Exam Seat No.________________

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who has worked for the year 2019-20 in the Computer Laboratory.

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Head of Department

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Examiner

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Practical No. 1

Limits & Continuity

1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$ 2) $\lim_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3) $\lim_{x \to \pi/6} \left(\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right)$ 4) $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

5) Examine the continuity of the following function at given points.

i) $$f(x) = \begin{cases} \dfrac{\sin x}{\sqrt{1-\cos 2x}} & \text{for } 0 < x \le \pi/2 \\ \dfrac{\cos x}{\pi - 2x} & \text{for } \pi/2 < x < \pi \end{cases} \quad \text{at } x = \pi/2$$

ii) $$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & 0 < x < 3 \\ x+3 & 3 \le x < 6 \\ \dfrac{x^2-9}{x+3} & 6 \le x < 9 \end{cases} \quad \text{at } x = 3 \ \& \ x = 6$$

(6) Find value of k, so that the function f(x) is cts at the indicated point.

i) $$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2} & x < 0 \\ k & x = 0 \end{cases} \quad \text{at } x = 0$$

ii) $$f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x} & x \ne 0 \\ k & x = 0 \end{cases} \quad \text{at } x = 0$$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \ne \pi/3$ }

$= k \quad x = \pi/3 \;\&\; \text{at } x = \pi/3$

(7) Discuss the continuity of the following functions which of these functions have a removable discontinuity? Redefine the function so as to remove the discontinuity

(i) $f(x) = \begin{cases} \dfrac{1-\cos 3x}{x \tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases}$ at $x = 0$

(ii) $f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \pi/60 & x = 0 \end{cases}$ at $x = 0$

(8) If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is cts at $x = 0$ find $f(0)$

(9) If $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$ is cts at $x = \pi/2$ find $f(\pi/2)$

1) $\lim\limits_{x\to a} \dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$

$= \lim\limits_{x\to a} \dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \dfrac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}}$

$= \lim\limits_{x\to a} \dfrac{(a+2x-3x)}{(3a+x-4x)} \times \dfrac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{3a+2x} + \sqrt{3x})}$

$= \lim\limits_{x\to a} \dfrac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \dfrac{1}{3} \lim\limits_{x\to a} \dfrac{(a-x)}{(a-x)} \times \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \dfrac{1}{3} \times \dfrac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}} = \dfrac{1}{3} \times \dfrac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$= \dfrac{1}{3} \times \dfrac{2\sqrt{a} + 2\sqrt{a}}{2\sqrt{3a}}$

$= \dfrac{1}{3} \times \dfrac{2}{\sqrt{3}} = \dfrac{2}{3\sqrt{3}}$

2. $\lim\limits_{y\to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}}\right]$

$= \lim\limits_{y\to 0} \dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \dfrac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}}$

$= \lim\limits_{y\to 0} \dfrac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})} = \lim\limits_{y\to 0} \dfrac{y}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$

$= \dfrac{1}{\sqrt{a+0}\,(\sqrt{a+0} + \sqrt{a})} = \dfrac{1}{2a}$

3. $\lim\limits_{x\to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By Substituting $x - \pi/6 = h$

$x = h + \pi/6$

where $h \to 0$

$\lim\limits_{h\to 0} \dfrac{\cos(h+\pi/6) - \sqrt{3}\sin(h+\pi/6)}{\pi - 6(h+\pi/6)}$

using

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$$= \lim_{h\to 0} \frac{\cancel{\cos(h+\pi} \cosh.\cos\pi/6 - \sinh.\sin\pi/6 - \sqrt{3}\sinh.\cos\pi/6 + \cosh.\sin\pi/6}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$\cos\pi/6 = \cos 30° = \sqrt{3}/2$

$\sin\pi/6 = 1/\sqrt{2}$

$$= \lim_{h\to 0} \frac{\cosh.\sqrt{3}/2 - \sinh.1/\sqrt{2} - \sqrt{3}(\sinh\sqrt{3}/2 + \cosh.1/\sqrt{2})}{\pi - 6h + \pi}$$

$$= \lim_{h\to 0} \frac{\cos.\sqrt{3}/2\,h - \sin h/2 - \sin 3h/2 - \cos\sqrt{3}/2\,h}{-6h}$$

$$= \lim_{h\to 0} \frac{-\sin 4h/2}{-6h}$$

$$= \lim_{h\to 0} \frac{\sin 4h}{h}$$

$$= 1/3 \lim_{h\to 0} \frac{\sin h}{h} = 1/3$$

4) $\lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

By rationalizing numerator & denominator.

$$= \lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right] \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}$$

$$= \lim_{x\to\infty} \frac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+4})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$= \lim_{x\to\infty} \frac{8(\sqrt{x^2+3} + \sqrt{x^2+1})}{2(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$= 4 \lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

* After applying limit we get, $= 4$

5) i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$
$= \frac{\cos x}{\pi - 2x}$, for $\pi/2 < x \le \pi$ } at $x = \pi/2$

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{\cancel{\cos}1 - \cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \pi/2$ define

ii) $\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2^+} \frac{\cos x}{\pi - 2x}$

By substituting method

$x - \pi/2 = h$

$x = h + \pi/2$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$$

$\lim_{h\to 0} \dfrac{\cos(h+\pi/2)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$

$\lim_{h\to 0} \dfrac{\cos(h+\pi/2)}{-2h}$

using $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\lim_{h\to 0} \dfrac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$

$\lim_{h\to 0} \dfrac{\cos h(0) - \sin h}{-2h} \quad \dfrac{\cos h(0) - \sin h}{-2h}$

$\lim_{h\to 0} \dfrac{-\sin h}{-2 \times h}$

$= \dfrac{1}{2}$

(iv) $\lim_{x\to \pi/2^-} f(x) = \lim_{x\to \pi/2^-} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x\to \pi/2^-} \dfrac{2 \sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x\to \pi/2^-} \dfrac{2\cos x}{\sqrt{2}}$

$\lim_{x\to \pi/2} \dfrac{2}{\sqrt{2}} \cdot \cos x$

$\therefore$ LHL $\neq$ RHL

$\therefore$ f is not continous at $x = \pi/2$

5.(ii) $f(x) = \dfrac{x^2-9}{x-3} \quad 0 < x \le 3$

$= x+3 \quad 3 \le x < 6$

$= \dfrac{x^2-9}{x+3} \quad 6 \le x < 9$ at $x=3$ & $x=6$

at $x = 3$

if $f(3) = \dfrac{x^2-9}{x-3} = 0$ f at $x = 3$ define

$\therefore \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x = 3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3 = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{(x-3)}$

$= 6$

LHL = RHL

f is continous at $x = 3$

for $x = 6$

$f(6) = \dfrac{x^2-9}{x+3} = \dfrac{36-9}{6+3} = 3$

$\lim_{x\to 6^+} \dfrac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \dfrac{(x-3)(x+3)}{(x+3)} = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL

function is not continous.

