Hardware Implementation of Relay Control with Enhanced Autotune Identification Using Machine Learning

Manish Kumar
Electronics and Electrical Engineering
Indian Institute of Technology Guwahati
Assam, India
rajmanishsingh47@gmail.com

Somanath Majhi
Electronics and Electrical Engineering
Indian Institute of Technology Guwahati
Assam, India
smajhi@iitg.ac.in

***Abstract*—Relay-based parameter identification of dynamical systems ensures robust and effective control strategies in automatic control systems. It ensures stable, sustained oscillatory output, which is vital for the parameter identification of unknown process models. Describing function-based sets of expressions are derived to identify the parameters of the first order plus dead time (FOPDT) transfer function model for single and multiple poles. A hardware model is constructed with a transportation delay of at least one second and consists of a relay circuit, a process model, and a delay circuit. Machine learning models are integrated to enhance the accuracy of parameter identification. Real-time data from the hardware model, obtained by modifying the physical components of the process model, is utilized to train the machine learning model for improved autotune identification.**

***Index Terms*—Relay control system, Describing function, transportation delay, Machine Learning**

I. Introduction

In industrial setups, numerous controllers are deployed to manage processes, each requiring individual tuning to match specific process dynamics, ensuring satisfactory and robust control performance. However, manual adjustment of these controllers is unreliable and time-consuming. Consequently, automatic tuning techniques have garnered attention among engineers. Relay-based automatic tuning features are commonly integrated into many industrial controllers. These controllers employ a relay feedback mechanism to induce sustained oscillations within the system, enabling the identification of system parameters through explicit expressions derived from the describing function approach [1]. These estimated parameters are subsequently employed for the automatic tuning of controllers. Parameter identification of complex models is very important for autotuning controllers and designing model-based controllers. It identifies an unknown process in terms of the transfer function, using measurements of input and output signals based on the Astrom-Hagglund relay feedback system [2]; Luyben suggested an approach for detecting process transfer functions, also called original ATV (autotune variation) method [3]. The procedure is divided into two phases. First, a relay feedback experiment is performed, and the gain and frequency are recorded from the limit cycle. Second, this information is fitted to a typical process control transfer function, such as a first-, second-, or third-order plus dead time system. Several methods are present to identify the process parameters, but relay feedback-based identification is widely used in industrial processes due to its simple and time-efficient approach. Despite its apparent effectiveness in industrial applications, the ATV approach may be greatly enhanced to estimate system characteristics better. The reason for this is that the nonlinear behavior of the relay leads to inaccuracy when we try to back-calculate the parameters of the linear models. So, the describing function's projected ultimate gain N and estimated ultimate frequency approximates information at critical frequency. When numerous higher harmonics are ignored in favor of merely accounting for fundamental harmonics, severe inaccuracies in the ultimate gain N and ultimate frequency for a typical transfer function result.

Researchers devised many approaches to improve the accuracy and autotune controllers utilizing relay-based identification. Kumar and Padhy described analytical expressions to determine measurement sensitivity, which is the variation in relative error between the time constant (T) and the time delay (D) concerning the limit cycle amplitude (A) and frequency (ω) [4]. Sharma et al. used neural networks to identify stable (first order plus dead time) FOPDT transfer function parameters [5]. Bajarangbali and Majhi have taken a relay with hysteresis to identify the FOPDT transfer function parameters [6]. Pandey and Majhi developed a proportional-integral (PI) controller-based relay feedback technique to improve the identification [7]. Krzystof S. Kula did an online identification of an analog second order plus dead time (SOPDT) with a simulation study [8]. Anees Peringal et al. identify the parameters of aerodynamic and delay sensor dynamics using relay-based identification by selecting a discrete set of values [9]. A. Ayyad et al. used a modified relay feedback test and deep neural network, but this method requires higher mathematical computation [10]. However, these methods did not improve the accuracy after a certain extent for various dynamic errors during real-time process operations.

This paper presents our viewpoint on the evolving field of dynamic systems, focusing on machine learning-based identification and autotuning. We introduce a new autotune identification method that relies on a machine learning algorithm to estimate the parameters of dynamical physical systems accurately. The machine learning model is trained using real-time data collected from the constructed hardware model.

