Tracking Control of Spacecraft Attitude on Time Dependent Trajectories

Term project (AE-778A) report

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16 November, 2022

Acknowledgement

I would like to express my deepest appreciation to all those who provided me the possibility to complete this report. A special gratitude I give to our term year project guide Dr Dipak Giri, whose contribution in stimulating suggestions and encouragement, helped us to coordinate my project especially in writing this report.

Abstract

A spacecraft attitude control which uses the quaternion parameterizations, is used. The derivative of the to-go quaternion is obtained where the desired attitude is a time dependent function. Using this new attitude formulation, a Lyapunov function based feedback control law that takes the time derivative of the desired attitude into account is derived. The simulation results demonstrate the success of the new algorithm in tracking the desired attitude trajectory.

Keywords - Spacecraft**,** quaternion, To-go quaternions, Lyapunov Function, PD control, Sliding mode control (SMC)

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Nomenclature

d = desired attitude quaternion

q = spacecraft current attitude quaternion

q, q4 = vector and scalar parts of the quaternion

t **=** *to-go* quaternion, additional rotation needed to reach the desired attitude

$()^{x}$ = skew-symmetric matrix to carry out vector cross Product

λ = Eigen-axis of the rotation

α = rotation angle

Chapter 1
INTRODUCTION

A spacecraft is a vehicle or machine designed to fly in outer space. A type of artificial satellite, spacecraft are used for a variety of purposes, including communications, Earth observation, meteorology, navigation, space colonization, planetary exploration, and transportation of humans and cargo.
A Spacecraft needs an attitude control subsystem to be correctly oriented in space and respond to external torques and forces properly. The attitude control subsystem consists of sensors and actuators, together with controlling algorithms. The attitude-control subsystem permits proper pointing for the science objective, sun pointing for power to the solar arrays and earth pointing for communications.

Next generation satellites with centimeter scale ground sampling distances will have more stringent attitude control requirements. They are expected to be lighter and more flexible with large solar arrays. Similarly, spacecraft with interplanetary missions will have large and flexible solar arrays, and/or solar sails. Solar sails may also be used for de-orbiting as well as orbit rising of satellites. Attitude control of such a satellite was previously addressed.**[1]** These flexible satellites require smooth, low jerk, attitude maneuvers.

The attitude control of spacecraft is a **nonlinear problem** that requires nonlinear control methods. The most common nonlinear control approach uses a Lyapunov function.[2, 3] The State De-pendent Riccati Equation (SDRE) approach is another nonlinear control approach that factors the equations in the state dependent coefficient form and continuously solves the algebraic Riccati equation for changing state values to obtain the feedback gain.[4] This method was previously applied to the attitude control of a spacecraft carrying a camera, that is mounted to the spacecraft through piezoelectric actuators.[5] The combined attitude control of the camera and the spacecraft[5] that uses inner and outer control loops, was addressed. A single and two loop satellite attitude control based on the SDRE methods were also compared and evaluated previously.[6]

However, these approaches do not yield sufficiently smooth, low jerk, attitude maneuvers over a properly planned attitude trajectory. The desired attitude trajectory may be defined using polynomials,[7] or based on a cost function with constraints on control torques, employing an optimization routine.[8,9]
The trajectory planning may also be carried out with the intention of avoiding certain attitudes.[7] The advantage of planning a smooth trajectory for a two craft formation was addressed in a recent manuscript of the authors as well. In the named papers, a tracking attitude control of the formation was carried out using the approximating sequence of Riccati equations control method.[10]

Here, a novel trajectory tracking attitude control approach is presented. It is based on the *to-go* quaternion formulation that takes the time dependent desired attitude trajectory into account.[11] A new Lyapunov function is defined based on this new *to-go* quaternion kinematic equations.

The formulation adds new terms containing derivatives of the desired attitude to the well known PD-control. The success of the approach is demonstrated through simulations.

1.1 Quaternion

A quaternion of the form $a + 0\,\mathrm{i} + 0\,\mathrm{j} + 0\,\mathrm{k}$, where a is a real number, is called scalar, and a quaternion of the form $0 + b\,\mathrm{i} + c\,\mathrm{j} + d\,\mathrm{k}$, where b, c, and d are real numbers, and at least one of b, c or d is nonzero, is called a vector quaternion. If $a + b\,\mathrm{i} + c\,\mathrm{j} + d\,\mathrm{k}$ is any quaternion, then a is called its scalar part and $b\,\mathrm{i} + c\,\mathrm{j} + d\,\mathrm{k}$ is called its vector part. Even though every quaternion can be viewed as a vector in a four-dimensional vector space, it is common to refer to the vector part as vectors in three-dimensional space. With this convention, a vector is the same as an element of the vector space.

1.2 Lyapunov Function

A Lyapunov's stability theory is one of the most fundamental pillars in control theory. Although this method was introduced more than hundred years ago, it remains popular among control researchers. This success is owed to its simplicity, generality, and usefulness. The Lyapunov stability is a method that was developed for analysis purposes. However, it has become of equal importance for control designs over the last decades. The Lyapunov stability theory can be generalized as follows. Let us consider the problem of solving the stability for an equilibrium of a dynamical system dx/dt=F(x) using the Lyapunov function method. It is clear that to find a stability using the Lyapunov method, we need to find a positive definite Lyapunov function V(x) defined in some region of the state space containing the equilibrium point whose derivative dV/dt is negative semi definite along the system trajectories.

1.3 PD (Proportional Derivative) control

- **Proportional term**

The proportional term produces an output value that is proportional to the current error value. The proportional response can be adjusted by multiplying the error by a constant K_p, called the proportional gain constant.

A high proportional gain results in a large change in the output for a given change in the error. If the proportional gain is too high, the system can become unstable. In contrast, a small gain results in a small output response to a large input error, and a less responsive or less sensitive controller. If the proportional gain is too low, the control action may be too small when responding to system disturbances. Tuning theory and industrial practice indicate that the proportional term should contribute the bulk of the output change

- **Derivative term**

The derivative of the process error is calculated by determining the slope of the error over time and multiplying this rate of change by the derivative gain K_d. The magnitude of the contribution of the derivative term to the overall control action is termed the derivative gain, K_d.

Derivative action predicts system behavior and thus improves settling time and stability of the system. An ideal derivative is not causal, so that implementations of PID controllers include an additional low-pass filtering for the derivative term to limit the high-frequency gain and noise. Derivative action is seldom used in practice though – by one estimate in only 25% of deployed controllers because of its variable impact on system stability in real-world applications.

1.4 Sliding Mode Control (SMC)

In control systems, **sliding mode control** (**SMC**) is a nonlinear control method that alters the dynamics of a nonlinear system by applying a discontinuous control signal (or more rigorously, a set-valued control signal) that forces the system to "slide" along a cross-section of the system's normal behavior. The state-feedback control law is not a continuous function of time. Instead, it can switch from one continuous structure to another based on the current position in the state space.

Hence, sliding mode control is a variable structure control method. The multiple control structures are designed so that trajectories always move toward an adjacent region with a different control structure, and so the ultimate trajectory will not exist entirely within one control structure. Instead, it will *slide* along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a *sliding mode* and the geometrical locus consisting of the boundaries is called the *sliding (hyper)surface*. In the context of modern control theory, any variable structure system, like a system under SMC, may be viewed as a special case of a hybrid dynamical system as the system both flows through a continuous state space but also moves through different discrete control modes.

