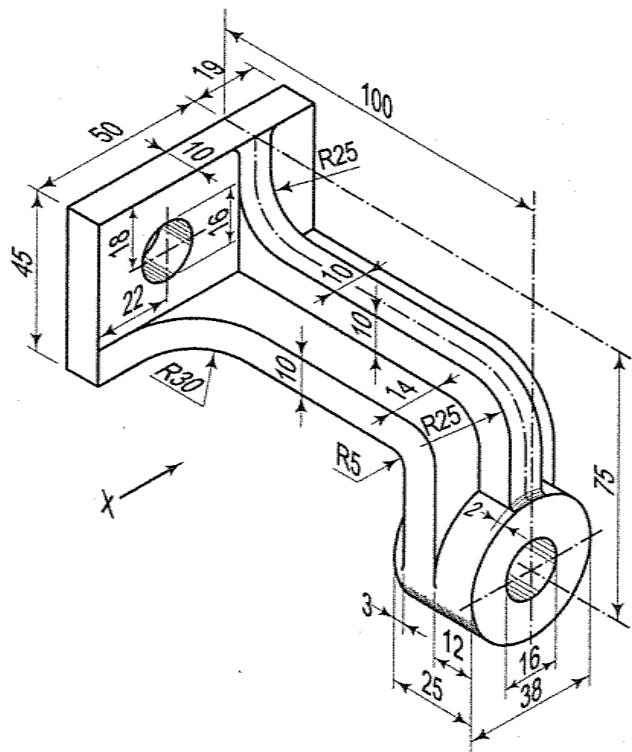


FIG. 20-62

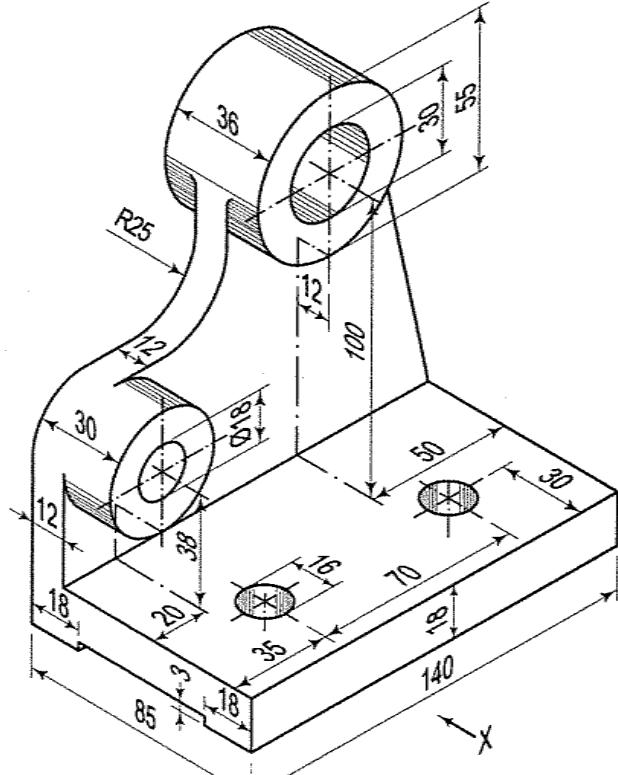


FIG. 20-63

31. Fig. 20-63: (i) Front view. (ii) Side view from the left. (iii) Top view.
32. Fig. 20-64: (i) Front view. (ii) Left-hand side view. (iii) Top view.
Use third-angle projection method.