II. Original ATV Method

The original ATV method proposed by Luyben involves one autotune test to determine the ultimate frequency (ω) and amplitude (A) of the limit cycle to determine the parameters of the transfer function model. The ultimate frequency of the limit cycle output is $\omega = 2\pi/T$, where T is the ultimate time period. Fourier series expansion determines the amplitude (A) from fundamental harmonics of limit cycle output. Now, the equivalent gain of nonlinearity (relay) is approximated as $N = 4h/\pi A$, where h is relay height.

A. Identification Method

A typical non-minimum phase stable transfer function model in process control is often assumed to be a stable time-delay process model with no zeros and single or multiple poles.
The general expression for the transfer function model is given below:

$$G(s) = \frac{Ke^{-Ds}}{(T_1 s + 1)^p} = \frac{K(-\lambda)^p e^{-Ds}}{(s-\lambda)^p} \quad (1)$$

where $p = 1, 2, 3, ...$ and $\lambda = -1/T_1$.

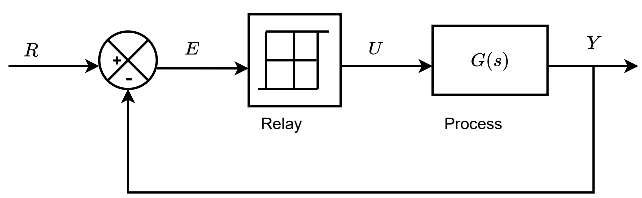

Fig. 1. offline identification technique

B. Analytical Expressions

There are three unknown parameters for each model. Steady-state gain is estimated from the steady-state analysis of the closed-loop system. Other unknown parameters, delay and time constant of the process, are identified from the analytical expressions suggested by Majhi et al. [11].
Condition for sustained stable limit cycle output is:

$$NG(j\omega) = -1 \quad (2)$$

where N is the equivalent gain of the relay.
Analytical expressions for transfer function model with single pole $(p = 1)$:

$$T_1 = \frac{\sqrt{(\frac{4hK}{\pi A_p})^2 - 1}}{\omega} \quad (3)$$

$$D = \frac{\pi - arctan(\omega T_1)}{\omega} \quad (4)$$

Now, the analytical expression for the transfer function model with multiple pole $(p = 2)$:

$$D = \frac{Tarcsin(1 - \pi A_p/2Kh)}{\pi} \quad (5)$$

$$T_1 = -\frac{Ttan((\pi D/T - \pi)/2)}{\pi} \quad (6)$$

III. Hardware Implementation

We require a specific hardware circuit to showcase the behavior of a dynamical system for parameter identification. Various electrical and electronic components are used to implement hardware. The hardware development part starts with various design calculations like the design of a relay circuit for square wave input to the system model, the design of an RC circuit to show the real-time system model, and the most difficult part of the hardware, the design of a transportation delay circuit to provide enough starting delay in seconds in the output of the system model.

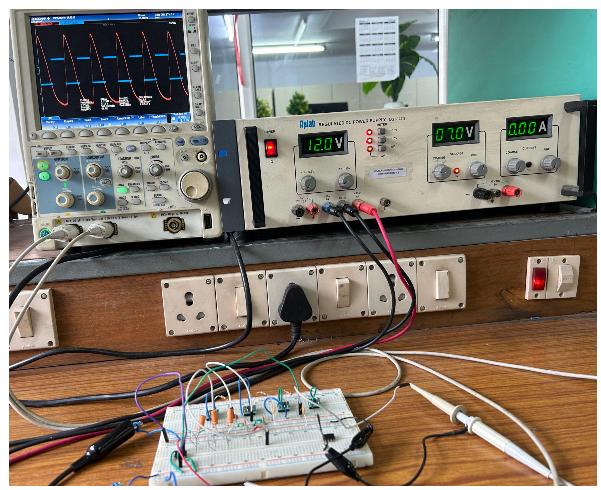

Fig. 2. Hardware setup

Relay is designed based on the fundamental principle of a comparator circuit that revolves around comparing a signal voltage at one input of the operational amplifier (OP747) with a reference voltage [12]. The process model I've developed embodies the system dynamics and is constructed using resistors and capacitors by forming a low-pass filter circuit.
Implementing a time delay within the process model proves essential for capturing lower-frequency datasets, enabling a deeper understanding of process dynamics. I have designed a hardware delay circuit using a voltage buffer amplifier configuration using an AD8031 rail-to-rail operational amplifier known for its high gain bandwidth of 80MHz. While ideal buffer circuits typically introduce minimal delay, additional delay in seconds is achieved by incorporating resistors and capacitors at the output of the buffer.

