Effective Force On The Mast of Racing Sailboat

Group - 10

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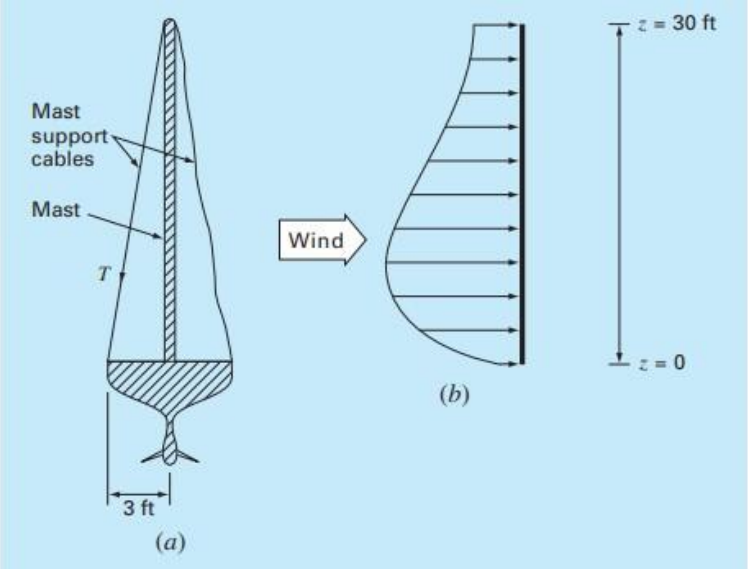
ANM Term Project Presentation

Problem Statement

A Cross section of a racing sailboat is shown. Wind forces (f) exerted per foot of mast from the sails very as a function of distance above the deck of the boat (Z).
Calculate the tensile force T in the left mast support cable, assuming that the right support cable is completely slack and the mast joins the deck in a manner that transmit horizontal or vertical forces but no moments. Assume that the mast remains vertical.



Methodology

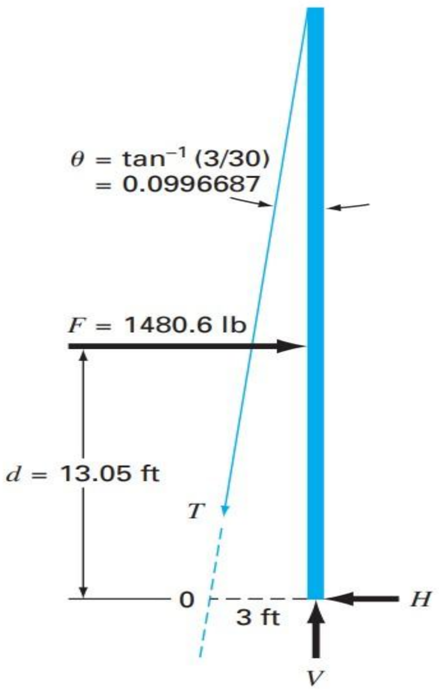
It is required that the distributed force "f" be converted to an equivalent total force F its effective location above the deck be calculated.
The total force exerted on the mast can be expressed as the internal of a continuous function:

$$F = \int_0^{30} 200\left(\frac{z}{5+z}\right)e^{-2z/30}\,dz$$

The effective line of action of d can be calculated by evaluation of the integral.

$$d = \frac{\int_0^{30} z f(z)\,dz}{\int_0^{30} f(z)\,dz}$$

$$d = \frac{\int_0^{30} 200z[z/(5+z)]e^{-2z/30}\,dz}{1480.6}$$



Simpson's ⅓ rule with a step size of 0.5 gives d=19,326.9/1480.6 = 13.05 ft.
With F and d known from numerical methods,a free-body diagram is used to develop Force and moment balance equations.

Methodology

$$V = \frac{Fd}{3} = \frac{(1480.6)(13.05)}{3} = 6440.6 \text{ lb}$$

$$T = \frac{V}{\cos\theta} = \frac{6440.6}{0.995} = 6473 \text{ lb}$$

$$H = F - T\sin\theta = 1480.6 - (6473)(0.0995) = 836.54 \text{ lb}$$

$$\sum F_H = 0 = F - T\sin\theta - H$$
$$\sum F_V = 0 = V - T\cos\theta$$
$$\sum M_0 = 0 = 3V - Fd$$

T= tension in the cable
H,V = unknown reactions on the mast transmitted by the deck.