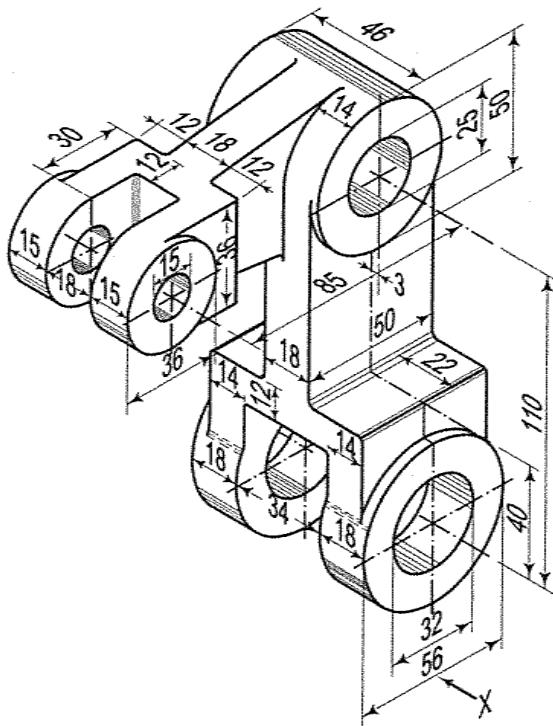


FIG. 20-64

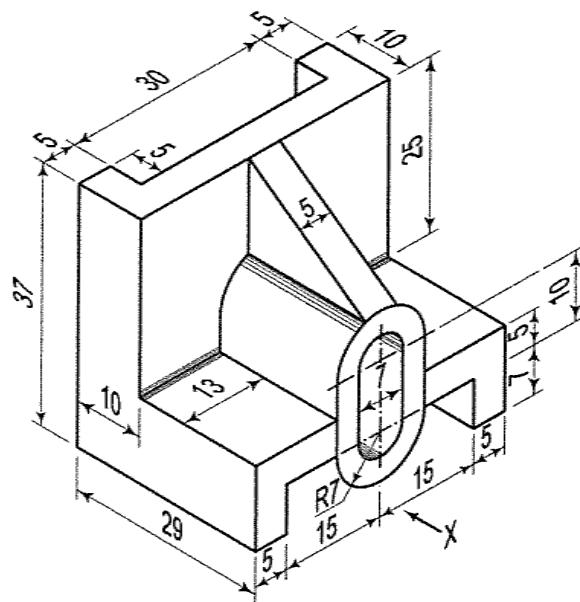


FIG. 20-65

33. Fig. 20-65: (i) Front view. (ii) Left-hand side view. (iii) Top view.

34. Fig. 20-66: (i) Front view. (ii) Right-hand side view. (iii) Top view.

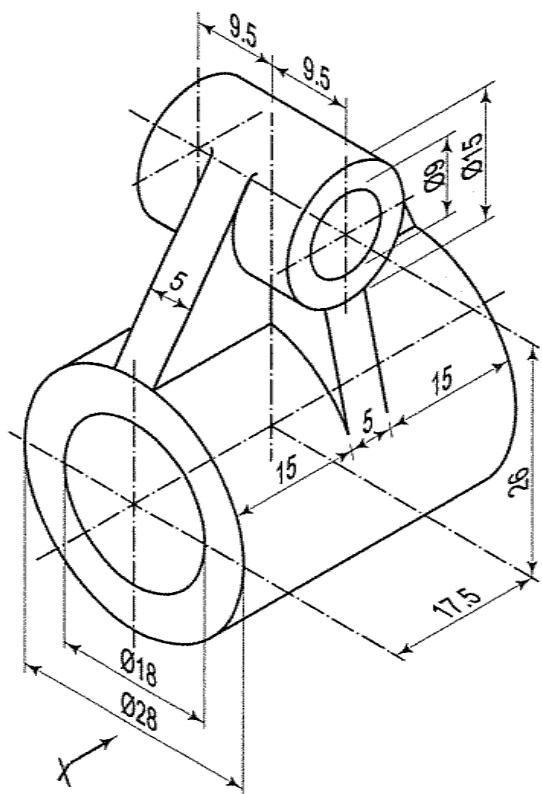


FIG. 20-66

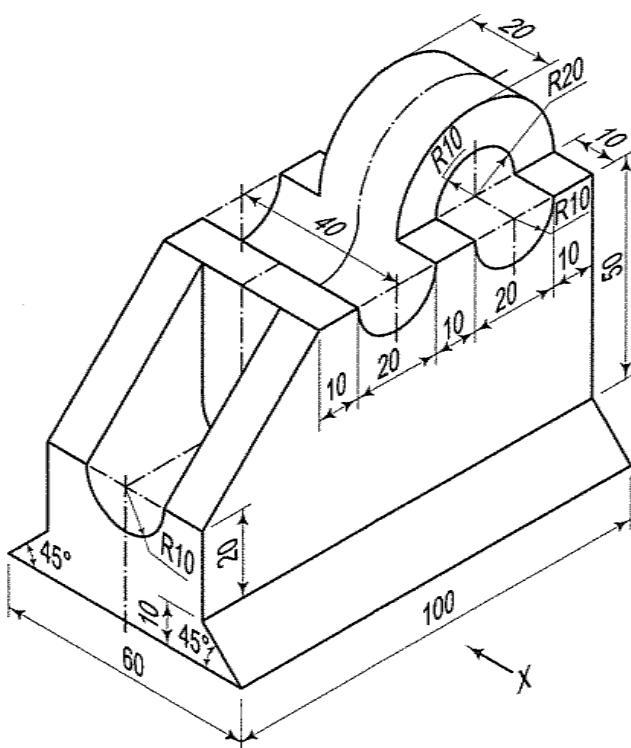


FIG. 20-67

35. Fig. 20-67: (i) Front view. (ii) Left-hand side view. (iii) Top view.
 36. Fig. 20-68: (i) Front view. (ii) Side view. (iii) Top view.

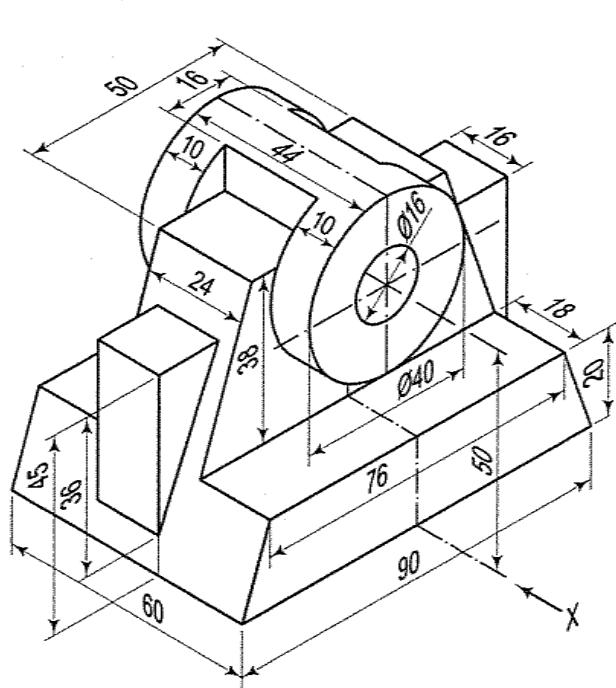


FIG. 20-68

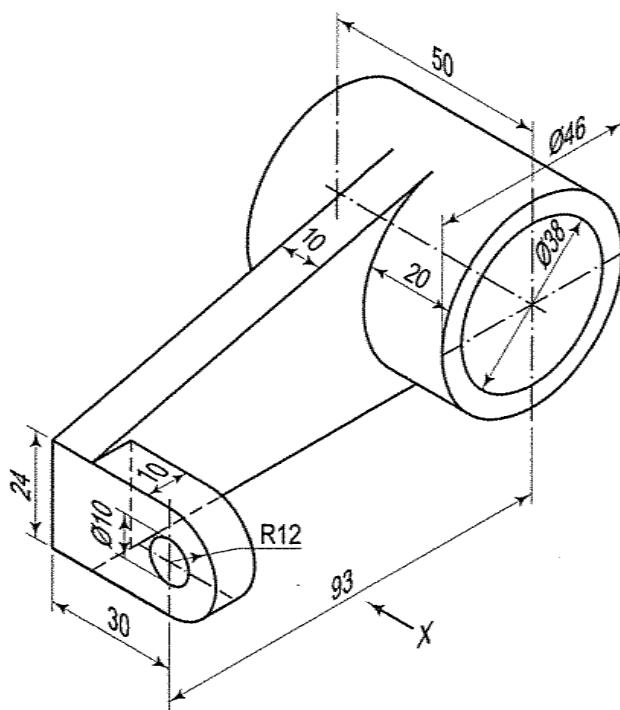


FIG. 20-69

37. Fig. 20-69: (i) Front view. (ii) Left-hand side view. (iii) Top view.

38. Fig. 20-70: (i) Front view. (ii) Left-hand side view. (iii) Top view.

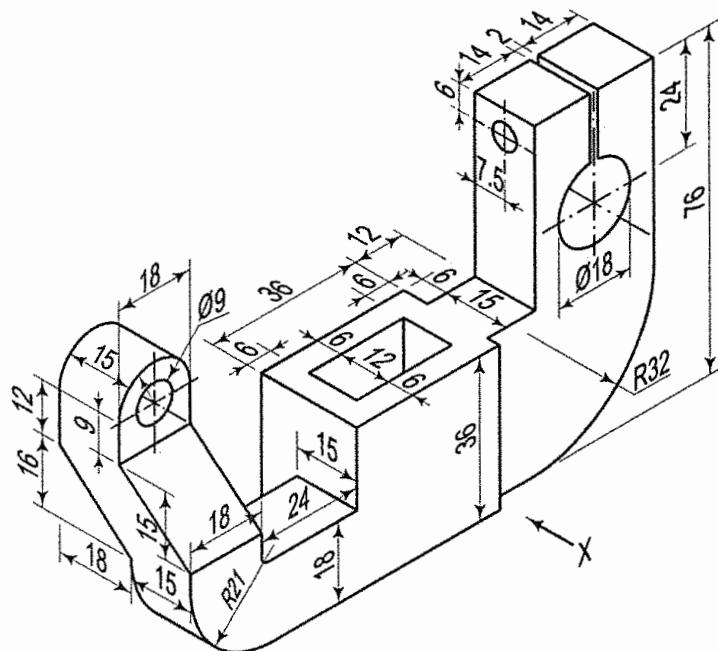
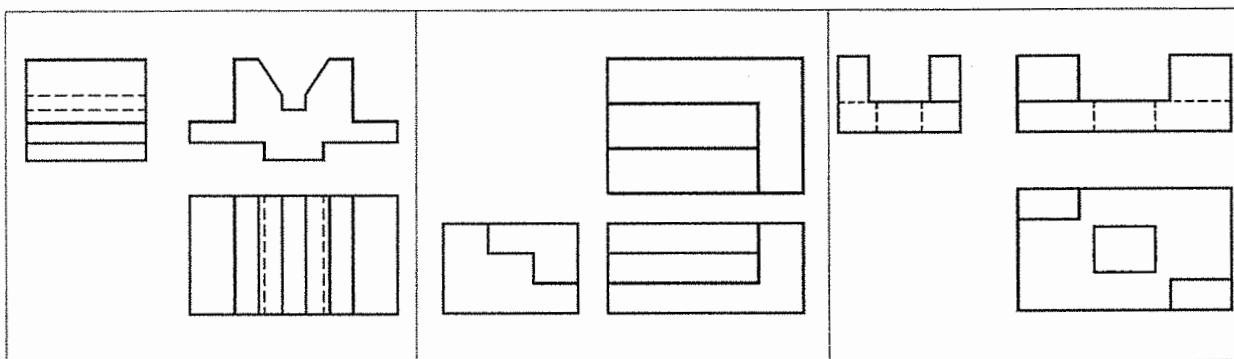