IV. Improved Autotune Using Machine Learning Algorithm

In this research, describing function-based analytical expressions are derived by approximation of relay gain, so there may be some error in estimated parameters. Here, optimization-based machine learning algorithms are used to predict accurate parameters of the unknown process model input parameters provided. Accurate identification of parameters simplifies the work of process engineers, especially when autotuning the controllers.

A. Input/Output Data Generation and Analysis

1) Data Generation: Data sets play a crucial role in machine learning. Typically, hardware is constructed to collect real-time data. To generate the real-time data, $\pm V_(sat)$ dc voltage supply is provided to the relay circuit op-amp, and negative feedback is given as input of the relay circuit op-amp. The output from this setup is a square wave signal, as depicted in Fig 3. A Square wave signal is given to the process model as input. Due to the delay in the circuit, a low-frequency sustained oscillatory output signal is generated, shown in Fig.4. To identify the process parameters, the amplitude (A_p) and time period (T) of sustained oscillatory output are recorded. Considering the parameters relay height (h), amplitude (A_p), and time period (T) as input parameters, the parameters of the FOPDT transfer function model Delay (D) and time constant (T_1) are identified by using the analytical expressions shown in section II.
To create a robust data set for machine learning training, it's essential to introduce variation in the process model. The variation is achieved by varying the process model components resistors and capacitors.

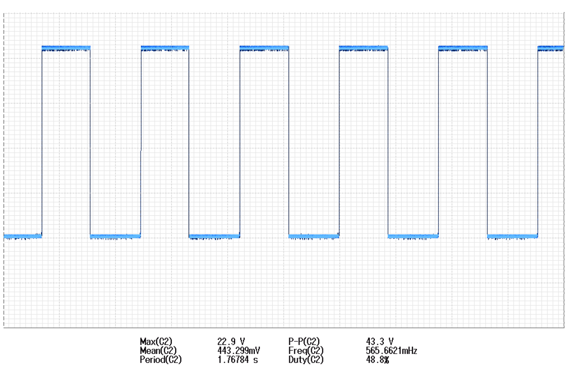

Fig. 3. Hardware relay output signal

2) Data Analysis: Machine learning facilitates the learning process for machines by utilizing training data. The learning process begins with thorough data analysis and establishing the mathematical relationship between the input and output variables, which can take linear or nonlinear forms.

Upon analyzing the graphical relationship between input and output features within the training dataset, as depicted in Fig 5-10, it becomes evident that a highly nonlinear relationship is observed in certain cases. Specifically, the