Chapter 2
DERIVATION OF TO-GO QUATERNION

Defining the quaternion associated with the current attitude using, q, and desired attitude with, d, the to-go attitude, t, may be written as,

d = qt

According to this definition, to-go quaternion is the conjugate or inverse of the more commonly used error quaternion. Then, the following may also be written:

$$t = q^{-1}d$$

$$\begin{aligned} t \quad &= (-\mathbf{q} + \; q_4)(\mathbf{d} + \; d_4) \\ &= -\mathbf{q} \times \mathbf{d} - d_4\mathbf{q} + q_4\mathbf{d} + \mathbf{q} \cdot \mathbf{d} + d_4\, q_4 \end{aligned}$$

In vector matrix form, the above multiplication yields

$$\left\{ \begin{array}{c} t_1 \\ t_2 \\ t_3 \\ t_4 \end{array} \right\} = \begin{bmatrix} -d_4 & -d_3 & d_2 & d_1 \\ d_3 & -d_4 & -d_1 & d_2 \\ -d_2 & d_1 & -d_4 & d_3 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} \left\{ \begin{array}{c} q_1 \\ q_2 \\ q_3 \\ q_4 \end{array} \right\}$$

$$\left\{ \begin{array}{c} \mathbf{t} \\ t_4 \end{array} \right\} = \mathbf{D} \left\{ \begin{array}{c} \mathbf{q} \\ q_4 \end{array} \right\}$$

Using the chain rule, the derivative of the to-go may be written as,

$$\left\{ \begin{array}{c} \dot{\mathbf{t}} \\ \dot{t}_4 \end{array} \right\} = \dot{\mathbf{D}} \left\{ \begin{array}{c} \mathbf{q} \\ q_4 \end{array} \right\} + \frac{1}{2}\mathbf{D}\boldsymbol{\Omega} \left\{ \begin{array}{c} \mathbf{q} \\ q_4 \end{array} \right\}$$

$$\left\{ \begin{array}{c} \dot{\mathbf{t}} \\ \dot{t}_4 \end{array} \right\} = \left[\dot{\mathbf{D}}\mathbf{D}_1 + \frac{1}{2}\mathbf{D}\boldsymbol{\Omega}\mathbf{D}_1 \right] \left\{ \begin{array}{c} \mathbf{t} \\ t_4 \end{array} \right\}$$

Where,

$$\left\{ \begin{array}{c} \dot{\mathbf{t}} \\ \dot{t}_4 \end{array} \right\} = \dot{\mathbf{D}} \left\{ \begin{array}{c} \mathbf{q} \\ q_4 \end{array} \right\} + \mathbf{D} \left\{ \begin{array}{c} \dot{\mathbf{q}} \\ \dot{q}_4 \end{array} \right\}$$

$$\begin{aligned} \left\{ \begin{array}{c} \dot{\mathbf{q}} \\ \dot{q}_4 \end{array} \right\} &= \tfrac{1}{2} \left[\begin{array}{cc} -\omega^{\mathbf{x}} & \omega \\ -\omega^T & 0 \end{array} \right] \left\{ \begin{array}{c} \mathbf{q} \\ q_4 \end{array} \right\} \\ &= \tfrac{1}{2} \mathbf{\Omega} \left\{ \begin{array}{c} \mathbf{q} \\ q_4 \end{array} \right\} \end{aligned}$$

$$\omega^{\mathbf{x}} = \left[\begin{array}{ccc} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{array} \right]$$

$$\mathbf{D}_1 = \left[\begin{array}{cccc} -d_4 & d_3 & -d_2 & d_1 \\ -d_3 & -d_4 & d_1 & d_2 \\ d_2 & -d_1 & -d_4 & d_3 \\ d_1 & d_2 & d_3 & d_4 \end{array} \right]$$

After some simplifications, to-go quaternion equation may also be written as

$$\left\{ \begin{array}{c} \dot{\mathbf{t}} \\ \dot{t}_4 \end{array} \right\} = \left[\left[\begin{array}{cc} -\mathbf{s}^{\mathbf{x}} & \mathbf{s} \\ -\mathbf{s}^T & 0 \end{array} \right] + \mathbf{I} s_4 + \frac{1}{2} \left[\begin{array}{cc} -\omega^{\mathbf{x}} & -\omega \\ \omega^T & 0 \end{array} \right] \right] \left\{ \begin{array}{c} \mathbf{t} \\ t_4 \end{array} \right\}$$

Where S,

$$\mathbf{s} = \left\{ \begin{array}{c} \dot{d}_1 d_4 + \dot{d}_2 d_3 - \dot{d}_3 d_2 - \dot{d}_4 d_1 \\ -\dot{d}_1 d_3 + \dot{d}_2 d_4 + \dot{d}_3 d_1 - \dot{d}_4 d_2 \\ \dot{d}_1 d_2 - \dot{d}_2 d_1 + \dot{d}_3 d_4 - \dot{d}_4 d_3 \end{array} \right\}$$

$$s_4 = \dot{d}_1 d_1 + \dot{d}_2 d_2 + \dot{d}_3 d_3 + \dot{d}_4 d_4$$

and I is the identity matrix of appropriate dimension. Since, s4 = 0, the vector and scalar parts of the to-go quaternion derivatives may be written as,

$$
\begin{aligned}
\dot{\mathbf{t}} &= -\left(\mathbf{s}^{\mathbf{x}} + \tfrac{1}{2}\omega^{\mathbf{x}}\right)\mathbf{t} + \left(\mathbf{s} - \tfrac{1}{2}\omega\right)t_4 \\
\dot{t}_4 &= \left(-\mathbf{s}^{T} + \tfrac{1}{2}\omega^{T}\right)\mathbf{t}
\end{aligned}
$$

These differential equations of attitude propagation of the to-go quaternion are used for spacecraft attitude control.

Chapter 3

Control algorithms

The nonlinear control may be realized using a properly selected Lyapunov function. Select a positive definite Lyapunov function as follows:

$$V(\mathbf{s},\omega,(1-t_4)) = \frac{1}{2}(-2\mathbf{s}+\omega)^T\mathbf{K}_p^{-1}\mathbf{J}\left(-2\mathbf{s}+\omega\right)+2(1-t_4)$$

Where, V ($\mathbf{s} = \mathbf{0}$; $\omega = \mathbf{0}$; $(1 - t4) = 0$) = 0, as the satellite attitude reaches the desired attitude. Taking the derivative of the Lyapunov function,

$$\dot{V} = (-2\mathbf{s}+\omega)^T\mathbf{K}_p^{-1}\mathbf{J}\left(-2\dot{\mathbf{s}}+\dot{\omega}\right)-2\dot{t}_4$$

The satellite attitude dynamics may be written as,

$$\dot{\omega} = \mathbf{J}^{-1}\left(-\omega^{\mathbf{x}}\mathbf{J}\omega+\mathbf{u}+\mathbf{M}_{ext}\right)$$

Where, $M_{ext} = 0$.

Using the dynamic equations,

$$\dot{V} = (-2\mathbf{s}+\omega)^T\mathbf{K}_p^{-1}\left(-2\mathbf{J}\dot{\mathbf{s}}+-\omega^{\mathbf{x}}\mathbf{J}\omega+\mathbf{u}\right)-(-2\mathbf{s}+\omega)^T\mathbf{t}$$

For asymptotic stability the derivative must be decaying. Equating the derivative to a negative definite function as,

$$\dot{V} = -(-2\mathbf{s}+\omega)^T\mathbf{K}_p^{-1}\mathbf{K}_d\left(-2\mathbf{s}+\omega\right)$$
$$\mathbf{K}_p^{-1}\left(-2\mathbf{J}\dot{\mathbf{s}}+-\omega^{\mathbf{x}}\mathbf{J}\omega+\mathbf{u}\right)-\mathbf{t} = -\mathbf{K}_p^{-1}\mathbf{K}_d\left(-2\mathbf{s}+\omega\right)$$

Finally, the control law is obtained

$$\mathbf{u} = \omega^{\mathbf{x}}\mathbf{J}\omega+\mathbf{K}_p\mathbf{t}-\mathbf{K}_d\omega+2(\mathbf{K}_d\mathbf{s}+\mathbf{J}\dot{\mathbf{s}})$$

The above formulation is different than the so called PD-control commonly used in satellite attitude control:

$$\mathbf{u} = \omega^{\mathbf{x}}\mathbf{J}\omega+\mathbf{K}_p\mathbf{t}-\mathbf{K}_d\omega$$

The positive definite weight matrices may be chosen as,[3]

$$\mathbf{K}_p = \omega_n^2 \mathbf{J}, \quad \mathbf{K}_d = 2\xi\omega_n \mathbf{J}$$

Where, ωn and ξ may be selected according to a particular response performance, such as the settling time.[3]