FIG. 20-70

SOLUTIONS TO EXERCISES 20



(Ex. 1. FIG. 20-33)

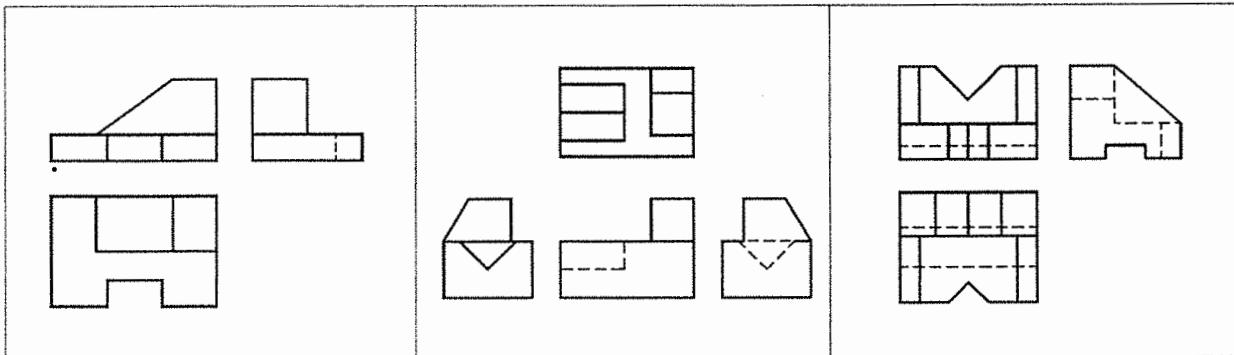
FIG. 20-71

(Ex. 2. FIG. 20-34)

FIG. 20-72

(Ex. 3. FIG. 20-35)

FIG. 20-73



(Ex. 4. FIG. 20-36)

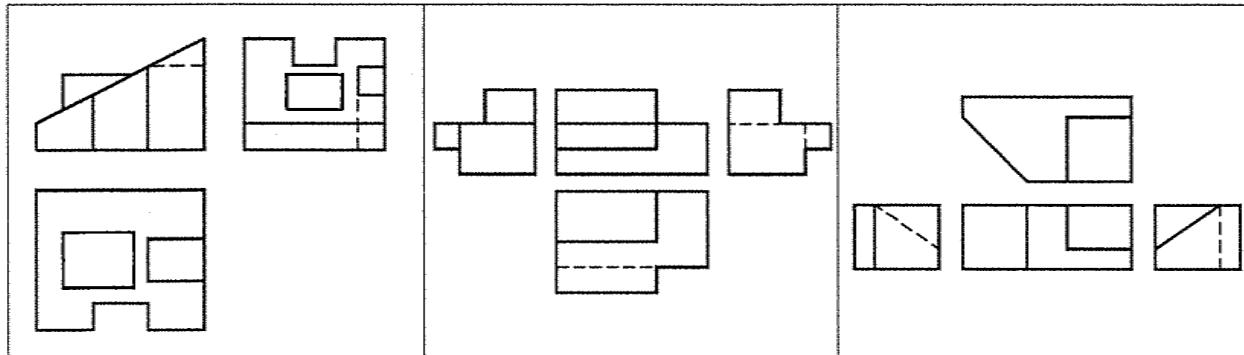
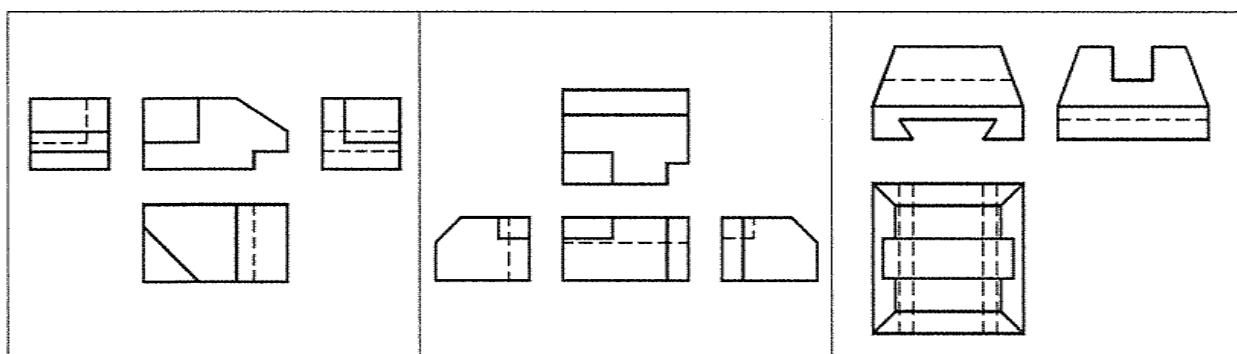
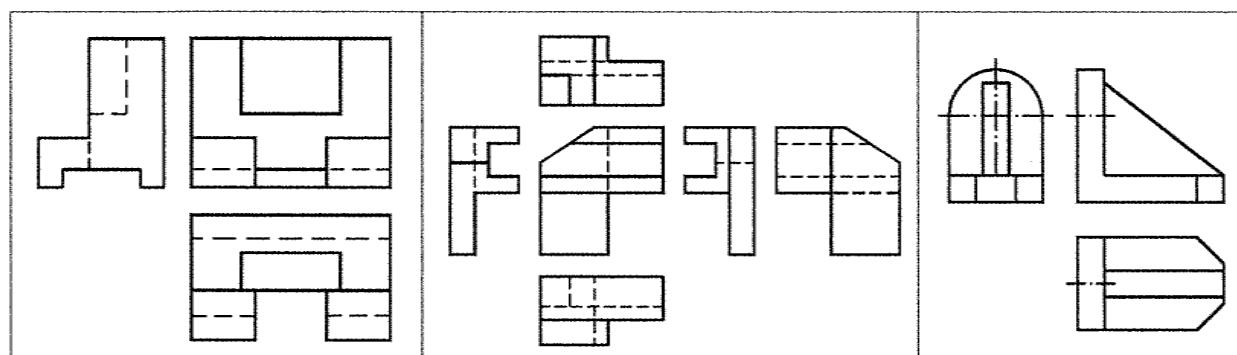
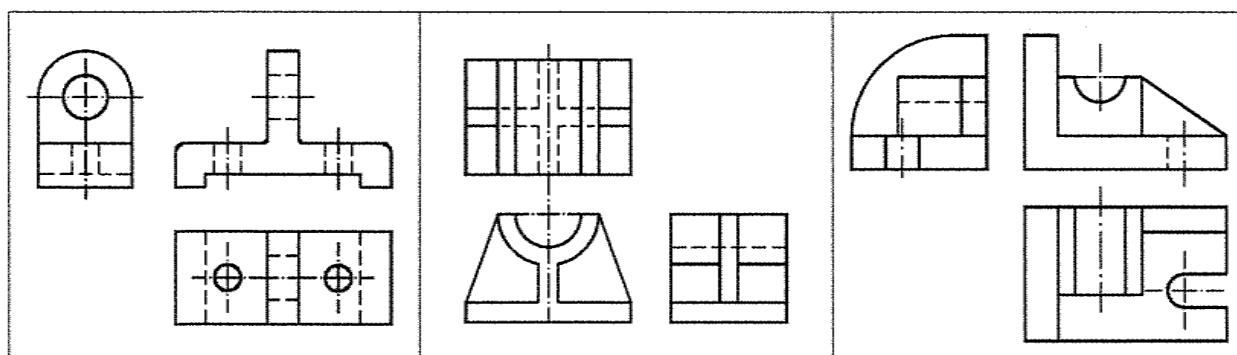
FIG. 20-74