6 i. $f(x) = \dfrac{1-\cos 4x}{x^2}$ $\quad x < 0$ $\quad$ at $x = 0$

$= k$ $\quad x = 0$

Sol^n f is continous at $x = 0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$$

$$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$\therefore k = 8$$

ii $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$ $\quad$ at $x = 0$

$= k$ $\quad x = 0$

$$f(x) = (\sec^2 x)^{\cot^2 x}$$

$$= \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

using $\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

$\cot^2 x = 1/\tan^2 x$

$$= \lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$$

we, know that

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$$\therefore k = e$$

iii $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$ $\quad$ at $x = \pi/3$

$= k$ $\quad x = \pi/3$

$x - \pi/3 = h$

$x = h + \pi/3$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 . \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}(1 - \tan \pi/3 . \tan h) - (\tan \pi/3 + \tan h)}{1 - \tan \pi/3 . \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3}\times\sqrt{3} . \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 . \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}.\tan h}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3} \frac{1}{(1-\sqrt{3}(0))}$$

$$= \frac{4}{3}$$

7 i. $f(x) = \frac{1-\cos 3x}{x\tan x}$ $x \neq 0$ $\Big\}$ at $x = 0$

$= 9$ $x = 0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$= \lim_{x\to 0} \frac{1-\cos 3x}{x\tan x} = \lim_{x\to 0} \frac{2\sin^2 3/2\, x}{x\tan x}$$

$$= \lim_{x\to 0} \frac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{x \cdot \dfrac{\tan x}{x^2} \times x^2}$$

$$= 2\lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1} = 2 \times 9/4 = 9/2$$

$\lim_{x\to 0} f(x) = 9/2$ $9 = f(0)$

$\therefore f$ is not continous at $x = 0$

Redefine function

$f(x) = \Big\{ \frac{1-\cos 3x}{x\tan x}$ $x \neq 0$

$9/2$ $x = 0$ $\Big\}$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removeable discontinuity at $x = 0$

7 ii $f(x) = \frac{(e^{3x}-1)\sin x^{\circ}}{x^2}$ $x \neq 0$ $\Big\}$ at $x = 0$

$= \pi/6$ $x = 0$

$$= \lim_{x\to 0} \frac{(e^{3x}-1) \cdot \sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$= \lim_{x\to 0} \frac{e^{3x}-1}{x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= \lim_{x\to 0} 3 \cdot \frac{e^{3x}-1}{3x} \lim_{x\to 0} \sin\left(\frac{\pi x}{180}\right)/x$$

$$= 3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \sin(\pi x/180)/x$$

$= \log e\ \pi/60 = \pi/60 = f(0)$

f is continous at $x = 0$

8. $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continous at $x = 0$

∴ Given, f is continous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$= \lim_{x\to 0} \dfrac{e^{x^2} - \cos x}{x^2} = \lim_{x\to 0} \dfrac{e^{x^2} - \cos x + 1 - 1}{x^2}$

$= \lim_{x\to 0} \dfrac{e^{x^2} - 1}{x^2} + \lim_{x\to 0} \dfrac{1 - \cos x}{x^2}$

$= \log e + \lim_{x\to 0} \dfrac{2\sin^2 x/2}{x^2}$

$= \log e + 2\lim_{x\to 0} \left(\dfrac{\sin x/2}{x}\right)^2$

$= \log e + 2\lim_{x\to 0} \left(\dfrac{\sin x/2}{x/2}\right)^2 \cdot 2\,(1/4)$

$= \cancel{\log e +}\ 1 + \dfrac{1}{2} = 3/2$

9. $f(x) = \dfrac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(0)$ is continous at $x = \pi/2$

$\lim_{x\to \pi/2} \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$\lim_{x\to \pi/2} \dfrac{2 - 1 + \sin x}{\cos^2 x(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to \pi/2} \dfrac{1 + \sin x}{1 - \sin^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to \pi/2} \dfrac{1 + \sin x}{(1 - \sin x)(1 + \sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to \pi/2} \dfrac{1}{(1 - \sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \dfrac{1}{2(\sqrt{2} + \sqrt{2})} = \dfrac{1}{4\sqrt{2}}$

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Derivative.

1) Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.
(i) $\cot x$ (ii) $\operatorname{cosec} x$ (iii) $\sec x$

2) If $f(x) = 4x+1$, $x \leq 2$
$= x^2+5$, $x > 0$, at $x = 2$,
then find f is differentiable or not?

3) If $f(x) = 4x+7$, $x < 3$
$= x^2+3x+1$, $x \geq 3$ then find f is differentiable or not?

4) If $f(x) = 8x-5$, $x \leq 2$
$= 3x^2-4x+7$, $x > 2$ at $x = 2$
then find f is differentiable or not.

1) i $\cot x$

$f(x) = \cot x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$

$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$

$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x-a}$

$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a) \tan x \tan a}$

put $x - a = h$
$x = a+h$
as $x \to a$, $h \to 0$

$Df(a) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a) \tan(a+h) \tan a}$

$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$

$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$\tan A - \tan B = \tan(A-B)(1+\tan A \tan B)$

$= \lim_{h \to 0} \frac{\tan(a-a-h) \cdot (1+\tan a \tan(a+h))}{h \times \tan(a+h) \tan a}$

$= \lim_{h \to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h) \tan a}$

$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$

$= \frac{-\sec^2 a}{\tan^2 a} = -1/\cos^2 a \times \cos^2 a/\sin^2 a$

$= -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\operatorname{cosec}^2 a$

$\therefore f$ is differentiable $\forall a \in \mathbb{R}$

ii) cosec x

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\sin x - 1/\sin a}{x - a} = \lim_{x \to a} \frac{\sin a - \sin x}{(x - a)/\sin a \sin x}$$

$= \lim_{x \to a}$ Put $x - a = h$

$x = a + h$

as $x \to a, h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)(\sin a \cdot \sin(a+h))}$$

formula:

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \cdot \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} -\frac{\sin \frac{h}{2}}{h/2} \times 1/2 \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -1/2 \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin a \sin(a+0)}$$

$$= -\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

iii) sec x

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a, h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

Formula: $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin((2a+h)/2)\sin(-h/2)}{\cos a \cos(a+h) \times (-h/2)} \times (-1/2)$$

$$= \lim_{h \to 0} \frac{-1/2 \times -2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= \frac{\sin a}{\cos^2 a} = \tan a \sec^2 a$$

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2)
LHD:
$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2^-} \frac{4x+1-(4\times 2)}{x-2}$$
$$= \lim_{x \to 2^-} \frac{4x+-9}{x-2} = \lim_{x \to 2^-} \frac{4x+8}{x-2}$$
$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