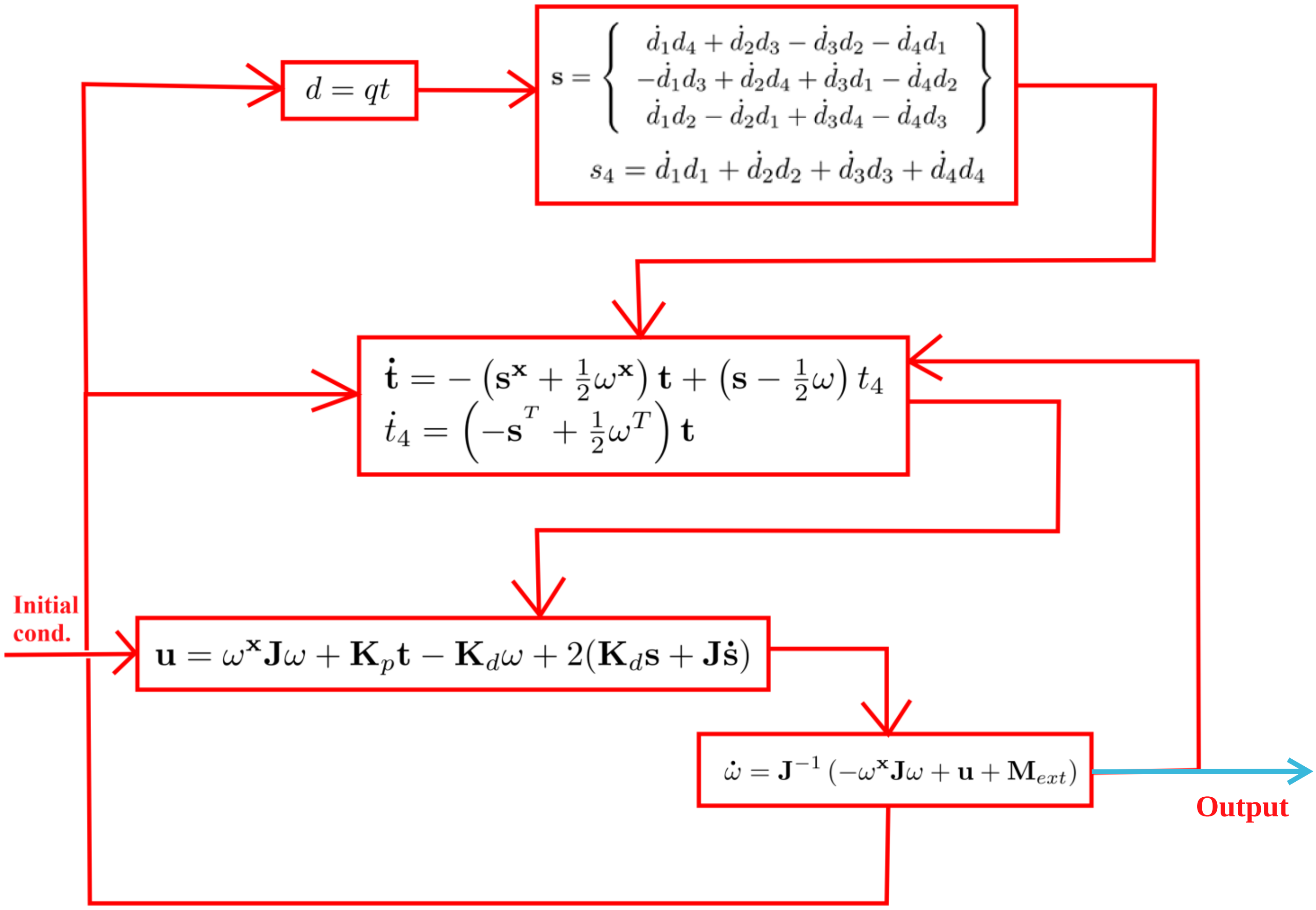
To-go quarternion dynamics can be written as,

$$\begin{bmatrix} t1\_dot \\ t2\_dot \\ t3\_dot \\ t4\_dot \end{bmatrix} = \left\{ \begin{bmatrix} 0 & s3 & -s2 & s1 \\ -s3 & 0 & s1 & s2 \\ s2 & -s1 & 0 & s3 \\ -s1 & -s2 & -s3 & 0 \end{bmatrix} + \begin{bmatrix} s1 \\ s2 \\ s3 \\ 0 \end{bmatrix} + \right.$$

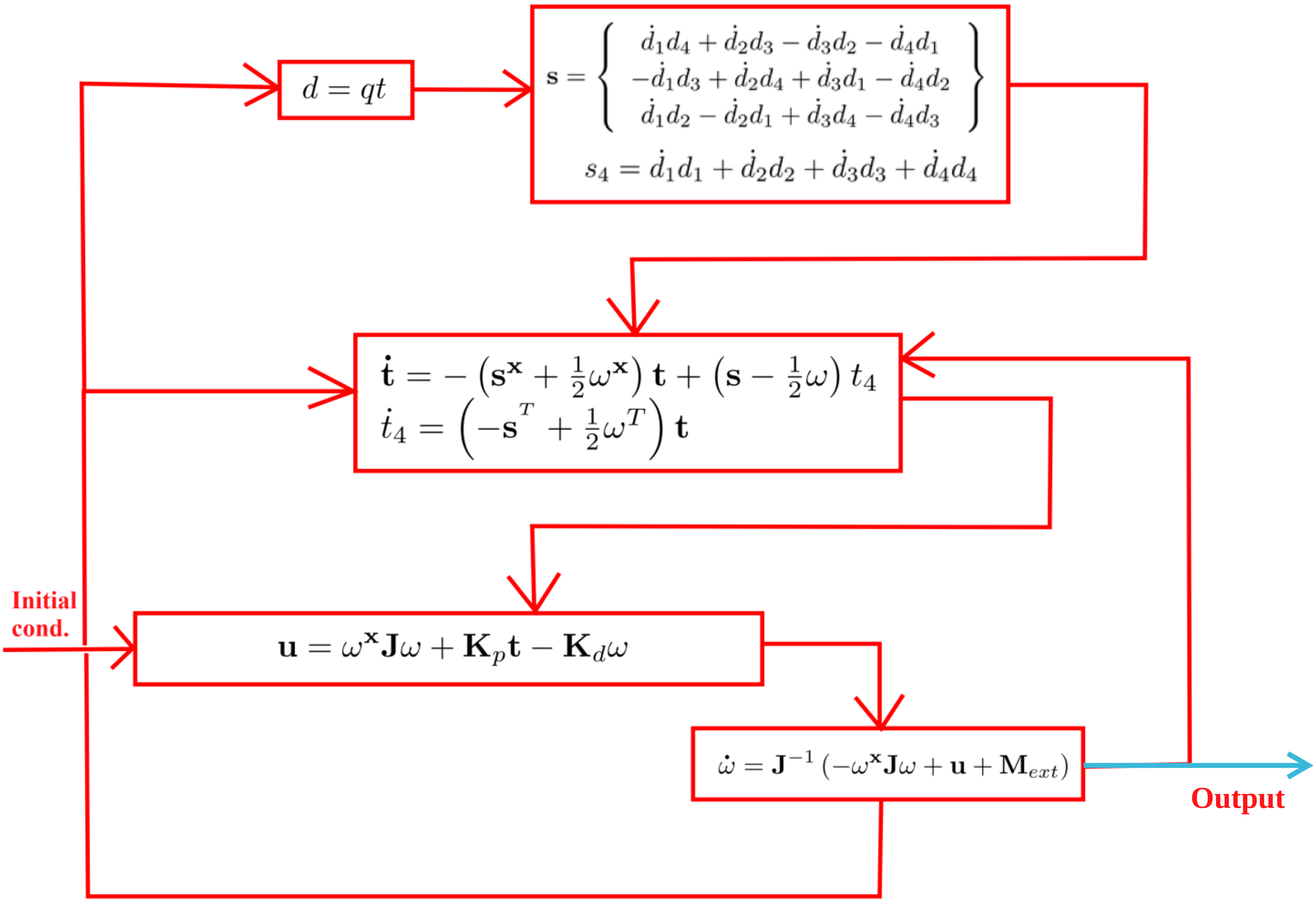
$$\left. \begin{bmatrix} s4 & 0 & 0 & 0 \\ 0 & s4 & 0 & 0 \\ 0 & 0 & s4 & 0 \\ 0 & 0 & 0 & s4 \end{bmatrix} + \begin{bmatrix} \cdot\, 0.5* & 0 & w3 & -w2 & -w1 \\ & -w3 & 0 & w1 & -w2 \\ & w2 & -w1 & 0 & -w3 \\ & w1 & w2 & w3 & 0 \end{bmatrix} \right\} +$$