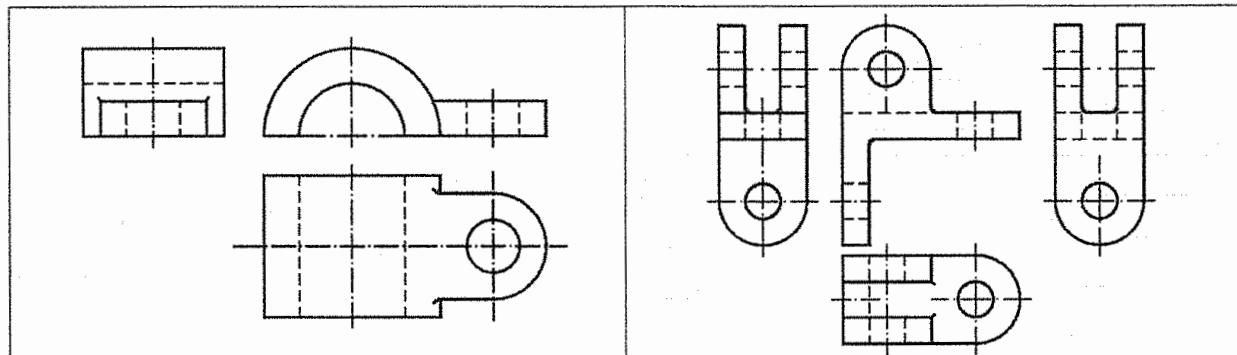
(Ex. 5. FIG. 20-37)

FIG. 20-75

(Ex. 6. FIG. 20-38)

FIG. 20-76

(Ex. 7. FIG. 20-39)
FIG. 20-77(Ex. 8. FIG. 20-40)
FIG. 20-78(Ex. 9. FIG. 20-41)
FIG. 20-79(Ex. 10. FIG. 20-42)
FIG. 20-80(Ex. 11. FIG. 20-43)
FIG. 20-81(Ex. 12. FIG. 20-44)
FIG. 20-82(Ex. 13. FIG. 20-45)
FIG. 20-83(Ex. 14. FIG. 20-46)
FIG. 20-84(Ex. 15. FIG. 20-47)
FIG. 20-85(Ex. 16. FIG. 20-48)
FIG. 20-86(Ex. 17. FIG. 20-49)
FIG. 20-87(Ex. 18. FIG. 20-50)
FIG. 20-88

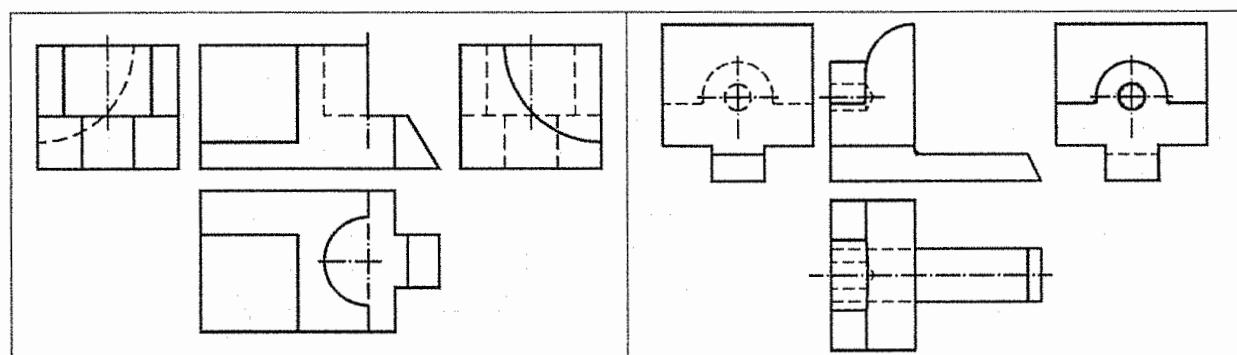


(Ex. 19, FIG. 20-51)

FIG. 20-89

(Ex. 20, FIG. 20-52)

FIG. 20-90

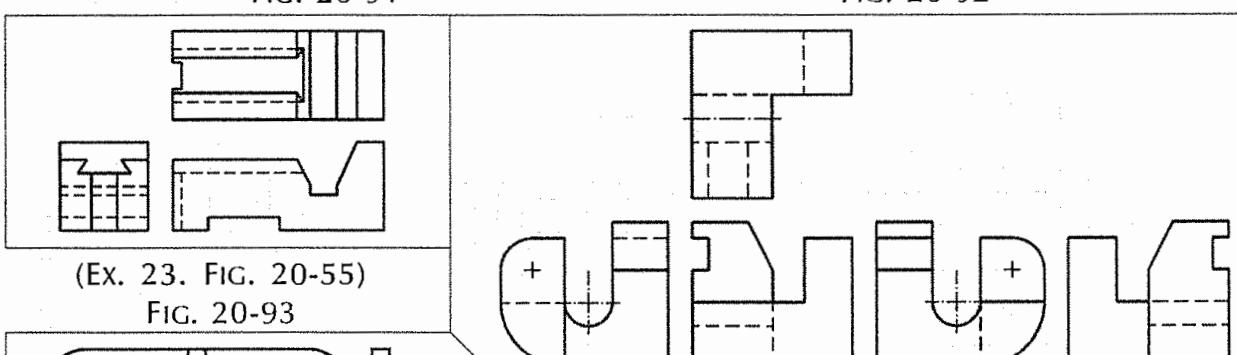


(Ex. 21, FIG. 20-53)

FIG. 20-91

(Ex. 22, FIG. 20-54)

