

Final Project
Fixed-Income Valuation — Due May 21

• For these problems use the Salomon model and take the relative volatility of the short rate to be 25% annually. We will consider only flat par yield curves, i.e., of the form $c(t) = r$ for all t . I have given you starter code `DandC.cpp` and some TeX files for graphical display. (You should probably also refer to the code `SalomonModel.cpp` in the Tool Box — specifically to the callable bond valuation function.)

1. Suppose Dollars & Cents Finance wants to sell you a security that works as follows. D&C will make a payment into an account every 6 months starting immediately. The first payment (made today) is \$300. Subsequent payments decline by \$5 every 6 months to 0 at the end of 30 years. The account initially has nothing in it and it earns no interest on its balance. You have the option to withdraw the entire balance at any time — but when you withdraw the balance the payments stop and the arrangement is terminated. Calculate and plot the fair price for this security as a function of the rate r for values of r ranging from 0 to 10% in steps of 1%. For $r = 5\%$ graphically display the nodes in the lattice at which it is optimal to withdraw (as we did in class for when to call a callable bond). Explain the results. [Hint: when $r = 5\%$ the fair value is 3737.87.]

2. *Duration* is a measure of a fixed-income security's price sensitivity to changing interest rates. Its units are years and it is defined by $D = -\frac{1}{P} \frac{dP}{dr}$, where P is the security's value and r is the rate level. Numerically estimate the duration of the security in problem 1 if $r = 5\%$ as follows. Shift the entire par curve up by an amount Δr and re-price the security — call this price P^+ . Now shift the entire par curve down by Δr and compute P^- . If P is the price with the 5% par curve, we have

$$D \approx -\frac{1}{P} \frac{P^+ - P^-}{2\Delta r}.$$

Estimate the duration first by taking Δr to be 100 basis points and then with $\Delta r = 1$ bp. Which estimate do you think is best?

3. A fixed-income security's *convexity* is defined by $C = \frac{1}{P} \frac{d^2P}{dr^2}$ and may be similarly numerically approximated:

$$C \approx \frac{1}{P} \frac{\frac{P^+ - P}{\Delta r} - \frac{P - P^-}{\Delta r}}{\Delta r} = \frac{1}{P} \frac{P^+ - 2P + P^-}{\Delta r^2}.$$

Estimate the convexity of the security in problem 1 if $r = 5\%$ by taking Δr to be 100 basis points and also 1 bp. Which estimate do you think is best?

4. Superimpose on your price plot in problem 1 a quadratic fit of the form $\hat{P}(r) = a(r - .05)^2 + b(r - .05) + c$. Take $c = P(.05) = 3737.87$; b should be your numerical estimate of $P'(.05)$ from problem 2; a should be $\frac{1}{2}$ of your estimate of $P''(.05)$ from problem 3. Do this first for estimates coming from the 100 bp shift and then for the estimates coming from the 1 bp shift. Which produces the better fit near $r = .05$? Explain.

I challenge you to solve this!:

5. Suppose the security permits you withdraw the money k times before D&C stops making the payments and terminates the arrangement. Here $1 \leq k \leq 60$. For example, if $k = 3$, if you so choose you could withdraw $300 + 295 + 290 = 885$ at time 1 year, then 285 at time 1.5 years, and then $280 + 275 = 555$ at time 2.5 years at which point the arrangement would terminate. Show the fair price as a function of k .

• In addition to numerical and graphical data, please carefully write up the algorithm that you use to solve problem 1 and, hopefully, problem 4. Your write-up should be sufficiently detailed and clear that I could code it up! Submit your code in a separate file.