$$\begin{bmatrix} t1 \\ t2 \\ t3 \\ t4 \end{bmatrix}$$

3.1 Control algorithms Block diagram of Non-linear controlled system



3.2 Control algorithms Block diagram of PD Classical controlled system



Chapter 4

Simulation

Here the desired attitude is defined as a time dependent eigen-axis rotation, i.e.,

$$\mathbf{q} = \lambda \sin(\alpha/2), \quad q_4 = \cos(\alpha/2)$$

The rotation angle is taken as a cubic function of time

$$\alpha = a + bt + ct^2 + et^3.$$

The coefficients may be calculated from the initial and terminal conditions specified.
In this manuscript, the following initial and final conditions are used

$$\alpha_0 = 0, \dot{\alpha_0} = 0, \alpha_f = 2\pi/3, \dot{\alpha_f} = 0 \ ,$$

We get, a = 0, b = 0, c = , d = 2π * 10e-4, d = (4π/3)*10e-6

The time histories of the resulting desired quaternion components are plotted.
The feedback control parameters, on the other hand are chosen as

$$\omega_n = 0.1 rad/s, \quad \xi = 0.7.$$

$\lambda = (1; 2; 3)^T \, sqrt(14)$

A non diagonal inertia matrix is selected

$$\mathbf{J} = \begin{bmatrix} 10 & -3 & -7 \\ -3 & 18 & 2 \\ -7 & 2 & 8 \end{bmatrix}$$