Trapezoidal Rule VS Simpson ⅓ rule

Trapezoidal Rule

Divide the integration interval from a to b into n. So, there are n+1 equally spaced base points (X0,X1,X2,.....Xn).

$$I = \int_{x_0}^{x_1} f(x)\,dx + \int_{x_1}^{x_2} f(x)\,dx + \cdots + \int_{x_{n-1}}^{x_n} f(x)\,dx$$

$$h = \frac{b-a}{n}$$

Substituting the trapezoidal rule for each integral yields

$$I = h\,\frac{f(x_0) + f(x_1)}{2} + h\,\frac{f(x_1) + f(x_2)}{2} + \cdots + h\,\frac{f(x_{n-1}) + f(x_n)}{2}$$

$$I = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$$

Simpson ⅓ Rule

The Total integral can be represented as:-

$$I = \int_{x_0}^{x_2} f(x)\,dx + \int_{x_2}^{x_4} f(x)\,dx + \cdots + \int_{x_{n-2}}^{x_n} f(x)\,dx$$

$$I \cong (b-a)\underbrace{}_{\text{Width}}\;\underbrace{\frac{f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)}{3n}}_{\text{Average height}}$$

$$I \cong 2h\frac{f(x_0) + 4f(x_1) + f(x_2)}{6} + 2h\frac{f(x_2) + 4f(x_3) + f(x_4)}{6} + \cdots + 2h\frac{f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)}{6}$$

Results

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raja@Rajagopalans-MacBook-Air Documents % cd "/Users/raja/Documents/" && g++ ANM.cpp -o ANM && "/Users/raja/Documents/"ANM
Enter lower limit of integration: 0
Enter upper limit of integration: 30
Enter number of sub intervals: 590
You wanna use Trapezoidal method (1) or Simpson's 1/3 method (2) ?1

Technique                  Step size(ft)        Segments           F(lb)
-----------------------------------------------------------------------------------------------------------------------------
Trapezoidal rule       0.0508475              590              1480.56
Simpson's 1/3 rule           0.5               60              1480.56
'd' known from numerical methods: 13.0422

Summing forces in the horizontal and vertical direction and taking moments about point 0 gives,

F-Tsinθ-H = 0

V-Tcosθ = 0

3V-Fd = 0

Enter the angle made by the left support cable with the mast: 0.0996687
H and V = the unknown reactions on the mast transmitted by the deck: 836.9 & 6436.59
T = the tension in the cable: 6468.7
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Discussion

This problem illustrates nicely two uses of numerical integration that may be encountered during the engineering design of structures. It is seen that both the trapezoidal rule and Simpson’s 1/3 rule are easy to apply and are practical problem-solving tools. Simpson’s 1y3 rule is more accurate than the trapezoidal rule for the same step size and thus may often be preferred

Limitation

The tensile force T in the left mast support cable is calculated assuming that the right support cable is completely slack and the mast joins the deck in a manner that transmits horizontal or vertical forces but no moments. And also the mast is assumed to be vertical.

It is also important to note that torques are present in the sailboat system. There is a torque caused by the force of the sail/wind on the mast, a torque caused by the sailors sitting on the edge of the boat, and a torque caused by the centerboard/water interactions. The calculation of these torques is beyond the scope of this project as the forces affecting a sailboat can be calculated and understood without complicating matters further by including the torque calculations.

Future Scope

In this problem statement, we can also add Rotational forces on sailing craft and Effect of coefficient of lift and drag on forces which will make our result more accurate and will able to find exact solution.
With those accurate results we can make the sailboat stronger to withhold all types of impacts and internal forces.
We combine elements of hydrodynamic forces (mainly drag) and aerodynamic forces (lift and drag) to predict sailboat performance at various wind speed for all points of sail

Thank You