README.md

Compte rendu TP5 - Interpolation polynomiale : Base d'Hermite

I. Base d'Hermite

	ϕ_1	ϕ_2	ϕ_3	ϕ_4
$\phi(0)$	1	0	0	0
$\phi(1)$	0	1	0	0
$\phi^1(0)$	0	0	1	0
$\phi^1(1)$	0	0	0	1

On obtient:

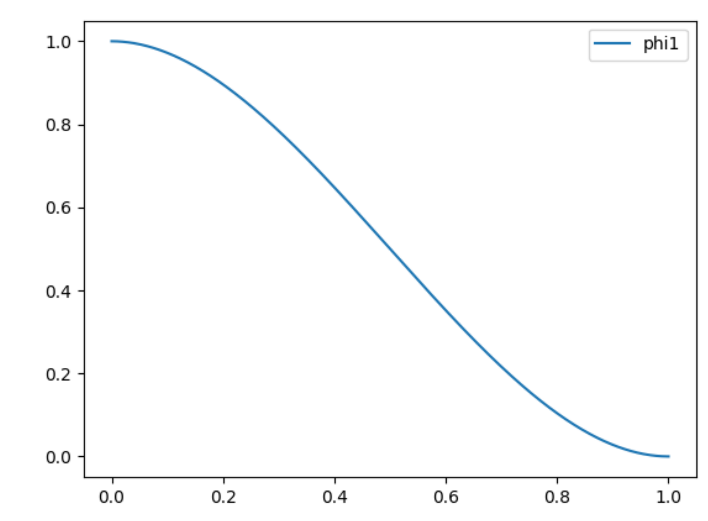
$\phi_1(x) = (x-1)^2(1+2x)$

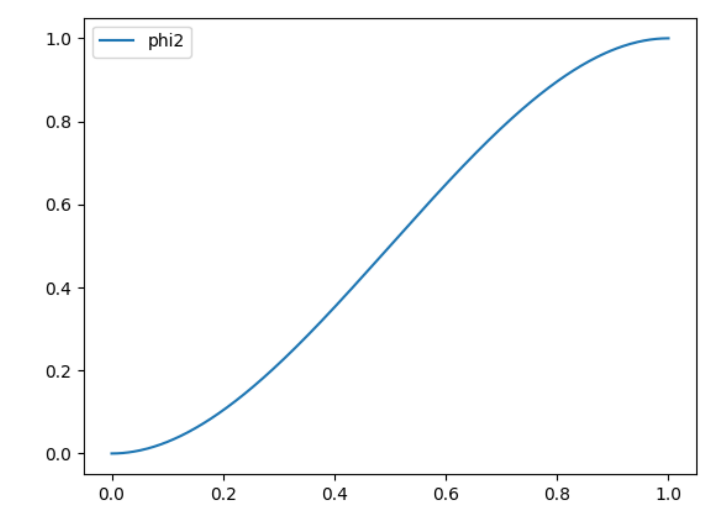
$\phi_2(x) = x^2(3-2x)$

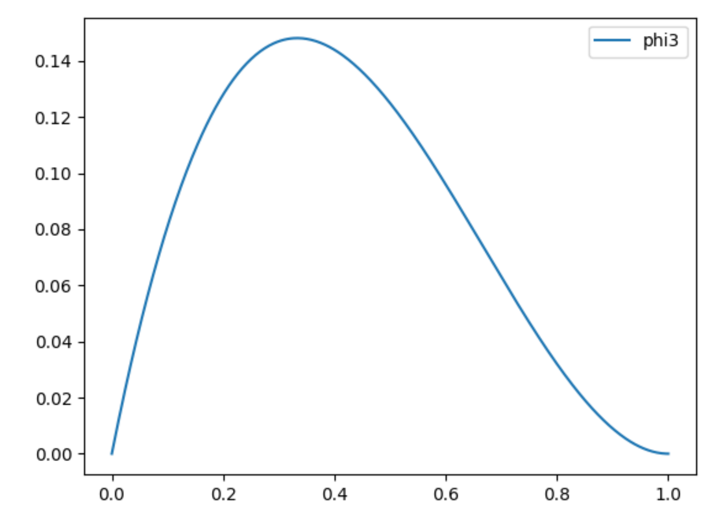
$\phi_3(x) = x(x-1)^2$

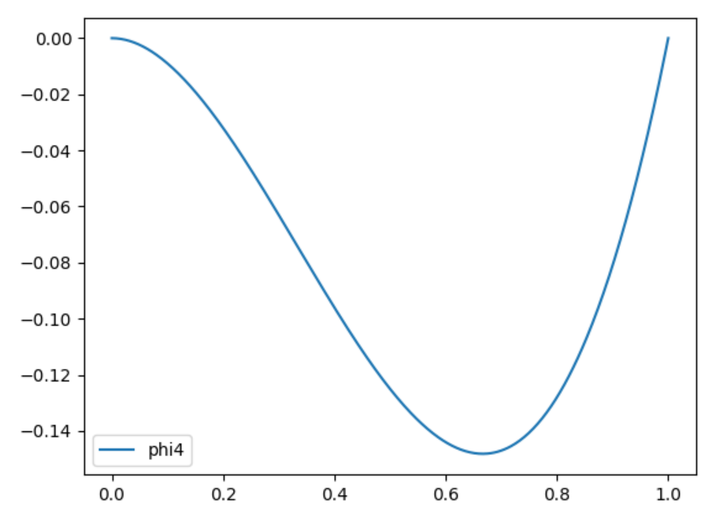
$\phi_4(x) = (x-1)x^2$

Allure des courbes de la base d'Hermite:









1. On cherche un polynome P de degré 3 tel que :

$$\begin{cases} P(0) = Y_0 \\ P(1) = Y_1 \\ P'(0) = V_0 \\ P'(1) = V_1 \end{cases}$$

$$P(x) = Y_0\phi_1(x) + Y_1\phi_2(x) + V_0\phi_3(x) + V_1\phi_4(x)$$

car

$$\begin{cases} P(0) = Y_0 \times 1 + Y_1 \times 0 + V_0 \times 0 + V_1 \times 0 = Y_0 \\ P(1) = Y_0 \times 0 + Y_1 \times 1 + V_0 \times 0 + V_1 \times 0 = Y_1 \\ P'(0) = Y_0 \times 0 + Y_1 \times 0 + V_0 \times 1 + V_1 \times 0 = V_0 \\ P'(1) = Y_0 \times 0 + Y_1 \times 0 + V_0 \times 0 + V_1 \times 1 = V_1 \end{cases}$$

1. On cherche un polynome P de degré 3 qui passe par deux points A et B et dont on connaît les dérivées VA et VB en XA et XB.

$$\begin{cases} P(X_A) = Y_A \\ P(X_B) = Y_B \\ P'(X_A) = V_A \\ P'(X_B) = V_B \end{cases}$$

On pose