FIG. 20-92

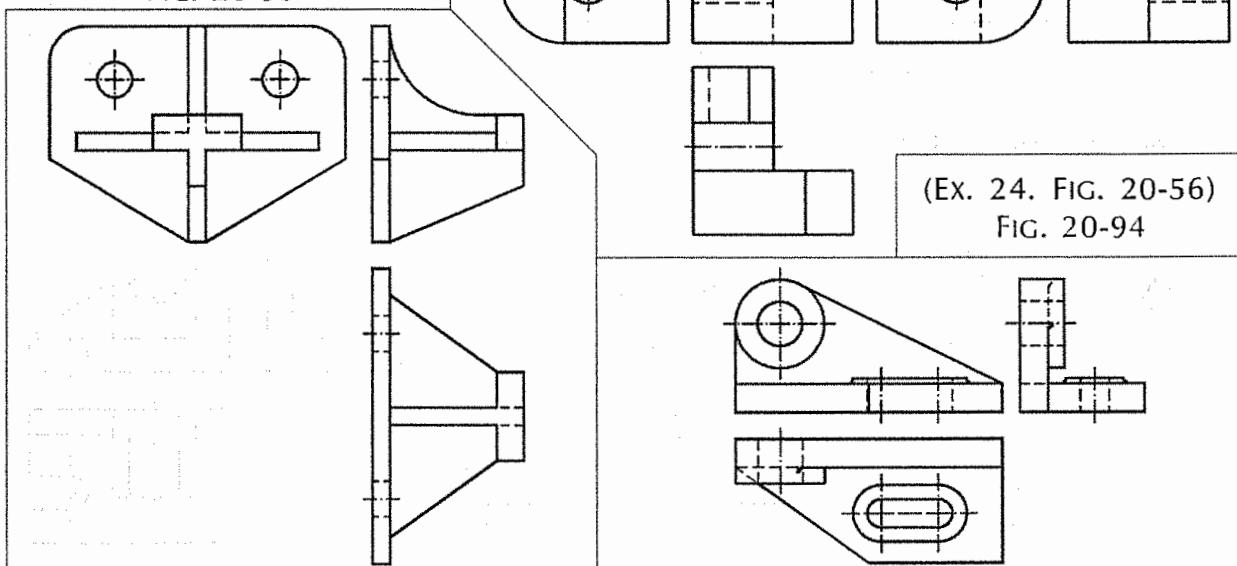


(Ex. 23, FIG. 20-55)

FIG. 20-93

(Ex. 24, FIG. 20-56)

FIG. 20-94

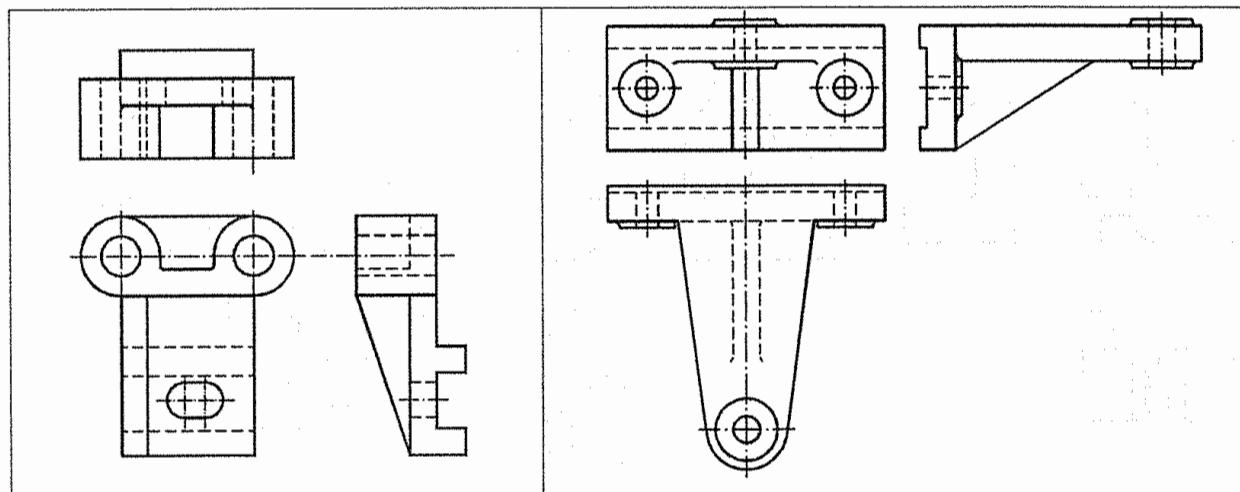
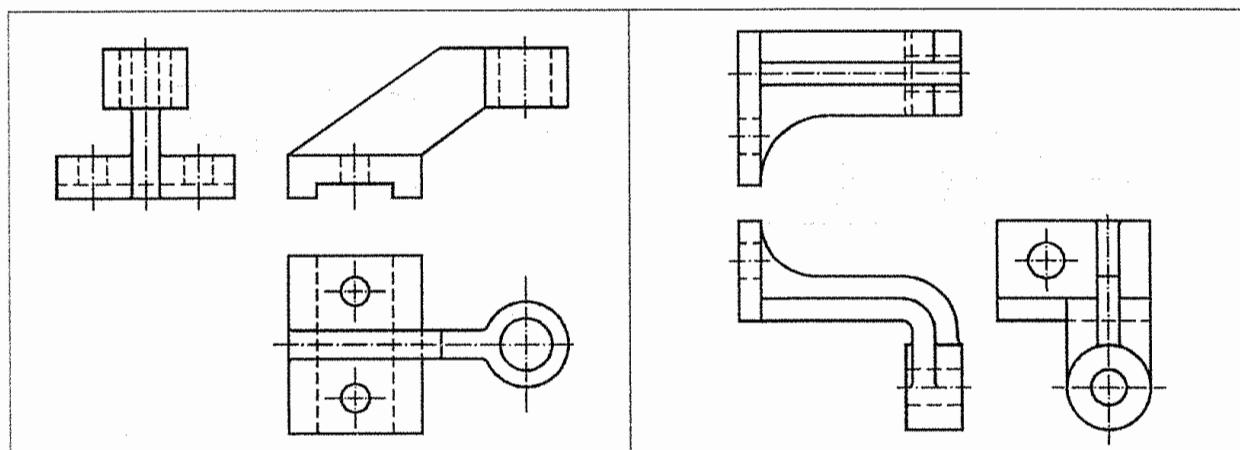
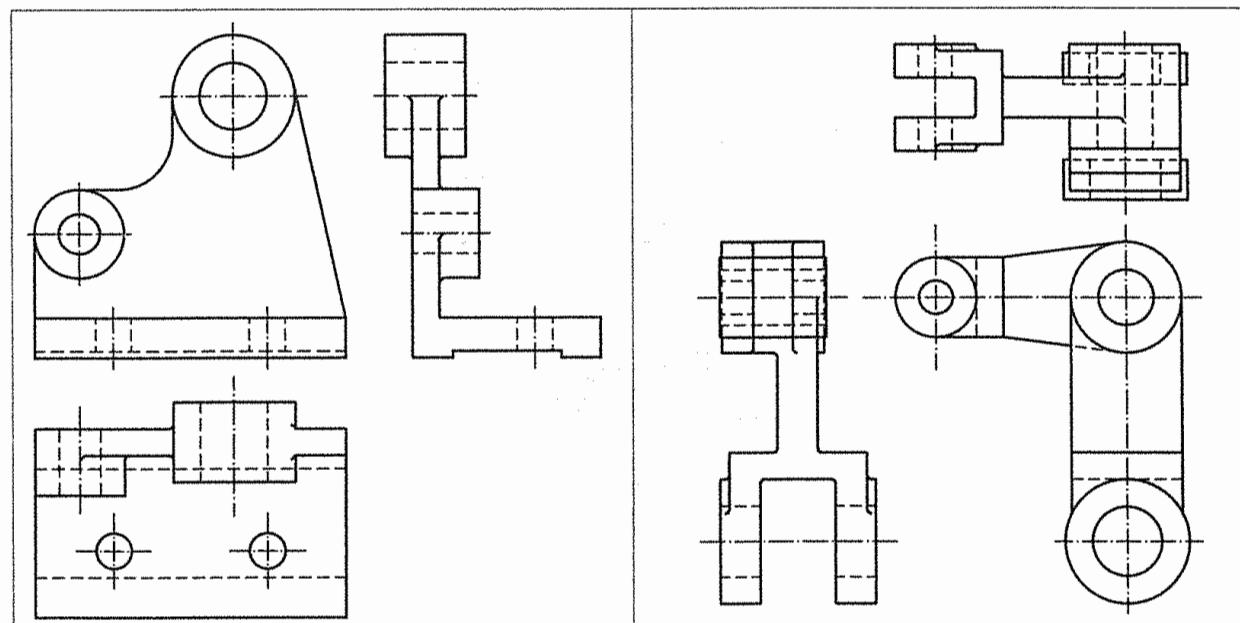