RHD:
$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2} = \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$
$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)} = 2+2$$
$Df(2^+) = 4$

LHD = RHD

f is differentiable at $x = 2$

3) RHD:
$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$
$$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3} = \lim_{x \to 3^+} \frac{x^2+3x+1\cdot}{x-3}$$
$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3} = \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$
$$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3} = \lim_{x \to 3^+} \frac{(x-3)}{(x-3)}(x+6)$$
$= 3+6$
$= 9$
$Df(3^+) = 9$

LHD:

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$$Df(3^-3) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3} = \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$$
$$= \lim_{x \to 3^-} \frac{4x-12}{x-3} = \lim_{x \to 3^-} \frac{4(x-3)}{(x-3)}$$
$Df(3^-) = 4$

RHD = LHD

f is not differentiable at $x = 3$

4) $f(2) = 8\times 2 - 5 = 16 - 5 = 11$

RHD:
$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$$
$$= \lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2} = \lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$
$$= \lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$
$$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)} = 3\times 2+2 = 8$$
$Df(2^+) = 8$

LHD:
$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2^-} \frac{8x-5-11}{x-2}$$
$$= \lim_{x \to 2^-} \frac{8x-16}{x-2} = \lim_{x \to 2^-} 8\frac{(x-2)}{(x-2)}$$
$= 8$
$Df(2^-) = 8$

LHD = RHD

f is differentiable at $x = 3$

Practical No. 3

Application of Derivative

Q1 Find the intervals in which functions is increasing or decreasing.

(i) $f(x) = x^3 - 5x - 11$ (ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$ (iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24 - 9x^2 + 2x^3$

Q2. Find the intervals in which function is concave upwards concave downwards.

(i) $y = 3x^2 - 2x^3$ ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

(iii) $y = x^3 - 27x + 5$ (iv) $y = 69 - 24x - 9x^2 + 2x^3$

(v) $y = 2x^3 + x^2 - 20x + 4$

1)

i) $f(x) = x^3 - 5x - 11$

$f(x) = x^2 - 5$

f is increasing iff $f'(x) > 0$

$\therefore 3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > 5/3$

$x > \pm\sqrt{5/3}$

$+ \quad -\sqrt{5/3} \quad - \quad \sqrt{5/3} \quad +$

$\therefore x \in \left(-\sqrt{5/3}, \sqrt{5/3}\right)$

ii) $f(x) = x^2 - 4x$

$f(x) = 2x - 4$

$\therefore$ f is increasing iff $f(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x > 2$

$\therefore x \in (2, \infty)$

$\therefore$ f is increasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x < 2$

$\therefore x \in (-\infty, 2)$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x+2) > 0$

$+ \quad -2 \quad - \quad 10/6 \quad +$

$\therefore x \in (-\infty, -2) \cup (10/6, \infty)$

f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x^2 + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$(6x - 10)(x + 2) < 0$

$+ \quad - \quad +$

$-2 \qquad 10/6$

$\therefore x \in (-2, 10/6)$

(iv) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$= 3x(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \qquad 3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

f is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

$\therefore x \in (-3, 3)$

$+ \quad - \quad +$

$-3 \qquad 3$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

i.e. $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

f is increasing iff $f'(x) > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x+1)(x-4) > 0$

$+ \quad - \quad +$

$-1 \qquad 4$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

f is decreasing iff $f'(x) < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$x^2 - 3x - 4 < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x+1)(x-4) < 0$

$+ \quad - \quad +$

$-1 \qquad 4$

$\therefore x \in (1, 4)$

2) i) $y = 3x^2 - 2x^3$

let, $f(x) = y = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$= 6(1-2x)$

$f''(x)$ is concave upward iff,

$f''(x) > 0$

$6(1-2x) > 0$

$1 - 2x > 0$

$-2x > -1$

$2x < 1$

$x < 1/2$

$\therefore x \in (-\infty, 1/2)$

$f''(x)$ is concave dowards iff,

$f''(x) < 0$

$6(1-2x) < 0$

$1 - 2x < 0$

$-2x < -1$

$2x > 1$

$x > 1/2$

$\therefore x \in (1/2, \infty)$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

let

$f(x) = y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$f''(x)$ is concave upwards iff,

$f''(x) > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$x^2 - x - 2x + 2 > 0$

$x(x-1) - 2(x-1) > 0$

$(x-1)(x-2) > 0$

+   −   +

1   2

$\therefore x \in (-\infty, -1) \cup (2, \infty)$

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$12(x^2 - 3x + 2) < 0$

$x^2 - 3x + 2 < 0$

$x^2 - x - 2x + 2 < 0$

$x(x-1) - 2(x-1) < 0$

$(x-2)(x-1) < 0$

+   −   +

1   2

$\therefore x \in (1, 2)$

(iii) $y = x^3 - 27x + 5$

let,

$f(x) = y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is concave upwards iff,

$f''(x) > 0$

$6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

$f''(x)$ is concave downwards iff,

$f''(x) < 0$

$6x < 0$

$x < 0$

$\therefore x \in (-\infty, 0)$

(iv) $y = 69 - 24x - 9x^2 + 2x^3$

let

$f(x) = y$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore f''(x) = -18 + 12x$

$f''(x)$ is concave upward iff,

$f''(x) > 0$

$-18 + 12x > 0$

$12x > 18$

$x > 18/12$

$\therefore x \in (3/2, \infty)$

$f''(x)$ is concave downwards iff,

$f''(x) < 0$

$-18 + 12x < 0$

$12x < 18$

$x < 18/12$

$\therefore x \in (\infty, 3/2)$

v) $y = 2x^3 + x^2 - 20x + 4$

let,

$f(x) = y$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$= 2(6x+1)$

$\therefore f''(x)$ is concave upwards iff,

$f''(x) > 0$

$2(6x-1) > 0$

$6x - 1 > 0$

$6x > 1$

$x > 1/6$

$\therefore x \in (1/6, \infty)$

$\therefore f''(x)$ is concave downwards iff,

$f''(x) < 0$

$2(6x+1) < 0$

$6x + 1 < 0$

$x < 1/6$

$\therefore x \in (-\infty, 1/6)$

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Practical NO:- 4

Application of Derivative & Newton's method

1) Find maximum and minimum value of following function.

i) $f(x) = x^2 + 16/x^2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ is $\left[\frac{-1}{2}, 4\right]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

2) Find the root of following equation by Newton (Take 4 iteration only). Correct upto 4 Decimal.

