MTH 4115 – Numerical Methods for Differential Equations in Finance

Project 4

Honesty Statement. My submission of the project files constitutes my pledge that all of the submission is my own work. I understand that if plagiarism is found in my submission, my professor will follow the procedures on academic dishonesty set forth by Baruch College.

1. Approximate $\mathcal{L}[f] = c_1(x)f''(x) + c_2(x)f'(x) + c_3(x)f(x)$ on

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b, \quad x_i = a + ih, \quad h = \frac{b-a}{n}, \quad 0 \le i \le n.$$

   - `[Lf A] = fd_matrix(a,b,c1,c2,c3,f)`.
   - `c1, c2, c3, f` are column vectors of length $n+1$, denoting function values.
   - `Lf` is a column vector of length $n+1$, denoting the approximations of $\mathcal{L}[f](x_i)$.
   - `A` is a matrix of size $(n+1) \times (n+1)$, denoting the differentiation matrix.
   - Use three-point centered-difference formulas for the interior points.
   - Ignore the top and bottom elements of `Lf` and set them to 0.
   - Ignore the top and bottom rows of `A` and set them to 0.

2. Construct the differentiation matrix for

$$\begin{aligned} &\mathcal{L}[u](x) = r(x), \quad x \in (a,b), \\ &a_1 u(a) + b_1 u'(a) = g_1, \quad a_2 u(b) + b_2 u'(b) = g_2. \end{aligned} \tag{1}$$

   - `[A L U P] = bvp_matrix(a,b,c1,c2,c3,a1,b1,a2,b2)`.
   - `A` is a matrix of size $(n+1) \times (n+1)$, denoting the differentiation matrix.
   - Use the FDF2 formula for the first row of `A` and BDF2 for the last.
   - `L, U, P` are the $PA = LU$ decomposition of A.

3. Solve (1).

   - `[u] = bvp_solve(r,g1,g2,L,U,P)`.
   - `r` is a column vector of length $n+1$, representing the right hand side.
   - The top and bottom elements of `r` are not used and shall be replace by `g1` and `g2`.
   - Solve it with forward and backward substitutions. You may simply use the backslash to solve the triangular systems.