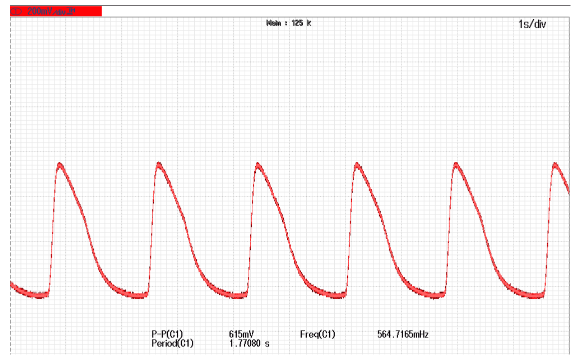

Fig. 4. hardware process output signal

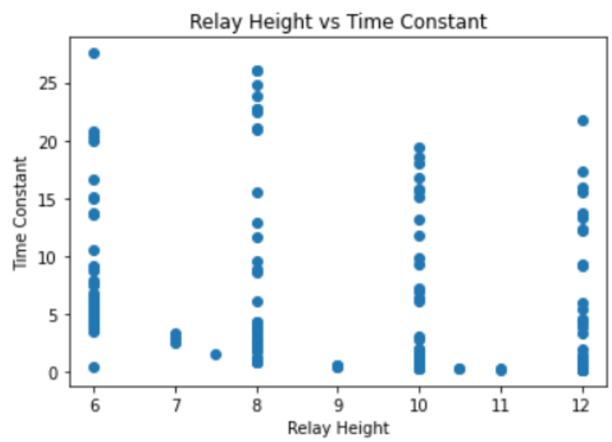

Fig. 5. Relay height vs Time constant plot

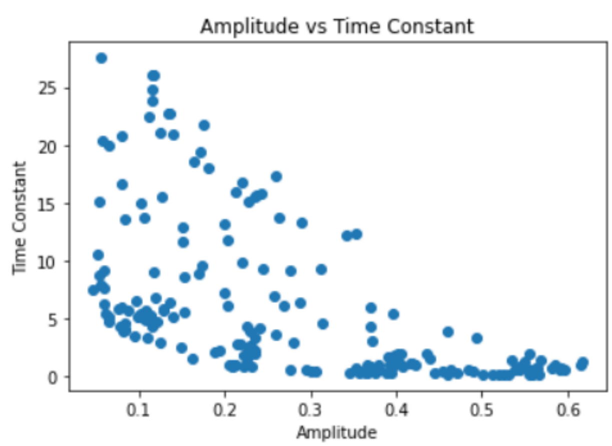

Fig. 6. Amplitude vs Time constant plot

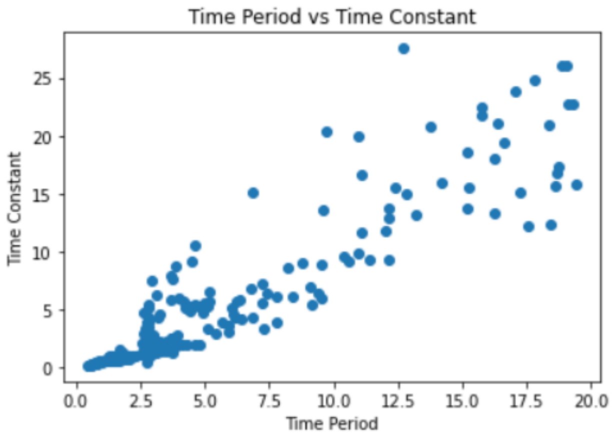

Fig. 7. Time period vs Time constant plot

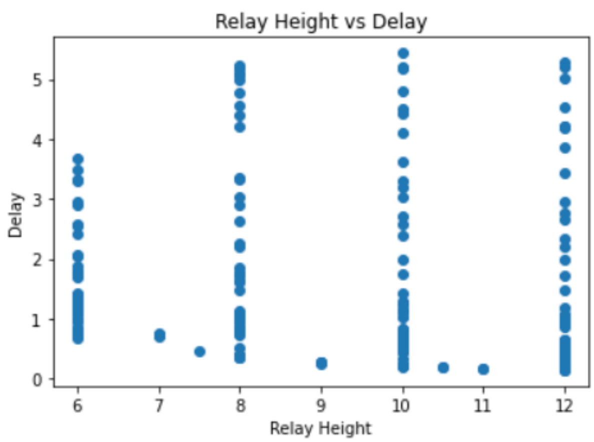

Fig. 8. Relay height vs Delay plot

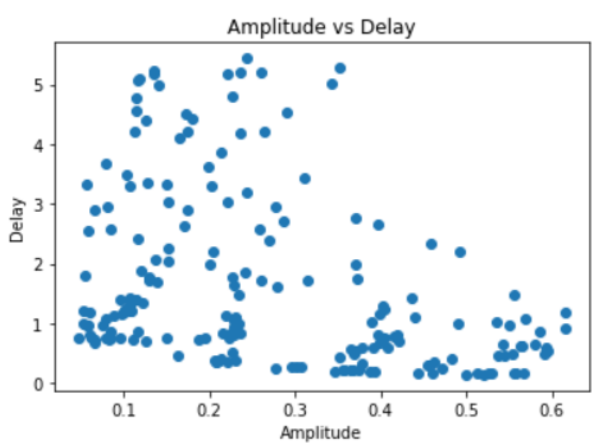

Fig. 9. Amplitude vs Delay plot

TABLE I
I/O DATASET OF FOPDT TRANSFER FUNCTION MODEL FOR $p = 1$

$S.N.$	H	A_p	T	T_1	D
0	6	0.0655	2.5988	4.806584	0.685187
1	6	0.083	2.72	3.961135	0.727106
2	6	0.096	2.8004	3.518842	0.756236
3	6	0.084	2.824	4.063042	0.755499
4	6	0.0625	2.8216	5.47097	.74216
...	...	...	...	...	...
191	12	0.353	18.444	12.362503	5.295248
192	12	.26	18.76	17.290839	5.20043
193	12	.277	10.5952	9.147655	2.956137
194	12	.3115	12.1608	9.294435	3.437489
195	12	0.2635	15.1928	13.811416	4.217194

TABLE II
I/O DATASET OF FOPDT TRANSFER FUNCTION MODEL FOR $p = 2$

$S.N.$	H	A_p	T	D	T_1
0	6	0.0655	2.5988	1.233084	0.89633
1	6	0.083	2.72	1.28187	0.947678
2	6	0.096	2.8004	1.313668	0.982397
3	6	0.084	2.824	1.330381	0.984449
4	6	0.0625	2.8216	1.340469	0.971360
...	...	...	...	...	...
191	12	0.353	18.444	9.085651	6.00876
192	12	.26	18.76	9.261017	6.091572
193	12	.277	10.5952	5.228234	3.442586
194	12	.3115	12.1608	5.995961	3.956208
195	12	0.2635	15.1928	7.499394	4.933925

