DESCRETE UNIT SQUARE COVER PROBLEM

A Project Report Submitted

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by

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CERTIFICATE

This is to certify that the work contained in this project report entitled "**Descrete unit square cover problem**" submitted by **Raj Kamal** (**Roll No.: 09012321**) to Indian Institute of Technology Guwahati for partial requirement of **MA498 Project-I** in Mathematics and Computing has been carried out by him under my supervision.

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ABSTRACT

Given a set A of n red points and a set B of m blue points on a 2-dimensional plane, covering points with unit squares is about finding a subset D of B of minimal cardinality such that if we draw unit squares with centres around points in D all the red points from A will lie inside the unions of squares in D. The purpose of this project is to come up with an efficient approximation scheme to cover points with unit squares. This is a variant of set cover problem. The decision version of set covering is NP-complete. Lund and Yannakakis [6] showed that set covering cannot be approximated in polynomial time within a factor of $0.72 \ln n$.

Contents

Chapter 1

Introduction

A set A of red points and another set B of blue points are given in 2-dimensional plane, unit squares are drawn taking blue point as center. Now descrete unit square cover problem is finding minimal subset $D \subseteq B$, such that all red points lie with in the unit squares drawn taking blue points from D. This is an instance covering problem. In combinatorics and computer science, covering problems are computational problems that ask whether a certain combinatorial structure covers another, or how large the structure has to be to do that. Covering problems are minimization problems. Packing problems are dual of covering problems. Wireless sensor networks are formed by connected sensors that each have the ability to collect, process, and store environmental information as well as communicate with others via inter-sensor wireless communication. These characteristics allow wireless sensor networks to be used in a wide range of applications. Covering points with unit disk finds its application in designing wireless sensor networks. Another problem where covering with squares or rectangle, has an important application is related to image processing, discussed in Tanimoto and Fowler [7]. None of the above problems was reported to have bounded error ratio approximation algorithm. We shall call an algorithm a δ-approximation, $\delta > 0$, for a certain problem if the error of the value of the optimal solution does not exceed δ. Our main focus is to identify a δ approximation algorithm such that δ is as small as possible. In some case one specify a family of algorithm such that for each $\epsilon > 0$ there is an ϵ-approximation algorithm in the family that solves a given problem instance with in relative error ϵ. Such a family is called an approximation scheme. The running time of approximation algorithm increases monotonically with $\frac{1}{\epsilon}$. If the functional dependency of the running time is on the size of the input and $\frac{1}{\epsilon}$ is polynomial, then the scheme is said to be fully polynomail, if on the other hand, it is polynomial only in the input scheme, the scheme is called polynomal. All the problems descibed and consequently their extensions are NP-Complete. As such there is no fully polynomial scheme for these problems, unless NP=P [4].

Chapter 2

Related work

- **Minimum geometric disk cover** Here we are given set of points and we have to find unit disk of minimum cardinality whose union covers the points. Here disk centers are not restricted to the set but can be arbitary points from the plane. Again, this problem is NP-hard [3] and has a PTAS solution [[8], [1]].
- **Descrete K centres** Given two sets of point in the plane P and Q and an integer K and we have to find a set of K disks centered on points in P whose union covers Q such that the radius of the largest disk is minimized. This is contraint optimization problem. We have to get a set Q that consists of K disks whose centers are from points in P and constraint is that disks can have atmost radius one. This problem is NP-Hard.
- **Descrete unit disk cover Problem** Given a set P of n points and a set D of m unit disks and DUDC problem is finding minimum cardinality subset D^* covering all the points in P[5]. The research was done by Das et al., where they proposed constant factor approximation algorithm to the problem in O(nlog n + m log m + mn). They brought down the approximation approximation factor to 18 which was previously 22 by Claude et al., [2].

Chapter 3

Preliminaries

Restricted line separable descrete unit disk cover(RLSDUDC):Suppose we are given a set P of n points and another set D of m points such that $\forall p_i \in P$ $\exists$ at least one point d_i from D which covers it i,e if we draw unit disk center around d_i, then p_i will be inside the disk and points in P and center of disks in D are separated by line. If such kind of condition exist then there exist an optimal set $D^* \subseteq D$ such that each point in P is covered by drawing unit disks with centeres at points in D^*. RLSDUDC is a problem of finding that optimal D^* [2].

Line Separable Descrete Unit Disk Cover(LSDUDC):Suppose we have set P of n point and a set D of m points in a plane. Line ℓ divides the plane into two halves ℓ^+ and ℓ^-. Set P lies in ℓ^+. Set D which contains center of m unit disks lies in $\ell^+ \cup \ell^-$. $\forall p_i \in P$ $\exists$ at least one point d_i from D which covers it i,e if we draw unit disk center around d_i, then p_i will be inside the disk and points in P. LSDUDC is problem of finding minimum sized set $D^* \subseteq D$ that cover all points in P with the constriants describe above [2].

