Homework 5

Option Valuation with Stochastic Volatility, Due April 30

In this project you will value call options via Monte Carlo simulation using a GARCH stochastic volatility model. Take initial stock price $S_0 = 100$, risk-free rate $r = 5\%$ per year, time to expiration $T = 0.5$ years, and use the model of Chapter 16, §3, to generate the stock price paths. The starter code `StoVol.cpp` will generate illustrative paths for you. Note that in this code the long-run volatility is 30% annually and the stochastic volatility starts off at 35% annually — use these values in your simulations. The values of α, β, and γ are those that maximum likelihood estimation produced in Chapter 16 for Exxon Mobil.

1. Use your model to value a call option with strike $K = 0$. The payoff at time T is $\max(S_T - 0, 0) = S_T$, so this "option" replicates the payoff of the stock at time T. Its value today should therefore be S_0 (taken to be 100). If it is not, there's a mistake somewhere in your code.

2. Value call options on XOM with strikes of 60, 70, 80, 90, 100, 110, 120, 130, 140, 150 and 160.

3. The code `ImpliedVol.cpp` computes a call option's implied volatility in the Black-Scholes (constant volatility) framework. Once you have computed the value of these eleven call options in the GARCH stochastic volatility framework, calculate their B-S implied volatility. Plot the results with the option's strike on the horizontal axis and the implied volatility on the vertical axis. What do you notice? Sanity check: the B-S implied volatilities should all be between 30 and 35. Use this as the vertical scale.