Chapter 5

Simulation & Results

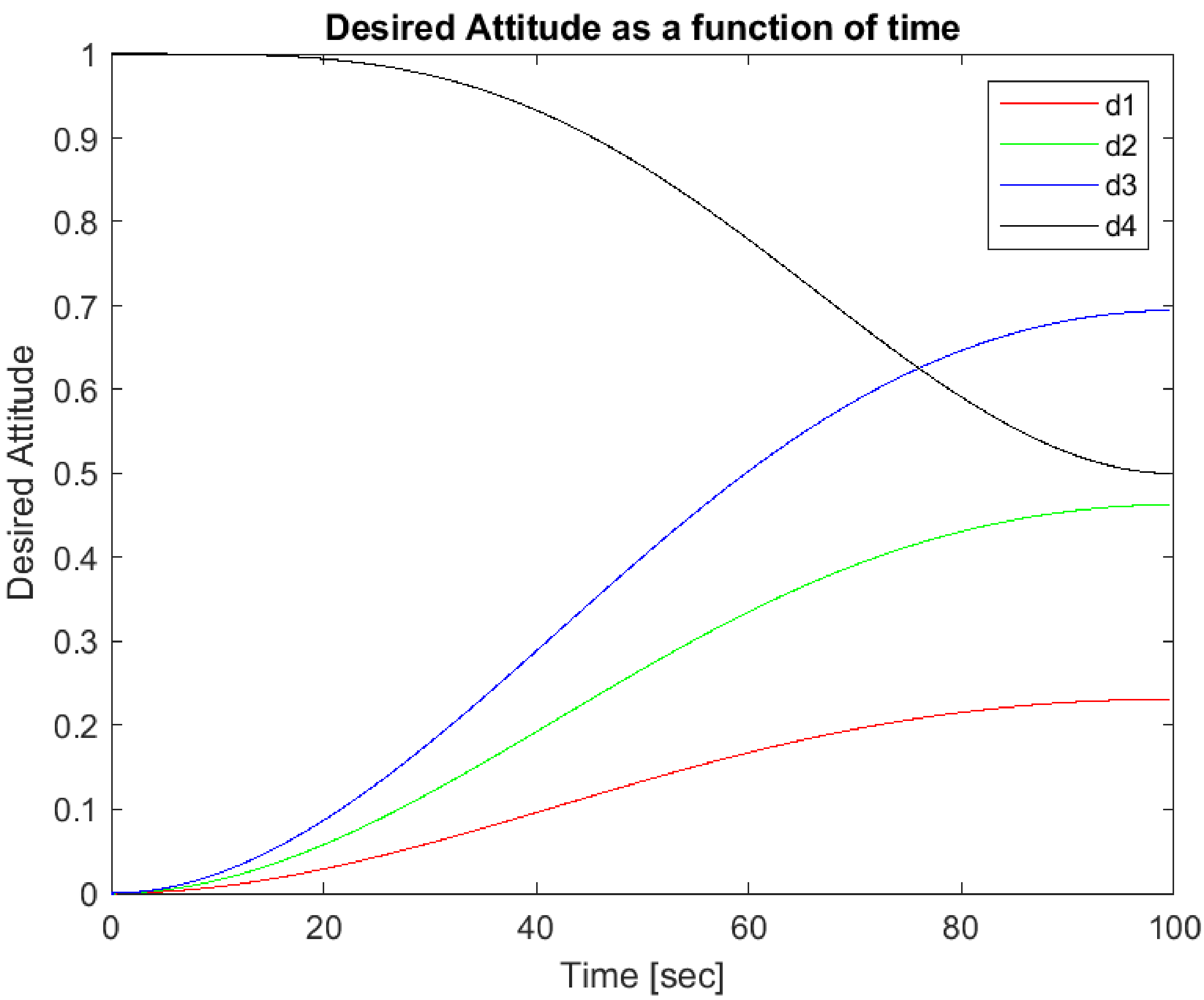


Fig. 1. Desired Attitude as a function of time

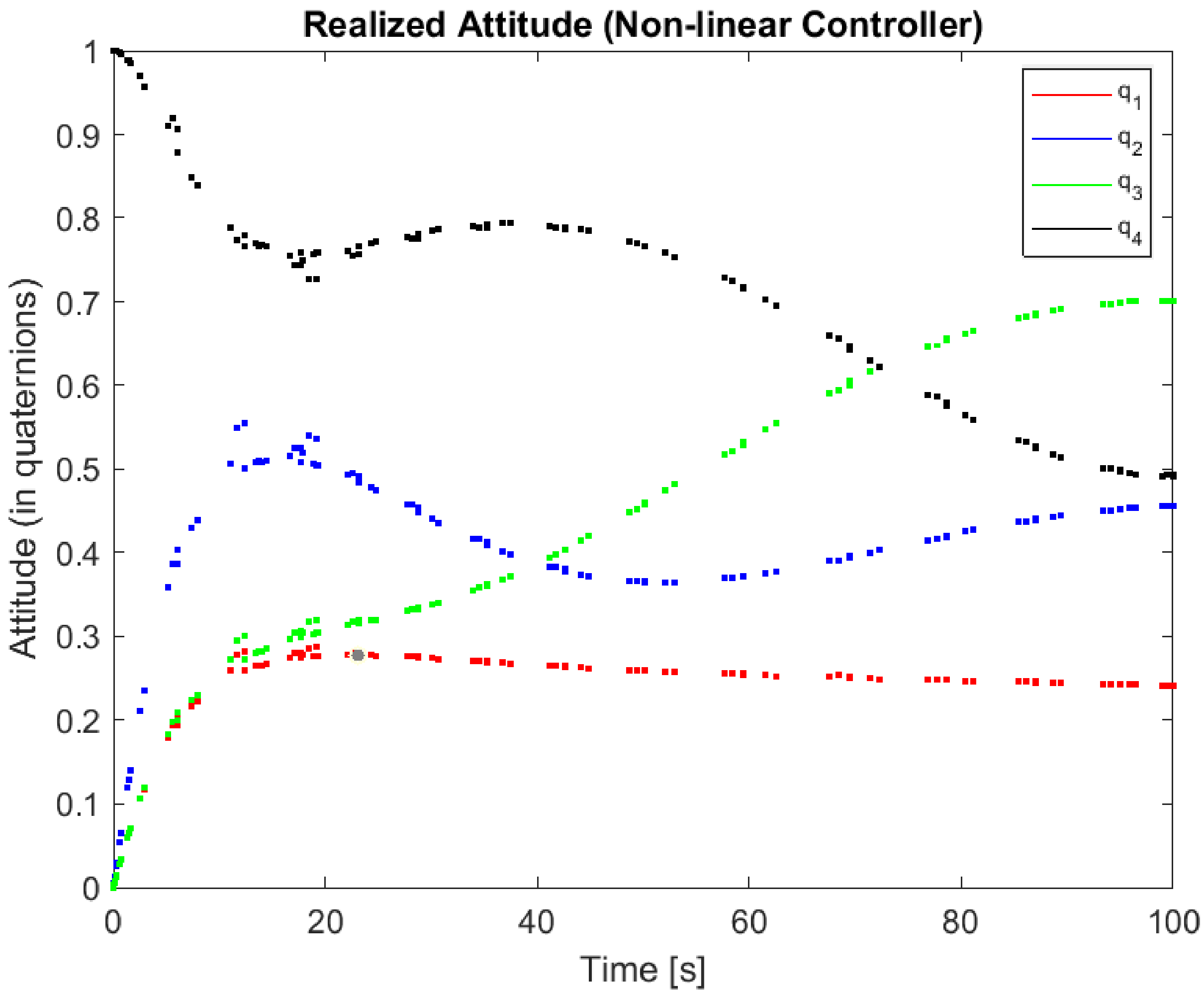


Fig. 2. Realized Attitude (Non-linear Controller)

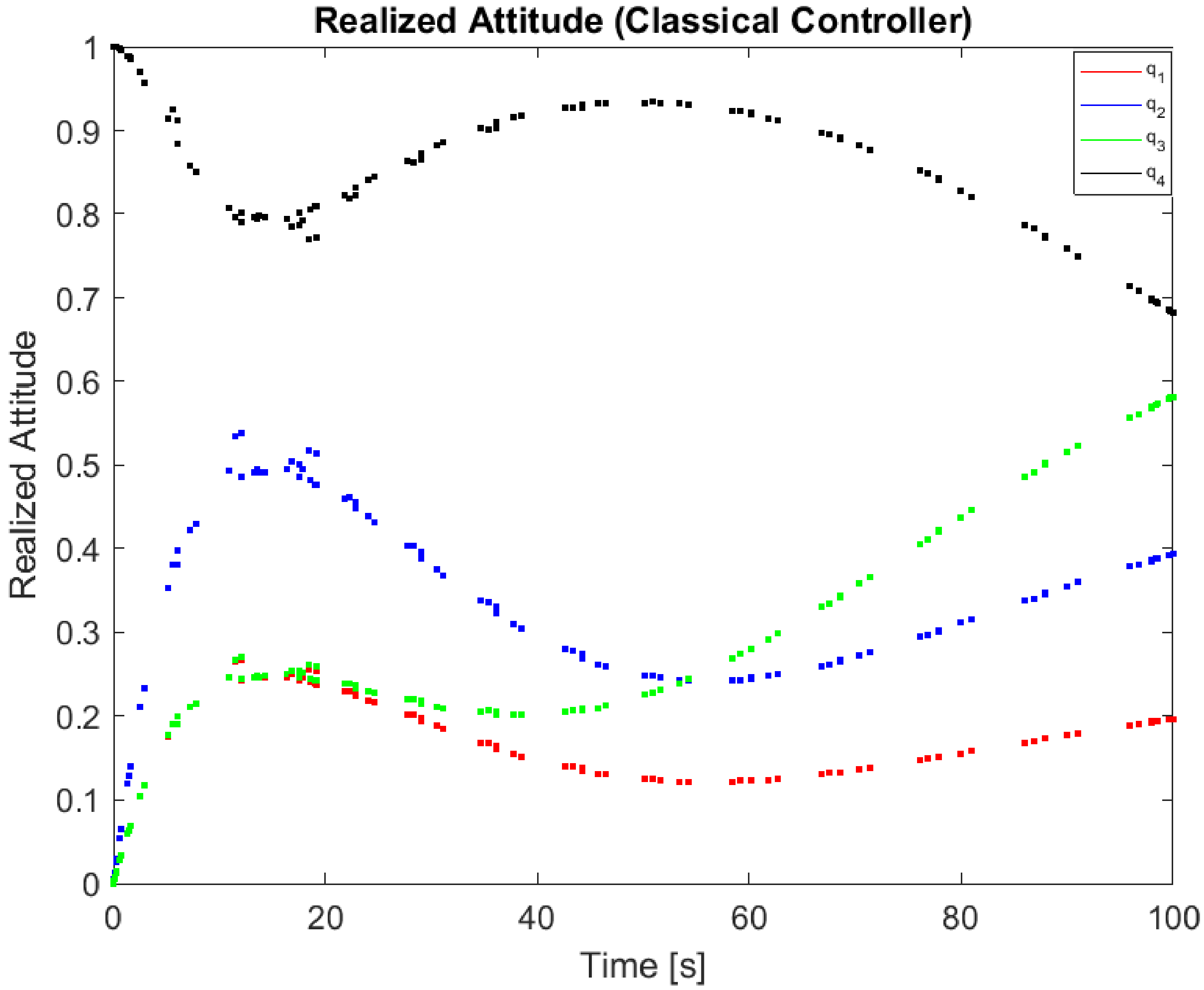


Fig. 3. Realized Attitude (PD Controller)

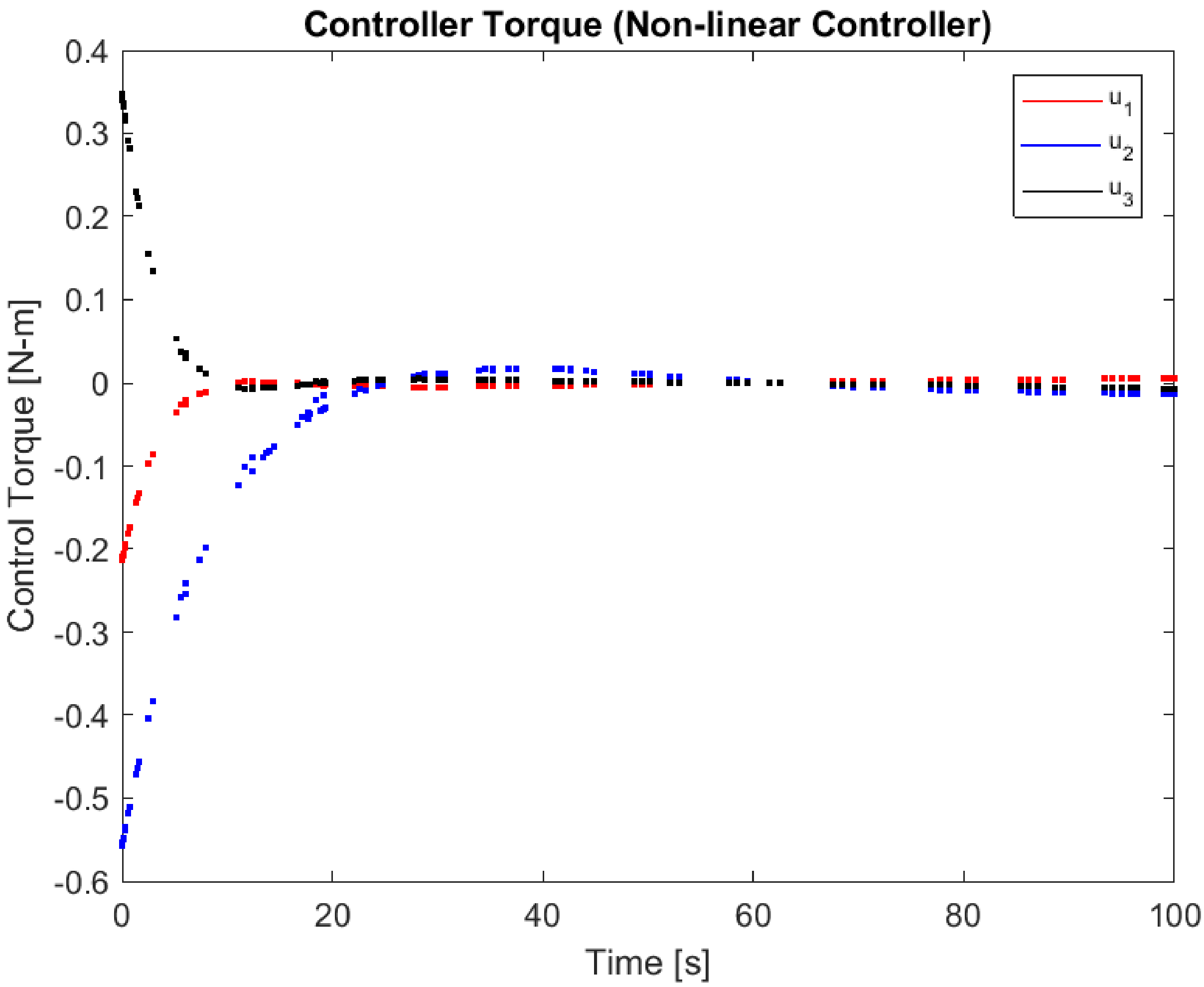


Fig. 4. Control Torque (Non-linear Controller)