$$t = \frac{x - X_A}{X_B - X_A}$$

$$P(x) = Y_A\phi_1(t) + Y_B\phi_2(t) + V_A\phi_3(t) + V_B\phi_4(t)$$

car

$$\begin{cases} P(X_A) = Y_A \times 1 + Y_B \times 0 + V_A \times 0 + V_B \times 0 = Y_A \\ P(X_B) = Y_A \times 0 + Y_B \times 1 + V_A \times 0 + V_B \times 0 = Y_B \\ P'(X_A) = Y_A \times 0 + Y_B \times 0 + V_A \times 1 + V_B \times 0 = V_A \\ P'(X_B) = Y_A \times 0 + Y_B \times 0 + V_A \times 0 + V_B \times 1 = V_B \end{cases}$$

Exemple:

$$\begin{cases} P(1) = 6 \\ P(5) = 2 \\ P'(1) = \frac{3}{2} \\ P'(5) = -3 \end{cases}$$

$$P(x) = \frac{1}{2}\left(\frac{x-1}{4}\right)^3 + \frac{1}{2}\left(\frac{x-5}{-4}\right)^3 + \frac{3}{4}\left(\frac{x-1}{4}\right)^2 + \frac{3}{4}\left(\frac{x-5}{-4}\right)^2$$

3. Faire la même chose

Soit

$$\begin{cases} P(5) = 2 \\ P(7) = -1 \\ P'(5) = -3 \\ P'(7) = 0 \end{cases}$$

alors

$$P(x) = \frac{1}{2}\left(\frac{x-5}{2}\right)^3 + \frac{1}{2}\left(\frac{x-7}{-2}\right)^3 + \frac{3}{4}\left(\frac{x-5}{2}\right)^2 + \frac{3}{4}\left(\frac{x-7}{-2}\right)^2$$

Soit

$$\begin{cases} P(7) = -1 \\ P(8) = 1 \\ P'(7) = 0 \\ P'(8) = 4 \end{cases}$$

alors

$$P(x) = \frac{1}{2}\left(\frac{x-7}{1}\right)^3 + \frac{1}{2}\left(\frac{x-8}{-1}\right)^3 + \frac{3}{4}\left(\frac{x-7}{1}\right)^2 + \frac{3}{4}\left(\frac{x-8}{-1}\right)^2$$