1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x = 0$)
2) $f(x) = x^3 - 4x - 9$ in $[2,3]$
3) $f(x) = x^3 - 1.8x^2 - 10x + 7$ in $[1,2]$

1) i) $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider,

$f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$\therefore 2x = \frac{32}{x^3}$

$\therefore x^4 = 16$

$\therefore x = \pm 2$

$\therefore x = +2, -2$

Now,

$f''(x) = 2 + \frac{96}{x^4}$

Now

$f''(2) = 2 + \frac{96}{(2)^4} = 2 + \frac{96}{(16)}$

$= 2 + 6 = 8$

$f''(2) = 8 > 0$

$\therefore$ at $x = 2$ f has minimum value

$f(2) = (2)^2 + 16/(2)^2$

$= 4 + 16/4$

$= 4 + 4 = 8$

$f''(-2) = 2 + \frac{96}{(-2)^4}$

$= 2 + \frac{96}{16}$

$= 8 > 0$

$f(-2) = (-2)^2 + 16/(-2)^2$

$= 4 + 4 = 8$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$, and $x = -2$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider

$f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore x^4 = x^2$

$\therefore x^2 = 1 \Rightarrow x = \pm 1$

Now,

$f''(x) = -30x + 60x^3$

Now,

$f(1) = -30 + 60 = 30 > 0$

$\therefore$ f has the minimum value at $x = 1$

$\therefore f(1) = 3 - 5 + 3 = 1$

for,

$f''(-1) = 30 - 60$

$= -30 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = +3 + 5 - 3$

$= 5$

$\therefore$ f has the maximum value 5 at $x = -1$ & has minimum value 1 at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ & $x - 2 = 0$

$\therefore x = 0$ & $x = 2$

So, Here,

$f''(x) = 6x - 6$

Now,

$f''(0) = 6(0) - 6 = -6 < 0$

$\therefore f$ has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

Now,

$f''(2) = 6(2) - 6 = 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1 = 8 - 12 + 1$

$= -3$

$\therefore f$ has minimum value -3 at $x = 2$

& maximum value 1 at $x = 0$.

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore (x+1)(x-2) = 0$

$\therefore x = -1$ & $x = 2$

Here,

$f''(x) = 12x - 6$

So,

$f''(-1) = 12(-1) - 6 = -12 - 6 = -18 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = -2 - 3 + 12 + 1 = -5 + 13 = 8$

$f''(2) = 24 - 6 = 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -8 - 12 + 1$

$= -19$

$\therefore f$ has maximum value 8 at $x = -1$

& minimum value -19 at $x = 2$.

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Q2) i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0 \Rightarrow$ Given

$f'(x) = 3x^2 - 6x - 55$

By Newton's method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$\therefore x_1 = 0 + \frac{9.5}{55}$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \frac{0.0829}{55.9467}$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{0.1712 + 0.0011}{55.9393}$

$= 0.1712$

The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$ $\quad [2,3]$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9 = 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9 = 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation,

$\therefore$ By Newtons method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{6}{23}$

$\therefore x_1 = 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9 = 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4 = 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.7392 - \frac{0.596}{18.5096}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9 = 19.8386 - 10.8284 - 9$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4 = 21.9851 - 4$

$= 17.9851$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7071 - \frac{0.002}{17.9851}$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9 = 19.7158 - 10.806 ...$

$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4 = 21.8943 - 4$

$= 17.8943$

$\therefore x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$= 2.7065$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $\quad [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) ...$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17$

$= -2.2$

Let $x_0 = 2$ be initial approximation

By Newton's method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$= 2 - \frac{2.2}{5.2} \quad = 2 - 0.4230$

$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= \cancel{0.6755}\ 7.4608 - 5.6772 - 10$

$= -8.2164$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f(x_2)} = 1.577 + \frac{0.6755}{8.2164}$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.99312 - 10$

$= -7.7143$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.6592 + \frac{0.0204}{7.7143}$

$\cancel{f(x_3) = f}$ $\therefore x_3 = 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9702 - 16.618 + 17$

$= 0.0004$

$f'(x) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.6618 + \frac{0.0004}{7.6977}$

$x_4 = 1.6618$

$\therefore$ The root of equation is 1.6618

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Practical - 5

Topic - Integration.

① Solve the following Integration

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ (ii) $\int (4e^{3x}+1)\,dx$

(iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

(iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$ (v) $\int t^7 \sin(2t^4)\,dt$

vi) $\int \sqrt{x}(x^2-1)\,dx$ (vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin x}}$ (ix) $\int e^{\cos^2 x}\sin 2x\,dx$

(x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

① i

$\rightarrow I = \int \frac{dx}{\sqrt{x^2+2x-3}} = \int \frac{dx}{\sqrt{x^2+2x+1-3-1}}$

$= \int \frac{dx}{\sqrt{x^2+2x+1-4}} = \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

By using the formula

$\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x+\sqrt{x^2-a^2}| + c$

$\therefore I = \log|x+1+\sqrt{(x+1)^2-2^2}| + c$

$\therefore I = \log|x+1+\sqrt{x^2+2x-3}| + c$

ii)

$\rightarrow I = \int (4e^{3x}+1)\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= \frac{4e^{3x}}{3} + x + c$

$= \frac{4e^{3x}}{3} + x + c$

iii)

$\rightarrow I = \int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$= 2\int x^2\,dx - 3\int \sin x\,dx + 5\int x^{1/2}\,dx$

$= \frac{2}{3}x^3 + 3\cos x + \frac{2}{3}\times 5x^{3/2} + c$

$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2} + c$

iv)

$\rightarrow I = \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \left(\frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\right)dx$

$= \int \left(x^{3-1/2} + 3x^{1-1/2} + 4x^{-1/2}\right)dx$

$= \int \left(x^{5/2} + 3x^{1/2} + 4x^{-1/2}\right)dx$

$= \int x^{5/2}\,dx + 3\int x^{1/2}\,dx + 4\int x^{-1/2}\,dx$

$= \frac{x^{7/2}}{7/2} + \frac{2\,3x^{3/2}}{3/2} + \frac{4x^{1/2}}{1/2} + c$

$\therefore J = \frac{2}{7}x^{7/2} + 2x^{3/2} + 8x^{1/2} + c$

v)

$\rightarrow I = \int t^7 \sin(2t^4)\, dt$

let, $t^4 = x$

$\therefore 4t^3 dt = dx \Rightarrow$

$\therefore I = \frac{1}{4}\int x \sin 2x \, dx$

$= \frac{1}{4}\left[x\int \sin 2x\, dx - \int\left[\frac{x\,d}{dx}\int \sin 2x\, dx\right]dx\right]$

$= \frac{1}{4}\left[x\left(-\frac{\cos 2x}{2}\right) - \int\left[\left(-\frac{\cos 2x}{2}\right)\right]dx\right]$

$= \frac{1}{4}\left[x\left(-\frac{\cos 2x}{2}\right) + \frac{\sin 2x}{4}\right] + c$

$\therefore I = -\frac{1}{8}x\cdot\cos 2x + \frac{1}{16}\sin 2x + c$

VI)