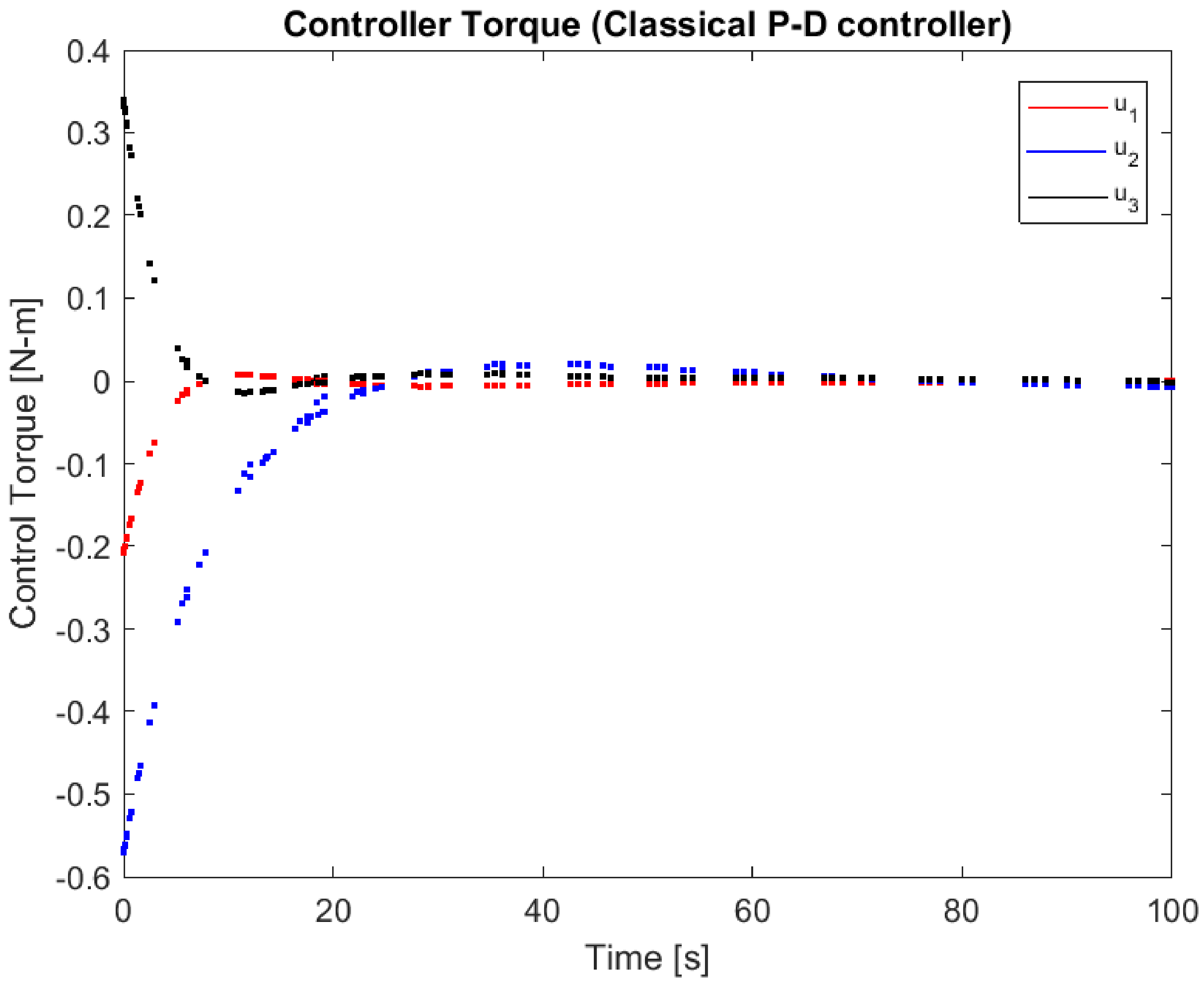


Fig. 5. Control Torque (PD Controller)

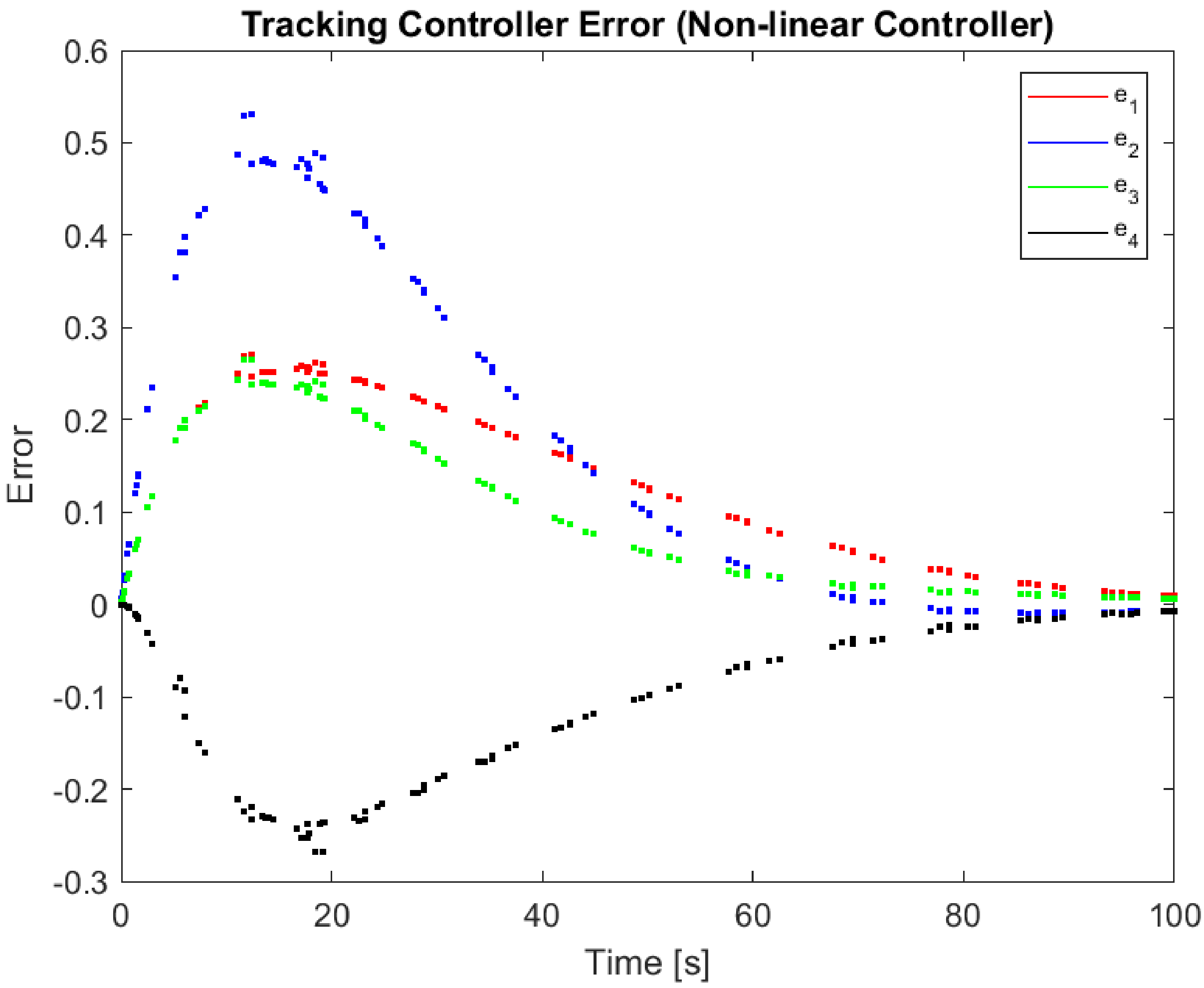


Fig. 6. Control Torque (Non-linear Controller)