Soit

$$\begin{cases} P(8) = 1 \\ P(10) = 2 \\ P'(8) = 4 \\ P'(10) = 1 \end{cases}$$

alors

$$P(x) = \frac{1}{2}\left(\frac{x-8}{2}\right)^3 + \frac{1}{2}\left(\frac{x-10}{-2}\right)^3 + \frac{3}{4}\left(\frac{x-8}{2}\right)^2 + \frac{3}{4}\left(\frac{x-10}{-2}\right)^2$$

II. Interpolation polynomiale

1. On écrit les fonction phi1, phi2, phi3, phi4 nulles en dehor de l'intervalle [0,1]:

```
def phi1(x):
    return 0 if x < 0 or x > 1 else (x-1)**2*(1+2*x)


def phi2(x):
    return 0 if x < 0 or x > 1 else x**2*(3-2*x)


def phi3(x):
    return 0 if x < 0 or x > 1 else x*(x-1)**2


def phi4(x):
    return 0 if x < 0 or x > 1 else (x-1)*x**2
```

2. Stockage des tableaux X,Y et V:

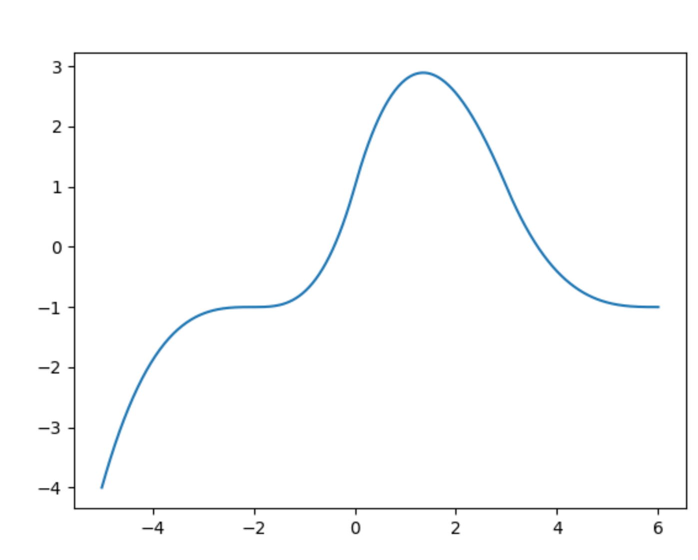
```
X = [-5, -2, 0, 3, 6]
Y = [-4, -1, 1, 1, -1]
V = [3, 0, 3, -2, 0]
```

3. Ecriture de foncHermite qui renvoie le polynome de Hermite:

```
def foncHermite(X, Y, V, x):
    n = len(X)
    P = 0
    for k in range(n-1):
        d = X[k+1]-X[k]
        t = (x-X[k])/d
        P += Y[k] * phi1(t) + Y[k+1]*phi2(t) + d*(V[k]*phi3(t)+V[k+1]*phi4(t))
    return P
```

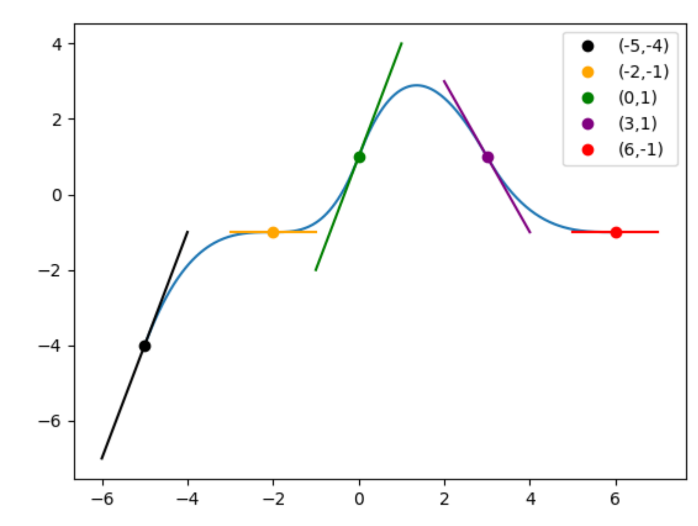
4. Affichage de la courbe représentative de P:

```
import numpy
import matplotlib.pyplot as pyplot

x = numpy.linspace(X[0], X[-1], 500)
pyplot.plot(x, [foncHermite(X, Y, V, i) for i in x])
pyplot.show()
```



5. Ajout des tangentes:

```
import numpy
import matplotlib.pyplot as pyplot

x = numpy.linspace(0, 10, 500)
pyplot.plot(x, [foncHermite(X, Y, V, i) for i in x])
for k in range(len(X)):
    pyplot.plot((X[k]-1, X[k]+1), (Y[k]-V[k], Y[k]+V[k]))
pyplot.show()
```



voila.