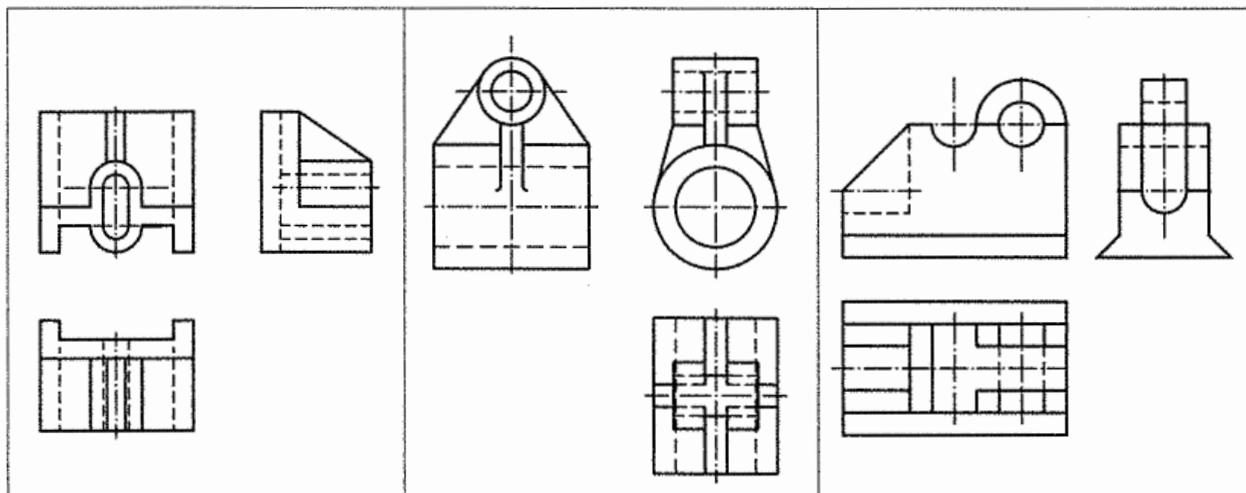
(Ex. 25, FIG. 20-57)

FIG. 20-95

(Ex. 26, FIG. 20-58)

FIG. 20-96

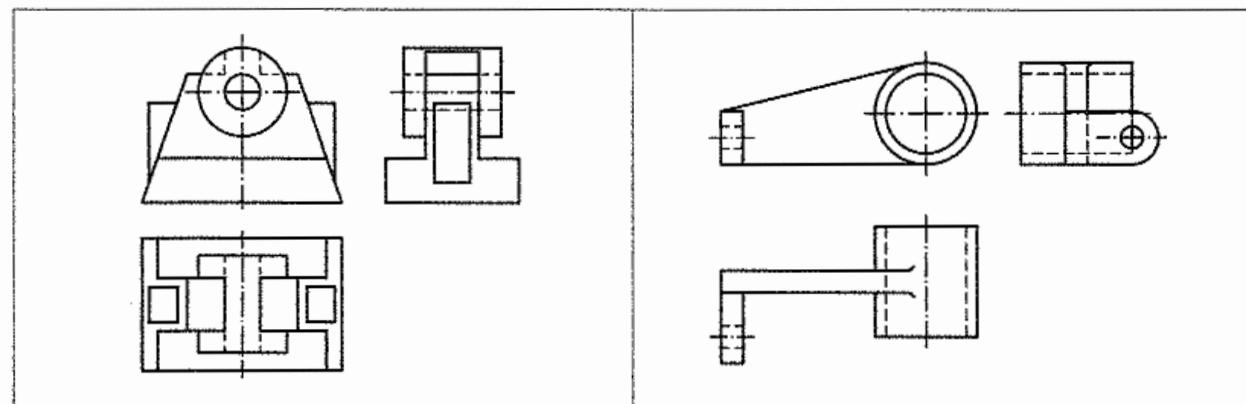
(Ex. 27. FIG. 20-59)
FIG. 20-97(Ex. 28. FIG. 20-60)
FIG. 20-98(Ex. 29. FIG. 20-61)
FIG. 20-99(Ex. 30. FIG. 20-62)
FIG. 20-100(Ex. 31. FIG. 20-63)
FIG. 20-101(Ex. 32. FIG. 20-64)
FIG. 20-102



(Ex. 33. FIG. 20-65)
FIG. 20-103

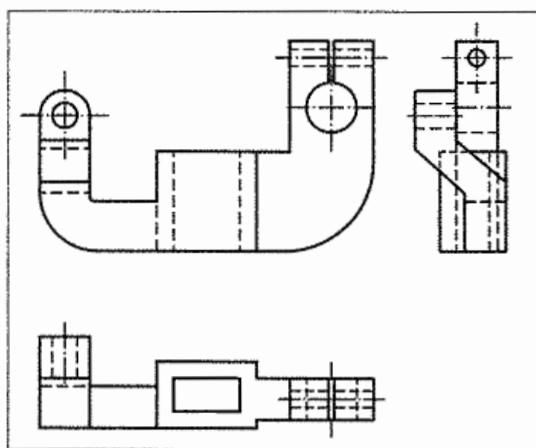
(Ex. 34. FIG. 20-66)
FIG. 20-104

(Ex. 35. FIG. 20-67)
FIG. 20-105



(Ex. 36. FIG. 20-68)
FIG. 20-106

(Ex. 37. FIG. 20-69)
FIG. 20-107



(Ex. 38. FIG. 20-70)
FIG. 20-108



CENTRES OF GRAVITY AND MOMENTS OF INERTIA OF AREAS

21-0. INTRODUCTION

Centres of gravity and moments of inertia of areas of plane surfaces are frequently used in engineering practice. Therefore, we shall study in this chapter, about these two parameters as shown below:

- (1) Centre of gravity of symmetrical and unsymmetrical areas
- (2) Moment of inertia of plane surfaces.

21-1. CENTRE OF GRAVITY

A body consists of numerous particles on which the pull of the earth, i.e. the forces of gravity act. The resultant of these forces acts through a point. This point is called the *centre of gravity* of the body. It can also be defined as a point about which the weights of all the particles balance. It may not necessarily lie within the body.