relationship between time constant and amplitude in Fig 6 as well as the relationship between delay and amplitude in Fig 9, exhibits significant non-linearity. Given these findings, it is reasonable to conclude that a linear regression model within machine learning performs poorly when applied to this dataset. Consequently, non-linear machine learning models are being utilized to predict the output parameters accurately.

B. Identification Procedure and Algorithm

Once the primary data set is prepared for training the machine learning model, various machine learning algorithms

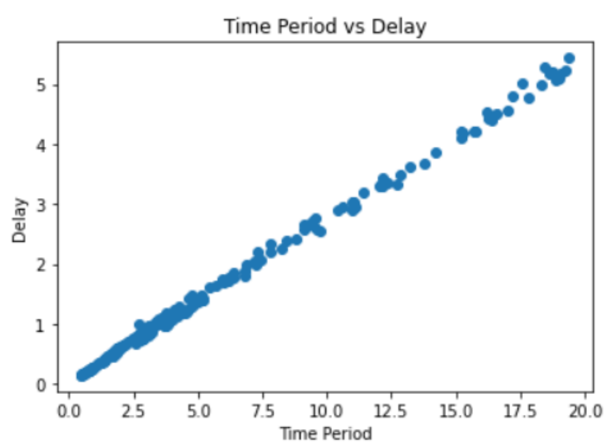

Fig. 10. Time period vs Delay plot

TABLE III
DATA STATICS OF FOPDT TRANSFER FUNCTION MODEL FOR SINGLE POLE

$S.N.$	H	A_p	T	T_1	D
$count$	196	196	196	196	196
$mean$	8.936224	0.268494	5.803212	5.962614	1.617129
std	2.221056	0.161311	5.350515	6.650002	1.458391
min	6	0.047	0.45896	0.210698	0.139115
25%	6.75	0.1215	1.9693	1.001309	0.593868
50%	8.5	0.2295	3.6785	3.55304	1.025237
75%	10.125	0.3925	8.2626	8.095556	2.34071
max	12	0.616	19.384	27.537659	5.43828

TABLE IV
DATA STATICS OF FOPDT TRANSFER FUNCTION MODEL FOR MULTIPLE POLES

$S.N.$	H	A_p	T	T_1	D
$count$	196	196	196	196	196
$mean$	8.936224	0.268494	5.803212	2.798092	1.95456
std	2.221056	0.161311	5.350515	2.604345	1.777781
min	6	0.047	0.45896	0.225439	0.150187
25%	6.75	0.1215	1.9693	0.960448	.654735
50%	8.5	0.2295	3.6785	1.744853	1.243441
75%	10.125	0.3925	8.2626	3.983031	2.781734
max	12	0.616	19.384	9.454015	6.500788

are trained, and their performance is evaluated. In this analysis, we found that different machine learning models showed varying levels of accuracy and how well they fit the data.