$\rightarrow I = \int \sqrt{x}\,(x^2 - 1)\, dx$

$= \int (x^{2+1/2} - x^{1/2})\, dx$

$= \int x^{5/2} dx - \int x^{1/2} dx$

$= \frac{x^{5/2+1}}{5/2+1} - \frac{x^{1/2+1}}{1/2+1} + c$

$\therefore J = \frac{2}{7}x^{7/2} + - \frac{2}{3}x^{3/2} + c$

VII)

$\rightarrow J = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right) dx$

Let, $\frac{1}{x^2} = t$

$\therefore -\frac{1}{x^3}\, dx = dt$

$= -\frac{2}{2}\int \sin t\, dt$

$= \frac{1}{2}\cos t + c$

$\therefore J = \frac{1}{2}\cos\left(\frac{1}{x^2}\right) + c$

VIII)

$\rightarrow J = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

Let, $\sin x = t$

$\cos x\, dx = dt$

$= \int \frac{dt}{\sqrt[3]{t^2}} = \int \frac{dt}{t^{2/3}} = \int t^{-2/3} dt$

$= 3t^{1/3} + c$

$\therefore I = 3(\sin x)^{1/3} + c$

(IX)

$\rightarrow I = \int e^{\cos^2 x} \sin 2x \, dx$

let, $\cos^2 x = t$

$\therefore -2\cos x \sin x \, dx = dt$

$\therefore -\sin 2x \, dx = dt$

$= -\int e^t \, dt$

$= -e^{\cos^2 x} + c$

(X)

$\rightarrow I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$

let $x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x) dx = dt$

$= \frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log t$

$\therefore I = \frac{1}{3} \log |x^3 - 3x^2 + 1| + C$

03/01/2020

Practical - 6

Topic: Application of Integration

Q.1 Find the length of the following curve

(1) $x = t - \sin t$, $y = 1 - \cos t$, $t \in [0, 2\pi]$

(2) $y = \sqrt{4 - x^2}$ $x \in [-2, 2]$

(3) $y = x^{3/2}$ in $[0, 4]$

(4) $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

(5) $x = \frac{1}{6} \times y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q.2 Using Simpson's Rule solve the following

(1) $\int_0^2 e^{x^2} dx$ with $n = 4$

(2) $\int_0^4 x^2 dx$ with $n = 4$

(3) $\int_0^{\pi/3} \sqrt{\sin x} \, dx$ with $n = 6$

Q.1 Ans.

(1) $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

$x = t - \sin t \Rightarrow \frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t \Rightarrow \frac{dy}{dt} = 0 - (-\sin t) = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t)^2 + (\sin t)^2} \, dt$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt = \int_0^{2\pi} \sqrt{2 - 2\cos t}$$

$$\cancel{= \int_0^{2\pi} \sqrt{2 \sin t/2}\, dt}$$ using identity $\sin^2 \frac{t}{2} = \frac{1-\cos t}{2}$

$$= \int_0^{2\pi} 2 \sin t/2 \, dt$$

$$= \left[-4\cos t/2\right]_0^{2\pi} = \left[-4\cos 2\pi\right] - \left[-4\cos 0\right]$$

$$= 4 + 4$$

$$= 8$$

(2) $y = \sqrt{4 - x^2}$ $x \in [2, -2]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$y = \sqrt{4-x^2}$ $\Rightarrow \therefore \frac{dy}{dx} = \frac{1}{2}(4-x^2)^{-1/2} \frac{d}{dx}(4-x^2)$

$$\therefore \frac{dy}{dx} = \frac{1}{2}(4-x^2)^{-1/2}(-2x)$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$\therefore \frac{dy}{dx} = 2\int_0^2 1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2 dx$$

$$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\, dx = 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left(\sin^{-1}(x/2)\right)_0^2$$

$$= 2\pi$$

(3) $y = x^{3/2}$ in $[0, 4]$

$$f'(x) = 3/2\, x^{1/2}$$

$$[f'(x)]^2 = 9/4\, x$$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int_0^4 \sqrt{1 + 9/4\, x}\, dx$$

$$= \int_0^4 \sqrt{\frac{4 + 9x}{4}}\, dx$$

$$= 1/2 \int_0^4 \sqrt{4 + 9x}\, dx$$

$$= 1/2 \left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 \cancel{dx}$$

$$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$$

$$= \frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$$= 1/27\left[(4)^{3/2} - (40)^{3/2}\right]$$

(4) $x = 3\sin t$ $y = 3\cos t$

$\frac{dx}{dt} = 3\cos t$ $\frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt.$$

$$= \int_0^{2\pi} \sqrt{9(1)}\ dt$$

$$= \int_0^{2\pi} 3\, dt$$

$$= 3\left[t\right]_0^{2\pi} = 3[2\pi - 0]$$

$$= 6\pi$$

(5) $x = \frac{1}{6} y^2 + \frac{1}{2y}$ on $y \in [1, 2]$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4 - 1}{2y^2}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\left(\frac{y^2+1}{2y}\right)^2}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-3}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{2}$$

Q2

(1) $\int_0^2 e^{x^2} dx$ with $n = 4$

$a = 0,\ b = 2,\ n = 4$

$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\Big[(1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182 + 54.5981)\Big]$$

$$= \frac{0.5}{3}\left[55.5981 + 43.0868 + 114.6326\right]$$

$$= 1.779$$

(2) $\int_0^4 x^2 dx$

$h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2 dx = \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= 64/3$$

$$\int_0^4 x^2 dx = 21.533$$

(3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n = 6$

$L = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$6\pi/18$	$8\pi/18$
y	0	0.4166	0.58	0.70	0.80087	0.8727	0.9[illegible]

$\int_0^{\pi/3} \sqrt{\sin x} = \pi/54 \times 12.1163$

$= \int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049.$

Practical - 7

Topic - Differential Equation

Q.1 Solve the following differential equation.