In case of an area, the figure is assumed to be a lamina of negligible thickness so that its centre of gravity will be practically on the surface. As the area has no weight this point is also called the *centroid*.

21-1-1. CENTRES OF GRAVITY OF SYMMETRICAL AREAS

When an area is symmetrical about both its axes, the centre of gravity will be at the point of intersection of these axes. Thus, the centre of gravity of a square, rectangle, parallelogram or rhombus will be at the point of intersection of its diagonals. The centre of gravity of a circle will be at the point of intersection of its any two diameters, i.e. at its centre.

When an area is symmetrical about one axis, the centre of gravity will lie on that axis.

The centre of gravity of a triangle lies at the point of intersection of its medians (fig. 21-1). The centre of gravity of a quadrilateral is determined by dividing it into triangles and as explained in problem 21-1.

Problem 21-1. Find the centre of gravity of the quadrilateral ABCD shown in fig. 21-2.

- (i) Draw the diagonal BD and locate the centres of gravity G_1 and G_2 of triangles BCD and ABD respectively. Similarly, draw the diagonal AC and determine the centres of gravity G_3 and G_4 of triangles ABC and ADC respectively.

- (ii) Draw lines joining G_1 with G_2 , and G_3 with G_4 . G , the point of intersection of lines G_1G_2 and G_3G_4 , is the centre of gravity of the quadrilateral $ABCD$.

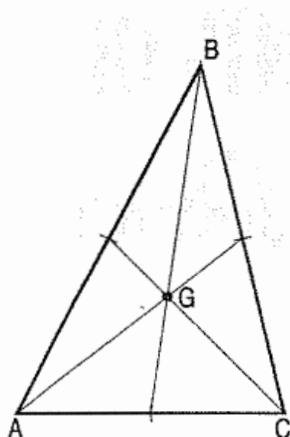


FIG. 21-1

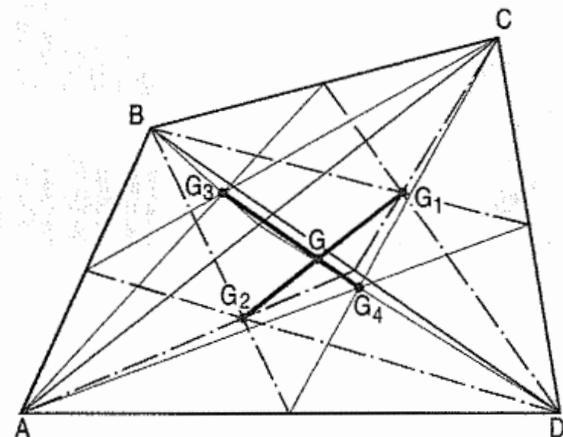


FIG. 21-2

A simple method of finding the centre of gravity of a trapezoid is illustrated in problem 21-2. It can also be located as explained in problem 21-3.

Problem 21-2. Determine the position of the centre of gravity of the trapezoid $ABCD$ shown in fig. 21-3.

- Draw a line joining the mid-points P and Q of the parallel sides AB and DC respectively.
- Produce AB to a point E so that $BE = DC$. Similarly, produce CD to a point F so that $FD = AB$.
- Draw the line EF intersecting the line PQ at a point G . Then G is the required centre of gravity.

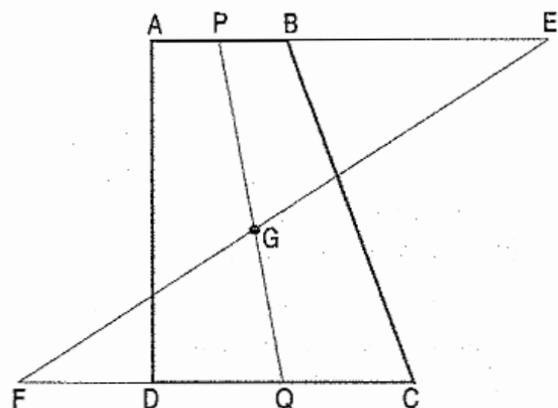


FIG. 21-3

21-1-2. CENTRES OF GRAVITY OF UNSYMMETRICAL AREAS

For locating the position of the centre of gravity of an area of unsymmetrical shape, it is first divided into a number of smaller areas of symmetrical shapes such as triangles, squares, rectangles, trapezoids etc. so that the centre of gravity of each such area is easily determined. Each small area is then treated as a force proportional to the area and acting at the centre of gravity of the area.

A resultant of these forces acting in one direction in the plane of the area is first determined. It will pass through the centre of gravity of the whole area. Another resultant of the same forces acting in the same plane but in another direction is then found out. This will also pass through the centre of gravity of the whole area. The point of intersection of these two resultants will be the centre of gravity of the whole area. The position of the resultant of parallel forces acting in a plane is graphically determined by means of *polar diagram* and *funicular polygon* or *link polygon* as explained in the following illustrative problems.

21-1-3. ILLUSTRATIVE PROBLEMS ON CENTRE OF GRAVITY



This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 50 for the following problem.

Problem 21-3. Find graphically the centre of gravity of the I-section shown in fig. 21-4.

As the section is symmetrical about its vertical axis yy' , the centre of gravity must lie on that axis.

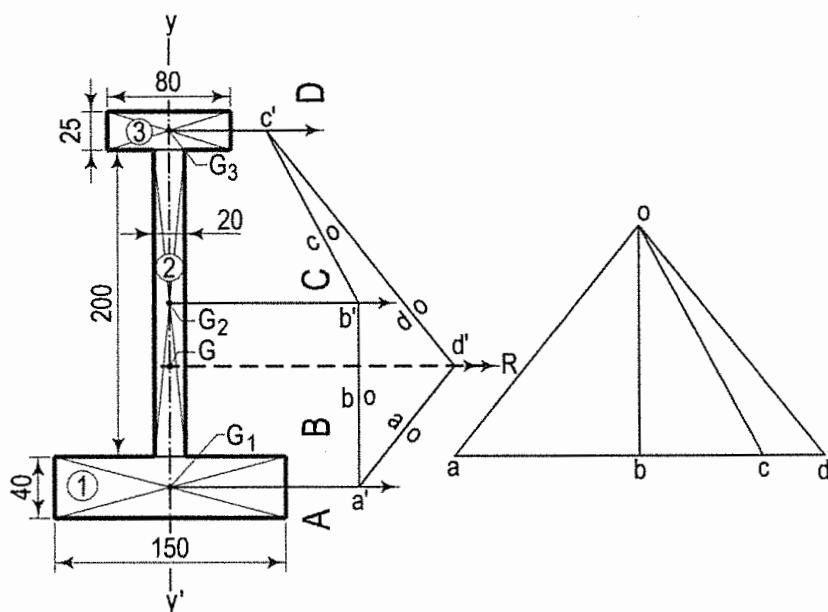


FIG. 21-4

- Draw the given section to a convenient scale. Divide it into three rectangles viz. 1, 2 and 3 and locate their centres of gravity G_1 , G_2 and G_3 respectively at the points of intersection of their respective diagonals. Through these points, draw parallel lines of action of forces representing the area of each rectangle. These lines may be drawn in any direction except vertical, as the centre of gravity lies in the vertical axis. Name these forces as AB , BC and CD respectively according to Bow's Notation. This figure is called the *space diagram*.
- Calculate the areas of the rectangles 1, 2 and 3. They will be 6000, 4000 and 2000 sq. mm respectively.
- Draw a line parallel to the lines of action of forces and on it mark a point a . Select any convenient scale, say $1 \text{ cm} = 1000 \text{ sq. mm}$ and cut-off lengths $ab = 6 \text{ cm}$, $bc = 4 \text{ cm}$ and $cd = 2 \text{ cm}$ in the direction of the forces. The line ad is called the *load line or force diagram*.
- Mark a point o at any convenient position either above or below the line ad and draw lines ao , bo , co and do . The point o is called the *pole* and the whole figure is called the *polar diagram*.
- Mark any point a' on the line of action of the force AB and draw a line parallel to ao . Through the same point, draw a line parallel to bo to intersect the line of action of the force BC at a point b' . Through b' , draw a line parallel to co to intersect