Theorem 3.0.1. *The RLSDUDC problem can be solved optimally where authors [2] solves the LSDUDC problem with a 2-factor approximation. The running time for both solutions is* $O(nlogn + mn)$ *where* $m = |D|$ *and* $n = |P|$ *.*

Theorem 3.0.2. *Descrete unit disk cover problem has an 18-approximation result in* $O(m \log m + n \log n + mn)$ *time [5].*

Chapter 4

Testing Feasiblity

Checking feasiblity is an important step while solving an optimization problem. In descrete unit square cover problem we have to find minimal subset set of blue points from a given set of blue points in 2 dimensional plane such that unit squares drawn around points in subset covers all the red points which are also given in 2 dimensional plane. So here we are searching for minimal subset.In order to proceed for the solution first we have to be sure whether the solution exists or not i.e, checking the feasiblity of problem. For testing feasiblity, we first sort m red point and n blue points in 2 dimensional plane with respect to x-coordinate to get A and B respectively. For each red point $r_i \in A$, where A is sorted set of n red points with respect to x-coordinate, first collect blue points $b_i \in B$ where B is sorted set of blue points with respect to x-coordinate, whose x-coordinate lies in the $[x_{r_i} - 0.5, x_{r_i} + 0.5]$. Suppose we get a set B_i of blue points then for each blue point $b_i \in B_i$ we calculate euclidean distance between $b_i$and r_i, if the distance calculated is found out to be less than one then r_i can be covered by b_i. Similarly we do for each red points $r_i \in A$. If we are able to get atleast a blue point for each red point in A satisfying euclidean distance less than one then the descrete unit square cover problem has a solution, otherwise not. Time Complexity $(m \log m + n \log n + mn)$.

Chapter 5

Descrete unit square cover problem

Descrete unit square cover problem is solvable using the descrete unit disc cover [5] in 18 approximation. I have tried solving problem combining shifting strategy [1] and using the result of line separable descrete unit disc cover [2] but the approximation was 32.Here is my approach. Given a set A of n red points and a set B of m blue points in two dimensional plane.

- m red points are sorted in X and Y direction separetely and stored in R_x and R_y array where R_x and R_y contains points in sorted X and Y coordinate respectively.

- n blue points are sorted in X and Y direction separetely and stored in B_x and B_y array where B_x and B_y contains points in sorted X and Y coordinate respectively.

- a $m * n$ bit matrix BM is created and each cell is initialized to 0. For each red point r_i from A we consider all the points that lies in the unit square drawn taking r_i as center. Out of those points if we find a blue point b_j from set B then set $BM[j][i] = 1$. Thus we prepare a bit matrix BM by which we can see what all blue points $bj \in B$ that covers the red point r_i by seeing the ith column of BM and all those blue points corresponding to row marked 1 in the ith column are points that can cover r_i.

- we also create a boolean array Rb whose each entry cooresponds to a $r_i \in A$, each entry initialized to false.

Now I partioned the 2-dimensional plane into $1 * 1$ square grids. Suppose k grids are formed.Now I tried finding cover of red points in each square grid with the blue points that are also in same grid.

- In each grid, we draw a line ℓ_1 that divides it into two rectangles of dimension each $0.5 * 1$. ℓ_0 and ℓ_2 define the upper and lower horizontal boundary of grid. Define upper rectangle as rectangle formed by $[\ell_0, \ell_1]$ as upper and lower boundary and similarly lower reactangle as rectangle formed by $[\ell_1, \ell_2]$.

- we take each red points in upper rectangle that can be covered by squares drawn by taking blue points in lower rectangle as center of the square and make a set $P1$ from those red points. Again we taken each red point in lower rectangle that can be cover by square drawn taking blue point that are in upper rectangle as center and create a set $P2$ from those red points.

- Now we take $P1$ and apply LSDUDC with respect to line ℓ_1 to cover points in P1. Suppose we get a set $S1$ as the result of LSDUDC. Now same we apply for $P2$. We apply LSDUDC with respect to line ℓ_1 to cover points in $P2$, from it we get a set $S2$. Now in the process of applying LSDUDC we mark Rb_i as true for each red point r_i that are covered.

- we create a set $cover1 = S1 \cup S2$ that corresponds to all those blue points that cover red points that are in grid.
- Similarly we do the same for each grid and get $cover1, cover2, ..., coverk$ and create a set $cover = cover1 \cup cover2 \cup cover3.... \cup coverk$.
- Now we scan the the boolean array Rb and check entries which are marked false, for those entries, scan the BM matrix and check all those blue points that can cover it.Suppose Rb_i is marked false which implies r_i is not yet covered so we check the ith column of BM matrix and create a set $temp$ containing all blue points that covers it. If out of blue points in set $temp$, there exist a blue point that is already in cover then make that Rb_i true, otherwise take an arbitary blue point from the $temp$ that covers r_i and add to set $cover$ and make Rb_i true.
- Finally we obtain a $cover$ containing blue points that can cover all red points given.

I am applying LSDUDC twice in each grid which creates 4 approximation [2]. Now for each red points in grid,it may be cover by blue points that are in left,right grids and may be also by upper and lower grids, which overall adds 8 to the approximation factor. Since we are applying shifting strategy so overall approximation becomes $8 * 4 = 32$ [1] which is much higher approximation than DUDC approach so the whole amount of work is not benefecial. My future work will be going to reduce the approximation factor .

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