Homework 2, Due March 3
The Role of Correlation

• "The correlations between financial quantities are notoriously unstable" — Paul Wilmott, quant guru.

• "Co-association between securities is not measurable using correlation... Anything that relies on correlation is charlatanism" — Nassim Nicholas Taleb, hedge fund manager.

• "The most dangerous part is when people believe everything coming out of it" — David Li, inventor of the Gaussian copula model for CDO valuation.

• "The corporate CDO world relied almost exclusively on this copula-based correlation model" — Darrell Duffie, Stanford University finance professor.

• "...it was not obvious that a pool of mortgage-backed securities rated BBB could be transformed into a new security that is mostly rated triple-A. But math made it so." — from the Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States. The link to the full report is `www.gpo.gov/fdsys/pkg/GPO-FCIC/pdf/GPO-FCIC.pdf`.

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1. Read Wired Magazine's 2-23-09 article *Recipe for Disaster: The Formula that Killed Wall Street*, by Felix Salmon. You can find it at `www.wired.com/2009/02/wp-quant/`. The first four quotes above appear in this article.

• The code `RiskyAnnuity.cpp` values a promised stream of monthly cash flow in the amount of \$100/month for 30 years (the term of a typical mortgage). The paying entity's time to default is modeled as exponentially distributed with a mean that depends on the entity's credit (see table below). If it enters default its promised payments cease. The code computes the expected present value at a risk-free rate of 3% compounded continuously. Based on this computed present value, a continuously compounding yield-to-maturity (YTM) of the promised cash flow is computed. This is done for six credit ratings of the entity. Run the code and verify that the resulting YTMs are:

Credit	Expected Years to Default	YTM (%)	Spread Over Risk-free (bps)
Risk-free	NA	3.00	0
AAA	200	3.50	50
AA	100	4.00	100
A	67	4.50	150
BBB	50	5.00	200
Speculative	33	6.00	300

• Here we assign a credit rating to the five tranches of a Collateralized Debt Obligation (CDO) using a very simple copula-based correlation model. The collateral of the CDO will consist of 20 entities, referred to as *names*, each promising payment into the CDO of \$100 per month for 30 years. The names each have a credit rating of BBB and their times to default $T_i, 1 \leq i \leq 20$, are exponential random variables with mean 50 years. If and when a name enters default it stops making its promised payment into the CDO. Each tranche is promised $\$100 \times 20 \div 5 = \$400$ per month from the CDO. Tranche number 1 is paid first, then tranche number 2 is paid with remaining funds, and so forth in what is called a *cash flow waterfall*. As names default the lower tranches are the first to lose out and may eventually receive no cash flow at all.

2. Use Monte Carlo simulation to value the steam of cash flow to each of the five tranches. For each realization of the CDO, generate uniforms $U_1, U_2, \ldots, U_{20}$ from a Gaussian copula model as follows. Each $U_i = \Psi(N_i)$, where $N_i \sim \mathrm{Normal}\,(0,1)$ and

$$\mathrm{Cov}\,(N_i, N_j) \;=\; \begin{cases} \rho & \text{if } i \neq j \\ 1 & \text{if } i = j, \end{cases}$$

where the correlation ρ is a parameter of the model. Then put $T_i = -50 \ln(U_i)$; this is the time of default for the i^{th} name. Using that the i^{th} name pays \$100 into the CDO at month m if and only if $T_i > m/12.0$, calculate the month-by-month stream of cash flow into the CDO. Calculate how it is then awarded to the each of the five tranches and discount the flows to each tranche at the risk-free rate (3% annually, compounded continuously) to estimate the expected present value (EPV) of the flows going to each of the five tranches. Convert this EPV into a yield-to-maturity, just as in the code `RiskyAnnuity.cpp`. Based on the table above, assign an implied credit rating to each tranche. Do this exercise for values of ρ ranging from 0 to 1 in steps of 0.1.

3. Explain your results in problem 2.