the line of action of the force CD at a point c' . Through c' , draw a line parallel to do to intersect the line drawn parallel to ao at a point d' . Through d' , draw a line parallel to the lines of action to intersect the axis yy' at a point G . This line is the line of action of the resultant R of the forces and G is the centre of gravity of the section. Figure $a'b'c'd'$ is called the *funicular polygon or link polygon*.

Problem 21-4. Find graphically the centre of gravity of the channel section shown in fig. 21-5.

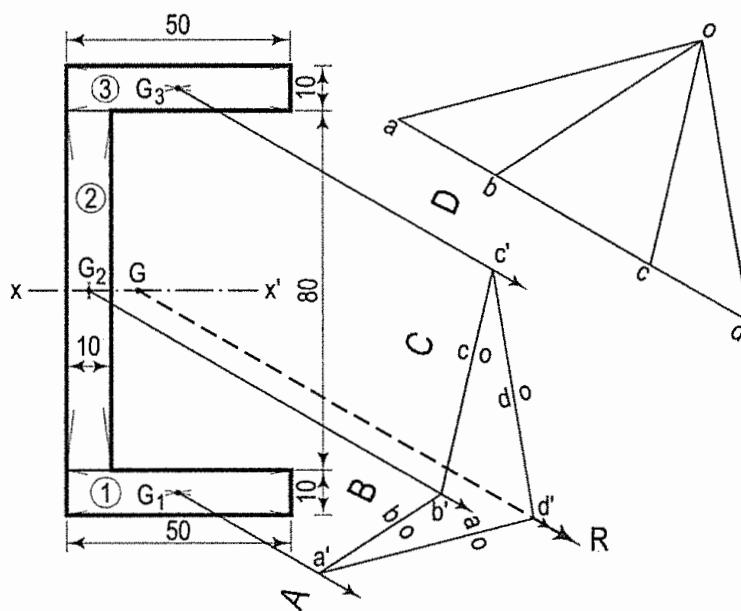


FIG. 21-5

As the section is symmetrical about its horizontal axis xx' , the centre of gravity must lie on this axis.

- Draw the section to any convenient scale. Divide it into three rectangles 1, 2 and 3 and locate their centres of gravity G_1 , G_2 and G_3 respectively. Through these points, draw (in any direction except horizontal) parallel lines of action of forces representing the area of each rectangle.
- Draw the polar diagram and funicular polygon as explained in problem 21-3 and obtain the point G where the line of action of the resultant R of the forces intersects the axis xx' . G is the centre of gravity of the section.

Note that the line of action of the resultant is drawn through d the point of intersection between the lines drawn through points on the first and the last lines of action and parallel to ao and do respectively.

Problem 21-5. Determine graphically the position of the centre of gravity of the Z-section shown in fig. 21-6.

As the section is not symmetrical about any axis, it will be necessary to obtain two resultants.

- Draw the section to a convenient scale and divide it into three rectangles 1, 2 and 3. (The division can also be done in a different way.)

- (ii) Mark the centres of gravity G_1 , G_2 and G_3 of these rectangles. Through these centres, draw lines of action of the forces in any one direction, say vertical. Calculate the areas of the three rectangles and draw the polar diagram and funicular polygon as explained in problem 21-3 and obtain the position of the resultant R .

Similarly, through points G_1 , G_2 and G_3 , draw horizontal or inclined parallel lines of action of forces and determine the position of the resultant R_1 . The point of intersection G , between the lines of action of the resultants R and R_1 is the required centre of gravity of the section.

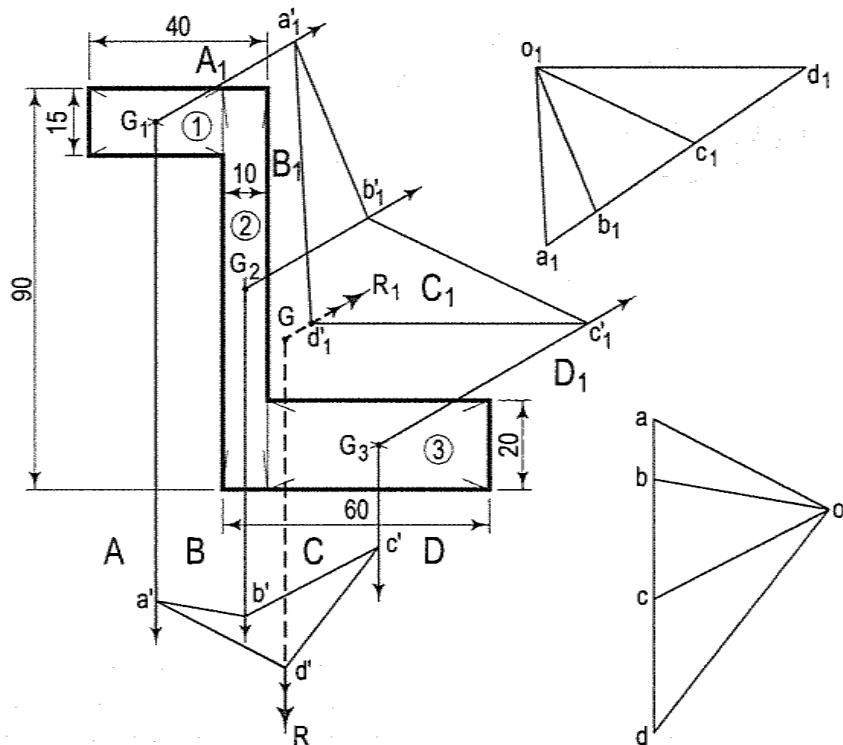


FIG. 21-6

Problem 21-6. Find graphically the centre of gravity of the plate PQRS of negligible thickness having a circular hole in it as shown in fig. 21-7.

- Draw the plate to a convenient scale. Complete the rectangle PTRS.
- Find the centres of gravity of the circle, the rectangle PTRS and the triangle QTR and name them as G_1 , G_2 and G_3 respectively. Through these points, draw vertical lines of action of forces representing the areas.

As the areas of the circle and the triangle are removed from the area of the rectangle, the direction of action (i.e. the sense) of the forces representing them, viz. AB and CD will be opposite (as shown by the arrows) to that of the force BC representing the area of the rectangle. Calculate the areas of the circle, the rectangle and the triangle. They will be 707, 6000 and 1200 sq. mm respectively.

- Draw a line parallel to the lines of action. Select a convenient scale, say $1 \text{ cm} = 1000 \text{ sq. mm}$, and on this line mark a length $ab = 0.7 \text{ cm}$ in