(1) $x \frac{dy}{dx} + y = e^x$

(2) $e^x \frac{dy}{dx} + 2e^x y = 1$

(3) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

(4) $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

(5) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

(6) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

(7) $\frac{dy}{dx} = \sin^2(x - y + 1)$

(8) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

Q.1

(1) $x \frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = 1/x \quad Q(x) = e^x/x$

$IF = e^{\int 1/x\, dx} = e^{\int 1/x\, dx} = e^{\log x} = x$

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$y(IF) = \int Q(x)\,(I.F)\,dx + c$

$= \int \frac{e^x}{x}\, x\,.dx + c$

$\therefore x.y = e^x + c$

(2) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = 1/e^x$

$P(x) = 2 \qquad Q = e^{-x}$

$IF = e^{\int 2dx}$

$IF = e^{2x}$

$y(IF) = \int Q(IF)\,dx + c = \int \frac{1}{e^x}(e^{2x})\,dx + c$

$= \int \frac{1}{e^x}(e^x.e^x)\,dx + c$

$= \int e^x\,dx + c$

$\therefore y.e^{2x} = e^x + c$

(3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\frac{x}{x}\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$\frac{dy}{dx} + 2/x \times y = \cos x/x^2$

$P = 2/x \qquad Q = \cos x/x^2$

$IF = e^{\int Pdx} = e^{\int 2/x\,dx} = e^{2\log x} = e^{\log x^2} = x^2$

(4) ~~$x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$~~

$y(IF) = \int Q(IF)\,dx + c$

$\therefore y(x^2) = \int \frac{\cos x}{x^2}.x^2\,dx + c = \sin x + c$

$\therefore y.x^2 = \sin x + c$

(4) $x\,dy/dx + 3y = \frac{\sin x}{x^2}$

$\therefore dy/dx + 3/x \times y = \frac{\sin x}{x^3}$

$P = 3/x \qquad Q = \frac{\sin x}{x^3}$

$(IF) = e^{\int Pdx} = e^{\int 3/x\,dx}$

$\therefore IF = e^{\log x^3} = x^3$

$\therefore y(IF) = \int Q(IF)\,dx + c. = \int \frac{\sin x}{x^3}.x^3\,dx + c$

$\therefore yx^3 = \int \sin x\,dx + c = -\cos x + c$

(5) $e^{2x}\,dy/dx + 2e^{2x}y = 2x$

$\therefore dy/dx + 2y = \frac{2x}{e^{2x}}$ dividing both sides by e^{2x}

$\therefore P = 2 \qquad Q = 2x/e^{2x}$

$IF = e^{\int Pdx} = e^{\int 2dx + c} = e^{2x}$

$\therefore IF = e^{2x}$

$y(IF) = \int Q(IF)\,dx + c = \int 2x/e^{2x}.e^{2x}\,dx + c$

$\therefore ye^{2x} = x + c$

(6) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy$

$\sec^2 x \tan y\, dx = -\sec^2 y \tan x\, dy$

$\frac{\sec^2 x\, dx}{\tan x} = -\frac{\sec^2 y\, dy}{\tan y}$

$\int \frac{\sec^2 x\, dx}{\tan x} = -\int \frac{\sec^2 y}{\tan y} dx$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\therefore \log|\tan x \cdot \tan y| = c$

$\therefore \tan x \cdot \tan y = e^c$

(7) $dy/dx = \sin^2(x-y+1)$

Put $x-y+1 = v$

differentiating on both sides

$x - y + 1 = v$

$1 - dy/dx = dv/dx$

$\frac{1-dv}{dx} = dy/dx$

$1 - dv/dx = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v\, dx = \int dx$

$\tan v = x + c$

$\therefore \tan(x+y-1) = x + c$

(8) $dy/dx = \frac{2x+3y-1}{6x+ay+6}$

Put $2x+3y = v$

$2 + 3dy/dx = dv/dx$

$dy/dx = 1/3\,(dv/dx - 2)$

$1/3\,(dv/dx - 2) = 1/3 \frac{(v-1)}{(v+2)}$

$dv/dx = \frac{v-1}{v+2} + 2$

$dv/dx = \frac{v-1+2v+4}{v+2}$

$= \frac{3v+2}{v+2}$

$= \frac{3(v+1)}{v+2}$

$\int \frac{(v+2)}{(v+1)} dv = 3dx$

$\int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = 3x$

$\therefore v + \log|x| = 3x + c$

$\therefore 2x + 3y + \log|2x+3y+1| = 3x + c$

$\therefore 3y = x - \log|2x+3y+1| + c$

10/01/2020

Practical - 8

Euler's Method

1) $\frac{dy}{dx} = y + e^x - 2$

$f(x,y) = y + e^x - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.57435
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h.f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28304
4	2	9.2831		

$\therefore$ By Euler's Formula
$y(2) = 9.2831$

2) $\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$; $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + n.f(x_n, y_n)$

n	x_0	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	0.104	0.408
2	0.4	0.408	1.1603	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

$\therefore$ By Euler's formula
$y(2) = 1.2942$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$, $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + n.f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

~~∴ By Euler's formula~~
~~y(2) = 285~~

iv) $dy/dx = 3x^2+1$. $y_0 = 2$ $x_0 = 1$ $h = 0$

For $h = 0.5$

$y_{n+1} = y_n + h.f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	3.9048
4	2	8.9048		

$y_{n+1} = y_n + h.f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

$\therefore$ By Euler's formula

$y(2) = 28.5$

AV
13/01/2020

Topic: Limits & Partial order derivatives

(1) Evaluate the following limits

(i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$ (ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

(iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

(2) find f_x, f_y for each of the following f.

(i) $f(x,y) = xye^{x^2+y^2}$ (ii) $f(x,y) = e^x \cos y$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

(3) Using defination find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

(4) find all second order partial derivatives of f. Also verify wether $f_{xy} = f_{yx}$

(i) $f(x,y) = \frac{y^2 - xy}{x^2}$ (v) $f(x,y) = x^3 + 3x^2y^2x - \log(x^2+1)$

(ii) $f(x,y) = \sin(xy) + e^{x+y}$

(5) Find the linearization of $f(x,y)$ at given point.

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

(ii) $f(x,y) = 1 - x + y \sin x$ at $(\pi/2, 0)$

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

Q1
(i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$ Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{-4(-1)+5}$

$= \frac{-64+3+1-1}{4+5}$

$= \frac{61}{9}$

(ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit,

$= \frac{(0+1)(2^2+0-4(2))}{2}$

$= \frac{1(4+0-8)}{2}$

$= -4/2$

$= -2$

(iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$, Denominator $= 0$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x+yz)}{x^2} \times \frac{(x-yz)}{(x-yz)}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

On Applying limit

$= \frac{1+1(1)}{(1)^2} = 2$

Q.2
(i) $f(x,y) = xye^{x^2+y^2}$

$\therefore f(x) = \partial/\partial x \, (f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\therefore f(x) = 2xye^{x^2+y^2}$

$f(y) = \partial/\partial y \, (f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2}) = xe^{x^2+y^2}(2y)$

(ii) $f(x,y) = e^x \cos y$

$\therefore f(x) = \frac{\partial}{\partial x}(e^x \cos y)$

$\therefore f(x) = e^x \cos y$

$f(y) = \frac{\partial}{\partial y}(f(x,y))$

$\therefore f(y) = \frac{\partial}{\partial y}(e^x \cos y) = -e^x \sin y$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f(x) = 3x^2y^2 - 6xy$

$f(y) = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f(y) = 2x^3y - 3x^2 + 3y^2$

Q3.