Among these models, the decision tree regressor model demonstrated the highest accuracy in estimating the Time-Constant parameter. On the other hand, the random forest model exhibited the best accuracy for estimating the delay parameter.

1) Decision Tree Regression Model: A decision tree regressor (DTR) is a predictive model used to estimate an output feature based on input feature parameters [13]. The model works by splitting the dataset into smaller sub-datasets, aiming to reduce the standard deviation within each subset. The process of splitting and the fundamental terminology of the DTR model are illustrated in Fig 5. The root node serves as the primary node of DTR, initially containing the entire dataset. The splitting process commences from the root node. Each decision node, which is a sub-node of the tree, further divides into additional subnodes. Ultimately, leaf nodes are reached, which contain the final decision, and no further splitting occurs beyond this point.

In the DTR model, recursive partitioning of the data set commences from the root node. The criterion for splitting a node is based on the information gain, representing the difference between the impurity of the parent node and the sum of impurities of its child node. Thus, the objective function of the DTR model is to maximize information gain. Ultimately, the data is considered pure or free from any impurities at each node.

2) Random Forest Regression Model: Random forest regression model (RFR) contains various decision trees, and the prediction of output feature is based on the average of all outputs obtained from each decision tree shown in Fig.11. It

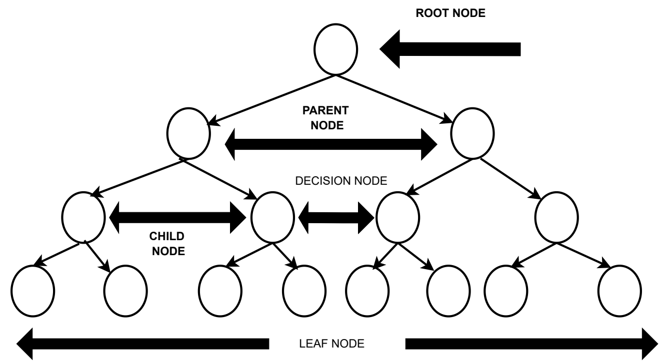

Fig. 11. Decision Tree Regression model Terminology

belongs to the family of ensemble machine learning algorithms that predicts a response by creating multiple decision trees and aggregating their results. Each tree in the forest is independently constructed using a unique bootstrap sample of the training data. RFR requires no assumption of the probability distribution of the target predictors as with linear regression and is robust against nonlinearity and over-fitting, although over-fitting may occur in instances where noisy data are being modeled.

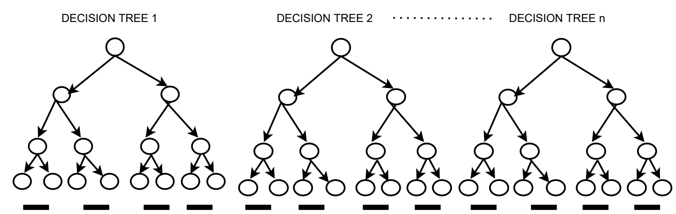

Fig. 12. Random Forest Regression Model Terminology

V. RESULT

Random forest regressor model and Decision tree regressor are carried out to predict the time constant and delay of the unknown process model. The R2 score of both models is shown in Table.V. The R2 score is a statistical measure that tells how much data fits well in the respective model. The closer the value of R2 score 1, the better the model fitted. Mean square error (MSE) is calculated of output features shown in Table V. MSE provides a more comprehensive view of prediction accuracy by penalizing larger errors more heavily. Lower MSE values indicate better model performance. Actual vs predicted values plots are shown in Fig.13-14. Prediction error plots provide a graphical comparison between actual values and predicted values and a best-fit line.