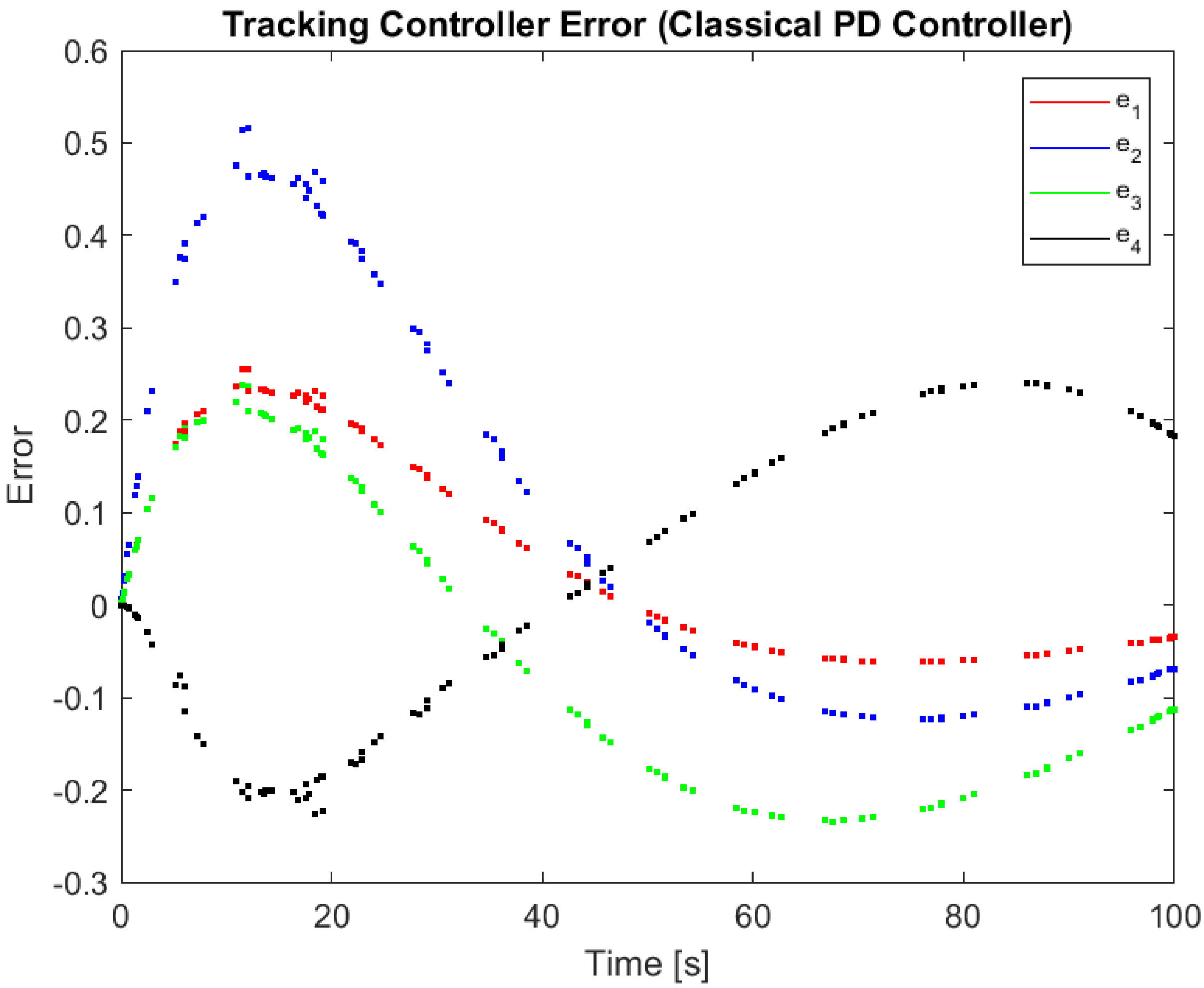


Fig. 7. Control Torque (PD Controller)

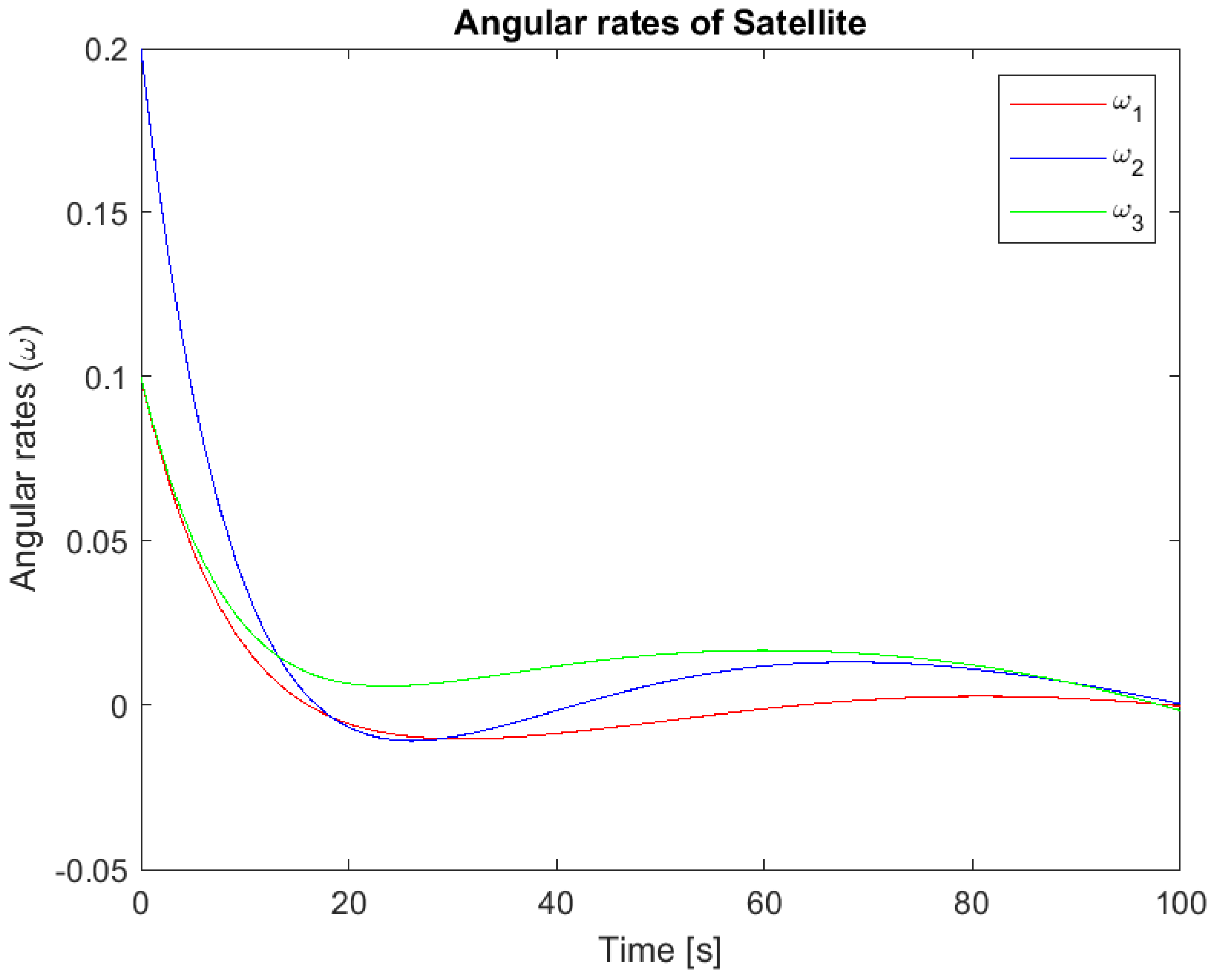


Fig. 8. Angular rates of satellite (Non-linear Controller)