(i) $f(x,y) = \frac{2x}{1+y^2}$

$f(x) = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2-0}{(1+y^2)^2}$

$= \frac{2(1+y^2)}{(1+y^2)(1+y)^2} = \frac{2}{1+y^2}$

At $(0,0)$

$\therefore f(x) = \frac{2}{1+0} = 2$

$f(y) = \frac{\partial}{\partial y}\frac{(2x)}{(1+y^2)}$

$= \frac{1+y^2\frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

At $(0,0)$

$= \frac{-4(0)(0)}{(1+0)^2}$

$\therefore f(y) = 0$

Q4) i) $f(x,y) = \frac{y^2-xy}{x^2}$

$f(x) = \frac{x^2\frac{\partial}{\partial x}(y^2-xy) - (y^2-xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$

$= \frac{x^2(-y) - (y^2-xy)(2x)}{x^4}$

$= \frac{-x^2y - 2x(y^2-xy)}{x^4}$

$f(y) = \frac{2y-x}{x^2}$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{d}{dx}(x^4)}{(x^4)^2} \quad - 1$$

$$f(yy) = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2} \quad - 2$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^3}{x^4} \quad - 3$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2\frac{d}{dx}(2y - x) - (2y - x)\frac{\partial}{\partial x}(x^2)}{(x^2)^3}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad - 4$$

from 3 & 4

$f(xy) = f(yx)$

(11) $f(x, y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f(x) = \frac{\partial}{\partial x}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f(y) = \frac{\partial}{\partial y}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$$

$$= 0 + 6x^2y - 0$$

$$= 6x^2y$$

$$f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1\,\frac{\partial(2x)}{\partial x} - 2x\,\frac{\partial(x^2+1)}{\partial x}}{(x^2+1)^2}\right)$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) - 1$$

$$f_{yy} = \frac{\partial}{\partial y}(6x^2y)$$

$$= 6x^2 \quad - 2$$

$$f(xy) = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 0 + 12xy - 0 = 12xy \quad - 4$$

$$f(yx) = \frac{\partial}{\partial x}6x^2y$$

$$= \cancel{tx}\,12xy \quad - 3$$

from 3 & 4

$f_{xy} = f_{yx}$

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f(x) = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f(y) = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}$

$= -y^2\sin(xy) + e^{x+y}$ — (1)

$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + e^{x+y}$ — (2)

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — (3)

$f_{yx} = \frac{d}{\partial x}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — (4)

from (3) & (4)

$f_{xy} \neq f_{yx}$

Q5)

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\quad$ $f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad$ $= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\quad$ f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + 1/\sqrt{2}\,(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \pi/2 + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$ $\quad$ $f_y = 0 - 0 + \sin x$

~~$f_x = 0 - 1$~~

f_x at $(\pi/2, 0) = -1 + 0 = -1$ $\quad$ f_y at $(\pi/2, 0) = \sin\pi/2 = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-0)$

$= 1 - \pi/2 - x + \pi/2 + y$

$= 1 - x + y$

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = 1/x + 0$ $\qquad$ $f_y = 1/y + 0$

f_x at $(1,1) = 1$ $\qquad$ f_y at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

24/01/2020

Practical-10

Topic: Directional derivative, maxima, minima, gradient vector, tangent & normal vector.

Q1) (i) $f(x,y) = x + 2y - 3$ $\qquad$ $a = (1,-1)$ $\quad$ $u = 3i - j$

Here,

$u = 3i - j$ is not a unit vector

$\bar{u} = 3i - j$

$|u| = \sqrt{10}$

$\therefore$ unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3i - j)$

$= 1/\sqrt{10}\,(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, -1/\sqrt{10}\right)$

Now,

$f(a+hu) = f((1,-1) + h(3/\sqrt{10}, -1/\sqrt{10}))$

$= 1 + 2(-1) - 3 + \left(\frac{3h}{\sqrt{10}}, \frac{-h}{\sqrt{10}}\right) + \left(1 + \frac{3h}{\sqrt{10}}, \frac{-1-h}{\sqrt{10}}\right)$

$= 4 + \frac{3h}{\sqrt{10}} - 4 - 1 + \frac{3}{\sqrt{10}} + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 - 2 - 3 + 3h/\sqrt{10} - 2h/\sqrt{10}$

$= -4 + h/\sqrt{10}$

$\therefore D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{-4 + \frac{h}{\sqrt{10}} - (-4)}{h}$

$= \lim_{h \to 0} \frac{h}{h\sqrt{10}}$

$= \frac{1}{\sqrt{10}}$

ii) $f(x,y) = y^2 - 4x + 1 \quad a = (3,4) \quad u = i + 5j$

Here,

$u = i+5j$ is not a unit vector

$\bar{u} = \bar{i} + 5\bar{j}$

$|\bar{u}| = \sqrt{26}$

∴ unit vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{26}}(\bar{i}+5\bar{j})$

$= \frac{1}{\sqrt{26}}(1,5)$

$= (1/\sqrt{26}, 5/\sqrt{26})$

Now,

$f(a+hu) = f\left((3,4) + h\left(1/\sqrt{26} + 5/\sqrt{26}\right)\right)$

$= f\left(3 + h/\sqrt{26}, 4 + \frac{5h}{\sqrt{26}}\right)$

$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$= \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}}}{h}$

$= \lim_{h \to 0} \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$

$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$

$= \frac{36}{\sqrt{26}}$

iii) $f(x,y) = 2x + 3y$

$a = (1,2)$, $u = 3i + 4j$

Here,

$u = 3i + 4j$ is not a unit vector.

$\bar{u} = 3\bar{i} + 4\bar{j}$

$|\bar{u}| = \sqrt{25} = 5$

∴ unit vector along $u = \frac{\bar{u}}{|\bar{u}|} = \frac{1}{5}(3\bar{i} + 4\bar{j})$

$= 1/5 \, (3,4)$

$= (3/5, 4/5)$

Now;

$f(a+hu) = f((1,2) + h(3/5, 4/5))$

$= f(1 + 3h/5,\ 2+4h/5)$

$= 2(1+3h/5) + 3(2 + 4h/5)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= 8 + \frac{18h}{5}$

$\therefore D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(u)}{h}$

$= \lim_{h\to 0} \frac{8 + \frac{18h}{5} - 8}{h}$

$= \lim_{h \to 0} \frac{18h}{5h}$

$= 18/5$

Q2

i $f(x,y) = x^y + y^x \qquad a = (1,1)$

$f_x = y(x^{y-1}) + y^x \log y$

$f_y = x(y^{x-1}) + x^y \log x$

$\nabla f(x,y) = f_x, f_y$

~~$= (y(x^{y-1}) + y^x$~~

$= (yx^{y-1} + y^x \log y,\ xy^{x-1} + x^y \log x)$

$\nabla f(x,y)$ at $(1,1)$

$= (1(1)^0 + 1 \log(1),\ 1(1)^{1-1} + 1 \log 1$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1} x) \cdot y^2$

