MTH 4115 – Numerical Methods for Differential Equations in Finance

Final Project

Honesty Statement. My submission of the project files constitutes my pledge that all of the submission is my own work. I understand that if plagiarism is found in my submission, my professor will follow the procedures on academic dishonesty set forth by Baruch College.

1. Solve the Black-Scholes PDE for a European put option with the backward Euler method.

$$\begin{aligned} &v_t = \frac{\sigma^2 s^2}{2} v_{ss} + (r-q) s v_s - rv \triangleq \mathcal{L}v, \quad (s,t) \in (0, S_m) \times (0, T], \\ &v(s,0) = \max(K-s, 0) \triangleq g(s), \quad \text{(initial condition)} \\ &v(0,t) = e^{-rt}K, \quad v(S_m, t) = 0, \quad \text{(boundary condition).} \end{aligned} \tag{1}$$

   - `v=bs_eu_be(N, M, K, T, r, q, sigma, Sm)`.
   - `M` denotes the number of sub-intervals in t.
   - `N` denotes the number of sub-intervals in s.
   - `v` is a column vector of length $N+1$, representing the option prices at T.

2. Solve (1) with the Crank-Nicolson method.

   - `v=bs_eu_cn(N, M, K, T, r, q, sigma, Sm)`.

3. Solve the Black-Scholes PDE for an American put option with the backward Euler method.

$$\begin{aligned} &v_t \geq \mathcal{L}v, \\ &v \geq g, \quad (v_t - \mathcal{L}v)(v-g) = 0, \\ &v(s,0) = g(s), \quad \text{(initial condition)} \\ &v(0,t) = K, \quad v(S_m, t) = 0, \quad \text{(boundary condition).} \end{aligned}$$

   - `v=bs_am_be(N, M, K, T, r, q, sigma, Sm)`.