A. Example 1

Let us take a higher-order closed-loop transfer function $G(s) = \frac{1}{(0.1s+1)^3(0.01s+1)^2(0.001s+1)^3+1}$ is identify as a stable FOPDT process model $G_1(s) = \frac{e^{-2.564s}}{5.461s+1}$ using a relay feedback test, generates a limit cycle output with the parameter

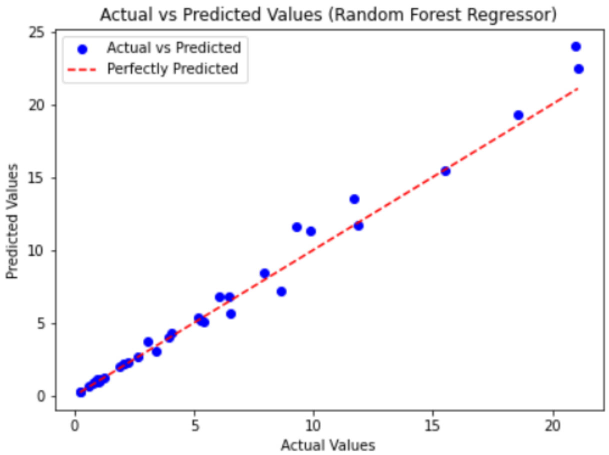

Fig. 13. Prediction error plot for Time Constant

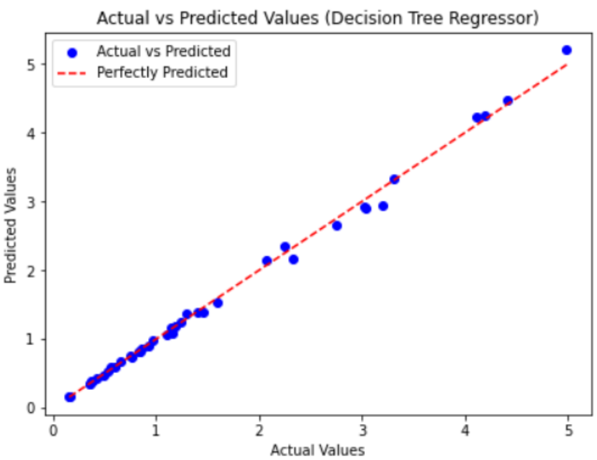

Fig. 14. Prediction error plot for Delay

$A = 2.223$ and time period $T = 8.838$ sec with the relay setting $h = 7$. Using the proposed machine learning model, we obtain an estimated FOPDT transfer function $G_2(s) = \frac{e^{-2.697s}}{5.306s+1}$ with %error in time constant is 2.921% and in delay is 4.931%.

B. Example 2

Considering closed-loop transfer function $G(s)$ in example-1. Now $G(s)$ is identified as SOPDT process model $G_1(s) = \frac{e^{-3.185s}}{(2.312s+1)^2}$ using a relay feedback test, generates a limit cycle output with the parameter $A = 5.16$ and time period $T = 6.903$ sec with the relay setting $h = 11$. Using the proposed machine learning model, we obtain an estimated SOPDT transfer function $G_2(s) = \frac{e^{-3.256s}}{(2.287s+1)^2}$ with %error in time constant is 1.093% and in delay is 2.149%.

VI. Conclusion

A machine learning-based method is implemented to model the complex higher-order systems to FOPDT models with reduced identification error with simple experiments without any mathematical computation. A hardware model is constructed with physical delay, and real-time data have been collected to make the proposed method more accurate for real-time system identifications and autotune the controllers precisely. We have proposed decision tree regression and a random forest model to rain the data sets. Finally, it concludes that the proposed method provides better estimation accuracy with real-time data.

TABLE V
Data statics of FOPDT transfer function model for one pole

$ProcessModel$	$Parameter$	$MLModel$	$\%MSE$	$R2Score$
FOPDT for $p = 1$	T_1	DTR	1.4709	0.9532
		RFR	0.6690	0.9187
	D	DTR	0.00519	0.9947
		RFR	0.00614	0.9978
FOPDT for $p = 2$	T_1	DTR	0.00756	0.9946
		RFR	0.00728	0.9964
	D	DTR	0.00771	0.998
		RFR	0.00942	0.997

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