$a = (1,-1)$

$f_x = y^2 (1/(1+x^2)) = y^2/(1+x^2)$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2},\ 2y \tan^{-1} x\right)$

$\nabla f(x,y)$ at $(1,-1)$

$= \left(\frac{(-1)^2}{1+(1)^2},\ 2(-1)\tan^{-1}(1)\right)$

$= (1/2,\ -2\pi/4)$

$= (1/2,\ \pi/2)$

(iii) $f(x,y,z) = xyz - e^{x+y+z} \qquad a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z},\ xz - e^{x+y+z},\ xy - e^{x+y+z})$

$\nabla f(x,y,z)$ at $(1,-1,0)$

$= (-1(0) - e^{1-1+0},\ 1(0) - e^{1-1+0},\ 1(-1) - e^{1-1+0})$

$= (0-1,\ 0-1,\ -1-1) = (-1,-1,-2)$

Q3) 70

i) $x^2\cos y + e^{xy} = 2$

at $(1,0)$

$f(x,y) = x^2\cos y + e^{xy} - 2$

$f_x = 2x\cos y + ye^{xy}$

$f_y = -x^2\sin y + xe^{xy}$

$(x_0, y_0) = (1,0)$

f_x at $(1,0) = 2(1)\cos 0 + 0$
$= 2$

f_y at $(1,0) = -(1)^2\sin(0) + 1(e)^{(10)}$
$= 1$

$f_x(x - x_0) + f_y(y - y_0) = 0$

$\therefore 2(x-1) + 1(y-0) = 0$

$\therefore 2x - 2 + y = 0$

$\therefore 2x + y - 2 = 0 \rightarrow$ eq^n of tangent

Now, for eq^n of normal

$bx + ay + d = 0$

$x + 2y + d = 0$

$1 + 2(0) + d = 0$

$\therefore d = -1$

$x + 2y - 1 = 0 \rightarrow$ eq^n of normal

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$
at $(2,-2)$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2$

$f_x = 2x + 0 - 2 + 0 + 0$
$= 2x - 2$

f_x at $(2,-2) = 2(2) - 2$
$= 2$

$f_y = 0 + 2y - 0 + 3 + 0$
$= 2y + 3$

f_y at $(2,-2) = 2(-2) + 3$
$= -1$

$\therefore$ eq^n of tangent.

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-1)(y+2) = 0$

$2x - 4 - y - 2 = 0$

$2x - y - 6 = 0 \rightarrow$ eq^n of tangent.

for equation of normal.

$bx + ay + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-(2) + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$d = 6$

$\therefore -x + 2y + 6 = 0 \rightarrow$ eq^n of normal.

Q4) i) $x^2 - 2yz + 3y + xz = 7$
at $(2,1,0)$

$f(x,y,z) = x^2 - 2yz + 3y + xz - 7$

$f(x) = 2x - 0 + 0 + z - 0$
$= 2x + z$

$\therefore f_x$ at $(2,1,0) = 2(2) + 0$
$= 4$

$f_y = -2z+3+0-0$
$= -2z+3$
f_y at $(2,1,0) = -2(0)+3$
$= 3$
$f_z = 0-2y+0+x-0$
$= -2y+x$
f_z at $(2,1,0) = -2(1)+2$
$= 0$

eqn of tangent;
$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$
$4(x-2)+3(y-1)+0(z-0)=0$
$4x-8+3y-3=0$
$\therefore 4x+3y-11=0$
eqn of normal
$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$
eqn of normal
$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z-0}{0}$

ii) $3xyz-x-y+z=4$
at $(1,-1,2)$
$f(x,y,z) = 3xyz-x-y+z+4$
$f_x = 3yz-1-0+0+0$
$= 3yz-1$
f_x at $(1,-1,2) = 3(-1)(2)-1$
$= -7$
$f_y = 3xz-0-1+0+0$
$= 3xz-1$
f_y at $(1,-1,2) = 3(1)(2)-1 = 5$

$f_z = 3xy-0-0+1-0$
$= 3xy+1$
f_z at $(1,-1,2) = 3(1)(-1)+1 = -2$
eqn of tangent.
$f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$
$\therefore -7(x-1)+5(y+1)+(-2)(z-2)=0$
$\therefore -7x+7+5y+5-2z+4=0$
$\therefore -7x+5y-2z+16=0$
eqn of normal;
$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$

$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$

Q5)
i) $f(x,y) = 3x^2+y^2-3xy+6x-4y$
$\therefore f_x = 6x+0-3y+6-0$
$= 6x-3y+6$ —— ①
$f_y = 2y-3x+0-4$
$= 2y-3x-4$ —— ②

$f_x = 0$
$6x-3y+6=0$
$3(2x-y+2)=0$
$2x-y+2=0$
$2x-y=-2$ —— ③

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ —— (4)

multiplying (3) by (2) & subtract (4) from (3)

$$4x - 2y = -4$$
$$-\ 2y - 3x = 4$$
$$7x = 0$$
$$x = 0$$

Substituting value of x in 3

$$2(0) - y = -2$$
$$-y = -2$$
$$y = 2$$

$\therefore$ critical points are $(0, 2)$

Now,

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12 + 9$

$= 33 > 0$

here, $r > 0$ and $rt - s^2 > 0$

$\therefore f$ has minimum at $(0, 2)$

$\therefore f(0, 2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) \ldots$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

$f(x, y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy - 0$

$= 8x^3 + 6xy$

$f_y = 0 + 3x^2 - 2y$

$= 3x^2 - 2y$

Now,

$f_x = 0$

$8x^3 + 6xy = 0$

$\Rightarrow 2x(4x^2 + 6xy) = 0$

$4x^2 + 6xy = 0$ — (1)

$f_y = 0$

$3x^2 - 2y = 0$ — (2)

multiplying eq^n (1) by (3) & (2) by (4) & subtracting (2) from (1)

$$12x^2 + 18y = 0$$
$$-\ 12x^2 - 8y = 0$$
$$24y = 0$$
$$y = 0 \quad \text{— (3)}$$

Substituting (3) in (2), we get.

$$3x^2 - 2(0) = 0$$
$$3x^2 = 0$$
$$x^2 = 0$$
$$x = 0 \quad \text{— (4)}$$

$\therefore$ critical point are $(0, 0)$

Now,

$r = f_{xx} = 24x^2 + 6y$

$t = f_{yy} = -2$

$s = f_{xy} = 6x$

$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$

$2 - 48x^2 - 12y - 36x^2$

$= -84x^2 - 12y$

At $(0,0)$

$r = 24(0)^2 + 6(0)$

$= 0$

$S = 6(0) = 0$

$rt - s^2 = -84(0)^2 - 12(0) = 0$

$r = 0$ & $rt - S^2 = 0$

$\therefore$ nothing can be said.

A
07/01/2021