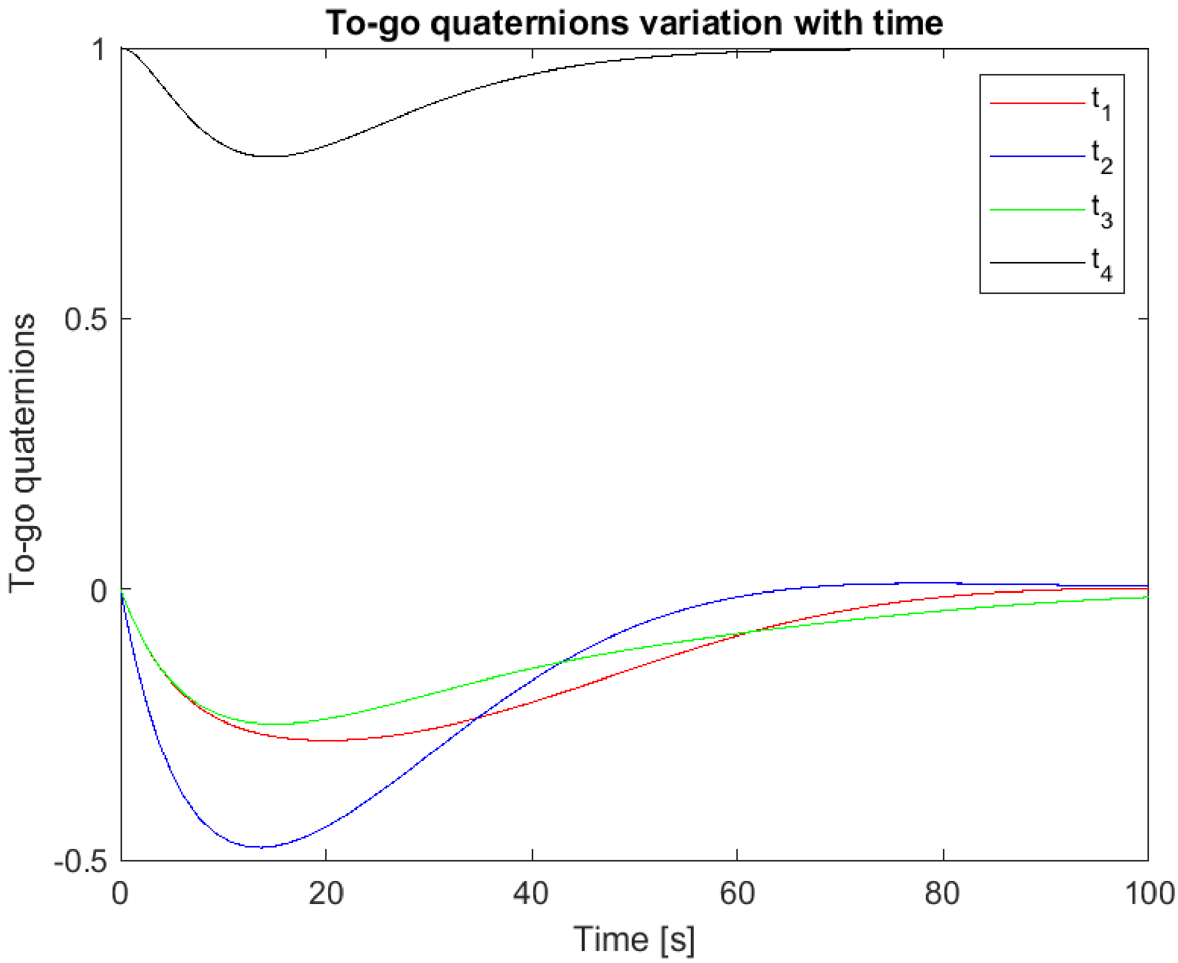


Fig. 9. To-go quaternion variation with time (Non-linear Controller)

- The desired attitude trajectory is presented in Figure (1).
- The attitude history obtained from the tracking controller by Non-linear controller and PD controller is presented in Figure (2) & Figure (3), respectively.
- From the figure it may be observed that this plot is very close to the desired attitude given in Figure (1).

- The difference between the desired and realized time histories are obtained by subtracting the quaternion components from each other. The resulting plot for Non-linear Controller & Classical PD controller is shown in Figure (6) & Figure (7), respectively.
- Figure shows that the error between these components is very small.
- As it may be observed from this figure that the differences are quite large compared with the new tracking controller, showing the excellent performance of the new tracking controller proposed.

- The corresponding control torque employed for Non-linear Controller & Classical PD controller is shown in Figure (4) & Figure (5).
- It may be observed from Figure (5) is that the torque histories and levels are quite similar to those obtained by the new Non-linear tracking controller shown in Figure (4).

- The angular rates of satellite is shown on Figure (8) & to-go quaternion history is shown in Figure (9).

Chapter 6

Conclusion

The kinematic equations for the *to-go* quaternion where the desired attitude quaternion is a continuous time dependent function is derived. The formulation is used to derive a nonlinear feedback control law using a new Lyapunov function.

The feedback law differs from the classical PD controller with additional terms due to the desired attitude trajectory and its derivative.

The simulation results shows that the new tracking control law tracks the trajectory quite precisely, as compared with the classical PD controller

Chapter 7

References

1] Tekinalp, O., and Atas,O., \Attitude Control Mechanization to De-Orbit Satellites Using Solar Sails," *2nd International Academy of Astronautics Conference on Dynamics and Control of Space Systems*, Roma, Italy, March, 24-26, 2014.

2] Wie, B., Weiss, H., Arapostathis, A., \Quaternion Feedback Regulator for Spacecraft Eigenaxis Rotations," J Guidance, Control and Dynamics, vol. 12, No. 3, 1989.

3] Wie, B. *Space Vehicle Dynamics and Control*, AIAA, Reston, VA, 1998.

4] Cimen, T. \State Dependent Riccati Equation(SDRE) Control:A Survey," Proceedings of the 17th World Con-gress of the International Federation of Automatic Control, South Korea, July 6-11, 2008, pp. 3761-3763.

5] Kayastha, S., Tekinalp, O., and Ozgoren, K. \Pointing Performance Control of a Spacecraft Camera using Piezoelectric Actuators," Advances in the Astronautical Sciences, Vol. 136, pp. 661-674, 2010.

6] Choukroun, D. and Tekinalp, O. \Spacecraft Attitude and Rate Estimation and Attitude Control Using the SDRE Method," 5th European Conference on Aeronautical and Space Sciences (EUCASS), Munich, Germany, 1-5 July, 2013.

7] Caubet, A., and Biggs, J.D. \Optimal Attitude Motion Planner for Large Slew Maneuvers Using a Shape-Based Meth-od," AIAA Guidance Navigation Control Conference, Boston, MA, August, 19-22, 2013.

8] Bilimoria, K.D., and Wie, B. \Time-Optimal Three-Axis Reorientation of a Rigid Spacecraft," Journal of Guidance Control and Dynamics, Vol. 16, No. 3, 1993, pp. 446-452.

9] Boyarko, G.A., Romano, M., and Yakimenko, O.A. \Time-Optimal Reorientation of a Spacecraft Using an Inverse Dynamics Optimization Method," Journal of Guidance Control and Dynamics, Vol. 16, No. 3, 2011, pp. 1197-1207.

10] Gomroki, M.M., and Tekinalp,O. \Maneuvering of Two Craft Coulomb Formation Using ASRE Method," AIAA/AAS Astrodynamics Specialist Conference, San Diego, CA, July 4-7, 2014.

11] Tekinalp, O., Gomroki, M.M., Atas, O., Nonlinear Tracking Attitude Control of Spacecraft on Time Dependent Trajectories, AAS/AIAA Astrodynamics Specialist Conference, Vail, CO, August 9-13, 2015